# Mark Scheme (Results)

October 2020

Pearson Edexcel International A Level
In Further Pure Mathmatics F3
(WFM03/01)

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **1(a)** | $4\sinh^3 x + 3\sinh x = 4\left(\frac{e^x - e^{-x}}{2}\right)^3 + 3\left(\frac{e^x - e^{-x}}{2}\right)$<br>$= 4\left(\frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{8}\right) + 3\left(\frac{e^x - e^{-x}}{2}\right)$<br>Uses $\sinh x = \frac{e^x - e^{-x}}{2}$ on both sinh terms and attempts to cube the bracket<br>(min accepted is a linear x a quadratic bracket) | | M1 |
| | $= \frac{1}{2}e^{3x} - \frac{3}{2}e^x + \frac{3}{2}e^{-x} - \frac{1}{2}e^{-3x} + \frac{3}{2}e^x - \frac{3}{2}e^{-x}$<br>$= \frac{e^{3x} - e^{-3x}}{2} = \sinh 3x$* | | A1* |
| | | | **(2)** |
| **(b)** | $\sinh 3x = 19\sinh x \Rightarrow 4\sinh^3 x + 3\sinh x = 19\sinh x$<br>$\Rightarrow 4\sinh^3 x - 16\sinh x = 0$<br>Uses the result from (a) and combines terms | | M1 |
| | ($\sinh x = 0$ or) $\sinh^2 x = 4$ | $\sinh^2 x = 4$ or $\sinh x = (\pm)2$ | A1 |
| | (0, 0) | States the origin as one intersection | B1 |
| | $\ln\left(2+\sqrt{5}\right)$ **and** $-\ln\left(2+\sqrt{5}\right)$ | Two correct non-zero $x$ values(allow e.g. $\ln\left(-2+\sqrt{5}\right)$ for $-\ln\left(2+\sqrt{5}\right)$) | A1 |
| | $\left(\ln\left(2+\sqrt{5}\right), 38\right)$ **and** $\left(-\ln\left(2+\sqrt{5}\right), -38\right)$ | Two correct **points** (allow e.g. $\ln\left(-2+\sqrt{5}\right)$ for $-\ln\left(2+\sqrt{5}\right)$) | A1 |
| | | | **(5)** |
| | **Alternative for (b) using exponentials** | | |
| | $\sinh 3x = 19\sinh x \Rightarrow \frac{e^{3x} - e^{-3x}}{2} = \frac{19\left(e^x - e^{-x}\right)}{2} \Rightarrow \ldots$<br>Substitutes the correct exponential forms and collects terms to one side | | M1 |
| | $\Rightarrow e^{6x} - 19e^{4x} + 19e^{2x} - 1 = 0$ | Correct equation (or equivalent) | A1 |
| | (0, 0) | States the origin as one intersection | B1 |
| | $\frac{1}{2}\ln\left(9+4\sqrt{5}\right)$ **or** $\frac{1}{2}\ln\left(9-4\sqrt{5}\right)$ | Two correct non-zero $x$ values (oe) | A1 |
| | $\left(\frac{1}{2}\ln\left(9+4\sqrt{5}\right), 38\right)$ **and** $\left(\frac{1}{2}\ln\left(9-4\sqrt{5}\right), -38\right)$ | Two correct **points** (oe) | A1 |
| | | | **Total 7** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **2(i)** | $3x^2+12x+24=3\left(x^2+4x+8\right)$ <br> $=3\left(\left(x+2\right)^2+4\right)$ | Obtains $3\left(\left(x+2\right)^2+...\right)$ or $3\left(x+2\right)^2+...$ <br> Must include 3 now or later | M1 |
| | $3\left(\left(x+2\right)^2+4\right)$ or $3\left(x+2\right)^2+12$ | | A1 |
| | $\int \frac{1}{3x^2+12x+24}\,dx=\frac{1}{3}\int \frac{1}{\left(x+2\right)^2+4}\,dx=\frac{1}{6}\arctan\frac{x+2}{2}(+c)$ <br> M1: Use of arctan <br> A1: Fully correct expression (condone omission of $+\ c$) | | M1A1 |
| | | | **(4)** |
| **(ii)** | $27-6x-x^2=-\left(x^2+6x-27\right)$ <br> $=-\left(\left(x+3\right)^2-36\right)$ | Obtains $-\left(\left(x+3\right)^2+...\right)$ or $-\left(x+3\right)^2+...$ | M1 |
| | $-\left(\left(x+3\right)^2-36\right)$ or $36-\left(x+3\right)^2$ | | A1 |
| | $\int \frac{1}{\sqrt{27-6x-x^2}}\,dx=\int \frac{1}{\sqrt{36-\left(x+3\right)^2}}\,dx=\arcsin\left(\frac{x+3}{6}\right)(+c)$ <br> (Or $=-\arccos\left(\frac{x+3}{6}\right)(+c)$) <br> M1: Use of arcsin (or – arccos) <br> A1: Fully correct expression (condone omission of $+\ c$) | | M1A1 |
| | | | **(4)** |
| | | | **Total 8** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **3** | $\mathbf{M}=\begin{pmatrix}3 & -4 & k\\ 1 & -2 & k\\ 1 & -5 & 5\end{pmatrix}$ | | |
| **(a)** | $\lvert\mathbf{M}-\lambda\mathbf{I}\rvert=\mathbf{M}=\begin{vmatrix}3-\lambda & -4 & k\\ 1 & -2-\lambda & k\\ 1 & -5 & 5-\lambda\end{vmatrix}=\begin{vmatrix}0 & -4 & k\\ 1 & -5 & k\\ 1 & -5 & 2\end{vmatrix}$ <br> $(0)+4[2-k]+k[-5+5]$ <br> Attempts $\lvert\mathbf{M}-\lambda\mathbf{I}\rvert$ using $\lambda$= 3 | | M1 |
| | $(0)+4[2-k]+k[-5+5]=0\Rightarrow k=...$ <br> Uses $\lvert\mathbf{M}-\lambda\mathbf{I}\rvert=0$ and solves for $k$ | | M1 |
| | $k=2$ | Cao | A1 |
| | | | **(3)** |
| **(b)** | $(3-\lambda)\left[(\lambda+2)(\lambda-5)+10\right]+4(5-\lambda-2)+2(-5+2+\lambda)=0$ <br> Attempts $\lvert\mathbf{M}-\lambda\mathbf{I}\rvert=0$ using their value of $k$ | | M1 |
| | $\Rightarrow(3-\lambda)\left[(\lambda+2)(\lambda-5)+12\right]=0$ <br> $(\lambda+2)(\lambda-5)+12\Rightarrow\lambda^2-3\lambda+2=0\Rightarrow(\lambda-2)(\lambda-1)=0\Rightarrow\lambda=...$ <br> Uses $\lambda$ = 3as a factor to obtain and solve a 3TQ to find the other eigenvalues <br> (Alternatively may use calculator to solve $\lambda^3-6\lambda^2+11\lambda-6=0$) | | M1 |
| | $\lambda=1,\ 2$ | Correct values | A1 |
| | | | **(3)** |
| **(c)** | $\begin{pmatrix}3 & -4 & 2\\ 1 & -2 & 2\\ 1 & -5 & 5\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=3\begin{pmatrix}x\\ y\\ z\end{pmatrix}\Rightarrow\begin{matrix}3x-4y+2z=3x\\ x-2y+2z=3y\\ x-5y+5z=3z\end{matrix}$ | Uses the eigenvalue 3 and their $k$ to form at least 2 equations in $x$, $y$ and $z$ | M1 |
| | $\alpha\begin{pmatrix}1\\ 1\\ 2\end{pmatrix}$ (α a constant) | Any correct eigenvector. Allow any constant multiple of **i** + **j** + 2**k** | A1 |
| | $\frac{1}{\sqrt{6}}\begin{pmatrix}1\\ 1\\ 2\end{pmatrix}$ | Correct normalised vector | A1 |
| | | | **(3)** |
| | | | **Total 9** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **4.** | $I_n = \int x^n \cos x \, dx$ | | |
| **(a)** | $\int x^n \cos x \, dx = x^n \sin x - \int n x^{n-1} \sin x \, dx$ <br> M1: Parts in the correct direction <br> A1: Correct expression | | M1A1 |
| | $= x^n \sin x - \left\{ -n x^{n-1} \cos x + \int n(n-1) x^{n-2} \cos x \, dx \right\}$ <br> Uses integration by parts again (**dependent on the first M**) | | dM1 |
| | $= x^n \sin x + n x^{n-1} \cos x - n(n-1) I_{n-2}$ * <br> Fully correct proof with no errors | | A1* |
| | | | **(4)** |
| **ALT** | | | |
| | $I_n = \int x^n \cos x \, dx = \int x^{n-1} (x \cos x) \, dx$ | | |
| | $= x^n \sin x + x^{n-1} \cos x - (n-1) \int x^{n-2} (x \sin x + \cos x) \, dx$ <br> M1: Parts in the correct direction <br> A1: Correct expression | | M1A1 |
| | $= x^n \sin x + x^{n-1} \cos x - (n-1) \int x^{n-1} \sin x dx - (n-1) I_{n-2}$ | | |
| | $= x^n \sin x + x^{n-1} \cos x - (n-1)\left\{ -x^{n-1} \cos x + (n-1) I_{n-2} \right\} - (n-1) I_{n-2}$ <br> Uses integration by parts again (**dependent on the first M**) | | dM1 |
| | $= x^n \sin x + n x^{n-1} \cos x - n(n-1) I_{n-2}$ * <br> Fully correct proof with no errors | | A1* |
| | | | |
| **(b)** | $I_0 = \sin x \ \ (+k)$ | | B1 |
| | $I_4 = x^4 \sin x + 4x^3 \cos x - 12 I_2$ | Applies the reduction formula once for $I_4$ or $I_2$ | M1 |
| | $= x^4 \sin x + 4x^3 \cos x - 12\left( x^2 \sin x + 2x \cos x - 2 I_0 \right)$ <br> Applies the reduction formula again and obtains an expression for $I_4$ which can include $I_0$ but not $I_2$ | | M1 |
| | $= \left( x^4 - 12x^2 + 24 \right) \sin x + \left( 4x^3 - 24x \right) \cos x + c$ <br> Award A1 for either bracket and A1 for the other <br> If the answer is not factorised but is otherwise correct, award A1A0 | | A1A1 |
| | | | **(5)** |
| | | | **Total 9** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **5** | $\frac{x^2}{25}-\frac{y^2}{4}=1 \quad y=mx+c$ | | |
| **(a)** | $\frac{x^2}{25}-\frac{(mx+c)^2}{4}=1 \Rightarrow 4x^2-25\left(m^2x^2+2cmx+c^2\right)=100$<br>Substitutes to obtain a quadratic in $x$ and eliminates fractions | | M1 |
| | $4x^2-25\left(m^2x^2+2cmx+c^2\right)=100$<br>$\left(\Rightarrow\left(25m^2-4\right)x^2+50cmx+25c^2+100=0\right)$<br>Correct 3TQ | | A1 |
| | $"b^2=4ac" \Rightarrow (50cm)^2=4\left(25m^2-4\right)\left(25c^2+100\right)$<br>Uses '$b^2=4ac$' or equivalent | | M1 |
| | $2500c^2m^2=2500c^2m^2+10000m^2-400c^2-1600$<br>$10000m^2=400c^2+1600$<br>$25m^2=c^2+4$*<br>Fully correct proof with no errors | | A1* |
| | | | **(4)** |
| **ALT 1** | Using hyperbolic parameters: | | |
| | $x=5\cosh t, y=2\sinh t \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\cosh t}{5\sinh t}$ | | |
| | $\frac{2\cosh t}{5\sinh t}(x-5\cosh t)=y-2\sinh t$<br>M1: Attempts the equation of the tangent A1: Correct equation (no simplification needed) | | M1A1 |
| | $y=\frac{2\cosh t}{5\sinh t}x-\frac{2\cosh^2 t-25\sinh^2 t}{\sinh t}$ | | |
| | $25m^2=\frac{4\cosh^2 t}{\sinh^2 t},\ 4+c^2=4+\frac{4}{\sinh^2 t}=\frac{4\left(\sinh^2 t+1\right)}{\sinh^2 t}=\frac{4\cosh^2 t}{\sinh^2 t}$<br>Extracts $25m^2$ and $4+c^2$ from their equation | | M1 |
| | $\therefore 25m^2=4+c^2$ *<br>Fully correct proof with no errors | | A1* |
| | | | **(4)** |
| **ALT 2** | Using trigonometric parameters: | | |
| | $x=5\sec t, y=2\tan t \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\sec t}{5\tan t}$ | | |
| | $\frac{2\sec t}{5\tan t}(x-5\sec t)=y-2\tan t$<br>M1: Attempts the equation of the tangent A1: Correct equation (no simplification needed) | | M1A1 |
| | $y=\frac{2\sec t}{5\tan t}x+\frac{2\tan^2 t-2\sec^2 t}{\tan t}$ | | |
| | $25m^2=\frac{4\sec^2 t}{\tan^2 t}=\frac{4}{\sin^2 t} \quad 4+c^2=4\left(1+\frac{1}{\tan^2 t}\right)=4\left(\frac{\sin^2 t+\cos^2 t}{\sin^2 t}\right)=\frac{4}{\sin^2 t}$ Extracts $25m^2$ and $4+c^2$ from their equation | | M1 |
| | $\therefore 25m^2=4+c^2$ *<br>Fully correct proof with no errors | | A1* |
| | | | **(4)** |

| | | | |
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| **(b)** | $25m^2 = c^2 + 4$ and $2 = m + c$<br>$25m^2 = (2-m)^2 + 4$ or $25(2-c)^2 = c^2 + 4$<br>Uses the given hyperbola and the straight line with the result from (a) to obtain an equation in *m* or *c* | | M1 |
| | $24m^2 + 4m - 8 = 0$<br>or<br>$24c^2 - 100c + 96 = 0$ | Correct 3TQ in *m* or *c* | A1 |
| | $24m^2 + 4m - 8 = 0 \Rightarrow m = \frac{1}{2}, -\frac{2}{3}$<br>Or<br>$24c^2 - 100c + 96 = 0 \Rightarrow c = \frac{3}{2}, \frac{8}{3}$ | Solves their 3TQ in *m* or *c* | M1 |
| | $y = \frac{1}{2}x + \frac{3}{2}$ **or** $y = -\frac{2}{3}x + \frac{8}{3}$ | One correct tangent | A1 |
| | $y = \frac{1}{2}x + \frac{3}{2}$ **and** $y = -\frac{2}{3}x + \frac{8}{3}$ | Both correct tangents | A1 |
| | | | **(5)** |
| **(c)** | $m = \frac{1}{2},\ c = \frac{3}{2} \Rightarrow \frac{9}{4}x^2 + \frac{75}{2}x + \frac{625}{4} = 0 \Rightarrow x = ...$<br>**or**<br>$m = -\frac{2}{3},\ c = \frac{8}{3} \Rightarrow \frac{64}{9}x^2 - \frac{800}{9}x + \frac{2500}{9} = 0 \Rightarrow x = ...$<br>Uses one of their *m* and *c* pairs and solves for *x* | | M1 |
| | $x = -\frac{25}{3}, y = -\frac{8}{3}$ **or** $x = \frac{25}{4}, y = -\frac{3}{2}$ | One correct point | A1 |
| | $x = -\frac{25}{3}, y = -\frac{8}{3}$ **and** $x = \frac{25}{4}, y = -\frac{3}{2}$ | Both correct points | A1 |
| | | | **(3)** |
| | | | **Total 12** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| 6(a) | $\mathbf{A}=\begin{pmatrix}1&-1&1\\1&1&1\\1&2&a\end{pmatrix}$ | | |
| | $\lvert\mathbf{A}\rvert=a-2+a-1+2-1(=2a-2)$ | Correct determinant in any form | B1 |
| | $\begin{pmatrix}1&-1&1\\1&1&1\\1&2&a\end{pmatrix}\to\begin{pmatrix}a-2&a-1&1\\-a-2&a-1&3\\-2&0&2\end{pmatrix}\to\begin{pmatrix}a-2&1-a&1\\a+2&a-1&-3\\-2&0&2\end{pmatrix}$<br>Applies the correct method to reach at least a matrix of cofactors<br>2 correct rows or 2 correct columns needed | | M1 |
| | $\begin{pmatrix}a-2&1-a&1\\a+2&a-1&-3\\-2&0&2\end{pmatrix}\to\begin{pmatrix}a-2&a+2&-2\\1-a&a-1&0\\1&-3&2\end{pmatrix}$<br>Correct transpose of cofactors | | A1 |
| | $\mathbf{A}^{-1}=\frac{1}{2a-2}\begin{pmatrix}a-2&a+2&-2\\1-a&a-1&0\\1&-3&2\end{pmatrix}$ | Correct inverse | A1 |
| | | | **(4)** |
| (b) | $a=4\Rightarrow\mathbf{A}^{-1}=\frac{1}{6}\begin{pmatrix}2&6&-2\\-3&3&0\\1&-3&2\end{pmatrix}$ | Correct inverse (follow through their matrix from (a)) | B1ft |
| | $=\frac{1}{6}\begin{pmatrix}2&6&-2\\-3&3&0\\1&-3&2\end{pmatrix}\begin{pmatrix}12-6\lambda\\4+2\lambda\\6+3\lambda\end{pmatrix}=...$ | Attempt to multiply the parametric form of $l_2$ by their inverse | M1 |
| | $=\begin{pmatrix}6-\lambda\\-4+4\lambda\\2-\lambda\end{pmatrix}$ | Correct parametric form | A1 |
| | $\mathbf{r}=\begin{pmatrix}6\\-4\\2\end{pmatrix}+\lambda\begin{pmatrix}-1\\4\\-1\end{pmatrix}$ | Correct equation (allow equivalent forms) but if given as $l$ = ... award A0 | A1 |
| | | | **(4)** |
| | | | **Total 8** |
| | | | |

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| | **Alternatives for (b)** | | |
| | | | |
| **(i)** | $a=4 \Rightarrow \mathbf{A}^{-1}=\frac{1}{6}\begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}$ | Correct inverse (follow through their matrix from (a)) | B1ft |
| | $\frac{1}{6}\begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}\begin{pmatrix} 12 \\ 4 \\ 6 \end{pmatrix}=\frac{1}{6}\begin{pmatrix} 36 \\ -24 \\ 12 \end{pmatrix}$ <br> $\frac{1}{6}\begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}\begin{pmatrix} -6 \\ 2 \\ 3 \end{pmatrix}=\frac{1}{6}\begin{pmatrix} -6 \\ 24 \\ -6 \end{pmatrix}$ | Attempt $\mathbf{A}^{-1}$(point on $l_2$) **and** $\mathbf{A}^{-1}$(direction of $l_2$) <br> **Both** correct (NB No ft) | M1 <br> A1 |
| | $\mathbf{r}=\begin{pmatrix} 6 \\ -4 \\ 2 \end{pmatrix}+\lambda\begin{pmatrix} -1 \\ 4 \\ -1 \end{pmatrix}$ | Correct equation (allow equivalent forms) but if given as $l$ = ... award A0 | A1 |
| | | | **(4)** |
| **(ii)** | $a=4 \Rightarrow \mathbf{A}^{-1}=\frac{1}{6}\begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}$ | Correct inverse (follow through their matrix from (a)) | B1ft |
| | $\frac{1}{6}\begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}\begin{pmatrix} 12 \\ 4 \\ 6 \end{pmatrix}=\frac{1}{6}\begin{pmatrix} 36 \\ -24 \\ 12 \end{pmatrix}$ <br> $\frac{1}{6}\begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}\begin{pmatrix} 6 \\ 6 \\ 9 \end{pmatrix}=\frac{1}{6}\begin{pmatrix} 30 \\ 0 \\ 6 \end{pmatrix}$ | Attempt $\mathbf{A}^{-1}$(point on $l_2$) for 2 points <br> **Both** correct (NB No ft) | M1 <br> A1 |
| | $\mathbf{r}=\begin{pmatrix} 6 \\ -4 \\ 2 \end{pmatrix}+\lambda\begin{pmatrix} -1 \\ 4 \\ -1 \end{pmatrix}$ | Obtain the direction vector and deduce correct equation (allow equivalent forms) but if given as $l$ = ... award A0 | A1 |
| | | | **(4)** |
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|---|---|---|---|
| **7** | $x = \cosh t + t, \quad y = \cosh t - t$ | | |
| **(a)** | $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \sinh t + 1, \quad \dfrac{\mathrm{d}y}{\mathrm{d}t} = \sinh t - 1$ | Correct derivatives | B1 |
| | $\left(\dfrac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = \sinh^2 t + 2\sinh t + 1 + \sinh^2 t - 2\sinh t + 1$ <br> $= 2\sinh^2 t + 2$ <br> M1: Squares correctly, cancels and collects terms | | M1 |
| | $= 2\left(1 + \sinh^2 t\right) = 2\cosh^2 t$ * | Uses $\cosh^2 t = 1 + \sinh^2 t$ to complete the proof with no errors | A1* |
| | | | **(3)** |
| **(b)** | $S = 2\pi\int y\,\mathrm{d}s = 2\pi\int\left(\cosh t - t\right)\sqrt{2}\cosh t\,\mathrm{d}t$ | Uses $S = 2\pi\int y\,\mathrm{d}s$ with the given $y$ and the result from part (a) | M1 |
| | $= 2\sqrt{2}\pi\int_0^{\ln 3}\left(\cosh^2 t - t\cosh t\right)\mathrm{d}t$ * | Correct proof with no errors | A1* |
| | | | **(2)** |
| **(c)** | $\int \cosh^2 t\,\mathrm{d}t = \int \pm\frac{1}{2} \pm \frac{1}{2}\cosh 2t\,\mathrm{d}t$ | Uses $\cosh^2 t = \pm\frac{1}{2} \pm \frac{1}{2}\cosh 2t$ | M1 |
| | $\int t\cosh t\,\mathrm{d}t = t\sinh t - \int \sinh t\,\mathrm{d}t$ | Attempts integration by parts the right way round on $t\cosh t$ | M1 |
| | | Correct expression | A1 |
| | $S = \left(2\sqrt{2}\pi\right)\int\left(\cosh^2 t - t\cosh t\right)\mathrm{d}t = \left(2\sqrt{2}\pi\right)\left[\frac{1}{2}t + \frac{1}{4}\sinh 2t - t\sinh t + \cosh t\right]$ <br> A1: 2 correct terms <br> A1: All correct | | A1A1 |
| | $(S =)\,2\sqrt{2}\pi\left\{\left(\frac{1}{2}\ln 3 + \frac{10}{9} - \frac{4}{3}\ln 3 + \frac{5}{3}\right) - (1)\right\}$ <br> dM1: Correct use of limits 0 and ln3 depends on both preceding M marks | | dM1 |
| | $S = \frac{1}{9}\sqrt{2}\pi\left(32 - 15\ln 3\right)$ | cao | A1 **(7)** |
| | | | **Total 12** |
| | **Alternative for (c)** | | |
| | $\int \cosh^2 t\,\mathrm{d}t = \int\left(\frac{\mathrm{e}^t + \mathrm{e}^{-t}}{2}\right)^2 \mathrm{d}t$ <br> $= \frac{1}{4}\int\left(\mathrm{e}^{2t} + 2 + \mathrm{e}^{-2t}\right)\mathrm{d}t$ | Substitutes the exponential form and attempts to square | M1 |
| | $\int t\cosh t\,\mathrm{d}t = \frac{1}{2}\int t\left(\mathrm{e}^t + \mathrm{e}^{-t}\right)\mathrm{d}t$ <br> $= \frac{1}{2}t\mathrm{e}^t - \frac{1}{2}\int t\mathrm{e}^t\mathrm{d}t - \left\{\frac{1}{2}t\mathrm{e}^{-t} - \frac{1}{2}\int \mathrm{e}^{-t}\mathrm{d}t\right\}$ | Substitutes the exponential form and attempts integration by parts the right way round <br> Correct expression | M1 <br> A1 |
| | $(S =)\left(2\sqrt{2}\pi\right)\left\{\frac{1}{4}\left(\frac{1}{2}\mathrm{e}^{2t} + 2t - \frac{1}{2}\mathrm{e}^{-2t}\right) - \frac{1}{2}t\mathrm{e}^t + \frac{1}{2}\mathrm{e}^t + \frac{1}{2}t\mathrm{e}^{-t} - \frac{1}{2}\mathrm{e}^{-t}\right\}$ <br> A1: either integral correct A1: other integral correct but both must be in a complete expression for $S$ | | A1A1 |
| | Depends on both M marks above | Correct use of limits 0 and ln3 | dM1 |
| | $S = \frac{1}{9}\sqrt{2}\pi\left(32 - 15\ln 3\right)$ | cao | A1 |

| | | | |
|---|---|---|---|
| | **Alternative for the first 3 marks of (c)** | | |
| | $=2\sqrt{2}\pi\int\left(\cosh^2 t - t\cosh t\right)\mathrm{d}t$<br>$=2\sqrt{2}\pi\int \cosh t\left(\cosh t - t\right)\mathrm{d}t$ | | |
| | $2\sqrt{2}\pi\left(\left[\sinh t\left(\cosh t - t\right)\right]-\int \sinh t\left(\sinh t - 1\right)\mathrm{d}t\right)$ | | |
| | $2\sqrt{2}\pi\left(\left[\sinh t\left(\cosh t - t\right)\right]-\left[\cosh t\left(\sinh t - 1\right)\right]+\int \cosh^2 t\mathrm{d}t\right)$<br>M1 (2nd on e-PEN): Use parts twice A1 Correct expression | | M1A1 |
| | $\int \cosh^2 t\,\mathrm{d}t = \int \pm\frac{1}{2}\pm\frac{1}{2}\cosh 2t\,\mathrm{d}t$ | Uses $\cosh^2 t = \pm\frac{1}{2}\pm\frac{1}{2}\cosh 2t$ | M1 (1st on e-PEN) |
| | Rest as main scheme | | |
| | | | |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **8(a)** | $\mathbf{n}=\begin{pmatrix}1\\-5\\3\end{pmatrix}\times\begin{pmatrix}3\\-2\\2\end{pmatrix}=\begin{pmatrix}-10+6\\-(2-9)\\-2+15\end{pmatrix}$ | Attempt vector product between normal vectors | M1 |
| | $=\begin{pmatrix}-4\\7\\13\end{pmatrix}$ | Correct vector | A1 |
| | $x=0\Rightarrow -5y+3z=11,\quad -2y+2z=7$ $\Rightarrow y=-\frac{1}{4},z=\frac{13}{4}$ or $y=0\Rightarrow x+3z=11,\quad 3x+2z=7$ | Correct strategy to find a point on $l$ | M1 |
| | $\Rightarrow x=-\frac{1}{7},z=\frac{26}{7}$ or $z=0\Rightarrow x-5y=11,3x-2y=7$ $\Rightarrow x=1,y=-2$ | Correct position vector of point on $l$ | A1 |
| | $\mathbf{r}=\mathbf{i}-2\mathbf{j}+\lambda(-4\mathbf{i}+7\mathbf{j}+13\mathbf{k})$ | Correct equation. (follow through their position and direction vectors but must be "$\mathbf{r}$ =") | A1ft |
| | | | **(5)** |
| **ALT** | $x=11+5y-3z$ | | |
| | $3x-2y+2z=7\Rightarrow 3(11+5y-3z)-2y+2z=7$ $\Rightarrow y-\frac{7z}{13}=-\frac{26}{13}\quad\left(z=\frac{13y+26}{7}\right)$ Eliminate one variable | | M1 |
| | $x=11+5\left(-\frac{26}{13}+\frac{7z}{13}\right)\Rightarrow z=\frac{13-13x}{4}$ | Obtain 2 correct expressions for one of the variables | A1 |
| | $\frac{x-1}{-\frac{4}{13}}=\frac{y+2}{\frac{7}{13}}=z$ | M1 Obtain a Cartesian equation for $l$ <br> A1 Correct equation | M1A1 |
| | $\mathbf{r}=(\mathbf{i}-2\mathbf{j})+\lambda\left(-\frac{4}{13}\mathbf{i}+\frac{7}{13}\mathbf{j}+\mathbf{k}\right)$ oe | Deduce a vector equation for $l$ Follow through their Cartesian equation | A1ft |
| | | | **(5)** |

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| **(b)** | $\begin{pmatrix}3\\2\\1\end{pmatrix}-\begin{pmatrix}2\\0\\3\end{pmatrix}=\begin{pmatrix}1\\2\\-2\end{pmatrix}$ | Correct vector joining $P$ to $Q$ | B1 |
| | $\begin{pmatrix}-4\\7\\13\end{pmatrix}\times\begin{pmatrix}1\\2\\-2\end{pmatrix}=\begin{pmatrix}-40\\5\\-15\end{pmatrix}$ | Attempt vector product between the direction of $l$ and their $\mathbf{i}+2\mathbf{j}-2\mathbf{k}$ | M1 |
| | | Correct vector | A1 |
| | $\sin\theta=\dfrac{\lvert-40\mathbf{i}+5\mathbf{j}-15\mathbf{k}\rvert}{\lvert-4\mathbf{i}+7\mathbf{j}+13\mathbf{k}\rvert\lvert\mathbf{i}+2\mathbf{j}-2\mathbf{k}\rvert}$ | Angle between $PQ$ and line $n$ | |
| | $d=\lvert\overrightarrow{PQ}\rvert\sin\theta$ | | |
| | $d=\dfrac{\lvert-40\mathbf{i}+5\mathbf{j}-15\mathbf{k}\rvert}{\lvert-4\mathbf{i}+7\mathbf{j}+13\mathbf{k}\rvert}=\dfrac{1}{\sqrt{234}}\sqrt{40^2+5^2+15^2}$ | Fully correct method for the distance | M1 |
| | $d=\dfrac{5\sqrt{481}}{39}$ | Cao Allow equivalent **exact** forms e.g. $d=\dfrac{5\sqrt{74}}{\sqrt{234}}$ | A1 |
| | | | **(5)** |
| **ALT 1** | $\mathbf{r}_m=\begin{pmatrix}2\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}-\frac{4}{7}\\1\\\frac{13}{7}\end{pmatrix}$ or $\mathbf{r}_n=\begin{pmatrix}3\\2\\1\end{pmatrix}+\mu\begin{pmatrix}-\frac{4}{7}\\1\\\frac{13}{7}\end{pmatrix}$ | Vector equation for either line with their direction vector from (a) | B1ft |
| | $\overrightarrow{OP}=\begin{pmatrix}2\\0\\3\end{pmatrix}$ $\overrightarrow{ON}=\begin{pmatrix}3-\frac{4}{7}\mu\\2+\mu\\1+\frac{13}{7}\mu\end{pmatrix}$ $\overrightarrow{NP}=\begin{pmatrix}-1+\frac{4}{7}\mu\\-2-\mu\\2-\frac{13}{7}\mu\end{pmatrix}$ | Uses either $P$ and the parametric form of a point on $n$ OR $Q$ and the parametric form of a point on $m$ | |
| | $\begin{pmatrix}-1+\frac{4}{7}\mu\\-2-\mu\\2-\frac{13}{7}\mu\end{pmatrix}.\begin{pmatrix}-\frac{4}{7}\\1\\\frac{13}{7}\end{pmatrix}=0$ | M1: Forms scalar product of vector $NP$ and direction vector of $l$ and equates to zero<br>A1: Correct vectors | M1A1 |
| | $\Rightarrow\mu=\dfrac{56}{117}$ | Solves | M1 |
| | $\Rightarrow d=\sqrt{\left(-\dfrac{85}{117}\right)^2+\left(-\dfrac{290}{117}\right)^2+\left(\dfrac{10}{9}\right)^2}=\dfrac{5\sqrt{481}}{39}$ | Obtains the correct distance | A1 |
| | | | **(5)** |
| | **Alternative for M1A1M1** | | |
| | $\overrightarrow{NP}=\begin{pmatrix}-1+\frac{4}{7}\mu\\-2-\mu\\2-\frac{13}{7}\mu\end{pmatrix}\Rightarrow d=\sqrt{\left(-1+\frac{4}{7}\mu\right)^2+(-2-\mu)^2+\left(2-\frac{13}{7}\mu\right)^2}\Rightarrow d$ is min when $\Rightarrow\mu=\dfrac{56}{117}$<br>M1: Find $d$ in terms of a parameter<br>A1: correct expression<br>M1: use calculus (or simplify and complete the square) to find the parameter corresponding to the min $d$ | | |

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|---|---|---|---|
| **ALT 2** | Correct vector *PQ* | | B1 |
| | $\begin{pmatrix}1\\2\\-2\end{pmatrix}.\begin{pmatrix}-4\\7\\13\end{pmatrix}=\begin{vmatrix}1\\2\\-2\end{vmatrix}\begin{vmatrix}-4\\7\\13\end{vmatrix}\cos\theta$ | Forms the scalar product and attempts to evaluate the LHS | M1 |
| | $\cos\theta=\frac{-16}{3\sqrt{234}}$ | Correct value for $\cos\theta$ exact or decimal | A1 |
| | $d=\lvert PQ\rvert\sin\theta=3\sqrt{1-\left(\frac{-16}{3\sqrt{234}}\right)^2}=\frac{5\sqrt{74}}{\sqrt{234}}$ | M1: Correct method for the distance.<br>A1: Correct **EXACT** distance | M1A1 |
| | | | **(5)** |
| | | | **Total 10** |