# Constructing Entire Functions (a summary)

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**Theorem** *(Bishop)***:** Any continua can be $\epsilon$-approximated in the Hausdorff metric by some $p^{-1}[-1, 1]$.

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infinite trees $\Longleftrightarrow$ Transcendental Functions

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$\mathcal{S}_{2,0}$ - transcendental functions with two critical values $\pm 1$ and no asymptotic values

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critical values: $\pm 1$

Put in somewhere notion of conformally balanced
are inverse images in $\mathcal{S}_{2,0}$ always trees?
I am adding a new line to see if GitHub sees the change.

# References

Chris Bishop (2014)
Constructing Entire Functions By Quasiconformal Folding
*Acta Mathematica*

Nuria Fagella, Sebastien Godillion, and Xavier Jarque (2014)
Wandering domains for composition of entire functions
*Journal of Mathematical Analysis and Applications*