Pearson Edexcel

Mark Scheme (Results)

January 2019

Pearson Edexcel International Advanced Level In Pure Mathematics P1 (WMA11/01)

Question Number	Scheme	Marks
1.	$\int \frac{2}{3}x^3 - \frac{1}{2x^3} + 5\,dx = \frac{2}{3}\times\frac{x^4}{4} - \frac{1}{2}\times\frac{x^{-2}}{-2} + 5x + c$	M1 A1
	$= \frac{1}{6}x^4 + \frac{1}{4}x^{-2} + 5x + c$	A1 A1
		(4 marks)

M1 For raising any power by 1 eg. $x^3 \to x^4$, $x^{-3} \to x^{-2}$, $5 \to 5x$ or eg. $x^3 \to x^{3+1}$

A1 For two of $\frac{2}{3}\times\frac{x^4}{4}$, $-\frac{1}{2}\times\frac{x^{-2}}{-2}$, $+5x$ correct (un-simplified) . Accept $5x^1$

This may be implied by a correct simplified answer

A1 For two of $\frac{1}{6}x^4$, $+\frac{1}{4}x^{-2}$, $+5x$ correct and in simplest form. Accept forms such as $\frac{x^4}{6}$, $\frac{1}{4x^2}$,

CONDONE $+\frac{0.25}{x^2}$ but NOT $\frac{1/4}{x^2}$, $\frac{5x}{1}$, $-\left(-\frac{1}{4}x^{-2}\right)$

A1 Fully correct and simplified with $+c$ all on one line. Accept simplified equivalents (see above) and ignore any spurious notation. ISW after a correct simplified answer is achieved.

A common mistake is writing $-\frac{1}{2x^3} = -2x^{-3} \to -x^{-2}$ this can still get the method mark for increasing the power by 1.

Question Number	Scheme	Marks
2.	Attempts both sides as powers of 3 $\dfrac{3^x}{3^{4y}} = 3^3 \times 3^{0.5} \Rightarrow 3^{x-4y} = 3^{"3.5"}$	M1
	Sets powers equal and attempts to makes y the subject : $x - 4y = "3.5" \Rightarrow y = ...$	dM1
	$y = \dfrac{1}{4}x - \dfrac{7}{8}$	A1
		(3) **(3 marks)**
Alt1	Multiplies by 3^{4y} first:	
	Attempts both sides as powers of 3 (Addition law on RHS) $3^x = 27\sqrt{3} \times 3^{4y} \Rightarrow 3^x = 3^{"3.5"+4y}$	M1
	Sets powers equal and makes y the subject $x = "3.5" + 4y \Rightarrow y = ...$	dM1
	$y = \dfrac{1}{4}x - \dfrac{7}{8}$	A1
Alt2	Divides by $27\sqrt{3}$ first:	
	Attempts both sides as powers of 3 (Subtraction law on LHS) $\dfrac{3^x}{3^{4y} \times 27\sqrt{3}} = 1 \Rightarrow 3^{x-4y-"3.5"} = 3^0$	M1
	Sets powers equal and makes y the subject $x - "3.5" - 4y = 0 \Rightarrow y = ...$	dM1
	$y = \dfrac{1}{4}x - \dfrac{7}{8}$	A1
Alt3	Takes logs of both sides	
	Eg Base 3: $\log_3\left(\dfrac{3^x}{3^{4y}}\right) = \log_3\left(27\sqrt{3}\right)$	
	$\log_3 3^x - \log_3 3^{4y} = \log_3 27\sqrt{3}$	M1
	$x - 4y = 3.5 \Rightarrow y = ...$	dM1
	$y = \dfrac{1}{4}x - \dfrac{7}{8}$	A1

M1 Attempts to use the subtraction law on the LHS and the addition law on the RHS to achieve a form of $3^{\cdots} = 3^{\cdots}$

Condone errors writing $27\sqrt{3}$ as a single power of 3 but it must be clear what the two indices are before adding if they make an error ($27 = 3^a$ and $\sqrt{3} = 3^b$ so $27\sqrt{3} = 3^{a+b}$)
A common mistake is to write $27\sqrt{3} \Rightarrow \sqrt{9} \times \sqrt{3} \times \sqrt{3} = 3^2$ which can be condoned and they can still get M1M1A0.

They may rearrange the equation first so look for attempts at the appropriate index laws being applied (see alternatives). In Alt2 allow the RHS=1.

They may use logs on both sides (the most likely would be base 3 - see Alt3) To score M1 they would need to take logs and then apply the laws of logs to either add or subtract.

dM1 Dependent upon the previous M mark, it is for an attempt to make y the subject. For this mark follow through their power for $27\sqrt{3}$ but they must have 3 terms in their equation relating to the powers and they cannot "lose" one in the rearrangement. (i.e $ax + by + c = 0$ oe where $a,b,c \neq 0$) Do not award this mark if they rearrange to make x the subject. Condone sign slips only.

A1 $y = \frac{1}{4}x - \frac{7}{8}$ or exact simplified equivalent eg $y = \frac{2x-7}{8}$, $y = 0.25x - 0.875$

DO NOT ACCEPT $y = \frac{x - \frac{7}{2}}{4}$ or $y = \frac{x-3.5}{4}$

Question Number	Scheme	Marks
3.(a)	Attempts to make y the subject	M1
	States $-\frac{3}{5}$ or exact equivalent	A1
		(2)
(b)	Uses perpendicular gradients rule $\Rightarrow$ gradient $l_2 = \frac{5}{3}$	M1
	Forms equation of l_2 using (6,-2) $y+2 = "\frac{5}{3}"(x-6)$	M1
	$y = \frac{5}{3}x - 12$	A1
		(3) **(5 marks)**
Alt1(a)	Eg Coordinates of two points on the line $(0,1.4)$ and $(1,0.8)$ Gradient $= \frac{0.8-1.4}{1-0}$ Gradient $= -0.6$	M1 A1

(a)

M1 For an attempt to rearrange $3x+5y-7=0$ and make y the subject.

Expect to see $\pm 5y = ...$ followed by $y = ...$ or equivalent. $\frac{3}{5}$ on its own is M0.

Alternatively they may find two pairs of coordinates and find the gradient between those two points.

Allow one slip in calculating the coordinates and it must be clear that they are attempting $\frac{y_2 - y_1}{x_2 - x_1}$

A1 For stating $-\frac{3}{5}$ or exact equivalent in (a). A correct answer implies both marks and isw after a correct gradient is stated.. (The value of c does not need to be correct).

Do not allow $y = -\frac{3}{5}x + \frac{7}{5}$ without some statement for 'm'. Do not allow $-\frac{3}{5}x$.

(b)

M1 Uses the perpendicular gradient rule following through on their gradient from (a).
If a gradient is not given, follow through on their 'm'

M1 For the equation of a straight line with a **changed** gradient using $(6,-2)$.
So if (a) was $-\frac{3}{5}$, then $(y+2)=\frac{3}{5}(x-6)$ would score this. At least one bracket must be correct. If the form $y=mx+c$ is used they must proceed as far as finding c. They must either have shown their gradient and $(6,-2)$ substituted into $y=mx+c$ and rearrange (maybe with errors) to find c or if they show no working then their c must be correct.

A1 $y=\frac{5}{3}x-12$ Allow exact equivalent values for their constants eg $y=\frac{10}{6}x-\frac{36}{3}$, $y=1.\dot{6}x-12$, $y=\frac{5x}{3}-12$ but do not allow equations such as $y=1.67x-12$. ISW after a correct equation in the correct form is found.

Question Number	Scheme	Marks
4.	When ---- represents < or > and —— represents $\le$ or $\ge$ Either $2y \le x$ or $y \ge 2x - \frac{1}{8}x^2$	B1
	$2x - \frac{1}{8}x^2 = 0 \Rightarrow x = 16 \Rightarrow x < \ldots$ or $x \le \ldots$	M1
	$x < 16$, $2y \le x$ and $y \ge 2x - \frac{1}{8}x^2$	A1
		(3) **(3 marks)**
Alt1	When ---- represents $\le$ or $\ge$ and —— represents < or > Either $2y < x$ or $y > 2x - \frac{1}{8}x^2$	B1
	$2x - \frac{1}{8}x^2 = 0 \Rightarrow x = 16 \Rightarrow x < \ldots$ or $x \le \ldots$	M1
	$x \le 16$, $2y < x$ and $y > 2x - \frac{1}{8}x^2$	A1
		(3)

B1 Sight of $2y \le x$ or $y \ge 2x - \frac{1}{8}x^2$. Either inequality is sufficient for B1 and they may be written in an equivalent correct form (see NB below)

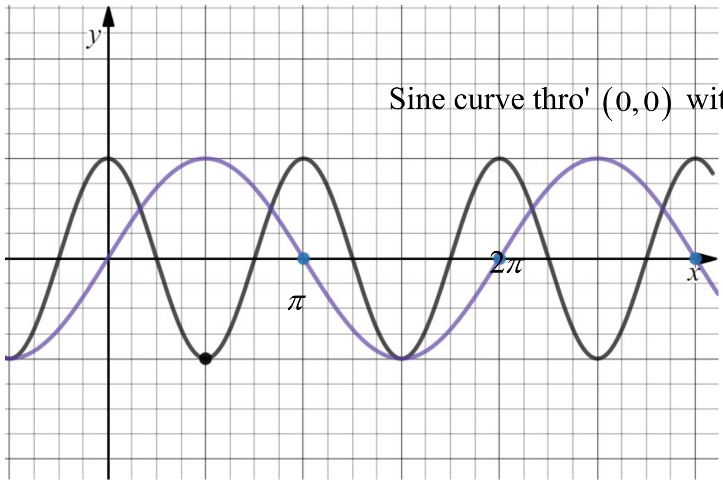
NB Inequalities cannot be in terms of R

M1 Attempts to find the upper bound for x to define R. Solves to find where the quadratic intersects the x-axis and then uses their value to write $x <\ldots$ or $x \le \ldots$. Use general principles for solving a quadratic equation (page 5). They do not need to find or state $x = 0$ and ignore any lower bound eg $0 < x < \ldots$

A1 $2y \le x$, $y \ge 2x - \frac{1}{8}x^2$ and $x < 16$ (Allow $A \le x < 16$ where $A \le 12$).
Candidates may write more than one inequality for a particular boundary. In these cases mark the last one. Correct inequalities labelled on the graph are also acceptable, however, an inequality written below takes precedence.

NB You may see $y \leqslant \frac{x}{2}$ for $2y \leqslant x$ or even $2x - \frac{1}{8}x^2 \le y \le \frac{x}{2}$ oe

Alternatively, some candidates may express their inequalities involving a boundary for a dashed line using $\le$ or $\ge$ and a boundary for a solid line using < or >. It may not always be clear so mark positively. See Alt1

Question Number	Scheme	Marks
5. (a)	$\left(\dfrac{\pi}{2},-1\right)$	B1 B1
		(2)
(b)		
	Sine curve thro' $(0,0)$ with max/min of ± 1	M1
	Fully correct	A1
		(2)
(c)	(i) 30 but follow through on $10\times$ the number of their solutions $0 \to 2\pi$ (ii) 32	B1ft B1 (2) **(6 marks)**

(a)

B1 Either coordinate correct. They may state the coordinates separately or condone the lack of brackets for this mark. (Accept the x-coordinate as 90° or awrt 1.57 radians for this mark) If only one coordinate is stated, it must be clear if it is the x or y coordinate.

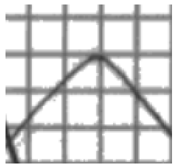
B1 For $\left(\dfrac{\pi}{2},-1\right)$ or $x=\dfrac{\pi}{2}, y=-1$ Allow $\left[\dfrac{\pi}{2},-1\right]$

SC $\left(-1,\dfrac{\pi}{2}\right)$ B1B0 (coordinates the wrong way round)

(b)

M1 For a sketch of a sine curve with at least one cycle starting at (or going through) the origin with the same maximum/minimum y-values as the $\cos 2x$ curve.
Condone poor/incorrect period and poor symmetry
Condone turning points appearing V shaped for this mark. If drawn on a separate diagram the maximum and minimum must appear to be ± 1 according to their axes and a complete cycle must be in the positive domain. Condone slight inaccuracies of the amplitude of their sine curve.

A1 A correct sketch of $\sin x$ between $-\frac{\pi}{2}$ and 3π. Labelling where the graph crosses the x-axis is not required. Turning points must appear curved. If multiple attempts are drawn and it is not clear which is their final attempt then withhold the A1.



Do not accept linear looking graphs so unless it is a clear V shape at one maximum or minimum then allow any curvature at the turning points. As a guide the curve should not go diagonally across the square either side of turning points. See graph on the right showing what is not acceptable. Where the graph crosses the x-axis, it must be within half a square of the correct points

(c) (i)

B1ft 30 or follow through on $10\times$ the number of their solutions between 0 and 2π (where 2π should be on the graph - see mark scheme for position) . The question said hence or otherwise so they may get B1 for 30 even if their graph does not suggest that number of solutions.

(ii)

B1 32

Question Number	Scheme	Marks
6.(a)	$\mathrm{f}'(x)=5x^{\frac{3}{2}}-40$	M1A1
	Attempts $5x^{\frac{"3"}{2}}-40=0\Rightarrow x^{\frac{"3"}{2}}=..$	M1
	$x=4$	A1 cao
		(4)
(b)	$\mathrm{f}''(x)=\frac{15}{2}x^{\frac{1}{2}}=5$	M1
	$\Rightarrow x^{\frac{1}{2}}=..\Rightarrow x=..^2$ $\qquad x=\frac{4}{9}$	M1 A1
		(3) **(7 marks)**

(a)

M1 For reducing the power by one on either x term ($x^{\frac{5}{2}}\to x^{\frac{3}{2}}$ or $-40x\to -40$)

A1 Correct (but may be unsimplified) $\left[\mathrm{f}'(x)\right]=5x^{\frac{3}{2}}-40$

M1 Attempts to solve their $\mathrm{f}'(x)=0\Rightarrow x^{\frac{"3"}{2}}=...$ by making their $x^{\frac{"3"}{2}}$ the subject. Their $\mathrm{f}''(x)$ must be a changed function. This can be implied by their final answer. If their $\mathrm{f}'(x)=0$ is of the form $\mathrm{f}'(x)=Ax^{B}+Cx^{D}$ then this mark can be awarded for taking a factor out eg. $x^{B}(A+Cx^{D-B})$ and doing the same as above on the terms in their bracket.

A1 $x=4$ cao (do not accept ± 4)

(b)

M1 For reducing the power by one on one of their terms in $\mathrm{f}'(x)$ and setting their $\mathrm{f}''(x)=5$

M1 For a correct method leading to $x=\ldots$ from an equation of the form $Ax^{\frac{1}{2}}=5$.
Eg for $\frac{15}{2}x^{\frac{1}{2}}=5$ either makes $x^{\frac{1}{2}}$ the subject and squares or squares both sides and makes x the subject. This can be implied by their final answer. Do not allow slips on the power.

A1 $x=\frac{4}{9}$ or exact equivalent

The question states that solutions based entirely on graphical or numerical methods are not acceptable. Therefore if no differentiation is shown then this will score no marks.

Question Number	Scheme	Marks
7.(a)	Attempts $\dfrac{\sin\angle ACB}{6.5}=\dfrac{\sin 35}{4.7}$	M1
	$\angle ACB=\text{awrt}(52\text{ or }53)°\quad\text{or}\quad\text{awrt}(127\text{or}128)°$	A1
	$\angle ACB=127.5°$	A1
		(3)
(b)	Eg $\dfrac{(AC)}{\sin\text{"}17.5°\text{"}}=\dfrac{6.5}{\sin\text{"}127.5°\text{"}}$ or $=\dfrac{4.7}{\sin 35°}$	M1
	$\left[\dfrac{(CD)}{\sin\text{"}75°\text{"}}=\dfrac{4.7}{\sin\text{"}127.5°\text{"}}\Rightarrow(CD)=...\Rightarrow(AC)+(CD)\right]$=awrt 8.2	A1
	Total length of wood $=8.1+6.5+4.7+4.7=\text{awrt}\,24.1$	A1
		(3)
		(6 marks)
Alt1(a)	$\cos 35=\dfrac{AC^2+6.5^2-4.7^2}{2\times6.5\times AC}\Rightarrow AC^2-13\cos(35)AC+20.16=0\Rightarrow AC=...$	
	$\cos\angle ACB=\dfrac{\text{"}AC\text{"}^2+4.7^2-6.5^2}{2\times\text{"}AC\text{"}\times4.7}$ oe	M1

(a)

M1 Uses the sine rule with the angles and sides in the correct positions.

Alternatively they may use the cosine rule on *ACB* and then solve the subsequent quadratic to find *AC* and then use the cosine rule again

A1 $\angle ACB=\text{awrt}(52\text{ or }53)°\quad\text{or}\quad\text{awrt}(127\text{or}128)°$

A1 $\angle ACB=127.5°$ only

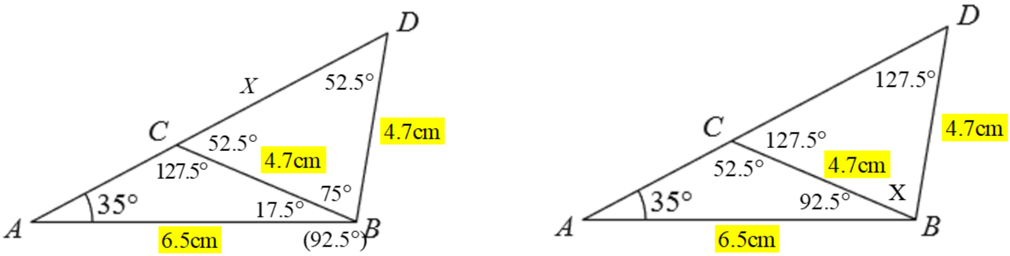
(b) Working for (b) may be found in (a) which is acceptable

M1 Uses a formula that finds **part or all of the length *AD* (eg *AC, CD* , *AX, XD, AD*)**.

The minimum required for this mark is the use of angle(s) and lengths in the correct places in the formula (which may have been rearranged to an alternative form). Condone mislabelling of the unknown length. This is usually the sine or cosine rule but they could split the triangle into two right angled triangles. See WAYS for additional guidance on methods. Sight of awrt8.2, awrt2.46 or awrt5.72 would imply this mark.

Condone angles in their triangles which do not add up to $180°$ and condone angles found with no working shown. For reference below these are the angles that would be found with $\angle ACB = 127.5°$ and $\angle ACB = 52.5°$ although they may "restart" so check their diagram as this may help.

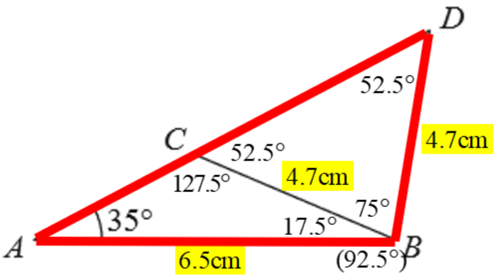
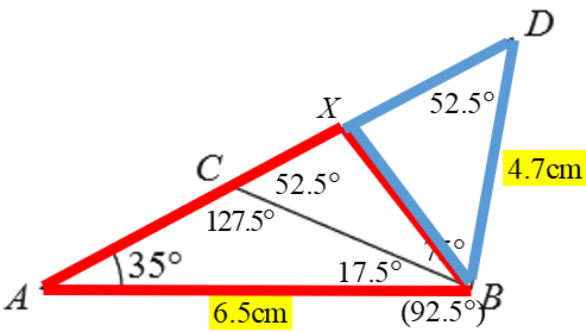
$\angle ACB = 127.5°$ $\angle ACB = 52.5°$

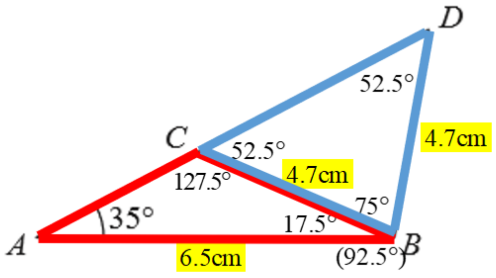


A1 awrt 8.2 Sight of awrt8.2 implies the length *AD* has been found. Ignore any labelling of lengths in their intermediate working and ignore any reference to *AC* (Accept $"..." = 8.2$). May be implied by a sum that totals awrt8.2 (eg awrt2.46+awrt5.72)

A1 awrt 24.1 ISW. Do not accept 24 or 25 (the length to the nearest metre) without seeing awrt24.1 or a calculation that totals awrt24.1 (eg $4.7+4.7+6.5+8.2\ (=24.1) \Rightarrow 24$)

Candidates who assumed $\angle ACB = 52.5°$ (acute) in (a):
Full marks can still be achieved as candidates may have restarted in (b) or not used the acute angle in their calculation which is often unclear. We are condoning any reference to $AC = 8.2$ so ignore any labelling of the lengths they are finding.

WAY 1 (b)	Uses triangle *ABD*:  **Option 1**: Cosine rule $4.7^2+6.5^2-2\times4.7\times6.5\cos(180°-35°-"52.5°")$ **Option 2**: Sine rule $\frac{(AD)}{\sin"92.5°"}=\frac{4.7}{\sin35°}$ or $=\frac{6.5}{\sin"52.5°"}$ **Option 3**: Cosine rule using $35°$ to form a quadratic Eg. $\cos35°=\frac{6.5^2+(AD)^2-4.7^2}{2\times6.5\times(AD)}$	M1 M1 M1
WAY 2 (b)	Forms two right angled triangles ABX and BDX :  Either of $6.5\cos35°$ or $4.7\cos"52.5°"$	M1

WAY 3 (b)	Uses the triangles *ABC* and *BCD*: 	
	Length *AC* **Option 1:** Cosine rule $(AC)^2 = 4.7^2 + 6.5^2 - 2\times4.7\times6.5\cos(180° - 35° - "127.5°")$	M1
	Option 2: Sine rule $\frac{(AC)}{\sin"17.5°"} = \frac{6.5}{\sin"127.5°"}$ or $\frac{4.7}{\sin 35°}$	M1
	Option 3: Cosine rule using $35°$ to form a quadratic in *AC* $\cos 35° = \frac{6.5^2 + (AC)^2 - 4.7^2}{2\times6.5\times(AC)}$	M1
	Length *CD* **Option 1:** Cosine rule $(CD)^2 = 4.7^2 + 4.7^2 - 2\times4.7\times4.7\cos(180° - 2\times"52.5°")$	
	Option 2: Sine rule $\frac{(CD)}{\sin"75°"} = \frac{4.7}{\sin"127.5°"}$	M1
	Option 3: Cosine rule using $52.5°$ to produce a quadratic in *CD* $\cos"52.5°" = \frac{4.7^2 + (CD)^2 - 4.7^2}{2\times4.7\times(CD)}$	M1
	Option 4: Using isosceles triangle property on *BCD* to find half of *CD* $4.7\cos"52.5°"$ (they do not have to double to find *CD* for M1)	M1

Question Number	Scheme	Marks
8 (a)	States $y=4$	B1 (1)
(b)	States $(16,9)$ only	B1 (1)
(c)	$k\leqslant 4, k=9$	B1, B1 (2)
(d) (i)	$a=6$	B1
(ii)	y=$\mathrm{f}(x-3)$	B1 (2)
		(6 marks)

(a)

B1 $y=4$ only. May be written on a new graph

(b)

B1 $(16,9)$ condone lack of brackets and must be the only answer (or clearly their final answer). May be written on a graph again.

(c)

B1 Sight of either one of $k\leqslant 4, k=9$ Must be in terms of *k*. Ignore any others for this mark

B1 Both of $k\leqslant 4, k=9$ ONLY Where several inequalities or equations are given only mark what appears to be their final answer for the $k=...$ and their final answer for $k\leqslant ...$ Ignore any inequalities which are within $k\leqslant 4$

SC $y\leqslant 4, y=9$ (They never write in terms of k so score B1B0)

(d)(i)

B1 $a=6$ (6 on its own is sufficient). Also allow (y=) $\mathrm{f}(x)-6$ and isw if they proceed to state $a=-6$

B1 y=$\mathrm{f}(x-3)$ Also accept $y=-\mathrm{f}(x)+6$, $y=\mathrm{f}(x+4)-9$ or even rearrangements such as $\mathrm{f}(x)=6-y$ Do not accept combinations of different transformations as the question asked for a single transformation.

Question Number	Scheme	Marks
9.	$\frac{3}{x}+5=-2x+c$ Multiplies through by x $\quad 3+5x=-2x^2+cx \Rightarrow \pm 2x^2 ....(=0)$	M1
	and writes in quadratic form $\quad \Rightarrow 2x^2+(5-c)x+3(=0)$ oe	A1
	Attempts $"b^2-4ac"=(5-c)^2-24$	M1
	Attempts $"b^2-4ac"=0 \Rightarrow (5-c)^2-24=0 \;\Rightarrow\; c=...$	dM1
	$(c=)\ 5\pm 2\sqrt{6}$ oe	A1
	Attempt at inside region	M1
	$5-2\sqrt{6}<c<5+2\sqrt{6}$ oe	A1
		(7) (7 marks)

M1 Attempts to multiply through by x and moves all terms to one side. This may be implied by later work. Condone one term not being multiplied by x for this mark but all four terms must be on one side. You do not have to see the "=0".

A1 Correct quadratic with the terms in x factorised, or correct values of a, b and c. Accept the equivalent form where terms have been collected on the other side. The "=0" is not needed for this mark and ignore the use of any inequalities.

M1 Attempts b^2-4ac using their values. You may see $"b^2...4ac"$. It is sufficient to see the values substituted in correctly for this mark and you can condone invisible brackets. They must have achieved a quadratic in x to calculate the discriminant so do not allow if eg a is their coefficient of x^{-1} but do allow this mark if they have a quadratic from incorrect working.

dM1 Attempts to solve their $b^2-4ac=0$ to find at least one of the critical values for c. Must have achieved a quadratic in c. Apply general marking principles for solving a quadratic. This is dependent on the previous method mark and may be implied by solutions correct to 1dp for their quadratic.

A1 Correct critical values $(c=)\ 5\pm\sqrt{24}$ or exact equivalent of the form $\frac{a\pm\sqrt{b}}{c}$ (the critical values may appear within inequalities)

M1 Finds inside region for their critical values. They may draw a diagram but they must proceed to an $...<c<...$ (allow use of $\leq$ for one or both inequalities for this mark and they may even be separate statements) May be in terms of x or any other variable for this mark.

A1 $5-2\sqrt{6}<c<5+2\sqrt{6}$ Must be in terms of c and must be exact

Accept others such as $5-\sqrt{24}<c<5+\sqrt{24}$, $c>5-2\sqrt{6}$ AND $c<5+2\sqrt{6}$ $c>5-2\sqrt{6}, c<5+2\sqrt{6}$
$5+2\sqrt{6}>c>5-2\sqrt{6}$, $\{c:5-2\sqrt{6}<c<5+2\sqrt{6}\}$, $\frac{10-\sqrt{96}}{2}<c<\frac{10+\sqrt{96}}{2}$

Do NOT accept $c>5-2\sqrt{6}$ OR $c<5+2\sqrt{6}$

Question Number	Scheme	Marks
10. (a)	Correct equations $\frac{1}{2}r^2\theta = 6, \quad 2r + r\theta = 10$	B1 B1
	Eliminates $r = \frac{10}{2+\theta} \Rightarrow \frac{1}{2}\left(\frac{10}{2+\theta}\right)^2 \theta = 6$	M1
	$\Rightarrow 50\theta = 6\left(4 + 4\theta + \theta^2\right) \Rightarrow \; 3\theta^2 - 13\theta + 12 = 0 \quad$ *	A1*
		(4)
(b)	$(3\theta - 4)(\theta - 3) = 0 \Rightarrow \theta = \frac{4}{3}, 3$	B1
	$\theta = \frac{4}{3}, r = 3 \qquad \theta = 3, r = 2$	M1 A1
		(3) (7 marks)

(a)

B1 One correct equation $\frac{1}{2}r^2\theta = 6$ or $2r + r\theta = 10$ (may use φ instead of θ which is fine)

B1 Two correct equations $\frac{1}{2}r^2\theta = 6$ and $2r + r\theta = 10$ ($r + r + r\theta = 10$ is acceptable)

Note you may see one of both of these equations in (b) which you can award the marks retrospectively. They may be implied from later work:

Eg. $r^2 = \frac{12}{\theta} \Rightarrow 2\sqrt{\frac{12}{\theta}} + \sqrt{\frac{12}{\theta}}\theta = 10$ implies B1B1

M1 Scored for eliminating r and reaching an equation in θ only. The initial equations must be of a similar form to the area of the sector and the perimeter but possibly with errors (look for $...r^2\theta = 6$ and $2r + ...r\theta = 10$). Condone errors when rearranging but their subsequent substitution must be correct for their rearrangement. Eg. $10 = r\theta + 2r \Rightarrow \frac{10}{\theta} = 3r$

A1* Reaches the given answer with no errors. There must be at least one intermediate line of manipulation following the elimination stage and $(\theta + 2)^2$ must be multiplied out correctly before achieving the final answer to score full marks.

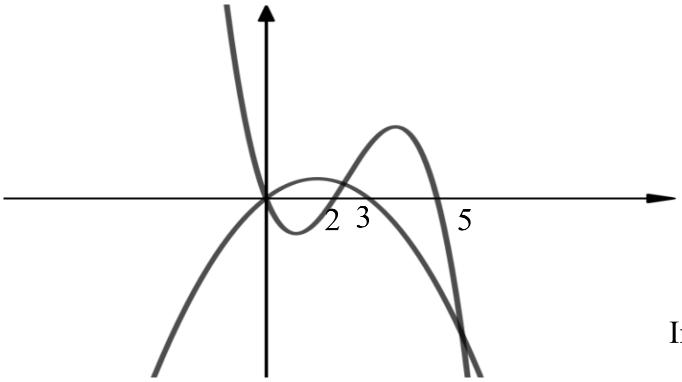
(b)

B1 $\theta = \frac{4}{3}, 3$ (They may just appear from a calculator)

M1 Substitutes one of their values of θ into one of the previous equations of the allowable form and proceeds as far as $r = \ldots$

A1 Both results $\theta = \frac{4}{3}, r = 3$ $\quad \theta = 3, r = 2$

Values must be exact. Withold the final mark if they do not rule out $r = -3 / r = -2$ from using the area of a sector equation and withhold the final mark if the corresponding values of r are incorrectly paired to the other values of θ ($\theta = \frac{4}{3}, r = 2$ and $\theta = 3, r = 3$)

Question Number	Scheme	Marks
11. (a)		
	$\cap$ shaped quadratic	B1
	Intercepts at O and 3	B1
	-ve cubic	B1
	Intercepts at O, 2 and5	B1
		(4)
(b)	Sets $x\left(x-2\right)(5-x)=x(3-x)$	M1
	$3x-x^2=-x^3+7x^2-10x\Rightarrow\pm(x^3-8x^2+13x)\ \ (=0)$ OR $\pm x\left\{\left(x-2\right)(5-x)-(3-x)\right\}\ \ (=0)$	dM1
	Proceeds to $x\left(x^2-8x+13\right)=0$ *	A1*
		(3)
(c)	Solves $x^2-8x+13=0\Rightarrow x=4\pm\sqrt{3}$	M1 A1
	Substitutes $x=\text{"}4-\sqrt{3}\text{"}$ into $y=x(3-x)$ oe	M1
	$y=\left(4-\sqrt{3}\right)\left(-1+\sqrt{3}\right)=-4+4\sqrt{3}+\sqrt{3}-3=\ldots$	M1
	$y=-7+5\sqrt{3}$	A1
		(5)
		(12 marks)

(a) **For both parts they must have a graph**

(i)

B1 $\cap$ shaped quadratic appearing anywhere on the graph. This is for the general shape so do not be concerned with any parts which appear linear.

B1 A quadratic which crosses the x-axis at O and 3. Accept a mark of 3 on the x-axis. The origin does not need to be labelled as a point of intersection.

(ii)

B1 -ve cubic appearing anywhere on the graph with a maximum and a minimum. This is for the general shape so do not be concerned with any parts which appear linear.

B1 A cubic which crosses the x-axis at O, 2 and 5. Accept 2 and 5 marked on the x-axis. The origin does not need to be labelled as a point of intersection.

It is not a requirement that the curves meet in quadrant 4. The relative heights of the maximum points are not important either. Points of intersection with the coordinate axes may be listed separately. Condone the lack of brackets but the x and y coordinates must be the correct way round. They cannot simply state eg $x = 2, x = 5$

(b)

M1 Sets the equations equal to each other

dM1 Multiples out and collects terms on one side (unsimplified). Condone errors in multiplying out and slips in collecting like terms but x must be a factor of each term. Condone invisible brackets for this mark and condone the absence of "=0"

Alternatively takes the factorised forms to one side and factors out the x term.

A1* Proceeds to $x\left(x^2 - 8x + 13\right) = 0$ with no errors including brackets. As a minimum you must see $(x-2)(5-x)$ being multiplied out, terms being collected on one side and a factor of x being taken out.

(c)

M1 Solves $x^2 - 8x + 13 = 0$ by completing the square or formula. Apply general marking principles for quadratics for this mark. Their solutions do not need to be exact for this mark.

A1 Either (or both) of $(x =)\ 4 \pm \sqrt{3}$ (oe but must be of the form $\frac{a \pm \sqrt{b}}{c}$ or simplified further)

M1 Substitutes $x = 4 - \sqrt{3}$ or their **lower** value of x into either equation to find y. This mark can be awarded if they had rounded decimal solutions to their quadratic. If they substitute both values in then this mark can still be awarded. You may need to check their y value on a calculator to imply this method mark if their x value is incorrect.

M1 Evidence of using the rules of surds to form a y coordinate that is exact and simplified. They must show evidence of working with surds before simplifying to two terms of the form $d + f\sqrt{g}$. Eg one of the bold terms from $y = (a - \sqrt{b})(c + \sqrt{b}) = ac \mathbf{+ a\sqrt{b} - c\sqrt{b} - b}$ would be sufficient. If no working shown then M0 and A0 will follow this.

A1 $(4 - \sqrt{3}, -7 + 5\sqrt{3})$ or exact equivalent. This must be the only coordinate stated as their final answer.

Note that the question stated using algebra and showing your working. If they simply state the solutions of the quadratic then they can only get M0A0M1M1A0.

If they simply state the solutions of the quadratic and show no surd work then the maximum they may be able to get is M0A0M1M0A0.

Question Number	Scheme	Marks
12.(a)	Substitutes $x=4$ in $\dfrac{dy}{dx}=3x\sqrt{x}-10x^{-\frac{1}{2}}=3\times4\times2-\dfrac{10}{2}=19$	M1A1
	Attempts $(y-(-2))="19"\times(x-4)\Rightarrow y=19x-78$	M1A1 cao
		(4)
(b)	$\mathrm{f}'(x)=3x^{\frac{3}{2}}-10x^{-\frac{1}{2}}\Rightarrow \mathrm{f}(x)=\dfrac{6}{5}x^{\frac{5}{2}}-20x^{\frac{1}{2}}+c$	M1 A1 A1
	$x=4,\mathrm{f}(x)=-2\Rightarrow$	
	$-2=38.4-40+c\Rightarrow c=...(-0.4)$	M1
	$[\mathrm{f}(x)=]\ \dfrac{6}{5}x^{\frac{5}{2}}-20x^{\frac{1}{2}}-0.4$	A1 cso
		(5) **(9 marks)**

(a)

M1 Substitutes $x=4$ into $\dfrac{dy}{dx}=3x\sqrt{x}-10x^{-\frac{1}{2}}$. Do not award this mark if they attempt to differentiate the expression first (look at the $-10x^{-\frac{1}{2}}$ for evidence of the power decreasing) but do condone an error made on the power of the first x term if they try to write it as a single power of x.

A1 Gradient = 19

M1 Attempts an equation of a tangent using their $\mathrm{f}'(4)$ and $(4,-2)$. If they attempt $(y+2)="19"(x-4)$ at least one of the brackets must be correct. If the form $y=mx+c$ is used they must proceed as far as finding *c*. They must either have shown their gradient and $(4,-2)$ substituted into $y=mx+c$ and rearrange (maybe with errors) to find *c* or if they show no working then their *c* must be correct.

A1 $y=19x-78$ cao

(b)

M1 Raises the power of any term by one $x^{-\frac{1}{2}}\rightarrow x^{\frac{1}{2}}, x^{\frac{3}{2}}\rightarrow x^{\frac{5}{2}}$ Accept eg $x^{\frac{3}{2}}\rightarrow x^{\frac{3}{2}+1}$

A1 Any term correct (may be un-simplified) with or without +*c*

A1 Both terms correct (may be un-simplified) with or without +*c*

M1 Substitutes $x=4, y=-2$ into their $\mathrm{f}(x)$ containing +*c* to obtain *c*. Condone errors in evaluating and rearranging

A1 $[\mathrm{f}(x)=]\ \dfrac{6}{5}x^{\frac{5}{2}}-20x^{\frac{1}{2}}-0.4$ or equivalent including $(y=)$ ... cso