Pearson Edexcel

Mark Scheme (Results)

Summer 2021

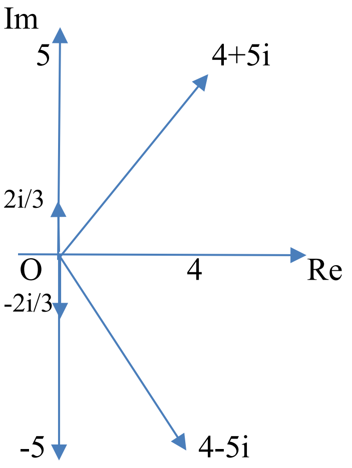
Pearson Edexcel International Advanced Level
In Further Pure Mathematics F1
(WFM01/01)

Question Number	Scheme	Notes	Marks
1	$\mathrm{f}(x)=x^3+4x-6$		
1(i)(a)	$\mathrm{f}(1)=-1$ $\mathrm{f}(1.5)=3.375\left(=\frac{27}{8}\right)$	Attempts to evaluate at both end points. If substitution not seen accept $\mathrm{f}(1)=-1$ **or** $\mathrm{f}(1.5)=3.375$ as evidence, with any value for the other end.	M1
	Sign change and $\mathrm{f}(x)$ continuous therefore α is between $x=1$ and $x=1.5$	$\mathrm{f}(1)=-1$ **and** $\mathrm{f}(1.5)=3.375$ <u>both correct</u> and mentions/indicates <u>sign change</u>, <u>continuous</u> and <u>conclusion</u>.	A1
			(2)
1(i)(b)	$\mathrm{f}'(x)=3x^2+4$	Correct derivative. Can be implied by a correct expression seen later such as $3(1.5)^2+4$	B1
	$x_2=1.5-\frac{\mathrm{f}(1.5)}{\mathrm{f}'(1.5)}=1.5-\frac{3.375}{10.75}$	Attempt Newton-Raphson using the **correct** formula.	M1
	$x_2=1.186...$	Accurate first application either awrt 1.186, $\frac{51}{43}$ or a correct numerical expression.	A1
	$x_3=\left(1.186...-\frac{0.412...}{8.220...}\right)=1.1358...$		
	$\alpha\approx 1.136$	cao	A1
			(4)
1(ii)	$\mathrm{g}(1.4)=3.442116...$ **or** $\mathrm{g}(1.5)=-3.601419...$	Evidence of at least one value correct to 3 d.p. or better. May be implied by correct answer if never seen.	B1
	$\frac{1.5-\beta}{\beta-1.4}=\frac{0-\mathrm{g}(1.5)}{\mathrm{g}(1.4)-0}$ or $\frac{\beta-1.4}{0-\mathrm{g}(1.4)}=\frac{1.5-1.4}{\mathrm{g}(1.5)-\mathrm{g}(1.4)}$ oe	A correct linear interpolation statement as shown oe (with correct signs). May omit the zeroes or use evaluated values (e.g 0.1 instead of 1.5 – 1.4). Other forms are possible.	B1
	$\Rightarrow \beta\approx 1.448869...$	Evaluates β from an attempt at a linear interpolation statement, allow if signs are incorrect. $\beta\approx$ awrt 1.449	M1 A1
			(4)
			Total 10

Qn No	Scheme	Notes	Marks
2. (a)	$\dfrac{z_2z_3}{z_1}=\dfrac{(p-\mathrm{i})(p+\mathrm{i})(2+\mathrm{i})}{(2-\mathrm{i})(2+\mathrm{i})}$	Multiply top and bottom by complex conjugate of their denominator. (The two M's may be scored if the given numbers are wrongly placed.)	M1
	$=\dfrac{(p^2+1)(2+\mathrm{i})}{5}$	Simplifies numerator with evidence that $\mathrm{i}^2=-1$ and denominator real. Accept any equivalent form in the numerator as long as there are not i^2 terms if expanded.	M1
	$=\dfrac{2(p^2+1)}{5}+\dfrac{(p^2+1)}{5}\mathrm{i}$	Correct real +imaginary form with i factored out. Accept as single fraction with numerator in correct form. Accept $'a='$ and $'b='$.	A1
			(3)
ALT	$\dfrac{z_2z_3}{z_1}=\dfrac{(p-\mathrm{i})(p+\mathrm{i})}{(2-\mathrm{i})}=a+b\mathrm{i}$ $p^2+1=(a+b\mathrm{i})(2-\mathrm{i})$ $\left.\begin{array}{l}2a+b=p^2+1\\2b-a=0\end{array}\right\}$	Cross multiplies by $2-\mathrm{i}$ (or their denominator), expands and equates real and imaginary parts. (The two M's may be scored if the given numbers are wrongly placed.)	M1
	$\left.\begin{array}{l}2a+b=p^2+1\\2b-a=0\end{array}\right\}\Rightarrow a=...,b=...$	Attempt to solve their equations.	M1
	$a+b\mathrm{i}=\dfrac{2(p^2+1)}{5}+\dfrac{(p^2+1)}{5}\mathrm{i}$	Correct real +imaginary form with i factored out. Accept as single fraction with numerator in correct form. Accept $'a='$ and $'b='$.	A1
			(3)
2(b)	$\left\|\dfrac{z_2z_3}{z_1}\right\|^2=\dfrac{4(p^2+1)^2}{25}+\dfrac{(p^2+1)^2}{25}$	Correct attempt at the modulus or modulus squared. Accept with their answers to part (a). Any erroneous i or i^2 is M0.	M1
	$\dfrac{4(p^2+1)^2}{25}+\dfrac{(p^2+1)^2}{25}=\left(2\sqrt{5}\right)^2$	Their $\left\|\dfrac{z_2z_3}{z_1}\right\|^2=\left(2\sqrt{5}\right)^2$	dM1
	$(p^2+1)^2=100\Rightarrow p=\pm3$	Attempt to solve and achieves $p=...$ (may be scored from use of $\|..\|^2=2\sqrt{5}$) $p=\pm3$	M1 A1
			(4)
ALT 1	$\left\|\dfrac{z_2z_3}{z_1}\right\|=2\sqrt{5}\Rightarrow\left\|z_2z_3\right\|=\sqrt{4+1}\times2\sqrt{5}$	Cross multiplies and attempts $\|z_1\|$	M1
	$\Rightarrow\left\|z_2\right\|^2=\sqrt{4+1}\times2\sqrt{5}\Rightarrow p^2+1=...$	Attempts $\|z_2z_3\|$ either directly or using $\left\|z_2z_2^*\right\|=\left\|z_2\right\|^2$ to get an equation in p.	dM1
	$(p^2+1)=10\Rightarrow p=\pm3$	Attempt to solve and achieves $p=...$ $p=\pm3$	M1 A1
			(4)
			Total 7

Question Number	Scheme	Notes	Marks
3(a)	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 2 & 0 \\ 1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 1 \\ -2 & -2 & 0 \end{pmatrix}$	Attempt to multiply in the correct order with at least four correct elements. May be done as three separate calculations, so look for at least 4 correct values.	M1
	(1,−2), (3,−2) and (1,0)	Accept as individual column vectors but not as a single 2x3 matrix.	A1
			(2)
3(b)	Rotation	Accept rotate or turn oe.	B1
	270° (anticlockwise) **about the origin**	Accept −90° (anticlockwise) or 90° clockwise (must be stated) and (0,0) or *O*. *Assume anticlockwise unless otherwise stated.*	B1
			(2)
3(c)	$\mathbf{Q} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	One correct, both correct	B1,B1
			(2)
3(d)	$\mathbf{RQ} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	Multiplication in correct order for their matrices and at least 1 row or 1 column correct.	M1
	$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	Correct matrix	A1
			(2)
3(e)	Reflection	Correct type identified.	B1
	in (the line) $y = x$	Correct line of reflection specified, accepting equivalent forms (e.g. line at angle 45° (anticlockwise) to the (positive) x-axis).	B1
			(2)
			Total 10

Question Number	Scheme	Notes	Marks
4(a)	$y=25x^{-1}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-25x^{-2}$ $xy=25\Rightarrow y+x\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}x}=-\frac{5}{t^2}.\frac{1}{5}$	Attempts a derivative expression, such as $\frac{\mathrm{d}y}{\mathrm{d}x}=kx^{-2}$ or $x\frac{\mathrm{d}y}{\mathrm{d}x}=ky$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\text{their }\frac{\mathrm{d}y}{\mathrm{d}t}}{\text{their }\frac{\mathrm{d}x}{\mathrm{d}t}}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-25x^{-2}$ or $\frac{-y}{x}$ or $\frac{-1}{t^2}$	Correct derivative.	A1
	$y-\frac{5}{t}=-\frac{1}{t^2}(x-5t)$	Uses $y-\frac{5}{t}=$ (their gradient) x $(x-5t)$ or $y=$ (their gradient)$x+c$ using $x=5t, y=\frac{5}{t}$ in an attempt to find c. Their gradient must be a function of t for marks to be awarded.	M1
	$t^2y+x=10t$ *	cso	A1
			(4)
Alt 4(a)	Intersect when $(10t-t^2y)y=25$	Substitutes given line equation into the curve equation to find intersections	M1
	$\Rightarrow t^2y^2-10ty+25=0$	Correct equation	A1
	$\Rightarrow (ty-5)^2=0\Rightarrow y=\frac{5}{t}$ is single root.	Shows the equation has a single root.	M1
	Single root when $y=\frac{5}{t}$ means the line is the tangent to H at P.	All work correct, with correct conclusion.	A1
			(4)
4(b)	$t^2(-5)+15=10t$	Substitute $(15,-5)$ into equation of tangent	M1
	$(t+3)(t-1)=0\Rightarrow t=..$	Solves via any valid means	M1
	$\Rightarrow t=-3, t=1$ (or one correct point)	Both values of t **or** one correct coordinate pair.	A1
	$\left(-15,-\frac{5}{3}\right)$ and $(5,5)$	Both correct sets of coordinates.	A1
			(4)
ALT	${t_1}^2y+x=10t_1$ ${t_2}^2y+x=10t_2$	Form equations with their t_1,t_2 and attempt to solve for x and y	M1
	$(x=)\frac{10t_1t_2}{(t_1+t_2)}=15, (y=)\frac{10}{(t_1+t_2)}=-5$	Equates to coordinate and attempt to solve for t_1,t_2	M1
	$t_1=-3, t_2=1$ (or one correct point)	Both values of t **or** one correct coordinate pair.	A1
	$\left(-15,-\frac{5}{3}\right)$ and $(5,5)$	Both correct sets of coordinates.	A1
			(4)
			Total 8

Question Number	Scheme	Notes	Marks
	$\mathrm{f}(x)=\left(9x^2+d\right)\left(x^2-8x+(10d+1)\right)$		
5(a)	$9x^2+d=0\Rightarrow x=\pm\sqrt{-\dfrac{d}{9}}$ or $\pm\dfrac{\mathrm{i}\sqrt{d}}{3}$	or exact equivalents	B1
	or $x=\dfrac{8\pm\sqrt{64-4(10d+1)}}{2}$ or $(x-4)^2-16+10d+1=0\Rightarrow x=\ldots$	Solve $x^2-8x+(10d+1)=0$ by formula or completing the square. Must have complete constant term.	M1
	$x=4+\sqrt{15-10d}$ and $x=4-\sqrt{15-10d}$	oe with discriminant simplified. Mark final answer, do not isw.	A1
			(3)
5(b)	$x=\pm\dfrac{2\mathrm{i}}{3}$ or f.t. their roots	Correct roots, or f.t.their answer for the $9x^2+d$	B1ft
	$x=4\pm5\mathrm{i}$ or f.t. their roots	Correct roots for the given quadratic, or f.t. their 3TQ	B1ft
	SC Award B1ftB0 if only one of **each** pair is given.		
			(2)
5(c)		Two roots on imaginary axis the same distance from *O*. Follow through their *imaginary* roots from (b). B0 if real roots found. Their two *complex* roots with real and imaginary parts, one the conjugate of the other, so reflected in the real axis. Must be correct relative scale compared with the imaginary roots **if the first B1 in (c) has been awarded** (ie clearly further from *O* if correct, or f.t. their answers). But if first B0 has been given, ignore scales. Accept points or vectors. Complex numbers must be labelled in some way e.g. via scales or coordinates or vectors.	B1ft B1ft
			(2)
			Total 7

Question Number	Scheme	Notes	Marks
6(a)	$y=\sqrt{8}x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx}=\frac{1}{2}\sqrt{8}x^{-\frac{1}{2}}=\sqrt{2}x^{-\frac{1}{2}}$ $y^2=8x \Rightarrow 2y\frac{dy}{dx}=8$ or $\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}=4.\frac{1}{4t}$	Attempts a derivative expression, such as $\frac{dy}{dx}=kx^{-\frac{1}{2}}$ or $ky\frac{dy}{dx}=c$ or their $\frac{dy}{dt}\times\left(\frac{1}{\text{their }\frac{dx}{dt}}\right)$	M1
	$\frac{dy}{dx}=\frac{1}{2}\sqrt{8}x^{-\frac{1}{2}}\left(=\sqrt{2}x^{-\frac{1}{2}}\right)$ or $2y\frac{dy}{dx}=8$ or $\frac{dy}{dx}=4.\frac{1}{4t}\left(=\frac{1}{t}\right)$	Correct differentiation (need not have substituted for t etc)	A1
	At P, gradient of normal is $m_N=-p$	Correct gradient for the normal.	A1
	$y-4p=-p(x-2p^2)$	$y-4p=(\text{their } m_N)\times(x-2p^2)$ or $y=(\text{their } m_N)x+c$ using $x=2p^2, y=4p$ in an attempt to find c. Their gradient must be a function of p for marks to be awarded. Must use a changed gradient, not tangent gradient.	M1
	$y+px=2p^3+4p$ *	cso	A1
			(5)
6(b)	$y+qx=2q^3+4q$	oe	B1
			(1)
6(c)			
	$y+px=2p^3+4p$ $\underline{y+qx=2q^3+4q}$ $px-qx=2p^3+4p-2q^3-4q$	Attempt to solve simultaneous equations. A correct equation in only one variable	M1 A1
	$(p-q)x=2((p-q)(p^2+pq+q^2)+2(p-q))$	Attempts to simplify the expression to required form. E.g. factorise difference of two cubes, $p^3-q^3=(p-q)(p^2+pq+q^2)$ or equivalent work to enable $p-q$ to cancel.	M1
	$x=2(p^2+pq+q^2+2)$*	cso for reaching the correct x coordinate.	A1
	$y+2p^3+2p^2q+2pq^2+4p=2p^3+4p$ $y=-2pq(p+q)$*	cso for reaching both coordinates correctly.	A1
			(5)
ALT	$qy+pqx=2p^3q+4pq$ $\underline{py+pqx=2pq^3+4pq}$ $py-qy=2pq^3-2p^3q$	Attempt to solve simultaneous equations. A correct equation in only one variable	M1 A1

	$(p-q)y=-2pq(p^2-q^2)$ $(p-q)y=-2pq(p-q)(p+q)$	Attempts to simplify the expression to required form. E.g. Attempt to factorise difference of two squares. $p^2-q^2=(p-q)(p+q)$ or equivalent work to enable $p-q$ to cancel.	M1
	$y=-2pq(p+q)$*	cso for reaching the correct y coordinate.	A1
	$-2p^2q^2-2pq^3+pqx=2p^3q+4pq$ $x=2(p^2+pq+q^2+2)$*	cso for reaching both coordinates correctly.	A1
			(5)
ALT 2	$-2pq(p+q)+2p(p^2+pq+q^2+2)=$ $-2p^2q-2pq^2+2p^3+2p^2q+2pq^2+4p$	Substitutes both coordinates into the equation for normal at P and expands brackets.	M1
	$=2p^3+4p$ so N is on normal at P	Simplifies correctly to $2p^3+4p$ and deduces N is on the normal at P	A1
	$-2pq(p+q)+2q(p^2+pq+q^2+2)=$ $-2p^2q-2pq^2+2p^2q+2pq^2+2q^3+4q$	Substitutes both coordinates into the equation for normal at Q and expands brackets.	M1
	$=2q^3+4q$ so N is on normal at Q	Simplifies correctly to $2q^3+4q$ and deduces N is on the normal at Q but penalise only the first instance for missing the deduction.	A1
	As N is on both normals it is therefore the intersection point of them.	Makes a suitable conclusion	A1
			(5)
6(d)	Grad $PQ=\dfrac{4q-4p}{2q^2-2p^2}\left(=\dfrac{2}{p+q}\right)$	Use of $\dfrac{y_2-y_1}{x_2-x_1}$ and substituting.	M1A1
	Grad $ON=\dfrac{-2pq(p+q)}{2(p^2+pq+q^2+2)}$	Can be unsimplified.	B1
	Grad $PQ\times$Grad ON $=\dfrac{2}{p+q}\times\dfrac{-2pq(p+q)}{2(p^2+pq+q^2+2)}=-1$	Using product of their gradients $=-1$	M1
	$(p+q)^2-3pq=-2$	-2	A1
			(5)
			Total 16

Question Number	Scheme	Notes	Marks
7	$\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$		
7(a)	$n=1, \text{LHS}=1^2=1, \ \text{RHS}=\frac{1}{6}.2.3=1$	Shows both LHS=1 and RHS=1 Accept LHS = 1 but must see at least $\frac{1}{6}.2.3=1$ for RHS.	B1
	Assume true for $n=k$		
	When $n=k+1$ $\sum_{r=1}^{k+1} r^2 = \frac{k}{6}(k+1)(2k+1)+(k+1)^2$	Adds $(k+1)^2$ to result for $n=k$	M1
	$=\frac{(k+1)}{6}(k(2k+1)+6(k+1))$	Attempt to factorise by $\frac{(k+1)}{6}$	dM1
	$=\frac{(k+1)}{6}(k+2)(2k+3)$ $=\frac{(k+1)}{6}((k+1)+1))((2(k+1)+1)$	Either factorised form. SC allow dM1A0 for fully factorising to a cubic expression and going direct to the fully factorised expression with no intermediate quadratic seen.	A1
	True for $n=1$. If true for $n=k$ then true for $n=k+1$ therefore true for all n.	Complete proof with no errors and these 4 statements seen anywhere. Depends on both M's and the A, but may be scored if the B is lost as long as some indication of true for n = 1 is given.	A1cso
			(5)
7(b)	$\sum_{r=1}^{n}(r^2+2)=\sum_{r=1}^{n} r^2+\sum_{r=1}^{n} 2$ $=\frac{n}{6}(n+1)(2n+1)+\ldots$	Split into the addition of 2 sums and applies the result of (a).	M1
	$=\frac{n}{6}(n+1)(2n+1)+2n$	Correct expression.	A1
	$=\frac{n}{6}(2n^2+3n+13)$	Factorises out the $\frac{n}{6}$ - must have a common factor n to achieve this mark; Simplifies to correct answer.	M1; A1
	$(a=2, b=3, c=13)$		
			(4)
7(c)	$\sum_{r=10}^{25}(r^2+2)=S_{25}-S_9$ $=\frac{25}{6}.(2\times 25^2+3\times 25+13)-\frac{9}{6}(2\times 9^2+3\times 9+13)$	Attempts $S_{25}-S_9$ or $S_{25}-S_{10}$ with some substitution.	M1
	$=\frac{25}{6}\times 1338-\frac{9}{6}\times 202=5575-303=5272$	For 5272	A1
	Note: Answer only (from calculator) is M0A0 as question requires use of part (b).		
			(2)
			Total 11

Question Number	Scheme	Notes	Marks
	$\mathrm{f}(n) = 4^{n+2} + 5^{2n+1}$ divisible by 21		
8	$n = 1, 4^3 + 5^3 = 189 = 9 \times 21$ (Or $n = 0$, $4^2 + 5^1 = 21$)	$\mathrm{f}(1) = 21 \times 9$ Accept f(0) = 21 as an alternative starting point.	B1
	Assume that for $n = k$, $\mathrm{f}(k) = (4^{k+2} + 5^{2k+1})$ is divisible by 21 for $k \in \mathbb{Z}^+$.		
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 4^{k+3} + 5^{2k+3} - (4^{k+2} + 5^{2k+1})$	Applies $\mathrm{f}(k+1)$ with at least 1 power correct. May be just as f(k + 1), or as part of an expression in f(k + 1) and f(k).	M1
	$= 4.4^{k+2} + 25.5^{2k+1} - 4^{k+2} - 5^{2k+1}$	For a correct expression in f(k + 1), and possibly f(k), with powers reduced to those of f(k).	A1
	$= 3.4^{k+2} + 24.5^{2k+1}$		
	$= 3\mathrm{f}(k) + 21.5^{2k+1}$ or $= 24\mathrm{f}(k) - 21.4^{k+2}$	For one of these expression or equivalent with obvious factor of 21 in each.	A1
	$\mathrm{f}(k+1) = 4\mathrm{f}(k) + 21.5^{2k+1}$	Makes $\mathrm{f}(k+1)$ the subject or gives clear reasoning of each term other than $\mathrm{f}(k+1)$ being divisible by 21. Dependent on at least one of the previous accuracy marks being awarded.	dM1
	{ $\mathrm{f}(k+1)$ is divisible by 21 as both $\mathrm{f}(k)$ and 21 are both divisible by 21}		
	If the result is **true for *n* = *k***, then it is now **true for *n* = *k* + 1**. As the result has shown to be **true for *n* = 1 (or 0)**, then the result is **true for all *n*** ($\in \mathbb{Z}^+$).	Correct conclusion seen at the end. Condone true for n = 1 stated earlier**.** Depends on both M's andA's, but may be scored if the B is lost as long as at least f(1) = 189 was reached (so e.g. if the 21×9 was not shown)	A1 cso
			(6)
ALT for first 4 marks	$n = 1, 4^3 + 5^3 = 189 = 9 \times 21$ (Or $n = 0$, $4^2 + 5^1 = 21$)	As main scheme.	B1
	$\mathrm{f}(k+1) - \alpha \mathrm{f}(k) = 4^{k+3} + 5^{2k+3} - \alpha(4^{k+2} + 5^{2k+1})$	Attempts f(k + 1) in any equation (as main scheme).	M1
	$\mathrm{f}(k+1) - \alpha \mathrm{f}(k) = (4-\alpha)4^{k+2} + (25-\alpha)5^{2k+1}$	For a correct expression with any α, with powers reduced to match f(k).	A1
	$\mathrm{f}(k+1) - \alpha \mathrm{f}(k) = (4-\alpha)(4^{k+2} + 5^{2k+1}) + 21.5^{2k+1}$ $\mathrm{f}(k+1) - \alpha \mathrm{f}(k) = (25-\alpha)(4^{k+2} + 5^{2k+1}) - 21.4^{k+2}$	Any suitable equation with powers sorted appropriately to match f(k)	A1
	NB: $\alpha = 0, \alpha = 4, \alpha = 25$ will make relevant terms disappear, but marks should be awarded accordingly.		
			Total 6