

Mark Scheme (Results)

October 2020

Pearson Edexcel International Advanced Level In Further Pure Mathematics F2 (WFM02/01)

Question Number	Scheme	Marks
1(a)	$\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} + 3\frac{\mathrm{d}y}{\mathrm{d}x} + 3x\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -2\sin x$	M1M1
	$\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} = -2\sin x - 3\frac{\mathrm{d}y}{\mathrm{d}x} - 3x\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$	A1 (3)
(b)	$\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} = -3 \times 5 = -15$	B1 (1)
(c)	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -3 \times 0 \times 5 + 2 = 2$	B1
	$y = 2 + 5x + x^2 - \frac{5}{2}x^3$	M1A1 (3)
		()
(a) M1	Accept the dashed notation throughout this question. Differentiate $3x \frac{dy}{dx}$ with respect to x . The product rule must be used for $x \frac{dy}{dx}$ one term correct	with at least
M1	Differentiate $\frac{d^2y}{dx^2}$ and $2\cos x$. $\frac{d^2y}{dx^2} \rightarrow \frac{d^3y}{dx^3}$ $2\cos x \rightarrow \pm 2\sin x$	
A1	$\frac{d^3y}{dx^3} = -3\left(x\frac{d^2y}{dx^2} + \frac{dy}{dx}\right) - 2\sin x$. Give A0 if not rearranged to have $\frac{d^3y}{dx^3} =$	
(b) B1	$\frac{d^3y}{dx^3} = -15$ provided 3 terms in result in (a)	
(c) B1	$\frac{d^2y}{dx^2} = 2$ can be implied by a correct x^2 term in the expansion	
M1	Use of a correct Taylor expansion with their values for $\frac{d^3y}{dx^3}$ and $\frac{d^2y}{dx^2}$ 2! or	
A1	$y = 2 + 5x + x^2 - \frac{5}{2}x^3$ Must include $y =$ or $f(x) =$ provided $f(x)$ has been somewhere in the work.	defined to be y

Question Number	Scheme	Marks
2 (a)	$\frac{3r+1}{r(r-1)(r+1)} = \frac{A}{r} + \frac{B}{r-1} + \frac{C}{r+1}$	
	$r(r-1)(r+1)^{-1}r^{-1}r-1^{-1}r+1$	
	$\frac{3r+1}{r(r-1)(r+1)} = -\frac{1}{r} + \frac{2}{r-1} - \frac{1}{r+1}$	M1A1 (2)
	r(r-1)(r+1) $r(r-1)(r+1)$	MIAI (2)
(b)	$\frac{2}{1} - \frac{1}{1} - \frac{1}{1}$	
	$\begin{bmatrix} 1 & 2 & 3 & \frac{2}{2} - \frac{1}{2} - \frac{1}{2} \end{bmatrix}$	
	$\begin{bmatrix} \frac{2}{2} - \frac{1}{3} - \frac{1}{4} & \frac{n-3}{2} & \frac{n-2}{2} & \frac{n-1}{2} \\ \frac{n-1}{2} & \frac{n-1}{2} & \frac{n-1}{2} & \frac{n-1}{2} \end{bmatrix}$	
	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	M1
	$\frac{3}{3} - \frac{4}{4} - \frac{5}{5}$ 2 1 1	
	$\frac{2}{4} - \frac{1}{5} - \frac{1}{6} \qquad \frac{n-1}{n-1} - \frac{n}{n+1}$	
	4 5 6	
	$=2-\frac{1}{2}+\frac{2}{2}-\frac{1}{n}-\frac{1}{n}-\frac{1}{n+1}$	dM1A1
		M1, A1 cso
	$\frac{5}{2} - \frac{2}{n} - \frac{1}{n+1} = \frac{5n(n+1) - 4(n+1) - 2n}{2n(n+1)}, = \frac{5n^2 - n - 4}{2n(n+1)}$	(5)
(c)		
	$\sum_{2}^{20} - \sum_{2}^{14}$	
	$= \frac{5 \times 20^2 - 20 - 4}{2 \times 20 \times 21} - \frac{5 \times 14^2 - 14 - 4}{2 \times 14 \times 15}$	M1
		IVII
	$=\frac{13}{210}$	A1 (2)
	210	[9]
(a)		
M1 A1	Correct method for obtaining the PFs Correct PFs	
(b)	Concertis	
M1	Show sufficient terms at both ends (eg 3 at start and 2 at end) to demon	strate the
1411	cancelling. (This can be implied by correct work at the next line) Must be using PFs of the correct form and start at $r = 2$ unless extra terms are ignored	
	at next stage. Can be split into $\sum \left(\frac{1}{r-1} - \frac{1}{r}\right) + \sum \left(\frac{1}{r-1} - \frac{1}{r+1}\right)$	
	at first stage. Can be split line $\sum \left(\frac{r-1}{r-1} - \frac{r}{r}\right)^{+} \sum \left(\frac{r-1}{r-1} - \frac{r}{r+1}\right)$	
dM1	Extract the non-cancelled terms (min 4 correct terms but 5/2 counts as	3 correct)
A1	Depends on first M of (b) Correct terms extracted	
M1	Write terms using the common denominator, numerator need not be sir	nplified. Must
Alcso	start with a min of 3 terms inc terms with denominators n and $(n + 1)$ Correct answer from correct working	
(c)	Coffeet unswer from coffeet working	
M1	Form and use the difference of the 2 summations shown using their res	ult from (b) or
A1	an earlier form seen in (b) Correct exact answer, as shown or equivalent	
111	Correct Cauce and wer, as shown or equivarent	

Question Number	Scheme	Marks
3	$\frac{x^2 + 3x + 10}{x + 2} = 7 - x$	This sketch on its own scores no marks, but it may be seen in the work
	$\begin{cases} x+2 \\ x^2+3x+10=14+5x-x^2 \end{cases}$	M1
	$x^{2}-x-2=0 (x-2)(x+1)=0$ CVs 2,-1 $-(x^{2}+3x+10)$	dM1 A1A1
	$\frac{-(x^2 + 3x + 10)}{x + 2} = 7 - x$ $-x^2 - 3x - 10 = 14 + 5x - x^2$ $8x = -24 \text{CV} - 3$ $x < -3 -1 < x < 2$	M1 A1 dddM1A1A1 [9]
NB	No algebra implies no marks	(*)
M1 dM1 A1 A1 M1 A1 dddM1 A1 A1	Form a quadratic equation or inequality, no simplification needed Solve the 3TQ any valid method Depends on the first M mark. Either CV Both CVs Change the sign of LHS or RHS and obtain an equation (quadratic or linear, no simplification needed) Correct CV from solving the linear equation $x < \text{their smallest CV}$ and $x = x < x < x < x < x < x < x < x < x < $	

Question Number	Scheme	Marks	
4 (a)	$\left 18\sqrt{3} - 18i \right = 18\sqrt{(3+1)} = 36$	B1	
	$ \begin{vmatrix} 18\sqrt{3} - 18i = 18\sqrt{(3+1)} = 36 \\ \tan \theta = \frac{-18}{18\sqrt{3}} \theta = -\frac{\pi}{6}, 18\sqrt{3} - 18i = 36\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right) $	M1,A1cao (3)	
(b)	$z^{4} = 36\left(\cos{-\frac{\pi}{6}} + i\sin{-\frac{\pi}{6}}\right) = 36\left(\cos\left(2k\pi - \frac{\pi}{6}\right) + i\sin\left(2k\pi - \frac{\pi}{6}\right)\right)$	M1	
	$z = \sqrt{6} \left(\cos \left(\frac{12k\pi - \pi}{24} \right) + i \sin \left(\frac{12k\pi - \pi}{24} \right) \right)$	M1	
	$k = 0 z_0 = \sqrt{6} \left(\cos \left(\frac{-\pi}{24} \right) + i \sin \left(\frac{-\pi}{24} \right) \right) = \sqrt{6} e^{i \left(-\frac{\pi}{24} \right)}$	B1	
	$k = 1$ $z_1 = \sqrt{6} \left(\cos \left(\frac{11\pi}{24} \right) + i \sin \left(\frac{11\pi}{24} \right) \right) = \sqrt{6} e^{i\frac{11\pi}{24}}$	A1ft	
	$k = 2$ $z_2 = \sqrt{6} \left(\cos \left(\frac{23\pi}{24} \right) + i \sin \left(\frac{23\pi}{24} \right) \right) = \sqrt{6} e^{i\frac{23\pi}{24}}$		
	$k = -1 z_3 = \sqrt{6} \left(\cos \left(-\frac{13\pi}{24} \right) + i \sin \left(-\frac{13\pi}{24} \right) \right) = \sqrt{6} e^{i \left(-\frac{13\pi}{24} \right)}$	A1ft (5) [8]	
(a) B1	Correct modulus		
M1	Attempt argument using $\tan \theta = \frac{\pm 18}{18\sqrt{3}}$ or other valid method. Can be in	mplied by	
	$\theta = \pm \frac{\pi}{6}$		
A1cao (b)	Correct answer in the required form.		
M1	Valid method for generating at least 2 roots, rotation through $\frac{\pi}{2}$ accept	ted	
M1	Apply de Moivre or use the rotation method		
B1 A1ft	Any one correct root Second root in required form		
A1ft	All 4 roots in the required form		
NB	Follow through their $\sqrt[4]{36}$ but 36 not acceptable. Argument in degrees – M1M1B1A0A0 (ie treat as mis-read) Incorrect argument: B0A1ftA1ft available		
	Answers in $r(\cos\theta + i\sin\theta)$ form – deduct final A marks		

Question Number	Scheme	Marks	
5	$w = \frac{z - 3i}{z + 2i}$		
	$z + 2i$ $w(z + 2i) = z - 3i z = \frac{i(2w + 3)}{1 - w}$ $ z = 1 \left \frac{i(2w + 3)}{1 - w} \right = 1$ $ i(2w + 3) = 1 - w $ $w = u + iv (2u + 3)^2 + 4v^2 = (1 - u)^2 + v^2$	M1	
	$\left z \right = 1 \left \frac{i(2w+3)}{1-w} \right = 1$	dM1	
	$\left i(2w+3) \right = \left 1 - w \right $		
	$w = u + iv$ $(2u + 3)^2 + 4v^2 = (1 - u)^2 + v^2$	ddM1	
	$4u^2 + 12u + 9 + 4v^2 = 1 - 2u + u^2 + v^2$		
	$3u^2 + 3v^2 + 14u + 8 = 0$	dddM1	
	$3u^{2} + 3v^{2} + 14u + 8 = 0$ $u^{2} + v^{2} + \frac{14}{3}u + \frac{8}{3} = 0$	A1	
	$\left(u + \frac{7}{3}\right)^2 + v^2 = -\frac{8}{3} + \frac{49}{9} = \frac{25}{9}$		
(i)	Centre $\left(-\frac{7}{3},0\right)$ Radius $\frac{5}{3}$	A1	
(ii)	Radius $\frac{5}{2}$	A1 (7)	
	3	[7]	
(a) M1	re-arrange to $z = \dots$		
dM1	dep (on first M1) using $ z =1$ with their previous result		
ddM1	dep (on both previous M marks) use $w = u + iv$ (or any other pair of legand find the moduli (or square of it)	etters inc (x, y)	
dddM1	and find the moduli (or square of it) dep (on all previous M marks) re-arrange to the form of the equation of a circle (same coeffs for the squared terms)		
A1	for a correct equation in u and v with coeffs of u^2 and v^2 both 1		
A1	Correct centre, must be in coordinate brackets. Completion of square no shown.	eed not be	
A1	Correct radius		
	Centre and radius must come from a correct circle equation for the	e A marks	

Question Number	Scheme	Marks	
6.	$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{\left(x\cot x + 2\right)}{x}y = \frac{4\sin x}{x^2}$	B1	
	$IF = e^{\int \frac{(x\cot x + 2)}{x} dx}$	M1	
	$= e^{(\ln \sin x + 2 \ln x)}$	A1	
	$=x^2\sin x$	A1	
	$\frac{\mathrm{d}}{\mathrm{d}x}\big(\text{their IF} \times y\big) = \text{their IF} \times \frac{4\sin x}{x^2}$	M1	
	$yx^{2} \sin x = \int 4\sin^{2} x dx = 4 \int \frac{1 - \cos 2x}{2} dx = 4 \left(\frac{x}{2} - \frac{1}{4} \sin 2x \right) $ (+C)	dM1A1	
	$y = \frac{2x - \sin 2x + C}{x^2 \sin x} \text{oe}$	A1cao [8]	
B1	Divide through by x^2		
M1	Attempt an IF of the form $e^{\int \frac{k(x\cot x+2)}{x} dx}$		
A1	$(\ln \sin x + 2 \ln x)$		
A1	Correct IF		
M1	Multiply through by their IF and write LHS in form shown – can be implied by next line. Allow if IF is seen instead of their function provided an IF has been attempted. Allow use of their RHS		
dM1	Attempt to integrate $\sin^2 x$, including using $\sin^2 x = \frac{1}{2} (1 \pm \cos 2x) \cos 2x$	$2x \to k \sin 2x$	
A1 A1	depends on previous M mark Correct integration, constant not needed Include the constant and treat it correctly. Must have $y =$		

Question Number	Scheme	Marks
7 (a)	$r\sin\theta = 2a\sin\theta + 2a\sin\theta\cos\theta \text{OR} r\sin\theta = 2a\sin\theta + a\sin2\theta$ $\frac{d(r\sin\theta)}{d\theta} = 2a\cos\theta + 2a\cos^2\theta \qquad \frac{d(r\sin\theta)}{d\theta} = 2a\cos\theta + 2a\cos2\theta$	B1 M1 A1
	$2\cos^{2}\theta + \cos\theta - 1 = 0 \text{terms in any order}$ $(2\cos\theta - 1)(\cos\theta + 1) = 0$	
	$\cos \theta = \frac{1}{2}$ $\theta = \frac{\pi}{3}$ ($\theta = \pi$ need not be seen)	dM1A1
	$r = 2a \times \frac{3}{2} = 3a$	A1 (6)
(b)	Area = $\frac{1}{2} \int r^2 d\theta = \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 4a^2 (1 + \cos \theta)^2 d\theta$	
	$=2a^2 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(1 + 2\cos\theta + \cos^2\theta\right) d\theta$	M1
	$=2a^2 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(1+2\cos\theta+\frac{1}{2}(\cos 2\theta+1)\right) d\theta$	M1
	$=2a^{2}\left[\theta+2\sin\theta+\frac{1}{2}\left(\frac{1}{2}\sin2\theta+\theta\right)\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	dM1A1
	$=2a^{2}\left[\frac{\pi}{3}+\sqrt{3}+\frac{1}{4}\times\frac{\sqrt{3}}{2}+\frac{\pi}{6}-\left(\frac{\pi}{6}+1+\frac{1}{4}\times\frac{\sqrt{3}}{2}+\frac{\pi}{12}\right)\right]$	M1 NB: A1 on e-PEN
	$=2a^2\left(\frac{\pi}{4}+\sqrt{3}-1\right)$	
	Area of $\triangle OAB = \frac{1}{2} \times 3a \times (2 + \sqrt{3})a \times \sin \frac{\pi}{6} \left(= \frac{3}{4}a^2(2 + \sqrt{3}) \right)$	
	Shaded area = $2a^2 \left(\frac{\pi}{4} + \sqrt{3} - 1\right) - \frac{3}{4}a^2 \left(2 + \sqrt{3}\right) = \frac{a^2}{4} \left(2\pi - 14 + 5\sqrt{3}\right)$	M1A1cao (7)
		[13]

Question Number	Scheme	Marks
(a) B1	Multiply r by $\sin \theta$ Award if not seen explicitly but a correct result following use of double is seen	
M1	Differentiate $r \sin \theta$ or $r \cos \theta$ (using product rule or using double ang	le formula first)
A1 dM1	Correct derivative for $r \sin \theta$	1 1'1
UIVI I	Use $\sin^2 \theta + \cos^2 \theta = 1$ to form a 3TQ in $\cos \theta$ and attempt its solution method	i by a vand
A1	Correct value for θ	
A1	Correct r	
(b)		
M1	Use area $=\frac{1}{2}\int r^2 d\theta$ with $r = 2a + 2a\cos\theta$, no limits needed,	
M1	Use a double angle formula to obtain a function ready for integrating (Alt method uses integration by parts – may be seen)	
dM1	Attempt the integration $\cos 2\theta \rightarrow \frac{1}{k} \sin 2\theta \ k = \pm 2 \text{ or } \pm 1$	
A1	Correct integration,	
M1	Substitute the limits (need not be simplified). Limits $\frac{\pi}{6}$ and their θ from this is $>\frac{\pi}{6}$	om (a) provided
	NB: A1 on e-PEN	
M1	Obtain the area of $\triangle OAB$ and subtract from their previous area	
A1	Correct answer	

Question Number	Scheme	Marks	
8 (a)	$x = e^{u} \frac{dx}{du} = e^{u} \text{ or } \frac{du}{dx} = e^{-u} \text{ or } \frac{dx}{du} = x$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^{-u} \frac{dy}{du}$ $\frac{d^{2}y}{dx^{2}} = -e^{-u} \frac{du}{dx} \frac{dy}{du} + e^{-u} \frac{d^{2}y}{du^{2}} \frac{du}{dx} = e^{-2u} \left(-\frac{dy}{du} + \frac{d^{2}y}{du^{2}} \right)$ $x^{2} \frac{d^{2}y}{dx^{2}} + 3x \frac{dy}{dx} - 8y = 4 \ln x$ $e^{2u} \times e^{-2u} \left(-\frac{dy}{du} + \frac{d^{2}y}{du^{2}} \right) + 3e^{u} \times e^{-u} \frac{dy}{du} - 8y = 4 \ln \left(e^{u} \right)$ $\frac{d^{2}y}{du^{2}} + 2 \frac{dy}{du} - 8y = 4u$ *	B1 M1 M1A1 dM1 A1*cso (6)	
B1	$\frac{\mathrm{d}x}{\mathrm{d}u} = \mathrm{e}^u$ oe as shown seen explicitly or used	1	
M1	Obtaining $\frac{dy}{dx}$ using chain rule here or seen later		
M1	Obtaining $\frac{d^2y}{dx^2}$ using product rule (penalise lack of chain rule by the A mark)		
A1	Correct expression for $\frac{d^2y}{dx^2}$ any equivalent form		
dM1 A1*cso	Substituting in the equation to eliminate x (u and y only). Depends on the 2^{nd} M mark Obtaining the given result from completely correct work		
	ALTERNATIVE 1		
	$x = e^u \frac{dx}{du} = e^u = x$	B1	
	$\frac{dy}{dx} = \frac{dy}{dx} \times \frac{dx}{dx} = x\frac{dy}{dx}$	M1	
	$\frac{d^2 y}{du^2} = 1 \frac{dx}{du} \times \frac{dy}{dx} + x \frac{d^2 y}{dx^2} \times \frac{dx}{du} = x \frac{dy}{dx} + x^2 \frac{d^2 y}{dx^2}$ $x^2 \frac{d^2 y}{dx^2} = \frac{d^2 y}{du^2} - \frac{dy}{du}$	M1A1	
	$\left(\frac{\mathrm{d}^2 y}{\mathrm{d}u^2} - \frac{\mathrm{d}y}{\mathrm{d}u}\right) + 3x \times \frac{1}{x} \frac{\mathrm{d}y}{\mathrm{d}u} - 8y = 4\ln\left(\mathrm{e}^u\right)$		
	$\frac{\mathrm{d}^2 y}{\mathrm{d}u^2} + 2\frac{\mathrm{d}y}{\mathrm{d}u} - 8y = 4u$	dM1A1*cso (6)	

Question Number	Scheme	Marks
B1	$\frac{dx}{du} = e^u$ oe as shown seen explicitly or used	
M1	Obtaining $\frac{dy}{du}$ using chain rule here or seen later	
M1	Obtaining $\frac{d^2y}{du^2}$ using product rule (penalise lack of chain rule by the A	A mark)
A1	Correct expression for $\frac{d^2y}{du^2}$ any equivalent form	
dM1 A1*cso	Substituting in the equation to eliminate x (u and y only). Depends on to Obtaining the given result from completely correct work	he 2 nd M mark
	ALTERNATIVE 2: $u = \ln x \frac{du}{dx} = \frac{1}{x}$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{x} \frac{dy}{du}$ $\frac{d^{2}y}{dx^{2}} = -\frac{1}{x^{2}} \frac{dy}{du} + \frac{1}{x} \frac{d^{2}y}{du^{2}} \times \frac{du}{dx} = -\frac{1}{x^{2}} \frac{dy}{du} + \frac{1}{x^{2}} \frac{d^{2}y}{du^{2}}$ $x^{2} \left(-\frac{1}{x^{2}} \frac{dy}{du} + \frac{1}{x^{2}} \frac{d^{2}y}{du^{2}} \right) + 3x \times \frac{1}{x} \frac{dy}{du} - 8y = 4u$ $\frac{d^{2}y}{du^{2}} + 2 \frac{dy}{du} - 8y = 4u$	B1 M1 M1A1 M1A1
	Notes as for main scheme	

There are also **other solutions** which will appear, either starting from equation II and obtaining equation I, or mixing letters x, y and u until the final stage. Mark as follows:

B1 as shown in schemes above

M1 obtaining a first derivative with chain rule

M1 obtaining a second derivative with product rule

A1 correct second derivative with 2 or 3 variables present

dM1 Either substitute in equation I or substitute in equation II according to method chosen and obtain an equation with only y and u (following sub in eqn I) or with only x and y (following sub in eqn II)

Alcso Obtaining the required result from completely correct work

Question Number	Scheme	Ма	rks
(b)	$m^2 + 2m - 8 = 0$		
	(m+4)(m-2)=0, m=-4.2	M1A1	
	(m+4)(m-2) = 0, m = -4,2 $CF = Ae^{-4u} + Be^{2u}$	A1	
	PI: try $y = au + b$ (or $y = cu^2 + au + b$ different derivatives, $c = 0$)		
	$\frac{\mathrm{d}y}{\mathrm{d}u} = a \frac{\mathrm{d}^2 y}{\mathrm{d}u^2} = 0$	M1	
	0+2a-8(au+b)=4u		
	$0 + 2a - 8(au + b) = 4u$ $a = -\frac{1}{2} b = -\frac{1}{8}$	dM1A	1
	$\therefore y = Ae^{-4u} + Be^{2u} - \frac{1}{2}u - \frac{1}{8}$	B1ft	(7)
(c)	$y = Ax^{-4} + Bx^2 - \frac{1}{2}\ln x - \frac{1}{8}$	B1	(1) [14]
			[]
(b) M1	Writing down the correct aux equation and solving to $m =$ (usual ru	ıles)	
A1	Correct solution $(m = -4, 2)$		
A1	Correct CF – can use any (single) variable $dy = d^2y$ d^2y d^2y	M0	
M1	Using an appropriate PI and finding $\frac{dy}{du}$ and $\frac{d^2y}{du^2}$ Use of $y = \lambda u$ scores M0		
dM1	Substitute in the equation to obtain values for the unknowns. Depends on the second M1		
A1 B1ft	Correct unknowns two or three (with $c = 0$) A complete solution, follow through their CF and a non-zero PI. Must have $y = a$ function of u		
	Allow recovery of incorrect variables.		
(c) B1	Reverse the substitution to obtain a correct expression for y in terms of x^{-4} or $e^{-4 \ln x}$ and x^2 or $e^{2 \ln x}$ allowed. Must start $y = \dots$	x No f	t here