Pearson Edexcel

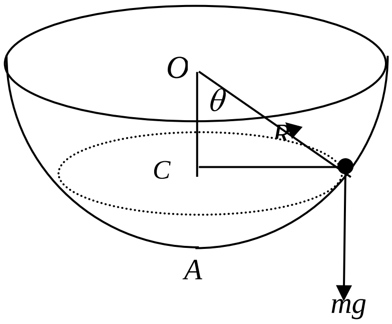
Mark Scheme (Results)

Summer 2022

Pearson Edexcel International Advanced Level
In Mechanics 3 (WME03) Paper 01

Question Number	Scheme	Marks
1(a)	$\frac{2\pi}{\omega}=\frac{1}{2} \quad \Rightarrow \omega = ...$	M1
	$\omega = 4\pi$	A1
	$v = "\omega" \times 0.3$	M1
	$v = 1.2\pi$, 3.8 or better (m s^{-1})	A1 (4)
(b)	$x = a\sin\omega t \Rightarrow 0.15 = 0.3\sin 4\pi t \quad \Rightarrow t = ...$	M1
	$t = \frac{1}{4\pi}\times\frac{\pi}{6}=\frac{1}{24}$ (s) $0.04166... = 0.042$ or better	A1 (2) **[6]**

	Notes
(a)	
M1	Use period = 1/frequency to find a value for ω . Must be correct way up.
A1	Correct value for ω
M1	Use of $v = a\omega$ or $v^2 = \omega^2(a^2 - x^2)$ with x=0 .
A1	cao
(b)	
M1	Use $0.15 = a\sin\omega t$ to obtain a value for t . Use *their a* and ω.
A1	Correct value, 0.042 or better
ALT 1(b)	**Using cos** Complete method using $x = a\cos\omega t$ AND $\frac{T}{4}$ to obtain a value for t
MI	$x = a\cos\omega t \Rightarrow 0.15 = 0.3\cos 4\pi t \quad \Rightarrow t = ...$ $\frac{T}{4} - t = \frac{0.5}{4} - t = ...$
A1	Correct value, 0.042 or better

Question Number	Scheme	Marks
2.		
	$R\sin\theta = m\times 6r\sin\theta\times\dfrac{g}{4r}$	M1A1A1
	$R=\dfrac{3}{2}mg$	
	$R\cos\theta = mg$	M1A1
	$\dfrac{3}{2}mg\cos\theta = mg$	DM1
	$\cos\theta = \dfrac{2}{3}$	A1
	$OC = 6r\cos\theta = 6r\times\dfrac{2}{3} = 4r$	M1A1
		[9]

	Notes
M1	Attempt NL2 along *CP* with correct number of terms and forces resolved.
A1	Either side correct
A1	Fully correct equation
	Note:
	If R is not resolved then M0 **but** do allow if $\sin\theta$ is cancelled from **both** sides: $R = m\times 6r\times\dfrac{g}{4r}$ would score M1A1A1
	If r is used instead of the radius: $R\sin\theta = m\times r\times\dfrac{g}{4r}$ would score M1A1A0 (force resolved correctly on LHS but error in radius on RHS)
M1	Resolve vertically
A1	Correct equation
DM1	Eliminate *R* between the two equations. Depends on both M marks above
A1	Correct value for $\cos\theta$ seen or implied
M1	Attempt to obtain *OC* (allow sin/cos confusion)
A1	$OC = 4r$
	Note: If θ is the angle with the horizontal then all equations above will appear with $\sin\theta$ and $\cos\theta$ reversed.

Question Number	Scheme	Marks
ALT 1	**Case: using trig ratios where radius, L, and ω2 are never replaced** M1 A1 A1: R sinθ = m L ω2 M1 A1: R cosθ = mg DM1 A1: tan θ = $\frac{L\omega^2}{g} = \frac{L}{4r}$ M1 A1: tan θ = $\frac{L}{OC} \Rightarrow OC = 4r$	
ALT 2	**Case: resolving tangentially where R is never seen** $mg\sin\theta = m\times(6r\sin\theta)\times\frac{g}{4r}\cos\theta$ scores M1A1A1 M1A1 DM1 leads straight to $\cos\theta = \frac{2}{3}$ A1	

Question Number	Scheme	Marks
3(a)	$v=\dfrac{50}{2x+3}$	
	$\dfrac{dv}{dt}=\dfrac{dv}{dx}\times\dfrac{dx}{dt}$	M1
	$=\dfrac{-100}{(2x+3)^2}\times\dfrac{50}{2x+3}\quad\left(=\dfrac{-5000}{(2x+3)^3}\right)$	DM1A1
	$x=12\quad \dfrac{dv}{dt}=-\dfrac{5000}{27^3}=-0.2540...=-0.25\ \text{ or }\ -0.254\ \mathrm{m\,s^{-2}}$	M1
	deceleration $=0.25$ (m s $^{-2}$) or better	A1 (5)
(b)	$v=\dfrac{dx}{dt}=\dfrac{50}{2x+3}$	M1
	$\int(2x+3)\,dx=\int 50\,dt$	
	$x^2+3x=50t\ \ (+c)$	M1A1
	$t=1,\ x=4\Rightarrow 28=50+c,\ \ c=-22$	A1
	$x=12\Rightarrow 50t=12^2+36+22\quad t=\dfrac{202}{50}=4.04$ (accept 4.0)	A1 (5)
		[10]
	Notes	
(a)		
M1	Uses chain rule of the form $\dfrac{dv}{dt}=\dfrac{dv}{dx}\times\dfrac{dx}{dt}$ or $\dfrac{d(\frac{1}{2}v^2)}{dx}$	
	Note, $\dfrac{1}{2}v^2=\dfrac{1250}{(2x+3)^2}\ \Rightarrow\ \text{acc}=\dfrac{d(\frac{1}{2}v^2)}{dx}=-\dfrac{2500}{(2x+3)^3}\times 2$	
	However, **M0** for acc = $\dfrac{1}{2}v^2$	
DM1	Differentiate v wrt x	
A1	Correct differentiation.	
M1	Sub x = 12 into their expression for acceleration to obtain the deceleration. Must have attempted to differentiate.	
A1	Correct deceleration – must be positive	
(b)		
M1	Use $v=\dfrac{dx}{dt}$	
M1	Attempt at integration	
A1	Correct integration but c may be missing	
A1	Use $t=1,\ x=4$ to obtain the correct value of c for their correct integration	
A1	Sub x = 12 to obtain the correct value of t	

ALT 3(b)	Using definite integration: $\int_4^{12} (2x+3)\mathrm{dx} = \int_1^T 50\,\mathrm{dt}$
M1	Integrate $\left[x^2+3x\right]_4^{12} = \left[50t\right]_1^T$
A1	Correct integration
A1	Sub in limits $12^2+3(12)-4^2-3(4)=50T-50$
A1	Obtain correct value

Question Number	Scheme	Marks
4(a)	Energy from C to D	
	$$mg\frac{l}{4}\sin 30° = \frac{\lambda}{2l}\left(\frac{l}{4}\right)^2$$	M1A1A1
	$$\lambda = 4mg \ *$$	A1* **(4)**
(b)	The greatest speed is when the acceleration of B is zero	
	$(\nwarrow)$ $\quad T = mg\sin 30° = \dfrac{4mge}{l}$	M1
	$$e = \frac{l}{8}$$	A1
	Energy: $\quad \dfrac{1}{2}mv^2 + \dfrac{4mg}{2l}\left(\dfrac{l}{8}\right)^2 = mg\dfrac{l}{8}\sin 30°$	M1A1A1
	$$v = \sqrt{\left(\frac{gl}{16}\right)} = \frac{\sqrt{gl}}{4}$$	DM1A1 **(7)**
		[11]
	Notes	
(a)		
M1	Attempt the energy equation from C to D. Must use a vertical height for PE. EPE must have the form kx^2. Must have 1 PE term and 1 EPE term.	
A1	Correct loss of PE	
A1	Correct final EPE	
A1*	Correct answer correctly obtained	
(b)		
M1	Resolve along the plane using HL to find T	
A1	Correct value for the extension	
M1	Form the energy equation with an extension they have found. **M0** if $l/4$ is used for the extension. Must use a vertical height for PE. EPE must have the form kx^2 Must have 1 PE term, 1 KE term and 1 EPE term.	
A1	Two correct terms	
A1	Completely correct equation	
DM1	Solve for v. Dependent on previous M.	
A1	Correct expression for v	
4(b) ALT 1	**Using integration**	
M1 A1	As above, for finding correct value for e. This may be embedded in a complete method.	
M1	Uses F=ma to and attempts to integrate. Must have the correct number of terms and weight resolved, $\int g\sin 30 - \dfrac{4gx}{l}\,\mathrm{d}x = \int v\,\mathrm{d}v$ leading to $\dfrac{gx}{2} - \dfrac{2gx^2}{l} = \dfrac{v^2}{2} + c$	
A1	Correct integration with at most one slip/error	
A1	Completely correct integration but c may be missing	
DM1	Find value for c (when $x = \frac{l}{4}$, v=0 gives c=0) **and** sub in e to find an expression for v	
A1	Correct expression for v	

4(b) **ALT 2** M1 A1 M1 A1 A1 M1 A1	**Using SHM** As above, for finding correct value for e. This may be embedded in a complete method. Correctly uses F=ma to show that the motion is SHM Correct proof of SHM Uses $v = aw$ to find an expression for v Correct expression for v

Question Number	Scheme	Marks
5(a)	$(\pi\rho)\int_0^r xy^2\mathrm{d}x$	
	$=(\pi\rho)\int_0^r x\left(r^2-x^2\right)\mathrm{d}x$	M1
	$=(\pi\rho)\left[\frac{1}{2}x^2r^2-\frac{x^4}{4}\right]_0^r$	A1
	$=(\pi\rho)\frac{r^4}{4}$	A1
	$\frac{2\pi\rho r^3}{3}\bar{x}=\pi\rho\int xy^2\mathrm{d}x$	M1
	$\bar{x}=\frac{\pi\rho r^4}{4}\div\frac{2\pi\rho r^3}{3}=\frac{3}{8}r$ *	A1* (5)
(b)	Hemisphere / Cone	
	Mass: $\frac{2}{3}\pi r^3$ / $\frac{1}{3}\pi k r^3$	B1
	Dist of c of m from centre of common plane: $\frac{3}{8}r$ / $\frac{1}{4}kr$	B1
	$\frac{2}{3}\times\frac{3}{8}r=\frac{k}{3}\times\frac{1}{4}kr$	M1A1ft
	$k^2=3 \quad k=\sqrt{3}$	A1 (5)
		[10]
(a)		
M1	Use of $(\pi\rho)\int_0^r xy^2\mathrm{d}x$ with $y^2=r^2-x^2$ and attempt the integration. Limits not needed.	
A1	Correct integration – limits not needed	
A1	Sub correct (upper) limit. (Sub of 0 not needed)	
M1	Use of $V\rho\bar{x}=\pi\rho\int xy^2\mathrm{d}x$ with their result to obtain $\bar{x}=\ldots$ where V is the volume of the hemisphere or sphere (π , p must be on both sides or neither)	
A1*	$\bar{x}=\frac{3}{8}r$	
(b)		
B1	Correct mass ratio for hemisphere and cone. Total mass not needed for this mark.	
B1	Correct distances of c of m for cone and hemisphere from centre of common plane (or another point). Both can be positive or one can be negative. Distances from vertex of cone (H) $kr+\frac{3}{8}r$ (C) $\frac{3}{4}kr$ Distances from peak of hemisphere (H) $\frac{5}{8}r$ (C) $r+\frac{1}{4}kr$	
M1	Form a dimensionally correct moments equation **with the correct value for $\bar{x}$ depending on where they have taken moments**. (0 from plane face, *kr* from vertex of cone, *r* from peak of hemisphere) Allow even if formula for sphere is used. Ignore signs.	

A1ft **A1**	Correct equation, follow through their masses and distances, signs to be correct here. Correct exact result.

Question Number	Scheme	Marks
6(a)	$S - mg\cos\theta = \dfrac{mv^2}{a}$	M1A1
	$\dfrac{1}{2}\times mv^2 - \dfrac{1}{2}\times m\times\dfrac{9ag}{5} = mga\cos\theta$	M1A1
	$mv^2 = 2mga\cos\theta + \dfrac{9}{5}mga$	
	$S = mg\cos\theta + 2mg\cos\theta + \dfrac{9}{5}mg$	DM1
	$S = \dfrac{3}{5}mg\left(5\cos\theta + 3\right)$ *	A1* cso (6)
(b)	$S = 0 \quad \cos\theta = -\dfrac{3}{5}$	B1
	$v^2 = \dfrac{3ag}{5} \quad v = \sqrt{\dfrac{3ag}{5}}$ *	M1A1*
		(3)
(c)	vert comp = $\sqrt{\dfrac{3ag}{5}}\times\dfrac{4}{5}$	M1
	Vert distance to highest point: $0 = \dfrac{16}{25}\times\dfrac{3ag}{5} - 2gs$	M1
	$s = \dfrac{24}{125}a$	A1
	Total distance above $O = \dfrac{24}{125}a + \dfrac{3}{5}a = \dfrac{99}{125}a$, 0.79$a$ or better	A1ft
		(4)
	Notes	**[13]**
(a) M1	Equation of motion along the radius. Must have 3 terms with weight resolved. Acceleration in either form.	
A1	Fully correct equation with acceleration v^2/r	
M1	Energy equation from A to general position. Difference of 2 KE terms and loss of PE (one or two terms) required. M0 for $v^2 = u^2 + 2as$	
A1	Fully correct equation	
DM1	Eliminate v^2 between the 2 equations. Depends on both preceding M marks	
A1 *cso	Obtain the **given** result from fully correct working.	
(b)		
B1	$\cos\theta = -\dfrac{3}{5}$ seen explicitly or used	
M1	Use their value of $\cos\theta$ to obtain the value of v^2 or v	
A1*	Correct answer from correct working	
(c)		
M1	Use their values for θ and v to obtain the vertical comp of velocity (allow sin/cos confusion)	
M1	Correct method to find the vertical distance to highest point using their vertical comp of vel	
A1	Correct expression for this vertical distance (may be implied)	
A1ft	Find the total distance above O by adding $\dfrac{3a}{5}$ to their previous answer. Both M marks needed.	

ALT 1	**Conservation of Energy from <u>slack</u> to find vertical height**
M1	Uses their value of θ and v to obtain the horizontal component at the highest point $\sqrt{\frac{3ag}{5}}\cos\theta$
M1	Forms an energy equation. **Must** have 2 KE terms and gain in PE $\frac{1}{2}m\frac{3ag}{5}-\frac{1}{2}m\frac{3ag}{5}\left(\frac{3}{5}\right)^2=mgs$
A1	Correct expression for this vertical distance $s=\frac{24}{125}a$
A1ft	Find the total distance above O by adding $\frac{3a}{5}$ to their previous answer. Both M marks needed. $\frac{99}{125}a$, $0.79a$ or better
ALT 2	**Conservation of Energy from <u>initial position</u> (A) to find vertical height**
M1	Uses their value of θ and v to obtain the horizontal component at the highest point $\sqrt{\frac{3ag}{5}}\cos\theta$
M1 A1	Forms an energy equation. **Must** have 2 KE terms and gain in PE $\frac{1}{2}m\frac{9ag}{5}-\frac{1}{2}m\frac{3ag}{5}\left(\frac{3}{5}\right)^2=mgh$
A1	Gives the total distance above O as $h=\frac{99}{125}a$ (do not isw)

Question Number	Scheme	Marks
7(a)	$(T=)\dfrac{20(1)}{2}=\dfrac{\lambda\times 0.8}{1.2}$	M1A1
	$\lambda=15$ *	A1* (3)
(b)	Either $1.25\ddot{x}=\dfrac{15(0.8-x)}{1.2}-\dfrac{20(1+x)}{2}$	M1A1A1
	Or $1.25\ddot{x}=\dfrac{20(1-x)}{2}-\dfrac{15(0.8+x)}{1.2}$	
	$\ddot{x}=-18x$	A1* (4)
(c)	$10=a\sqrt{18}\Rightarrow a=\dfrac{10}{\sqrt{18}}$ oe	B1
	When string PB becomes slack $v^2=18\left(\left(\dfrac{10}{\sqrt{18}}\right)^2-0.8^2\right)$	M1
	$v=9.4063...$ $v=9.4$ or 9.41 m s^{-1}	A1 (3)
(d)	$0.8=\dfrac{10}{\sqrt{18}}\sin\sqrt{18}t_1$	M1A1
	$t_1=\dfrac{1}{\sqrt{18}}\sin^{-1}\left(0.8\dfrac{\sqrt{18}}{10}\right)$ $(=0.0816...)$	A1
	PA becomes slack when $x=-1$	
	$(\pm 1)=\dfrac{10}{\sqrt{18}}\sin\sqrt{18}t_2$	M1
	$t_2=\dfrac{1}{\sqrt{18}}\sin^{-1}\left(\dfrac{\sqrt{18}}{10}\right)$ $(=0.1032...)$	A1
	$T=2(t_1+t_2)=2\left(\dfrac{1}{\sqrt{18}}\sin^{-1}\left(0.8\dfrac{\sqrt{18}}{10}\right)+\dfrac{1}{\sqrt{18}}\sin^{-1}\left(\dfrac{\sqrt{18}}{10}\right)\right)$	A1 (6)
	$=0.3697...=0.37$ or 0.370	
	Notes	**[16]**
(a)		
M1	Form an equation by equating the 2 tensions (found using HL)	
A1	Equation correct	
A1*	Correct answer correctly obtained	
(b)		
M1	Equation of motion for P. Acceleration can be a	
A1	Correct equation of motion with at most one error, acceleration may be a	
A1	Fully correct equation of motion, acceleration may be a	
A1*	Correct **given** equation, correctly obtained	

(c)	
B1	Correct amplitude, $a = \frac{10}{\sqrt{18}}, \frac{5\sqrt{2}}{3}, \frac{\sqrt{50}}{3}$, 2.4 oe
M1	Use $v^2 = \omega^2\left(a^2 - x^2\right)$ with x = 0.8 and their a and ω
A1	Correct speed when x = 0.8
(d)	
M1	Use x = 0.8 to find the time until *PB* becomes slack using their a and ω
A1	Correct equation
A1	Correct time (seen or implied) Allow consistent use of degrees.
	NB There are alternative method for finding this time but a complete method for the time until *PB* becomes slack must be used for the M mark to be awarded.
M1	Use $x = \pm 1$ to find the time until *PA* becomes slack (as before, alternative methods must be complete) using their a and ω
A1	Correct time obtained. Ignore consistent use of degrees.
A1	Complete to obtain the **correct** value of T
ALT (c)	**Conservation of Energy, O to slack**
M1	Dimensionally correct energy equation with **3 EPE** terms and **2 KE** terms
B1 (treat as A1)	$\frac{20\times1^2}{2\times2} + \frac{1.25\times10^2}{2} + \frac{15\times0.8^2}{2\times1.2} = \frac{20\times1.8^2}{2\times2} + \frac{1.25\times v^2}{2}$
A1	Correct answer. $v = 9.4063...$ $v = 9.4$ or $9.41\ \mathrm{m\,s^{-1}}$
ALT 7 (d)	**Using cos**
M1 A1	$0.8 = \frac{10}{\sqrt{18}}\cos\sqrt{18}t_1$
A1	$t_1 = \frac{1}{\sqrt{18}}\cos^{-1}\left(0.8\frac{\sqrt{18}}{10}\right) \quad (= 0.2886...)$
M1	$-1 = \frac{10}{\sqrt{18}}\cos\sqrt{18}t_2$
A1	$t_2 = \frac{1}{\sqrt{18}}\cos^{-1}\left(-\frac{\sqrt{18}}{10}\right) \quad (= 0.4735...)$
A1	$T = 2(t_2 - t_1) = 2\left(\frac{1}{\sqrt{18}}\cos^{-1}\left(-\frac{\sqrt{18}}{10}\right) - \frac{1}{\sqrt{18}}\cos^{-1}\left(0.8\frac{\sqrt{18}}{10}\right)\right)$ $= 0.3697... = 0.37$ or 0.370