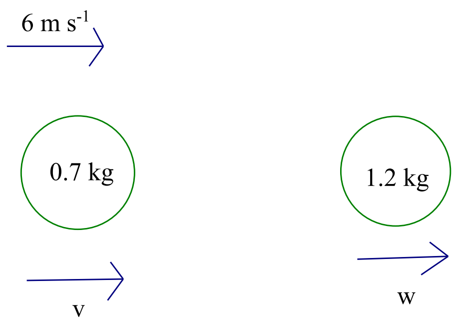
Mechanics M2 Mark scheme

Question	Scheme		Marks
1(a)	Resolving parallel to the plane	Condone trig confusion	M1
	$D = 900g\sin\theta + 800$		A1
	$\frac{900}{25}g + 800(=1152.8)$ (N)		
	Work done : Their $D \times$ distance $= 1152.8 \times 14 \times 10$	Independent. For use of 14 x 10 x their D	M1
	$= 161392 = 161$ kJ (160)	Accept 161000 (J), 160000 (J). Ignore incorrect units.	A1
			(4)
	Alternative using energy		
	Work done $= 900gd\sin\theta + 800d$	Allow with incorrect d	M1A1
	Use of $d = 14 \times 10$	Independent – allow in an incorrect expression	M1
	$= 161392 = 161$ kJ (160)		A1
			(4)
1(b)	Equation of motion	All terms required. Condone trig confusion and sign errors. Allow with $900a$	M1
	$D - 900g\sin\theta - 800 = 900 \times 0.7$	Correct unsimplified with $a = 0.7$ used Accept with their 1152.8 arising from a 2 term expression in (a)	A1
	($D - 1152.8 = 900 \times 0.7$)		
	$D = 1782.8$ (N)		
	Use of $P = Fv$ $\quad P = 14 \times \frac{\text{their } D}{1000}$	Independent Treat missing 1000 as misread, so allow for $14 \times \text{their}D$ Allow for $\frac{1000P}{14}$ (or $\frac{P}{14}$) in their equation of motion	M1
	$P = 25.0$ (25)	cao	A1
			(4)
			(8 marks)

Question	Scheme		Marks
2(a)			
	CLM: $0.7\times6=0.7\times v+1.2w$	Requires all terms & dimensionally correct	M1
	$(42=7v+12w)$	Correct unsimplified	A1
	Impact:	Used the right way round Condone sign errors	M1
	$w-v=6e$		A1
	Equation in e and v only: $42-72e=19v$	Dependent on the two previous M marks	DM1
	Use direction to form an inequality:	Independent. Applied correctly for their v	M1
	$42-72e>0 \quad \Rightarrow \quad e<\frac{7}{12}$	***Given answer***	A1
			(7)
2(b)	Impulse on Q: $I=w\times1.2$		M1
	Solve for w : $w=v+6e=\dfrac{42-72\times\frac{1}{4}}{19}+6\times\frac{1}{4}$	Accept unsimplified with e substituted. Have to be using w in part (b) $w=\frac{105}{38}=2.763.....$ seen or implied	B1
	$I=1.2\times\frac{42}{19}\times\frac{5}{4}=\frac{63}{19}(=3.32)$ (N s)	3.3 or better	A1
			(3)
	Alternative		
	Impulse on Q $=-$ impulse on P		
	$=-0.7(v-6)$	Accept negative here	M1
	$=-0.7\left(\dfrac{42-\frac{1}{4}\times72}{19}-6\right)$	Substitute for e in their v $v=\frac{24}{19}=1.263....$ seen or implied Accept negative here.	B1

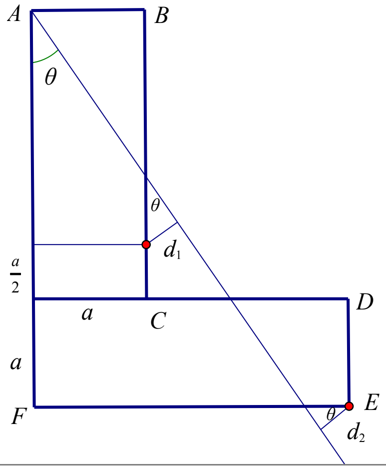
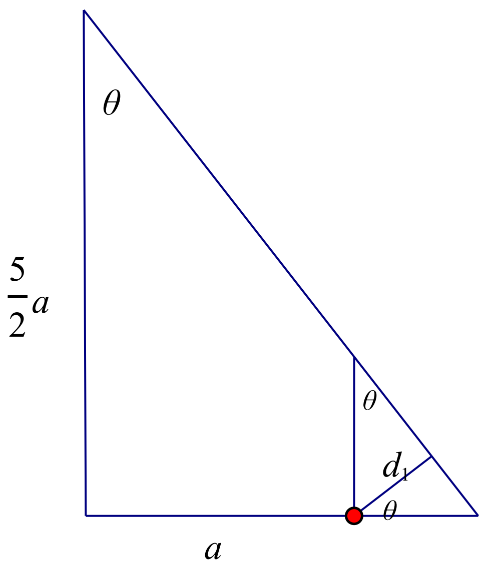
Question	Scheme		Marks
2(b) ***continued***	$=\dfrac{63}{19}$	Final answer must be positive. 3.3 or better	A1
			(3)
			(10 marks)

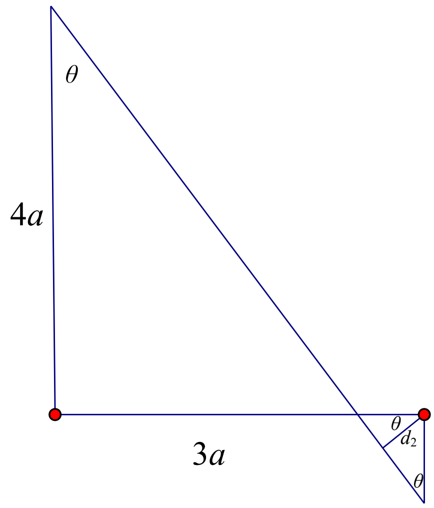
Question	Scheme		Marks
3(a)	Use $\mathbf{v}=\lambda(\mathbf{i}+\mathbf{j})$: $6T^2+6T=3T^2+24$	Form an equation in t, T or λ $\lambda^2-108\lambda+2592=0$	M1
	Solve for T $3T^2+6T-24=0$,	Simplify to quadratic in t, T or λ and solve.	M1
	$(T+4)(T-2)=0$, $T=2$	$T=2$ only	A1
	If they score M1 and then state $T=2$ allow 3/3		
	If they guess $T=2$ and show that it works then allow 3/3.		
	If all we see is $T=2$ with no equation then 0/3 for (a) but full marks are available for (b) and (c).		
			(3)
3(b)	Differentiate: $\mathbf{a}=(12t+6)\mathbf{i}+6t\mathbf{j}$	Majority of powers going down Need to be considering both components	M1
		Correct in t or T	A1
	$=30\mathbf{i}+12\mathbf{j}$ (m s^{-2})	Cao	A1
			(3)
3(c)	Integrate : $\mathbf{r}=\left(2t^3+3t^2(+A)\right)\mathbf{i}+\left(t^3+24t(+B)\right)\mathbf{j}$	Clear evidence of integration. Need to be considering both components. Do not need to see the constant(s).	M1
	-1 each error		A2
	If the integration is seen in part (a) it scores no marks at that stage, but if the result is used in part (c) then the M1A2 is available in part (c)		
	$\boldsymbol{OA}=28\mathbf{i}+56\mathbf{j}$ Use their T		
	Distance $=28\sqrt{5}=62.6$ (m)	Dependent on previous M1 Use of Pythagoras on their $\boldsymbol{OA}$	DM1
	63 or better , $\sqrt{3920}$		A1
	NB: Incorrect T can score 2/3 in (b) and 4/5 in (c)		
			(5)
			(11 marks)

Question	Scheme		Marks
4(a)	Resolve perpendicular to the plane: $R = 2g\cos\alpha$		B1
	Use $F = \mu R$: $F = \frac{1}{4}\times 2g\times\frac{4}{5}\left(=\frac{2g}{5}\right)$	with $\frac{1}{4}$ and their R (3.92)	M1
	Work done: WD $= 2.5\times F$	For their F	dM1
	$= 2.5\times\frac{2g}{5} = 9.8\,(\text{J})$	Accept g	A1
	If a candidate has found the total work done but you can see the correct terms/processes for finding the work done against friction, give B1M1DM1A0 (3/4)		
			(4)
4(b)	Change in PE : $\pm(4g\times 2.5 - 2g\times 2.5\sin\alpha)$	Requires one gaining and one losing Condone trig confusion	M1
	$=\pm(4g\times 2.5 - 2g\times 1.5)$	$\pm$ (correct unsimplified)	A1
	PE lost $= 7g = 68.6$ (J)	or 69 (J) Accept $7g$	A1
			(3)
4(c)	KE gained + WD = loss in GPE	The question requires the use of work-energy. Alternative methods score 0/4. Requires all terms but condone sign errors (must be considering both particles)	M1
	$\frac{1}{2}\times 4v^2 + \frac{1}{2}\times 2v^2 + (\text{their (a)}) = (\text{their (b)})$	Correct unsimplified. -1 each error	A2
	$3v^2 = 6g$		
	$v = \sqrt{2g} = 4.43\,(\text{m s}^{-1})$	or 4.4. Accept $\sqrt{2g}$	A1
			(4)
	Alternative		
	Equations of motion for each particle leading to $T = \frac{12g}{5} = 23.52$ followed by a W-E equation for P: $2.5T = \frac{1}{2}\times 2v^2 + 2g\times 2.5\sin\alpha + (a)$ M1A2	Equations of motion for each particle leading to $T = \frac{12g}{5} = 23.52$ followed by a W-E equation for Q: $\frac{1}{2}\times 4v^2 + 2.5T = 4g\times 2.5$	
	$v = \sqrt{2g} = 4.43\,(\text{m s}^{-1})$		A1

Question	Scheme	Marks
4(c) ***continued***	Use of $\alpha = 36.9$ gives correct answers to 3 sf	
	Use of $\alpha = 37$ gives correct answers to 2 sf and more than this is not justified, so A0 if they give 3 sf in this case.	
		(11 marks)

Question	Scheme		Marks
5	Moments about **vertical** axis (AF):	Requires all terms and dimensionally correct but condone g missing	M1
	$\frac{Mg}{2}\times\frac{1}{2}a+\frac{Mg}{2}\times1.5a+3akMg=Mg\left(1+k\right)\bar{x}$	-1 each error Accept with M and/or g not seen.	A2
	$\left(\bar{x}=\frac{1+3k}{1+k}a\right)$		
	Moments about **horizontal** axis (AB or FE):	Requires all terms and dimensionally correct but condone g missing	M1
	$\frac{Mg}{2}\times1.5a+\frac{Mg}{2}\times3.5a+4akMg=Mg\left(1+k\right)\bar{y}$	-1 each error. Accept with M and/or g not seen. Do not penalise repeated errors.	A2
	$\left(\bar{y}=\frac{2.5+4k}{1+k}a\right)$		
		Working with axes through F gives $\bar{x}=\frac{1+3k}{1+k}a$ and $\bar{y}=\frac{1.5}{1+k}a$	
	SR: A candidate working with a mixture of mass and mass ratio can score 4/6 M1A0A0M1A2		
	Use of $\tan\theta$ with their distances from AF & AB	Must be considering the whole system. Allow for inverted ratio.	M1
	$\tan\theta=\frac{M+3kM}{2.5M+4kM}\left(=\frac{4}{7}\right)$	or exact equivalent	A1
	Equate their $\tan\theta$ to $\frac{4}{7}$ and solve for k : $7M+21kM=10M+16kM$		M1
	$k=\frac{3}{5}$	cso	A1
			(10)
	Alternative for the people who start by considering only the L shape.		

Question	Scheme		Marks
5 ***continued***	$\bar{x} = a$ and $\bar{y} = \frac{5}{2}a$ or $\frac{3}{2}a$	M1 (for either) requires all terms and dimensionally correct but condone g/M missing. A1 for each correct.	M1A2
	Combine with the particle	M1 (for both) requires all terms and dimensionally correct but condone g missing. A1 for each correct.	M1A2
	See over for a more geometrical approach		
		Candidate starts by finding centre of mass at $\left(a, \frac{3}{2}a\right)$ relative to F (or equivalent), M1A2 scored	
		Use of $\tan\theta$ with their distances for finding d_1 or d_2.	M1
		Obtain length of a side in a triangle containing d_1 $\left(\frac{5}{2}a\right)\tan\theta - a\left(= \frac{3}{7}a\right)$ Correct for their centre of mass	A1

Question	Scheme		Marks
5 ***continued***		$d_1 = \left(\frac{3}{7}a\right)\cos\theta$ Correct for their centre of mass	A1
		Use of $\tan\theta$ to find second distance $3a - 4a\tan\theta = \frac{5}{7}a$	M1
		$d_2 = \frac{5}{7}a\cos\theta$	A1
	Moments about A: $Md_1 = kMd_2$		M1
	$\frac{3}{7}a\cos\theta = k \times \frac{5}{7}a\cos\theta \Rightarrow k = \frac{3}{5}$		A1
			(10)
			(10 marks)

Question	Scheme		Marks
6(a)	Taking moments about A:	Requires all terms - condone trig confusion and sign errors	M1
	$bF = 3mga\cos\theta + mg \times 2a\cos\theta$	-1 each error	A2
	$bF = 5mga\cos\theta$ $F = \dfrac{5mga}{b}\cos\theta$	***Given answer***	A1
			(4)
6(b)	Component of **R** parallel to AB: $\left(R\cos(\phi-\theta)\right)$	Requires all terms - condone trig confusion	M1
	$= 3mg\sin\theta + mg\sin\theta = 4mg\sin\theta$	Correct unsimplified	A1
	Component of **R** perpendicular to AB:	Requires all terms - condone consistent trig confusion and sign errors	M1
	$\left(R\sin(\phi-\theta)\right) + F = 4mg\cos\theta$	Correct unsimplified	A1
	Alternatives for: $M(B)$	$2aR\sin(\phi-\theta) + 3mga\cos\theta = F(2a-b)$	M1A1
	$M(C)$	$bR\sin(\phi-\theta) + (2a-b)mg\cos\theta = 3mg(b-a)\cos\theta$	
	$\left(R\sin(\phi-\theta)\right) = 4mg\cos\theta - \dfrac{5mga}{b}\cos\theta$	Correct with F substituted.	A1
	ISW for incorrect work after correct components seen		**(5)**
	Alternative		
	$X = F\sin\theta = \dfrac{5mga}{b}\cos\theta\sin\theta$	Allow with F. Requires all terms - condone trig confusion	M1
	F substituted		A1
	$Y = 4mg - F\cos\theta = 4mg - \dfrac{5mga}{b}\cos^2\theta$	Allow with F. Requires all terms - condone trig confusion and sign errors.	M1
	Correct unsimplified		A1
	Correct substituted		A1
			(5)
6(c)	Use of $R\sin(\phi-\theta) > 0$		M1
	Solve for b in terms of a: $4 > \dfrac{5a}{b}$, $(2a \geq)b > \dfrac{5}{4}a$	$2a$ not required CSO	A1
			(2)
	Special case:		
	Misread of directions in (b)	**NB:** This MR can score full marks	**(2)**

Question	Scheme		Marks
6(c) *continued*	**Alternative**		
	For $\varphi > \theta, \tan\phi > \tan\theta$		
	$\tan\varphi = \frac{Y}{X} = \frac{4 - \frac{5a}{b}\cos^2\theta}{\frac{5a}{b}\cos\theta\sin\theta} > \tan\theta$		M1
	$4 - \frac{5a}{b}\cos^2\theta > \frac{5a}{b}\sin^2\theta$		
	$4 > \frac{5a}{b}\left(\cos^2\theta + \sin^2\theta\right) \quad \Rightarrow b > \frac{5}{4}a$	cso	A1
			(2)
			(11 marks)

Question	Scheme		Marks
7(a)	Equate horizontal components of speeds:		M1
	$u\cos\theta° = 6\cos 45°\left(=3\sqrt{2}\right)\ (4.24....)$	Correct unsimplified	A1
	Use suvat for vertical speeds: $u\sin\theta° - 2g = -6\sin 45°$	Condone sign errors	M1
	$\left(u\sin\theta = 2g - 3\sqrt{2}\right)$	Correct unsimplified	A1
	Divide to find $\tan\theta$: $\tan\theta = \dfrac{2g - 6\sin 45}{6\cos 45}$	Dependent on previous 2 Ms. Follow their components.	DM1
	$\left(= \dfrac{2g - 3\sqrt{2}}{3\sqrt{2}} = 3.61..\right) \Rightarrow$ $\theta = 74.6\ (75)$	$(u = 15.93....)$	A1
			(6)
7(b)	At max height, speed $= u\cos\theta (= 3\sqrt{2}$ (m s^{-1}))		B1
	KE $= \dfrac{1}{2}\times 0.7\times\left(3\sqrt{2}\right)^2$ (J)	Correct for their v at the top, $v \neq 0$	M1
	$= 6.3$ (J)	accept awrt 6.30. CSO	A1
			(3)
7(c)	When P is moving upwards at 6 m s^{-1}	Use suvat to find first time $v = 6$	M1
	$u\sin\theta - gt = 3\sqrt{2}$		A1
	$2g - 3\sqrt{2} - gt = 3\sqrt{2}$	Solve for t	M1
	$t = \dfrac{2g - 6\sqrt{2}}{g} = 1.13....$	Sensitive to premature approximation. Allow 1.14.	A1
	$T = 2 - 1.13 = 0.87$	CAO accept awrt 0.87	A1
			(5)
	Alternative		
	$6\sin 45 = 0 + gt$	find time from top to A:	M1A1
	$T = 2t = \dfrac{\frac{12\sqrt{2}}{2}}{g} = 0.87$	Correct strategy Correct unsimplified	M1 A1 A1
			(5)

Question	Scheme		Marks
7(c) *continued*	**Alternative**		
	: $u\sin\theta = gt$ (their u, θ)	Time to top	M1
	$t = 1.567....$		A1
	$T = 2(2 - 1.567...)$		M1A1
	$= 0.87$		A1
			(5)
	Alternative		
	Vertical speed at A = - (vertical speed at B) = $= \sqrt{36 - (3\sqrt{2})^2} = 3\sqrt{2}$	Or use the 45° angle	M1 A1
	Use $v = u + at$ for $A \to B$	Correct use for their values	M1
	$-3\sqrt{2} = 3\sqrt{2} - gT$		A1
	$T = 0.87$		A1
	See below for alt 7d		**(5)**
	Alternative 7d		
	$v^2 = (3\sqrt{2})^2 + (u\sin\theta - gt)^2 \le 36$	Form expression for v^2. Inequality not needed at this stage	M1
		Correct inequality for v^2 .	A1
	$-\sqrt{18} \le u\sin\theta - gt \le \sqrt{18}$		M1
	$\dfrac{u\sin\theta - \sqrt{18}}{g} \le t \le \dfrac{u\sin\theta + \sqrt{18}}{g}$		A1
	$T = \dfrac{u\sin\theta + \sqrt{18}}{g} - \dfrac{u\sin\theta - \sqrt{18}}{g} = \dfrac{2\sqrt{18}}{g} = 0.866$		A1
			(5)
			(14 marks)