Mark Scheme (Results)

January 2021

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F1
Paper WFM01/01

Question Number	Scheme	Marks
1.(a)	$f(0.2)=...$ and $f(0.6)=...$	M1
	$f(0.2)=-0.5973...$ and $f(0.6)=0.2707...$	
	Continuous function with change of sign so root (in given interval)	A1 (2)
(b)	$f(0.4)=-0.1788$	B1
	$f(0.5)=...$	M1
	$f(0.5)=0.04114... \Rightarrow 0.4\leqslant\alpha\leqslant 0.5$	A1 (3)
		[5]

	Notes
	Must see correct values for the accuracy marks. But allow signs only for attempts at values for the method marks.
(a)	
M1	Attempts both values, accept f(0.2)=... and f(0.6) =… with any values. (NB f(0.2) = −0.2069…, f(0.6) = 1.379 are the values with calculator in degrees mode.)
A1	Both values correct (rounded or truncated to 1d.p.) and a correct conclusion (continuous and sign change). Allusion to continuity must be mentioned somewhere in the solution. Allow other ways to show sign change e.g. <0, >0 etc.
(b)	
B1	Correct value of f (0.4); may be rounded or truncated to 1 dp
M1	Attempt value of f (0.5) or attempt value of f(0.3) if relevant for their sign of f(0.4).
A1	Correct value of f (0.5) which may be rounded to 2 dp and correct interval. Allow as open or closed interval. Accept any valid notation for the interval. Accept e.g. $0.4 < x < 0.5$

Question Number	Scheme	Marks
2		
(a)	$\frac{3}{8}-\frac{\sqrt{71}}{8}\mathrm{i}$	B1 (1)
(b)	$\left(x-\frac{3}{8}-\frac{\sqrt{71}}{8}\mathrm{i}\right)\left(x-\frac{3}{8}+\frac{\sqrt{71}}{8}\mathrm{i}\right)\ \left((x-4)=0\right)$	
	$\left(x^2-\frac{3}{4}x+\frac{5}{4}\right)\ \left((x-4)=0\right)$	M1A1
	$x^3-\frac{19}{4}x^2+\frac{17}{4}x-5\ \ (=0)$	dM1
	$4x^3-19x^2+17x-20\ \ (=0)\quad p=17,\ q=-20$	A1 (4) **[5]**

(a) **B1**	Correct answer only
(b)	
M1	Attempt the multiplication of the 2 brackets with the complex terms. Allow $(x\pm\text{root})$ for the brackets. Allow "invisible" brackets.
A1	Correct quadratic obtained - may have multiplied by the 4 (or other constant factor) and this is fine. (Need not be fully simplified but must have real terms)
dM1	Attempt to multiply their quadratic by $(x-4)$ or may divide their quadratic into the cubic or other full method leading to at least one of p or q.
A1	Correct values. Values of p and q need not be shown explicitly but may been seen in a cubic, provided the cubic starts $4x^3-19x^2$ (isw after a correct cubic)
	Note if a candidate uses a hybrid method, mark under main scheme unless an Alt scores more marks.

Alt 1 (b)	$-\frac{q}{4}=4\times\left(\frac{3}{8}+\frac{\sqrt{71}}{8}\mathrm{i}\right)\times\left(\frac{3}{8}-\frac{\sqrt{71}}{8}\mathrm{i}\right)=\ldots\to q=..$ **or** $\frac{p}{4}=4\left(\frac{3}{8}+\frac{\sqrt{71}}{8}\mathrm{i}\right)+4\left(\frac{3}{8}-\frac{\sqrt{71}}{8}\mathrm{i}\right)+\left(\frac{3}{8}+\frac{\sqrt{71}}{8}\mathrm{i}\right)\left(\frac{3}{8}-\frac{\sqrt{71}}{8}\mathrm{i}\right)\to p=\ldots$	M1
	$\Rightarrow q=-16\times\left(\frac{9}{64}+\frac{71}{64}\right)=-20$ **or** $p=17$	A1
	E.g. $\mathrm{f}(4)=0\Rightarrow 4(4)^3-19(4)^2+4p-"20"=0\Rightarrow p=\ldots$ or $\frac{p}{4}=4\left(\frac{3}{8}+\frac{\sqrt{71}}{8}\mathrm{i}\right)+4\left(\frac{3}{8}-\frac{\sqrt{71}}{8}\mathrm{i}\right)+\left(\frac{3}{8}+\frac{\sqrt{71}}{8}\mathrm{i}\right)\left(\frac{3}{8}-\frac{\sqrt{71}}{8}\mathrm{i}\right)\to p=\ldots$	dM1
	$p=17,\ q=-20$	A1 (4)

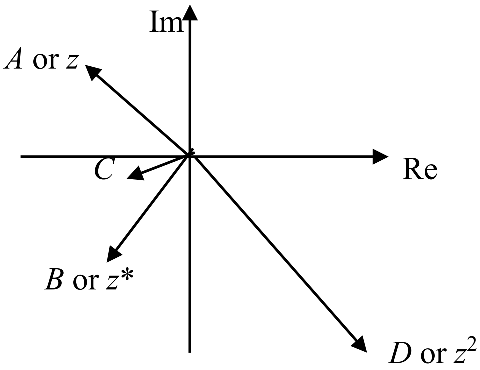
Question Number	Scheme	Marks
M1	A correct attempt to use product of roots is $-\frac{q}{4}$ to find a value for q **or** pair sum is $\frac{p}{4}$ to find a value of p.	
A1	Correct value for p or q	
dM1	Correct full method to find both p and q.	
A1	Correct values for both	
Alt 2	Attempts at using the factor theorem are possible but unlikely to succeed. Score as follows: M1: Uses the factor theorem to generate two equations in the two unknowns (note they will need to use a complex root to achieve this and equate real and imaginary parts.). A1: Correct equations. dM1: Solves their two equations to find values for p and q. A1: Correct values Send to review if unsure.	
Alt 3 (b)	$4x^2-3x+p-12$ $x-4\overline{)4x^3-19x^2+px+q}$ $4x^3-16x^2$ $-3x^2+px+q$ $-3x^2+12x$ $(p-12)x+q$ $(p-12)x-4(p-12)$ $\Rightarrow q+4(p-12)=0\ \ \&\ \frac{p-12}{4}=\left(\frac{3}{8}+\frac{\sqrt{71}}{8}\mathrm{i}\right)\left(\frac{3}{8}-\frac{\sqrt{71}}{8}\mathrm{i}\right)=\frac{5}{4}$ $P=17,\ q=-20$	M1 A1 dM1 A1 (4)
(b) **M1**	Divides $x-4$ into the cubic to achieve a 3TQ quotient and a remainder	
A1	Correct quotient and remainder	
dM1	Correct full method to find p or q	
A1	Correct values	

Question Number	Scheme	Marks
3(a)	$k(k+5)-6=0$	M1
	$k^2+5k-6=0$	
	$\left((k-1)(k+6)=0 \Rightarrow\right) \quad k=1,\ -6$	A1 (2)
(b)	$\dfrac{1}{\text{"}k^2+5k-6\text{"}}\begin{pmatrix} k & 2 \\ 3 & k+5 \end{pmatrix}$	M1A1 (2)
		[4]
(a)		
M1	Attempts determinant and sets equal to zero (or equivalent method) to obtain an unsimplified quadratic equation	
A1	Correct values for k (may solve the quadratic by any valid means)	
(b)		
M1	Forms the matrix of signed minors (must have at least three correct elements) divided or multiplied by an attempt at the determinant	
A1	Fully correct inverse	

Question Number	Scheme	Marks
4 (a)	$\alpha+\beta=-\frac{5}{2} \quad \alpha\beta=\frac{7}{2}$	B1
	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=\left(-\frac{5}{2}\right)^3-3\left(\frac{7}{2}\right)\left(-\frac{5}{2}\right)$	M1
	$=\frac{85}{8}$	A1 (3)
(b)	$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}=\ldots$	M1
	$=\left(\frac{85}{8}\right)\times\left(\frac{2}{7}\right)=\frac{85}{28}$	A1
	$\frac{\alpha^2}{\beta}\times\frac{\beta^2}{\alpha}=\alpha\beta=\frac{7}{2}$	B1ft
	$x^2-"\frac{85}{28}"x+"\frac{7}{2}"\ (=0)$	M1
	$28x^2-85x+98=0$	A1 (5) **[8]**

	Note: if a candidate solves the equation and uses the roots to answer the question, then send to review.
(a)	
B1	Both correct. (Seen anywhere in the working)
M1	Uses their sum and product of roots in a correct expression for $\alpha^3+\beta^3$.
A1	Correct value. Must be exact. Accept 10.625
(b)	
M1	substitutes their values for $\alpha^3+\beta^3$ and $\alpha\beta$ into $\frac{\alpha^3+\beta^3}{\alpha\beta}=\ldots$ (allow slips in substitution).
A1	Correct sum as a single fraction (may be seen or implied in their equation)
B1ft	Correct product or follow through their product
M1	Use $x^2-\text{sum of roots}\times x+\text{product of roots}$ with their values for sum and product. "= 0" may be missing.
A1	A correct final equation as shown or any integer multiple of this. " = 0" must be included.

Question Number	Scheme	Marks
5(a)	$\sum_{r=1}^{n}(r+1)(r+5)=\sum_{r=1}^{n}(r^2+6r+5)$	B1
	$=\sum_{r=1}^{n}r^2+6\sum_{r=1}^{n}r+5n$	
	$=\frac{n}{6}(n+1)(2n+1)+6\frac{n}{2}(n+1)+5n$	M1A1
	$=\frac{n}{6}(2n^2+3n+1+18n+18+30)$	dM1
	$=\frac{n}{6}(2n^2+21n+49)=\frac{n}{6}(n+7)(2n+7)$ *	A1 * (5)
(b)	$\sum_{r=n+1}^{2n}=\sum_{r=1}^{2n}-\sum_{r=1}^{n}=\frac{2n}{6}(2n+7)(4n+7)-\frac{n}{6}(n+7)(2n+7)$	M1
	$=\frac{n}{6}(2n+7)\{8n+14-(n+7)\}$	
	$=\frac{7n}{6}(2n+7)(n+1)$	A1 (2)
		[7]
(a)		
B1	Brackets multiplied out correctly. Summation signs not needed.	
M1	Use at least two correct formulae from $\sum_{r=1}^{n}r$, $\sum_{r=1}^{n}r^2$ and $\sum_{r=1}^{n}1=n$.	
A1	Fully correct expression.	
dM1	Attempt to remove factor $\frac{n}{6}$ from an expression with common factor n present. (if "5n" is just 5 then this mark will not be scored). Must be seen before the given answer is quoted. No need to simplify the remaining quadratic factor.	
A1*	Obtain the correct 3 term quadratic and factorise. This is a "show that" question, so the 3 TQ must be seen. No errors seen.	
(b)		
M1	Use $\sum_{r=n+1}^{2n}=\sum_{r=1}^{2n}-\sum_{r=1}^{n}$	
A1	Simplify to the correct answer.	

Question Number	Scheme	Marks
6(a)	$\lambda = 4$	B1 (1)
(b)	$\arctan\dfrac{3}{"4"}$ or $\arctan\dfrac{-3}{"4"}$	M1
	(Second quadrant so arg z = 2.498...) = 2.5 (rad)	A1 (2)
(c)(i)	$\dfrac{z+3\mathrm{i}}{2-4\mathrm{i}} = \dfrac{-4+6\mathrm{i}}{2-4\mathrm{i}} \times \dfrac{2+4\mathrm{i}}{2+4\mathrm{i}}$ or $\dfrac{z+3\mathrm{i}}{2-4\mathrm{i}} = a+\mathrm{i}b \Rightarrow -4+6\mathrm{i} = (a+\mathrm{i}b)(2-4\mathrm{i})$	M1
	$= \dfrac{-8+12\mathrm{i}-16\mathrm{i}+24\mathrm{i}^2}{4+16} = -\dfrac{8}{5}-\dfrac{1}{5}\mathrm{i}$ Accept e.g. $\dfrac{-32-4\mathrm{i}}{20}$ Or $2a+4b=-4, 2b-4a=6 \Rightarrow a=..,b=..$	dM1A1
(ii)	$z^2 = (-4+3\mathrm{i})^2 = 16-24\mathrm{i}+9\mathrm{i}^2 = 16-24\mathrm{i}-9$	M1
	$= 7-24\mathrm{i}$	A1ft (5)
(d)		B1 B1ft B1ft (3) **[11]**

(a)
B1 Correct answer. No working needed.

(b)
M1 For $\arctan\left(\pm\dfrac{3}{"4"}\right)$ with their "4". Can be awarded from $\tan\theta = \pm\dfrac{3}{"4"} \Rightarrow \theta = ...$ or by implication if correct value for either arctan or correct final answer (rounded or not rounded, may be degrees) is seen.

A1 Cao 2.5

(c)(i)
M1 Multiplies numerator and denominator by complex conjugate of denominator. Award if denominator of 4+16 or 20 is seen instead of product. May still have *z* at this stage, or even allow with $\lambda + 3\mathrm{i}$ as numerator.
Alternatively, sets equal to *a* + i*b* and cross multiplies.

dM1 Using their λ or 4 substitutes **correctly** for *z*, fully expands the numerator and uses $\mathrm{i}^2 = -1$
Alt, uses $\mathrm{i}^2 = -1$, equates real and imaginary terms and solves their equations for *a* and *b*

A1 Correct answer only, as shown or single fraction accepting equivalent fractions or with exact decimals (−1.6 − 0.2i).

(ii) M1 Squaring an expression of form $k + 3\mathrm{i}$ (with a real value for *k*) to get 3 terms (may be implied) and uses $\mathrm{i}^2 = -1$

A1ft Correct answer, follow through their $\lambda > 0$ (ie for $"\lambda^2 - 9" - "6\lambda"\mathrm{i}$ must be negative i term)

(d) NB: Penalise once only (in the first mark due) for mislabelling or failing to label points as long as they look to be placed correctly. Award if lines/arrows not included. Points may be labelled by letter, name or their Cartesian coordinates (which may be given on the axes).

B1 Plots *z* in second quadrant and *z** as mirror image in the Real axis. Both must be labelled.

B1ft Plot and label *C* for their solution to (c)(ii) It must be the correct side of *B* (for their

	answers) and a correct relative scale (so noticeably closer to O than their B if correct values).
B1ft	Plot and label their D (- 24 need not be to scale, but should be further from O than their B).

Question Number	Scheme	Marks
7(a)	$\begin{vmatrix} 4 & -5 \\ -3 & 2 \end{vmatrix} = 8-15=-7 \Rightarrow \text{Area } T' = \text{"}\pm 7\text{"} \times 23 = ...$	M1
	$\text{Area } T' = 161$	A1 (2)
(b)	$\begin{pmatrix} 4 & -5 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 3p+2 \\ 2p-1 \end{pmatrix} = \begin{pmatrix} 17 \\ -18 \end{pmatrix}$ or $\begin{pmatrix} 3p+2 \\ 2p-1 \end{pmatrix} = \frac{1}{8-15}\begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 17 \\ -18 \end{pmatrix}$	
	$4(3p+2)-5(2p-1)=17$ or $-3(3p+2)+2(2p-1)=-18$ or $3p+2=-\frac{1}{7}(34-90)$ or $2p-1=-\frac{1}{7}(51-72)$ $(\text{e.g. } 2p+13=17 \Rightarrow .....)$	M1
	$p=2$	A1 (2)
(c)	Rotation; through 90° clockwise (or 270° anticlockwise) about origin	B1;B1 (2)
(d)	**CA** = **B**	
	$\mathbf{A}^{-1} = -\frac{1}{7}\begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}$ or $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 4 & -5 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 4a-3b & -5a+2b \\ 4c-3d & -5c+2d \end{pmatrix}$	B1
	$\mathbf{C} = -\frac{1}{7}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix} = ...$ or $\begin{cases} 4a-3b=0 & -5a+2b=1 \\ 4c-3d=-1 & -5c+2d=0 \end{cases} \Rightarrow ...$	M1
	$\mathbf{C} = -\frac{1}{7}\begin{pmatrix} 3 & 4 \\ -2 & -5 \end{pmatrix}$ or $\begin{pmatrix} -\frac{3}{7} & -\frac{4}{7} \\ \frac{2}{7} & \frac{5}{7} \end{pmatrix}$ oe	A1 (3)
		[9]

(a)	
M1	Attempts to find the determinant of **M** and use as a scale factor. Accept if a slip in calculation is made and accept if negative is used for this mark. Dividing by the determinant is M0.
A1	Correct answer only. No working needed (correct answer implies the method).
(b)	
M1	Form a matrix equation using either **A** or an attempt at $\mathbf{A}^{-1}$, obtain a linear equation and solve for p.
A1	Correct value for p, obtained from a correct equation. (No need to check in other equation.)
(c)	
B1	Rotation, rotates, rotate or rotating (oe) Accept "turn"
B1	Correct angle (degrees or radians) with direction specified and about origin or (0, 0)
(d)	
B1	Correct matrix for $\mathbf{A}^{-1}$ May have been found in (b) but must be used in (d). Alternatively, correct **CA** with unknowns for entries of **C**.
M1	Multiply **B** by $\mathbf{A}^{-1}$ **on the right.** Alternatively, sets **CA** equal to **B** and solves equations.
A1	Correct matrix **C**(isw after a correct answer).

Question Number	Scheme	Marks
8(a)	$200t^3 = 25$ or $\left(\dfrac{25}{x}\right)^2 = 40x$ or $y^2 = 40\left(\dfrac{25}{y}\right)$	M1
	$t = \dfrac{1}{2}$ or $x = \dfrac{5}{2}$ or $y = 10$	A1
	$\left(\dfrac{5}{2}, 10\right)$	A1 (3)
(b)	$y^2 = 40x \Rightarrow x = \dfrac{y^2}{40} \Rightarrow \dfrac{dx}{dy} = \dfrac{2y}{40}$ or $2y\dfrac{dy}{dx} = 40 \Rightarrow \dfrac{dy}{dx} = \dfrac{40}{2y}$ or $\dfrac{dy}{dx} = \dfrac{\sqrt{10}}{\sqrt{x}}$ or by parametric differentiation: $\dfrac{dy}{dx} = \dfrac{1}{t}$	B1
	at $(10, 20)$: $\dfrac{dy}{dx} = 1$ or $\dfrac{dx}{dy} = 1$	M1
	Grad normal $= -1$	A1
	$y - 20 = -(x - 10)$	M1
	$x + y - 30 = 0$	A1 (5)
(c)	$xy = 25 \quad x + y - 30 = 0$	
	$x + \dfrac{25}{x} - 30 = 0$ or $\dfrac{25}{y} + y - 30 = 0$ or $\dfrac{5}{t} + 5t - 30 = 0$	M1
	$x^2 - 30x + 25 = 0$ or $y^2 - 30y + 25 = 0$ or $5t^2 - 30t + 5 = 0$	dM1
	$x = 15 \pm \sqrt{200}$ or $y = 15 \pm \sqrt{200}$ or $t = 3 \pm 2\sqrt{2}$ (or exact equivalents)	A1
	eg $x = 15 + \sqrt{200} \Rightarrow y = 15 - \sqrt{200}$	ddM1
	$\left(15 + 10\sqrt{2}, 15 - 10\sqrt{2}\right) \quad \left(15 - 10\sqrt{2}, 15 + 10\sqrt{2}\right)$	A1A1 (6)
		[14]

(a)	
M1	Attempt an equation in a single variable.
A1	Correct value for x, y or t
A1	Correct values for x and y. Need not be in coordinate brackets. No other points seen.
(b)	
B1	Any correct expression involving the derivative, $\dfrac{dy}{dx}$ or $\dfrac{dx}{dy}$, for P
M1	Attempt to obtain value of their derivative at (10, 20). May be from an incorrect curve.
A1	Correct gradient of normal.
M1	Equation of normal by any complete method. Must involve a sign change of their derivative and have numerical gradient. Cam be scored from an incorrect starting equation/point.
A1	Correct equation in form demanded (though terms may be in different order).
(c)	
M1	Use the equation of H and their equation of the normal from (b) to obtain an equation in a single variable
dM1	Obtains a 3TQ and attempts to solve by any valid means.
A1	Correct values for x or y or t. Must be exact but need not be fully simplified (but discriminant must be evaluated).
ddM1	Use at least one of their values for x or y to obtain a value for the other coordinate or t to find at least one set of coordinates. (Can be scored with inexact values.)
A1	Either pair of coordinates correct. Allow if unsimplified.
A1	Second pair of coordinates correct and no extra solutions and both pairs in simplest form (as shown in scheme). Need not be coordinates as long as correctly paired. Award A1A0 if $x = 15 \pm 10\sqrt{2}, y = 15 \pm 10\sqrt{2}$ is given

Question Number	Scheme	Marks
9(i)	$n=1 \quad u_1 = 3\times\frac{2}{3}-1=1$ (so true for $n = 1$ (†))	B1
	Assume true for $n=k$ ie $u_k = 3\left(\frac{2}{3}\right)^k - 1$ (†)	
	$u_{k+1} = \frac{1}{3}(2u_k - 1) = \frac{1}{3}\left(2\left(3\left(\frac{2}{3}\right)^k - 1\right) - 1\right) = \frac{1}{3}\left(6\left(\frac{2}{3}\right)^k - 2 - 1\right)$	M1A1
	$= \frac{1}{3}\left(2\times3\left(\frac{2}{3}\right)^{k+1}\times\left(\frac{3}{2}\right) - 2 - 1\right)$	dM1
	$= \frac{1}{3}\times2\times3\left(\frac{2}{3}\right)^{k+1}\times\left(\frac{3}{2}\right) + \frac{1}{3}(-2-1)$	
	$= 3\left(\frac{2}{3}\right)^{k+1} - 1$	A1
	$\therefore$ if true for $n=k$, also true for $n=k+1$ (†)	
	(True for $n=1$) so $u_n = 3\left(\frac{2}{3}\right)^n - 1$ is true for all $n\in\mathbb{Z}^+$	A1cso (6)
(ii)	$\mathrm{f}(1) = 2^3 + 3^3 = 8 + 27 = 35$ (Multiple of 7) (so true for $n = 1$ (†))	B1
	Assume $\mathrm{f}(k)$ is a multiple of 7 $\mathrm{f}(k) = 2^{k+2} + 3^{2k+1}$ is a multiple of 7 (†)	
	$\mathrm{f}(k+1) - M\mathrm{f}(k) = 2^{k+3} + 3^{2k+3} - M\left(2^{k+2} + 3^{2k+1}\right)$	M1
	$= 2^{k+2}(2-M) + 3^{2k+1}\left(3^2 - M\right)$	A1
	$= (2-M)\left(2^{k+2} + 3^{2k+1}\right) + 3^{2k+1}\times7$ or $(9-M)\left(2^{k+2} + 3^{2k+1}\right) - 7\times2^{k+2}$ oe	dM1
	$\therefore \mathrm{f}(k+1) = 2\mathrm{f}(k) + 7\times3^{2k+1}$ oe e.g. $9\mathrm{f}(k) - 7\times2^{k+2}$ Or e.g. $7\times3^{2k+1}$ is a multiple of 7, so if f(k) is a multiple of 7 <u>then f(k+1) is also a multiple of 7</u>	A1
	If the result is true for $n = k$ it is also true for $n = k + 1$ (†)	
	As the result has been shown to be true for $n = 1$, it is true for all $n\in\mathbb{Z}^+$	A1 cso (6) **[12]**

9(i)	
B1	Check that the formula gives 1 when $n = 1$ Working must be shown. (Need not state true for n =1 for this mark – but see final A)
M1	(Assume true for $n = k$ and) attempts to substitute the formula for u_k into $u_{k+1} = \frac{1}{3}(2u_k - 1)$ or equivalent with suffixes increased. Allow slips.
A1	Correct substitution.
dM1	Obtain an expression with $\left(\frac{2}{3}\right)^{k+1}$ and no other k. Alternatively, expands u_{k+1} to a matching expression (ie work from both directions).
A1	Correct expression when $n = k + 1$ At least one intermediate stage of working must be shown and no errors (though notational slips may be condoned). If working from both directions, it is for correct work to reach matching expressions.
A1cso	Correct concluding statements following correct solution which has included each of the points (†) at some stage during the working. Depends on all except the first B mark (e.g. if they think they have checked n =1 but have really checked n = 2). Note: Allow the M's and first two A's for students who go from k+1 to k+2 but treat it as k to k + 1.
(ii)	
B1	Checks the case $n = 1$. Minimum statement of f(1) = 35
M1	Attempts an expression for $f(k + 1) - Mf(k)$ with any value of M. Need not be simplified. Most likely with $M = 1$ but may be seen with other values of M. With $M = 0$, $f(k+1) = 2^{k+3} + 3^{2k+3}$ is all that is required.
A1	A correct expression with terms 2^{k+2} and 3^{2k+1} clearly identified.
dM1	Attempts to extract/identify f(k) within a correct expression to give terms divisible by 7. With $M = 0$ look for $f(k+1) = 2\times\left(2^{k+2} + 3^{3k+1}\right) + 7\times 3^{2k+1}$ or $9\times\left(2^{k+2} + 3^{3k+1}\right) - 7\times 2^{k+2}$ oe and similar for other value of M.
A1	One of the correct expressions for f(k+1) shown (or with powers of 2 and 3) or full reason why f(k+1) is divisible by 7, following a suitable expression.
A1cso	Correct concluding statements following correct solution which has included each of the points (†) at some stage during the working. Depends on all previous marks.