Pearson Edexcel

Mark Scheme (Results)

January 2022

Pearson Edexcel International Advanced Level In Mechanics M3 (WME03) Paper 01

Question Number	Scheme	Marks
1.	Area $=\int_0^a\left(x^2+ax\right)\mathrm{d}x=\left[\frac{1}{3}x^3+\frac{1}{2}ax^2\right]_0^a=\frac{5a^3}{6}$	M1A1
	$\int \frac{1}{2}y^2\,\mathrm{d}x=\int_0^a\frac{1}{2}\left(x^4+2ax^3+a^2x^2\right)\mathrm{d}x$	M1
	$=\frac{1}{2}\left[\frac{1}{5}x^5+\frac{a}{2}x^4+\frac{a^2}{3}x^3\right]_0^a\ \left(=\frac{31a^5}{60}\right)$	DM1A1
	$\overline{y}=\dfrac{\int \frac{1}{2}y^2\,\mathrm{d}x}{\int y\,\mathrm{d}x}=\frac{31a^5}{60}\div\frac{5a^3}{6}=\frac{31a^2}{50}$	M1A1 (7)
		[7]
M1 **A1**	Attempt the **area** by integration. Powers of both terms to increase by 1. Correct area.	
M1	Use $\int\frac{1}{2}\,y^2\,dx$ to give $\int_0^2\frac{1}{2}(x^4+2ax^3+a^2x^2)dx$. Limits not needed. Squaring to be correct. For method mark, condone missing $\frac{1}{2}$ or any multiple.	
DM1 **A1**	Attempt the integration (powers of at least 2 terms to increase by 1). Depends on second M mark. Correct integration and correct limits shown. Limits needed but substitution does not need to be seen.	
M1	Use $\overline{y}=\dfrac{\int\frac{1}{2}y^2\,\mathrm{d}x}{\int y\,\mathrm{d}x}$ Note: This independent method mark is for use of the correct formula.	
A1	Correct answer.	

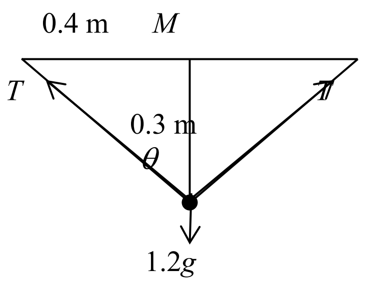
Question Number	Scheme	Marks
2	Any correct sin or cos ratio.	B1
	$T\cos 60° + N = mg$	M1A1
	$T\sin 60° = mr\omega^2 = m\omega^2 \times 2l\sin 60°$	M1A1
	$\frac{1}{2}T + N = mg \qquad \frac{1}{2}T = ml\omega^2$	
	$\Rightarrow\ ml\omega^2 + N = mg$	DM1
	$N \geq 0 \Rightarrow l\omega^2 \leq g$	DM1
	$\boldsymbol{\omega} \leq \sqrt{\frac{g}{l}}$ *	A1 * (8) **[8]**
B1	May be seen explicitly or used in an equation.	
M1	Attempt at vertical resolution, 3 terms needed.	
A1	Correct equation.	
M1	Attempt an equation for NL2 along the radius, acceleration in either form but not $'a'$. May have r and v in the equation.	
A1	Fully correct equation with the acceleration in $r\omega^2$ form and radius in terms of l.	
DM1	Eliminate T Depends on both M marks above. Must see an equation still involving N.	
DM1	Use $N \geq 0$ Depends on all 3 M marks above. Must see correct inequality stated, not $N = 0$ or $N > 0$.	
A1*	Reach the **given** result from fully correct working.	
ALT	For solutions that do not use vertical equilibrium but go straight to a vertical inequality.	
B1	As above.	
M1 A1 DM1	Forming a correct inequality $T\cos 60 \leq mg$	
M1 A1	Attempt NL2 as the main mark scheme.	
DM1	Eliminate T Depends on M marks above.	
A1*	Reach the given result from fully correct working.	

Question Number	Scheme	Marks
3		
(a)	$mv\dfrac{\mathrm{d}v}{\mathrm{d}x} = mg\sin\alpha - \dfrac{1}{3}mx^2$	M1A1
	$\dfrac{1}{2}v^2 = xg\sin\alpha - \dfrac{1}{9}x^3 \ \left(+c\right)$	DM1A1
	$x = 2 \quad \dfrac{1}{2}v^2 = 2g\sin\alpha - \dfrac{8}{9}$	DM1
	(v = 3.728...)	
	$v = 3.7$ or $3.73 \ \left(\mathrm{m\,s^{-1}}\right)$	A1cso (6)
ALT	By energy:	
	$mg\sin\alpha x = \int \dfrac{1}{3}mx^2\mathrm{d}x + \dfrac{1}{2}mv^2$	M1A1
	$xg\sin\alpha = \dfrac{1}{9}x^3 + \dfrac{1}{2}v^2\left(+c\right)$	DM1A1
	$x = 2 \quad \dfrac{1}{2}v^2 = 2g\sin\alpha - \dfrac{8}{9}$	DM1
	$v = 3.7$ or $3.73 \ \left(\mathrm{m\,s^{-1}}\right)$	A1
(b)	$v = 0 \Rightarrow x^2 = 9g\sin\alpha = 9g\times\dfrac{2}{5} \ \left(x \neq 0\right)$	
	$x = 5.939... \Rightarrow OA = 5.9$ or $5.94 \ \left(\mathrm{m}\right)$	M1A1 (2) **[8]**

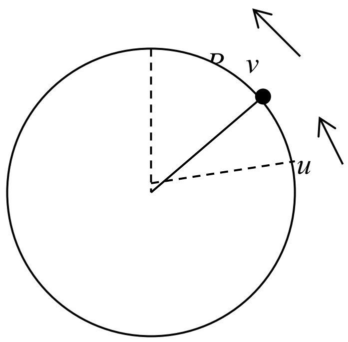
Question Number	Scheme	Marks
(a)		
M1	Attempt an equation of motion parallel to the plane with acceleration in any form (including a)	
A1	Correct equation with the acceleration in $v\frac{\mathrm{d}v}{\mathrm{d}x}$ form	
DM1	Attempt the integration, powers increase by 1 in 2 terms – the constant may be missing. Acceleration must be in $v\frac{\mathrm{d}v}{\mathrm{d}x}$ form.	
A1	Correct integration.	
DM1	Sub $x = 2$ in their expression for v^2 Depends on all previous M marks.	
A1	Correct value for v and $+c$ should be dealt with. Must be 2 or 3 sf	
ALT		
M1	Attempt a 3 term energy equation – KE, GPE, work done. Integral form is not required here.	
A1	Fully correct equation with integral form for work done. Rest as main scheme.	
(b)		
M1	Use $v = 0$ in their expression for v and obtain a value of x	
A1	Correct value of length OA. (Allow if x instead of OA) Must be 2 or 3 sf (unless already penalised in (a)	
ALT (b)	Start again with energy and integrate to obtain a value of x	
M1	See mark scheme.	
A1		

Question Number	Scheme	Marks
4(a)	Ratio of masses: $4\pi a^2$ $4\pi a\times ka$ $8\pi a^2$ $12\pi a^2+4k\pi a^2$	B1
	Distances: (0) $\frac{k}{2}a$ $(1+k)a$ $\bar{y}$	B1
	$(0+)k\times\frac{k}{2}a+2(1+k)a=(k+3)\bar{y}$	M1A1ft
	$\left(\frac{k^2}{2}+2+2k\right)a=(k+3)\bar{y}$	
	$\bar{y}=\frac{(k^2+4k+4)}{2(k+3)}a$ *	A1 * (5)
(b)	$\tan 60°=\frac{(k^2+4k+4)}{2(k+3)}a\div 2a$	M1
	$k^2+4k(1-\sqrt{3})+(4-12\sqrt{3})=0$	A1
	$k>0\Rightarrow k=5.8147...=5.8$ or 5.81 or better	A1 (3) **[8]**
(a)		
B1	Correct ratio of masses – any equivalent to that shown	
B1	Correct distances from O or a parallel axis.	
M1	Attempt a moments equation. Must be dimensionally correct (not using volumes) and have no extra terms.	
A1ft	Correct equation, follow through their ratio of masses and distances	
A1*	Correct **given** expression with sufficient working	
(b)		
M1	Use $\tan 60=\frac{\bar{y}}{2a}$ or $\frac{2a}{\bar{y}}$ May also use $\tan 30$	
A1	Obtain the correct 3TQ	
A1	Correct value for k.	
	Note for (a): The distance from O for the **combined** cylinder and base is $\frac{ak^2}{2(1+k)}$.	

Question Number	Scheme	Marks
5(a)	$x = 4\cos\left(\frac{1}{5}\pi t\right) \quad \dot{x} = -4\times\frac{\pi}{5}\sin\left(\frac{1}{5}\pi t\right)$	
	$\ddot{x} = -4\times\left(\frac{\pi}{5}\right)^2 \cos\left(\frac{1}{5}\pi t\right)$	M1A1
	$\ddot{x} = -\left(\frac{\pi}{5}\right)^2 x \quad \therefore \text{SHM}$	A1 (3)
(b)	period $= \dfrac{2\pi}{\frac{\pi}{5}} = 10$ (s)	M1A1 (2)
(c)	amplitude $= 4$ (m)	B1 (1)
(d)	$\dot{x} = -4\times\frac{\pi}{5}\sin\left(\frac{1}{5}\pi t\right)$ or $\lvert\dot{x}_{\max}\rvert = a\omega$	M1
	Max speed $= 4\times\frac{\pi}{5} = \frac{4\pi}{5}$ or 0.8π (ms^{-1})	A1 (2)
(e)	At A $x = 1.5 \quad 1.5 = 4\cos\left(\frac{1}{5}\pi t\right) \Rightarrow t_A = \frac{5}{\pi}\cos^{-1}\left(\frac{1.5}{4}\right)$	M1A1
	At B $x = -2.5 \quad -2.5 = 4\cos\left(\frac{1}{5}\pi t\right) \Rightarrow t_B = \frac{5}{\pi}\cos^{-1}\left(\frac{-2.5}{4}\right)$	A1
	Time A to B $= t_B - t_A = \frac{5}{\pi}\cos^{-1}\left(\frac{-2.5}{4}\right) - \frac{5}{\pi}\cos^{-1}\left(\frac{1.5}{4}\right) = 1.6862... = 1.7$ or better (s)	A1 (4)
		[12]
(a)		
M1	Differentiate the given expression for x twice (**Both derivatives must be shown)** Need to see: cos to sin to cos (ignore signs)	
A1	Both derivatives correct	
A1	Rewrite in the standard form for SHM and give the conclusion.	
(b)		
M1	Correct method	
A1	Correct period	
(c)		
B1	Correct amplitude	
(d)		
M1	Use either method to obtain the max speed	
A1	Correct max speed	
(e)		
M1	Find the time from the start to either A or B	
A1	One correct time	
A1	Second relevant time	
A1	Correct time from A to B. 1.7 (s) or better	

Question Number	Scheme	Marks
ALT (e)	$1.5 = 4\sin\left(\frac{1}{5}\pi t\right) \quad \Rightarrow t_A = \frac{5}{\pi}\sin^{-1}\left(\frac{1.5}{4}\right)$ M1A1 $2.5 = 4\sin\left(\frac{1}{5}\pi t\right) \quad \Rightarrow t_B = \frac{5}{\pi}\sin^{-1}\left(\frac{2.5}{4}\right)$ A1 ${}_At_B = t_B + t_A = \frac{5}{\pi}\sin^{-1}\left(\frac{2.5}{4}\right) + \frac{5}{\pi}\sin^{-1}\left(\frac{1.5}{4}\right) = 1.6862..$ 1.7 or better A1	
6(a)	$T = \frac{\lambda x}{l} \Rightarrow 30 = \frac{\lambda \times 0.3}{0.5}$	M1A1
	$\lambda = 50$ *	A1* (3)
(b)		
	Extension = 0.5 m (used in (b) or (c))	B1
	$T = \frac{50 \times 0.5}{0.5} = (50)$	M1A1ft
	$\mathrm{R}(\uparrow) \quad 2T\cos\theta - 1.2g = 1.2a$	M1
	$100 \times \frac{3}{5} - 1.2 \times 9.8 = 1.2a$	A1ft
	$a = 40.2 \quad a = 40$ or 40.2 m s^{-2} (positive)	A1 (6)
(c)	$\text{E.P.E.} = \frac{1}{2} \times 50 \times \frac{0.5^2}{0.5}$	B1ft (any correct EPE)
	$1.2g \times 0.3 + \frac{1}{2} \times 1.2v^2 = \frac{1}{2} \times 50 \times \frac{0.5^2}{0.5} - \frac{1}{2} \times 50 \times \frac{0.3^2}{0.5}$	M1A1A1
	$v^2 = \frac{1}{0.6}\left(25 \times \frac{0.5^2}{0.5} - 25 \times \frac{0.3^2}{0.5} - 1.2g \times 0.3\right) \quad (= 7.452)$	DM1
	$v = 2.730... \quad = 2.7$ or $2.73\,\mathrm{m\,s^{-1}}$	A1 (6)

Question Number	Scheme	Marks
		[15]
(a) **M1** **A1** **A1*** **(b)** **B1** **M1** **A1ft** **M1** **A1ft** **A1** **(c)** **B1ft** **M1** **A1** **A1** **DM1** **A1**	 Use HL with $T = 30$ Correct equation Obtain **given** value for λ from fully correct working Correct extension, seen explicitly or used in (b) or (c) Extension 0.25 if half string used. Use HL to form an equation using $\lambda = 50$ and their extension. Correct equation, ft their extension Attempt a vertical equation of motion. Must have 3 terms with *T* resolved *a* could be negative. Correct equation with their value of *T* Correct value of the acceleration (positive). Must be 2 or 3 sf. Either EPE correct, follow through their extension. Energy equation, from *C* to the ceiling. PE, KE and 2 EPE terms required. EPE of the form $k\frac{x^2}{l}$. Both EPE terms correct. Completely correct equation Solve for v^2 or v Correct value for v. Must be 2 or 3 sf (unless penalised in (b)).	

Question Number	Scheme	Marks
7		
(a)	$mgl(\cos\theta - \cos\alpha) = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$	M1A1A1
	$v^2 = u^2 - 2gl(\cos\theta - \cos\alpha)$ *	A1* (4)
(b)	$\cos\alpha = \frac{2}{5}$ $\quad v^2 = 3gl - 2gl\left(\cos\theta - \frac{2}{5}\right)$	M1 A1
	At top $\theta = 0°$ $\quad v^2 = 3gl - 2gl \times \frac{3}{5}$	M1
	$v^2 = \frac{9gl}{5}$	
	$v^2 > 0 \Rightarrow$ complete circle *	A1* (4)
(c)	Equation of motion along radius at lowest point: $kT - mg = \frac{mw^2}{l}$	M1A1
	$\theta = 180$ $\quad w^2 = 3gl - 2gl\left(-1 - \frac{2}{5}\right) = \frac{29gl}{5}$	M1
	$kT = \frac{m}{l} \times \frac{29gl}{5} + mg = \frac{34mg}{5}$	M1A1
	At highest point: $\quad T_2 + mg = \frac{mv^2}{l}$	M1
	$\theta = 0$ $\quad T = \frac{9mg}{5} - mg = \frac{4mg}{5}$	M1 A1
	$k\frac{4mg}{5} = \frac{34mg}{5} \Rightarrow k = \frac{17}{2}$	A1
		[17]

Question Number	Scheme	Marks

(a)	
M1	Attempt energy equation from A to general position. Must have a difference of 2 PE terms and a difference of 2 KE terms.
A1	Correct gain in PE or loss of KE
A1	Fully correct equation
A1*	Reach the **given** result from fully correct working
(b)	
M1	Sub $u = \sqrt{3gl}$ and $cos\alpha = \frac{2}{5}$ in the result in (a)
A1	Correct equation
M1	Put $\theta = 0$ to find an expression for v^2 at the top (maybe finding KE)
A1*	Fully correct working and conclusion with reason eg reference to $v^2,\ v,\ KE$
(c)	
M1	Form an equation of motion along the radius at the lowest point. Acceleration in either form.
A1	Correct equation with acceleration in v^2/r form
M1	Use $\theta = 180$ in result from (a) to obtain an expression for w^2
M1	Eliminate w^2 and obtain an expression for kT
A1	Correct expression for kT
M1	Form an equation of motion along the radius at the highest point. Acceleration in either form.
M1	Sub $\theta = 0$ and obtain an expression for T
A1	Correct expression for T
A1	Correct value of k. Must be exact.
NB	The equation of motion at the top may be seen first. Award M1A1 for either equation correct and M1 for the second.

Question Number	Scheme	Marks
ALT1 7(c)	Better equation of motion at top **or** bottom: $$T - mg = \frac{mv^2}{l}$$ $$T + mg = \frac{mv^2}{l}$$	M1 A1
	Other equation of motion – see above	M1
	Finding speed at the bottom: $\theta = 180 \qquad w^2 = 3gl - 2gl\left(-1 - \frac{2}{5}\right) = \frac{29gl}{5}$	M1
	Finding maximum Tension (lowest point) $\theta = 180,\ T = \frac{m}{l} \times \frac{29gl}{5} + mg = \frac{34mg}{5}$	M1 A1
	Finding minimum Tension (highest point) $\theta = 0 \qquad T = \frac{9mg}{5} - mg = \frac{4mg}{5}$	M1 A1
	Dividing Tensions to reach the correct answer $k\frac{4mg}{5} = \frac{34mg}{5} \Rightarrow k = \frac{17}{2}$	A1

Question Number	Scheme	Marks
ALT 2 7 (c)	General equation of motion: $$T + mgcos\theta = \frac{mv^2}{l}$$	M1 A1
	Use of $u = \sqrt{3gl}$ and $cos\alpha = \frac{2}{5}$ to replace v^2 in their equation of motion	M1
	Finding speed at the lowest point: $\theta = 180 \qquad w^2 = 3gl - 2gl\left(-1 - \frac{2}{5}\right) = \frac{29gl}{5}$	M1
	Finding maximum Tension (lowest point) $\theta = 180,\ T = \frac{m}{l} \times \frac{29gl}{5} + mg = \frac{34mg}{5}$	M1 A1
	Finding minimum Tension (highest point) $\theta = 0,\ T = \frac{9mg}{5} - mg = \frac{4mg}{5}$	M1 A1
	Dividing Tensions to reach the correct answer $k\frac{4mg}{5} = \frac{34mg}{5} \Rightarrow k = \frac{17}{2}$	A1