Mark Scheme (Unused)

January 2022

Pearson Edexcel International Advanced Level
in Pure Mathematics P4 (WMA14)
Paper 01

Question Number	Scheme	Notes	Marks
1(a)	$\dfrac{2}{\sqrt{9-2x}}=\dfrac{2}{3\sqrt{\left(1-\frac{2}{9}x\right)}}$ or $\dfrac{2}{\sqrt{9-2x}}=2(9-2x)^{-\frac{1}{2}}=2\times\dfrac{1}{3}\left(1-\dfrac{2}{9}x\right)^{-\frac{1}{2}}$	Obtains $\sqrt{9-2x}=3\sqrt{(1-...)}$	B1
	$\left(1-\dfrac{2}{9}x\right)^{-\frac{1}{2}}=1+\left(-\tfrac{1}{2}\right)\left(-\tfrac{2}{9}x\right)+\dfrac{-\frac{1}{2}\left(-\frac{1}{2}-1\right)}{2!}\left(-\tfrac{2}{9}x\right)^2+\dfrac{-\frac{1}{2}\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!}\left(-\tfrac{2}{9}x\right)^3+...$ M1: Attempts the binomial expansion of $(1 + kx)^n$ to get the third and/or fourth term with an acceptable structure. The correct binomial coefficient must be combined with the correct power of x and the correct power of 2. A1: Correct simplified or unsimplified expansion (NB simplified is $=1+\dfrac{1}{9}x+\dfrac{1}{54}x^2+\dfrac{5}{1458}x^3+...$)		M1 A1
	$\dfrac{2}{\sqrt{9-2x}}=\dfrac{2}{3}+\dfrac{2}{27}x+\dfrac{1}{81}x^2+\dfrac{5}{2187}x^3+...$	2 correct simplified terms	A1
		All correct	A1
			(5)
(b)	$x=1\Rightarrow\dfrac{2}{\sqrt{9-2}}=\dfrac{2}{3}+\dfrac{2}{27}+\dfrac{1}{81}+\dfrac{5}{2187}+...$ $\Rightarrow\sqrt{7}\approx2\div\text{"}\dfrac{1652}{2187}\text{"}$ or $2\times\text{"}\dfrac{2187}{1652}\text{"}$ Substitutes x = 1 and divides into 2 or equivalent		M1
	$=2.6477$	Correct approximation	A1
			(2)
	Alternative for (b):		
	$x=1\Rightarrow\dfrac{2}{\sqrt{9-2}}=\dfrac{2}{3}+\dfrac{2}{27}+\dfrac{1}{81}+\dfrac{5}{2187}+...$ $\dfrac{2}{\sqrt{7}}=\dfrac{2\sqrt{7}}{7}\Rightarrow\sqrt{7}\approx\dfrac{7}{2}\times\text{"}\dfrac{1652}{2187}\text{"}$ Substitutes x = 1 and multiplies by $\dfrac{7}{2}$		M1
	$=2.6438$	Correct approximation	A1
			Total 7

Question Number	Scheme	Notes	Marks
2(a)	$\dfrac{x}{y}=t$	Cao	B1
			(1)
(b)	$y=\dfrac{\left(\dfrac{x}{y}\right)^3}{2\left(\dfrac{x}{y}\right)+1}$ or $x=\dfrac{\left(\dfrac{x}{y}\right)^4}{2\left(\dfrac{x}{y}\right)+1}$	Uses the y coordinate to obtain y in terms of x and y or uses the x coordinate to obtain x in terms of y and x	M1
	$y=\dfrac{x^3}{2xy^2+y^3}\Rightarrow y\left(2xy^2+y^3\right)=x^3$ or $x=\dfrac{x^4}{2xy^3+y^4}\Rightarrow x\left(2xy^3+y^4\right)=x^4$	Uses correct algebra to eliminate the fractions	M1
	$x^3-2xy^3-y^4=0$*	Cso	A1*
			(3)
			Total 4

Question Number	Scheme	Notes	Marks
3(a)	$3y^2-11x^2+11xy=20y-36x+28$ $\Rightarrow \underline{6y\frac{dy}{dx}}-22x+\underline{\underline{11x\frac{dy}{dx}+11y}}=20\frac{dy}{dx}-36$ M1: $y^2\to Ay\frac{dy}{dx}$ M1: $11xy\to px\frac{dy}{dx}+qy$ A1: All correct		M1M1A1
	$(6y+11x-20)\frac{dy}{dx}=22x-11y-36\Rightarrow\frac{dy}{dx}=...$ Collects terms in $\frac{dy}{dx}$ (must be 3 and from the appropriate terms) and makes $\frac{dy}{dx}$ the subject		M1
	$\frac{dy}{dx}=\frac{22x-11y-36}{6y+11x-20}$	Correct expression or correct equivalent	A1
			(5)
(b)	$x=4\Rightarrow 3y^2-176+44y=20y-144+28$	Substitutes $x = 4$ into C to obtain a 3TQ in y	M1
	$3y^2+24y-60=0\Rightarrow y=...$	Solves for y	M1
	$y=-10\ \left(,2\right)$	Correct value	A1
	$(4,-10)\to\frac{dy}{dx}=\frac{88+110-36}{-60+44-20}$	Substitutes $x = 4$ and their negative y into their $\frac{dy}{dx}$	M1
	$\frac{dy}{dx}=-\frac{9}{2}$	Correct value	A1
			(5)
			Total 10

Question Number	Scheme	Notes	Marks
4(a)	$\frac{4-4x}{x(x-2)^2} \equiv \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$	Correct form for the partial fractions	B1
	$4-4x = A(x-2)^2 + Bx(x-2) + Cx$ $\Rightarrow A = \ldots$ or $B = \ldots$ or $C = \ldots$	Uses a correct strategy to find at least one of their constants	M1
	$\frac{4-4x}{x(x-2)^2} \equiv \frac{1}{x} - \frac{1}{x-2} - \frac{2}{(x-2)^2}$	2 correct constants	A1
		All correct	A1
			(4)
(b)	$\int \left(\frac{1}{x} - \frac{1}{x-2} - \frac{2}{(x-2)^2} \right) \mathrm{d}x = \ln x - \ln(x-2) + \frac{2}{x-2} (+c)$		M1
	M1 for $\int \frac{\alpha}{x}\,\mathrm{d}x = \beta \ln x$ or $\int \frac{\alpha}{x-2}\,\mathrm{d}x = \beta \ln(x-2)$		M1
	M1 for $\int \frac{\alpha}{(x-2)^2}\,\mathrm{d}x = \frac{\beta}{x-2}$ A1: All correct		A1
			(3)
(c)	$\left[\ln x - \ln(x-2) + \frac{2}{x-2} \right]_3^5 = \left(\ln 5 - \ln 3 + \frac{2}{3} \right) - (\ln 3 - \ln 1 + 2)$		M1
	$= \ln \frac{5}{9} - \frac{4}{3}$ M1: Correct use of limits and reaches the required form using log rules A1: Correct answer		A1
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
5(a)	$4+2\lambda=13+5\mu$ $4-3\lambda=-1+\mu$ $-5+6\lambda=4-3\mu$	For writing down any 2 of these equations.	M1
	E.g. $4+2\lambda=13+5\mu$ $4-3\lambda=-1+\mu$ $\Rightarrow \lambda=\ldots$ or $\mu=\ldots$	Full method for finding λ or μ	M1
	$\lambda=2,\ \mu=-1$	Both correct values	A1
	$-5+6\lambda=-5+12=7$ $4-3\mu=4+3=7$ So lines intersect	Shows that the parameters satisfy the third equation and makes a conclusion.	B1
	$\lambda=2 \rightarrow (4+4)\mathbf{i}+(4-6)\mathbf{j}+(-5+12)\mathbf{k}$ **or** $\mu=-1 \rightarrow (13-5)\mathbf{i}+(-1-1)\mathbf{j}+(4+3)\mathbf{k}$	Uses their λ or μ to find A.	M1
	$8\mathbf{i}-2\mathbf{j}+7\mathbf{k}$	Correct vector or coordinates	A1
			(6)
(b)	$\begin{pmatrix}2\\-3\\6\end{pmatrix}\bullet\begin{pmatrix}5\\1\\-3\end{pmatrix}=10-3-18=\sqrt{2^2+3^2+6^2}\sqrt{5^2+1^2+3^2}\cos\theta$ Full attempt at the scalar product between the direction vectors		M1
	$\cos\theta=\pm\dfrac{11}{7\sqrt{35}}$	Correct magnitude for cos θ (may be implied by e.g. $\theta=105.4\ldots$ or $74.6\ldots$	A1
	$\theta=74.6°$	Awrt 74.6	A1
			(3)
(c)	$\lvert 2\mathbf{i}-3\mathbf{j}+6\mathbf{k}\rvert=\sqrt{2^2+3^2+6^2}=7$	Finds the magnitude of the direction of l_1	M1
	$35\div 7=5 \Rightarrow \lambda=5$ $8\mathbf{i}-2\mathbf{j}+7\mathbf{k}\pm 5(2\mathbf{i}-3\mathbf{j}+6\mathbf{k})$	Correct strategy for one of the points	M1
	$(18,-17,37)$ or $(-2,13,-23)$	One correct point (ignore labels)	A1
	$P(18,-17,37)$ and $Q(-2,13,-23)$	Correct points with correct labels	A1
			(4)
			Total 13

Question Number	Scheme	Notes	Marks
6 Way 1	$\int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\int e^{2x}\sin 3x\,dx\,(+c)$ M1: For applying parts to obtain $\alpha e^{2x}\sin 3x \pm \beta\int e^{2x}\sin 3x\,dx\,(+c)$ A1: Correct expression		M1A1
	$\int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\left\{-\frac{1}{3}e^{2x}\cos 3x + \frac{2}{3}\int e^{2x}\cos 3x\,dx\right\}(+c)$ Applies parts again to $\int e^{2x}\sin 3x\,dx$ and obtains $\alpha e^{2x}\cos 3x \pm \beta\int e^{2x}\cos 3x\,dx$		M1
	$\int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x - \frac{4}{9}\int e^{2x}\cos 3x\,dx\,(+c)$ Fully correct application of parts twice		A1
	$\int e^{2x}\cos 3x\,dx + \frac{4}{9}\int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x\,(+c)$ $\Rightarrow \frac{13}{9}\int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x\,(+c) \Rightarrow \int e^{2x}\cos 3x\,dx = ...$ Fully correct strategy for finding $\int e^{2x}\cos 3x\,dx$		M1
	$= \frac{3}{13}e^{2x}\sin 3x + \frac{2}{13}e^{2x}\cos 3x + k$	Cao	A1
			(6)
Way 2	$\int e^{2x}\cos 3x\,dx = \frac{1}{2}e^{2x}\cos 3x + \frac{3}{2}\int e^{2x}\sin 3x\,dx\,(+c)$ M1: For applying parts to obtain $\alpha e^{2x}\cos 3x \pm \beta\int e^{2x}\sin 3x\,dx\,(+c)$ A1: Correct expression		M1A1
	$\int e^{2x}\cos 3x\,dx = \frac{1}{2}e^{2x}\cos 3x + \frac{3}{2}\left\{\frac{1}{2}e^{2x}\sin 3x - \frac{3}{2}\int e^{2x}\cos 3x\,dx\right\}(+c)$ Applies parts again to $\int e^{2x}\sin 3x\,dx$ and obtains $\alpha e^{2x}\sin 3x \pm \beta\int e^{2x}\cos 3x\,dx$		M1
	$\int e^{2x}\cos 3x\,dx = \frac{1}{2}e^{2x}\cos 3x + \frac{3}{4}e^{2x}\sin 3x - \frac{9}{4}\int e^{2x}\cos 3x\,dx\,(+c)$ Fully correct application of parts twice		A1
	$\int e^{2x}\cos 3x\,dx + \frac{9}{4}\int e^{2x}\cos 3x\,dx = \frac{1}{2}e^{2x}\cos 3x + \frac{3}{4}e^{2x}\sin 3x\,(+c)$ $\Rightarrow \frac{13}{4}\int e^{2x}\cos 3x\,dx = \frac{1}{2}e^{2x}\cos 3x + \frac{3}{4}e^{2x}\sin 3x\,(+c) \Rightarrow \int e^{2x}\cos 3x\,dx = ...$ Fully correct strategy for finding $\int e^{2x}\cos 3x\,dx$		M1
	$= \frac{3}{13}e^{2x}\sin 3x + \frac{2}{13}e^{2x}\cos 3x + k$	Cao	A1
			Total 6

Question Number	Scheme	Notes	Marks
7(a)	$\frac{dV}{dt}=300-kV \Rightarrow \int \frac{dV}{300-kV}=\int dt$	Correct separation of variables	B1
	$\int \frac{dV}{300-kV}=-\frac{1}{k}\ln(300-kV)$	$\int \frac{dV}{300-kV}=\alpha\ln(300-kV)$	M1
	$-\frac{1}{k}\ln(300-kV)=t+c$	Correct equation including a constant of integration	A1
	$-\frac{1}{k}\ln(300-kV)=t+c \Rightarrow \ln(300-kV)=-kt+d$ $\Rightarrow 300-kV=e^{-kt+d}$ Correct processing to remove the "ln"		M1
	$kV=300-e^{-kt+d} \Rightarrow V=\frac{300}{k}-Be^{-kt}$ $V=\frac{300}{k}+Ae^{-kt}$ *	Correct proof	A1*
			(5)
(b)	$V=0,t=0 \Rightarrow 0=\frac{300}{k}+A \Rightarrow A=-\frac{300}{k}$	Uses $V=0$ when $t=0$ to find A in terms of k	M1
	$V=\frac{300}{k}-\frac{300}{k}e^{-kt} \Rightarrow \frac{dV}{dt}=300e^{-kt}$	$\frac{dV}{dt}=\alpha e^{-kt}$	M1
	$300e^{-10k}=200 \Rightarrow e^{-10k}=\frac{2}{3} \Rightarrow k=...$	Uses $\frac{dV}{dt}=200$ when $t=10$ and correct processing to find k	M1
	$k=-\frac{1}{10}\ln\frac{2}{3}$	Oe e.g. $\frac{1}{10}\ln\frac{3}{2}$	A1
			(4)
(b) Way 2	$V=0,t=0 \Rightarrow 0=\frac{300}{k}+A \Rightarrow A=-\frac{300}{k}$	Uses $V=0$ when $t=0$ to find A in terms of k	M1
	$\frac{dV}{dt}=200, t=10 \Rightarrow 200=300-kV$ $\Rightarrow kV=100$	Uses $\frac{dV}{dt}=200$ when $t=10$ to find a value for kV	M1
	$V=\frac{300}{k}+Ae^{-kt} \Rightarrow kV=300-300e^{-10k}$ $\Rightarrow 100=300-300e^{-kt} \Rightarrow e^{-10k}=\frac{2}{3} \Rightarrow k=...$	Substitutes for kV, kA and $t=10$ and uses correct processing to find k	M1
	$k=-\frac{1}{10}\ln\frac{2}{3}$	Oe e.g. $\frac{1}{10}\ln\frac{3}{2}$	A1
(c)	$6000=\frac{3000}{\ln 1.5}-\frac{3000}{\ln 1.5}e^{-\frac{t}{10}\ln 1.5}$ $\Rightarrow e^{-\frac{t}{10}\ln 1.5}=1-2\ln 1.5$ $\Rightarrow -\frac{t}{10}\ln 1.5=\ln(1-2\ln 1.5)$	Correct strategy using V = 6000 to reach $\alpha t=\ldots$	M1
	$t=41$	Correct value	A1
			(2)
			Total 11

Question Number	Scheme	Notes	Marks
8	Assume that there exist positive real numbers x and y such $\dfrac{9x}{y}+\dfrac{y}{x}<6$	Starts the proof by contradicting the given statement	B1
	$\dfrac{9x}{y}+\dfrac{y}{x}<6 \Rightarrow 9x^2+y^2<6xy$ as x and y are both positive	Multiplies through by xy	M1
	$\Rightarrow 9x^2+y^2-6xy<0$ $\Rightarrow (3x-y)^2<0$	Reaches a correct contradictory statement	A1
	As x and y are positive real numbers, this is a contradiction and so $\dfrac{9x}{y}+\dfrac{y}{x}<6$ must be incorrect and so $\dfrac{9x}{y}+\dfrac{y}{x} \ldots 6$*	Makes a suitable conclusion	A1*
			(4)
			Total 4

Question Number	Scheme	Notes	Marks
9(a)	$V=\pi\int y^2\,\mathrm{d}x=\pi\int y^2\frac{\mathrm{d}x}{\mathrm{d}\theta}\mathrm{d}\theta$ $=\pi\int(3\sin\theta-\sin 2\theta)^2(-5\sin\theta)\,\mathrm{d}\theta$	Applies $V=\pi\int y^2\frac{\mathrm{d}x}{\mathrm{d}\theta}\mathrm{d}\theta$ with or without the π	M1
	$=\pi\int(3\sin\theta-2\sin\theta\cos\theta)^2(-5\sin\theta)\,\mathrm{d}\theta$	Applies $\sin 2\theta=2\sin\theta\cos\theta$	M1
		Fully correct integral in terms of $\sin\theta$ and $\cos\theta$ only (π not needed)	A1
	$=\pi\int\sin^2\theta(3-2\cos\theta)^2(-5\sin\theta)\,\mathrm{d}\theta$ $V=-5\pi\int\sin^3\theta(3-2\cos\theta)^2\,\mathrm{d}\theta$ $V=-5\pi\int_{\pi}^{0}\sin^3\theta(3-2\cos\theta)^2\,\mathrm{d}\theta$ $V=5\pi\int_{0}^{\pi}\sin^3\theta(3-2\cos\theta)^2\,\mathrm{d}\theta$ *	Completes correctly with correct limits and no incorrect statements previously. The factor of π must be present throughout.	A1*
			(4)
(b)	$u=\cos\theta\Rightarrow V=5\pi\int\sin^3\theta(3-2u)^2\frac{\mathrm{d}u}{-\sin\theta}$	Applies the substitution correctly	M1
	$\theta=0\Rightarrow u=1,\ \theta=\pi\Rightarrow u=-1$	Attempts to change θ limits to u limits	M1
	$V=-5\pi\int\sin^2\theta(3-2u)^2\,\mathrm{d}u=-5\pi\int(1-u^2)(3-2u)^2\mathrm{d}u$ Correct integral in terms of u only		A1
	$(1-u^2)(3-2u)^2=(1-u^2)(9-12u+4u^2)$	Attempt to expand	M1
	$=9-12u-5u^2+12u^3-4u^4$	Correct expansion	A1
	$V=5\pi\int_{-1}^{1}(9-12u-5u^2+12u^3-4u^4)\,\mathrm{d}u$ $=5\pi\left[9u-6u^2-\frac{5u^3}{3}+3u^4-\frac{4u^5}{5}\right]_{-1}^{1}=\ldots$ Integrates and applies their u limits		M1
	$=\frac{196}{3}\pi$	Cao	A1
			(7)
			Total 11