



Mark Scheme (Results)

January 2021

Pearson Edexcel International Advanced Level
In Statistics 3 (WST03/01)

Question Number	Scheme	Marks
1. (a)	[In QP: 33, 15, 23] 29, 34, 39, 06, 31, 13, 42	M1A1 (2)
(b)	This will give 4 girls with numbers 15, 23, 06, 13 This will give 6 boys with numbers 33, 29, 34, 39, 31, 42	B1 B1 (2)
(c)	Since the highest number is 42 ...therefore may miss <u>older players</u>	M1 A1 (2)
[6 marks]		
Notes		
(a)	M1 for 7 numbers (at least 4 correct in any order) (Condone repeats but only count once towards the “4”) e.g. <u>29</u> , 33, <u>34</u> , <u>39</u> , 15, 29, <u>31</u> The 33 and 15 are repeats of those in QP and 29 is a repeat but all will count for the “7” This will score M1 as there are 4 of the correct numbers listed: 29, 34, 39 and 31 A1 for all 7 correct with no repeats	
(b)	1 st B1 for showing the 4 girls in sample (No ft for incorrect random numbers) 2 nd B1 for showing the 6 boys in the sample (No ft for incorrect random numbers)	
(c)	M1 for mention of highest number of 42 (or ft their highest number as long as < 60) A1 for stating that this means older players may be missing from the sample This can be awarded if their highest number is stated for M1 and is < 42	

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2. (a)	<table><tr><td>Student</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td><td>I</td><td>J</td><td>K</td></tr><tr><td>Objects rank</td><td>9</td><td>6</td><td>8</td><td>2</td><td>1</td><td>10</td><td>7</td><td>3</td><td>5</td><td>4</td><td>11</td></tr><tr><td>Maths rank</td><td>11</td><td>4</td><td>5</td><td>1</td><td>2</td><td>9</td><td>3</td><td>7</td><td>8</td><td>6</td><td>10</td></tr></table>											Student	A	B	C	D	E	F	G	H	I	J	K	Objects rank	9	6	8	2	1	10	7	3	5	4	11	Maths rank	11	4	5	1	2	9	3	7	8	6	10	M1
	Student	A	B	C	D	E	F	G	H	I	J	K																																				
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												M1																																				
$\sum d^2 = 4 + 4 + 9 + 1 + 1 + 1 + 16 + 16 + 9 + 4 + 1 = 66$											M1																																					
$r_s = 1 - \frac{6 \times "66"}{11(11^2 - 1)} \quad ; = \underline{0.7}$											dM1; A1																																					
(b)	<p>$H_0 : \rho = 0 \quad H_1 : \rho > 0$</p> <p>Critical value ($n = 11$ 5% one-tail) is 0.5364</p> <p>(Significant result so) there is evidence to support the teacher’s belief</p> <p><u>or</u> there is evidence of a positive correlation between short term memory and mathematical ability (o.e.)</p> <p><u>or</u> evidence that students with strong maths ability also have good short term memory (o.e.)</p>											B1 B1 B1																																				
(c)	<p>Data shows positive correlation but does not necessarily imply that enhanced short term memory <u>causes</u> increase in mathematical ability.</p>											B1																																				
												(5) 																																				

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3. (a)	All expected frequencies are $(88 \div 4) = \underline{22}$ Degrees of freedom = 3, so critical value $\chi^2_3(5\%) = 7.815$ (Not significant so) insufficient evidence to suggest <u>not</u> uniformly distributed	B1 B1, B1ft B1 (4)
(b)	e.g. H_0 : School is independent of club chosen H_1 : Club chosen depends on which school a student is from	B1 (1)
(c)	$\frac{28 \times 17}{88} = 5.409...$ awrt <u>5.41</u>	B1 (1)
(d)	Expected frequency for Music and School C = $4.77 < 5$ (Allow $\frac{105}{22}$ for 4.77) So combine Music column with another column giving 3x3 table so 4 df	B1 B1 (2)
(e)	Critical value $\chi^2_4(5\%) = 9.488$ [Not significant so] insufficient evidence of an association between school and choice of club	B1 B1 (2)
[10 marks]		
Notes		
(a)	<p>Ignore values of any test statistics calculated in (a) or (e)</p> <p>1st B1 for 22 2nd B1 for degrees of freedom = 3 (can be implied by sight of 7.815 as cv) 3rd B1ft for 7.815 (or better - cal: 7.814727910... <u>or</u> correct 5% cv for their d.f.) 4th B1 for comment suggesting uniform distribution is a suitable model. Must follow from comparing 6.09 with their cv. Do not allow contradictory statements e.g. “significant” so uniform dist’ is suitable</p>	
(b)	<p>B1 for both hypotheses with some context (“club” and “school” mentioned at least once) Use of “independence” or “association”</p>	
(c)	<p>B1 for a correct expression or awrt 5.41 (allow $\frac{119}{22}$)</p>	
(d)	<p>1st B1 for identifying that Music & School C has E_i that is < 5 (a value to 2 sf should be seen, may be in (c), but must state <u>this</u> $E_i < 5$ as well) 2nd B1 for pooling <u>music</u> with another <u>column</u> leading to 3x3 table and 4 degrees of freedom Must clearly state the pooling and evidence for 4 df e.g. allow $(3-1) \times (4-1-1)$ [NB pooling with Art gives 4.3987..., with Sports 4.3247..., with Computers 7.2879...]</p>	
(e)	<p>1st B1 for 9.488 (or awrt 9.488) 2nd B1 for a correct, not significant, conclusion mentioning <u>school</u> and <u>clubs</u></p>	

Question Number	Scheme	Marks
4. (a)	Use of $\bar{x} \pm z \times \frac{18}{\sqrt{25}}$; $z = 2.3263$ (or better) $= (44.0253..., 60.7746...)$ awrt (44.0, 60.8)	M1;B1 A1, A1 (4)
(b)	$H_0 : \mu_A = \mu_B$ $H_1 : \mu_B > \mu_A$ $z = (\pm) \frac{57.8 - 52.4}{18 \sqrt{\frac{1}{25} + \frac{1}{30}}}$ $= (\pm) 1.1078...$ awrt (±) 1.11 5% one-tail critical value is 1.6449 (or p -value = 0.13396... i.e. awrt 0.134) (not sig') so insufficient evidence (in these data) to support newspaper's claim	B1 M1dM1 A1 B1 A1 (6)
(c)	Require $\frac{\bar{x} - \mu}{\frac{18}{\sqrt{n}}} > z$ where $z = -1.6449$ (o.e.) $\mu < 52.4 + 1.64(49) \times \frac{18}{5}$ or $\mu < 57.8 + 1.64(49) \times \frac{18}{\sqrt{30}}$ i.e. $\mu < 58.3216...$ and $\mu < 63.2056...$ So $\mu = \mathbf{58.3}$	M1 A1 M1 A1 (4) [14 marks]
Notes		
(a)	M1 for use of correct expression with 18, 25 and $1 < z < 3$ (Ignore \bar{x} for this mark) B1 for $z = 2.3263$ or better (calc: 2.32634787...) 1 st A1 for awrt 44.0 (ans only of 44.02...or awrt 44.03 scores M1B1 implied) 2 nd A1 for awrt 60.8 (ans only of 60.77... or awrt 60.77 scores M1B1 implied)	
(b)	1 st B1 for both hypotheses in terms of μ s (If using μ_1 etc they must define which is which) 1 st M1 for a correct denominator (18 needn't be outside square root) [4.87(44...)] 2 nd dM1 for a correct expression for test statistic 1 st A1 for awrt (±) 1.11 2 nd B1 for critical value of 1.6449 or better (If B0 in (a) for 2.33 allow 1.64 or 1.645 here) [Allow p -value of awrt 0.134 and condone awrt 0.866 if compared with 0.95] 2 nd A1 Correct contextual conclusion, ft comparing their "1.11" with 1.64 (or their cv) but must be not significant and mention "claim" or "score in town A" and "score in town B"	
(c)	1 st M1 for a correct starting <u>inequality</u> with any z such that $ z > 1$ (Allow \geq) 1 st A1 for either correct <u>inequality</u> for μ , allow $z = 1.64$ or better 2 nd M1 for both cases of $\bar{x} + z \frac{18}{\sqrt{n}}$ ($z > 1$) can allow "=" or inequality, may be in CI 2 nd A1 (dep on both Ms) for sight of both awrt 58.3 and awrt 63.2 and selecting awrt 58.3	

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5. (a)	<p>$H_0 : N(6, 0.75^2)$ is a suitable model for the length of fallen pine cones $H_1 : N(6, 0.75^2)$ is NOT a suitable model for the lengths of the pine cones</p> <p>e.g. $E_i : 5 \leq x < 5.5 = 80 \times P(5 \leq X < 5.5) = 80 \times P(-\frac{4}{3} \leq Z < -\frac{2}{3}) [= \mathbf{12.77 \sim 12.90}]$ <u>or</u> $E_i : 6 \leq x < 6.5 = 80 \times P(0 \leq Z < \frac{2}{3}) [= \mathbf{19.80 \sim 19.89}]$ $E_i : 5.5 \leq x < 6 = \mathbf{19.80 \sim 19.89}$ or $x \geq 6.5 = 40 - "19.80" = \mathbf{20.11 \sim 20.20}$</p> <table><tr><td></td><td>$x < 5$</td><td>$5 \leq x < 5.5$</td><td>$5.5 \leq x < 6$</td><td>$6 \leq x < 6.5$</td><td>$x \geq 6.5$</td></tr><tr><td>E_i</td><td>7.30~7.43</td><td>12.77~12.90</td><td>19.80~19.89</td><td>19.80~19.89</td><td>20.11~20.20</td></tr><tr><td>$\frac{(O-E)^2}{E}$</td><td>0.23~0.28</td><td>0.093~0.12</td><td>0.84~0.90</td><td>1.87~1.95</td><td>5.08~5.16</td></tr></table> <p>$\sum \frac{(O_i - E_i)^2}{E_i}$ or $\sum \frac{O_i^2}{E_i} - 80 = 8.308... ;$ answer in [8.15 ~ 8.4]</p> <p>$\nu = 5 - 1 = 4 \Rightarrow \chi_4^2(10\%) = 7.779$ (significant result so) the data do not support Chrystal's belief</p>		$x < 5$	$5 \leq x < 5.5$	$5.5 \leq x < 6$	$6 \leq x < 6.5$	$x \geq 6.5$	E_i	7.30~7.43	12.77~12.90	19.80~19.89	19.80~19.89	20.11~20.20	$\frac{(O-E)^2}{E}$	0.23~0.28	0.093~0.12	0.84~0.90	1.87~1.95	5.08~5.16	B1 M1 A1 M1 A1 dM1; A1 B1; B1ft A1ft (10)
	$x < 5$	$5 \leq x < 5.5$	$5.5 \leq x < 6$	$6 \leq x < 6.5$	$x \geq 6.5$															
E_i	7.30~7.43	12.77~12.90	19.80~19.89	19.80~19.89	20.11~20.20															
$\frac{(O-E)^2}{E}$	0.23~0.28	0.093~0.12	0.84~0.90	1.87~1.95	5.08~5.16															
(b)	<p>$\hat{\mu} = \frac{464}{80} = \mathbf{5.8}$ (cm); $s^2 = \frac{2722.59 - 80 \times "5.8^2"}{79}$ $s^2 = 0.39734... \text{ awrt } \mathbf{0.397}$ (cm²)</p>	B1; M1 A1 (3)																		
(c)	<p>$\nu = 5 - 3 = 2 ;$ so $\chi_2^2(10\%) = 4.605$ (Not sig') so a normal distribution is a plausible model for length of pine cones</p>	B1; B1ft B1ft (3)																		
(d)	<p>$P(X > 7 \mu = 5.8 \text{ and } s = \sigma = 0.63035...) = P\left(Z > \frac{7 - "5.8"}{\sqrt{0.397..}}\right) = P(Z > 1.90..)$ $= \mathbf{0.028 \sim 0.029}$</p>	M1 A1 (2) [18m'ks]																		
Notes																				
(a)	<p>1st B1 for both hypotheses. Must include the model and mention “length(s)” and “cones” 1st M1 for correct use of normal to find E_i for one cell 1st A1 for a middle value e.g. awrt 12.77~12.90 inclusive (12.77 is from tables, 12.90 calc) 2nd M1 for use of symmetry to get E_i for $5.5 \leq x < 6$ (same as $6 \leq x < 6.5$) or $x \geq 6.5$ (40 - ...) 2nd A1 for a correct set of expected frequencies (all awrt in given ranges) 3rd dM1 (dep on 1st M1) for a correct attempt to find test statistic...at least one correct term 3rd A1 for answer in the range 8.15-8.4 (inclusive) 2nd B1 for degrees of freedom = 4 3rd B1ft for a correct 10% critical value using their degrees of freedom 4th A1ft dep on M3 and cv = awrt 7.78 for contextual conclusion: length, cones, N (μ, σ not needed) <u>or</u> Chrystal's belief</p>																			
(b)	<p>B1 for 5.8 M1 for a correct expression (ft their mean) A1 for awrt 0.397 (Condone $\frac{3139}{7900}$)</p>																			
(c)	<p>1st B1 for degrees of freedom = 2 2nd B1ft for a correct cv (different from their part (a)) ft their df 3rd B1ft for a correct conclusion in context ft cv (“length” and “cones”) Ignore any μ or σ</p>																			
(d)	<p>M1 for standardising with 7, their 5.8 ($\neq 6$) and their s.d. from (b). Ignore any $\times 80$ A1 for a correct proportion of 0.028 or 0.029. (ISW if correct ans followed by $\times 80$)</p>																			

Question Number	Scheme	Marks
6. (a)	Let $D = Y - R$ then $E(D) = -3$; $\text{Var}(D) = 0.8^2 + 1.5^2$ or 1.7^2 or 2.89 $P(D > 0) = P\left(Z > \frac{0 - (-3)}{1.7}\right)$ <u>or</u> $P(Z > 1.7647\dots)$ $= 0.03880655\dots$ <u>or</u> $1 - 0.9608 = 0.0392$ awrt <u>0.039</u>	B1, M1 M1 A1 (4)
(b)	$(R_1 + R_2 + R_3) \sim N\left(45, \sqrt{3 \times 1.5^2}\right)$; $4Y \sim N\left(48, \sqrt{4^2 \times 0.8^2}\right)$ $L = 4Y - (R_1 + R_2 + R_3) \Rightarrow L \sim N\left(3, \sqrt{16.99}\right)$ $P(L > 0) = P\left(Z > \frac{0 - 3}{\sqrt{16.99}}\right)$ <u>or</u> $P(Z > 0 - 0.7278\dots)$ [use 0 - 0.73 in tables] $=$ awrt <u>0.767</u>	M1A1A1 M1A1 dM1 A1 (7)
(c)	$E(X) = 780$ gives $15a + 12b = 780$ $[\text{Var}(X) =] 1.5^2 \times a^2 + 0.8^2 \times b^2$ Sub for a : $\text{Var}(X) = 2.25(52 - 0.8b)^2 + 0.64 \times b^2$ <u>or</u> $2.08b^2 - 187.2b + 6084$ $\frac{d}{db}[\text{Var}(X)] = 0 \Rightarrow 4.16b - 187.2 = 0$ <u>$b = 45$</u> So $a = 52 - 0.8 \times 45 = 52 - 36$ <u>$a = 16$</u>	M1A1 M1 M1 M1 A1 A1 (7)
[18 marks]		
Notes		
(a)	B1 for $E(D) = -3$ (or +3 if using $R - Y$) and 1 st M1 for $\text{Var}(D) = 0.8^2 + 1.5^2$ o.e. 2 nd M1 for attempt at $P(D > 0)$ must standardise with their -3 and their 1.7 and inequality A1 for awrt 0.039	
(b)	1 st M1 for correct mean or variance for either $R_1 + R_2 + R_3$ or $4Y$ 1 st A1 for $(R_1 + R_2 + R_3) \sim N(45, \sqrt{6.75^2})$ 2 nd A1 for $4Y \sim N(48, \sqrt{10.24^2})$ 2 nd M1 for attempting a suitable L (condone $3R - 4L$ etc) Must have L with mean of ± 3 and $\sigma_L^2 = "6.75" + "10.24" = (4.1218\dots)^2$ 3 rd A1 for a correct mean and variance. Sight of $N(\pm 3, 16.99)$ scores 1st 5 marks 3 rd dM1 (dep on 2 nd M1) for attempting a prob (\rightarrow ans > 0.5) using $\mu_L = \pm 3$ and their σ_L 4 th A1 for awrt 0.767 (Calc: 0.7666384... or tables 0.7673)	
(c)	1 st M1 for an attempt to use $E(X) = 780$ must see a linear equation in a and b using 780 1 st A1 for $15a + 12b = 780$ o.e. e.g. $5a + 4b = 260$ or $a + 0.8b = 52$ etc 2 nd M1 for an attempt to find an expression for $\text{Var}(X)$ (condone a and b wrong way around) 3 rd M1 for forming a quadratic expression for $\text{Var}(X)$ in terms of a or b only (M0 for $= k, k \neq 0$) 4 th M1 suitable method for finding min (e.g. differentiation, or completing square or calc) e.g. $\frac{13}{4}(a^2 - 32a + 832)$ [3 rd M1] then $k[(a - 16)^2 + m]$ would score 4 th M1 2 nd A1 for $b = 45$ <u>or</u> $a = 16$ 3 rd A1 for both $b = 45$ <u>and</u> $a = 16$	Correct answers should be accompanied by evidence for 1 st 4 marks