| Please check the examination details below before ent | ering your candidate information |
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| Candidate surname | Other names |
| Centre Number Candidate Number | |
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| Pearson Edexcel Internation | nal Advanced Level |
| Thursday 18 May 2023 | |
| Morning (Time: 1 hour 30 minutes) Paper reference | wMA12/01 |
| Mathematics | ₾ • |
| International Advanced Subsidian Pure Mathematics P2 | ry/Advanced Level |
| You must have: Mathematical Formulae and Statistical Tables (Ye | ellow), calculator |

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
 - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets
 - use this as a guide as to how much time to spend on each guestion.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.



Turn over ▶



1. The continuous curve C has equation y = f(x).

A table of values of x and y for y = f(x) is shown below.

| x | 4.0 | 4.2 | 4.4 | 4.6 | 4.8 | 5.0 |
|---|-----|--------|--------|--------|--------|-----|
| y | 9.2 | 8.4556 | 3.8512 | 5.0342 | 7.8297 | 8.6 |

Use the trapezium rule with all the values of y in the table to find an approximation for

$$\int_{4}^{5} f(x) \, \mathrm{d}x$$

giving your answer to 3 decimal places.

(3)



| Question 1 continued | |
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| | (Total for Question 1 is 3 marks) |



In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

$$f(x) = 4x^3 - 8x^2 + 5x + a$$

where a is a constant.

Given that (2x - 3) is a factor of f(x),

(a) use the factor theorem to show that a = -3

(2)

(b) Hence show that the equation f(x) = 0 has only one real root.

(4)

| Question 2 continued | | |
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| | Total for Question 2 is 6 marks) | |
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| 3. | A circle C has centre (2, 5) | |
|----|---|-----|
| | Given that the point $P(8, -3)$ lies on C | |
| | (a) (i) find the radius of C | |
| | (ii) find an equation for C | |
| | (n) mu un oquation for o | (3) |
| | (b) Find the equation of the tangent to C at P giving your answer in the form | |
| | ax + by + c = 0 where a, b and c are integers to be found. | (4) |
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| (Te | otal for Question 3 is 7 marks) |
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4. The binomial expansion, in ascending powers of x, of

$$(3 + px)^5$$

where p is a constant, can be written in the form

$$A + Bx + Cx^2 + Dx^3 \dots$$

where A, B, C and D are constants.

(a) Find the value of A

(1)

Given that

- B = 18D
- p < 0
- (b) find
 - (i) the value of p
 - (ii) the value of C

(6)



| Question 4 continued |
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| | (Total for Question 4 is 7 marks) |



| 5. | Use the laws of logarithms to solve |
|----|-------------------------------------|
| | $1_{0.0}$ (16w) + 1 |

$$\log_2(16x) + \log_2(x+1) = 3 + \log_2(x+6)$$

(5)

| Question 5 continued | |
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| (Total for Question 5 is 5 | marks) |



6. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A software developer released an app to download.

The numbers of downloads of the app each month, in thousands, for the first three months after the app was released were

$$2k-15$$
 k $k+4$

where k is a constant.

Given that the numbers of downloads each month are modelled as a geometric series,

(a) show that $k^2 - 7k - 60 = 0$

(2)

(b) predict the number of downloads in the 4th month.

(4)

The **total** number of all downloads of the app is predicted to exceed 3 million for the first time in the *N*th month.

(c) Calculate the value of *N* according to the model.

(3)



| Question 6 continued |
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| (Total for Question 6 is 9 ma | rks) |
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7. The height of a river above a fixed point on the riverbed was monitored over a 7-day period.

The height of the river, H metres, t days after monitoring began, was given by

$$H = \frac{\sqrt{t}}{20}(20 + 6t - t^2) + 17 \qquad 0 \le t \le 7$$

Given that *H* has a stationary value at $t = \alpha$

(a) use calculus to show that α satisfies the equation

$$5\alpha^2 - 18\alpha - 20 = 0$$

(5)

(b) Hence find the value of α , giving your answer to 3 decimal places.

(1)

(c) Use further calculus to prove that H is a maximum at this value of α .

(2)



| Question 7 continued | |
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| 8. | (i) | A | student | writes | the | foll | owing | statement |
|----|-----|---|---------|--------|-----|------|-------|-----------|
|----|-----|---|---------|--------|-----|------|-------|-----------|

"When a and b are consecutive **prime** numbers, $a^2 + b^2$ is never a multiple of 10" Prove by counter example that this statement is **not** true.

(2)

(ii) Given that x and y are even integers greater than 0 and less than 6, prove by exhaustion, that

$$1 < x^2 - \frac{xy}{4} < 15$$

(3)

| Question 8 continued | |
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| | (Total for Question 8 is 5 marks) |



9. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$3\cos\theta(\tan\theta\sin\theta+3) = 11 - 5\cos\theta$$

may be written as

$$3\cos^2\theta - 14\cos\theta + 8 = 0$$

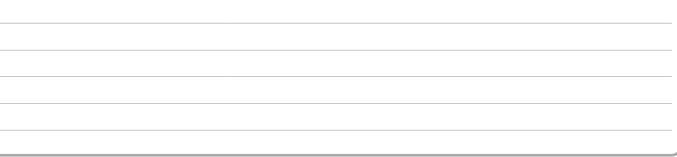
(3)

(b) Hence solve, for $0 < x < 360^{\circ}$

$$3\cos 2x(\tan 2x \sin 2x + 3) = 11 - 5\cos 2x$$

giving your answers to one decimal place.

(4)



| Question 9 continued |
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10. The curve C has equation

$$y = \frac{(x-k)^2}{\sqrt{x}} \qquad x > 0$$

where k is a **positive** constant.

(a) Show that

$$\int_{1}^{16} \frac{(x-k)^2}{\sqrt{x}} \, \mathrm{d}x = ak^2 + bk + \frac{2046}{5}$$

where a and b are integers to be found.

C A R C X

Figure 1

Figure 1 shows a sketch of the curve C and the line l.

Given that l intersects C at the point A(1, 9) and at the point B(16, q) where q is a constant,

(b) show that
$$k = 4$$

(2)

(5)

The region R, shown shaded in Figure 1, is bounded by C and l

Using the answers to parts (a) and (b),

(c) find the area of region R

(3)



| Question 10 continued |
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11. A sequence u_1, u_2, u_3, \dots is defined by

$$u_{n+1} = b - au_n$$

$$u_1 = 3$$

where a and b are constants.

- (a) Find, in terms of a and b,
 - (i) u_2
 - (ii) u_3

(2)

Given

- b = a + 9
- (b) show that

$$a^2 - 5a - 66 = 0$$

(3)

(c) Hence find the larger possible value of \boldsymbol{u}_2

(3)



| Question 11 continued | | |
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| | (Total for Overtion 11 to 0 |
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