Pearson Edexcel

Mark Scheme (Results)

October 2021

Pearson Edexcel International A Level
In Further Pure Mathematics F3 (WFM03)
Paper 01

Question	Scheme	Marks
1	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\times\frac{2}{\sqrt{(2x)^2-1}}$	M1
	$1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=1+\frac{1}{4x^2-1}=\frac{4x^2}{4x^2-1}$	M1
	$\int\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x=\int\sqrt{\frac{4x^2}{4x^2-1}}\,\mathrm{d}x=2\int\frac{x}{\sqrt{4x^2-1}}\mathrm{d}x$	A1
	$=\frac{2\left(4x^2-1\right)^{\frac{1}{2}}}{8\times \frac{1}{2}}$	M1
	$s=\left[\frac{\left(4x^2-1\right)^{\frac{1}{2}}}{2}\right]_{\frac{7}{2}}^{13}=\frac{1}{2}\left(\sqrt{4\times169-1}-\sqrt{4\times\frac{49}{4}-1}\right)=\ldots$	dM1
	$=\frac{1}{2}\left(15\sqrt{3}-4\sqrt{3}\right)=\frac{11}{2}\sqrt{3}$	A1
		(6)
		(6 marks)

Notes:

M1: Attempts $\frac{\mathrm{d}y}{\mathrm{d}x}$, accept the form $\frac{A}{\sqrt{(2x)^2-1}}$. Allow $\frac{A}{\sqrt{2x^2-1}}$ (condone missing brackets)

Alternative 1:

Writes $\frac{1}{2}\operatorname{ar}\cosh 2x$ as $\frac{1}{2}\ln\left(2x+\sqrt{4x^2-1}\right)$ leading to

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\times\frac{1}{2x+\sqrt{4x^2-1}}\times\left(2+\frac{4x}{\sqrt{4x^2-1}}\right)=\frac{2x+\sqrt{4x^2-1}}{\sqrt{4x^2-1}\left(2x+\sqrt{4x^2-1}\right)}=\frac{1}{\sqrt{4x^2-1}}$$

Alternative 2:

$$y=\frac{1}{2}\operatorname{ar}\cosh 2x\Rightarrow 2y=\operatorname{ar}\cosh 2x\Rightarrow\cosh 2y=2x\rightarrow 4\sinh 2y\frac{\mathrm{d}y}{\mathrm{d}x}=2x\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\sinh 2y}=\frac{1}{\sqrt{4x^2-1}}$$

If either approach is taken then the same condition for the form of the derivative applies.

Note that this differentiation may be seen in an attempt by parts of $\int y\,\mathrm{d}x$

M1: Attempts to find $1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ using their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and attempts common denominator.

A1: Reaches a correct simplified integral with $\sqrt{x^2}$ replaced with x as shown in the scheme.

Allow equivalent forms e.g. $2\int x\sqrt{\frac{1}{4x^2-1}}\mathrm{d}x$, $\frac{1}{2}\int\frac{4x}{\sqrt{(2x)^2-1}}\mathrm{d}x$

This may be implied by subsequent work.

M1: Attempts the integration and reaches the form $\alpha\left(\beta x^2-1\right)^{\frac{1}{2}}$. α and/or β may be 1

This may be implied by e.g.

$$u=4x^2-1\to k\int\frac{1}{\sqrt{u}}\,\mathrm{d}u=\alpha\sqrt{u}\ \text{ or }\ u=x^2\to k\int\frac{1}{\sqrt{4u-1}}\,\mathrm{d}u=\alpha\sqrt{4u-1}$$

dM1: Applies the limits to their integral. **Depends on the previous 2 method marks.**
Any attempts at substitution requires use of changed limits e.g.

$$u=4x^2-1\to\frac{1}{4}\int\frac{1}{\sqrt{u}}\,\mathrm{d}u\to\frac{1}{2}\left[\sqrt{u}\right]_{48}^{675}=\ldots$$

A1: cao Accept equivalents in the correct form, such as $\frac{1}{2}\sqrt{363}$

<u>Examples of alternative for the final 3 marks:</u>

$$x=\frac{1}{2}\cosh u\Rightarrow 2\int\frac{x}{\sqrt{4x^2-1}}\,\mathrm{d}x=\int\frac{\cosh u}{\sqrt{\cosh^2 u-1}}\frac{1}{2}\sinh u\,\mathrm{d}u$$

$$\int\frac{1}{2}\cosh u\,\mathrm{d}u=\frac{1}{2}\left[\sinh u\right]_{\operatorname{arcosh}7}^{\operatorname{arcosh}26}=\frac{1}{2}\left(\frac{\mathrm{e}^{\ln\left(26+15\sqrt{3}\right)}-\mathrm{e}^{-\ln\left(26+15\sqrt{3}\right)}}{2}-\frac{\mathrm{e}^{\ln\left(7+4\sqrt{3}\right)}-\mathrm{e}^{-\ln\left(7+4\sqrt{3}\right)}}{2}\right)$$

$$=\frac{1}{2}\left(15\sqrt{3}-4\sqrt{3}\right)=\frac{11}{2}\sqrt{3}$$

Score M1 for a complete method for the substitution leading to $k\sinh u$ and then dM1 for applying changed limits (or reverts back to x) and A1 as above

$$x=\frac{1}{2}\sec u\Rightarrow 2\int\frac{x}{\sqrt{4x^2-1}}\,\mathrm{d}x=\int\frac{\sec u}{\sqrt{\sec^2 u-1}}\frac{1}{2}\sec u\tan u\,\mathrm{d}u$$

$$\int\frac{1}{2}\sec^2 u\,\mathrm{d}u=\frac{1}{2}\left[\tan u\right]_{\operatorname{arcosh}\frac{1}{7}}^{\operatorname{arcosh}\frac{1}{26}}$$

$$=\frac{1}{2}\left(15\sqrt{3}-4\sqrt{3}\right)=\frac{11}{2}\sqrt{3}$$

Score M1 for a complete method for the substitution leading to $k\tan u$ and then dM1 for applying changed limits (or reverts back to x) and A1 as above

Special Case if no integration is attempted:
Note that if candidates do not attempt the integration but obtain the correct exact answer then a special case of **M1M1A1M0A0A1** (4/6) should be awarded.

Question	Scheme	Marks
2.	$\cosh y = x, y < 0 \Rightarrow y = \ln\left[x - \sqrt{x^2 - 1}\right]$	
	$\cosh y = x \Rightarrow x = \dfrac{e^y + e^{-y}}{2}$	**B1**
	$\Rightarrow 2xe^y = e^{2y} + 1$	**M1**
	$\Rightarrow e^{2y} - 2xe^y + 1 = 0 \Rightarrow e^y = \dfrac{2x \pm \sqrt{(2x)^2 - 4\times1\times1}}{2}$ or $\Rightarrow e^{2y} - 2xe^y + 1 = 0 \Rightarrow (e^y - x)^2 + 1 - x^2 = 0 \Rightarrow e^y = \ldots$	**M1**
	$= x \pm \sqrt{x^2 - 1}$	**A1**
	So $y = \ln\left[x - \sqrt{x^2 - 1}\right]$ *	**A1***
	since $y < 0 \Rightarrow e^y < 1$ so need $x - \sqrt{x^2 - 1}$ (as x >1 so must subtract)	**B1**
		(6)
		(6 marks)

Notes:

B1: Correct statement for x in terms of exponentials. $\cosh y = \dfrac{e^x + e^{-x}}{2}$ scores B0.

M1: Multiplies through by e^y to achieve a quadratic in e^y. (Terms need not be gathered.)

M1: Uses the quadratic formula or other valid method (e.g. completing the square) to solve for e^y.

A1: Correct solution(s) for e^y. Accept if only the negative one is given. Accept $\dfrac{2x \pm \sqrt{4x^2 - 4}}{2}$

A1*: Completely correct work leading to the given answer regardless of the justification why the negative root is taken (correct or incorrect). Must be no errors seen.

B1: Suitable justification for taking the negative root given.

E.g. $y < 0$ so $y = \ln\left[x - \sqrt{x^2 - 1}\right]$. Condone $x \pm \sqrt{x^2 - 1} < 1$ so $y = \ln\left[x - \sqrt{x^2 - 1}\right]$.

Note that the B1 can only be awarded if all previous marks have been awarded.

But the reason may be given before or after ln has been taken.

E.g. $(e^y - x)^2 + 1 - x^2 = 0 \Rightarrow e^y - x = \pm\sqrt{x^2 - 1}$ but $y < 0$ so $e^y - x = -\sqrt{x^2 - 1}$

<u>Working backwards:</u>

$$y = \ln\left[x - \sqrt{x^2 - 1}\right] \Rightarrow e^y = x - \sqrt{x^2 - 1}\,(\text{B1}) \Rightarrow e^y + e^{-y} = x - \sqrt{x^2 - 1} + \frac{1}{x - \sqrt{x^2 - 1}}\,(\text{M1})$$

$$x - \sqrt{x^2 - 1} + \frac{1}{x - \sqrt{x^2 - 1}} = \frac{2x\left(x - \sqrt{x^2 - 1}\right)}{x - \sqrt{x^2 - 1}}\,(\text{M1}) = 2x\,(\text{A1}) \Rightarrow x = \frac{e^y + e^{-y}}{2} = \cosh y\,(\text{A1})$$

Final B1 unlikely to be available.

Question	Scheme	Marks
3(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{6\cos\theta}{-8\sin\theta}$ or $\frac{2x}{64}+\frac{2y}{36}\frac{\mathrm{d}y}{\mathrm{d}x}=0$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{4}\times\frac{1}{2}\left(576-9x^2\right)^{-\frac{1}{2}}\times -18x$	B1
	$m_T=-\frac{3\cos\theta}{4\sin\theta}\Rightarrow m_N=-\frac{1}{m_T}=\frac{4\sin\theta}{3\cos\theta}$	M1
	So normal is $y-6\sin\theta=\frac{4\sin\theta}{3\cos\theta}\left(x-8\cos\theta\right)$ or $y=\frac{4\sin\theta}{3\cos\theta}x+c,\ c=6\sin\theta-\frac{4\sin\theta}{3\cos\theta}\times 8\cos\theta$	dM1
	$\Rightarrow 3y\cos\theta-18\sin\theta\cos\theta=4x\sin\theta-32\sin\theta\cos\theta$ $\Rightarrow 4x\sin\theta-3y\cos\theta=14\sin\theta\cos\theta$ *	A1*
		(4)
(b)	A is $\left(\frac{7}{2}\cos\theta, 0\right)$ and B is $\left(0, -\frac{14}{3}\sin\theta\right)$	B1
	M is $\left(\frac{\frac{7}{2}\cos\theta}{2}, -\frac{\frac{14}{3}\sin\theta}{2}\right)=\left(\frac{7}{4}\cos\theta, -\frac{7}{3}\sin\theta\right)$	M1
	$\sin^2\theta+\cos^2\theta=1\Rightarrow\left(-\frac{3}{7}y\right)^2+\left(\frac{4}{7}x\right)^2=1$	dM1 A1
	$\Rightarrow 16x^2+9y^2=49$	A1
		(5)
		(9 marks)

Notes:

(a)

B1: A correct statement for, or involving, $\frac{\mathrm{d}y}{\mathrm{d}x}$. See examples in scheme for parametric, implicit and direct forms.

M1: Finds $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ and applies the perpendicular condition to find gradient of the normal.

dM1: Uses their normal gradient and P to find the equation of the normal

A1*: Correct answer from correct work with at least one intermediate step and no errors seen.

(b)

B1: Correct coordinates for A and B or correct intercepts of l seen or implied by working. Allow in any form simplified or unsimplified.

M1: Uses their A and B to attempt the midpoint, M. May be implied by at least one correct coordinate.

dM1: Uses $\sin^2\theta+\cos^2\theta=1$ with their M to form an equation in x and y only.

Depends on the previous mark.

A1: A correct unsimplified equation.

A1: Correct equation in the required form. Allow any integer multiple.

Special Case: If M is found as e.g. $\left(\frac{7}{4}\cos\theta, \frac{7}{3}\sin\theta\right)$ withhold the final mark only if otherwise correct.

Question	Scheme	Marks
4(a)	$\begin{vmatrix} 2 & 0 & -1 \\ k & 3 & 2 \\ -2 & 1 & k \end{vmatrix} = 2\begin{vmatrix} 3 & 2 \\ 1 & k \end{vmatrix} - 0\begin{vmatrix} k & 2 \\ -2 & k \end{vmatrix} + (-1)\begin{vmatrix} k & 3 \\ -2 & 1 \end{vmatrix} = 2(3k-2)-(k+6) = \ldots$	**M1**
	$= 6k-4-k-6 = 5k-10*$	**A1***
		(2)
(b)	$\mathbf{M}^T = \begin{pmatrix} 2 & k & -2 \\ 0 & 3 & 1 \\ -1 & 2 & k \end{pmatrix}$ or minors $\begin{pmatrix} 3k-2 & k^2+4 & k+6 \\ 1 & 2k-2 & 2 \\ 3 & 4+k & 6 \end{pmatrix}$ or cofactors $\begin{pmatrix} 3k-2 & -k^2-4 & k+6 \\ -1 & 2k-2 & -2 \\ 3 & -4-k & 6 \end{pmatrix}$	**M1**
	Adjugate matrix is $\begin{pmatrix} 3k-2 & -1 & 3 \\ -k^2-4 & 2k-2 & -4-k \\ k+6 & -2 & 6 \end{pmatrix}$ (≥ 6 entries correct)	**M1**
	Hence $\mathbf{M}^{-1} = \dfrac{1}{5k-10}\begin{pmatrix} 3k-2 & -1 & 3 \\ -k^2-4 & 2k-2 & -4-k \\ k+6 & -2 & 6 \end{pmatrix}$	**dM1A1**
		(4)
(c)	Images of A, B and C are $(5,4k-18,3k-16)$, $(0,7-2k,9-4k)$ and $(0,4k-2,8k-14)$	**M1** **A1**
	$(\pm)50 = \dfrac{1}{6}\begin{vmatrix} 5 & 4k-18 & 3k-16 \\ 0 & 7-2k & 9-4k \\ 0 & 4k-2 & 8k-14 \end{vmatrix} \Rightarrow (\pm)300 = 5(\ldots)(=200k-400) \Rightarrow k = \ldots$	**M1**
	$(300 = 200k-400 \Rightarrow)\, k = \dfrac{7}{2}$ or $(-300 = 200k-400 \Rightarrow)\, k = \dfrac{1}{2}$	**A1**
	$k = \dfrac{1}{2}$ and $k = \dfrac{7}{2}$	**A1**
		(5)
Alt method	Using volume scale factor. Attempts $\mathbf{a}.(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} 4 & -8 & 3 \\ -2 & 5 & -4 \\ 4 & -6 & 8 \end{vmatrix} = 4(40-24)+8(-16+16)+3(12-20) = \ldots$	**M1**
	Volume of T is $\dfrac{1}{6}\lvert\mathbf{a}.(\mathbf{b}\times\mathbf{c})\rvert = \left\lvert\dfrac{1}{6}\begin{vmatrix} 4 & -8 & 3 \\ -2 & 5 & -3 \\ 4 & 6 & -8 \end{vmatrix}\right\rvert = \ldots\ \dfrac{20}{3}$	**A1**
	Volume image of $T = \lvert\det\mathbf{M}\rvert \times \dfrac{20}{3} \Rightarrow \dfrac{20}{3}\lvert 5k-10\rvert = 50 \Rightarrow k = \ldots$	**M1**

	$\left(\frac{20}{3}(5k-10)=50 \Rightarrow\right) k=\frac{7}{2}$ or $\left(\frac{20}{3}(10-5k)=50 \Rightarrow\right) k=\frac{1}{2}$	**A1**
	$k=\frac{1}{2}$ and $k=\frac{7}{2}$	**A1**
		(5)
		(11 marks)

Notes:

(a)

M1: Correct method for expanding the determinant to reach a linear expression in k. Expect expansion along the top row, but may expand along any row or column. Sarrus gives $6 + k - (6 + 4)$.

A1*: Correct expression from correct work.

(b)

M1: Begins the process of finding the inverse by attempting either the transpose, or the matrix of minors or cofactors. Look for at least 6 correct entries.

M1: Proceeds to find the adjugate matrix (may include the reciprocal determinant). Again look for 6 correct entries.

dM1: Full method to find the inverse matrix, so divides their adjugate by the determinant.

Depends on both previous marks.

A1: Fully correct inverse.

(c)

M1: Attempts to find the image vectors of A, B and C under the transformation. (O mapping to O may be assumed). May be implied by at least two correct entries in one of the three vectors – but must be finding all three.

A1: Correct image vectors. Allow unsimplified and isw if necessary.

M1: Use their image vectors in a suitable scalar triple product to find the volume, and set volume equal to 50 and attempts to solve for k. Must include the 1/6 but may appear later.

$$\text{Usually } \frac{1}{6}(200k-400)=50 \text{ leading to } k=\frac{7}{2}$$

A1: One correct value for k obtained, either $k=\frac{7}{2}$ or $k=\frac{1}{2}$

A1: Both values of k correctly found. $k=\frac{7}{2}$ and $k=\frac{1}{2}$

Alt method using determinant as volume scale factor.

M1: Attempts an appropriate scalar triple product. May have rows in different order.

A1: Correct volume for tetrahedron T. Need not be simplified, so $\frac{40}{6}$ is fine here.

M**1:** Uses the determinant as the volume scale factor to set up at least one equation in k using their volume and the given volume and attempts to solve for k. The 1/6 may have been missing.

$$\text{Usually } \frac{20}{3}(5k-10)=50 \text{ leading to } k=\frac{7}{2}$$

A1: One correct value for k obtained, either $k=\frac{7}{2}$ or $k=\frac{1}{2}$

A1: Both values of k correctly found. $k=\frac{7}{2}$ and $k=\frac{1}{2}$

Question	Scheme	Marks
5(a)	$(5\mathbf{i}+\mathbf{j})\times(8\mathbf{i}-2\mathbf{j}+3\mathbf{k})=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 5 & 1 & 0\\ 8 & -2 & 3\end{vmatrix}=\ldots$ Or $\left.\begin{array}{r}(u\mathbf{i}+v\mathbf{j}+w\mathbf{k}).(5\mathbf{i}+\mathbf{j})=0\\ (u\mathbf{i}+v\mathbf{j}+w\mathbf{k}).(8\mathbf{i}-2\mathbf{j}+3\mathbf{k})=0\end{array}\right\}\Rightarrow\begin{array}{c}5u+v=0\\ 8u-2v+3w=0\end{array}\Rightarrow u,v,w=\ldots$	**M1**
	$\mathbf{n}=3\mathbf{i}-15\mathbf{j}-18\mathbf{k}$ or $\alpha(\mathbf{i}-5\mathbf{j}-6\mathbf{k})$ for any $\alpha\neq 0$	**A1**
		(2)
(b)	(i) $\mathbf{r}=(2\mathbf{i}-4\mathbf{j}+4\mathbf{k})+s(8\mathbf{i}-2\mathbf{j}+3\mathbf{k})+t(5\mathbf{i}+\mathbf{j})$	**B1**
		(1)
	(ii) $(2\mathbf{i}-4\mathbf{j}+4\mathbf{k}).(3\mathbf{i}-15\mathbf{j}-18\mathbf{k})=\ldots(=-6)$	**M1**
	So $\mathbf{r}.(3\mathbf{i}-15\mathbf{j}-18\mathbf{k})=-6$ oe such as $\mathbf{r}.(-\mathbf{i}+5\mathbf{j}+6\mathbf{k})=2$	**A1**
		(2)
(c) **Way 1**	Distance from plane in (b) to origin is $\dfrac{\pm 6}{\sqrt{3^2+15^2+18^2}}$ oe e.g. $\dfrac{2}{\sqrt{1^2+5^2+6^2}}$ Or attempts similar for parallel plane containing l_1, e.g. $\dfrac{(\mathbf{i}+2\mathbf{j}-5\mathbf{k}).(3\mathbf{i}-15\mathbf{j}-18\mathbf{k})}{\sqrt{3^2+15^2+18^2}}=\ldots$	**M1**
	$=\pm\dfrac{2}{\sqrt{62}}$ (oe evaluated) or $\mp\dfrac{21}{\sqrt{62}}$ if considering other plane.	**A1**
	Both $\dfrac{\pm 6}{\sqrt{3^2+15^2+18^2}}$ oe and $\dfrac{(\mathbf{i}+2\mathbf{j}-5\mathbf{k}).(3\mathbf{i}-15\mathbf{j}-18\mathbf{k})}{\sqrt{3^2+15^2+18^2}}=\ldots$ attempted	**M1**
	Hence shortest distance between lines is $\dfrac{2}{\sqrt{62}}+\dfrac{21}{\sqrt{62}}=\ldots$	**M1**
	$=\dfrac{23}{\sqrt{62}}$ or $\dfrac{23\sqrt{62}}{62}$	**A1**
		(5)
Way 2	$\overrightarrow{AB}=\pm\big((\mathbf{i}+2\mathbf{j}-5\mathbf{k})-(2\mathbf{i}-4\mathbf{j}+4\mathbf{k})\big)=\pm(-\mathbf{i}+6\mathbf{j}-9\mathbf{k})$	**M1 A1**
	$d=AB\cos\theta=\dfrac{\overrightarrow{AB}.\mathbf{n}}{\lvert\mathbf{n}\rvert}=\dfrac{\pm(-\mathbf{i}+6\mathbf{j}-9\mathbf{k}).(3\mathbf{i}-15\mathbf{j}-18\mathbf{k})}{\sqrt{3^2+15^2+18^2}}$ oe	**M1**
	$=\dfrac{\pm(-3-90+162)}{\sqrt{558}}=\dfrac{\pm 69}{\sqrt{558}}=\ldots$	**M1**
	$=\dfrac{23}{\sqrt{62}}$ or $\dfrac{23\sqrt{62}}{62}$	**A1**
		(5)

Way 3	$\left(2\mathbf{i}-4\mathbf{j}+4\mathbf{k}\right)+\mu\left(8\mathbf{i}-2\mathbf{j}+3\mathbf{k}\right)-\left(\left(\mathbf{i}+2\mathbf{j}-5\mathbf{k}\right)+\lambda\left(5\mathbf{i}+\mathbf{j}\right)\right)$ $=\left(1+8\mu-5\lambda\right)\mathbf{i}+\left(-6-2\mu-\lambda\right)\mathbf{j}+\left(9+3\mu\right)\mathbf{k}$	**M1 A1**
	$\left(\left(1+8\mu-5\lambda\right)\mathbf{i}+\left(-6-2\mu-\lambda\right)\mathbf{j}+\left(9+3\mu\right)\mathbf{k}\right).\left(5\mathbf{i}+\mathbf{j}\right)=0$ $\Rightarrow 38\mu-26\lambda=1$ $\left(\left(1+8\mu-5\lambda\right)\mathbf{i}+\left(-6-2\mu-\lambda\right)\mathbf{j}+\left(9+3\mu\right)\mathbf{k}\right).\left(8\mathbf{i}-2\mathbf{j}+3\mathbf{k}\right)=0$ $\Rightarrow 77\mu-38\lambda=-47$ $\Rightarrow \lambda=-\frac{207}{62},\ \mu=-\frac{70}{31}$	**M1**
	$\left(2\mathbf{i}-4\mathbf{j}+4\mathbf{k}\right)+\mu\left(8\mathbf{i}-2\mathbf{j}+3\mathbf{k}\right)-\left(\left(\mathbf{i}+2\mathbf{j}-5\mathbf{k}\right)+\lambda\left(5\mathbf{i}+\mathbf{j}\right)\right)$ $=-\frac{23}{62}\mathbf{i}+\frac{115}{62}\mathbf{j}+\frac{69}{31}\mathbf{k}$ $d=\sqrt{\left(\frac{23}{62}\right)^2+\left(\frac{115}{62}\right)^2+\left(\frac{69}{31}\right)^2}$	**M1**
	$=\frac{23}{\sqrt{62}}$ or $\frac{23\sqrt{62}}{62}$	**A1**
		(5)
		(10 marks)

Notes:

Accept equivalent vector notation, e.g. column vectors, throughout.

(a)

M1: Any correct method to find a vector perpendicular to the two direction vectors of the lines. Look for the cross product between the two direction vectors, but may use dot products and solving equations. In the latter case the method should lead to values for u, v and w.

For the vector product, if no method is shown look for at least 2 correct components.

A1: Any correct vector, a scalar multiple of $-\mathbf{i}+5\mathbf{j}+6\mathbf{k}$

(b)

B1: Any correct equation. Must have **r** = ... or e.g. $\begin{pmatrix} x \\ y \\ z \end{pmatrix}=...$

M1: Uses their normal vector from (a) with any point on the plane (probably $\left(2\mathbf{i}-4\mathbf{j}+4\mathbf{k}\right)$ to find p

Condone slips with the calculation so $\left(2\mathbf{i}-4\mathbf{j}+4\mathbf{k}\right).\left(3\mathbf{i}-15\mathbf{j}-18\mathbf{k}\right)$ evaluated as a scalar is sufficient for M1. May also be implied by $p=-6$

A1: Any correct equation of the correct form.

(c)

Way 1

M1: Uses the plane equation from (b) (or otherwise) OR the parallel plane containing l_1 to find the distance of one of these planes to the origin.

A1: Correct distance between one of the planes and the origin, accept ± here.

M1: Attempts distance of both the parallel planes containing l_1 and l_2 from the origin.

M1: Correct method for finding the distance between lines – i.e. subtracts their distances either way round.

A1: Correct answer. Accept $\frac{23}{\sqrt{62}}$ or $\frac{23\sqrt{62}}{62}$

Way 2

M1: Subtracts position vectors of points on the lines (either way around). Implied by two correct coordinates if method not shown. (Forms suitable hypotenuse.)

A1: Correct vector or as coordinates, either direction.

M1: Correct formula for the distance using their vectors, $d = AB\cos\theta = \frac{\overrightarrow{AB}.\mathbf{n}}{|\mathbf{n}|}$ with their $\overrightarrow{AB}$ and **n**.

M1: Complete evaluation of the formula.

A1: Correct answer. Accept $\frac{23}{\sqrt{62}}$ or $\frac{23\sqrt{62}}{62}$ but must be positive.

Way 3

M1: Subtracts position vectors of general points on each line (either way around). Implied by two correct coordinates if method not shown.

A1: Correct vector or as coordinates, either direction.

M1: Forms scalar product of the general vector with both direction vectors, sets = 0 and solves simultaneously

M1: Substitutes the values of their parameters back into the general vector and attempts its magnitude

A1: Correct answer. Accept $\frac{23}{\sqrt{62}}$ or $\frac{23\sqrt{62}}{62}$ but must be positive.

Question	Scheme	Marks
6(a) **Way 1**	$I_n=\int_0^{\sqrt{\frac{\pi}{2}}} x^{n-1}.x\cos\left(x^2\right)\,\mathrm{d}x=\left[x^{n-1}.\frac{1}{2}\sin\left(x^2\right)\right]_0^{\sqrt{\frac{\pi}{2}}}-\int_0^{\sqrt{\frac{\pi}{2}}}(n-1)x^{n-2}.\frac{1}{2}\sin\left(x^2\right)\,\mathrm{d}x$	**M1A1**
	$=\left[x^{n-1}.\frac{1}{2}\sin\left(x^2\right)\right]_0^{\sqrt{\frac{\pi}{2}}}-\frac{1}{2}(n-1)\int_0^{\sqrt{\frac{\pi}{2}}}x^{n-3}.x\sin\left(x^2\right)\,\mathrm{d}x$ $=\left[x^{n-1}.\frac{1}{2}\sin\left(x^2\right)\right]_0^{\sqrt{\frac{\pi}{2}}}-\frac{1}{2}(n-1)\left(\underline{\left[x^{n-3}.-\frac{1}{2}\cos\left(x^2\right)\right]_0^{\sqrt{\frac{\pi}{2}}}-\int_0^{\sqrt{\frac{\pi}{2}}}(n-3)x^{n-4}.-\frac{1}{2}\cos\left(x^2\right)\,\mathrm{d}x}\right)$	**<u>dM1A1</u>**
	$=\left(\frac{1}{2}\left(\sqrt{\frac{\pi}{2}}\right)^{n-1}\sin\frac{\pi}{2}-0\right)-\frac{1}{2}(n-1)\left[(0-0)+\frac{1}{2}(n-3)I_{n-4}\right]$	**dM1**
	$=\frac{1}{2}\left(\frac{\pi}{2}\right)^{\frac{n-1}{2}}-\frac{1}{4}(n-1)(n-3)I_{n-4}$ *	**A1***
		(6)
Way 2	$I_n=\left[\frac{x^{n+1}}{n+1}.\cos\left(x^2\right)\right]_0^{\sqrt{\frac{\pi}{2}}}-\int_0^{\sqrt{\frac{\pi}{2}}}\frac{x^{n+1}}{n+1}.-2x\sin\left(x^2\right)\,\mathrm{d}x$	**M1A1**
	$=\left[\frac{x^{n+1}}{n+1}.\cos\left(x^2\right)\right]_0^{\sqrt{\frac{\pi}{2}}}+\frac{2}{n+1}\int_0^{\sqrt{\frac{\pi}{2}}}x^{n+2}\sin\left(x^2\right)\,\mathrm{d}x$ $=\left[\frac{x^{n+1}}{n+1}.\cos\left(x^2\right)\right]_0^{\sqrt{\frac{\pi}{2}}}+\frac{2}{n+1}\left(\underline{\left[\frac{x^{n+3}}{n+3}.\sin\left(x^2\right)\right]_0^{\sqrt{\frac{\pi}{2}}}-\int_0^{\sqrt{\frac{\pi}{2}}}\frac{x^{n+3}}{n+3}.2x\cos\left(x^2\right)\,\mathrm{d}x}\right)$	**<u>dM1A1</u>**
	$=(0-0)+\frac{2}{n+1}\left(\frac{1}{n+3}\left(\sqrt{\frac{\pi}{2}}\right)^{n+3}\sin\frac{\pi}{2}-0-\frac{2}{n+3}I_{n+4}\right)$	**dM1**
	$\Rightarrow I_{n+4}=\frac{1}{2}\left(\frac{\pi}{2}\right)^{\frac{n+3}{2}}-\frac{1}{4}(n+1)(n+3)I_n$ so replacing n by $n-4$ gives $I_n=\frac{1}{2}\left(\frac{\pi}{2}\right)^{\frac{n-1}{2}}-\frac{1}{4}(n-1)(n-3)I_{n-4}$ *	**A1***
		(6)
(b)	$I_1=\int_0^{\sqrt{\frac{\pi}{2}}}x\cos\left(x^2\right)\,\mathrm{d}x=\left[\frac{1}{2}\sin\left(x^2\right)\right]_0^{\sqrt{\frac{\pi}{2}}}=\frac{1}{2}$	**B1**
	$I_5=\frac{1}{2}\left(\frac{\pi}{2}\right)^{\frac{5-1}{2}}-\frac{1}{4}(5-1)(5-3)\times"\frac{1}{2}"$	**M1**
	$=\frac{\pi^2}{8}-1$ oe e.g. $\frac{\pi^2-8}{8},\ \frac{1}{2}\left(\frac{\pi}{2}\right)^2-1$	**A1**
		(3)

(9 marks)

Notes:

(a) Way 1

M1: Applies integration by parts in the correct direction having made the 'split' and obtains:

$$\left[\pm\alpha x^{n-1}\sin\left(x^2\right)\right] \pm\beta\int x^{n-2}.\sin\left(x^2\right)\,\mathrm{d}x$$

A1: Fully correct expression

dM1: Applies integration by parts in the correct direction to $\beta\int x^{n-2}.\sin\left(x^2\right)\,\mathrm{d}x$ and obtains:

$$\left[\pm\alpha x^{n-3}\cos\left(x^2\right)\right] \pm\beta\int x^{n-4}\cos\left(x^2\right)\,\mathrm{d}x$$

Depends on the previous M mark.

A1: Correct second application of parts e.g.

$$\int x^{n-2}.\sin\left(x^2\right)\,\mathrm{d}x = \left[x^{n-3}.-\frac{1}{2}\cos\left(x^2\right)\right] - \int (n-3)x^{n-4}.-\frac{1}{2}\cos\left(x^2\right)\,\mathrm{d}x$$

dM1: Applies the limits completely to their result and replaces final integral by I_{n-4}. The substitution of limits may have been carried out in stages throughout the work, or may be applied after integration by parts twice has been carried out. **Depends on both previous M marks. There must some explicit evidence that the limits have been applied but this may be taken from either the** $\left[x^{n-1}.\frac{1}{2}\sin\left(x^2\right)\right]_0^{\sqrt{\frac{\pi}{2}}}$ = e.g. $\sqrt{\frac{\pi}{2}}^{\,n-1}.\frac{1}{2}\sin\left(\sqrt{\frac{\pi}{2}}^{\,2}\right)$, $\sqrt{\frac{\pi}{2}}^{\,n-1}.\frac{1}{2}$, $\frac{1}{2}\left(\frac{\pi}{2}\right)^{\frac{n-1}{2}} - 0$

or $\left[x^{n-3}.-\frac{1}{2}\cos\left(x^2\right)\right]_0^{\sqrt{\frac{\pi}{2}}}$ = e.g. $0-0$, 0

A1*: Achieves the printed answer from completely correct work with no errors seen and evidence of the given limits being applied.

Way 2

M1: Applies integration by parts in the correct direction and obtains:

$$\left[\pm\alpha x^{n+1}\cos\left(x^2\right)\right] \pm\beta\int x^{n+1}.x\sin\left(x^2\right)\,\mathrm{d}x$$

A1: Fully correct expression

dM1: Applies integration by parts in the correct direction to $\beta\int x^{n+1}.x\sin\left(x^2\right)\,\mathrm{d}x$ and obtains:

$$\left[\pm\alpha x^{n+3}\sin\left(x^2\right)\right] \pm\beta\int x^{n+3}.x\cos\left(x^2\right)\,\mathrm{d}x$$

Depends on the previous M mark.

A1: Correct second application of parts e.g.

$$\int x^{n+2}.\sin\left(x^2\right)\,\mathrm{d}x = \left[\frac{x^{n+3}}{n+3}.\sin\left(x^2\right)\right] - \int \frac{x^{n+3}}{n+3}.2x\cos\left(x^2\right)\,\mathrm{d}x$$

dM1: Applies the limits completely to their result and replaces final integral by I_{n+4}. The substitution of limits may have been carried out in stages throughout the work, or may be applied after integration by parts twice has been carried out. **Depends on both previous M marks. There must some explicit evidence that the limits have been applied but this may be taken from either the** $\left[\frac{x^{n+1}}{n+1}.\cos\left(x^2\right)\right]_0^{\sqrt{\frac{\pi}{2}}} =$ e.g. $0-0, 0$ or

$$\left[\frac{x^{n+3}}{n+3}.\sin\left(x^2\right)\right]_0^{\sqrt{\frac{\pi}{2}}} = \text{e.g.} \frac{\sqrt{\frac{\pi}{2}}^{n+3}}{n+3}.\sin\left(\sqrt{\frac{\pi}{2}}^2\right), \frac{\sqrt{\frac{\pi}{2}}^{n+3}}{n+3}.\sin\left(\frac{\pi}{2}\right), \frac{\sqrt{\frac{\pi}{2}}^{n+3}}{n+3}.(1), \frac{\left(\frac{\pi}{2}\right)^{\frac{n+3}{2}}}{n+3}-0$$

A1*: Achieves the printed answer from completely correct work with no errors and evidence of the given limits being applied with a clear statement that n is replaced by $n-4$

(b)

B1: Correct I_1. May be seen after attempting the reduction.

M1: Applies the reduction formula with their I_1 and $n = 5$ to reach a value. Condone slips with evaluating $\frac{1}{4}(n-1)(n-3)$ as long as the intention is clear.

A1: Correct answer.

Note: Beware incorrect work in (a) leading to what appears to be a correct form e.g.

$$I_n = \int_0^{\sqrt{\frac{\pi}{2}}} x^n \cos\left(x^2\right) \mathrm{d}x = \left[x^n.\frac{\sin\left(x^2\right)}{2x}\right]_0^{\sqrt{\frac{\pi}{2}}} - \int_0^{\sqrt{\frac{\pi}{2}}} nx^{n-1}.\frac{\sin\left(x^2\right)}{2x}\,\mathrm{d}x$$

This scores M0 at the start and hence will usually score no marks in part (a)

Question	Scheme	Marks
7(a)	$b^2 = a^2\left(e^2-1\right) \Rightarrow e^2 = \frac{25}{a^2}+1 = \frac{25+a^2}{a^2}$ oe	B1
		(1)
(b)	$x = (\pm)\frac{a}{e}$ $\quad\quad$ $\frac{x}{a} = (\pm)\frac{y}{5}$	B1
	$\frac{a}{e}\times\frac{1}{a} = \pm\frac{y}{5} \Rightarrow y = \pm\frac{5}{e} \Rightarrow AA' = 2\times\frac{5}{e}$ or $\frac{5}{e}-\left(-\frac{5}{e}\right)$	M1
	$= \frac{10}{e}$	A1
		(3)
(c)	$\frac{1}{2}\times\frac{"10"}{e}\times\left(ae+\frac{a}{e}\right)$ or e.g. $\frac{1}{2}\times\frac{10a}{\sqrt{25+a^2}}\times\left(\sqrt{25+a^2}+\frac{a^2}{\sqrt{25+a^2}}\right)$	M1
	$\frac{1}{2}\frac{10}{e}\left(ae+\frac{a}{e}\right) = \frac{164}{3} \Rightarrow 15\left(a+\frac{a}{e^2}\right) = 164$ or $\frac{1}{2}\times\frac{10a}{\sqrt{25+a^2}}\times\left(\sqrt{25+a^2}+\frac{a^2}{\sqrt{25+a^2}}\right) = \frac{164}{3}$	M1
	$\Rightarrow 15a\left(1+\frac{a^2}{25+a^2}\right) = 164$	A1 (M1 on EPEN)
	$\Rightarrow 15a\left(\frac{25+2a^2}{25+a^2}\right) = 164 \Rightarrow 375a+30a^3 = 164\left(25+a^2\right)$ $\Rightarrow 30a^3-164a^2+375a-4100 = 0$*	A1*
		(4)
(d)	$30a^3-164a^2+375a-4100 = (3a-20)\left(10a^2+12a+205\right)$	B1 (M1 on EPEN)
	$12^2-4(10)(205) = \ldots$ $10a^2+12a+205 = 10\left(\left(a+\frac{12}{20}\right)^2-\frac{144}{400}\right)+205$	M1
	E.g. $12^2-4(10)(205)<0$ so there are no other roots of the equation. Hence $a = \frac{20}{3}$ is only possible value.	A1
		(3)
		(11 marks)

Notes:

(a)
B1: Correct expression.

(b)
B1: Identifies at least one correct equation for a directrix and at least one asymptote, stated or used – including the $b = 5$.
M1: Solves to find y coordinates of A and A' or just one of these and doubles to get length. Allow if b is used rather than 5.
A1: Correct length (from subtracting or doubling). Must be positive.
(c)
M1: Uses focus $(-ae, 0)$ and directrix $x = \frac{a}{e}$ (allow if the alternative pair is used) with their length from (b), to form a **correct or correct ft** expression for the area of triangle AFA'.
M1: Sets their area equation equal to $\frac{164}{3}$ to obtain an equation in e^2 and a.

Their attempt at the area must be of the form $\frac{1}{2}\times\frac{"10"}{e}\times\pm\left(ae\pm\frac{a}{e}\right)$

Alternatively, allow an equation in just a^2 if $e=\sqrt{\frac{25+a^2}{a^2}}$ is substituted first.

A1(M1 on EPEN): Correct equation in terms of a only. Allow any correct form.
A1*: Correct result achieved with no errors seen and sufficient working shown.
(d)
B1(M1 on EPEN): A correct method for showing that $a = \frac{20}{3}$ is a solution of the equation.

Examples:

$$30a^3-164a^2+375a-4100=(3a-20)(10a^2+12a+205)$$

$$30a^3-164a^2+375a-4100=\left(a-\frac{20}{3}\right)(30a^2+36a+615)$$

$$\mathrm{f}\left(\frac{20}{3}\right)=\frac{80000}{9}-\frac{65600}{9}+2500-4100=0$$

Or e.g. long division and obtains correct quotient and no remainder

M1: A correct method for showing there are no other roots. May use completing the square (as in scheme) or attempt discriminant or differentiation,

e.g. $\frac{\mathrm{d}}{\mathrm{d}a}(eqn)=90a^2-328a+375=90\left(a-\frac{82}{45}\right)^2+\frac{3427}{45}>0$ so strictly increasing hence only one solution.

If using discriminant then values must be used i.e. not just $b^2 - 4ac < 0$
An attempt at the discriminant may be seen as part of the quadratic formula e.g.

$$a=\frac{-12\pm\sqrt{12^2-4(10)(205)}}{2(10)}$$

A1: All work correct with **reason** and **conclusion** made that $a=\frac{20}{3}$ is the only possible value. If the discriminant is evaluated then it must be correct. For reference $12^2-4(10)(205)=-8056$ and $36^2-4(30)(615)=-72504$ but note that e.g. $12^2-4(10)(205)<0$ with a conclusion is acceptable.

Note that just using a calculator to solve the cubic generally scores no marks.

Question	Scheme	Marks
8(a)	$\frac{dy}{dx}=\pm\frac{1}{\sqrt{1-k\sqrt{x}^2}}\times ...x^{-\frac{1}{2}}$ or $\cos y=2x^{\frac{1}{2}}\Rightarrow \pm\sin y\frac{dy}{dx}=..x^{-\frac{1}{2}}$	M1
	$\frac{dy}{dx}=\pm\frac{1}{\sqrt{1-4x}}\times\left(Kx^{-\frac{1}{2}}\right)$ or $\frac{dy}{dx}=\pm\frac{Kx^{-\frac{1}{2}}}{\sqrt{1-\left(2\sqrt{x}\right)^2}}$	dM1
	$\frac{dy}{dx}=-\frac{1}{\sqrt{x}\sqrt{1-4x}}$ oe e.g. $\frac{dy}{dx}=-\frac{1}{\sqrt{x-4x^2}}$	A1
		(3)
(b) Way 1	$\int y\,dx=\int 1\times\arccos\left(2\sqrt{x}\right)dx = x\arccos\left(2\sqrt{x}\right)-\int x\frac{-1}{\sqrt{x}\sqrt{1-4x}}dx$	M1
	$=x\arccos\left(2\sqrt{x}\right)+\int\frac{\sqrt{x}}{\sqrt{1-4x}}dx$*	A1*
		(2)
Way 2	$\frac{d}{dx}\left(x\arccos\left(2\sqrt{x}\right)\right)=1.\arccos\left(2\sqrt{x}\right)+x.\frac{-1}{\sqrt{x}\sqrt{1-4x}}$	M1
	$\Rightarrow\int\arccos\left(2\sqrt{x}\right)dx=x\arccos\left(2\sqrt{x}\right)+\int\frac{\sqrt{x}}{\sqrt{1-4x}}\;dx$*	A1*
		(2)
(c)	$\frac{1}{2\sqrt{x}}\frac{dx}{d\theta}=-\frac{1}{2}\sin\theta,\ dx=-\sqrt{x}\sin\theta\,d\theta,\ \frac{dx}{d\theta}=-\frac{1}{2}\sin\theta\cos\theta$ $\frac{dx}{d\theta}=-\frac{1}{4}\sin 2\theta$	B1
	$\int\frac{\sqrt{x}}{\sqrt{1-4x}}dx=\int\frac{-\left(\frac{1}{2}\cos\theta\right)^2\sin\theta}{\sqrt{1-4\left(\frac{1}{2}\cos\theta\right)^2}}d\theta$	M1
	$=-\frac{1}{4}\int\frac{\cos^2\theta\sin\theta}{\sqrt{1-\cos^2\theta}}d\theta=-\frac{1}{4}\int\cos^2\theta\,d\theta$	A1
	$x=0\Rightarrow\theta=\frac{\pi}{2}$ $x=\frac{1}{8}\Rightarrow\theta=\frac{\pi}{4}$ So $\int_0^{\frac{1}{8}}\frac{\sqrt{x}}{\sqrt{1-4x}}dx=\frac{1}{4}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cos^2\theta\,d\theta$	A1
		(4)

(d)	$$\frac{1}{4}\int\frac{1}{2}(1+\cos 2\theta)\,\mathrm{d}\theta = K\left(\theta \pm \frac{1}{2}\sin 2\theta\right)$$	**M1**
	$$\int_0^{\frac{1}{8}}\frac{\sqrt{x}}{\sqrt{1-4x}}\,\mathrm{d}x = \frac{1}{8}\left[\theta+\frac{1}{2}\sin 2\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \ldots\left(=\frac{\pi}{32}-\frac{1}{16}\right)$$ or e.g. $$\int_0^{\frac{1}{8}}\frac{\sqrt{x}}{\sqrt{1-4x}}\,\mathrm{d}x = \frac{1}{8}\left[\theta+\frac{1}{2}\sin 2\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = -\frac{1}{8}\left[\arccos 2\sqrt{x}+\frac{1}{2}\sin 2\arccos 2\sqrt{x}\right]_0^{\frac{1}{8}}$$ $$=\ldots\left(=-\frac{1}{8}\left(\frac{\pi}{4}+\frac{1}{2}-\frac{\pi}{2}\right)\right)$$	**dM1**
	$$\Rightarrow \int_0^{\frac{1}{8}}\arccos\left(2\sqrt{x}\right)\mathrm{d}x = \left[x\arccos 2\sqrt{x}\right]_0^{\frac{1}{8}}+\frac{\pi}{32}-\frac{1}{16}=\frac{1}{8}\arccos\frac{1}{\sqrt{2}}-0+\frac{\pi}{32}-\frac{1}{16}$$	**dM1**
	$$=\frac{\pi}{16}-\frac{1}{16}\text{ oe}$$	**A1**
		(4)
		(13 marks)

Notes:

(a)

M1: Attempts to apply the arccos derivative formula together with chain rule. Look for $\frac{\mathrm{d}y}{\mathrm{d}x}=\pm\frac{1}{\sqrt{1-k\sqrt{x}^2}}\times \mathrm{f}(x)$ where f (x) is an attempt at differentiating $2\sqrt{x}$ where $\mathrm{f}(x)\neq\alpha\sqrt{x}$

Note that k may be 1 for this mark.

Alternatively, takes cosine of both sides and differentiates to the form shown in the scheme.

dM1: Correct form for the overall derivative achieved, may be errors in sign or constants with $k \neq 1$

Alternatively, divides through by sin y and applies Pythagorean identity to achieve derivative in terms of x.

A1: Correct derivative, but need not be simplified. Award when first seen and isw.

(b) Way 1

M1: Attempts to apply integration by parts to $1\times\arccos\left(2\sqrt{x}\right)$.

Look for $x\arccos\left(2\sqrt{x}\right)-\int x\text{"their (a)"}\,\mathrm{d}x$ or $u=\arccos\left(2\sqrt{x}\right)\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}x}=\text{part}(a),\ \frac{\mathrm{d}v}{\mathrm{d}x}=1\Rightarrow v=x$

A1*: Correct work leading to the printed answer. There must be a clear statement for the integration by parts before the given answer is stated.

So e.g. $u=\arccos\left(2\sqrt{x}\right)\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}x}=\text{part}(a),\ \frac{\mathrm{d}v}{\mathrm{d}x}=1\Rightarrow v=x$

$\Rightarrow\int\arccos\left(2\sqrt{x}\right)\mathrm{d}x = x\arccos\left(2\sqrt{x}\right)+\int\frac{\sqrt{x}}{\sqrt{1-4x}}\,\mathrm{d}x^*$ scores M1A0

You can condone $\int\arccos\left(2\sqrt{x}\right)\mathrm{d}x = x\arccos\left(2\sqrt{x}\right)+\int\frac{x^{\frac{1}{2}}}{\sqrt{1-4x}}\,\mathrm{d}x^*$

Way 2

M1: Applies the product rule to $x\arccos\left(2\sqrt{x}\right)$, look for $1.\arccos\left(2\sqrt{x}\right)+x.$"their (a)".

A1*: Rearranges and integrates to achieve the given result, with no errors seen.

(c)

B1: Any correct expression involving $\mathrm{d}x$ and $\mathrm{d}\theta$, see examples in scheme.

M1: Makes a complete substitution in the integral $\int \frac{\sqrt{x}}{\sqrt{1-4x}}\,\mathrm{d}x$ to achieve an integral in θ only.

Ignore attempts at substitution into the $x\arccos\left(2\sqrt{x}\right)$.

A1: A correct simplified integral aside from limits. May be implied by e.g. $\frac{1}{4}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2\theta\,\mathrm{d}\theta$

Note that this mark depends on the B mark.

A1: Finds correct limits for θ and applies to the integral by reversing the sign – i.e. correct answer with limits and sign all correct. Accept equivalent limits e.g. $-\frac{\pi}{4}$ to $-\frac{\pi}{4}$ or $\frac{\pi}{2}$ to $\frac{3\pi}{4}$

Note that this mark depends on the B mark.

(d)

M1: Applies double angle identity to get the integral in a suitable form and attempts to integrate. Accept $\cos^2\theta=\frac{1}{2}(\pm1\pm\cos2\theta)$ used as identity and look for $1\to\theta$ and $\cos2\theta\to\pm\frac{1}{2}\sin2\theta$

dM1: Applies their limits (either way round) to their integral in θ or reverse substitution and applies limits 0 and $\frac{1}{8}$.

Depends on the previous method mark.

dM1: Applies limits of 0 and $\frac{1}{8}$ to the $x\arccos\left(2\sqrt{x}\right)$ to obtain a value (or their limits either way round if they applied the substitution to this to obtain a value) and combines with the result of the other integral.

Depends on both previous method marks.

A1: Correct final answer.