Generate artificial data

```
clear all;

x = -4:0.05:4;
y = -4:0.05:4;

rng(123456);
```

```
figure;
[X, Y] = meshgrid(x, y);
fkk = 3; % frequency of test function
freq=2*fkk; fk=1;
f1 = @(x,y) sin(freq*sqrt((x-fk).^2+y.^2));%./(freq*sqrt((x-fk).^2+y.^2));
freq=1*fkk; fk=-1;
f2 = @(x,y) -sin(freq*sqrt((x-fk).^2+y.^2));%./(freq*sqrt((x-fk).^2+y.^2));
freq = 0.1*fkk;
f3 = @(x,y) -sin(freq*sqrt(x.^2+y.^2));
ff = @(x,y) f1(x,y)+f2(x,y)+f3(x,y);
Z = ff(X, Y);
%Z = peaks(X, Y);
surf(X, Y, Z);
```
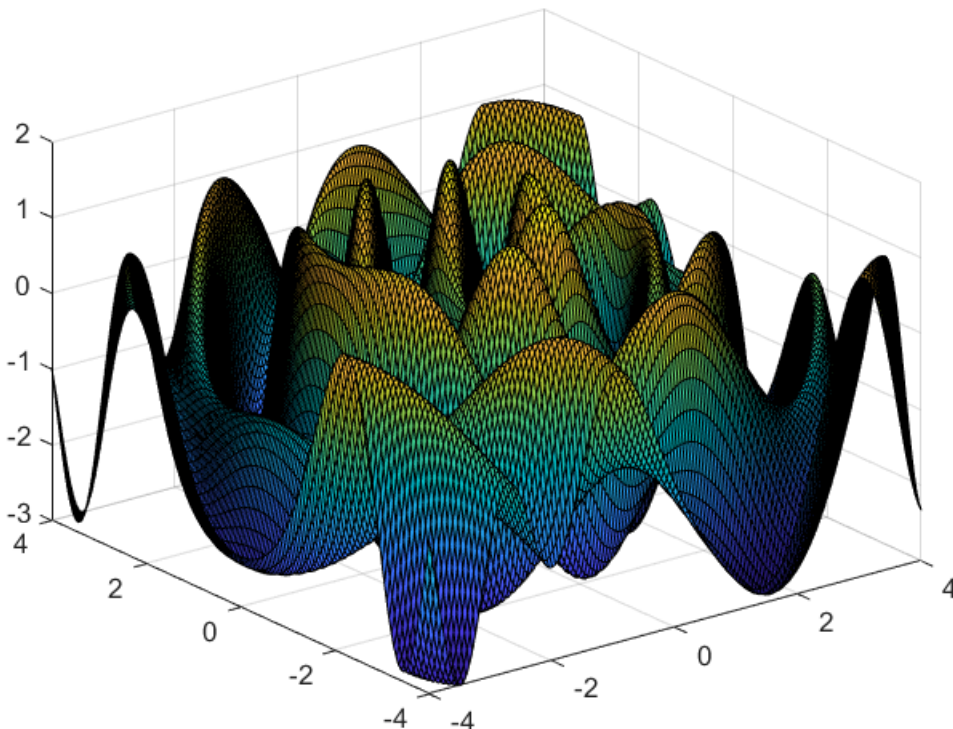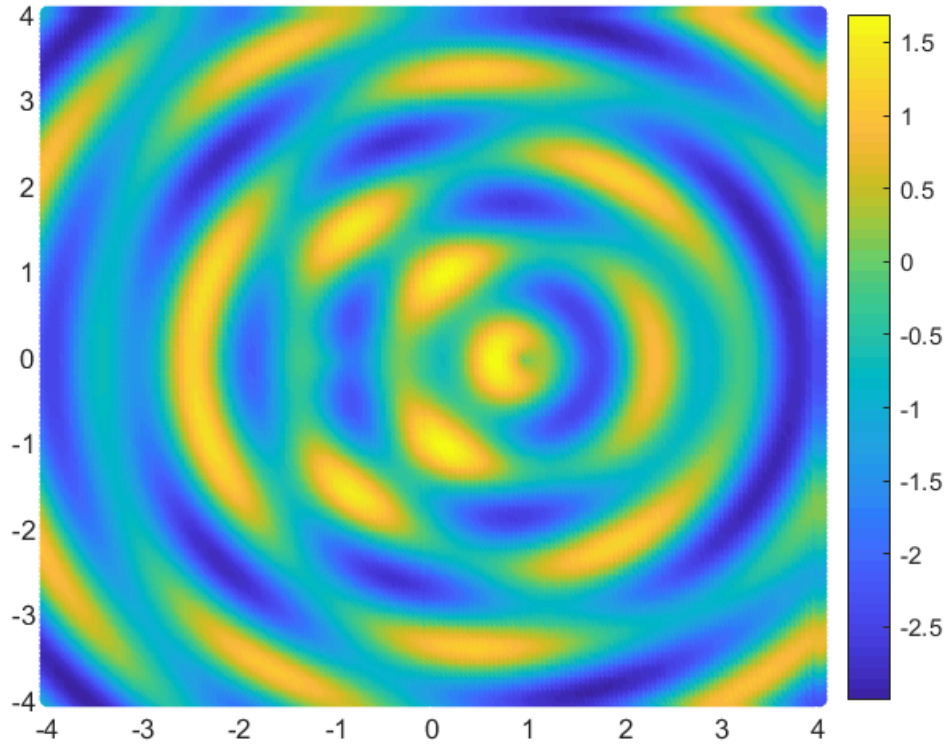


```
figure;
scatter(X(:), Y(:), [], Z(:), "filled");
```

```
colorbar;
```



```
xnorm = 1; ynorm = 1; % whether use normalization
yns = 0.1; % noise scale

n = 1e4; % size of training data
ttcs = 300;
ttm = round(n / (ttcs/2));
```

```
% represent_ratio = 5;
% dcs = 300; % size of the communication set
% mn = round(n / dcs); % mn is the number of experts (normal)
% m = round(n / (dcs*represent_ratio)); % m is the number of experts (sparse representative)
% lamds = 0:0.5:3.0;
```

```
testsize = 2000; % size of test data
valsize = 1000;
induce_step = 100;

% training input
ori_all_trainxs = 8*(rand(n, 1)-0.5);
ori_all_trainys = 8*(rand(n, 1)-0.5);

figure;
plot(ori_all_trainxs, ori_all_trainys, 'b.');
```
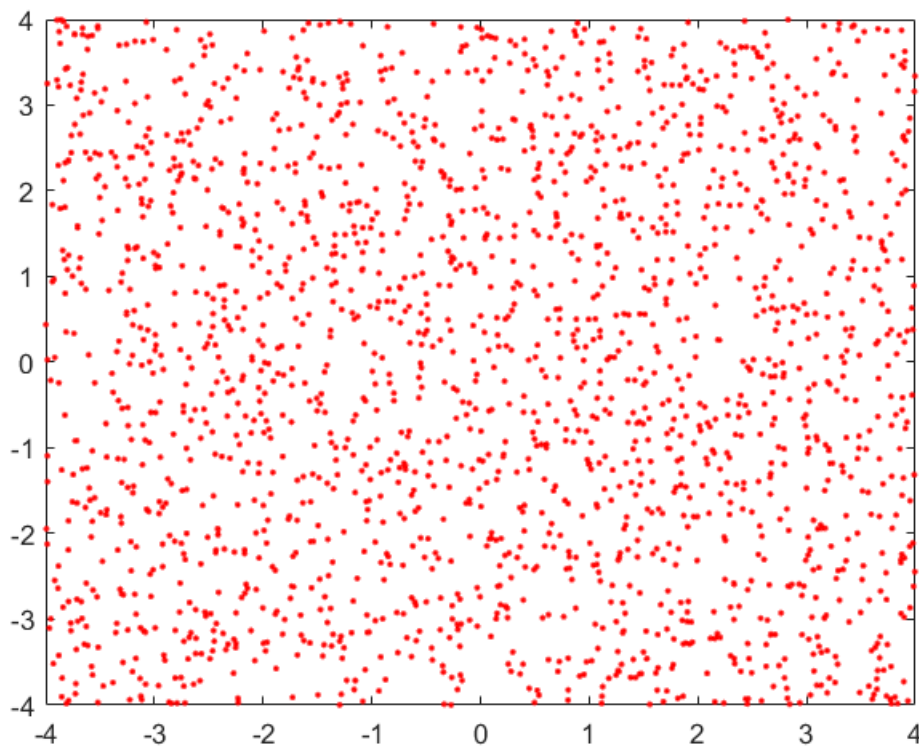
2
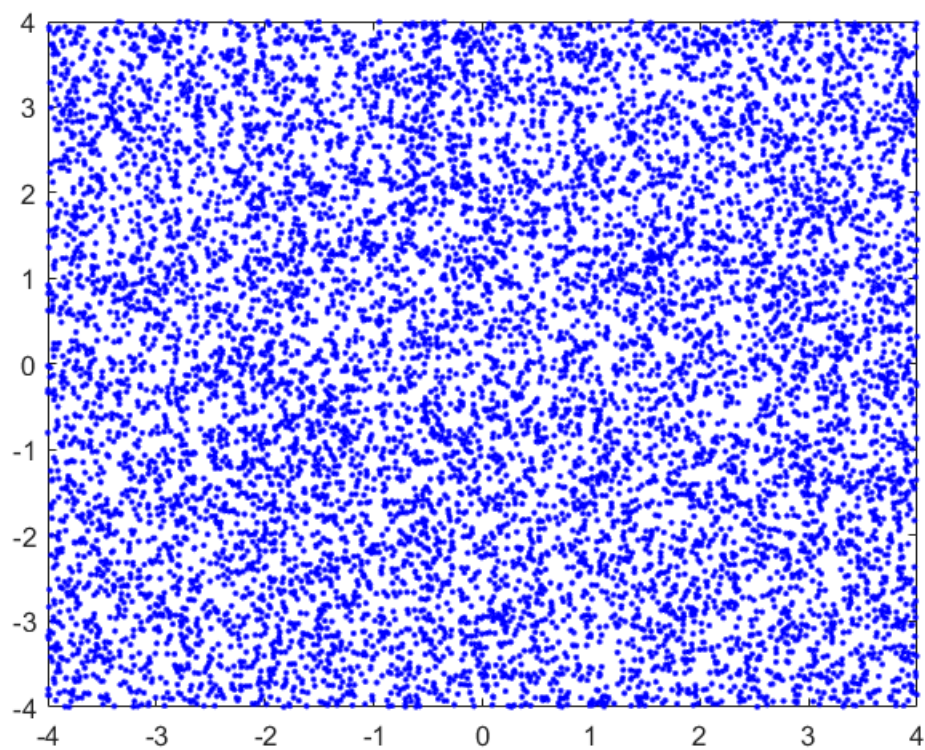
```
% test input
ori_testxs = 8*(rand(testsize, 1)-0.5);
ori_testys = 8*(rand(testsize, 1)-0.5);

% validation input
ori_valxs = 8*(rand(valsize, 1)-0.5);
ori_valys = 8*(rand(valsize, 1)-0.5);

figure;
plot(ori_testxs, ori_testys, 'r.');
```

```matlab
% ininput normalization
if xnorm == 1
    norm_xmean = mean(ori_all_trainxs);
    norm_xstd = std(ori_all_trainxs);
    all_trainxs = (ori_all_trainxs - norm_xmean) / norm_xstd;
    norm_ymean = mean(ori_all_trainys);
    norm_ystd = std(ori_all_trainys);
    all_trainys = (ori_all_trainys - norm_ymean) / norm_ystd;

    testxs = (ori_testxs - norm_xmean) / norm_xstd;
    testys = (ori_testys - norm_ymean) / norm_ystd;

    valxs = (ori_valxs - norm_xmean) / norm_xstd;
    valys = (ori_valys - norm_ymean) / norm_ystd;
else
    all_trainxs = ori_all_trainxs;
    all_trainys = ori_all_trainys;
    testxs = ori_testxs;
    testys = ori_testys;
    valxs = ori_valxs;
    valys = ori_valys;
end
```

```matlab
% training output
ln = length(all_trainxs);
xvec = zeros(ln, 2); ori_xvec = zeros(ln, 2);
```

```
for i = 1:ln
    xvec(i, :) = [all_trainxs(i), all_trainys(i)];
    ori_xvec(i, :) = [ori_all_trainxs(i), ori_all_trainys(i)];
end
ori_yvec = ff(ori_all_trainxs, ori_all_trainys) + yns*randn(ln, 1);

% test output
ln = length(testxs);
xvec_test = zeros(ln, 2); ori_xvec_test = zeros(ln, 2);
for i = 1:ln
    xvec_test(i, :) = [testxs(i), testys(i)];
    ori_xvec_test(i, :) = [ori_testxs(i), ori_testys(i)];
end
ori_yvec_test = ff(ori_testxs, ori_testys) + yns*randn(ln, 1);

% validation output
ln = length(valxs);
xvec_val = zeros(ln, 2); ori_xvec_val = zeros(ln, 2);
for i = 1:ln
    xvec_val(i, :) = [valxs(i), valys(i)];
    ori_xvec_val(i, :) = [ori_valxs(i), ori_valys(i)];
end
ori_yvec_val = ff(ori_valxs, ori_valys) + yns*randn(ln, 1);

% output normalization
if ynorm == 1
    norm_fmean = mean(ori_yvec);
    norm_fstd = std(ori_yvec);
    all_trainxs = (ori_all_trainxs - norm_xmean) / norm_xstd;
    yvec = (ori_yvec - norm_fmean) / norm_fstd;
    yvec_test = (ori_yvec_test - norm_fmean) / norm_fstd;
    yvec_val = (ori_yvec_val - norm_fmean) / norm_fstd;
else
    yvec = ori_yvec;
    yvec_test = ori_yvec_test;
    yvec_val = ori_yvec_val;
end
```

store results

```
kti = 5; % average over kti runs
grls = 0.3:0.05:1.0; % percentage of remaining data to be test
grbcm0_smse_rec = zeros(kti, 1);
rbcm0_smse_rec = zeros(kti, 1);
bcm0_smse_rec = zeros(kti, 1);
gpoe0_smse_rec = zeros(kti, 1);
poe0_smse_rec = zeros(kti, 1);
vfe0_smse_rec = zeros(kti, 1);
spgp0_smse_rec = zeros(kti, 1);
grbcm0_msll_rec = zeros(kti, 1);
rbcm0_msll_rec = zeros(kti, 1);
bcm0_msll_rec = zeros(kti, 1);
gpoe0_msll_rec = zeros(kti, 1);
```

```
poe0_msll_rec = zeros(kti, 1);
vfe0_msll_rec = zeros(kti, 1);
spgp0_msll_rec = zeros(kti, 1);
grbcm_gr_smse = zeros(kti, length(grls));
grbcm_gr_msll = zeros(kti, length(grls));
grbcm2_gr_smse = zeros(kti, length(grls));
grbcm2_gr_msll = zeros(kti, length(grls));
grbcm2_spgp_gr_smse = zeros(kti, length(grls));
grbcm2_spgp_gr_msll = zeros(kti, length(grls));
```

Experiment I: remove data from the training set

```
sf2 = 1 ; ell = 1 ; sn2 = 0.1 ;
d = size(xvec,2);
opts.Xnorm = 'N' ; opts.Ynorm = 'N' ;
opts.Ms = ttm+1;
opts.ell = ell ; opts.sf2 = sf2 ; opts.sn2 = sn2 ;
opts.meanfunc = []; opts.covfunc = @covSEard; opts.likfunc = @likGauss; opts.inffunc = @infGaus

meanfunc = [];                    % empty: don't use a mean function
covfunc = opts.covfunc;              % Squared Exponental covariance function
likfunc = opts.likfunc;              % Gaussian likelihood
inffunc = opts.inffunc;
km_iters = 1e4;
opts.induce_step = induce_step ;
```

```
% hyp.cov = log([ones(d,1)*ell;sf2]); hyp.lik = log(sn2); hyp.mean = [];
opts.numOptFC = 50 ;
opts.Ms = ttm+1;
opts.xvec = xvec;
opts.yvec = yvec;
opts.grbcm_baseline = 0;
opts.global_index = ones(n,1);
% opts.inffunc = @infGaussLik; opts.meanfunc = meanfunc; opts.likfunc = likfunc;
opts.covfunc = covfunc;
covfuncF = {@apxSparse, {opts.covfunc}, []};
opts.covfuncF = covfuncF;
opts.compute_hyp = 0;
```

```
% default partition
dcs_ecs_r = 0.5;
dcs = round(ttcs*dcs_ecs_r) % size of the communication set
```

```
dcs = 150
```

```
ecs = ttcs - dcs % size of other experts
```

```
ecs = 150
```

```
n_per = dcs ; % size of Dc
```

```
mn = round(n / ecs); % mn is the number of experts (normal)
Indics = randperm(n) ;
I_com = Indics(1:n_per) ; % randomly select communication set
[idx, C] = kmeans(xvec, mn,'MaxIter',km_iters);
```

Baselines of VFE and SPGP.

```
% hyp.cov = log([ones(d,1)*ell;sf2]); hyp.lik = log(sn2); hyp.mean = [];
opts.numOptFC = 30 ;
opts.Ms = mn+1;
opts.xvec = xvec;
opts.yvec = yvec;
opts.induce_size = dcs;
opts.grbcm_baseline = 0;
opts.global_index = ones(n,1);
opts.I_com = I_com;
% opts.inffunc = @infGaussLik; opts.meanfunc = meanfunc; opts.likfunc = likfunc;
opts.covfunc = covfunc;
covfuncF = {@apxSparse, {opts.covfunc}, xvec(I_com,:)};
opts.covfuncF = covfuncF;
opts.compute_hyp = 0;
```

```
g_opts = opts;
g_opts.compute_hyp = 1;
g_opts.grbcm_baseline = 1;
g_opts.global_index = ones(n,1);
g_models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,g_opts);
```

```
Optimizing hyps in training...
Linesearch      0;  Value 1.514893e+04
Linesearch      1;  Value 9.591731e+03
Linesearch      2;  Value 1.008327e+03
Linesearch      3;  Value 5.366056e+02
Linesearch      4;  Value -5.009841e+03
Linesearch      5;  Value -5.188800e+03
Linesearch      6;  Value -5.290960e+03
Linesearch      7;  Value -5.518916e+03
Linesearch      8;  Value -5.820404e+03
Linesearch      9;  Value -5.865758e+03
Linesearch     10;  Value -5.879238e+03
Linesearch     11;  Value -5.880771e+03
Linesearch     12;  Value -5.881934e+03
Linesearch     13;  Value -5.882696e+03
Linesearch     14;  Value -5.883075e+03
Linesearch     15;  Value -5.883098e+03
Linesearch     16;  Value -5.883102e+03
Linesearch     17;  Value -5.883103e+03
Linesearch     18;  Value -5.883105e+03
Linesearch     19;  Value -5.883105e+03
Linesearch     20;  Value -5.883106e+03
Linesearch     21;  Value -5.883106e+03
Linesearch     22;  Value -5.883106e+03
Linesearch     23;  Value -5.883106e+03
Linesearch     24;  Value -5.883106e+03
Linesearch     25;  Value -5.883106e+03
Linesearch     26;  Value -5.883106e+03
```

```
Linesearch     27;  Value -5.883106e+03
Linesearch     28;  Value -5.883106e+03
Linesearch     29;  Value -5.883106e+03
Linesearch     30;  Value -5.883106e+03
```

```
opts.hyp = g_models{1}.hyp;
g_opts.hyp = g_models{1}.hyp;
[tmu,ts2, ~] =  aggregation_predict(xvec_test,g_models,'GRBCM', 1, g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[grbcmMSE,grbcmSMSE,grbcmMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tm
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'GRBCM', n_per, grbcmMSE,grbc
```

```
GRBCM (Dc size 150): MSE 0.01194456, SMSE 0.01073117, MSLL -2.04631920
```

```
g_opts.compute_hyp = 0;
```

When testing baseline, VFE and SPGP use all partition budget (ttcs = dcs + ecs).

```
I_com2 = Indics(1:ttcs) ; % randomly select communication set
```

```
% % VFE Baseline
vfe_opts = opts;
vfe_opts.induce_type = 'VFE_opt';
xu = xvec(I_com2, :);
inffunc = @(varargin) infGaussLik(varargin{:}, struct('s', 0.0));
vfe_hyp = opts.hyp;
vfe_hyp.xu = xu;
[vfe_hyp, tmp_nlzs] = minimize(vfe_hyp,@sp_gp,-vfe_opts.induce_step,inffunc,meanfunc,covfuncF,l
```

```
Function evaluation      0;  Value 1.043977e+05
Function evaluation     12;  Value 9.409866e+04
Function evaluation     14;  Value 5.968357e+04
Function evaluation     16;  Value 4.313461e+04
Function evaluation     19;  Value 3.856822e+04
Function evaluation     22;  Value 3.200742e+04
Function evaluation     24;  Value 2.562515e+04
Function evaluation     27;  Value 2.324601e+04
Function evaluation     28;  Value 2.105492e+04
Function evaluation     30;  Value 1.949654e+04
Function evaluation     32;  Value 1.825844e+04
Function evaluation     34;  Value 1.749235e+04
Function evaluation     36;  Value 1.704403e+04
Function evaluation     37;  Value 1.657409e+04
Function evaluation     39;  Value 1.622103e+04
Function evaluation     41;  Value 1.592669e+04
Function evaluation     42;  Value 1.564913e+04
Function evaluation     44;  Value 1.545176e+04
Function evaluation     46;  Value 1.530488e+04
Function evaluation     47;  Value 1.514473e+04
Function evaluation     48;  Value 1.500742e+04
Function evaluation     50;  Value 1.489445e+04
Function evaluation     52;  Value 1.476606e+04
```

```
Function evaluation     53;  Value 1.462310e+04
Function evaluation     54;  Value 1.450422e+04
Function evaluation     56;  Value 1.441189e+04
Function evaluation     58;  Value 1.430048e+04
Function evaluation     59;  Value 1.419716e+04
Function evaluation     61;  Value 1.411689e+04
Function evaluation     63;  Value 1.407135e+04
Function evaluation     65;  Value 1.401122e+04
Function evaluation     67;  Value 1.396307e+04
Function evaluation     69;  Value 1.392483e+04
Function evaluation     71;  Value 1.389268e+04
Function evaluation     72;  Value 1.386004e+04
Function evaluation     74;  Value 1.381816e+04
Function evaluation     76;  Value 1.378063e+04
Function evaluation     78;  Value 1.374883e+04
Function evaluation     79;  Value 1.371692e+04
Function evaluation     80;  Value 1.368823e+04
Function evaluation     82;  Value 1.366439e+04
Function evaluation     83;  Value 1.364154e+04
Function evaluation     85;  Value 1.361486e+04
Function evaluation     87;  Value 1.358312e+04
Function evaluation     89;  Value 1.355767e+04
Function evaluation     90;  Value 1.353221e+04
Function evaluation     92;  Value 1.351080e+04
Function evaluation     94;  Value 1.348465e+04
Function evaluation     95;  Value 1.345886e+04
Function evaluation     97;  Value 1.343603e+04
Function evaluation     98;  Value 1.341409e+04
Function evaluation    100;  Value 1.339499e+04
```

```matlab
vfe_opts.hyp = opts.hyp;
vfe_opts.xu = vfe_hyp.xu;
vfe_opts.inffunc = @infGaussLik; vfe_opts.meanfunc = meanfunc; vfe_opts.covfuncF = covfuncF; vf
vfe_opts.covfunc = covfunc;
[tmu, ts2] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, xvec_test);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[vfeMSE,vfeSMSE,vfeMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'VFE baseline', n_per, vfeMSE
```

```
VFE baseline (Dc size 150): MSE 0.01403693, SMSE 0.01261098, MSLL -1.91573823
```

```matlab
% vfe0_smse_rec(ki) = vfeSMSE; vfe0_msll_rec(ki) = vfeMSLL;
[yu, su] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, vfe_opts.xu);
vfe_opts.yu = yu; vfe_opts.su = su;
```

```matlab
sp_opts = opts;
sp_opts.induce_size = ttcs;
sp_opts.induce_type = 'SPGP_opt';
hyp_init(1:d,1) = -2*opts.hyp.cov(1:d);
hyp_init(d+1,1) = 2*opts.hyp.cov(d+1);
hyp_init(d+2,1) = 2*opts.hyp.lik;

xu = xvec(I_com2, :);
w_init = [reshape(xu,sp_opts.induce_size*d,1);hyp_init];
[w,tmp_nlzs] = minimize(w_init,'spgp_lik_nohyp',-sp_opts.induce_step,yvec,xvec,sp_opts.induce_s
```

```
Function evaluation      0;  Value 3.090109e+02
Function evaluation      9;  Value -2.676229e+02
Function evaluation     11;  Value -6.314090e+02
Function evaluation     13;  Value -1.283239e+03
Function evaluation     15;  Value -1.878377e+03
Function evaluation     16;  Value -2.238519e+03
Function evaluation     18;  Value -2.540664e+03
Function evaluation     19;  Value -2.888074e+03
Function evaluation     21;  Value -3.162569e+03
Function evaluation     23;  Value -3.389267e+03
Function evaluation     25;  Value -3.584066e+03
Function evaluation     27;  Value -3.702786e+03
Function evaluation     29;  Value -3.793048e+03
Function evaluation     30;  Value -3.884369e+03
Function evaluation     32;  Value -3.948634e+03
Function evaluation     34;  Value -4.019338e+03
Function evaluation     36;  Value -4.076558e+03
Function evaluation     37;  Value -4.129050e+03
Function evaluation     39;  Value -4.165458e+03
Function evaluation     41;  Value -4.196223e+03
Function evaluation     43;  Value -4.221447e+03
Function evaluation     45;  Value -4.243060e+03
Function evaluation     47;  Value -4.261385e+03
Function evaluation     49;  Value -4.277067e+03
Function evaluation     50;  Value -4.292116e+03
Function evaluation     52;  Value -4.304426e+03
Function evaluation     54;  Value -4.315548e+03
Function evaluation     56;  Value -4.324937e+03
Function evaluation     58;  Value -4.331313e+03
Function evaluation     60;  Value -4.336710e+03
Function evaluation     62;  Value -4.340194e+03
Function evaluation     63;  Value -4.343589e+03
Function evaluation     65;  Value -4.346428e+03
Function evaluation     67;  Value -4.350556e+03
Function evaluation     68;  Value -4.355125e+03
Function evaluation     70;  Value -4.362218e+03
Function evaluation     72;  Value -4.365342e+03
Function evaluation     74;  Value -4.371384e+03
Function evaluation     75;  Value -4.377034e+03
Function evaluation     77;  Value -4.380611e+03
Function evaluation     78;  Value -4.383885e+03
Function evaluation     80;  Value -4.387135e+03
Function evaluation     82;  Value -4.389352e+03
Function evaluation     84;  Value -4.390743e+03
Function evaluation     86;  Value -4.391925e+03
Function evaluation     88;  Value -4.392766e+03
Function evaluation     89;  Value -4.393538e+03
Function evaluation     91;  Value -4.394539e+03
Function evaluation     93;  Value -4.396138e+03
Function evaluation     95;  Value -4.398161e+03
Function evaluation     96;  Value -4.400000e+03
Function evaluation     98;  Value -4.401525e+03
Function evaluation     99;  Value -4.403141e+03
```

```
xb = reshape(w(1:sp_opts.induce_size*d,1),sp_opts.induce_size,d);
sp_opts.xu = xb;
sp_opts.sp_hyp = w(sp_opts.induce_size*d+1:end,1);
sp_opts.hyp = opts.hyp;
[tmu,ts2] = spgp_pred(sp_opts.yvec,sp_opts.xvec,sp_opts.xu,xvec_test,sp_opts.sp_hyp);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
```

```
    end
    [spgpMSE,spgpSMSE,spgpMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu,
    fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'SPSG baseline', n_per, spgpM
```

SPSG baseline (Dc size 150): MSE 0.02807007, SMSE 0.02521857, MSLL -1.89692687

```
    % spgp0_smse_rec(ki) = spgpSMSE; spgp0_msll_rec(ki) = spgpMSLL;
    [yu,su] = spgp_pred(sp_opts.yvec,sp_opts.xvec,sp_opts.xu,sp_opts.xu,sp_opts.sp_hyp);
    sp_opts.yu = yu; sp_opts.su = su;
```

Average over 5 runs.

```
    kti = 5;
    m = mn;

    for ki=1:kti
    fprintf("=============%d================", ki)
    n_per = dcs ; % size of Dc
    Indics = randperm(n) ;
    I_com = Indics(1:n_per) ; % randomly select communication set
    [idx, C] = kmeans(xvec, m,'MaxIter',km_iters);

    % hyp.cov = log([ones(d,1)*ell;sf2]); hyp.lik = log(sn2); hyp.mean = [];
    opts.numOptFC = 30 ;
    opts.Ms = m+1;
    opts.xvec = xvec;
    opts.yvec = yvec;
    opts.induce_size = dcs;
    opts.grbcm_baseline = 0;
    opts.global_index = ones(n,1);
    opts.I_com = I_com;
    % opts.inffunc = @infGaussLik; opts.meanfunc = meanfunc; opts.likfunc = likfunc;
    opts.covfunc = covfunc;
    covfuncF = {@apxSparse, {opts.covfunc}, xvec(I_com,:)};
    opts.covfuncF = covfuncF;
    opts.compute_hyp = 0;

    g_opts = opts;
    g_opts.compute_hyp = 1;
    g_opts.grbcm_baseline = 1;
    g_opts.global_index = ones(n,1);
    g_models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,g_opts);
    opts.hyp = g_models{1}.hyp;
    g_opts.hyp = g_models{1}.hyp;
    [tmu,ts2, ~] =  aggregation_predict(xvec_test,g_models,'GRBCM', 1, g_opts);
    if ynorm==1
        tmu = tmu * norm_fstd + norm_fmean;
        ts2 = ts2 * norm_fstd^2;
    end
    [grbcmMSE_bl,grbcmSMSE_bl,grbcmMSLL_bl] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yve
    fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'GRBCM', n_per, grbcmMSE_bl,g
    grbcm0_smse_rec(ki) = grbcmSMSE_bl; grbcm0_msll_rec(ki) = grbcmMSLL_bl;
```

```matlab
g_opts.compute_hyp = 0;


% % VFE Baseline
vfe_opts = opts;
vfe_opts.induce_type = 'VFE_opt';
xu = xvec(I_com, :);
inffunc = @(varargin) infGaussLik(varargin{:}, struct('s', 0.0));
vfe_hyp = opts.hyp;
vfe_hyp.xu = xu;
vfe_hyp = minimize(vfe_hyp,@sp_gp,-vfe_opts.induce_step,inffunc,meanfunc,covfuncF,likfunc,xvec,
vfe_opts.hyp = opts.hyp;
vfe_opts.xu = vfe_hyp.xu;
vfe_opts.inffunc = @infGaussLik; vfe_opts.meanfunc = meanfunc; vfe_opts.covfuncF = covfuncF; vf
vfe_opts.covfunc = covfunc;
[tmu, ts2] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, xvec_test);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[vfeMSE_bl,vfeSMSE_bl,vfeMSLL_bl] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test,
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'VFE', n_per, vfeMSE_bl,vfeSM
vfe0_smse_rec(ki) = vfeSMSE_bl; vfe0_msll_rec(ki) = vfeMSLL_bl;
[yu, su] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, vfe_opts.xu);
vfe_opts.yu = yu; vfe_opts.su = su;

sp_opts = opts;
sp_opts.induce_type = 'SPGP_opt';
hyp_init(1:d,1) = -2*opts.hyp.cov(1:d);
hyp_init(d+1,1) = 2*opts.hyp.cov(d+1);
hyp_init(d+2,1) = 2*opts.hyp.lik;

xu = xvec(I_com, :);
w_init = [reshape(xu,sp_opts.induce_size*d,1);hyp_init];
[w,f] = minimize(w_init,'spgp_lik_nohyp',-sp_opts.induce_step,yvec,xvec,sp_opts.induce_size);
xb = reshape(w(1:sp_opts.induce_size*d,1),sp_opts.induce_size,d);
sp_opts.xu = xb;
sp_opts.sp_hyp = w(sp_opts.induce_size*d+1:end,1);
sp_opts.hyp = opts.hyp;

[tmu,ts2] = spgp_pred(sp_opts.yvec,sp_opts.xvec,sp_opts.xu,xvec_test,sp_opts.sp_hyp);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[spgpMSE_bl,spgpSMSE_bl,spgpMSLL_bl] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_te
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'SPSG', n_per, spgpMSE_bl,spg
spgp0_smse_rec(ki) = spgpSMSE_bl; spgp0_msll_rec(ki) = spgpMSLL_bl;
[yu,su] = spgp_pred(sp_opts.yvec,sp_opts.xvec,sp_opts.xu,sp_opts.xu,sp_opts.sp_hyp);
sp_opts.yu = yu; sp_opts.su = su;

vfe_opts.grbcm_baseline = 0;
vfe_opts.global_index = ones(n,1);
models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,vfe_opts); % use hyp of vfe
```

```matlab
[tmu,ts2] =  aggregation_predict_GRBCM_VS_apx(xvec_test,models,vfe_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'GRBCM (VFE)', n_per, MSE,SMS

sp_opts.grbcm_baseline = 0;
sp_opts.global_index = ones(n,1);
models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,sp_opts); % use hyp of vfe
[tmu,ts2] =  aggregation_predict_GRBCM_VS_apx(xvec_test,models,sp_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'GRBCM (SPGP)', n_per, MSE,SM


criterion = 'RBCM';
[tmu,ts2,t_dGP_predict] = aggregation_predict(xvec_test,g_models,criterion, 1, g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[rbcmMSE,rbcmSMSE,rbcmMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu,
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', criterion, n_per, rbcmMSE,rbc
rbcm0_smse_rec(ki) = rbcmSMSE; rbcm0_msll_rec(ki) = rbcmMSLL;

criterion = 'BCM';
[tmu,ts2,t_dGP_predict] = aggregation_predict(xvec_test,g_models,criterion, 1, g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[bcmMSE,bcmSMSE,bcmMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', criterion, n_per, bcmMSE,bcmS
bcm0_smse_rec(ki) = bcmSMSE; bcm0_msll_rec(ki) = bcmMSLL;

criterion = 'PoE';
[tmu,ts2,t_dGP_predict] = aggregation_predict(xvec_test,g_models,criterion, 1, g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[poeMSE,poeSMSE,poeMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', criterion, n_per, poeMSE,poeS
poe0_smse_rec(ki) = poeSMSE; poe0_msll_rec(ki) = poeMSLL;

criterion = 'GPoE';
[tmu,ts2,t_dGP_predict] = aggregation_predict(xvec_test,g_models,criterion, 1, g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
```

```matlab
        ts2 = ts2 * norm_fstd^2;
end
[gpoeMSE,gpoeSMSE,gpoeMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu,
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', criterion, n_per, gpoeMSE,gpo
gpoe0_smse_rec(ki) = gpoeSMSE; gpoe0_msll_rec(ki) = gpoeMSLL;

% compute informatic importance
ds = zeros(n, m);
for i=1:n
    for j=1:m
        %ds(i,j) = muti1(i)-muti2(j,i)+muti1(j)-muti2(i,j);
        ds(i,j) = norm(xvec(i, :) - C(j,:));
    end
end
% 1st closest cluster centers
[~, fcc] = min(ds, [], 2);
% set to inf
for i=1:n
    ds(i, fcc(i)) = 1e10;
end
% 2st closest cluster centers
[~, scc] = min(ds, [], 2);
rk = zeros(n, 1);
for iik=1:n
    if mod(iik, 1e2)==0
        fprintf('processing distance: %d/%d\n', iik, n);
    end
    i = fcc(iik);
    [tmp_mu, tmp_sig2] = gp(vfe_hyp,inffunc,meanfunc, ...
        covfunc,likfunc,models{i+1}.X,models{i+1}.Y,xvec(iik,:));
    h1 = 0.5*log(tmp_sig2);
    j = scc(iik);
%      [tmp_mu, tmp_sig2] = gp(vfe_hyp,inffunc,meanfunc, ...
%           covfunc,likfunc,[models{i+1}.X;models{j+1}.X],[models{i+1}.Y;models{j+1}.Y],xvec(iik,
    [tmp_mu, tmp_sig2] = gp(vfe_hyp,inffunc,meanfunc, ...
        covfunc,likfunc,[models{j+1}.X],[models{j+1}.Y],xvec(iik,:));
    h2 = 0.5*log(tmp_sig2);
    rk(iik) = h2 - h1; % importance is the difference between mutual information
end

if ki==1
    figure;
    scatter(xvec(:,1), xvec(:,2), 3, rk, "filled"); colorbar;
    figure; hold on;
    plot(xvec(I_com,1), xvec(I_com,2), 'r.');
    plot(vfe_opts.xu(:,1), vfe_opts.xu(:,2), 'bo');
    plot(sp_opts.xu(:,1), sp_opts.xu(:,2), 'go');
    legend('initial points', 'VFE induced', 'SPGP induced');
end

for kj=1:length(grls) % test/validate different remaining percentage
    kj
    gr=grls(kj)
    crk = rk;
```

```matlab
        crk(I_com) = -1e10;
        [~, crk_idx] = sort(crk, 'descend');
        rn = round(n*gr);
        global_index = zeros(n,1);
        global_index(crk_idx(1:rn)) = 1; % select remaining data according to the importance

        % only remove data from subsets, in RBGCM
        g_opts.global_index = global_index;
        models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,g_opts); % use hyp of vfe
        [tmu,ts2] =  aggregation_predict_GRBCM_VS_apx(xvec_test,models,g_opts);
        if ynorm==1
            tmu = tmu * norm_fstd + norm_fmean;
            ts2 = ts2 * norm_fstd^2;
        end
        [grbcmMSE,grbcmSMSE,grbcmMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test
        fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM', n_per, grbcmMSE,
        grbcm_gr_smse(ki,kj) = grbcmSMSE; grbcm_gr_msll(ki,kj) = grbcmMSLL;

        vfe_opts.global_index = global_index;
        models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,vfe_opts); % use hyp of vfe
        [tmu,ts2] =  aggregation_predict_GRBCM_VS_apx(xvec_test,models,vfe_opts);
        if ynorm==1
            tmu = tmu * norm_fstd + norm_fmean;
            ts2 = ts2 * norm_fstd^2;
        end
        [MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
        fprintf('%s (Dc size %d): \r\nMSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM++ (VFE)', n_pe
        grbcm2_gr_smse(ki,kj) = SMSE; grbcm2_gr_msll(ki,kj) = MSLL;

        sp_opts.global_index = global_index;
        models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,sp_opts); % use hyp of vfe
        [tmu,ts2] =  aggregation_predict_GRBCM_VS_apx(xvec_test,models,sp_opts);
        if ynorm==1
            tmu = tmu * norm_fstd + norm_fmean;
            ts2 = ts2 * norm_fstd^2;
        end
        [MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
        fprintf('%s (Dc size %d): \r\nMSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM++ (SPGP)', n_p
        grbcm2_spgp_gr_smse(ki,kj) = SMSE; grbcm2_spgp_gr_msll(ki,kj) = MSLL;
    end
end
```

```
==============1================
Optimizing hyps in training...
Linesearch       0;  Value 1.426666e+04
Linesearch       1;  Value 9.336957e+03
Linesearch       2;  Value 8.431051e+02
Linesearch       3;  Value 2.820384e+02
Linesearch       4;  Value -4.944810e+03
Linesearch       5;  Value -5.147537e+03
Linesearch       6;  Value -5.301380e+03
Linesearch       7;  Value -5.728193e+03
Linesearch       8;  Value -5.843524e+03
Linesearch       9;  Value -5.855129e+03
Linesearch      10;  Value -5.856859e+03
Linesearch      11;  Value -5.859369e+03
```
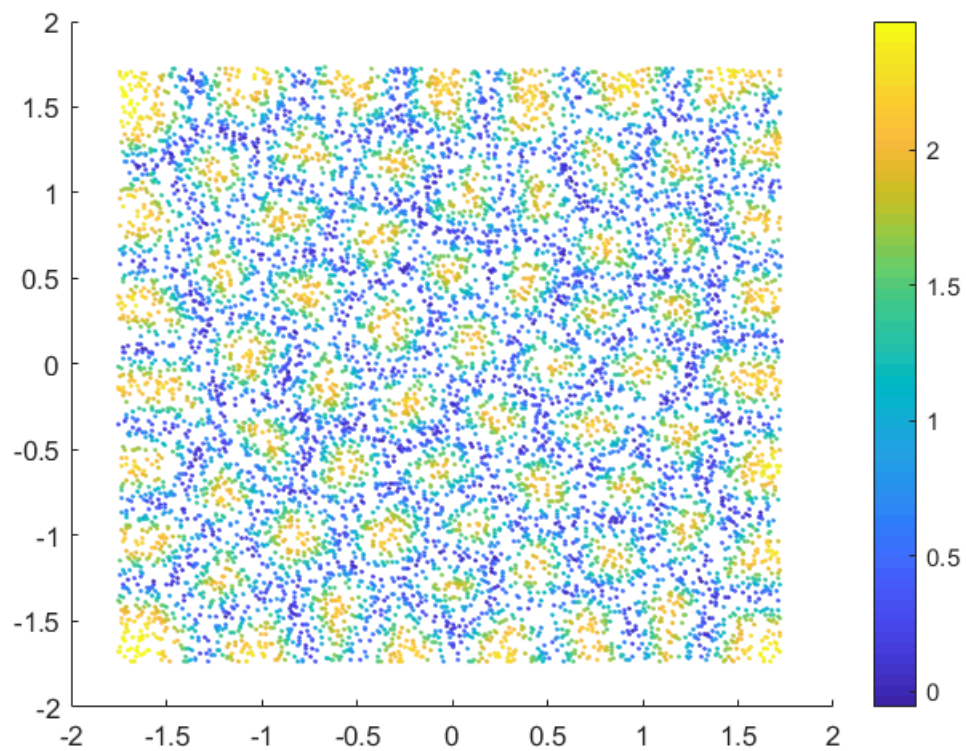
```
Linesearch      12;  Value -5.860839e+03
Linesearch      13;  Value -5.860898e+03
Linesearch      14;  Value -5.860903e+03
Linesearch      15;  Value -5.860904e+03
Linesearch      16;  Value -5.860904e+03
Linesearch      17;  Value -5.860905e+03
Linesearch      18;  Value -5.860905e+03
Linesearch      19;  Value -5.860905e+03
Linesearch      20;  Value -5.860905e+03
Linesearch      21;  Value -5.860905e+03
Linesearch      22;  Value -5.860905e+03
Linesearch      23;  Value -5.860905e+03
Linesearch      24;  Value -5.860905e+03
Linesearch      25;  Value -5.860905e+03
GRBCM (Dc size 150): MSE 0.01172563, SMSE 0.01053448, MSLL -2.06695450
Function evaluation      0;  Value 4.505934e+05
Function evaluation     20;  Value 4.393988e+05
Function evaluation     24;  Value 3.632041e+05
Function evaluation     28;  Value 3.336858e+05
Function evaluation     30;  Value 3.261053e+05
Function evaluation     33;  Value 3.045514e+05
Function evaluation     35;  Value 2.877013e+05
Function evaluation     37;  Value 2.706024e+05
Function evaluation     39;  Value 2.600876e+05
Function evaluation     40;  Value 2.489169e+05
Function evaluation     42;  Value 2.427538e+05
Function evaluation     44;  Value 2.361274e+05
Function evaluation     46;  Value 2.301285e+05
Function evaluation     48;  Value 2.259706e+05
Function evaluation     50;  Value 2.224674e+05
Function evaluation     52;  Value 2.193868e+05
Function evaluation     53;  Value 2.159179e+05
Function evaluation     54;  Value 2.132083e+05
Function evaluation     56;  Value 2.108453e+05
Function evaluation     58;  Value 2.081035e+05
Function evaluation     60;  Value 2.065092e+05
Function evaluation     61;  Value 2.046772e+05
Function evaluation     62;  Value 2.029571e+05
Function evaluation     64;  Value 2.021361e+05
Function evaluation     66;  Value 2.007831e+05
Function evaluation     68;  Value 2.000909e+05
Function evaluation     70;  Value 1.991170e+05
Function evaluation     71;  Value 1.981152e+05
Function evaluation     72;  Value 1.972485e+05
Function evaluation     74;  Value 1.964333e+05
Function evaluation     76;  Value 1.959284e+05
Function evaluation     78;  Value 1.952260e+05
Function evaluation     79;  Value 1.944768e+05
Function evaluation     81;  Value 1.939016e+05
Function evaluation     82;  Value 1.932620e+05
Function evaluation     83;  Value 1.926295e+05
Function evaluation     85;  Value 1.922733e+05
Function evaluation     87;  Value 1.913319e+05
Function evaluation     89;  Value 1.908746e+05
Function evaluation     91;  Value 1.899308e+05
Function evaluation     93;  Value 1.893999e+05
Function evaluation     94;  Value 1.888423e+05
Function evaluation     96;  Value 1.884229e+05
Function evaluation     97;  Value 1.879804e+05
Function evaluation     99;  Value 1.875268e+05
VFE (Dc size 150): MSE 0.08042591, SMSE 0.07225584, MSLL -1.10863768
Function evaluation      0;  Value 8.615221e+03
Function evaluation     14;  Value 7.954539e+03
Function evaluation     16;  Value 7.147066e+03
Function evaluation     19;  Value 6.801060e+03
```
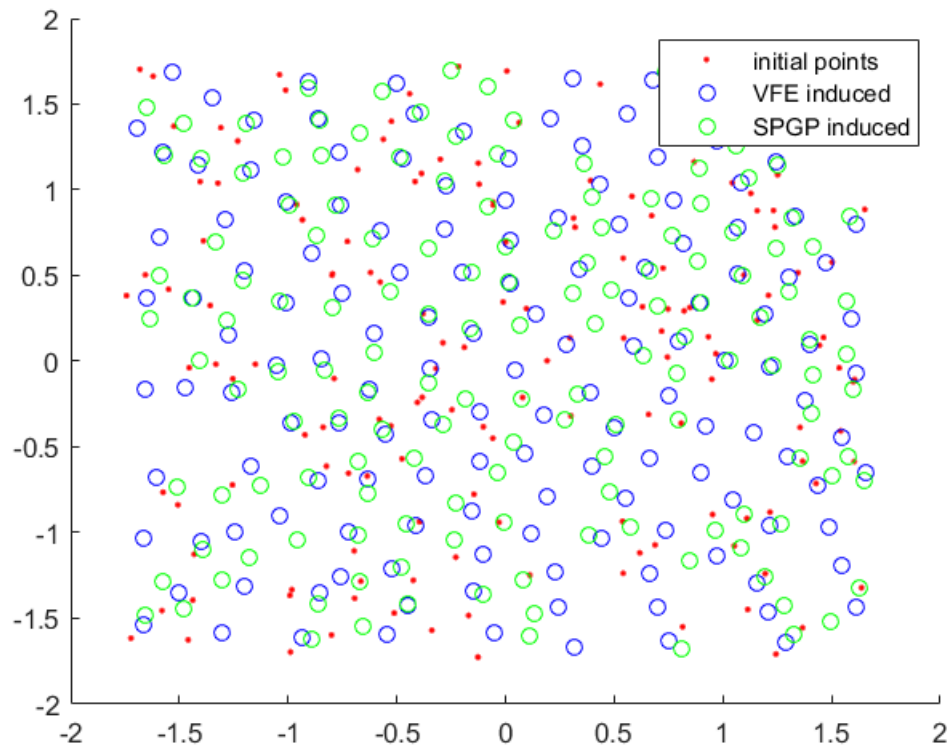
```
Function evaluation      21;   Value 6.080931e+03
Function evaluation      23;   Value 5.664354e+03
Function evaluation      25;   Value 5.183877e+03
Function evaluation      27;   Value 4.840719e+03
Function evaluation      29;   Value 4.613344e+03
Function evaluation      31;   Value 4.461130e+03
Function evaluation      32;   Value 4.303491e+03
Function evaluation      34;   Value 4.178215e+03
Function evaluation      36;   Value 4.086046e+03
Function evaluation      37;   Value 4.002733e+03
Function evaluation      39;   Value 3.945485e+03
Function evaluation      40;   Value 3.889130e+03
Function evaluation      42;   Value 3.839898e+03
Function evaluation      44;   Value 3.783169e+03
Function evaluation      46;   Value 3.730944e+03
Function evaluation      48;   Value 3.693001e+03
Function evaluation      50;   Value 3.665170e+03
Function evaluation      52;   Value 3.632164e+03
Function evaluation      54;   Value 3.607882e+03
Function evaluation      56;   Value 3.585900e+03
Function evaluation      58;   Value 3.559826e+03
Function evaluation      60;   Value 3.539388e+03
Function evaluation      61;   Value 3.519312e+03
Function evaluation      62;   Value 3.497111e+03
Function evaluation      63;   Value 3.474326e+03
Function evaluation      64;   Value 3.452027e+03
Function evaluation      66;   Value 3.436526e+03
Function evaluation      67;   Value 3.420103e+03
Function evaluation      69;   Value 3.406376e+03
Function evaluation      70;   Value 3.392435e+03
Function evaluation      72;   Value 3.382998e+03
Function evaluation      73;   Value 3.373856e+03
Function evaluation      75;   Value 3.366411e+03
Function evaluation      76;   Value 3.358929e+03
Function evaluation      78;   Value 3.353175e+03
Function evaluation      80;   Value 3.346166e+03
Function evaluation      82;   Value 3.337561e+03
Function evaluation      84;   Value 3.330489e+03
Function evaluation      85;   Value 3.322404e+03
Function evaluation      87;   Value 3.310611e+03
Function evaluation      89;   Value 3.303367e+03
Function evaluation      91;   Value 3.295513e+03
Function evaluation      92;   Value 3.287072e+03
Function evaluation      93;   Value 3.278970e+03
Function evaluation      94;   Value 3.270690e+03
Function evaluation      96;   Value 3.267028e+03
Function evaluation      97;   Value 3.263511e+03
Function evaluation      99;   Value 3.260548e+03
Function evaluation     100;   Value 3.257682e+03
SPSG (Dc size 150): MSE 0.15849426, SMSE 0.14239361, MSLL -1.07684218
GRBCM (VFE) (Dc size 150): MSE 0.01164928, SMSE 0.01046589, MSLL -2.11182698
GRBCM (SPGP) (Dc size 150): MSE 0.01169228, SMSE 0.01050451, MSLL -2.10289377
RBCM (Dc size 150): MSE 0.01196550, SMSE 0.0107, MSLL -1.8052
BCM (Dc size 150): MSE 0.01230899, SMSE 0.0111, MSLL -2.2205
PoE (Dc size 150): MSE 0.09443472, SMSE 0.0848, MSLL 3.8836
GPoE (Dc size 150): MSE 0.01187461, SMSE 0.0107, MSLL -1.8069
processing distance: 100/10000
processing distance: 200/10000
processing distance: 300/10000
processing distance: 400/10000
processing distance: 500/10000
processing distance: 600/10000
processing distance: 700/10000
processing distance: 800/10000
processing distance: 900/10000
```

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processing distance: 1000/10000
processing distance: 1100/10000
processing distance: 1200/10000
processing distance: 1300/10000
processing distance: 1400/10000
processing distance: 1500/10000
processing distance: 1600/10000
processing distance: 1700/10000
processing distance: 1800/10000
processing distance: 1900/10000
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processing distance: 2100/10000
processing distance: 2200/10000
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processing distance: 2500/10000
processing distance: 2600/10000
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processing distance: 2800/10000
processing distance: 2900/10000
processing distance: 3000/10000
processing distance: 3100/10000
processing distance: 3200/10000
processing distance: 3300/10000
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processing distance: 6400/10000
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processing distance: 6600/10000
processing distance: 6700/10000
processing distance: 6800/10000
processing distance: 6900/10000
processing distance: 7000/10000
processing distance: 7100/10000
processing distance: 7200/10000
processing distance: 7300/10000
processing distance: 7400/10000
```

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processing distance: 7500/10000
processing distance: 7600/10000
processing distance: 7700/10000
processing distance: 7800/10000
processing distance: 7900/10000
processing distance: 8000/10000
processing distance: 8100/10000
processing distance: 8200/10000
processing distance: 8300/10000
processing distance: 8400/10000
processing distance: 8500/10000
processing distance: 8600/10000
processing distance: 8700/10000
processing distance: 8800/10000
processing distance: 8900/10000
processing distance: 9000/10000
processing distance: 9100/10000
processing distance: 9200/10000
processing distance: 9300/10000
processing distance: 9400/10000
processing distance: 9500/10000
processing distance: 9600/10000
processing distance: 9700/10000
processing distance: 9800/10000
processing distance: 9900/10000
processing distance: 10000/10000
```

```
kj = 1
gr = 0.3000
GRBCM (Dc size 150): MSE 0.03802091, SMSE 0.0342, MSLL -1.7650
GRBCM++ (VFE) (Dc size 150):
MSE 0.02404988, SMSE 0.0216, MSLL -1.9025
GRBCM++ (SPGP) (Dc size 150):
MSE 0.02489501, SMSE 0.0224, MSLL -1.8908
kj = 2
gr = 0.3500
GRBCM (Dc size 150): MSE 0.02823575, SMSE 0.0254, MSLL -1.8512
GRBCM++ (VFE) (Dc size 150):
MSE 0.02121783, SMSE 0.0191, MSLL -1.9663
GRBCM++ (SPGP) (Dc size 150):
MSE 0.02172222, SMSE 0.0195, MSLL -1.9568
kj = 3
gr = 0.4000
GRBCM (Dc size 150): MSE 0.02375748, SMSE 0.0213, MSLL -1.9011
GRBCM++ (VFE) (Dc size 150):
MSE 0.01867057, SMSE 0.0168, MSLL -2.0054
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01925549, SMSE 0.0173, MSLL -1.9985
kj = 4
gr = 0.4500
GRBCM (Dc size 150): MSE 0.02069351, SMSE 0.0186, MSLL -1.9512
GRBCM++ (VFE) (Dc size 150):
MSE 0.01705732, SMSE 0.0153, MSLL -2.0423
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01712109, SMSE 0.0154, MSLL -2.0381
kj = 5
gr = 0.5000
GRBCM (Dc size 150): MSE 0.01809162, SMSE 0.0163, MSLL -2.0041
GRBCM++ (VFE) (Dc size 150):
MSE 0.01604061, SMSE 0.0144, MSLL -2.0789
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01575286, SMSE 0.0142, MSLL -2.0820
```

```
kj = 6
gr = 0.5500
GRBCM (Dc size 150): MSE 0.01639901, SMSE 0.0147, MSLL -2.0343
GRBCM++ (VFE) (Dc size 150):
MSE 0.01501805, SMSE 0.0135, MSLL -2.0976
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01447486, SMSE 0.0130, MSLL -2.1083
kj = 7
gr = 0.6000
GRBCM (Dc size 150): MSE 0.01490709, SMSE 0.0134, MSLL -2.0651
GRBCM++ (VFE) (Dc size 150):
MSE 0.01420645, SMSE 0.0128, MSLL -2.1156
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01399847, SMSE 0.0126, MSLL -2.1170
kj = 8
gr = 0.6500
GRBCM (Dc size 150): MSE 0.01386693, SMSE 0.0125, MSLL -2.0909
GRBCM++ (VFE) (Dc size 150):
MSE 0.01346957, SMSE 0.0121, MSLL -2.1298
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01364434, SMSE 0.0123, MSLL -2.1187
kj = 9
gr = 0.7000
GRBCM (Dc size 150): MSE 0.01331684, SMSE 0.0120, MSLL -2.0984
GRBCM++ (VFE) (Dc size 150):
MSE 0.01280239, SMSE 0.0115, MSLL -2.1498
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01296685, SMSE 0.0116, MSLL -2.1384
kj = 10
gr = 0.7500
GRBCM (Dc size 150): MSE 0.01315274, SMSE 0.0118, MSLL -2.0862
GRBCM++ (VFE) (Dc size 150):
MSE 0.01275284, SMSE 0.0115, MSLL -2.1386
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01299896, SMSE 0.0117, MSLL -2.1190
kj = 11
gr = 0.8000
GRBCM (Dc size 150): MSE 0.01272787, SMSE 0.0114, MSLL -2.0828
GRBCM++ (VFE) (Dc size 150):
MSE 0.01255101, SMSE 0.0113, MSLL -2.1324
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01268324, SMSE 0.0114, MSLL -2.1139
kj = 12
gr = 0.8500
GRBCM (Dc size 150): MSE 0.01245223, SMSE 0.0112, MSLL -2.0790
GRBCM++ (VFE) (Dc size 150):
MSE 0.01224547, SMSE 0.0110, MSLL -2.1321
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01232473, SMSE 0.0111, MSLL -2.1164
kj = 13
gr = 0.9000
GRBCM (Dc size 150): MSE 0.01199292, SMSE 0.0108, MSLL -2.0864
GRBCM++ (VFE) (Dc size 150):
MSE 0.01189973, SMSE 0.0107, MSLL -2.1356
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01196291, SMSE 0.0107, MSLL -2.1202
kj = 14
gr = 0.9500
GRBCM (Dc size 150): MSE 0.01188138, SMSE 0.0107, MSLL -2.0703
GRBCM++ (VFE) (Dc size 150):
MSE 0.01182995, SMSE 0.0106, MSLL -2.1175
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01186167, SMSE 0.0107, MSLL -2.1059
kj = 15
gr = 1
```

GRBCM (Dc size 150): MSE 0.01172563, SMSE 0.0105, MSLL -2.0670
GRBCM++ (VFE) (Dc size 150):
MSE 0.01164928, SMSE 0.0105, MSLL -2.1118
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01169228, SMSE 0.0105, MSLL -2.1029
==============2================
Optimizing hyps in training...
Linesearch   0; Value 1.444916e+04
Linesearch   1; Value 9.386262e+03
Linesearch   2; Value 6.585001e+02
Linesearch   3; Value 6.431182e+01
Linesearch   4; Value -4.956066e+03
Linesearch   5; Value -5.169826e+03
Linesearch   6; Value -5.602236e+03
Linesearch   7; Value -5.814159e+03
Linesearch   8; Value -5.880528e+03
Linesearch   9; Value -5.887417e+03
Linesearch   10; Value -5.888584e+03
Linesearch   11; Value -5.889148e+03
Linesearch   12; Value -5.889847e+03
Linesearch   13; Value -5.889972e+03
Linesearch   14; Value -5.890024e+03
Linesearch   15; Value -5.890031e+03
Linesearch   16; Value -5.890033e+03
Linesearch   17; Value -5.890035e+03
Linesearch   18; Value -5.890036e+03
Linesearch   19; Value -5.890036e+03
Linesearch   20; Value -5.890036e+03
Linesearch   21; Value -5.890036e+03
Linesearch   22; Value -5.890036e+03
Linesearch   23; Value -5.890036e+03
Linesearch   24; Value -5.890036e+03
Linesearch   25; Value -5.890036e+03
Linesearch   26; Value -5.890036e+03
Linesearch   27; Value -5.890036e+03
Linesearch   28; Value -5.890036e+03
Linesearch   29; Value -5.890036e+03
Linesearch   30; Value -5.890036e+03
GRBCM (Dc size 150): MSE 0.01195815, SMSE 0.01074338, MSLL -2.01397980
Function evaluation   0; Value 4.536107e+05
Function evaluation   17; Value 4.357326e+05
Function evaluation   19; Value 3.764044e+05
Function evaluation   22; Value 3.401485e+05
Function evaluation   25; Value 3.089513e+05
Function evaluation   27; Value 2.885467e+05
Function evaluation   29; Value 2.703042e+05
Function evaluation   31; Value 2.556791e+05
Function evaluation   33; Value 2.477159e+05
Function evaluation   34; Value 2.389489e+05
Function evaluation   36; Value 2.335224e+05
Function evaluation   37; Value 2.276805e+05
Function evaluation   39; Value 2.250578e+05
Function evaluation   41; Value 2.231022e+05
Function evaluation   43; Value 2.201958e+05
Function evaluation   44; Value 2.175262e+05
Function evaluation   46; Value 2.160835e+05
Function evaluation   48; Value 2.139468e+05
Function evaluation   50; Value 2.122865e+05
Function evaluation   52; Value 2.107884e+05
Function evaluation   54; Value 2.087419e+05
Function evaluation   56; Value 2.075597e+05
Function evaluation   58; Value 2.059366e+05
Function evaluation   60; Value 2.046613e+05
Function evaluation   62; Value 2.030003e+05
Function evaluation   64; Value 2.017150e+05

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Function evaluation      66;   Value 2.008321e+05
Function evaluation      68;   Value 1.998137e+05
Function evaluation      69;   Value 1.987254e+05
Function evaluation      71;   Value 1.981819e+05
Function evaluation      73;   Value 1.972526e+05
Function evaluation      75;   Value 1.965534e+05
Function evaluation      77;   Value 1.959371e+05
Function evaluation      78;   Value 1.952707e+05
Function evaluation      79;   Value 1.946647e+05
Function evaluation      81;   Value 1.943275e+05
Function evaluation      83;   Value 1.937275e+05
Function evaluation      85;   Value 1.933493e+05
Function evaluation      87;   Value 1.928727e+05
Function evaluation      89;   Value 1.922865e+05
Function evaluation      91;   Value 1.920298e+05
Function evaluation      93;   Value 1.916859e+05
Function evaluation      94;   Value 1.913452e+05
Function evaluation      95;   Value 1.909771e+05
Function evaluation      97;   Value 1.907995e+05
Function evaluation      99;   Value 1.905167e+05
Function evaluation     100;   Value 1.902154e+05
VFE (Dc size 150): MSE 0.10831815, SMSE 0.09731465, MSLL -1.10248528
Function evaluation       0;   Value 8.451065e+03
Function evaluation       9;   Value 8.147864e+03
Function evaluation      11;   Value 7.341941e+03
Function evaluation      13;   Value 6.856384e+03
Function evaluation      15;   Value 6.297188e+03
Function evaluation      16;   Value 5.972830e+03
Function evaluation      18;   Value 5.639324e+03
Function evaluation      20;   Value 5.373101e+03
Function evaluation      21;   Value 5.118634e+03
Function evaluation      23;   Value 4.971459e+03
Function evaluation      25;   Value 4.785635e+03
Function evaluation      26;   Value 4.604961e+03
Function evaluation      28;   Value 4.448009e+03
Function evaluation      30;   Value 4.328221e+03
Function evaluation      32;   Value 4.263366e+03
Function evaluation      33;   Value 4.191963e+03
Function evaluation      35;   Value 4.096453e+03
Function evaluation      37;   Value 4.016314e+03
Function evaluation      39;   Value 3.957116e+03
Function evaluation      41;   Value 3.911625e+03
Function evaluation      43;   Value 3.856342e+03
Function evaluation      45;   Value 3.809176e+03
Function evaluation      47;   Value 3.769407e+03
Function evaluation      48;   Value 3.724088e+03
Function evaluation      50;   Value 3.671794e+03
Function evaluation      52;   Value 3.625117e+03
Function evaluation      53;   Value 3.587473e+03
Function evaluation      55;   Value 3.560744e+03
Function evaluation      56;   Value 3.534394e+03
Function evaluation      58;   Value 3.512997e+03
Function evaluation      59;   Value 3.490956e+03
Function evaluation      61;   Value 3.475934e+03
Function evaluation      62;   Value 3.461914e+03
Function evaluation      64;   Value 3.449411e+03
Function evaluation      65;   Value 3.436742e+03
Function evaluation      67;   Value 3.429811e+03
Function evaluation      69;   Value 3.417443e+03
Function evaluation      71;   Value 3.408570e+03
Function evaluation      72;   Value 3.400221e+03
Function evaluation      73;   Value 3.391541e+03
Function evaluation      75;   Value 3.380306e+03
Function evaluation      77;   Value 3.375474e+03
Function evaluation      79;   Value 3.367169e+03
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Function evaluation      80;   Value 3.359393e+03
Function evaluation      82;   Value 3.354958e+03
Function evaluation      84;   Value 3.346719e+03
Function evaluation      85;   Value 3.339302e+03
Function evaluation      87;   Value 3.334769e+03
Function evaluation      89;   Value 3.326719e+03
Function evaluation      90;   Value 3.318691e+03
Function evaluation      91;   Value 3.311253e+03
Function evaluation      93;   Value 3.304163e+03
Function evaluation      94;   Value 3.297823e+03
Function evaluation      95;   Value 3.290885e+03
Function evaluation      97;   Value 3.285919e+03
Function evaluation      98;   Value 3.281004e+03
Function evaluation     100;   Value 3.276750e+03
SPSG (Dc size 150): MSE 0.17101853, SMSE 0.15364560, MSLL -1.09171269
GRBCM (VFE) (Dc size 150): MSE 0.01181869, SMSE 0.01061809, MSLL -2.09521551
GRBCM (SPGP) (Dc size 150): MSE 0.01175277, SMSE 0.01055886, MSLL -2.08620979
RBCM (Dc size 150): MSE 0.01218529, SMSE 0.0109, MSLL -1.7726
BCM (Dc size 150): MSE 0.01249961, SMSE 0.0112, MSLL -2.2050
PoE (Dc size 150): MSE 0.10268905, SMSE 0.0923, MSLL 4.6703
GPoE (Dc size 150): MSE 0.01221013, SMSE 0.0110, MSLL -1.7530
processing distance: 100/10000
processing distance: 200/10000
processing distance: 300/10000
processing distance: 400/10000
processing distance: 500/10000
processing distance: 600/10000
processing distance: 700/10000
processing distance: 800/10000
processing distance: 900/10000
processing distance: 1000/10000
processing distance: 1100/10000
processing distance: 1200/10000
processing distance: 1300/10000
processing distance: 1400/10000
processing distance: 1500/10000
processing distance: 1600/10000
processing distance: 1700/10000
processing distance: 1800/10000
processing distance: 1900/10000
processing distance: 2000/10000
processing distance: 2100/10000
processing distance: 2200/10000
processing distance: 2300/10000
processing distance: 2400/10000
processing distance: 2500/10000
processing distance: 2600/10000
processing distance: 2700/10000
processing distance: 2800/10000
processing distance: 2900/10000
processing distance: 3000/10000
processing distance: 3100/10000
processing distance: 3200/10000
processing distance: 3300/10000
processing distance: 3400/10000
processing distance: 3500/10000
processing distance: 3600/10000
processing distance: 3700/10000
processing distance: 3800/10000
processing distance: 3900/10000
processing distance: 4000/10000
processing distance: 4100/10000
processing distance: 4200/10000
processing distance: 4300/10000
processing distance: 4400/10000
```

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processing distance: 4500/10000
processing distance: 4600/10000
processing distance: 4700/10000
processing distance: 4800/10000
processing distance: 4900/10000
processing distance: 5000/10000
processing distance: 5100/10000
processing distance: 5200/10000
processing distance: 5300/10000
processing distance: 5400/10000
processing distance: 5500/10000
processing distance: 5600/10000
processing distance: 5700/10000
processing distance: 5800/10000
processing distance: 5900/10000
processing distance: 6000/10000
processing distance: 6100/10000
processing distance: 6200/10000
processing distance: 6300/10000
processing distance: 6400/10000
processing distance: 6500/10000
processing distance: 6600/10000
processing distance: 6700/10000
processing distance: 6800/10000
processing distance: 6900/10000
processing distance: 7000/10000
processing distance: 7100/10000
processing distance: 7200/10000
processing distance: 7300/10000
processing distance: 7400/10000
processing distance: 7500/10000
processing distance: 7600/10000
processing distance: 7700/10000
processing distance: 7800/10000
processing distance: 7900/10000
processing distance: 8000/10000
processing distance: 8100/10000
processing distance: 8200/10000
processing distance: 8300/10000
processing distance: 8400/10000
processing distance: 8500/10000
processing distance: 8600/10000
processing distance: 8700/10000
processing distance: 8800/10000
processing distance: 8900/10000
processing distance: 9000/10000
processing distance: 9100/10000
processing distance: 9200/10000
processing distance: 9300/10000
processing distance: 9400/10000
processing distance: 9500/10000
processing distance: 9600/10000
processing distance: 9700/10000
processing distance: 9800/10000
processing distance: 9900/10000
processing distance: 10000/10000
kj = 1
gr = 0.3000
GRBCM (Dc size 150): MSE 0.03446682, SMSE 0.0310, MSLL -1.7938
GRBCM++ (VFE) (Dc size 150):
MSE 0.02183968, SMSE 0.0196, MSLL -1.9488
GRBCM++ (SPGP) (Dc size 150):
MSE 0.02225236, SMSE 0.0200, MSLL -1.9235
kj = 2
gr = 0.3500
```

GRBCM (Dc size 150): MSE 0.02538968, SMSE 0.0228, MSLL -1.8871
GRBCM++ (VFE) (Dc size 150):
MSE 0.01884228, SMSE 0.0169, MSLL -2.0071
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01988925, SMSE 0.0179, MSLL -1.9757
kj = 3
gr = 0.4000
GRBCM (Dc size 150): MSE 0.02226980, SMSE 0.0200, MSLL -1.9291
GRBCM++ (VFE) (Dc size 150):
MSE 0.01773210, SMSE 0.0159, MSLL -2.0451
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01803864, SMSE 0.0162, MSLL -2.0235
kj = 4
gr = 0.4500
GRBCM (Dc size 150): MSE 0.01911881, SMSE 0.0172, MSLL -1.9908
GRBCM++ (VFE) (Dc size 150):
MSE 0.01645857, SMSE 0.0148, MSLL -2.0831
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01659627, SMSE 0.0149, MSLL -2.0672
kj = 5
gr = 0.5000
GRBCM (Dc size 150): MSE 0.01758823, SMSE 0.0158, MSLL -2.0089
GRBCM++ (VFE) (Dc size 150):
MSE 0.01548590, SMSE 0.0139, MSLL -2.1036
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01560160, SMSE 0.0140, MSLL -2.0869
kj = 6
gr = 0.5500
GRBCM (Dc size 150): MSE 0.01613889, SMSE 0.0145, MSLL -2.0390
GRBCM++ (VFE) (Dc size 150):
MSE 0.01457448, SMSE 0.0131, MSLL -2.1270
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01438594, SMSE 0.0129, MSLL -2.1242
kj = 7
gr = 0.6000
GRBCM (Dc size 150): MSE 0.01464278, SMSE 0.0132, MSLL -2.0719
GRBCM++ (VFE) (Dc size 150):
MSE 0.01384114, SMSE 0.0124, MSLL -2.1431
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01381176, SMSE 0.0124, MSLL -2.1374
kj = 8
gr = 0.6500
GRBCM (Dc size 150): MSE 0.01350580, SMSE 0.0121, MSLL -2.0956
GRBCM++ (VFE) (Dc size 150):
MSE 0.01294306, SMSE 0.0116, MSLL -2.1657
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01297714, SMSE 0.0117, MSLL -2.1506
kj = 9
gr = 0.7000
GRBCM (Dc size 150): MSE 0.01298525, SMSE 0.0117, MSLL -2.0966
GRBCM++ (VFE) (Dc size 150):
MSE 0.01276897, SMSE 0.0115, MSLL -2.1603
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01281554, SMSE 0.0115, MSLL -2.1419
kj = 10
gr = 0.7500
GRBCM (Dc size 150): MSE 0.01278875, SMSE 0.0115, MSLL -2.0811
GRBCM++ (VFE) (Dc size 150):
MSE 0.01261766, SMSE 0.0113, MSLL -2.1465
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01266732, SMSE 0.0114, MSLL -2.1279
kj = 11
gr = 0.8000
GRBCM (Dc size 150): MSE 0.01247461, SMSE 0.0112, MSLL -2.0667
GRBCM++ (VFE) (Dc size 150):

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MSE 0.01243300, SMSE 0.0112, MSLL -2.1326
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01243696, SMSE 0.0112, MSLL -2.1132
kj = 12
gr = 0.8500
GRBCM (Dc size 150): MSE 0.01232257, SMSE 0.0111, MSLL -2.0528
GRBCM++ (VFE) (Dc size 150):
MSE 0.01221458, SMSE 0.0110, MSLL -2.1316
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01215209, SMSE 0.0109, MSLL -2.1144
kj = 13
gr = 0.9000
GRBCM (Dc size 150): MSE 0.01222423, SMSE 0.0110, MSLL -2.0333
GRBCM++ (VFE) (Dc size 150):
MSE 0.01212136, SMSE 0.0109, MSLL -2.1087
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01208131, SMSE 0.0109, MSLL -2.0952
kj = 14
gr = 0.9500
GRBCM (Dc size 150): MSE 0.01200543, SMSE 0.0108, MSLL -2.0272
GRBCM++ (VFE) (Dc size 150):
MSE 0.01190662, SMSE 0.0107, MSLL -2.1090
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01189820, SMSE 0.0107, MSLL -2.0897
kj = 15
gr = 1
GRBCM (Dc size 150): MSE 0.01195815, SMSE 0.0107, MSLL -2.0140
GRBCM++ (VFE) (Dc size 150):
MSE 0.01181869, SMSE 0.0106, MSLL -2.0952
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01175277, SMSE 0.0106, MSLL -2.0862
=============3================
Optimizing hyps in training...
Linesearch      0;  Value 1.505384e+04
Linesearch      1;  Value 9.567417e+03
Linesearch      2;  Value 6.544292e+02
Linesearch      3;  Value 2.393672e+01
Linesearch      4;  Value -4.875612e+03
Linesearch      5;  Value -5.182447e+03
Linesearch      6;  Value -5.279436e+03
Linesearch      7;  Value -5.486112e+03
Linesearch      8;  Value -5.829361e+03
Linesearch      9;  Value -5.880189e+03
Linesearch     10;  Value -5.899879e+03
Linesearch     11;  Value -5.900717e+03
Linesearch     12;  Value -5.902174e+03
Linesearch     13;  Value -5.902746e+03
Linesearch     14;  Value -5.902831e+03
Linesearch     15;  Value -5.902838e+03
Linesearch     16;  Value -5.902838e+03
Linesearch     17;  Value -5.902838e+03
Linesearch     18;  Value -5.902838e+03
Linesearch     19;  Value -5.902838e+03
Linesearch     20;  Value -5.902838e+03
Linesearch     21;  Value -5.902838e+03
Linesearch     22;  Value -5.902838e+03
Linesearch     23;  Value -5.902838e+03
Linesearch     24;  Value -5.902838e+03
Linesearch     25;  Value -5.902838e+03
GRBCM (Dc size 150): MSE 0.01196406, SMSE 0.01074869, MSLL -2.01800756
Function evaluation      0;  Value 4.748386e+05
Function evaluation     14;  Value 4.018706e+05
Function evaluation     20;  Value 3.598953e+05
Function evaluation     22;  Value 3.503557e+05
Function evaluation     25;  Value 3.203713e+05
```

```
Function evaluation      27;   Value 2.865523e+05
Function evaluation      30;   Value 2.719933e+05
Function evaluation      31;   Value 2.569471e+05
Function evaluation      33;   Value 2.445929e+05
Function evaluation      35;   Value 2.330227e+05
Function evaluation      37;   Value 2.246474e+05
Function evaluation      39;   Value 2.191468e+05
Function evaluation      41;   Value 2.147156e+05
Function evaluation      43;   Value 2.107777e+05
Function evaluation      45;   Value 2.085263e+05
Function evaluation      47;   Value 2.054717e+05
Function evaluation      49;   Value 2.031486e+05
Function evaluation      51;   Value 2.017933e+05
Function evaluation      53;   Value 2.007113e+05
Function evaluation      55;   Value 1.993983e+05
Function evaluation      57;   Value 1.983656e+05
Function evaluation      59;   Value 1.976381e+05
Function evaluation      61;   Value 1.971528e+05
Function evaluation      63;   Value 1.962913e+05
Function evaluation      65;   Value 1.957405e+05
Function evaluation      67;   Value 1.953481e+05
Function evaluation      69;   Value 1.950946e+05
Function evaluation      71;   Value 1.947641e+05
Function evaluation      73;   Value 1.945300e+05
Function evaluation      75;   Value 1.941951e+05
Function evaluation      77;   Value 1.939408e+05
Function evaluation      78;   Value 1.936846e+05
Function evaluation      80;   Value 1.934647e+05
Function evaluation      82;   Value 1.932255e+05
Function evaluation      84;   Value 1.930296e+05
Function evaluation      86;   Value 1.927336e+05
Function evaluation      88;   Value 1.924869e+05
Function evaluation      89;   Value 1.922459e+05
Function evaluation      91;   Value 1.920344e+05
Function evaluation      93;   Value 1.916358e+05
Function evaluation      95;   Value 1.912978e+05
Function evaluation      97;   Value 1.909075e+05
Function evaluation      99;   Value 1.907273e+05
VFE (Dc size 150): MSE 0.08820018, SMSE 0.07924035, MSLL -1.11115774
Function evaluation       0;   Value 8.559161e+03
Function evaluation      10;   Value 7.551960e+03
Function evaluation      18;   Value 7.130636e+03
Function evaluation      21;   Value 6.624339e+03
Function evaluation      23;   Value 6.272498e+03
Function evaluation      24;   Value 5.743157e+03
Function evaluation      26;   Value 5.376038e+03
Function evaluation      28;   Value 5.128521e+03
Function evaluation      30;   Value 4.903643e+03
Function evaluation      32;   Value 4.770005e+03
Function evaluation      33;   Value 4.626269e+03
Function evaluation      35;   Value 4.514055e+03
Function evaluation      36;   Value 4.396472e+03
Function evaluation      38;   Value 4.305283e+03
Function evaluation      40;   Value 4.187415e+03
Function evaluation      42;   Value 4.101512e+03
Function evaluation      44;   Value 4.041397e+03
Function evaluation      46;   Value 3.972600e+03
Function evaluation      48;   Value 3.925758e+03
Function evaluation      49;   Value 3.881641e+03
Function evaluation      50;   Value 3.834603e+03
Function evaluation      52;   Value 3.807005e+03
Function evaluation      54;   Value 3.770857e+03
Function evaluation      56;   Value 3.740846e+03
Function evaluation      58;   Value 3.715687e+03
Function evaluation      59;   Value 3.688124e+03
```

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Function evaluation      60;   Value 3.663636e+03
Function evaluation      62;   Value 3.641743e+03
Function evaluation      63;   Value 3.619274e+03
Function evaluation      65;   Value 3.590859e+03
Function evaluation      67;   Value 3.566084e+03
Function evaluation      69;   Value 3.547963e+03
Function evaluation      71;   Value 3.532949e+03
Function evaluation      72;   Value 3.518327e+03
Function evaluation      74;   Value 3.506497e+03
Function evaluation      75;   Value 3.495647e+03
Function evaluation      77;   Value 3.486499e+03
Function evaluation      78;   Value 3.476991e+03
Function evaluation      80;   Value 3.462942e+03
Function evaluation      82;   Value 3.452476e+03
Function evaluation      84;   Value 3.442851e+03
Function evaluation      86;   Value 3.435580e+03
Function evaluation      88;   Value 3.426270e+03
Function evaluation      90;   Value 3.415144e+03
Function evaluation      92;   Value 3.407596e+03
Function evaluation      93;   Value 3.399392e+03
Function evaluation      95;   Value 3.390055e+03
Function evaluation      97;   Value 3.384448e+03
Function evaluation      99;   Value 3.379730e+03
Function evaluation     100;   Value 3.375293e+03
SPSG (Dc size 150): MSE 0.20853875, SMSE 0.18735432, MSLL -1.11060237
GRBCM (VFE) (Dc size 150): MSE 0.01185004, SMSE 0.01064625, MSLL -2.07041622
GRBCM (SPGP) (Dc size 150): MSE 0.01174659, SMSE 0.01055331, MSLL -2.07619676
RBCM (Dc size 150): MSE 0.01189211, SMSE 0.0107, MSLL -1.8093
BCM (Dc size 150): MSE 0.01228173, SMSE 0.0110, MSLL -2.2227
PoE (Dc size 150): MSE 0.09660415, SMSE 0.0868, MSLL 4.0535
GPoE (Dc size 150): MSE 0.01187685, SMSE 0.0107, MSLL -1.7986
processing distance: 100/10000
processing distance: 200/10000
processing distance: 300/10000
processing distance: 400/10000
processing distance: 500/10000
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processing distance: 3000/10000
processing distance: 3100/10000
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processing distance: 3500/10000
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processing distance: 3900/10000
processing distance: 4000/10000
processing distance: 4100/10000
processing distance: 4200/10000
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processing distance: 9400/10000
processing distance: 9500/10000
processing distance: 9600/10000
processing distance: 9700/10000
processing distance: 9800/10000
processing distance: 9900/10000
```

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processing distance: 10000/10000
kj = 1
gr = 0.3000
GRBCM (Dc size 150): MSE 0.03623360, SMSE 0.0326, MSLL -1.7645
GRBCM++ (VFE) (Dc size 150):
MSE 0.02275566, SMSE 0.0204, MSLL -1.9211
GRBCM++ (SPGP) (Dc size 150):
MSE 0.02322467, SMSE 0.0209, MSLL -1.9323
kj = 2
gr = 0.3500
GRBCM (Dc size 150): MSE 0.02838956, SMSE 0.0255, MSLL -1.8574
GRBCM++ (VFE) (Dc size 150):
MSE 0.01963122, SMSE 0.0176, MSLL -1.9878
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01958823, SMSE 0.0176, MSLL -1.9895
kj = 3
gr = 0.4000
GRBCM (Dc size 150): MSE 0.02314256, SMSE 0.0208, MSLL -1.9262
GRBCM++ (VFE) (Dc size 150):
MSE 0.01782820, SMSE 0.0160, MSLL -2.0405
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01741111, SMSE 0.0156, MSLL -2.0350
kj = 4
gr = 0.4500
GRBCM (Dc size 150): MSE 0.02025490, SMSE 0.0182, MSLL -1.9740
GRBCM++ (VFE) (Dc size 150):
MSE 0.01653493, SMSE 0.0149, MSLL -2.0770
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01627736, SMSE 0.0146, MSLL -2.0711
kj = 5
gr = 0.5000
GRBCM (Dc size 150): MSE 0.01812104, SMSE 0.0163, MSLL -2.0103
GRBCM++ (VFE) (Dc size 150):
MSE 0.01576439, SMSE 0.0142, MSLL -2.0971
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01547517, SMSE 0.0139, MSLL -2.0917
kj = 6
gr = 0.5500
GRBCM (Dc size 150): MSE 0.01640594, SMSE 0.0147, MSLL -2.0393
GRBCM++ (VFE) (Dc size 150):
MSE 0.01475320, SMSE 0.0133, MSLL -2.1144
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01486537, SMSE 0.0134, MSLL -2.0949
kj = 7
gr = 0.6000
GRBCM (Dc size 150): MSE 0.01527336, SMSE 0.0137, MSLL -2.0510
GRBCM++ (VFE) (Dc size 150):
MSE 0.01390148, SMSE 0.0125, MSLL -2.1338
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01396084, SMSE 0.0125, MSLL -2.1116
kj = 8
gr = 0.6500
GRBCM (Dc size 150): MSE 0.01415870, SMSE 0.0127, MSLL -2.0658
GRBCM++ (VFE) (Dc size 150):
MSE 0.01312247, SMSE 0.0118, MSLL -2.1459
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01313498, SMSE 0.0118, MSLL -2.1309
kj = 9
gr = 0.7000
GRBCM (Dc size 150): MSE 0.01372573, SMSE 0.0123, MSLL -2.0600
GRBCM++ (VFE) (Dc size 150):
MSE 0.01287062, SMSE 0.0116, MSLL -2.1405
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01284139, SMSE 0.0115, MSLL -2.1259
kj = 10
```

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gr = 0.7500
GRBCM (Dc size 150): MSE 0.01314646, SMSE 0.0118, MSLL -2.0634
GRBCM++ (VFE) (Dc size 150):
MSE 0.01263282, SMSE 0.0113, MSLL -2.1287
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01247930, SMSE 0.0112, MSLL -2.1218
kj = 11
gr = 0.8000
GRBCM (Dc size 150): MSE 0.01262302, SMSE 0.0113, MSLL -2.0638
GRBCM++ (VFE) (Dc size 150):
MSE 0.01236220, SMSE 0.0111, MSLL -2.1251
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01223071, SMSE 0.0110, MSLL -2.1204
kj = 12
gr = 0.8500
GRBCM (Dc size 150): MSE 0.01247962, SMSE 0.0112, MSLL -2.0499
GRBCM++ (VFE) (Dc size 150):
MSE 0.01215675, SMSE 0.0109, MSLL -2.1158
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01218114, SMSE 0.0109, MSLL -2.1003
kj = 13
gr = 0.9000
GRBCM (Dc size 150): MSE 0.01218132, SMSE 0.0109, MSLL -2.0506
GRBCM++ (VFE) (Dc size 150):
MSE 0.01198098, SMSE 0.0108, MSLL -2.1072
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01198217, SMSE 0.0108, MSLL -2.0989
kj = 14
gr = 0.9500
GRBCM (Dc size 150): MSE 0.01205797, SMSE 0.0108, MSLL -2.0276
GRBCM++ (VFE) (Dc size 150):
MSE 0.01189529, SMSE 0.0107, MSLL -2.0852
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01183704, SMSE 0.0106, MSLL -2.0845
kj = 15
gr = 1
GRBCM (Dc size 150): MSE 0.01196406, SMSE 0.0107, MSLL -2.0180
GRBCM++ (VFE) (Dc size 150):
MSE 0.01185004, SMSE 0.0106, MSLL -2.0704
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01174659, SMSE 0.0106, MSLL -2.0762
=============4================
Optimizing hyps in training...
Linesearch      0;  Value 1.565467e+04
Linesearch      1;  Value 9.736418e+03
Linesearch      2;  Value 6.814167e+02
Linesearch      3;  Value 1.397998e+02
Linesearch      4;  Value -5.008644e+03
Linesearch      5;  Value -5.233680e+03
Linesearch      6;  Value -5.321393e+03
Linesearch      7;  Value -5.521026e+03
Linesearch      8;  Value -5.799991e+03
Linesearch      9;  Value -5.873890e+03
Linesearch     10;  Value -5.885955e+03
Linesearch     11;  Value -5.888980e+03
Linesearch     12;  Value -5.894184e+03
Linesearch     13;  Value -5.895648e+03
Linesearch     14;  Value -5.895648e+03
Linesearch     15;  Value -5.895649e+03
Linesearch     16;  Value -5.895650e+03
Linesearch     17;  Value -5.895650e+03
Linesearch     18;  Value -5.895650e+03
Linesearch     19;  Value -5.895650e+03
Linesearch     20;  Value -5.895650e+03
Linesearch     21;  Value -5.895650e+03
```

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Linesearch     22;  Value -5.895650e+03
GRBCM (Dc size 150): MSE 0.01217717, SMSE 0.01094015, MSLL -2.00467919
Function evaluation      0;  Value 4.801970e+05
Function evaluation     20;  Value 4.046670e+05
Function evaluation     22;  Value 3.529593e+05
Function evaluation     25;  Value 3.406056e+05
Function evaluation     27;  Value 3.239685e+05
Function evaluation     29;  Value 2.979557e+05
Function evaluation     31;  Value 2.800586e+05
Function evaluation     33;  Value 2.702228e+05
Function evaluation     35;  Value 2.575403e+05
Function evaluation     37;  Value 2.483187e+05
Function evaluation     38;  Value 2.388547e+05
Function evaluation     40;  Value 2.329330e+05
Function evaluation     42;  Value 2.288227e+05
Function evaluation     43;  Value 2.245643e+05
Function evaluation     45;  Value 2.214203e+05
Function evaluation     46;  Value 2.179326e+05
Function evaluation     48;  Value 2.160095e+05
Function evaluation     50;  Value 2.138234e+05
Function evaluation     52;  Value 2.118103e+05
Function evaluation     54;  Value 2.092475e+05
Function evaluation     56;  Value 2.075592e+05
Function evaluation     58;  Value 2.061701e+05
Function evaluation     60;  Value 2.041349e+05
Function evaluation     62;  Value 2.023943e+05
Function evaluation     64;  Value 2.011890e+05
Function evaluation     66;  Value 1.997894e+05
Function evaluation     68;  Value 1.981652e+05
Function evaluation     70;  Value 1.961380e+05
Function evaluation     72;  Value 1.946078e+05
Function evaluation     74;  Value 1.934955e+05
Function evaluation     76;  Value 1.921602e+05
Function evaluation     78;  Value 1.912489e+05
Function evaluation     80;  Value 1.901867e+05
Function evaluation     82;  Value 1.894135e+05
Function evaluation     84;  Value 1.885458e+05
Function evaluation     86;  Value 1.875090e+05
Function evaluation     88;  Value 1.868101e+05
Function evaluation     89;  Value 1.861469e+05
Function evaluation     90;  Value 1.854990e+05
Function evaluation     92;  Value 1.850699e+05
Function evaluation     94;  Value 1.845303e+05
Function evaluation     96;  Value 1.841055e+05
Function evaluation     97;  Value 1.837195e+05
Function evaluation     99;  Value 1.834777e+05
VFE (Dc size 150): MSE 0.08492627, SMSE 0.07629903, MSLL -1.09895602
Function evaluation      0;  Value 9.176698e+03
Function evaluation     19;  Value 7.558707e+03
Function evaluation     23;  Value 7.167834e+03
Function evaluation     26;  Value 6.498872e+03
Function evaluation     27;  Value 6.128718e+03
Function evaluation     29;  Value 5.658701e+03
Function evaluation     31;  Value 5.311499e+03
Function evaluation     33;  Value 5.013407e+03
Function evaluation     35;  Value 4.796584e+03
Function evaluation     36;  Value 4.709402e+03
Function evaluation     37;  Value 4.513974e+03
Function evaluation     38;  Value 4.306162e+03
Function evaluation     40;  Value 4.130651e+03
Function evaluation     41;  Value 3.967741e+03
Function evaluation     43;  Value 3.878914e+03
Function evaluation     44;  Value 3.779870e+03
Function evaluation     45;  Value 3.690486e+03
Function evaluation     47;  Value 3.610116e+03
```

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Function evaluation     48;  Value 3.526583e+03
Function evaluation     50;  Value 3.481456e+03
Function evaluation     51;  Value 3.436139e+03
Function evaluation     53;  Value 3.407138e+03
Function evaluation     54;  Value 3.374996e+03
Function evaluation     56;  Value 3.350992e+03
Function evaluation     57;  Value 3.325188e+03
Function evaluation     58;  Value 3.297816e+03
Function evaluation     59;  Value 3.274679e+03
Function evaluation     61;  Value 3.262067e+03
Function evaluation     62;  Value 3.248375e+03
Function evaluation     64;  Value 3.234282e+03
Function evaluation     66;  Value 3.225193e+03
Function evaluation     67;  Value 3.216500e+03
Function evaluation     69;  Value 3.209922e+03
Function evaluation     71;  Value 3.200334e+03
Function evaluation     72;  Value 3.190846e+03
Function evaluation     73;  Value 3.181101e+03
Function evaluation     74;  Value 3.171073e+03
Function evaluation     76;  Value 3.156641e+03
Function evaluation     78;  Value 3.144656e+03
Function evaluation     79;  Value 3.132762e+03
Function evaluation     81;  Value 3.125510e+03
Function evaluation     83;  Value 3.115889e+03
Function evaluation     85;  Value 3.108457e+03
Function evaluation     87;  Value 3.103339e+03
Function evaluation     88;  Value 3.098209e+03
Function evaluation     90;  Value 3.094253e+03
Function evaluation     92;  Value 3.091080e+03
Function evaluation     93;  Value 3.087949e+03
Function evaluation     95;  Value 3.085169e+03
Function evaluation     97;  Value 3.081971e+03
Function evaluation     99;  Value 3.079617e+03
SPSG (Dc size 150): MSE 0.15535826, SMSE 0.13957618, MSLL -1.10775617
GRBCM (VFE) (Dc size 150): MSE 0.01166735, SMSE 0.01048212, MSLL -2.09381373
GRBCM (SPGP) (Dc size 150): MSE 0.01165665, SMSE 0.01047251, MSLL -2.11163624
RBCM (Dc size 150): MSE 0.01211432, SMSE 0.0109, MSLL -1.7714
BCM (Dc size 150): MSE 0.01259098, SMSE 0.0113, MSLL -2.1973
PoE (Dc size 150): MSE 0.10116991, SMSE 0.0909, MSLL 4.5106
GPoE (Dc size 150): MSE 0.01209238, SMSE 0.0109, MSLL -1.7601
processing distance: 100/10000
processing distance: 200/10000
processing distance: 300/10000
processing distance: 400/10000
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processing distance: 2100/10000
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processing distance: 2500/10000
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processing distance: 2600/10000
processing distance: 2700/10000
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processing distance: 2900/10000
processing distance: 3000/10000
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processing distance: 9100/10000
processing distance: 9200/10000
processing distance: 9300/10000
processing distance: 9400/10000
processing distance: 9500/10000
processing distance: 9600/10000
processing distance: 9700/10000
processing distance: 9800/10000
processing distance: 9900/10000
processing distance: 10000/10000
kj = 1
gr = 0.3000
GRBCM (Dc size 150): MSE 0.03667155, SMSE 0.0329, MSLL -1.7936
GRBCM++ (VFE) (Dc size 150):
MSE 0.02019413, SMSE 0.0181, MSLL -1.9487
GRBCM++ (SPGP) (Dc size 150):
MSE 0.02178315, SMSE 0.0196, MSLL -1.9445
kj = 2
gr = 0.3500
GRBCM (Dc size 150): MSE 0.03229905, SMSE 0.0290, MSLL -1.8325
GRBCM++ (VFE) (Dc size 150):
MSE 0.02105571, SMSE 0.0189, MSLL -1.9753
GRBCM++ (SPGP) (Dc size 150):
MSE 0.02125684, SMSE 0.0191, MSLL -1.9787
kj = 3
gr = 0.4000
GRBCM (Dc size 150): MSE 0.02781187, SMSE 0.0250, MSLL -1.8811
GRBCM++ (VFE) (Dc size 150):
MSE 0.01858508, SMSE 0.0167, MSLL -2.0303
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01999398, SMSE 0.0180, MSLL -2.0154
kj = 4
gr = 0.4500
GRBCM (Dc size 150): MSE 0.02351944, SMSE 0.0211, MSLL -1.9409
GRBCM++ (VFE) (Dc size 150):
MSE 0.01718783, SMSE 0.0154, MSLL -2.0675
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01792712, SMSE 0.0161, MSLL -2.0618
kj = 5
gr = 0.5000
GRBCM (Dc size 150): MSE 0.01927899, SMSE 0.0173, MSLL -1.9891
GRBCM++ (VFE) (Dc size 150):
MSE 0.01586740, SMSE 0.0143, MSLL -2.0844
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01589025, SMSE 0.0143, MSLL -2.0891
kj = 6
gr = 0.5500
GRBCM (Dc size 150): MSE 0.01676823, SMSE 0.0151, MSLL -2.0367
GRBCM++ (VFE) (Dc size 150):
MSE 0.01475222, SMSE 0.0133, MSLL -2.1114
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01456498, SMSE 0.0131, MSLL -2.1191
kj = 7
gr = 0.6000
GRBCM (Dc size 150): MSE 0.01537908, SMSE 0.0138, MSLL -2.0548
GRBCM++ (VFE) (Dc size 150):
MSE 0.01393915, SMSE 0.0125, MSLL -2.1321
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01402920, SMSE 0.0126, MSLL -2.1206
kj = 8
gr = 0.6500
GRBCM (Dc size 150): MSE 0.01481765, SMSE 0.0133, MSLL -2.0498
GRBCM++ (VFE) (Dc size 150):
MSE 0.01345841, SMSE 0.0121, MSLL -2.1332
GRBCM++ (SPGP) (Dc size 150):
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MSE 0.01339112, SMSE 0.0120, MSLL -2.1285
kj = 9
gr = 0.7000
GRBCM (Dc size 150): MSE 0.01371856, SMSE 0.0123, MSLL -2.0709
GRBCM++ (VFE) (Dc size 150):
MSE 0.01293609, SMSE 0.0116, MSLL -2.1409
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01270848, SMSE 0.0114, MSLL -2.1464
kj = 10
gr = 0.7500
GRBCM (Dc size 150): MSE 0.01337078, SMSE 0.0120, MSLL -2.0667
GRBCM++ (VFE) (Dc size 150):
MSE 0.01268965, SMSE 0.0114, MSLL -2.1354
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01243284, SMSE 0.0112, MSLL -2.1465
kj = 11
gr = 0.8000
GRBCM (Dc size 150): MSE 0.01302682, SMSE 0.0117, MSLL -2.0606
GRBCM++ (VFE) (Dc size 150):
MSE 0.01243839, SMSE 0.0112, MSLL -2.1285
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01220774, SMSE 0.0110, MSLL -2.1460
kj = 12
gr = 0.8500
GRBCM (Dc size 150): MSE 0.01241926, SMSE 0.0112, MSLL -2.0689
GRBCM++ (VFE) (Dc size 150):
MSE 0.01200297, SMSE 0.0108, MSLL -2.1336
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01182333, SMSE 0.0106, MSLL -2.1501
kj = 13
gr = 0.9000
GRBCM (Dc size 150): MSE 0.01214732, SMSE 0.0109, MSLL -2.0627
GRBCM++ (VFE) (Dc size 150):
MSE 0.01175437, SMSE 0.0106, MSLL -2.1295
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01163558, SMSE 0.0105, MSLL -2.1482
kj = 14
gr = 0.9500
GRBCM (Dc size 150): MSE 0.01218780, SMSE 0.0109, MSLL -2.0273
GRBCM++ (VFE) (Dc size 150):
MSE 0.01171632, SMSE 0.0105, MSLL -2.1097
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01170307, SMSE 0.0105, MSLL -2.1231
kj = 15
gr = 1
GRBCM (Dc size 150): MSE 0.01217717, SMSE 0.0109, MSLL -2.0047
GRBCM++ (VFE) (Dc size 150):
MSE 0.01166735, SMSE 0.0105, MSLL -2.0938
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01165665, SMSE 0.0105, MSLL -2.1116
==============5================
Optimizing hyps in training...
Linesearch      0;  Value 1.516349e+04
Linesearch      1;  Value 9.615361e+03
Linesearch      2;  Value 8.085738e+02
Linesearch      3;  Value 2.506699e+02
Linesearch      4;  Value -4.954405e+03
Linesearch      5;  Value -5.160917e+03
Linesearch      6;  Value -5.369256e+03
Linesearch      7;  Value -5.752272e+03
Linesearch      8;  Value -5.873357e+03
Linesearch      9;  Value -5.877073e+03
Linesearch     10;  Value -5.879312e+03
Linesearch     11;  Value -5.882820e+03
Linesearch     12;  Value -5.885232e+03
```

```
Linesearch     13;  Value -5.885277e+03
Linesearch     14;  Value -5.885284e+03
Linesearch     15;  Value -5.885285e+03
Linesearch     16;  Value -5.885287e+03
Linesearch     17;  Value -5.885287e+03
Linesearch     18;  Value -5.885287e+03
Linesearch     19;  Value -5.885287e+03
Linesearch     20;  Value -5.885287e+03
Linesearch     21;  Value -5.885287e+03
Linesearch     22;  Value -5.885287e+03
Linesearch     23;  Value -5.885287e+03
GRBCM (Dc size 150): MSE 0.01195039, SMSE 0.01073641, MSLL -2.03962653
Function evaluation      0;  Value 4.528892e+05
Function evaluation     11;  Value 4.453990e+05
Function evaluation     12;  Value 4.380699e+05
Function evaluation     15;  Value 3.803871e+05
Function evaluation     17;  Value 3.410258e+05
Function evaluation     19;  Value 3.086824e+05
Function evaluation     21;  Value 2.806371e+05
Function evaluation     23;  Value 2.675512e+05
Function evaluation     25;  Value 2.569464e+05
Function evaluation     27;  Value 2.444878e+05
Function evaluation     29;  Value 2.368295e+05
Function evaluation     30;  Value 2.301019e+05
Function evaluation     32;  Value 2.249293e+05
Function evaluation     34;  Value 2.205749e+05
Function evaluation     36;  Value 2.175336e+05
Function evaluation     37;  Value 2.142961e+05
Function evaluation     39;  Value 2.117575e+05
Function evaluation     41;  Value 2.097364e+05
Function evaluation     43;  Value 2.080378e+05
Function evaluation     45;  Value 2.060634e+05
Function evaluation     47;  Value 2.049364e+05
Function evaluation     49;  Value 2.039763e+05
Function evaluation     51;  Value 2.031651e+05
Function evaluation     53;  Value 2.024947e+05
Function evaluation     55;  Value 2.014957e+05
Function evaluation     57;  Value 2.007432e+05
Function evaluation     59;  Value 2.001795e+05
Function evaluation     61;  Value 1.996619e+05
Function evaluation     62;  Value 1.991953e+05
Function evaluation     64;  Value 1.988048e+05
Function evaluation     66;  Value 1.982715e+05
Function evaluation     68;  Value 1.978580e+05
Function evaluation     70;  Value 1.972770e+05
Function evaluation     71;  Value 1.966966e+05
Function evaluation     72;  Value 1.961186e+05
Function evaluation     74;  Value 1.955830e+05
Function evaluation     76;  Value 1.949541e+05
Function evaluation     77;  Value 1.943231e+05
Function evaluation     78;  Value 1.937456e+05
Function evaluation     80;  Value 1.932478e+05
Function evaluation     82;  Value 1.926190e+05
Function evaluation     84;  Value 1.920831e+05
Function evaluation     86;  Value 1.914848e+05
Function evaluation     88;  Value 1.910621e+05
Function evaluation     90;  Value 1.905293e+05
Function evaluation     91;  Value 1.900035e+05
Function evaluation     93;  Value 1.896772e+05
Function evaluation     95;  Value 1.892603e+05
Function evaluation     96;  Value 1.888620e+05
Function evaluation     98;  Value 1.885291e+05
Function evaluation    100;  Value 1.882388e+05
VFE (Dc size 150): MSE 0.09548850, SMSE 0.08578829, MSLL -1.08399580
Function evaluation      0;  Value 8.558743e+03
```

```
Function evaluation      10;   Value 7.804210e+03
Function evaluation      12;   Value 6.949061e+03
Function evaluation      14;   Value 6.309153e+03
Function evaluation      16;   Value 5.974922e+03
Function evaluation      18;   Value 5.763835e+03
Function evaluation      19;   Value 5.537288e+03
Function evaluation      21;   Value 5.292204e+03
Function evaluation      23;   Value 5.143255e+03
Function evaluation      25;   Value 4.944884e+03
Function evaluation      26;   Value 4.748552e+03
Function evaluation      27;   Value 4.576369e+03
Function evaluation      29;   Value 4.491134e+03
Function evaluation      30;   Value 4.397877e+03
Function evaluation      32;   Value 4.296903e+03
Function evaluation      34;   Value 4.214909e+03
Function evaluation      35;   Value 4.130701e+03
Function evaluation      37;   Value 4.063390e+03
Function evaluation      39;   Value 4.021743e+03
Function evaluation      40;   Value 3.979304e+03
Function evaluation      42;   Value 3.924692e+03
Function evaluation      43;   Value 3.877499e+03
Function evaluation      45;   Value 3.847730e+03
Function evaluation      47;   Value 3.814279e+03
Function evaluation      48;   Value 3.778477e+03
Function evaluation      49;   Value 3.748153e+03
Function evaluation      51;   Value 3.719768e+03
Function evaluation      53;   Value 3.694969e+03
Function evaluation      55;   Value 3.673367e+03
Function evaluation      56;   Value 3.651655e+03
Function evaluation      58;   Value 3.638777e+03
Function evaluation      59;   Value 3.626318e+03
Function evaluation      60;   Value 3.614855e+03
Function evaluation      62;   Value 3.605751e+03
Function evaluation      64;   Value 3.599017e+03
Function evaluation      66;   Value 3.593980e+03
Function evaluation      67;   Value 3.588472e+03
Function evaluation      69;   Value 3.581409e+03
Function evaluation      71;   Value 3.575757e+03
Function evaluation      72;   Value 3.570248e+03
Function evaluation      74;   Value 3.566579e+03
Function evaluation      76;   Value 3.560309e+03
Function evaluation      78;   Value 3.556136e+03
Function evaluation      80;   Value 3.550154e+03
Function evaluation      81;   Value 3.544779e+03
Function evaluation      83;   Value 3.537421e+03
Function evaluation      85;   Value 3.531138e+03
Function evaluation      87;   Value 3.522995e+03
Function evaluation      89;   Value 3.516671e+03
Function evaluation      90;   Value 3.511052e+03
Function evaluation      92;   Value 3.505849e+03
Function evaluation      94;   Value 3.499269e+03
Function evaluation      95;   Value 3.491977e+03
Function evaluation      97;   Value 3.486843e+03
Function evaluation      99;   Value 3.483207e+03
Function evaluation     100;   Value 3.479804e+03
SPSG (Dc size 150): MSE 0.13861333, SMSE 0.12453229, MSLL -1.09108737
GRBCM (VFE) (Dc size 150): MSE 0.01176017, SMSE 0.01056551, MSLL -2.10075918
GRBCM (SPGP) (Dc size 150): MSE 0.01166815, SMSE 0.01048284, MSLL -2.08165763
RBCM (Dc size 150): MSE 0.01216250, SMSE 0.0109, MSLL -1.7755
BCM (Dc size 150): MSE 0.01276918, SMSE 0.0115, MSLL -2.1903
PoE (Dc size 150): MSE 0.10103713, SMSE 0.0908, MSLL 4.5045
GPoE (Dc size 150): MSE 0.01213032, SMSE 0.0109, MSLL -1.7646
processing distance: 100/10000
processing distance: 200/10000
processing distance: 300/10000
```

```
processing distance: 400/10000
processing distance: 500/10000
processing distance: 600/10000
processing distance: 700/10000
processing distance: 800/10000
processing distance: 900/10000
processing distance: 1000/10000
processing distance: 1100/10000
processing distance: 1200/10000
processing distance: 1300/10000
processing distance: 1400/10000
processing distance: 1500/10000
processing distance: 1600/10000
processing distance: 1700/10000
processing distance: 1800/10000
processing distance: 1900/10000
processing distance: 2000/10000
processing distance: 2100/10000
processing distance: 2200/10000
processing distance: 2300/10000
processing distance: 2400/10000
processing distance: 2500/10000
processing distance: 2600/10000
processing distance: 2700/10000
processing distance: 2800/10000
processing distance: 2900/10000
processing distance: 3000/10000
processing distance: 3100/10000
processing distance: 3200/10000
processing distance: 3300/10000
processing distance: 3400/10000
processing distance: 3500/10000
processing distance: 3600/10000
processing distance: 3700/10000
processing distance: 3800/10000
processing distance: 3900/10000
processing distance: 4000/10000
processing distance: 4100/10000
processing distance: 4200/10000
processing distance: 4300/10000
processing distance: 4400/10000
processing distance: 4500/10000
processing distance: 4600/10000
processing distance: 4700/10000
processing distance: 4800/10000
processing distance: 4900/10000
processing distance: 5000/10000
processing distance: 5100/10000
processing distance: 5200/10000
processing distance: 5300/10000
processing distance: 5400/10000
processing distance: 5500/10000
processing distance: 5600/10000
processing distance: 5700/10000
processing distance: 5800/10000
processing distance: 5900/10000
processing distance: 6000/10000
processing distance: 6100/10000
processing distance: 6200/10000
processing distance: 6300/10000
processing distance: 6400/10000
processing distance: 6500/10000
processing distance: 6600/10000
processing distance: 6700/10000
processing distance: 6800/10000
```

```
processing distance: 6900/10000
processing distance: 7000/10000
processing distance: 7100/10000
processing distance: 7200/10000
processing distance: 7300/10000
processing distance: 7400/10000
processing distance: 7500/10000
processing distance: 7600/10000
processing distance: 7700/10000
processing distance: 7800/10000
processing distance: 7900/10000
processing distance: 8000/10000
processing distance: 8100/10000
processing distance: 8200/10000
processing distance: 8300/10000
processing distance: 8400/10000
processing distance: 8500/10000
processing distance: 8600/10000
processing distance: 8700/10000
processing distance: 8800/10000
processing distance: 8900/10000
processing distance: 9000/10000
processing distance: 9100/10000
processing distance: 9200/10000
processing distance: 9300/10000
processing distance: 9400/10000
processing distance: 9500/10000
processing distance: 9600/10000
processing distance: 9700/10000
processing distance: 9800/10000
processing distance: 9900/10000
processing distance: 10000/10000
kj = 1
gr = 0.3000
GRBCM (Dc size 150): MSE 0.03415921, SMSE 0.0307, MSLL -1.8041
GRBCM++ (VFE) (Dc size 150):
MSE 0.02296870, SMSE 0.0206, MSLL -1.9197
GRBCM++ (SPGP) (Dc size 150):
MSE 0.02438858, SMSE 0.0219, MSLL -1.9110
kj = 2
gr = 0.3500
GRBCM (Dc size 150): MSE 0.02738856, SMSE 0.0246, MSLL -1.8672
GRBCM++ (VFE) (Dc size 150):
MSE 0.02116914, SMSE 0.0190, MSLL -1.9676
GRBCM++ (SPGP) (Dc size 150):
MSE 0.02061068, SMSE 0.0185, MSLL -1.9654
kj = 3
gr = 0.4000
GRBCM (Dc size 150): MSE 0.02218323, SMSE 0.0199, MSLL -1.9351
GRBCM++ (VFE) (Dc size 150):
MSE 0.01789279, SMSE 0.0161, MSLL -2.0312
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01797282, SMSE 0.0161, MSLL -2.0158
kj = 4
gr = 0.4500
GRBCM (Dc size 150): MSE 0.01913175, SMSE 0.0172, MSLL -1.9951
GRBCM++ (VFE) (Dc size 150):
MSE 0.01596187, SMSE 0.0143, MSLL -2.0860
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01610888, SMSE 0.0145, MSLL -2.0703
kj = 5
gr = 0.5000
GRBCM (Dc size 150): MSE 0.01657046, SMSE 0.0149, MSLL -2.0512
GRBCM++ (VFE) (Dc size 150):
MSE 0.01492240, SMSE 0.0134, MSLL -2.1203
```

GRBCM++ (SPGP) (Dc size 150):
MSE 0.01459078, SMSE 0.0131, MSLL -2.1116
kj = 6
gr = 0.5500
GRBCM (Dc size 150): MSE 0.01503855, SMSE 0.0135, MSLL -2.0863
GRBCM++ (VFE) (Dc size 150):
MSE 0.01369683, SMSE 0.0123, MSLL -2.1596
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01353569, SMSE 0.0122, MSLL -2.1418
kj = 7
gr = 0.6000
GRBCM (Dc size 150): MSE 0.01440895, SMSE 0.0129, MSLL -2.0993
GRBCM++ (VFE) (Dc size 150):
MSE 0.01376001, SMSE 0.0124, MSLL -2.1532
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01334786, SMSE 0.0120, MSLL -2.1403
kj = 8
gr = 0.6500
GRBCM (Dc size 150): MSE 0.01368246, SMSE 0.0123, MSLL -2.1045
GRBCM++ (VFE) (Dc size 150):
MSE 0.01310295, SMSE 0.0118, MSLL -2.1602
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01301808, SMSE 0.0117, MSLL -2.1391
kj = 9
gr = 0.7000
GRBCM (Dc size 150): MSE 0.01318951, SMSE 0.0118, MSLL -2.1056
GRBCM++ (VFE) (Dc size 150):
MSE 0.01275745, SMSE 0.0115, MSLL -2.1545
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01271688, SMSE 0.0114, MSLL -2.1341
kj = 10
gr = 0.7500
GRBCM (Dc size 150): MSE 0.01298532, SMSE 0.0117, MSLL -2.0971
GRBCM++ (VFE) (Dc size 150):
MSE 0.01255840, SMSE 0.0113, MSLL -2.1490
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01250969, SMSE 0.0112, MSLL -2.1292
kj = 11
gr = 0.8000
GRBCM (Dc size 150): MSE 0.01232110, SMSE 0.0111, MSLL -2.1174
GRBCM++ (VFE) (Dc size 150):
MSE 0.01208703, SMSE 0.0109, MSLL -2.1587
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01194638, SMSE 0.0107, MSLL -2.1456
kj = 12
gr = 0.8500
GRBCM (Dc size 150): MSE 0.01218807, SMSE 0.0109, MSLL -2.0984
GRBCM++ (VFE) (Dc size 150):
MSE 0.01194362, SMSE 0.0107, MSLL -2.1421
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01183894, SMSE 0.0106, MSLL -2.1295
kj = 13
gr = 0.9000
GRBCM (Dc size 150): MSE 0.01197369, SMSE 0.0108, MSLL -2.0898
GRBCM++ (VFE) (Dc size 150):
MSE 0.01177533, SMSE 0.0106, MSLL -2.1357
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01168794, SMSE 0.0105, MSLL -2.1221
kj = 14
gr = 0.9500
GRBCM (Dc size 150): MSE 0.01192180, SMSE 0.0107, MSLL -2.0675
GRBCM++ (VFE) (Dc size 150):
MSE 0.01175136, SMSE 0.0106, MSLL -2.1179
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01164143, SMSE 0.0105, MSLL -2.1036

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```
kj = 15
gr = 1
GRBCM (Dc size 150): MSE 0.01195039, SMSE 0.0107, MSLL -2.0396
GRBCM++ (VFE) (Dc size 150):
MSE 0.01176017, SMSE 0.0106, MSLL -2.1008
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01166815, SMSE 0.0105, MSLL -2.0817
```

Display results

```
grbcm0_smse = mean(grbcm0_smse_rec(1:kti)); grbcm0_msll = mean(grbcm0_msll_rec(1:kti));
rbcm0_smse = mean(rbcm0_smse_rec(1:kti)); rbcm0_msll = mean(rbcm0_msll_rec(1:kti));
bcm0_smse = mean(bcm0_smse_rec(1:kti)); bcm0_msll = mean(bcm0_msll_rec(1:kti));
poe0_smse = mean(poe0_smse_rec(1:kti)); poe0_msll = mean(poe0_msll_rec(1:kti));
gpoe0_smse = mean(gpoe0_smse_rec(1:kti)); gpoe0_msll = mean(gpoe0_msll_rec(1:kti));
vfe0_smse = mean(vfe0_smse_rec(1:kti)); vfe0_msll = mean(vfe0_msll_rec(1:kti));
spgp0_smse = mean(spgp0_smse_rec(1:kti)); spgp0_msll = mean(spgp0_msll_rec(1:kti));
```

```
fprintf('GRBCM: %6.8f, %6.4f', grbcm0_smse, grbcm0_msll);
```

GRBCM: 0.01074062, -2.0286

```
fprintf('RBCM: %6.8f, %6.4f', rbcm0_smse, rbcm0_msll);
```

RBCM: 0.01083843, -1.7868

```
fprintf('BCM: %6.8f, %6.4f', bcm0_smse, bcm0_msll);
```

BCM: 0.01122129, -2.2072

```
fprintf('GPoE: %6.8f, %6.4f', gpoe0_smse, gpoe0_msll);
```

GPoE: 0.01081409, -1.7767

```
fprintf('PoE: %6.8f, %6.4f', poe0_smse, poe0_msll);
```

PoE: 0.08911107, 4.3245

```
fprintf('VFE: %6.8f, %6.4f', vfe0_smse, vfe0_msll);
```

VFE: 0.08217963, -1.1010

```
fprintf('SPGP: %6.8f, %6.4f', spgp0_smse, spgp0_msll);
```

SPGP: 0.14950040, -1.0956

```
aamse = grbcm2_gr_smse(1:kti,:);
aamsll = grbcm2_gr_msll(1:kti,:);
mmse = mean(aamse);
mstd = std(aamse);
mmse_ro = mean(grbcm_gr_smse(1:kti,:));
mstd_ro = std(grbcm_gr_smse(1:kti,:));
mmse_sp = mean(grbcm2_spgp_gr_smse(1:kti,:));
mstd_sp = std(grbcm2_spgp_gr_smse(1:kti,:));
```

```
figure; hold on;
plot([min(grls), max(grls)], [grbcm0_smse,grbcm0_smse]);
plot([min(grls), max(grls)], [vfeSMSE_bl,vfeSMSE_bl]);
plot([min(grls), max(grls)], [spgpSMSE_bl,spgpSMSE_bl]);
errorbar(grls, mmse, mstd);
errorbar(grls, mmse_sp, mstd_sp);
errorbar(grls, mmse_ro, mstd_ro);
legend('GRBCM', 'VFE', 'SPGP', 'H-GPR (VFE)', 'H-GPR (SPGP)', 'GRBCM (only remove)');
xlabel('remain\_r'); ylabel('SMSE');
```



```
aamse = grbcm2_gr_smse(1:kti,:);
aamsll = grbcm2_gr_msll(1:kti,:);
mmse = mean(aamse);
mstd = std(aamse);
mmse_ro = mean(grbcm_gr_smse(1:kti,:));
mstd_ro = std(grbcm_gr_smse(1:kti,:));
mmse_sp = mean(grbcm2_spgp_gr_smse(1:kti,:));
mstd_sp = std(grbcm2_spgp_gr_smse(1:kti,:));

figure; hold on;
plot([min(grls), max(grls)], [grbcm0_smse,grbcm0_smse]);
plot([min(grls), max(grls)], [vfeSMSE_bl,vfeSMSE_bl]);
plot([min(grls), max(grls)], [spgpSMSE_bl,spgpSMSE_bl]);
errorbar(grls, mmse, mstd);
errorbar(grls, mmse_sp, mstd_sp);
errorbar(grls, mmse_ro, mstd_ro);
```

```
legend('GRBCM', 'VFE', 'SPGP', 'H-GPR (VFE)', 'H-GPR (SPGP)', 'GRBCM (only remove)');
xlabel('remain\_r'); ylabel('SMSE');
ylim([0.01, 0.016]);
```
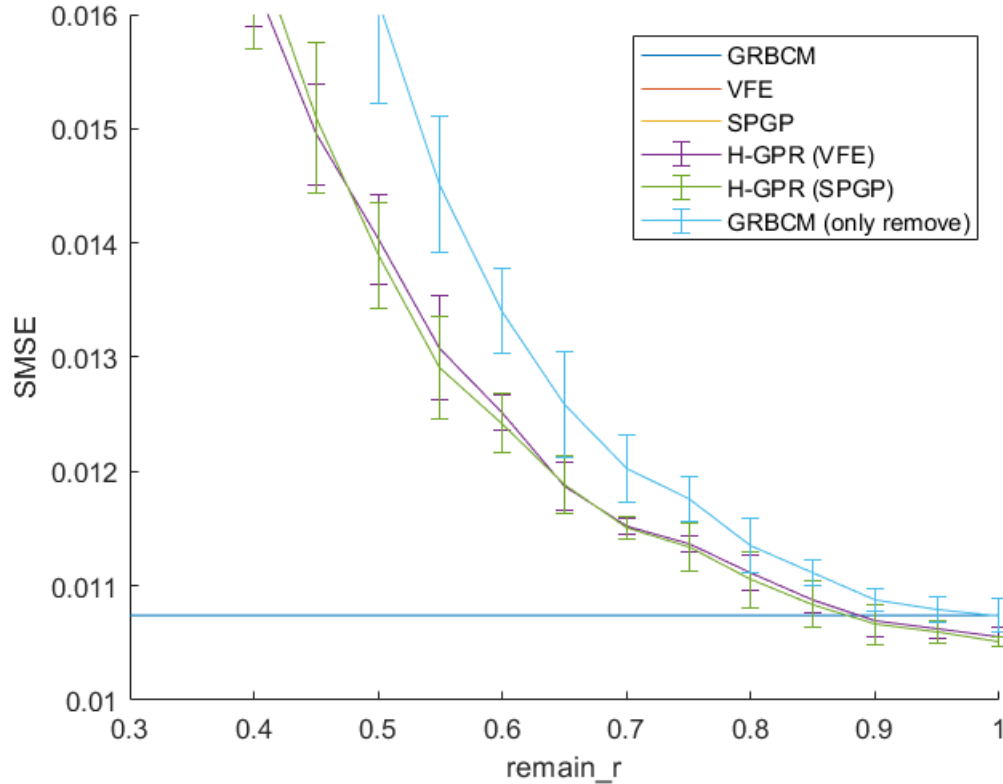


```
mmsll = mean(aamsll);
mstd = std(aamsll);
mmsll_ro = mean(grbcm_gr_msll(1:kti,:));
mstd_ro = std(grbcm_gr_msll(1:kti,:));
mmsll_sp = mean(grbcm2_spgp_gr_msll(1:kti,:));
mstd_sp = std(grbcm2_spgp_gr_msll(1:kti,:));
figure; hold on;
plot([min(grls), max(grls)], [grbcm0_msll,grbcm0_msll]);
plot([min(grls), max(grls)], [vfeMSLL_bl,vfeMSLL_bl]);
plot([min(grls), max(grls)], [spgpMSLL_bl,spgpMSLL_bl]);
errorbar(grls, mmsll, mstd);
errorbar(grls, mmsll_sp, mstd_sp);
errorbar(grls, mmsll_ro, mstd_ro);
legend('GRBCM', 'VFE', 'SPGP', 'H-GPR (VFE)', 'H-GPR (SPGP)', 'GRBCM (only remove)');
xlabel('remain\_r'); ylabel('MSLL');
ylim([-2.3, -0.4]);
```
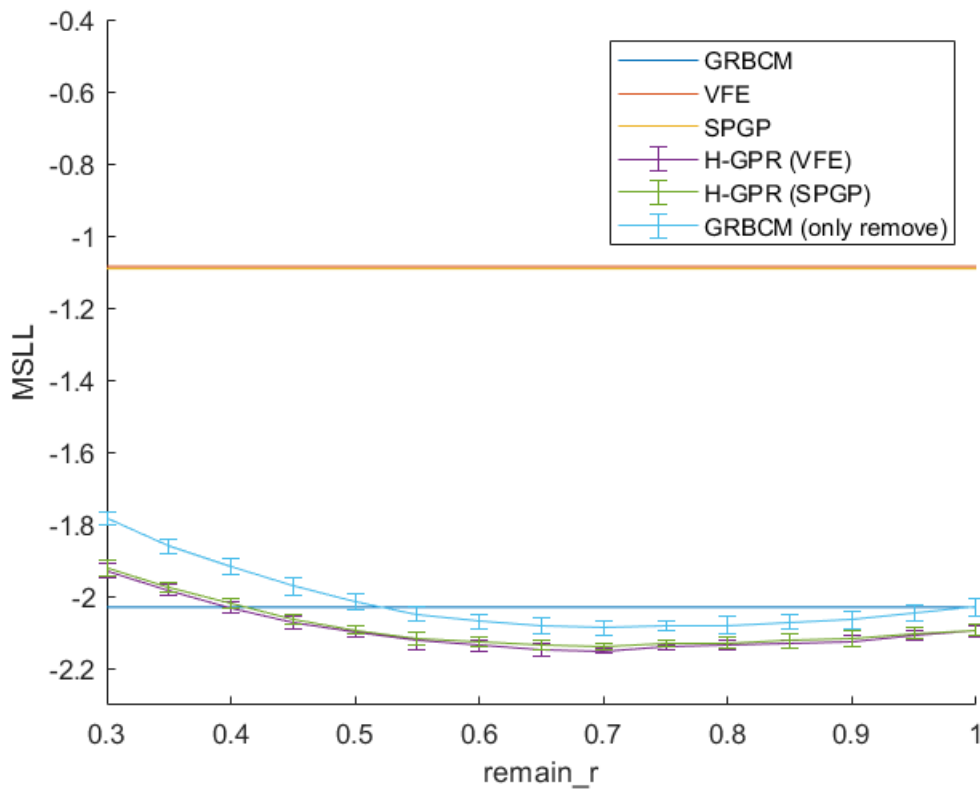
```
fprintf('Best SMSE (GRBCM+VFE, dcs %d, ecs %d): %6.8f\n', dcs, ecs, min(mmse));
```

Best SMSE (GRBCM+VFE, dcs 150, ecs 150): 0.01055557

```
fprintf('Best MSLL (GRBCM+VFE, dcs %d, ecs %d): %6.8f\n', dcs, ecs, min(mmsll));
```

Best MSLL (GRBCM+VFE, dcs 150, ecs 150): -2.14919562

```
fprintf('Best SMSE (GRBCM+SPGP, dcs %d, ecs %d): %6.8f\n', dcs, ecs, min(mmse_sp));
```

Best SMSE (GRBCM+SPGP, dcs 150, ecs 150): 0.01051441

```
fprintf('Best MSLL (GRBCM+SPGP, dcs %d, ecs %d): %6.8f\n', dcs, ecs, min(mmsll_sp));
```

Best MSLL (GRBCM+SPGP, dcs 150, ecs 150): -2.13733784

Experiment II: re-balancing sizes of dcs and ecs

```
% hyp.cov = log([ones(d,1)*ell;sf2]); hyp.lik = log(sn2); hyp.mean = [];
opts.numOptFC = 50 ;
opts.xvec = xvec;
opts.yvec = yvec;
opts.grbcm_baseline = 0;
opts.global_index = ones(n,1);
% opts.inffunc = @infGaussLik; opts.meanfunc = meanfunc; opts.likfunc = likfunc;
```

```
opts.covfunc = covfunc;
covfuncF = {@apxSparse, {opts.covfunc}, []};
opts.covfuncF = covfuncF;
opts.compute_hyp = 0;
```

Heuristically rebalancing GRBCM and VFE/SPGP budget based on validating performance.

Obtain results on the validation set.

```
% default partition
dcs_ecs_r = 0.5;
dcs = round(ttcs*dcs_ecs_r) % size of the communication set
```

```
dcs = 150
```

```
ecs = ttcs - dcs % size of other experts
```

```
ecs = 150
```

```
n_per = dcs ; % size of Dc
mn = round(n / ecs); % mn is the number of experts (normal)
Indics = randperm(n) ;
I_com = Indics(1:n_per) ; % randomly select communication set
[idx, C] = kmeans(xvec, mn,'MaxIter',km_iters);
```

```
% hyp.cov = log([ones(d,1)*ell;sf2]); hyp.lik = log(sn2); hyp.mean = [];
opts.numOptFC = 30 ;
opts.Ms = mn+1;
opts.xvec = xvec;
opts.yvec = yvec;
opts.induce_size = dcs;
opts.grbcm_baseline = 0;
opts.global_index = ones(n,1);
opts.I_com = I_com;
% opts.inffunc = @infGaussLik; opts.meanfunc = meanfunc; opts.likfunc = likfunc;
opts.covfunc = covfunc;
covfuncF = {@apxSparse, {opts.covfunc}, xvec(I_com,:)};
opts.covfuncF = covfuncF;
opts.compute_hyp = 0;
```

```
g_opts = opts;
g_opts.compute_hyp = 1;
g_opts.grbcm_baseline = 1;
g_opts.global_index = ones(n,1);
g_models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,g_opts);
```

```
Optimizing hyps in training...
Linesearch      0;  Value 1.520749e+04
Linesearch      1;  Value 9.617101e+03
Linesearch      2;  Value 8.546503e+02
```

```
Linesearch      3;  Value 3.267713e+02
Linesearch      4;  Value -4.974412e+03
Linesearch      5;  Value -5.207574e+03
Linesearch      6;  Value -5.296999e+03
Linesearch      7;  Value -5.452391e+03
Linesearch      8;  Value -5.834078e+03
Linesearch      9;  Value -5.895270e+03
Linesearch     10;  Value -5.900734e+03
Linesearch     11;  Value -5.900877e+03
Linesearch     12;  Value -5.901010e+03
Linesearch     13;  Value -5.901030e+03
Linesearch     14;  Value -5.901030e+03
Linesearch     15;  Value -5.901030e+03
Linesearch     16;  Value -5.901030e+03
Linesearch     17;  Value -5.901030e+03
Linesearch     18;  Value -5.901030e+03
Linesearch     19;  Value -5.901030e+03
Linesearch     20;  Value -5.901030e+03
```

```
opts.hyp = g_models{1}.hyp;
g_opts.hyp = g_models{1}.hyp;
[tmu,ts2, ~] =  aggregation_predict(xvec_val,g_models,'GRBCM', 1, g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[grbcmMSE,grbcmSMSE,grbcmMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_val, ori_yvec_val, tmu,
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'GRBCM', n_per, grbcmMSE,grbc
```

```
GRBCM (Dc size 150): MSE 0.01164162, SMSE 0.01061083, MSLL -2.05446337
```

```
g_opts.compute_hyp = 0;
```

```
% % VFE Baseline
vfe_opts = opts;
vfe_opts.induce_type = 'VFE_opt';
xu = xvec(I_com, :);
inffunc = @(varargin) infGaussLik(varargin{:}, struct('s', 0.0));
vfe_hyp = opts.hyp;
vfe_hyp.xu = xu;
[vfe_hyp, tmp_nlzs] = minimize(vfe_hyp,@sp_gp,-vfe_opts.induce_step,inffunc,meanfunc,covfuncF,]
```

```
Function evaluation     0;  Value 4.717579e+05
Function evaluation    13;  Value 4.420991e+05
Function evaluation    14;  Value 4.014932e+05
Function evaluation    15;  Value 3.573228e+05
Function evaluation    17;  Value 3.345072e+05
Function evaluation    19;  Value 3.134580e+05
Function evaluation    21;  Value 2.867571e+05
Function evaluation    23;  Value 2.652266e+05
Function evaluation    25;  Value 2.514846e+05
Function evaluation    27;  Value 2.389698e+05
Function evaluation    29;  Value 2.304899e+05
Function evaluation    31;  Value 2.234305e+05
Function evaluation    33;  Value 2.185293e+05
Function evaluation    35;  Value 2.144861e+05
Function evaluation    37;  Value 2.114519e+05
Function evaluation    39;  Value 2.076748e+05
Function evaluation    41;  Value 2.050223e+05
```

```
Function evaluation      43;  Value 2.029074e+05
Function evaluation      45;  Value 2.010522e+05
Function evaluation      46;  Value 1.993414e+05
Function evaluation      47;  Value 1.976699e+05
Function evaluation      49;  Value 1.954688e+05
Function evaluation      51;  Value 1.929131e+05
Function evaluation      53;  Value 1.904211e+05
Function evaluation      54;  Value 1.881514e+05
Function evaluation      56;  Value 1.864938e+05
Function evaluation      58;  Value 1.850230e+05
Function evaluation      60;  Value 1.842018e+05
Function evaluation      62;  Value 1.834750e+05
Function evaluation      64;  Value 1.829942e+05
Function evaluation      66;  Value 1.823637e+05
Function evaluation      68;  Value 1.819990e+05
Function evaluation      70;  Value 1.816677e+05
Function evaluation      71;  Value 1.813273e+05
Function evaluation      73;  Value 1.810636e+05
Function evaluation      74;  Value 1.808105e+05
Function evaluation      76;  Value 1.806249e+05
Function evaluation      77;  Value 1.804450e+05
Function evaluation      79;  Value 1.803137e+05
Function evaluation      80;  Value 1.801735e+05
Function evaluation      81;  Value 1.800476e+05
Function evaluation      83;  Value 1.799324e+05
Function evaluation      85;  Value 1.798594e+05
Function evaluation      87;  Value 1.797736e+05
Function evaluation      89;  Value 1.796985e+05
Function evaluation      91;  Value 1.796322e+05
Function evaluation      93;  Value 1.795559e+05
Function evaluation      94;  Value 1.794864e+05
Function evaluation      95;  Value 1.794071e+05
Function evaluation      97;  Value 1.793496e+05
Function evaluation      99;  Value 1.792689e+05
```

```matlab
vfe_opts.hyp = opts.hyp;
vfe_opts.xu = vfe_hyp.xu;
vfe_opts.inffunc = @infGaussLik; vfe_opts.meanfunc = meanfunc; vfe_opts.covfuncF = covfuncF; vf
vfe_opts.covfunc = covfunc;
[tmu, ts2] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, xvec_val);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[vfeMSE,vfeSMSE,vfeMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_val, ori_yvec_val, tmu, ts2);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'VFE', n_per, vfeMSE,vfeSMSE,
```

```
VFE (Dc size 150): MSE 0.06615567, SMSE 0.06029797, MSLL -1.12535485
```

```matlab
% vfe0_smse_rec(ki) = vfeSMSE; vfe0_msll_rec(ki) = vfeMSLL;
[yu, su] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, vfe_opts.xu);
vfe_opts.yu = yu; vfe_opts.su = su;
```

```matlab
models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,vfe_opts); % use hyp of vfe
[tmu,ts2] =  aggregation_predict_GRBCM_VS_apx(xvec_val,models,vfe_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
```

```
[MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_val, ori_yvec_val, tmu, ts2);
fprintf('%s (Dc size %d): \r\nMSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM++ (VFE)', n_per, M
```

```
GRBCM++ (VFE) (Dc size 150):
MSE 0.01124902, SMSE 0.0103, MSLL -2.1378
```

```
sig_temp = 10;
dcs_ecs_r = sigmoid(-MSLL+grbcmMSLL, sig_temp)
```

```
dcs_ecs_r = 0.6971
```

```
dcs = round(ttcs*dcs_ecs_r)
```

```
dcs = 209
```

```
ecs = ttcs - dcs
```

```
ecs = 91
```

```
m = round(n / ecs)
```

```
m = 110
```

```
n_per = dcs ; % size of Dc
Indics = randperm(n) ;
I_com = Indics(1:n_per) ; % randomly select communication set
[idx, C] = kmeans(xvec, m,'MaxIter',km_iters);

% hyp.cov = log([ones(d,1)*ell;sf2]); hyp.lik = log(sn2); hyp.mean = [];
opts.numOptFC = 30 ;
opts.Ms = m+1;
opts.xvec = xvec;
opts.yvec = yvec;
opts.induce_size = dcs;
opts.grbcm_baseline = 0;
opts.global_index = ones(n,1);
opts.I_com = I_com;
% opts.inffunc = @infGaussLik; opts.meanfunc = meanfunc; opts.likfunc = likfunc;
opts.covfunc = covfunc;
covfuncF = {@apxSparse, {opts.covfunc}, xvec(I_com,:)};
opts.covfuncF = covfuncF;
opts.compute_hyp = 0;
```

```
g_opts = opts;
g_opts.compute_hyp = 0;
g_opts.grbcm_baseline = 1;
g_opts.global_index = ones(n,1);
g_models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,g_opts);
opts.hyp = g_models{1}.hyp;
g_opts.hyp = g_models{1}.hyp;
[tmu,ts2, ~] =  aggregation_predict(xvec_test,g_models,'GRBCM', 1, g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
```

```
        ts2 = ts2 * norm_fstd^2;
end
[grbcmMSE,grbcmSMSE,grbcmMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tm
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'GRBCM', n_per, grbcmMSE,grbc
```

```
GRBCM (Dc size 209): MSE 0.01194803, SMSE 0.01073429, MSLL -2.09346765
```

```
% grbcm0_smse_rec(ki) = grbcmSMSE; grbcm0_msll_rec(ki) = grbcmMSLL;
g_opts.compute_hyp = 0;
```

```
% % VFE Baseline
vfe_opts = opts;
vfe_opts.induce_type = 'VFE_opt';
xu = xvec(I_com, :);
vfe_hyp = opts.hyp;
vfe_hyp.xu = xu;
vfe_hyp = minimize(vfe_hyp,@sp_gp,-vfe_opts.induce_step,inffunc,meanfunc,covfuncF,likfunc,xvec,
```

```
Function evaluation      0;  Value 2.642170e+05
Function evaluation     12;  Value 2.425455e+05
Function evaluation     14;  Value 2.065878e+05
Function evaluation     16;  Value 1.563441e+05
Function evaluation     18;  Value 1.299826e+05
Function evaluation     20;  Value 1.139659e+05
Function evaluation     22;  Value 1.030235e+05
Function evaluation     24;  Value 9.410919e+04
Function evaluation     26;  Value 8.747297e+04
Function evaluation     28;  Value 8.260454e+04
Function evaluation     30;  Value 7.899434e+04
Function evaluation     32;  Value 7.659052e+04
Function evaluation     34;  Value 7.453704e+04
Function evaluation     36;  Value 7.285295e+04
Function evaluation     38;  Value 7.172924e+04
Function evaluation     40;  Value 7.082931e+04
Function evaluation     42;  Value 7.007815e+04
Function evaluation     43;  Value 6.934832e+04
Function evaluation     45;  Value 6.882614e+04
Function evaluation     46;  Value 6.835123e+04
Function evaluation     48;  Value 6.805647e+04
Function evaluation     49;  Value 6.774481e+04
Function evaluation     50;  Value 6.743817e+04
Function evaluation     51;  Value 6.712339e+04
Function evaluation     52;  Value 6.681410e+04
Function evaluation     53;  Value 6.651699e+04
Function evaluation     55;  Value 6.629188e+04
Function evaluation     56;  Value 6.607126e+04
Function evaluation     58;  Value 6.589944e+04
Function evaluation     60;  Value 6.576589e+04
Function evaluation     62;  Value 6.566135e+04
Function evaluation     64;  Value 6.557802e+04
Function evaluation     66;  Value 6.551073e+04
Function evaluation     68;  Value 6.545585e+04
Function evaluation     69;  Value 6.540614e+04
Function evaluation     71;  Value 6.537231e+04
Function evaluation     73;  Value 6.532066e+04
Function evaluation     75;  Value 6.528097e+04
Function evaluation     76;  Value 6.524283e+04
Function evaluation     77;  Value 6.520691e+04
Function evaluation     79;  Value 6.517829e+04
Function evaluation     81;  Value 6.514292e+04
Function evaluation     82;  Value 6.510817e+04
```

```
Function evaluation      83;  Value 6.507332e+04
Function evaluation      84;  Value 6.503556e+04
Function evaluation      86;  Value 6.499435e+04
Function evaluation      87;  Value 6.495654e+04
Function evaluation      89;  Value 6.492228e+04
Function evaluation      91;  Value 6.487643e+04
Function evaluation      93;  Value 6.483567e+04
Function evaluation      94;  Value 6.479456e+04
Function evaluation      96;  Value 6.475743e+04
Function evaluation      97;  Value 6.471720e+04
Function evaluation      99;  Value 6.469000e+04
Function evaluation     100;  Value 6.466097e+04
```

```
vfe_opts.hyp = opts.hyp;
vfe_opts.xu = vfe_hyp.xu;
vfe_opts.inffunc = @infGaussLik; vfe_opts.meanfunc = meanfunc; vfe_opts.covfuncF = covfuncF; vf
vfe_opts.covfunc = covfunc;
[tmu, ts2] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, xvec_test);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[vfeMSE,vfeSMSE,vfeMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'VFE baseline', n_per, vfeMSE
```

```
VFE baseline (Dc size 209): MSE 0.02818299, SMSE 0.02532002, MSLL -1.49612437
```

```
% vfe0_smse_rec(ki) = vfeSMSE; vfe0_msll_rec(ki) = vfeMSLL;
[yu, su] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, vfe_opts.xu);
vfe_opts.yu = yu; vfe_opts.su = su;
```

```
sp_opts = opts;
sp_opts.induce_type = 'SPGP_opt';
hyp_init(1:d,1) = -2*opts.hyp.cov(1:d);
hyp_init(d+1,1) = 2*opts.hyp.cov(d+1);
hyp_init(d+2,1) = 2*opts.hyp.lik;

sp_opts.induce_size = dcs;
xu = xvec(I_com, :);
w_init = [reshape(xu,sp_opts.induce_size*d,1);hyp_init];
[w,f] = minimize(w_init,'spgp_lik_nohyp',-sp_opts.induce_step,yvec,xvec,sp_opts.induce_size);
```

```
Function evaluation       0;  Value 5.427874e+03
Function evaluation      10;  Value 4.657592e+03
Function evaluation      11;  Value 4.006937e+03
Function evaluation      12;  Value 2.988510e+03
Function evaluation      14;  Value 2.314853e+03
Function evaluation      16;  Value 1.764070e+03
Function evaluation      17;  Value 1.471049e+03
Function evaluation      19;  Value 1.174218e+03
Function evaluation      21;  Value 1.045482e+03
Function evaluation      23;  Value 8.514789e+02
Function evaluation      25;  Value 6.989717e+02
Function evaluation      26;  Value 5.481385e+02
Function evaluation      28;  Value 4.225518e+02
Function evaluation      30;  Value 3.122134e+02
Function evaluation      31;  Value 1.908940e+02
Function evaluation      33;  Value 1.008356e+02
Function evaluation      34;  Value 1.155951e+01
```

```
Function evaluation     36;  Value -5.192299e+01
Function evaluation     38;  Value -1.029726e+02
Function evaluation     39;  Value -1.544956e+02
Function evaluation     41;  Value -1.987702e+02
Function evaluation     43;  Value -2.334568e+02
Function evaluation     44;  Value -2.713714e+02
Function evaluation     46;  Value -2.968104e+02
Function evaluation     48;  Value -3.189461e+02
Function evaluation     49;  Value -3.411076e+02
Function evaluation     51;  Value -3.548797e+02
Function evaluation     52;  Value -3.678048e+02
Function evaluation     54;  Value -3.774485e+02
Function evaluation     56;  Value -3.884200e+02
Function evaluation     58;  Value -3.969647e+02
Function evaluation     60;  Value -4.071284e+02
Function evaluation     62;  Value -4.146471e+02
Function evaluation     63;  Value -4.223360e+02
Function evaluation     65;  Value -4.290719e+02
Function evaluation     67;  Value -4.366761e+02
Function evaluation     69;  Value -4.425793e+02
Function evaluation     71;  Value -4.474680e+02
Function evaluation     72;  Value -4.527923e+02
Function evaluation     74;  Value -4.584574e+02
Function evaluation     76;  Value -4.622254e+02
Function evaluation     77;  Value -4.662047e+02
Function evaluation     78;  Value -4.703279e+02
Function evaluation     80;  Value -4.732622e+02
Function evaluation     82;  Value -4.770642e+02
Function evaluation     83;  Value -4.808539e+02
Function evaluation     84;  Value -4.849466e+02
Function evaluation     85;  Value -4.885176e+02
Function evaluation     87;  Value -4.914513e+02
Function evaluation     88;  Value -4.941045e+02
Function evaluation     90;  Value -4.965238e+02
Function evaluation     91;  Value -4.990347e+02
Function evaluation     93;  Value -5.023620e+02
Function evaluation     95;  Value -5.047941e+02
Function evaluation     97;  Value -5.066537e+02
Function evaluation     99;  Value -5.087829e+02
```

```matlab
xb = reshape(w(1:sp_opts.induce_size*d,1),sp_opts.induce_size,d);
sp_opts.xu = xb;
sp_opts.sp_hyp = w(sp_opts.induce_size*d+1:end,1);
sp_opts.hyp = opts.hyp;

[tmu,ts2] = spgp_pred(sp_opts.yvec,sp_opts.xvec,sp_opts.xu,xvec_test,sp_opts.sp_hyp);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[spgpMSE,spgpSMSE,spgpMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu,
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'SPSG baseline', n_per, spgpM
```

```
SPSG baseline (Dc size 209): MSE 0.03907018, SMSE 0.03510123, MSLL -1.49856836
```

```matlab
% spgp0_smse_rec(ki) = spgpSMSE; spgp0_msll_rec(ki) = spgpMSLL;
[yu,su] = spgp_pred(sp_opts.yvec,sp_opts.xvec,sp_opts.xu,sp_opts.xu,sp_opts.sp_hyp);
sp_opts.yu = yu; sp_opts.su = su;
```

```matlab
[~, minidx] = min(mmsll);
```

```matlab
best_msll_gr = grls(minidx)
```

best_msll_gr = 0.7000

```matlab
[~, minidx] = min(mmse);
best_smse_gr = grls(minidx)
```

best_smse_gr = 1

```matlab
kti = 1;
rb_vfe_msll = zeros(kti, 1);
rb_vfe_smse = zeros(kti, 1);

rb_sp_msll = zeros(kti, 1);
rb_sp_smse = zeros(kti, 1);
```

```matlab
for ki=1:kti
    gr = best_msll_gr
    crk = rk;
    crk(I_com) = -1e10;
    [~, crk_idx] = sort(crk, 'descend');
    rn = round(n*gr);
    global_index = zeros(n,1);
    global_index(crk_idx(1:rn)) = 1; % select remaining data according to the importance

    vfe_opts.global_index = global_index;
    models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,vfe_opts); % use hyp of vfe
    [tmu,ts2] =  aggregation_predict_GRBCM_VS_apx(xvec_test,models,vfe_opts);
    if ynorm==1
        tmu = tmu * norm_fstd + norm_fmean;
        ts2 = ts2 * norm_fstd^2;
    end
    [MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
    fprintf('%s (Dc size %d): \r\nMSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM++ (VFE)', n_pe
    rb_vfe_msll(ki) = MSLL;

    sp_opts.global_index = global_index;
    models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,sp_opts); % use hyp of vfe
    [tmu,ts2] =  aggregation_predict_GRBCM_VS_apx(xvec_test,models,sp_opts);
    if ynorm==1
        tmu = tmu * norm_fstd + norm_fmean;
        ts2 = ts2 * norm_fstd^2;
    end
    [MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
    fprintf('%s (Dc size %d): \r\nMSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM++ (SPGP)', n_p
    rb_sp_msll(ki) = MSLL;


    %%%==========================================================================%%

    gr = best_smse_gr
    crk = rk;
```

```
    crk(I_com) = -1e10;
    [~, crk_idx] = sort(crk, 'descend');
    rn = round(n*gr);
    global_index = zeros(n,1);
    global_index(crk_idx(1:rn)) = 1; % select remaining data according to the importance

    vfe_opts.global_index = global_index;
    models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,vfe_opts); % use hyp of vfe
    [tmu,ts2] =  aggregation_predict_GRBCM_VS_apx(xvec_test,models,vfe_opts);
    if ynorm==1
        tmu = tmu * norm_fstd + norm_fmean;
        ts2 = ts2 * norm_fstd^2;
    end
    [MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
    fprintf('%s (Dc size %d): \r\nMSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM++ (VFE)', n_pe
    rb_vfe_smse(ki) = SMSE;

    sp_opts.global_index = global_index;
    models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,sp_opts); % use hyp of vfe
    [tmu,ts2] =  aggregation_predict_GRBCM_VS_apx(xvec_test,models,sp_opts);
    if ynorm==1
        tmu = tmu * norm_fstd + norm_fmean;
        ts2 = ts2 * norm_fstd^2;
    end
    [MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
    fprintf('%s (Dc size %d): \r\nMSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM++ (SPGP)', n_p
    rb_sp_smse(ki) = SMSE;
end
```

```
gr = 0.7000
GRBCM++ (VFE) (Dc size 209):
MSE 0.01210498, SMSE 0.0109, MSLL -2.1799
GRBCM++ (SPGP) (Dc size 209):
MSE 0.01218951, SMSE 0.0110, MSLL -2.1800
gr = 1
GRBCM++ (VFE) (Dc size 209):
MSE 0.01152754, SMSE 0.0104, MSLL -2.1667
GRBCM++ (SPGP) (Dc size 209):
MSE 0.01151470, SMSE 0.0103, MSLL -2.1839
```

```
fprintf('Best SMSE (GRBCM+VFE, dcs %d, ecs %d): %6.8f\n', dcs, ecs, mean(rb_vfe_smse));
```

```
Best SMSE (GRBCM+VFE, dcs 209, ecs 91): 0.01035652
```

```
fprintf('Best MSLL (GRBCM+VFE, dcs %d, ecs %d): %6.8f\n', dcs, ecs, mean(rb_vfe_msll));
```

```
Best MSLL (GRBCM+VFE, dcs 209, ecs 91): -2.17988576
```

```
fprintf('Best SMSE (GRBCM+SPGP, dcs %d, ecs %d): %6.8f\n', dcs, ecs, mean(rb_sp_smse));
```

```
Best SMSE (GRBCM+SPGP, dcs 209, ecs 91): 0.01034498
```

```
fprintf('Best MSLL (GRBCM+SPGP, dcs %d, ecs %d): %6.8f\n', dcs, ecs, mean(rb_sp_msll));
```

```
Best MSLL (GRBCM+SPGP, dcs 209, ecs 91): -2.17995699
```