

Generate artificial data

```
clear all;
```

```
x = -4:0.05:4;
```

```
y = -4:0.05:4;
```

```
rng(123456);
```

```
figure;
```

```
[X, Y] = meshgrid(x, y);
```

```
fkk = 2; % frequency of test function
```

```
freq=2*fkk; fk=1;
```

```
f1 = @(x,y) sin(freq*sqrt((x-fk).^2+y.^2));%./(freq*sqrt((x-fk).^2+y.^2));
```

```
freq=1*fkk; fk=-1;
```

```
f2 = @(x,y) -sin(freq*sqrt((x-fk).^2+y.^2));%./(freq*sqrt((x-fk).^2+y.^2));
```

```
freq = 0.1*fkk;
```

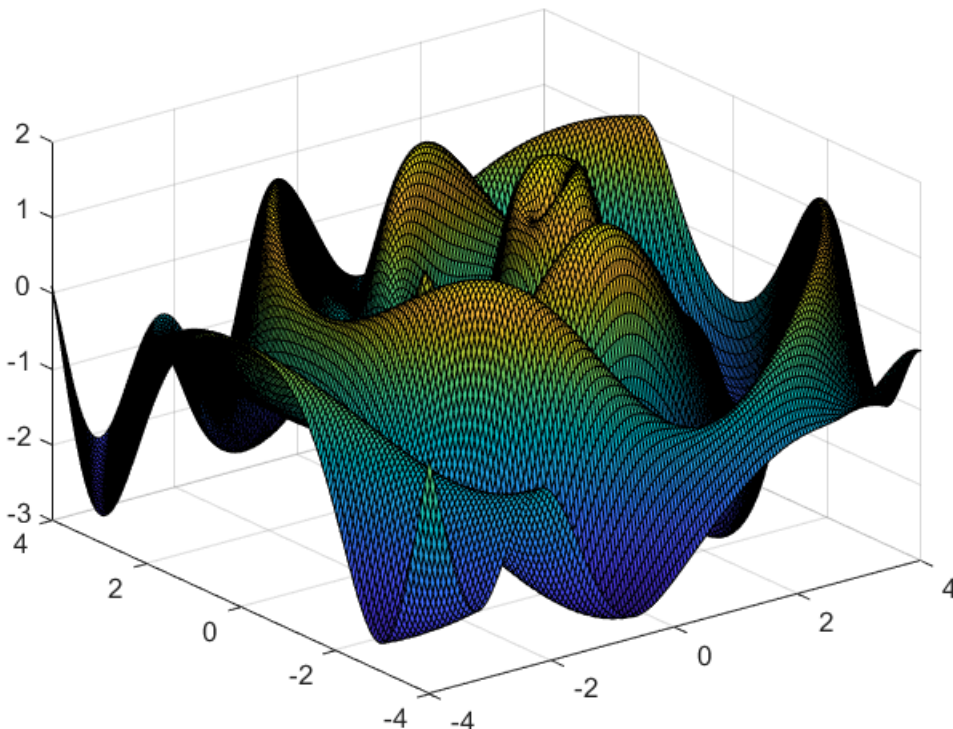
```
f3 = @(x,y) -sin(freq*sqrt(x.^2+y.^2));
```

```
ff = @(x,y) f1(x,y)+f2(x,y)+f3(x,y);
```

```
Z = ff(X, Y);
```

```
%Z = peaks(X, Y);
```

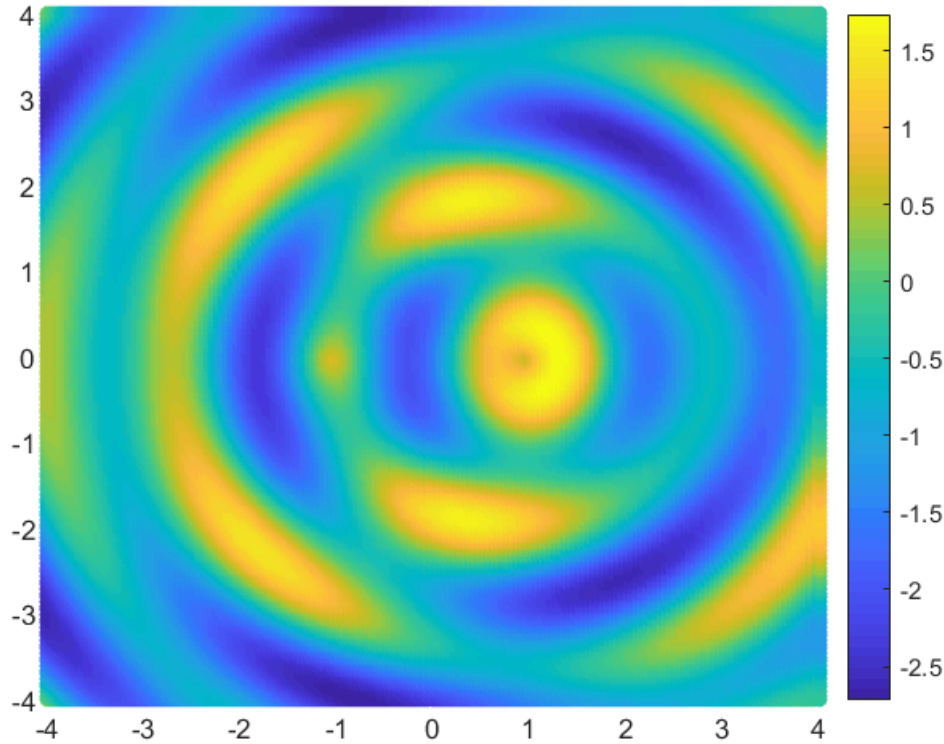
```
surf(X, Y, Z);
```



```
figure;
```

```
scatter(X(:), Y(:), [], Z(:), "filled");
```

```
colorbar;
```



```
xnorm = 1; ynorm = 1; % whether use normalization  
yns = 0.1; % noise scale
```

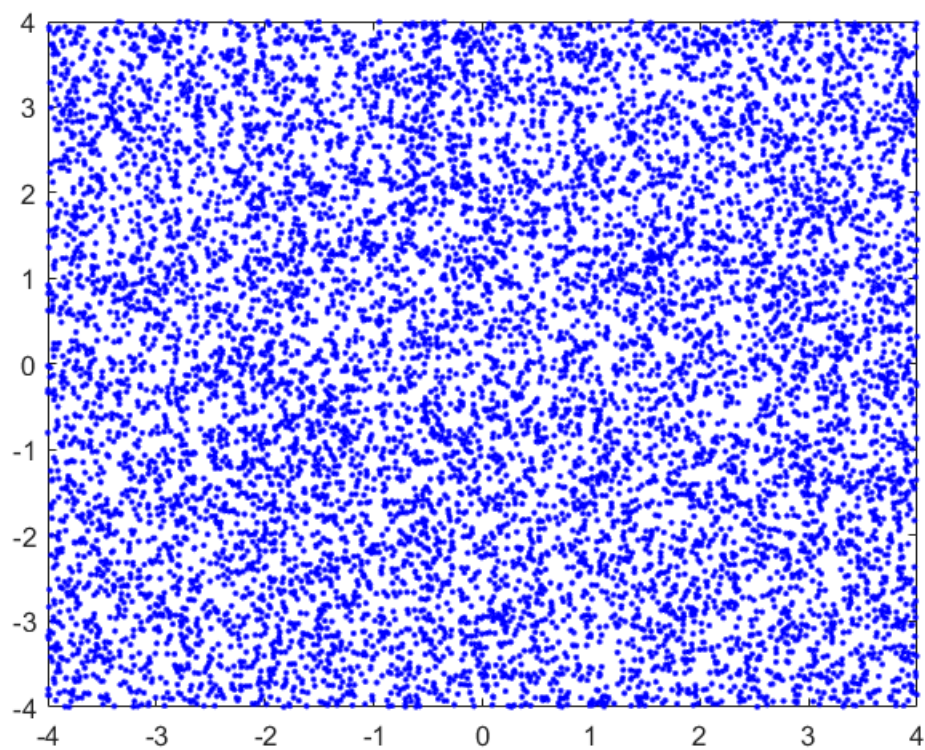
```
n = 1e4; % size of training data  
ttcs = 300;  
ttm = round(n / (ttcs/2));
```

```
% represent_ratio = 5;  
% dcs = 300; % size of the communication set  
% mn = round(n / dcs); % mn is the number of experts (normal)  
% m = round(n / (dcs*represent_ratio)); % m is the number of experts (sparse representative)  
% lamds = 0:0.5:3.0;
```

```
testsize = 2000; % size of test data  
valsize = 1000;  
induce_step = 100;
```

```
% training input  
ori_all_trainxs = 8*(rand(n, 1)-0.5);  
ori_all_trainys = 8*(rand(n, 1)-0.5);
```

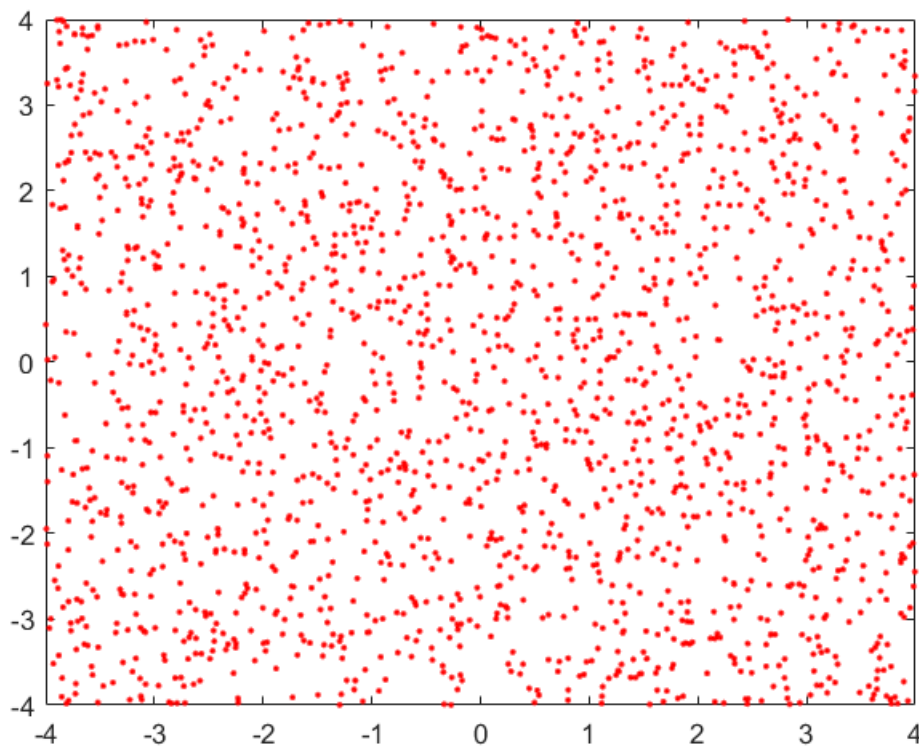
```
figure;  
plot(ori_all_trainxs, ori_all_trainys, 'b.');
```



```
% test input
ori_testxs = 8*(rand(testsize, 1)-0.5);
ori_testys = 8*(rand(testsize, 1)-0.5);

% validation input
ori_valxs = 8*(rand(valsize, 1)-0.5);
ori_valys = 8*(rand(valsize, 1)-0.5);

figure;
plot(ori_testxs, ori_testys, 'r.');
```



```
% input normalization
if xnorm == 1
    norm_xmean = mean(ori_all_trainxs);
    norm_xstd = std(ori_all_trainxs);
    all_trainxs = (ori_all_trainxs - norm_xmean) / norm_xstd;
    norm_ymean = mean(ori_all_trainys);
    norm_ystd = std(ori_all_trainys);
    all_trainys = (ori_all_trainys - norm_ymean) / norm_ystd;

    testxs = (ori_testxs - norm_xmean) / norm_xstd;
    testys = (ori_testys - norm_ymean) / norm_ystd;

    valxs = (ori_valxs - norm_xmean) / norm_xstd;
    valys = (ori_valys - norm_ymean) / norm_ystd;
else
    all_trainxs = ori_all_trainxs;
    all_trainys = ori_all_trainys;
    testxs = ori_testxs;
    testys = ori_testys;
    valxs = ori_valxs;
    valys = ori_valys;
end
```

```
% training output
ln = length(all_trainxs);
xvec = zeros(ln, 2); ori_xvec = zeros(ln, 2);
```

```

for i = 1:ln
    xvec(i, :) = [all_trainxs(i), all_trainys(i)];
    ori_xvec(i, :) = [ori_all_trainxs(i), ori_all_trainys(i)];
end
ori_yvec = ff(ori_all_trainxs, ori_all_trainys) + yns*randn(ln, 1);

% test output
ln = length(testxs);
xvec_test = zeros(ln, 2); ori_xvec_test = zeros(ln, 2);
for i = 1:ln
    xvec_test(i, :) = [testxs(i), testys(i)];
    ori_xvec_test(i, :) = [ori_testxs(i), ori_testys(i)];
end
ori_yvec_test = ff(ori_testxs, ori_testys) + yns*randn(ln, 1);

% validation output
ln = length(valxs);
xvec_val = zeros(ln, 2); ori_xvec_val = zeros(ln, 2);
for i = 1:ln
    xvec_val(i, :) = [valxs(i), valys(i)];
    ori_xvec_val(i, :) = [ori_valxs(i), ori_valys(i)];
end
ori_yvec_val = ff(ori_valxs, ori_valys) + yns*randn(ln, 1);

% output normalization
if ynorm == 1
    norm_fmean = mean(ori_yvec);
    norm_fstd = std(ori_yvec);
    all_trainxs = (ori_all_trainxs - norm_xmean) / norm_xstd;
    yvec = (ori_yvec - norm_fmean) / norm_fstd;
    yvec_test = (ori_yvec_test - norm_fmean) / norm_fstd;
    yvec_val = (ori_yvec_val - norm_fmean) / norm_fstd;
else
    yvec = ori_yvec;
    yvec_test = ori_yvec_test;
    yvec_val = ori_yvec_val;
end
end

```

store results

```

kti = 5; % average over kti runs
grls = 0.3:0.05:1.0; % percentage of remaining data to be test
grbcm0_smse_rec = zeros(kti, 1);
rbcm0_smse_rec = zeros(kti, 1);
bcm0_smse_rec = zeros(kti, 1);
gpoe0_smse_rec = zeros(kti, 1);
poe0_smse_rec = zeros(kti, 1);
vfe0_smse_rec = zeros(kti, 1);
spgp0_smse_rec = zeros(kti, 1);
grbcm0_msll_rec = zeros(kti, 1);
rbcm0_msll_rec = zeros(kti, 1);
bcm0_msll_rec = zeros(kti, 1);
gpoe0_msll_rec = zeros(kti, 1);

```

```

poe0_msll_rec = zeros(kti, 1);
vfe0_msll_rec = zeros(kti, 1);
spgp0_msll_rec = zeros(kti, 1);
grbcm_gr_smse = zeros(kti, length(grls));
grbcm_gr_msll = zeros(kti, length(grls));
grbcm2_gr_smse = zeros(kti, length(grls));
grbcm2_gr_msll = zeros(kti, length(grls));
grbcm2_spgp_gr_smse = zeros(kti, length(grls));
grbcm2_spgp_gr_msll = zeros(kti, length(grls));

```

Experiment I: remove data from the training set

```

sf2 = 1 ; ell = 1 ; sn2 = 0.1 ;
d = size(xvec,2);
opts.Xnorm = 'N' ; opts.Ynorm = 'N' ;
opts.Ms = ttm+1;
opts.ell = ell ; opts.sf2 = sf2 ; opts.sn2 = sn2 ;
opts.meanfunc = []; opts.covfunc = @covSEard; opts.likfunc = @likGauss; opts.inffunc = @infGauss;

meanfunc = []; % empty: don't use a mean function
covfunc = opts.covfunc; % Squared Exponential covariance function
likfunc = opts.likfunc; % Gaussian likelihood
inffunc = opts.inffunc;
km_iters = 1e4;
opts.induce_step = induce_step ;

```

```

% hyp.cov = log([ones(d,1)*ell;sf2]); hyp.lik = log(sn2); hyp.mean = [];
opts.numOptFC = 50 ;
opts.Ms = ttm+1;
opts.xvec = xvec;
opts.yvec = yvec;
opts.grbcm_baseline = 0;
opts.global_index = ones(n,1);
% opts.inffunc = @infGaussLik; opts.meanfunc = meanfunc; opts.likfunc = likfunc;
opts.covfunc = covfunc;
covfuncF = {@apxSparse, {opts.covfunc}, []};
opts.covfuncF = covfuncF;
opts.compute_hyp = 0;

```

```

% default partition
dcs_ecs_r = 0.5;
dcs = round(ttcs*dcs_ecs_r) % size of the communication set

```

```

dcs = 150

```

```

ecs = ttcs - dcs % size of other experts

```

```

ecs = 150

```

```

n_per = dcs ; % size of Dc

```

```

mn = round(n / ecs); % mn is the number of experts (normal)
Indics = randperm(n) ;
I_com = Indics(1:n_per) ; % randomly select communication set
[idx, C] = kmeans(xvec, mn, 'MaxIter', km_iters);

```

Baselines of VFE and SPGP.

```

% hyp.cov = log([ones(d,1)*ell;sf2]); hyp.lik = log(sn2); hyp.mean = [];
opts.numOptFC = 30 ;
opts.Ms = mn+1;
opts.xvec = xvec;
opts.yvec = yvec;
opts.induce_size = dcs;
opts.grbcm_baseline = 0;
opts.global_index = ones(n,1);
opts.I_com = I_com;
% opts.inffunc = @infGaussLik; opts.meanfunc = meanfunc; opts.likfunc = likfunc;
opts.covfunc = covfunc;
covfuncF = {@apxSparse, {opts.covfunc}, xvec(I_com,:)};
opts.covfuncF = covfuncF;
opts.compute_hyp = 0;

```

```

g_opts = opts;
g_opts.compute_hyp = 1;
g_opts.grbcm_baseline = 1;
g_opts.global_index = ones(n,1);
g_models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,g_opts);

```

```

Optimizing hyps in training...
Linesearch      0; Value 5.162781e+03
Linesearch      1; Value 2.449878e+03
Linesearch      2; Value -1.331980e+03
Linesearch      3; Value -5.249277e+03
Linesearch      4; Value -5.962682e+03
Linesearch      5; Value -6.032907e+03
Linesearch      6; Value -6.069304e+03
Linesearch      7; Value -6.274396e+03
Linesearch      8; Value -6.315859e+03
Linesearch      9; Value -6.320591e+03
Linesearch     10; Value -6.322875e+03
Linesearch     11; Value -6.323489e+03
Linesearch     12; Value -6.324257e+03
Linesearch     13; Value -6.325137e+03
Linesearch     14; Value -6.325151e+03
Linesearch     15; Value -6.325154e+03
Linesearch     16; Value -6.325154e+03
Linesearch     17; Value -6.325155e+03
Linesearch     18; Value -6.325155e+03
Linesearch     19; Value -6.325155e+03
Linesearch     20; Value -6.325155e+03
Linesearch     21; Value -6.325155e+03
Linesearch     22; Value -6.325155e+03
Linesearch     23; Value -6.325155e+03
Linesearch     24; Value -6.325155e+03
Linesearch     25; Value -6.325155e+03
Linesearch     26; Value -6.325155e+03

```



```

Linesearch      27; Value -6.325155e+03
Linesearch      28; Value -6.325155e+03
Linesearch      29; Value -6.325155e+03

```

```

opts.hyp = g_models{1}.hyp;
g_opts.hyp = g_models{1}.hyp;
[tmu,ts2, ~] = aggregation_predict(xvec_test,g_models,'GRBCM', 1, g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[grbcmMSE,grbcmSMSE,grbcmMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tm
fprintf('%s (Dc size %d): MSE %.6f, SMSE %.6f, MSLL %.6f\r\n', 'GRBCM', n_per, grbcmMSE,grbcm

```

```
GRBCM (Dc size 150): MSE 0.01179634, SMSE 0.01238287, MSLL -2.04868360
```

```
g_opts.compute_hyp = 0;
```

When testing baseline, VFE and SPGP use all partition budget (ttcs = dcs + ecs).

```
I_com2 = Indics(1:ttcs) ; % randomly select communication set
```

```

% % VFE Baseline
vfe_opts = opts;
vfe_opts.induce_type = 'VFE_opt';
xu = xvec(I_com2, :);
inffunc = @(varargin) infGaussLik(varargin{:}, struct('s', 0.0));
vfe_hyp = opts.hyp;
vfe_hyp.xu = xu;
[vfe_hyp, tmp_nllzs] = minimize(vfe_hyp,@sp_gp,-vfe_opts.induce_step,inffunc,meanfunc,covfuncF,I

```

```

Function evaluation      0; Value 6.697854e+02
Function evaluation     10; Value -8.441569e+02
Function evaluation     13; Value -2.343632e+03
Function evaluation     14; Value -4.484990e+03
Function evaluation     15; Value -4.940360e+03
Function evaluation     18; Value -5.556287e+03
Function evaluation     19; Value -6.084505e+03
Function evaluation     21; Value -6.381782e+03
Function evaluation     23; Value -6.572633e+03
Function evaluation     24; Value -6.739983e+03
Function evaluation     27; Value -6.819390e+03
Function evaluation     28; Value -6.895779e+03
Function evaluation     30; Value -6.933462e+03
Function evaluation     32; Value -6.952915e+03
Function evaluation     35; Value -6.980646e+03
Function evaluation     37; Value -6.998711e+03
Function evaluation     39; Value -7.014689e+03
Function evaluation     41; Value -7.025193e+03
Function evaluation     43; Value -7.032373e+03
Function evaluation     45; Value -7.038789e+03
Function evaluation     47; Value -7.043647e+03
Function evaluation     49; Value -7.047403e+03
Function evaluation     51; Value -7.050251e+03
Function evaluation     53; Value -7.053806e+03

```



```

Function evaluation    55; Value -7.056408e+03
Function evaluation    57; Value -7.057545e+03
Function evaluation    59; Value -7.059650e+03
Function evaluation    61; Value -7.060955e+03
Function evaluation    63; Value -7.061563e+03
Function evaluation    66; Value -7.063734e+03
Function evaluation    68; Value -7.064890e+03
Function evaluation    70; Value -7.065401e+03
Function evaluation    72; Value -7.066691e+03
Function evaluation    74; Value -7.067739e+03
Function evaluation    77; Value -7.067925e+03
Function evaluation    80; Value -7.068874e+03
Function evaluation    82; Value -7.069685e+03
Function evaluation    85; Value -7.069748e+03
Function evaluation    88; Value -7.070221e+03
Function evaluation    90; Value -7.070539e+03
Function evaluation    92; Value -7.070724e+03
Function evaluation    94; Value -7.071053e+03
Function evaluation    96; Value -7.071088e+03
Function evaluation   100; Value -7.071435e+03

```

```

vfe_opts.hyp = opts.hyp;
vfe_opts.xu = vfe_hyp.xu;
vfe_opts.inffunc = @infGaussLik; vfe_opts.meanfunc = meanfunc; vfe_opts.covfuncF = covfuncF; vfe_opts.covfunc = covfunc;
[tmu, ts2] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, xvec_test);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[vfeMSE,vfeSMSE,vfeMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'VFE baseline', n_per, vfeMSE, vfeSMSE, vfeMSLL);

```

VFE baseline (Dc size 150): MSE 0.01095326, SMSE 0.01149786, MSLL -2.22928219

```

% vfe0_smse_rec(ki) = vfeSMSE; vfe0_msll_rec(ki) = vfeMSLL;
[yu, su] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, vfe_opts.xu);
vfe_opts.yu = yu; vfe_opts.su = su;

```

```

sp_opts = opts;
sp_opts.induce_size = ttcs;
sp_opts.induce_type = 'SPGP_opt';
hyp_init(1:d,1) = -2*opts.hyp.cov(1:d);
hyp_init(d+1,1) = 2*opts.hyp.cov(d+1);
hyp_init(d+2,1) = 2*opts.hyp.lik;

xu = xvec(I_com2, :);
w_init = [reshape(xu,sp_opts.induce_size*d,1);hyp_init];
[w,tmp_nlzs] = minimize(w_init,'spgp_lik_nohyp',-sp_opts.induce_step,yvec,xvec,sp_opts.induce_size);

```

```

Function evaluation    0; Value -6.615935e+03
Function evaluation    8; Value -6.773176e+03
Function evaluation   10; Value -7.129234e+03
Function evaluation   12; Value -7.258895e+03
Function evaluation   16; Value -7.359168e+03
Function evaluation   18; Value -7.453648e+03
Function evaluation   19; Value -7.520448e+03
Function evaluation   21; Value -7.564845e+03
Function evaluation   23; Value -7.587883e+03

```

```

Function evaluation      26; Value -7.617321e+03
Function evaluation      29; Value -7.624374e+03
Function evaluation      31; Value -7.638049e+03
Function evaluation      33; Value -7.645932e+03
Function evaluation      35; Value -7.658067e+03
Function evaluation      37; Value -7.662863e+03
Function evaluation      39; Value -7.663976e+03
Function evaluation      42; Value -7.669416e+03
Function evaluation      46; Value -7.670158e+03
Function evaluation      50; Value -7.680442e+03
Function evaluation      53; Value -7.681455e+03
Function evaluation      56; Value -7.685747e+03
Function evaluation      59; Value -7.686161e+03
Function evaluation      64; Value -7.697916e+03
Function evaluation      67; Value -7.698685e+03
Function evaluation      69; Value -7.700381e+03
Function evaluation      71; Value -7.704202e+03
Function evaluation      74; Value -7.704879e+03
Function evaluation      78; Value -7.707114e+03
Function evaluation      81; Value -7.707263e+03
Function evaluation      85; Value -7.710498e+03
Function evaluation      88; Value -7.710618e+03
Function evaluation      92; Value -7.712813e+03
Function evaluation      95; Value -7.712938e+03
Function evaluation     100; Value -7.716003e+03

```

```

xb = reshape(w(1:sp_opts.induce_size*d,1),sp_opts.induce_size,d);
sp_opts.xu = xb;
sp_opts.sp_hyp = w(sp_opts.induce_size*d+1:end,1);
sp_opts.hyp = opts.hyp;
[tmu,ts2] = spgp_pred(sp_opts.yvec,sp_opts.xvec,sp_opts.xu,xvec_test,sp_opts.sp_hyp);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[spgpMSE,spgpSMSE,spgpMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu,
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'SPSG baseline', n_per, spgpM

```

SPSG baseline (Dc size 150): MSE 0.01103506, SMSE 0.01158373, MSLL -2.21967138

```

% spgp0_smse_rec(ki) = spgpSMSE; spgp0_msll_rec(ki) = spgpMSLL;
[yu,su] = spgp_pred(sp_opts.yvec,sp_opts.xvec,sp_opts.xu,sp_opts.xu,sp_opts.sp_hyp);
sp_opts.yu = yu; sp_opts.su = su;

```

Average over 5 runs.

```

kti = 5;
m = mn;

for ki=1:kti
fprintf("=====kd=====", ki)
n_per = dcs ; % size of Dc
Indics = randperm(n) ;
I_com = Indics(1:n_per) ; % randomly select communication set
[idx, C] = kmeans(xvec, m, 'MaxIter',km_iters);

% hyp.cov = log([ones(d,1)*ell;sf2]); hyp.lik = log(sn2); hyp.mean = [];

```

```

opts.numOptFC = 30 ;
opts.Ms = m+1;
opts.xvec = xvec;
opts.yvec = yvec;
opts.induce_size = dcs;
opts.grbcm_baseline = 0;
opts.global_index = ones(n,1);
opts.I_com = I_com;
% opts.inffunc = @infGaussLik; opts.meanfunc = meanfunc; opts.likfunc = likfunc;
opts.covfunc = covfunc;
covfuncF = {@apxSparse, {opts.covfunc}, xvec(I_com,:)};
opts.covfuncF = covfuncF;
opts.compute_hyp = 0;

g_opts = opts;
g_opts.compute_hyp = 1;
g_opts.grbcm_baseline = 1;
g_opts.global_index = ones(n,1);
g_models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,g_opts);
opts.hyp = g_models{1}.hyp;
g_opts.hyp = g_models{1}.hyp;
[tmu,ts2, ~] = aggregation_predict(xvec_test,g_models,'GRBCM', 1, g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[grbcmMSE_bl,grbcmSMSE_bl,grbcmMSLL_bl] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'GRBCM', n_per, grbcmMSE_bl,grbcmSMSE_bl,grbcmMSLL_bl);
grbcm0_smse_rec(ki) = grbcmSMSE_bl; grbcm0_msll_rec(ki) = grbcmMSLL_bl;
g_opts.compute_hyp = 0;

% % VFE Baseline
vfe_opts = opts;
vfe_opts.induce_type = 'VFE_opt';
xu = xvec(I_com, :);
inffunc = @(varargin) infGaussLik(varargin{:}, struct('s', 0.0));
vfe_hyp = opts.hyp;
vfe_hyp.xu = xu;
vfe_hyp = minimize(vfe_hyp,@sp_gp,-vfe_opts.induce_step,inffunc,meanfunc,covfuncF,likfunc,xvec,yvec);
vfe_opts.hyp = vfe_hyp;
vfe_opts.xu = vfe_hyp.xu;
vfe_opts.inffunc = @infGaussLik; vfe_opts.meanfunc = meanfunc; vfe_opts.covfuncF = covfuncF; vfe_opts.covfunc = covfunc;
[tmu, ts2] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, xvec_test);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[vfeMSE_bl,vfeSMSE_bl,vfeMSLL_bl] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'VFE', n_per, vfeMSE_bl,vfeSMSE_bl,vfeMSLL_bl);
vfe0_smse_rec(ki) = vfeSMSE_bl; vfe0_msll_rec(ki) = vfeMSLL_bl;
[yu, su] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, vfe_opts.xu);
vfe_opts.yu = yu; vfe_opts.su = su;

```

```

sp_opts = opts;
sp_opts.induce_type = 'SPGP_opt';
hyp_init(1:d,1) = -2*opts.hyp.cov(1:d);
hyp_init(d+1,1) = 2*opts.hyp.cov(d+1);
hyp_init(d+2,1) = 2*opts.hyp.lik;

xu = xvec(I_com, :);
w_init = [reshape(xu,sp_opts.induce_size*d,1);hyp_init];
[w,f] = minimize(w_init,'spgp_lik_nohyp',-sp_opts.induce_step,yvec,xvec,sp_opts.induce_size);
xb = reshape(w(1:sp_opts.induce_size*d,1),sp_opts.induce_size,d);
sp_opts.xu = xb;
sp_opts.sp_hyp = w(sp_opts.induce_size*d+1:end,1);
sp_opts.hyp = opts.hyp;

[tmu,ts2] = spgp_pred(sp_opts.yvec,sp_opts.xvec,sp_opts.xu,xvec_test,sp_opts.sp_hyp);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[spgpMSE_bl,spgpSMSE_bl,spgpMSLL_bl] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'SPSG', n_per, spgpMSE_bl,spgpSMSE_bl,spgpMSLL_bl);
spgp0_smse_rec(ki) = spgpSMSE_bl; spgp0_msll_rec(ki) = spgpMSLL_bl;
[yu,su] = spgp_pred(sp_opts.yvec,sp_opts.xvec,sp_opts.xu,sp_opts.xu,sp_opts.sp_hyp);
sp_opts.yu = yu; sp_opts.su = su;

vfe_opts.grbcm_baseline = 0;
vfe_opts.global_index = ones(n,1);
models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,vfe_opts); % use hyp of vfe
[tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_test,models,vfe_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'GRBCM (VFE)', n_per, MSE,SMSE,MSLL);

sp_opts.grbcm_baseline = 0;
sp_opts.global_index = ones(n,1);
models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,sp_opts); % use hyp of vfe
[tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_test,models,sp_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'GRBCM (SPGP)', n_per, MSE,SMSE,MSLL);

criterion = 'RCBM';
[tmu,ts2,t_dGP_predict] = aggregation_predict(xvec_test,g_models,criterion, 1, g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end

```

end

```
[rbcmMSE,rbcmSMSE,rbcmMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu,  
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', criterion, n_per, rbcmMSE,rbcmSMSE,rbcmMSLL;  
rbcm0_smse_rec(ki) = rbcmSMSE; rbcm0_msll_rec(ki) = rbcmMSLL;
```

criterion = 'BCM';

```
[tmu,ts2,t_dGP_predict] = aggregation_predict(xvec_test,g_models,criterion, 1, g_opts);
```

if ynorm==1

```
    tmu = tmu * norm_fstd + norm_fmean;
```

```
    ts2 = ts2 * norm_fstd^2;
```

end

```
[bcmMSE,bcmSMSE,bcmMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2,
```

```
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', criterion, n_per, bcmMSE,bcmSMSE,bcmMSLL;  
bcm0_smse_rec(ki) = bcmSMSE; bcm0_msll_rec(ki) = bcmMSLL;
```

criterion = 'PoE';

```
[tmu,ts2,t_dGP_predict] = aggregation_predict(xvec_test,g_models,criterion, 1, g_opts);
```

if ynorm==1

```
    tmu = tmu * norm_fstd + norm_fmean;
```

```
    ts2 = ts2 * norm_fstd^2;
```

end

```
[poeMSE,poeSMSE,poeMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2,
```

```
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', criterion, n_per, poeMSE,poeSMSE,poeMSLL;  
poe0_smse_rec(ki) = poeSMSE; poe0_msll_rec(ki) = poeMSLL;
```

criterion = 'GPoE';

```
[tmu,ts2,t_dGP_predict] = aggregation_predict(xvec_test,g_models,criterion, 1, g_opts);
```

if ynorm==1

```
    tmu = tmu * norm_fstd + norm_fmean;
```

```
    ts2 = ts2 * norm_fstd^2;
```

end

```
[gpoeMSE,gpoeSMSE,gpoeMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2,
```

```
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', criterion, n_per, gpoeMSE,gpoeSMSE,gpoeMSLL;  
gpoe0_smse_rec(ki) = gpoeSMSE; gpoe0_msll_rec(ki) = gpoeMSLL;
```

% compute informatic importance

```
ds = zeros(n, m);
```

for i=1:n

```
    for j=1:m
```

```
        %ds(i,j) = muti1(i)-muti2(j,i)+muti1(j)-muti2(i,j);
```

```
        ds(i,j) = norm(xvec(i, :) - C(j,:));
```

```
    end
```

end

% 1st closest cluster centers

```
[~, fcc] = min(ds, [], 2);
```

% set to inf

for i=1:n

```
    ds(i, fcc(i)) = 1e10;
```

end

% 2st closest cluster centers

```
[~, scc] = min(ds, [], 2);
```

```
rk = zeros(n, 1);
```

for iik=1:n

```
    if mod(iik, 1e2)==0
```

```

        fprintf('processing distance: %d/%d\n', iik, n);
    end
    i = fcc(iik);
    [tmp_mu, tmp_sig2] = gp(vfe_hyp,inffunc,meanfunc, ...
        covfunc,likfunc,models{i+1}.X,models{i+1}.Y,xvec(iik,:));
    h1 = 0.5*log(tmp_sig2);
    j = scc(iik);
    % [tmp_mu, tmp_sig2] = gp(vfe_hyp,inffunc,meanfunc, ...
    % covfunc,likfunc,[models{i+1}.X;models{j+1}.X],[models{i+1}.Y;models{j+1}.Y],xvec(iik,
    [tmp_mu, tmp_sig2] = gp(vfe_hyp,inffunc,meanfunc, ...
        covfunc,likfunc,[models{j+1}.X],[models{j+1}.Y],xvec(iik,:));
    h2 = 0.5*log(tmp_sig2);
    rk(iik) = h2 - h1; % importance is the difference between mutual information
end

if ki==1
    figure;
    scatter(xvec(:,1), xvec(:,2), 3, rk, "filled"); colorbar;
    figure; hold on;
    plot(xvec(I_com,1), xvec(I_com,2), 'r. ');
    plot(vfe_opts.xu(:,1), vfe_opts.xu(:,2), 'bo');
    plot(sp_opts.xu(:,1), sp_opts.xu(:,2), 'go');
    legend('initial points', 'VFE induced', 'SPGP induced');
end

for kj=1:length(grls) % test/validate different remaining percentage
    kj
    gr=grls(kj)
    crk = rk;
    crk(I_com) = -1e10;
    [~, crk_idx] = sort(crk, 'descend');
    rn = round(n*gr);
    global_index = zeros(n,1);
    global_index(crk_idx(1:rn)) = 1; % select remaining data according to the importance

    % only remove data from subsets, in RBGCM
    g_opts.global_index = global_index;
    models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,g_opts); % use hyp of vfe
    [tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_test,models,g_opts);
    if ynorm==1
        tmu = tmu * norm_fstd + norm_fmean;
        ts2 = ts2 * norm_fstd^2;
    end
    [grbcmMSE,grbcmSMSE,grbcmMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test);
    fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM', n_per, grbcmMSE,
    grbcm_gr_smse(ki,kj) = grbcmSMSE; grbcm_gr_msll(ki,kj) = grbcmMSLL;

    vfe_opts.global_index = global_index;
    models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,vfe_opts); % use hyp of vfe
    [tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_test,models,vfe_opts);
    if ynorm==1
        tmu = tmu * norm_fstd + norm_fmean;
        ts2 = ts2 * norm_fstd^2;
    end
end

```

```

[MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): \r\nMSE %.8f, SMSE %.4f, MSLL %.4f\r\n', 'GRBCM++ (VFE)', n_pe
grbcm2_gr_smse(ki,kj) = SMSE; grbcm2_gr_msll(ki,kj) = MSLL;

sp_opts.global_index = global_index;
models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,sp_opts); % use hyp of vfe
[tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_test,models,sp_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): \r\nMSE %.8f, SMSE %.4f, MSLL %.4f\r\n', 'GRBCM++ (SPGP)', n_pe
grbcm2_spgp_gr_smse(ki,kj) = SMSE; grbcm2_spgp_gr_msll(ki,kj) = MSLL;
end
end

```

```

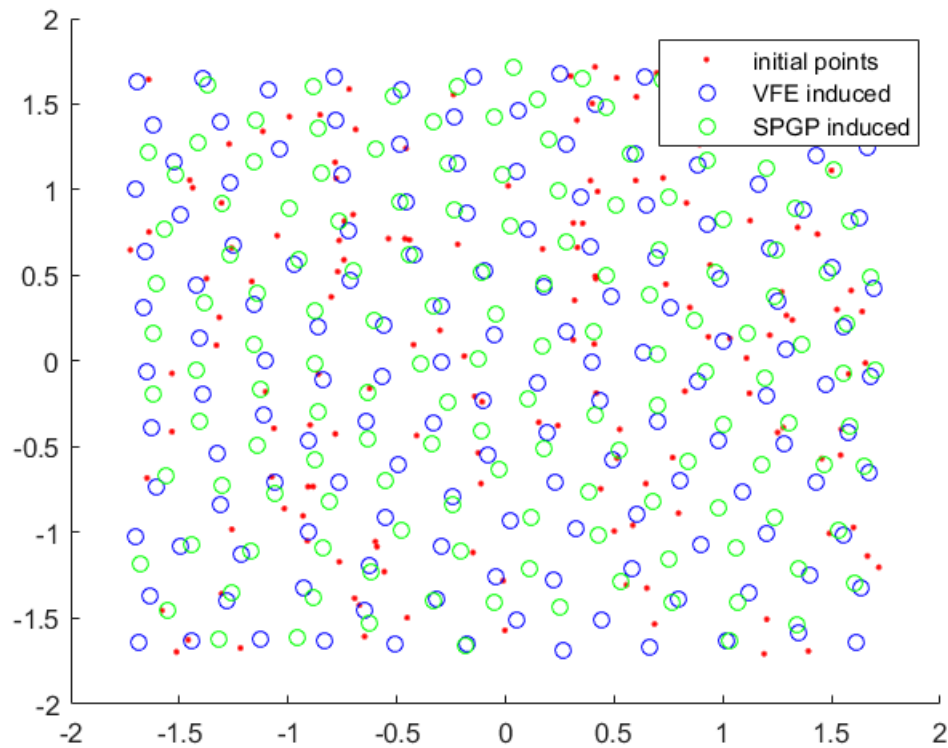
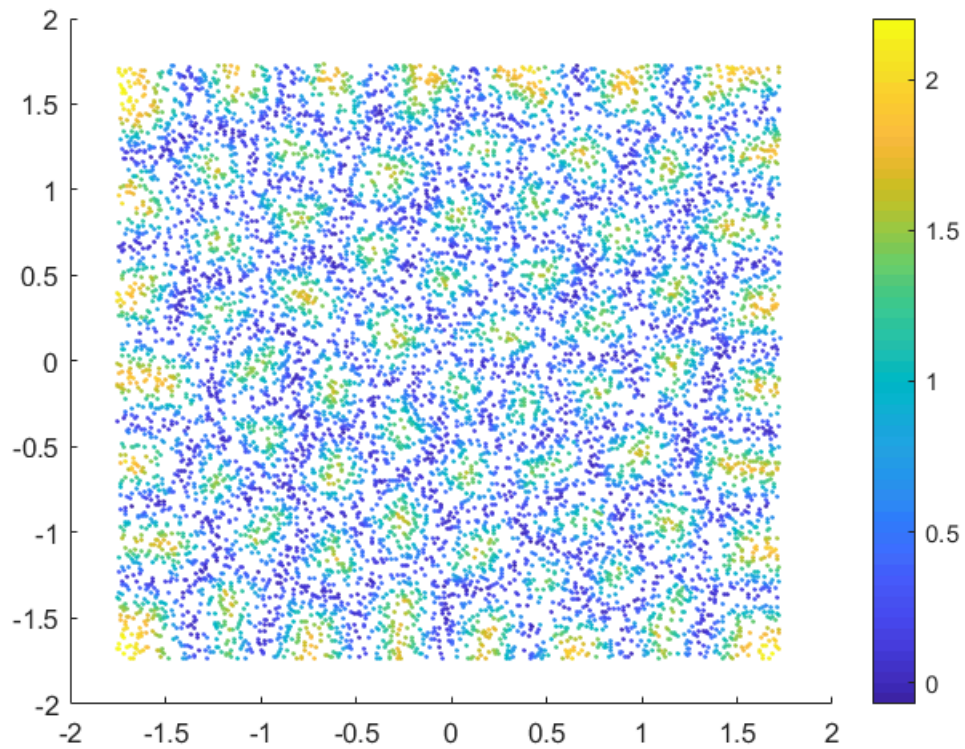
=====1=====
Optimizing hyps in training...
Linesearch    0; Value 5.385029e+03
Linesearch    1; Value 3.234932e+03
Linesearch    2; Value -2.707768e+02
Linesearch    3; Value -5.190398e+03
Linesearch    4; Value -5.965677e+03
Linesearch    5; Value -6.046373e+03
Linesearch    6; Value -6.226080e+03
Linesearch    7; Value -6.329636e+03
Linesearch    8; Value -6.371351e+03
Linesearch    9; Value -6.373932e+03
Linesearch   10; Value -6.374328e+03
Linesearch   11; Value -6.374465e+03
Linesearch   12; Value -6.374489e+03
Linesearch   13; Value -6.374495e+03
Linesearch   14; Value -6.374495e+03
Linesearch   15; Value -6.374495e+03
Linesearch   16; Value -6.374495e+03
Linesearch   17; Value -6.374495e+03
Linesearch   18; Value -6.374495e+03
Linesearch   19; Value -6.374495e+03
Linesearch   20; Value -6.374495e+03
Linesearch   21; Value -6.374495e+03
Linesearch   22; Value -6.374495e+03
Linesearch   23; Value -6.374495e+03
Linesearch   24; Value -6.374495e+03
GRBCM (Dc size 150): MSE 0.01139128, SMSE 0.01195766, MSLL -2.08934137
Function evaluation    0; Value 8.367137e+04
Function evaluation   13; Value 6.853488e+04
Function evaluation   14; Value 4.447068e+04
Function evaluation   16; Value 3.114094e+04
Function evaluation   18; Value 2.511706e+04
Function evaluation   20; Value 2.221299e+04
Function evaluation   22; Value 1.972557e+04
Function evaluation   24; Value 1.818714e+04
Function evaluation   26; Value 1.731422e+04
Function evaluation   28; Value 1.678572e+04
Function evaluation   29; Value 1.622786e+04
Function evaluation   31; Value 1.576174e+04
Function evaluation   33; Value 1.537683e+04
Function evaluation   35; Value 1.505397e+04
Function evaluation   36; Value 1.473428e+04
Function evaluation   38; Value 1.460210e+04
Function evaluation   39; Value 1.446488e+04

```


Function evaluation	41;	Value 1.436124e+04
Function evaluation	43;	Value 1.430701e+04
Function evaluation	45;	Value 1.424575e+04
Function evaluation	46;	Value 1.418720e+04
Function evaluation	48;	Value 1.414586e+04
Function evaluation	50;	Value 1.406019e+04
Function evaluation	52;	Value 1.401346e+04
Function evaluation	54;	Value 1.393500e+04
Function evaluation	56;	Value 1.384607e+04
Function evaluation	58;	Value 1.379413e+04
Function evaluation	60;	Value 1.367429e+04
Function evaluation	62;	Value 1.358521e+04
Function evaluation	64;	Value 1.351267e+04
Function evaluation	66;	Value 1.337979e+04
Function evaluation	68;	Value 1.331737e+04
Function evaluation	70;	Value 1.323090e+04
Function evaluation	72;	Value 1.317335e+04
Function evaluation	74;	Value 1.312647e+04
Function evaluation	75;	Value 1.308293e+04
Function evaluation	77;	Value 1.304819e+04
Function evaluation	79;	Value 1.302029e+04
Function evaluation	81;	Value 1.299818e+04
Function evaluation	83;	Value 1.298084e+04
Function evaluation	85;	Value 1.296805e+04
Function evaluation	87;	Value 1.295845e+04
Function evaluation	89;	Value 1.294653e+04
Function evaluation	91;	Value 1.293802e+04
Function evaluation	93;	Value 1.293105e+04
Function evaluation	95;	Value 1.292169e+04
Function evaluation	97;	Value 1.291677e+04
Function evaluation	99;	Value 1.290995e+04
Function evaluation	100;	Value 1.290374e+04
VFE (Dc size 150): MSE 0.01369386, SMSE 0.01437472, MSLL -1.86004928		
Function evaluation	0;	Value 1.441805e+02
Function evaluation	10;	Value -8.265688e+02
Function evaluation	13;	Value -1.715202e+03
Function evaluation	14;	Value -2.269937e+03
Function evaluation	16;	Value -2.640938e+03
Function evaluation	18;	Value -3.000908e+03
Function evaluation	20;	Value -3.304308e+03
Function evaluation	22;	Value -3.546761e+03
Function evaluation	24;	Value -3.722466e+03
Function evaluation	26;	Value -3.829921e+03
Function evaluation	28;	Value -3.912823e+03
Function evaluation	29;	Value -3.994168e+03
Function evaluation	31;	Value -4.049572e+03
Function evaluation	32;	Value -4.105239e+03
Function evaluation	34;	Value -4.144733e+03
Function evaluation	35;	Value -4.185232e+03
Function evaluation	37;	Value -4.220547e+03
Function evaluation	39;	Value -4.242051e+03
Function evaluation	41;	Value -4.247524e+03
Function evaluation	43;	Value -4.261855e+03
Function evaluation	47;	Value -4.263981e+03
Function evaluation	49;	Value -4.269542e+03
Function evaluation	51;	Value -4.272887e+03
Function evaluation	53;	Value -4.275487e+03
Function evaluation	55;	Value -4.277079e+03
Function evaluation	57;	Value -4.277891e+03
Function evaluation	59;	Value -4.278283e+03
Function evaluation	61;	Value -4.279036e+03
Function evaluation	65;	Value -4.279261e+03
Function evaluation	69;	Value -4.280582e+03
Function evaluation	73;	Value -4.280740e+03
Function evaluation	75;	Value -4.281086e+03

Function evaluation 78; Value -4.281155e+03
 Function evaluation 83; Value -4.282223e+03
 Function evaluation 85; Value -4.282284e+03
 Function evaluation 87; Value -4.282408e+03
 Function evaluation 89; Value -4.282429e+03
 Function evaluation 91; Value -4.282434e+03
 Function evaluation 94; Value -4.282465e+03
 Function evaluation 97; Value -4.282474e+03
 SPSPG (Dc size 150): MSE 0.02530876, SMSE 0.02656713, MSLL -1.84315299
 GRBCM (VFE) (Dc size 150): MSE 0.01095005, SMSE 0.01149450, MSLL -2.20379153
 GRBCM (SPGP) (Dc size 150): MSE 0.01140266, SMSE 0.01196961, MSLL -2.15665419
 RBCM (Dc size 150): MSE 0.01159382, SMSE 0.0122, MSLL -1.5360
 BCM (Dc size 150): MSE 0.01207724, SMSE 0.0127, MSLL -2.0767
 PoE (Dc size 150): MSE 0.08200900, SMSE 0.0861, MSLL 4.9382
 GPoE (Dc size 150): MSE 0.01182763, SMSE 0.0124, MSLL -1.4719
 processing distance: 100/10000
 processing distance: 200/10000
 processing distance: 300/10000
 processing distance: 400/10000
 processing distance: 500/10000
 processing distance: 600/10000
 processing distance: 700/10000
 processing distance: 800/10000
 processing distance: 900/10000
 processing distance: 1000/10000
 processing distance: 1100/10000
 processing distance: 1200/10000
 processing distance: 1300/10000
 processing distance: 1400/10000
 processing distance: 1500/10000
 processing distance: 1600/10000
 processing distance: 1700/10000
 processing distance: 1800/10000
 processing distance: 1900/10000
 processing distance: 2000/10000
 processing distance: 2100/10000
 processing distance: 2200/10000
 processing distance: 2300/10000
 processing distance: 2400/10000
 processing distance: 2500/10000
 processing distance: 2600/10000
 processing distance: 2700/10000
 processing distance: 2800/10000
 processing distance: 2900/10000
 processing distance: 3000/10000
 processing distance: 3100/10000
 processing distance: 3200/10000
 processing distance: 3300/10000
 processing distance: 3400/10000
 processing distance: 3500/10000
 processing distance: 3600/10000
 processing distance: 3700/10000
 processing distance: 3800/10000
 processing distance: 3900/10000
 processing distance: 4000/10000
 processing distance: 4100/10000
 processing distance: 4200/10000
 processing distance: 4300/10000
 processing distance: 4400/10000
 processing distance: 4500/10000
 processing distance: 4600/10000
 processing distance: 4700/10000
 processing distance: 4800/10000
 processing distance: 4900/10000
 processing distance: 5000/10000

processing distance: 5100/10000
processing distance: 5200/10000
processing distance: 5300/10000
processing distance: 5400/10000
processing distance: 5500/10000
processing distance: 5600/10000
processing distance: 5700/10000
processing distance: 5800/10000
processing distance: 5900/10000
processing distance: 6000/10000
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processing distance: 6200/10000
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processing distance: 6400/10000
processing distance: 6500/10000
processing distance: 6600/10000
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processing distance: 6800/10000
processing distance: 6900/10000
processing distance: 7000/10000
processing distance: 7100/10000
processing distance: 7200/10000
processing distance: 7300/10000
processing distance: 7400/10000
processing distance: 7500/10000
processing distance: 7600/10000
processing distance: 7700/10000
processing distance: 7800/10000
processing distance: 7900/10000
processing distance: 8000/10000
processing distance: 8100/10000
processing distance: 8200/10000
processing distance: 8300/10000
processing distance: 8400/10000
processing distance: 8500/10000
processing distance: 8600/10000
processing distance: 8700/10000
processing distance: 8800/10000
processing distance: 8900/10000
processing distance: 9000/10000
processing distance: 9100/10000
processing distance: 9200/10000
processing distance: 9300/10000
processing distance: 9400/10000
processing distance: 9500/10000
processing distance: 9600/10000
processing distance: 9700/10000
processing distance: 9800/10000
processing distance: 9900/10000
processing distance: 10000/10000



$k_j = 1$
 $gr = 0.3000$
 GRBCM (Dc size 150): MSE 0.01739541, SMSE 0.0183, MSLL -2.0064
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01254776, SMSE 0.0132, MSLL -2.1049
 GRBCM++ (SPGP) (Dc size 150):

MSE 0.01291186, SMSE 0.0136, MSLL -2.0967
kj = 2
gr = 0.3500
GRBCM (Dc size 150): MSE 0.01564784, SMSE 0.0164, MSLL -2.0501
GRBCM++ (VFE) (Dc size 150):
MSE 0.01243585, SMSE 0.0131, MSLL -2.1266
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01238654, SMSE 0.0130, MSLL -2.1220
kj = 3
gr = 0.4000
GRBCM (Dc size 150): MSE 0.01457483, SMSE 0.0153, MSLL -2.0805
GRBCM++ (VFE) (Dc size 150):
MSE 0.01219412, SMSE 0.0128, MSLL -2.1488
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01205852, SMSE 0.0127, MSLL -2.1457
kj = 4
gr = 0.4500
GRBCM (Dc size 150): MSE 0.01389724, SMSE 0.0146, MSLL -2.1011
GRBCM++ (VFE) (Dc size 150):
MSE 0.01191639, SMSE 0.0125, MSLL -2.1686
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01181586, SMSE 0.0124, MSLL -2.1611
kj = 5
gr = 0.5000
GRBCM (Dc size 150): MSE 0.01343202, SMSE 0.0141, MSLL -2.1106
GRBCM++ (VFE) (Dc size 150):
MSE 0.01181086, SMSE 0.0124, MSLL -2.1800
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01168053, SMSE 0.0123, MSLL -2.1720
kj = 6
gr = 0.5500
GRBCM (Dc size 150): MSE 0.01270172, SMSE 0.0133, MSLL -2.1330
GRBCM++ (VFE) (Dc size 150):
MSE 0.01157062, SMSE 0.0121, MSLL -2.1918
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01139965, SMSE 0.0120, MSLL -2.1861
kj = 7
gr = 0.6000
GRBCM (Dc size 150): MSE 0.01259985, SMSE 0.0132, MSLL -2.1247
GRBCM++ (VFE) (Dc size 150):
MSE 0.01157882, SMSE 0.0122, MSLL -2.1942
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01135532, SMSE 0.0119, MSLL -2.1907
kj = 8
gr = 0.6500
GRBCM (Dc size 150): MSE 0.01234027, SMSE 0.0130, MSLL -2.1193
GRBCM++ (VFE) (Dc size 150):
MSE 0.01152467, SMSE 0.0121, MSLL -2.1965
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01148786, SMSE 0.0121, MSLL -2.1844
kj = 9
gr = 0.7000
GRBCM (Dc size 150): MSE 0.01211184, SMSE 0.0127, MSLL -2.1235
GRBCM++ (VFE) (Dc size 150):
MSE 0.01140105, SMSE 0.0120, MSLL -2.1995
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01149870, SMSE 0.0121, MSLL -2.1813
kj = 10
gr = 0.7500
GRBCM (Dc size 150): MSE 0.01196623, SMSE 0.0126, MSLL -2.1149
GRBCM++ (VFE) (Dc size 150):
MSE 0.01138711, SMSE 0.0120, MSLL -2.1971
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01156105, SMSE 0.0121, MSLL -2.1743
kj = 11

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gr = 0.8000
GRBCM (Dc size 150): MSE 0.01170724, SMSE 0.0123, MSLL -2.1174
GRBCM++ (VFE) (Dc size 150):
MSE 0.01129221, SMSE 0.0119, MSLL -2.1983
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01159206, SMSE 0.0122, MSLL -2.1661
kj = 12
gr = 0.8500
GRBCM (Dc size 150): MSE 0.01165361, SMSE 0.0122, MSLL -2.1041
GRBCM++ (VFE) (Dc size 150):
MSE 0.01123925, SMSE 0.0118, MSLL -2.1983
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01155159, SMSE 0.0121, MSLL -2.1625
kj = 13
gr = 0.9000
GRBCM (Dc size 150): MSE 0.01153679, SMSE 0.0121, MSLL -2.1005
GRBCM++ (VFE) (Dc size 150):
MSE 0.01113399, SMSE 0.0117, MSLL -2.2009
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01151773, SMSE 0.0121, MSLL -2.1600
kj = 14
gr = 0.9500
GRBCM (Dc size 150): MSE 0.01153869, SMSE 0.0121, MSLL -2.0857
GRBCM++ (VFE) (Dc size 150):
MSE 0.01109318, SMSE 0.0116, MSLL -2.1985
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01153632, SMSE 0.0121, MSLL -2.1513
kj = 15
gr = 1
GRBCM (Dc size 150): MSE 0.01139128, SMSE 0.0120, MSLL -2.0893
GRBCM++ (VFE) (Dc size 150):
MSE 0.01095005, SMSE 0.0115, MSLL -2.2038
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01140266, SMSE 0.0120, MSLL -2.1567
=====2=====
Optimizing hyps in training...
Lineasearch      0; Value 5.638740e+03
Lineasearch      1; Value 3.605276e+03
Lineasearch      2; Value -6.005281e+02
Lineasearch      3; Value -4.724894e+03
Lineasearch      4; Value -5.926020e+03
Lineasearch      5; Value -5.970024e+03
Lineasearch      6; Value -6.050656e+03
Lineasearch      7; Value -6.173519e+03
Lineasearch      8; Value -6.285687e+03
Lineasearch      9; Value -6.328349e+03
Lineasearch     10; Value -6.329906e+03
Lineasearch     11; Value -6.331703e+03
Lineasearch     12; Value -6.333470e+03
Lineasearch     13; Value -6.334108e+03
Lineasearch     14; Value -6.334721e+03
Lineasearch     15; Value -6.334746e+03
Lineasearch     16; Value -6.334751e+03
Lineasearch     17; Value -6.334752e+03
Lineasearch     18; Value -6.334753e+03
Lineasearch     19; Value -6.334754e+03
Lineasearch     20; Value -6.334755e+03
Lineasearch     21; Value -6.334755e+03
Lineasearch     22; Value -6.334755e+03
Lineasearch     23; Value -6.334755e+03
Lineasearch     24; Value -6.334755e+03
Lineasearch     25; Value -6.334755e+03
Lineasearch     26; Value -6.334755e+03
Lineasearch     27; Value -6.334755e+03
Lineasearch     28; Value -6.334755e+03

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Linesearch      29; Value -6.334755e+03
Linesearch      30; Value -6.334755e+03
GRBCM (Dc size 150): MSE 0.01163393, SMSE 0.01221238, MSLL -2.02731277
Function evaluation      0; Value 1.073122e+05
Function evaluation     12; Value 9.321011e+04
Function evaluation     14; Value 6.269181e+04
Function evaluation     17; Value 4.860754e+04
Function evaluation     19; Value 3.865753e+04
Function evaluation     21; Value 3.284925e+04
Function evaluation     23; Value 2.909480e+04
Function evaluation     25; Value 2.581498e+04
Function evaluation     27; Value 2.369526e+04
Function evaluation     29; Value 2.213212e+04
Function evaluation     31; Value 2.113722e+04
Function evaluation     33; Value 2.024226e+04
Function evaluation     35; Value 1.967130e+04
Function evaluation     37; Value 1.918537e+04
Function evaluation     39; Value 1.884627e+04
Function evaluation     41; Value 1.842534e+04
Function evaluation     43; Value 1.808744e+04
Function evaluation     45; Value 1.780546e+04
Function evaluation     47; Value 1.760369e+04
Function evaluation     49; Value 1.731485e+04
Function evaluation     51; Value 1.710214e+04
Function evaluation     53; Value 1.684405e+04
Function evaluation     55; Value 1.662729e+04
Function evaluation     57; Value 1.644687e+04
Function evaluation     59; Value 1.629690e+04
Function evaluation     61; Value 1.619072e+04
Function evaluation     63; Value 1.611091e+04
Function evaluation     65; Value 1.604448e+04
Function evaluation     67; Value 1.599525e+04
Function evaluation     69; Value 1.595169e+04
Function evaluation     71; Value 1.589429e+04
Function evaluation     72; Value 1.583319e+04
Function evaluation     74; Value 1.577086e+04
Function evaluation     76; Value 1.573179e+04
Function evaluation     77; Value 1.569614e+04
Function evaluation     79; Value 1.566987e+04
Function evaluation     80; Value 1.564331e+04
Function evaluation     82; Value 1.562420e+04
Function evaluation     84; Value 1.561324e+04
Function evaluation     85; Value 1.560287e+04
Function evaluation     87; Value 1.558588e+04
Function evaluation     89; Value 1.555653e+04
Function evaluation     91; Value 1.551765e+04
Function evaluation     93; Value 1.545419e+04
Function evaluation     94; Value 1.539005e+04
Function evaluation     95; Value 1.532702e+04
Function evaluation     96; Value 1.526123e+04
Function evaluation     98; Value 1.520988e+04
Function evaluation    100; Value 1.517294e+04
VFE (Dc size 150): MSE 0.01421180, SMSE 0.01491842, MSLL -1.82636187
Function evaluation      0; Value 7.511621e+02
Function evaluation     10; Value -5.742617e+02
Function evaluation     11; Value -1.396250e+03
Function evaluation     13; Value -1.954734e+03
Function evaluation     16; Value -2.326029e+03
Function evaluation     19; Value -2.709197e+03
Function evaluation     21; Value -3.018381e+03
Function evaluation     24; Value -3.078099e+03
Function evaluation     26; Value -3.264271e+03
Function evaluation     28; Value -3.444762e+03
Function evaluation     30; Value -3.558030e+03
Function evaluation     32; Value -3.629687e+03

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Function evaluation 34; Value -3.714286e+03
 Function evaluation 36; Value -3.776445e+03
 Function evaluation 38; Value -3.813070e+03
 Function evaluation 39; Value -3.847079e+03
 Function evaluation 41; Value -3.872985e+03
 Function evaluation 43; Value -3.895947e+03
 Function evaluation 45; Value -3.911625e+03
 Function evaluation 47; Value -3.923151e+03
 Function evaluation 48; Value -3.936021e+03
 Function evaluation 49; Value -3.948857e+03
 Function evaluation 50; Value -3.961642e+03
 Function evaluation 52; Value -3.975863e+03
 Function evaluation 53; Value -3.991111e+03
 Function evaluation 55; Value -4.014272e+03
 Function evaluation 57; Value -4.026489e+03
 Function evaluation 59; Value -4.034557e+03
 Function evaluation 61; Value -4.041097e+03
 Function evaluation 62; Value -4.048267e+03
 Function evaluation 64; Value -4.052750e+03
 Function evaluation 66; Value -4.056072e+03
 Function evaluation 68; Value -4.058230e+03
 Function evaluation 70; Value -4.059852e+03
 Function evaluation 71; Value -4.061436e+03
 Function evaluation 73; Value -4.062710e+03
 Function evaluation 74; Value -4.064075e+03
 Function evaluation 76; Value -4.065035e+03
 Function evaluation 78; Value -4.065817e+03
 Function evaluation 79; Value -4.066543e+03
 Function evaluation 81; Value -4.067189e+03
 Function evaluation 83; Value -4.067601e+03
 Function evaluation 85; Value -4.067944e+03
 Function evaluation 86; Value -4.068274e+03
 Function evaluation 88; Value -4.068507e+03
 Function evaluation 90; Value -4.068711e+03
 Function evaluation 92; Value -4.068862e+03
 Function evaluation 94; Value -4.068994e+03
 Function evaluation 96; Value -4.069166e+03
 Function evaluation 98; Value -4.069416e+03
 Function evaluation 100; Value -4.069713e+03
 SPSPG (Dc size 150): MSE 0.02292318, SMSE 0.02406293, MSLL -1.81821628
 GRBCM (VFE) (Dc size 150): MSE 0.01119413, SMSE 0.01175071, MSLL -2.18946377
 GRBCM (SPGP) (Dc size 150): MSE 0.01124675, SMSE 0.01180595, MSLL -2.17114343
 RBCM (Dc size 150): MSE 0.01185232, SMSE 0.0124, MSLL -1.5092
 BCM (Dc size 150): MSE 0.01226770, SMSE 0.0129, MSLL -2.0627
 PoE (Dc size 150): MSE 0.08605954, SMSE 0.0903, MSLL 5.2101
 GPoE (Dc size 150): MSE 0.01193050, SMSE 0.0125, MSLL -1.4729
 processing distance: 100/10000
 processing distance: 200/10000
 processing distance: 300/10000
 processing distance: 400/10000
 processing distance: 500/10000
 processing distance: 600/10000
 processing distance: 700/10000
 processing distance: 800/10000
 processing distance: 900/10000
 processing distance: 1000/10000
 processing distance: 1100/10000
 processing distance: 1200/10000
 processing distance: 1300/10000
 processing distance: 1400/10000
 processing distance: 1500/10000
 processing distance: 1600/10000
 processing distance: 1700/10000
 processing distance: 1800/10000
 processing distance: 1900/10000

processing distance: 8500/10000
 processing distance: 8600/10000
 processing distance: 8700/10000
 processing distance: 8800/10000
 processing distance: 8900/10000
 processing distance: 9000/10000
 processing distance: 9100/10000
 processing distance: 9200/10000
 processing distance: 9300/10000
 processing distance: 9400/10000
 processing distance: 9500/10000
 processing distance: 9600/10000
 processing distance: 9700/10000
 processing distance: 9800/10000
 processing distance: 9900/10000
 processing distance: 10000/10000
 kj = 1
 gr = 0.3000
 GRBCM (Dc size 150): MSE 0.01710950, SMSE 0.0180, MSLL -1.9981
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01206910, SMSE 0.0127, MSLL -2.1174
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01253507, SMSE 0.0132, MSLL -2.1047
 kj = 2
 gr = 0.3500
 GRBCM (Dc size 150): MSE 0.01597488, SMSE 0.0168, MSLL -2.0242
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01186532, SMSE 0.0125, MSLL -2.1436
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01270068, SMSE 0.0133, MSLL -2.1099
 kj = 3
 gr = 0.4000
 GRBCM (Dc size 150): MSE 0.01525040, SMSE 0.0160, MSLL -2.0413
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01178270, SMSE 0.0124, MSLL -2.1624
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01269104, SMSE 0.0133, MSLL -2.1184
 kj = 4
 gr = 0.4500
 GRBCM (Dc size 150): MSE 0.01441597, SMSE 0.0151, MSLL -2.0600
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01161554, SMSE 0.0122, MSLL -2.1788
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01242633, SMSE 0.0130, MSLL -2.1324
 kj = 5
 gr = 0.5000
 GRBCM (Dc size 150): MSE 0.01388002, SMSE 0.0146, MSLL -2.0694
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01153083, SMSE 0.0121, MSLL -2.1911
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01230669, SMSE 0.0129, MSLL -2.1347
 kj = 6
 gr = 0.5500
 GRBCM (Dc size 150): MSE 0.01338500, SMSE 0.0141, MSLL -2.0763
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01169234, SMSE 0.0123, MSLL -2.1885
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01217082, SMSE 0.0128, MSLL -2.1420
 kj = 7
 gr = 0.6000
 GRBCM (Dc size 150): MSE 0.01285115, SMSE 0.0135, MSLL -2.0852
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01143049, SMSE 0.0120, MSLL -2.2015
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01175538, SMSE 0.0123, MSLL -2.1633

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kj = 8
gr = 0.6500
GRBCM (Dc size 150): MSE 0.01254187, SMSE 0.0132, MSLL -2.0879
GRBCM++ (VFE) (Dc size 150):
MSE 0.01135387, SMSE 0.0119, MSLL -2.2061
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01155790, SMSE 0.0121, MSLL -2.1752
kj = 9
gr = 0.7000
GRBCM (Dc size 150): MSE 0.01230063, SMSE 0.0129, MSLL -2.0898
GRBCM++ (VFE) (Dc size 150):
MSE 0.01130950, SMSE 0.0119, MSLL -2.2073
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01147323, SMSE 0.0120, MSLL -2.1792
kj = 10
gr = 0.7500
GRBCM (Dc size 150): MSE 0.01219888, SMSE 0.0128, MSLL -2.0756
GRBCM++ (VFE) (Dc size 150):
MSE 0.01135642, SMSE 0.0119, MSLL -2.2035
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01143491, SMSE 0.0120, MSLL -2.1808
kj = 11
gr = 0.8000
GRBCM (Dc size 150): MSE 0.01218183, SMSE 0.0128, MSLL -2.0575
GRBCM++ (VFE) (Dc size 150):
MSE 0.01138244, SMSE 0.0119, MSLL -2.1988
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01150544, SMSE 0.0121, MSLL -2.1714
kj = 12
gr = 0.8500
GRBCM (Dc size 150): MSE 0.01194258, SMSE 0.0125, MSLL -2.0588
GRBCM++ (VFE) (Dc size 150):
MSE 0.01129191, SMSE 0.0119, MSLL -2.2000
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01134323, SMSE 0.0119, MSLL -2.1792
kj = 13
gr = 0.9000
GRBCM (Dc size 150): MSE 0.01174444, SMSE 0.0123, MSLL -2.0555
GRBCM++ (VFE) (Dc size 150):
MSE 0.01116425, SMSE 0.0117, MSLL -2.2019
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01121860, SMSE 0.0118, MSLL -2.1833
kj = 14
gr = 0.9500
GRBCM (Dc size 150): MSE 0.01166532, SMSE 0.0122, MSLL -2.0405
GRBCM++ (VFE) (Dc size 150):
MSE 0.01116838, SMSE 0.0117, MSLL -2.1959
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01123717, SMSE 0.0118, MSLL -2.1764
kj = 15
gr = 1
GRBCM (Dc size 150): MSE 0.01163393, SMSE 0.0122, MSLL -2.0273
GRBCM++ (VFE) (Dc size 150):
MSE 0.01119413, SMSE 0.0118, MSLL -2.1895
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01124675, SMSE 0.0118, MSLL -2.1711
=====3=====
Optimizing hyps in training...
Linesearch      0; Value 5.386432e+03
Linesearch      1; Value 2.984077e+03
Linesearch      2; Value -9.523606e+02
Linesearch      3; Value -5.161134e+03
Linesearch      4; Value -5.971739e+03
Linesearch      5; Value -6.004446e+03
Linesearch      6; Value -6.146076e+03

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LineSearch      7; Value -6.269819e+03
LineSearch      8; Value -6.292849e+03
LineSearch      9; Value -6.305531e+03
LineSearch     10; Value -6.308167e+03
LineSearch     11; Value -6.310531e+03
LineSearch     12; Value -6.310671e+03
LineSearch     13; Value -6.310680e+03
LineSearch     14; Value -6.310684e+03
LineSearch     15; Value -6.310684e+03
LineSearch     16; Value -6.310684e+03
LineSearch     17; Value -6.310684e+03
LineSearch     18; Value -6.310684e+03
LineSearch     19; Value -6.310684e+03
LineSearch     20; Value -6.310684e+03
LineSearch     21; Value -6.310684e+03
LineSearch     22; Value -6.310684e+03
LineSearch     23; Value -6.310684e+03
GRBCM (Dc size 150): MSE 0.01158586, SMSE 0.01216192, MSLL -1.99965990
Function evaluation    0; Value 1.080256e+05
Function evaluation   11; Value 1.068845e+05
Function evaluation   16; Value 9.061339e+04
Function evaluation   18; Value 6.013886e+04
Function evaluation   21; Value 4.948223e+04
Function evaluation   22; Value 4.000395e+04
Function evaluation   24; Value 3.485884e+04
Function evaluation   25; Value 3.074590e+04
Function evaluation   27; Value 2.699422e+04
Function evaluation   29; Value 2.419657e+04
Function evaluation   31; Value 2.251128e+04
Function evaluation   33; Value 2.130937e+04
Function evaluation   34; Value 2.015717e+04
Function evaluation   36; Value 1.920895e+04
Function evaluation   38; Value 1.843460e+04
Function evaluation   40; Value 1.784670e+04
Function evaluation   42; Value 1.731294e+04
Function evaluation   44; Value 1.689626e+04
Function evaluation   45; Value 1.651631e+04
Function evaluation   47; Value 1.627488e+04
Function evaluation   49; Value 1.605678e+04
Function evaluation   51; Value 1.590251e+04
Function evaluation   52; Value 1.574227e+04
Function evaluation   54; Value 1.562997e+04
Function evaluation   55; Value 1.552284e+04
Function evaluation   57; Value 1.544198e+04
Function evaluation   59; Value 1.531584e+04
Function evaluation   61; Value 1.523844e+04
Function evaluation   63; Value 1.517803e+04
Function evaluation   65; Value 1.514298e+04
Function evaluation   67; Value 1.511534e+04
Function evaluation   69; Value 1.509839e+04
Function evaluation   71; Value 1.507784e+04
Function evaluation   72; Value 1.505859e+04
Function evaluation   74; Value 1.504068e+04
Function evaluation   75; Value 1.502369e+04
Function evaluation   77; Value 1.500926e+04
Function evaluation   79; Value 1.500069e+04
Function evaluation   80; Value 1.499135e+04
Function evaluation   82; Value 1.498375e+04
Function evaluation   83; Value 1.497605e+04
Function evaluation   84; Value 1.496832e+04
Function evaluation   86; Value 1.495899e+04
Function evaluation   88; Value 1.494767e+04
Function evaluation   90; Value 1.493326e+04
Function evaluation   92; Value 1.491229e+04
Function evaluation   94; Value 1.489440e+04

```

Function evaluation 96; Value 1.487897e+04
 Function evaluation 97; Value 1.486405e+04
 Function evaluation 98; Value 1.485004e+04
 Function evaluation 100; Value 1.484138e+04
 VFE (Dc size 150): MSE 0.01338109, SMSE 0.01404641, MSLL -1.83488690
 Function evaluation 0; Value 6.146704e+02
 Function evaluation 5; Value 5.828182e+02
 Function evaluation 11; Value -3.320418e+02
 Function evaluation 15; Value -7.489599e+02
 Function evaluation 16; Value -1.281569e+03
 Function evaluation 17; Value -1.833580e+03
 Function evaluation 19; Value -2.280788e+03
 Function evaluation 22; Value -2.439660e+03
 Function evaluation 24; Value -2.824487e+03
 Function evaluation 26; Value -3.076901e+03
 Function evaluation 28; Value -3.279065e+03
 Function evaluation 30; Value -3.433164e+03
 Function evaluation 32; Value -3.539743e+03
 Function evaluation 34; Value -3.602123e+03
 Function evaluation 35; Value -3.663897e+03
 Function evaluation 37; Value -3.707388e+03
 Function evaluation 39; Value -3.738265e+03
 Function evaluation 41; Value -3.767340e+03
 Function evaluation 43; Value -3.802769e+03
 Function evaluation 45; Value -3.826875e+03
 Function evaluation 46; Value -3.852755e+03
 Function evaluation 47; Value -3.874243e+03
 Function evaluation 49; Value -3.892847e+03
 Function evaluation 51; Value -3.913308e+03
 Function evaluation 53; Value -3.927880e+03
 Function evaluation 55; Value -3.935672e+03
 Function evaluation 57; Value -3.947845e+03
 Function evaluation 59; Value -3.963575e+03
 Function evaluation 61; Value -3.969283e+03
 Function evaluation 63; Value -3.983152e+03
 Function evaluation 65; Value -3.989687e+03
 Function evaluation 67; Value -3.997984e+03
 Function evaluation 68; Value -4.005866e+03
 Function evaluation 70; Value -4.008972e+03
 Function evaluation 72; Value -4.014366e+03
 Function evaluation 74; Value -4.016758e+03
 Function evaluation 76; Value -4.022873e+03
 Function evaluation 78; Value -4.024324e+03
 Function evaluation 80; Value -4.026483e+03
 Function evaluation 82; Value -4.029459e+03
 Function evaluation 84; Value -4.031878e+03
 Function evaluation 85; Value -4.034209e+03
 Function evaluation 86; Value -4.036470e+03
 Function evaluation 88; Value -4.038375e+03
 Function evaluation 90; Value -4.040707e+03
 Function evaluation 92; Value -4.044364e+03
 Function evaluation 94; Value -4.047045e+03
 Function evaluation 96; Value -4.049128e+03
 Function evaluation 98; Value -4.050661e+03
 Function evaluation 99; Value -4.052075e+03
 SP5G (Dc size 150): MSE 0.02339898, SMSE 0.02456240, MSLL -1.81421902
 GRBCM (VFE) (Dc size 150): MSE 0.01109536, SMSE 0.01164703, MSLL -2.18553293
 GRBCM (SPGP) (Dc size 150): MSE 0.01105575, SMSE 0.01160545, MSLL -2.18009869
 RBCM (Dc size 150): MSE 0.01166361, SMSE 0.0122, MSLL -1.5217
 BCM (Dc size 150): MSE 0.01211051, SMSE 0.0127, MSLL -2.0758
 PoE (Dc size 150): MSE 0.07710818, SMSE 0.0809, MSLL 4.2973
 GPoE (Dc size 150): MSE 0.01170504, SMSE 0.0123, MSLL -1.4961
 processing distance: 100/10000
 processing distance: 200/10000
 processing distance: 300/10000

processing distance: 6900/10000
 processing distance: 7000/10000
 processing distance: 7100/10000
 processing distance: 7200/10000
 processing distance: 7300/10000
 processing distance: 7400/10000
 processing distance: 7500/10000
 processing distance: 7600/10000
 processing distance: 7700/10000
 processing distance: 7800/10000
 processing distance: 7900/10000
 processing distance: 8000/10000
 processing distance: 8100/10000
 processing distance: 8200/10000
 processing distance: 8300/10000
 processing distance: 8400/10000
 processing distance: 8500/10000
 processing distance: 8600/10000
 processing distance: 8700/10000
 processing distance: 8800/10000
 processing distance: 8900/10000
 processing distance: 9000/10000
 processing distance: 9100/10000
 processing distance: 9200/10000
 processing distance: 9300/10000
 processing distance: 9400/10000
 processing distance: 9500/10000
 processing distance: 9600/10000
 processing distance: 9700/10000
 processing distance: 9800/10000
 processing distance: 9900/10000
 processing distance: 10000/10000
 kj = 1
 gr = 0.3000
 GRBCM (Dc size 150): MSE 0.01834617, SMSE 0.0193, MSLL -1.9849
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01270275, SMSE 0.0133, MSLL -2.1068
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01259191, SMSE 0.0132, MSLL -2.1065
 kj = 2
 gr = 0.3500
 GRBCM (Dc size 150): MSE 0.01595394, SMSE 0.0167, MSLL -2.0405
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01239972, SMSE 0.0130, MSLL -2.1355
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01221854, SMSE 0.0128, MSLL -2.1342
 kj = 3
 gr = 0.4000
 GRBCM (Dc size 150): MSE 0.01487661, SMSE 0.0156, MSLL -2.0650
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01216192, SMSE 0.0128, MSLL -2.1562
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01211535, SMSE 0.0127, MSLL -2.1486
 kj = 4
 gr = 0.4500
 GRBCM (Dc size 150): MSE 0.01422002, SMSE 0.0149, MSLL -2.0749
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01201862, SMSE 0.0126, MSLL -2.1690
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01207273, SMSE 0.0127, MSLL -2.1572
 kj = 5
 gr = 0.5000
 GRBCM (Dc size 150): MSE 0.01352306, SMSE 0.0142, MSLL -2.0869
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01194920, SMSE 0.0125, MSLL -2.1768

GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01188293, SMSE 0.0125, MSLL -2.1664
 kj = 6
 gr = 0.5500
 GRBCM (Dc size 150): MSE 0.01304433, SMSE 0.0137, MSLL -2.0927
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01173448, SMSE 0.0123, MSLL -2.1890
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01166347, SMSE 0.0122, MSLL -2.1786
 kj = 7
 gr = 0.6000
 GRBCM (Dc size 150): MSE 0.01263689, SMSE 0.0133, MSLL -2.0925
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01152358, SMSE 0.0121, MSLL -2.1984
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01147454, SMSE 0.0120, MSLL -2.1871
 kj = 8
 gr = 0.6500
 GRBCM (Dc size 150): MSE 0.01231041, SMSE 0.0129, MSLL -2.0928
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01133219, SMSE 0.0119, MSLL -2.2050
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01126155, SMSE 0.0118, MSLL -2.1952
 kj = 9
 gr = 0.7000
 GRBCM (Dc size 150): MSE 0.01218308, SMSE 0.0128, MSLL -2.0733
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01131044, SMSE 0.0119, MSLL -2.2029
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01125213, SMSE 0.0118, MSLL -2.1937
 kj = 10
 gr = 0.7500
 GRBCM (Dc size 150): MSE 0.01206722, SMSE 0.0127, MSLL -2.0626
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01127482, SMSE 0.0118, MSLL -2.2021
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01116588, SMSE 0.0117, MSLL -2.1950
 kj = 11
 gr = 0.8000
 GRBCM (Dc size 150): MSE 0.01192459, SMSE 0.0125, MSLL -2.0498
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01121857, SMSE 0.0118, MSLL -2.1999
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01112935, SMSE 0.0117, MSLL -2.1935
 kj = 12
 gr = 0.8500
 GRBCM (Dc size 150): MSE 0.01199940, SMSE 0.0126, MSLL -2.0175
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01126378, SMSE 0.0118, MSLL -2.1924
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01116024, SMSE 0.0117, MSLL -2.1868
 kj = 13
 gr = 0.9000
 GRBCM (Dc size 150): MSE 0.01174158, SMSE 0.0123, MSLL -2.0255
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01115692, SMSE 0.0117, MSLL -2.1931
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01106563, SMSE 0.0116, MSLL -2.1874
 kj = 14
 gr = 0.9500
 GRBCM (Dc size 150): MSE 0.01166940, SMSE 0.0122, MSLL -2.0088
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01111561, SMSE 0.0117, MSLL -2.1894
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01106897, SMSE 0.0116, MSLL -2.1828

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kj = 15
gr = 1
GRBCM (Dc size 150): MSE 0.01158586, SMSE 0.0122, MSLL -1.9997
GRBCM++ (VFE) (Dc size 150):
MSE 0.01109536, SMSE 0.0116, MSLL -2.1855
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01105575, SMSE 0.0116, MSLL -2.1801
=====4=====
Optimizing hyps in training...
Linesearch      0; Value 5.549886e+03
Linesearch      1; Value 3.928008e+03
Linesearch      2; Value 2.240696e+02
Linesearch      3; Value -4.652802e+03
Linesearch      4; Value -5.874838e+03
Linesearch      5; Value -5.989814e+03
Linesearch      6; Value -6.012587e+03
Linesearch      7; Value -6.059889e+03
Linesearch      8; Value -6.274910e+03
Linesearch      9; Value -6.302532e+03
Linesearch     10; Value -6.304613e+03
Linesearch     11; Value -6.305444e+03
Linesearch     12; Value -6.305536e+03
Linesearch     13; Value -6.305787e+03
Linesearch     14; Value -6.305890e+03
Linesearch     15; Value -6.305892e+03
Linesearch     16; Value -6.305893e+03
Linesearch     17; Value -6.305893e+03
Linesearch     18; Value -6.305893e+03
Linesearch     19; Value -6.305893e+03
Linesearch     20; Value -6.305893e+03
Linesearch     21; Value -6.305893e+03
Linesearch     22; Value -6.305893e+03
Linesearch     23; Value -6.305893e+03
Linesearch     24; Value -6.305893e+03
Linesearch     25; Value -6.305893e+03
Linesearch     26; Value -6.305893e+03
Linesearch     27; Value -6.305893e+03
Linesearch     28; Value -6.305893e+03
Linesearch     29; Value -6.305893e+03
Linesearch     30; Value -6.305893e+03
GRBCM (Dc size 150): MSE 0.01176037, SMSE 0.01234510, MSLL -2.01008836
Function evaluation  0; Value 9.432406e+04
Function evaluation 20; Value 9.034200e+04
Function evaluation 22; Value 6.987268e+04
Function evaluation 23; Value 5.025063e+04
Function evaluation 25; Value 3.943308e+04
Function evaluation 28; Value 3.324968e+04
Function evaluation 29; Value 2.723544e+04
Function evaluation 31; Value 2.375363e+04
Function evaluation 33; Value 2.140762e+04
Function evaluation 35; Value 1.950485e+04
Function evaluation 37; Value 1.824203e+04
Function evaluation 39; Value 1.738354e+04
Function evaluation 41; Value 1.678572e+04
Function evaluation 43; Value 1.625110e+04
Function evaluation 44; Value 1.573579e+04
Function evaluation 45; Value 1.520250e+04
Function evaluation 47; Value 1.472536e+04
Function evaluation 49; Value 1.435396e+04
Function evaluation 51; Value 1.406640e+04
Function evaluation 53; Value 1.383135e+04
Function evaluation 55; Value 1.367242e+04
Function evaluation 57; Value 1.356030e+04
Function evaluation 58; Value 1.344000e+04
Function evaluation 59; Value 1.333705e+04

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Function evaluation 61; Value 1.327862e+04
 Function evaluation 63; Value 1.320837e+04
 Function evaluation 65; Value 1.315172e+04
 Function evaluation 67; Value 1.310636e+04
 Function evaluation 68; Value 1.306359e+04
 Function evaluation 70; Value 1.303160e+04
 Function evaluation 72; Value 1.299565e+04
 Function evaluation 74; Value 1.297184e+04
 Function evaluation 76; Value 1.295070e+04
 Function evaluation 77; Value 1.292831e+04
 Function evaluation 78; Value 1.290712e+04
 Function evaluation 80; Value 1.288159e+04
 Function evaluation 82; Value 1.285894e+04
 Function evaluation 84; Value 1.283886e+04
 Function evaluation 86; Value 1.282091e+04
 Function evaluation 88; Value 1.280084e+04
 Function evaluation 89; Value 1.278082e+04
 Function evaluation 91; Value 1.276237e+04
 Function evaluation 92; Value 1.274457e+04
 Function evaluation 94; Value 1.272882e+04
 Function evaluation 96; Value 1.271492e+04
 Function evaluation 98; Value 1.270313e+04
 Function evaluation 100; Value 1.269416e+04
 VFE (Dc size 150): MSE 0.01398996, SMSE 0.01468555, MSLL -1.85715006
 Function evaluation 0; Value 4.669355e+02
 Function evaluation 31; Value -6.285982e+02
 Function evaluation 32; Value -1.334184e+03
 Function evaluation 35; Value -1.747342e+03
 Function evaluation 36; Value -2.097567e+03
 Function evaluation 38; Value -2.593933e+03
 Function evaluation 40; Value -2.862139e+03
 Function evaluation 42; Value -3.166353e+03
 Function evaluation 44; Value -3.297140e+03
 Function evaluation 46; Value -3.514057e+03
 Function evaluation 48; Value -3.678245e+03
 Function evaluation 50; Value -3.802950e+03
 Function evaluation 52; Value -3.884495e+03
 Function evaluation 54; Value -3.931832e+03
 Function evaluation 55; Value -3.978131e+03
 Function evaluation 57; Value -4.004820e+03
 Function evaluation 59; Value -4.045383e+03
 Function evaluation 61; Value -4.079520e+03
 Function evaluation 63; Value -4.106814e+03
 Function evaluation 64; Value -4.134852e+03
 Function evaluation 66; Value -4.156296e+03
 Function evaluation 68; Value -4.171897e+03
 Function evaluation 70; Value -4.193521e+03
 Function evaluation 71; Value -4.216776e+03
 Function evaluation 73; Value -4.243311e+03
 Function evaluation 75; Value -4.261552e+03
 Function evaluation 77; Value -4.276184e+03
 Function evaluation 78; Value -4.289651e+03
 Function evaluation 80; Value -4.295598e+03
 Function evaluation 82; Value -4.302551e+03
 Function evaluation 84; Value -4.308561e+03
 Function evaluation 86; Value -4.313391e+03
 Function evaluation 87; Value -4.318369e+03
 Function evaluation 89; Value -4.324420e+03
 Function evaluation 91; Value -4.328638e+03
 Function evaluation 93; Value -4.332377e+03
 Function evaluation 95; Value -4.337013e+03
 Function evaluation 97; Value -4.340318e+03
 Function evaluation 99; Value -4.342659e+03
 Function evaluation 100; Value -4.345091e+03
 SPSG (Dc size 150): MSE 0.02690281, SMSE 0.02824044, MSLL -1.83614458

GRBCM (VFE) (Dc size 150): MSE 0.01108963, SMSE 0.01164102, MSLL -2.19555611
GRBCM (SPGP) (Dc size 150): MSE 0.01116398, SMSE 0.01171906, MSLL -2.17877549
RBCM (Dc size 150): MSE 0.01175487, SMSE 0.0123, MSLL -1.4909
BCM (Dc size 150): MSE 0.01221997, SMSE 0.0128, MSLL -2.0661
PoE (Dc size 150): MSE 0.07332335, SMSE 0.0770, MSLL 3.7752
GPoE (Dc size 150): MSE 0.01170156, SMSE 0.0123, MSLL -1.4797
processing distance: 100/10000
processing distance: 200/10000
processing distance: 300/10000
processing distance: 400/10000
processing distance: 500/10000
processing distance: 600/10000
processing distance: 700/10000
processing distance: 800/10000
processing distance: 900/10000
processing distance: 1000/10000
processing distance: 1100/10000
processing distance: 1200/10000
processing distance: 1300/10000
processing distance: 1400/10000
processing distance: 1500/10000
processing distance: 1600/10000
processing distance: 1700/10000
processing distance: 1800/10000
processing distance: 1900/10000
processing distance: 2000/10000
processing distance: 2100/10000
processing distance: 2200/10000
processing distance: 2300/10000
processing distance: 2400/10000
processing distance: 2500/10000
processing distance: 2600/10000
processing distance: 2700/10000
processing distance: 2800/10000
processing distance: 2900/10000
processing distance: 3000/10000
processing distance: 3100/10000
processing distance: 3200/10000
processing distance: 3300/10000
processing distance: 3400/10000
processing distance: 3500/10000
processing distance: 3600/10000
processing distance: 3700/10000
processing distance: 3800/10000
processing distance: 3900/10000
processing distance: 4000/10000
processing distance: 4100/10000
processing distance: 4200/10000
processing distance: 4300/10000
processing distance: 4400/10000
processing distance: 4500/10000
processing distance: 4600/10000
processing distance: 4700/10000
processing distance: 4800/10000
processing distance: 4900/10000
processing distance: 5000/10000
processing distance: 5100/10000
processing distance: 5200/10000
processing distance: 5300/10000
processing distance: 5400/10000
processing distance: 5500/10000
processing distance: 5600/10000
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processing distance: 5900/10000

processing distance: 6000/10000
 processing distance: 6100/10000
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 processing distance: 6300/10000
 processing distance: 6400/10000
 processing distance: 6500/10000
 processing distance: 6600/10000
 processing distance: 6700/10000
 processing distance: 6800/10000
 processing distance: 6900/10000
 processing distance: 7000/10000
 processing distance: 7100/10000
 processing distance: 7200/10000
 processing distance: 7300/10000
 processing distance: 7400/10000
 processing distance: 7500/10000
 processing distance: 7600/10000
 processing distance: 7700/10000
 processing distance: 7800/10000
 processing distance: 7900/10000
 processing distance: 8000/10000
 processing distance: 8100/10000
 processing distance: 8200/10000
 processing distance: 8300/10000
 processing distance: 8400/10000
 processing distance: 8500/10000
 processing distance: 8600/10000
 processing distance: 8700/10000
 processing distance: 8800/10000
 processing distance: 8900/10000
 processing distance: 9000/10000
 processing distance: 9100/10000
 processing distance: 9200/10000
 processing distance: 9300/10000
 processing distance: 9400/10000
 processing distance: 9500/10000
 processing distance: 9600/10000
 processing distance: 9700/10000
 processing distance: 9800/10000
 processing distance: 9900/10000
 processing distance: 10000/10000
 kj = 1
 gr = 0.3000
 GRBCM (Dc size 150): MSE 0.01633777, SMSE 0.0172, MSLL -2.0232
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01239572, SMSE 0.0130, MSLL -2.1130
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01414331, SMSE 0.0148, MSLL -2.0680
 kj = 2
 gr = 0.3500
 GRBCM (Dc size 150): MSE 0.01511984, SMSE 0.0159, MSLL -2.0561
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01228758, SMSE 0.0129, MSLL -2.1365
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01354831, SMSE 0.0142, MSLL -2.0881
 kj = 3
 gr = 0.4000
 GRBCM (Dc size 150): MSE 0.01415581, SMSE 0.0149, MSLL -2.0792
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01210955, SMSE 0.0127, MSLL -2.1559
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01319598, SMSE 0.0139, MSLL -2.1100
 kj = 4
 gr = 0.4500
 GRBCM (Dc size 150): MSE 0.01380110, SMSE 0.0145, MSLL -2.0865

GRBCM++ (VFE) (Dc size 150):
 MSE 0.01193365, SMSE 0.0125, MSLL -2.1716
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01221223, SMSE 0.0128, MSLL -2.1460
 kj = 5
 gr = 0.5000
 GRBCM (Dc size 150): MSE 0.01305027, SMSE 0.0137, MSLL -2.1076
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01173746, SMSE 0.0123, MSLL -2.1855
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01195424, SMSE 0.0125, MSLL -2.1596
 kj = 6
 gr = 0.5500
 GRBCM (Dc size 150): MSE 0.01251162, SMSE 0.0131, MSLL -2.1167
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01159548, SMSE 0.0122, MSLL -2.1934
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01171635, SMSE 0.0123, MSLL -2.1733
 kj = 7
 gr = 0.6000
 GRBCM (Dc size 150): MSE 0.01239106, SMSE 0.0130, MSLL -2.1105
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01161800, SMSE 0.0122, MSLL -2.1949
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01175566, SMSE 0.0123, MSLL -2.1755
 kj = 8
 gr = 0.6500
 GRBCM (Dc size 150): MSE 0.01232301, SMSE 0.0129, MSLL -2.0982
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01148113, SMSE 0.0121, MSLL -2.1981
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01157093, SMSE 0.0121, MSLL -2.1809
 kj = 9
 gr = 0.7000
 GRBCM (Dc size 150): MSE 0.01205195, SMSE 0.0127, MSLL -2.1067
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01133317, SMSE 0.0119, MSLL -2.2062
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01142513, SMSE 0.0120, MSLL -2.1877
 kj = 10
 gr = 0.7500
 GRBCM (Dc size 150): MSE 0.01190888, SMSE 0.0125, MSLL -2.0971
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01128846, SMSE 0.0118, MSLL -2.2063
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01135605, SMSE 0.0119, MSLL -2.1886
 kj = 11
 gr = 0.8000
 GRBCM (Dc size 150): MSE 0.01183023, SMSE 0.0124, MSLL -2.0834
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01123365, SMSE 0.0118, MSLL -2.2063
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01130874, SMSE 0.0119, MSLL -2.1869
 kj = 12
 gr = 0.8500
 GRBCM (Dc size 150): MSE 0.01170517, SMSE 0.0123, MSLL -2.0728
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01112283, SMSE 0.0117, MSLL -2.2085
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01118271, SMSE 0.0117, MSLL -2.1906
 kj = 13
 gr = 0.9000
 GRBCM (Dc size 150): MSE 0.01172131, SMSE 0.0123, MSLL -2.0503
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01114490, SMSE 0.0117, MSLL -2.2017


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GRBCM++ (SPGP) (Dc size 150):
MSE 0.01120661, SMSE 0.0118, MSLL -2.1831
kj = 14
gr = 0.9500
GRBCM (Dc size 150): MSE 0.01173146, SMSE 0.0123, MSLL -2.0282
GRBCM++ (VFE) (Dc size 150):
MSE 0.01115160, SMSE 0.0117, MSLL -2.1952
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01124236, SMSE 0.0118, MSLL -2.1759
kj = 15
gr = 1
GRBCM (Dc size 150): MSE 0.01176037, SMSE 0.0123, MSLL -2.0101
GRBCM++ (VFE) (Dc size 150):
MSE 0.01108963, SMSE 0.0116, MSLL -2.1956
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01116398, SMSE 0.0117, MSLL -2.1788
=====5=====
Optimizing hyps in training...
Linesearch      0; Value 5.550688e+03
Linesearch      1; Value 3.405172e+03
Linesearch      2; Value -2.541012e+02
Linesearch      3; Value -5.043024e+03
Linesearch      4; Value -5.999837e+03
Linesearch      5; Value -6.028911e+03
Linesearch      6; Value -6.226571e+03
Linesearch      7; Value -6.280282e+03
Linesearch      8; Value -6.337892e+03
Linesearch      9; Value -6.341289e+03
Linesearch     10; Value -6.345062e+03
Linesearch     11; Value -6.345620e+03
Linesearch     12; Value -6.345710e+03
Linesearch     13; Value -6.345728e+03
Linesearch     14; Value -6.345754e+03
Linesearch     15; Value -6.345756e+03
Linesearch     16; Value -6.345758e+03
Linesearch     17; Value -6.345758e+03
Linesearch     18; Value -6.345758e+03
Linesearch     19; Value -6.345758e+03
Linesearch     20; Value -6.345758e+03
Linesearch     21; Value -6.345758e+03
Linesearch     22; Value -6.345758e+03
Linesearch     23; Value -6.345758e+03
Linesearch     24; Value -6.345758e+03
Linesearch     25; Value -6.345758e+03
GRBCM (Dc size 150): MSE 0.01161221, SMSE 0.01218957, MSLL -2.05599705
Function evaluation      0; Value 8.020045e+04
Function evaluation     12; Value 6.721576e+04
Function evaluation     13; Value 5.000233e+04
Function evaluation     15; Value 4.094570e+04
Function evaluation     17; Value 3.380988e+04
Function evaluation     19; Value 2.844933e+04
Function evaluation     21; Value 2.541513e+04
Function evaluation     22; Value 2.280156e+04
Function evaluation     24; Value 2.124715e+04
Function evaluation     26; Value 1.990066e+04
Function evaluation     28; Value 1.903699e+04
Function evaluation     29; Value 1.815899e+04
Function evaluation     30; Value 1.730276e+04
Function evaluation     32; Value 1.682544e+04
Function evaluation     34; Value 1.641654e+04
Function evaluation     36; Value 1.590782e+04
Function evaluation     38; Value 1.551695e+04
Function evaluation     40; Value 1.517366e+04
Function evaluation     42; Value 1.497110e+04
Function evaluation     43; Value 1.476692e+04

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Function evaluation	44;	Value 1.454719e+04
Function evaluation	46;	Value 1.434730e+04
Function evaluation	47;	Value 1.415249e+04
Function evaluation	48;	Value 1.393257e+04
Function evaluation	49;	Value 1.373960e+04
Function evaluation	51;	Value 1.363941e+04
Function evaluation	53;	Value 1.352142e+04
Function evaluation	55;	Value 1.341569e+04
Function evaluation	57;	Value 1.322246e+04
Function evaluation	59;	Value 1.308855e+04
Function evaluation	61;	Value 1.298637e+04
Function evaluation	63;	Value 1.291005e+04
Function evaluation	65;	Value 1.285747e+04
Function evaluation	67;	Value 1.282154e+04
Function evaluation	69;	Value 1.279079e+04
Function evaluation	71;	Value 1.276326e+04
Function evaluation	73;	Value 1.274050e+04
Function evaluation	75;	Value 1.272033e+04
Function evaluation	77;	Value 1.270654e+04
Function evaluation	79;	Value 1.269624e+04
Function evaluation	81;	Value 1.268814e+04
Function evaluation	83;	Value 1.268191e+04
Function evaluation	84;	Value 1.267534e+04
Function evaluation	86;	Value 1.266800e+04
Function evaluation	87;	Value 1.265985e+04
Function evaluation	89;	Value 1.264468e+04
Function evaluation	91;	Value 1.262785e+04
Function evaluation	93;	Value 1.261480e+04
Function evaluation	95;	Value 1.260381e+04
Function evaluation	97;	Value 1.259478e+04
Function evaluation	98;	Value 1.258639e+04
Function evaluation	100;	Value 1.258208e+04
VFE (Dc size 150): MSE 0.01482629, SMSE 0.01556346, MSLL -1.85681013		
Function evaluation	0;	Value -4.469338e+02
Function evaluation	9;	Value -8.830146e+02
Function evaluation	11;	Value -1.491578e+03
Function evaluation	13;	Value -2.203616e+03
Function evaluation	16;	Value -2.530794e+03
Function evaluation	17;	Value -2.922168e+03
Function evaluation	18;	Value -3.155587e+03
Function evaluation	20;	Value -3.299616e+03
Function evaluation	22;	Value -3.509429e+03
Function evaluation	24;	Value -3.632192e+03
Function evaluation	26;	Value -3.768931e+03
Function evaluation	28;	Value -3.863080e+03
Function evaluation	31;	Value -3.895983e+03
Function evaluation	33;	Value -3.956336e+03
Function evaluation	35;	Value -3.996001e+03
Function evaluation	37;	Value -4.018319e+03
Function evaluation	39;	Value -4.051448e+03
Function evaluation	41;	Value -4.076007e+03
Function evaluation	42;	Value -4.098604e+03
Function evaluation	43;	Value -4.126166e+03
Function evaluation	45;	Value -4.167285e+03
Function evaluation	47;	Value -4.188102e+03
Function evaluation	49;	Value -4.205346e+03
Function evaluation	50;	Value -4.222553e+03
Function evaluation	52;	Value -4.235536e+03
Function evaluation	54;	Value -4.240341e+03
Function evaluation	56;	Value -4.252185e+03
Function evaluation	58;	Value -4.262013e+03
Function evaluation	60;	Value -4.269431e+03
Function evaluation	62;	Value -4.274864e+03
Function evaluation	64;	Value -4.283447e+03
Function evaluation	66;	Value -4.295603e+03

Function evaluation 68; Value -4.301723e+03
 Function evaluation 70; Value -4.309288e+03
 Function evaluation 72; Value -4.315219e+03
 Function evaluation 74; Value -4.326949e+03
 Function evaluation 76; Value -4.335449e+03
 Function evaluation 78; Value -4.339678e+03
 Function evaluation 80; Value -4.347273e+03
 Function evaluation 82; Value -4.352910e+03
 Function evaluation 84; Value -4.354854e+03
 Function evaluation 86; Value -4.359757e+03
 Function evaluation 90; Value -4.361323e+03
 Function evaluation 92; Value -4.364364e+03
 Function evaluation 94; Value -4.365825e+03
 Function evaluation 96; Value -4.367879e+03
 Function evaluation 98; Value -4.369542e+03
 Function evaluation 100; Value -4.370570e+03
 SPSPG (Dc size 150): MSE 0.02325426, SMSE 0.02441047, MSLL -1.86440011
 GRBCM (VFE) (Dc size 150): MSE 0.01104233, SMSE 0.01159137, MSLL -2.19742805
 GRBCM (SPGP) (Dc size 150): MSE 0.01119618, SMSE 0.01175286, MSLL -2.17064650
 RBCM (Dc size 150): MSE 0.01146434, SMSE 0.0120, MSLL -1.5262
 BCM (Dc size 150): MSE 0.01179649, SMSE 0.0124, MSLL -2.0933
 PoE (Dc size 150): MSE 0.07271323, SMSE 0.0763, MSLL 3.8225
 GPoE (Dc size 150): MSE 0.01152724, SMSE 0.0121, MSLL -1.4957
 processing distance: 100/10000
 processing distance: 200/10000
 processing distance: 300/10000
 processing distance: 400/10000
 processing distance: 500/10000
 processing distance: 600/10000
 processing distance: 700/10000
 processing distance: 800/10000
 processing distance: 900/10000
 processing distance: 1000/10000
 processing distance: 1100/10000
 processing distance: 1200/10000
 processing distance: 1300/10000
 processing distance: 1400/10000
 processing distance: 1500/10000
 processing distance: 1600/10000
 processing distance: 1700/10000
 processing distance: 1800/10000
 processing distance: 1900/10000
 processing distance: 2000/10000
 processing distance: 2100/10000
 processing distance: 2200/10000
 processing distance: 2300/10000
 processing distance: 2400/10000
 processing distance: 2500/10000
 processing distance: 2600/10000
 processing distance: 2700/10000
 processing distance: 2800/10000
 processing distance: 2900/10000
 processing distance: 3000/10000
 processing distance: 3100/10000
 processing distance: 3200/10000
 processing distance: 3300/10000
 processing distance: 3400/10000
 processing distance: 3500/10000
 processing distance: 3600/10000
 processing distance: 3700/10000
 processing distance: 3800/10000
 processing distance: 3900/10000
 processing distance: 4000/10000
 processing distance: 4100/10000
 processing distance: 4200/10000

```

processing distance: 4300/10000
processing distance: 4400/10000
processing distance: 4500/10000
processing distance: 4600/10000
processing distance: 4700/10000
processing distance: 4800/10000
processing distance: 4900/10000
processing distance: 5000/10000
processing distance: 5100/10000
processing distance: 5200/10000
processing distance: 5300/10000
processing distance: 5400/10000
processing distance: 5500/10000
processing distance: 5600/10000
processing distance: 5700/10000
processing distance: 5800/10000
processing distance: 5900/10000
processing distance: 6000/10000
processing distance: 6100/10000
processing distance: 6200/10000
processing distance: 6300/10000
processing distance: 6400/10000
processing distance: 6500/10000
processing distance: 6600/10000
processing distance: 6700/10000
processing distance: 6800/10000
processing distance: 6900/10000
processing distance: 7000/10000
processing distance: 7100/10000
processing distance: 7200/10000
processing distance: 7300/10000
processing distance: 7400/10000
processing distance: 7500/10000
processing distance: 7600/10000
processing distance: 7700/10000
processing distance: 7800/10000
processing distance: 7900/10000
processing distance: 8000/10000
processing distance: 8100/10000
processing distance: 8200/10000
processing distance: 8300/10000
processing distance: 8400/10000
processing distance: 8500/10000
processing distance: 8600/10000
processing distance: 8700/10000
processing distance: 8800/10000
processing distance: 8900/10000
processing distance: 9000/10000
processing distance: 9100/10000
processing distance: 9200/10000
processing distance: 9300/10000
processing distance: 9400/10000
processing distance: 9500/10000
processing distance: 9600/10000
processing distance: 9700/10000
processing distance: 9800/10000
processing distance: 9900/10000
processing distance: 10000/10000
kj = 1
gr = 0.3000
GRBCM (Dc size 150): MSE 0.01773401, SMSE 0.0186, MSLL -2.0058
GRBCM++ (VFE) (Dc size 150):
MSE 0.01248909, SMSE 0.0131, MSLL -2.1156
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01275721, SMSE 0.0134, MSLL -2.1114

```

kj = 2
 gr = 0.3500
 GRBCM (Dc size 150): MSE 0.01726470, SMSE 0.0181, MSLL -2.0195
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01228587, SMSE 0.0129, MSLL -2.1378
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01240649, SMSE 0.0130, MSLL -2.1305
 kj = 3
 gr = 0.4000
 GRBCM (Dc size 150): MSE 0.01587415, SMSE 0.0167, MSLL -2.0471
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01208217, SMSE 0.0127, MSLL -2.1578
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01198582, SMSE 0.0126, MSLL -2.1515
 kj = 4
 gr = 0.4500
 GRBCM (Dc size 150): MSE 0.01393708, SMSE 0.0146, MSLL -2.0965
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01169699, SMSE 0.0123, MSLL -2.1808
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01165020, SMSE 0.0122, MSLL -2.1698
 kj = 5
 gr = 0.5000
 GRBCM (Dc size 150): MSE 0.01353297, SMSE 0.0142, MSLL -2.1004
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01181062, SMSE 0.0124, MSLL -2.1842
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01166028, SMSE 0.0122, MSLL -2.1763
 kj = 6
 gr = 0.5500
 GRBCM (Dc size 150): MSE 0.01309312, SMSE 0.0137, MSLL -2.1054
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01171548, SMSE 0.0123, MSLL -2.1911
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01154204, SMSE 0.0121, MSLL -2.1837
 kj = 7
 gr = 0.6000
 GRBCM (Dc size 150): MSE 0.01266661, SMSE 0.0133, MSLL -2.1203
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01162990, SMSE 0.0122, MSLL -2.1959
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01147969, SMSE 0.0121, MSLL -2.1894
 kj = 8
 gr = 0.6500
 GRBCM (Dc size 150): MSE 0.01272251, SMSE 0.0134, MSLL -2.1019
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01172978, SMSE 0.0123, MSLL -2.1907
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01160159, SMSE 0.0122, MSLL -2.1822
 kj = 9
 gr = 0.7000
 GRBCM (Dc size 150): MSE 0.01235299, SMSE 0.0130, MSLL -2.1056
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01156571, SMSE 0.0121, MSLL -2.1936
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01158887, SMSE 0.0122, MSLL -2.1772
 kj = 10
 gr = 0.7500
 GRBCM (Dc size 150): MSE 0.01217254, SMSE 0.0128, MSLL -2.0972
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01138643, SMSE 0.0120, MSLL -2.1993
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01152078, SMSE 0.0121, MSLL -2.1748
 kj = 11
 gr = 0.8000

```

GRBCM (Dc size 150): MSE 0.01188725, SMSE 0.0125, MSLL -2.0984
GRBCM++ (VFE) (Dc size 150):
MSE 0.01115434, SMSE 0.0117, MSLL -2.2085
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01135872, SMSE 0.0119, MSLL -2.1773
kj = 12
gr = 0.8500
GRBCM (Dc size 150): MSE 0.01178595, SMSE 0.0124, MSLL -2.0919
GRBCM++ (VFE) (Dc size 150):
MSE 0.01106921, SMSE 0.0116, MSLL -2.2099
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01129438, SMSE 0.0119, MSLL -2.1785
kj = 13
gr = 0.9000
GRBCM (Dc size 150): MSE 0.01171388, SMSE 0.0123, MSLL -2.0789
GRBCM++ (VFE) (Dc size 150):
MSE 0.01104761, SMSE 0.0116, MSLL -2.2059
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01123929, SMSE 0.0118, MSLL -2.1756
kj = 14
gr = 0.9500
GRBCM (Dc size 150): MSE 0.01165230, SMSE 0.0122, MSLL -2.0657
GRBCM++ (VFE) (Dc size 150):
MSE 0.01102911, SMSE 0.0116, MSLL -2.2022
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01122680, SMSE 0.0118, MSLL -2.1703
kj = 15
gr = 1
GRBCM (Dc size 150): MSE 0.01161221, SMSE 0.0122, MSLL -2.0560
GRBCM++ (VFE) (Dc size 150):
MSE 0.01104233, SMSE 0.0116, MSLL -2.1974
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01119618, SMSE 0.0118, MSLL -2.1706

```

Display results

```

grbcm0_smse = mean(grbcm0_smse_rec(1:kti)); grbcm0_msll = mean(grbcm0_msll_rec(1:kti));
rbcm0_smse = mean(rbcm0_smse_rec(1:kti)); rbcm0_msll = mean(rbcm0_msll_rec(1:kti));
bcm0_smse = mean(bcm0_smse_rec(1:kti)); bcm0_msll = mean(bcm0_msll_rec(1:kti));
poe0_smse = mean(poe0_smse_rec(1:kti)); poe0_msll = mean(poe0_msll_rec(1:kti));
gpoe0_smse = mean(gpoe0_smse_rec(1:kti)); gpoe0_msll = mean(gpoe0_msll_rec(1:kti));
vfe0_smse = mean(vfe0_smse_rec(1:kti)); vfe0_msll = mean(vfe0_msll_rec(1:kti));
spgp0_smse = mean(spgp0_smse_rec(1:kti)); spgp0_msll = mean(spgp0_msll_rec(1:kti));

```

```
fprintf('GRBCM: %.8f, %.4f', grbcm0_smse, grbcm0_msll);
```

```
GRBCM: 0.01208953, -2.0410
```

```
fprintf('RBCM: %.8f, %.4f', rbcm0_smse, rbcm0_msll);
```

```
RBCM: 0.01214952, -1.5484
```

```
fprintf('BCM: %.8f, %.4f', bcm0_smse, bcm0_msll);
```

```
BCM: 0.01263528, -2.0850
```

```
fprintf('GPoE: %.8f, %.4f', gpoe0_smse, gpoe0_msll);
```

GPoE: 0.01227057, -1.5083

```
fprintf('PoE: %.8f, %.4f', poe0_smse, poe0_msll);
```

PoE: 0.08400372, 4.5370

```
fprintf('VFE: %.8f, %.4f', vfe0_smse, vfe0_msll);
```

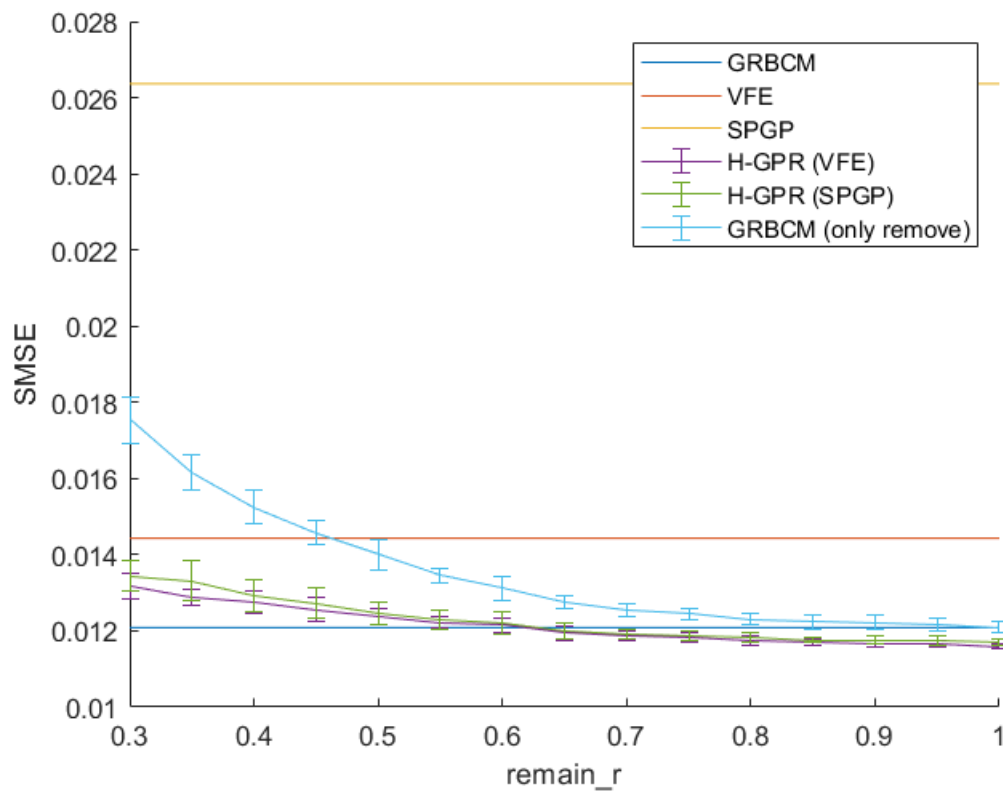
VFE: 0.01457434, -1.8528

```
fprintf('SPGP: %.8f, %.4f', spgp0_smse, spgp0_msll);
```

SPGP: 0.02590087, -1.8408

```
aamse = grbcm2_gr_smse(1:kti,:);
aamsll = grbcm2_gr_msll(1:kti,:);
mmse = mean(aamse);
mstd = std(aamse);
mmse_ro = mean(grbcm_gr_smse(1:kti,:));
mstd_ro = std(grbcm_gr_smse(1:kti,:));
mmse_sp = mean(grbcm2_spgp_gr_smse(1:kti,:));
mstd_sp = std(grbcm2_spgp_gr_smse(1:kti,:));

figure; hold on;
plot([min(grls), max(grls)], [grbcm0_smse,grbcm0_smse]);
plot([min(grls), max(grls)], [vfeSMSE_b1,vfeSMSE_b1]);
plot([min(grls), max(grls)], [spgpSMSE_b1,spgpSMSE_b1]);
errorbar(grls, mmse, mstd);
errorbar(grls, mmse_sp, mstd_sp);
errorbar(grls, mmse_ro, mstd_ro);
legend('GRBCM', 'VFE', 'SPGP', 'H-GPR (VFE)', 'H-GPR (SPGP)', 'GRBCM (only remove)');
xlabel('remain\_r'); ylabel('SMSE');
```

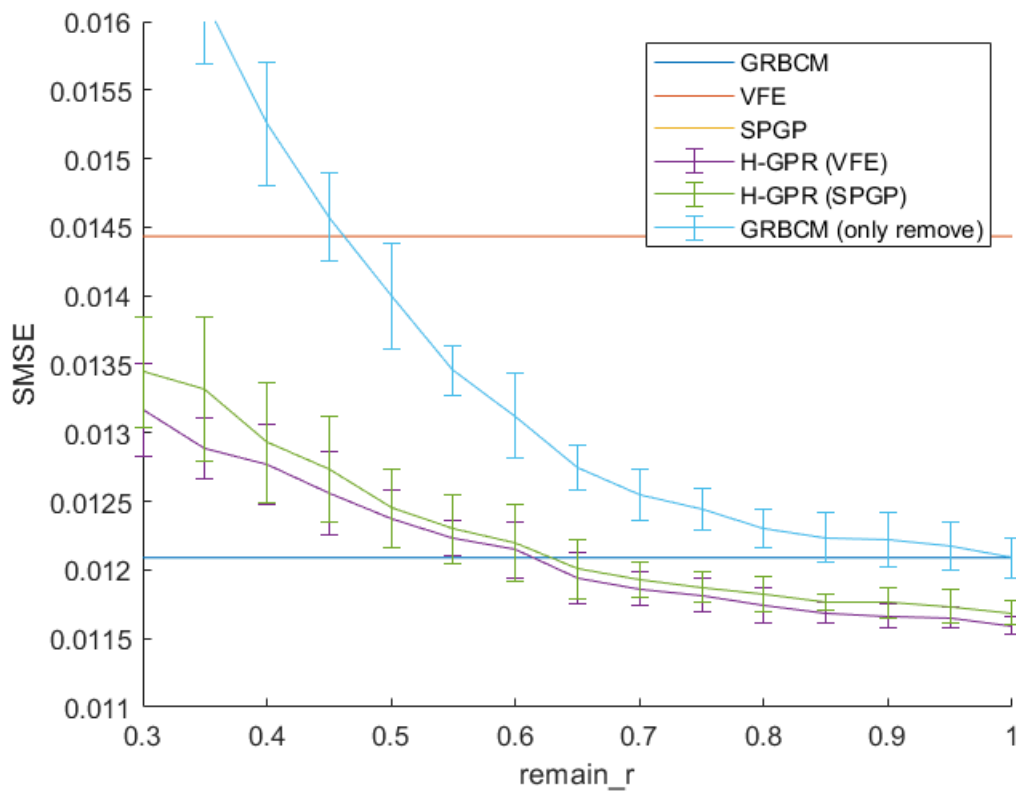


```

aamse = grbcm2_gr_smse(1:kti,:);
aamsll = grbcm2_gr_msll(1:kti,:);
mmse = mean(aamse);
mstd = std(aamse);
mmse_ro = mean(grbcm_gr_smse(1:kti,:));
mstd_ro = std(grbcm_gr_smse(1:kti,:));
mmse_sp = mean(grbcm2_spgp_gr_smse(1:kti,:));
mstd_sp = std(grbcm2_spgp_gr_smse(1:kti,:));

figure; hold on;
plot([min(grls), max(grls)], [grbcm0_smse,grbcm0_smse]);
plot([min(grls), max(grls)], [vfeSMSE_b1,vfeSMSE_b1]);
plot([min(grls), max(grls)], [spgpSMSE_b1,spgpSMSE_b1]);
errorbar(grls, mmse, mstd);
errorbar(grls, mmse_sp, mstd_sp);
errorbar(grls, mmse_ro, mstd_ro);
legend('GRBCM', 'VFE', 'SPGP', 'H-GPR (VFE)', 'H-GPR (SPGP)', 'GRBCM (only remove)');
xlabel('remain\_r'); ylabel('SMSE');
ylim([0.011, 0.016]);

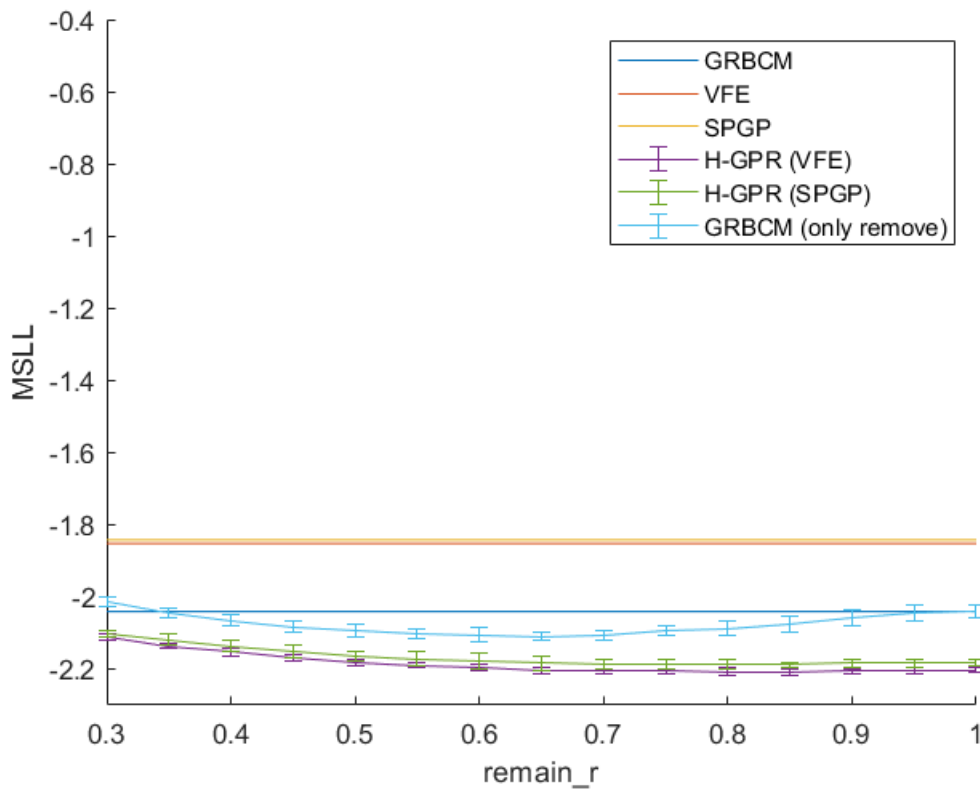
```

```

mmsll = mean(aamsll);
mstd = std(aamsll);
mmsll_ro = mean(grbcm_gr_msll(1:kti,:));
mstd_ro = std(grbcm_gr_msll(1:kti,:));
mmsll_sp = mean(grbcm2_spgp_gr_msll(1:kti,:));
mstd_sp = std(grbcm2_spgp_gr_msll(1:kti,:));
figure; hold on;
plot([min(grls), max(grls)], [grbcm0_msll,grbcm0_msll]);
plot([min(grls), max(grls)], [vfeMSLL_b1,vfeMSLL_b1]);
plot([min(grls), max(grls)], [spgpMSLL_b1,spgpMSLL_b1]);
errorbar(grls, mmsll, mstd);
errorbar(grls, mmsll_sp, mstd_sp);
errorbar(grls, mmsll_ro, mstd_ro);
legend('GRBCM', 'VFE', 'SPGP', 'H-GPR (VFE)', 'H-GPR (SPGP)', 'GRBCM (only remove)');
xlabel('remain\_r'); ylabel('MSLL');
ylim([-2.3, -0.4]);

```



```
fprintf('Best SMSE (GRBCM+VFE, dcs %d, ecs %d): %6.8f\n', dcs, ecs, min(mmse));
```

```
Best SMSE (GRBCM+VFE, dcs 150, ecs 150): 0.01159231
```

```
fprintf('Best MSLL (GRBCM+VFE, dcs %d, ecs %d): %6.8f\n', dcs, ecs, min(mmsll));
```

```
Best MSLL (GRBCM+VFE, dcs 150, ecs 150): -2.20840538
```

```
fprintf('Best SMSE (GRBCM+SPGP, dcs %d, ecs %d): %6.8f\n', dcs, ecs, min(mmse_sp));
```

```
Best SMSE (GRBCM+SPGP, dcs 150, ecs 150): 0.01168647
```

```
fprintf('Best MSLL (GRBCM+SPGP, dcs %d, ecs %d): %6.8f\n', dcs, ecs, min(mmsll_sp));
```

```
Best MSLL (GRBCM+SPGP, dcs 150, ecs 150): -2.18825519
```

Experiment II: re-balancing sizes of dcs and ecs

```
% hyp.cov = log([ones(d,1)*ell;sf2]); hyp.lik = log(sn2); hyp.mean = [];
opts.numOptFC = 50 ;
opts.xvec = xvec;
opts.yvec = yvec;
opts.grbcm_baseline = 0;
opts.global_index = ones(n,1);
% opts.inffunc = @infGaussLik; opts.meanfunc = meanfunc; opts.likfunc = likfunc;
```

```

opts.covfunc = covfunc;
covfuncF = {@apxSparse, {opts.covfunc}, []};
opts.covfuncF = covfuncF;
opts.compute_hyp = 0;

```

Heuristically rebalancing GRBCM and VFE/SPGP budget based on validating performance.

Obtain results on the validation set.

```

% default partition
dcs_ecs_r = 0.5;
dcs = round(ttcs*dcs_ecs_r) % size of the communication set

```

```

dcs = 150

```

```

ecs = ttcs - dcs % size of other experts

```

```

ecs = 150

```

```

n_per = dcs ; % size of Dc
mn = round(n / ecs); % mn is the number of experts (normal)
Indics = randperm(n) ;
I_com = Indics(1:n_per) ; % randomly select communication set
[idx, C] = kmeans(xvec, mn, 'MaxIter', km_iters);

```

```

% hyp.cov = log([ones(d,1)*ell;sf2]); hyp.lik = log(sn2); hyp.mean = [];
opts.numOptFC = 30 ;
opts.Ms = mn+1;
opts.xvec = xvec;
opts.yvec = yvec;
opts.induce_size = dcs;
opts.grbcm_baseline = 0;
opts.global_index = ones(n,1);
opts.I_com = I_com;
% opts.inffunc = @infGaussLik; opts.meanfunc = meanfunc; opts.likfunc = likfunc;
opts.covfunc = covfunc;
covfuncF = {@apxSparse, {opts.covfunc}, xvec(I_com,:)};
opts.covfuncF = covfuncF;
opts.compute_hyp = 0;

```

```

g_opts = opts;
g_opts.compute_hyp = 1;
g_opts.grbcm_baseline = 1;
g_opts.global_index = ones(n,1);
g_models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,g_opts);

```

```

Optimizing hyps in training...
Linesearch      0; Value 5.656439e+03
Linesearch      1; Value 3.912485e+03
Linesearch      2; Value 1.453360e+01

```

```

Linesearch      3; Value -4.871853e+03
Linesearch      4; Value -5.935522e+03
Linesearch      5; Value -6.021155e+03
Linesearch      6; Value -6.062374e+03
Linesearch      7; Value -6.099711e+03
Linesearch      8; Value -6.202108e+03
Linesearch      9; Value -6.313987e+03
Linesearch     10; Value -6.348283e+03
Linesearch     11; Value -6.351989e+03
Linesearch     12; Value -6.352783e+03
Linesearch     13; Value -6.353207e+03
Linesearch     14; Value -6.353433e+03
Linesearch     15; Value -6.353554e+03
Linesearch     16; Value -6.353573e+03
Linesearch     17; Value -6.353573e+03
Linesearch     18; Value -6.353573e+03
Linesearch     19; Value -6.353573e+03
Linesearch     20; Value -6.353573e+03
Linesearch     21; Value -6.353573e+03
Linesearch     22; Value -6.353573e+03
Linesearch     23; Value -6.353573e+03
Linesearch     24; Value -6.353573e+03
Linesearch     25; Value -6.353573e+03
Linesearch     26; Value -6.353573e+03
Linesearch     27; Value -6.353573e+03
Linesearch     28; Value -6.353573e+03

```

```

opts.hyp = g_models{1}.hyp;
g_opts.hyp = g_models{1}.hyp;
[tmu,ts2, ~] = aggregation_predict(xvec_val,g_models,'GRBCM', 1, g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[grbcmMSE,grbcmSMSE,grbcmMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_val, ori_yvec_val, tmu,
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'GRBCM', n_per, grbcmMSE,grbcmSMSE,grbcmMSLL);

```

```
GRBCM (Dc size 150): MSE 0.01141714, SMSE 0.01154193, MSLL -2.07614533
```

```
g_opts.compute_hyp = 0;
```

```

% % VFE Baseline
vfe_opts = opts;
vfe_opts.induce_type = 'VFE_opt';
xu = xvec(I_com, :);
inffunc = @(varargin) infGaussLik(varargin{:}, struct('s', 0.0));
vfe_hyp = opts.hyp;
vfe_hyp.xu = xu;
[vfe_hyp, tmp_nlz] = minimize(vfe_hyp,@sp_gp,-vfe_opts.induce_step,inffunc,meanfunc,covfuncF,I);

```

```

Function evaluation      0; Value 9.713131e+04
Function evaluation     12; Value 9.523192e+04
Function evaluation     14; Value 9.461697e+04
Function evaluation     18; Value 8.908681e+04
Function evaluation     19; Value 8.818066e+04
Function evaluation     21; Value 8.685544e+04
Function evaluation     23; Value 8.595105e+04
Function evaluation     25; Value 8.558632e+04
Function evaluation     28; Value 8.459281e+04

```

```

Function evaluation      34; Value 5.531429e+04
Function evaluation      36; Value 4.157103e+04
Function evaluation      38; Value 3.313305e+04
Function evaluation      40; Value 2.675902e+04
Function evaluation      42; Value 2.393547e+04
Function evaluation      44; Value 2.228229e+04
Function evaluation      46; Value 2.010133e+04
Function evaluation      48; Value 1.898583e+04
Function evaluation      49; Value 1.781364e+04
Function evaluation      51; Value 1.678928e+04
Function evaluation      53; Value 1.617900e+04
Function evaluation      55; Value 1.576131e+04
Function evaluation      57; Value 1.544028e+04
Function evaluation      59; Value 1.528818e+04
Function evaluation      60; Value 1.512241e+04
Function evaluation      61; Value 1.496021e+04
Function evaluation      63; Value 1.482118e+04
Function evaluation      65; Value 1.466520e+04
Function evaluation      67; Value 1.456075e+04
Function evaluation      68; Value 1.445371e+04
Function evaluation      70; Value 1.437666e+04
Function evaluation      71; Value 1.429490e+04
Function evaluation      73; Value 1.423908e+04
Function evaluation      75; Value 1.417368e+04
Function evaluation      76; Value 1.410870e+04
Function evaluation      78; Value 1.403148e+04
Function evaluation      80; Value 1.394687e+04
Function evaluation      82; Value 1.388260e+04
Function evaluation      83; Value 1.381530e+04
Function evaluation      85; Value 1.376057e+04
Function evaluation      87; Value 1.370898e+04
Function evaluation      88; Value 1.365767e+04
Function evaluation      90; Value 1.361370e+04
Function evaluation      92; Value 1.357779e+04
Function evaluation      94; Value 1.354602e+04
Function evaluation      96; Value 1.352213e+04
Function evaluation      98; Value 1.350245e+04
Function evaluation     99; Value 1.348168e+04

```

```

vfe_opts.hyp = opts.hyp;
vfe_opts.xu = vfe_hyp.xu;
vfe_opts.inffunc = @infGaussLik; vfe_opts.meanfunc = meanfunc; vfe_opts.covfuncF = covfuncF; vfe_opts.covfunc = covfunc;
[tmu, ts2] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, xvec_val);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[vfeMSE,vfeSMSE,vfeMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_val, ori_yvec_val, tmu, ts2);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'VFE', n_per, vfeMSE,vfeSMSE,vfeMSLL);

```

```
VFE (Dc size 150): MSE 0.01364978, SMSE 0.01379897, MSLL -1.86333239
```

```

% vfe0_smse_rec(ki) = vfeSMSE; vfe0_msll_rec(ki) = vfeMSLL;
[yu, su] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, vfe_opts.xu);
vfe_opts.yu = yu; vfe_opts.su = su;

```

```

models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,vfe_opts); % use hyp of vfe
[tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_val,models,vfe_opts);
if ynorm==1

```

```

    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_val, ori_yvec_val, tmu, ts2);
fprintf('%s (Dc size %d): \r\nMSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM++ (VFE)', n_per, M
GRBCM++ (VFE) (Dc size 150):
MSE 0.01076750, SMSE 0.0109, MSLL -2.2371

```

```

sig_temp = 10;
dcs_ecs_r = sigmoid(-MSLL+grbcmMSLL, sig_temp)

```

```

dcs_ecs_r = 0.8334

```

```

dcs = round(ttcs*dcs_ecs_r)

```

```

dcs = 250

```

```

ecs = ttcs - dcs

```

```

ecs = 50

```

```

m = round(n / ecs)

```

```

m = 200

```

```

n_per = dcs ; % size of Dc
Indics = randperm(n) ;
I_com = Indics(1:n_per) ; % randomly select communication set
[idx, C] = kmeans(xvec, m, 'MaxIter', km_iters);

% hyp.cov = log([ones(d,1)*ell;sf2]); hyp.lik = log(sn2); hyp.mean = [];
opts.numOptFC = 30 ;
opts.Ms = m+1;
opts.xvec = xvec;
opts.yvec = yvec;
opts.induce_size = dcs;
opts.grbcm_baseline = 0;
opts.global_index = ones(n,1);
opts.I_com = I_com;
% opts.inffunc = @infGaussLik; opts.meanfunc = meanfunc; opts.likfunc = likfunc;
opts.covfunc = covfunc;
covfuncF = {@apxSparse, {opts.covfunc}, xvec(I_com,:)};
opts.covfuncF = covfuncF;
opts.compute_hyp = 0;

```

```

g_opts = opts;
g_opts.compute_hyp = 0;
g_opts.grbcm_baseline = 1;
g_opts.global_index = ones(n,1);
g_models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,g_opts);
opts.hyp = g_models{1}.hyp;
g_opts.hyp = g_models{1}.hyp;

```

```
[tmu,ts2, ~] = aggregation_predict(xvec_test,g_models,'GRBCM', 1, g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[grbcmMSE,grbcmSMSE,grbcmMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tm
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'GRBCM', n_per, grbcmMSE,grbcm
```

```
GRBCM (Dc size 250): MSE 0.01157380, SMSE 0.01214926, MSLL -2.14087613
```

```
% grbcm0_smse_rec(ki) = grbcmSMSE; grbcm0_msll_rec(ki) = grbcmMSLL;
g_opts.compute_hyp = 0;
```

```
% % VFE Baseline
vfe_opts = opts;
vfe_opts.induce_type = 'VFE_opt';
xu = xvec(I_com, :);
vfe_hyp = opts.hyp;
vfe_hyp.xu = xu;
vfe_hyp = minimize(vfe_hyp,@sp_gp,-vfe_opts.induce_step,inffunc,meanfunc,covfuncF,likfunc,xvec,
```

```
Function evaluation      0; Value 4.095793e+03
Function evaluation      7; Value 4.044684e+03
Function evaluation     12; Value 2.153300e+03
Function evaluation     16; Value -1.446170e+03
Function evaluation     20; Value -2.178237e+03
Function evaluation     21; Value -3.626735e+03
Function evaluation     23; Value -4.443031e+03
Function evaluation     25; Value -4.830984e+03
Function evaluation     27; Value -5.061232e+03
Function evaluation     29; Value -5.239347e+03
Function evaluation     31; Value -5.381591e+03
Function evaluation     33; Value -5.490788e+03
Function evaluation     35; Value -5.562676e+03
Function evaluation     37; Value -5.605160e+03
Function evaluation     39; Value -5.639820e+03
Function evaluation     41; Value -5.671277e+03
Function evaluation     43; Value -5.691736e+03
Function evaluation     45; Value -5.707315e+03
Function evaluation     46; Value -5.721022e+03
Function evaluation     48; Value -5.734093e+03
Function evaluation     50; Value -5.742686e+03
Function evaluation     52; Value -5.749166e+03
Function evaluation     54; Value -5.757408e+03
Function evaluation     55; Value -5.766263e+03
Function evaluation     57; Value -5.771008e+03
Function evaluation     59; Value -5.776933e+03
Function evaluation     60; Value -5.782334e+03
Function evaluation     62; Value -5.786774e+03
Function evaluation     63; Value -5.791444e+03
Function evaluation     65; Value -5.796838e+03
Function evaluation     67; Value -5.801830e+03
Function evaluation     69; Value -5.805324e+03
Function evaluation     71; Value -5.810269e+03
Function evaluation     73; Value -5.812992e+03
Function evaluation     75; Value -5.814946e+03
Function evaluation     77; Value -5.817301e+03
Function evaluation     79; Value -5.818760e+03
Function evaluation     80; Value -5.820234e+03
Function evaluation     82; Value -5.821918e+03
```

```

Function evaluation      84; Value -5.823893e+03
Function evaluation      86; Value -5.826184e+03
Function evaluation      88; Value -5.828213e+03
Function evaluation      90; Value -5.829774e+03
Function evaluation      91; Value -5.831309e+03
Function evaluation      93; Value -5.832477e+03
Function evaluation      95; Value -5.833470e+03
Function evaluation      96; Value -5.834384e+03
Function evaluation      98; Value -5.835236e+03
Function evaluation     100; Value -5.836416e+03

```

```

vfe_opts.hyp = opts.hyp;
vfe_opts.xu = vfe_hyp.xu;
vfe_opts.inffunc = @infGaussLik; vfe_opts.meanfunc = meanfunc; vfe_opts.covfuncF = covfuncF; vfe_opts.covfunc = covfunc;
[tmu, ts2] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, xvec_test);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[vfeMSE,vfeSMSE,vfeMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'VFE baseline', n_per, vfeMSE, vfeSMSE, vfeMSLL);

```

```
VFE baseline (Dc size 250): MSE 0.01120477, SMSE 0.01176188, MSLL -2.20525114
```

```

% vfe0_smse_rec(ki) = vfeSMSE; vfe0_msll_rec(ki) = vfeMSLL;
[yu, su] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, vfe_opts.xu);
vfe_opts.yu = yu; vfe_opts.su = su;

```

```

sp_opts = opts;
sp_opts.induce_type = 'SPGP_opt';
hyp_init(1:d,1) = -2*opts.hyp.cov(1:d);
hyp_init(d+1,1) = 2*opts.hyp.cov(d+1);
hyp_init(d+2,1) = 2*opts.hyp.lik;

sp_opts.induce_size = dcs;
xu = xvec(I_com, :);
w_init = [reshape(xu,sp_opts.induce_size*d,1);hyp_init];
[w,f] = minimize(w_init,'sppg_lik_nohyp',-sp_opts.induce_step,yvec,xvec,sp_opts.induce_size);

```

```

Function evaluation      0; Value -5.763649e+03
Function evaluation     10; Value -6.000609e+03
Function evaluation     11; Value -6.253401e+03
Function evaluation     12; Value -6.523034e+03
Function evaluation     13; Value -6.759506e+03
Function evaluation     18; Value -6.774817e+03
Function evaluation     23; Value -6.968110e+03
Function evaluation     25; Value -7.084927e+03
Function evaluation     27; Value -7.188545e+03
Function evaluation     31; Value -7.231472e+03
Function evaluation     32; Value -7.256834e+03
Function evaluation     33; Value -7.290383e+03
Function evaluation     36; Value -7.304290e+03
Function evaluation     39; Value -7.323292e+03
Function evaluation     41; Value -7.337716e+03
Function evaluation     43; Value -7.346195e+03
Function evaluation     45; Value -7.357249e+03
Function evaluation     46; Value -7.367864e+03
Function evaluation     48; Value -7.377567e+03

```



```

Function evaluation    50; Value -7.384789e+03
Function evaluation    52; Value -7.393556e+03
Function evaluation    54; Value -7.399465e+03
Function evaluation    56; Value -7.407218e+03
Function evaluation    58; Value -7.412209e+03
Function evaluation    60; Value -7.415975e+03
Function evaluation    62; Value -7.420214e+03
Function evaluation    64; Value -7.423479e+03
Function evaluation    66; Value -7.426126e+03
Function evaluation    67; Value -7.428695e+03
Function evaluation    69; Value -7.430148e+03
Function evaluation    71; Value -7.433258e+03
Function evaluation    73; Value -7.435869e+03
Function evaluation    75; Value -7.437667e+03
Function evaluation    77; Value -7.439478e+03
Function evaluation    82; Value -7.440130e+03
Function evaluation    84; Value -7.440940e+03
Function evaluation    87; Value -7.441295e+03
Function evaluation    89; Value -7.442162e+03
Function evaluation    94; Value -7.442442e+03
Function evaluation    96; Value -7.443289e+03
Function evaluation    99; Value -7.443473e+03

```

```

xb = reshape(w(1:sp_opts.induce_size*d,1),sp_opts.induce_size,d);
sp_opts.xu = xb;
sp_opts.sp_hyp = w(sp_opts.induce_size*d+1:end,1);
sp_opts.hyp = opts.hyp;

[tmu,ts2] = spgp_pred(sp_opts.yvec,sp_opts.xvec,sp_opts.xu,xvec_test,sp_opts.sp_hyp);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[spgpMSE,spgpSMSE,spgpMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu,
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'SPSG baseline', n_per, spgpM

```

```
SPSG baseline (Dc size 250): MSE 0.01127628, SMSE 0.01183695, MSLL -2.18665098
```

```

% spgp0_smse_rec(ki) = spgpSMSE; spgp0_msll_rec(ki) = spgpMSLL;
[yu,su] = spgp_pred(sp_opts.yvec,sp_opts.xvec,sp_opts.xu,sp_opts.xu,sp_opts.sp_hyp);
sp_opts.yu = yu; sp_opts.su = su;

```

```

[~, minidx] = min(mmsll);
best_msll_gr = grls(minidx)

```

```
best_msll_gr = 0.8500
```

```

[~, minidx] = min(mmse);
best_smse_gr = grls(minidx)

```

```
best_smse_gr = 1
```

```

kti = 1;
rb_vfe_msll = zeros(kti, 1);
rb_vfe_smse = zeros(kti, 1);

```

```
rb_sp_msll = zeros(kti, 1);
rb_sp_smse = zeros(kti, 1);
```

```
for ki=1:kti
    gr = best_msll_gr
    crk = rk;
    crk(I_com) = -1e10;
    [~, crk_idx] = sort(crk, 'descend');
    rn = round(n*gr);
    global_index = zeros(n,1);
    global_index(crk_idx(1:rn)) = 1; % select remaining data according to the importance

    vfe_opts.global_index = global_index;
    models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,vfe_opts); % use hyp of vfe
    [tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_test,models,vfe_opts);
    if ynorm==1
        tmu = tmu * norm_fstd + norm_fmean;
        ts2 = ts2 * norm_fstd^2;
    end
    [MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
    fprintf('%s (Dc size %d): \r\nMSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM++ (VFE)', n_p
    rb_vfe_msll(ki) = MSLL;

    sp_opts.global_index = global_index;
    models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,sp_opts); % use hyp of vfe
    [tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_test,models,sp_opts);
    if ynorm==1
        tmu = tmu * norm_fstd + norm_fmean;
        ts2 = ts2 * norm_fstd^2;
    end
    [MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
    fprintf('%s (Dc size %d): \r\nMSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM++ (SPGP)', n_p
    rb_sp_msll(ki) = MSLL;

    %%%=====%%

    gr = best_smse_gr
    crk = rk;
    crk(I_com) = -1e10;
    [~, crk_idx] = sort(crk, 'descend');
    rn = round(n*gr);
    global_index = zeros(n,1);
    global_index(crk_idx(1:rn)) = 1; % select remaining data according to the importance

    vfe_opts.global_index = global_index;
    models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,vfe_opts); % use hyp of vfe
    [tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_test,models,vfe_opts);
    if ynorm==1
        tmu = tmu * norm_fstd + norm_fmean;
        ts2 = ts2 * norm_fstd^2;
    end
    [MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
```

```

fprintf('%s (Dc size %d): \r\nMSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM++ (VFE)', n_pe
rb_vfe_smse(ki) = SMSE;

sp_opts.global_index = global_index;
models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,sp_opts); % use hyp of vfe
[tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_test,models,sp_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): \r\nMSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM++ (SPGP)', n_p
rb_sp_smse(ki) = SMSE;
end

```

```

gr = 0.8500
GRBCM++ (VFE) (Dc size 250):
MSE 0.01097973, SMSE 0.0115, MSLL -2.2169
GRBCM++ (SPGP) (Dc size 250):
MSE 0.01095480, SMSE 0.0115, MSLL -2.2184
gr = 1
GRBCM++ (VFE) (Dc size 250):
MSE 0.01089448, SMSE 0.0114, MSLL -2.2233
GRBCM++ (SPGP) (Dc size 250):
MSE 0.01088845, SMSE 0.0114, MSLL -2.2230

```

```
fprintf('Best SMSE (GRBCM+VFE, dcs %d, ecs %d): %6.8f\n', dcs, ecs, mean(rb_vfe_smse));
```

```
Best SMSE (GRBCM+VFE, dcs 250, ecs 50): 0.01143616
```

```
fprintf('Best MSLL (GRBCM+VFE, dcs %d, ecs %d): %6.8f\n', dcs, ecs, mean(rb_vfe_msll));
```

```
Best MSLL (GRBCM+VFE, dcs 250, ecs 50): -2.21693520
```

```
fprintf('Best SMSE (GRBCM+SPGP, dcs %d, ecs %d): %6.8f\n', dcs, ecs, mean(rb_sp_smse));
```

```
Best SMSE (GRBCM+SPGP, dcs 250, ecs 50): 0.01142983
```

```
fprintf('Best MSLL (GRBCM+SPGP, dcs %d, ecs %d): %6.8f\n', dcs, ecs, mean(rb_sp_msll));
```

```
Best MSLL (GRBCM+SPGP, dcs 250, ecs 50): -2.21840471
```