

Generate artificial data

```
clear all;
```

```
x = -4:0.05:4;
```

```
y = -4:0.05:4;
```

```
rng(123456);
```

```
figure;
```

```
[X, Y] = meshgrid(x, y);
```

```
fkk = 4; % frequency of test function
```

```
freq=2*fkk; fk=1;
```

```
f1 = @(x,y) sin(freq*sqrt((x-fk).^2+y.^2));%./(freq*sqrt((x-fk).^2+y.^2));
```

```
freq=1*fkk; fk=-1;
```

```
f2 = @(x,y) -sin(freq*sqrt((x-fk).^2+y.^2));%./(freq*sqrt((x-fk).^2+y.^2));
```

```
freq = 0.1*fkk;
```

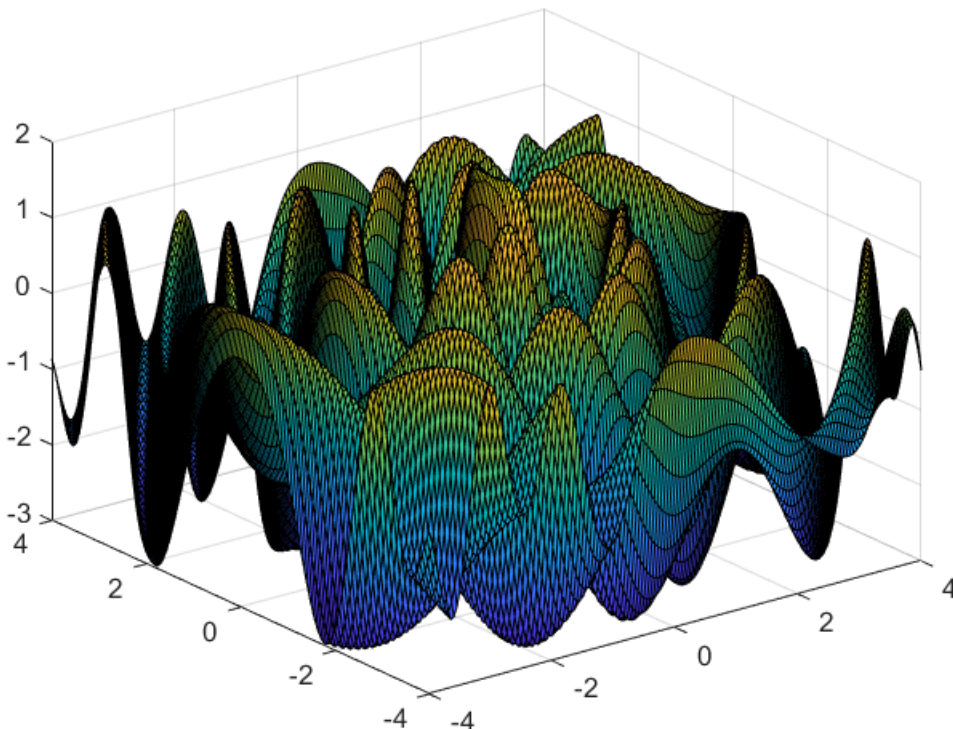
```
f3 = @(x,y) -sin(freq*sqrt(x.^2+y.^2));
```

```
ff = @(x,y) f1(x,y)+f2(x,y)+f3(x,y);
```

```
Z = ff(X, Y);
```

```
%Z = peaks(X, Y);
```

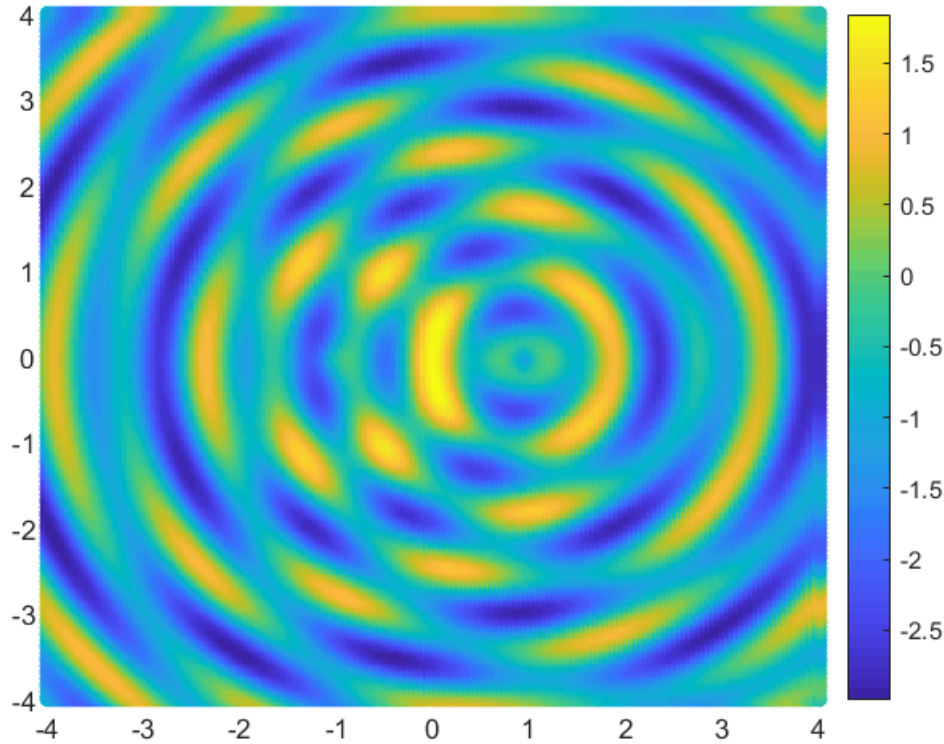
```
surf(X, Y, Z);
```



```
figure;
```

```
scatter(X(:), Y(:), [], Z(:), "filled");
```

```
colorbar;
```



```
xnorm = 1; ynorm = 1; % whether use normalization  
yns = 0.1; % noise scale
```

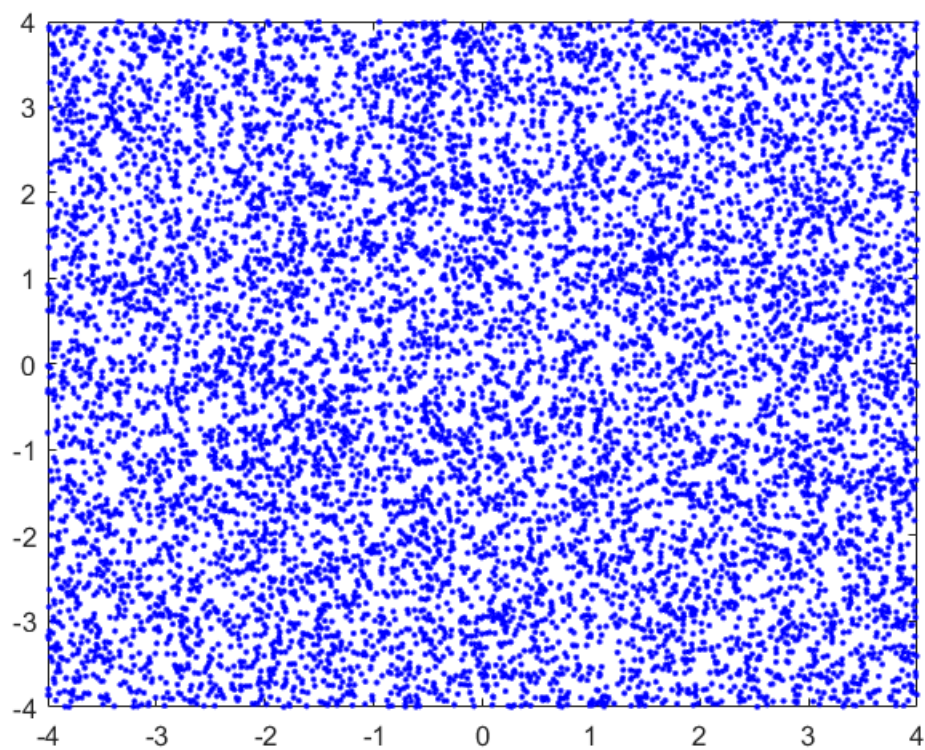
```
n = 1e4; % size of training data  
ttcs = 300;  
ttm = round(n / (ttcs/2));
```

```
% represent_ratio = 5;  
% dcs = 300; % size of the communication set  
% mn = round(n / dcs); % mn is the number of experts (normal)  
% m = round(n / (dcs*represent_ratio)); % m is the number of experts (sparse representative)  
% lamds = 0:0.5:3.0;
```

```
testsize = 2000; % size of test data  
valsize = 1000;  
induce_step = 100;
```

```
% training input  
ori_all_trainxs = 8*(rand(n, 1)-0.5);  
ori_all_trainys = 8*(rand(n, 1)-0.5);
```

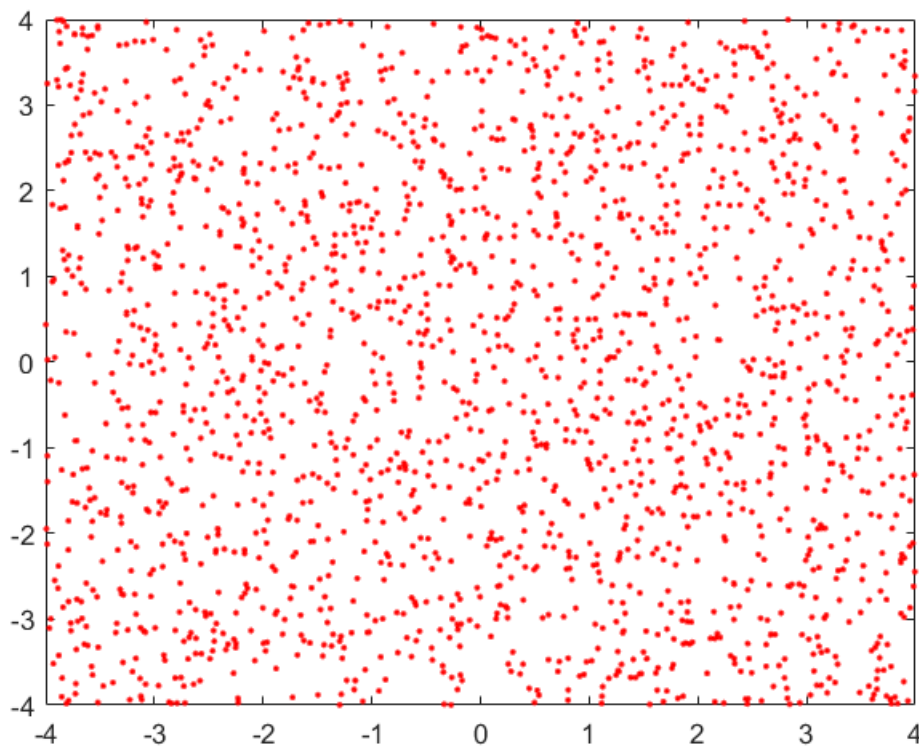
```
figure;  
plot(ori_all_trainxs, ori_all_trainys, 'b.');
```



```
% test input
ori_testxs = 8*(rand(testsize, 1)-0.5);
ori_testys = 8*(rand(testsize, 1)-0.5);

% validation input
ori_valxs = 8*(rand(valsize, 1)-0.5);
ori_valys = 8*(rand(valsize, 1)-0.5);

figure;
plot(ori_testxs, ori_testys, 'r.');
```



```
% input normalization
if xnorm == 1
    norm_xmean = mean(ori_all_trainxs);
    norm_xstd = std(ori_all_trainxs);
    all_trainxs = (ori_all_trainxs - norm_xmean) / norm_xstd;
    norm_ymean = mean(ori_all_trainys);
    norm_ystd = std(ori_all_trainys);
    all_trainys = (ori_all_trainys - norm_ymean) / norm_ystd;

    testxs = (ori_testxs - norm_xmean) / norm_xstd;
    testys = (ori_testys - norm_ymean) / norm_ystd;

    valxs = (ori_valxs - norm_xmean) / norm_xstd;
    valys = (ori_valys - norm_ymean) / norm_ystd;
else
    all_trainxs = ori_all_trainxs;
    all_trainys = ori_all_trainys;
    testxs = ori_testxs;
    testys = ori_testys;
    valxs = ori_valxs;
    valys = ori_valys;
end
```

```
% training output
ln = length(all_trainxs);
xvec = zeros(ln, 2); ori_xvec = zeros(ln, 2);
```

```

for i = 1:ln
    xvec(i, :) = [all_trainxs(i), all_trainys(i)];
    ori_xvec(i, :) = [ori_all_trainxs(i), ori_all_trainys(i)];
end
ori_yvec = ff(ori_all_trainxs, ori_all_trainys) + yns*randn(ln, 1);

% test output
ln = length(testxs);
xvec_test = zeros(ln, 2); ori_xvec_test = zeros(ln, 2);
for i = 1:ln
    xvec_test(i, :) = [testxs(i), testys(i)];
    ori_xvec_test(i, :) = [ori_testxs(i), ori_testys(i)];
end
ori_yvec_test = ff(ori_testxs, ori_testys) + yns*randn(ln, 1);

% validation output
ln = length(valxs);
xvec_val = zeros(ln, 2); ori_xvec_val = zeros(ln, 2);
for i = 1:ln
    xvec_val(i, :) = [valxs(i), valys(i)];
    ori_xvec_val(i, :) = [ori_valxs(i), ori_valys(i)];
end
ori_yvec_val = ff(ori_valxs, ori_valys) + yns*randn(ln, 1);

% output normalization
if ynorm == 1
    norm_fmean = mean(ori_yvec);
    norm_fstd = std(ori_yvec);
    all_trainxs = (ori_all_trainxs - norm_xmean) / norm_xstd;
    yvec = (ori_yvec - norm_fmean) / norm_fstd;
    yvec_test = (ori_yvec_test - norm_fmean) / norm_fstd;
    yvec_val = (ori_yvec_val - norm_fmean) / norm_fstd;
else
    yvec = ori_yvec;
    yvec_test = ori_yvec_test;
    yvec_val = ori_yvec_val;
end
end

```

store results

```

kti = 5; % average over kti runs
grls = 0.3:0.05:1.0; % percentage of remaining data to be test
grbcm0_smse_rec = zeros(kti, 1);
rbcm0_smse_rec = zeros(kti, 1);
bcm0_smse_rec = zeros(kti, 1);
gpoe0_smse_rec = zeros(kti, 1);
poe0_smse_rec = zeros(kti, 1);
vfe0_smse_rec = zeros(kti, 1);
spgp0_smse_rec = zeros(kti, 1);
grbcm0_msll_rec = zeros(kti, 1);
rbcm0_msll_rec = zeros(kti, 1);
bcm0_msll_rec = zeros(kti, 1);
gpoe0_msll_rec = zeros(kti, 1);

```

```

poe0_msll_rec = zeros(kti, 1);
vfe0_msll_rec = zeros(kti, 1);
spgp0_msll_rec = zeros(kti, 1);
grbcm_gr_smse = zeros(kti, length(grls));
grbcm_gr_msll = zeros(kti, length(grls));
grbcm2_gr_smse = zeros(kti, length(grls));
grbcm2_gr_msll = zeros(kti, length(grls));
grbcm2_spgp_gr_smse = zeros(kti, length(grls));
grbcm2_spgp_gr_msll = zeros(kti, length(grls));

```

Experiment I: remove data from the training set

```

sf2 = 1 ; ell = 1 ; sn2 = 0.1 ;
d = size(xvec,2);
opts.Xnorm = 'N' ; opts.Ynorm = 'N' ;
opts.Ms = ttm+1;
opts.ell = ell ; opts.sf2 = sf2 ; opts.sn2 = sn2 ;
opts.meanfunc = []; opts.covfunc = @covSEard; opts.likfunc = @likGauss; opts.inffunc = @infGauss;

meanfunc = []; % empty: don't use a mean function
covfunc = opts.covfunc; % Squared Exponential covariance function
likfunc = opts.likfunc; % Gaussian likelihood
inffunc = opts.inffunc;
km_iters = 1e4;
opts.induce_step = induce_step ;

```

```

% hyp.cov = log([ones(d,1)*ell;sf2]); hyp.lik = log(sn2); hyp.mean = [];
opts.numOptFC = 50 ;
opts.Ms = ttm+1;
opts.xvec = xvec;
opts.yvec = yvec;
opts.grbcm_baseline = 0;
opts.global_index = ones(n,1);
% opts.inffunc = @infGaussLik; opts.meanfunc = meanfunc; opts.likfunc = likfunc;
opts.covfunc = covfunc;
covfuncF = {@apxSparse, {opts.covfunc}, []};
opts.covfuncF = covfuncF;
opts.compute_hyp = 0;

```

```

% default partition
dcs_ecs_r = 0.5;
dcs = round(ttcs*dcs_ecs_r) % size of the communication set

```

```

dcs = 150

```

```

ecs = ttcs - dcs % size of other experts

```

```

ecs = 150

```

```

n_per = dcs ; % size of Dc

```



```

mn = round(n / ecs); % mn is the number of experts (normal)
Indics = randperm(n) ;
I_com = Indics(1:n_per) ; % randomly select communication set
[idx, C] = kmeans(xvec, mn, 'MaxIter', km_iters);

```

Baselines of VFE and SPGP.

```

% hyp.cov = log([ones(d,1)*ell;sf2]); hyp.lik = log(sn2); hyp.mean = [];
opts.numOptFC = 30 ;
opts.Ms = mn+1;
opts.xvec = xvec;
opts.yvec = yvec;
opts.induce_size = dcs;
opts.grbcm_baseline = 0;
opts.global_index = ones(n,1);
opts.I_com = I_com;
% opts.inffunc = @infGaussLik; opts.meanfunc = meanfunc; opts.likfunc = likfunc;
opts.covfunc = covfunc;
covfuncF = {@apxSparse, {opts.covfunc}, xvec(I_com,:)};
opts.covfuncF = covfuncF;
opts.compute_hyp = 0;

```

```

g_opts = opts;
g_opts.compute_hyp = 1;
g_opts.grbcm_baseline = 1;
g_opts.global_index = ones(n,1);
g_models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,g_opts);

```

```

Optimizing hyps in training...
Linesearch      0; Value 2.393679e+04
Linesearch      1; Value 1.176214e+04
Linesearch      2; Value 6.163332e+03
Linesearch      3; Value 5.854543e+03
Linesearch      4; Value -1.986843e+03
Linesearch      5; Value -2.496178e+03
Linesearch      6; Value -3.208067e+03
Linesearch      7; Value -3.350145e+03
Linesearch      8; Value -4.258142e+03
Linesearch      9; Value -4.488851e+03
Linesearch     10; Value -4.528923e+03
Linesearch     11; Value -4.531934e+03
Linesearch     12; Value -4.534338e+03
Linesearch     13; Value -4.535504e+03
Linesearch     14; Value -4.535505e+03
Linesearch     15; Value -4.535505e+03
Linesearch     16; Value -4.535505e+03
Linesearch     17; Value -4.535505e+03
Linesearch     18; Value -4.535505e+03
Linesearch     19; Value -4.535505e+03
Linesearch     20; Value -4.535505e+03
Linesearch     21; Value -4.535505e+03
Linesearch     22; Value -4.535505e+03
Linesearch     23; Value -4.535505e+03
Linesearch     24; Value -4.535505e+03
Linesearch     25; Value -4.535505e+03

```

```

opts.hyp = g_models{1}.hyp;
g_opts.hyp = g_models{1}.hyp;
[tmu,ts2, ~] = aggregation_predict(xvec_test,g_models,'GRBCM', 1, g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[grbcmMSE,grbcmSMSE,grbcmMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tm
fprintf('%s (Dc size %d): MSE %.8f, SMSE %.8f, MSLL %.8f\r\n', 'GRBCM', n_per, grbcmMSE,grbcm

```

```
GRBCM (Dc size 150): MSE 0.01257547, SMSE 0.01196988, MSLL -1.96780742
```

```
g_opts.compute_hyp = 0;
```

When testing baseline, VFE and SPGP use all partition budget (ttcs = dcs + ecs).

```
I_com2 = Indics(1:ttcs) ; % randomly select communication set
```

```

% % VFE Baseline
vfe_opts = opts;
vfe_opts.induce_type = 'VFE_opt';
xu = xvec(I_com2, :);
inffunc = @(varargin) infGaussLik(varargin{:}, struct('s', 0.0));
vfe_hyp = opts.hyp;
vfe_hyp.xu = xu;
[vfe_hyp, tmp_nlz] = minimize(vfe_hyp,@sp_gp,-vfe_opts.induce_step,inffunc,meanfunc,covfuncF,I

```

```

Function evaluation      0; Value 3.707323e+05
Function evaluation     16; Value 3.304338e+05
Function evaluation     17; Value 2.883060e+05
Function evaluation     20; Value 2.542481e+05
Function evaluation     22; Value 2.198459e+05
Function evaluation     24; Value 2.087152e+05
Function evaluation     25; Value 1.892107e+05
Function evaluation     27; Value 1.756530e+05
Function evaluation     29; Value 1.629768e+05
Function evaluation     31; Value 1.563370e+05
Function evaluation     32; Value 1.493191e+05
Function evaluation     34; Value 1.443786e+05
Function evaluation     36; Value 1.403163e+05
Function evaluation     38; Value 1.373652e+05
Function evaluation     40; Value 1.352177e+05
Function evaluation     42; Value 1.325530e+05
Function evaluation     44; Value 1.308722e+05
Function evaluation     45; Value 1.291648e+05
Function evaluation     46; Value 1.276831e+05
Function evaluation     48; Value 1.264237e+05
Function evaluation     49; Value 1.252144e+05
Function evaluation     50; Value 1.240994e+05
Function evaluation     51; Value 1.228284e+05
Function evaluation     53; Value 1.219665e+05
Function evaluation     55; Value 1.212392e+05
Function evaluation     57; Value 1.206543e+05
Function evaluation     59; Value 1.202240e+05
Function evaluation     61; Value 1.197307e+05

```



```

Function evaluation    63; Value 1.193133e+05
Function evaluation    65; Value 1.189587e+05
Function evaluation    67; Value 1.185597e+05
Function evaluation    69; Value 1.182326e+05
Function evaluation    71; Value 1.179690e+05
Function evaluation    73; Value 1.177373e+05
Function evaluation    74; Value 1.174870e+05
Function evaluation    75; Value 1.172653e+05
Function evaluation    77; Value 1.170635e+05
Function evaluation    78; Value 1.168573e+05
Function evaluation    80; Value 1.166933e+05
Function evaluation    81; Value 1.165188e+05
Function evaluation    82; Value 1.163506e+05
Function evaluation    84; Value 1.161472e+05
Function evaluation    86; Value 1.159938e+05
Function evaluation    88; Value 1.158684e+05
Function evaluation    89; Value 1.157414e+05
Function evaluation    91; Value 1.156294e+05
Function evaluation    93; Value 1.154994e+05
Function evaluation    95; Value 1.153994e+05
Function evaluation    97; Value 1.152857e+05
Function evaluation    99; Value 1.151418e+05
Function evaluation   100; Value 1.150073e+05

```

```

vfe_opts.hyp = opts.hyp;
vfe_opts.xu = vfe_hyp.xu;
vfe_opts.inffunc = @infGaussLik; vfe_opts.meanfunc = meanfunc; vfe_opts.covfuncF = covfuncF; vfe_opts.covfunc = covfunc;
[tmu, ts2] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, xvec_test);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[vfeMSE,vfeSMSE,vfeMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'VFE baseline', n_per, vfeMSE, vfeSMSE, vfeMSLL);

```

VFE baseline (Dc size 150): MSE 0.05015171, SMSE 0.04773660, MSLL -1.24505517

```

% vfe0_smse_rec(ki) = vfeSMSE; vfe0_msll_rec(ki) = vfeMSLL;
[yu, su] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, vfe_opts.xu);
vfe_opts.yu = yu; vfe_opts.su = su;

```

```

sp_opts = opts;
sp_opts.induce_size = ttcs;
sp_opts.induce_type = 'SPGP_opt';
hyp_init(1:d,1) = -2*opts.hyp.cov(1:d);
hyp_init(d+1,1) = 2*opts.hyp.cov(d+1);
hyp_init(d+2,1) = 2*opts.hyp.lik;

xu = xvec(I_com2, :);
w_init = [reshape(xu,sp_opts.induce_size*d,1);hyp_init];
[w,tmp_nlz] = minimize(w_init,'spgp_lik_nohyp',-sp_opts.induce_step,yvec,xvec,sp_opts.induce_size);

```

```

Function evaluation    0; Value 8.003027e+03
Function evaluation   10; Value 6.905067e+03
Function evaluation   11; Value 5.751114e+03
Function evaluation   14; Value 5.255732e+03
Function evaluation   15; Value 4.824455e+03
Function evaluation   17; Value 4.564236e+03

```

Function evaluation	18;	Value 4.297099e+03
Function evaluation	20;	Value 4.105163e+03
Function evaluation	21;	Value 3.896262e+03
Function evaluation	23;	Value 3.713547e+03
Function evaluation	24;	Value 3.527465e+03
Function evaluation	26;	Value 3.369774e+03
Function evaluation	28;	Value 3.258013e+03
Function evaluation	30;	Value 3.161050e+03
Function evaluation	32;	Value 3.092476e+03
Function evaluation	34;	Value 3.010800e+03
Function evaluation	36;	Value 2.959714e+03
Function evaluation	37;	Value 2.900693e+03
Function evaluation	39;	Value 2.872686e+03
Function evaluation	41;	Value 2.833612e+03
Function evaluation	42;	Value 2.792681e+03
Function evaluation	43;	Value 2.753610e+03
Function evaluation	44;	Value 2.715740e+03
Function evaluation	46;	Value 2.682596e+03
Function evaluation	48;	Value 2.644126e+03
Function evaluation	49;	Value 2.608797e+03
Function evaluation	51;	Value 2.580390e+03
Function evaluation	52;	Value 2.549528e+03
Function evaluation	53;	Value 2.520048e+03
Function evaluation	54;	Value 2.490776e+03
Function evaluation	56;	Value 2.467377e+03
Function evaluation	57;	Value 2.444507e+03
Function evaluation	59;	Value 2.424828e+03
Function evaluation	61;	Value 2.409531e+03
Function evaluation	63;	Value 2.391967e+03
Function evaluation	64;	Value 2.374766e+03
Function evaluation	66;	Value 2.361894e+03
Function evaluation	68;	Value 2.350428e+03
Function evaluation	70;	Value 2.336174e+03
Function evaluation	72;	Value 2.324117e+03
Function evaluation	74;	Value 2.314301e+03
Function evaluation	75;	Value 2.304762e+03
Function evaluation	77;	Value 2.296561e+03
Function evaluation	79;	Value 2.290126e+03
Function evaluation	81;	Value 2.280730e+03
Function evaluation	82;	Value 2.272192e+03
Function evaluation	84;	Value 2.264525e+03
Function evaluation	86;	Value 2.255512e+03
Function evaluation	88;	Value 2.245337e+03
Function evaluation	90;	Value 2.237007e+03
Function evaluation	91;	Value 2.228327e+03
Function evaluation	92;	Value 2.219474e+03
Function evaluation	94;	Value 2.212685e+03
Function evaluation	96;	Value 2.206675e+03
Function evaluation	97;	Value 2.200582e+03
Function evaluation	99;	Value 2.195336e+03
Function evaluation	100;	Value 2.189720e+03

```

xb = reshape(w(1:sp_opts.induce_size*d,1),sp_opts.induce_size,d);
sp_opts.xu = xb;
sp_opts.sp_hyp = w(sp_opts.induce_size*d+1:end,1);
sp_opts.hyp = opts.hyp;
[tmu,ts2] = spgp_pred(sp_opts.yvec,sp_opts.xvec,sp_opts.xu,xvec_test,sp_opts.sp_hyp);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[spgpMSE,spgpSMSE,spgpMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu,

```

```
fprintf('%s (Dc size %d): MSE %.8f, SMSE %.8f, MSLL %.8f\r\n', 'SPSG baseline', n_per, spgpM
```

```
SPSG baseline (Dc size 150): MSE 0.09106134, SMSE 0.08667618, MSLL -1.22127648
```

```
% spgp0_smse_rec(ki) = spgpSMSE; spgp0_msll_rec(ki) = spgpMSLL;
[yu,su] = spgp_pred(sp_opts.yvec,sp_opts.xvec,sp_opts.xu,sp_opts.xu,sp_opts.sp_hyp);
sp_opts.yu = yu; sp_opts.su = su;
```

Average over 5 runs.

```
kti = 5;
m = mn;

for ki=1:kti
fprintf("=====%d=====", ki)
n_per = dcs ; % size of Dc
Indics = randperm(n) ;
I_com = Indics(1:n_per) ; % randomly select communication set
[idx, C] = kmeans(xvec, m, 'MaxIter', km_iters);

% hyp.cov = log([ones(d,1)*ell;sf2]); hyp.lik = log(sn2); hyp.mean = [];
opts.numOptFC = 30 ;
opts.Ms = m+1;
opts.xvec = xvec;
opts.yvec = yvec;
opts.induce_size = dcs;
opts.grbcm_baseline = 0;
opts.global_index = ones(n,1);
opts.I_com = I_com;
% opts.inffunc = @infGaussLik; opts.meanfunc = meanfunc; opts.likfunc = likfunc;
opts.covfunc = covfunc;
covfuncF = {@apxSparse, {opts.covfunc}, xvec(I_com,:)};
opts.covfuncF = covfuncF;
opts.compute_hyp = 0;

g_opts = opts;
g_opts.compute_hyp = 1;
g_opts.grbcm_baseline = 1;
g_opts.global_index = ones(n,1);
g_models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,g_opts);
opts.hyp = g_models{1}.hyp;
g_opts.hyp = g_models{1}.hyp;
[tmu,ts2, ~] = aggregation_predict(xvec_test,g_models,'GRBCM', 1, g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[grbcmMSE_bl,grbcmSMSE_bl,grbcmMSLL_bl] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, g_opts);
fprintf('%s (Dc size %d): MSE %.8f, SMSE %.8f, MSLL %.8f\r\n', 'GRBCM', n_per, grbcmMSE_bl,grbcmSMSE_bl,grbcmMSLL_bl);
grbcm0_smse_rec(ki) = grbcmSMSE_bl; grbcm0_msll_rec(ki) = grbcmMSLL_bl;
g_opts.compute_hyp = 0;
```

```

%% VFE Baseline
vfe_opts = opts;
vfe_opts.induce_type = 'VFE_opt';
xu = xvec(I_com, :);
inffunc = @(varargin) infGaussLik(varargin{:}, struct('s', 0.0));
vfe_hyp = opts.hyp;
vfe_hyp.xu = xu;
vfe_hyp = minimize(vfe_hyp,@sp_gp,-vfe_opts.induce_step,inffunc,meanfunc,covfuncF,likfunc,xvec,yvec);
vfe_opts.hyp = vfe_hyp;
vfe_opts.xu = vfe_hyp.xu;
vfe_opts.inffunc = @infGaussLik; vfe_opts.meanfunc = meanfunc; vfe_opts.covfuncF = covfuncF; vfe_opts.likfunc = likfunc;
vfe_opts.covfunc = covfunc;
[tmu, ts2] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, xvec_test);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[vfeMSE_bl,vfeSMSE_bl,vfeMSLL_bl] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, vfe_opts);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'VFE', n_per, vfeMSE_bl,vfeSMSE_bl,vfeMSLL_bl);
vfe0_smse_rec(ki) = vfeSMSE_bl; vfe0_msll_rec(ki) = vfeMSLL_bl;
[yu, su] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, vfe_opts.xu);
vfe_opts.yu = yu; vfe_opts.su = su;

sp_opts = opts;
sp_opts.induce_type = 'SPGP_opt';
hyp_init(1:d,1) = -2*opts.hyp.cov(1:d);
hyp_init(d+1,1) = 2*opts.hyp.cov(d+1);
hyp_init(d+2,1) = 2*opts.hyp.lik;

xu = xvec(I_com, :);
w_init = [reshape(xu,sp_opts.induce_size*d,1);hyp_init];
[w,f] = minimize(w_init,'spgp_lik_nohyp',-sp_opts.induce_step,yvec,xvec,sp_opts.induce_size);
xb = reshape(w(1:sp_opts.induce_size*d,1),sp_opts.induce_size,d);
sp_opts.xu = xb;
sp_opts.sp_hyp = w(sp_opts.induce_size*d+1:end,1);
sp_opts.hyp = opts.hyp;

[tmu,ts2] = spgp_pred(sp_opts.yvec,sp_opts.xvec,sp_opts.xu,xvec_test,sp_opts.sp_hyp);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[spgpMSE_bl,spgpSMSE_bl,spgpMSLL_bl] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, sp_opts);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'SPSG', n_per, spgpMSE_bl,spgpSMSE_bl,spgpMSLL_bl);
spgp0_smse_rec(ki) = spgpSMSE_bl; spgp0_msll_rec(ki) = spgpMSLL_bl;
[yu,su] = spgp_pred(sp_opts.yvec,sp_opts.xvec,sp_opts.xu,sp_opts.xu,sp_opts.sp_hyp);
sp_opts.yu = yu; sp_opts.su = su;

vfe_opts.grbcm_baseline = 0;
vfe_opts.global_index = ones(n,1);
models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,vfe_opts); % use hyp of vfe
[tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_test,models,vfe_opts);
if ynorm==1

```

```

    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'GRBCM (VFE)', n_per, MSE,SMSE,MSLL);

sp_opts.grbcm_baseline = 0;
sp_opts.global_index = ones(n,1);
models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,sp_opts); % use hyp of vfe
[tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_test,models,sp_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'GRBCM (SPGP)', n_per, MSE,SMSE,MSLL);

criterion = 'RBCM';
[tmu,ts2,t_dGP_predict] = aggregation_predict(xvec_test,g_models,criterion, 1, g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[rbcmMSE,rbcmSMSE,rbcmMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', criterion, n_per, rbcmMSE,rbcmSMSE,rbcmMSLL);
rbcm0_smse_rec(ki) = rbcmSMSE; rbcm0_msll_rec(ki) = rbcmMSLL;

criterion = 'BCM';
[tmu,ts2,t_dGP_predict] = aggregation_predict(xvec_test,g_models,criterion, 1, g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[bcmMSE,bcmSMSE,bcmMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', criterion, n_per, bcmMSE,bcmSMSE,bcmMSLL);
bcm0_smse_rec(ki) = bcmSMSE; bcm0_msll_rec(ki) = bcmMSLL;

criterion = 'PoE';
[tmu,ts2,t_dGP_predict] = aggregation_predict(xvec_test,g_models,criterion, 1, g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[poeMSE,poeSMSE,poeMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', criterion, n_per, poeMSE,poeSMSE,poeMSLL);
poe0_smse_rec(ki) = poeSMSE; poe0_msll_rec(ki) = poeMSLL;

criterion = 'GPoE';
[tmu,ts2,t_dGP_predict] = aggregation_predict(xvec_test,g_models,criterion, 1, g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end

```

```

[gpoemSE,gpoesMSE,gpoemSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu,
fprintf('%s (Dc size %d): MSE %.8f, SMSE %.4f, MSL %6.4f\r\n', criterion, n_per, gpoemSE,gpo
gpoes0_smse_rec(ki) = gpoesMSE; gpoes0_msl_rec(ki) = gpoemSLL;

% compute informatic importance
ds = zeros(n, m);
for i=1:n
    for j=1:m
        %ds(i,j) = muti1(i)-muti2(j,i)+muti1(j)-muti2(i,j);
        ds(i,j) = norm(xvec(i, :) - C(j,:));
    end
end
% 1st closest cluster centers
[~, fcc] = min(ds, [], 2);
% set to inf
for i=1:n
    ds(i, fcc(i)) = 1e10;
end
% 2st closest cluster centers
[~, scc] = min(ds, [], 2);
rk = zeros(n, 1);
for iik=1:n
    if mod(iik, 1e2)==0
        fprintf('processing distance: %d/%d\n', iik, n);
    end
    i = fcc(iik);
    [tmp_mu, tmp_sig2] = gp(vfe_hyp,inffunc,meanfunc, ...
        covfunc,likfunc,models{i+1}.X,models{i+1}.Y,xvec(iik,:));
    h1 = 0.5*log(tmp_sig2);
    j = scc(iik);
    % [tmp_mu, tmp_sig2] = gp(vfe_hyp,inffunc,meanfunc, ...
    % covfunc,likfunc,[models{i+1}.X;models{j+1}.X],[models{i+1}.Y;models{j+1}.Y],xvec(iik,
    [tmp_mu, tmp_sig2] = gp(vfe_hyp,inffunc,meanfunc, ...
        covfunc,likfunc,[models{j+1}.X],[models{j+1}.Y],xvec(iik,:));
    h2 = 0.5*log(tmp_sig2);
    rk(iik) = h2 - h1; % importance is the difference between mutual information
end

if ki==1
    figure;
    scatter(xvec(:,1), xvec(:,2), 3, rk, "filled"); colorbar;
    figure; hold on;
    plot(xvec(I_com,1), xvec(I_com,2), 'r. ');
    plot(vfe_opts.xu(:,1), vfe_opts.xu(:,2), 'bo');
    plot(sp_opts.xu(:,1), sp_opts.xu(:,2), 'go');
    legend('initial points', 'VFE induced', 'SPGP induced');
end

for kj=1:length(grls) % test/validate different remaining percentage
    kj
    gr=grls(kj)
    crk = rk;
    crk(I_com) = -1e10;
    [~, crk_idx] = sort(crk, 'descend');

```



```

rn = round(n*gr);
global_index = zeros(n,1);
global_index(crk_idx(1:rn)) = 1; % select remaining data according to the importance

% only remove data from subsets, in RBGCM
g_opts.global_index = global_index;
models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,g_opts); % use hyp of vfe
[tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_test,models,g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[grbcmMSE,grbcmSMSE,grbcmMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM', n_per, grbcmMSE, grbcm_gr_smse(ki,kj) = grbcmSMSE; grbcm_gr_msll(ki,kj) = grbcmMSLL;

vfe_opts.global_index = global_index;
models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,vfe_opts); % use hyp of vfe
[tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_test,models,vfe_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): \r\nMSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM++ (VFE)', n_per, MSE, grbcm2_gr_smse(ki,kj) = SMSE; grbcm2_gr_msll(ki,kj) = MSLL;

sp_opts.global_index = global_index;
models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,sp_opts); % use hyp of vfe
[tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_test,models,sp_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): \r\nMSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM++ (SPGP)', n_per, MSE, grbcm2_spgp_gr_smse(ki,kj) = SMSE; grbcm2_spgp_gr_msll(ki,kj) = MSLL;

end
end

```

```

=====1=====
Optimizing hyps in training...
Linesearch    0; Value 2.422999e+04
Linesearch    1; Value 1.184364e+04
Linesearch    2; Value 5.467047e+03
Linesearch    3; Value 5.163614e+03
Linesearch    4; Value -2.886670e+03
Linesearch    5; Value -3.221907e+03
Linesearch    6; Value -3.316389e+03
Linesearch    7; Value -4.193879e+03
Linesearch    8; Value -4.487277e+03
Linesearch    9; Value -4.534952e+03
Linesearch   10; Value -4.541689e+03
Linesearch   11; Value -4.541862e+03
Linesearch   12; Value -4.541866e+03
Linesearch   13; Value -4.541867e+03
Linesearch   14; Value -4.541867e+03

```

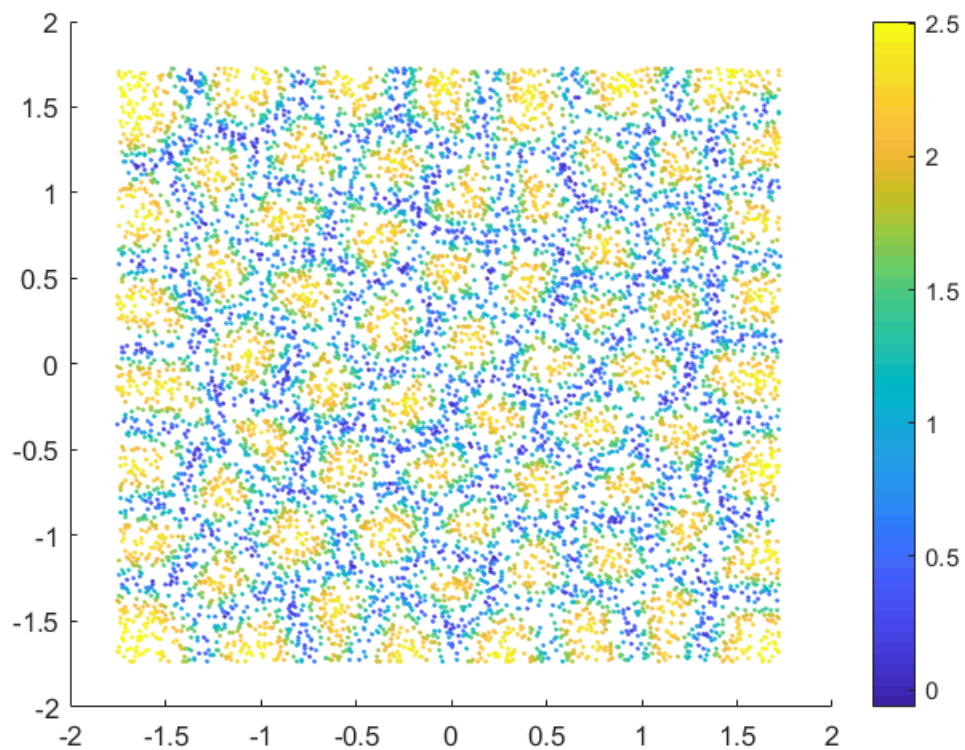
```

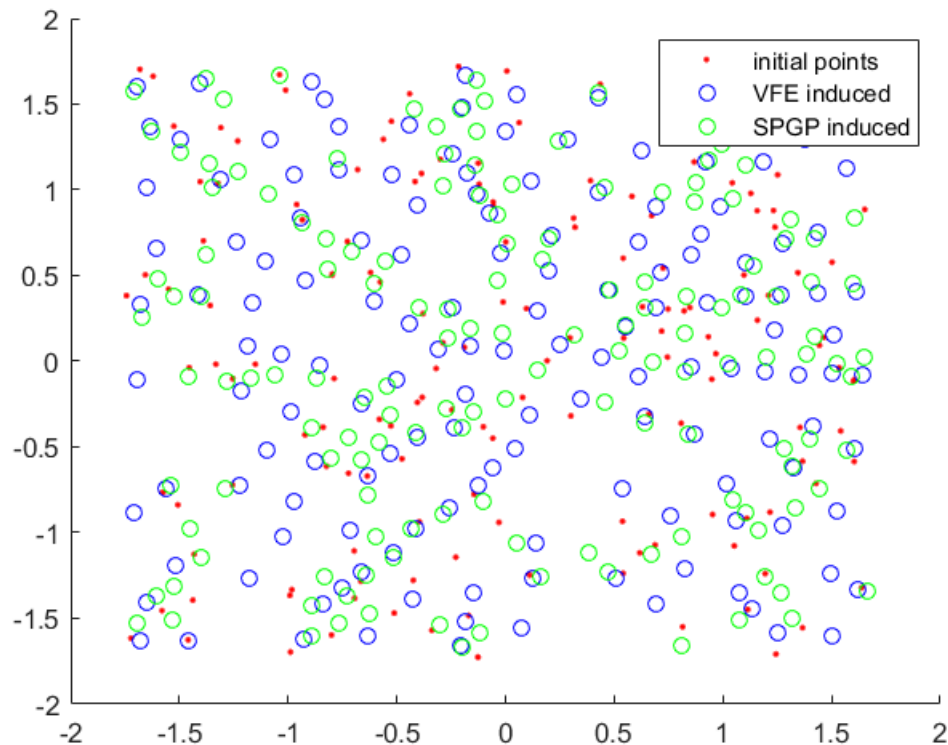
Linesearch      15; Value -4.541867e+03
Linesearch      16; Value -4.541867e+03
Linesearch      17; Value -4.541867e+03
Linesearch      18; Value -4.541867e+03
Linesearch      19; Value -4.541867e+03
Linesearch      20; Value -4.541867e+03
Linesearch      21; Value -4.541867e+03
Linesearch      22; Value -4.541867e+03
Linesearch      23; Value -4.541867e+03
Linesearch      24; Value -4.541867e+03
Linesearch      25; Value -4.541867e+03
GRBCM (Dc size 150): MSE 0.01244503, SMSE 0.01184573, MSLL -1.99268697
Function evaluation    0; Value 7.167835e+05
Function evaluation    13; Value 7.135049e+05
Function evaluation    18; Value 6.435179e+05
Function evaluation    20; Value 5.967653e+05
Function evaluation    22; Value 5.630492e+05
Function evaluation    24; Value 5.530678e+05
Function evaluation    25; Value 5.374005e+05
Function evaluation    27; Value 5.284854e+05
Function evaluation    29; Value 5.202562e+05
Function evaluation    31; Value 5.149289e+05
Function evaluation    33; Value 5.125709e+05
Function evaluation    35; Value 5.102987e+05
Function evaluation    37; Value 5.082959e+05
Function evaluation    38; Value 5.063377e+05
Function evaluation    40; Value 5.046794e+05
Function evaluation    42; Value 5.037219e+05
Function evaluation    44; Value 5.013091e+05
Function evaluation    46; Value 4.999898e+05
Function evaluation    48; Value 4.994813e+05
Function evaluation    50; Value 4.981589e+05
Function evaluation    52; Value 4.963566e+05
Function evaluation    54; Value 4.957092e+05
Function evaluation    56; Value 4.942480e+05
Function evaluation    58; Value 4.934651e+05
Function evaluation    60; Value 4.923376e+05
Function evaluation    63; Value 4.920022e+05
Function evaluation    65; Value 4.914547e+05
Function evaluation    67; Value 4.909454e+05
Function evaluation    68; Value 4.904229e+05
Function evaluation    70; Value 4.897538e+05
Function evaluation    72; Value 4.887040e+05
Function evaluation    74; Value 4.883728e+05
Function evaluation    76; Value 4.879430e+05
Function evaluation    78; Value 4.872966e+05
Function evaluation    80; Value 4.868571e+05
Function evaluation    81; Value 4.864628e+05
Function evaluation    83; Value 4.856775e+05
Function evaluation    85; Value 4.854210e+05
Function evaluation    87; Value 4.849095e+05
Function evaluation    89; Value 4.844932e+05
Function evaluation    91; Value 4.842415e+05
Function evaluation    93; Value 4.838571e+05
Function evaluation    95; Value 4.834296e+05
Function evaluation    97; Value 4.831679e+05
Function evaluation    98; Value 4.828921e+05
Function evaluation   100; Value 4.824912e+05
VFE (Dc size 150): MSE 0.32919013, SMSE 0.31333761, MSLL -0.57504772
Function evaluation    0; Value 1.214558e+04
Function evaluation    7; Value 1.207189e+04
Function evaluation   13; Value 1.103764e+04
Function evaluation   16; Value 1.057551e+04
Function evaluation   17; Value 1.027865e+04
Function evaluation   20; Value 1.003783e+04

```

Function evaluation 21; Value 9.796230e+03
 Function evaluation 23; Value 9.646574e+03
 Function evaluation 24; Value 9.500744e+03
 Function evaluation 26; Value 9.375495e+03
 Function evaluation 28; Value 9.288683e+03
 Function evaluation 29; Value 9.199393e+03
 Function evaluation 31; Value 9.130564e+03
 Function evaluation 33; Value 9.076951e+03
 Function evaluation 34; Value 9.018486e+03
 Function evaluation 35; Value 8.962642e+03
 Function evaluation 37; Value 8.922706e+03
 Function evaluation 39; Value 8.897277e+03
 Function evaluation 41; Value 8.868925e+03
 Function evaluation 43; Value 8.824209e+03
 Function evaluation 45; Value 8.799162e+03
 Function evaluation 47; Value 8.777987e+03
 Function evaluation 49; Value 8.741484e+03
 Function evaluation 51; Value 8.715477e+03
 Function evaluation 52; Value 8.690008e+03
 Function evaluation 53; Value 8.664735e+03
 Function evaluation 55; Value 8.646478e+03
 Function evaluation 56; Value 8.630031e+03
 Function evaluation 58; Value 8.615245e+03
 Function evaluation 60; Value 8.602265e+03
 Function evaluation 62; Value 8.593461e+03
 Function evaluation 64; Value 8.575936e+03
 Function evaluation 66; Value 8.560834e+03
 Function evaluation 68; Value 8.549487e+03
 Function evaluation 70; Value 8.534458e+03
 Function evaluation 72; Value 8.517056e+03
 Function evaluation 73; Value 8.499191e+03
 Function evaluation 75; Value 8.477129e+03
 Function evaluation 77; Value 8.462288e+03
 Function evaluation 78; Value 8.447233e+03
 Function evaluation 80; Value 8.434142e+03
 Function evaluation 81; Value 8.421770e+03
 Function evaluation 82; Value 8.408549e+03
 Function evaluation 84; Value 8.398404e+03
 Function evaluation 86; Value 8.385793e+03
 Function evaluation 88; Value 8.377677e+03
 Function evaluation 90; Value 8.367731e+03
 Function evaluation 92; Value 8.360111e+03
 Function evaluation 93; Value 8.352232e+03
 Function evaluation 95; Value 8.345477e+03
 Function evaluation 97; Value 8.337344e+03
 Function evaluation 99; Value 8.331682e+03
 SPSPG (Dc size 150): MSE 0.42024017, SMSE 0.40000302, MSLL -0.57563766
 GRBCM (VFE) (Dc size 150): MSE 0.01268806, SMSE 0.01207705, MSLL -1.98482061
 GRBCM (SPGP) (Dc size 150): MSE 0.01247200, SMSE 0.01187140, MSLL -1.98605977
 RBCM (Dc size 150): MSE 0.01281609, SMSE 0.0122, MSLL -1.8143
 BCM (Dc size 150): MSE 0.01320222, SMSE 0.0126, MSLL -2.1741
 PoE (Dc size 150): MSE 0.10237897, SMSE 0.0974, MSLL 3.7793
 GPoE (Dc size 150): MSE 0.01268349, SMSE 0.0121, MSLL -1.8173
 processing distance: 100/10000
 processing distance: 200/10000
 processing distance: 300/10000
 processing distance: 400/10000
 processing distance: 500/10000
 processing distance: 600/10000
 processing distance: 700/10000
 processing distance: 800/10000
 processing distance: 900/10000
 processing distance: 1000/10000
 processing distance: 1100/10000
 processing distance: 1200/10000

processing distance: 7800/10000
processing distance: 7900/10000
processing distance: 8000/10000
processing distance: 8100/10000
processing distance: 8200/10000
processing distance: 8300/10000
processing distance: 8400/10000
processing distance: 8500/10000
processing distance: 8600/10000
processing distance: 8700/10000
processing distance: 8800/10000
processing distance: 8900/10000
processing distance: 9000/10000
processing distance: 9100/10000
processing distance: 9200/10000
processing distance: 9300/10000
processing distance: 9400/10000
processing distance: 9500/10000
processing distance: 9600/10000
processing distance: 9700/10000
processing distance: 9800/10000
processing distance: 9900/10000
processing distance: 10000/10000





```

kj = 1
gr = 0.3000
GRBCM (Dc size 150): MSE 0.08618565, SMSE 0.0820, MSLL -1.4412
GRBCM++ (VFE) (Dc size 150):
MSE 0.06124814, SMSE 0.0583, MSLL -1.5020
GRBCM++ (SPGP) (Dc size 150):
MSE 0.06089531, SMSE 0.0580, MSLL -1.5222
kj = 2
gr = 0.3500
GRBCM (Dc size 150): MSE 0.05622199, SMSE 0.0535, MSLL -1.5696
GRBCM++ (VFE) (Dc size 150):
MSE 0.04410065, SMSE 0.0420, MSLL -1.6180
GRBCM++ (SPGP) (Dc size 150):
MSE 0.04424603, SMSE 0.0421, MSLL -1.6143
kj = 3
gr = 0.4000
GRBCM (Dc size 150): MSE 0.04352921, SMSE 0.0414, MSLL -1.6616
GRBCM++ (VFE) (Dc size 150):
MSE 0.03543223, SMSE 0.0337, MSLL -1.6891
GRBCM++ (SPGP) (Dc size 150):
MSE 0.03619204, SMSE 0.0344, MSLL -1.6972
kj = 4
gr = 0.4500
GRBCM (Dc size 150): MSE 0.03348525, SMSE 0.0319, MSLL -1.7430
GRBCM++ (VFE) (Dc size 150):
MSE 0.02964483, SMSE 0.0282, MSLL -1.7639
GRBCM++ (SPGP) (Dc size 150):
MSE 0.02935344, SMSE 0.0279, MSLL -1.7629
kj = 5
gr = 0.5000
GRBCM (Dc size 150): MSE 0.02715576, SMSE 0.0258, MSLL -1.8230
GRBCM++ (VFE) (Dc size 150):
MSE 0.02581090, SMSE 0.0246, MSLL -1.8305
GRBCM++ (SPGP) (Dc size 150):
MSE 0.02450188, SMSE 0.0233, MSLL -1.8414

```


kj = 6
 gr = 0.5500
 GRBCM (Dc size 150): MSE 0.02326546, SMSE 0.0221, MSLL -1.8596
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.02198821, SMSE 0.0209, MSLL -1.8761
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.02152415, SMSE 0.0205, MSLL -1.8743
 kj = 7
 gr = 0.6000
 GRBCM (Dc size 150): MSE 0.01956203, SMSE 0.0186, MSLL -1.9070
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01950488, SMSE 0.0186, MSLL -1.9216
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01969445, SMSE 0.0187, MSLL -1.9034
 kj = 8
 gr = 0.6500
 GRBCM (Dc size 150): MSE 0.01721117, SMSE 0.0164, MSLL -1.9515
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01733029, SMSE 0.0165, MSLL -1.9694
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01721185, SMSE 0.0164, MSLL -1.9497
 kj = 9
 gr = 0.7000
 GRBCM (Dc size 150): MSE 0.01600575, SMSE 0.0152, MSLL -1.9756
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01589735, SMSE 0.0151, MSLL -1.9952
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01590606, SMSE 0.0151, MSLL -1.9706
 kj = 10
 gr = 0.7500
 GRBCM (Dc size 150): MSE 0.01540785, SMSE 0.0147, MSLL -1.9767
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01572814, SMSE 0.0150, MSLL -1.9853
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01555823, SMSE 0.0148, MSLL -1.9675
 kj = 11
 gr = 0.8000
 GRBCM (Dc size 150): MSE 0.01457902, SMSE 0.0139, MSLL -1.9773
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01454465, SMSE 0.0138, MSLL -1.9947
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01453589, SMSE 0.0138, MSLL -1.9723
 kj = 12
 gr = 0.8500
 GRBCM (Dc size 150): MSE 0.01368763, SMSE 0.0130, MSLL -1.9900
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01375648, SMSE 0.0131, MSLL -2.0046
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01374192, SMSE 0.0131, MSLL -1.9863
 kj = 13
 gr = 0.9000
 GRBCM (Dc size 150): MSE 0.01286398, SMSE 0.0122, MSLL -2.0089
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01300729, SMSE 0.0124, MSLL -2.0169
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01299667, SMSE 0.0124, MSLL -2.0010
 kj = 14
 gr = 0.9500
 GRBCM (Dc size 150): MSE 0.01264567, SMSE 0.0120, MSLL -1.9953
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01296308, SMSE 0.0123, MSLL -1.9877
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01282924, SMSE 0.0122, MSLL -1.9801
 kj = 15
 gr = 1

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GRBCM (Dc size 150): MSE 0.01244503, SMSE 0.0118, MSLL -1.9927
GRBCM++ (VFE) (Dc size 150):
MSE 0.01268806, SMSE 0.0121, MSLL -1.9848
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01247200, SMSE 0.0119, MSLL -1.9861
=====2=====
Optimizing hyps in training...
Linesearch      0; Value 2.547383e+04
Linesearch      1; Value 1.206295e+04
Linesearch      2; Value 5.996622e+03
Linesearch      3; Value 5.463941e+03
Linesearch      4; Value -2.589468e+03
Linesearch      5; Value -3.182883e+03
Linesearch      6; Value -3.266081e+03
Linesearch      7; Value -4.183049e+03
Linesearch      8; Value -4.471594e+03
Linesearch      9; Value -4.527934e+03
Linesearch     10; Value -4.545090e+03
Linesearch     11; Value -4.546736e+03
Linesearch     12; Value -4.547707e+03
Linesearch     13; Value -4.548013e+03
Linesearch     14; Value -4.548060e+03
Linesearch     15; Value -4.548060e+03
Linesearch     16; Value -4.548061e+03
Linesearch     17; Value -4.548061e+03
Linesearch     18; Value -4.548061e+03
Linesearch     19; Value -4.548061e+03
Linesearch     20; Value -4.548061e+03
Linesearch     21; Value -4.548061e+03
Linesearch     22; Value -4.548061e+03
Linesearch     23; Value -4.548061e+03
Linesearch     24; Value -4.548061e+03
Linesearch     25; Value -4.548061e+03
Linesearch     26; Value -4.548061e+03
Linesearch     27; Value -4.548061e+03
Linesearch     28; Value -4.548061e+03
Linesearch     29; Value -4.548061e+03
Linesearch     30; Value -4.548061e+03
GRBCM (Dc size 150): MSE 0.01267743, SMSE 0.01206694, MSLL -1.93005199
Function evaluation      0; Value 7.357831e+05
Function evaluation     15; Value 6.801936e+05
Function evaluation     17; Value 6.398666e+05
Function evaluation     19; Value 6.008916e+05
Function evaluation     21; Value 5.896451e+05
Function evaluation     23; Value 5.624808e+05
Function evaluation     25; Value 5.495049e+05
Function evaluation     27; Value 5.423183e+05
Function evaluation     29; Value 5.362841e+05
Function evaluation     30; Value 5.300302e+05
Function evaluation     32; Value 5.249865e+05
Function evaluation     34; Value 5.205486e+05
Function evaluation     36; Value 5.182306e+05
Function evaluation     38; Value 5.152773e+05
Function evaluation     40; Value 5.133927e+05
Function evaluation     42; Value 5.117631e+05
Function evaluation     44; Value 5.105312e+05
Function evaluation     45; Value 5.094888e+05
Function evaluation     46; Value 5.083345e+05
Function evaluation     48; Value 5.072469e+05
Function evaluation     50; Value 5.064098e+05
Function evaluation     51; Value 5.056333e+05
Function evaluation     53; Value 5.044853e+05
Function evaluation     55; Value 5.039100e+05
Function evaluation     57; Value 5.036035e+05
Function evaluation     59; Value 5.032219e+05

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Function evaluation	61;	Value	5.025387e+05
Function evaluation	63;	Value	5.022263e+05
Function evaluation	65;	Value	5.015046e+05
Function evaluation	67;	Value	5.012301e+05
Function evaluation	69;	Value	5.009891e+05
Function evaluation	70;	Value	5.007693e+05
Function evaluation	72;	Value	5.005620e+05
Function evaluation	74;	Value	5.003067e+05
Function evaluation	76;	Value	4.999898e+05
Function evaluation	78;	Value	4.999149e+05
Function evaluation	81;	Value	4.996628e+05
Function evaluation	83;	Value	4.995140e+05
Function evaluation	85;	Value	4.993366e+05
Function evaluation	87;	Value	4.990830e+05
Function evaluation	89;	Value	4.989268e+05
Function evaluation	91;	Value	4.987007e+05
Function evaluation	93;	Value	4.985645e+05
Function evaluation	94;	Value	4.984130e+05
Function evaluation	96;	Value	4.982260e+05
Function evaluation	98;	Value	4.981173e+05
VFE (Dc size 150): MSE 0.35244205, SMSE 0.33546980, MSLL -0.54369302			
Function evaluation	0;	Value	1.159471e+04
Function evaluation	6;	Value	1.157571e+04
Function evaluation	11;	Value	1.094861e+04
Function evaluation	13;	Value	1.069584e+04
Function evaluation	14;	Value	1.041539e+04
Function evaluation	15;	Value	1.013506e+04
Function evaluation	17;	Value	9.931751e+03
Function evaluation	19;	Value	9.789063e+03
Function evaluation	20;	Value	9.643138e+03
Function evaluation	22;	Value	9.497941e+03
Function evaluation	24;	Value	9.413487e+03
Function evaluation	25;	Value	9.331380e+03
Function evaluation	27;	Value	9.269602e+03
Function evaluation	29;	Value	9.216081e+03
Function evaluation	31;	Value	9.174860e+03
Function evaluation	33;	Value	9.139270e+03
Function evaluation	34;	Value	9.104792e+03
Function evaluation	36;	Value	9.064375e+03
Function evaluation	37;	Value	9.024947e+03
Function evaluation	39;	Value	8.990141e+03
Function evaluation	41;	Value	8.961579e+03
Function evaluation	43;	Value	8.927697e+03
Function evaluation	45;	Value	8.898536e+03
Function evaluation	47;	Value	8.874411e+03
Function evaluation	49;	Value	8.835088e+03
Function evaluation	51;	Value	8.802722e+03
Function evaluation	53;	Value	8.776029e+03
Function evaluation	55;	Value	8.757344e+03
Function evaluation	57;	Value	8.731980e+03
Function evaluation	58;	Value	8.706249e+03
Function evaluation	60;	Value	8.690137e+03
Function evaluation	62;	Value	8.675572e+03
Function evaluation	64;	Value	8.661820e+03
Function evaluation	65;	Value	8.648380e+03
Function evaluation	67;	Value	8.637130e+03
Function evaluation	68;	Value	8.625857e+03
Function evaluation	70;	Value	8.618136e+03
Function evaluation	72;	Value	8.607149e+03
Function evaluation	74;	Value	8.594772e+03
Function evaluation	76;	Value	8.584526e+03
Function evaluation	79;	Value	8.580423e+03
Function evaluation	81;	Value	8.573184e+03
Function evaluation	83;	Value	8.564867e+03
Function evaluation	85;	Value	8.561837e+03

Function evaluation 87; Value 8.554990e+03
 Function evaluation 89; Value 8.547029e+03
 Function evaluation 91; Value 8.543273e+03
 Function evaluation 93; Value 8.537216e+03
 Function evaluation 95; Value 8.533100e+03
 Function evaluation 97; Value 8.530273e+03
 Function evaluation 99; Value 8.523963e+03
 Function evaluation 100; Value 8.517746e+03
 SPSPG (Dc size 150): MSE 0.45748500, SMSE 0.43545429, MSLL -0.54394736
 GRBCM (VFE) (Dc size 150): MSE 0.01289921, SMSE 0.01227804, MSLL -1.95077649
 GRBCM (SPGP) (Dc size 150): MSE 0.01297273, SMSE 0.01234802, MSLL -1.94537717
 RBCM (Dc size 150): MSE 0.01304278, SMSE 0.0124, MSLL -1.7706
 BCM (Dc size 150): MSE 0.01348841, SMSE 0.0128, MSLL -2.1561
 PoE (Dc size 150): MSE 0.09763217, SMSE 0.0929, MSLL 3.4752
 GPoE (Dc size 150): MSE 0.01288216, SMSE 0.0123, MSLL -1.7763
 processing distance: 100/10000
 processing distance: 200/10000
 processing distance: 300/10000
 processing distance: 400/10000
 processing distance: 500/10000
 processing distance: 600/10000
 processing distance: 700/10000
 processing distance: 800/10000
 processing distance: 900/10000
 processing distance: 1000/10000
 processing distance: 1100/10000
 processing distance: 1200/10000
 processing distance: 1300/10000
 processing distance: 1400/10000
 processing distance: 1500/10000
 processing distance: 1600/10000
 processing distance: 1700/10000
 processing distance: 1800/10000
 processing distance: 1900/10000
 processing distance: 2000/10000
 processing distance: 2100/10000
 processing distance: 2200/10000
 processing distance: 2300/10000
 processing distance: 2400/10000
 processing distance: 2500/10000
 processing distance: 2600/10000
 processing distance: 2700/10000
 processing distance: 2800/10000
 processing distance: 2900/10000
 processing distance: 3000/10000
 processing distance: 3100/10000
 processing distance: 3200/10000
 processing distance: 3300/10000
 processing distance: 3400/10000
 processing distance: 3500/10000
 processing distance: 3600/10000
 processing distance: 3700/10000
 processing distance: 3800/10000
 processing distance: 3900/10000
 processing distance: 4000/10000
 processing distance: 4100/10000
 processing distance: 4200/10000
 processing distance: 4300/10000
 processing distance: 4400/10000
 processing distance: 4500/10000
 processing distance: 4600/10000
 processing distance: 4700/10000
 processing distance: 4800/10000
 processing distance: 4900/10000
 processing distance: 5000/10000

processing distance: 5100/10000
 processing distance: 5200/10000
 processing distance: 5300/10000
 processing distance: 5400/10000
 processing distance: 5500/10000
 processing distance: 5600/10000
 processing distance: 5700/10000
 processing distance: 5800/10000
 processing distance: 5900/10000
 processing distance: 6000/10000
 processing distance: 6100/10000
 processing distance: 6200/10000
 processing distance: 6300/10000
 processing distance: 6400/10000
 processing distance: 6500/10000
 processing distance: 6600/10000
 processing distance: 6700/10000
 processing distance: 6800/10000
 processing distance: 6900/10000
 processing distance: 7000/10000
 processing distance: 7100/10000
 processing distance: 7200/10000
 processing distance: 7300/10000
 processing distance: 7400/10000
 processing distance: 7500/10000
 processing distance: 7600/10000
 processing distance: 7700/10000
 processing distance: 7800/10000
 processing distance: 7900/10000
 processing distance: 8000/10000
 processing distance: 8100/10000
 processing distance: 8200/10000
 processing distance: 8300/10000
 processing distance: 8400/10000
 processing distance: 8500/10000
 processing distance: 8600/10000
 processing distance: 8700/10000
 processing distance: 8800/10000
 processing distance: 8900/10000
 processing distance: 9000/10000
 processing distance: 9100/10000
 processing distance: 9200/10000
 processing distance: 9300/10000
 processing distance: 9400/10000
 processing distance: 9500/10000
 processing distance: 9600/10000
 processing distance: 9700/10000
 processing distance: 9800/10000
 processing distance: 9900/10000
 processing distance: 10000/10000
 kj = 1
 gr = 0.3000
 GRBCM (Dc size 150): MSE 0.08096680, SMSE 0.0771, MSLL -1.5180
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.05164956, SMSE 0.0492, MSLL -1.5842
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.06048498, SMSE 0.0576, MSLL -1.5711
 kj = 2
 gr = 0.3500
 GRBCM (Dc size 150): MSE 0.05529933, SMSE 0.0526, MSLL -1.6292
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.04066314, SMSE 0.0387, MSLL -1.6747
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.04345079, SMSE 0.0414, MSLL -1.6624
 kj = 3

gr = 0.4000
 GRBCM (Dc size 150): MSE 0.03834810, SMSE 0.0365, MSLL -1.7161
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.03253585, SMSE 0.0310, MSLL -1.7476
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.03454680, SMSE 0.0329, MSLL -1.7241
 kj = 4
 gr = 0.4500
 GRBCM (Dc size 150): MSE 0.03182218, SMSE 0.0303, MSLL -1.7858
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.02941045, SMSE 0.0280, MSLL -1.8077
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.02881540, SMSE 0.0274, MSLL -1.7949
 kj = 5
 gr = 0.5000
 GRBCM (Dc size 150): MSE 0.02640182, SMSE 0.0251, MSLL -1.8311
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.02447352, SMSE 0.0233, MSLL -1.8666
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.02597059, SMSE 0.0247, MSLL -1.8256
 kj = 6
 gr = 0.5500
 GRBCM (Dc size 150): MSE 0.02325934, SMSE 0.0221, MSLL -1.8648
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.02243629, SMSE 0.0214, MSLL -1.8903
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.02347236, SMSE 0.0223, MSLL -1.8576
 kj = 7
 gr = 0.6000
 GRBCM (Dc size 150): MSE 0.01981496, SMSE 0.0189, MSLL -1.9182
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01947579, SMSE 0.0185, MSLL -1.9369
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.02037404, SMSE 0.0194, MSLL -1.9072
 kj = 8
 gr = 0.6500
 GRBCM (Dc size 150): MSE 0.01727225, SMSE 0.0164, MSLL -1.9512
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01746546, SMSE 0.0166, MSLL -1.9700
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01736781, SMSE 0.0165, MSLL -1.9541
 kj = 9
 gr = 0.7000
 GRBCM (Dc size 150): MSE 0.01603982, SMSE 0.0153, MSLL -1.9679
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01659480, SMSE 0.0158, MSLL -1.9801
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01647180, SMSE 0.0157, MSLL -1.9657
 kj = 10
 gr = 0.7500
 GRBCM (Dc size 150): MSE 0.01514401, SMSE 0.0144, MSLL -1.9695
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01552512, SMSE 0.0148, MSLL -1.9826
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01548643, SMSE 0.0147, MSLL -1.9754
 kj = 11
 gr = 0.8000
 GRBCM (Dc size 150): MSE 0.01441326, SMSE 0.0137, MSLL -1.9492
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01486647, SMSE 0.0142, MSLL -1.9661
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01480535, SMSE 0.0141, MSLL -1.9511
 kj = 12
 gr = 0.8500
 GRBCM (Dc size 150): MSE 0.01348158, SMSE 0.0128, MSLL -1.9630


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GRBCM++ (VFE) (Dc size 150):
MSE 0.01387903, SMSE 0.0132, MSLL -1.9829
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01388269, SMSE 0.0132, MSLL -1.9677
kj = 13
gr = 0.9000
GRBCM (Dc size 150): MSE 0.01316646, SMSE 0.0125, MSLL -1.9460
GRBCM++ (VFE) (Dc size 150):
MSE 0.01351569, SMSE 0.0129, MSLL -1.9650
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01354017, SMSE 0.0129, MSLL -1.9562
kj = 14
gr = 0.9500
GRBCM (Dc size 150): MSE 0.01283052, SMSE 0.0122, MSLL -1.9393
GRBCM++ (VFE) (Dc size 150):
MSE 0.01307901, SMSE 0.0124, MSLL -1.9637
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01325807, SMSE 0.0126, MSLL -1.9463
kj = 15
gr = 1
GRBCM (Dc size 150): MSE 0.01267743, SMSE 0.0121, MSLL -1.9301
GRBCM++ (VFE) (Dc size 150):
MSE 0.01289921, SMSE 0.0123, MSLL -1.9508
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01297273, SMSE 0.0123, MSLL -1.9454
=====3=====
Optimizing hyps in training...
Linesearch      0; Value 2.582557e+04
Linesearch      1; Value 1.209224e+04
Linesearch      2; Value 6.421659e+03
Linesearch      3; Value 5.726784e+03
Linesearch      4; Value -2.649959e+03
Linesearch      5; Value -3.306634e+03
Linesearch      6; Value -3.394757e+03
Linesearch      7; Value -4.407108e+03
Linesearch      8; Value -4.546302e+03
Linesearch      9; Value -4.572753e+03
Linesearch     10; Value -4.573757e+03
Linesearch     11; Value -4.573838e+03
Linesearch     12; Value -4.573848e+03
Linesearch     13; Value -4.573849e+03
Linesearch     14; Value -4.573850e+03
Linesearch     15; Value -4.573850e+03
Linesearch     16; Value -4.573850e+03
Linesearch     17; Value -4.573850e+03
Linesearch     18; Value -4.573850e+03
Linesearch     19; Value -4.573850e+03
Linesearch     20; Value -4.573850e+03
Linesearch     21; Value -4.573850e+03
GRBCM (Dc size 150): MSE 0.01260542, SMSE 0.01199839, MSLL -1.95563677
Function evaluation    0; Value 7.541337e+05
Function evaluation    12; Value 7.494185e+05
Function evaluation    16; Value 6.758437e+05
Function evaluation    20; Value 6.397898e+05
Function evaluation    22; Value 6.197247e+05
Function evaluation    23; Value 5.891737e+05
Function evaluation    25; Value 5.733100e+05
Function evaluation    27; Value 5.644798e+05
Function evaluation    28; Value 5.553467e+05
Function evaluation    30; Value 5.504239e+05
Function evaluation    32; Value 5.437676e+05
Function evaluation    33; Value 5.372461e+05
Function evaluation    35; Value 5.314527e+05
Function evaluation    37; Value 5.286821e+05
Function evaluation    39; Value 5.249675e+05

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Function evaluation	41;	Value	5.199421e+05
Function evaluation	44;	Value	5.179744e+05
Function evaluation	45;	Value	5.158672e+05
Function evaluation	47;	Value	5.132880e+05
Function evaluation	49;	Value	5.117526e+05
Function evaluation	50;	Value	5.102715e+05
Function evaluation	52;	Value	5.080196e+05
Function evaluation	54;	Value	5.061600e+05
Function evaluation	56;	Value	5.050121e+05
Function evaluation	58;	Value	5.033504e+05
Function evaluation	60;	Value	5.020283e+05
Function evaluation	62;	Value	5.010531e+05
Function evaluation	64;	Value	4.995675e+05
Function evaluation	66;	Value	4.987955e+05
Function evaluation	68;	Value	4.983364e+05
Function evaluation	70;	Value	4.975856e+05
Function evaluation	72;	Value	4.971312e+05
Function evaluation	74;	Value	4.966238e+05
Function evaluation	76;	Value	4.962110e+05
Function evaluation	78;	Value	4.955189e+05
Function evaluation	80;	Value	4.952065e+05
Function evaluation	82;	Value	4.944729e+05
Function evaluation	84;	Value	4.940885e+05
Function evaluation	86;	Value	4.938006e+05
Function evaluation	88;	Value	4.931225e+05
Function evaluation	90;	Value	4.928994e+05
Function evaluation	92;	Value	4.925891e+05
Function evaluation	93;	Value	4.922708e+05
Function evaluation	95;	Value	4.918862e+05
Function evaluation	97;	Value	4.914243e+05
Function evaluation	99;	Value	4.911690e+05
VFE (Dc size 150): MSE 0.34377903, SMSE 0.32722395, MSL -0.56560340			
Function evaluation	0;	Value	1.209042e+04
Function evaluation	8;	Value	1.201105e+04
Function evaluation	12;	Value	1.098944e+04
Function evaluation	15;	Value	1.056781e+04
Function evaluation	17;	Value	1.030808e+04
Function evaluation	19;	Value	1.012572e+04
Function evaluation	20;	Value	9.965014e+03
Function evaluation	21;	Value	9.788319e+03
Function evaluation	23;	Value	9.621219e+03
Function evaluation	25;	Value	9.500257e+03
Function evaluation	26;	Value	9.412316e+03
Function evaluation	28;	Value	9.335965e+03
Function evaluation	30;	Value	9.282144e+03
Function evaluation	32;	Value	9.215456e+03
Function evaluation	34;	Value	9.179156e+03
Function evaluation	36;	Value	9.125963e+03
Function evaluation	38;	Value	9.055797e+03
Function evaluation	40;	Value	9.017487e+03
Function evaluation	41;	Value	8.979078e+03
Function evaluation	42;	Value	8.941869e+03
Function evaluation	44;	Value	8.911965e+03
Function evaluation	46;	Value	8.891290e+03
Function evaluation	48;	Value	8.873503e+03
Function evaluation	50;	Value	8.848262e+03
Function evaluation	52;	Value	8.824835e+03
Function evaluation	53;	Value	8.804194e+03
Function evaluation	54;	Value	8.783320e+03
Function evaluation	55;	Value	8.761653e+03
Function evaluation	57;	Value	8.746327e+03
Function evaluation	59;	Value	8.711508e+03
Function evaluation	61;	Value	8.689693e+03
Function evaluation	63;	Value	8.672106e+03
Function evaluation	65;	Value	8.649198e+03

Function evaluation 67; Value 8.638913e+03
 Function evaluation 69; Value 8.624933e+03
 Function evaluation 71; Value 8.609114e+03
 Function evaluation 73; Value 8.599317e+03
 Function evaluation 75; Value 8.582910e+03
 Function evaluation 77; Value 8.571080e+03
 Function evaluation 79; Value 8.564983e+03
 Function evaluation 81; Value 8.553440e+03
 Function evaluation 83; Value 8.539701e+03
 Function evaluation 85; Value 8.533634e+03
 Function evaluation 87; Value 8.524018e+03
 Function evaluation 89; Value 8.516663e+03
 Function evaluation 90; Value 8.508922e+03
 Function evaluation 92; Value 8.503472e+03
 Function evaluation 94; Value 8.496414e+03
 Function evaluation 95; Value 8.489380e+03
 Function evaluation 97; Value 8.484390e+03
 Function evaluation 99; Value 8.477674e+03
 SPSPG (Dc size 150): MSE 0.46082627, SMSE 0.43863466, MSLL -0.56903986
 GRBCM (VFE) (Dc size 150): MSE 0.01257153, SMSE 0.01196613, MSLL -1.98402330
 GRBCM (SPGP) (Dc size 150): MSE 0.01231244, SMSE 0.01171952, MSLL -1.98805066
 RBCM (Dc size 150): MSE 0.01254621, SMSE 0.0119, MSLL -1.8346
 BCM (Dc size 150): MSE 0.01314275, SMSE 0.0125, MSLL -2.1769
 PoE (Dc size 150): MSE 0.10196945, SMSE 0.0971, MSLL 3.7554
 GPoE (Dc size 150): MSE 0.01246089, SMSE 0.0119, MSLL -1.8357
 processing distance: 100/10000
 processing distance: 200/10000
 processing distance: 300/10000
 processing distance: 400/10000
 processing distance: 500/10000
 processing distance: 600/10000
 processing distance: 700/10000
 processing distance: 800/10000
 processing distance: 900/10000
 processing distance: 1000/10000
 processing distance: 1100/10000
 processing distance: 1200/10000
 processing distance: 1300/10000
 processing distance: 1400/10000
 processing distance: 1500/10000
 processing distance: 1600/10000
 processing distance: 1700/10000
 processing distance: 1800/10000
 processing distance: 1900/10000
 processing distance: 2000/10000
 processing distance: 2100/10000
 processing distance: 2200/10000
 processing distance: 2300/10000
 processing distance: 2400/10000
 processing distance: 2500/10000
 processing distance: 2600/10000
 processing distance: 2700/10000
 processing distance: 2800/10000
 processing distance: 2900/10000
 processing distance: 3000/10000
 processing distance: 3100/10000
 processing distance: 3200/10000
 processing distance: 3300/10000
 processing distance: 3400/10000
 processing distance: 3500/10000
 processing distance: 3600/10000
 processing distance: 3700/10000
 processing distance: 3800/10000
 processing distance: 3900/10000
 processing distance: 4000/10000

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processing distance: 4100/10000
processing distance: 4200/10000
processing distance: 4300/10000
processing distance: 4400/10000
processing distance: 4500/10000
processing distance: 4600/10000
processing distance: 4700/10000
processing distance: 4800/10000
processing distance: 4900/10000
processing distance: 5000/10000
processing distance: 5100/10000
processing distance: 5200/10000
processing distance: 5300/10000
processing distance: 5400/10000
processing distance: 5500/10000
processing distance: 5600/10000
processing distance: 5700/10000
processing distance: 5800/10000
processing distance: 5900/10000
processing distance: 6000/10000
processing distance: 6100/10000
processing distance: 6200/10000
processing distance: 6300/10000
processing distance: 6400/10000
processing distance: 6500/10000
processing distance: 6600/10000
processing distance: 6700/10000
processing distance: 6800/10000
processing distance: 6900/10000
processing distance: 7000/10000
processing distance: 7100/10000
processing distance: 7200/10000
processing distance: 7300/10000
processing distance: 7400/10000
processing distance: 7500/10000
processing distance: 7600/10000
processing distance: 7700/10000
processing distance: 7800/10000
processing distance: 7900/10000
processing distance: 8000/10000
processing distance: 8100/10000
processing distance: 8200/10000
processing distance: 8300/10000
processing distance: 8400/10000
processing distance: 8500/10000
processing distance: 8600/10000
processing distance: 8700/10000
processing distance: 8800/10000
processing distance: 8900/10000
processing distance: 9000/10000
processing distance: 9100/10000
processing distance: 9200/10000
processing distance: 9300/10000
processing distance: 9400/10000
processing distance: 9500/10000
processing distance: 9600/10000
processing distance: 9700/10000
processing distance: 9800/10000
processing distance: 9900/10000
processing distance: 10000/10000
kj = 1
gr = 0.3000
GRBCM (Dc size 150): MSE 0.08985974, SMSE 0.0855, MSLL -1.4800
GRBCM++ (VFE) (Dc size 150):
MSE 0.04926116, SMSE 0.0469, MSLL -1.5744

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GRBCM++ (SPGP) (Dc size 150):
 MSE 0.05589050, SMSE 0.0532, MSLL -1.5869
 kj = 2
 gr = 0.3500
 GRBCM (Dc size 150): MSE 0.06235844, SMSE 0.0594, MSLL -1.5740
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.04233645, SMSE 0.0403, MSLL -1.6429
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.04609913, SMSE 0.0439, MSLL -1.6544
 kj = 3
 gr = 0.4000
 GRBCM (Dc size 150): MSE 0.04557645, SMSE 0.0434, MSLL -1.6642
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.03567895, SMSE 0.0340, MSLL -1.7192
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.03605901, SMSE 0.0343, MSLL -1.7309
 kj = 4
 gr = 0.4500
 GRBCM (Dc size 150): MSE 0.03401892, SMSE 0.0324, MSLL -1.7423
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.02900204, SMSE 0.0276, MSLL -1.7888
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.02861337, SMSE 0.0272, MSLL -1.7982
 kj = 5
 gr = 0.5000
 GRBCM (Dc size 150): MSE 0.02808145, SMSE 0.0267, MSLL -1.8143
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.02426234, SMSE 0.0231, MSLL -1.8566
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.02448703, SMSE 0.0233, MSLL -1.8586
 kj = 6
 gr = 0.5500
 GRBCM (Dc size 150): MSE 0.02355599, SMSE 0.0224, MSLL -1.8763
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.02266813, SMSE 0.0216, MSLL -1.8828
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.02204703, SMSE 0.0210, MSLL -1.8959
 kj = 7
 gr = 0.6000
 GRBCM (Dc size 150): MSE 0.02056007, SMSE 0.0196, MSLL -1.9134
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.02026632, SMSE 0.0193, MSLL -1.9233
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01918001, SMSE 0.0183, MSLL -1.9403
 kj = 8
 gr = 0.6500
 GRBCM (Dc size 150): MSE 0.01814541, SMSE 0.0173, MSLL -1.9379
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01771062, SMSE 0.0169, MSLL -1.9600
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01724226, SMSE 0.0164, MSLL -1.9680
 kj = 9
 gr = 0.7000
 GRBCM (Dc size 150): MSE 0.01700347, SMSE 0.0162, MSLL -1.9408
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01698256, SMSE 0.0162, MSLL -1.9554
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01627094, SMSE 0.0155, MSLL -1.9748
 kj = 10
 gr = 0.7500
 GRBCM (Dc size 150): MSE 0.01542608, SMSE 0.0147, MSLL -1.9677
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01562202, SMSE 0.0149, MSLL -1.9768
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01493181, SMSE 0.0142, MSLL -1.9965

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kj = 11
gr = 0.8000
GRBCM (Dc size 150): MSE 0.01405632, SMSE 0.0134, MSLL -1.9770
GRBCM++ (VFE) (Dc size 150):
MSE 0.01458083, SMSE 0.0139, MSLL -1.9806
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01380011, SMSE 0.0131, MSLL -2.0075
kj = 12
gr = 0.8500
GRBCM (Dc size 150): MSE 0.01360595, SMSE 0.0130, MSLL -1.9752
GRBCM++ (VFE) (Dc size 150):
MSE 0.01372576, SMSE 0.0131, MSLL -1.9965
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01326084, SMSE 0.0126, MSLL -2.0101
kj = 13
gr = 0.9000
GRBCM (Dc size 150): MSE 0.01312055, SMSE 0.0125, MSLL -1.9728
GRBCM++ (VFE) (Dc size 150):
MSE 0.01342821, SMSE 0.0128, MSLL -1.9853
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01277363, SMSE 0.0122, MSLL -2.0111
kj = 14
gr = 0.9500
GRBCM (Dc size 150): MSE 0.01277594, SMSE 0.0122, MSLL -1.9576
GRBCM++ (VFE) (Dc size 150):
MSE 0.01272917, SMSE 0.0121, MSLL -1.9920
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01250465, SMSE 0.0119, MSLL -1.9913
kj = 15
gr = 1
GRBCM (Dc size 150): MSE 0.01260542, SMSE 0.0120, MSLL -1.9556
GRBCM++ (VFE) (Dc size 150):
MSE 0.01257153, SMSE 0.0120, MSLL -1.9840
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01231244, SMSE 0.0117, MSLL -1.9881
=====4=====
Optimizing hyps in training...
Linesearch      0; Value 2.495245e+04
Linesearch      1; Value 1.192853e+04
Linesearch      2; Value 6.000879e+03
Linesearch      3; Value 5.608080e+03
Linesearch      4; Value -3.245131e+03
Linesearch      5; Value -3.334369e+03
Linesearch      6; Value -3.535008e+03
Linesearch      7; Value -4.404544e+03
Linesearch      8; Value -4.531659e+03
Linesearch      9; Value -4.545213e+03
Linesearch     10; Value -4.550063e+03
Linesearch     11; Value -4.553632e+03
Linesearch     12; Value -4.553737e+03
Linesearch     13; Value -4.553741e+03
Linesearch     14; Value -4.553741e+03
Linesearch     15; Value -4.553741e+03
Linesearch     16; Value -4.553741e+03
Linesearch     17; Value -4.553741e+03
Linesearch     18; Value -4.553741e+03
Linesearch     19; Value -4.553741e+03
Linesearch     20; Value -4.553741e+03
Linesearch     21; Value -4.553741e+03
Linesearch     22; Value -4.553741e+03
Linesearch     23; Value -4.553741e+03
Linesearch     24; Value -4.553741e+03
Linesearch     25; Value -4.553741e+03
GRBCM (Dc size 150): MSE 0.01257367, SMSE 0.01196817, MSLL -1.95072472
Function evaluation      0; Value 7.273286e+05

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Function evaluation	12;	Value 7.229906e+05
Function evaluation	17;	Value 6.764411e+05
Function evaluation	20;	Value 6.425866e+05
Function evaluation	22;	Value 6.148942e+05
Function evaluation	24;	Value 5.979267e+05
Function evaluation	26;	Value 5.844341e+05
Function evaluation	27;	Value 5.685099e+05
Function evaluation	29;	Value 5.576400e+05
Function evaluation	31;	Value 5.523321e+05
Function evaluation	33;	Value 5.485740e+05
Function evaluation	35;	Value 5.439960e+05
Function evaluation	37;	Value 5.412452e+05
Function evaluation	39;	Value 5.388291e+05
Function evaluation	41;	Value 5.378023e+05
Function evaluation	43;	Value 5.347418e+05
Function evaluation	45;	Value 5.336436e+05
Function evaluation	47;	Value 5.318177e+05
Function evaluation	49;	Value 5.309035e+05
Function evaluation	51;	Value 5.295981e+05
Function evaluation	53;	Value 5.286946e+05
Function evaluation	55;	Value 5.270474e+05
Function evaluation	57;	Value 5.258224e+05
Function evaluation	58;	Value 5.244926e+05
Function evaluation	60;	Value 5.233882e+05
Function evaluation	62;	Value 5.224714e+05
Function evaluation	64;	Value 5.212862e+05
Function evaluation	66;	Value 5.204005e+05
Function evaluation	67;	Value 5.195583e+05
Function evaluation	69;	Value 5.190111e+05
Function evaluation	70;	Value 5.184747e+05
Function evaluation	71;	Value 5.179287e+05
Function evaluation	72;	Value 5.173529e+05
Function evaluation	74;	Value 5.166995e+05
Function evaluation	76;	Value 5.163443e+05
Function evaluation	78;	Value 5.160383e+05
Function evaluation	80;	Value 5.156470e+05
Function evaluation	82;	Value 5.153595e+05
Function evaluation	84;	Value 5.151004e+05
Function evaluation	86;	Value 5.147765e+05
Function evaluation	88;	Value 5.143805e+05
Function evaluation	90;	Value 5.141736e+05
Function evaluation	92;	Value 5.139354e+05
Function evaluation	94;	Value 5.136125e+05
Function evaluation	96;	Value 5.133783e+05
Function evaluation	98;	Value 5.131947e+05
Function evaluation	100;	Value 5.129058e+05
VFE (Dc size 150): MSE 0.37368778, SMSE 0.35569242, MSLL -0.52557347		
Function evaluation	0;	Value 1.161394e+04
Function evaluation	8;	Value 1.139851e+04
Function evaluation	10;	Value 1.128056e+04
Function evaluation	12;	Value 1.072488e+04
Function evaluation	15;	Value 1.039484e+04
Function evaluation	17;	Value 1.020227e+04
Function evaluation	19;	Value 1.000486e+04
Function evaluation	21;	Value 9.847490e+03
Function evaluation	24;	Value 9.775554e+03
Function evaluation	26;	Value 9.647100e+03
Function evaluation	28;	Value 9.540964e+03
Function evaluation	30;	Value 9.476288e+03
Function evaluation	32;	Value 9.404359e+03
Function evaluation	34;	Value 9.361512e+03
Function evaluation	36;	Value 9.318913e+03
Function evaluation	38;	Value 9.286628e+03
Function evaluation	40;	Value 9.265383e+03
Function evaluation	42;	Value 9.232850e+03

Function evaluation 44; Value 9.213001e+03
 Function evaluation 46; Value 9.189394e+03
 Function evaluation 47; Value 9.165399e+03
 Function evaluation 48; Value 9.140205e+03
 Function evaluation 50; Value 9.124080e+03
 Function evaluation 52; Value 9.094737e+03
 Function evaluation 54; Value 9.075354e+03
 Function evaluation 55; Value 9.057167e+03
 Function evaluation 56; Value 9.039078e+03
 Function evaluation 58; Value 9.012061e+03
 Function evaluation 60; Value 8.993667e+03
 Function evaluation 62; Value 8.966937e+03
 Function evaluation 64; Value 8.949001e+03
 Function evaluation 65; Value 8.929314e+03
 Function evaluation 67; Value 8.907024e+03
 Function evaluation 69; Value 8.892433e+03
 Function evaluation 71; Value 8.865242e+03
 Function evaluation 73; Value 8.852590e+03
 Function evaluation 75; Value 8.835598e+03
 Function evaluation 77; Value 8.822373e+03
 Function evaluation 79; Value 8.803841e+03
 Function evaluation 81; Value 8.791773e+03
 Function evaluation 83; Value 8.775844e+03
 Function evaluation 85; Value 8.767526e+03
 Function evaluation 87; Value 8.754516e+03
 Function evaluation 89; Value 8.745433e+03
 Function evaluation 90; Value 8.736602e+03
 Function evaluation 91; Value 8.727602e+03
 Function evaluation 92; Value 8.719303e+03
 Function evaluation 93; Value 8.710081e+03
 Function evaluation 95; Value 8.699875e+03
 Function evaluation 97; Value 8.687176e+03
 Function evaluation 98; Value 8.672891e+03
 Function evaluation 100; Value 8.656721e+03
 SPSP (Dc size 150): MSE 0.44279123, SMSE 0.42146811, MSLL -0.56228035
 GRBCM (VFE) (Dc size 150): MSE 0.01278409, SMSE 0.01216846, MSLL -1.95670773
 GRBCM (SPGP) (Dc size 150): MSE 0.01240935, SMSE 0.01181177, MSLL -1.98198955
 RBCM (Dc size 150): MSE 0.01268894, SMSE 0.0121, MSLL -1.8057
 BCM (Dc size 150): MSE 0.01318720, SMSE 0.0126, MSLL -2.1704
 PoE (Dc size 150): MSE 0.09685275, SMSE 0.0922, MSLL 3.3851
 GPoE (Dc size 150): MSE 0.01261829, SMSE 0.0120, MSLL -1.8029
 processing distance: 100/10000
 processing distance: 200/10000
 processing distance: 300/10000
 processing distance: 400/10000
 processing distance: 500/10000
 processing distance: 600/10000
 processing distance: 700/10000
 processing distance: 800/10000
 processing distance: 900/10000
 processing distance: 1000/10000
 processing distance: 1100/10000
 processing distance: 1200/10000
 processing distance: 1300/10000
 processing distance: 1400/10000
 processing distance: 1500/10000
 processing distance: 1600/10000
 processing distance: 1700/10000
 processing distance: 1800/10000
 processing distance: 1900/10000
 processing distance: 2000/10000
 processing distance: 2100/10000
 processing distance: 2200/10000
 processing distance: 2300/10000
 processing distance: 2400/10000

[illegible]

processing distance: 9000/10000
 processing distance: 9100/10000
 processing distance: 9200/10000
 processing distance: 9300/10000
 processing distance: 9400/10000
 processing distance: 9500/10000
 processing distance: 9600/10000
 processing distance: 9700/10000
 processing distance: 9800/10000
 processing distance: 9900/10000
 processing distance: 10000/10000
 kj = 1
 gr = 0.3000
 GRBCM (Dc size 150): MSE 0.07373933, SMSE 0.0702, MSLL -1.5141
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.05390936, SMSE 0.0513, MSLL -1.5772
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.05560037, SMSE 0.0529, MSLL -1.5794
 kj = 2
 gr = 0.3500
 GRBCM (Dc size 150): MSE 0.06144136, SMSE 0.0585, MSLL -1.5868
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.04650714, SMSE 0.0443, MSLL -1.6459
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.04651279, SMSE 0.0443, MSLL -1.6555
 kj = 3
 gr = 0.4000
 GRBCM (Dc size 150): MSE 0.04587743, SMSE 0.0437, MSLL -1.6749
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.03676567, SMSE 0.0350, MSLL -1.7217
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.03723318, SMSE 0.0354, MSLL -1.7236
 kj = 4
 gr = 0.4500
 GRBCM (Dc size 150): MSE 0.03501903, SMSE 0.0333, MSLL -1.7589
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.02974144, SMSE 0.0283, MSLL -1.8085
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.02948178, SMSE 0.0281, MSLL -1.8056
 kj = 5
 gr = 0.5000
 GRBCM (Dc size 150): MSE 0.02938119, SMSE 0.0280, MSLL -1.8033
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.02683626, SMSE 0.0255, MSLL -1.8503
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.02585180, SMSE 0.0246, MSLL -1.8593
 kj = 6
 gr = 0.5500
 GRBCM (Dc size 150): MSE 0.02440563, SMSE 0.0232, MSLL -1.8731
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.02269089, SMSE 0.0216, MSLL -1.9023
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.02269183, SMSE 0.0216, MSLL -1.8987
 kj = 7
 gr = 0.6000
 GRBCM (Dc size 150): MSE 0.02049987, SMSE 0.0195, MSLL -1.9163
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01962400, SMSE 0.0187, MSLL -1.9416
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01954882, SMSE 0.0186, MSLL -1.9407
 kj = 8
 gr = 0.6500
 GRBCM (Dc size 150): MSE 0.01829367, SMSE 0.0174, MSLL -1.9428
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01794791, SMSE 0.0171, MSLL -1.9615

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GRBCM++ (SPGP) (Dc size 150):
MSE 0.01790830, SMSE 0.0170, MSLL -1.9627
kj = 9
gr = 0.7000
GRBCM (Dc size 150): MSE 0.01664741, SMSE 0.0158, MSLL -1.9591
GRBCM++ (VFE) (Dc size 150):
MSE 0.01654960, SMSE 0.0158, MSLL -1.9732
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01620193, SMSE 0.0154, MSLL -1.9799
kj = 10
gr = 0.7500
GRBCM (Dc size 150): MSE 0.01547138, SMSE 0.0147, MSLL -1.9730
GRBCM++ (VFE) (Dc size 150):
MSE 0.01566137, SMSE 0.0149, MSLL -1.9803
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01548781, SMSE 0.0147, MSLL -1.9833
kj = 11
gr = 0.8000
GRBCM (Dc size 150): MSE 0.01424892, SMSE 0.0136, MSLL -1.9911
GRBCM++ (VFE) (Dc size 150):
MSE 0.01427557, SMSE 0.0136, MSLL -2.0045
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01406679, SMSE 0.0134, MSLL -2.0063
kj = 12
gr = 0.8500
GRBCM (Dc size 150): MSE 0.01352382, SMSE 0.0129, MSLL -1.9934
GRBCM++ (VFE) (Dc size 150):
MSE 0.01352553, SMSE 0.0129, MSLL -2.0136
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01331280, SMSE 0.0127, MSLL -2.0127
kj = 13
gr = 0.9000
GRBCM (Dc size 150): MSE 0.01292721, SMSE 0.0123, MSLL -1.9883
GRBCM++ (VFE) (Dc size 150):
MSE 0.01304856, SMSE 0.0124, MSLL -2.0038
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01273720, SMSE 0.0121, MSLL -2.0116
kj = 14
gr = 0.9500
GRBCM (Dc size 150): MSE 0.01266939, SMSE 0.0121, MSLL -1.9692
GRBCM++ (VFE) (Dc size 150):
MSE 0.01291541, SMSE 0.0123, MSLL -1.9763
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01257218, SMSE 0.0120, MSLL -1.9895
kj = 15
gr = 1
GRBCM (Dc size 150): MSE 0.01257367, SMSE 0.0120, MSLL -1.9507
GRBCM++ (VFE) (Dc size 150):
MSE 0.01278409, SMSE 0.0122, MSLL -1.9567
GRBCM++ (SPGP) (Dc size 150):
MSE 0.01240935, SMSE 0.0118, MSLL -1.9820
=====5=====
Optimizing hyps in training...
Linesearch      0; Value 2.435948e+04
Linesearch      1; Value 1.184032e+04
Linesearch      2; Value 5.975864e+03
Linesearch      3; Value 5.618184e+03
Linesearch      4; Value -2.346979e+03
Linesearch      5; Value -3.169518e+03
Linesearch      6; Value -3.305710e+03
Linesearch      7; Value -3.426029e+03
Linesearch      8; Value -3.844988e+03
Linesearch      9; Value -4.349581e+03
Linesearch     10; Value -4.552160e+03
Linesearch     11; Value -4.563185e+03

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LineSearch      12; Value -4.564395e+03
LineSearch      13; Value -4.566200e+03
LineSearch      14; Value -4.566593e+03
LineSearch      15; Value -4.566595e+03
LineSearch      16; Value -4.566595e+03
LineSearch      17; Value -4.566595e+03
LineSearch      18; Value -4.566595e+03
LineSearch      19; Value -4.566595e+03
LineSearch      20; Value -4.566595e+03
LineSearch      21; Value -4.566595e+03
LineSearch      22; Value -4.566595e+03
GRBCM (Dc size 150): MSE 0.01240993, SMSE 0.01181231, MSLL -1.96659440
Function evaluation      0; Value 7.413348e+05
Function evaluation      13; Value 7.003969e+05
Function evaluation      14; Value 6.393704e+05
Function evaluation      16; Value 6.063719e+05
Function evaluation      18; Value 5.820468e+05
Function evaluation      20; Value 5.665873e+05
Function evaluation      22; Value 5.548630e+05
Function evaluation      24; Value 5.477949e+05
Function evaluation      26; Value 5.423127e+05
Function evaluation      28; Value 5.377375e+05
Function evaluation      30; Value 5.349731e+05
Function evaluation      32; Value 5.321149e+05
Function evaluation      34; Value 5.305747e+05
Function evaluation      36; Value 5.285404e+05
Function evaluation      37; Value 5.264078e+05
Function evaluation      39; Value 5.249596e+05
Function evaluation      41; Value 5.223597e+05
Function evaluation      43; Value 5.215322e+05
Function evaluation      45; Value 5.206437e+05
Function evaluation      47; Value 5.193371e+05
Function evaluation      49; Value 5.182169e+05
Function evaluation      51; Value 5.172460e+05
Function evaluation      53; Value 5.166703e+05
Function evaluation      55; Value 5.158963e+05
Function evaluation      57; Value 5.153477e+05
Function evaluation      59; Value 5.147297e+05
Function evaluation      60; Value 5.140551e+05
Function evaluation      62; Value 5.137048e+05
Function evaluation      63; Value 5.133182e+05
Function evaluation      65; Value 5.124616e+05
Function evaluation      67; Value 5.118959e+05
Function evaluation      69; Value 5.112912e+05
Function evaluation      70; Value 5.106275e+05
Function evaluation      72; Value 5.102242e+05
Function evaluation      74; Value 5.093772e+05
Function evaluation      76; Value 5.087734e+05
Function evaluation      78; Value 5.082126e+05
Function evaluation      80; Value 5.071913e+05
Function evaluation      82; Value 5.068480e+05
Function evaluation      84; Value 5.064209e+05
Function evaluation      86; Value 5.061573e+05
Function evaluation      88; Value 5.057403e+05
Function evaluation      90; Value 5.054242e+05
Function evaluation      92; Value 5.051421e+05
Function evaluation      94; Value 5.047945e+05
Function evaluation      96; Value 5.040619e+05
Function evaluation      98; Value 5.036710e+05
Function evaluation     100; Value 5.033298e+05
VFE (Dc size 150): MSE 0.35400207, SMSE 0.33695470, MSLL -0.54679777
Function evaluation      0; Value 1.184067e+04
Function evaluation      9; Value 1.141055e+04
Function evaluation     10; Value 1.098025e+04
Function evaluation     11; Value 1.077812e+04

```

Function evaluation 13; Value 1.033302e+04
 Function evaluation 15; Value 1.012674e+04
 Function evaluation 17; Value 9.947207e+03
 Function evaluation 19; Value 9.763196e+03
 Function evaluation 20; Value 9.593534e+03
 Function evaluation 22; Value 9.467632e+03
 Function evaluation 24; Value 9.355456e+03
 Function evaluation 26; Value 9.303363e+03
 Function evaluation 28; Value 9.239611e+03
 Function evaluation 29; Value 9.198911e+03
 Function evaluation 31; Value 9.115554e+03
 Function evaluation 33; Value 9.055232e+03
 Function evaluation 34; Value 8.998185e+03
 Function evaluation 36; Value 8.965161e+03
 Function evaluation 38; Value 8.935549e+03
 Function evaluation 39; Value 8.904013e+03
 Function evaluation 41; Value 8.869141e+03
 Function evaluation 43; Value 8.846734e+03
 Function evaluation 44; Value 8.823088e+03
 Function evaluation 45; Value 8.800176e+03
 Function evaluation 47; Value 8.779470e+03
 Function evaluation 49; Value 8.761893e+03
 Function evaluation 50; Value 8.744939e+03
 Function evaluation 52; Value 8.730483e+03
 Function evaluation 53; Value 8.716081e+03
 Function evaluation 55; Value 8.706714e+03
 Function evaluation 57; Value 8.691402e+03
 Function evaluation 59; Value 8.680112e+03
 Function evaluation 61; Value 8.673599e+03
 Function evaluation 63; Value 8.664435e+03
 Function evaluation 65; Value 8.659968e+03
 Function evaluation 67; Value 8.656622e+03
 Function evaluation 69; Value 8.651797e+03
 Function evaluation 71; Value 8.648041e+03
 Function evaluation 73; Value 8.643016e+03
 Function evaluation 74; Value 8.638731e+03
 Function evaluation 75; Value 8.633826e+03
 Function evaluation 77; Value 8.626460e+03
 Function evaluation 78; Value 8.619653e+03
 Function evaluation 80; Value 8.613915e+03
 Function evaluation 82; Value 8.606866e+03
 Function evaluation 84; Value 8.601356e+03
 Function evaluation 86; Value 8.592415e+03
 Function evaluation 87; Value 8.582642e+03
 Function evaluation 90; Value 8.571336e+03
 Function evaluation 92; Value 8.562548e+03
 Function evaluation 94; Value 8.556984e+03
 Function evaluation 96; Value 8.552513e+03
 Function evaluation 98; Value 8.545036e+03
 Function evaluation 100; Value 8.542187e+03
 SPSPG (Dc size 150): MSE 0.44057428, SMSE 0.41935792, MSLL -0.56608380
 GRBCM (VFE) (Dc size 150): MSE 0.01267798, SMSE 0.01206746, MSLL -1.96755443
 GRBCM (SPGP) (Dc size 150): MSE 0.01243119, SMSE 0.01183256, MSLL -1.96220512
 RBCM (Dc size 150): MSE 0.01276805, SMSE 0.0122, MSLL -1.8032
 BCM (Dc size 150): MSE 0.01332427, SMSE 0.0127, MSLL -2.1660
 PoE (Dc size 150): MSE 0.09983053, SMSE 0.0950, MSLL 3.6090
 GPoE (Dc size 150): MSE 0.01268752, SMSE 0.0121, MSLL -1.8034
 processing distance: 100/10000
 processing distance: 200/10000
 processing distance: 300/10000
 processing distance: 400/10000
 processing distance: 500/10000
 processing distance: 600/10000
 processing distance: 700/10000
 processing distance: 800/10000

[illegible]

processing distance: 7400/10000
 processing distance: 7500/10000
 processing distance: 7600/10000
 processing distance: 7700/10000
 processing distance: 7800/10000
 processing distance: 7900/10000
 processing distance: 8000/10000
 processing distance: 8100/10000
 processing distance: 8200/10000
 processing distance: 8300/10000
 processing distance: 8400/10000
 processing distance: 8500/10000
 processing distance: 8600/10000
 processing distance: 8700/10000
 processing distance: 8800/10000
 processing distance: 8900/10000
 processing distance: 9000/10000
 processing distance: 9100/10000
 processing distance: 9200/10000
 processing distance: 9300/10000
 processing distance: 9400/10000
 processing distance: 9500/10000
 processing distance: 9600/10000
 processing distance: 9700/10000
 processing distance: 9800/10000
 processing distance: 9900/10000
 processing distance: 10000/10000
 kj = 1
 gr = 0.3000
 GRBCM (Dc size 150): MSE 0.08438643, SMSE 0.0803, MSLL -1.4802
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.05430907, SMSE 0.0517, MSLL -1.5503
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.05778295, SMSE 0.0550, MSLL -1.5659
 kj = 2
 gr = 0.3500
 GRBCM (Dc size 150): MSE 0.05639249, SMSE 0.0537, MSLL -1.6176
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.04073539, SMSE 0.0388, MSLL -1.6645
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.04276373, SMSE 0.0407, MSLL -1.6792
 kj = 3
 gr = 0.4000
 GRBCM (Dc size 150): MSE 0.03973317, SMSE 0.0378, MSLL -1.7186
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.03279330, SMSE 0.0312, MSLL -1.7434
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.03312330, SMSE 0.0315, MSLL -1.7530
 kj = 4
 gr = 0.4500
 GRBCM (Dc size 150): MSE 0.03171281, SMSE 0.0302, MSLL -1.7991
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.02924390, SMSE 0.0278, MSLL -1.8135
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.02843066, SMSE 0.0271, MSLL -1.8291
 kj = 5
 gr = 0.5000
 GRBCM (Dc size 150): MSE 0.02636145, SMSE 0.0251, MSLL -1.8583
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.02496073, SMSE 0.0238, MSLL -1.8731
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.02397708, SMSE 0.0228, MSLL -1.8876
 kj = 6
 gr = 0.5500
 GRBCM (Dc size 150): MSE 0.02291914, SMSE 0.0218, MSLL -1.8983

GRBCM++ (VFE) (Dc size 150):
 MSE 0.02117163, SMSE 0.0202, MSLL -1.9231
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.02086516, SMSE 0.0199, MSLL -1.9320
 kj = 7
 gr = 0.6000
 GRBCM (Dc size 150): MSE 0.01906484, SMSE 0.0181, MSLL -1.9412
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01843045, SMSE 0.0175, MSLL -1.9565
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01831077, SMSE 0.0174, MSLL -1.9600
 kj = 8
 gr = 0.6500
 GRBCM (Dc size 150): MSE 0.01712515, SMSE 0.0163, MSLL -1.9636
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01734981, SMSE 0.0165, MSLL -1.9669
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01654562, SMSE 0.0157, MSLL -1.9774
 kj = 9
 gr = 0.7000
 GRBCM (Dc size 150): MSE 0.01556757, SMSE 0.0148, MSLL -1.9896
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01594446, SMSE 0.0152, MSLL -1.9848
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01528047, SMSE 0.0145, MSLL -1.9904
 kj = 10
 gr = 0.7500
 GRBCM (Dc size 150): MSE 0.01481407, SMSE 0.0141, MSLL -1.9927
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01499191, SMSE 0.0143, MSLL -1.9976
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01478674, SMSE 0.0141, MSLL -1.9893
 kj = 11
 gr = 0.8000
 GRBCM (Dc size 150): MSE 0.01376916, SMSE 0.0131, MSLL -2.0158
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01390779, SMSE 0.0132, MSLL -2.0179
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01387972, SMSE 0.0132, MSLL -2.0094
 kj = 12
 gr = 0.8500
 GRBCM (Dc size 150): MSE 0.01291976, SMSE 0.0123, MSLL -2.0246
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01322800, SMSE 0.0126, MSLL -2.0256
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01302738, SMSE 0.0124, MSLL -2.0154
 kj = 13
 gr = 0.9000
 GRBCM (Dc size 150): MSE 0.01261762, SMSE 0.0120, MSLL -2.0122
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01279980, SMSE 0.0122, MSLL -2.0167
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01264687, SMSE 0.0120, MSLL -2.0068
 kj = 14
 gr = 0.9500
 GRBCM (Dc size 150): MSE 0.01238591, SMSE 0.0118, MSLL -1.9952
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01275656, SMSE 0.0121, MSLL -1.9850
 GRBCM++ (SPGP) (Dc size 150):
 MSE 0.01246825, SMSE 0.0119, MSLL -1.9856
 kj = 15
 gr = 1
 GRBCM (Dc size 150): MSE 0.01240993, SMSE 0.0118, MSLL -1.9666
 GRBCM++ (VFE) (Dc size 150):
 MSE 0.01267798, SMSE 0.0121, MSLL -1.9676


```
GRBCM++ (SPGP) (Dc size 150):  
MSE 0.01243119, SMSE 0.0118, MSLL -1.9622
```

Display results

```
grbcm0_smse = mean(grbcm0_smse_rec(1:kti)); grbcm0_msll = mean(grbcm0_msll_rec(1:kti));  
rbcm0_smse = mean(rbcm0_smse_rec(1:kti)); rbcm0_msll = mean(rbcm0_msll_rec(1:kti));  
bcm0_smse = mean(bcm0_smse_rec(1:kti)); bcm0_msll = mean(bcm0_msll_rec(1:kti));  
poe0_smse = mean(poe0_smse_rec(1:kti)); poe0_msll = mean(poe0_msll_rec(1:kti));  
gpoe0_smse = mean(gpoe0_smse_rec(1:kti)); gpoe0_msll = mean(gpoe0_msll_rec(1:kti));  
vfe0_smse = mean(vfe0_smse_rec(1:kti)); vfe0_msll = mean(vfe0_msll_rec(1:kti));  
spgp0_smse = mean(spgp0_smse_rec(1:kti)); spgp0_msll = mean(spgp0_msll_rec(1:kti));
```

```
fprintf('GRBCM: %6.8f, %6.4f', grbcm0_smse, grbcm0_msll);
```

```
GRBCM: 0.01193831, -1.9591
```

```
fprintf('RBCM: %6.8f, %6.4f', rbcm0_smse, rbcm0_msll);
```

```
RBCM: 0.01215734, -1.8057
```

```
fprintf('BCM: %6.8f, %6.4f', bcm0_smse, bcm0_msll);
```

```
BCM: 0.01262999, -2.1687
```

```
fprintf('GPoE: %6.8f, %6.4f', gpoe0_smse, gpoe0_msll);
```

```
GPoE: 0.01205650, -1.8071
```

```
fprintf('PoE: %6.8f, %6.4f', poe0_smse, poe0_msll);
```

```
PoE: 0.09493003, 3.6008
```

```
fprintf('VFE: %6.8f, %6.4f', vfe0_smse, vfe0_msll);
```

```
VFE: 0.33373569, -0.5513
```

```
fprintf('SPGP: %6.8f, %6.4f', spgp0_smse, spgp0_msll);
```

```
SPGP: 0.42298360, -0.5634
```

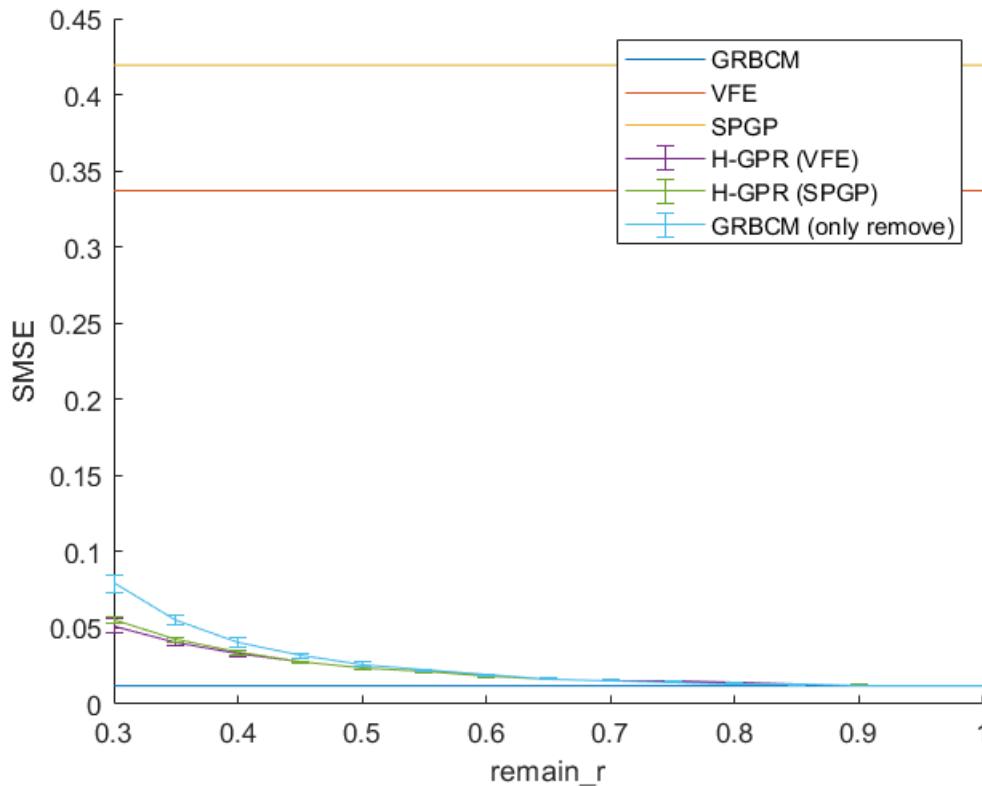
```
aamse = grbcm2_gr_smse(1:kti,:);  
aamsll = grbcm2_gr_msll(1:kti,:);  
mmse = mean(aamse);  
mstd = std(aamse);  
mmse_ro = mean(grbcm_gr_smse(1:kti,:));  
mstd_ro = std(grbcm_gr_smse(1:kti,:));  
mmse_sp = mean(grbcm2_spgp_gr_smse(1:kti,:));  
mstd_sp = std(grbcm2_spgp_gr_smse(1:kti,:));
```

```
figure; hold on;  
plot([min(grls), max(grls)], [grbcm0_smse,grbcm0_smse]);  
plot([min(grls), max(grls)], [vfeSMSE_b1,vfeSMSE_b1]);
```

```

plot([min(grls), max(grls)], [spgpSMSE_b1,spgpSMSE_b1]);
errorbar(grls, mmse, mstd);
errorbar(grls, mmse_sp, mstd_sp);
errorbar(grls, mmse_ro, mstd_ro);
legend('GRBCM', 'VFE', 'SPGP', 'H-GPR (VFE)', 'H-GPR (SPGP)', 'GRBCM (only remove)');
xlabel('remain\_r'); ylabel('SMSE');

```

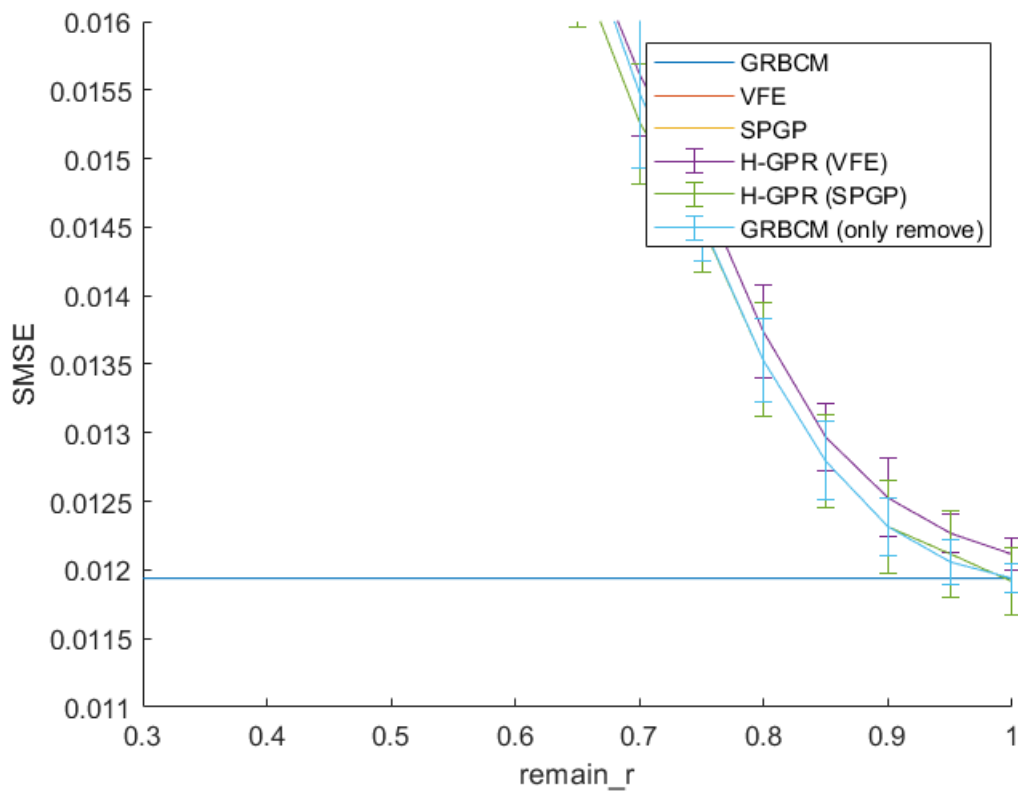


```

aamse = grbcm2_gr_smse(1:kti,:);
aamsll = grbcm2_gr_msl1(1:kti,:);
mmse = mean(aamse);
mstd = std(aamse);
mmse_ro = mean(grbcm_gr_smse(1:kti,:));
mstd_ro = std(grbcm_gr_smse(1:kti,:));
mmse_sp = mean(grbcm2_spgp_gr_smse(1:kti,:));
mstd_sp = std(grbcm2_spgp_gr_smse(1:kti,:));

figure; hold on;
plot([min(grls), max(grls)], [grbcm0_smse,grbcm0_smse]);
plot([min(grls), max(grls)], [vfeSMSE_b1,vfeSMSE_b1]);
plot([min(grls), max(grls)], [spgpSMSE_b1,spgpSMSE_b1]);
errorbar(grls, mmse, mstd);
errorbar(grls, mmse_sp, mstd_sp);
errorbar(grls, mmse_ro, mstd_ro);
legend('GRBCM', 'VFE', 'SPGP', 'H-GPR (VFE)', 'H-GPR (SPGP)', 'GRBCM (only remove)');
xlabel('remain\_r'); ylabel('SMSE');
ylim([0.011, 0.016]);

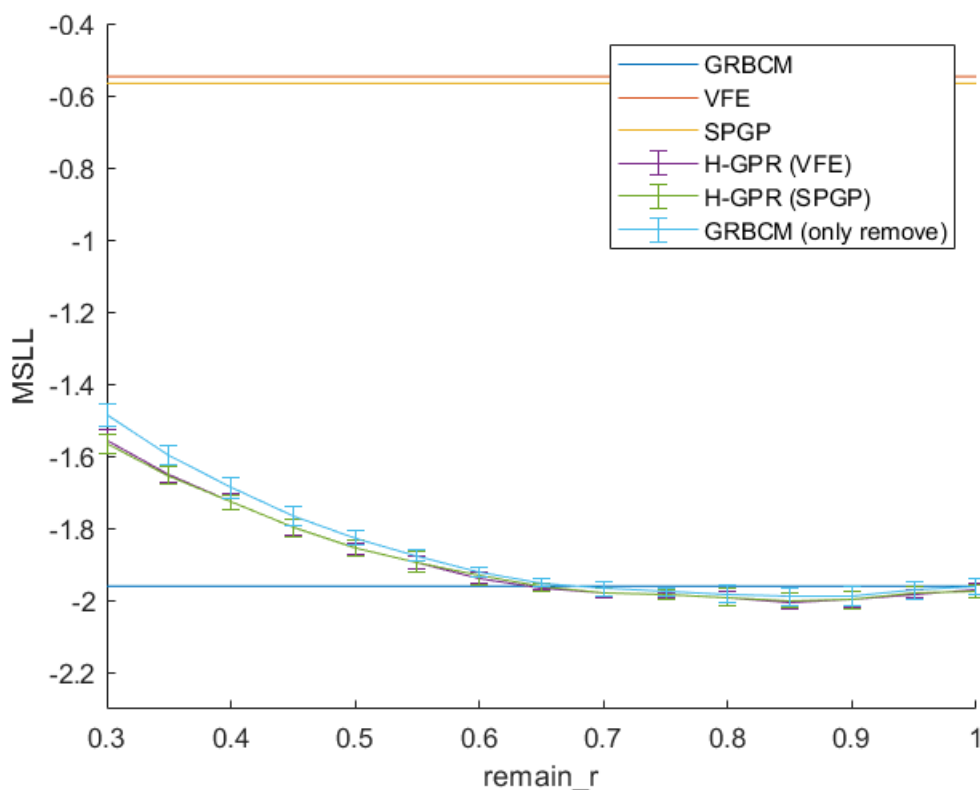
```



```

mmsll = mean(aamsll);
mstd = std(aamsll);
mmsll_ro = mean(grbcm_gr_msl1(1:kti,:));
mstd_ro = std(grbcm_gr_msl1(1:kti,:));
mmsll_sp = mean(grbcm2_spgp_gr_msl1(1:kti,:));
mstd_sp = std(grbcm2_spgp_gr_msl1(1:kti,:));
figure; hold on;
plot([min(grls), max(grls)], [grbcm0_msl1,grbcm0_msl1]);
plot([min(grls), max(grls)], [vfeMSLL_b1,vfeMSLL_b1]);
plot([min(grls), max(grls)], [spgpMSLL_b1,spgpMSLL_b1]);
errorbar(grls, mmsll, mstd);
errorbar(grls, mmsll_sp, mstd_sp);
errorbar(grls, mmsll_ro, mstd_ro);
legend('GRBCM', 'VFE', 'SPGP', 'H-GPR (VFE)', 'H-GPR (SPGP)', 'GRBCM (only remove)');
xlabel('remain\_r'); ylabel('MSLL');
ylim([-2.3, -0.4]);

```



```
fprintf('Best SMSE (GRBCM+VFE, dcs %d, ecs %d): %.6f\n', dcs, ecs, min(mmse));
```

```
Best SMSE (GRBCM+VFE, dcs 150, ecs 150): 0.01211143
```

```
fprintf('Best MSLL (GRBCM+VFE, dcs %d, ecs %d): %.6f\n', dcs, ecs, min(mmsll));
```

```
Best MSLL (GRBCM+VFE, dcs 150, ecs 150): -2.00461553
```

```
fprintf('Best SMSE (GRBCM+SPGP, dcs %d, ecs %d): %.6f\n', dcs, ecs, min(mmse_sp));
```

```
Best SMSE (GRBCM+SPGP, dcs 150, ecs 150): 0.01191665
```

```
fprintf('Best MSLL (GRBCM+SPGP, dcs %d, ecs %d): %.6f\n', dcs, ecs, min(mmsll_sp));
```

```
Best MSLL (GRBCM+SPGP, dcs 150, ecs 150): -1.99843931
```

Experiment II: re-balancing sizes of dcs and ecs

```
% hyp.cov = log([ones(d,1)*ell;sf2]); hyp.lik = log(sn2); hyp.mean = [];
opts.numOptFC = 50 ;
opts.xvec = xvec;
opts.yvec = yvec;
opts.grbcm_baseline = 0;
opts.global_index = ones(n,1);
% opts.inffunc = @infGaussLik; opts.meanfunc = meanfunc; opts.likfunc = likfunc;
```

```

opts.covfunc = covfunc;
covfuncF = {@apxSparse, {opts.covfunc}, []};
opts.covfuncF = covfuncF;
opts.compute_hyp = 0;

```

Heuristically rebalancing GRBCM and VFE/SPGP budget based on validating performance.

Obtain results on the validation set.

```

% default partition
dcs_ecs_r = 0.5;
dcs = round(ttcs*dcs_ecs_r) % size of the communication set

```

```

dcs = 150

```

```

ecs = ttcs - dcs % size of other experts

```

```

ecs = 150

```

```

n_per = dcs ; % size of Dc
mn = round(n / ecs); % mn is the number of experts (normal)
Indics = randperm(n) ;
I_com = Indics(1:n_per) ; % randomly select communication set
[idx, C] = kmeans(xvec, mn, 'MaxIter', km_iters);

```

```

% hyp.cov = log([ones(d,1)*ell;sf2]); hyp.lik = log(sn2); hyp.mean = [];
opts.numOptFC = 30 ;
opts.Ms = mn+1;
opts.xvec = xvec;
opts.yvec = yvec;
opts.induce_size = dcs;
opts.grbcm_baseline = 0;
opts.global_index = ones(n,1);
opts.I_com = I_com;
% opts.inffunc = @infGaussLik; opts.meanfunc = meanfunc; opts.likfunc = likfunc;
opts.covfunc = covfunc;
covfuncF = {@apxSparse, {opts.covfunc}, xvec(I_com,:)};
opts.covfuncF = covfuncF;
opts.compute_hyp = 0;

```

```

g_opts = opts;
g_opts.compute_hyp = 1;
g_opts.grbcm_baseline = 1;
g_opts.global_index = ones(n,1);
g_models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,g_opts);

```

```

Optimizing hyps in training...
Linesearch      0; Value 2.554155e+04
Linesearch      1; Value 1.203778e+04
Linesearch      2; Value 6.205577e+03

```

```

Linesearch      3; Value 5.647150e+03
Linesearch      4; Value -2.856793e+03
Linesearch      5; Value -3.374244e+03
Linesearch      6; Value -4.417077e+03
Linesearch      7; Value -4.537702e+03
Linesearch      8; Value -4.552633e+03
Linesearch      9; Value -4.555153e+03
Linesearch     10; Value -4.555334e+03
Linesearch     11; Value -4.555496e+03
Linesearch     12; Value -4.555586e+03
Linesearch     13; Value -4.555596e+03
Linesearch     14; Value -4.555596e+03
Linesearch     15; Value -4.555596e+03
Linesearch     16; Value -4.555596e+03
Linesearch     17; Value -4.555596e+03
Linesearch     18; Value -4.555596e+03
Linesearch     19; Value -4.555596e+03
Linesearch     20; Value -4.555596e+03
Linesearch     21; Value -4.555596e+03

```

```

opts.hyp = g_models{1}.hyp;
g_opts.hyp = g_models{1}.hyp;
[tmu,ts2, ~] = aggregation_predict(xvec_val,g_models,'GRBCM', 1, g_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[grbcmMSE,grbcmSMSE,grbcmMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_val, ori_yvec_val, tmu,
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'GRBCM', n_per, grbcmMSE,grbcmSMSE,grbcmMSLL);

```

```
GRBCM (Dc size 150): MSE 0.01239359, SMSE 0.01188868, MSLL -1.98765624
```

```
g_opts.compute_hyp = 0;
```

```

% % VFE Baseline
vfe_opts = opts;
vfe_opts.induce_type = 'VFE_opt';
xu = xvec(I_com, :);
inffunc = @(varargin) infGaussLik(varargin{:}, struct('s', 0.0));
vfe_hyp = opts.hyp;
vfe_hyp.xu = xu;
[vfe_hyp, tmp_nlls] = minimize(vfe_hyp,@sp_gp,-vfe_opts.induce_step,inffunc,meanfunc,covfuncF,I_com);

```

```

Function evaluation      0; Value 7.229444e+05
Function evaluation     11; Value 7.212137e+05
Function evaluation     17; Value 6.823985e+05
Function evaluation     18; Value 6.495202e+05
Function evaluation     20; Value 5.965361e+05
Function evaluation     22; Value 5.740223e+05
Function evaluation     24; Value 5.571046e+05
Function evaluation     26; Value 5.415440e+05
Function evaluation     28; Value 5.316005e+05
Function evaluation     30; Value 5.265335e+05
Function evaluation     32; Value 5.230467e+05
Function evaluation     33; Value 5.196960e+05
Function evaluation     35; Value 5.142292e+05
Function evaluation     38; Value 5.122838e+05
Function evaluation     40; Value 5.098819e+05
Function evaluation     41; Value 5.074446e+05

```

```

Function evaluation    43; Value 5.063233e+05
Function evaluation    45; Value 5.055247e+05
Function evaluation    47; Value 5.044619e+05
Function evaluation    49; Value 5.035190e+05
Function evaluation    51; Value 5.026623e+05
Function evaluation    53; Value 5.020834e+05
Function evaluation    55; Value 5.011335e+05
Function evaluation    57; Value 5.008378e+05
Function evaluation    59; Value 5.001978e+05
Function evaluation    61; Value 4.997586e+05
Function evaluation    63; Value 4.994321e+05
Function evaluation    65; Value 4.984389e+05
Function evaluation    67; Value 4.981917e+05
Function evaluation    68; Value 4.979481e+05
Function evaluation    70; Value 4.975324e+05
Function evaluation    71; Value 4.971176e+05
Function evaluation    73; Value 4.969174e+05
Function evaluation    75; Value 4.966389e+05
Function evaluation    76; Value 4.963923e+05
Function evaluation    78; Value 4.960597e+05
Function evaluation    80; Value 4.956749e+05
Function evaluation    82; Value 4.954818e+05
Function evaluation    84; Value 4.951766e+05
Function evaluation    86; Value 4.950957e+05
Function evaluation    88; Value 4.948565e+05
Function evaluation    90; Value 4.945397e+05
Function evaluation    92; Value 4.943325e+05
Function evaluation    94; Value 4.938526e+05
Function evaluation    96; Value 4.936584e+05
Function evaluation    98; Value 4.932595e+05
Function evaluation   100; Value 4.931425e+05

```

```

vfe_opts.hyp = opts.hyp;
vfe_opts.xu = vfe_hyp.xu;
vfe_opts.inffunc = @infGaussLik; vfe_opts.meanfunc = meanfunc; vfe_opts.covfuncF = covfuncF; vfe_opts.covfunc = covfunc;
[tmu, ts2] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, xvec_val);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[vfeMSE,vfeSMSE,vfeMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_val, ori_yvec_val, tmu, ts2);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'VFE', n_per, vfeMSE,vfeSMSE,vfeMSLL);

```

```
VFE (Dc size 150): MSE 0.34919698, SMSE 0.33497076, MSLL -0.53739727
```

```

% vfe0_smse_rec(ki) = vfeSMSE; vfe0_msll_rec(ki) = vfeMSLL;
[yu, su] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, vfe_opts.xu);
vfe_opts.yu = yu; vfe_opts.su = su;

```

```

models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,vfe_opts); % use hyp of vfe
[tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_val,models,vfe_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_val, ori_yvec_val, tmu, ts2);
fprintf('%s (Dc size %d): \r\nMSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM++ (VFE)', n_per, MSE, SMSE, MSLL);

```

GRBCM++ (VFE) (Dc size 150):
MSE 0.01226882, SMSE 0.0118, MSLL -2.0070

```
sig_temp = 10;  
dcs_ecs_r = sigmoid(-MSLL+grbcmMSLL, sig_temp)
```

```
dcs_ecs_r = 0.5483
```

```
dcs = round(ttcs*dcs_ecs_r)
```

```
dcs = 164
```

```
ecs = ttcs - dcs
```

```
ecs = 136
```

```
m = round(n / ecs)
```

```
m = 74
```

```
n_per = dcs ; % size of Dc  
Indics = randperm(n) ;  
I_com = Indics(1:n_per) ; % randomly select communication set  
[idx, C] = kmeans(xvec, m, 'MaxIter', km_iters);  
  
% hyp.cov = log([ones(d,1)*e11;sf2]); hyp.lik = log(sn2); hyp.mean = [];  
opts.numOptFC = 30 ;  
opts.Ms = m+1;  
opts.xvec = xvec;  
opts.yvec = yvec;  
opts.induce_size = dcs;  
opts.grbcm_baseline = 0;  
opts.global_index = ones(n,1);  
opts.I_com = I_com;  
% opts.inffunc = @infGaussLik; opts.meanfunc = meanfunc; opts.likfunc = likfunc;  
opts.covfunc = covfunc;  
covfuncF = {@apxSparse, {opts.covfunc}, xvec(I_com,:)};  
opts.covfuncF = covfuncF;  
opts.compute_hyp = 0;
```

```
g_opts = opts;  
g_opts.compute_hyp = 0;  
g_opts.grbcm_baseline = 1;  
g_opts.global_index = ones(n,1);  
g_models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,g_opts);  
opts.hyp = g_models{1}.hyp;  
g_opts.hyp = g_models{1}.hyp;  
[tmu,ts2, ~] = aggregation_predict(xvec_test,g_models,'GRBCM', 1, g_opts);  
if ynorm==1  
    tmu = tmu * norm_fstd + norm_fmean;  
    ts2 = ts2 * norm_fstd^2;  
end
```



```
[grbcmMSE,grbcmSMSE,grbcmMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tn
fprintf('%s (Dc size %d): MSE %.8f, SMSE %.8f, MSLL %.8f\r\n', 'GRBCM', n_per, grbcmMSE,grbcm
```

```
GRBCM (Dc size 164): MSE 0.01256733, SMSE 0.01196213, MSLL -1.96530623
```

```
% grbcm0_smse_rec(ki) = grbcmSMSE; grbcm0_msll_rec(ki) = grbcmMSLL;
g_opts.compute_hyp = 0;
```

```
% % VFE Baseline
```

```
vfe_opts = opts;
vfe_opts.induce_type = 'VFE_opt';
xu = xvec(I_com, :);
vfe_hyp = opts.hyp;
vfe_hyp.xu = xu;
vfe_hyp = minimize(vfe_hyp,@sp_gp,-vfe_opts.induce_step,inffunc,meanfunc,covfuncF,likfunc,xvec,
```

```
Function evaluation      0; Value 6.771657e+05
Function evaluation     16; Value 5.998275e+05
Function evaluation     19; Value 5.581266e+05
Function evaluation     22; Value 5.449867e+05
Function evaluation     24; Value 5.147507e+05
Function evaluation     26; Value 5.060681e+05
Function evaluation     28; Value 4.943599e+05
Function evaluation     30; Value 4.857384e+05
Function evaluation     32; Value 4.806604e+05
Function evaluation     34; Value 4.774503e+05
Function evaluation     37; Value 4.757455e+05
Function evaluation     39; Value 4.732163e+05
Function evaluation     41; Value 4.703673e+05
Function evaluation     44; Value 4.693126e+05
Function evaluation     46; Value 4.685113e+05
Function evaluation     48; Value 4.680056e+05
Function evaluation     49; Value 4.675208e+05
Function evaluation     51; Value 4.660696e+05
Function evaluation     53; Value 4.654481e+05
Function evaluation     55; Value 4.646997e+05
Function evaluation     57; Value 4.637241e+05
Function evaluation     59; Value 4.631622e+05
Function evaluation     61; Value 4.626585e+05
Function evaluation     63; Value 4.620575e+05
Function evaluation     65; Value 4.617130e+05
Function evaluation     67; Value 4.614442e+05
Function evaluation     69; Value 4.609135e+05
Function evaluation     71; Value 4.604544e+05
Function evaluation     73; Value 4.601040e+05
Function evaluation     74; Value 4.597679e+05
Function evaluation     76; Value 4.588266e+05
Function evaluation     79; Value 4.587191e+05
Function evaluation     81; Value 4.584269e+05
Function evaluation     83; Value 4.580557e+05
Function evaluation     85; Value 4.578701e+05
Function evaluation     87; Value 4.574517e+05
Function evaluation     89; Value 4.573179e+05
Function evaluation     92; Value 4.568743e+05
Function evaluation     93; Value 4.564465e+05
Function evaluation     96; Value 4.563604e+05
Function evaluation     98; Value 4.560946e+05
Function evaluation    100; Value 4.559315e+05
```

```
vfe_opts.hyp = opts.hyp;
vfe_opts.xu = vfe_hyp.xu;
```

```

vfe_opts.inffunc = @infGaussLik; vfe_opts.meanfunc = meanfunc; vfe_opts.covfuncF = covfuncF; vfe_opts.covfunc = covfunc;
[tmu, ts2] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, xvec_test);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[vfeMSE,vfeSMSE,vfeMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'VFE baseline', n_per, vfeMSE);

```

VFE baseline (Dc size 164): MSE 0.35266320, SMSE 0.33568030, MSLL -0.56279104

```

% vfe0_smse_rec(ki) = vfeSMSE; vfe0_msll_rec(ki) = vfeMSLL;
[yu, su] = gp(vfe_hyp, @infGaussLik, meanfunc, covfuncF, likfunc, xvec, yvec, vfe_opts.xu);
vfe_opts.yu = yu; vfe_opts.su = su;

```

```

sp_opts = opts;
sp_opts.induce_type = 'SPGP_opt';
hyp_init(1:d,1) = -2*opts.hyp.cov(1:d);
hyp_init(d+1,1) = 2*opts.hyp.cov(d+1);
hyp_init(d+2,1) = 2*opts.hyp.lik;

sp_opts.induce_size = dcs;
xu = xvec(I_com, :);
w_init = [reshape(xu,sp_opts.induce_size*d,1);hyp_init];
[w,f] = minimize(w_init,'spgp_lik_nohyp',-sp_opts.induce_step,yvec,xvec,sp_opts.induce_size);

```

```

Function evaluation      0; Value 1.167502e+04
Function evaluation     11; Value 1.087010e+04
Function evaluation     13; Value 1.030906e+04
Function evaluation     15; Value 9.917304e+03
Function evaluation     17; Value 9.509369e+03
Function evaluation     19; Value 9.348561e+03
Function evaluation     21; Value 9.230657e+03
Function evaluation     22; Value 9.100081e+03
Function evaluation     24; Value 9.013301e+03
Function evaluation     25; Value 8.931787e+03
Function evaluation     27; Value 8.862468e+03
Function evaluation     28; Value 8.794968e+03
Function evaluation     30; Value 8.748461e+03
Function evaluation     32; Value 8.682862e+03
Function evaluation     34; Value 8.631602e+03
Function evaluation     36; Value 8.553610e+03
Function evaluation     38; Value 8.500209e+03
Function evaluation     39; Value 8.439062e+03
Function evaluation     40; Value 8.384485e+03
Function evaluation     42; Value 8.345989e+03
Function evaluation     44; Value 8.316136e+03
Function evaluation     45; Value 8.291159e+03
Function evaluation     46; Value 8.260202e+03
Function evaluation     48; Value 8.227162e+03
Function evaluation     50; Value 8.208258e+03
Function evaluation     52; Value 8.185340e+03
Function evaluation     54; Value 8.164158e+03
Function evaluation     56; Value 8.147718e+03
Function evaluation     58; Value 8.129279e+03
Function evaluation     60; Value 8.114842e+03
Function evaluation     62; Value 8.098203e+03
Function evaluation     63; Value 8.081950e+03

```

```

Function evaluation      65; Value 8.067945e+03
Function evaluation      67; Value 8.051477e+03
Function evaluation      69; Value 8.037831e+03
Function evaluation      71; Value 8.026086e+03
Function evaluation      73; Value 8.012333e+03
Function evaluation      75; Value 7.995382e+03
Function evaluation      77; Value 7.981854e+03
Function evaluation      79; Value 7.970177e+03
Function evaluation      80; Value 7.958857e+03
Function evaluation      82; Value 7.948716e+03
Function evaluation      84; Value 7.934061e+03
Function evaluation      86; Value 7.921501e+03
Function evaluation      88; Value 7.904418e+03
Function evaluation      90; Value 7.890768e+03
Function evaluation      92; Value 7.881398e+03
Function evaluation      94; Value 7.866760e+03
Function evaluation      96; Value 7.857361e+03
Function evaluation      98; Value 7.849550e+03
Function evaluation     100; Value 7.842555e+03

```

```

xb = reshape(w(1:sp_opts.induce_size*d,1),sp_opts.induce_size,d);
sp_opts.xu = xb;
sp_opts.sp_hyp = w(sp_opts.induce_size*d+1:end,1);
sp_opts.hyp = opts.hyp;

[tmu,ts2] = spgp_pred(sp_opts.yvec,sp_opts.xvec,sp_opts.xu,xvec_test,sp_opts.sp_hyp);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[spgpMSE,spgpSMSE,spgpMSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu,
fprintf('%s (Dc size %d): MSE %6.8f, SMSE %6.8f, MSLL %6.8f\r\n', 'SPSG baseline', n_per, spgpM

```

SPSG baseline (Dc size 164): MSE 0.38672866, SMSE 0.36810530, MSLL -0.60175932

```

% spgp0_smse_rec(ki) = spgpSMSE; spgp0_msll_rec(ki) = spgpMSLL;
[yu,su] = spgp_pred(sp_opts.yvec,sp_opts.xvec,sp_opts.xu,sp_opts.xu,sp_opts.sp_hyp);
sp_opts.yu = yu; sp_opts.su = su;

```

```

[~, minidx] = min(mmsll);
best_msll_gr = grls(minidx)

```

best_msll_gr = 0.8500

```

[~, minidx] = min(mmse);
best_smse_gr = grls(minidx)

```

best_smse_gr = 1

```

kti = 1;
rb_vfe_msll = zeros(kti, 1);
rb_vfe_smse = zeros(kti, 1);

rb_sp_msll = zeros(kti, 1);
rb_sp_smse = zeros(kti, 1);

```

```

for ki=1:kti
    gr = best_msll_gr
    crk = rk;
    crk(I_com) = -1e10;
    [~, crk_idx] = sort(crk, 'descend');
    rn = round(n*gr);
    global_index = zeros(n,1);
    global_index(crk_idx(1:rn)) = 1; % select remaining data according to the importance

    vfe_opts.global_index = global_index;
    models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,vfe_opts); % use hyp of vfe
    [tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_test,models,vfe_opts);
    if ynorm==1
        tmu = tmu * norm_fstd + norm_fmean;
        ts2 = ts2 * norm_fstd^2;
    end
    [MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
    fprintf('%s (Dc size %d): \r\nMSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM++ (VFE)', n_p);
    rb_vfe_msll(ki) = MSLL;

    sp_opts.global_index = global_index;
    models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,sp_opts); % use hyp of vfe
    [tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_test,models,sp_opts);
    if ynorm==1
        tmu = tmu * norm_fstd + norm_fmean;
        ts2 = ts2 * norm_fstd^2;
    end
    [MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
    fprintf('%s (Dc size %d): \r\nMSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM++ (SPGP)', n_p);
    rb_sp_msll(ki) = MSLL;

    %%%=====%%

    gr = best_smse_gr
    crk = rk;
    crk(I_com) = -1e10;
    [~, crk_idx] = sort(crk, 'descend');
    rn = round(n*gr);
    global_index = zeros(n,1);
    global_index(crk_idx(1:rn)) = 1; % select remaining data according to the importance

    vfe_opts.global_index = global_index;
    models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,vfe_opts); % use hyp of vfe
    [tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_test,models,vfe_opts);
    if ynorm==1
        tmu = tmu * norm_fstd + norm_fmean;
        ts2 = ts2 * norm_fstd^2;
    end
    [MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
    fprintf('%s (Dc size %d): \r\nMSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM++ (VFE)', n_p);
    rb_vfe_smse(ki) = SMSE;

```

```

sp_opts.global_index = global_index;
models = aggregation_train_GRBCM_VS_apx(xvec,yvec,idx,sp_opts); % use hyp of vfe
[tmu,ts2] = aggregation_predict_GRBCM_VS_apx(xvec_test,models,sp_opts);
if ynorm==1
    tmu = tmu * norm_fstd + norm_fmean;
    ts2 = ts2 * norm_fstd^2;
end
[MSE,SMSE,MSLL] = evaluate2(ori_xvec, ori_yvec, ori_xvec_test, ori_yvec_test, tmu, ts2);
fprintf('%s (Dc size %d): \r\nMSE %6.8f, SMSE %6.4f, MSLL %6.4f\r\n', 'GRBCM++ (SPGP)', n_p,
rb_sp_smse(ki) = SMSE;
end

```

```

gr = 0.8500
GRBCM++ (VFE) (Dc size 164):
MSE 0.01350363, SMSE 0.0129, MSLL -1.9789
GRBCM++ (SPGP) (Dc size 164):
MSE 0.01292443, SMSE 0.0123, MSLL -1.9922
gr = 1
GRBCM++ (VFE) (Dc size 164):
MSE 0.01291452, SMSE 0.0123, MSLL -1.9694
GRBCM++ (SPGP) (Dc size 164):
MSE 0.01234696, SMSE 0.0118, MSLL -1.9771

```

```
fprintf('Best SMSE (GRBCM+VFE, dcs %d, ecs %d): %6.8f\n', dcs, ecs, mean(rb_vfe_smse));
```

```
Best SMSE (GRBCM+VFE, dcs 164, ecs 136): 0.01229260
```

```
fprintf('Best MSLL (GRBCM+VFE, dcs %d, ecs %d): %6.8f\n', dcs, ecs, mean(rb_vfe_msll));
```

```
Best MSLL (GRBCM+VFE, dcs 164, ecs 136): -1.97891661
```

```
fprintf('Best SMSE (GRBCM+SPGP, dcs %d, ecs %d): %6.8f\n', dcs, ecs, mean(rb_sp_smse));
```

```
Best SMSE (GRBCM+SPGP, dcs 164, ecs 136): 0.01175238
```

```
fprintf('Best MSLL (GRBCM+SPGP, dcs %d, ecs %d): %6.8f\n', dcs, ecs, mean(rb_sp_msll));
```

```
Best MSLL (GRBCM+SPGP, dcs 164, ecs 136): -1.99219990
```