Mark Scheme (Results)

Summer 2023

Pearson Edexcel International Advanced Level
In Statistics S2 (WST02)
Paper 01

Question Number	Scheme	Marks
1. (a)(i)	$X \sim B(50, 0.4)$ $P(X = 26) = 0.9686 - 0.9427$ or ${}^{50}C_{26}\,(0.4)^{26}(0.6)^{24}$ awrt **0.0259**	M1 A1 (2)
(ii)	$P(X \geqslant 26) = 1 - P(X \leqslant 25)$ $= 1 - 0.9427 =$ awrt **0.0573**	M1 A1 (2)
(iii)	(From tables) $k =$ **19**	B1 (1)
(b)(i)	$J \sim N(240, 144)$ $P(X \leqslant 222) \sim P(J < 222.5) = P\left(Z < \frac{222.5 - 240}{\sqrt{144}}\right)$ $P(Z < -1.46) = 1 - 0.9279 =$ awrt **0.0721 – 0.0724**	M1A1 M1M1 A1 (5)
(ii)	n is large (oe) and p is close to 0.5	B1 (1) **[11 marks]**

	Notes
(a)(i)	M1 Use of tables or ${}^{50}C_{26}(p)^{26}(1-p)^{24}$ with $0 < p < 1$ allow alternative notations for ${}^{50}C_{26}$
	A1 awrt 0.0259 (correct answer scores 2 out of 2)
(ii)	M1 writing or using $1 - P(X \leqslant 25)$
	A1 awrt 0.0573 (calc 0.0573437….) (correct answer scores 2 out of 2)
(iii)	B1 19 cao $k \leqslant 19$ or $k \geqslant 19$ is B0
(b)(i)	1st M1 For writing or using N(240,…) (May be seen in standardisation)
	1st A1 For writing or using N(240, 144) (May be seen in standardisation)
	2nd M1 use of continuity correction 222 ± 0.5
	3rd M1 $\pm\left(\frac{222 \text{ or } 222.5 \text{ or } 221.5 - \textit{their mean}}{\textit{their sd}}\right)$ if distribution not clearly stated, then the mean and sd must be correct in the standardisation to score this mark
	2nd A1 awrt 0.0721 through to awrt 0.0724 (calc 0.0723743…)
	Answer in the range implies all previous marks unless clearly comes from wrong method
	[NB: Use of binomial distribution gives 0.0719]
(ii)	B1 both conditions required
	for n is large allow in words e.g. 'sample is large'
	allow 0.4 in place of p
	condone '$n > 30$' (or any number > 30)
	Ignore comments about np

Question Number	Scheme	Marks
2. (a)	e.g. Population is small	B1 (1)
(b)(i)	list/register/database of **all** members (of the leisure centre)	B1
(ii)	A member (of the leisure centre)	B1 (2)
(c)	*C* is the statistic as it is (a quantity) based only on values (oe) taken from the sample/it contains no unknown parameters/population values	B1 (1) **[4 marks]**
	Notes	
(a)	B1 any correct characteristic of the population that makes a census a practical alternative to a sample (accessible, finite, well-defined)	
(b)(i)	B1 idea of list (oe) and idea of **all** members (e.g. list of each member of the leisure centre))	
(ii)	B1 a single member Condone members Also condone One of the members in the sample The opinion/view of one of the members is B0	
(c)	B1 choosing *C* (or clearly identifying *C* in words) only with a correct supporting reason which must include value (oe) and sample or no unknown parameters For values allow e.g. information, observations, calculations, function, numerical data, etc.	

Question Number	Scheme	Marks
3. (a)	$\int_2^5 \frac{1}{48}\left(x^2-8x+c\right)\mathrm{d}x=1$	M1
	$1=\frac{1}{48}\left[\frac{x^3}{3}-4x^2+cx\right]_2^5$	M1
	$1=\frac{1}{48}\left(\left(\frac{5^3}{3}-4(5^2)+5c\right)-\left(\frac{2^3}{3}-4(2^2)+2c\right)\right)$ <u>or</u> $48=39-84+3c$	
	$(\Rightarrow 3c=93\Rightarrow)c=31$*	A1cso*
		(3)
(b)	$\mathrm{P}(2<X<3)=\frac{1}{48}\left[\frac{x^3}{3}-4x^2+31x\right]_2^3$	M1
	$\frac{1}{48}\left(\left(\frac{3^3}{3}-4(3^2)+31(3)\right)-\left(\frac{2^3}{3}-4(2^2)+31(2)\right)\right)=\frac{13}{36}$ (=awrt 0.361)	A1
		(2)
(c)	Less than 3 since "$\frac{13}{36}$" > 0.25	B1
		(1)
(d)	x = 4 leads to the minimum/lowest value of f(x) / f(x) is a positive quadratic	B1
		(1)
(e)	Considers x = 2 and x = 5 by e.g. • $\mathrm{f}(2)=0.39(583\dot{)}\,[=\frac{19}{48}]$ and $\mathrm{f}(5)=0.\dot{3}\,[=\frac{16}{48}]$ (so f(2) > f(5)) • Sketch of f(x) from x = 2 to x = 5 • x = 2 is further than x = 4 (then x = 5)	M1
	Mode is x = 2	A1
		(2) **[9 marks]**

	Notes
(a)	1st M1 setting up integral and equating to 1 (condone missing dx) limits not needed 2nd M1 attempting to integrate f(x) at least one term $x^n \to x^{n+1}$ (need not be = 1) Use of integration of f(x) with F(2) = 0 and F(5) = 1 can score M1M1 A1* cso including use of correct limits. There should be at least one line of working between scoring the 2nd M1 and arriving at the given answer. Allow a verification method 1st M1 setting up integral 2nd M1 attempting to integrate A1cso use of correct limits to show that it integrates to 1 and concluding that c = 31
(b)	M1 for use of integration of f(x) $x^n \to x^{n+1}$ with correct limits 2 and 3 (ft from their (a)) A1 allow awrt 0.361 (correct answer scores 2 out of 2)
(c)	B1 less than 3 with correct reasoning. May use their part (b), but must be consistent with 'less than 3' If the lower quartile is found awrt 2.67, allow LQ/2.67 < 3
(d)	B1 correct reason why the method does not give the correct mode. Allow a sketch of f(x). Also allow, e.g. 'Kei's method did not consider the end-points'
(e)	M1 considers end-points A1 mode is 2 cao Answer only scores M0A0. Must have some justification.

Question Number	Scheme	Marks
4. (a)	p is small	B1 (1)
(b)	Let N = number of candles **not** suitable for sale	
	$N \sim B(125, 0.02)$	M1
	$\approx C \sim Po(2.5)$	A1
	$P(C \leqslant 6)$	M1
	$= 0.9858$ awrt **0.986**	A1 (4)
(c)(i)	$H_0 : p = 0.05$ $H_1 : p < 0.05$	B1
	$D \sim B(30, 0.05)$	M1
	$P(D = 0) = 0.2146$	A1
	Do not reject H_0 / not significant	M1
	The manufacturer's claim **is not** supported/There **is not** enough evidence to suggest that the proportion(oe) of candle holders with minor defects is less than 5%/ Charlie's claim **is** supported	A1 (5)
(ii)	Impossible to reject H_0 (since $P(D = 0) > 0.05$)	B1 (1)
(d)	0.95^{50} [=0.0769…] …. or $X \sim B(50, 0.05)$, $P(X = 0)$ (is still) > 0.05	M1
	(so still not possible to reject H_0) hence Ashley's change does not make the test appropriate.	A1 (2)
		[13 marks]

	Notes
(a)	B1 correct condition allow 'p is close to 0' allow '$p < 0.1$' or any value less than 0.1 (condone $np < 10$ or $np \leqslant 10$)
(b)	1st M1 recognising Binomial distribution (may be implied by Po(2.5))
	1st A1 correct distribution Po(2.5)
	2nd M1 writing or using $P(C \leqslant 6)$ from Poisson distribution
	2nd A1 awrt 0.986 from correct distribution used (calc : 0.9858126….)
	[NB : Use of binomial gives 0.98678…] Answer only 0.9858 or better scores 4 out of 4, but answer of 0.986 must see Po(2.5) to award full marks.
(c)(i)	B1 correct hypotheses in terms of p or π
	1st M1 writing or using B(30, 0.05) (may be implied by 1st A1)
	1st A1 awrt 0.215
	2nd M1 a correct ft statement consistent with their p-value and 0.05 No context needed but do not allow contradicting non contextual comments.
	2nd A1 correct conclusion in context which must be **not rejecting** H_0.
	Must use underlined words (oe) No hypotheses then A0
	Condone e.g. '5% of candle holders have minor defects'
(ii)	B1 correct reasoning which implies there is no critical region/ H_0 cannot be rejected
	Sample size is too small on its own is B0.
(d)	M1 for 0.95^{50} or for $X \sim B(50, 0.05)$ and $P(X = 0) > 0.05$
	A1 test is (still) not appropriate with M1 scored

Question Number	Scheme	Marks
5. (a)	$F(3) = 0 \rightarrow \frac{1}{16}\left(3^2 - 6(3) + a\right) = 0$	M1
	$a = 9$	A1
	$F(10) = 1 \rightarrow \frac{1}{12}\left(100(10) - (5)10^2 + c\right) = 1$	M1
	$c = -488$	A1 (4)
(b)	$\frac{1}{16}\left(5^2 - 6(5) + "9"\right) = \frac{1}{12}\left(5 + b\right) \quad \left| \quad \frac{1}{12}(9+b) = \frac{1}{12}\left(100(9) - 5(9^2) + "-488"\right)\right.$	M1
	$b = -2$	A1 (2)
(c)	$P(6 < Y \leqslant 9) = F(9) - F(6)$	M1
	$= \frac{1}{12}\left(9 + "-2"\right) - \frac{1}{12}\left(6 + "-2"\right)$	M1
	$= \frac{1}{4}$	A1 (3)
(d)	$f(y) = \frac{1}{12}$	B1 (1)
(e)	$E(6Y - 5) = [26.5+]\int_5^9 (6y - 5)"\frac{1}{12}"dy$	M1
	$= [26.5+]\frac{1}{12}[(3y^2 - 5y)]_5^9$	dM1
	$= 26.5 + \frac{1}{12}[(3(9^2) - 5(9)) - (3(5^2) - 5(5))]$	dM1
	$= \frac{233}{6}$	A1 (4)
		[Total 14]

	Notes
(a)	1st M1 writing or use of F(3) = 0 1st A1 $a = 9$ cao 2nd M1 writing or use of F(10) = 1 2nd A1 $c = -488$ cao
(b)	M1 use of F(5) = F(5) [$= \frac{1}{4}$] or F(9) = F(9) [$= \frac{7}{12}$] ft their values from (a) A1 $b = -2$ cao
(c)	1st M1 writing or using F(9) – F(6) (may be implied by 2nd M1) 2nd M1 substituting 9 and 6 into F(x) with their value of b allow $\frac{1}{12}\left(100(9) + 5(9^2) + "-488"\right) - \frac{1}{12}\left(6 + "-2"\right)$ with their value of b and their value of c A1 $\frac{1}{4}$ oe
(d)	B1 $\frac{1}{12}$
(e)	1st M1 use of $\int_5^9 (6y - 5)"\frac{1}{12}"dy$ (ignore limits) 2nd M1 (dep on 1st M1) attempt to integrate $(6y - 5)"\frac{1}{12}"$ with at least one $y^n \rightarrow y^{n+1}$ 3rd M1 (dep on 1st M1) $26.5 + \int_5^9 (6y - 5)"\frac{1}{12}"dy$ A1 awrt 38.8
SC:	Answer only or correct answer not using given information scores M0M1M1A1

Question Number	Scheme	Marks
6. (a)	$P(17 < W < k) = P(W < k) - P(W < 17) = \frac{53}{60} - \left(1 - \frac{1}{5}\right) = \frac{1}{12}$	M1 A1 (2)
(b)(i)	$\frac{(b-a)^2}{12} = 75$, $\frac{b-17}{b-a} = \frac{1}{5}$ or $\frac{17-a}{b-a} = \frac{4}{5}$	B1, B1
	$\frac{(b-a)^2}{12} = 75 \rightarrow (b-a) = 30$ $\frac{b-17}{30} = \frac{1}{5}$	M1
	$b = 23$ and $a = -7$	A1 (4)
(ii)	$P(W < k) = \frac{k-("-7")}{"23"-("-7")} = \frac{53}{60}$ or $P(17 < W < k) = \frac{k-17}{30} = \frac{1}{12}$ or $P(W > k) = \frac{"23"-k}{"23"-("-7")} = \frac{7}{60}$	M1
	$k = 19.5$	A1 (2)
(c)	$P(-5 < W < 5) = \frac{5-(-5)}{"23"-("-7")} = \frac{1}{3}$	M1A1ft (2)
(d)	$E(W^2) = Var(W) + E(W)^2 = 75 + \left(\frac{"23"+"-7"}{2}\right)^2 = 139$	M1 A1 (2) **[Total 12]**

	Notes
(a)	M1 for writing or using $P(W < k) - P(W < 17)$ allow < or $\leqslant$
	Allow equivalent expressions e.g. $P(W > 17) - P(W > k) = \frac{1}{5} - \left(1 - \frac{53}{60}\right)$
	A1 oe condone awrt 0.0833 condone $\frac{1}{12}$ coming from $\frac{13}{12} - 1$ or $\left\lvert -\frac{1}{12} \right\rvert$
(b) (i)	1st B1 correct equation for variance
	2nd B1 either correct probability equation Allow e.g. k in place of $(b - a)$
	1st M1 eliminating $(b - a)$ which must appear in both equations.
	A1 both $b = 23$ and $a = -7$ correct answers imply all 4 marks
(ii)	M1 probability expression using uniform distribution ft their values
	A1 $k = 19.5$ oe cao
(c)	M1 for 10/(their b – their a)
	A1ft $\frac{1}{3}$ oe condone awrt 0.333 (Allow ft $\frac{10}{their(b-a)}$ as exact fraction or evaluated to 3sf or better provided $a < -5$ and $b > 5$)
(d)	M1 use of $E(W^2) = Var(W) + (E(W))^2$ with values substitued for $Var(W)$ and $E(W)$
	ft their values of a and b allow any rearrangement. Must have a correct (ft) expression or value for $E(W)$
	Also allow $\int_{"-7"}^{"23"} \frac{1}{"23"-"-7"} w^2 dw$
	A1 139 cao

Question Number	Scheme	Marks
7. (a)	$R \sim \text{Po}(8)$ $\text{P}(4 \leqslant R \leqslant 8) = \text{P}(R \leqslant 8) - \text{P}(R \leqslant 3) = 0.5925 - 0.0424$ $= 0.5501$ = awrt **<u>0.550</u>**	B1 M1 A1 (3)
(b)	$H \sim \text{Po}(4)$ $\text{P}(H \leqslant 2) = 0.2381$ $Y \sim \text{B}(5, \text{“0.2381”})$ $\text{P}(Y = 2) = {}^5\text{C}_2(\text{"0.2381"})^2(1-\text{"0.2381"})^3$ $= 0.25073\ldots$ = awrt **<u>0.251</u>**	B1 B1 M1 M1 A1 (5)
(c)	W = number sold in first fifteen minutes X = number sold in last forty five minutes $\text{P}(W > X \mid R = 4) = \dfrac{\text{P}(W=4)\text{P}(X=0) + \text{P}(W=3)\text{P}(X=1)}{\text{P}(R=4)}$ $= \dfrac{\frac{\text{e}^{-2}2^4}{4!}\frac{\text{e}^{-6}6^0}{0!} + \frac{\text{e}^{-2}2^3}{3!}\frac{\text{e}^{-6}6^1}{1!}}{\frac{\text{e}^{-8}8^4}{4!}}$ $= \dfrac{13}{256}$ (awrt 0.0508 or awrt 0.0509) F = number of muffins sold in first 15 minutes $F \sim \text{B}(4, 0.25)$ $\text{P}(F > 2) =$ $\text{P}(F = 3) + \text{P}(F = 4)$ $= {}^4\text{C}_3(0.25)^3(0.75) + 0.25^4$	M1 M1 M1 A1 (4) **[Total 12]**
	Notes	
(a)	B1 writing or using Po(8) (may be implied by one correct probability from 0.5925, 0.0424 0.4530 or 0.0996) M1 writing or using $\text{P}(R \leqslant 8) - \text{P}(R \leqslant 3)$ A1 awrt 0.550 (calc: 0.55016….) correct answer scores 3 out of 3	
(b)	1st B1 writing or using Po(4) 2nd B1 awrt 0.238 1st M1 choosing binomial distribution with n = 5 and their p 2nd M1 ${}^5\text{C}_2\, p^2(1-p)^3$ with $0 < p < 1$ A1 awrt 0.251	
(c)	1st M1 attempt at either correct product $\text{P}(W=4)\text{P}(X=0)$ or $\text{P}(W=3)\text{P}(X=1)$ from $W \sim \text{Po}(2)$ and $X \sim \text{Po}(6)$ implied by awrt 0.0902×awrt 0.0025 or awrt 0.180×awrt 0.0149 or awrt 0.0029 2nd M1 conditional probability with $\text{P}(R = 4)$ from $R \sim \text{Po}(8)$ on denominator implied by awrt 0.0573 seen in the denominator of a probability expression 3rd M1 complete expression for the required probability implied (awrt 0.0902×awrt 0.0025+awrt 0.180×awrt 0.0149)/awrt 0.0573 for 3rd M1 A1 allow awrt 0.0508 or awrt 0.0509 from use of tables	
ALT	1st M1 identifying B(4, 0.25) 2nd M1 $\text{P}(F = 3) + \text{P}(F = 4)$ from B(4, 0.25) 3rd M1 $4p^3q + p^4$ from B(4, 0.25)	