# Mark Scheme (Results)

## Summer 2021

Pearson Edexcel International Advanced Subsidiary/Advanced Level
In Pure Mathematics P4 (WMA14/01)

| Question Number | Scheme | Marks |
|---|---|---|
| **1 (a)** | $(1+kx)^{\frac{1}{2}} = 1+\left(\frac{1}{2}\right)\times(kx)+\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)}{2!}\times(kx)^2+\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{3!}\times(kx)^3 \ldots$ | |
| **(i)** | $\frac{1}{2}k=\frac{1}{8}\Rightarrow k=\frac{1}{4}$ | M1A1 |
| **(ii)** | $A=\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)}{2!}\times "k"^2=-\frac{1}{128}$ $\quad B=\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{3!}\times "k"^3=\frac{1}{1024}$ | M1 A1 A1 |
| | | **(5)** |
| **(b)** | Substitutes $x=0.6\Rightarrow\sqrt{1.15}=1+\frac{1}{8}\times0.6-"\frac{1}{128}"\times0.6^2+"\frac{1}{1024}"\times0.6^3=1.072398$ | M1 A1 |
| | | **(2)** |
| | | **(7 marks)** |
| **1(b) alt** | $(1+kx)^{\frac{1}{2}}=1+\left(\frac{1}{2}\right)\times(kx)-\left(\frac{1}{8}\right)\times(kx)^2+\left(\frac{1}{16}\right)\times(kx)^3$ <br> Substitutes $"kx"=0.15$ <br> $\Rightarrow\sqrt{1.15}=1+\frac{1}{2}\times0.15-\frac{1}{8}\times0.15^2+\frac{1}{16}\times0.15^3=1.072398$ | M1A1 |
| | | **(2)** |

(a)(i)

M1: Sets $\frac{1}{2}k=\frac{1}{8}$ or $\frac{1}{2}kx=\frac{1}{8}x$ and proceeds to find *k*. Implied by a correct value for *k*

A1: $k=\frac{1}{4}$ oe such as $\frac{2}{8}$ or 0.25

(a)(ii)

M1: Correct attempt at 3rd **or** 4th term. Eg. $Ax^2=\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)}{2!}\times(kx)^2$ or $Bx^3=\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{3!}\times(kx)^3$ with an attempt at substituting in their value for *k* to find a value for *A* or a value for *B*. Condone a missing bracket around the $kx$ terms so allow with *k* instead of $k^2$ or $k^3$ respectively. These may be simplified so award for $A=-\frac{1}{8}\times("k")^2$ or $B=\frac{1}{16}\times("k")^3$ again, condoning *k* instead of $k^2$ or $k^3$ respectively.

A1: $A=-\frac{1}{128}$ o.e. There is no requirement to simplify the fraction

A1: $B=\frac{1}{1024}$ o.e You may occasionally see $B=\frac{3}{3072}$ which is fine.

(b)

M1: For an attempt to substitute

- either $kx=0.15$ into an expansion of the form $1\pm p\times(kx)\pm q\times(kx)^2\pm r\times(kx)^3$
- or $x=\frac{0.15}{"k"}$ into their $1+\frac{1}{8}x+"A"x^2+"B"x^3$

A1: 1.072398 Must be to 6 decimal places.

| Question Number | Scheme | Marks |
|---|---|---|
| **2** | Achieves a lower limit of 2 | B1 |
| | Attempts $\alpha\int y^2\,\mathrm{d}x = \alpha\int \frac{81}{(2x-3)^{2.5}}\mathrm{d}x = \alpha\times\frac{-27}{(2x-3)^{1.5}}$ | M1, A1 |
| | Correct attempt at volume of solid generated **under** curve = | |
| | $=\pi\int_{"2"}^{6} y^2\mathrm{d}x = \left[\frac{k}{(2x-3)^{1.5}}\right]_{"2"}^{6} = \left(\left(-\frac{27}{27}\right)-\left(-\frac{27}{1}\right)\right)$ | dM1 |
| | Volume $= 26\pi$ | A1 |
| | Correct attempt at volume of solid $= \pi\times 9^2\times(6-"2")-"26\pi"$ | ddM1 |
| | $= 298\pi$ | A1 |
| | | **(7 marks)** |

B1: Finds the lower limit. This may be awarded anywhere. Accept on the diagram or in the limits of an integral

M1: Integrates an expression of the form $\int \frac{...}{(2x-3)^{2.5}}\mathrm{d}x$ and achieves $\frac{...}{(2x-3)^{1.5}}$ oe

A1: Correct integration of $\alpha\int y^2\,\mathrm{d}x = \alpha\int \frac{81}{(2x-3)^{2.5}}\mathrm{d}x$ giving a solution $\alpha\times\frac{-27}{(2x-3)^{1.5}}$ o.e.

following through on their $\alpha$. Typical values of $\alpha$ will be 1, $\pi$ or $2\pi$. No need to simplify here

dM1: Attempts to find the volume of the solid generated by rotating the area under the curve.

Look for $\left[\frac{...}{(2x-3)^{1.5}}\right]_{"2"}^{6} = ...$ It is dependent upon the previous M1.

The top limit must be 6 and the bottom limit must be their solution to $\frac{9}{(2x-3)^{1.25}} = 9$

A1: Correct exact volume for the solid generated by rotating the area under the curve. Volume $= 26\pi$

This may be the second part of a larger/complete expression. E.g. $....-26\pi$

ddM1: Correct method for the volume of the solid formed by rotating the area above the curve.

Accept $\pi\times 9^2\times(6-"2")-"26\pi"$

It is dependent both previous M's

A1: $298\pi$

| Question Number | Scheme | Marks |
|---|---|---|
| **3 (a)** | Attempts to find $\dfrac{\frac{1}{3}\times 20^2\times 24}{160}=20$ seconds | M1 A1 |
| | | **(2)** |
| **(b)** | Attempts $\dfrac{\mathrm{d}V}{\mathrm{d}h}=h^2+\dfrac{8}{3}h$ | M1 |
| | Attempts to use $\dfrac{\mathrm{d}V}{\mathrm{d}t}=\dfrac{\mathrm{d}V}{\mathrm{d}h}\times\dfrac{\mathrm{d}h}{\mathrm{d}t}\Rightarrow 160=\left(h^2+\dfrac{8}{3}h\right)\times\dfrac{\mathrm{d}h}{\mathrm{d}t}$ | M1 A1 |
| | Substitutes $h=5\Rightarrow 160=\left(5^2+\dfrac{40}{3}\right)\times\dfrac{\mathrm{d}h}{\mathrm{d}t}\Rightarrow\dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{96}{23}$ (4.2) cm s $^{-1}$ | dM1 A1 |
| | | **(5)** |
| | | **(7 marks)** |

(a)

M1: Attempts to find the volume, using the given formula, at $h=20$ and dividing by 160. Implied by 20
Condone slips. For example they may have incorrectly multiplied out/calculated the expression for $V$

A1: 20 seconds. This requires the correct units as well

(b)

M1: For an attempt to differentiate $V$

Scored for an attempt to multiply out and then differentiate term by term to achieve $\dfrac{\mathrm{d}V}{\mathrm{d}h}=\alpha h^2+\beta h$

or via the product rule to achieve $V=\dfrac{1}{3}h^2(h+4)\Rightarrow\dfrac{\mathrm{d}V}{\mathrm{d}h}=(h+4)\times ph+qh^2$

M1: Attempts to use $\dfrac{\mathrm{d}V}{\mathrm{d}t}=\dfrac{\mathrm{d}V}{\mathrm{d}h}\times\dfrac{\mathrm{d}h}{\mathrm{d}t}$ (or equivalent) with $\dfrac{\mathrm{d}V}{\mathrm{d}t}=160$ and their $\dfrac{\mathrm{d}V}{\mathrm{d}h}$.

A1: Correct expression involving $\dfrac{\mathrm{d}h}{\mathrm{d}t}$ and $h$ E.g. $160=\left(h^2+\dfrac{8}{3}h\right)\times\dfrac{\mathrm{d}h}{\mathrm{d}t}$

dM1: Dependent upon the previous M only. It is for substituting $h=5$ and proceeding to a value for $\dfrac{\mathrm{d}h}{\mathrm{d}t}=...$

A1: Either $\dfrac{96}{23}$ or awrt 4.2 cm s $^{-1}$ There is no requirement for the units here

| Question | Scheme | Marks |
| --- | --- | --- |
| **4.** | $\displaystyle\int_1^4 \frac{10}{5x+2x\sqrt{x}}\,\mathrm{d}x$ | |
| | $u=\sqrt{x}\Rightarrow x=u^2\Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}u}=2u$ | B1 |
| | $\displaystyle\int \frac{10}{5x+2x\sqrt{x}}\,\mathrm{d}x=\int \frac{10}{5u^2+2u^3}2u\mathrm{d}u$ | M1 A1 |
| | $\displaystyle=\int \frac{20}{5u+2u^2}\mathrm{d}u=\int \frac{4}{u}-\frac{8}{5+2u}\mathrm{d}u$ | dM1 A1 |
| | $=4\ln u-4\ln(5+2u)$ | ddM1 |
| | Limits $=\left[4\ln u-4\ln(5+2u)\right]_1^2=4\ln 2-4\ln 9+4\ln 7=...$ | M1 |
| | $=4\ln\left(\dfrac{14}{9}\right)$ | A1 **(8 marks)** |

B1: For $\dfrac{\mathrm{d}x}{\mathrm{d}u}=2u$ or $\dfrac{\mathrm{d}u}{\mathrm{d}x}=\dfrac{1}{2}x^{-\frac{1}{2}}$ or equivalent

M1: Attempts to write all terms in the integral in terms of $u$ (inc the d$x$)

Condone slips for this mark BUT the d$x$ CANNOT just be replaced by d$u$.

Look for either $x\to u^2$ or $x\sqrt{x}\to u^3$ with either $\mathrm{d}x\to \mathrm{f}(u)\mathrm{d}u$ or $\mathrm{d}x\to k\times \mathrm{d}u,\ k\neq 1$

A1: Correct integrand in terms of just $u$ which may be unsimplified. E.g $\displaystyle\int \frac{10}{5u^2+2u^3}2u\mathrm{d}u$

dM1: Attempts to use PF and writes their integrand in terms of its component fractions.

Look for an integral of the form $\displaystyle\int \frac{P}{Qu+Ru^2}\mathrm{d}u\to\int \frac{p}{u}+\frac{q}{Q+Ru}\mathrm{d}u$

A1: Correct PF $\displaystyle\int \frac{4}{u}-\frac{8}{5+2u}(\mathrm{d}u)$

ddM1: For $...\ln u\pm...\ln(5+2u)$ but follow through on their PF's which must be of a similar form

M1: Uses the limits 1 and 2 within their attempted integral. The integration may be incorrect.

Alternatively substitutes $u=\sqrt{x}$ and uses the limits 1 and 4 within their attempted integral

A1: $4\ln\left(\dfrac{14}{9}\right)$

| Question Number | Scheme | Marks |
| --- | --- | --- |
| **5 (a)** | $\underline{\underline{2y\frac{dy}{dx}}} = \underline{e^{-2x}\frac{dy}{dx} - 2ye^{-2x}} - 3$ | $\underline{\underline{B1}}\,\underline{M1}$ A1 |
| | $\left(e^{-2x} - 2y\right)\frac{dy}{dx} = 2ye^{-2x} + 3 \Rightarrow \frac{dy}{dx} = \frac{2ye^{-2x} + 3}{e^{-2x} - 2y}$ * | A1* |
| | | **(4)** |
| **(b)** | Puts $x = 0$ into the equation of the curve $\Rightarrow y = y^2 \Rightarrow y = 1$ | B1 |
| | Attempts tangent at $(0,0)$ **or** $(0,1)$ $\quad y = 3x$ **or** $y = -5x + 1$ | M1 A1 |
| | Solves $y = 3x$ with $y = -5x + 1 \Rightarrow R = \left(\frac{1}{8}, \frac{3}{8}\right)$ | dM1 A1 |
| | | **(5)** |
| | | **(9 marks)** |

(a) Originally scored M1 B1 A1 A1, now B1 M1 A1 A1

B1: Correct differentiation using the chain rule $y^2 \to 2y\frac{dy}{dx}$ .

M1: Attempts to apply the product rule of differentiation on $ye^{-2x}$ to give $e^{-2x}\frac{dy}{dx} \pm ...ye^{-2x}$

A1: Correct differentiation $2y\frac{dy}{dx} = e^{-2x}\frac{dy}{dx} - 2ye^{-2x} - 3$

Allow $2y\,dy = e^{-2x}dy - 2ye^{-2x}dx - 3dx$

A1*: Proceeds to the given answer via an intermediate line equivalent to $\left(e^{-2x} - 2y\right)\frac{dy}{dx} = 2ye^{-2x} + 3$ with correct bracketing.

(b)

B1: Deduces or implies that $x = 0, y = 1$ at $P$ . May state or use coordinates of $P$ (0,1)

M1: Scored for an attempt to find the equation of the tangent at $O$ **or** the equation of the tangent at $P$

E.g. Substitutes $(0,0)$ into $\frac{dy}{dx} = \frac{2ye^{-2x} + 3}{e^{-2x} - 2y} = \beta$ and states $y = \beta x$

Alternatively substitutes $(0,"1")$ into $\frac{dy}{dx} = \frac{2ye^{-2x} + 3}{e^{-2x} - 2y} \to "k"$ and attempts $y - 1 = "k"x$ or equivalent

A1: Achieves a correct equation for **either** tangent. Look for **either** $y = 3x$ **or** $y = -5x + 1$ o.e.

dM1: Correct attempt **at both** tangents with an attempt to solve simultaneously.

For the attempt to solve accept $y = 3x$ , $y = -5x + 1 \Rightarrow x = ..., y = ...$

A1: Correct coordinates for $R = \left(\frac{1}{8}, \frac{3}{8}\right)$ oe

| Question Number | Scheme | Marks |
|---|---|---|
| **6 (a)(i)** | Area $R = \int y \dfrac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t = \int 4\sin t \times -4\sin 2t\,\mathrm{d}t$ | M1 A1 |
| | $= -\int 32\sin^2 t\cos t\,\mathrm{d}t$ | dM1 |
| | $x = 0 \Rightarrow t = \dfrac{\pi}{4}$ and $y = 0 \Rightarrow t = 0 \Rightarrow$ Area $= -\int_{\frac{\pi}{4}}^{0} 32\sin^2 t\cos t\,\mathrm{d}t = \int_{0}^{\frac{\pi}{4}} 32\sin^2 t\cos t\,\mathrm{d}t$ * | A1* |
| | | **(4)** |
| **(ii)** | Area $= \left[\dfrac{32}{3}\sin^3 t\right]_0^{\frac{\pi}{4}} = \dfrac{32}{3}\times\dfrac{2\sqrt{2}}{8} = \dfrac{8\sqrt{2}}{3}$ | M1 A1 |
| | | **(2)** |
| **(b)** | Attempts to use $\cos 2t = 1 - 2\sin^2 t \Rightarrow \dfrac{x}{2} = 1 - 2\left(\dfrac{y}{4}\right)^2$ | M1A1 |
| | $y = \sqrt{8-4x}$ | A1 |
| | | **(3)** |
| **(c)** | Range is $0 \leqslant \mathrm{f} \leqslant 4$ | B1 |
| | | **(1)** |
| | | **(10 marks)** |

(a)(i)

M1: Attempts to multiply $y$ by $\frac{\mathrm{d}x}{\mathrm{d}t}$ to achieve an integrand of the form $\pm k\sin t\sin 2t$

Look for area $R = \int y\frac{\mathrm{d}x}{\mathrm{d}t}(\mathrm{d}t) = \int (4)\sin t \times \pm k\sin 2t\ (\mathrm{d}t)$ but condone a missing 4 or d$t$

A1: Correct integrand. Achieves area $R = \int y\frac{\mathrm{d}x}{\mathrm{d}t}(\mathrm{d}t) = \int 4\sin t \times -4\sin 2t\,(\mathrm{d}t)$

Condone a missing d$t$ and do not be concerned by the limits. Allow unsimplified.

dM1: Substitutes $\sin 2t = 2\sin t\cos t \to$ seen or implied $= \pm\int A\sin^2 t\cos t\ \mathrm{d}t$

Dependent upon previous M. Condone a missing d$t$ and do not be concerned by the limits.

A1*: Achieves area $\left(= \int_0^2 y\ \mathrm{d}x\right) = -\int_{\frac{\pi}{4}}^{0} 32\sin^2 t\cos t\ \mathrm{d}t \to \int_0^{\frac{\pi}{4}} 32\sin^2 t\cos t\ \mathrm{d}t$

This is a given answer so you must see

- evidence of the d$t$ in at least one line other than the given answer
- correct application of the limits as seen above with as a minimum $-\int_{\frac{\pi}{4}}^{0} \ldots\,\mathrm{d}t \to \int_0^{\frac{\pi}{4}} \ldots\,\mathrm{d}t$

(a)(ii)

M1: Integrates to a form $\left[A\sin^3 t\right]$ with some attempt to apply the limit(s)

You may see a substitution $u = \sin t$ which is fine. Just look for $Au^3$ with some attempt to apply the adapted limits

A1: Correct answer $\frac{8\sqrt{2}}{3}$ o.e. following correct **algebraic** integration.

Cannot be scored without the M mark.

(b)

M1: Attempts to use a double angle formula of the form $\cos 2t = \pm 1 \pm 2\sin^2 t$ with the parametric equations to get a Cartesian equation.

If the parametric equations are substituted into the given form of the answer $y = \sqrt{ax+b}$ , marks are only scored when the double angle formula is used

A1: Any correct un-simplified equation $\frac{x}{2} = 1 - 2\left(\frac{y}{4}\right)^2$

On the alternative method, this A mark is scored when the candidate writes down $a + 2b = 0. -4b = 16 \Rightarrow a = \ldots, b = \ldots$

A1: $y = \sqrt{8-4x}$ or $y = \sqrt{-4x+8}$ ONLY

(c)

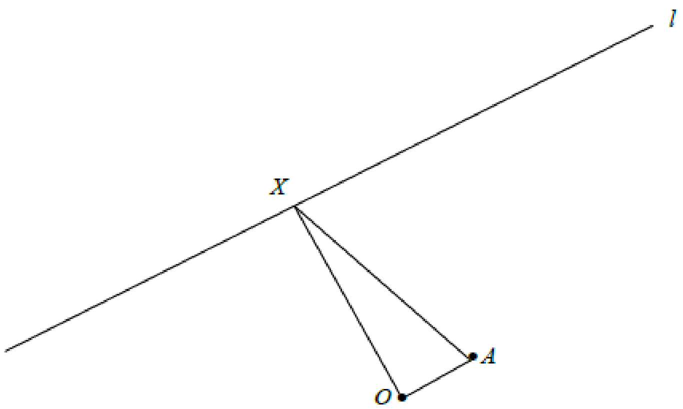
B1: Range is $0 \leqslant \mathrm{f} \leqslant 4$

Allow other acceptable forms such as $0 \leqslant y \leqslant 4$, $0 \leqslant \mathrm{f}(x) \leqslant 4$ and $[0,4]$

Examples of unacceptable forms are $0 \leqslant x \leqslant 4$, $[0,4)$

| Question Number | Scheme | Marks |
|---|---|---|
| **7 (a)** | Finds $\left\|\begin{pmatrix}4\\4\\2\end{pmatrix}\right\| = \sqrt{4^2+4^2+2^2} = 6$ and attempts $\frac{1}{"6"}\begin{pmatrix}4\\4\\2\end{pmatrix}$ | M1 |
| | $\overrightarrow{OA} = \begin{pmatrix}2/3\\2/3\\1/3\end{pmatrix}$ oe | A1 |
| | | **(2)** |
| **(b)** | Co-ordinates or position vector of point $X = \begin{pmatrix}1+4\lambda\\-10+4\lambda\\-9+2\lambda\end{pmatrix}$ | M1 |
| | $\overrightarrow{OX}.\begin{pmatrix}4\\4\\2\end{pmatrix} = 0 \Rightarrow 4(1+4\lambda)+4(-10+4\lambda)+2(-9+2\lambda)=0$ | dM1 |
| | $36\lambda = 54 \Rightarrow \lambda = 1.5$ | ddM1 A1 |
| | $X = (7,-4,-6)$ | A1 |
| | | **(5)** |
| **(c)** | Finds $OX = \sqrt{7^2+(-4)^2+(-6)^2} = \sqrt{101}$ and $OA = 1$ | M1 |
| | Area $OXA = \frac{1}{2}\times 1\times\sqrt{101} = \frac{\sqrt{101}}{2}$ | dM1 A1 |
| | | **(3)** |
| | | **(10 marks)** |

Handy diagram




(a)

M1: Correct attempt at the unit vector. Look for an attempt at $\sqrt{4^2+4^2+2^2}$ and use of $\frac{\mathbf{r}}{|\mathbf{r}|}$ where $\mathbf{r} = \begin{pmatrix}4\\4\\2\end{pmatrix}$

or equivalent vector form such as $\mathbf{r} = (4\mathbf{i}+4\mathbf{j}+2\mathbf{k})$. Condone an attempt to find the coordinates of $A$

A1: $\overrightarrow{OA}=\begin{pmatrix} 2/3 \\ 2/3 \\ 1/3 \end{pmatrix}$ or equivalent such as $\overrightarrow{OA}=\frac{1}{6}\begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix}$ or $\overrightarrow{OA}=\frac{1}{6}(4\mathbf{i}+4\mathbf{j}+2\mathbf{k})$ but **must be in vector form and not in coordinate form**.

(b)

M1: For an attempt at the co-ordinates or position vector of point $X=(1+4\lambda,-10+4\lambda,-9+2\lambda)$ **or** $\begin{pmatrix} 1+4\lambda \\ -10+4\lambda \\ -9+2\lambda \end{pmatrix}$

dM1: For using $\overrightarrow{OX}.\begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix}=0$ to set up an equation in $\lambda$

Alternatively finds $OX^2=(1+4\lambda)^2+(-10+4\lambda)^2+(-9+2\lambda)^2$ and attempts to differentiate and set = 0

May use $OX^2+OA^2=AX^2$ to set up an equation in $\lambda$

ddM1: Solves for $\lambda$ This is dependent upon having scored both previous M's

A1: Correct value for $\lambda=1.5$

A1: Correct coordinates for $X=(7,-4,-6)$. Condone position vector form $7\mathbf{i}-4\mathbf{j}-6\mathbf{k}$ o.e.

(c)

M1: Finds all elements required to calculate the area.

In the main scheme this would be the distance $OX$ or $OX^2$ AND distance $OA$ or $OA^2$ (which is 1)

dM1: Correct method of finding the area of $OXA=\frac{1}{2}\times OX\times 1$

A1: Area = $\frac{\sqrt{101}}{2}$ o.e

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There are various alternatives for part (c). Amongst others are;

Alt I **via vector product**.

$$\frac{1}{2}\times\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -4 & -6 \\ 2/3 & 2/3 & 1/3 \end{vmatrix}=\frac{1}{2}\times\left|\frac{8}{3}\mathbf{i}-\frac{19}{3}\mathbf{j}+\frac{22}{3}\mathbf{i}\right|=\frac{1}{2}\times\sqrt{\left(\frac{8}{3}\right)^2+\left(-\frac{19}{3}\right)^2+\left(\frac{22}{3}\right)^2}=\frac{\sqrt{101}}{2}$$

M1: For an attempt at $\frac{1}{2}\times\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -4 & -6 \\ 2/3 & 2/3 & 1/3 \end{vmatrix}=\frac{1}{2}\times\left|\frac{8}{3}\mathbf{i}-\frac{19}{3}\mathbf{j}+\frac{22}{3}\mathbf{i}\right|$

dM1: Followed by an attempt at finding the modulus of the resulting vector multiplied by 1/2

Alt II **via scalar products**

M1: Attempts to find all three components required to find the area triangle $OXA$

E.g. Angle $OXA$ with length of side $OX$ and $XA$

Alternatively angle $OAX$ with length of side $OA$ and $XA$

For this to be scored

- appropriate gradient vectors need to be attempted by subtracting
- a correct attempt at using $\mathbf{a}.\mathbf{b}=|\mathbf{a}||\mathbf{b}|\cos\theta$ to find $\cos\theta$ or $\theta$

dM1: Full attempt at area of triangle using $\frac{1}{2}|OX||XA|\sin(OXA)$ or equivalent

| Question Number | Scheme | Marks |
|---|---|---|
| 8 (a) | $\int y^{-\frac{1}{3}}\,dy = \int 6xe^{-2x}\,dx$ | B1 |
| | $\frac{3}{2}y^{\frac{2}{3}} = -3xe^{-2x} + \int 3e^{-2x}\,dx$ | M1 M1 |
| | $\frac{3}{2}y^{\frac{2}{3}} = -3xe^{-2x} - \frac{3}{2}e^{-2x} + c$ | dM1 A1 |
| | Substitutes $(0,1) \Rightarrow c = 3$ | M1 |
| | $y^2 = \left(-2xe^{-2x} - e^{-2x} + 2\right)^3$ | A1 |
| | | **(7)** |
| (b) | As $x \to \infty, e^{-2x} \to 0$ and $y^2 = (2)^3 \Rightarrow y = 2^{\frac{3}{2}}$ | M1 A1 |
| | | **(2)** |
| | | **(9 marks)** |

(a)

B1: Separates the variables either $\int y^{-\frac{1}{3}}\,dy = \int 6xe^{-2x}\,dx$ or $\int \frac{1}{6}y^{-\frac{1}{3}}\,dy = \int xe^{-2x}\,dx$

Condone with missing integral signs but the d$x$ and d$y$ **must be present** and **in the correct positions**

M1: For integrating the lhs $y^{-\frac{1}{3}} \to y^{\frac{2}{3}}$

M1: For integrating the rhs by parts the right way around. Look for $\int xe^{-2x}\,dx \to ..xe^{-2x} \pm \int ...e^{-2x}\,dx$

dM1: For fully integrating the rhs to obtain $..xe^{-2x} \pm ...e^{-2x}$. Depending upon the previous M

A1: Correct integration with or without '+ $c$'.

Look for $\frac{3}{2}y^{\frac{2}{3}} = -3xe^{-2x} - \frac{3}{2}e^{-2x} + c$ or equivalent such as $3y^{\frac{2}{3}} = -6xe^{-2x} - 3e^{-2x} + a$

dM1: Must have a "+$c$" now. Substitutes $(0,1) \Rightarrow c = ...$

It is dependent upon having a correct attempt to integrate one side so M1 or M2 must have been awarded.

A1: CSO $y^2 = \left(-2xe^{-2x} - e^{-2x} + 2\right)^3$

(b)

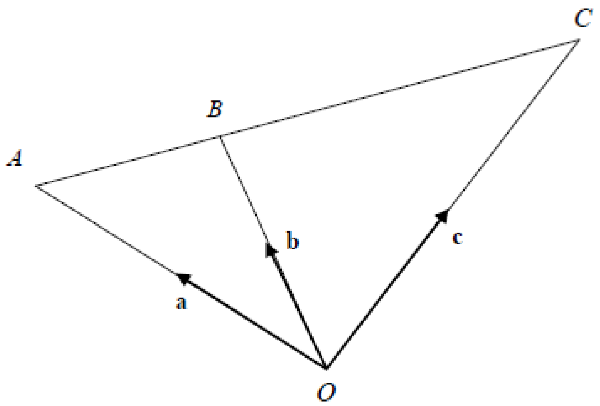
M1: For $e^{-2x} \to 0$ $y^2 = ("2")^3$

Follow through on their $g(x)$ in $y^2 = g(x)$ but $g(x)$ must be a function of $e^{-kx}$ with $g(0) \neq 0$

Implied by a correct decimal answer for their $y^2 = g(x)$

A1: $y = 2^{\frac{3}{2}}$ o.e such as $y = \sqrt{8}$ cso. ISW after sight of this. Condone $y = \pm 2^{\frac{3}{2}}$ o.e.

| Question Number | Scheme | Marks |
| --- | --- | --- |
| **9 (i)** | Attempts two of $\overrightarrow{AB}=\mathbf{b}-\mathbf{a}$, $\overrightarrow{AC}=\mathbf{c}-\mathbf{a}$ **and** $\overrightarrow{BC}=\mathbf{c}-\mathbf{b}$ either way around | M1 |
| | Attempts $\mathbf{c}-\mathbf{b}=2\times(\mathbf{b}-\mathbf{a})$ oe such as $\mathbf{c}-\mathbf{a}=3\times(\mathbf{b}-\mathbf{a})$ | dM1 |
| | $\Rightarrow \mathbf{c}=3\mathbf{b}-2\mathbf{a}$ * | A1 * |
| | | **(3)** |
| **(ii)** | Assume that there exists a number $n$ that isn't a multiple of 3 yet $n^2$ is a multiple of 3 | B1 |
| | If $n$ is not a multiple of 3 then $m=3p+1$ or $m=3p+2$ $(p\in\mathbb{N})$ giving | |
| | $m^2=(3p+1)^2=9p^2+6p+1$ | M1 |
| | Or $m^2=(3p+2)^2=9p^2+12p+4=3\left(3p^2+4p+1\right)+1$ | M1 A1 |
| | $(3p+1)^2=9p^2+6p+1\left(=3\left(3p^2+2p\right)+1\right)$ is one more than a multiple of 3 | |
| | $(3p+2)^2=9p^2+12p+4$ is not a multiple of 3 as 3 does not divide into 4 (exactly) | |
| | Hence if $n$ is a multiple of 3 then $n^2$ is a multiple of 3 | A1 |
| | | **(5)** |
| | | **(8 marks)** |




(i)

M1: Attempts any two of $\overrightarrow{AB}$, $\overrightarrow{AC}$ **and** $\overrightarrow{BC}$**.**

Condone the wrong way around but it must be subtraction.

Allow marked in the correct place on a diagram

dM1: Uses the given information.

Accept $\overrightarrow{AB}=\frac{1}{3}\overrightarrow{AC}$, $\overrightarrow{BC}=2\times\overrightarrow{AB}$, $\overrightarrow{BC}=\frac{2}{3}\times\overrightarrow{AC}$ etc condoning slips as in previous M1.

A1*: Fully correct work inc bracketing leading to the given answer $\mathbf{c}=3\mathbf{b}-2\mathbf{a}$

Expect to see the brackets multiplied out. So $\mathbf{c}-\mathbf{b}=2\times(\mathbf{b}-\mathbf{a})\Rightarrow\mathbf{c}-\mathbf{b}=2\mathbf{b}-2\mathbf{a}\Rightarrow\mathbf{c}=3\mathbf{b}-2\mathbf{a}$ is fine.

(ii)

B1: For setting up the contradiction.

Eg Assume that there exists a number $n$ that isn't a multiple of 3, yet $n^2$ is a multiple of 3

As a minimum accept something like " define a number $n$ such that $n$ is not a multiple of 3 but $n^2$ is"

M1: States that $m = 3p+1$ or $m = 3p+2$ and attempts to square.

Alternatives exist such as $m = 3p+1$ or $m = 3p-1$

Using modulo 3 arithmetic it would be $1 \to 1$ and $2 \to 4 = 1$

M1: States that $m = 3p+1$ AND $m = 3p+2$ and attempts to square o.e.

A1: Achieves forms that can be argued as to why they are NOT a multiple of 3

E.g. $m^2 = (3p+1)^2 = 3\left(3p^2 + 2p\right) + 1$ or even $9p^2 + 6p + 1$

and $m^2 = (3p+2)^2 = 3\left(3p^2 + 4p + 1\right) + 1$ or even $9p^2 + 12p + 4$

A1: Correct proof which requires

- Correct calculations
- Correct reasons. E.g. $9p^2 + 12p + 4$ is not a multiple of 3 as 4 is not a multiple of 3

  There are many ways to argue these. E.g $m^2 = (3p+1)^2 = 3\left(3p^2 + 2p\right) + 1$ is sufficient as long as followed (or preceded by) "not a multiple of 3"
- Minimal conclusion such as ✓. Note that B0 M1 M1 M1 A1 is possible