# **Pure Mathematics P1 Mark scheme**

Question	Scheme	Marks
1(a)	$y = 4x^3 - \frac{5}{x^2}$	
	$x^n \to x^{n-1}$ e.g. sight of $x^2$ or $x^{-3}$ or $\frac{1}{x^3}$	M1
	$3 \times 4x^2$ or $-5 \times -2x^{-3}$ (o.e.) (Ignore + c for this mark)	A1
	$12x^2 + \frac{10}{x^3} \text{ or } 12x^2 + 10x^{-3}$	<b>A</b> 1
	<u>all on one line</u> and no + c	(3)
(b)	$x^n \to x^{n+1}$ e.g. sight of $x^4$ or $x^{-1}$ or $\frac{1}{x^1}$	M1
	Do <u>not</u> award for integrating their answer to part (a) $4 \frac{x^4}{4} \qquad \text{or} \qquad -5 \times \frac{x^{-1}}{-1}$	A1
	For fully correct and simplified answer with + c <u>all on one line</u> . Allow $\Rightarrow \text{Allow } x^4 + 5 \times \frac{1}{x} + c$ $\Rightarrow \text{Allow } 1x^4 \text{ for } x^4$	A1
		(3)
		6 marks)

Question	Scheme	Marks
2(a)	$3^{-1.5} = \frac{1}{3\sqrt{3}}  \left(\frac{\times\sqrt{3}}{\times\sqrt{3}}\right)$	
	$=\frac{\sqrt{3}}{9}  \text{so}  a = \frac{1}{9}$	A1
		(2)
	Alternative	
	$3^{-1.5} = a\sqrt{3} \Rightarrow a = \frac{3^{-1.5}}{3^{0.5}} = 3^{-1.5-0.5}$	M1
	$\Rightarrow a = 3^{-2} = \frac{1}{9}$	A1
(b)	$\left(2x^{\frac{1}{2}}\right)^3 = 2^3 x^{\frac{3}{2}}$ One correct power either 2 <sup>3</sup> or $x^{\frac{3}{2}}$ .	M1
	$\frac{8x^{\frac{3}{2}}}{4x^2} = 2x^{-\frac{1}{2}} \text{ or } \frac{2}{\sqrt{x}}$	dM1 A1
		(3)
	(1)	5 marks)

# Notes:

(a)

M1: Scored for a full attempt to write  $3^{-1.5}$  in the form  $a\sqrt{3}$  or, as an alternative, makes a the subject and attempts to combine the powers of 3

A1: For  $a = \frac{1}{9}$  Note: A correct answer with no working scores full marks

**(b)** 

**M1:** For an attempt to expand  $\left(2x^{\frac{1}{2}}\right)^3$  Scored for one correct power either  $2^3$  or  $x^{\frac{3}{2}}$ .  $\left(2x^{\frac{1}{2}}\right) \times \left(2x^{\frac{1}{2}}\right) \times \left(2x^{\frac{1}{2}}\right)$  on its own is not sufficient for this mark.

**dM1:** For dividing their coefficients of x and subtracting their powers of x. Dependent upon the previous M1

**A1:** Correct answer  $2x^{-\frac{1}{2}}$  or  $\frac{2}{\sqrt{x}}$ 

Question	Scheme		Marks
3	y = -4x - 1 $\Rightarrow (-4x - 1)^2 + 5x^2 + 2x = 0$	Attempts to makes y the subject of the linear equation and substitutes into the other equation.	M1
	$21x^2 + 10x + 1 = 0$	Correct 3 term quadratic	A1
		dM1: Solves a <b>3 term</b> quadratic by the usual rules	
	$(7x+1)(3x+1) = 0 \Rightarrow (x = )-\frac{1}{7}, -\frac{1}{3}$	A1: $(x = ) -\frac{1}{7}, -\frac{1}{3}$	dM1A1
	3 1	M1: Substitutes to find at least one <i>y</i> value	
	$y = -\frac{3}{7}, \frac{1}{3}$	A1: $y = -\frac{3}{7}, \frac{1}{3}$	M1 A1
			(6)
	Alternative		
	$x = -\frac{1}{4}y - \frac{1}{4}$	Attempts to makes <i>x</i> the subject of the linear equation and substitutes into the other equation.	M1
	$\Rightarrow y^2 + 5\left(-\frac{1}{4}y - \frac{1}{4}\right)^2 + 2\left(-\frac{1}{4}y - \frac{1}{4}\right) = 0$	the other equation.	IVII
	$\frac{21}{16}y^2 + \frac{1}{8}y - \frac{3}{16} = 0$ $(21y^2 + 2y - 3 = 0)$	Correct 3 term quadratic	A1
	$(7y+3)(3y-1) = 0 \Rightarrow (y=)-\frac{3}{7}, \frac{1}{3}$	Solves a 3 term quadratic	dM1
		$(y=)-\frac{3}{7}, \frac{1}{3}$	A1
	r - 1 1	Substitutes to find at least one <i>x</i> value.	M1
	$x = -\frac{1}{7}, -\frac{1}{3}$	$x = -\frac{1}{7}, -\frac{1}{3}$	A1
			(6)
		(	6 marks)

Question	Scheme	Marks
4	Sets $2x^2 + 8x + 3 = 4x + c$ and collects x terms together	M1
	Obtains $2x^2 + 4x + 3 - c = 0$ o.e.	A1
	States that $b^2 - 4ac = 0$	dM1
	$4^2 - 4 \times 2 \times (3 - c) = 0$ and so $c =$	dM1
	c = 1 cso	A1
		(5)
	Alternative 1A	
	Sets derivative " $4x + 8$ " = $4 \implies x =$	M1
	x = -1	A1
	Substitute $x = -1$ in $y = 2x^2 + 8x + 3 \ (\implies y = -3)$	dM1
	Substitute $x = -1$ and $y = -3$ in $y = 4x + c$ or into $(y + 3) = 4(x + 1)$ and expand	dM1
	c = 1 or writing $y = 4x + 1$ cso	A1
		(5)
	Alternative 1B	'
	Sets derivative " $4x + 8$ " = $4 \Rightarrow x =$ ,	M1
	x = -1	A1
	Substitute $x = -1$ in $2x^2 + 8x + 3 = 4x + c$	dM1
	Attempts to find value of <i>c</i>	dM1
	c = 1 or writing $y = 4x + 1$ cso	A1
		(5)
	Alternative 2	·
	Sets $2x^2 + 8x + 3 = 4x + c$ and collects x terms together	M1
	Obtains $2x^2 + 4x + 3 - c = 0$ or equivalent	A1
	States that $b^2 - 4ac = 0$	dM1
	$4^2 - 4 \times 2 \times (3 - c) = 0$ and so $c =$	dM1
	c = 1 cso	A1
		(5)
	Alternative 3	
	Sets $2x^2 + 8x + 3 = 4x + c$ and collects x terms together	M1
	Obtains $2x^2 + 4x + 3 - c = 0$ or equivalent	A1
	Uses $2(x+1)^2 - 2 + 3 - c = 0$ or equivalent	dM1
	Writes $-2 + 3 - c = 0$	dM1
	So $c = 1$ cso	A1
		(5)
		(5 marks)

# **Question 4** continued

#### **Notes:**

### Method 1A

- **M1:** Attempts to solve their  $\frac{dy}{dx} = 4$ . They must reach x = ... (Just differentiating is M0 A0).
- A1: x = -1 (If this follows  $\frac{dy}{dx} = 4x + 8$ , then give M1 A1 by implication).
- **dM1:** (Depends on previous M mark) Substitutes their x = -1 into f(x) or into "their f(x) from (b)" to find y.
- **dM1:** (Depends on both previous M marks) Substitutes their x = -1 and their y = -3 values into y = 4x + c to find c or uses equation of line is (y + "3") = 4(x + "1") and rearranges to y = mx + c
- **A1:** c = 1 or allow for y = 4x + 1 cso.

# Method 1B

- M1A1: Exactly as in Method 1A above.
- **dM1:** (Depends on previous M mark) Substitutes **their** x = -1 into  $2x^2 + 8x + 3 = 4x + c$
- **dM1:** Attempts to find value of c then A1 as before.

# Method 2

- M1: Sets  $2x^2 + 8x + 3 = 4x + c$  and tries to collect x terms together.
- **A1:** Collects terms e.g.  $2x^2 + 4x + 3 c = 0$  or  $-2x^2 4x 3 + c = 0$  or  $2x^2 + 4x + 3 = c$  or even  $2x^2 + 4x = c 3$ . Allow "=0" to be missing on RHS.
- **dM1:** Then use completion of square  $2(x+1)^2 2 + 3 c = 0$  (Allow  $2(x+1)^2 k + 3 c = 0$ ) where k is non zero. It is enough to give the correct or almost correct (with k) completion of the square.
- **dM1:** -2 + 3 c = 0 AND leading to a solution for c (Allow -1 + 3 c = 0) (x = -1 has been used)
- A1:  $c = 1 \cos \theta$

#### Method 3

- M1: Sets  $2x^2 + 8x + 3 = 4x + c$  and tries to collect x terms together. May be implied by  $2x^2 + 8x + 3 4x \pm c$  on one side.
- **A1:** Collects terms e.g.  $2x^2 + 4x + 3 c = 0$  or  $-2x^2 4x 3 + c = 0$  or  $2x^2 + 4x + 3 = c$  even  $2x^2 + 4x = c 3$ . Allow "=0" to be missing on RHS.
- **dM1:** Then use completion of square  $2(x+1)^2 k + 3 c = 0$  (Allow  $2(x+1)^2 k + 3 c = 0$ ) where k is non zero. It is enough to give the correct or almost correct (with k) completion of the square.
- **dM1:** -2 + 3 c = 0 AND leading to a solution for c (Allow -1 + 3 c = 0) (x = -1 has been used)
- **A1:**  $c = 1 \cos \theta$

Question			Marks
5(a)	1 1	Straight line, positive gradient positive intercept	B1
		Curve 'U' shape anywhere	B1
		Correct y intercepts 2, $-6$	B1
	-2 $-6$ $3$	Correct x-intercepts of $-2$ and 3 with intersection shown at $(-2, 0)$	B1
			(4)
(b)	Finite region between line and curv	ve shaded	B1
			(1)
(c)	$(x^2 - x - 6 < x + 2) \Rightarrow x^2 - 2x - 8$	< 0	
	$(x-4)(x+2) < 0 \implies \text{Line}$	and curve intersect at $x = 4$ and $x = -2$	M1 A1
		-2 < x < 4	A1
			(3)
		(1)	8 marks)
Notes:			

# **Notes:**

- (a) As scheme.
- **(b)** As scheme.

(c)

**M1:** For a valid attempt to solve the equation  $x^2 - 2x - 8 = 0$ 

**A1:** For x = 4 and x = -2

**A1:** -2 < x < 4

Question	Scheme		Marks
6(a)	37	Shape through (0, 0)	B1
		(3, 0)	В1
		(1.5, -1)	B1
			(3)
(b)	, in	Shape , not through $(0, 0)$	B1
		Minimum in 4 <sup>th</sup> quadrant	B1
		(-p, 0) and $(6-p, 0)$	B1
		(3-p,-1)	B1
			(4)
		(*	7 marks)

#### **Notes:**

(a)

**B1:** U shaped parabola through origin.

**B1:** (3,0) stated or 3 labelled on x - axis (even (0,3) on x - axis).

**B1:** (1.5, -1) or equivalent e.g. (3/2, -1) labelled or stated and matching minimum point on the graph.

**(b)** 

B1: Is for any translated curve to left or right or up or down not through origin

**B1:** Is for minimum in  $4^{th}$  quadrant and x intercepts to left and right of y axis (i.e. correct position).

**B1:** Coordinates stated or shown on x axis (Allow (0 - p, 0) instead of (-p, 0))

**B1:** Coordinates stated.

Note: If values are taken for p, then it is possible to give M1A1B0B0 even if there are several attempts. (In this case none of the curves should go through the origin for M1 and all minima should be in fourth quadrant and all x intercepts need to be to left and right of y axis for A1

Question	Scheme	Marks
7	$f(x) = \int \left(\frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1\right) dx$	
	$x^{n} \to x^{n+1} \Rightarrow f(x) = \frac{3}{8} \times \frac{x^{3}}{3} - 10 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + x(+c)$	M1 A1 A1
	Substitute $x = 4$ , $y = 25 \implies 25 = 8 - 40 + 4 + c$ $\implies c =$	M1
	$f(x) = \frac{x^3}{8} - 20x^{\frac{1}{2}} + x + 53$	A1
		(5)

(5 marks)

#### **Notes:**

**M1:** Attempt to integrate  $x^n \to x^{n+1}$ 

A1: Term in  $x^3$  or term in  $x^{\frac{1}{2}}$  correct, coefficient need not be simplified, no need for +x nor +c

A1: ALL three terms correct, coefficients need not be simplified, no need for +c

M1: For using x = 4, y = 25 in their f(x) to form a linear equation in c and attempt to find c

A1:  $=\frac{x^3}{8}-20x^{\frac{1}{2}}+x+53$  cao (all coefficients and powers must be simplified to give this answer- do not need a left hand side and if there is one it may be f(x) or y). Need full expression with 53. These marks need to be scored in part (a).

Question	Sc	heme	Marks
8(a)	$2x + 3y = 26 \Rightarrow 3y = 26 \pm 2x \text{ an}$	d attempt to find $m$ from $y = mx + c$	M1
	$( \Rightarrow y = \frac{26}{3} - \frac{2}{3}$	$(\frac{2}{3}x)$ so gradient = $-\frac{2}{3}$	A1
	Gradient of perpendicular = $\frac{-1}{\text{their gradient}}$ (= $\frac{3}{2}$ )		M1
	Line goes	through (0, 0) so $y = \frac{3}{2}x$	A1
			(4)
(b)	Solves their $y = \frac{3}{2}x$ with their $2x + 3y = 26$ to form equation in x or in y		M1
	Solves their equation in x or in y to obtain $x = \mathbf{or} \ y = \mathbf{or} $		
	$x = 4$ or any equivalent e.g. $\frac{156}{39}$ or $y = 6$ o.a.e		A1
	$B=(0,\frac{26}{3})$ used or stated in (b)		В1
	<u> </u>	Area = $\frac{1}{2}$ ×"4"× $\frac{"26"}{3}$	dM1
	$\frac{26}{3}$ $x=4$	$= \frac{52}{3}$ (o.e. with integer numerator and denominator)	A1
			(6)

(10 marks)

### **Notes:**

(a)

M1: Complete method for finding gradient. (This may be implied by later correct answers.) e.g. Rearranges  $2x + 3y = 26 \Rightarrow y = mx + c$  so m = 1

Or finds coordinates of two points on line and finds gradient e.g.

(13,0) and (1,8) so 
$$m = \frac{8-0}{1-13}$$

A1: States or implies that gradient  $=-\frac{2}{3}$  condone  $=-\frac{2}{3}x$  if they continue correctly. Ignore errors in constant term in straight line equation.

**M1:** Uses  $m_1 \times m_2 = -1$  to find the gradient of  $l_2$ . This can be implied by the use of  $\frac{-1}{\text{their gradient}}$ 

**A1:**  $y = \frac{3}{2}x$  or 2y - 3x = 0 Allow  $y = \frac{3}{2}x + 0$  Also accept 2y = 3x,  $y = \frac{39}{26}x$  or even  $y - 0 = \frac{3}{2}(x - 0)$  and isw.

### **Question 8 notes** continued

(b)

M1: Eliminates variable between their  $y = \frac{3}{2}x$  and their (possibly rearranged) 2x + 3y = 26 to form an equation in x or y. (They may have made errors in their rearrangement).

**dM1:** (Depends on previous M mark) Attempts to solve their equation to find the value of x or y

**A1:** x = 4 or equivalent or y = 6 or equivalent

**B1:** y coordinate of B is  $\frac{26}{3}$  (stated or implied) - isw if written as  $(\frac{26}{3}, 0)$ .

Must be used or stated in (b)

**dM1:** (Depends on previous M mark) Complete method to find area of triangle *OBC* (using their values of x and/or y at point C and their  $\frac{26}{3}$ )

**A1:** Cao  $\frac{52}{3}$  or  $\frac{104}{6}$  or  $\frac{1352}{78}$  o.e

# Alternative 1

Uses the area of a triangle formula  $\frac{1}{2} \times OB \times (x \text{ coordinate of } C)$ 

**Alternative methods**: Several Methods are shown below. The only mark which differs from Alternative 1 is the last M mark and its use in each case is described below:

### Alternative 2

In 8(b) using 
$$\frac{1}{2} \times BC \times OC$$

**dM1:** Uses the area of a triangle formula  $\frac{1}{2} \times BC \times OC$  Also finds OC (= $\sqrt{52}$ ) and BC= ( $\frac{4}{3}\sqrt{13}$ )

#### Alternative 3

In 8(b) using 
$$\frac{1}{2} \begin{vmatrix} 0.4 & 0.0 \\ 0.6 & \frac{26}{3} & 0 \end{vmatrix}$$

**dM1:** States the area of a triangle formula  $\frac{1}{2}\begin{vmatrix} 0 & 4 & 0 & 0 \\ 0 & 6 & \frac{26}{3} & 0 \end{vmatrix}$  or equivalent with their values

# **Alternative 4**

In 8(b) using area of triangle *OBX* – area of triangle *OCX* where *X* is point (13, 0)

**dM1:** Uses the correct subtraction  $\frac{1}{2} \times 13 \times "\frac{26}{3}" - \frac{1}{2} \times 13 \times "6"$ 

#### Alternative 5

In 8(b) using area =  $\frac{1}{2}$  (6 × 4) +  $\frac{1}{2}$  (4 × 8/3) drawing a line from C parallel to the x axis and dividing triangle into two right angled triangles

**dM1:** For correct method area =  $\frac{1}{2}$  ("6" × "4") +  $\frac{1}{2}$  ("4" × ["26/3"-"6"])

# Method 6 Uses calculus

**dM1:**  $\int_{0}^{4} \frac{26}{3} - \frac{2x}{3} - \frac{3x}{2} dx = \left[ \frac{26}{3} x - \frac{x^{2}}{3} - \frac{3x^{2}}{4} \right]_{0}^{4}$ 

Question	Scheme	Marks
9(a)	Substitutes $x = 2$ into $y = 20 - 4 \times 2 - \frac{18}{2}$ and gets 3	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -4 + \frac{18}{x^2}$	M1 A1
	Substitute $x = 2 \Rightarrow \frac{dy}{dx} = \left(\frac{1}{2}\right)$ then finds negative reciprocal (-2)	dM1
	States or uses $y - 3 = -2(x - 2)$ or $y = -2x + c$ with their (2, 3)	ddM1
	to deduce that $y = -2x + 7$	A1*
		(6)
(b)	Put $20-4x-\frac{18}{x} = -2x+7$ and simplify to give $2x^2 - 13x + 18 = 0$ Or put $y = 20-4\left(\frac{7-y}{2}\right) - \frac{18}{\left(\frac{7-y}{2}\right)}$ to give $y^2 - y - 6 = 0$	M1 A1
	(2x-9)(x-2) = 0 so $x =$ or $(y-3)(y+2) = 0$ so $y =$	dM1
	$\left(\frac{9}{2},-2\right)$	A1 A1
		(5)
		1 manka)

# (11 marks)

# **Notes:**

(a)

B1: Substitutes x = 2 into expression for y and gets 3 cao (must be in part (a) and must use curve equation – not line equation). This must be seen to be substituted.

M1: For an attempt to differentiate the negative power with  $x^{-1}$  to  $x^{-2}$ .

**A1:** Correct expression for  $\frac{dy}{dx} = -4 + \frac{18}{x^2}$ 

**dM1:** Dependent on **first** M1 substitutes x = 2 into their derivative to obtain a numerical gradient and find negative reciprocal or states that  $-2 \times \frac{1}{2} = -1$ 

# Alternative 1

**dM1:** Dependent on **first** M1. Finds equation of line using changed gradient (not their  $\frac{1}{2}$  but  $-\frac{1}{2}$  2 or -2) e.g. y - "3" = -"2"(x - 2) or y = "-2" x + c and use of (2, "3") to find c =

A1\*: cso. This is a given answer y = -2x + 7 obtained with no errors seen and equation should be stated.

Alternative 2 – checking given answer

**dM1:** Uses given equation of line and checks that (2, 3) lies on the line.

A1\*: cso. This is a given answer y = -2x + 7 so statement that normal and line have the same gradient and pass through the same point must be stated.

# Question 9 notes continued

**(b)** 

M1: Equate the two given expressions, collect terms and simplify to a 3TQ. There may be sign errors when collecting terms but putting for example  $20x - 4x^2 - 18 = -2x + 7$  is M0 here.

**A1:** Correct 3TQ = 0 (need = 0 for A mark)  $2x^2 - 13x + 18 = 0$ 

**dM1:** Attempt to solve an appropriate quadratic by factorisation, use of formula, or completion of the square (see general instructions).

A1:  $x = \frac{9}{2}$  o.e or y = -2 (allow second answers for this mark so ignore x = 2 or y = 3)

**A1:** Correct solutions only so both  $x = \frac{9}{2}$ , y = -2 or  $\left(\frac{9}{2}, -2\right)$ 

If x = 2, y = 3 is included as an answer and point B is not identified then last mark is A0. Answer only – with no working – send to review. The question stated 'use algebra'.

Question	Scheme		Marks
10(a)	$9^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \cos \alpha \Rightarrow \cos \alpha = \dots$	Correct use of cosine rule leading to a value for $\cos \alpha$	M1
	$\cos \alpha = \frac{4^2 + 6^2 - 9^2}{2 \times 4 \times 6} \left( = -\frac{2}{4} \right)$	$\frac{29}{48} = -0.604$	
	$\alpha$ = 2.22 * $\alpha$	eso	A1
			(2)
	Alternative		
	$XY^2 = 4^2 + 6^2 - 2 \times 4 \times 6\cos 2.22 \Rightarrow XY^2 =$	Correct use of cosine rule leading to a value for $XY^2$	M1
	XY = 9.00		A1
			(2)
(b)	$2\pi - 2.22 (= 4.06366)$	$2\pi - 2.22$ or $2\pi - 2.2$ or awrt 4.06 (May be implied)	B1
	$\frac{1}{2} \times 4^2 \times "4.06"$	Correct method for major sector area. Allow $\pi - 2.22$ for the major sector angle.	M1
	32.5	Awrt 32.5	A1
		·	(3)
	Alternative – Circle Minor – sector		
	$\pi \times 4^2$	Correct expression for circle area	B1
	$\pi \times 4^2 - \frac{1}{2} \times 4^2 \times 2.22 = 32.5$	Correct method for circle - minor sector area	M1
	= 32.5	Awrt 32.5	A1
			(3)
(c)	Area of triangle = $\frac{1}{2} \times 4 \times 6 \times \sin 2.22 (= 9.56)$	Correct expression for the area of triangle XYZ (allow 2.2 or awrt 2.22)	B1
	So area required = "9.56" + "32.5"	Their Triangle XYZ + part (b) or correct attempt at major sector (Not triangle ZXW)	M1
	Area of logo = $42.1 \text{ cm}^2 \text{ or } 42.0 \text{ cm}^2$	Awrt 42.1 or 42.0 (or <u>just</u> 42)	A1
			(3)
(d)	Arc length = $4 \times 4.06 (= 16.24)$	M1: $4 \times their(2\pi - 2.22)$	M1
	or	<b>or</b> circumference – minor arc	A1ft
	$8\pi-4\times2.22$	A1: Correct ft expression	
	Perimeter = $ZY + WY + $ Arc Length	9 + 2 + Any Arc	M1
	Perimeter of logo = 27.2 or 27.3	Awrt 27.2 <b>or</b> awrt 27.3	A1
	1 crimeter of logo 27.2 of 27.3	Awit 27.2 or awit 27.3	