Mark Scheme (Results)

Summer 2023

Question Number		Scheme	Marks
1 (a)		When the data is ordinal e.g. Judges' ranks	B1
		When a non-linear relationship might be expected	B1
			(2)
(b)		$H_0 : \rho = 0$, $H_1 : \rho \neq 0$	B1
		Critical value $r_s = -0.6485$ or CR: $r_s \leqslant -0.6485$ $\left(\text{and } r_s \geqslant 0.6485\right)$	B1
		Reject H_0 or significant or lies in the critical region	M1
		The Spearman's rank correlation coefficient shows there is sufficient evidence of a correlation [between the length and maximum diameter of the melons]	A1
			(4)
(c)		$H_0 : \rho = 0$, $H_1 : \rho < 0$	B1
		Critical value $r = -0.5494$ or CR: $r \leqslant -0.5494$	B1
		The product moment correlation coefficient shows there is insufficient evidence of a **negative** correlation [between the length and maximum diameter of the melons]	B1
			(3)
		Notes	**Total 9**
(a)	**B1**	For one correct condition	
	B1	For a second correct condition. Condone not underlying normal	
(b)	**B1**	For both hypotheses correct. Must be in terms of ρ. Must be attached to H_0 and H_1	
	B1	For critical value of –0.6485 (Allow –0.5636 if a one tailed test is stated for H_1) Condone 0.6485 if compared with 0.673	
	M1	A correct statement – no context needed but do not allow contradicting non contextual comments. ft their CV provided the CV is negative (May be implied by a correct conclusion) Condone a positive CV if a comparison with 0.673 seen	
	A1	For a correct conclusion which is rejecting H_0 Allow negative correlation This mark is independent of the hypotheses	
(c)	**B1**	For both hypotheses correct. Must be in terms of ρ. Must be attached to H_0 and H_1	
	B1	For critical value of –0.5494 (Allow –0.6319 if a two tailed test is stated for H_1) Condone 0.5494 if compared with 0.525	
	B1	For a correct conclusion which is not rejecting H_0	

Question Number	Scheme	Marks
2 (a)	$\dfrac{60 \times 60}{240}$ or $\dfrac{60 \times 84}{240}$ or $\dfrac{60 \times 96}{240}$	M1
	15 and 21 and 24	A2
		(3)

(b)	H_0 : There is no association between the payment amount and payment method used H_1 : There is an association between the payment amount and payment method used	B1

Observed	Expected	$\dfrac{(O-E)^2}{E}$	
23	15	$\dfrac{(23-'15')^2}{'15'} = 4.2667$	M1
21	21	$\dfrac{(21-'21')^2}{'21'} = 0$	
16	24	$\dfrac{(16-'24')^2}{'24'} = 2.6667$	

$\chi^2 = 2.4048 + '4.2667' + '0' + '2.6667'$		M1
$= 9.3381\ldots$ awrt 9.34		A1
$v = (3-1)(3-1) = 4$ $\chi^2_4(0.05) = 9.488 \Rightarrow$ CR: $X^2 \geqslant 9.488$		B1 B1ft
[Not in the CR/Not significant/Do not reject H_0] There is no evidence of an association between **the payment amount and payment method used**		dA1
		(7)

Notes			**Total 10**
(a)	**M1**	For a correct method for finding one expected value	
	A2	For all 3 answers correct (A1 for 2 correct answers or 1 correct and 3 values that sum to 60)	
(b)	**B1**	Both hypotheses correct. Must mention method **and** amount with payment at least once. (may be written in terms of independence)	
	M1	For a correct method for finding all three contributions to the χ^2 value ft their part a May be implied by 3 correct values If expected values are incorrect then working must be shown	
	M1	For adding their values to 2.4048 (If all 9 values are calculated the 6 values not found in part (a) must have working shown or the correct values seen or awrt 9.34)	
	A1	awrt 9.34	
	B1	$v = 4$ This mark can be implied by a correct critical value of 9.488	
	B1ft	9.488 or better ft their DoF	
	dA1	Dependent on both M marks. A correct contextualised conclusion which is not rejecting H_0 Must mention **method** and **amount**. If no hypotheses or they are the wrong way round, then A0 here. Contradictory statements score A0. e.g. "Significant, do not reject H_0".Condone "relationship" or "connection" here but **not** "correlation".	

Question Number		Scheme	Marks
3 (a)		It is not a statistic as it involves <u>unknown</u> [population] parameter	B1
			(1)
(b)		$E(S) = E\left(\dfrac{3}{5}X_1 + \dfrac{5}{7}X_2\right) = \dfrac{3}{5}E(X_1) + \dfrac{5}{7}E(X_2)$	M1
		$= \dfrac{3}{5}\mu + \dfrac{5}{7}\mu = \dfrac{46}{35}\mu \neq \mu$ So S is a biased estimator for μ	A1
			(2)
(c)		$'\dfrac{46}{35}\mu' - \mu = \dfrac{11}{35}\mu$	B1ft
			(1)
(d)		$E(Y) = aE(X_1) + bE(X_2) = \mu$ $\Rightarrow (a+b)\mu = \mu$	M1
		$a + b = 1$	A1
			(2)
(e)		$\mathrm{Var}\left(Y\right) = a^2\mathrm{Var}(X_1) + b^2\mathrm{Var}(X_2) = \left(a^2 + b^2\right)\sigma^2$	M1
		$\mathrm{Var}\left(Y\right) = \left(a^2 + {}'(1-a)'^2\right)\sigma^2$	M1
		$\mathrm{Var}\left(Y\right) = \left(2a^2 - 2a + 1\right)\sigma^2$ *	A1*
			(3)
		Notes	**Total 9**
(a)	**B1**	For a correct explanation Allow σ is unknown (Do not allow σ is unknown variance)	
(b)	**M1**	For writing or using $E(S) = aE(X_1) + bE(X_2)$ Condone missing subscripts	
	A1	cao (Allow $1.31\mu \neq \mu$)	
(c)	**B1ft**	Follow through their part (a) $- \mu$	
(d)	**M1**	For writing or using $E(Y) = aE(X_1) + bE(X_2) = \mu$ (May be implied by $a + b = 1$) Condone missing subscripts	
	A1	Cao	
(e)	**M1**	For writing or using $\mathrm{Var}\left(Y\right) = a^2\mathrm{Var}(X_1) + b^2\mathrm{Var}(X_2)$ Condone missing subscripts	
	M1	For substitution of $b = 1 - a$ ft their part (d) into their expression for Var(Y)	
	A1*	Answer is given so no incorrect working must be seen	

Question Number	Scheme	Mark
4 (a)	$\left[\int_a^{a+1} \dfrac{2}{25}t\,\mathrm{d}t\right] = \dfrac{2}{25}\left[\dfrac{t^2}{2}\right]_a^{a+1}$ or $\mathrm{F}(t) = \begin{cases} 0 & t < 0 \\ \dfrac{1}{25}t^2 & 0 \leqslant t < 5 \\ 1 & t > 5 \end{cases}$ or	M1
	$\dfrac{1}{2}\left(\dfrac{2}{25}(a+1) + \dfrac{2}{25}a\right)(a+1-a)$	
	$\dfrac{1}{25}\left((a+1)^2 - a^2\right)$ or $\dfrac{1}{25}(a+1)^2 - \dfrac{1}{25}a^2$ or $\left(\dfrac{1}{25}a + \dfrac{1}{25} + \dfrac{1}{25}a\right)$	M1
	$\dfrac{1}{25}\left(a^2 + 2a + 1 - a^2\right)$ oe $\left[= \dfrac{1}{25}(2a+1)\right]$*	A1*
		(3)
(b)	H_0 : The data could be modelled by the p.d.f	B1
	H_1 : The data could not be modelled by the p.d.f	
	Expected frequencies: 6, 18, 30, 42, 54	M1 A1
	$\sum \dfrac{(O-E)^2}{E} = \dfrac{(10-{}'6')^2}{'6'} + \ldots + \dfrac{(68-{}'54')^2}{'54'}$	M1
	or $\sum \dfrac{O^2}{E} - N = \dfrac{10^2}{'6'} + \ldots + \dfrac{68^2}{'54'} - 150$ or $2.666\ldots + 1.388\ldots + 1.2 + 1.166\ldots + 3.629$	
	$= 10.05\ldots$ awrt 10.1	A1
	$v = 4$	B1
	$\chi_4^2(0.05) = 9.488 \Rightarrow \mathrm{CR} \geqslant 9.488$	B1ft
	[In the CR so there is sufficient evidence to reject H_0]	
	Sufficient evidence to say that data does not fit the given p.d.f	dA1
		(8)
	Notes	**Total 11**
(a)	**M1**	For correct integration, ignore limits or finding the area of a trapezium
	M1	For substitution of the limits. May be implied by $\dfrac{1}{25}\left(a^2 + 2a + 1 - a^2\right)$ or simplifying the expression for the area of the trapezium
	A1*	Answer is given so no incorrect working should be seen. At least one correct line of working from the method mark to the final answer should be seen
(b)	**B1**	Both hypotheses correct. Allow H_0 : The p.d.f/$\mathrm{f}(t)$ is a suitable model H_1 : The p.d.f/$\mathrm{f}(t)$ is not a suitable model
	M1	For a correct method to find at least one expected frequency e.g. $\dfrac{1}{25} \times 150$ Ignore any reference to limits
	A1	For all 5 expected frequencies correct
	M1	For an attempt at the test statistic, at least 2 correct expressions/values ft their expected frequencies
	A1	awrt 10.1
	B1	$v = 4$ This mark can be implied by a correct critical value of 9.488
	B1ft	9.488 or better ft their DoF
	dA1	Dependent on 2$^{\mathrm{nd}}$ M1. A correct conclusion based on their χ^2 critical value If no hypotheses or they are the wrong way round, then A0 here.

Question Number	Scheme	Marks
5 (a)	$\bar{x} \pm 1.6449 \times \dfrac{5}{\sqrt{10}}$	M1 B1
	$\bar{x} \pm 2.60 \Rightarrow (\bar{x} - 2.60,\ \bar{x} + 2.60)\,*$	A1*
		(3)
(b)	$\bar{y} \pm 1.96 \times \dfrac{3}{\sqrt{20}}$	M1 B1
	$\bar{y} \pm 1.31 \Rightarrow (\bar{y} - 1.31,\ \bar{y} + 1.31)$	A1
		(3)
(c)(i)	$\bar{X} - \bar{Y} \sim N\left(\mu - \mu,\ \dfrac{5^2}{10} + \dfrac{3^2}{20}\right) \Rightarrow \bar{X} - \bar{Y} \sim N(0,\ 2.95)$	M1 A1
(ii)	Do not overlap when either	
	$\bar{x} - 2.60 > \bar{y} + '1.31'$ or $\bar{x} + 2.60 < \bar{y} - '1.31'$	M1
	$\bar{x} - \bar{y} > 3.91$ or $\bar{x} - \bar{y} < -3.91$	A1ft
	$2 \times P(\bar{X} - \bar{Y} > 3.91) = 2 \times P\left(Z > \dfrac{'3.91' - '0'}{'\sqrt{2.95}'}\right) = \left[2 \times P(Z > 2.276...)\right]$	M1 M1
	$[2 \times 0.0113] = 0.0226$ (calculator gives $[2 \times 0.0114...] = 0.0228$)	A1
		(7)
Notes		**Total 3**
(a)	**M1**	For use of $\bar{x} \pm z$ value $\times \dfrac{5}{\sqrt{10}}$
	B1	For use of $z = 1.6449$ or better
	A1*	Answer is given so no incorrect working should be seen (condone use of 1.645)
(b)	**M1**	For use of $\bar{y} \pm z$ value $\times \dfrac{3}{\sqrt{20}}$
	B1	For use of $z = 1.96$ or better
	A1	For $(\bar{y} - \text{awrt}1.31,\ \bar{y} + \text{awrt}1.31)$ Allow 1.315
(c)(i)	**M1**	For a correct method to find the variance (May be seen in a standardisation expression)
	A1	For $N(0,\ 2.95)$ (May be seen in a standardisation expression) Allow $N\left(0,\ \dfrac{5^2}{10} + \dfrac{3^2}{20}\right)$ oe
(ii)	**M1**	For $\bar{x} - 2.60 > \bar{y} + 1.31$ oe or $\bar{x} + 2.60 > \bar{y} - 1.31$ oe ft part (b)
	A1ft	For $\bar{x} - \bar{y} > '3.91'$ or $\bar{x} - \bar{y} < -'3.91'$ ft part (b)
	M1	For multiplying by 2 (may be seen at any stage of their working)
	M1	For standardising ft their 3.91, their mean and their standard deviation (Do not allow use of 2.6 or 1.31 as their 3.91)
	A1	For answers in the range awrt 0.0226 – awrt 0.0228

Question Number		Scheme	Marks
6 (a)		$\alpha = 5.1$	B1
		$\beta = \sqrt{\dfrac{1694.65 - 65 \times ('5.1')^2}{64}}$	M1
		$= 0.25$	A1
			(3)
(b)		$H_0 : \mu_A = \mu_B$ $H_1 : \mu_A < \mu_B$	B1
		$z = \pm \dfrac{5.0 - '5.1'}{\sqrt{\dfrac{0.24^2}{70} + \dfrac{'0.25'^2}{65}}}$	M1 M1
		$= -2.367...$ awrt -2.37	A1
		One tailed c.v. $z = -1.6449$ **or** CR: $z \leqslant -1.6449$	B1
		In CR/Significant/Reject H_0	M1
		Sufficient evidence to support Roxane's claim	A1
			(7)
(c)		Since the sample is **large** the **CLT** applies.	M1
		No [need to assume that the fat content is normally distributed]	A1
			(2)
(d)		Assumed that $s^2 = \sigma^2$ in **both** groups	B1
			(1)
		Notes	**Total 13**
(a)	**B1**	cao	
	M1	For a correct method to find β using their α	
	A1	Cao	
(b)	**B1**	Both hypotheses correct. Allow equivalent hypotheses. Must be in terms of μ	
	M1	For correct standard error ft their s in part a	
	M1	For an attempt to find the test statistic, ft their SE and their α	
	A1	awrt -2.37 (Allow 2.37)	
	B1	-1.6449 or better (seen) (Allow 1.6449 or better if comparing to their 2.37)	
	M1	A correct statement – need not be contextual but do not allow contradicting non contextual comments. ft their CV and test statistic	
	A1	A correct contextual statement e.g sufficient evidence to support that crisps from **brand A** have a **lower fat** content than the crisps from **brand B** (must include the words in bold)	
(c)	**M1**	A suitable comment that mentions large and CLT	
	A1	A correct answer, context not required.	
(d)	**B1**	For the assumption that sample variance = population variance for **both** groups	

Question Number	Scheme	Marks
7 (a)	$E(X) = 4 \times 15 - 3 \times 10 [= 30]$	M1
	$Var(X) = 4^2 \times 5^2 + 3^2 \times 4^2 [= 544]$	M1
	So $X \sim N(30, \ 544)$	
	$P(X < 40) = P\left(Z < \dfrac{40 - '30'}{'\sqrt{544}'}\right)\left[= P(Z < 0.428...)\right]$	M1
	$= 0.6664$ \qquad (Calculator gives 0.6659…) \qquad\qquad awrt 0.666	A1
		(4)
(b)	$E(A + B + D) = 15 + 10 + 3 \times 20 = [85]$	M1
	$Var(A + B + D) = 5^2 + 4^2 + 3 \times \sigma^2 = [41 + 3\sigma^2]$	M1
	So $A + B + D \sim N(85, \ 41+3\sigma^2)$	
	$P(A + B + D < 76) = P\left(Z < \dfrac{76 - 85}{\sqrt{41 + 3\sigma^2}}\right) = 0.242$	
	So $\dfrac{-9}{\sqrt{41 + 3\sigma^2}} = -0.7$ \qquad or \qquad $\dfrac{9}{\sqrt{41 + 3\sigma^2}} = 0.7$ \ (Calculator gives $-0.69988…$)	M1 A1
	$3\sigma^2 = \left(\dfrac{-9}{-0.7}\right)^2 - 41$	dM1
	$\sigma = 6.437...$ \qquad\qquad\qquad\qquad\qquad\qquad awrt 6.44	A1
		(6)
	Notes	**Total 10**
(a)	**M1**	For a correct method to find $E(X)$. May be implied by a correct standardisation expression.
	M1	For a correct method to find $Var(X)$ Allow $\sqrt{544}$ oe or 23.3^2 or better. May be implied by a correct standardisation expression.
	M1	For standardising $(\pm)$ using their mean and their variance
	A1	awrt 0.666
(b)	**M1**	For a correct method to find $E(A + B + D)$
	M1	For a correct method to find $Var(A + B + D)$
	M1	For standardising $(\pm)$ using their mean and their standard deviation which is in terms of σ^2 and setting equal to -0.7 or better. Allow $+0.7$
	A1	For the correct equation
	dM1	Dependent on the previous M mark. For squaring and rearranging leading to an equation in σ^2
	A1	awrt 6.44 (Do not award if previous A mark was not awarded)