Constructing Entire Functions (a summary)

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Theorem *(Bishop)***:** Any continua can be ϵ-approximated in the Hausdorff metric by some $p^{-1}[-1,1]$.

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$\mathcal{S}_{2,0}$ - transcendental functions with two critical values ± 1 and no asymptotic values

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critical values: ± 1

Put in somewhere notion of conformally balanced
are inverse images in $\mathcal{S}_{2,0}$ always trees?

References

Chris Bishop (2014)
Constructing Entire Functions By Quasiconformal Folding
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