Mark Scheme (Results)

Summer 2021

Pearson Edexcel International Advanced Level
In Statistics S2 Paper WST02/01

Question Number	Scheme	Marks
Throughout the paper the candidates may use different letters to the ones given in the mark scheme.		
1. (a)	[X ~ the number of pansy seeds that do not germinate or Y = the number…that do germinate]	
	$X \sim \mathrm{B}(20, 0.05)$ or $Y \sim \mathrm{B}(20, 0.95)$	B1
(i)	$\mathrm{P}(X \leqslant 4) - \mathrm{P}(X \leqslant 2) = 0.9974 - 0.9245$ or	
	$\binom{20}{3}0.05^3 \times 0.95^{17} + \binom{20}{4}0.05^4 \times 0.95^{16} = 0.05958... + 0.01332...$	M1
	$= 0.072909\ldots$ awrt **0.0729**	A1
(ii)	$\mathrm{P}(X \leqslant 1)$ or $\mathrm{P}(Y \geqslant 19) = 20 \times (0.95)^{19}(0.05) + (0.95)^{20}$	M1
	$= 0.7358$ $= 0.735839\ldots$ awrt **0.736**	A1 (5)
(b)	[Let W = no. of packets where $Y > 18$] $\mathrm{P}(W = 5) = (\text{"0.7358..."})^5$	M1
	$= 0.21573\ldots$ awrt **0.216**	A1 (2)
(c)	$\mathrm{H}_0 : p = 0.05$ $\mathrm{H}_1 : p > 0.05$	B1 (1)
(d)	[V= no. of seeds that do not germinate $V \sim \mathrm{B}(100, 0.05)$ approximates to] $V \sim \mathrm{Po}(5)$	M1A1
	CR for 1-tail in (c): $\mathrm{P}(V \geqslant 9) = 0.0681$, $\mathrm{P}(V \geqslant 10) = 0.0318$, CR $V \geqslant 10$ oe CR for 2-tail in (c): $\mathrm{P}(V \geqslant 10) = 0.0318$, $\mathrm{P}(V \geqslant 11) = 0.0137$, CR $V \geqslant 11$ oe	
	$\mathrm{P}(V \geqslant 8) = 1 - \mathrm{P}(V \leqslant 7)$ $= 1 - 0.8666$ $= 0.1334$	M1 A1
	Accept H_0 or not significant or 8 does not lie in the critical region	dM1
	Data consistent with ***Spany*'s claim** or Insufficient evidence for ***Jem's* belief** or insufficient evidence that **percentage** of **seeds not germinating** is more than **5%** (o.e.)	A1cso (6)
		Total 14

Notes

(a) **B1:** writing or using $\mathrm{B}(20, 0.05)$ [Allow $Y \sim \mathrm{B}(20, 0.95)$ if Y is clearly defined]. Implied by 1 correct prob.

(i) **M1:** for $\mathrm{P}(X \leqslant 4) - \mathrm{P}(X \leqslant 2)$ and one correct prob. or $\mathrm{P}(X = 3) + \mathrm{P}(X = 4)$ and 1 correct prob.

(ii) **M1:** for $\mathrm{P}(X \leqslant 1)$ or $[20] \times (0.95)^{19}(0.05) + (0.95)^{20}$ - condone missing 20

(b) **M1:** for $(\text{their(a)(ii)})^5$

(c) **B1:** both hypotheses correct with p or π

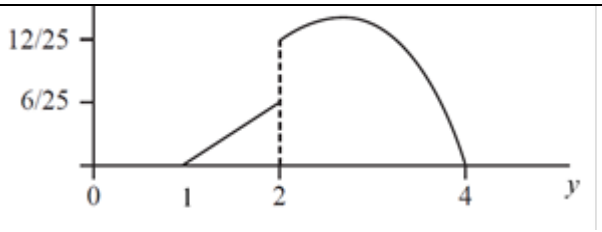
(d) **1st M1:** for realising a Poisson approximation is appropriate. NB Po(95) is M0A0
1st A1: writing or using $V \sim \mathrm{Po}(5)$ i.e correct mean for the Poisson.
2nd M1: for writing or using $1 - \mathrm{P}(V \leqslant 7)$ or $\mathrm{P}(V \leqslant 7) = 0.8666$
or writing $\mathrm{P}(V \geqslant 10) = 0.0318$ or $\mathrm{P}(V \geqslant 9) = 0.0681$ or $\mathrm{P}(V \geqslant 11) = 0.0137$ **leading to a CR**.
Implied by correct CR or probability = awrt 0.133
2nd A1: for awrt 0.133 or $V \geqslant 10$ oe (e.g. $V > 9$) or $V \geqslant 11$ oe allow any letter but CR must match part(c)
3rd dM1: dep on 2nd M1. ft their CR or probability. A correct statement based on comparing 8 with their CR or their prob with 0.05 or 0.025 [condone $0.866 < 0.95$]– contradicting non-contextual comments M0
3rd A1 cso: all previous marks must be awarded. A **correct** statement in context. Need **Bold words.**
NB award M1A1 for a correct contextual statement on its own.
If there are no hypotheses or they are the wrong way around, then 3rd M0 3rd A0

SC1 **Normal approximation:** Award marks in pairs with 2, 4 or 6 marks available
Sight of $\mathrm{N}(5 \text{ or } 95, \sqrt{4.75}^2)$ M1A1; probability awrt 0.125/6 M1A1; Correct contextual concl' dM1A1

SC2 **No approximation:** Use of $\mathrm{B}(100, 0.05)$ M0A0; probability awrt 0.128 or CR $\geqslant 10$ M1A1; then M0A0

Question Number		Scheme	Marks
2.	(a)	[X = number of faults in 4 m^2 so $X \sim$ Po(3)]	
		$P(X=5) = P(X \leqslant 5) - P(X \leqslant 4)$ [= 0.9161 – 0.8153] or $\frac{e^{-3}3^5}{5!}$ (allow λ instead of 3)	M1
		= 0.1008 or 0.100818… awrt **0.101**	A1
			(2)
	(b)	[Y = number of faults in 6 m^2 so] $Y \sim$Po(4.5) and $\left[P(Y>5)\right] = 1 - P(Y \leqslant 5)$ [= 1 – 0.7029]	M1
		= 0.2971 or (calc) 0.29706956… awrt **0.297**	A1
			(2)
	(c)	**0.101** (or ft their answer to (a))	B1ft
		Faults occur **independently/ randomly**	B1
			(2)
	(d)	[F = number of faults in a small rug] $F \sim$ Po(0.9)	B1
		$e^{-"0.9"}n \times 80 + \left(1 - e^{-"0.9"}\right)n \times 60 \geqslant 4000$ or $(\text{awrt } 0.407)n \times 80 + (\text{awrt } 0.593)n \times 60 \geqslant 4000$	M1
		$n \geqslant \frac{4000}{20e^{-"0.9"} + 60} = 58.71\ldots$	M1
		$n =$ **59**	A1
			(4)
	(e)	$H_0 : \lambda = 9$ $H_1 : \lambda > 9$	B1
		$R \sim$ Po("0.9"×10) and [P($R \geqslant 13$)] = 1 – P($R \leqslant 12$) [= 1 – 0.8758]	
		P($R \leqslant 13$) = 0.9261 or P($R \geqslant 14$) = 0.0739 or P($R \leqslant 14$) = 0.9585 or P($R \geqslant 15$) = 0.0415	M1
		[P($R \geqslant 13$)] = 0.1242 awrt 0.124 or CR $R \geqslant 15$ (oe)	A1
		so insufficient evidence to reject H_0 /not significant/ not in critical region	M1
		There is insufficient evidence that the **rate** at which **faults** occur is higher for **Rhiannon**	A1
			(5)
			Total 15

	Notes
(a)	**M1:** for using or writing $P(X \leqslant 5) - P(X \leqslant 4)$ or $\frac{e^{-\lambda}\lambda^5}{5!}$ (Accept letter λ or any value of λ)
(b)	**M1:** writing or using Po(4.5) and sight of $\left[P(Y>5)\right] = 1 - P(Y \leqslant 5)$ Implied by sight of 1 – 0.7029
(c)	**2nd B1:** for a comment about faults occurring **randomly/independently** or Poisson has "**no memory**"
(d)	**B1:** writing or using Po(0.9) May be implied by sight of 0.407 or 0.593 **1st M1:** for $e^{-\lambda}n \times 80 + \left(1 - e^{-\lambda}\right)n \times 60 > 4000$ any value for λ. Allow = 4000 **2nd M1:** for solving their equation leading to a positive value of n. Allow any value of λ and allow n = … **A1:** for an answer of 59 only
(e)	**B1:** both hypotheses correct with λ or μ. Allow 3 or 0.75 or 0.9 instead of 9 **1st M1:** for writing or using Po("9") and writing or using 1 – P($R \leqslant 12$) (implied by 1 – 0.8758) or one of: P($R \leqslant 13$) = 0.9261, P($R \geqslant 14$) = 0.0739, P($R \leqslant 14$) = 0.9585, P($R \geqslant 15$) = 0.0415 **leading to a CR** **1st A1:** for probability = awrt 0.124 or CR of $R \geqslant 15$ oe e.g. $R > 14$ **2nd M1:** for a correct conclusion based on their prob & 0.05 or their CR & 13. Assume correct hypotheses. Do not allow contradicting conclusions **2nd A1:** dep on both Ms for a correct contextual comment including the words in bold.

Question Number	Scheme	Marks
3. (a)		M1 A1 (2)
(b)	$\frac{\mathrm{d}\left(\frac{3}{50}\left(4y^2-y^3\right)\right)}{\mathrm{d}y}=\frac{3}{50}\left(8y-3y^2\right)$	M1
	$\frac{3}{50}\left(8y-3y^2\right)=0$; $\underline{y=\frac{8}{3}}$ oe	M1; A1 (3)
(c)	$\mathrm{E}(Y^2)=\int_1^2\left(\frac{6}{25}y^3-\frac{6}{25}y^2\right)\mathrm{d}y+\int_2^4\left(\frac{12}{50}y^4-\frac{3}{50}y^5\right)\mathrm{d}y$	M1
	$=\left[\frac{6}{100}y^4-\frac{6}{75}y^3\right]_1^2+\left[\frac{12}{250}y^5-\frac{3}{300}y^6\right]_2^4$	A1
	$=\left[\left(\frac{8}{25}\right)-\left(-\frac{1}{50}\right)\right]+\left[\left(\frac{1024}{125}\right)-\left(\frac{112}{125}\right)\right]$; $=\underline{\frac{1909}{250}}$ or **<u>7.636</u>** or **<u>7.64</u>**	dM1; A1 (4)
(d)	$\mathrm{Var}(Y)=\text{"}\frac{1909}{250}\text{"}-2.696^2$	M1
	$=0.367584$ awrt **<u>0.368</u>**	A1 (2)
(e)	$\frac{1}{2}(y-1)\times\frac{6}{25}(y-1)=0.1$ <u>or</u> $\int_1^x\frac{6}{25}(y-1)\,\mathrm{d}y=0.1$	M1
	$\frac{1}{2}(y-1)\times\frac{6}{25}(y-1)=0.1$ <u>or</u> $\frac{6}{25}\left[\left(\frac{x^2}{2}-x\right)+\frac{1}{2}\right]=0.1$ <u>or</u> $\frac{6}{50}(x-1)^2=0.1$	A1
	$(y-1)^2=\frac{5}{6}$ <u>or</u> $y=1\pm\sqrt{\frac{5}{6}}$; $y=1.9128...$ awrt **<u>1.91</u>**	dM1; A1 (4) **Total 15**

	Notes
(a)	**M1:** the two parts must be the right shape and not joined. Ignore labels and condone if it goes below x - axis **A1:** for 6/25, 12/25, 1, 2 and 4 and must not go beyond 4 or < 1 [Can allow "freehand" straight line]
(b)	**1st M1:** for attempting to differentiate $y^n \to y^{n-1}$ for n = 2 <u>or</u> 3 **2nd M1:** for equating their differential ($\neq \mathrm{f}(y)$) to zero and an attempt at solving so must reach y = … **A1:** for $\frac{8}{3}$ oe and allow awrt 2.67 If $y = 0$ is seen it must be rejected.
(c)	**1st M1:** for using $\int y^2\mathrm{f}(y)$ for both parts, <u>and</u> an attempt at integration (some $y^n \to y^{n+1}$) Ignore limits. **1st A1:** for correct integration for both parts. Ignore limits. **2nd dM1 :** dep on 1st M1for adding the 2 parts together <u>and</u> substituting the correct limits in to each part. **2nd A1:** allow 7.64 or 7.636 You will need to check that they have used algebraic integration.
(d)	**M1:** for "their part(c)" – 2.696 ^{2} **A1:** for awrt 0.368
(e)	**1st M1:** allow $\frac{1}{2}t\times\frac{6}{25}(t-1)=0.1$ <u>or</u> $\int_1^x\frac{6}{25}(y-1)\,\mathrm{d}y=0.1$ <u>and</u> some integration and sub' of 1 and x **1st A1:** for a correct equation in any form **2nd dM1:** dependent on 1st M1 for a correct method for solving their equation. Implied by correct answer. **2nd A1:** for awrt 1.91 (second solution should be rejected)

Question Number	Scheme	Marks
4.	[A = the number on the ball] $\mathrm{P}(A=1)=\frac{2}{9}$ $\mathrm{P}(A=2)=\frac{1}{3}$ $\mathrm{P}(A=5)=\frac{4}{9}$	B1
(i)	Possible samples with a range of 4 are: (1,1,5) (1,2,5) (1,5,5)	M1
	(1,1,5) $"\frac{2}{9}" \times "\frac{2}{9}" \times "\frac{4}{9}" \times 3 = \frac{16}{243}$ or (1,5,5) $"\frac{2}{9}" \times "\frac{4}{9}" \times "\frac{4}{9}" \times 3 = \frac{32}{243}$	M1
	(1,2,5) $"\frac{2}{9}" \times "\frac{1}{3}" \times "\frac{4}{9}" \times 6 = \frac{16}{81}$	M1
	$\mathrm{P}(B=4)=\frac{16}{243}+\frac{32}{243}+\frac{16}{81} = \underline{\underline{\frac{32}{81}}}$	A1
(ii)	$\mathrm{P}(B=0)=\left("\frac{2}{9}"\right)^3+\left("\frac{1}{3}"\right)^3+\left("\frac{4}{9}"\right)^3=\frac{11}{81}$	M1
	$\mathrm{P}(B=1)=3\times\frac{2}{9}\times\left(\frac{1}{3}\right)^2+3\times\frac{1}{3}\times\left(\frac{2}{9}\right)^2=\frac{10}{81}$ or $\mathrm{P}(B=3)=3\times\frac{1}{3}\times\left(\frac{4}{9}\right)^2+3\times\frac{4}{9}\times\left(\frac{1}{3}\right)^2=\frac{28}{81}$	M1
	$1-"\frac{11}{81}"-"\frac{10}{81}"-"\frac{32}{81}"=\frac{28}{81}$ or $1-"\frac{11}{81}"-"\frac{28}{81}"-"\frac{32}{81}"=\frac{10}{81}$	M1
	b | 0 | 1 | 3 | 4 $\mathrm{P}(B=b)$ | $\frac{11}{81}$ | $\frac{10}{81}$ | $\frac{28}{81}$ | $\frac{32}{81}$	B1 A1
		(10)
		Total 10

	Notes
(i)	**B1:** for writing or using the 3 correct probabilities
	1st M1: for identifying the 3 possible samples
	2nd M1: for $p\times p\times q\times 3$ or $p\times q\times q\times 3$ where p and q are probabilities with $(p+q)<1$
	3rd M1: for $p\times q\times r\times 6$ where p, q and r are probabilities with $(p+q+r)=1$
	A1: for $\frac{32}{81}$ or awrt 0.395 [Calc: 0.3950617…]
(ii)	**1st M1:** for $p^3+q^3+r^3$ (for their p, q and r)
	2nd M1: for $3\times p\times(q)^2+3\times q\times(p)^2$ or $3\times q\times(r)^2+3\times r\times(q)^2$ (for their p, q and r)
	3rd M1: for use of all probabilities of P(B = b) adding to 1 [Must have 3, 4 or 5 values for b]
	B1: for ranges 0, 1, 3 and 4 with none omitted and no extras. Allow extras if assigned probability of 0
	A1: for a fully correct probability distribution.
SC A0 in (i)	If A0 scored in (i) and all other marks scored in (ii) and correct prob's for 2 values of b : award A1 in (ii)

Question Number	Scheme	Marks
5 (a)(i)	If $y = 0$ then $1-(\alpha+\beta y^2)=0 \quad \therefore \ \alpha = 1$ *	B1cso
(ii)	If $y = 5$ then $1-(\alpha+\beta y^2)=1$	
	$1+25\beta=0 \quad \therefore \beta=-\frac{1}{25}$ *	B1cso
		(2)
(b)	$F(y)=\frac{1}{25}y^2$ so $f(y)=\frac{dF(y)}{dy}=\frac{2}{25}y$	M1
	$\therefore [f(y)=]\begin{cases}\frac{2}{25}y & 0\leqslant y\leqslant 5\\ 0 & \text{otherwise}\end{cases}$	A1
		(2)
(c)	$\left[P\left(R>\frac{11}{5}\right)=P\left(Y>\frac{5}{3}\right)=1-\frac{1}{25}\times\left(\frac{5}{3}\right)^2=\right]\ \underline{\underline{\frac{8}{9}}}$ oe	B1
	$\frac{3d-\frac{11}{5}}{3d-d}=\frac{8}{9}$ oe or $\frac{\frac{11}{5}-d}{3d-d}=\frac{1}{9}$ oe	M1
	$d=\underline{\underline{\frac{9}{5}}}$ oe	A1
		(3)
(d)	$P\left(Y<\frac{11}{5}\right)=\frac{121}{625}$ or 0.1936	B1
	[Let G = the number of spins with distance < 2.2 m]	
	$[P(G\geqslant 5)=]$	
	$\left(\frac{"1"}{9}\right)^3\times\left(\frac{"121"}{625}\right)^3+3\times\left(\frac{"1"}{9}\right)^2\times\left(\frac{"8"}{9}\right)\times\left(\frac{"121"}{625}\right)^3+3\times\left(\frac{"1"}{9}\right)^3\times\left(\frac{"121"}{625}\right)^2\times\left(\frac{"504"}{625}\right)$	M1, M1
	= 0.000 373226 awrt **0.000 373**	A1
		(4)
		Total 11

Notes

(a) (i) **B1:** for stating or using the fact that when $y = 0$ then $\alpha+\beta y^2 = 1$

(ii) **B1:** for stating or using that when $y = 5$ then $\alpha+\beta y^2 = 0$ and setting up the equation leading to $\beta=-\frac{1}{25}$

(b) **M1:** for differentiating. Implied by $\pm\frac{2}{"25"}y$ can ft their value of β

A1: for a fully correct f(y) defined for the whole range.

(c) **B1:** for using F(y) and $\frac{5}{3}$ to find P($Y > \frac{5}{3}$). Allow $\frac{8}{9}$ or any exact equivalent.

M1: for LHS = p where $0 < p < 1$

A1: for $\frac{9}{5}$ or any exact equivalent e.g. 1.8

(d) **B1:** for $\frac{121}{625}$ or awrt 0.194 This mark could be implied by a correct answer.

1st M1: for $p^3q^3+np^2(1-p)q^3+np^3q^2(1-q)$ where p and q are probabilities and n is an integer > 0

2nd M1: for $p^3q^3+3p^2(1-p)q^3+3p^3q^2(1-q)$ where p and q are probabilities.

A1: for awrt 0.000 373

Question Number	Scheme	Marks
6. (i)	$z = 1.25$	B1
	$\frac{187.5-\mu}{\sigma}=1.25$	M1 M1 A1
	$187.5-\mu=1.25\sigma$	
	$\mu = 225p$	M1
	$\sigma=\sqrt{225p(1-p)}$	M1
	$(187.5-225p)^2=(1.25)^2\times225p(1-p)$ or $(150-180p)^2=225p(1-p)$ (o.e.)	M1
	e.g. $900(5-6p)^2=225(p-p^2) \Rightarrow 4(25-60p+36p^2)=p-p^2$	A1*
	Leading to $145p^2-241p+100=0$ *	
(ii)	$[(29p-25)(5p-4)=0\Rightarrow]$ $p=0.8$ or $p=\frac{25}{29}$ (accept: 0.862(0689…))	M1
	[p =] 0.8 because 0.862 gives a mean greater than 188 (oe)	A1
		(10)
		Total 10

	Notes
(i)	**B1:** for 1.25 or better (calculator gives: 1.25027…)
	1st M1: for attempting to use a continuity correction i.e. for sight of 188 ± 0.5
	2nd M1: for standardising using μ and σ or np and $\sqrt{np(1-p)}$ (Condone letter n or any integer > 0)
	1st A1: for a correct equation with compatible signs, allow 1.250... If using a value for n it must be 225
	3rd M1: for $\mu = 225p$ seen at any stage in the working.
	4th M1: for $\sigma=\sqrt{225p(1-p)}$ seen at any stage in the working. Must be for σ not $\sigma^2 = 225p(1-p)$
	5th M1: for squaring to get a quadratic equation in p
	2nd A1*: dep on all previous Ms and use of 1.25 (with correct sign) for at least 1correct intermediate step from a correct quadratic equation e.g one of those in scheme for 5th M1
(ii)	**M1:** for solving the quadratic correctly–leading to p = … or implied by 0.8 or awrt 0.862
	A1: for 0.8 and a correct reason to eliminate 0.862