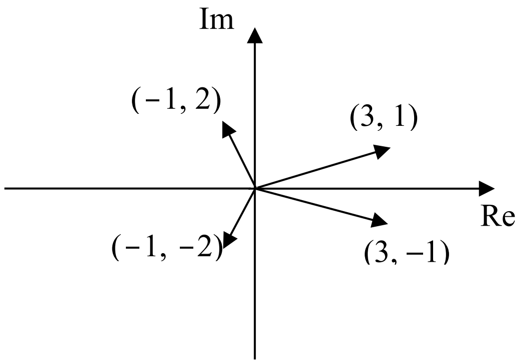
Mark Scheme (Results)

October 2020

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F1
(WFM01/01)

Question Number	Scheme	Notes	Marks
1(a)	$\mathrm{f}(x)=x^3-\dfrac{10\sqrt{x}-4x}{x^2} \qquad x>0$		
	$\mathrm{f}(1.4)=-0.435673...$ $\mathrm{f}(1.5)=0.598356...$	Attempts both f(1.4) and f(1.5)	M1
	Sign change (positive, negative) (and $\mathrm{f}(x)$ is continuous) therefore (a **root**) α is between $x=1.4$ and $x=1.5$	Both f(1.4) = awrt −0.4 and f(1.5) = awrt 0.6, sign change and conclusion. For 'sign change' indication that f(1.4) < 0 and f(1.5) > 0 is sufficient. Also f(1.4) f(1.5) < 0 is sufficient. 'Therefore root' (without mention of the interval) is a sufficient conclusion. Mention of 'continuous' is not required.	A1
			(2)
(b)	$\left(\mathrm{f}(x)=x^3-\dfrac{10\sqrt{x}-4x}{x^2}=x^3-10x^{-\frac{3}{2}}+4x^{-1}\right)$		
	$\mathrm{f}'(x)=3x^2+15x^{-\frac{5}{2}}-4x^{-2}$ (or equivalent, see below)	$x^n\to x^{n-1}$ for one term	M1
		2 correct terms simplified or unsimplified	A1
		All correct simplified or unsimplified	A1
			(3)
(c)	$(x_1)=1.4-\dfrac{\mathrm{f}(1.4)}{\mathrm{f}'(1.4)}$ $\left(=1.4-\dfrac{-0.43567...}{10.30720...}\right)=...$	Correct application of N-R leading to an answer. <u>Values</u> of f(1.4) and f '(1.4) need not be seen before their final answer.	M1
	$=1.442$	cao (must be corrected to 3 d.p.) isw if x_2, etc. have been found, but the answer for 'one use of N-R' must be seen as 1.442 to score this mark.	A1
			(2)
			Total 7
(b)	Equivalent unsimplified versions are acceptable, e.g. (using quotient rule); $3x^2-\dfrac{\left(5x^{\frac{3}{2}}-4x^2\right)-20x^{\frac{3}{2}}+8x^2}{x^4}$	The 'two correct terms' still applies for the first A1. Here a 'term' would be, for example, the $x^{-\frac{5}{2}}$ terms in unsimplified form. Isw after a correct unsimplified form.	
(b)(c)	A common error in (b) is to have $+4x^{-2}$ instead of $-4x^{-2}$, giving 1.430 in (c). This, if otherwise correct, would score (b) 110 and (c) 10		

Question Number	Scheme	Notes	Marks
2	$5x^2-2x+3=0$		
(a)	$\alpha+\beta=\dfrac{2}{5},\quad \alpha\beta=\dfrac{3}{5}$	Both correct	B1
			(1)
(b)(i)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	Uses a correct identity	M1
	$=\left(\dfrac{2}{5}\right)^2-2\left(\dfrac{3}{5}\right)=-\dfrac{26}{25}$	Correct value (allow -1.04), even after $\alpha+\beta=-\frac{2}{5}$ in (a)	A1
(ii)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ or $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$	Uses a correct identity	M1
	$=\left(\dfrac{2}{5}\right)^3-3\left(\dfrac{3}{5}\right)\left(\dfrac{2}{5}\right)=-\dfrac{82}{125}$	Correct value (allow -0.656)	A1
			(4)
(c)	$\text{Sum}=\alpha+\beta+\alpha^2+\beta^2=\dfrac{2}{5}-\dfrac{26}{25}\left(=-\dfrac{16}{25}\right)$	Attempts value of sum	M1
	$\text{Product}=\alpha\beta+\alpha^3+\beta^3+(\alpha\beta)^2=\dfrac{3}{5}-\dfrac{82}{125}+\left(\dfrac{3}{5}\right)^2\left(=\dfrac{38}{125}\right)$	Attempts value of product, using the <u>correct</u> expansion of $(\alpha+\beta^2)(\beta+\alpha^2)$	M1
	$x^2+\dfrac{16}{25}x+\dfrac{38}{125}(=0)$	Applies $x^2-(\text{their sum})x+\text{their product}$ Accept unsimplified versions. The '= 0' is not required	M1
	$125x^2+80x+38=0$	Allow any integer multiple. Must be a fully correct equation, including the '= 0' Not just $p=125$, $q=80$, $r=38$	A1
			(4)
			Total 9

Question Number	Scheme	Notes	Marks
3	$\mathrm{f}(z) = z^4 + az^3 + bz^2 + cz + d$		
(a)	$(z =)3 - \mathrm{i}$ or $(z =) -1 + 2\mathrm{i}$		B1
	$(z =)3 - \mathrm{i}$ and $(z =) -1 + 2\mathrm{i}$		B1
			(2)
(b)		$3 \pm \mathrm{i}$ correctly plotted with vectors or dots or crosses etc. or $-1 \pm 2\mathrm{i}$ correctly plotted with vectors or dots or crosses etc.	B1
		All 4 correct roots correctly plotted with scaling approximately correct (e.g. (-1, 2) higher than (3, 1), etc.) There should be approximate symmetry about the real axis, but be generous	B1
			(2)
(c)	$z = 3 \pm \mathrm{i} \Rightarrow (z-(3+\mathrm{i}))(z-(3-\mathrm{i})) = ...$ or $z = -1 \pm 2\mathrm{i} \Rightarrow (z-(-1+2\mathrm{i}))(z-(-1-2\mathrm{i})) = ...$	Correct strategy to find at least one quadratic factor. Throughout this part ignore the use of x (or other variable) instead of z	M1
	$z^2 - 6z + 10$ or $z^2 + 2z + 5$	One correct quadratic	A1
	$z^2 - 6z + 10$ and $z^2 + 2z + 5$	Both correct	A1
	$(z^2 - 6z + 10)(z^2 + 2z + 5) = ...$	Attempts product of their two <u>3-term</u> quadratic factors... no 'missing terms' in the expansion	M1
	$a = -4, b = 3, c = -10, d = 50$ or $\mathrm{f}(z) = z^4 - 4z^3 + 3z^2 - 10z + 50$	All correct values or correct quartic	A1
			(5)
			Total 9

(c) **Way 2**	$(z-(3+\mathrm{i}))(z-(-1\pm 2i))=\cdots$ or $(z-(3-\mathrm{i}))(z-(-1\pm 2\mathrm{i}))=\cdots$	Correct strategy to find at least one quadratic factor. Throughout this part ignore the use of x (or other variable) instead of z	M1
	$z^2+z(-2+\mathrm{i})+(-1-7\mathrm{i})$ (i) or $z^2+z(-2-\mathrm{i})+(-1+7\mathrm{i})$ (ii) or $z^2+z(-2+3\mathrm{i})+(-5-5\mathrm{i})$ (iii) or $z^2+z(-2-3\mathrm{i})+(-5+5\mathrm{i})$ (iv)	One correct quadratic	A1
	(i) and (ii) correct or (iii) and (iv) correct	A correct pair	A1
	e,g $[z^2+z(-2+i)+(-1-7i)]\times$ $[z^2+z(-2-\mathrm{i})+(-1+7\mathrm{i})]=$...	Attempts product of their two <u>3-term</u> quadratic factors... no 'missing terms' in the expansion	M1
	$a=-4, b=3, c=-10, d=50$ or $\mathrm{f}(z)=z^4-4z^3+3z^2-10z+50$	All correct values or correct quartic	A1

(c) **Way 3**	$\sum\propto=(3+\mathrm{i})+(3-\mathrm{i})+(-1-2\mathrm{i})+(-1+2\mathrm{i})=..$ or $\alpha\beta\gamma\delta=(3+\mathrm{i})(3-\mathrm{i})(-1-2\mathrm{i})(-1+2\mathrm{i})=..$	Attempts one of these	M1
	a = - 4 or $d=50$		A1
	Both a = - 4 and $d=50$		A1
	$\sum\alpha\beta=$ ... and $\sum\alpha\beta\gamma=$ ...	Attempts $\sum\alpha\beta$ (all 6 terms) and $\sum\alpha\beta\gamma$ (all 4 terms)	M1
	$a=-4, b=3, c=-10, d=50$ or $\mathrm{f}(z)=z^4-4z^3+3z^2-10z+50$	All correct values or correct quartic	A1

(c) **Way 4**	$(3+\mathrm{i})^4+a(3+\mathrm{i})^3+b(3+\mathrm{i})^2+c(3+\mathrm{i})+d=0$ (......) + (......) i = 0	Substitutes one of the roots into the given quartic and fully multiplies out	M1
	$(28+18a+8b+3c+d)+\mathrm{i}(96+26a+6b+c)$ or $(-7+11a-3b-c+d)+\mathrm{i}(-24+2a+4b-2c)$	One correct expansion	A1
	$(28+18a+8b+3c+d)+\mathrm{i}(96+26a+6b+c)$ and $(-7+11a-3b-c+d)+\mathrm{i}(-24+2a+4b-2c)$	Obtains a second correct expansion using another root.	A1
	$(28+18a+8b+3c+d)=0$, etc leading to $a=$, $b=$, $c=$, $d=$	Solves 4 simultaneous equation to find values of a, b, c and d	M1
	$a=-4, b=3, c=-10, d=50$ or $\mathrm{f}(z)=z^4-4z^3+3z^2-10z+50$	All correct values or correct quartic	A1

Question Number	Scheme	Notes	Marks
4(a)	$(2r-1)^2 = 4r^2 - 4r + 1$	Correct expansion	B1
	$\sum_{r=1}^{n}(4r^2-4r+1) = 4\times\frac{1}{6}n(n+1)(2n+1) - 4\times\frac{1}{2}n(n+1) + n$ M1: Attempt to use at least one of the standard results correctly A1: Correct expression		M1A1
	$= \frac{1}{3}n\left[2(n+1)(2n+1) - 6(n+1) + 3\right]$	Attempt to factorise $\frac{1}{3}n(\ldots.)$ Condone one slip but there must have been + n, not +1 in their expression for the sum	M1
	$= \frac{1}{3}n\left[4n^2-1\right]$*	Correct proof with no errors. There should be an intermediate step showing the expansion of $(n+1)(2n+1)$, or equivalent	A1*
	Condone poor or incorrect use of notation, e.g. Σ used at every step of the proof		
			(5)
(b)	$2r-1 = 499 \Rightarrow r = 250$	Identifies the correct upper limit (may be implied)	B1
	$2r-1 = 201 \Rightarrow r = 101$	Identifies the correct lower limit (may be implied)	B1
	$\sum_{r=101}^{250}(2r-1)^2 = \frac{1}{3}\times 250\left(4\times 250^2-1\right) - \frac{1}{3}\times 100\left(4\times 100^2-1\right)$ Uses the result from part (a) together with their upper limit and their lower limit – 1. A common mistake is to assume 500 and 200 are the limits, and in this case the mark is scored if 199 is used		M1
	= 19 499 950	Cao	A1
			(4)
			Total 9

Question Number	Scheme	Notes	Marks
5(a)	$xy=64 \Rightarrow y=64x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-64x^{-2}$ or $xy=64 \Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x}+y=0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{y}{x}$ or $x=8p, y=\frac{8}{p} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-8p^{-2}}{8}$	Correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ This can be in any form, simplified or unsimplified. The parameter could be a different variable, e.g. t	B1
	$m_T=-\frac{64}{64p^2} \Rightarrow m_N=p^2$ or $m_T=-\frac{8/p}{8p} \Rightarrow m_N=p^2$ or $m_T=-p^{-2} \Rightarrow m_N=p^2$	Correct use of the perpendicular gradient rule and the point P to obtain the normal gradient	M1
		Correct normal gradient of p^2	A1
	$y-\frac{8}{p}=p^2(x-8p)$ or $y=p^2x+c, \frac{8}{p}=p^2\times 8p+c \Rightarrow c=...$	Correct straight line method for normal	M1
	$p^3x-py=8(p^4-1)$*	cso. (No errors, but possibly direct from the version in line 1 above)	A1*
			(5)
(b)	$p^3x-py=8(p^4-1), xy=64 \Rightarrow$ $p^3x-p\frac{64}{x}=8(p^4-1)$ or $p^3\frac{64}{y}-py=8(p^4-1)$	Uses both equations to obtain an equation in one variable	M1
	$p^3x^2+8(1-p^4)x-64p=0$ or $py^2+8(p^4-1)y-64p^3=0$	Correct quadratic. Must have the x^2 or y^2 term, but the x or y terms need not be combined. The terms do not need to be 'all on one side', and the coefficients could involve fractions, e.g. $p^2x^2+\frac{8x}{p}-8p^3x=64$	A1
	$(x-8p)(p^3x+8)=0 \Rightarrow x=...$ or $(py-8)(y+8p^3)=0 \Rightarrow y=...$	Solves their 3TQ (usual rules) to obtain the <u>other</u> value of x or y. The <u>other</u> value must be picked out as a solution. This could be done by algebraic division... (see below)	dM1
	$x=-\frac{8}{p^3}$ $y=-8p^3$ or $\left(-\frac{8}{p^3}, -8p^3\right)$	Correct coordinates (ignore coordinates of P if they are also given as an answer). $-8p^{-3}$ may be seen rather than $-\frac{8}{p^3}$	A1
			(4)
			Total 9

5(b)	Rather than solving the 3TQ for the dM1, algebraic division can be used. To score the mark the division should follow the usual rules for solution by factorisation, so in the first case, e.g. if the quadratic is correct, the quotient should be $\pm p^3x \pm 8$, then this must lead to the other value $x_2 = \cdots$

5(b)	Note that another way to find the other value for the dM1 is to use the 'sum of roots' = $-\frac{b}{a}$, e.g. $8p + x_2 = \dfrac{-8(1-p^4)}{p^3} \qquad x_2 = \cdots$

(b) **Way 2**	$p^3x - py = 8\left(p^4-1\right), \left(8q,\ \dfrac{8}{q}\right) \Rightarrow$ $p^3 8q - p\dfrac{8}{q} = 8\left(p^4-1\right)$	Uses the given normal equation and the parametric form for Q to form an equation in p and q	M1
	$p^3q^2 - p = qp^4 - q$	Correct quadratic. Must have the q^2 term, but the q terms need not be combined. The terms do not need to be 'all on one side', and the coefficients could involve fractions.	A1
	$(p-q)\left(p^3q+1\right) = 0 \Rightarrow q = \ldots$	Solves their 3TQ (usual rules) to obtain the value of q. This could be done by algebraic division (condition as for main scheme)	dM1
	$q = -\dfrac{1}{p^3} \Rightarrow x = -\dfrac{8}{p^3} \quad y = -8p^3$ or $\left(-\frac{8}{p^3}, -8p^3\right)$	Correct coordinates (ignore coordinates of P if they are also given as an answer). $-8p^{-3}$ may be seen rather than $-\frac{8}{p^3}$	A1
			(4)

Question Number	Scheme	Notes	Marks
6(i)(a)	Stretch scale factor 3 parallel to the y-axis	Stretch (<u>not</u> enlargement)	B1
		Scale factor 3 parallel to the y-axis. Allow, e.g. '3 times y values', 'y increased by 3 factor', or similar. Allow, e.g. 'direction of y', 'along y', 'vertical', or similar. Ignore any mention of the origin. If additional transformations are included, send to Review	B1
			(2)
(b)	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$	Correct matrix. $\frac{1}{\sqrt{2}}$ may be seen rather than $\frac{\sqrt{2}}{2}$	B1
			(1)
(c)	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$	Attempt to multiply the right way round, i.e. **BA**, not **AB** At least two correct terms (for their matrix **B**) are needed to indicate a correct multiplication attempt	M1
	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{3\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{3\sqrt{2}}{2} \end{pmatrix}$ or equiv. e.g. $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 3 \\ -1 & 3 \end{pmatrix}$	Correct matrix	A1
			(2)
(ii)	Trapezium area$=\frac{1}{2}(5+2)(k+8)$	Correct method for the area of the trapezium	M1
	$\begin{vmatrix} 5 & 1 \\ -2 & 3 \end{vmatrix} = 5\times3-(-2)\times1=17$	Correct method for the determinant	M1
		17 (Allow $\pm$ 17)	A1
	$\frac{1}{2}(5+2)(k+8)\times17=510 \Rightarrow k=\ldots$	Multiplies their trapezium area by their determinant, sets equal to 510 and solves for k. Or equivalently: Equates their trapezium area to (510 $\div$ determinant) and solves for k	M1
	$k=\frac{4}{7}$	$\frac{4}{7}$ or exact equivalent. If additional answers such as $-\frac{4}{7}$ are given and not rejected, this is A0	A1
			(5)

(ii) Way 2	$\begin{pmatrix}5 & 1\\-2 & 3\end{pmatrix}\begin{pmatrix}-2 & -2 & 5 & 5\\0 & k & 8 & 0\end{pmatrix}$ $=\begin{pmatrix}-10 & -10+k & 33 & 25\\4 & 4+3k & 14 & -10\end{pmatrix}$	Multiplies correct matrices to find the coordinates for T'	2nd M
		Correct coordinates (can be left in matrix form)	A1
	$\frac{1}{2}[-10(4+3k)+14(-10+k)-330+100-4(-10+k)-33(4+3k)-350-100]$	Correct method for area of T' ('shoelace rule' with or without a modulus), using their coordinates for T'	1st M
	$\pm\frac{1}{2}(952+119k)=510$, $k=\ldots$	Sets area of T' equal to 510 and solves for k	M1
	$k=\frac{4}{7}$	$\frac{4}{7}$ or exact equivalent. If additional answers such as $-\frac{4}{7}$ are given and not rejected, this is A0	A1
			Total 10

Question Number	Scheme	Notes	Marks
7(a) Way 1	$3x-4y+48=0 \Rightarrow x=\dfrac{4y-48}{3}$ $y^2=4ax \Rightarrow y^2=4a\left(\dfrac{4y-48}{3}\right)$ or $3x-4y+48=0 \Rightarrow y=\dfrac{3x+48}{4}$ $y^2=4ax \Rightarrow \left(\dfrac{3x+48}{4}\right)^2=4ax$ or $x=\dfrac{y^2}{4a} \Rightarrow \dfrac{3y^2}{4a}-4y+48=0$	Uses both equations to obtain an equation in one variable.	M1
	$3y^2-16ay+192a=0$ or $9x^2+(288-64a)x+2304=0$ or $3x-8\sqrt{a}\sqrt{x}+48=0$	Correct 3TQ (Coefficients could be 'fractional') (This could be a quadratic in $\sqrt{x}$)	A1
	Equal roots: $(16a)^2=4\times3\times192a$ or $(288-64a)^2=4\times9\times2304$ $\Rightarrow a=\ldots$	Uses "$b^2=4ac$" to find a value for a	M1
	$a=9$ *	cso	A1*
	Beware the use of the given result $a = 9$, but there may be cases where 'working backwards' deserves merit (if in doubt, send to Review).		
			(4)

(a) **Way 2**	$y^2 = 4ax \Rightarrow 2y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 4a$ $3x - 4y + 48 = 0 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3}{4}$ $\Rightarrow 2y \times \dfrac{3}{4} = 4a$	Uses differentiation to obtain the gradient of C and substitutes the gradient of l to obtain an equation connecting y and a, or connecting x and a, or an equation in t	M1
	$y = 2a^{\frac{1}{2}}x^{\frac{1}{2}} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = a^{\frac{1}{2}}x^{-\frac{1}{2}} \Rightarrow a^{\frac{1}{2}}x^{-\frac{1}{2}} = \dfrac{3}{4}$		
	$x = at^2,\ y = 2at \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{t}$ $\dfrac{1}{t} = \dfrac{3}{4}$		
	$y = \frac{8a}{3}$ or $x = \frac{16a}{9}$	Correct y value, or correct x value (possibly implied in subsequent work, particularly if using the parametric equations)	A1
	$y^2 = 4ax \Rightarrow \dfrac{64a^2}{9} = 4ax \Rightarrow x = \dfrac{16a}{9}$ $3 \times \dfrac{16a}{9} - 4 \times \dfrac{8a}{3} + 48 = 0 \Rightarrow a = \ldots$	Uses $y^2 = 4ax$ or l to find a value for x (or y) and substitutes their x and y into the other equation to find a value for a If using parameter t, substitutes their value for t into $3(at^2) - 4(2at) + 48 = 0$ and solves to find a value for a	M1
	$x = \dfrac{4y - 48}{3} = \dfrac{32a}{9} - 16$ $\dfrac{64a^2}{9} = 4a\left(\dfrac{32a}{9} - 16\right) \Rightarrow a = \cdots$		
	$y^2 = 4ax \Rightarrow y^2 = \frac{64a^2}{9} \Rightarrow y = \frac{8a}{3}$ $3 \times \dfrac{16a}{9} - 4 \times \dfrac{8a}{3} + 48 = 0 \Rightarrow a = \ldots$		
	$y = \dfrac{3x + 48}{4} = \dfrac{4a}{3} + 12$ $\left(\dfrac{4a}{3} + 12\right)^2 = 4a\left(\dfrac{16a}{9}\right) \Rightarrow a = \cdots$		
	$3(at^2) - 4(2at) + 48 = 0$ $3\left(\dfrac{16a}{9}\right) - 4\left(\dfrac{8a}{3}\right) + 48 = 0 \Rightarrow a = \cdots$		
	$a = 9$ *	cso	A1*

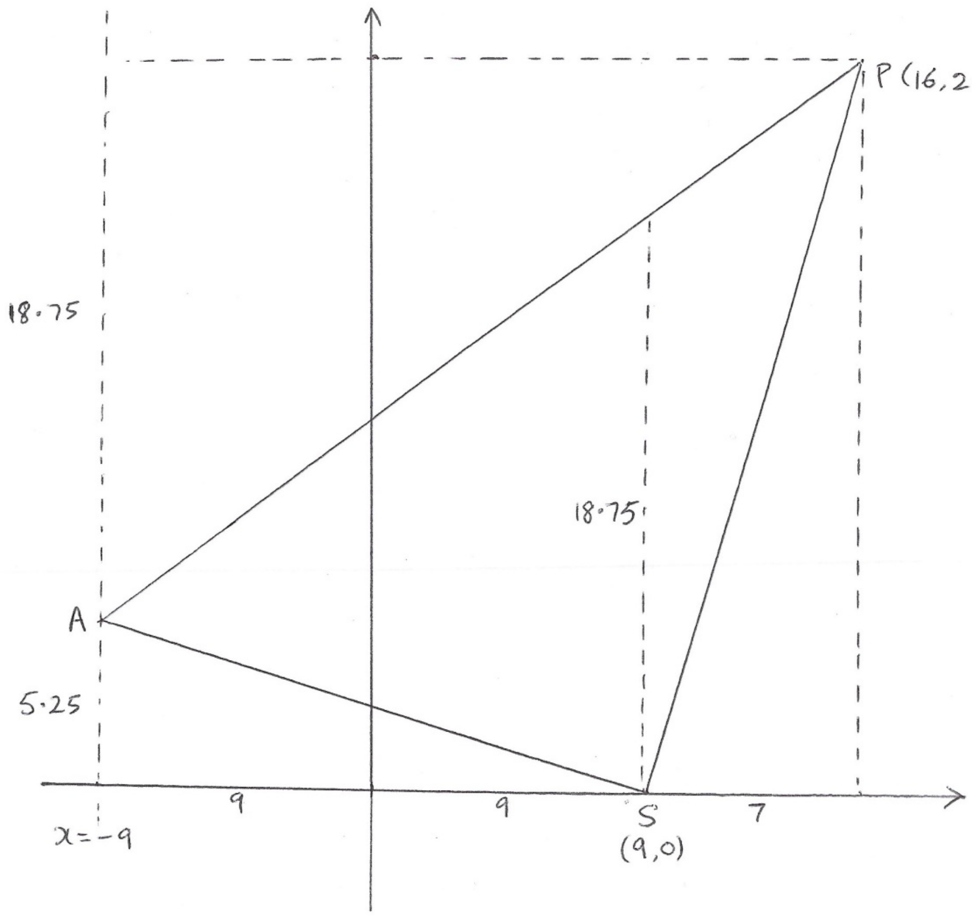
(b)	$a=9 \Rightarrow 3y^2-144y+1728=0 \Rightarrow y=24$ $9x^2-288x+2304=0 \Rightarrow x=16$	Uses $a = 9$ to solve their 3TQ to obtain the repeated root for x or y.	M1
	$x=16$ and $y=24$	Correct values or coordinates.	A1
			(2)

(b) Way 2 follows (a)Way2	$a=9 \Rightarrow x=\cdots$ or $y=\cdots$	Substitutes $a = 9$ into their expression for x or y, OR substitutes $a = 9$ into at^2 to find x , or into $2at$ to find y.	M1
	$x=16$ and $y=24$	Correct values or coordinates.	A1

(c) Way 1	Focus is at (9, 0)	Correct focus (could be seen on a sketch or implied in working)	B1
	$x=-9 \Rightarrow 3(-9)-4y+48=0 \Rightarrow y=5.25$	Correct method with the correct directrix to find the y coordinate of A	M1
	E.g.Trapezium – 2 triangles $=\frac{1}{2}\left(\frac{21}{4}+24\right)\times 25-\frac{1}{2}\times 18\times\frac{21}{4}-\frac{1}{2}\times 7\times 24=\frac{1875}{8}$ Fully correct triangle area method (condone one slip if the intention seems clear)		dM1
	$=\frac{1875}{8}\ (234.375)$	Correct area (exact)	A1
			(4)
			Total 10

(c) Way 2	Focus is at $(9,0)$	Correct focus (could be seen on a sketch or implied in working)	B1
	$x=9 \Rightarrow 3(9)-4y+48=0 \Rightarrow y=18.75$	Correct method to find the y coordinate when $x = 9$, but also requires correct directrix at some stage of the solution	M1
	E.g. $A=\frac{1}{2}(18.75\times 18)+\frac{1}{2}(18.75\times(16-9))$	Fully correct triangle area method (condone one slip if the intention seems clear)	dM1
	$=\frac{1875}{8}\ (234.375)$	Correct area (exact)	A1
(c) Way 3	Focus is at (9, 0)	Correct focus (could be seen on a sketch or implied in working)	B1
	$x=-9 \Rightarrow 3(-9)-4y+48=0 \Rightarrow y=5.25$	Correct method with the correct directrix to find the y coordinate of A	M1
	E.g. $\frac{1}{2}\begin{vmatrix} 9 & -9 & 16 & 9 \\ 0 & 5.25 & 24 & 0 \end{vmatrix}$ $=\frac{1}{2}\lvert 47.25-216-84-216 \rvert$	Fully correct area method (condone one slip if the intention seems clear)	dM1
	$=\frac{1875}{8}\ (234.375)$	Correct area (exact)	A1

(c) **Way 4**	Focus is at (9, 0)	Correct focus (could be seen on a sketch or implied in working)	B1
	$x = -9 \Rightarrow 3(-9) - 4y + 48 = 0 \Rightarrow y = 5.25$	Correct method with the correct directrix to find the y coordinate of A	M1
	E.g.Rectangle – 3 triangles $(25 \times 24) - \frac{1}{2}(18 \times 5.25) - \frac{1}{2}(7 \times 24) - \frac{1}{2}(25 \times 18.75)$ Fully correct triangle area method (condone one slip if the intention seems clear)		dM1
	$= \frac{1875}{8}\ (234.375)$	Correct area (exact)	A1



Question Number	Scheme	Notes	Marks
8(i)	$\sum_{r=1}^{n}\frac{2r^2-1}{r^2(r+1)^2}=\frac{n^2}{(n+1)^2}$		
	$\frac{2(1)^2-1}{1^2(1+1)^2}=\frac{1}{4},\ \frac{1^2}{(1+1)^2}=\frac{1}{4}$	Getting $\frac{1}{2^2}$ or $\frac{1}{4}$ from each is the minimum	B1
	Assume $\sum_{r=1}^{k}\frac{2r^2-1}{r^2(r+1)^2}=\frac{k^2}{(k+1)^2}$		
	$\sum_{r=1}^{k+1}\frac{2r^2-1}{r^2(r+1)^2}=\frac{k^2}{(k+1)^2}+\frac{2(k+1)^2-1}{(k+1)^2(k+2)^2}$ Assumes the result is true for say $n = k$ and adds the next term		M1
	$\frac{k^2(k+2)^2+2(k+1)^2-1}{(k+1)^2(k+2)^2}$	Attempts common denominator	dM1
		Correct expression	A1
	$\frac{k^4+4k^3+4k^2+2k^2+4k+1}{(k+1)^2(k+2)^2}=\frac{k^4+4k^3+6k^2+4k+1}{(k+1)^2(k+2)^2}=\frac{(k+1)^4}{(k+1)^2(k+2)^2}$		
	$\frac{(k+1)^2}{(k+2)^2}$	Achieves this result with intermediate working and no errors	A1
	If the result is true for $n=k$, then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso
			(6)
(ii)	$f(n)=12^n+2\times5^{n-1}$		
	$f(1)=12+2\times1=14$	This is sufficient	B1
	$f(k+1)=12^{k+1}+2\times5^k$	Attempt f(k + 1)	M1
	$f(k+1)-f(k)=12^{k+1}+2\times5^k-12^k-2\times5^{k-1}$ $f(k+1)-f(k)=11\times12^k+22\times5^{k-1}+10\times5^{k-1}-24\times5^{k-1}$	Working with f(k + 1) – f(k)	
	$=11\times\left(12^k+2\times5^{k-1}\right)-14\times5^{k-1}$	$11\times\left(12^k+2\times5^{k-1}\right)$ or $11f(k)$	A1
		$-14\times5^{k-1}$	A1
	$f(k+1)=12f(k)-14\times5^{k-1}$	Makes f(k + 1) the subject **Dependent on at least one of the A marks**	dM1
	If the result is true for $n=k$, then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso
			(6)
			Total 12

ALT 1	$\mathrm{f}(1)=12+2\times1=14$	This is sufficient	B1
	$\mathrm{f}(k+1)=12^{k+1}+2\times5^{k}$	Attempt f(k+1)	M1
	$\mathrm{f}(k+1)=12\left(12^{k}+2\times5^{k-1}\right)+2\times5\times5^{k-1}-12\times2\times5^{k-1}$		
	$\mathrm{f}(k+1)=12\left(12^{k}+2\times5^{k-1}\right)-14\times5^{k-1}$	$12\left(12^{k}+2\times5^{k-1}\right)$ or $12\mathrm{f}(k)$	A1
		$-14\times5^{k-1}$	A1
	$\mathrm{f}(k+1)=12\mathrm{f}\left(k\right)-14\times5^{k-1}$	**Dependent on at least one of the A marks**	dM1
	If the result is <u>true for $n=k$</u>, then it is <u>true for $n=k+1$</u>. As the result has been shown to be <u>true for $n=1$</u>, then the result is <u>true for all n</u>.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso
ALT 2	$\mathrm{f}(1)=12+2\times1=14$	This is sufficient	B1
	Let $12^{k}+2\times5^{k-1}=7M$		
	$\mathrm{f}(k+1)=12^{k+1}+2\times5^{k}$	Attempt f(k+1)	M1
	$\mathrm{f}(k+1)=12\left(7M-2\times5^{k-1}\right)+2\times5^{k}$	OR: $\mathrm{f}(k+1)=5(7M)+7\times12^{k}$	
	$\mathrm{f}(k+1)=84M-14\times5^{k-1}$ OR: $\mathrm{f}(k+1)=35M+7\times12^{k}$	$84M$ OR: $35M$	A1
		$-14\times5^{k-1}$ $+7\times12^{k}$	A1
	$\mathrm{f}(k+1)=12\mathrm{f}\left(k\right)-14\times5^{k-1}$ OR: $\mathrm{f}(k+1)=5\mathrm{f}(k)+7\times12^{k}$	**Dependent on at least one of the A marks**	dM1
	If the result is <u>true for $n=k$</u>, then it is <u>true for $n=k+1$</u>. As the result has been shown to be <u>true for $n=1$</u>, then the result is <u>true for all n</u>.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso
ALT 3	$\mathrm{f}(1)=12+2\times1=14$	This is sufficient	B1
	$\mathrm{f}(k+1)=12^{k+1}+2\times5^{k}$	Attempts f(k + 1)	M1
	Working with f(k + 1) – mf(k) $\mathrm{f}(k+1)-m\mathrm{f}(k)=12^{k+1}+2\times5^{k}-m\left(12^{k}+2\times5^{k-1}\right)$ $\mathrm{f}(k+1)-\mathrm{f}(k)=\left(12-m\right)\times12^{k}+2\times\left(12-m\right)\times5^{k-1}+10\times5^{k-1}-24\times5^{k-1}$		
	$=\left(12-m\right)\times\left(12^{k}+2\times5^{k-1}\right)-14\times5^{k-1}$	$\left(12-m\right)\times\left(12^{k}+2\times5^{k-1}\right)$ or $\left(12-m\right)\mathrm{f}(k)$	A1
		$-14\times5^{k-1}$	A1
	$\mathrm{f}\left(k+1\right)=12\mathrm{f}\left(k\right)-14\times5^{k-1}$	Makes f(k + 1) the subject **Dependent on at least one of the A marks**	dM1
	If the result is <u>true for $n=k$</u>, then it is <u>true for $n=k+1$</u>. As the result has been shown to be <u>true for $n=1$</u>, then the result is <u>true for all n</u>.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso

ALT 4	$\mathrm{f}(1)=12+2\times1=14$	This is sufficient	B1
	$\mathrm{f}(k+1)=12^{k+1}+2\times5^{k}$	Attempts f(k + 1)	M1
	$\mathrm{f}(k+1)-5\mathrm{f}(k)=12^{k+1}+2\times5^{k}-5\left(12^{k}+2\times5^{k-1}\right)$	Working with f(k + 1) – 5f(k)	
	$=7\times12^{k}+2\times5^{k}-2\times5^{k}$	7×12^{k}	A1
		$2\times5^{k}-2\times5^{k}$ (or zero)	A1
	$\mathrm{f}\left(k+1\right)=5\mathrm{f}\left(k\right)+7\times12^{k}$	Makes f(k + 1) the subject **Dependent on at least one of the A marks**	dM1
	If the result is true for $n=k$, then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso

NOTES:

Part (i)
This approach may be seen:
Assume result is true for $n = k$ and $n = k + 1$
Subtract: (sum to (k + 1) terms) minus (sum to k terms)
Show that this is equal to the (k + 1)th term
Please send any such response to Review.

Part (ii)
Apart from the given alternatives, other versions will work and can be marked equivalently.
If in any doubt, send to Review.