Pearson Edexcel

Mark Scheme (Results)

January 2024

Pearson Edexcel International Advanced Level in Pure Mathematics P2 (WMA12) Paper 01

Question Number	Scheme	Marks
1	$\mathrm{f}(x)=ax^3+3x^2-8x+2$	
	Sets $\mathrm{f}(2)=3\Rightarrow 8a+12-16+2=3$	M1
	$\Rightarrow 8a=5\Rightarrow a=\dfrac{5}{8}$	M1A1
		(3)
		(3 marks)

M1: Attempts to set $\mathrm{f}(2)=3$. The values embedded and the expression set equal to 3 is sufficient. May be implied by further work. If 2 embedded in the expression is not seen, condone slips in their evaluation provided there is still evidence that the intention was to substitute in 2.
May also be seen as $\left(\mathrm{f}(2)-3=\right)\ a(2)^3+3(2)^2-8(2)-1=0$

M1: Solves a linear equation in *a* arising from setting $\mathrm{f}(\pm 2)=\pm 3$. Condone slips in their rearrangement proceeding to $a=...$ (May be implied by further work).

A1: $a=\dfrac{5}{8}$ or exact equivalent

Answers of $a=\dfrac{5}{8}$ may appear with very little or no working, possibly via trial and improvement. If so, then marks can only be allocated if evidence is shown.

e.g. $\mathrm{f}(x)=\dfrac{5}{8}\times x^3+3x^2-8x+2\Rightarrow\ \mathrm{f}(2)=5+3(2)^2-8(2)+2=3\Rightarrow \text{dividing by}\,(x-2)$ gives a remainder of 3

More difficult alternative methods may be seen:
Alt I: e.g. algebraic division (note they may also divide ax^3+3x^2-8x-1 by $(x-2)$ and set the remainder equal to 0)

$$\begin{array}{r l}
 & ax^2+(3+2a)x+(-2+4a) \\
x-2\,\big) & ax^3 \quad +3x^2 \quad -8x \quad +2 \\
 & ax^3 \quad -2ax^2 \\
 & (3+2a)x^2 \quad -8x \\
 & (3+2a)x^2+(-6-4a)x \\
 & (-2+4a)x \quad +2 \\
 & (-2+4a)x \quad +4-8a \\
 & -2+8a\Rightarrow -2+8a=3\Rightarrow a=\dfrac{5}{8}
\end{array}$$

M1: Divides the cubic by $(x-2)$, leading to a quadratic quotient where both the coefficient of *x* and the constant term are in terms of *a*. They should then have a linear remainder in *a* which is then set equal to 3

M1: Solves an equation resulting from setting a linear remainder in *a* equal to ± 3. It is dependent on the first method mark via this route.

A1: Completely correct with $a=\dfrac{5}{8}$

Alt II: You may also see a grid or an attempt at factorisation of ax^3+3x^2-8x-1 via inspection

M1: For an attempt at factorising e.g. $ax^3+3x^2-8x-1=(x-2)\left(ax^2+bx+\frac{1}{2}\right)$

M1: Forms two correct simultaneous equations and proceeds to find a value for a. Condoning slips in their solving.

$x^2: 3=-2a+b$

$x: -8=\frac{1}{2}-2b \Rightarrow b=\frac{17}{4} \Rightarrow a=\frac{3-\frac{17}{4}}{-2}=\frac{5}{8}$

A1: Completely correct with $a=\frac{5}{8}$

Question Number	Scheme	Marks
2	$\left(\frac{3}{8}+4x\right)^{12}$	
	Term in x^7 is ${}^{12}C_7\left(\frac{3}{8}\right)^5(4x)^7$ or coefficient of x^7 is ${}^{12}C_7\left(\frac{3}{8}\right)^5 4^7$	M1 A1
	Coefficient is 96228	A1
		(3)
		(3 marks)

Note that you do not need to see x^7 in their solution

M1: Combines a correct binomial coefficient with $\left(\frac{3}{8}\right)^5$ and either $(4x)^7$, 4^7 or x^7

Look for the binomial coefficient of the form e.g. $\begin{pmatrix}12\\7\end{pmatrix}$, ${}^{12}C_7$ or 792 o.e. Condone $\begin{pmatrix}12\\5\end{pmatrix}$ or ${}^{12}C_5$.

May be implied by further work.

Alternatively $\left(\frac{3}{8}+4x\right)^{12}=\left(\frac{3}{8}\right)^{12}\left(1+\frac{32}{3}x\right)^{12}\Rightarrow\left(\frac{3}{8}\right)^{12}\times\frac{12\times11\times10\times9\times8\times7\times6}{7!}\times\left(\frac{32}{3}x\right)^7$

Condone invisible brackets for this mark. e.g. ${}^{12}C_7\times\frac{3}{8}^5\times4x^7$

A1: Correct unsimplified term or coefficient e.g. ${}^{12}C_7\left(\frac{3}{8}\right)^5\times4^7$ or e.g. $\frac{12!}{5!7!}\left(\frac{3}{8}\right)^5\times4^7$. Invisible brackets may be implied by further work. Do not be concerned by the presence or absence of x^7

Note ${}^{12}C_7\left(\frac{3}{8}\right)^5(4x)^7$ or ${}^{12}C_5\left(\frac{3}{8}\right)^5(4x)^7$ scores M1A1 (may be seen as part of a list or several terms found in the expansion)

A1: For 96228 but condone 96 228 x^7 isw once a correct answer is seen. The term or coefficient must be identified if they have more than one term. A correct answer on its own with no incorrect working seen can score full marks.

Note: $\left({}^{12}C_7\left(\frac{3}{8}\right)^5\times4x^7=23.493...\right)\approx23.5$ will score M1A0A0 (where they multiply by 4 instead of 4^7)

Question Number	Scheme	Marks
3(a)	Attempts $(8-3)^2+(5--7)^2=...$	M1
	Writes $(x-3)^2+(y-5)^2=k$	M1
	$(x-3)^2+(y-5)^2=169$	A1
		(3)
(b)	Attempts $d^2+\left(2\sqrt{22}\right)^2=169\Rightarrow d=...$	M1
	States or uses $(y=)\ 5+d$	dM1
	$y=14$	A1
		(3)
		(6 marks)

(a)

M1: Attempts to find the radius or radius ^{2}. Must proceed to find a value. Condone a sign slip if attempting $5--7$ if this is seen as $5-7$ o.e. May be implied by 13 or 169. Do not be concerned with the labelling of the expression or value as r or r^2

M1: Writes the equation of the circle in the form $(x-3)^2+(y-5)^2=k$, where $k>0$, o.e.

$$\text{e.g. } x^2+y^2-6x-10y+c=0 \text{ where } c<0$$

Invisible brackets may be implied by further work. Condone $(x-3)^2+(y-5)^2=r^2$ where no attempt has been made to find the radius or radius^2

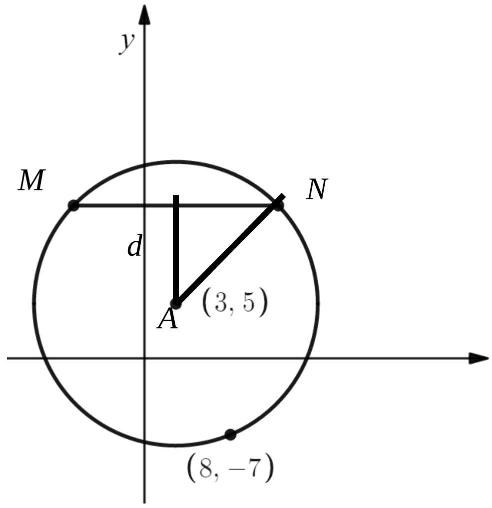
A1: $(x-3)^2+(y-5)^2=169$ o.e. e.g. $x^2+y^2-6x-10y-135=0$ isw once a correct unsimplified equation for C is found. Condone $(x-3)^2+(y-5)^2=13^2$. A correct equation scores M1M1A1 but it must be seen in (a).

(b) **Note that** $(3+2\sqrt{22},\,14)$ **as the final answer is A0**

M1: Attempts Pythagoras' theorem with "13" (as the hypotenuse) and $2\sqrt{22}$ to find d. Not just d^2 (See diagram)

dM1: States or uses the equation for MN, $(y=)\ 5+d$ but also condone stating/using $(y=)\ 5-d$ (may be implied by their value for y)

A1: $y=14$ only (Must be exactly 14 e.g. $13.99...=14$ is A0). **Further work leading to a different equation of a line is A0.**



Alternative using their centre and the x coordinates of M and N are equidistant from the centre

M1: Attempts to find the x coordinates of M or N e.g. $3\pm 2\sqrt{22}$, uses their equation of the circle with one of $x=3\pm 2\sqrt{22}$ and forms a three term quadratic in y (e.g. $y^2-10y-56=0$)

dM1: Attempts to solve their quadratic proceeding to a value for y. Usual rules for solving a quadratic. Accept via a calculator. It is dependent on the previous method mark.

A1: $y=14$ only (Must be exactly 14 e.g. $13.99...=14$ is A0) **Further work leading to a different equation of a line is A0.**

Alternative finding an angle in the triangle MNA, then using the angle to find d

M1: e.g. attempts the cosine rule to find the angle MNA, and then trigonometry to find d

$$\cos(\angle MNA)=\frac{13^2+\left(4\sqrt{22}\right)^2-13^2}{2\times 13\times 4\sqrt{22}}\Rightarrow \angle MNA=43.8...^\circ$$

$d=\sin 43.8...\times 13=...$

dM1: Same as main scheme

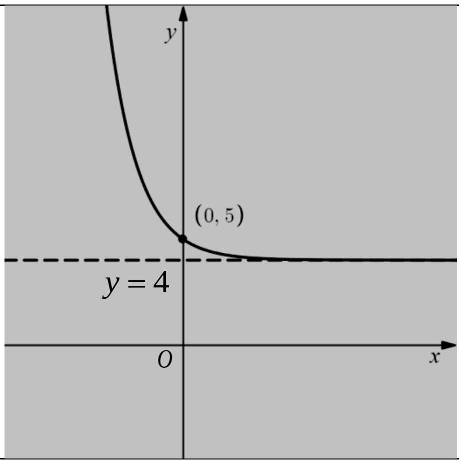
A1: $y=14$ only (Must be exactly 14 e.g. $13.99...=14$ is A0) **Further work leading to a different equation of a line is A0.**

Note: There may be other more complex attempts (including use of area) Send to review if unsure.

e.g. Attempting to find the line which passes through AN $\left(y=\frac{9}{2\sqrt{22}}x+\frac{220-27\sqrt{22}}{44}\right)$ and solving simultaneously with the equation of the circle $\Rightarrow y=14$

This typically will score M1 for attempting Pythagoras' theorem, but can score for attempting the gradient of the line AN.

The dM1 mark would be for the full attempt to find where the line through AN intersects the equation of the circle.

Question Number	Scheme	Marks
4(a)	 Correct shape Correct intercept Correct equation for asymptote	B1 B1 B1
		(3)
(b)	Strip width = 2.5	B1
	$\int_{-4}^{8.5}\left(3^{-\frac{1}{2}x}+4\right)\mathrm{d}x \approx \frac{5}{4}\{13+4.009+2\times(6.280+4.577+4.146+4.037)\}$ $=68.86125\left(=\text{awrt } 69\right)$	M1 A1
		(3)
(c)	(i) $\int_{-4}^{8.5}\left(3^{-\frac{1}{2}x}\right)\mathrm{d}x = 69-4\times\left(8.5--4\right)=\text{awrt } 19$	M1, A1ft
	(ii) $\int_{-4}^{8.5}\left(3^{-\frac{1}{2}x}+4\right)\mathrm{d}x + \int_{-8.5}^{4}\left(3^{\frac{1}{2}x}+4\right)\mathrm{d}x = \text{awrt } 138$	B1ft
		(3)
		(9 marks)

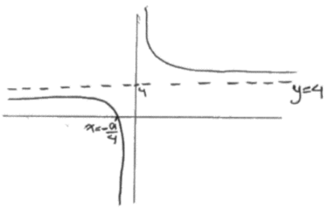
(a) **If there are multiple attempts then mark the most complete attempt. If coordinates/the equation of the asymptote are stated by the question and on their diagram, then the sketch takes precedence.**

B1: Correct shape in quadrants 1 and 2 and does not appear in quadrants 3 or 4 or touch the *x*-axis. Do not be concerned about the y intercept or asymptote. Mark the intention to draw an exponentially decreasing graph – do not penalise parts of the curve which may appear linear.

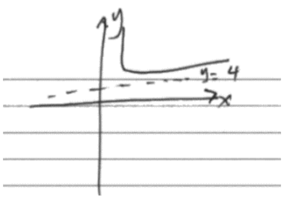
B1: Intercept at (0, 5). Condone invisible brackets, or may just have 5 marked on the *y*-axis, or $x=0,\ y=5$. If the point is labelled as $(5,\ 0)$ then condone provided it is a *y* intercept which is above the *x*-axis.

B1: Graph with asymptote at $y=4$ (asymptote does not need to be drawn) Must be an equation. There must be a graph with the intention of a horizontal asymptote above the *x*-axis to score the mark. If more than one horizontal asymptote is given, then B0. Where the asymptote is not drawn (e.g. with dashed lines) it must be clear that $y=4$ refers to the asymptote of the graph and not the *y*-intercept.

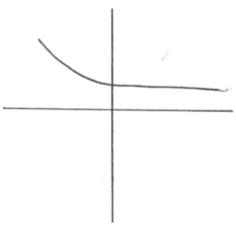
Examples:



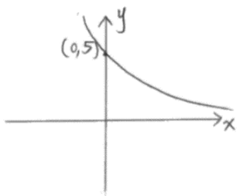
B0B0B1



B0B0B1



B1B0B0



B1B1B0

(b)

B1: For a correct strip width of $h = 2.5$ o.e. May be implied by 1.25 or equivalent in front of the bracket

M1: Correct application of the trapezium rule with all the *y* values and their *h* (which may be 1 and condone if negative)

Condone a missing trailing bracket. Condone other bracketing errors only if followed by a correct answer for their values. Condone miscopying of the *y* values from the table but the intended values must be used in the correct places.

Also award for attempts at forming individual trapezia and adding these together.

e.g.

$$2.5\left(\frac{13+6.280}{2}\right)+2.5\left(\frac{6.280+4.577}{2}\right)+2.5\left(\frac{4.577+4.146}{2}\right)+2.5\left(\frac{4.146+4.037}{2}\right)+2.5\left(\frac{4.037+4.009}{2}\right)$$

A1: awrt 69 (exact answer is 68.86125) isw once a correct answer is seen. awrt 69 following a correct calculation.

Note that the calculator answer for $\int_{-4}^{8.5}\left(3^{-\frac{1}{2}x}+4\right)\mathrm{d}x$ is 66.367... which scores 0 marks.

(c) They may use their 69 or a more accurate value of their 69 which is acceptable.

(i) **Attempts at redoing the trapezium rule with new values is M0A0**

M1: Attempts $"69"\pm 4\times(8.5--4)$ or $"69"\pm\int_{-4}^{8.5} 4\,\mathrm{d}x$ or $(\text{b})\pm 50$

May also be implied by just stating the value of $(\text{b})\pm 50$

A1ft: awrt 19 but follow through on $(\text{b})-50$.

(c)(ii)

B1ft: awrt 138 but follow through on $2\times(\text{b})$

Part (c) when the answer to part (b) is awrt 66 (where (b) has been done using the calculator)

(c)(i) **If a method is shown** then M1A1ft is possible e.g. $66\pm 4\times(8.5--4)$ or e.g. $66-50$ for awrt 16.

awrt 16 with no working is M0A0ft (as this could have been done using the calculator)

(c)(ii) awrt 132 scores B1ft (condoned)

Question Number	Scheme	Marks
5(i)	$r = 0.25$	B1
	$S_\infty = \frac{1.5}{1-0.25} = 2$	M1 A1
		(3)
(ii)	(a) $\left(u_1 = 3,\right)\ u_2 = 0, u_3 = \frac{3}{2}, u_4 = 3, \ldots$	M1
	$\left(u_1 = 3\right), u_2 = 0, u_3 = \frac{3}{2}, u_4 = 3, \ldots$ and e.g. states that $u_4 = u_1$ so sequence will repeat (is periodic)	A1
		(2)
	(b) (Order) 3	A1
		(1)
	(c) $\sum_{n=1}^{70} u_n = 23\times\left(3+0+\frac{3}{2}\right)+3 = 106.5$	M1, A1
		(2)
		(8 marks)

(i)

B1: States or uses $r = 0.25$ (may be implied by using $r = 0.25$ in the sum to infinity formula). 6×0.25 or any other term in the sequence is not sufficient evidence that this is a geometric sequence.
Alternatively, shows an understanding of the sigma notation by writing at least the first three terms in the sequence e.g. $6\times\frac{1}{4}+6\times\left(\frac{1}{4}\right)^2+6\times\left(\frac{1}{4}\right)^3+\ldots$

Condone invisible brackets and may be written as a list.

M1: Attempts $S_\infty = \frac{a}{1-r}$ with $a = 1.5$ (or 6×0.25), $r = 0.25$ or $a = 6, r = 0.25$

You may also see $S_\infty = \frac{a}{1-r} - a$ with $a = 6, r = 0.25$ which is acceptable

Beware of other variants such as taking a factor of 6 out and using $a = 0.25, r = 0.25$

The values embedded in the expression is sufficient to score this mark.

A1: 2 cao **following evidence of the use of the sum to infinity formula**

Note $6\times\frac{1}{4}+6\times\left(\frac{1}{4}\right)^2+6\times\left(\frac{1}{4}\right)^3+\ldots = 2$ is B1M0A0 (no evidence of using the sum to infinity formula)

(ii)(a)

M1: Uses the recurrence relation correctly at least once. If they do not achieve a correct $u_2 = 0$ then check e.g. their u_3 is a correct follow through for their u_2

A1: $\left(u_1 = 3\right), u_2 = 0, u_3 = \frac{3}{2}, u_4 = 3, \ldots$ with a statement or conclusion such as $u_4 = u_1$ /repeats/cycles (is periodic) As a minimum accept (3), $0, \frac{3}{2}, 3$ (hence) periodic

(ii)(b) **This mark can only be scored following correct values in (ii)(a)** (or a restart in (ii)(b))

A1: (Order is) 3 (do not accept (repeats every) third). Cannot be for just listing the terms of the sequence.

(ii)(c)

M1: Establishes a correct method of finding $\sum_{n=1}^{70} u_n$ for their periodic sequence of order 3 found in (a).

If they did not have a periodic sequence of order 3 in (a) and they do not restart then this mark cannot be scored.
Seeing the values embedded in the expression is sufficient to score this mark.
Attempts to use the sum of a geometric or an arithmetic series is M0A0

A1: 106.5 or exact equivalent e.g. $\frac{213}{2}$ ignore any subsequent incorrect attempts to round.

Question Number	Scheme	Marks
6 (a)	$2\log_4(x+3)=\log_4(x+3)^2$ or $\frac{1}{2}=\log_4 2$ o.e	B1
	Combines two terms e.g. $2\log_4(x+3)+\log_4 x=\log_4 x(x+3)^2$	M1
	e.g. $x(x+3)^2=2(4x+2)$	A1
	e.g. $x\left(x^2+6x+9\right)=8x+4\Rightarrow x^3+6x^2+x-4=0$ *	A1*
		(4)
(b) (i)	$x^3+6x^2+x-4=(x+1)\left(x^2+5x-4\right)$	M1
	$x=\dfrac{-5\pm\sqrt{25+16}}{2}\Rightarrow x=\dfrac{-5\pm\sqrt{41}}{2}$	dM1 A1
		(3)
(ii)	$x=\dfrac{-5+\sqrt{41}}{2}$	B1
		(1)
		(8 marks)

(a) **Do not penalise the omission of base 4 in their working provided the terms and values are consistent with base 4.**

B1: Writes (or may be implied by further work)

- $2\log_4(x+3)$ as $\log_4(x+3)^2$ **or**
- $\frac{1}{2}$ as $=\log_4 2$

M1: Correctly combines (at least) **two of the original terms**. e.g.

- $2\log_4(x+3)+\log_4 x=\log_4 x(x+3)^2$ is M1 but $2\log_4(x+3)+\log_4 x=2\log_4 x(x+3)$ is M0
- $\log_4(4x+2)-\log_4 x=\log_4\left(\dfrac{4x+2}{x}\right)$ is M1
- $\log_4(4x+2)+\frac{1}{2}=\log_4(8x+4)$ is M1

Do not penalise if they subsequently make errors or apply laws incorrectly. May be implied if they proceed to an equation not involving logs e.g. $x(x+3)^2=2(4x+2)$ (but not the given answer)

Beware that $2\log(x+3)+\log x=2\log x(x+3)=\log x(x+3)^2$ is B1M0A0A0

A1: A correct intermediate equation not involving logs (but not the given answer).

Must follow correct log work. e.g. $\dfrac{\log_4 x(x+3)^2}{\log_4(4x+2)}=\dfrac{1}{2}\Rightarrow\dfrac{x(x+3)^2}{(4x+2)}=2$ is A0

A1*: Correct proof. Expect to see

- a correct equation not involving logs which is not the final answer
- the brackets multiplied out $(x+3)^2$ or $(x^2+3x)(x+3)$ before proceeding to the given answer.

Do not allow this mark for recovery from incorrect log work but allow invisible brackets to be "recovered" or implied if the intention is clear.

(b) (i) **Answers with no working score 0 marks in (b)(i)**

M1: Divides by $(x+1)$. May be implied by a correct quadratic. May be seen as the quotient if attempting algebraic division. If via division look for $(x+1)\left(x^2 \pm 5x....\right)$ or via inspection $(x+1)\left(x^2 \pm ....x \pm 4\right)$

dM1: Attempts to find the roots of their quadratic factor (usual rules for solving a quadratic). Accept solving the quadratic via a calculator (you may need to check this). Condone decimals to 2sf for this mark. e.g. may see 0.70 or -5.7
Note that if there are multiple attempts to solve the quadratic (including via a calculator so roots may be stated) then mark the one which scores the most marks).

A1: $(x=)\ \dfrac{-5 \pm \sqrt{41}}{2}$ or exact equivalent e.g. $-2.5 \pm \sqrt{10.25}$ (Do not accept decimal answers but isw if seen after an exact answer is seen)
Ignore the presence or absence of $x=-1$

(b)(ii) **This mark can only be scored provided $(x=)\ \dfrac{-5+\sqrt{41}}{2}$ (or awrt 0.702) is found in (i), even if this is via solving the cubic on a calculator**

B1: $x = \dfrac{-5+\sqrt{41}}{2}$ or exact equivalent ONLY (accept awrt 0.702 if given in (b)(i)) If both answers are present then they must indicate they have chosen $x = \dfrac{-5+\sqrt{41}}{2}$ e.g circling or ticking (or putting a cross by the other one)

Question Number	Scheme	Marks
7.(a)	Attempts to use $4000 = 300 + 11d$ to find 'd'	M1
	Uses $300 + 3"d"$	M1
	Wheat production in year 4 is awrt 1310 (to the nearest 10) (tonnes)	A1
		(3)
(b)	Attempts to use $4000 = 300r^{11}$ to find 'r'	M1
	Finds $r = (1.266...)$ and MULTIPLIES this by 300	M1
	Wheat production in year 2 is awrt 380 (to the nearest 10) (tonnes)	A1
		(3)
(c)	Attempts $\frac{12}{2}\{300+4000\}$ or $\frac{300\left("1.266..."^{12}-1\right)}{"1.266..."-1}$	M1
	Finds $\frac{12}{2}\{300+4000\} - \frac{300\left("1.266..."^{12}-1\right)}{"1.266..."-1} = (25800 - 17935)$	dM1
	Difference = 7860 but allow 7870 (tonnes) (not AWRT)	A1
		(3)
		(9 marks)

Condone slips copying 300 and 4000 if they lose or gain an extra 0 in all parts

(a)

M1: Attempts to use the AP formula in an attempt to find 'd'
Accept an attempt at $4000 = 300 + 11d$ resulting in a value for d.
Accept the calculation $\frac{4000-300}{11}$ proceeding to a value (condoning slips)
May be implied by $\frac{3700}{11}$ or awrt 336

M1: Attempts to find the **fourth term** (do not allow a misread) by attempting $300 + 3\times"336"$.
You may award this following an "incorrect" AP formula with $12d$ being used instead of $11d$. e.g.
$4000 = 300 + 12d$ or more likely $\frac{4000-300}{12}$ usually leading to an answer of 1225.
If no method seen to find d then it must be correct.

A1: awrt 1310 (**to the nearest 10**) (tonnes) isw once a correct answer is seen.

Alt (a)

M1: Forms the equation $\frac{x-300}{4000-300} = \frac{4-1}{12-1}$ or may be implied by further work

M1: Rearranges the equation to $x = 300 + \frac{4-1}{12-1}\times(4000-300)$ or allow this mark for $x = 300 + \frac{4}{12}\times(4000-300)$

A1: awrt 1310 (**to the nearest 10**) (tonnes) isw once a correct answer is seen.

(b)

M1: Attempts to use the GP formula in an attempt to find 'r'

Look for $4000 = 300r^{11} \Rightarrow r^{11} = \frac{4000}{300} \Rightarrow r = \sqrt[11]{\frac{4000}{300}}$ condoning slips. Implied by r = awrt 1.3 provided they do not start from an incorrect equation.

M1: A correct attempt to find the **second term** (do not allow a misread) by multiplying 300 by their $"1.266..."$. It is dependent on an attempt to find r from either $4000 = 300r^{11}$ or $4000 = 300r^{12}$ condoning slips. If no method is seen to find r then it must be correct. e.g. following $4000 = 300r^{12}$ or $\sqrt[12]{\frac{4000}{300}}$.

Condone slips.

A1: awrt 380 (to the nearest 10) (tonnes) isw once a correct answer is seen.

(c) **Condone use of their *d* and their *r* even if they have come from incorrect methods**

M1: A correct method to find the sum of either the AP or the GP

For the AP accept an attempt at either $\frac{12}{2}\{300+4000\}$ or $\frac{12}{2}\{2\times300+11\times"336"\}$

For the GP accept an attempt at either $\frac{300\left("1.266..."^{12}-1\right)}{"1.266..."-1}$ or $\frac{300\left(1-"1.266..."^{12}\right)}{1-"1.266..."}$

dM1: Both formulae must be attempted "correctly" (see above) and the difference taken

FYI if d and r are correct, the sums are 25 800 tonnes and 17 935

A1: Difference = 7860 or 7870 (tonnes) **This is NOT AWRT**.

(Note that use of $r = 1.266$ will lead to a difference of 7810 which is max M1dM1A0)

Alt (c) Listing values

Accept listing the values

M1: Attempts to add the 12 terms for the arithmetic or geometric series. Must include 300 and 4000, but condone one omission of one of the middle terms. May be implied by calculating the difference between the AP and GP values for each year and adding those together.

dM1: Attempts to add the 12 terms for the arithmetic and geometric series. Must include 300 and 4000 for both sums, but condone one omission of one of the middle terms in each of the summations. May be implied.

A1: Difference = 7860 or 7870 **This is NOT AWRT**.

Year	AP	GP	Difference
1	300	300	0
2	636.3636	379.6544	256.7093
3	972.7273	480.4582	492.2691
4	1309.091	608.0268	701.0641
5	1645.455	769.4668	875.9877
6	1981.818	973.7715	1008.047
7	2318.182	1232.322	1085.86
8	2654.545	1559.521	1095.024
9	2990.909	1973.597	1017.312
10	3327.273	2497.616	829.6567
11	3663.636	3160.77	502.8668
12	4000	4000	0
Sum	25800	17935.2	7864.796

Question Number	Scheme	Marks
8 (i)	Substitutes a value e.g. $n=6$ into n^2+3n+1 where n^2+3n+1 is **not prime**	M1
	Correct calculation for that value e.g $n^2+3n+1=55$ And conclusion "which is not prime"	A1
		(2)
(ii)	Attempts to find n^2-2 for either odds or evens E.g Attempts $(2p+1)^2-2$ or $(2p)^2-2$	M1
	Achieves either $(2p+1)^2-2=4p^2+4p-1$ or $(2p)^2-2=4p^2-2$ and shows or gives a reason why the expression is not a multiple of 4 where required (see notes)	A1
	Attempts to find n^2-2 for both odds and evens (See above)	dM1
	Achieves both $(2p+1)^2-2=4p^2+4p-1$ and $(2p)^2-2=4p^2-2$ and shows or gives reasons why these are not multiples of 4 where required (see notes) With a conclusion that they are not multiples of 4. *	A1*
		(4)
		(6 marks)

(i)

M1: Substitutes a value into n^2+3n+1 where n^2+3n+1 is not prime. (Does not have to be evaluated) Possible values are $n=6,11,13,16$ etc. Condone slips substituting in or if they have evaluated incorrectly provided the intention was to substitute in a valid value for *n*. (Note that $n=0$ is not acceptable)

A1: A correct calculation for the value and a conclusion. There must be some reference to not being prime or they show that the number is divisible by e.g. 5 and state that it is false/not true

e.g. $6^2+3\times6+1=55$ which is not prime (so not true) is M1A1

e.g. when $n=11$ then $n^2+3n+1=155$ $155\div5=31$ so false is M1A1

e.g. $6^2+3\times6+1=55$ so false is M1A0

Values of *n* up to 50 for which n^2+3n+1 is not prime:

n	n^2+3n+1	n	n^2+3n+1
6	55	31	1055
11	155	32	1121
13	209	33	1189
16	305	35	1331
17	341	36	1405
21	505	39	1639
22	551	41	1805
24	649	42	1891
26	755	46	2255
28	869	48	2449
		50	2651

(ii) **There should be no errors in the algebra but allow e.g. invisible brackets to be "recovered". Withhold the final mark if n is used instead of k**

M1: Uses algebra to describe odds e.g. $n = 2p+1$ or evens e.g. $n = 2p$ and attempts $n^2 - 2$ (allow equivalent representation of odd or even e.g. $n = 2k+2$ **or** $2n \pm 5$)

Condone arithmetical slips and condone the use of e.g. $n = 2n$ and $n = 2n \pm 1$

A1: They must

- achieve a correct expanded $n^2 - 2$ for either odds or evens
- show or give an explanation why the expression is not a multiple of 4 (other than the trivial case when $n = 2p$.

e.g. $(2p+1)^2 - 2 = 4p^2 + 4p - 1$ which is 1 less than a multiple of 4 is A1 (explanation)

Do not accept e.g. "you cannot take 4 out as a common factor" as this is insufficient and should be shown, e.g. $(2p+1)^2 - 2 = 4p^2 + 4p - 1 = 4\left(p^2 + p\right) - 1$ is A1 (shows)

For the case when $n = 2p \Rightarrow (2p)^2 - 2 = 4p^2 - 2$ it does not need to be followed by an explanation as it is clearly not a multiple of 4 but if one is given then it must be correct. e.g. when divided by 4 gives $p^2 - \frac{1}{2}$

They must have evaluated numbers with indices so do not accept $2^2 p^2 - 2$

dM1: Attempts to find $n^2 - 2$ for both odds and evens (See above)

A1: Fully correct proof. They must

- achieve a correct expanded $n^2 - 2$ for both odds and evens
- show or gives a reason as to why each expression is not a multiple of 4 (other than for $n = 2p$)
- make a concluding overall statement. "Hence not a multiple of 4 (for all $n \in \mathbb{N}$)_" Accept "hence proven", "statement proved", "QED" **if they have stated for each separate case that the expression is not a multiple of 4.**

Attempts using $4k,\ 4k+1,\ 4k+2,\ 4k+3$ should be applied similarly to the main scheme above:

M1: Attempts **at least two of the four cases** from e.g. $4k,\ 4k+1,\ 4k+2,\ 4k+3$ (but could be others e.g. $4k-2,\ 4k-1,\ 4k,\ 4k+1$

A1: They must

- achieve two correct expanded $n^2 - 2$
- show or give a reason as to why the two expressions are not a multiple of 4 (same examples as in notes above for main scheme).

	$n^2 - 2$
$4k-3$	$16k^2 - 24k + 7$
$4k-2$	$16k^2 - 16k + 2$
$4k-1$	$16k^2 - 8k - 1$
$4k$	$16k^2 - 2$
$4k+1$	$16k^2 + 8k - 1$
$4k+2$	$16k^2 + 16k + 2$
$4k+3$	$16k^2 + 24k + 7$

dM1: Attempts to find $n^2 - 2$ for all four cases (see above)

A1: Fully correct proof (see above)

Question Number	Scheme	Marks
9 (i)	States or uses $\tan x = \dfrac{\sin x}{\cos x}$	B1
	$\sin x \tan x = 5 \Rightarrow \sin^2 x = 5\cos x \Rightarrow 1 - \cos^2 x = 5\cos x$	M1A1
	$\cos^2 x + 5\cos x - 1 = 0 \Rightarrow \left(\cos x =\right) \dfrac{-5 \pm \sqrt{29}}{2} \Rightarrow x = \text{awrt } 78.9°, 281.1°$	M1dM1A1
		(6)
(ii)	(a) $A = 5$	B1
		(1)
	(b) $2\theta - \dfrac{3\pi}{8} = \dfrac{3\pi}{2} \Rightarrow \theta = ...$	M1
	$\theta = \dfrac{15\pi}{16}$	A1
	y coordinate $Q = -3$ (or $2 - "A"$)	B1ft
		(3)
	(c) Sets $0 = "5"\sin\left(2\theta - \dfrac{3\pi}{8}\right) + 2 \Rightarrow \sin\left(2\theta - \dfrac{3\pi}{8}\right) = \pm\dfrac{2}{"5"}$	M1
	$\sin\left(2\theta - \dfrac{3\pi}{8}\right) = \pm\dfrac{2}{5} \Rightarrow \left(2\theta - \dfrac{3\pi}{8}\right) = \arcsin\left(\pm\dfrac{2}{5}\right) = ...$	dM1
	One of $\theta = 0.38,\ 2.4,\ 3.5,\ 5.5,\ 6.7,\ 8.6,\ 9.8....$	A1
	$\theta = \text{awrt } 5.51$	A1
		(4)
		Total 14

(i) **Do not penalise working in another variable or if the variable is omitted at times in their working.**
e.g. $\sin$ instead of $\sin x$ is condoned as a slip or $\cos^2 x$ as $\cos x^2$. They are required to show the stages of working but not to the same level of accuracy in their presentation as a "show" or "prove" question.

B1: States or uses $\tan x = \dfrac{\sin x}{\cos x}$ (may be seen as $\sin x \tan x \Rightarrow \tan^2 x \cos x$)

M1: Uses $\pm\sin^2 x \pm \cos^2 x = \pm 1$ to set up a quadratic equation in cos *x* only. If terms have been collected on one side condone the omission of = 0. May form the equation $\tan^2 x \cos x = 5$ and use the identity $\pm 1 \pm \tan^2 x = \pm\sec^2 x$ proceeding to a quadratic in cos *x* only

$$\text{e.g.} \Rightarrow \cos x(\sec^2 - 1) = 5 \Rightarrow \frac{1}{\cos x} - \cos x = 5 \Rightarrow 1 - \cos^2 x = 5\cos x$$

or alternatively, square both sides and proceed to a quartic in cos *x* only.

A1: Correct 3TQ (or quartic) in cos *x*. The three terms do not need to be on the same side of the equation. The =0 may be implied by further work. e.g. attempting to solve the quadratic. Alternatively award for the quartic equation $\cos^4 x - 27\cos^2 x + 1 = 0$

M1: Attempts to solve their 3TQ (or quartic) in cos *x*. Usual rules apply for solving the quadratic. Accept via a calculator but at least one root must be correct (you may need to check this)
They cannot proceed from the 3TQ directly to an angle for *x*; this is M0 and no further marks can be scored.
Accept either root as a rounded decimal to 2sf e.g. for the correct quadratic it would be awrt 0.19 or awrt -5.2

dM1: Attempts to find one value for *x* in the given range using their root. You may need to check this. It is dependent on the previous method mark. Accept in radians. It is acceptable to proceed from the roots of their quadratic directly to an angle in the given range to score this mark. May be implied by awrt 79 or awrt 281 (or in radians awrt 1.4 or awrt 4.9)

A1: awrt $x = 78.9°, 281.1°$ (must be in degrees) and no others in the range.

(ii)(a)

B1: $A=5$ Check by the question. If there are multiple answers, mark the answer in the main body of the work.

(ii)(b) **Solutions with no working:** Note if the two coordinates are given with no working then max M0A0B1

$\left(-3, \frac{15\pi}{16}\right)$ (or coordinates given the wrong way round) with no working seen then SC001

M1: Attempts to solve $2\theta-\frac{3\pi}{8}=\frac{3\pi}{2}$ or any other minimum value e.g. $2\theta-\frac{3\pi}{8}=-\frac{\pi}{2}$.

They must proceed using the correct order of operations e.g allow $\theta=\dfrac{\frac{3\pi}{2}\pm\frac{3\pi}{8}}{2}$, which may be implied by their answer. Answer only with no working though is M0A0.

If they have a mixture of radians and degrees within an equation and they do not "recover" then the method mark cannot be scored.

A1: $(\theta=)\ \frac{15\pi}{16}$ must be exact following a correct equation. May be seen as $\left(\frac{15\pi}{16}, -3\right)$ isw if they proceed to write the coordinates the wrong way round.

B1ft: y coordinate $Q=-3$ or follow through on $2-"A"$ (may be seen on the diagram or by the question). If there is a contradiction, then the answer in the main body of the work takes precedence. If they proceed to write the coordinates the wrong way round then isw. May be seen as a pair of coordinates.

(ii)(c) **In EPEN this is M1A1dM1A1 but we are marking this M1dM1A1A1**

Solutions with no working in (c) scores 0 marks.

M1: Sets $0="5"\sin\left(2\theta-\frac{3\pi}{8}\right)+2$ (which may be implied) and proceeds to $\sin\left(2\theta-\frac{3\pi}{8}\right)=\pm\frac{2}{"5"}$ o.e.

May be implied by e.g. $2\theta-\frac{3\pi}{8}="-0.41..."$ or any other equivalent angle in radians or $2\theta-67.5="-23.5..."$ or any other equivalent if working in degrees. You may need to check this. Allow use of X for 2θ or $2\theta-\frac{3\pi}{8}$

dM1: Proceeds from $\sin\left(2\theta-\frac{3\pi}{8}\right)=\pm\frac{2}{5}$ to $2\theta-\frac{3\pi}{8}=\arcsin\left(\pm\frac{2}{5}\right)=...$ which is one of the values below:

$2\theta-\frac{3\pi}{8}=\arcsin\left(-\frac{2}{5}\right)=-0.41, 3.6, 5.9, 9.8, 12.2$ $2\theta-\frac{3\pi}{8}=\arcsin\left(\frac{2}{5}\right)=0.41, 2.7, 6.7, 9.0, 13.0, 15.3$

May be implied by $2\theta=\arcsin\left(\pm\frac{2}{5}\right)+1.17...$ or allow the expression $=\dfrac{\arcsin\left(\pm\frac{2}{5}\right)+\frac{3\pi}{8}}{2}$

The sign slip is only condoned before they take arcsin(…)

A1: Any one of the values in the table provided M1dM1 has been scored. Do not withhold this mark if other incorrect angles are seen.

	Radians (awrt)	Degrees (awrt)
θ	0.38, 2.4, 3.5, 5.5, 6.7, 8.6, 9.8, 11.8, 12.9	22, 136, 202, 316, 382, 496, 562, 676, 742

A1: $\theta=$ awrt 5.51 only (must be in radians). **(Can only be scored from correct working and all previous marks are scored)**

Question Number	Scheme	Marks
10 (a)	$\left(\frac{dy}{dx}=\right) x-2187x^{-\frac{5}{2}}$	M1, A1
	Sets $x-2187x^{-\frac{5}{2}}=0\Rightarrow x^{\frac{7}{2}}=2187\left(\text{or e.g. } x=\left(\sqrt[7]{2187}\right)^2\right)\Rightarrow x=9$ *	dM1A1*
		(4)
(b)	e.g. $\int\left\{\frac{1}{2}x^2+1458x^{-\frac{3}{2}}-74\right\}dx=\frac{1}{6}x^3-2916x^{-\frac{1}{2}}-74x$ or $\int\left\{\frac{1}{2}x^2+1458x^{-\frac{3}{2}}-\text{"}94.5\text{"}\right\}dx=\frac{1}{6}x^3-2916x^{-\frac{1}{2}}-94.5x$	M1A1ft
	y value at P is 20.5	B1
	e.g. Area $R=\left[\frac{1}{6}x^3-2916x^{-\frac{1}{2}}-74x\right]_4^9-(9-4)\times\text{"}20.5\text{"}$ $=\left(\frac{1}{6}\times9^3-2916\times9^{-\frac{1}{2}}-74\times9\right)-\left(\frac{1}{6}\times4^3-2916\times4^{-\frac{1}{2}}-74\times4\right)-(9-4)\times\text{"}20.5\text{"}$	dM1
	$\left(-1516\frac{1}{2}+1743\frac{1}{3}-102\frac{1}{2}\right)=124\frac{1}{3}$	A1
		(5)
		Total 9

(a)

M1: Attempts to differentiate with one index correct e.g. $...x^2\rightarrow...x$ or $x^{-\frac{3}{2}}\rightarrow x^{-\frac{3}{2}-1}$ but not for $74\rightarrow0$.

A1: Correct differentiation. May be left unsimplified but indices must be processed.

dM1: Either solves their $\frac{dy}{dx}=0$ of the form $x-...x^m=0$ where m is a fraction via $x^n=A$ or $x=A^{\frac{1}{n}}$ It must be a solvable equation and they must correctly deal with the indices for their $\frac{dy}{dx}$. Look out for attempts where $\frac{dy}{dx}$ is manipulated before being set equal to zero which is acceptable.

e.g. $\frac{dy}{dx}=x^{\frac{7}{2}}-2187=0\Rightarrow x^{\frac{7}{2}}=2187\Rightarrow x=...$

Alternatively substitutes $x=9$ into their $\frac{dy}{dx}$ and finds its value.

A1*: Correct calculations and working leading to prove that the x coordinate of P is 9. It is sufficient to achieve $x^n=A$ and proceed directly to $x=9$ but e.g. $x-2187x^{-\frac{5}{2}}=0\Rightarrow x=9$ would be dM0A0.

It is also acceptable to proceed from $x-2187x^{-\frac{5}{2}}=0$ to an expression for x which is not the given answer before achieving $x=9$ e.g. $x=\left(\sqrt[7]{2187}\right)^2$

If using the verification method they should conclude that $x=9$ (or have a preamble that if $x=9$ is substituted in then $\frac{dy}{dx}=0$, then substitute in, achieves 0 followed by e.g. tick, QED etc.)

(b) **Note that if no algebraic integration is seen then maximum score is M0A0B1dM0A0**

M1: Attempts to integrate with one index correct e.g. $...x^2 \to ...x^3$ or $x^{-\frac{3}{2}} \to x^{-\frac{3}{2}+1}$ (indices do not need to be processed for this mark). Also accept $\pm 74 \to \pm 74x$ or $"-94.5" \to "-94.5"x$

A1ft: Correct integration. $\frac{1}{6}x^3 - 2916x^{-\frac{1}{2}} - 74x$ or follow through if curve – line is integrated proceeding to $\frac{1}{6}x^3 - 2916x^{-\frac{1}{2}} - "94.5"x$ (allowing for an error on the coefficient of x when attempting to subtract before integrating)
May be left unsimplified but indices must be processed (may be implied by further working – not just the final answer). Do not be concerned with spurious notation e.g. a $\mathrm{d}x$ or integral sign still present.

B1: y value at P is 20.5 or e.g. $\frac{41}{2}$. This may be seen as part of a wider calculation or on the diagram or next to the questions (can also be scored for sight in (a)) This mark can be scored for sight of -94.5 or $-94.5x$ when they are attempting to integrate curve – line.

dM1: Full method to find the area of R. They do not need to proceed as far as finding a value for the area, the values embedded is sufficient. It is dependent on the previous method mark.
The method for finding y at P must be correct (may be implied by e.g. 20.5 or attempting to substitute $x = 9$ into the equation of the curve).
If no integration is seen then this mark cannot be scored.
If they proceed from the integrated expression to the answer without showing some substitution of the limits (or evidence of this which is not the answer) then dM0A0.
e.g. $\left[\frac{1}{6}x^3 - 2916x^{-\frac{1}{2}} - 74x\right]_4^9 - (9-4)\times 20.5 = 124\frac{1}{3}$ scores dM0A0 (no evidence of limits substituted)
e.g. $-1516\frac{1}{2} + 1743\frac{1}{3} - 102\frac{1}{2} = 124\frac{1}{3}$ scores dM1A1 (evidence of limits substituted)
(**Note** $\frac{1361}{6} = 226\frac{5}{6}$ is only the area under the curve so this is not a full method to find the required area)

A1: Correct working and calculations seen leading to the answer of $124\frac{1}{3}$ o.e. $\frac{373}{3}$ including $124.\dot{3}$ or $124.\bar{3}$ 124.33… but not rounded answers of 124.3
Can only be scored provided all of the previous marks have been scored.