# **Statistics S3 Mark scheme**

Question	Scheme	Marks
1(a)	$\{w\} = 018 \text{ or } 18$	B1
		(1)
(b)	$\{x\} = 18$	B1
		(1)
(c)	$\{\text{prob}\}=0$	B1
		(1)
(d)	Advantage: Any one of:	B1
	<ul> <li><u>Simple or easy</u> to use also allow "quick" or "efficient" (o.e.)</li> <li>It is suitable for large samples (or populations)</li> <li>Gives a good spread of the data</li> </ul>	
	<ul> <li>Disadvantage: Any one of:</li> <li>The alphabetical list is (probably) not random</li> <li>Biased since the list is not (truly) random</li> <li>Some combinations of names are not possible</li> </ul>	B1
		(2)
		(5 marks)

# **Notes:**

(d) If no labels are given treat the 1<sup>st</sup> reason as an advantage and the 2<sup>nd</sup> as a disadvantage

**B1:** For advantage

**B1:** For disadvantage – "it requires a sampling frame" is 2<sup>nd</sup> B0 since the alphabetical list is given. Note: Do not score both B1 marks for opposing advantages and disadvantages.

Question					S	Schem	ıe					Marks	
2(a)		$\overline{A}$	В	C	L	N	R	S	T	Y			
	Judge 1	6	3	4	9	2	8	1	5	7			
	Judge 2	8	4	5	7	3	9	1	2	6			
	or											M1	
		S	N	В	C	T	$\boldsymbol{A}$	Y	R	L			
	Judge 1								8	9			
	Judge 2	1	3	4	5	2	8	6	9	7			
	$\sum d^2 = 4$	+ 1 + <sup>1</sup>	1 + 4 -	<b>⊥1</b> ⊥ 1	1 + 0 -	+ 9 + °	1					M1	
			+1+					4 = 2	22		$\sum d^2 = 22$	A1	
	6(22)												
	$r_s = 1 - \frac{602}{903}$	(80);	= 0.8	16666	66				$\frac{4}{6}$	$\frac{9}{0}$ or	awrt <b>0.817</b>	A1	
												(5)	
(b)	$H_0: \rho = 0$	, H <sub>1</sub> :	$\rho$ > (	)								B1	
	Critical Value = 0.7833 or CR: $r_s \ge 0.7833$ <b>0.7833</b>											B1	
	Since $r_s =$	0.816	6 i	t lies	in the	CR,	or r	eject	H <sub>0</sub> (0	.e.)		M1	
	The two judges (or "they") are in agreement or											Λ 1 Ω	
	there is a p	ositiv	e cor	relatio	on bet	ween	the ra	anks o	of the	two	judges.	A1ft	
												(4)	
											(9	9 marks)	

#### **Notes:**

(a)

M1: For an attempt to rank at least one row (at least 4 correct)

M1: For an attempt at  $d^2$  row (may be implied by sight of  $\sum d^2 = 22$  or 221 for reverse ranks)

A1: For  $\sum d^2 = 22$  (or 221 if reverse ranking is used) Can be implied by correct answer.

M1: For use of the correct formula with their  $\sum d^2$  (if it is clearly stated) If the answer is not correct then a correct expression is required

False Ranking - e.g. Alphabetic ranking: Gives

Judge 1: 7 5 2 3 8 1 9 6 4

Judge 2: 7 8 5 2 3 9 4 1 6  $\sum d^2 = 162$  and  $r_s = -0.35$ 

### **Question 2 notes** continued

Scores: M0(for ranking), M1(for attempt at  $d^2$  row), A0, M1 (for use of their  $\sum d^2$ ), A0 i.e. 2 out of 5. Can follow through their  $r_s$  in (b)

**(b)** 

- **B1:** For both hypotheses stated correctly in terms of  $\rho$  (allow  $\rho_s$ ) H<sub>1</sub> must be compatible with ranking.
- **B1:** For cv = 0.7833 (independent of their H<sub>1</sub> (no 2-tail value in tables) <u>but</u> compatible sign with their  $r_s$ ).
- **M1:** For a correct statement (in words) relating their  $r_s$  with their critical value. E.g. "reject  $H_0$ ", "in critical region", "significant", "positive correlation". May be implied by a correct contextual comment.
  - |cv|>1 If their cv is |cv|>1 (often from using normal tables) award M0A0
    - If |their | > |their cv| then "significant" (o.e.) for M1 and "judges are in
    - agreement" (o.e.) for A1ft
    - If |their | < |their cv| then "not significant" (o.e.) for M1 and "judges don't agree" (o.e.) for A1ft
- **A1ft:** For a correct follow through conclusion in context. "Positive correlation" alone scores M1 A0. For reverse ranking should still say "judges <u>are</u> in agreement"

Question				Scheme				Mark	
3(a)	$\widehat{\lambda} = \frac{0(47)}{*}$	+ 1(57) + 2	(46) + 3(35) 200	5) + 4(9) + 3	$\frac{5(6)}{2} = \frac{2}{2}$	$\frac{320}{200} = 1.6$	Full exp' or at least 2 products and 320/200 seen	B1 *	
								(1)	
(b)	$r = 200 \times \frac{e}{r}$	$\frac{e^{-1.6}(1.6)^2}{2!}$ {	= 51.68550	0861}		Using $r = 200 \times$	$\frac{e^{-1.6}(1.6)^2}{2!}$	M1	
	s = 200 - ( their $r + s =$		.61 + their	r + 27.57 +	-11.03)	{= 4.72449	139} <u>or</u>	M1	
	r = 51.685	50861 ar	s = 4.72	449139		r = a	wrt <b>51.69</b> and $s = \text{awrt } 4.72$	A1	
								(3)	
(c)	H <sub>0</sub> : Poisso	on (distribut	tion) is a su	itable/ sens	ible (mo	odel)		В1	
	H <sub>1</sub> : Poisson (distribution ) is not a suitable/ sensible (model).								
	Number of accidents	Observed	Expected	Combined Observed	Combin	<u> </u>	$\frac{2}{E}$		
	0	47	40.38	47	40.38				
	1	57	64.61	57	64.61				
	2	46	51.69	46	51.69				
	3	35	27.57	35	27.57	2.0024	44.4324		
	4 ≥ 5	9	11.03 4.72	15	15.75	0.0357	14.2857	M1	
			1.,2		Tota	als 4.6461	204.6461		
	<b>\</b>	$(2-F)^2$	$\mathbf{\Sigma}^{0^2}$					M1	
	$X^2 = \sum \frac{0}{2}$	$\frac{E}{E}$ 0:	$\sum \frac{S}{E}$	- 200 ;= <sup>2</sup>	1.6461		awrt <b>4.65</b>	A1	
	v = 5 - 1 -						3	B1 ft	
	$\chi_3^2(0.10) =$	6.251 ⇒ 9	$CR: X^2 \geqslant$	6.251			6.251	B1 ft	
	, ,				an than	is insuffici			
	[Since $X^2 = 4.6461$ does not lie in the CR, then there is insufficient evidence to reject $H_0$ ]								
	The number of <i>accidents</i> per day can be modelled by a Poisson distribution <u>or</u> the <i>supervisor's</i> belief is correct.								
								(7)	
							(1	1 marks	

**(b)** 

**Note:** Allow A1 for s = awrt 4.74 (fou as a result of using expected values to full accuracy.)

### **Question 3 notes** continued

(c)

**B1:** For <u>both</u> hypotheses and mentioning Poisson at least once. Allow Poisson is a "good fit/model" but <u>not</u> "good method". Inclusion of 1.6 for mean in hypotheses is B0 but condone in conclusion.

M1: For an attempt to pool 4 accidents and  $\geq 5$  accidents or pool when  $E_i < 5$  No pooling is M0

M1: For an attempt at the test statistic, at least 2 correct expressions/values (to awrt 2 d.p.)

A1: For awrt 4.65 (score M1M1A1 if awrt 4.65 seen). **No pooling:** If no pooling can allow  $2^{nd}$  M1 if  $X^2 = 5.33$  is seen

**B1ft:** For n-1-1 i.e. subtracting 2 from their n.

**B1ft:** For a correct ft for their  $\chi_k^2(0.10)$ , where k = n - 1 - 1 from their n. (B1B1 may be implied by 6.251 (if pooling) or 7.779 for no pooling)

**A1ft:** (*Dep. on the* 2<sup>nd</sup> M1) For correct comment in context based on their test statistic and their critical value that mentions *accidents* or *supervisor*. Condone mention of Po(1.6) in conclusion. Score A0 for inconsistencies e.g. "significant" followed by "supervisor's belief is justified"

Note: Full accuracy gives a combined expected frequency of 15.76,  $\frac{(O-E)^2}{E} = 0.0366$ ,

 $\frac{O^2}{E}$  = 14.2766,  $X^2$  = 4.64855... and p-value 0.199.

4(a) Let $X =$ weight of a sack of potatoes, $X \sim N(25.6, 0.24^2)$ So $D = X_1 - X_2 \sim N(0, 2(0.24)^2)$ or $D \sim N(0, 0.1152)$ $\begin{cases} P( D  > 0.5) = \begin{cases} 2P(D > 0.5) \\ = 2 \times P\left(Z > \frac{0.5}{\sqrt{0.1152}}\right) \end{cases}$ $= 2 \times P\left(Z > \frac{0.5}{\sqrt{0.1152}}\right)$ $= 0.1416$ Attempt at $D$ and $D \sim N(0,)$ $2 \times P(D > 0.5) \text{ can be implied}$ $dM1$ $= 2 \times P\left(Z > \frac{0.5}{\sqrt{0.1152}}\right)$ $= 0.1416$ awrt 0.141 or awrt 0.142  (6)  (b) Let $Y =$ weight of an empty pallet, $Y \sim N(20.0, 0.32^2)$ So $T = X_1 + X_2 + + X_{30} + Y$ $T \sim N(30(25.6) + 20, 30(0.24)^2 + 0.32^2)$ $T \sim N(788, 1.8304)$ $T \sim N(788, 1.8304)$ $\{P(T > 785) = \} P\left(Z > \frac{785 - 788}{\sqrt{1.8304}}\right)$ $= P(Z > -2.2174)$ $= 0.9868$ awrt 0.987  Al (5)	Question	Scheme			Marks				
So $D = X_1 - X_2 \sim N(0, 2(0.24)^2)$ or $D \sim N(0, 0.1152)$ $ \begin{cases} P( D  > 0.5) = \begin{cases} 2 \times P(D > 0.5) \\ = 2 \times P(Z > \frac{0.5}{\sqrt{0.1152}}) \end{cases} $ $= 2 \times P(Z > 1.4731) \text{ or } = 2(1 - 0.9292)$ $= 0.1416$ $= 0.1416$ $= 0.142$ $= 0.1416$ $= 0.142$ $= 0.1416$ $= 0.142$ $= 0.1416$ $= 0.142$ $= 0.1416$ $= 0.142$ $= 0.1416$ $= 0.142$ $= 0.1416$ $= 0.142$ $= 0.1416$ $= 0.142$ $= 0.1416$ $= 0.142$ $= 0.1416$ $= 0.142$ $= 0.1416$ $= 0.142$ $= 0.1416$ $= 0.142$ $= 0.1416$ $= 0.142$ $= 0.1416$ $= 0.141$ $= 0.142$ $= 0.141$ $= 0.141$ $= 0.142$ $= 0.141$	4(a)								
		So $D = X_1 - X_2 \sim N(0, 2(0.24)^2)$ or		and	M1				
$\{P( D  > 0.5) = \} \ P(D > 0.5) $ implied $= 2 \times P\left(Z > \frac{0.5}{\sqrt{0.1152}}\right) $ dM1 $= 2 \times P(Z > 1.4731) \text{ or } = 2(1 - 0.9292)$ $= 0.1416 $ awrt 0.141 or awrt 0.142  A1 $= 0.142 $ (6) $\text{(b)} $ Let $Y = \text{weight of an empty pallet}, $ $Y \sim N(20.0, 0.32^2) $ So $T = X_1 + X_2 + + X_{30} + Y$ $= X_1 + X_2 + + X_{30} + Y $ 30(25.6) + 20 or 788 B1 $= X_1 + X_2 + + X_{30} + Y $ 30(0.24) <sup>2</sup> + 0.32 <sup>2</sup> M1 $= X_1 + X_2 + + X_{30} + Y $ 1.83 A1 $= X_1 + X_2 + + X_{30} + Y $ 1.84 A1 $= X_1 + X_2 + + X_{30} + Y $ 1.85 A1 $= X_1 + X_2 + + X_{30} + Y $ 1.85 A1 $= X_1 + X_2 + + X_{30} + Y $ 1.85 A1 $= X_1 + X_2 + + X_{30} + Y $ 1.85 A1 $= X_1 + X_2 + + X_{30} + Y $ 1.85 A1 $= X_1 + X_2 + + X_{30} + Y $ 1.85 A1 $= X_1 + X_2 + + X_{30} + Y $ 1.85 A1 $= X_1 + X_2 + + X_{30} + Y $ 1.85 A1 $= X_1 + X_2 + + X_{30} + Y $ 1.85 A1 $= X_1 + X_2 + + X_{30} + Y $ 1.85 A1 $= X_1 + X_2 + + X_{30} + Y $ 1.85 A1 $= X_1 + X_2 + + X_{30} + Y $ 1.85 A1 $= X_1 + X_2 + + X_{30} + Y $ 1.85 A1 $= X_1 + X_2 + X_2 + + X_{30}$		$D \sim N(0, 0.1152)$		` ′ ` ′	A1 A1				
$= 2 \times P \left( Z > \frac{SS}{\sqrt{0.1152}} \right)$ $= 2 \times P \left( Z > \frac{SS}{\sqrt{0.1152}} \right)$ $= 0.1416$ $= 0.1416$ $= 0.141$ $= 0.141$ $= 0.142$ $= 0.1416$ $= 0.141$ $= 0.142$ $= 0.141$ $= 0.142$ $= 0.141$ $= 0.141$ $= 0.142$ $= 0.141$ $= 0.141$ $= 0.142$ $= 0.141$		${P( D  > 0.5) = } 2P(D > 0.5)$	2 × P(	` '	dM1				
= 0.1416 $= 0.1416$ $= 0.141  or awrt  0.141  or awrt  0.142$ $= 0.1416$ $= 0.1416$ $= 0.142$ $= 0.1416$ $= 0.142$ $= 0.1416$ $= 0.142$ $= 0.1416$ $= 0.141  or awrt  0.141  or awrt  0.142$ $= 0.142$ $= 0.1416$ $= 0.1416$ $= 0.142$ $= 0.1416$ $= 0.14$		$= 2 \times P\left(Z > \frac{0.5}{\sqrt{0.1152}}\right)$			dM1				
(b) Let $Y =$ weight of an empty pallet, $Y \sim N(20.0, 0.32^2)$ So $T = X_1 + X_2 + + X_{30} + Y$ $T \sim N(30(25.6) + 20, 30(0.24)^2 + 0.32^2)$ $T \sim N(788, 1.8304)$ $T \sim N(788, 1.8304)$ $\{P(T > 785) = \} P\left(Z > \frac{785 - 788}{\sqrt{1.8304}}\right)$ $= P(Z > -2.2174)$ $= 0.9868$ Aurt 0.987 A1 (5)		$= 2 \times P(Z > 1.4731) $ or $= 2(1 - 0.9292)$							
(b) Let $Y =$ weight of an empty pallet, $Y \sim N(20.0, 0.32^2)$ So $T = X_1 + X_2 + + X_{30} + Y$ $T \sim N(30(25.6) + 20, 30(0.24)^2 + 0.32^2)$ $T \sim N(788, 1.8304)$ $T \sim N(788, 1.8304)$ $\{P(T > 785) = \} P\left(Z > \frac{785 - 788}{\sqrt{1.8304}}\right)$ $= P(Z > -2.2174)$ $= 0.9868$ Aurt 0.987  A1  (5)		= 0.1416		141 or awrt	A1				
$ \begin{array}{c} Y \sim \mathrm{N}(20.0\;, 0.32^2) \\ \hline \mathrm{So}\; T = X_1 + X_2 + \ldots + X_{30} + Y \\ \hline \\ T \sim \mathrm{N}(30(25.6) + 20\;, 30(0.24)^2 + 0.32^2) \\ \hline \\ T \sim \mathrm{N}(788\;, 1.8304) \\ \hline \\ \{\mathrm{P}(T > 785) = \}  \mathrm{P}\left(Z > \frac{785 - 788}{\sqrt{1.8304}}\right) \\ \hline = \mathrm{P}(Z > -2.2174\ldots) \\ \hline = 0.9868 \\ \hline \end{array} \qquad \begin{array}{c} \mathrm{30}(25.6) + 20 \;\; \underline{\mathrm{or}}\; 788  \mathrm{B1} \\ \hline 30(0.24)^2 + 0.32^2  \mathrm{M1} \\ \hline \mathrm{N}\; \mathrm{and}\; 1.8304 \;\; \mathrm{or}\; \mathrm{awrt} \\ 1.83 \\ \hline \end{array} \qquad \begin{array}{c} \mathrm{A1} \\ \hline 1.83 \\ \hline \end{array} $			ı		(6)				
$T \sim N(30(25.6) + 20, 30(0.24)^{2} + 0.32^{2})$ $T \sim N(788, 1.8304)$ $\{P(T > 785) = \} P\left(Z > \frac{785 - 788}{\sqrt{1.8304}}\right)$ $= P(Z > -2.2174)$ $= 0.9868$ $30(25.6) + 20 \text{ or } 788$ $B1$ $30(0.24)^{2} + 0.32^{2}$ $M1$ $N \text{ and } 1.8304 \text{ or awrt}$ $1.83$ $M1$ $= P(Z > -2.2174)$ $= 0.9868$ $\text{awrt } 0.987$ $A1$ $(5)$	(b)								
$T \sim N(30(25.6) + 20, 30(0.24)^{2} + 0.32^{2})$ $T \sim N(788, 1.8304)$ $\{P(T > 785) = \} P\left(Z > \frac{785 - 788}{\sqrt{1.8304}}\right)$ $= P(Z > -2.2174)$ $= 0.9868$ $A1$ $(5)$		So $T = X_1 + X_2 + \dots + X_{30} + Y$							
$30(0.24)^{2} + 0.32^{2} \qquad M1$ $T \sim N(788, 1.8304) \qquad N \text{ and } 1.8304 \text{ or awrt} \qquad 1.83$ $\{P(T > 785) = \}  P\left(Z > \frac{785 - 788}{\sqrt{1.8304}}\right) \qquad M1$ $= P(Z > -2.2174)$ $= 0.9868 \qquad \text{awrt } 0.987 \qquad \text{A1}$ (5)		T N(20/25 () 20 20/02 () 2 20/2	30(25	.6) + 20 <u>or</u> <b>788</b>	B1				
$\{P(T > 785) = \} P\left(Z > \frac{785 - 788}{\sqrt{1.8304}}\right) $ $= P(Z > -2.2174)$ $= 0.9868$ <b>awrt 0.987 A1</b> (5)		$I \sim N(30(25.6) + 20, 30(0.24)^2 + 0.32^2)$	3	$30(0.24)^2 + 0.32^2$	M1				
$ \begin{cases} P(T > 785) =  \end{cases} P\left(Z > \frac{1}{\sqrt{1.8304}}\right) $ $ = P(Z > -2.2174) $ $ = 0.9868 $ awrt 0.987 A1 (5)		$T \sim N(788, 1.8304)$	N an		A1				
= 0.9868 awrt 0.987 A1 (5)		$\left\{ P(T > 785) = \right\}  P\left(Z > \frac{785 - 788}{\sqrt{1.8304}}\right)$			M1				
(5)		= P(Z > -2.2174)							
		= 0.9868		awrt 0.987					
					(5) Total 11)				

(Total 11)

#### **Notes:**

(a)

M1: For clear definition of D and normal distribution with mean of 0 (Can be implied by  $3^{rd}$  M1).

**A1:** For correct use of  $Var(X_1 - X_2)$  formula.

**A1:** For 0.1152

**dM1:** For realising need  $2 \times P(D > 0.5)$  (Dependent on 1<sup>st</sup> M1 i.e. must be using suitable D).

**dM1:** Dep on 1st M1 for standardising with 0.5, 0 and their s.d.( $\neq$  0.24)Must lead to P(Z > + ve) (o.e.). P(Z > 1.47) implies 1<sup>st</sup> M1 1<sup>st</sup> A1 2<sup>nd</sup> A1 and 3<sup>rd</sup> M1. Correct answer only will score 6 out of 6.

### **Question 4 notes** continued

**(b)** 

**B1:** For a mean of 30(25.6) + 20. Can be implied by 788.

M1: For  $30(0.24)^2 + 0.32^2$ . Can be implied by 1.8304 or awrt 1.83

Allow M1 for swapping error i.e.  $30 \times 0.32^2 + 0.24^2$  if the expression is seen

**A1:** For normal and correct variance of 1.8304 or awrt 1.83. Normality may be implied by standardisation

M1: For standardising with 785 with their mean and st. dev.. $(\neq 0.24)$  Must lead to P(Z > -ve)

**A1:** Awrt 0.987. Correct answer only will score 5 out of 5

Note: Calculator answers are (a) 0.14071..., (b) 0.98670...

Question					Scher	ne			Marks
5	H <sub>0</sub> : Grades associated) H <sub>1</sub> : Grades associated)				_		not	"grades" and "gender" mentioned at least once.	B1
	Observed Distinction		Mal		Female	<b>;</b>		An attempt to convert percentages to observed frequencies.	M1
	Merit Unsatisfacto	ory	36		96 20			All observed frequencies are correct.	A1
	1	Expected Male Distinction 45 Merit 123.889			Female 36 99.111	6	Totals 81 223	Some attempt at $\frac{\text{(Row Total)(Column Total)}}{\text{(Grand Total)}}$ Can be implied by a correct $E_i$	M1
	Unsatisfacto	ory		31.111 24 200 1			56 360	All expected frequencies are correct to nearest integer.	A1
	Observed  37  44  127	37 45 1. 44 36 1.					$ \frac{O^2}{E} $ 30.422 53.778 130.189	At least 2 correct terms for $\frac{(O-E)^2}{E}$ or $\frac{O^2}{E}$ or correct expressions with their $E_i$ . Accept 2 sf accuracy for the M1 mark.	M1
	96 99.111 0.098 92.987 36 31.111 0.768 41.657 20 24.889 0.960 16.071 Totals 5.104 365.104						41.657 16.071	All correct $\frac{(O-E)^2}{E} \text{ or } \frac{O^2}{E}$ terms to either 2 dp or better. Allow truncation. $(\Rightarrow \text{by awrt } 5.1 \text{ if } 3^{\text{rd}}$ M1 seen)	A1
	$X^2 = \sum \frac{(O)}{C}$	$\frac{(E-E)^2}{E}$	2 - or	$\sum \frac{C}{I}$	$\frac{0^2}{E} - 360$	;=	awrt 5.1	awrt 5.1	A1
	v = (3-1)(2	-1) =	= 2					$(\nu =)$ 2 (Can be implied by 5.991)	B1
	$\chi_2^2(0.05) = 5$							For <b>5.991</b> only	B1
	Since $X^2 = 5.1$ does not lie in the CR, then there is insufficient evidence to reject $H_0$								M1

Question	Scheme	Marks				
5	Business Studies grades and gender are independent or					
continued	There is no association between Business Studies grades and gender or	A1ft				
	Head of department's (belief) is correct					
		(4)				
	(1	2 marks)				
<b>Notes:</b>						
Final M1:	For a correct statement linking their test statistic and their critical value ( $> 3$ .	8)				
	Note: Contradictory statements score M0. E.g. "significant, do not reject I	Ι <sub>0</sub> ".				
Final A1ft:	For a correct ft statement in context –					
	must mention "grades" and "gender" or "sex" or "head of department"					
	Condone "relationship" or "connection" here but <b>not</b> "correlation".					
	e.g. "There is no evidence of a relationship between grades and gender"					
5.10 only	Just seeing 5.10 only can imply 1 <sup>st</sup> 3 Ms but loses 1 <sup>st</sup> 3 As so can score 4 out of 7 (Qu says show")					
<b>Note:</b> Full accuracy gives $X^2 = 5.104356$ and p-value 0.0779						

Question					Scher	ne			Marks
5	Mark Scheme for candidates who use percentages instead of observed values.								
	H <sub>0</sub> : Grades associa H <sub>1</sub> : Grades associ	ated)	ender	"grades" and "gender" mentioned at least once.	B1				
	Observed Distinction Merit		Male 18.5 63.5		27 60	Female 27.5 60.0		These marks cannot be obtained.	M0 A0
	Expecte Distinction	ed	M	ale 3	Femal 23		Totals 46	Some attempt at  (Row Total)(Column Total) (Grand Total)	M1
	Merit Unsatisfac	tory		.75 .25	61.75 15.25		$\longrightarrow$ one of these E		
	Totals					200	Expected frequencies are not correct.	A0	
	Observed	Expe	ected (O-		$\frac{(O-E)^2}{E}$		$\frac{O^2}{E}$	At least 2 "correct" terms for $\frac{(O-E)^2}{E}$	
	18.5 27.5	23 23			804		.8804	or $\frac{O^2}{E}$ or correct expressions with	M1
	63.5 60.0	61. 61.			496 496		.2996	their $E_i$ . Accept 2 sf	
	18.0 12.5	15			.959 .959		.2459	accuracy for the M1 mark.	
		То	otals	2.8	518	202	2.8518	This mark cannot be obtained.	A0
	$X^{2} = \sum \frac{(O-E)^{2}}{E}$ or $\sum \frac{O^{2}}{E} - 360 = 2.8518$							This mark cannot be obtained.	A0
	$\nu = (3-1)(2-1) = 2$							$(\nu =)$ <b>2</b> (Can be implied by 5.991)	B1
	$\chi_2^2(0.05) = 5$	.991 =	CR:	$X^2 \geqslant$	5.991			For <b>5.991</b> only	B1

Question	Scheme					
5 continued	Since $X^2 = 2.86$ does not lie in the CR, then there is insufficient evidence to reject $H_0$	M1				
	Not available since comes from incorrect	A0				
	·	(12)				
	(1)	2 marks)				
Notoge						

### **Notes:**

If a candidate uses percentages rather than observed values then they can obtain a maximum of **6 marks**. They can get B1 M0A0 M1A0 M1A0A0 B1B1M1A0.

Question	Scheme			Marks			
6(a)	$\left\{ \hat{\mu} = \frac{\sum x}{n} = \frac{1570}{50} = \right\} \ \overline{x} = 31.4$						
	$\left\{ \hat{\sigma}^2 = \frac{\sum x^2 - n\overline{x}^2}{n - 1} = \right\} s_x^2 = \frac{49467.58 - 50(31.4)^2}{50 - 1}$						
	= 3.460816		awrt <b>3.46</b>	A1			
				(4)			
<b>(b)</b>	[Let $Y = \text{time taken to complete obstacle coulons}]$	urse in the af	ternoon.]				
	$H_0: \mu_x = \mu_y, \ H_1: \mu_x > \mu_y$			B1			
	$(z =) \frac{"31.4" - 30.9}{\sqrt{\frac{"3.46"}{50} + \frac{3.03}{50}}}$			M1 A1ft			
	= 1.38781		awrt <b>1.39</b>	A1			
	CR: $Z \ge 1.6449$ or probability = awrt 0.082 or awrt 0.083						
	Since $z = 1.38781$ does not lie in the CR, the evidence to reject $H_0$	hen there is i	nsufficient	M1			
	Conclude that the <u>mean time</u> to complete the same for the early <u>morning</u> and late <u>afternoo</u>		urse is the	A1			
				(7)			
(c)	$\overline{X}$ and $\overline{Y}$ are both approx. normally distrib (Condone $\overline{x}$ and $\overline{y}$ )	outed or $\overline{X}$ –	$\overline{Y}$ normal	B1			
				(1)			
(d)	Have assumed $s^2 \simeq \sigma^2$ or variance of sampl population	e = variano	ce of	B1			
				(1)			
			(1:	3 marks)			

(13 marks)

## **Notes:**

(a)

31.4 cao. Allow 31 minutes, 24 seconds. **B1**:

A correct expression for either s or  $s^2$  (ignore label) M1:

**A1ft:** A correct expression for  $s^2$  with their ft  $\bar{x}$ .

**A1:** Awrt 3.46 (Correct answer scores 3 out of 3)

**(b)** 

Both hypotheses stated correctly, with some indication of which  $\mu$  is which. E.g. **B1**:  $\mu_{\scriptscriptstyle M}\,,\mu_{\scriptscriptstyle A}$ 

## Question 6 notes continued

- M1: For an attempt at  $\frac{a-b}{\sqrt{\frac{c}{50} + \frac{d}{50}}}$  with at least 3 of a, b, c or d correct. Allow  $\pm$
- **A1ft:** For  $\pm \frac{\text{their } 31.4 30.9}{\sqrt{\frac{\text{their } 3.46}{50} + \frac{3.03}{50}}}$

Allow 
$$D = \overline{x} - \overline{y}$$
 1.64 ~ 1.65 =  $\frac{D - 0}{\sqrt{\frac{"3.46"}{50} + \frac{3.03}{50}}}$  [SE = 0.360277..]

- A1: For awrt 1.39 (possibly  $\pm$ )(Allow for CV D = awrt 0.593) (NB d = 0.5) Correct answer scores M1A1ftA1 but  $0-(31.4-30.9) \rightarrow -1.39$  loses this 2<sup>nd</sup> A mark
- **B1:** Critical value of 1.6449 or better (seen). Allow for probability = awrt 0.082 or awrt 0.083.

Note: p-values are 0.0823 (tables) and 0.0826 (calculator).

- M1: For a correct statement linking their test statistic and their critical value.Note: Contradictory statements score M0. E.g. "significant, do not reject H<sub>0</sub>".
- **A1:** For a correct statement in context that accepts H<sub>0</sub> (no ft) Condone "no difference in mean times". Must mention "mean time", "morning" and "afternoon" or "both times of day"

(c)

**B1:** E.g.  $\overline{X} \sim N(...)$  need both. Allow in words e.g "sample means are normally distributed".

(d)

**B1:** Condone only mentioning "x" or "y" <u>but</u> watch out for  $s_x = s_y$  or  $\sigma_x = \sigma_y$  which scores B0.

Question	Scheme					
7(a)	Let $X =$ score on a	die				
	35	E(S) = 3.5	B1			
	$E(S) = 3.5$ , $Var(S) = \frac{35}{12}$	$Var(S) = \frac{35}{12}$ or awrt <b>2.92</b>	B1			
			(2)			
	So, $\overline{S} \sim N \left( "3.5", \frac{"\left(\frac{35}{12}\right)"}{40} \right)$ or $\overline{S} \sim N \left( "3.5", \frac{7}{96} \right)$					
(b)	$P(\overline{S} < 3) = P\left(Z < \frac{3 - "3.5"}{\sqrt{\frac{7}{96}}}\right) = P(Z < -1)$	.85164)}	M1			
	$\{=1-0.9678\} = 0.0322$ 0.032 to 0.0322					
			(3)			
		(:	5 marks)			

### **Notes:**

(a)

**B1:**  $(2^{nd})$  allow awrt 2.92

**(b)** 

**B1ft:** For  $\overline{S} \sim N\left("3.5", \frac{\left(\frac{35}{12}\right)"}{40}\right)$  seen or implied. Follow through their E(S) and their Var(S)

N.B 
$$\frac{7}{96} = 0.07291\dot{6}$$
 accept awrt 0.0729

M1: For an attempt to standardise with 3, their mean (>3) and  $\sqrt{\frac{\text{their Var}(S)}{40}}$ . Must lead to P(Z < -ve)

**A1:** For  $0.032 \sim 0.0322$ 

# Alternative ΣS

**B1ft:** For  $\sum S \sim N\left(140, \frac{350}{3}\right)$  where 140 is 40×their E(S) and variance is 40×their Var(S).

# **Question 7 notes** continued

**M1:** For 
$$P\left(Z < \frac{120 - "140"}{\sqrt{\frac{350}{3}}}\right)$$
 or  $P\left(Z < \frac{119.5 - "140"}{\sqrt{\frac{350}{3}}}\right) = P(Z < -1.8979...)$ 

**A1:** for 0.032~0.0322 or (with continuity correction) 0.0287 (tables) or 0.0289 (calculator).

Question	Scheme							
8(a)	$\left\{ \overline{x} = \frac{29.74 + 31.86}{2} \right\} \implies \overline{x} = 30.$	8	30.8 can be implied. See note.	B1				
	"1.96" $\left(\frac{\sigma}{\sqrt{n}}\right)$ = 31.86 - 30.8 or	$2("1.96")\left(\frac{c}{}\right)$	$\left(\frac{r}{n}\right) = 31.86 - 29.74$	M1				
	$SE_{\bar{x}} = \frac{31.86 - 30.8}{1.96} = 0.540816 = 0.54 (2 dp)$ awrt <b>0.54</b>							
				(3)				
(b)	A 90% CI for	$\mu$ is $\overline{x} \pm 1.6$	$449 \left( \frac{\sigma}{\sqrt{n}} \right)$	В1				
	$=30.8 \pm 1.6449(0.54)$	(their $\overline{x}$ ) $\pm$	(their $z$ )(their $SE_{\bar{x}}$ from (a))	M1				
	= (29.91, 31.69)		(awrt <b>29.9</b> , awrt <b>31.7</b> )	A1				
				(3)				
(c)	Let $X =$ number of confidence is	intervals con	taining $\mu$					
	or $Y =$ number of confidence	intervals not	containing $\mu$					
	So $X \sim Bin(4, 0.9)$ or $Y \sim Bin(4, 0.1)$							
	$P(X \ge 3) \text{ or } P(Y \le 1) = {}^{4}C_{3}(0.9)^{3}$	$(0.1) + (0.9)^4$	${}^{4}C_{3}(0.9)^{3}(0.1) + (0.9)^{4}$ oe	A1				
	= 0.2916 + 0.6561 = 0.9477		<b>0.9477</b> or <b>0.948</b>	A1				
				(3)				

(9 marls)

#### **Notes:**

(a)

**B1:** 
$$\overline{x} = 30.8 \text{ may be implied by } 1.96 \left( \frac{\sigma}{\sqrt{n}} \right) = [31.86 - 30.8] = 1.06 \text{ or}$$

$$2(1.96) \left( \frac{\sigma}{\sqrt{n}} \right) = 31.86 - 29.74$$

M1: A correct equation for either a width or a half-width involving a z-value  $1.5 \le z \le 2$  Eg: "their  $z''\left(\frac{\sigma}{\sqrt{n}}\right) = 31.86 - "30.8"$  ft their  $\overline{x}$  or  $2("\text{their }z")\left(\frac{\sigma}{\sqrt{n}}\right) = 31.86 - 29.74$  or "their  $z''(\text{SE}_{\overline{x}}) = 31.86 - "30.8"$  or  $2("\text{their }z")(\text{SE}_{\overline{x}}) = 31.86 - 29.74$  are fine for M1.

A1: 0.54 or awrt 0.54 Must be seen as final answer to (a) NB  $\frac{53}{98}$  as final answer is A0 Condone  $\bar{x} \pm 1.96\sigma = ...$  for B1 and M1 but A0 even if they say " $\sigma = \text{standard error} = 0.54$ ". Otherwise answer only of 0.54 scores 3 out of 3