Pearson Edexcel

Mark Scheme (Results)

October 2021

Pearson Edexcel International A Level
In Pure Mathematics P3 (WMA13) Paper 01

Question Number	Scheme	Marks
1. (a)	$\dfrac{5x}{x^2+7x+12}+\dfrac{5x}{x+4}=\dfrac{5x+5x(x+3)}{(x+3)(x+4)}$	M1 A1
	$=\dfrac{5x^2+20x}{(x+3)(x+4)}=\dfrac{5x(x+4)}{(x+3)(x+4)}=\dfrac{5x}{x+3}$ *	A1*
		(3)
(b)	$y=\dfrac{5x}{(x+3)}\Rightarrow xy+3y=5x\Rightarrow 5x-xy=3y$	M1
	$\Rightarrow x=\dfrac{3y}{5-y}$ So $\mathrm{f}^{-1}(x)=\dfrac{3x}{5-x}$	A1
	Domain $0<x<5$	A1
		(3)
(c) (i)	$\mathrm{f}(x)=\dfrac{5x}{(x+3)}\Rightarrow(\mathrm{f}'(x)=)\dfrac{5(x+3)-5x}{(x+3)^2}=\dfrac{15}{(x+3)^2}$	M1 A1
(ii)	States (f is an) increasing function with a suitable reason E.g. Since $(x+3)^2$ is positive	A1
		(3)
		(9 marks)

(a)

M1 Attempts to combine the two fractions using a common denominator.
Allow errors on the numerator but at least one of the terms must have been adapted.
Usual rules apply for factorising the quadratic denominator.
Condone invisible brackets and slips on the numerator when combining the two fractions.
Allow the two fractions to be written separately with the same denominator.

A1 For a correct un-simplified fraction with a quadratic numerator and denominator (which may be implied)

$$\frac{5x}{x^2+7x+12}+\frac{5x}{x+4}=\frac{5x(x+4)}{\cdots}+\frac{5x\left(x^2+7x+12\right)}{\cdots}=\frac{5x^3+40x^2+80x}{\left(x^2+7x+12\right)(x+4)}=\frac{x(5x+20)(x+4)}{\left(x^2+7x+12\right)(x+4)}=\frac{x(5x+20)}{\left(x^2+7x+12\right)}$$

A1* Correctly achieves the given answer of $\dfrac{5x}{x+3}$ showing intermediate steps (cso). Expect to see the two fractions combined and then both the numerator and denominator factorised before cancelling terms to achieve full marks. In the case of forming a cubic numerator and denominator you must see the terms collected before factorising. Bracket errors at some point in their working is A0*

(b)

M1 Attempts to change the subject on $y=\dfrac{5x}{x+3}$ (or $x=\dfrac{5y}{y+3}$). Look for cross multiplication with an attempt to collect terms. Do not follow through on their answer to part (a)

A1 $\quad \mathrm{f}^{-1}(x)=\dfrac{3x}{5-x}$ or $\mathrm{f}^{-1}(x)=\dfrac{-3x}{x-5}$ Must be in terms of x. Condone $\mathrm{f}^{-1}=\ldots$ (or $\mathrm{f}^{-1}=y=\ldots$) but **do not allow** just $y=\ldots$ or $\mathrm{f}^{-1}:\ y=\ldots$

A1 $\quad$ Correct domain $0<x<5$

(c) Mark (i) and (ii) together

M1 $\quad$ Attempts to use the quotient rule or product rule. Look for an expression of the form $\dfrac{A(x+3)-5x}{(x+3)^2}$ or $5x(x+3)^{-1}\to B(x+3)^{-1}\pm 5x(x+3)^{-2}$ where A and B are non-zero constants.

A1 $\quad \left(\mathrm{f}'(x)=\right)\dfrac{15}{(x+3)^2}$ or $15(x+3)^{-2}$ or $\dfrac{15}{x^2+6x+9}$ Do not allow $\dfrac{5}{x+3}-\dfrac{5x}{(x+3)^2}$ for this mark.

A1 $\quad$ They achieve $\left(\mathrm{f}'(x)=\right)\dfrac{15}{(x+3)^2}$ and states

- (that f is an) **increasing function**
- **since** $(x+3)^2$ **is positive**

Alt methods in (a)

Taking out common factors

M1A1: $\dfrac{5x}{x^2+7x+12}+\dfrac{5x}{x+4}=5x\left\{\dfrac{1+(x+3)}{(x+3)(x+4)}\right\}$ or $=\dfrac{5x}{(x+4)}\left\{\dfrac{1+(x+3)}{(x+3)}\right\}$

Methods where candidates "split up" fractions.

M1 $\quad$ Attempts $\dfrac{5x}{x^2+7x+12}$ OR $\dfrac{5}{x^2+7x+12}$ by partial fractions

A1 $\quad \dfrac{5x}{x+3}-\dfrac{5x}{x+4}+\dfrac{5x}{x+4}$

A1* $\quad \dfrac{5x}{x+3}$ You would expect to see $\dfrac{5x}{x+3}-\dfrac{5x}{x+4}+\dfrac{5x}{x+4}$ before proceeding to the given answer.

Alt method in (b)

M1 $\quad$ Writes $y=\dfrac{5x}{x+3}$ as $y=5\pm\dfrac{k}{x+3}$ and then attempts to make x the subject.

A1 $\quad \mathrm{f}^{-1}(x)=\dfrac{15}{5-x}-3$ or equivalent

A1 $\quad 0<x<5$

Alt method in (c)

M1 $\quad$ Writes $y=\dfrac{5x}{x+3}$ as $y=5\pm\dfrac{k}{x+3}$ and attempts the chain rule to get $\pm\dfrac{C}{(x+3)^2}$

A1 $\quad \left(\mathrm{f}'(x)=\right)\dfrac{15}{(x+3)^2}$ or exact equivalent

A1 States that since $\left(x+3\right)^2$ is positive so f is increasing function

Question Number	Scheme	Marks
2 (a)	$\left(\frac{13}{3},5\right)$	B1 B1
		(2)
(b)(i)	$\mathrm{f}(x)\ldots 5$	B1 ft
(ii)	10	B1
		(2)
(c)	Attempts to solve either $16-2x\ldots 3x-13+5 \Rightarrow$ a value or inequality in x Or $16-2x\ldots -3x+13+5 \Rightarrow$ a value or inequality in x	M1
	Both correct critical values $2, \frac{24}{5}$	A1
	Selects inside region for their critical values	dM1
	$2<x<\frac{24}{5}$	A1
		(4)
(d)	$a=4, b=\frac{1}{3}$	B1 ft B1
		(2)
		(10 marks)
Alt(c)	$(11-2x)^2\ldots(3x-13)^2 \Rightarrow 121-44x+4x^2\ldots 9x^2-78x+169$ $5x^2-34x+48\ldots 0 \Rightarrow$ a value or inequality in x	M1
	$2, \frac{24}{5}$	A1
	Selects inside region for their critical values	dM1
	$2<x<\frac{24}{5}$	A1

Note – Check the diagram and next to the questions for answers

(a)

B1 One correct coordinate. Allow as $x=\frac{13}{3}, y=5$ or exact equivalent and condone missing brackets.

B1 Both coordinates correct. Allow as $x=\frac{13}{3}, y=5$ or exact equivalent and condone missing brackets.

(b)(i)

B1ft $\mathrm{f}(x)\ldots 5$. Allow equivalent correct answers. E.g. $y\ldots 5$, $y\in[5,\infty)$, $\mathrm{f}\ldots 5$ or using set notation. Follow through on their y coordinate in (a). Do not allow "range $\ldots 5$"

(b)(ii)

B1 10

(c)

M1 Attempts to solve a correct equation or inequality. Look for the equation or inequality (ignore the direction) after the modulus signs have been removed allowing slips in their rearrangement.

A1 Correct critical values for $x ... 2, \frac{24}{5}$ or exact equivalent which may be part of an incorrect inequality

dM1 Selects inside region for their critical values or both correct inequalities seen for their values.
Allow this to be written separately and allow the critical values to be included within their region.

A1 $2 < x < \frac{24}{5}$ Allow other exact equivalent forms such as $2 < x < \frac{48}{10}$ or $x \in \left(2, \frac{24}{5}\right)$ or $\frac{24}{5} > x > 2$

The inequalities must be presented together on one line for this mark.

Accept two separate statements such as "$x > 2$ AND $x < \frac{24}{5}$" but not "$x > 2$ or $x < \frac{24}{5}$" or "$x > 2,\ x < \frac{24}{5}$"

(d)

B1ft One correct value. Either $a = 4$ or $b = \frac{1}{3}$ but ft on their part (a) so accept $a = \frac{20}{"5"}$ or $b = "\frac{13}{3}" - 4$

B1 $a = 4, b = \frac{1}{3}$ which may be embedded within $y = a\mathrm{f}(x + b)$

Question Number	Scheme	Marks
3.(a) (i)	$0=40-30\mathrm{e}^{1-0.05t} \Rightarrow 1-0.05t=\ln\frac{40}{30}$	M1
	$\Rightarrow (k=)$ awrt 14.2	A1
(ii)	March 1814	A1
		(3)
(b)	Attempts $G=40-30\mathrm{e}^{1-0.05\times 70}=37.5$ tonnes	M1 A1
		(2)
(c)	40 (tonnes)	B1
		(1)
		(6 marks)

Mark (a)(i) and (ii) together
(a) (i)

M1 Sets $G=0$ and proceeds to a linear equation in t or k using a correct method by taking logs of both sides, but condone slips in the rest of their rearrangement.

A1 $(k=)$ awrt 14.2 or 14.25 but allow the exact value $20\left(1-\ln\frac{4}{3}\right)$ or other equivalent exact expressions such as $\frac{\ln\frac{4}{3}-1}{-0.05}$ or $20\left(1-\ln\frac{4}{3}\right)$ or $20\ln\left(\frac{3}{4}\mathrm{e}\right)$. Isw after a correct answer.

(a)(ii)

A1 March 1814 but allow April 1814 following "correct" *k*.
Condone "the third month of 1814" or "the fourth month of 1814" following "correct" *k*
Withhold this mark if they have two answers of which one is incorrect.

(b)

M1 Attempts to find *G* with $t=70$ (condone $t=69$) which may be implied by awrt 37.5
The numbers embedded in the equation is sufficient eg $G=40-30\mathrm{e}^{1-0.05\times 70}$

A1 awrt 37.5 tonnes (requires units)

(c)

B1 40 (tonnes). Condone $G<40$ or $G\leqslant 40$. Do not allow $G>40$

Question Number	Scheme	Marks
4 (a)	$2\sin(\theta-30°)=5\cos\theta \Rightarrow 2\sin\theta\cos30°-2\cos\theta\sin30°=5\cos\theta$	M1
	$\div\cos\theta \qquad \Rightarrow 2\tan\theta\cos30°-2\sin30°=5$	dM1
	$\Rightarrow 2\tan\theta\times\frac{\sqrt{3}}{2}-2\times\frac{1}{2}=5$	A1
	$\Rightarrow \sqrt{3}\tan\theta=6 \Rightarrow \tan\theta=2\sqrt{3}$ *	A1*
		(4)
(b)	Attempts $\arctan 2\sqrt{3}$ ..and then subtracts $20°$	M1
	$\Rightarrow x=$ awrt $53.9°, 233.9°$	A1, A1
		(3)
		(7 marks)

(a)

M1 Attempts to use $\sin(\theta-30°)=\sin\theta\cos(\pm30°)\pm\cos\theta\sin(\pm30°)$ within the given equation

Condone the omission of a 2 on the second term and a slip on the 5 of $5\cos\theta$

dM1 Divides by $\cos\theta$ to set up an equation in just $\tan\theta$.

They may collect terms in $\sin\theta$ and $\cos\theta$ before dividing by $\cos\theta$ to set up an equation in just $\tan\theta$

An equation with $\cos 30°$ and $\sin 30°$ still not processed is acceptable.

A1 Fully correct equation in $\tan\theta$ with the $\cos 30°$ and $\sin 30°$ processed

($\sqrt{3}\sin\theta=6\cos\theta \Rightarrow \tan\theta=2\sqrt{3}$ is acceptable for both A marks)

(Note If they proceed directly to the final answer from

$\tan\theta=\frac{5+2\sin30°}{2\cos30°} \Rightarrow \tan\theta=2\sqrt{3}$ then maximum M1dM1A0A0 unless

$\tan\theta=\frac{5+1}{\sqrt{3}}$ or equivalent is seen before the final given answer.

A1* Correctly proceeds to given answer.

(b) Answers with no working scores 0 marks

M1 Attempts to find a value for x.

Allow $\arctan 2\sqrt{3}$ ...followed by adding or subtracting $20°$. Which may be implied by

$\tan(x+20)=2\sqrt{3} \Rightarrow x=\arctan(2\sqrt{3})\pm20=...$

Alternatively, attempts to use $\sin(x-10°)=\sin x\cos10°\pm\cos x\sin10°$ within the given equation, divides by $\cos x$ to set up an equation in just $\tan x$ and proceeds to find an angle for x

$\tan x=\frac{5\cos20+2\sin10}{2\cos10+5\sin20} \Rightarrow x=...$

A1 One value provided M1 has been scored. Allow either awrt $54°$ or $234°$ (or in radians awrt 0.94 or 4.08)

A1 $x=$ awrt $53.9°, 233.9°$ and no others inside the range provided M1 has been scored. Ignore any angles outside the range. Must be in degrees

Question Number	Scheme	Marks
5 (i)	$\int \frac{8}{(2x-3)^3}\,\mathrm{d}x = \frac{-2}{(2x-3)^2}\,(+c)$	M1 A1
	$\int_2^4 \frac{8}{(2x-3)^3}\,\mathrm{d}x = \left[\frac{-2}{(2x-3)^2}\right]_2^4 = -\frac{2}{25}+2=\frac{48}{25}$	dM1 A1
		(4)
(ii)	$\int x\left(x^2+3\right)^7\,\mathrm{d}x = \frac{1}{16}\left(x^2+3\right)^8+c$	M1 A1
		(2)
		(6 marks)
Alt(i)	Let $u=2x-3$ $\int \frac{8}{u^3}\times\frac{1}{2}\,\mathrm{d}u = -\frac{2}{u^2}\,(+c)$ $\int_2^4 \frac{8}{(2x-3)^3}\,\mathrm{d}x = \left[-\frac{2}{u^2}\right]_1^5 = -\frac{2}{25}+2=\frac{48}{25}$	M1 A1 dM1 A1
Alt(ii)	Let $u=x^2+3$ $\int x\left(x^2+3\right)^7\,\mathrm{d}x = \int \frac{u^7}{2}\,\mathrm{d}u = \frac{u^8}{16}+c = \frac{1}{16}\left(x^2+3\right)^8+c$	M1A1

(i)

M1 Achieves $\frac{A}{(2x-3)^2}$ or equivalent or in the alternative method $\frac{A}{u^2}$

A1 Achieves $\frac{-2}{(2x-3)^2}$ or in the alternative method $-\frac{2}{u^2}$ (which may be unsimplified but the indices must be processed). There is no requirement for the $+c$

dM1 Substitutes 2 and 4 into $\frac{A}{(2x-3)^2}$ or equivalent or 1 and 5 into $\frac{A}{u^2}$ and subtracts either way round. May be implied but M1 must have been scored.

A1 $\frac{48}{25}$ or 1.92 isw after a correct answer

(ii)

M1 Achieves $k\left(x^2+3\right)^8$ or equivalent or in the alternative method ku^8. Alternatively multiplies out the expression and integrates achieving an expression of the form

$\pm \ldots x^{16} \pm \ldots x^{14} \pm \ldots x^{12} \pm \ldots x^{10} \pm \ldots x^{8} \pm \ldots x^{6} \pm \ldots x^{4} \pm \ldots x^{2}$

A1 $\dfrac{1}{16}\left(x^2+3\right)^8+c$ Must be in terms of x and the + c must be present

Allow $\dfrac{x^{16}}{16}+\dfrac{3x^{14}}{2}+\dfrac{63x^{12}}{4}+\dfrac{189x^{10}}{2}+\dfrac{2835x^{8}}{8}+\dfrac{1701x^{6}}{2}+\dfrac{5103x^{4}}{4}+\dfrac{2187x^{2}}{2}+c$ or simplified equivalent

Question Number	Scheme	Marks
6 (i)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{6x}{x^2-5}-8x$	M1 A1
	Stationary point when $\frac{6x}{x^2-5}-8x=0 \Rightarrow x^2=\frac{23}{4} \Rightarrow x=\frac{\sqrt{23}}{2}$ only	dM1 A1
		(4)
(ii)(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)4-24\sin x\cos x$	M1
(b)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)4-12\sin 2x$	dM1 A1
	Maximum gradient = 16	A1
		(4)
		(8 marks)

(i)

M1 For differentiating $3\ln\left(x^2-5\right)\to\frac{Ax}{x^2-5}$ or $\frac{A}{x^2-5}$

A1 $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{6x}{x^2-5}-8x$ seen or implied by eg $\frac{6x}{x^2-5}=8x$

dM1 For proceeding from $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{Ax}{x^2-5}-Bx=0$ to a quadratic or cubic equation. Eg $6x-8x^3+40x=0$ oe
Do not be too concerned with the mechanics of their rearrangement.

A1 For $(x=)\frac{\sqrt{23}}{2}$ only provided dM1 has been scored. Solving the quadratic or cubic with a calculator is acceptable. Condone recovery of slips provided the method is correct.
$x=0$ and $x=-\frac{\sqrt{23}}{2}$ must be rejected if found or condone some minimal conclusion eg stating $p=23$

Mark (ii) (a) + (b) together
(ii) Way One

M1 For differentiating using the chain rule with $\sin^2 x\to ...\sin x\cos x$

dM1 .....and then using $\sin 2x=2\sin x\cos x$ to proceed to $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)A+B\sin 2x$. Condone the slip writing $A+B\sin x$ provided their coefficient of $\sin x\cos x$ has been halved or writing $2\sin\cos x$

A1 $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)4-12\sin 2x$ or $4+(-12)\sin 2x$ or $4-12\times\sin 2x$ (or states A and B following a correct method) Condone recovery of slips provided the method is correct.

A1 Maximum gradient = 16 (the previous A1 must have been scored)

(ii) Way Two

M1 Attempts to use $\cos 2x = \pm 1 \pm 2\sin^2 x$ in an attempt to write $y = 4x - 12\sin^2 x$ in terms of $\cos 2x$

dM1 ...and then differentiates $\cos 2x \to k\sin 2x$ to proceed to $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right) A + B\sin 2x$

A1 $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right) 4 - 12\sin 2x$ or $4 + (-12)\sin 2x$ or $4 - 12 \times \sin 2x$ (or states A and B following a correct method) Condone recovery of slips provided the method is correct.

A1 Maximum gradient = 16 (the previous A1 must have been scored)

(Note some candidates may find the second derivative and set equal to 0 and then substitute in their angle to find the maximum gradient.)

Question Number	Scheme		Marks
7.(a)	$\log_{10} M = 1.93\log_{10} 45 + 0.684 \Rightarrow \log_{10} M = 3.8747$		M1
	$(M =)$ awrt 7500 (kg)		A1
			(2)
(b)	$\log_{10} M = 1.93\log_{10} r + 0.684$	$M = p\,r^q$	
	$\log_{10} M = \log_{10} r^{1.93} + 0.684$	$\log_{10} M = \log_{10} pr^q$	
	$M = 10^{\log_{10} r^{1.93} + 0.684}$	$\log_{10} M = \log_{10} p + \log_{10} r^q$	
	$M = 10^{0.684} \times r^{1.93}$	$\log_{10} M = \log_{10} p + q\log_{10} r$	B1
	Either $p = 10^{0.684}$ or $q = 1.93$		M1
	Both $p =$ awrt 4.83 and $q = 1.93$		A1
			(3)
(c)	"p" is the mass (in kg) of a tree with radius 1 cm		B1
			(1)
			(6 marks)

(a)

M1 Substitutes $r = 45$ in $\log_{10} M = 1.93\log_{10} r + 0.684$ and proceeds to $\log_{10} M = ...$
(which must be a numerical value) (Condone the use of $r = 0.45$).
Implied by awrt 7500 for $r = 45$ or awrt 1.0 for $r = 0.45$

A1 $(M =)$ awrt 7500 (kg). Condone 7.5×10^3

Alt (a)

M1 Substitutes $r = 45$ in their $M = p\,r^q$ and finds M.

A1 $(M =)$ awrt 7500 (kg)

(b) Mark (b) and (c) together

B1 A correct proof showing that $\log_{10} M = 1.93\log_{10} r + 0.684 \Leftrightarrow M = p\,r^q$. Condone log or lg for $\log_{10}$ but use of natural logs ln is B0

Most are starting with $\log_{10} M = 1.93\log_{10} r + 0.684$. Expect to see the addition (or subtraction) law of logs explicitly used or index law being applied before proceeding to the final answer.

Eg $\log_{10} M = \log_{10} 4.83 + \log_{10} r^{1.93} = \log_{10}\left(4.83\times r^{1.93}\right) \Rightarrow M = 4.83r^{1.93}$ is B1

$\log_{10} M = \log_{10} r^{1.93} + 0.684 \Rightarrow M = 10^{1.93\log_{10} r + 0.684} \Rightarrow M = 10^{1.93\log_{10} r}\times 10^{0.684} = 4.83r^{1.93}$ is B1

$\log_{10} M = \log_{10} r^{1.93} + 0.684 \Rightarrow M = 10^{1.93\log_{10} r + 0.684} \Rightarrow M = r^{1.93}\times 10^{0.684} = 4.83r^{1.93}$ is B1

$\log_{10} M = \log_{10} r^{1.93} + 0.684 \Rightarrow M = 10^{1.93\log_{10} r}\times 10^{0.684} = 4.83r^{1.93}$ is B1

$\log_{10} M = \log_{10} 4.83 + \log_{10} r^{1.93} \Rightarrow M = 4.83r^{1.93}$ is B0 (addition law of logs not seen)

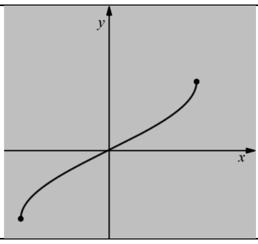
No incorrect working should be seen such as $10^{\log_{10} r^{1.93} + 0.684} = 10^{\log_{10} r^{1.93}} + 10^{0.684} = 10^{\log_{10} r^{1.93}} \times 10^{0.684}$ but condone the omission of the brackets around $\log_{10} 4.83\times r^{1.93}$ if seen in their working.

M1 Either $p = 10^{0.684}$ or $q = 1.93$ This may be implied by $M = p\,r^q$ with a "correct" p or q

A1 Both $p =$ awrt 4.83 and $q = 1.93$ (B0M1A1 can be scored) This may be implied by $M = 4.83\,r^{1.93}$

(c)

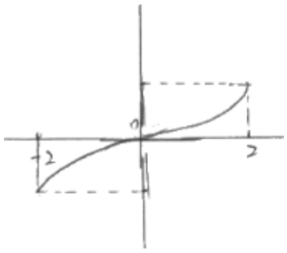
B1 "p" is the mass of a tree with radius 1 cm

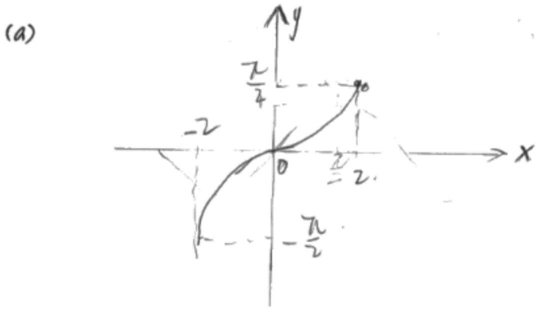
Question Number	Scheme	Marks
8 (a)	 Shape and position	B1
		(1)
(b)	$x = 2\sin y \Rightarrow \left(\frac{dx}{dy} = \right) 2\cos y$ and attempts to use **both** $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$ and $\cos y = \sqrt{1-\sin^2 y} = \sqrt{1-...x^2}$	M1
	$\left(\frac{dy}{dx} = \right) \frac{1}{2\sqrt{1-\frac{x^2}{4}}}$	A1
	$\left(\frac{dy}{dx} = \right) \frac{1}{\sqrt{4-x^2}}$	A1cso
		(3)
(c)	Substitutes $x = \sqrt{2}$ into their $\frac{dy}{dx} = \frac{1}{\sqrt{4-x^2}}$ $\left(= \frac{\sqrt{2}}{2} \right)$	M1
	Finds the equation of the tangent at P $y - \frac{\pi}{4} = "\frac{\sqrt{2}}{2}"\left(x - "\sqrt{2}"\right)$	dM1
	$y = \frac{\sqrt{2}}{2}x - 1 + \frac{\pi}{4}$	A1
		(3)
		(7 marks)
Alt(b)	$y = \arcsin\left(\frac{x}{2}\right) \Rightarrow \left(\frac{dy}{dx} = \right) \frac{1}{2} \times \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$	M1A1
	$\left(\frac{dy}{dx} = \right) \frac{1}{\sqrt{4-x^2}}$	A1

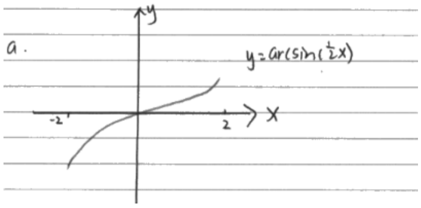
(a)

B1 Correct shape and position: Look for a curve in quadrants 1 and 3 with non zero gradient at the origin and gradient $\to \infty$ at both ends. Ignore values labelled on the axes.
If there is more than one attempt, mark the one which appears in the main body of the work as their fullest attempt. If there is more than one sketch on the same graph, then it must be clearly labelled which is the one to be marked.

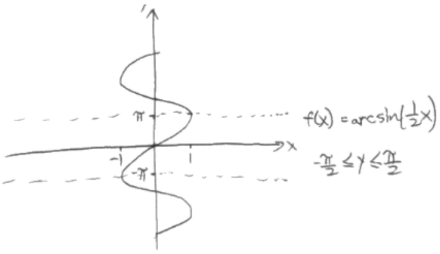
Examples of B1

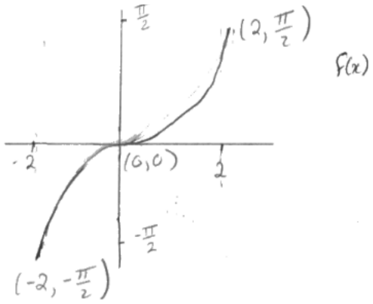






Examples of B0





(b) Note on EPEN it is M1M1A1 which is now marked as M1A1A1

M1 $\quad x = 2\sin y \Rightarrow \dfrac{dx}{dy} = \pm k\cos y$ where k is non zero and attempts to use both $\dfrac{dy}{dx} = 1 \div \dfrac{dx}{dy}$ and $\sin^2 y + \cos^2 y = 1$ in order to obtain $\dfrac{dy}{dx}$ in terms of x. Allow slips with dealing with their "2"

Allow working leading to $\dfrac{...}{...\sqrt{1-...x^2}}$ to score M1.

A1 $\quad$ A correct expression for $\dfrac{dy}{dx}$ as a function of x which may be unsimplified.

eg $\left(\dfrac{dy}{dx} =\right)\dfrac{1}{2\sqrt{1-\dfrac{x^2}{4}}}$ or $\dfrac{1}{\sqrt{\dfrac{4}{4}-\dfrac{x^2}{4}}}$ or $\dfrac{1}{2\sqrt{\dfrac{4-x^2}{4}}}$

A1cso $\left(\dfrac{dy}{dx} =\right)\dfrac{1}{\sqrt{4-x^2}}$

Alt (b)

M1 $\quad y = \arcsin\left(\dfrac{x}{2}\right) \Rightarrow \dfrac{dy}{dx} = k \times \dfrac{1}{\sqrt{1-\left(\dfrac{x}{2}\right)^2}}$ (where k can equal 1) then A1A1 as above

SC $\quad y = \arcsin\left(\dfrac{x}{2}\right) \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\sqrt{4-x^2}}$ with no correct intermediate working score as 100.

(c)

M1 Attempts to find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ at *P*. See scheme but allow sub of $y=\frac{\pi}{4}$ into their $\frac{\mathrm{d}x}{\mathrm{d}y}=\pm k\cos y$

dM1 Full method to find the equation of the tangent at *P*. Their $\frac{\mathrm{d}y}{\mathrm{d}x}$ may not be exact. If they use $y=mx+c$ they must proceed as far as $c=...$

A1 Correct equation and in the correct form. The coefficient of x and the constant must be exact. Isw

Question Number	Scheme	Marks
9 (a)	$\mathrm{f}(x)=\left(x^3-4x\right)\mathrm{e}^{-\frac{1}{2}x} \Rightarrow \mathrm{f}'(x)=\left(3x^2-4\right)\mathrm{e}^{-\frac{1}{2}x}-\frac{1}{2}\left(x^3-4x\right)\mathrm{e}^{-\frac{1}{2}x}$	M1 A1
		(2)
(b)	$\mathrm{f}'(0)=-4$ so equation of normal is $y=-\frac{1}{-4}x$	M1
	$y=\frac{1}{4}x$	B1
	Sets $\frac{1}{4}\cancel{x}=\cancel{x}\left(x^2-4\right)\mathrm{e}^{-\frac{1}{2}x} \Rightarrow x^2-4=\frac{1}{4}\mathrm{e}^{\frac{1}{2}x}$	M1
	$\Rightarrow x^2=\frac{16+\mathrm{e}^{\frac{1}{2}x}}{4} \Rightarrow x=-\frac{1}{2}\sqrt{16+\mathrm{e}^{\frac{1}{2}x}}$ *	A1*
		(4)
(c)	(i) $x_2=-\frac{1}{2}\sqrt{16+\mathrm{e}^{\frac{1}{2}\times -2}}=-2.0229$	M1 A1
	(ii) $(x=)-2.0226$	A1
		(3)
		(9 marks)

(a)

M1 Uses a valid method to differentiate. This could be:

(i) using the product rule on $\mathrm{f}(x)=\left(x^3-4x\right)\mathrm{e}^{-\frac{1}{2}x}$ so score for an expression of the form $(\mathrm{f}'(x)=)\ \pm A\left(x^3-4x\right)\mathrm{e}^{-\frac{1}{2}x}\pm\left(Bx^2\pm C\right)\mathrm{e}^{-\frac{1}{2}x}$. ($A, B, C\neq 0$) Condone the squared missing on the Bx^2 term

(ii) using the quotient rule on $\mathrm{f}(x)=\dfrac{x^3-4x}{\mathrm{e}^{\frac{1}{2}x}}$ so score for an expression of the form

$$(\mathrm{f}'(x)=)\ \frac{\pm\mathrm{e}^{\frac{1}{2}x}\left(Bx^2\pm C\right)-A\mathrm{e}^{\frac{1}{2}x}\left(x^3-4x\right)}{\left(\mathrm{e}^{\frac{1}{2}x}\right)^2}$$ ($A, B, C\neq 0$) Condone the squared

missing on the Bx^2 term or an attempt at (iii) $\mathrm{f}'(x)=uvw'+uv'w+u'vw$ which could look like:

$$(x^2-4)\mathrm{e}^{-\frac{1}{2}x}+x\frac{\mathrm{d}\left((x^2-4)\mathrm{e}^{-\frac{1}{2}x}\right)}{\mathrm{d}x}=\pm...(x^2-4)\mathrm{e}^{-\frac{1}{2}x}\pm x(...x)\mathrm{e}^{-\frac{1}{2}x}\pm...x(x^2-4)\mathrm{e}^{-\frac{1}{2}x}$$

A1 Correct $\mathrm{f}'(x)$ but may be unsimplified. Isw after a correct unsimplified expression

$$\left(3x^2-4\right)\mathrm{e}^{-\frac{1}{2}x}-\frac{1}{2}\left(x^3-4x\right)\mathrm{e}^{-\frac{1}{2}x} \text{ or } (x^2-4)\mathrm{e}^{-\frac{1}{2}x}+x\frac{\mathrm{d}\left((x^2-4)\mathrm{e}^{-\frac{1}{2}x}\right)}{\mathrm{d}x}=(x^2-4)\mathrm{e}^{-\frac{1}{2}x}+x(2x)\mathrm{e}^{-\frac{1}{2}x}-\frac{1}{2}x(x^2-4)\mathrm{e}^{-\frac{1}{2}x}$$

(b) Note on EPEN it is M1A1M1A1 but we are marking this M1B1M1A1

M1 Full method to find the equation of the normal through *O*.

Look for an attempt at $\mathrm{f}'(0)$ followed by the equation $y=-\dfrac{1}{\mathrm{f}'(0)}x$

B1 Equation of normal is $y=\dfrac{1}{4}x$ (seen or implied) (which may follow an incorrect $\mathrm{f}'(x)$ from part (a))

M1 Equates their $y=\dfrac{1}{4}x$ (which must be a straight line through the origin) with $\mathrm{f}(x)=x\left(x^2-4\right)\mathrm{e}^{-\frac{1}{2}x}$, divides through or factorises out the *x* term and attempts to make x^2 (or allow $4x^2$) the subject

A1* Full proof showing all steps. There is no requirement to justify the – sign. Note that A1* cannot be scored if A0 in part (a), unless they restart in (b).

(c)(i)

M1 Substitutes $x=-2$ into the iteration formula and finds x_2. May also be implied by –2.0228 or –2.0229

A1 awrt –2.0229

(ii)

A1 $(x=)-2.0226$ correct to 4 dp

Question Number	Scheme	Marks
10.(a)	$(1+2\cos 2x)^2 = 1+4\cos 2x+4\cos^2 2x$	
	Uses $\cos 4x = 2\cos^2 2x - 1 \Rightarrow (1+2\cos 2x)^2 = 1+4\cos 2x+2\cos 4x+2$	M1
	$= 3+4\cos 2x+2\cos 4x$	A1
		(2)
(b)	$a = \dfrac{2\pi}{3}$	B1
	$\displaystyle\int 3+4\cos 2x+2\cos 4x \,\mathrm{d}x = 3x+2\sin 2x+\frac{1}{2}\sin 4x$	M1 A1 ft
	Area $= \left[3x+2\sin 2x+\dfrac{1}{2}\sin 4x\right]_0^{\frac{2\pi}{3}} = 2\pi - \dfrac{3}{4}\sqrt{3}$	dM1 A1
		(5)
		(7 marks)

(a)

M1 Attempts to multiply out $(1+2\cos 2x)^2 = 1+\ldots\cos 2x+\ldots\cos^2 2x$ and use $\cos 4x = 2\cos^2 2x - 1$ to obtain $(1+2\cos 2x)^2$ in the form $p+q\cos 2x+r\cos 4x$.

Condone slips in the rearrangement of $\cos 4x = 2\cos^2 2x - 1$ but it must be clear that the identity was correct originally otherwise M0.

Beware of candidates who write $(1+2\cos 2x)^2 = 1+4\cos 2x+4\cos^2 4x$ which is M0A0

A1 $3+4\cos 2x+2\cos 4x$

(b)

B1 Deduces that $a = \dfrac{2\pi}{3}$ (allow 120° for this mark). If more than one angle is found, then look for which one is substituted into their integrated expression.

M1 Integrates $q\cos 2x + r\cos 4x \to \pm\ldots\sin 2x \pm\ldots\sin 4x$

A1ft Integrates $p+q\cos 2x+r\cos 4x \to px+\dfrac{q}{2}\sin 2x+\dfrac{r}{4}\sin 4x$ unsimplified where *p*, *q* and $r \neq 0$

dM1 Substitutes 0 and $a = \dfrac{2\pi}{3}$ (or awrt 2.09) (or equivalent) into a valid function (M1 must have been scored) and subtracts either way around. Note q, $r \neq 0$ but they do not need a *px* term.

Also allow $a = \dfrac{\pi}{3}$ (or awrt 1.05) or $a = \dfrac{4\pi}{3}$ (or awrt 4.19) *a* must be in radians to evaluate correctly.

This mark cannot be scored without a value for a. You do not have to explicitly see 0 substituted in and their answer may imply a correct substitution into their integrated expression.

A1 $2\pi - \frac{3}{4}\sqrt{3}$ or simplified equivalent