Mark Scheme (Results)

Summer 2023

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F2 (WFM02)
Paper 01

Question Number	Scheme	Marks
1(a)	$\dfrac{2}{\sqrt{r}+\sqrt{r-2}}\times\dfrac{\sqrt{r}-\sqrt{r-2}}{\sqrt{r}-\sqrt{r-2}}$	M1
	$\dfrac{2\left(\sqrt{r}-\sqrt{r-2}\right)}{r-(r-2)}=\sqrt{r}-\sqrt{r-2}$ *	A1*
		(2)
(b)	$r=2:\ \sqrt{2}-\sqrt{2-2}\ \left(=\sqrt{2}-0\right)$ ... $r=n-2:\ \sqrt{n-2}-\sqrt{n-4}$ $r=3:\ \sqrt{3}-\sqrt{3-2}\ \left(=\sqrt{3}-1\right)$ $r=n-1:\ \sqrt{n-1}-\sqrt{n-3}$ $r=4:\ \sqrt{4}-\sqrt{4-2}\ \left(=2-\sqrt{2}\right)$ $r=n:\ \sqrt{n}-\sqrt{n-2}$	M1
	$\left[\sum_{r=2}^{n}\dfrac{2}{\sqrt{r}+\sqrt{r-2}}=\right]\sqrt{n}+\sqrt{n-1}-1$	A1 A1
		(3)
(c)	$\left[\sum_{r=4}^{50}\dfrac{2}{\sqrt{r}+\sqrt{r-2}}=\right]\mathrm{f}(50)-\mathrm{f}(3)=\sqrt{50}+\sqrt{49}-1-\left(\sqrt{3}+\sqrt{2}-1\right)$	M1
	$\left(=5\sqrt{2}+7-1-\sqrt{3}-\sqrt{2}+1\right)=7+4\sqrt{2}-\sqrt{3}$	A1
		(2)
		Total 7

Notes

(a)

M1: Indicates intention to multiply either side by a correct fraction, may use $\dfrac{\sqrt{r-2}-\sqrt{r}}{\sqrt{r-2}-\sqrt{r}}$. The “2” may be missing. May work in reverse from right hand side to left, and this is fine.

Alternatively, may multiply the initial expression through by $\sqrt{r}+\sqrt{r-2}$ and use a sequence of equivalences (though accept with $\Rightarrow$ or nothing between lines).

A1: Fully correct proof. A result from rationalisation that is not the given answer must be seen. If using a sequences of equivalences there must be a minimal conclusion.

(b)

M1: Correct process of differences evidenced in their work, e.g. attempts any three of the 6 expression shown. There should be enough evidence of at least one pair of cancelling terms. Ignore any attempts at any of r = 0 or 1 if they start earlier than r = 2.

A1: Correct algebraic terms **or** correct constant term(s) extracted. Accept unsimplified expressions such as "$-\sqrt{1}-\sqrt{0}$"

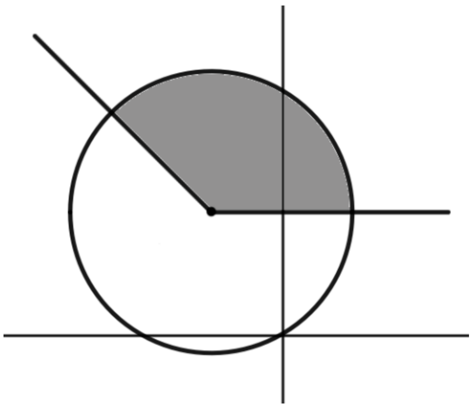
A1: Fully correct simplified expression. Must have simplified the $\sqrt{1}$ to 1

(c)

M1: Attempts f(50) – f(3) using **their answer to (b)**. Must be indication of subtraction. The '-1's may be omitted, and allow if a slip is made. They may subtract the sum of their results from r = 2 and r = 3 from (b) or by using (a) again to obtain f(3) but their answer to (b) must be used for f(50). Allow from attempts starting at r = 1 in their summations. f(50) −f(4) is M0.

A1: Correct expression or A = 7, B =4, C = –1 **following a correct answer to (b)**.

The decimal answer 10.92480344… without evidence of the M mark is 0/2

Question Number	Scheme	Marks
2(a)	$\left(\cos\frac{5\pi}{12}+\mathrm{i}\sin\frac{5\pi}{12}\right)^4=\cos\frac{20\pi}{12}+\mathrm{i}\sin\frac{20\pi}{12}$ or/and $\left(\cos\frac{\pi}{3}-\mathrm{i}\sin\frac{\pi}{3}\right)^3=\cos(-\pi)+\mathrm{i}\sin(-\pi)$	M1 / A1
	$(z_1=)\dfrac{\cos\frac{5\pi}{3}+\mathrm{i}\sin\frac{5\pi}{3}}{\cos(-\pi)+\mathrm{i}\sin(-\pi)}=\cos\left(\frac{5\pi}{3}-(-\pi)\right)+\mathrm{i}\sin\left(\frac{5\pi}{3}-(-\pi)\right)$ Alt : $(z_1=)\dfrac{\cos\frac{5\pi}{3}+\mathrm{i}\sin\frac{5\pi}{3}}{-1}=-\cos\frac{5\pi}{3}-\mathrm{i}\sin\frac{5\pi}{3}$	M1
	$=\cos\frac{8\pi}{3}+\mathrm{i}\sin\frac{8\pi}{3}\quad=\cos\frac{2\pi}{3}+\mathrm{i}\sin\frac{2\pi}{3}$ * Alt: if denominator -1 used via e.g. $\cos\left(\frac{5\pi}{3}-\pi\right)+\mathrm{i}\sin\left(-\frac{5\pi}{3}\right)$	A1*
		(4)
(b)	$\lvert z-z_1\rvert\leqslant 1\qquad 0\leqslant\arg(z-z_1)\leqslant\frac{3\pi}{4}$	
	 A circle in any position (may just see the minor arc)	M1
	A pair of **half**–lines in correct directions from their centre, one with negative gradient and one parallel to (but not) the x–axis. *If full lines are used the M marks can be implied by their shading*	M1
	Area shaded inside their circle between the two half lines from from the parallel one anticlockwise to the negative gradient line.	M1
	Fully correct shaded sector. See notes.	A1
		(4)
(c)	$\arctan\left(\dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)=\ldots\qquad\frac{\pi}{3}\quad(\text{or }60^\circ)$	M1 A1
		(2)
		Total 10

Notes

(a)

M1: One correct use of de Moivre in polar (or exponential) form. Allow use of $\mathrm{e}^{\mathrm{i}\theta}$ for $\cos\theta+\mathrm{i}\sin\theta$ throughout until the final A.

A1: Both correct (unsimplified) in polar form. Accept for both marks use of $\left(\cos\frac{\pi}{3}-\mathrm{i}\sin\frac{\pi}{3}\right)^3\to\cos\pi-\mathrm{i}\sin\pi$ but denominator directly to $\cos\pi+\mathrm{i}\sin\pi$ with no evidence of dealing with the negative between terms is A0.

M1: A correct method shown for the division of the two complex numbers. No need to simplify for this mark, a difference of argument in the trig terms is fine. Look for the subtraction of the arguments. Sight of the $\frac{8\pi}{3}$ can imply the mark if no incorrect work is seen. Note it is M0 if the arguments are added (so

$\cos\left(\frac{5\pi}{3}-\pi\right)+\mathrm{i}\sin\left(\frac{5\pi}{3}-\pi\right)$ is M0 unless the denominator has clearly been written as $\cos\pi+\mathrm{i}\sin\pi$ first.)

May write the denominator as -1 first, which is correct, score for $-\cos\frac{5\pi}{3}-\mathrm{i}\sin\frac{5\pi}{3}$.

Accept methods that convert both numbers into exact Cartesian form, apply a correct process to realise the denominator and convert back to polar form.

Do not allow going straight to $\cos\frac{2\pi}{3}+\mathrm{i}\,\sin\frac{2\pi}{3}$ for this mark as it is a given answer. Justification is required and an incorrect method is M0.

A1cso: Obtains $\cos\frac{2\pi}{3}+\mathrm{i}\,\sin\frac{2\pi}{3}$ with suitable intermediate step shown and no errors in the work. Must have scored the preceding 3 marks. This will usually be via $\cos\frac{8\pi}{3}+\mathrm{i}\,\sin\frac{8\pi}{3}$ unless equivalent suitable working has been shown to justify the correct modulus (e.g. proceeding via -1 in the denominator).

(b)

Do not be concerned about lines being dashed or dotted in this part. Accept either.

M1: A circle in any position. You may see just the minor arc of a circle, which is acceptable as long as it is clearly an arc of a circle (e.g. implied by their shading) and not just a angle demarcation.

M1: Draws or indicates a pair of half–lines from their centre (which need not be in quadrant 2) with one with negative gradient proceeding up and left and one parallel to the *x*–axis proceeding right but not the *x*-axis itself. If full lines are used this can be implied by the shading of the correct region between lines.

M1: Shades the area between their pair of rays (the second ray may have positive gradient for this mark) and inside their circle, anticlockwise from the horizontal line. The half line need not stem from the centre of the circle for this mark, and accept the *x*-axis as the horizontal line for this mark.

A1: Fully correct shaded sector. Must

- be in quadrants 1 and 2
- have approximately correct gradients for the half-lines (-1 and 0)
- have circle with centre in quadrant 2 and (if whole circle shown) passing roughly through the origin. If only an arc is shown apply bod as long as the position is reasonable.

Ignore any centre coordinates and axes intersections of any major sector. If extra regions are shaded they must make clear which their region *R* is in order to access the mark.

(c)

M1: Identifies the correct point for their sector, which must be from a circle with centre in quadrant 1 or 2 (above the real axis) and ray parallel to the real axis, and attempts the relevant angle. Look for selecting the point (c, d) at the "3 o'clock" position having identified a suitable sector and proceeding to find a relevant positive angle (e.g., allow if $\pi-\theta$ is found) (accept $\arctan\pm\frac{c}{d}$ or $\arctan\pm\frac{d}{c}$ as an attempt at the angle).

Their point must be in quadrants 1 or 2. If no shading was shown in (b) allow for attempts at the relevant point of horizontal ray and circle intersection. Could use other trig.

A1 : Either correct value. Mark final answer.

Question Number	Scheme	Marks
3	$\dfrac{x+2}{x+4} \leqslant \dfrac{x}{k(x-1)}$	
(a)	$\left[\times k\left(x+4\right)^2(x-1)^2 \Rightarrow\right] \quad k(x+4)(x-1)^2(x+2) \leqslant x(x+4)^2(x-1)$ $\Rightarrow k(x+4)(x-1)^2(x+2) - x(x+4)^2(x-1)\ \{\leqslant 0\}$ **OR** $\dfrac{x+2}{x+4} - \dfrac{x}{k(x-1)}\ \{\leqslant 0\} \Rightarrow \dfrac{k(x+2)(x-1)-x(x+4)}{k(x+4)(x-1)}\ \{\leqslant 0\}$	M1
	$\left(x+4\right)\left(x-1\right)\left[kx^2+kx-2k-x^2-4x\right]\{\leqslant 0\}$	**d**M1
	$\left(x+4\right)\left(x-1\right)\left[\left(k-1\right)x^2+\left(k-4\right)x-2k\right] \leqslant 0$ (oe in corerct form)	A1cso
		(3)
(b)	$k = 3 \Rightarrow 2x^2 - x - 6 = 0$ $[(2x + 3)(x - 2) = 0] \qquad \Rightarrow x = -\frac{3}{2}, 2$	M1
	$-4 < x \leqslant -\frac{3}{2} \qquad 1 < x \leqslant 2$	**d**M1 A1 A1
		(4)
		Total 7

Notes

(a)

M1: Attempts to multiply both sides by e.g. $k\left(x+4\right)^2(x-1)^2$ or $k^2\left(x+4\right)^2(x-1)^2$ and bring terms together OR bring terms together and attempt a common denominator. There may be slips but the intention must be clear - allow if e.g. one term is missing.

dM1: Factorises out $\left(x+4\right)\left(x-1\right)$ and/or multiplies by $\left(x+4\right)^2(x-1)^2$ (which could be implied) and expands remaining terms to get unsimplified 3 term quadratic (terms need not be collected but it must be equivalent to $px^2 + qx + r$ where at least one of p, q and r is a function of k and none are zero). **Dependent on previous M mark.**

A1cso: Correct statement or $p = k - 1$, $q = k - 4$, $r = -2k$ with no algebraic errors (but condone e.g. recovery of missing brackets if work is clear). It is A0 if any incorrect statement is seen e.g., use of an incorrect inequality sign. Accept with any positive multiples of the coefficients (including k or $1/k$ etc).

(b)

M1: Uses $k = 3$ **in their quadratic from (a)** (implied by 2 out of three terms correct) and solves to obtain a value. Apply usual rules (may be by calculator - may need to check). If no working, obtains one consistent solution which must be real. **Alternatively** restarts from $\dfrac{x+2}{x+4} \leqslant \dfrac{x}{3(x-1)}$ and uses a valid method to obtain and solve a quadratic (see above). Marks for (a) cannot be scored in (b).

dM1: With critical values -4, 1 and two different real solutions from their quadratic ($\neq -4$ or 1), chooses the region between the two smaller values and the region between the two larger values and no other region. Could be non-strict inequalities. **Dependent on previous M mark.**

A1: Both regions correct but condone e.g incorrect strict non-strict inequalities.

A1: Both completely correct regions. Allow equivalent notations. Ignore any word between the regions but withhold this last mark if $\cap$ used with set notation.

Question Number	Scheme	Marks
4	$\dfrac{d^2y}{dx^2}-8\dfrac{dy}{dx}+16y=48x^2-34$	
(a)	$(\text{AE:})\quad m^2-8m+16=0$ $\left[\Rightarrow (m-4)^2=0\right]$ $\Rightarrow m=4$	M1
	$(\text{CF: } y=)\quad (A+Bx)e^{4x}$	A1
	$(\text{PI: } y=)\quad \lambda x^2+\mu x+\nu$	B1
	$y'=2\lambda x+\mu \quad y''=2\lambda$ $2\lambda-8(2\lambda x+\mu)+16(\lambda x^2+\mu x+\nu)=48x^2-34$ $16\lambda x^2=48x^2\ \ (-16\lambda+16\mu)x=0\quad 2\lambda-8\mu+16\nu=-34$ $\lambda=3\quad \mu=3\quad \nu=-1$	M1
	$y=\text{"}(A+Bx)e^{4x}\text{"}+3x^2+3x-1$	A1ft
		(5)
(b)	$(0,4)\Rightarrow 4=A-1\quad (A=5)$	M1
	$\dfrac{dy}{dx}=4(A+Bx)e^{4x}+Be^{4x}+6x+3$	M1
	$21=4A+B+3\Rightarrow B=-2\ \ A=5$ $[y]=(5-2x)e^{4x}+3x^2+3x-1$	M1A1
		(4)
(c)	$(x=-2\Rightarrow y=)\ 9e^{-8}+5$	M1 A1
		(2)
		Total 11

Notes

(a)

M1: Forms the auxiliary equation (condone one slip/copying error) and solves 3TQ. Usual rules. One consistent solution if no working (could be complex). Implied by a correct CF if no incorrect working shown.

A1: Correct complementary function y = ... not required. May only be seen in final answer.

B1: Correct form for particular integral y = ... not required.

M1: Correct method to obtain value for constants (or constant - but PI must be a quadratic - but could have 1 or 2 terms) - so differentiates twice (powers reduced) and substitutes, equates terms and solves equations. Allow if there are minor slips in the process if the holistic approach is correct.

A1ft: A correct general solution following through on their CF only - the PI must be correct. Must have " y = " e.g., not " GS = "

(b)

M1: Uses (0, 4) in their answer to (a) and forms an equation in one or both of their constants.

M1: Differentiates their GS, which must contain a term "Bxe^{kx}", to obtain an expression of the correct form for their GS - product rule must be used, powers reduced, but may have errors in coefficients.

M1: Substitutes $x=0,\ \dfrac{dy}{dx}=21$ into their equation where their derivative is a changed function and finds values for their B (and their A if not found already).

A1: Any correct equation Condone e.g., " PS =... "

(c)

M1: Substitutes x = –2 into their particular solution and obtains an expression of the right form or non-zero values for p, q and r, which need not be integers for the M but should be gathered terms. Implied by a correct answer as long as it fits their answer to (b) if no method shown.

A1: Correct expression or values

Question Number	Scheme	Marks
5(a)	$w=\dfrac{z+1}{z-3}\Rightarrow wz-3w=z+1\Rightarrow wz-z=3w+1\Rightarrow z=\dfrac{3w+1}{w-1}$	M1 A1
	$(z=)\dfrac{(3u+1)+3\mathrm{i}v}{(u-1)+\mathrm{i}v}\times\dfrac{(u-1)-\mathrm{i}v}{(u-1)-\mathrm{i}v}=...$	**d**M1
	$\Rightarrow x=\dfrac{3u^2-2u+3v^2-1}{(u-1)^2+v^2}\quad y=\dfrac{-4v}{(u-1)^2+v^2}$ $(y=4x\Rightarrow)\quad -4v=12u^2-8u+12v^2-4$	**dd**M1
	$\Rightarrow 3u^2+3v^2-2u+v-1=0*$	A1cso*
		(5)
(b)	$u^2-\dfrac{2}{3}u+v^2+\dfrac{1}{3}v-\dfrac{1}{3}=0\Rightarrow\left(u-\dfrac{1}{3}\right)^2-\dfrac{1}{9}+\left(v+\dfrac{1}{6}\right)^2-\dfrac{1}{36}-\dfrac{1}{3}=0$ $\Rightarrow\left(u-\dfrac{1}{3}\right)^2+\left(v+\dfrac{1}{6}\right)^2=\dfrac{17}{36}$ $\Rightarrow$ centre: $\left(\dfrac{1}{3},-\dfrac{1}{6}\right)$ $\left[\text{allow } x=\dfrac{1}{3}, y=-\dfrac{1}{6}\right]$ radius: $\sqrt{\dfrac{17}{36}}$ or $\dfrac{\sqrt{17}}{6}$	B1 B1
		(2)
		Total 7

Notes

(a)

M1: Completes an attempt to make z the subject.

A1: Correct expression.

dM1: **Dependent on previous M mark.** Replaces w with u + iv and indicates an appropriate attempt to rationalise the denominator. Accept if x + iy is used instead for this mark.

ddM1: **Dependent on both previous M marks.** Equates real and imaginary parts to obtain expressions for x and y in u and/or v only and uses $y = 4x$ to form an equation in u and v only. Condone intermediate incorrect statements that are recovered for this mark, e.g. condone, missing or incorrect (real) denominator or slips with initial inclusion of i in the y if recovered. E.g. any of

$x=3u^2-2u+3v^2-1,\quad x=\dfrac{3u^2-2u+3v^2-1}{(u-1)^2-v^2},\quad y=\dfrac{-4v\mathrm{i}}{(u-1)^2+v^2}$ or similar may be recovered for the M's

A1cso*:obtains the equation (must be equal to 0) or states k = –1 following correct work. Do not allow recovery from incorrect statements as shown above for the final mark - all may lead to the correct answer but are A0. But if there are no incorrect statements e.g. cancelling of denominators may be implied for the A. Watch out for an incorrect $z=\dfrac{3w+1}{1-w}$ which also leads to a correct expression but loses both A marks.

Note: Alternatively methods are possible. Two are shown on the next page, but are not exhaustive.

(b)

Allow both marks from cases where $k=-1$ is guessed or achieved from fortuitous work.

B1: Correct centre **or** radius. Accept unsimplified fractions for this mark.

B1: Correct centre **and** radius - fractions must be simplified.

5(a) Alt	$z=x+\mathrm{i}y,\ y=4x \Rightarrow w=\dfrac{z+1}{z-3}=\dfrac{x+4x\mathrm{i}+1}{x+4x\mathrm{i}-3}=\dfrac{(x+1)+4\mathrm{i}x}{(x-3)+4\mathrm{i}x}\times\dfrac{(x-3)-4\mathrm{i}x}{(x-3)-4\mathrm{i}x}=\ldots$	M1
	$(u+\mathrm{i}v=)\dfrac{(17x^2-2x-3)-16\mathrm{i}x}{(x-3)^2+16x^2}$ oe	**A1**
	$\Rightarrow u=\dfrac{17x^2-2x-3}{(x-3)^2+16x^2} \quad v=\dfrac{-16x}{(x-3)^2+16x^2}$ $\Rightarrow 3u^2+3v^2-2u+v=\ldots$	**d**M1
	$\Rightarrow\ldots\Rightarrow 3u^2+3v^2-2u+v-1=0*$ or states $k=-1$	**dd**M1A1cso*
		(5)
5(a) Alt II	$z=x+\mathrm{i}y,\ y=4x \Rightarrow w=\dfrac{z+1}{z-3}=\dfrac{x+4x\mathrm{i}+1}{x+4x\mathrm{i}-3}$	
	$\Rightarrow(u+\mathrm{i}v)\big((x-3)+4\mathrm{i}x\big)=(x+1)+4\mathrm{i}x\Rightarrow\ldots+\ldots\mathrm{i}=\ldots$	M1
	$u(x-3)-4vx+\mathrm{i}\big(v(x-3)+4ux\big)=(x+1)+4\mathrm{i}x$	A1
	$\Rightarrow u(x-3)-4vx=(x+1) \quad v(x-3)+4ux=4x$ $\Rightarrow x=\dfrac{3u+1}{u-4v-1} \quad x=\dfrac{3v}{v+4u-4}\Rightarrow\dfrac{3u+1}{u-4v-1}=\dfrac{3v}{v+4u-4}$	**d**M1
	$(3u+1)(v+4u-4)=3v(u-4v-1)\Rightarrow 12u^2+12v^2-8u+4v-4=0$ $\Rightarrow 3u^2+3v^2-2u+v-1=0*$	**dd**M1A1cso*
		(5)

Notes

(a) Alt I

M1: Uses $y = 4x$ and $z = x + \mathrm{i}y$ in the expression for w and multiplies through by complex conjugate of denominator

A1: Correct expression in terms of x (oe)

dM1: Replaces w with $u + \mathrm{i}v$ and equates real and imaginary parts and substitutes into the LHS of given equation. **Dependent on previous M mark.**

ddM1: Proceeds simplify the expression putting over common denominator.They are unlikely to make much progress at this stage in this method. **Dependent on both previous M marks.**

A1cso*: Correctly shows the expression equates to 0 and gives minimal conclusion.

(a) Alt II

M1: Uses $y = 4x$ and $z = x + \mathrm{i}y$ in the expression for w and equates to $u + \mathrm{i}v$, cross multiples and expands.

A1: Correct expression with the i^2 terms simplified.

dM1: Equates real and imaginary parts and proceeds to eliminate x solving simultaneously. **Dependent on previous M mark.**

ddM1: Proceeds to simplify by cross multiplying and expanding to at least eliminate uv terms. **Dependent on both previous M marks.**

A1cso*: Correctly shows the expression required holds. As main scheme.

Question Number	Scheme	Marks
6(a)	$y=\sec x \Rightarrow \dfrac{dy}{dx}=\sec x\tan x$ $\dfrac{d^2y}{dx^2}=(\sec x\tan x)\tan x+\sec x(\sec^2 x)\ \left(=\sec x\tan^2 x+\sec^3 x\right)$	M1
	$\dfrac{d^2y}{dx^2}=\sec x(\sec^2 x-1)+\sec^3 x=2\sec^3 x-\sec x$ $\Rightarrow \dfrac{d^3y}{dx^3}=6\sec^2 x(\sec x\tan x)-\sec x\tan x$ **or** $\dfrac{d^3y}{dx^3}=2\sec x\tan x\sec^2 x+\tan^2 x\sec x\tan x+3\sec^2 x\sec x\tan x$	**d**M1
	$\left(\dfrac{d^3y}{dx^3}=5\sec^3 x\tan x+\sec x\tan x(\sec^2 x-1)=6\sec^3 x\tan x-\sec x\tan x\right)$ $\Rightarrow \dfrac{d^3y}{dx^3}=\sec x\tan x(6\sec^2 x-1)$	**dd**M1 A1
		(4)
(b)	$\sec\left(\dfrac{\pi}{3}\right)=2,\ \tan\left(\dfrac{\pi}{3}\right)=\sqrt{3}\Rightarrow f\left(\dfrac{\pi}{3}\right)=2,\ f'\left(\dfrac{\pi}{3}\right)=2\sqrt{3},\ f''\left(\dfrac{\pi}{3}\right)=14,\ f'''\left(\dfrac{\pi}{3}\right)=46\sqrt{3}$	M1
	$f(x)=f(a)+(x-a)f'(a)+\dfrac{(x-a)^2}{2!}f''(a)+\dfrac{(x-a)^3}{3!}f'''(a)+...$ $\Rightarrow 2+2\sqrt{3}\left(x-\dfrac{\pi}{3}\right)+7\left(x-\dfrac{\pi}{3}\right)^2+\dfrac{23\sqrt{3}}{3}\left(x-\dfrac{\pi}{3}\right)^3$	**d**M1 A1
		(3)
(c)	$\left(\sec\dfrac{7\pi}{24}\approx\right)=2+2\sqrt{3}\left(-\dfrac{\pi}{24}\right)+7\left(-\dfrac{\pi}{24}\right)^2+\dfrac{23\sqrt{3}}{3}\left(-\dfrac{\pi}{24}\right)^3=...$	M1
	$\sec\dfrac{7\pi}{24}\approx 1.636709263...\ \approx 1.637$	A1
		(2)
		Total 9

Notes

(a)

M1: Differentiates twice and obtains expression for second derivative allowing for sign errors only

dM1: **Dependent on previous M mark.** Differentiates again and obtains expression for third derivative allowing for sign errors only (which could come from incorrect signs in trig identities)

ddM1: **Dependent on both previous M marks.** Obtains an answer of the correct form or values for p and q. Allow slips from their third derivative but any trig identities used can only have errors in sign.

A1: Fully correct expression or values for p and q

Note: If they decide not to work in sec x and tan x the main scheme still applies – see Alt next page.

Allow "meet in the middle" approaches that determine the values of p and q

(b)

M1: Uses sec x and their three derivatives to attempt to find values for $f\left(\dfrac{\pi}{3}\right),\ f'\left(\dfrac{\pi}{3}\right),\ f''\left(\dfrac{\pi}{3}\right)$ and $f'''\left(\dfrac{\pi}{3}\right)$ evidenced by values for each with at least 2 correct if no method seen. May be recovered by substitution of values into formula if not all listed.

dM1: **Dependent on previous M mark.** Uses a correct Taylor series with all four of their values. Series must be correct for their values but allow slips if a correct formula is quoted.

A1: Correct series with coefficients in a simplest form e.g., allow $\sqrt{12}$ for $2\sqrt{3}$, $\frac{23}{\sqrt{3}}$ for $\frac{23}{3}\sqrt{3}$ Condone absence of sec x =... or y =… Must have had a correct third derivative in (a), though need not have been simplified to the form required (and allow if there was a correct third derivative which was incorrectly simplified if the correct answer is found).

(c)

M1: Shows evidence of substitution of $\frac{7\pi}{24}$ into their series of the right form (powers of $\left(x-\frac{\pi}{3}\right)$). If only a value is given score M0 unless it is the correct 4 s.f. value for their series (1.636709263... allowing awrt 1.637 if (b) is correct). You may need to check the answer.

A1: 1.637 only (not awrt) from a correct series.

Note that sec $\frac{7\pi}{24}$ = 1.642679632...

6(a) Alt	$y=\sec x \Rightarrow y=(\cos x)^{-1} \Rightarrow \frac{dy}{dx}=-(\cos x)^{-2}(-\sin x)=\frac{\sin x}{\cos^2 x}$ $\frac{d^2y}{dx^2}=\frac{\cos^2 x\cos x-2\sin x\cos x(-\sin x)}{\cos^4 x}$	M1
	$\frac{d^2y}{dx^2}=\frac{\cos^3 x+2\cos x(1-\cos^2 x)}{\cos^4 x}=\frac{2\cos x-\cos^3 x}{\cos^4 x}=2(\cos x)^{-3}-(\cos x)^{-1}$ $\Rightarrow \frac{d^3y}{dx^3}=-6(\cos x)^{-4}(-\sin x)+(\cos x)^{-2}(-\sin x)$	**d**M1
	$\frac{d^3y}{dx^3}=\frac{6\sin x}{\cos^4 x}-\frac{\sin x}{\cos^2 x}=\frac{\sin x}{\cos^2 x}\left(\frac{6}{\cos^2 x}-1\right)=\sec x\tan x(6\sec^2 x-1)$	**dd**M1 A1
		(4)

Notes
Mark as per main scheme allowing sign slips only - forms correct at each stage.

Question Number	Scheme	Marks
7(a)	$z = y^{-2} \Rightarrow \frac{dz}{dy} = -2y^{-3}$ or $1 = -2y^{-3}\frac{dy}{dz}$ $y^2 = z^{-1} \Rightarrow 2y\frac{dy}{dz} = -z^{-2}$ $y = z^{-\frac{1}{2}} \Rightarrow \frac{dy}{dz} = -\frac{1}{2}z^{-\frac{3}{2}}$	B1
	$\frac{dy}{dx} = \frac{dz}{dx}\cdot\frac{dy}{dz}$ (oe) $\Rightarrow$ e.g., $\frac{dy}{dx} = -\frac{1}{2}y^3\frac{dz}{dx}$, $\frac{dy}{dx} = -\frac{1}{2}z^{-\frac{3}{2}}\frac{dz}{dx}$	M1 A1
	$x\frac{dy}{dx} + y + 4x^2y^3\ln x = 0 \Rightarrow$ e.g., $-\frac{1}{2}xy^3\frac{dz}{dx} + y + 4x^2y^3\ln x = 0$ $-\frac{1}{2}xz^{-\frac{3}{2}}\frac{dz}{dx} + z^{-\frac{1}{2}} + 4x^2z^{-\frac{3}{2}}\ln x = 0$	**d**M1
	$\frac{dz}{dx} - \frac{2}{xy^2} - 8x\ln x = 0$ $\frac{dz}{dx} - \frac{2z}{x} - 8x\ln x = 0$ $\Rightarrow \frac{dz}{dx} - \frac{2z}{x} = 8x\ln x$ *	A1*
		(5)
(b)	$(\text{IF} =)\ e^{\int -\frac{2}{x}(dx)}$	M1
	$= e^{-2\ln x}\ \left(= x^{-2}\right)$	A1
	$x^{-2}z = \int x^{-2}(8x\ln x)[dx]$	M1
	E.g. Parts: $\int x^{-1}\ln x\,dx$: $\left(u = \ln x,\ u' = x^{-1}, v' = x^{-1}, v = \ln x\right)$ $\Rightarrow I = (\ln x)^2 - kI \Rightarrow I = p(\ln x)^2$ Or substitution: $t = \ln x,\ \frac{dt}{dx} = x^{-1} \Rightarrow I = k\int x^{-1}\ln x \cdot x\,dt = k\int t\,dt = pt^2$	M1
	$\int x^{-1}\ln x\,dx = \frac{1}{2}(\ln x)^2\ [+c]$	A1
	$x^{-2}z = 4(\ln x)^2 + k \Rightarrow z = 4x^2(\ln x)^2 + kx^2$ $\Rightarrow y^2 = \frac{1}{4x^2(\ln x)^2 + kx^2}$ oe	A1
		(6)
		Total 11

Notes

(a)

B1: Any correct equation following differentiation of the given substitution. Could be implied.

M1: Uses a correct chain rule to obtain an equation linking $\frac{dy}{dx}$ $\left(\text{or } \frac{dx}{dy}\right)$ **and** $\frac{dz}{dx}$ $\left(\text{or } \frac{dx}{dz}\right)$

A1: Any correct equation

Note that the first three marks could be scored by a first step of, e.g.:

$$\frac{dz}{dx} = -2y^{-3}\frac{dy}{dx} \quad\text{or}\quad 2y\frac{dy}{dx} = -z^{-2}\frac{dz}{dx} \quad\text{or}\quad \frac{dy}{dx} = -\frac{1}{2}z^{-\frac{3}{2}}\frac{dz}{dx}$$

dM1: **Dependent on previous M mark.** Substitutes their $\frac{dy}{dx}$ into differential equation (I). Need not replace fully for this mark.

A1*: Fully correct proof with no errors. There must be an intermediate line of working between the line where they substitute into DE(I) and the given answer.

Note: Use Review for any incorrect but potentially creditworthy attempts that "meet in the middle" or use (II) $\Rightarrow$ (I)

(b)

M1: Correct form for integrating factor. Allow $e^{\int \frac{k}{x}}$ and missing dx

A1: Correct IF in any form

M1: Obtains $\text{IF}\cdot z = \int \text{IF}\cdot(8x\ln x)[\mathrm{d}x]$ Correct for their IF and z not y

M1: Must have achieved an integral of form $k\int x^{-1}(\ln x)[\mathrm{d}x]$. Correct form following integration of $x^{-1}\ln x$ (constant not required):

$$\int x^{-1}\ln x\,\mathrm{d}x \Rightarrow p(\ln x)^2\ [+c]$$

The work in the scheme above is sufficient but you do not need to scrutinise the details, accept work that leads to the correct form.. Condone " $\ln x^2$ " for this mark

A1: Correct integration (constant not required) $\ln x^2$ is A0 unless recovered – must see $(\ln x)^2$ but allow $\ln^2 x$

A1: Any correct equation in $y^2 = ...$ form e.g., $y^2 = \left[x^2\left(4\ln^2 x + k\right)\right]^{-1}$ Constant may appear as a multiple, e.g., 8c

Question Number	Scheme	Marks
8	$r = 6(1+\cos\theta) \qquad 0 \leqslant \theta \leqslant \pi$	
(a)	$\theta = 0,\ r = 6(1+\cos 0) \Rightarrow 12$ or $(12, 0)$	B1
		(1)
(b)	$\dfrac{\mathrm{d}}{\mathrm{d}\theta}(r\sin\theta)$: $= 6\sin\theta(1+\cos\theta) \Rightarrow$ $6\sin\theta(-\sin\theta)+6\cos\theta(1+\cos\theta)$ $= 6\sin\theta + 6\sin\theta\cos\theta \Rightarrow$ $6\cos\theta + 6(\cos^2\theta - \sin^2\theta)$ $= 6\sin\theta + 3\sin 2\theta \Rightarrow$ $6\cos\theta + 6\cos 2\theta$	M1
	$\Rightarrow 2\cos^2\theta + \cos\theta - 1 = 0$ $[(2\cos\theta - 1)(\cos\theta + 1) = 0 \Rightarrow]\quad \cos\theta = \dfrac{1}{2}\quad [\text{or } -1]$	**d**M1
	$\theta = \dfrac{\pi}{3} \qquad r = 6\left(1+\cos\dfrac{\pi}{3}\right) = 9 \qquad$ or $\left(9, \dfrac{\pi}{3}\right)$	A1 A1
		(4)
8(c)	$[\tfrac{1}{2}]\int r^2\,\mathrm{d}\theta = [\tfrac{1}{2}]\int 36(1+\cos\theta)^2\ [\mathrm{d}\theta]$	M1
	$\int r^2\,\mathrm{d}\theta = 36\int(1+2\cos\theta+\cos^2\theta)[\mathrm{d}\theta] = 36\int\left(\tfrac{3}{2}+2\cos\theta+\tfrac{1}{2}\cos 2\theta\right)[\mathrm{d}\theta]$	M1
	$\int(a + b\cos\theta + c\cos 2\theta)[\mathrm{d}\theta] = a\theta \pm b\sin\theta \pm \tfrac{c}{2}\sin 2\theta$	M1
	$18\left[\tfrac{3}{2}\theta + 2\sin\theta + \tfrac{1}{4}\sin 2\theta\right]_{...}^{...}$	A1
	$18\left[\tfrac{3}{2}\theta + 2\sin\theta + \tfrac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{3}} = 18\left(\dfrac{\pi}{2}+\sqrt{3}+\dfrac{\sqrt{3}}{8}\right)\ [-0] \qquad \left[= 9\pi + \dfrac{81}{4}\sqrt{3}\right]$	**d**M1
	E.g. $OB = 6\left(1+\cos\dfrac{'\pi'}{3}\right) \Rightarrow BQ = 6\left(1+\cos\dfrac{'\pi'}{3}\right)\sin\dfrac{'\pi'}{3} \quad \left[= \dfrac{9\sqrt{3}}{2}\right]$	M1
	$BP = 12 - 9\cos\dfrac{\pi}{3} = 12 - \dfrac{9}{2} = \dfrac{15}{2}$ $\Rightarrow$ area $OBPA = \dfrac{1}{2}\left(12+\dfrac{15}{2}\right)\left(\dfrac{9\sqrt{3}}{2}\right)$ or $\dfrac{1}{2}\times\dfrac{9}{2}\times\dfrac{9\sqrt{3}}{2}+\dfrac{9\sqrt{3}}{2}\times\dfrac{15}{2} \quad \left[= \dfrac{351}{8}\sqrt{3}\right]$	A1
	area of region $R = \dfrac{351}{8}\sqrt{3} - \dfrac{81}{4}\sqrt{3} - 9\pi = \dfrac{189}{8}\sqrt{3} - 9\pi$	A1
		(8)
		Total 13

Notes

(a)

B0: Correct values for θ and r or correct coordinates. Condone (0, 12).

(b)

M1: Differentiates $r\sin\theta$. Allow sign errors only. The "6" may be missing

dM1:**Dependent on previous M mark.** Uses correct identity/identities to reach a 3TQ in $\cos\theta$ and solves.

Apply usual rules or by calculator must obtain at least one real consistent solution where $-1 \leqslant \cos\theta \leqslant 1$.

Accept $\theta = \dfrac{\pi}{3}$ following a correct derivative set equal to zero to imply the M if no incorrect working shown (by calculator or by inspection).

A1: Either r or θ correct.

A1: Both coordinates correct. Only accept $\dfrac{\pi}{3}$ and 9. Withhold last mark if additional answers offered and not rejected, but isw after correct answers seen if only a miscopy is made.

(c)

M1: Attempts $\int r^2 \, \mathrm{d}\theta$ which may include the $\frac{1}{2}$ for the area formula. The multiple 36 may be missing or wrong. Condone poor squaring - allow this mark if there are only two terms.

M1: Uses $\cos^2\theta = \pm\frac{1}{2}\cos 2\theta \pm \frac{1}{2}$ and obtains an integrand of the form $a + b\cos\theta + c\cos 2\theta$ (constants may be uncollected)

M1: Integrates and obtains a form $a\theta \pm b\sin\theta \pm \frac{c}{2}\sin 2\theta$ (sign errors on trig terms only). May be two terms in θ

A1: Fully correct expression for the area of the sector after integration. Ignore limits but $\frac{1}{2}$ must have been used or appear later.

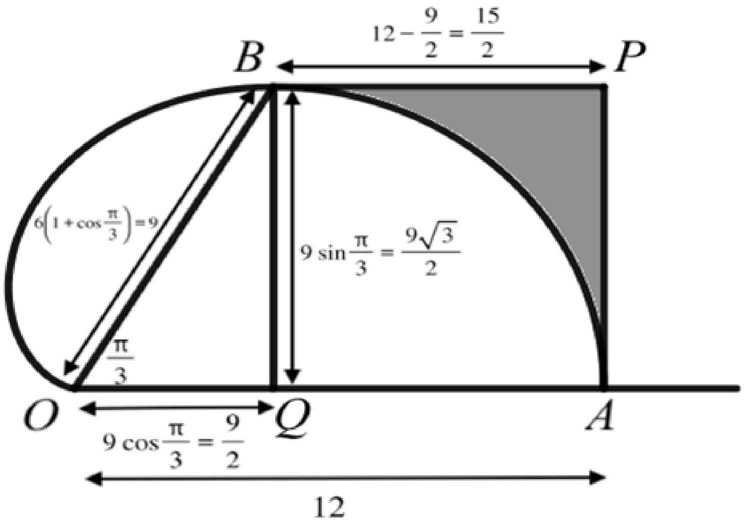
dM1: **Dependent on previous M mark.** Evidence of substitution of "correct" limits into the integral, so their $\frac{\pi}{3}\left(\text{provided } 0 < \theta < \frac{\pi}{2}\right)$ and the lower limit must be 0 and lead to zero but this can be implied by omission.

M1: Correct method for the perpendicular distance between l_1 and the initial line with their r and $\theta\left(\text{provided } 0 < \theta < \frac{\pi}{2}\right)$ for point *B*. This may only be seen embedded in their attempt at an area, so may be implied if not explicit. E.g. look for "their *r* sin their $\frac{\pi}{3}$" or may find *r* again from the equation of curve.

A1: Correct expression for area of trapezium *OBPA* - may be given as as a sum of separate areas, e.g. triangle *OBQ* + rectangle *QBPA*. Other formulations are possible.

A1: Correct answer in the correct form or $p = \frac{189}{8}$ or exact equivalent, $q = -9$

Useful diagram:



	TOTAL FOR PAPER: 75 MARKS