Pearson
Edexcel

Mark Scheme (Results)

January 2020

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F1
(WFM01) Paper 01

January 2020
WFM01/01 Further Pure Mathematics F1
Mark Scheme

Question Number	Scheme	Notes	Marks
1.	(a) $\mathbf{A}=\begin{pmatrix}p & -5\\ -2 & p+3\end{pmatrix}$ (b) $p=3$; $\mathbf{A}=\begin{pmatrix}a & -5\\ -2 & d\end{pmatrix}$		
(a)	$\det(\mathbf{A}) = p(p+3)-(-5)(-2)\ \{= p(p+3)-10\}$	Applies $p(p+3)\pm(-5)(-2)$	M1
	$p^2+3p-10=0 \Rightarrow (p+5)(p-2)=0 \Rightarrow p=...$	Obtains a correct expression for $\det(\mathbf{A})$, sets their $\det(\mathbf{A})=0$ and solves their 3TQ = 0 by any valid method to give $p=....$	M1
	$p=-5,\ 2$	$p=-5,\ 2$	A1
			(3)
(b)	$\left\{p=3\Rightarrow\mathbf{A}=\begin{pmatrix}3 & -5\\ -2 & 6\end{pmatrix}\right\}$		
	For either $\begin{pmatrix}6 & 5\\ 2 & 3\end{pmatrix}$ or $\det(\mathbf{A}) = 3(3+3)-10$ or 8	For **either** $\begin{pmatrix}6 & 5\\ 2 & 3\end{pmatrix}$ **or** a correct numerical expression or value for $\det(\mathbf{A})$, which can be seen or implied	B1
	$\mathbf{A}^{-1}=\dfrac{1}{3(3+3)-(-5)(-2)}\begin{pmatrix}6 & 5\\ 2 & 3\end{pmatrix}$	$\dfrac{1}{ad\pm(-5)(-2)}\text{Adj}(\mathbf{A})$, where a correct method has been employed for finding their $\text{Adj}(\mathbf{A})$	M1
	$\mathbf{A}^{-1}=\dfrac{1}{8}\begin{pmatrix}6 & 5\\ 2 & 3\end{pmatrix}$ or $=\begin{pmatrix}\frac{3}{4} & \frac{5}{8}\\ \frac{1}{4} & \frac{3}{8}\end{pmatrix}$ or $=\begin{pmatrix}0.75 & 0.625\\ 0.25 & 0.375\end{pmatrix}$ or $=\begin{pmatrix}\frac{6}{8} & \frac{5}{8}\\ \frac{2}{8} & \frac{3}{8}\end{pmatrix}$	Correct $\mathbf{A}^{-1}$	A1
			(3)
			6

Question 1 Notes

1. (b)	**Note**	$\mathbf{A}=\begin{pmatrix}a & b\\ c & d\end{pmatrix}\Rightarrow\text{Adj}(\mathbf{A})=\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}$ is a correct method for finding their $\text{Adj}(\mathbf{A})$
	Note	Allow B1 M1 A0 for just writing $\dfrac{1}{3(3+3)-(-5)(-2)}\begin{pmatrix}p+3 & 5\\ 2 & p\end{pmatrix}$
	Note	Allow B0 M1 A0 for just writing $\dfrac{1}{3(3+3)+(-5)(-2)}\begin{pmatrix}p+3 & 5\\ 2 & p\end{pmatrix}$
	Note	Allow B0 M1 A0 for just writing $\dfrac{1}{p(p+3)\pm(-5)(-2)}\begin{pmatrix}p+3 & 5\\ 2 & p\end{pmatrix}$
	Note	Allow M1 for evidence of a correct numerical expression for $\det\mathbf{A}=ad\pm(-5)(-2)$ followed by $\dfrac{1}{\text{their }\det(A)}\text{Adj}(\mathbf{A})$ where a correct method has been employed for finding their $\text{Adj}(\mathbf{A})$
	Note	Give final A0 for $\dfrac{1}{18-10}\begin{pmatrix}6 & 5\\ 2 & 3\end{pmatrix}$ without reference to $\dfrac{1}{8}\begin{pmatrix}6 & 5\\ 2 & 3\end{pmatrix}$ or any other acceptable answer
	Note	Give B1 M1 A1 for writing down a correct final answer for $\mathbf{A}^{-1}$ from no working

Question Number	Scheme	Notes	Marks
2.	Let $\mathrm{f}(x)=3x^3+kx^2+33x+13$; $k\in\mathbb{R}$; $x=-\frac{1}{3}$ is a root of $\mathrm{f}(x)=0$		
	Note: Ignore labelling of parts when marking Q2		
(a) **Way 1**	$3\left(-\frac{1}{3}\right)^3+k\left(-\frac{1}{3}\right)^2+33\left(-\frac{1}{3}\right)+13=0\Rightarrow k=...$	Some evidence of substituting $x=-\frac{1}{3}$ into the given equation and solves to find $k=...$	M1
	$\left\{-\frac{1}{9}+\frac{1}{9}k-11+13=0\Rightarrow -1+k+18=0\Rightarrow\right\}\ k=-17$	$k=-17$	A1
			(2)
(a) **Way 2**	$\mathrm{f}(x)=(3x+1)(x^2+Ax+13)$ $x:\ 3(13)+A=33\Rightarrow A=-6$ $x^2:\ k=1+(\text{"}-6\text{"})(3)$	Expresses $\mathrm{f}(x)=(3x\pm1)(x^2+Ax\pm13)$, equates x terms to find A and equates x^2 terms to find k	M1
	$k=-17$	$k=-17$	A1
			(2)
(b)	$\{\mathrm{f}(x)=\}\ (3x+1)(x^2-6x+13)$ or $\{\mathrm{f}(x)=\}\ \left(x+\frac{1}{3}\right)(3x^2-18x+39)$	Attempts to find the quadratic factor e.g. using long division to obtain $(3x\pm1)$ with $(x^2\pm qx+...)$ or $\left(x\pm\frac{1}{3}\right)$ with $(3x^2\pm qx+...)$; $q=$ value $\neq 0$ e.g. factorising/equating coefficients to obtain $\mathrm{f}(x)=(3x\pm1)(x^2\pm qx\pm r)$ or $\mathrm{f}(x)=\left(x\pm\frac{1}{3}\right)(3x^2\pm qx\pm r)$, $q=$ value $\neq 0$, r can be 0	M1
		$x^2-6x+13$ or $3x^2-18x+39$ seen in their working	A1
	$\{x^2-6x+13=0$ or $3x^2-18x+39=0\Rightarrow\}$		
	e.g. • $x=\dfrac{--6\pm\sqrt{(-6)^2-4(1)(13)}}{2(1)}$ • $(x-3)^2-9+13=0\Rightarrow x=...$	**dependent on the previous M mark** Correct method of applying the quadratic formula or completing the square **for solving their 3TQ** on their quadratic factor	dM1
	$\{x=\}\ 3\pm2\mathrm{i}$ (or $3\pm\mathrm{i}2$)	$3+2\mathrm{i}$ **and** $3-2\mathrm{i}$	A1
			(4)
			6

	Question 2 Notes	
2. (b)	**Note**	You can assume $z\equiv x$ for solutions in this part
	Note	Give final dM1A1 for $x^2-6x+13=0$ or $3x^2-18x+39=0\Rightarrow x=3+2\mathrm{i},\,3-2\mathrm{i}$ with no intermediate working.
	Note	Give M1 A1 dM1 A1 for $3x^3-17x^2+33x+13=0\Rightarrow x=3+2\mathrm{i},\,3-2\mathrm{i}$ with no intermediate working.
	Note	They must be solving a 3TQ $\text{"}A\text{"}x^2+\text{"}B\text{"}x+\text{"}C\text{"}$ where A, B, C ***are all numerical values*** $\neq 0$ for the final dM1 mark.
	Note	<u>**Special Case:**</u> If ***their quadratic*** factor $x^2+\text{"}B\text{"}x+\text{"}C\text{"}$ **can** be factorised then allow dM1 for correct factorisation leading to $x=...$ Otherwise, give dM0 for applying a method of factorisation to solve their 3TQ = 0.

	Question 2 Notes Continued			
2. (b)	**Note**	**Reminder: Method mark for solving a 3TQ = 0** **Formula:** $Ax^2 + Bx + C = 0 \Rightarrow$ Attempt to use the correct formula (with values for A, B, C) **Completing the Square:** $x^2 + Bx + C = 0 \Rightarrow \left(x \pm \frac{B}{2}\right)^2 \pm q \pm C = 0,\ q \neq 0,$ leading to $x = \ldots$		
	Note:	**Comparing coefficients:** $\mathrm{f}(x) = (3x+1)(x^2 + \alpha x + \beta) \equiv 3x^3 - 17x^2 + 33x + 13$ $x^2 : 3\alpha + 1 = -17 \Rightarrow \alpha = -6;\ \ z : 3\beta + \alpha = 33 \Rightarrow 3\beta - 6 = 33 \Rightarrow \beta = 13;\ \ \text{constant} : \beta = 13$ yielding quadratic factor = $x^2 - 6x + 13$		
	Note	The solutions $3 \pm 2\mathrm{i}$ need to follow on from a correct $x^2 - 6x + 13 = 0$ or $3x^2 - 18x + 39 = 0$ in order to gain the final A mark.		
	Note	Give final A0 for writing $\frac{6 \pm 4\mathrm{i}}{2}$ followed by either $3 \pm 4\mathrm{i}$ or $6 \pm 2\mathrm{i}$		
2. (a) **ALT 1**	**Note**	**Long division:** $\begin{array}{r\|l} & 3x^2 - 18x + 39 \\ \hline x+\frac{1}{3} & 3x^3 + kx^2 + 33x + 13 \\ & \underline{3x^3 + x^2} \\ & (k-1)x^2 + 33x \\ & \underline{-18x^2 - 6x} \\ & 39x + 13 \\ & \underline{39x + 13} \\ & 0 \end{array}$ or $\begin{array}{r\|l} & x^2 - 6x + 13 \\ \hline 3x+1 & 3x^3 + kx^2 + 33x + 13 \\ & \underline{3x^3 + x^2} \\ & (k-1)x^2 + 33x \\ & \underline{-18x^2 - 6x} \\ & 39x + 13 \\ & \underline{39x + 13} \\ & 0 \end{array}$		
		$(k-1) - -18 = 0 \Rightarrow k = \ldots$	Full complete method of dividing by either $x + \frac{1}{3}$ or $(3x+1)$, applying remainder = 0 and solving a relevant equation to find $k = \ldots$	M1
		$k = -17$	$k = -17$	A1
				(2)
	Note	Give M0 for dividing by either $x - \frac{1}{3}$ or $3x - 1$		

		Question 2 Notes Continued		
2. (a) **ALT 2**	**Note**	**Long division:** $\begin{array}{r l} & x^2 + \left(\frac{k-1}{3}\right)x + \left(\frac{100-k}{9}\right) \\ \hline 3x+1 \,\vert & 3x^3 + kx^2 + 33x + 13 \\ & \underline{3x^3 + x^2} \\ & (k-1)x^2 + 33x \\ & \underline{(k-1)x^2 + \left(\frac{k-1}{3}\right)x} \\ & \left(\frac{100-k}{3}\right)x + 13 \\ & \underline{\left(\frac{100-k}{3}\right)x + \left(\frac{100-k}{9}\right)} \\ & 13 - \left(\frac{100-k}{9}\right) \end{array}$ or $\begin{array}{r l} & 3x^2 + (k-1)x + \left(\frac{100-k}{3}\right) \\ \hline x+\frac{1}{3} \,\vert & 3x^3 + kx^2 + 33x + 13 \\ & \underline{3x^3 + x^2} \\ & (k-1)x^2 + 33x \\ & \underline{(k-1)x^2 + \left(\frac{k-1}{3}\right)x} \\ & \left(\frac{100-k}{3}\right)x + 13 \\ & \underline{\left(\frac{100-k}{3}\right)x + \left(\frac{100-k}{9}\right)} \\ & 13 - \left(\frac{100-k}{9}\right) \end{array}$		
		$13-\left(\frac{100-k}{9}\right)=0 \Rightarrow k=...$ or $33-\left(\frac{k-1}{3}\right)=39 \Rightarrow k=...$	Full complete method of dividing by either $x+\frac{1}{3}$ or $(3x+1)$, applying remainder $=0$ and solving a relevant equation to find $k=...$	M1
		$\left\{\frac{117-100+k}{9}=0 \Rightarrow\right\} k=-17$	$k=-17$	A1
				(2)
	Note	Give M0 for dividing by either $x-\frac{1}{3}$ or $3x-1$		

Question Number	Scheme	Notes	Marks
3. (a)	$\sum_{r=1}^{n} r^2(2r+3) = 2\sum_{r=1}^{n} r^3 + 3\sum_{r=1}^{n} r^2$		
	$= 2\left(\frac{1}{4}n^2(n+1)^2\right) + 3\left(\frac{1}{6}n(n+1)(2n+1)\right)$	Attempts to expand $r^2(2r+3)$ and attempts to substitute at least one correct formula for either $\sum_{r=1}^{n} r^3$ or $\sum_{r=1}^{n} r^2$ into their resulting expression	M1
		Obtains an expression of the form $\alpha n^2(n+1)^2 + \beta n(n+1)(2n+1);\ \alpha, \beta \neq 0$	M1
		$2\left(\frac{1}{4}n^2(n+1)^2\right) + 3\left(\frac{1}{6}n(n+1)(2n+1)\right)$ which can be simplified or un-simplified	A1
	$= \frac{1}{2}n(n+1)\big(n(n+1)+(2n+1)\big)$ $= \frac{1}{2}n(n+1)(n^2+3n+1)$ *	Achieves the given result via an appropriate intermediate step with no algebraic errors seen in their working	A1 * **cso**
			(4)
(b)	$\left\{\sum_{r=10}^{25} r^2(2r+3) = \right\}$	{**Note:** Let $\mathrm{f}(n) = \frac{n}{2}(n+1)(n^2+3n+1)$ or their answer to part (a) or their un-simplified expression (for $\mathrm{f}(n)$) of the form $\alpha n^2(n+1)^2 + \beta n(n+1)(2n+1);\ \alpha, \beta \neq 0$}	
	$= \frac{25}{2}(25+1)((25)^2+3(25)+1) - \frac{9}{2}(9+1)((9)^2+3(9)+1)$	Applies $\mathrm{f}(25) - \mathrm{f}(9)$ **Note:** Give M0 for applying $\mathrm{f}(25) - \mathrm{f}(10)$	M1
	$\left\{= \frac{25}{2}(26)(701) - \frac{9}{2}(10)(109) = 227825 - 4905\right\}$		
	$= 222920$	222920 **cao**	A1
			(2)
			6
	Question 3 Notes		
3. (a)	**Note**	**Final A mark:** $\text{LHS} = \frac{1}{2}n^2(n+1)^2 + \frac{1}{2}n(n+1)(2n+1) = \frac{1}{2}n^2(n^2+2n+1) + \frac{1}{2}n(2n^2+3n+1)$ $= \frac{1}{2}n^4 + n^3 + \frac{1}{2}n^2 + n^3 + \frac{3}{2}n^2 + \frac{1}{2}n \ = \frac{1}{2}n^4 + 2n^3 + 2n^2 + \frac{1}{2}n$ $\text{RHS} = \frac{n}{2}(n+1)(n^2+3n+1) = \frac{n}{2}(n^3+3n^2+n+n^2+3n+1) = \frac{n}{2}(n^3+4n^2+4n+1)$ $= \frac{1}{2}n^4 + 2n^3 + 2n^2 + \frac{1}{2}n$ Give final A1 cso for using algebra to show that the LHS and RHS are the same with some acknowledgment (e.g. 'proved', LHS = RHS, QED or □) that their proof is complete.	

		Question 3 Notes Continued
3. (a)	**Note**	Give final A0 for • jumping from $\frac{1}{2}n^4 + 2n^3 + 2n^2 + \frac{1}{2}n$ to $\frac{n}{2}(n+1)(n^2+3n+1)$ with no intermediate working
	Note	Condone final A1 for • jumping from $\frac{n}{2}(n^3 + 4n^2 + 4n + 1)$ to $\frac{n}{2}(n+1)(n^2+3n+1)$ with no intermediate working
	Note	Achieving the given result via an appropriate intermediate step with no algebraic errors seen in their working includes e.g. • $2\left(\frac{1}{4}n^2(n+1)^2\right) + 3\left(\frac{1}{6}n(n+1)(2n+1)\right) = \frac{1}{2}n^2(n+1)^2 + \frac{1}{2}n(n+1)(2n+1)$ $= \frac{1}{2}n(n+1)(n^2+3n+1)$ • $2\left(\frac{1}{4}n^2(n+1)^2\right) + 3\left(\frac{1}{6}n(n+1)(2n+1)\right) = \frac{1}{2}n(n+1)(n^2+n) + \frac{1}{2}n(n+1)(2n+1)$ $= \frac{1}{2}n(n+1)(n^2+3n+1)$ • $2\left(\frac{1}{4}n^2(n+1)^2\right) + 3\left(\frac{1}{6}n(n+1)(2n+1)\right) = \frac{1}{2}n(n+1)[n(n+1)] + \frac{1}{2}n(n+1)(2n+1)$ $= \frac{1}{2}n(n+1)(n^2+3n+1)$
3. (b)	**Note**	Allow M1 for $227825 - 4905$ and A1 for obtaining 222920
	Note	Allow M1 for $\left(\frac{1}{2}(25)^2(26)^2 + \frac{1}{2}(25)(26)(51)\right) - \left(\frac{1}{2}(9)^2(10)^2 + \frac{1}{2}(9)(10)(19)\right)$ $\{= (211250 + 16575) - (4050 + 855) = 227825 - 4905\}$ and A1 for obtaining 222920
	Note	Give M0 A0 for writing 222920 by itself with no supporting working
	Note	Allow M1 A1 for writing $\sum_{r=1}^{25} r^2(2r+3) - \sum_{r=1}^{9} r^2(2r+3) = 222920$
	Note	Give M0 A0 for listing individual terms i.e. $\sum_{r=10}^{25} r^2(2r+3) = (10)^2(23) + (11)^2(25) + (12)^2(27) + ... + (25)^2(53)$ $= 2300 + 3025 + 3888 + ... + 33125 = 222920$ by itself is M0 A0
	Note	Give M0 A0 for applying $\mathrm{f}(25) - \mathrm{f}(10) = \frac{25}{2}(25+1)((25)^2 + 3(25) + 1) - \frac{10}{2}(10+1)((10)^2 + 3(10) + 1)$ $= \frac{25}{2}(26)(701) - 5(11)(131) = 227825 - 7205 = 220620$
	Note	For M1 allow only one slip when substituting in $n = 25$ and $n = 9$
	Note	Give M0 for • $\frac{25}{2}(25+1)((25)^2 + 3(25) + 1) - \frac{9}{2}(9+1)((10)^2 + 3(10) + 1)$ $\{= 227825 - 5895 = 221930\}$

Question Number	Scheme	Notes	Marks
4.	$z_1 = p + 5\text{i},\ z_2 = 9 + 8\text{i},\ z_3 = \frac{z_1}{z_2}$; $\arg(z_1) = \frac{\pi}{3}$		
(a) **Way 1**	$z_3 = \frac{(p+5\text{i})}{(9+8\text{i})} \times \frac{(9-8\text{i})}{(9-8\text{i})}$	Multiplies numerator **and** denominator of z_3 by $9-8\text{i}$	M1
	$= \frac{9p - 8p\text{i} + 45\text{i} + 40}{81+64}$	Applies $\text{i}^2 = -1$ to give either • a correct expression in terms of p for the numerator **or** • a correct numerical expression or value for the denominator	A1
	$= \frac{9p+40}{145} + \left(\frac{-8p+45}{145}\right)\text{i}$	Correct answer written in the form $x + \text{i}y$ o.e. **or** writes a correct $x = \frac{9p+40}{145}$, $y = \frac{-8p+45}{145}$	A1
			(3)
(a) **Way 2**	$z_3 = \frac{(p+5\text{i})}{(9+8\text{i})} \times \frac{(-9+8\text{i})}{(-9+8\text{i})}$	Multiplies numerator **and** denominator of z_3 by $-9+8\text{i}$	M1
	$= \frac{-9p + 8p\text{i} - 45\text{i} - 40}{-81-64}$	Applies $\text{i}^2 = -1$ to give either • a correct expression in terms of p for the numerator **or** • a correct numerical expression or value for the denominator	A1
	$= \frac{-9p-40}{-145} + \left(\frac{8p-45}{-145}\right)\text{i}$	Correct answer written in the form $x + \text{i}y$ o.e. **or** writes a correct $x = \frac{-9p-40}{-145}$ and $y = \frac{8p-45}{-145}$	A1
			(3)
(b)	$\left\{\lvert z_2\rvert = \sqrt{9^2+8^2} \Rightarrow\right\}\ \lvert z_2\rvert = \sqrt{145}$	$\sqrt{145}$	B1
			(1)
(c)(i) **Way 1**	$\left\{\arg(z_1) = \frac{\pi}{3} \Rightarrow\right\}$		
	e.g. $\arctan\left(\frac{5}{p}\right) = \frac{\pi}{3}$ or $\tan\left(\frac{\pi}{3}\right) = \frac{5}{p}$ or $\sqrt{3} = \frac{5}{p}$	Uses trigonometry to form a correct equation in p	M1
	$p = \frac{5}{\sqrt{3}}$ or $\frac{5}{3}\sqrt{3}$ or $\sqrt{\frac{25}{3}}$	Correct exact value for p **Note:** You can apply isw	A1
(c)(i) **Way 2**	$\left\{z_1 = \sqrt{p^2+25}\left(\cos\frac{\pi}{3} + \text{i}\sin\frac{\pi}{3}\right) = p + 5\text{i} \Rightarrow\right\}$		
	e.g. $\sqrt{p^2+25}\left(\cos\frac{\pi}{3}\right) = p$ or $\sqrt{p^2+25}\left(\sin\frac{\pi}{3}\right) = 5$	Uses trigonometry to form a correct equation in p	M1
	$p = \frac{5}{\sqrt{3}}$ or $\frac{5}{3}\sqrt{3}$ or $\sqrt{\frac{25}{3}}$	Correct exact value for p **Note:** You can apply isw	A1
(ii)	• $\lvert z_3\rvert = \frac{\lvert z_1\rvert}{\lvert z_2\rvert} = \frac{\sqrt{\left(\frac{5}{\sqrt{3}}\right)^2 + (5)^2}}{\sqrt{145}} = \frac{\sqrt{\frac{100}{3}}}{\sqrt{145}}$ • $z_3 = \frac{8+3\sqrt{3}}{29} + \frac{27-8\sqrt{3}}{87} \Rightarrow \lvert z_3\rvert = \sqrt{\left(\frac{8+3\sqrt{3}}{29}\right)^2 + \left(\frac{27-8\sqrt{3}}{87}\right)^2}$		
	$\lvert z_3\rvert = \frac{10}{\sqrt{435}}$ or $\frac{10}{435}\sqrt{435}$ or $\frac{2}{87}\sqrt{435}$ or $\frac{2\sqrt{435}}{87}$	Correct exact answer written in the form $\frac{a}{\sqrt{b}}$ or $c\sqrt{b}$; $a,b \in \mathbb{Z},\ c \in \mathbb{Q}$	B1
	Note: Give B1 for $\lvert z_3\rvert = \sqrt{\frac{20}{87}}$		**(3)**
			7

	Question 4 Notes	
4. (a)	**Note**	Give 2nd A0 for $z_3 = \frac{9p+40}{81+64} + \left(\frac{-8p+45}{81+64}\right)\text{i}$ without reference to $z_3 = \frac{9p+40}{145} + \left(\frac{-8p+45}{145}\right)\text{i}$
	Note	$\frac{9p+40+(45-8p)\text{i}}{145}$ is not considered to be in the form $x + \text{i}y$
	Note	Allow final A1 for $z_3 = \frac{9p}{145} + \frac{8}{29} + \left(\frac{9}{29} - \frac{8p}{145}\right)\text{i}$
	Note	Allow final A1 for $z_3 = \frac{9p+40}{145} - \left(\frac{8p-45}{145}\right)\text{i}$
	Note	y written as $y = \left(\frac{-8p+45}{145}\right)\text{i}$ is incorrect
	Note	M1 A1 can be implied for writing $z_3 = \frac{(p+5\text{i})}{(9+8\text{i})} = \frac{9p-8p\text{i}}{145} + \frac{8+9\text{i}}{29}$ and final A1 is then given for $z_3 = \frac{9p}{145} + \frac{8}{29} + \left(\frac{9}{29} - \frac{8p}{145}\right)\text{i}$
(b)	**Note**	You can apply isw after seeing $\sqrt{145}$
	Note	Give B0 for writing $12, 12.0$ or awrt 12.0 without reference to $\sqrt{145}$
(c)(i)	**Note**	Give M1 for any of $\arctan\left(\frac{5}{p}\right) = 60$, $\tan 60 = \frac{5}{p}$, $\arctan\left(\frac{p}{5}\right) = \frac{\pi}{6}$, $\tan 30 = \frac{p}{5}$
	Note	Give M1 A0 for $p = 2.88$ (truncated) or $p =$ awrt 2.89 without reference to a correct exact value
	Note	Give A0 for $p = \pm\frac{5}{\sqrt{3}}$ with no evidence of rejecting the negative value of p
(c)(ii)	**Note**	Allow B1 for $\lvert z_3 \rvert = \frac{\sqrt{1740}}{87}$

Question Number	Scheme	Notes	Marks
5.	$f(x)=x^4-12x^{\frac{3}{2}}+7$; $x \geqslant 0$		
(a) **Way 1**	$f(2)=-10.9411255...$ $f(3)=\ \ 25.64617093...$	Attempts to evaluate both $f(2)$ and $f(3)$ **and** either $f(2)=-10$ (truncated) or awrt -11 **or** $f(3)=25$ (truncated) or awrt 26	M1
	Sign change {negative, positive} {and $f(x)$ is continuous} therefore a root $\{\alpha\}$ exists in the interval $\{[2, 3]\}$	Both values correct awrt (or truncated) to 2 sf, reason and a valid conclusion	A1 **cso**
			(2)
(b)	$f'(x)=4x^3-18x^{\frac{1}{2}}$	At least one of either $x^4 \to \pm Ax^3$ or $-12x^{\frac{3}{2}} \to \pm Bx^{\frac{1}{2}}$; $A, B \neq 0$	M1
		Correct differentiation, which can be un-simplified or simplified	A1
	$\left\{\alpha \simeq 2.5-\dfrac{f(2.5)}{f'(2.5)} \Rightarrow\right\} \alpha \simeq 2.5-\dfrac{(2.5)^4-12(2.5)^{\frac{3}{2}}+7}{4(2.5)^3-18(2.5)^{\frac{1}{2}}}$	**dependent on the previous M mark** Valid attempt at Newton-Raphson using the applied $f(2.5)$ and their applied $f'(2.5)$	dM1
	$\left\{\alpha \simeq 2.5-\dfrac{-1.3716649...}{34.0395011...} = 2.5+0.0402962...\right\}$		
	$\alpha = 2.54$ (2 dp)	**dependent on all 3 previous marks** 2.54 on first iteration (Ignore any subsequent iterations)	A1 **cao** **cso**
	Correct differentiation followed by 2.54 (with no working seen) scores full marks in part (b)		**(4)**
(c) **Way 1**	$f(2.535)=-0.137392933...$ $f(2.545)=\ \ 0.231219419...$	Chooses a suitable interval $[x_L, x_U]$ for x, which is within ± 0.005 and either side of their answer to (b) and attempts to find either $f(x_L)$ or $f(x_U)$	M1
	Sign change {negative, positive} {and $f(x)$ is continuous} therefore (a root) $\alpha = 2.54$ {2 dp}	Both values correct awrt 1 sf, reason and a valid conclusion	A1
			(2)
(c) **Way 2**	**Condoned Method: Applying Newton-Raphson again.** E.g. Using $\alpha = 2.54,\ 2.5402962...$		
	- $\alpha \simeq 2.54-\dfrac{0.046101609...}{36.8609766...} = 2.538751631...$ - $\alpha \simeq 2.5402962...-\dfrac{0.05693746...}{36.88822382...} = 2.538752436...$ So $\alpha = 2.54$ (2 dp)	Evidence of applying Newton-Raphson for a second time on their answer to part (b)	M1
		Obtains either a truncated 2.538 or awrt 2.539 and a valid conclusion	A1
	Note: Work for Way 2 can be recovered in part (b)		**(2)**
			8

	Question 5 Notes	
5. (a)	**Note**	**Way 1: A1, correct solution only** Required to state **both** values for $f(2)$ and $f(3)$ correct awrt (or truncated) to 2 sf along with **a reason and a conclusion.** Reference to change of sign **or** e.g. $f(2) \times f(3) < 0$ **or** $f(2) < 0 < f(3)$ **or** a diagram **or** < 0 and > 0 **or** one negative, one positive are sufficient reasons. There must be a conclusion, e.g. $\{x$ or$\}\ \alpha \in [2, 3]$ or $\{x$ or$\}\ \alpha \in (2, 3)$ or root lies between 2 and 3. Ignore the presence or absence of any reference to continuity.
	Note	A minimal acceptable reason and conclusion is "change of sign, so $\alpha \in [2, 3]$" **or** "change of sign, so root is between 2 and 3" **or** "change of sign, so root" **or** "$f(2) = -10.9 < 0,\ f(3) = 25.6 > 0$, so root" **or** "change of sign, so in the interval"

<table>
<tr><th></th><th colspan="2">Question 5 Notes Continued</th></tr>
<tr><td>5. (a)</td><td>Note</td><td>Give final A0 for writing as their conclusion “root lies between $\mathrm{f}(2)$ and $\mathrm{f}(3)$ ”</td></tr>
<tr><td>5. (a)</td><td>Note</td><td><u>ALT</u>
The root of $\mathrm{f}(x) = 0$ is $2.5388...,$ so they can choose x_1 which is less than $2.5388...,$
and choose x_2 which is greater than $2.5388...$ with both x_1 and x_2 lying in the interval $[2, 3]$.
M1: Finds $\mathrm{f}(x_1)$ and $\mathrm{f}(x_2)$ with one of these values correct awrt (or truncated) to 2 sf
A1: Both values correct awrt (or truncated) to 2 sf, reason (e.g. sign change) and conclusion</td></tr>
<tr><td></td><td>Note</td><td><u>Helpful Table</u>
<table>
<tr><th>x</th><th>$\mathrm{f}(x)$</th></tr>
<tr><td>2</td><td>$-10.9411255...$</td></tr>
<tr><td>2.1</td><td>$-10.0701694...$</td></tr>
<tr><td>2.2</td><td>$-8.731928012...$</td></tr>
<tr><td>2.3</td><td>$-6.873372451...$</td></tr>
<tr><td>2.4</td><td>$-4.439168148...$</td></tr>
<tr><td>2.5</td><td>$-1.371664903...$</td></tr>
<tr><td>2.6</td><td>$2.389111651...$</td></tr>
<tr><td>2.7</td><td>$6.90546741...$</td></tr>
<tr><td>2.8</td><td>$12.24204622...$</td></tr>
<tr><td>2.9</td><td>$18.46583545...$</td></tr>
<tr><td>3</td><td>$25.64617093...$</td></tr>
</table></td></tr>
<tr><td>(b)</td><td>dM1</td><td>This mark can be implied by applying at least one correct <i>value</i> of either $\mathrm{f}(2.5)$ or their $\mathrm{f}'(2.5)$ (where $\mathrm{f}'(2.5)$ is found using their $\mathrm{f}'(x)$) to awrt 2 significant figures in $2.5 - \frac{\mathrm{f}(2.5)}{\mathrm{f}'(2.5)}$.
So <i>just writing</i> $2.5 - \frac{\mathrm{f}(2.5)}{\mathrm{f}'(2.5)}$ with an incorrect ft answer on their $\mathrm{f}'(2.5)$ scores dM0 A0.</td></tr>
<tr><td></td><td>Note</td><td>Allow M1 A1 dM1 A1 for $2.5 - \frac{\mathrm{f}(2.5)}{\mathrm{f}'(2.5)} = 2.54$ with no algebraic differentiation</td></tr>
<tr><td></td><td>Note</td><td>Allow M1 A1 dM1 A1 for correct answer 2.54 given with no other working</td></tr>
<tr><td></td><td>Note</td><td>You can imply the M1 A1 marks for the absence of algebraic differentiation by either
• $\mathrm{f}'(2.5) = 4(2.5)^3 - 18(2.5)^{\frac{1}{2}}$
• $\mathrm{f}'(2.5)$ applied correctly in $\alpha \simeq 2.5 - \frac{(2.5)^4 - 12(2.5)^{\frac{3}{2}} + 7}{4(2.5)^3 - 18(2.5)^{\frac{1}{2}}}$
• $\mathrm{f}'(2.5) = $ awrt 34</td></tr>
<tr><td></td><td>Note</td><td>Differentiating INCORRECTLY to give $\mathrm{f}'(x) = 4x^3 + 18x^{\frac{1}{2}}$ leads to
$\alpha \simeq 2.5 - \frac{-1.3716649...}{90.9604989...} = 2.51507978... = 2.52$ (2 dp)
This response should be given M1 A0 dM1 A0</td></tr>
<tr><td></td><td>Note</td><td>Differentiating INCORRECTLY to give $\mathrm{f}'(x) = 4x^3 + 18x^{\frac{1}{2}}$ and
$\alpha \simeq 2.5 - \frac{\mathrm{f}(2.5)}{\mathrm{f}'(2.5)} = 2.52$ is M1 A0 dM1 A0</td></tr>
</table>

<table>
<tr><th></th><th colspan="2">Question 5 Notes Continued</th></tr>
<tr><td>5. (c)</td><td>Note</td><td>If they obtain a correct answer 2.54 by an incorrect method in part (b) then M1 A1 is allowed in part (c).</td></tr>
<tr><td></td><td>Note</td><td>Way 1: A1, correct solution only
Required to state both values for $\mathrm{f}(x_L)$ and $\mathrm{f}(x_U)$ correct awrt (or truncated) to 1 sf along with a reason and a conclusion. Reference to change of sign or e.g. $\mathrm{f}(2.535) \times \mathrm{f}(2.545) < 0$ or $\mathrm{f}(2.535) < 0 < \mathrm{f}(2.545)$ or a diagram or < 0 and > 0 or one negative, one positive are sufficient reasons. There must be a (minimal, not incorrect) conclusion e.g. $\alpha = 2.54$, root (or α to part (b)) is correct, QED or □ are all acceptable. Ignore the presence or absence of any reference to continuity.</td></tr>
<tr><td></td><td>Note</td><td>A minimal acceptable reason and conclusion is any of
• “change of sign, hence root”
• “change of sign, so $\alpha = 2.54$”
• “change of sign, so $x = 2.54$”
• “change of sign, so α is correct {to 2 decimal places}”
• “$\mathrm{f}(2.535) = -0.1 < 0,\ \mathrm{f}(2.545) = 0.2 > 0,$ so root”
• “$\mathrm{f}(2.535) = -0.1 < 0,\ \mathrm{f}(2.545) = 0.2 > 0,$ so $\alpha = 2.54$”</td></tr>
<tr><td></td><td>Note</td><td>No explicit reference to 2 decimal places is necessary for the conclusion</td></tr>
<tr><td></td><td>Note</td><td>Give A0 for stating “root is in between 2.535 and 2.545” or “root lies in the given interval” without reference to either $\alpha = 2.54$, root (or α to part (b)) is correct, QED or □</td></tr>
<tr><td>(c)</td><td>Note</td><td><u>Way 1: ALT</u>
The root of $\mathrm{f}(x) = 0$ is $2.5388...,$ so they can choose x_L which is less than $2.5388...,$ and choose x_U which is greater than $2.5388...$ with both x_L and x_U lying in the interval $[2.535, 2.545]$ and evaluate $\mathrm{f}(x_L)$ and $\mathrm{f}(x_U)$
M1: Chooses a suitable interval $[x_L, x_U]$ and attempts to find either $\mathrm{f}(x_L)$ or $\mathrm{f}(x_U)$
A1: Both values correct awrt (or truncated) to 1 sf, reason (e.g. sign change) and conclusion</td></tr>
<tr><td></td><td>Note</td><td><u>Helpful Table</u>
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<tr><td>2.535</td><td>$-0.137392933...$</td></tr>
<tr><td>2.536</td><td>$-0.100854301...$</td></tr>
<tr><td>2.537</td><td>$-0.064244144...$</td></tr>
<tr><td>2.538</td><td>$-0.027562401...$</td></tr>
<tr><td>2.539</td><td>0.00919099...</td></tr>
<tr><td>2.54</td><td>0.046016091...</td></tr>
<tr><td>2.541</td><td>0.082912964...</td></tr>
<tr><td>2.542</td><td>0.119881671...</td></tr>
<tr><td>2.543</td><td>0.156922274...</td></tr>
<tr><td>2.544</td><td>0.194034836...</td></tr>
<tr><td>2.545</td><td>0.231219419...</td></tr>
</table></td></tr>
<tr><td>(c)
Way 2</td><td>Note</td><td>If $\alpha = 2.54$ in part (b), then give M1 A1 in part (c) for any of
• “$\alpha_2 = 2.538 \Rightarrow \alpha_2 = 2.54$”
• “$\alpha_2 = 2.539 \Rightarrow \alpha_2 = 2.54$”
• “$\alpha_2 = 2.539,$ so answer to part (b) is correct”</td></tr>
<tr><td></td><td>Note</td><td>If $\alpha = 2.54$ in part (b), then give M1 A0 in part (c) for writing “$\alpha \simeq 2.54 - \frac{\mathrm{f}(2.54)}{\mathrm{f}'(2.54)} = 2.54$”</td></tr>
</table>

Question Number	Scheme	Notes	Marks
6.	$\mathbf{A}=\begin{pmatrix}2 & 3\\ 1 & -4\end{pmatrix}$; $A: R(3p-13,\ p-4)\ \mapsto\ R'(7,-2)$		
(a) **Way 1**	$\left\{\begin{pmatrix}x_{R'}\\ y_{R'}\end{pmatrix}=\begin{pmatrix}2 & 3\\ 1 & -4\end{pmatrix}\begin{pmatrix}3p-13\\ p-4\end{pmatrix}=\right\}$ $=\begin{pmatrix}2(3p-13)+3(p-4)\\ 1(3p-13)-4(p-4)\end{pmatrix}$	Correct method of multiplying out either $\begin{pmatrix}2 & 3\end{pmatrix}\begin{pmatrix}3p-13\\ p-4\end{pmatrix}$ or $\begin{pmatrix}1 & -4\end{pmatrix}\begin{pmatrix}3p-13\\ p-4\end{pmatrix}$ to give a linear expression in terms of p for either $x_{R'}$ or $y_{R'}$ **Note:** Allow one slip in their multiplication	M1
	• $2(3p-13)+3(p-4)=7\ \Rightarrow\ p=\dots$ • $1(3p-13)-4(p-4)=-2\ \Rightarrow\ p=\dots$	**dependent on the previous M mark** Solves either their $x_{R'}=7$ or their $y_{R'}=-2$ to give $p=\dots$	dM1
	$\{9p-38=7$ or $-p+3=-2\ \Rightarrow\}\ \ p=5$	$p=5$	A1
			(3)
(a) **Way 2**	$\{\mathbf{AR}=\mathbf{R}'\Rightarrow\mathbf{R}=\mathbf{A}^{-1}\mathbf{R}'\Rightarrow\}$		
	$\mathbf{R}=\dfrac{1}{-8-3}\begin{pmatrix}-4 & -3\\ -1 & 2\end{pmatrix}\begin{pmatrix}7\\ -2\end{pmatrix}=\begin{pmatrix}2\\ 1\end{pmatrix}$	Applies $\mathbf{A}^{-1}\begin{pmatrix}7\\ -2\end{pmatrix}$ to find the value for either x_R or y_R **Note:** Allow one slip in finding $\mathbf{A}^{-1}$	M1
	• $3p-13=2\ \Rightarrow\ p=\dots$ • $p-4=1\ \Rightarrow\ p=\dots$	**dependent on the previous M mark** Solves either $3p-13=$ their x_R or $p-4=$ their y_R to give $p=\dots$	dM1
	$p=5$	$p=5$	A1
			(3)
(a) **Way 3**	$\{\mathbf{AR}=\mathbf{R}'\Rightarrow\}\ \begin{pmatrix}2 & 3\\ 1 & -4\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix}=\begin{pmatrix}7\\ -2\end{pmatrix}$ $\begin{matrix}2a+3b=7\\ a-4b=-2\end{matrix}\ \ \Rightarrow a=2$ or $b=1$	Correct method of applying $\begin{pmatrix}2 & 3\\ 1 & -4\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix}=\begin{pmatrix}7\\ -2\end{pmatrix}$ to form a pair of simultaneous equations and attempts to find either $a=\dots$ or $b=\dots$ **Note:** Allow one slip in their multiplication	M1
	• $3p-13=2\ \Rightarrow\ p=\dots$ • $p-4=1\ \Rightarrow\ p=\dots$	**dependent on the previous M mark** Solves either $3p-13=$ their a or $p-4=$their b to give $p=\dots$	dM1
	$p=5$	$p=5$	A1
			(3)
(b) **Way 1**	$\{R(3(5)-13,\ 5-4)=R(2,1)\}$ $\{\text{Area}(ORS)=\}\ \dfrac{1}{2}(7)(\text{"}2\text{"})$	A correct method for finding their x_R **and** applies $\dfrac{1}{2}(7)(\text{their }x_R)$	M1
	$=7\ (\text{units})^2$	7	A1 **cao**
			(2)
(c)	$\{\text{Area}(OR'S')=\}\ \lvert 2(-4)-3(1)\rvert\times(7)$	$\pm\,(2(-4)-3(1))\times(\text{their area}(ORS))$	M1
	$=\ \ 77$	Correct answer of 77, which must be positive Only allow follow through of the value for $11\times$ their positive answer to (b)	A1 ft
			(2)
			7

Question Number	Scheme	Notes	Marks
6. (b) **Way 2**	{Area (ORS)} $= \frac{1}{2}\begin{vmatrix}0 & 2 & 0 & 0\\ 0 & 1 & 7 & 0\end{vmatrix} = \frac{1}{2}\lvert(0+14+0)-(0+0+0)\rvert$	A correct method for finding their $R(2, 1)$ with a complete applied method for finding area(ORS) using $S(0, 7)$ and their $R(2, 1)$	M1
	$= 7$ (units)^2	7	A1 **cao**
			(2)

		Question 6 Notes
6.	**Note**	$ORS \mapsto OR'S' \Rightarrow \begin{pmatrix}0 & 2 & 0\\ 0 & 1 & 7\end{pmatrix} \mapsto \begin{pmatrix}0 & 7 & 21\\ 0 & -2 & -28\end{pmatrix}$
(b) **Way 1**	**Note**	A correct method for finding their x_R includes any of • $x_R = 3(\text{"5"}) - 13 = 2$, where $p = \text{"5"}$ is found using part (a), Way 1 • their x_R found by applying $\mathbf{A}^{-1}\mathbf{R}'$ using part (a), Way 2 • $x_R =$ their a found using part (a), Way 3
(b) **Way 2**	**Note**	Give M1 A1 for $\frac{1}{2}\begin{vmatrix}2 & 0\\ 1 & 7\end{vmatrix} = \frac{1}{2}\lvert 14-0\rvert = 7$
	Note	Give M0 A0 for $\begin{vmatrix}0 & 2 & 0 & 0\\ 0 & 1 & 7 & 0\end{vmatrix} = \lvert\lvert(0+14+0)-(0+0+0)\rvert\rvert = 14$
	Note	There are other ways to find Area(ORS). All ways require a complete correct method for the M mark and a correct area of 7 for the A mark.
	Note	Give M1 for $\frac{1}{2}(1)(\text{"2"}) + \frac{1}{2}(6)(\text{"2"})$ as this method is equivalent to writing $\frac{1}{2}(7)(\text{"2"})$
	Note	Give M0 for the calculation $\frac{1}{2}(7)(7)$ $\left\{= \frac{49}{2}\right\}$
(c)	**Note**	Give M1 A0 for applying $(2(-4)-3(1)) \times (7)$ to give -77 with no reference to 77
	Note	Part (c) requires the use of the answer to part (b). So give M0 A0 for • Area $(OR'S') = \frac{1}{2}\begin{vmatrix}0 & 7 & 21 & 0\\ 0 & -2 & -28 & 0\end{vmatrix} = \frac{1}{2}\lvert(0-196+0)-(0-42+0)\rvert = \frac{1}{2}(154) = 77$ • Area $(OR'S') = \frac{1}{2}\begin{vmatrix}7 & 21\\ -2 & -28\end{vmatrix} = \frac{1}{2}\lvert(-196)-(-42)\rvert = \frac{1}{2}(154) = 77$ • Area $(OR'S') = (28)(21) - \frac{1}{2}(21)(28) - \frac{1}{2}(7)(2) - \frac{1}{2}(2+28)(14)$ $= 588 - 294 - 7 - 210 = 77$
	Note	Allow M1 A1 for • $\dfrac{\begin{vmatrix}7 & 21\\ -2 & -28\end{vmatrix}}{\begin{vmatrix}2 & 0\\ 1 & 7\end{vmatrix}} \times 7 = \dfrac{\lvert(-196)-(-42)\rvert}{\lvert 14-0\rvert} \times 7 = \dfrac{154}{14} \times 7 = 11 \times 7 = 77$

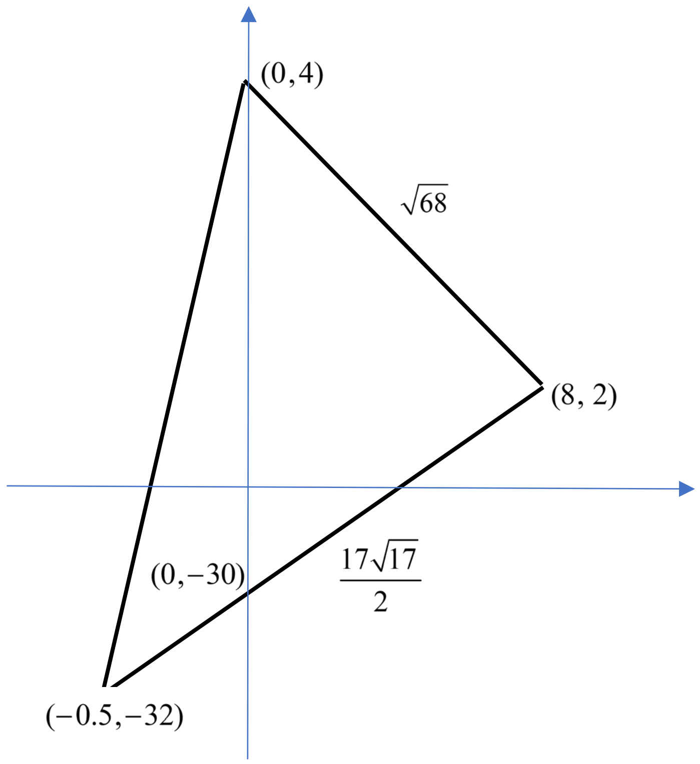
Question Number	Scheme	Notes	Marks
7.	$3x^2 + px - 5 = 0$ has roots α, β ; p is a constant (c) $\left(\alpha + \frac{1}{\beta}\right) + \left(\beta + \frac{1}{\alpha}\right) = 2\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$		
(a) (i)	$\alpha\beta = -\frac{5}{3}$	$\alpha\beta = -\frac{5}{3}$	B1
(ii)	$\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ $= \alpha\beta + 2 + \frac{1}{\alpha\beta} \quad = -\frac{5}{3} + 2 + \frac{1}{\left(-\frac{5}{3}\right)}$	Expands to give $\frac{1}{\alpha\beta} + 1 + 1 + \alpha\beta$; and uses their value of $\alpha\beta$ at least once in a resulting expression	M1
	$= -\frac{4}{15}$	$-\frac{4}{15}$	A1
			(3)
(b)(i)	$\alpha + \beta = -\frac{p}{3}$	$\alpha + \beta = -\frac{p}{3}$ (may be recovered from (a))	B1
(ii)	$\left(\alpha + \frac{1}{\beta}\right) + \left(\beta + \frac{1}{\alpha}\right) = \alpha + \beta + \frac{\alpha + \beta}{\alpha\beta}$	Evidence of $\frac{1}{\beta} + \frac{1}{\alpha}$ rewritten as $\frac{\alpha + \beta}{\alpha\beta}$ Can be implied	M1
	$= -\frac{p}{3} + \frac{-\frac{p}{3}}{-\frac{5}{3}}$ or $-\frac{p}{3} + \frac{p}{5}$ or $-\frac{2p}{15}$	$-\frac{p}{3} + \frac{-\frac{p}{3}}{-\frac{5}{3}}$ or $-\frac{p}{3} + \frac{p}{5}$ or $-\frac{2p}{15}$ or an equivalent fraction in terms of p **Note:** You can apply isw	A1
			(3)
(c)	$-\frac{2p}{15} = 2\left(-\frac{4}{15}\right) \Rightarrow p = 4$	Correctly obtains $p = 4$	B1
			(1)
(d)	$\sum = 2\left(-\frac{4}{15}\right) \quad = -\frac{8}{15}$; $\prod = -\frac{4}{15}$		
	$x^2 - -\frac{8}{15}x - \frac{4}{15} = 0$	Valid method for finding (their sum) **and applies** $x^2 - (\text{their sum})x + \text{their product}$ (can be implied), for their numerical values of the sum and product. **Note:** "=0" is not required for this mark **Note:** E.g. Using $(\text{their sum}) = \alpha + \beta = -\frac{p}{3} = -\frac{4}{3}$ is not considered a valid method for finding (their sum)	M1
	$15x^2 + 8x - 4 = 0$	***Any integer multiple*** of $15x^2 + 8x - 4 = 0$, including the "=0"	A1 **cso**
			(2)
			9

Question Number	Scheme	Notes	Marks
(a)(ii) **Way 2**	$\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$ $=\frac{(\alpha\beta+1)(\alpha\beta+1)}{\alpha\beta}=\frac{\left(-\frac{5}{3}+1\right)\left(-\frac{5}{3}+1\right)}{\left(-\frac{5}{3}\right)}=\frac{\frac{4}{9}}{-\frac{5}{3}}$	Expands to give $\frac{(\alpha\beta+1)(\alpha\beta+1)}{\alpha\beta}$ and uses their value of $\alpha\beta$ at least once in a resulting expression	M1
	$=-\frac{4}{15}$	$-\frac{4}{15}$	A1
(b)(ii) **Way 2**	$\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)$ $=\frac{(\alpha\beta+1)}{\beta}+\frac{(\alpha\beta+1)}{\alpha}=\frac{\alpha^2\beta+\alpha+\alpha\beta^2+\beta}{\alpha\beta}$	Embedded evidence of $\frac{1}{\beta}+\frac{1}{\alpha}$ rewritten as $\frac{\alpha+\beta}{\alpha\beta}$ Can be implied	M1
	$=\frac{\alpha\beta(\alpha+\beta)+\alpha+\beta}{\alpha\beta}$		
	$=\frac{(-\frac{5}{3})(-\frac{p}{3})+(-\frac{p}{3})}{(-\frac{5}{3})}$ or $\frac{\frac{5p}{9}-\frac{p}{3}}{-\frac{5}{3}}$ or $\frac{\frac{2p}{9}}{-\frac{5}{3}}$ or $-\frac{2p}{15}$	Correct expression in terms of p **Note:** You can apply isw	A1

	Question 7 Notes	
7. (d)	**Note**	Valid method for finding (their sum) includes • applying their $p=\ldots$ in (c) to $\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)=$ their $-\frac{2p}{15}$ found in (b)(ii) • applying $\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)=2\left(\text{their } -\frac{4}{15} \text{ from (a)(ii)}\right)$
	Note	Defining a quadratic equation $px^2+qx+r=0$ and a correct method leading to $p=15,\ q=8,\ r=-4$ without writing a final answer of $15x^2+8x-4=0$ is final M1 A0
	Note	Give M0 for $\sum=-\frac{8}{15}$, $\mathbf{\Pi}=-\frac{4}{15}$ leading to $x^2+\frac{8}{15}-\frac{4}{15}=0$ (without recovery)
	Note	Allow M1 for $\sum=-\frac{8}{15}$, $\mathbf{\Pi}=-\frac{4}{15}$ with $x^2-(\text{sum})x+(\text{product})$ leading to $x^2+\frac{8}{15}-\frac{4}{15}=0$
	Note	Give A1 for $15y^2+8y-4=0$ (i.e. writing their answer completely in another variable)
	Note	$\alpha,\beta=\frac{-2\pm\sqrt{19}}{3}$ and $\alpha+\frac{1}{\beta},\beta+\frac{1}{\alpha}=\frac{-4\pm2\sqrt{19}}{15}$ may be used in (d) to find the sum and product of $\alpha+\frac{1}{\beta}$ and $\beta+\frac{1}{\alpha}$

	Question 7 Notes Continued	
7.	**ALT**	For finding $\alpha, \beta = \frac{-p+\sqrt{p^2+60}}{6}, \frac{-p-\sqrt{p^2+60}}{6}$
(a) (i)	**Note**	Give B1 for $\alpha, \beta = \frac{-p+\sqrt{p^2+60}}{6}, \frac{-p-\sqrt{p^2+60}}{6}$ and then finding $\alpha\beta = -\frac{5}{3}$ or $-\frac{60}{36}$
(b) (i)	**Note**	Give B1 for $\alpha, \beta = \frac{-p+\sqrt{p^2+60}}{6}, \frac{-p-\sqrt{p^2+60}}{6}$ and then finding $\alpha+\beta = -\frac{p}{3}$
	Note	Allow B1 for writing $\alpha+\beta = \frac{-p+\sqrt{p^2+60}}{6} + \frac{-p-\sqrt{p^2+60}}{6}$
(b)(ii)	**Note**	Allow M1 A1 for writing $\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)$ as $\frac{-p+\sqrt{p^2+60}}{6} + \frac{-p-\sqrt{p^2+60}}{6} + \frac{6}{-p+\sqrt{p^2+60}} + \frac{6}{-p-\sqrt{p^2+60}}$

Question Number	Scheme	Notes	Marks
8.	$H: xy = 16;\ P\left(4t, \frac{4}{t}\right), t \neq 0,$ and $A: t = 2$ lies on H. $A(8, 2)$		
(a)	$y = \frac{16}{x} = 16x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -16x^{-2}$ or $-\frac{16}{x^2}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = \pm k x^{-2};\ k \neq 0$	M1
	$xy = 16 \Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$	Uses implicit differentiation to give $\pm x\frac{\mathrm{d}y}{\mathrm{d}x} \pm y$	
	$x = 4t,\ y = \frac{4}{t} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}.\frac{\mathrm{d}t}{\mathrm{d}x} = -\left(\frac{4}{t^2}\right)\left(\frac{1}{4}\right)$	their $\frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{1}{\text{their } \frac{\mathrm{d}y}{\mathrm{d}t}}$; **Condone** $p \equiv t$	
	So at P, $m_T = -\frac{1}{t^2}$	Correct calculus work leading to $m_T = -\frac{1}{t^2}$	A1
	So, $m_N = t^2$	Applies $m_N = \frac{-1}{m_T}$, where m_T is found using calculus	M1
	• $y - \frac{4}{t} = "t^2"(x - 4t)$ • $\frac{4}{t} = "t^2"(4t) + c \Rightarrow y = "t^2"x +$ their c	Correct straight line method for an equation of a normal where $m_N\ (\neq m_T)$ is found by using calculus	M1
	Correct algebra leading to $ty - t^3x = 4 - 4t^4$ *	Correct solution only	A1 cso
			(5)
(b)	$\{t = 2 \Rightarrow\}$ **N**$: 2y - 8x = 4 - 64\ \{\Rightarrow y = 4x - 30\}$	Uses $t = 2$ to find the equation of the normal to H at A	M1
	• $x(4x - 30) = 16\ \{\Rightarrow 2x^2 - 15x - 8 = 0\}$ • $\left(\frac{y + 30}{4}\right)y = 16\ \{\Rightarrow y^2 + 30y - 64 = 0\}$ • $\frac{4}{t} = 4(4t) - 30\ \{\Rightarrow 8t^2 - 15t - 2 = 0\}$	Substitutes the equation of the normal into the equation of the curve H to obtain an equation in x only or y only or t only	M1
	• $(x - 8)(2x + 1) = 0 \Rightarrow x_B = -\frac{1}{2}$ • $(y - 2)(y + 32) = 0 \Rightarrow y_B = -32$ • $(t - 2)(8t + 1) = 0 \Rightarrow t_B = -\frac{1}{8}$	**dependent on the first two M marks** Solves their 3 TQ = 0 to obtain a value for the x (or y) coordinate of B or a value of t at B	ddM1
	$B(-0.5, -32)$	Correct coordinates for B	A1
	$AB = \sqrt{(8 - -0.5)^2 + (2 - -32)^2}$	**dependent on the second M mark** Correct Pythagoras method to find the length of AB	dM1
	$= \frac{17\sqrt{17}}{2}$ or $\frac{\sqrt{4913}}{2}$ or $\sqrt{\frac{4913}{4}}$ or $\sqrt{1228.25}$	Correct exact length	A1
			(6)
(c)	$y - 2 = -\frac{1}{4}(x - 8)$ and $x = 0 \Rightarrow y_C = 2 + 2 = 4$	Finds the equation of the tangent at $(8, 2)$ to H, and sets $x = 0$ to find $y_C = ...$	M1
	$AC = \sqrt{(8 - 0)^2 + (2 - 4)^2}\ \{= \sqrt{68}\}$ Area $ABC = \frac{1}{2}\left(\frac{17\sqrt{17}}{2}\right)\left(\sqrt{68}\right)$	Uses the points $(8, 2)$, $(-0.5, -32)$ and $(0, 4)$ in a complete method to find the area of triangle ABC	M1
	$= 144.5$ or $\frac{289}{2}$	Correct answer	A1
			(3)
			14

	Question 8 Notes	
8. (b)	**Note**	The correct coordinates of B can be implied. e.g. embedded in the distance expression for AB
	Note	An incorrect **N**: $y = 4x + 30$ leads to the correct length AB for $A(-8, -2)$ and $B(0.5, 32)$
	Note	Condone final dM1 for $x_B = -\frac{1}{2}$ leading to $B(-2, -8)$ and $AB = \sqrt{(8 - -2)^2 + (2 - -8)^2}$
(c)	**Note**	Give 1st M0 for setting $x = 0$ in the equation of the normal to find $y_C = ...$
	Note	The 2nd M mark can only be gained by using all 3 correct points $(8, 2)$, $(-0.5, -32)$ and $(0, 4)$. Complete area methods include • Area $ABC = \frac{1}{2}\left(\frac{17\sqrt{17}}{2}\right)\left(\sqrt{68}\right)\{= 144.5\}$ • AB crosses y-axis at $(0, -30)$ and so Area $ABC = \frac{1}{2}(34)\left(\frac{1}{2}\right) + \frac{1}{2}(34)(8)\{= 8.5 + 136 = 144.5\}$ • Area $ABC = \frac{1}{2}\begin{vmatrix} 8 & -0.5 & 0 & 8 \\ 2 & -32 & 4 & 2 \end{vmatrix} = \frac{1}{2}\lvert(-256 - 2 + 0) - (-1 + 0 + 32)\rvert$ $\left\{= \frac{1}{2}\lvert(-289)\rvert = 144.5\right\}$ • Area $ABC = (32 + 4)\left(\frac{1}{2} + 8\right) - \frac{1}{2}(32 + 2)\left(\frac{1}{2} + 8\right) - \frac{1}{2}(32 + 4)\left(\frac{1}{2}\right) - \frac{1}{2}(2)(8)$ $\{= 306 - 144.5 - 9 - 8 = 144.5\}$ • Area $ABC = \frac{1}{2}(8 + 8.5)(36) - \frac{1}{2}(32 + 2)\left(\frac{1}{2} + 8\right) - \frac{1}{2}(2)(8)$ $\{= 297 - 144.5 - 8 = 144.5\}$
	Note	**Helpful Sketch** 

Question Number	Scheme	Notes	Marks
9.	$\mathrm{f}(n)=7^n(3n+1)-1$ is a multiple of 9	$u_1=2,\ u_2=6,\ u_{n+2}=3u_{n+1}-2u_n \Rightarrow u_n=2(2^n-1)$	
(i) **Way 1**	$\mathrm{f}(1)=7(4)-1=27$ {is a multiple of 9}	$\mathrm{f}(1)=27$ is the minimum	B1
	$\mathrm{f}(k+1)-\mathrm{f}(k)=\underline{7^{k+1}(3(k+1)+1)-1}-(7^k(3k+1)-1)$	Attempts $\mathrm{f}(k+1)-\mathrm{f}(k)$	M1
		A correct expression for $\underline{\mathrm{f}(k+1)}$	A1
	$=7^{k+1}(3k+4)-1-7^k(3k+1)+1=7^k(21k+28)-7^k(3k+1)$		
	$=18k(7^k)+27(7^k)$ or $7^k(18k+27)$	**dependent on the previous M mark** Uses correct algebra to achieve an expression where each term is an obvious multiple of 9	dM1
	$\mathrm{f}(k+1)=9(7^k)(2k+3)+7^k(3k+1)-1$ or $\mathrm{f}(k+1)=18k(7^k)+27(7^k)+\mathrm{f}(k)$	Correct algebra leading to either e.g. $\mathrm{f}(k+1)=9(7^k)(2k+3)+7^k(3k+1)-1$ or $\mathrm{f}(k+1)=18k(7^k)+27(7^k)+\mathrm{f}(k)$	A1
	If the result is true for $n=k$, then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n $(\in\mathbb{Z}^+)$		A1 cso
			(6)
(i) **Way 2**	$\mathrm{f}(1)=7(4)-1=27$ {is a multiple of 9}	$\mathrm{f}(1)=27$ is the minimum	B1
	$\mathrm{f}(k+1)=7^{k+1}(3(k+1)+1)-1$	Attempts $\mathrm{f}(k+1)$	M1
		A correct expression for $\mathrm{f}(k+1)$	A1
	$=7^{k+1}(3k+4)-1=7^k(21k+28)-1$		
	$=18k(7^k)+27(7^k)+7^k(3k+1)-1$ **or** $=(7^k)(18k+27)+7^k(3k+1)-1$ **or** $=9(7^k)(2k+3)+7^k(3k+1)-1$	**dependent on the previous M mark** Uses correct algebra to express $\mathrm{f}(k+1)=\mathrm{g}(k)+7^k(3k+1)-1$ or $\mathrm{f}(k+1)=\mathrm{g}(k)+\mathrm{f}(k)$ where each term in $\mathrm{g}(k)$ is an obvious multiple of 9	dM1
		Correct algebra leading to either e.g. $\mathrm{f}(k+1)=9(7^k)(2k+3)+7^k(3k+1)-1$ or $\mathrm{f}(k+1)=18k(7^k)+27(7^k)+\mathrm{f}(k)$	A1
	If the result is true for $n=k$, then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n $(\in\mathbb{Z}^+)$		A1 cso
			(6)
(ii)	$\{n=1,\}\ u_1=2(2^1-1)=2$;	Checks that the general formula works for either u_1 or u_2	M1
	$\{n=2,\}\ u_2=2(2^2-1)=6$	Checks that the general formula works for both u_1 and u_2	A1
	$\{u_{k+2}=3u_{k+1}-2u_k \Rightarrow\}$ $u_{k+2}=3(2(2^{k+1}-1))-2(2(2^k-1))$	Finds u_{k+2} by attempting to substitute $u_{k+1}=2(2^{k+1}-1)$ and $u_k=2(2^k-1)$ into $u_{k+2}=3u_{k+1}-2u_k$ Condone one slip	M1
	$\{u_{k+2}\}=6(2^{k+1})-6-4(2^k)+4$		
	$\{u_{k+2}\}=3(2^{k+2})-2^{k+2}-2$	Valid evidence of working in the same power of 2	M1
	$=2(2^{k+2})-2=2(2^{k+2}-1)$	***Uses algebra*** in a complete method to achieve this result with no errors	A1
	If the result is true for $n=k$ and for $n=k+1$, then it is true for $n=k+2$. As the result has been shown to be true for $n=1$ and $n=2$, then the result is true for all n $(\in\mathbb{Z}^+)$		A1 **cso**
			(6)
			12

	Question 9 Notes	
9. (i)	**Note**	**Final A1** is dependent on all previous marks being scored. It is gained by candidates conveying the ideas of **all** four underlined points **in part (i)** **either** at the end of their solution **or** as a narrative in their solution.
	Note	Shows $f(k+1)-f(k)=7^k(18k+27)$ **or** $f(k+1)-f(k)=9(7^k)(2k+3)$ and writing if $f(k+1)-f(k)=9(7^k)(2k+3)$ o.e. is a multiple of 9 then $f(k+1)$ is a multiple of 9 is acceptable for the penultimate A mark in part (i). This means that the final A mark can potentially be available.
	Note	Only showing $f(k+1)=\ 7f(k)\ +6+21(7^k)$ (see Way 4) does not get the final dM mark because $6+21(7^k)$ is not an obvious multiple of 9
	Note	Allow dM1 for obtaining e.g. $f(k+1)-f(k)=18k(7^k)-27(7^k)$ or $f(k+1)-f(k)=7^k(18k-27)$
	Note	Allow dM1 for obtaining $f(k+1)=18k(7^k)-27(7^k)+7^k(3k+1)-1$ or $f(k+1)=9(7^k)(2k-3)+f(k)$
(ii)	**Note**	**1st M1:** At least one check is correct. **1st A1:** Both checks are correct • Check 1: Shows $u_1=2$ by writing an intermediate step of e.g. $2(2^1-1)$ or 2×1 • Check 2: Shows $u_2=6$ by writing an intermediate step of e.g. $2(2^2-1)$ or 2×3
	Note	Ignore $u_3=3u_2-2u_1=3(6)-2(2)=14$ as part of their solution to (ii)
	Note	Ignore $\{n=3,\}$ $u_2=2(2^3-1)=14$ as part of their solution to (ii)
	Note	Valid evidence of working in the same power of 2 includes: • $6(2^{k+1})-4(2^k)\rightarrow6(2^{k+1})-2(2^{k+1})$ or $2(3(2^{k+1})-2^{k+1})$ • $3(2(2^{k+1}))-2(2(2^k))\rightarrow3(2^{k+2})-(2^{k+2})$ • $3(2(2^{k+1}))-2(2(2^k))\rightarrow12(2^k)-4(2^k)$ • $6(2^{k+1})-4(2^k)\rightarrow8(2^k)$ (by implication) • $6(2^{k+1})-4(2^k)\rightarrow4(2^{k+1})$ (by implication)
	Note	Writing $u_{k+2}=3(2(2^{k+1}-1))-2(2(2^k-1))=2(2^{k+2}-1)$ is 2nd M1, 3rd M0, 2nd A0
	Note	Showing {RHS = } $u_{k+2}=2(2^{k+2}-1)=8(2^k)-2$ and writing {LHS = } $u_{k+2}=3(2(2^{k+1}-1))-2(2(2^k-1))$ and using valid algebra to show that $u_{k+2}=8(2^k)-2$ {= RHS} is fine for the 2nd M, 3rd M and 2nd A marks
	Note	**Final A1** is dependent on all previous marks being scored. It is gained by candidates conveying the ideas of **all** four underlined points **in part (ii)** **either** at the end of their solution **or** as a narrative in their solution.
	Note	"Assume for $n=k$, $u_k=2(2^k-1)$ and for $n=k+1$, $u_{k+1}=2(2^{k+1}-1)$" satisfies the requirement "true for $\boldsymbol{n=k}$ and $n=k+1$"
	Note	"For $n\in\mathbb{Z}^+$, $u_n=2(2^n-1)$" satisfies the requirement "true for all *n*"
	Note	Full marks in (ii) can be obtained for an equivalent proof where e.g. • $n=k, n=k+1, \rightarrow n=k-2, n=k-1$; i.e. $k\equiv k-2$ • $n=k, n=k+1, \rightarrow n=k-1, n=k$; i.e. $k\equiv k-1$
(i), (ii)	**Note**	Allow as part of their conclusion "true for all positive values of *n*"
	Note	Allow as part of their conclusion "true for all values of *n*"
	Note	Allow as part of their conclusion "true for all $n\in\mathbb{N}$"
	Note	Condone referring to *n* as any integer in their conclusion for the final A1
	Note	Condone $n\in\mathbb{Z}^*$ as part of their conclusion for the final A1
	Note	Referring to *n* as a real number their conclusion is final A0

Question Number	Scheme	Notes	Marks
9.	$\mathrm{f}(n)=7^n(3n+1)-1$ is a multiple of 9; $P\in\mathbb{Z}^+$		
(i)	$\mathrm{f}(1)=7(4)-1=27$ {is a multiple of 9}	$\mathrm{f}(1)=27$ is the minimum	B1
Way 3	$\mathrm{f}(k+1)-(9P+1)\mathrm{f}(k)$	Attempts $\mathrm{f}(k+1)-(9P+1)\mathrm{f}(k)$	M1
	$=\underline{7^{k+1}(3(k+1)+1)-1}-(9P+1)(7^k(3k+1)-1)$	A correct expression for $\underline{\mathrm{f}(k+1)}$	A1
	$=7^k(21k+28-(9P+1)(3k+1))-1+9P+1$		
	$=7^k(21k+28-(27Pk+9P+3k+1))-1+9P+1$		
	$=7^k(21k+28-27Pk-9P-3k-1)+9P$		
	$=7^k(18k-27Pk-9P+27)+9P$	**dependent on the previous M mark** Uses correct algebra to achieve an expression where each term is an obvious multiple of 9	dM1
	$\mathrm{f}(k+1)=7^k(18k-27PK-9P+27)+9P+(9P+1)\mathrm{f}(k)$	Achieves a correct result for $\mathrm{f}(k+1)=\ldots$	A1
	If the result is $\underline{\text{true for } n=k}$, then it is $\underline{\text{true for } n=k+1.}$ As the result has been shown to be $\underline{\text{true for } n=1,}$ then the result $\underline{\text{is true for all } n}$ $(\in\mathbb{Z}^+)$		A1 cso
			(6)
	Note: $P=0\Rightarrow \mathrm{f}(k+1)-\mathrm{f}(k)=\ 7^k(18k+27)$ $P=1\Rightarrow \mathrm{f}(k+1)-10\mathrm{f}(k)=\ 7^k(18-9k)+9$ $P=2\Rightarrow \mathrm{f}(k+1)-19\mathrm{f}(k)=\ 7^k(9-36k)+18$ $P=3\Rightarrow \mathrm{f}(k+1)-28\mathrm{f}(k)=\ 7^k(-63k)+27=27-9k(7^{k+1})$		

Question Number	Scheme	Notes	Marks
9.	$\mathrm{f}(n) = 7^n(3n+1) - 1$ is a multiple of 9		
(i) **Way 4**	$\mathrm{f}(1) = 7(4) - 1 = 27$ {is a multiple of 9}	$\mathrm{f}(1) = 27$ is the minimum	B1
	$\mathrm{f}(k+1) = 7^{k+1}(3(k+1)+1) - 1$	Attempts $\mathrm{f}(k+1)$	M1
		A correct expression for $\mathrm{f}(k+1)$	A1
	$= 7(7^k)(3k+3+1) - 1$		
	$= 7(7^k)(3k+1) + 3(7)(7^k) - 1$		
	$= 7[(7^k)(3k+1) - 1] + 7 + 21(7^k) - 1$ $= 7\mathrm{f}(k) + 6 + 21(7^k)$ Let $\mathrm{g}(n) = 6 + 21(7^n)$ $\mathrm{g}(1) = 6 + 21(7^1) = 153$ {is a multiple of 9} {Assume the result is true for $\boldsymbol{n = k}$} $\mathrm{g}(k+1) = 6 + 21(7^{k+1})$ $= 6 + 147(7^k)$ $= 6 + 21(7^k) + 126(7^k)$ or $= \mathrm{g}(k) + 9(14)(7^k)$	**dependent on the previous M mark** Uses correct algebra to express $\mathrm{f}(k+1) = \alpha(7^k(3k+1) - 1) + \mathrm{g}(k)$ or $\mathrm{f}(k+1) = \alpha\mathrm{f}(k) + \mathrm{g}(k);\ \alpha \neq 0$ and uses correct algebra to achieve an expression for $\mathrm{g}(k+1)$ where each term is an obvious multiple of 9	M1
		Correct algebra leading to $\mathrm{f}(k+1) = 7\mathrm{f}(k) + 6 + 21(7^k)$ o.e. and $\mathrm{g}(k+1) = 6 + 21(7^k) + 126(7^k)$ where $\mathrm{g}(n) = 6 + 21(7^n)$	A1
	Proves that $\mathrm{g}(n) = 6 + 21(7^n)$ is a multiple of 9 and proves that for $\mathrm{f}(n)$ if the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all n $(\in \mathbb{Z}^+)$		A1 cso
			(6)
	Note: An alternative Way 4 method shows • $\mathrm{f}(k+1) = 7\mathrm{f}(k) + 6 + 21(7^k) = 7\mathrm{f}(k) + 9(7^k + 1) + 3(7^k) - 3$ • Defines $\mathrm{g}(n) = 3(7^n) - 3$ and proceeds to show that $\mathrm{g}(n)$ is also a multiple of 9		