Linear Combination.

When we add a multiple of one equation to the other we are making a **linear combination** of the equations.

The method of elimination is also called the **method of linear combinations**. Sometimes it is necessary to multiply both equations by suitable constants in order to eliminate one of the variables.

Example 4.33

Use linear combinations to solve the system

$$\begin{aligned} 5x - 2y &= 22 \\ 2x - 5y &= 13 \end{aligned}$$

Solution. This time we choose to eliminate the x-terms. We must arrange things so that the coefficients of the x-terms are opposites, so we look for the smallest integer that both 2 and 5 divide into evenly. (This number is called the **lowest common multiple**, or LCM, of 2 and 5.) The LCM of 2 and 5 is 10.

We want one of the coefficients of x to be 10, and the other to be -10. To achieve this, we multiply the first equation by 2 and the second equation by -5.

$$\begin{aligned} \mathbf{2}(5x - 2y = 22) &\qquad \rightarrow \qquad & 10x - 4y = 44 \\ \mathbf{-5}(2x - 5y = 13) &\qquad \rightarrow \qquad & -10x + 25y = -65 \end{aligned}$$

Adding these new equations eliminates the x-term and yields an equation in y.

$$\begin{array}{r} 10x - 4y = 44 \\ \underline{-10x + 25y = -65} \\ 21y = -21 \end{array}$$

We solve for y to find $y = -1$. Finally, we substitute $y = \mathbf{-1}$ into the first equation and solve for x.

$$\begin{aligned} 5x - 2(\mathbf{-1}) &= 22 \\ 5x + 2 &= 22 \\ x &= 4 \end{aligned}$$

The solution to the system is $(4, -1)$.

Reading Questions

RQ 4.34 What is a linear combination of expressions?

Answer. A sum of multiples of the expressions

RQ 4.35 What is the first step in the elimination method?

Answer. Write both equations into the general linear form, $Ax + By = C$.

Here are the steps for solving a system by elimination.