Pure Mathematics P2 Mark scheme

Question	Scheme	Marks
1(a)	$f(x)= x^4 + x^3 + 2x^2 + ax + b$	
	Attempting f(1) **or** f(−1)	M1
	f(1) = 1 + 1 + 2 + a + b = 7 **or** 4 + a + b = 7 $\Rightarrow$ a + b = 3 (as required) **AG**	A1* **cso**
		(2)
(b)	Attempting f(−2) **or** f(2)	M1
	f(−2) = 16 – 8 + 8 – 2a + b = –8 {⇒ – 2a + b = –24}	A1
	Solving both equations simultaneously to get as far as a =… **or** b =…	dM1
	Any one of a = 9 **or** b = −6	A1
	Both a = 9 and b = −6	A1
		(5)
		(7marks)

Notes:

(a)

M1: For attempting either $f(1)$ or $f(-1)$.

A1: For applying $f(1)$, setting the result equal to 7, and manipulating this correctly to give the result given on the paper as $a+b=3$. Note that the answer is given in part (a).

Alternative

M1: For long division by $(x-1)$ to give a remainder in a and b which is independent of x.

A1: Or $\{\text{Remainder} =\}$ $b+a+4=7$ leading to the correct result of $a+b=3$ (answer given).

(b)

M1: Attempting either $f(-2)$ or $f(2)$.

A1: correct underlined equation in a and b; e.g. $\underline{16-8+8-2a+b=-8}$ or equivalent, e.g. $-2a+b=-24$.

dM1: An attempt to eliminate one variable from 2 linear simultaneous equations in a and b. Note that this mark is dependent upon the award of the first method mark.

A1: Any one of $a=9$ or $b=-6$.

A1: Both $a=9$ and $b=-6$ and a correct solution only.

Alternative

M1: For long division by $(x+2)$ to give a remainder in a and b which is independent of x.

A1: For $\{\text{Remainder} =\}$ $\underline{b-2(a-8)=-8}$ $\{\Rightarrow -2a+b=-24\}$.

Then dM1A1A1 are applied in the same way as before.

Question	Scheme		Marks
2(a)	$S_\infty = \dfrac{20}{1-\frac{7}{8}}$; $= 160$	Use of a correct S_∞ formula	M1
		160	A1
			(2)
(b)	$S_{12} = \dfrac{20\left(1-\left(\frac{7}{8}\right)^{12}\right)}{1-\frac{7}{8}}$; $=127.77324...$ $= 127.8$ (1 dp)	M1: Use of a correct S_n formula with $n = 12$ (condone missing brackets around $\dfrac{7}{8}$)	M1 A1
		A1: **awrt** 127.8	
			(2)
(c)	$160 - \dfrac{20(1-(\frac{7}{8})^N)}{1-\frac{7}{8}} < 0.5$	Applies S_N (**GP only**) with $a = 20$, $r = \frac{7}{8}$ and "uses" 0.5 and their S_∞ at any point in their working.	M1
	$160\left(\dfrac{7}{8}\right)^N < (0.5)$ or $\left(\dfrac{7}{8}\right)^N < \left(\dfrac{0.5}{160}\right)$	Attempt to isolate $+160\left(\frac{7}{8}\right)^N$ or $\left(\frac{7}{8}\right)^N$	dM1
	$N\log\left(\dfrac{7}{8}\right) < \log\left(\dfrac{0.5}{160}\right)$	Uses the law of logarithms to obtain an equation or an inequality of the form $N\log\left(\dfrac{7}{8}\right) < \log\left(\dfrac{0.5}{\text{their } S_\infty}\right)$ or $N > \log_{0.875}\left(\dfrac{0.5}{\text{their } S_\infty}\right)$	M1
	$N > \dfrac{\log\left(\frac{0.5}{160}\right)}{\log\left(\frac{7}{8}\right)} = 43.19823...$ **cso** $\Rightarrow N = 44$	$N = 44$ (Allow $N \geq 44$ but no $N > 44$	A1 cso
	An incorrect **<u>inequality</u>** statement at any stage in a candidate's working loses the final mark. Some candidates do not realise that the direction of the inequality is reversed in the final line of their solution. **BUT** it is possible to gain full marks for using =, as long as no incorrect working seen.		
			(4)
	Alternative: Trial & Improvement Method in (c):		
	Attempts $160 - S_N$ or S_N with at least one value for $N > 40$		M1
	Attempts $160 - S_N$ or S_N with $N = 43$ or $N = 44$		dM1
	For evidence of examining $160 - S_N$ or S_N for **both** $N = 43$ **and** $N = 44$ with **both** values correct to 2 DP Eg: $160 - S_{43} =$ awrt 0.51 and $160 - S_{44} =$ awrt 0.45 or $S_{43} =$ awrt 159.49 and $S_{44} =$ awrt 159.55		M1
	$N = 44$		A1 cso
	Answer of $N = 44$ only with no working scores no marks		
			(4)
			(8 marks)

Question	Scheme	Marks
3(a)	<table><tr><td>x</td><td>0</td><td>0.25</td><td>0.5</td><td>0.75</td><td>1</td></tr><tr><td>y</td><td>1</td><td>1.251</td><td>**1.494**</td><td>**1.741**</td><td>2</td></tr></table>	B1 B1
		(2)
(b)	$\frac{1}{2}\times 0.25$, $\{(1+2)+2(1.251+1.494+1.741)\}$ o.e.	B1 M1 A1ft
	= 1.4965	A1
		(4)
(c)	Gives any valid reason including • Decrease the width of the strips • Use more trapezia • Increase the number of strips Do not accept use more decimal places	B1
		(1)
		(7 marks)

Notes:

(a)

B1: For 1.494

B1: For 1.741 (1.740 is **B0**). Wrong accuracy e.g. 1.49, 1.74 is B1B0

(b)

B1: Need ½ of 0.25 or 0.125 o.e.

M1: Requires first bracket to contain first plus last values **and** second bracket to include no additional values from the three in the table. If the only mistake is to omit one value from second bracket this may be regarded as a slip and **M** mark can be allowed (An extra repeated term forfeits the **M** mark however) x values: **M0** if values used in brackets are x values instead of y values

A1ft: Follows their answers to part (a) and is for {correct expression}

A1: Accept 1.4965, 1.497, or 1.50 only after correct work. (No follow through except one special case below following 1.740 in table).

Separate trapezia may be used: **B1** for 0.125, **M1** for $\frac{1}{2}h(a+b)$ used 3 or 4 times (and **A1**ft if it is all correct) e.g. 0.125(1+ 1.251) + 0.125(1.251+1.494) + 0.125(1.741 + 2) is **M1 A0** equivalent to missing one term in { } in main scheme.

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>4</td><td>A solution based around a table of results
<table>
<tr><td>n</td><td>n^2</td><td>n^2+2</td><td></td></tr>
<tr><td>1</td><td>1</td><td>3</td><td>Odd</td></tr>
<tr><td>2</td><td>4</td><td>6</td><td>Even</td></tr>
<tr><td>3</td><td>9</td><td>11</td><td>Odd</td></tr>
<tr><td>4</td><td>16</td><td>18</td><td>Even</td></tr>
<tr><td>5</td><td>25</td><td>27</td><td>Odd</td></tr>
<tr><td>6</td><td>36</td><td>38</td><td>Even</td></tr>
</table></td><td></td></tr>
<tr><td></td><td>When n is odd, n^2 is odd (odd $\times$odd = odd) so n^2+2 is also odd</td><td>M1</td></tr>
<tr><td></td><td>So for all odd numbers n, n^2+2 is also odd and so cannot be divisible by 4 (as all numbers in the 4 times table are even)</td><td>A1</td></tr>
<tr><td></td><td>When n is even, n^2 is even and a multiple of 4, so n^2+2 cannot be a multiple of 4</td><td>M1</td></tr>
<tr><td></td><td>Fully correct and exhaustive proof. Award for both of the cases above plus a final statement "So for all n, n^2+2 cannot be divisible by 4"</td><td>A1*</td></tr>
<tr><td></td><td></td><td>(4)</td></tr>
<tr><td></td><td colspan="2">Alternative - (algebraic) proof</td></tr>
<tr><td></td><td>If n is even, $n=2k$, so $\frac{n^2+2}{4}=\frac{(2k)^2+2}{4}=\frac{4k^2+2}{4}=k^2+\frac{1}{2}$</td><td>M1</td></tr>
<tr><td></td><td>If n is odd, $n=2k+1$, so $\frac{n^2+2}{4}=\frac{(2k+1)^2+2}{4}==\frac{4k^2+4k+3}{4}=k^2+k+\frac{3}{4}$</td><td>M1</td></tr>
<tr><td></td><td>For a partial explanation stating that
either of $k^2+\frac{1}{2}$ or $k^2+k+\frac{3}{4}$ are not a whole numbers.
with some valid reason stating why this means that n^2 +2 is not a multiple of 4.</td><td>A1</td></tr>
<tr><td></td><td>Full proof with no errors or omissions. This must include
The conjecture
Correct notation and algebra for both even and odd numbers
A full explanation stating why, for all n, n^2+2 is not divisible by 4</td><td>A1*</td></tr>
<tr><td></td><td></td><td>(4)</td></tr>
<tr><td colspan="3" align="right">(4 marks)</td></tr>
</table>

Question	Scheme		Marks
5(a)	$(S =)a + (a + d) + \ldots + [a+(n-1)d]$	B1: requires at least 3 terms, must include first and last terms, an adjacent term and dots!	B1
	$(S =)[a+(n-1)d] + \ldots\ldots + a$	M1: for reversing series (dots needed)	M1
	$2S = [2a+(n-1)d] + \ldots\ldots + [2a+(n-1)d]$	dM1: for adding, must have $2S$ and be a genuine attempt. Either line is sufficient. Dependent on 1st M1.	dM1
	$2S = n[2a+(n-1)d]$ $S = \frac{n}{2}[2a+(n-1)d]$ cso	(NB −Allow first 3 marks for use of l for last term but as given for final mark)	A1
			(4)
(b)	$600 = 200 + (N-1)20 \Rightarrow N = \ldots$	Use of 600 with a **<u>correct</u>** formula in an attempt to find N.	M1
	$N = 21$	cso	A1
			(2)
(c)	**Look for an AP first:**		
	$S = \frac{21}{2}(2 \times 200 + 20 \times 20)$ **or** $\frac{21}{2}(200 + 600)$	M1: Use of correct sum formula with their **integer** $n = N$ or $N - 1$ from part (b) where $3 < N < 52$ and $a = 200$ and $d = 20$.	M1A1
	$S = \frac{20}{2}(2 \times 200 + 19 \times 20)$ **or** $\frac{20}{2}(200 + 580)$ (= 8400 **or** 7800)	M1: Use of correct sum formula with their **integer** $n = N$ or $N - 1$ from part (b) where $3 < N < 52$ and $a = 200$ and $d = 20$.	
	Then for the constant terms:		
	$600 \times (52 - "N")$ (= 18600)	M1: $600 \times k$ where k is an integer and $3 < k < 52$	M1
		A1: A correct un-simplified follow through expression with their k consistent with n so that $n + k = 52$	A1ft
	So total is 27000	**cao**	A1
	There are no marks in (c) for just finding S_{52}		
			(5)
			(11 marks)

Question	Scheme		Marks
6(i)	$\log_2\left(\dfrac{2x}{5x+4}\right)=-3$ **or** $\log_2\left(\dfrac{5x+4}{2x}\right)=3$ **or** $\log_2\left(\dfrac{5x+4}{x}\right)=4$		M1
	$\left(\dfrac{2x}{5x+4}\right)=2^{-3}$ **or** $\left(\dfrac{5x+4}{2x}\right)=2^3$ **or** $\left(\dfrac{5x+4}{x}\right)=2^4$		M1
	$16x = 5x + 4 \Rightarrow x =$ (depends on Ms and must be this equation or equiv)		dM1
	$x = \dfrac{4}{11}$ or exact recurring decimal $0.\dot{3}\dot{6}$ after correct work		A1 **cso**
	Alternative		
	$\log_2(2x)+3=\log_2(5x+4)$		
	So $\log_2(2x)+\log_2(8)=\log_2(5x+4)$ earns 2^{nd} M1 (3 replaced by $\log_2 8$)		2^{nd} M1
	Then $\log_2(16x)=\log_2(5x+4)$ earns 1^{st} M1 (addition law of logs)		1^{st} M1
	Then final M1 A1 as before		dM1A1
			(4)
(ii)	$\log_a y + \log_a 2^3 = 5$		M1
	$\log_a 8y = 5$	Applies product law of logarithms	dM1
	$y=\dfrac{1}{8}a^5$ **cso**	$y=\dfrac{1}{8}a^5$ **cso**	A1
			(3)
			(7 marks)

Notes:

(i)

M1: Applying the subtraction or addition law of logarithms correctly to make **two** log **terms into one** log term .

M1: For RHS of either 2^{-3}, 2^3, 2^4 or $\log_2\left(\dfrac{1}{8}\right)$, $\log_2 8$ or $\log_2 16$ i.e. using connection between log base 2 and 2 to a power. This may follow an error. **Use of** 3^2 **is M0**

dM1: Obtains **correct** linear equation in x. usually the one in the scheme and attempts $x =$

A1: **cso**. Answer of 4/11 with **no** suspect log work preceding this.

(ii)

M1: Applies power law of logarithms to replace $3\log_a 2$ by $\log_a 2^3$ or $\log_a 8$

dM1: (Should not be following M0) Uses addition law of logs to give $\log_a 2^3 y = 5$ or $\log_a 8y = 5$

Question	Scheme	Marks
7(a)	Obtain $(x \pm 10)^2$ **and** $(y \pm 8)^2$	M1
	(10, 8)	A1
		(2)
(b)	See $(x \pm 10)^2 + (y \pm 8)^2 = 25\ (= r^2)$ **or** $(r^2 =)$ "100"+"64" − 139	M1
	$r = 5$*	A1
		(2)
(c)	Substitute $x = 13$ into the equation of circle and solve quadratic to give $y =$	M1
	e.g. $x = 13 \Rightarrow (13-10)^2 + (y-8)^2 = 25 \Rightarrow (y-8)^2 = 16$ so $y = 4$ **or** 12 **N.B. This can be attempted via a 3, 4, 5 triangle so spotting this and achieving one value for y is M1 A1. Both values scores M1 A1 A1**	A1 A1
		(3)
(d)	$OC = \sqrt{10^2 + 8^2} = \sqrt{164}$	M1
	Length of tangent $= \sqrt{164 - 5^2} = \sqrt{139}$	M1 A1
		(3)
		(10 marks)

Notes:

(a)

M1: Obtains $(x \pm 10)^2$ **and** $(y \pm 8)^2$ May be implied by one correct coordinate

A1: (10, 8) Answer only scores both marks.

Alternative: *Method 2:* From $x^2 + y^2 + 2gx + 2fy + c = 0$ centre is $(\pm g,\ \pm f)$

M1: Obtains $(\pm 10, \pm 8)$

A1: Centre is $(-g,\ -f)$**, and so centre is** (10, 8).

(b)

M1: For a correct method leading to $r = \ldots$, or $r^2 =$

Allow "100"+"64"−139 or an attempt at using $(x \pm 10)^2 + (y \pm 8)^2 = r^2$ form to identify $r=$

A1*: $r = 5$ This is a printed answer, so a correct method must be seen.

Alternative:

(b)

M1: Attempts to use $\sqrt{g^2 + f^2 - c}$ or $(r^2 =)$"100"+"64"−139

A1*: $r = 5$ following a correct method.

(c)

M1: Substitutes $x = 13$ into either form of the circle equation, forms and solves the quadratic equation in y

A1: Either $y = 4$ **or** 12

A1: Both $y = 4$ **and** 12

Question 7 notes *continued*
(d) **M1:** Uses Pythagoras' Theorem to find length OC using their $(10,8)$ **M1:** Uses Pythagoras' Theorem to find *OX*. Look for $\sqrt{OC^2 - r^2}$ **A1:** $\sqrt{139}$ only

Question	Scheme	Marks
8(a)	Substitutes $x=1$ in C_1: $y = 10x - x^2 - 8 = 10 - 1 - 8 = 1$ and in C_2: $y = x^3 = 1^3 = 1 \Rightarrow (1, 1)$ lies on both curves.	B1
		(1)
(b)	$10x - x^2 - 8 = x^3$ $x^3 + x^2 - 10x + 8 = 0$	B1
	$(x - 1)(x^2 + 2x - 8) = 0$	M1 A1
	$(x - 1)(x + 4)(x - 2) = 0 \quad x = 2$	M1 A1
	$(2, 8)$	A1
		(6)
(c)	$\int\left\{\left(10x - x^2 - 8\right) - x^3\right\}\mathrm{d}x$	M1
	$= 5x^2 - \frac{x^3}{3} - 8x - \frac{x^4}{4}$	M1 A1
	Using limits 2 and 1: $\left(20 - \frac{8}{3} - 16 - 4\right) - \left(5 - \frac{1}{3} - 8 - \frac{1}{4}\right)$	M1
	$= \frac{11}{12}$	A1
		(5)
		(12 marks)

Notes:

(a)

B1: Substitutes x =nto both $y = 10x - x^2 - 8$ and $y = x^3$ **AND** achieves y =1 in both.

(b)

B1: Sets equations equal to each other and proceeds to $x^3 + x^2 - 10x + 8 = 0$

M1: Divides by $(x - 1)$ to form a quadratic factor. Allow any suitable algebraic method including division or inspection.

A1: Correct quadratic factor $(x^2 + 2x - 8)$

M1: For factorising of their quadratic factor.

A1: Achieves $x = 2$

A1: Coordinates of $B = (2, 8)$

(c)

M1: For knowing that the area of R = $\int\left\{\left(10x - x^2 - 8\right) - x^3\right\}\mathrm{d}x$

This may also be scored for finding separate areas and subtracting.

M1: For raising the power of x seen in at least three terms.

A1: Correct integration. It may be left un-simplified. That is allow $\frac{10x^2}{2}$ for $5x^2$

Question 8 notes *continued*	
M1:	For using the limits "2" and 1 in their integrated expression. If separate areas have been attempted, "2" and 1 must be used in both integrated expressions.
A1:	For $\frac{11}{12}$ or exact equivalent.

Question	Scheme		Marks
9(i)	**Way 1** Divides by $\cos 3\theta$ to give $\tan 3\theta = \sqrt{3}$ so$\Rightarrow (3\theta) = \frac{\pi}{3}$	**Way 2** Or Squares both sides, uses $\cos^2 3\theta + \sin^2 3\theta = 1$, obtains $\cos 3\theta = \pm\frac{1}{2}$ or $\sin 3\theta = \pm\frac{\sqrt{3}}{2}$ so $(3\theta) = \frac{\pi}{3}$	M1
	Adds π **or** 2π to previous value of angle(to give $\frac{4\pi}{3}$ **or** $\frac{7\pi}{3}$)		M1
	So $\theta = \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}$ (all three, no extra in range)		A1
			(3)
(ii)(a)	$4(1 - \cos^2 x) + \cos x = 4 - k$	Applies $\sin^2 x = 1 - \cos^2 x$	M1
	Attempts to solve $4\cos^2 x - \cos x - k = 0$, to give $\cos x =$		dM1
	$\cos x = \frac{1 \pm \sqrt{1+16k}}{8}$ **or** $\cos x = \frac{1}{8} \pm \sqrt{\frac{1}{64} + \frac{k}{4}}$ **or** other correct equivalent		A1
			(3)
(b)	$\cos x = \frac{1 \pm \sqrt{49}}{8} = 1$ and $-\frac{3}{4}$ (see the note below if errors are made)		M1
	Obtains two solutions from 0 , 139 , 221 (0 **or** 2.42 **or** 3.86 in radians)		dM1
	$x = 0$ and 139 and 221 (allow awrt 139 and 221) must be in degrees		A1
			(3)
			(9 marks)

Notes:

(i)

M1: Obtains $\frac{\pi}{3}$. Allow $x = \frac{\pi}{3}$ or even $\theta = \frac{\pi}{3}$. Need not see working here. May be implied by $\theta = \frac{\pi}{9}$ in final answer (allow $(3\theta) = 1.05$ or $\theta = 0.349$ as decimals or $(3\theta) = 60$ or $\theta = 20$ as degrees for this mark). Do not allow $\tan 3\theta = -\sqrt{3}$ nor $\tan 3\theta = \pm\frac{1}{\sqrt{3}}$

M1: Adding π or 2π to a previous value however obtained. It is not dependent on the previous mark. (May be implied by final answer of $\theta = \frac{4\pi}{9}$ **or** $\frac{7\pi}{9}$). This mark may also be given for answers as decimals [4.19 or 7.33], or degrees (240 or 420).

Question 9 notes *continued*	
A1:	Need all three correct answers in terms of π and **no extras in range**.
NB:	$\theta = 20°$,$80°$, $140°$ **earns M1M1A0 and 0.349, 1.40 and 2.44 earns M1M1A0**
(ii)(a)	
M1:	Applies $\sin^2 x = 1 - \cos^2 x$ (allow even if brackets are missing e.g. $4 \times 1 - \cos^2 x$). This must be awarded in (ii) (a) for an expression with k not after $k = 3$ is substituted.
dM1:	Uses formula or completion of square to obtain cos x = expression in k (Factorisation attempt is M0)
A1:	cao - award for their final simplified expression
(ii)(b)	
M1:	**Either** attempts to substitute $k = 3$ into their answer to obtain two values for cos x **Or** restarts with $k = 3$ to find two values for cos x (They cannot earn marks in ii(a) for this). **In both cases** they need to have applied $\sin^2 x = 1 - \cos^2 x$ (brackets may be missing) **and** correct method for solving their quadratic (usual rules – see notes) The values for cos x may be >1 or < -1.
dM1:	Obtains **two correct** values for x
A1:	Obtains **all three correct values** in degrees (allow awrt 139 and 221) including 0. Ignore excess answers outside range (including 360 degrees) Lose this mark for excess answers in the range or radian answers.