



Mark Scheme (Results)

January 2023

Pearson Edexcel International Advanced Level
In Pure Mathematics P1 (WMA11) Paper 01

Question Number	Scheme	Marks
1	$y = 2 + 10x^{\frac{1}{2}} - 2x^{\frac{3}{2}}$	
(a)	$\frac{dy}{dx} = 5x^{-\frac{1}{2}} - 3x^{\frac{1}{2}}$	M1 A1 A1
		(3)
(b)	$x = 2 \Rightarrow \frac{dy}{dx} = \frac{5}{\sqrt{2}} - 3\sqrt{2}$	M1
	$\frac{5}{\sqrt{2}} - 3\sqrt{2} = \frac{5}{2}\sqrt{2} - 3\sqrt{2} = -\frac{1}{2}\sqrt{2}$	A1
		(2)
		(5 marks)

(a) Mark (a) and (b) together.

M1: For ANY one of: $10x^{\frac{1}{2}} \rightarrow \dots x^{-\frac{1}{2}}$ or $-2x^{\frac{3}{2}} \rightarrow \dots x^{\frac{1}{2}}$ or $2 \rightarrow 0$

A1: For one simplified term e.g. either $10x^{\frac{1}{2}} \rightarrow 5x^{-\frac{1}{2}}$ or $-2x^{\frac{3}{2}} \rightarrow -3x^{\frac{1}{2}}$

A1: $\left(\frac{dy}{dx} = \right) 5x^{-\frac{1}{2}} - 3x^{\frac{1}{2}}$. Allow e.g. $\left(\frac{dy}{dx} = \right) \frac{5}{\sqrt{x}} - 3\sqrt{x}$. The “ $\frac{dy}{dx} =$ ” is not required. Condone “+ 0” as part of their answer.

There must be no other terms e.g. “+ c” or e.g. $\left(\frac{dy}{dx} = \right) 2 + 5x^{-\frac{1}{2}} - 3x^{\frac{1}{2}}$ but apply isw if possible once a correct simplified derivative with no extra terms is seen.

(b)

M1: For an attempt to substitute $x = 2$ fully into their $\frac{dy}{dx}$ which is not y e.g. it must be a “changed” function which could even come from integration.

All that is required is the substitution so allow for e.g. $5(2)^{-\frac{1}{2}} - 3(2)^{\frac{1}{2}}$

This mark may be implied by their answer or e.g. a decimal answer of awrt -0.7 following a correct derivative.

Do **not** allow this mark if they have e.g. $\left(\frac{dy}{dx} = \right) 5x^{-\frac{1}{2}} - 3x^{\frac{1}{2}} + c$ in part (a) **AND** subsequently go on to try and establish a value for “c” using $x = 2$.

A1: $-\frac{1}{2}\sqrt{2}$ or exact simplified equivalent e.g. $-\frac{1}{\sqrt{2}}$, $-0.5\sqrt{2}$, $-2^{-\frac{1}{2}}$, $-2^{-0.5}$ **following at least one intermediate line of working.**

E.g. $\frac{5}{\sqrt{x}} - 3\sqrt{x} = \frac{5}{\sqrt{2}} - 3\sqrt{2} = -\frac{1}{\sqrt{2}}$ scores M1A0

but $\frac{5}{\sqrt{x}} - 3\sqrt{x} = \frac{5}{\sqrt{2}} - 3\sqrt{2} = \frac{5\sqrt{2}}{2} - 3\sqrt{2} = -\frac{\sqrt{2}}{2}$ or $-\frac{1}{\sqrt{2}}$ scores M1A1

Other examples of sufficient working:

$$\frac{5}{\sqrt{x}} - 3\sqrt{x} = \frac{5}{\sqrt{2}} - 3\sqrt{2} = \frac{5-6}{\sqrt{2}} = -\frac{1}{\sqrt{2}}, \quad \frac{5}{\sqrt{x}} - 3\sqrt{x} = \frac{5}{\sqrt{2}} - 3\sqrt{2} = \left(\frac{5}{2} - 3\right)\sqrt{2} = -\frac{\sqrt{2}}{2}$$

Apply isw once a correct exact answer is seen.

Question Number	Scheme	Marks
2(a)	$P(-3,7), Q(9,11)$ and $R(12,2)$	
Way 1	$\text{grad } PQ = \frac{11-7}{9-(-3)} = \frac{1}{3}, \text{ grad } QR = \frac{11-2}{9-12} = -3$	M1 A1
	$\frac{1}{3} \times -3 = -1$ so angle $PQR = 90^\circ$	A1
		(3)
Way 2	$PQ^2 = (9-(-3))^2 + (11-7)^2 = 160$ $QR^2 = (12-9)^2 + (2-11)^2 = 90$ $PR^2 = (12-(-3))^2 + (2-7)^2 = 250$	M1 A1
	$PQ^2 + QR^2 = PR^2$ (or e.g. $90 + 160 = 250$) so angle $PQR = 90^\circ$ or e.g. $\cos \theta = \frac{160+90-250}{2\sqrt{160}\sqrt{90}} = 0 \Rightarrow \theta = 90^\circ$	A1
		(3)
Way 3	$\overrightarrow{PQ} = \begin{pmatrix} 9-(-3) \\ 11-7 \end{pmatrix} = \begin{pmatrix} 12 \\ 4 \end{pmatrix}, \overrightarrow{QR} = \begin{pmatrix} 12-9 \\ 2-11 \end{pmatrix} = \begin{pmatrix} 3 \\ -9 \end{pmatrix}$	M1A1
	$\overrightarrow{PQ} \cdot \overrightarrow{QR} = \begin{pmatrix} 12 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -9 \end{pmatrix} = 36 - 36 = 0$ so angle $PQR = 90^\circ$	A1
		(3)

General Guidance for part (a)

M1: Requires some correct work depending on the method

A1: Correct work for the chosen method

A1: Depends on both previous marks and requires an explanation and a conclusion.

The conclusion must refer to angle PQR being 90° or equivalent e.g. “ PQ and QR are perpendicular”,

“ Q is 90° ”, “ Q is a right angle” etc. but condone e.g. “ $A = 90^\circ$ ” or e.g. “ $\theta = 90^\circ$ ” if the solution is otherwise correct.

This may occur in a preamble e.g. If $\text{grad } PQ \times \text{Grad } QR = -1$ then $PQR = 90^\circ$,

$$\frac{11-7}{9-3} = \frac{1}{3}, \frac{11-2}{9-12} = -3, \frac{1}{3} \times -3 = -1 \text{ hence proven.}$$

Scores full marks.

(a) Way 1

M1: Attempts both gradients with an attempt at $\frac{\text{difference in } y}{\text{difference in } x}$ seen at least once.

A1: Achieves both correct gradients which may be left unsimplified e.g. $\text{grad } PQ = \frac{4}{12}$ and

$$\text{grad } QR = \frac{9}{-3}$$

A1: e.g. $\frac{1}{3} \times -3 = -1 \Rightarrow PQR = 90^\circ$ or e.g. $\frac{1}{3}$ is the negative reciprocal of -3 so $Q = 90$.

Do not allow ambiguous statements e.g. $\frac{1}{3}$ is the opposite inverse of -3 for the explanation.

NB: $\frac{11-7}{9-3} = \frac{1}{3}, \frac{11-2}{9-12} = -3, \frac{1}{3} \times -3 = -1 \Rightarrow PQR = 90^\circ$ would be a minimum for M1A1A1

(a) Way 2

M1: Attempts all three lengths with an attempt at “the difference between the coordinates” and “squaring” seen at least twice. Note that the “differences” may be implied by their answers or their lengths.

A1: All three lengths or lengths² correct as single exact terms or as decimals – allow one d.p.

Lengths are $\sqrt{160}, \sqrt{90}, \sqrt{250}$ or $4\sqrt{10}, 3\sqrt{10}, 5\sqrt{10}$ or e.g. 12.6, 9.5 (or 9.4 truncated), 15.8

A1: via Pythagoras or cosine rule e.g.

$$PQ^2 + QR^2 = PR^2 \text{ so } PQR = 90^\circ \text{ or e.g. } \cos \theta = \frac{160 + 90 - 250}{2\sqrt{160}\sqrt{90}} = 0 \Rightarrow \theta = 90^\circ$$

If via the cosine rule, all values must be in the correct positions with correct signs.

“160, 90, 250 is a Pythagorean triple so $PQR = 90^\circ$ ” is acceptable for the final mark.

Values must appear as exact for this mark. E.g. $\cos \theta = \frac{12.6^2 + 9.5^2 - 15.8^2}{2 \times 12.6 \times 9.5} = 0 \Rightarrow \theta = 90^\circ$ scores

M1A1A0

(a) Way 3

M1: Attempts $\pm \overrightarrow{PQ}$ and $\pm \overrightarrow{QR}$ with an attempt at “the difference between the coordinates” seen at least twice

A1: Correct vectors for $\pm \overrightarrow{PQ}$ and $\pm \overrightarrow{QR}$

A1: $\overrightarrow{PQ} \cdot \overrightarrow{QR} = 0$ so angle $PQR = 90^\circ$

Part (b)		
(b)	E.g. $(-3, 7) + (3, -9) = \dots$ or $(12, 2) - (12, 4) = \dots$	M1
	$(0, -2)$	A1
		(2)
ALT 1	$\text{grad } PQ = \frac{11-7}{9-3} = \frac{1}{3} \Rightarrow \text{eqn } RS \text{ is } y-2 = \frac{1}{3}(x-12)$ $\text{grad } QR = \frac{11-2}{9-12} = -3 \Rightarrow \text{eqn } PS \text{ is } y-7 = -3(x+3)$ $\Rightarrow x = \dots, y = \dots$	M1
	$(0, -2)$	A1
ALT 2	Midpoint PR is $\left(\frac{9}{2}, \frac{9}{2}\right) \Rightarrow \frac{9+x}{2} = \frac{9}{2}, \frac{11+y}{2} = \frac{9}{2} \Rightarrow x = \dots, y = \dots$	M1
	$(0, -2)$	A1
		(5 marks)

(b)

M1: Any suitable method of finding at least x or y for S . It can be implied by one correct coordinate which may be seen on a diagram.

e.g. Via vectors $(-3, 7) + (3, -9) = (0, -2)$ or $(12, 2) - (12, 4) = (0, -2)$

A1: Correct coordinates $(0, -2)$ which may be written separately e.g. $x = 0, y = -2$ or as $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$

Alt 1(b)

M1: Attempts the equation of line PS , the equation of line RS and solves simultaneously to find x or y

A1: Correct coordinates $(0, -2)$ which may be written separately e.g. $x = 0, y = -2$ or as $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$

Alt 2(b)

M1: Attempts the midpoint of PR and uses Q to find S for at least one of x or y

A1: Correct coordinates $(0, -2)$ which may be written separately e.g. $x = 0, y = -2$ or as $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$

Question Number	Scheme	Marks
3	$\int \frac{4x^5 + 3}{2x^2} dx = \int 2x^3 + \frac{3}{2}x^{-2} dx$	M1 A1
	$= \frac{1}{2}x^4 - \frac{3}{2}x^{-1} + c$	dM1 A1 A1
		(5)
		(5 marks)

M1: Attempts to write as **a sum of two terms** with one processed index correct.

Award for $Px^3 + Qx^k$ or $Px^k + Qx^{-2}$ or $Px^k + \frac{Q}{x^2}$ where k could be 0 i.e. a constant term.

A1: Correct integrand written as a sum of two terms with indices processed. E.g. $2x^3 + \frac{3}{2}x^{-2}$ seen in one expression.

Award for any exact equivalent such as $\frac{1}{2}\left(4x^3 + \frac{3}{x^2}\right)$ or e.g. $\frac{4}{2}x^3 + \frac{3}{2x^2}$ or e.g. $\frac{4x^3 + 3x^{-2}}{2}$

dM1: Attempts to integrate an expression of the form $Px^m + Qx^n$, $m = 3$ or $n = -2$, raising one of the indices by one. Depends on the first M mark.

A1: Either term correct and simplified and from correct work: $\frac{1}{2}x^4 + Ax^n(+c)$ or $Bx^m - \frac{3}{2}x^{-1}(+c)$

A1: $\frac{1}{2}x^4 - \frac{3}{2}x^{-1} + c$. Allow simplified equivalents e.g. $\frac{1}{2}x^4 - \frac{3}{2x} + c$, $0.5x^4 - 1.5x^{-1} + c$ but **not** e.g.

$\frac{1}{2}x^4 - \frac{3}{x} + c$ and **not** $\frac{1}{2}x^4 + -\frac{3}{2}x^{-1} + c$.

Condone poor notation such as $\int \frac{1}{2}x^4 - \frac{3}{2x} + c(dx)$, $y = \frac{1}{2}x^4 - \frac{3}{2x} + c$

You can ignore subsequent working if necessary e.g. award the marks once a correct answer is seen as a correct single expression.

Correct answer only scores full marks.

Question Number	Scheme	Marks
4	$kx^2 + 6kx + 5 = 0$	
	$b^2 - 4ac = (6k)^2 - 4 \times k \times 5$	M1
	$b^2 - 4ac = (6k)^2 - 4 \times k \times 5 \dots 0 \Rightarrow k \dots$	dM1
	$k < \frac{5}{9}$	A1
	$0 < k < \frac{5}{9}$	A1
		(4 marks)

M1: Attempts to use $b^2 - 4ac$ for the given quadratic with $b = 6k$, $a = k$ and $c = 5$.

May be seen as part of the quadratic formula or may be implied by an attempt to solve e.g. $b^2 = 4ac$

Condone attempts where the "6" isn't squared e.g. $6k^2 - 4 \times k \times 5$ but the k must be squared.

dM1: Dependent upon the previous M mark, it is for setting $b^2 - 4ac \dots 0$ leading to a non-zero value for k from an "equation" of the form $\alpha k^2 - \beta k \dots 0$

Condone any of e.g. "=", "<", ">" etc. for "..." for this mark.

A1: For obtaining an upper limit for k of $\frac{5}{9}$ (not just the value) but condone $k \leq \frac{5}{9}$ which may be

implied by e.g. $0 < k < \frac{5}{9}$ or $0 < k \leq \frac{5}{9}$. Allow exact equivalents for $\frac{5}{9}$ e.g. $\frac{10}{18}$ etc. Condone the use

of x for this mark so allow e.g. $x \leq \frac{5}{9}$, $x < \frac{5}{9}$ Allow $0.\dot{5}$ for $\frac{5}{9}$

Allow the inequalities to be on separate lines e.g. $k < \frac{5}{9}$
 $k > 0$

Question Number	Scheme	Marks
5(a)	$9^x = p^2, \quad 3^{x+2} = 9p \quad \text{or} \quad 3^{x-1} = \frac{p}{3}$	B1
	$3 \times 9^x + 3^{x+2} = 1 + 3^{x-1} \Rightarrow 3p^2 + 3^2 \times p = 1 + \frac{p}{3}$	M1
	$9p^2 + 26p - 3 = 0 \quad \text{via} \quad 3p^2 + 9p = 1 + \frac{p}{3} *$	A1*
		(3)
(b)	$9p^2 + 26p - 3 = 0 \Rightarrow (9p - 1)(p + 3) = 0$	M1
	$3^x = \frac{1}{9}$	A1 M1 in EPEN
	$x = -2$	A1
		(3)
		(6 marks)

(a) Mark (a) and (b) together.

B1: Uses an index law and states or implies any of $9^x = p^2$, $3^{x+2} = 9p$ or $3^{x-1} = \frac{p}{3}$ or equivalent forms

e.g. $9^x = p \times p$, $3^{x+2} = 3^2 \times p$, $3^{x-1} = p \times 3^{-1}$

If awarding for the first term, then it must be from correct work so $3 \times 9^x = 3 \times 3^x \times 3^x = 3p^2$ is fine but $3 \times 9^x = 3 \times 3^2 \times 3^x = 3p^2$ or $3 \times 9^x = 3 \times 3 \times 3^x = 3p^2$ is not, but do check the other terms.

M1: Look for $3 \times 9^x + 3^{x+2} = 1 + 3^{x-1} \Rightarrow 3p^2 \pm kp = 1 \pm \frac{p}{3}$ oe **obtained from correct work**, with $k = 6$ or 9

So if e.g. $3p^2$ is obtained from $3 \times 9^x = 3 \times 3^2 \times 3^x = 3p^2$ or $3 \times 9^x = 3 \times 3 \times 3^x = 3p^2$ then score M0.

A1*: Proceeds to the given answer of $9p^2 + 26p - 3 = 0$ with no errors or omissions.

An intermediate line of $3p^2 + 9p = 1 + \frac{p}{3}$ o.e. must be seen.

Note that the following is common in part (a) and scores no marks: $3 \times 9^x + 3^{x+2} = 1 + 3^{x-1} \Rightarrow 3 \times 3p + p^2 = 1 + p^{-1}$

(b)

M1: Valid non-calculator attempt at solving $9p^2 + 26p - 3 = 0$ – see General Guidance.

It must be clear they are solving the given quadratic not their incorrect one.

Answers just written down scores M0

A1(M1 in EPEN): For $3^x = \frac{1}{9}$ seen. It must be clear that it is a value for 3^x and not a value for p or x .

May be implied by e.g. $p = \frac{1}{9} \Rightarrow x = -2$. You can ignore e.g. $3^x = -3$ for this mark.

A1: $x = -2$ only

Question Number	Scheme	Marks
6(a)	$A = \frac{1}{2} r^2 \theta \Rightarrow 40 = \frac{1}{2} r^2 \times 2.4 \Rightarrow r = \dots$	M1
	$r = \sqrt{\frac{80}{2.4}} \Rightarrow r = 5.77 \text{ (m)}$	A1
		(2)
(b)	States or uses that angle $AOB = \text{awrt } 0.37$	B1
		(1)
(c)	$\frac{1}{2} \times "5.77" \times 6.25 \times \sin "0.37" (= 6.5377\dots)$	M1
	Full method for area of stage = $40 + 2 \times \frac{1}{2} \times 5.77 \times 6.25 \times \sin 0.37$	dM1
	$= 53.1 \text{ m}^2$	A1
		(3)
(d)	$r\theta = "5.77" \times 2.4 = (13.848)$	M1
	$x^2 = 6.25^2 + "5.77"^2 - 2 \times 6.25 \times "5.77" \cos "0.37" \quad (x = 2.26)$	M1
	Full method for perimeter of stage = $12.5 + 2 \times "2.26" + "5.77" \times 2.4$	ddM1
	$= 30.9 \text{ m}$	A1
		(4)
		(10 marks)

Allow equivalent correct work in degrees but part (b) must be in radians.

(a)

M1: Attempts to use $A = \frac{1}{2} r^2 \theta$ with $A = 40$ and $\theta = 2.4$ to find a value for r

A1: Achieves awrt 5.77 (m). Apply isw if necessary e.g. if they subsequently write 5.77 as 5.8.

(b)

B1: Angle $AOB = \text{awrt } 0.37$. Allow this to score anywhere in their answer.

(c)

M1: Attempts area of triangle AOB (or triangle DOC) $\frac{1}{2} \times "a" \times 6.25 \times \sin "b"$ (May be seen "doubled")

NB: "doubled" is 13.075...

dM1: Attempts area of stage $40 + 2 \times \frac{1}{2} \times "a" \times 6.25 \times \sin "b"$

or e.g. $\frac{1}{2} \times "5.77" \times 2.4 + 2 \times \frac{1}{2} \times "a" \times 6.25 \times \sin "b"$

A1: Awrt 53.1 (m²). Condone awrt 53.0 but not just 53 unless awrt 53.1 or awrt 53.0 is seen earlier.

(d)

M1: Attempts $r\theta = a \times 2.4$

M1: Attempts to use the cosine rule to find length AB or AB^2 (or CD/CD^2) (Allow anywhere in their solution)

e.g. $x^2 = 6.25^2 + a^2 - 2 \times 6.25 \times a \cos b$

ddM1: Full method to find perimeter of stage $12.5 + 2 \times AB + a \times 2.4$

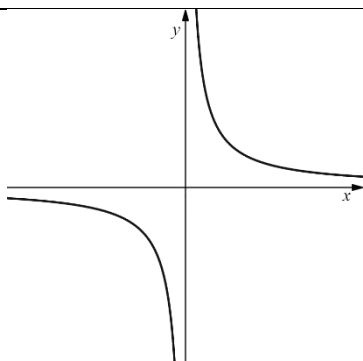
It must be clear they are using AB not AB^2

Depends on both previous M marks.

A1: Awrt 30.9 (m)

Beware in (d): $AB = 6.25 \sin(0.37) = 2.26$...erroneously and will lead to the correct answer 30.9

Generally, this will score M1M0ddM0A0 if the arc length calculation is correct.

Question Number	Scheme		Marks
7(a)		Correct Shape in quadrant 1 or 3	M1
		Fully correct shape and position	A1
(b)	E.g. Translate 2 (units) to the right (parallel to the x -axis)		B1, B1
			(2)
(c)	When $x = -4 \Rightarrow \frac{6}{x-2} = kx + 7 \Rightarrow \frac{6}{-6} = -4k + 7 \Rightarrow k = \dots$		M1
	$k = 2$		A1
			(2)
(d)	$\frac{6}{x-2} = "2"x + 7 \Rightarrow 6 = 2x^2 + 3x - 14$		M1
	$0 = 2x^2 + 3x - 20 \Rightarrow (2x - 5)(x + 4) = 0$		dM1
	$Q = \left(\frac{5}{2}, 12\right)$		A1, A1
			(4)
			(10 marks)

(a)

M1: For a monotonically decreasing function in quadrant 1 or 3 with no incorrect asymptotes in that quadrant.

It must not cross either axis but be tolerant of functions that don't go up or down as far as the x -axis and as far left or right as the y -axis.

The “ends” should not come back away from the axes significantly but be tolerant of “wobbles”.

A1: Correct shape and position with no incorrect asymptotes.

Remember to check both diagrams and score the best single attempt if both are used.

(b)

B1: Partial description that implies at least one of the two components but is not fully correct

E.g. "Translates 2 units left", "Shifts/moves 2 units right"

B1: Requires (1): Translate/translation **and** (2): $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ or “2 (units to the) right” or e.g. “+2 in the x direction”

A minimum could be e.g.: “Translate +2 on x ”

(c)

M1: Substitutes $x = \pm 4$ into $\frac{6}{x-2} = kx + 7$ and solves for k .

Allow equivalent work e.g. $x = -4 \Rightarrow y = -1 \Rightarrow -1 = k(-4) + 7 \Rightarrow k = \dots$

Note that some will rearrange before substitution which is fine.

E.g. $\frac{6}{x-2} = kx + 7 \Rightarrow 6 = (x-2)(kx+7) = kx^2 + (7-2k)x - 14, 6 = 24k - 42 \Rightarrow k = \dots$

A1: $k = 2$ or e.g. $k = \frac{4}{2}$

(d)

M1: Equates $\frac{6}{x-2}$ with $kx + 7$ using their **value** for k , cross multiplies to obtain a quadratic equation in x with terms not necessarily collected. Note that the rearrangement may have already been done in part (c).

dM1: Solves 3TQ by any acceptable method including via a calculator.

A1: $x = \frac{5}{2}$. Condone $Q = \frac{5}{2}$

A1: $Q = \left(\frac{5}{2}, 12\right)$. Must be as coordinates or $x = \dots, y = \dots$

Question Number	Scheme	Marks
8(a)	$y = 3x + "c" \text{ or } y = "m"x - 12'$	M1
	$y = 3x - 12$	A1
		(2)
(b)	$k = 10$	B1
		(1)
(c)	E.g. $y = A(x-4)(x-10)$ or $y = C(x-7)^2 - 18$	M1
	E.g. $-18 = A(7-4)(7-10) \Rightarrow A = \dots$ Or $0 = C(4-7)^2 - 18 \Rightarrow C = \dots$	dM1
	$y = 2(x-4)(x-10)$, $y = 2(x-7)^2 - 18$ o.e.	A1
		(3)
(d)	Two of $y > 3x - 12$, $y < 2(x-4)(x-10)$, $x > 0$, $x < 4$	M1
	E.g. $3x - 12 < y < 2(x-4)(x-10)$, $0 < x < 4$	A1
		(2)
		(8 marks)

(a)

M1: Attempts form $y = mx + c$ with m or c correct. May be implied by e.g. $m = 3$ (or $\frac{12}{4}$) or $c = -12$

A1: $y = 3x - 12$. A full correct equation is required including “y =”. Condone $\frac{12}{4}$ for 3.

(b)

B1: $k = 10$. Condone $x = 10$ and condone (10, 0)

(c)

M1: Attempts an equation of the form $(y =)A(x-4)(x-10)$ or $(y =)C(x-7)^2 - 18$

Condone with $A, C = 1$ or any other constant.

It is possible they could try with an attempt to use all three coordinates.

The “y =” is not needed unless the equation is attempted as e.g. $y + 18 = C(x-7)^2$

dM1: Full attempt at equation with an attempt at finding A or C . **Depends on the first mark.**

E.g. $y = A(x-4)(x-10)$ and uses $x = 7, y = -18$ to find A or $y = C(x-7)^2 - 18$ and uses $x = 4$ or $x = 10$ when $y = 0$ to find C

A1: $(y =)2(x-4)(x-10)$, $(y =)2(x-7)^2 - 18$ o.e. e.g. $(y =)2(x-7)^2 - 18$

The “y =” is **not** required here, just look for a correct expression.

(d)

M1: Two of $y > 3x - 12$, $y < 2(x-4)(x-10)$, $x > 0$, $x < 4$ Accept with $\leq \leftrightarrow <$ and $\geq \leftrightarrow >$

Follow through their answers to part (a) and part (c) provided (a) is linear and (c) is quadratic.

Do not allow e.g. $R > 3x - 12$, $R < 2(x-4)(x-10)$ etc. but allow $f(x)$ for y .

A1: Fully defines region correctly (**not ft here**).

E.g. $3x-12 < y < 2(x-4)(x-10)$, $0 < x < 4$ (or $x > 0, x < 4$)

Or e.g. $y > 3x-12$, $y < 2(x-4)(x-10)$, $x > 0$, $x < 4$

The right hand side of the inequality $0 < x < 4$ may be larger. Accept $0 < x < p$ as long as

$$4 \leq p \leq \frac{23}{2}$$

Allow **consistent** use of $> \leftrightarrow \geq$ for all of their inequalities.

Question Number	Scheme	Marks
9(a)	π	B1
		(1)
(b)(i)	3	B1
		(1)
(ii)	5	B1
		(1)
(iii)	201	B1
		(1)
		(4 marks)

(a)

B1: Period is π (radians) but condone 180° or just 180

(b)(i)

B1: 3

(ii)

B1: 5

(iii)

B1: 201

Question Number	Scheme	Marks
10(a)	One of $-\frac{20}{3} < x < -6, \quad x > \frac{3}{2}$	M1
	Both $-\frac{20}{3} < x < -6, \quad x > \frac{3}{2}$	A1
		(2)
(b)	$(3x+20)(x+6)(2x-3) = (3x+20)(2x^2+9x-18) =$	M1
	$= 6x^3 + 67x^2 + 126x - 360$	A1 A1
		(3)
(c)	$\frac{dy}{dx} = 18x^2 + 134x + 126 \Rightarrow$ Gradient at $x=0$ is 126	M1
	Equation of l is $y = 126x - 360$	A1ft
	l cuts C again when $6x^3 + 67x^2 + 126x - 360 = 126x - 360$	dM1
	$6x^3 + 67x^2 = 0 \Rightarrow x^2(6x+67) = 0$	ddM1
	$x = -\frac{67}{6}$	A1
		(5)
		(10 marks)

(a)

M1: One of $-\frac{20}{3} < x < -6, \quad x > \frac{3}{2}$. Condone with $\leftrightarrow \leq$ for this mark and allow equivalent notation

e.g. $\left(-\frac{20}{3}, -6\right), \left(\frac{3}{2}, \infty\right), \left[-\frac{20}{3}, -6\right], \left[\frac{3}{2}, \infty\right)$

Allow the first inequality to be written separately as e.g. $x > -\frac{20}{3}, \quad x < -6,$

$x > -\frac{20}{3}$ and $x < -6$

Condone incorrect notation e.g. $-\frac{20}{3} < f(x) < -6, \quad f(x) > \frac{3}{2}$

or e.g. $-\frac{20}{3} < y < -6, \quad y > \frac{3}{2}$

A1: Both $-\frac{20}{3} < x < -6, \quad x > \frac{3}{2}$.

Allow the first inequality to be written separately as e.g. $x > -\frac{20}{3}, \quad x < -6$ and allow equivalent

notation e.g. $\left(-\frac{20}{3}, -6\right), \left[\frac{3}{2}, \infty\right)$ but **not** e.g. $-\frac{20}{3} < f(x) < -6, \quad f(x) > \frac{3}{2}$

(b) Allow (b) marks to score anywhere in their solution.

M1: Attempts to multiply two brackets to create a quadratic before multiplying by the third to form a cubic

A1: $6x^3 + \alpha x^2 + \beta x \pm 360$ where α and β are not both zero.

A1: $6x^3 + 67x^2 + 126x - 360$. Condone a spurious “= 0” e.g. $6x^3 + 67x^2 + 126x - 360 = 0$ but do not isw e.g. $x^3 + \frac{67}{6}x^2 + 21x - 60$ is A0 but see note below.

(c)

M1: Attempts the gradient of l by differentiating and substituting $x = 0$

For the differentiation look for one of $\dots x^3 \rightarrow \dots x^2$, $\dots x^2 \rightarrow \dots x$, $kx \rightarrow k$

A1ft: Equation of l is $y = 126x - 360$. Follow through on their $y = ax^3 + bx^2 + cx + d \Rightarrow y = cx + d$

Allow equivalent correct equations e.g. $y + 360 = 126(x - 0)$

dm1: Sets the equation of their l to their answer for (b). **Depends on the first M mark.**

ddM1: Attempts to solve cubic of the form $ax^3 + bx^2 = 0$ by taking out a factor of x^2 or e.g. by division by x^2 leading to a value for x . **Depends on both previous M marks.**

May be implied by e.g. $ax^3 + bx^2 = 0 \Rightarrow x = -\frac{b}{a}$

A1: $x = -\frac{67}{6}$ and no other solutions apart from $x = 0$ which can be ignored. Ignore any attempts to find y .

Note that **just** an incorrect x coefficient in the expansion in (b) will result in $x = -\frac{67}{6}$ in (c) if the subsequent work is correct. In such cases allow full recovery in (c).

Similarly, if the expansion in part (b) is e.g. divided by 6 to give $x^3 + \frac{67}{6}x^2 + 21x - 60$, allow full

recovery in (c) as correct work should lead to the correct answer of $x = -\frac{67}{6}$

Question Number	Scheme	Marks
11(a)	States or implies that the gradient of the normal is $-\frac{1}{3}$	M1
	Correct equation of normal e.g. $y - 16 = -\frac{1}{3}(x - 4)$	A1
		(2)
(b)	$f''(x) = 4x + x^{-\frac{1}{2}} \Rightarrow f'(x) = 2x^2 + 2x^{\frac{1}{2}} + c$	M1
	$x = 4, f'(x) = 3 \Rightarrow 3 = 32 + 4 + c \Rightarrow c = \dots(-33)$	dM1
	$f'(x) = 2x^2 + 2x^{\frac{1}{2}} - 33$	A1
	$f'(x) = 2x^2 + 2x^{\frac{1}{2}} - 33 \Rightarrow f(x) = \frac{2}{3}x^3 + \frac{4}{3}x^{\frac{3}{2}} - 33x + d$	dM1
	$x = 4, f(x) = 16 \Rightarrow (f(x) =) \frac{2}{3}x^3 + \frac{4}{3}x^{\frac{3}{2}} - 33x + \frac{284}{3}$	ddM1A1
		(6)
		(8 marks)

(a)

M1: States or implies that the gradient of the normal is $-\frac{1}{3}$

A1: Finds the equation of the normal $y - 16 = -\frac{1}{3}(x - 4)$ o.e. e.g. $y = -\frac{1}{3}x + \frac{52}{3}$, $3y + x - 52 = 0$

But **not** $\frac{y-16}{x-4} = -\frac{1}{3}$.

Requires a full correct equation. Apply isw once a correct equation is seen.

(b) Award marks if work in (b) is seen in (a).

M1: Attempts to integrate $f''(x)$ once with one index correct. E.g. $4x \rightarrow \dots x^2$ or $\frac{1}{\sqrt{x}} \rightarrow \dots x^{\frac{1}{2}}$

dM1: Applies $f'(4) = 3$ and solves to find a constant of integration. **Depends on first M1.**

A1: $(f'(x) =) 2x^2 + 2x^{\frac{1}{2}} - 33$ or obtains $(f'(x) =) 2x^2 + 2x^{\frac{1}{2}} + c$ with c correctly calculated as -33

Ignore labelling of the function e.g. they may call it $f(x)$

dM1: Dependent upon first M1 only. For an attempt to integrate $f''(x)$ twice and achieve a form

$(f(x) =) ax^3 + bx^{\frac{3}{2}} + \dots$ where \dots could be 0. Ignore labelling of the function e.g. they may call it $f(x)$

dddM1: Dependent upon all previous M's. It is for using $f(4) = 16$ (may be implied) to find the constant of integration.

A1: $(f(x) =) \frac{2}{3}x^3 + \frac{4}{3}x^{\frac{3}{2}} - 33x + \frac{284}{3}$ or exact equivalent. Apply isw once a correct **expression** is seen.

The “ $f(x)=$ ” is not required and ignore any label they may have given it.

Also allow $(f(x)=)\frac{2}{3}x^3 + \frac{4}{3}x^{\frac{3}{2}} - 33x + c$ with c correctly calculated as $\frac{284}{3}$.