Mark Scheme (Results)

October 2023

Pearson Edexcel International Advanced Level in Pure Mathematics (WMA11) Paper 01

Question Number	Scheme	Marks
1a	$y=5x^3+\dfrac{3}{x^2}-7x=5x^3+3x^{-2}-7x$	
	$\left(\dfrac{dy}{dx}=\right)\ 15x^2-6x^{-3}-7$	M1A1A1
		(3)
b	$\left(\dfrac{d^2y}{dx^2}=\right)\ 30x+18x^{-4}$	M1A1
		(2)
		(5 marks)

Do not be concerned by the labelling of their answers in part (a) or part (b)

(a)

M1: Decreases the power by 1 on one of the terms $...x^3 \to ...x^2 \quad ...x^{-2} \to ...x^{-3} \quad \pm 7x \to \pm 7$
The index does not have to be processed. e.g. $...x^3 \to ...x^{3-1}$

A1: Two correct unsimplified terms (indices must be processed). May be seen on different lines or listed. Condone $7x^0$ as an unsimplified term. Do not withhold this mark if there is a $+c$ present.

A1: $15x^2-6x^{-3}-7$ or simplified equivalent. Condone any spurious notation around the expression but withhold this mark if there is a constant of integration. Isw once a correct answer is seen. Do not accept $15x^2-6x^{-3}-7x^0$ or e.g. $15x^2+-6x^{-3}-7$

(b)

M1: Decreases the power by 1 on one of the terms of their changed function. Do not allow this mark to be scored for a constant of integration in (a) $\to 0$ in (b)

A1: $30x+18x^{-4}$ or simplified equivalent e.g. $30x+\dfrac{18}{x^4}$. Condone e.g. $30x^1+\dfrac{18}{x^4}$. Isw once a correct answer is seen. Withhold this mark if there is a constant of integration and it has not already been penalised in (a).
Allow this mark to be scored following $15x^2-6x^{-3}\pm 7$ in (a). Also allow full marks to be scored if they have no constant.

Note: (a) $15x^2-6x^{-1}-7$ and (b) $30x+6$ will score M1A1A0 M1A0

Question Number	Scheme	Marks
2a	$\frac{1}{8}x$	B1
		(1)
b	$\frac{1}{256}x^{\frac{3}{2}}$	B1
		(1)
c	$\left(\frac{1}{2}\left(\frac{1}{64}x^2\times\frac{16}{\sqrt{x}}\right)\right)^{-\frac{4}{3}}=\left(\frac{1}{8}x^{\frac{3}{2}}\right)^{-\frac{4}{3}}=16x^{-2}$	M1A1
		(2)
		(4 marks)

(a)

B1: $\frac{1}{8}x$ or simplified equivalent e.g. $0.125x$ or $\frac{1}{8}x^1$ Accept e.g. $\frac{x}{8}$ but $\pm\frac{1}{8}x$ is B0.

Do not withhold this mark unless it is clear that they intend to write $\frac{1}{8x}$

(b)

B1: $\frac{1}{256}x^{\frac{3}{2}}$ or simplified equivalent e.g. $0.00390625x^{\frac{3}{2}}$ condone $\frac{x^{\frac{3}{2}}}{256}$. Isw after a correct answer is seen.

Do not accept e.g. $\frac{1}{256}\sqrt{x^3}$ or $\frac{1}{256}\left(\sqrt{x}\right)^3$ or $\frac{1}{256}x\sqrt{x}$ as n is not a simplified constant.

Do not accept $x^{\frac{3}{2}}\div 256$ or $\frac{1}{256x^{-\frac{3}{2}}}$ (not of the required form)

Do not withhold this mark unless it is clear that they intend to write $\frac{1}{256x^{\frac{3}{2}}}$

(c)

M1: Attempts to use the index laws and proceeds to either

- $\left(...x^{\frac{3}{2}}\right)^{-\frac{4}{3}}$ or $\left(...x^{-\frac{3}{2}}\right)^{\frac{4}{3}}$
- $...x^{-\frac{8}{3}}\times...x^{\frac{2}{3}}$ (or $...x^{-\frac{8}{3}}\div...x^{-\frac{2}{3}}$)
- $...x^{-2}$
- $16x^n$ where $n\neq-\frac{1}{2}$

Allow $\frac{16}{x^2}$ to score M1.

Be aware that incorrect understanding of indices such as $\frac{16}{\sqrt{x}}=16x^{-2}$ is M0A0

A1: $16x^{-2}$ (from a correct method)

(Correct answer with no incorrect working seen can score M1A1). Isw after a correct answer is seen. $\frac{16}{x^2}$ is A0

Question Number	Scheme	Marks
3a	$\dfrac{8-\sqrt{15}}{2\sqrt{3}+\sqrt{5}}\times\dfrac{2\sqrt{3}-\sqrt{5}}{2\sqrt{3}-\sqrt{5}}=\dfrac{16\sqrt{3}-8\sqrt{5}-2\sqrt{45}+\sqrt{75}}{12-5}$	M1
	e.g. $\dfrac{21\sqrt{3}-14\sqrt{5}}{7}$	dM1
	$3\sqrt{3}-2\sqrt{5}$	A1
		(3)
b	$\left(x+5\sqrt{3}\right)\sqrt{5}=40-2x\sqrt{3}\Rightarrow x\sqrt{5}+2x\sqrt{3}=40-5\sqrt{15}$	M1
	$(x=)\ \dfrac{40-5\sqrt{15}}{2\sqrt{3}+\sqrt{5}}$	A1
	$(x=)\ 15\sqrt{3}-10\sqrt{5}$	A1ft
		(3)
		(6 marks)

(a) Note that $3\sqrt{3}-2\sqrt{5}$ with no working scores M0dM0A0

M1: Attempts to rationalise the denominator by multiplying both numerator and denominator by $k\left(2\sqrt{3}-\sqrt{5}\right)$, where k is an integer usually 1, and proceeds to a fraction such as $\dfrac{...\sqrt{3}\pm...\sqrt{5}\pm...\sqrt{45}\pm...\sqrt{75}}{...}$ or $\dfrac{...}{12-5}$ or $\dfrac{...}{7}$

but not for e.g. $\dfrac{...}{12+2\sqrt{15}-2\sqrt{15}-5}$ or $\dfrac{...}{\left(2\sqrt{3}\right)^2-\left(\sqrt{5}\right)^2}$ as these still have surds on the denominator.

Allow $\sqrt{45}$ and $\sqrt{75}$ to be written in terms of $\sqrt{3}$ and $\sqrt{5}$ as well.
Condone slips in multiplying out as well as miscopying errors.

Note $\dfrac{21\sqrt{3}-14\sqrt{5}}{7}$ with no intermediate working can still score M1 for the denominator.

Attempting to multiply by a multiple of $\dfrac{2\sqrt{3}+\sqrt{5}}{2\sqrt{3}+\sqrt{5}}$ is M0dM0A0

dM1: Attempts to simplify surds and may collect terms to achieve a fraction where

- the numerator is in terms of $\sqrt{3}$ and $\sqrt{5}$ only.
- the denominator is a multiple of 7 (which may be unsimplified)

It is dependent on the previous method mark.
If they have not fully multiplied out the numerator (or implied) then this mark cannot be scored. e.g.

$\dfrac{(8-\sqrt{15})(2\sqrt{3}-\sqrt{5})}{...}=\dfrac{16\sqrt{3}+8\sqrt{5}}{...}$ scores a maximum of M1dM0A0.

Note $\dfrac{21\sqrt{3}-14\sqrt{5}}{7}$ **with no previous working seen** scores M1dM0A0 because they have not shown any multiplying out of the brackets on the numerator.

Note $\dfrac{8-\sqrt{15}}{2\sqrt{3}+\sqrt{5}}=\dfrac{16\sqrt{3}-8\sqrt{5}-2\sqrt{45}+\sqrt{75}}{7}=3\sqrt{3}-2\sqrt{5}$ is M1dM0A0 because they have not collected terms on the numerator (or changed them all to be in terms of $\sqrt{3}$ and $\sqrt{5}$ before simplifying to the final answer).

A1: $3\sqrt{3}-2\sqrt{5}$. The answer does not imply the method marks. Do not withhold this mark for slips in working such as invisible brackets provided they are recovered/implied by further work.

Note that a number of candidates are misreading $\frac{8-\sqrt{15}}{2\sqrt{3}+5}\times\frac{2\sqrt{3}-5}{2\sqrt{3}-5}=\frac{16\sqrt{3}-40-2\sqrt{45}+5\sqrt{15}}{12-25}$ this can score SC100 condoning one sign slip in their multiplying out of the brackets in the numerator.

(b) Note that $x=15\sqrt{3}-10\sqrt{5}$ with no working scores M0A0A0

M1: Multiplies out the brackets and isolates the *x* terms on one side. Condone for this mark if the surds are converted to rounded decimals.

e.g. $(\sqrt{5}+2\sqrt{3})x=5(8-\sqrt{15})$ scores M1

Alternatively divides both sides by $\sqrt{5}$ and isolates the *x* terms on one side.

e.g. $x+2x\frac{\sqrt{3}}{\sqrt{5}}=\frac{40}{\sqrt{5}}-5\sqrt{3}$

Condone slips in their working and invisible brackets, but there must be two terms on each side of the equation.

A1: $x=\frac{40-5\sqrt{15}}{2\sqrt{3}+\sqrt{5}}$ or exact equivalent (which cannot be $15\sqrt{3}-10\sqrt{5}$).

In the alternative method it may be seen as $x=\frac{\frac{40}{\sqrt{5}}-5\sqrt{3}}{1+2\frac{\sqrt{3}}{\sqrt{5}}}$ which can score this mark.

Do not accept proceeding from $(\sqrt{5}+2\sqrt{3})x=5(8-\sqrt{15})\Rightarrow x=15\sqrt{3}-10\sqrt{5}$ without any intermediate expression for *x*. This scores A0 as the use of calculator technology is not acceptable.
An expression in decimals only is A0.

A1ft: $x=15\sqrt{3}-10\sqrt{5}$ only (or exact simplified equivalent) e.g. $x=5\left(3\sqrt{3}-2\sqrt{5}\right)$ This mark cannot be awarded without the previous A mark being scored.

Follow through on their part (a) answer of the form $a\sqrt{3}+b\sqrt{5}$ so award for $5a\sqrt{3}+5b\sqrt{5}$ oe including $5\left(a\sqrt{3}+b\sqrt{5}\right)$ where *a* and *b* may be fractions. Isw once they have achieved the correct answer.

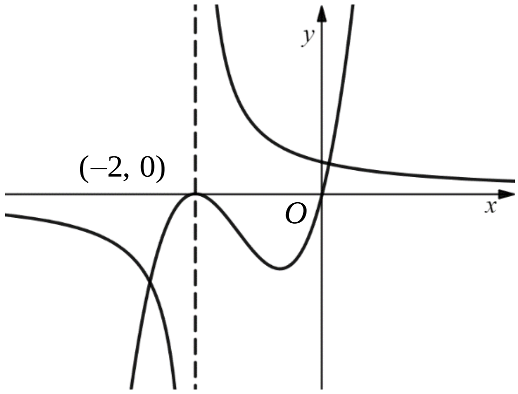
Note: If a candidate has an unsimplified answer in part (a) e.g. $\frac{21\sqrt{3}-14\sqrt{5}}{7}$ then do not withhold this mark in part (b) if they proceed to e.g. $\frac{105\sqrt{3}-70\sqrt{5}}{7}$ as they have already been penalised once for not giving an answer in simplest form.

Alternative method – squaring both sides (send to review if unsure)

M1: Attempts to square both sides and proceeds to a three term quadratic in *x* with terms collected on one side of the equation (condone slips in multiplying out and collecting terms.

A1: As above in main scheme

A1ft: As above in main scheme

Question Number	Scheme	Marks
4a	$x=-2$	B1
		(1)
b	$x^3+4x^2+4x=x(x+2)^2$	M1A1
		(2)
c		B1B1B1
		(3)
d	2 as the graphs intersect (each other) twice (since $(x+2)(x^3+4x^2+4x)=1$ is the same as $x^3+4x^2+4x=\dfrac{1}{x+2}$)	B1
		(1)
		(7 marks)

Note: Check for answers by the questions or on the Figure (rather than the diagram). If there is a contradiction between the answers by the question and the main body of the solution then the main body of the solution takes precedence.

(a)

B1: $x=-2$ only. Accept labelled on the diagram but do not accept just -2. If other equations are given (other than $y=0$) then B0.

Beware that $x=-2$ is one of the solutions to the cubic so the equation of the asymptote must be seen in (a), on the diagram or by the question.

(b)

M1: Attempts to take out a linear factor eg $x(x^2\pm4x\pm4)$ or $(x+2)(x^2\pm2x)$. May be implied by the correct answer.

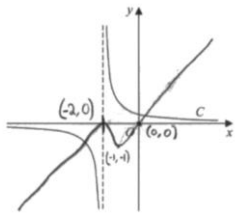
A1: $x(x+2)^2$ or $x(x+2)(x+2)$. Accept $(x+2)(x+2)x$ and condone a missing trailing bracket on the final linear factor e.g. $x(x+2)(x+2$ but $x(x+2^2$ is A0.

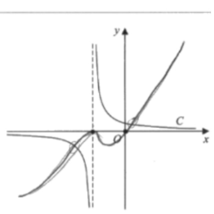
Condone $(x+0)(x+2)^2$. Isw once a correct expression is seen and ignore a spurious = 0

(c) **Note: If there are multiple attempts then score the highest one**

B1: Sketches a positive cubic anywhere on a set of axes. Do not penalise poor curvature provided the intention is clear and ignore if the curvature looks asymptotic. The cubic does not need to have two turning points for this mark.

Examples of acceptable cubic shapes (including where poor curvature would be condoned):





(If you are unsure then send to review)

B1: A cubic which

- has a turning point where the *x*-axis and vertical asymptote intersect
- passes through the origin (from quadrants 3 to 1 or from quadrants 2 to 4 – it cannot start or stop at the origin) It also cannot be a turning point.

Do not penalise poor curvature provided the intention is clear and ignore if the curvature looks asymptotic.

B1: $(-2,\ 0)$ **indicated on the graph** where their graph crosses or turns on the **negative *x*-axis** Do not be concerned regarding any other points where the graph crosses or turns on the *x*-axis including the origin.
Condone -2 labelled on the *x*-axis or the coordinates the wrong way round as $(0,\ -2)$ or with missing brackets.
The asymptote labelled $x=-2$ does not score this mark and do not be concerned with the point of intersection relative to the asymptote.

(d) **This mark can only be scored provided B1B1B0 or B1B1B1 is scored in (c)**

B1: **2** (real) roots **and** a valid reason e.g. (graphs/curves/functions/equations/they) **intersect/meet/cross/touch** (each other) **twice**. e.g. “intersect twice” or “2 intersections” can score this mark.
May also refer to an intersection in the first and third quadrants.

Only withhold the mark if it is clear that they are referring to intersections between the cubic graph and the *x*-axis.

Do not accept responses relating to the roots of the equation, use of the discriminant or other algebraic methods which do not use the graphs.

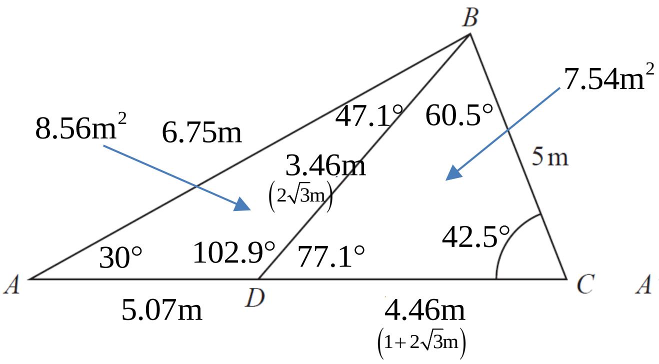
Question Number	Scheme	Marks
5a	e.g. $(\cos\theta =)\ \dfrac{5^2+(1+x)^2-x^2}{2\times5(1+x)}\ \left(=\dfrac{25+1+2x+x^2-x^2}{2\times5(1+x)}\right)$ or e.g. $x^2=5^2+(1+x)^2-2\times5(1+x)\cos\theta$	M1
	$\cos\theta=\dfrac{13+x}{5+5x}$ *	A1*
		(2)
b	$\theta=\text{awrt }42°\quad(42.470747\ldots)$	B1
	--- Attempts to find *AB*, *AD* or *AC*: e.g. $\dfrac{AB}{\sin"42"}=\dfrac{5}{\sin30}\Rightarrow AB=...$ or e.g. $\angle ABC=180-30-"42.5"="107.6"$ $\dfrac{\sin DBC}{1+2\sqrt{3}}=\dfrac{\sin"42.5"}{2\sqrt{3}}\Rightarrow\angle DBC="60.5",\ \angle ABD="107.6"-"60.5"="47.1"$ $\dfrac{AD}{\sin("47.1")}=\dfrac{2\sqrt{3}}{\sin30}\Rightarrow AD=...$ or e.g. $\dfrac{AC}{\sin"108"}=\dfrac{5}{\sin30}\Rightarrow AC=...$ ---	M1
	$AB=6.75...$ or $AD=5.07...$ or $AC=9.53...$	A1
	Area $=\dfrac{1}{2}\times5\times"6.75"\times\sin(180-30-"42.5")$ or $=\dfrac{1}{2}\times5\times"9.54"\times\sin("42.5")$	dM1
	$=\text{awrt}16.1\ (\text{m}^2)$	A1
		(5)
		(7 marks)

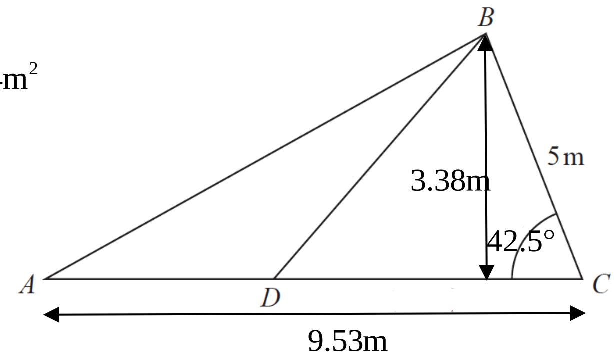
(a)

M1: Uses the cosine rule to form an equation for $\cos\theta$. Condone just an expression for $\cos\theta$.
(You may not see $\cos\theta=$)
Condone invisible brackets for this mark.
Be aware of longer versions which may involve splitting triangle *BCD* into two right angled triangles. They would still need to proceed to an equation involving $\cos\theta$ to score this mark.

A1*: Achieves the given answer with no errors seen including invisible brackets. There must be at least one stage of intermediate working between their starting equation and achieving the given answer. Condone $\cos\theta$ not appearing in the answer line, provided it is seen correctly as the subject on an earlier line.

(b) Note there are a variety of different methods to finding the area of triangle *ABC*





B1: $\theta = \text{awrt } 42°$ seen or implied. Accept 42.5. May work in radians (awrt 0.73/0.74 radians)

M1: Attempts to find *AB*, *AD* or *AC* using a correct method. Angles and lengths must be in the correct positions in the relevant formula or formulae. Condone slips in any rearrangement, calculations and substituting in $x = 2\sqrt{3}$ provided the method is correct. Condone working in radians provided the angles are consistently in degrees or radians within the expression or formula.

A1: awrt 6.75/6.76 or awrt 9.53/9.54 or awrt 5.07/5.08 (may be implied by further work).

dM1: Correct full method to find the total area (the expression is sufficient. It is dependent on the previous method mark. Condone use of incorrectly rounded angles/slips and may work in radians.

e.g. $\text{Area} = \frac{1}{2} \times 5 \times "6.75" \times \sin\left(\pi - \frac{\pi}{6} - "0.74"\right)$

May find the areas of the two separate triangles *ABD* and *BDC* and add them together.

e.g. $\text{Area} = \frac{1}{2} \times "5.07" \times "6.75" \times \sin 30 + \frac{1}{2} \times "4.46" \times 5 \times \sin "42.5"$

Use the diagrams above to help with the various methods. Invisible brackets may be implied by further work or their answer.

Note that if, as part of their method to find the total area, they find angle *ADB* but incorrectly deduce this as an acute angle then this is dM0

A1: awrt 16.1 m^2 (condone lack of units)

Question Number	Scheme	Marks
6a	$4(p-2x)=\frac{12+15p}{x+p}$ $8x^2+4px+12+15p-4p^2 \quad (=0)$	M1
	e.g. $a=8,\ b=4p,\ c=12+15p-4p^2$ $(4p)^2-4\times8\times(12+15p-4p^2) \quad (>0)$	dM1
	$3p^2-10p-8>0 \quad *$	A1*
		(3)
b	e.g. $(p-4)(3p+2)\ (=0)\Rightarrow 4,\ -\frac{2}{3}$	M1
	$p<-\frac{2}{3}$ or $p>4$	M1A1
		(3)
		(6 marks)

Allow part (b) to follow on from (a) without any labelling of specific parts

(a)

M1: Multiplies by $(x+p)$, multiplies out the brackets and collects terms on one side.
Condone slips and the omission of = 0, but there must be an $...x^2$ term.
May be implied by further work e.g. their attempt at the discriminant.

dM1: Attempts b^2-4ac for their quadratic in *x* with their *b* and their *c* both being in terms of *p*.
The values/expressions for *a*, *b* and *c* for their quadratic in *x* must be embedded in the correct positions in the expression but condone invisible brackets and sign slips miscopying.
It is dependent on the previous method mark.

Condone for this mark if they work in terms of *x* instead of *p* provided the coefficients were actually those in terms of *p* from their original quadratic. i.e.
$8x^2+4px+12+15p-4p^2=0 \Rightarrow a=8,\ b=4p,\ c=12+15p-4p^2$
$\Rightarrow (4x)^2-4\times8\times(12+15x-4x^2)\ (>0)$ (they have just written *x* instead of *p* which is fine)

A1*: Achieves $3p^2-10p-8>0$ with all stages of working shown:

- Multiplying out $(p-2x)(x+p)$
- Collecting terms on one side of an equation (=0 may be implied)
- Finding the discriminant
- At least one intermediate stage of working between the discriminant and the final answer

There must be no errors seen including invisible brackets but condone a missing trailing bracket if it does not alter the processing. e.g. $(4p)^2-4\times8\times(12+15p-4p^2$
Condone the > 0 appearing for the first time in their final answer, provided an incorrect inequality has not been used in earlier working.
Do not withhold this mark if they state e.g. "$b^2-4ac=0$" as part of their working.

(b)

M1: Attempts to find critical values by solving the given quadratic by either

- factorising (do not accept $(p-4)(p+\frac{2}{3})$ unless they have divided both sides of the inequality/equation by 3 first)
- completing the square $3\left(p-\frac{5}{3}\right)^2 \pm ... \Rightarrow (p=)\ ...$
- quadratic formula
  Usual rules apply (see general marking principles for guidance). May be in another variable. Allow this mark to be scored if seen in (a).
  This mark cannot be awarded from directly using a calculator and stating the roots.

M1: Attempts to find the outside region for their critical values. May use another variable e.g. x
May be implied by $p \leqslant "-\frac{2}{3}",\ p \geqslant "4"$ or incorrect use of inequalities e.g. $4 < p < -\frac{2}{3}$
This is **not** dependent on the first method mark so if a calculator has been used then this mark can still be scored.

A1: $p < -\frac{2}{3}$ or $p > 4$ (or equivalent). (may appear on separate lines). Isw once a correct answer is seen, provided there is no contradiction and no part of the range is rejected.
This can only be scored provided both previous method marks have been awarded.
Must be in terms of *p*
Accept e.g. $p < -\frac{2}{3},\ p > 4,\ p < -\frac{2}{3} \quad p > 4 \quad \left\{p : p < -\frac{2}{3} \cup p > 4\right\}$ or variations of these. Accept $\left(-\infty, -\frac{2}{3}\right),\ (4, \infty)$
Do not accept e.g. $p < -\frac{2}{3}$ and $p > 4 \quad \left\{p : p < -\frac{2}{3} \cap p > 4\right\}$ or variations of these
Do not accept $\left(-\infty, -\frac{2}{3}\right],\ [4, \infty) \qquad p \leqslant -\frac{2}{3},\ p \geqslant 4 \qquad 4 < p < -\frac{2}{3}$

Question Number	Scheme	Marks
7ai	$\mathrm{f}'(4)=\dfrac{4(4)^2+10-7(4)^{\frac{1}{2}}}{4(4)^{\frac{1}{2}}}=\dfrac{15}{2}$	B1
ii	$"\dfrac{15}{2}"\to-\dfrac{2}{15}$	M1
	$y+1="-\dfrac{2}{15}"(x-4)$	M1
	$2x+15y+7=0$	A1
		(4)
b	$\dfrac{4x^2+10-7x^{\frac{1}{2}}}{4x^{\frac{1}{2}}}=\pm\ldots x^{\frac{3}{2}}\pm\ldots x^{-\frac{1}{2}}\pm\ldots$	M1
	Two of the terms of $x^{\frac{3}{2}}+\dfrac{5}{2}x^{-\frac{1}{2}}-\dfrac{7}{4}$	A1
	$\displaystyle\int\left(x^{\frac{3}{2}}+\frac{5}{2}x^{-\frac{1}{2}}-\frac{7}{4}\right)\mathrm{d}x=\frac{2}{5}x^{\frac{5}{2}}+5x^{\frac{1}{2}}-\frac{7}{4}x\ \ (+c)$	dM1A1ft
	$\dfrac{2}{5}(4)^{\frac{5}{2}}+5(4)^{\frac{1}{2}}-\dfrac{7}{4}(4)+c=-1\Rightarrow c=\ldots$	ddM1
	$(\mathrm{f}(x)=)\ \dfrac{2}{5}x^{\frac{5}{2}}+5x^{\frac{1}{2}}-\dfrac{7}{4}x-\dfrac{84}{5}$	A1
		(6)
		(10 marks)

Mark (a) and (b) together so do not be concerned with labelling of the parts

(a)

(i)

B1: $\dfrac{15}{2}$ oe stated (as the gradient of f(x) at P).

(ii)

M1 Finds the negative reciprocal of their gradient in part (i). If they do not have a gradient in (i) then only allow $-\dfrac{2}{15}$ or an attempt at $-\dfrac{1}{\mathrm{f}'(4)}$

M1 Attempts to find the equation of the normal using a changed gradient to that found in (i) and $(4,\ -1)$ with the coordinates in the correct positions. If they do not have a gradient in (i) then allow any gradient $\neq\dfrac{15}{2}$. If they use $y=mx+c$ they must proceed as far as $c=\ldots$

A1: $2x+15y+7=0$ or any multiple of this where all the coefficients are integers and all terms are on the same side of the equation. e.g. $30y+14+4x=0$ scores A1

(b)

M1: Splits into **three** separate terms with at least one term with the correct index. The index does not need to be processed. e.g. $x^{2-\frac{1}{2}}$

A1: Two of $x^{\frac{3}{2}}+\frac{5}{2}x^{-\frac{1}{2}}-\frac{7}{4}$ (unsimplified but the indices must be processed). May appear as a list of terms on different lines. May be implied by a correctly integrated expression.

dM1: Attempts to increase the power by one on at least one term. It is dependent on the previous method mark. The index does not need to be processed for this mark. It cannot be scored for attempting to integrate individual terms on the numerator or denominator.

A1ft: $\frac{2}{5}x^{\frac{5}{2}}+5x^{\frac{1}{2}}-\frac{7}{4}x$ or exact unsimplified equivalent (indices processed). e.g. $\dfrac{x^{\frac{5}{2}}}{\frac{5}{2}}+\dfrac{\frac{5}{2}x^{\frac{1}{2}}}{\frac{1}{2}}-\frac{7}{4}x$

Follow through on an expression of the form $Ax^{\frac{3}{2}}+Bx^{-\frac{1}{2}}+C \to \dfrac{Ax^{\frac{5}{2}}}{\frac{5}{2}}+\dfrac{Bx^{\frac{1}{2}}}{\frac{1}{2}}+Cx$ where *A*, *B* and *C* are all non-zero.
Do not be concerned with the presence or omission of the constant of integration.
Accept x^1 for *x*.
Ignore any spurious notation including the presence of the integral sign.

ddM1: Attempts to substitute $x=4$ into their changed expression (condone one slip in substituting in), sets equal to -1 and attempts to find *c*.
This mark cannot be scored if they do not have a constant of integration.
It is dependent on both of the previous method marks.
Do not be concerned by the mechanics of the rearrangement and condone arithmetical slips, but they must achieve a value.

The substitution may be implied by a correct value for *c* for their integrated expression if no working is shown (you may need to check this), or allow for $\pm\frac{84}{5}$

A1: $(\mathrm{f}(x)=)\ \frac{2}{5}x^{\frac{5}{2}}+5x^{\frac{1}{2}}-\frac{7}{4}x-\frac{84}{5}$ or any equivalent expression e.g. $0.4x^{\frac{5}{2}}+5x^{\frac{1}{2}}-1.75x-16.8$
Accept x^1 for *x*. Condone equivalent fractions for the coefficients provided both numerator and denominator are integers. e.g. $(\mathrm{f}(x)=)\ \dfrac{x^{\frac{5}{2}}}{2.5}+5x^{\frac{1}{2}}-\frac{7}{4}x-\frac{84}{5}$ is A0
Withhold the final mark if there is still integration notation around the answer or if it is set equal to 0

Question Number	Scheme	Marks
8a	$x\left(x^3-\frac{7}{2}x-5\right)=\frac{15}{2}-5x$ $\Rightarrow 2x^4-7x^2-15=0$ *	M1 A1*
		(2)
b	$(2x^2+3)(x^2-5)=0\Rightarrow\left(x^2=\right)...$	M1
	$(x=)\ \ (\pm)\sqrt{5}$	B1
	$y=\frac{15}{2\times"\pm\sqrt{5}"}-5=...$ or $y=\left("\pm\sqrt{5}"\right)^3-\frac{7}{2}\left("\pm\sqrt{5}"\right)-5$	M1
	$\left(-\sqrt{5},\ -\frac{3}{2}\sqrt{5}-5\right)\quad\left(\sqrt{5},\ \frac{3}{2}\sqrt{5}-5\right)$	
	$PQ=\sqrt{("2\sqrt{5}")^2+("3\sqrt{5}")^2}=...$	dM1
	$PQ=\sqrt{65}$	A1cao
		(5)
		(7 marks)

(a)

M1: Sets up a correct equation in *x*. Invisible brackets may be implied by further work.

e.g. $x^4-\frac{7}{2}x^2-5x=\frac{15}{2}-5x$

A1*: Multiplies out and rearranges the equation to achieve the given answer with no errors seen including invisible brackets. Condone the implied = 0 in their working, provided it is present in their final answer. There must be at least one stage of intermediate working between their starting equation and proceeding to the given answer.

$x^4-\frac{7}{2}x^2-5x=\frac{15}{2}-5x\Rightarrow 2x^4-7x^2-15=0$ is M1A0*

(b) **Work for (b) may be seen in (a) which can score**

M1: Attempts to solve the quadratic equation in x^2 by either

- factorising
- completing the square
- quadratic formula

Usual rules apply. Do not be too concerned by the labelling e.g. it is acceptable to use a different variable as well as condoning $y=x^2$ and $x=x^2$

This mark cannot be awarded from directly using a calculator and stating the roots.

B1: At least one of $\sqrt{5}$ and $-\sqrt{5}$ (and no others)

M1: **Note that if they do not square root their solution to the quadratic in x^2 then this mark cannot be scored (and no further marks).**

Attempts to substitute in $\pm\sqrt{"5"}$ (which must be of the form $+k\sqrt{b}$ and $-k\sqrt{b}$ where *k* may be 1) into either of the equations of the curves and proceeds to find irrational expressions for the *y* coordinates of *P* and *Q* of the form $\pm5\pm n\sqrt{"5"}$ (which may be the same for both). They cannot be decimals.

May be seen as e.g. $\left(-\sqrt{5},\ \frac{-3\sqrt{5}-10}{2}\right)\quad\left(\sqrt{5},\ \frac{3\sqrt{5}-10}{2}\right)$

dM1: A full attempt to find the length PQ. It is dependent on the previous method mark.

- They must be using $(k\sqrt{b}, \pm 5 \pm n\sqrt{b}),\ (-k\sqrt{b}, \pm 5 \pm n\sqrt{b})$ or equivalent.
- Look for an expression for PQ of the form e.g.

$$\sqrt{\left((k\sqrt{b} - -k\sqrt{b})\right)^2 + \left((\pm 5 \pm n\sqrt{b}) - (\pm 5 \pm n\sqrt{b})\right)^2}$$

- They must be attempting to find $(x_2 - x_1)^2$ for their x coordinates which would lead to $\pm 2k\sqrt{b}$ but condone ambiguity in the location of negatives when calculating $(y_2 - y_1)^2$.

  e.g. $x = \pm\sqrt{5}$: $\sqrt{\left((\sqrt{5} - -\sqrt{5})\right)^2 + \left(-\dfrac{10+3\sqrt{5}}{2} - \dfrac{-10+3\sqrt{5}}{2}\right)^2}$ scores B1M1dM1

  e.g. $x = \pm\sqrt{11}$ $\sqrt{\left((-\sqrt{11} - \sqrt{11})\right)^2 + \left(\dfrac{110+15\sqrt{11}}{22} - \dfrac{-110-15\sqrt{11}}{22}\right)^2}$ scores B0M1dM1

  (as their $(x_2 - x_1)^2$ is correct for their $\pm\sqrt{"5"}$ and we condone sign errors when finding $(y_2 - y_1)^2$ using their y coordinates of P and Q)
- If the subtractions are not shown then they must achieve either of the following:
  - $\sqrt{(2k\sqrt{b})^2 + (2n\sqrt{b})^2}$ (n does not have to be an integer) or equivalent
  - $\sqrt{(2k\sqrt{b})^2 + 0^2}$ (where the y coordinates were the same)

    (They must have still found the y coordinates to score this mark)

Do not condone invisible brackets unless it does not affect the processing. This mark may be implied by their answer.

A1: $\sqrt{65}$ cao (**this cannot be scored if the first method mark has not been scored**)

Question Number	Scheme	Marks
9a	$0.72 = OP \times 0.6 \Rightarrow OP = 1.2$ **or** $\dfrac{0.72}{0.6} = 1.2$ **or** $0.72 = 1.2 \times 0.6 \Rightarrow OP = 1.2$ *	B1*
		(1)
b	$\frac{1}{2} \times (x+1.2)^2 \times 0.6$	B1
	$\frac{1}{2} \times (x+1.2)^2 \times 0.6 - \frac{1}{2} \times 1.2^2 \times 0.6 = 90 \Rightarrow 5x^2 + 12x - 1500 = 0$ *	M1A1*
		(3)
c	$x = \text{awrt } 16.2 \quad \left(= \dfrac{-6+4\sqrt{471}}{5} \right)$	B1
	Perimeter $= 0.72 + "16.2" + "16.2" + 0.6 \times ("16.2" + 1.2) = \text{awrt } 43 / \text{awrt } 44$ m (see notes)	M1A1
		(3)
		(7 marks)

(a)

B1*: Either

- forms an equation using $l = r\theta$ with $l = 0.72$ and $\theta = 0.6$ and concludes $OP = 1.2$ e.g. $0.72 = r \times 0.6 \Rightarrow r = 1.2$
- states $\dfrac{0.72}{0.6} = 1.2$
- forms the equation using $0.72 = 1.2 \times 0.6$ and concludes that $OP = 1.2$

Condone the use of *r* or any other variable instead of *OP*. Condone lack of units or incorrect units.

Allow the angle to be in degrees instead or attempts to find the diameter first, but they must not round the value at any point in their working.

e.g. $0.72 = \dfrac{\frac{0.6}{\pi} \times 180}{360} \pi d \Rightarrow d = 2.4 \Rightarrow r = 1.2$ scores B1 (exact angle used)

e.g. $\dfrac{0.72}{\left(\frac{34.4}{360}\right)} = 2\pi r \Rightarrow r = 1.2$ scores B0 (34.4 is a rounded angle)

(b)

B1: Correct expression for the area of sector *OPQRSO*

$\frac{1}{2} \times (x+1.2)^2 \times 0.6$ Missing brackets may be implied by further work.

May appear within an equation e.g. $\frac{1}{2} \times 0.6\left[(x+1.2)^2 - 1.2^2\right] = 90$

Condone incorrect equations i.e. $\frac{1}{2} \times (x+1.2)^2 \times 0.6 = 90$

M1: Proceeds from an equation of the form $...(x \pm 1.2)^2 \times 0.6 - ... \times 1.2^2 \times 0.6 = 90$ or equivalent, (i.e. both should be positive but may be missing e.g. ½) and attempts to multiply out the brackets. Condone slips and invisible brackets.

A1*: Achieves the given answer with no errors seen including invisible brackets. They must have multiplied out any brackets before proceeding to the given answer to score this mark.

e.g. $\frac{54}{125}+\frac{18}{25}x+\frac{3}{10}x^2-0.432=90 \Rightarrow 5x^2+12x-1500=0$ is acceptable as a final step to score A1*

(c)

B1: $x=$ awrt 16.2 $\left(=\frac{-6+4\sqrt{471}}{5}\right)$ (ignore the negative root) May be seen on the diagram or next to the quadratic in the question. It may also be implied in their working to find the perimeter. It cannot be for the values embedded in the quadratic formula.

M1: A correct method to find the perimeter using their value for x which must be positive. They must proceed as far as a value for the perimeter to score this mark. Do not condone the lengths 0.72 and 1.2 to be rounded, but condone for example their "16.2" to be "16".

May see exact values used in the expression e.g. $2\left(\frac{-6+4\sqrt{471}}{5}\right)+0.72+\left(\frac{12\sqrt{471}}{25}\right)=...$

A1: awrt 43 m or awrt 44 m, **including units**, **following a correct method. Typically look for 43.4…m or 43.5…m**

Note some incorrect methods lead to awrt 43 which is maximum B1M0A0.

e.g. $"16.2"\times 0.6+"16.2"+"16.2"+0.72=42.7$ which is B1M0A0 (uses 16.2 instead of 16.2+1.2)

e.g. $"16.2+1.2"\times 0.6+"16.2"+"16.2"=42.8$ which is B1M0A0 (missing arc *PS*)

Question Number	Scheme	Marks
10ai	$(n=)\ 3$	B1
ii	1080	B1
		(2)
b	$(1620,\ -3)$	B1B1
		(2)
c	e.g. $k=\frac{1}{2}\left(\frac{12}{5}-\frac{3}{5}\right)=\frac{9}{10}$	M1A1
		(2)
		(6 marks)

Note: answers may appear next to questions or on the diagram.
If there is a contradiction between the answer by the question/on the diagram and the main body of the work then the answer in the main body of the work takes precedence.

(a)
(i)
B1: $(n=)\ 3$

(ii)
B1: 1080 (must be in degrees). Do not accept e.g. $(1080,\ 3)$

(b)
B1: One of the two coordinates $(1620,\ -3)$ Condone missing brackets. Allow $x=\ldots$, $y=\ldots$.
Condone 9π instead of 1620 for this mark and condone the coordinates to be the wrong way round e.g. $(-3,\ 1620)$ scores B1B0

B1: Both coordinates $(1620,\ -3)$ Condone missing brackets. Allow $x=\ldots$, $y=\ldots$.

(c)
M1: Attempts to find a value for k by either

- attempting to find $k=\frac{1}{2}\left(\frac{12}{5}-\frac{3}{5}\right)=\ldots$.
- attempting to find $k=\frac{12}{5}-\frac{1}{2}\left(\frac{12}{5}+\frac{3}{5}\right)=\ldots$ or $k=-\frac{3}{5}+\frac{1}{2}\left(\frac{12}{5}+\frac{3}{5}\right)=\ldots$
- forming two **correct** simultaneous equations and eliminating sin a (may be labelled as a different variable, condone a for sin a)
  $2\sin(a)+k=\frac{12}{5},\ -2\sin(a)+k=-\frac{3}{5}\Rightarrow k=\ldots$ scores M1
  They must have correctly used $\sin(-a)=-\sin(a)$. If they eliminate k instead, they must proceed to find a or $\sin a$ and then substitute back into one of the equations to find a value for k
  i.e. $4\sin a=3\Rightarrow a=48.59\ldots\Rightarrow 2\sin(48.59\ldots)+k=\frac{12}{5}\Rightarrow k=\ldots$

A1: $\frac{9}{10}$ or 0.9 (this is not awrt 0.9 so if they achieve a rounded value e.g. 0.899… and round to 0.9 then M1A0)

Question Number	Scheme	Marks
11a	$2(x\pm...)^2$	B1
	$...(x\pm3)^2...$	M1
	$2(x-3)^2-4$	A1
		(3)
b	$(3,\ -4)$	B1ft
		(1)
c	$m=\dfrac{28--4}{-1-3}\ \ (=-8)$	M1
	$y-28=-8(x+1)$	dM1
	$y=-8x+20$	A1
		(3)
d	$y\leqslant "-8x+20"$ and $y\geqslant 2x^2-12x+14$ (or $y\geqslant "2(x-3)^2-4"$)	B1ftB1ft
	$y\leqslant -8x+20\qquad y\geqslant 2x^2-12x+14\qquad y\geqslant 0,\ x\geqslant 0$	B1cso
		(3)
		(10 marks)

Note: answers may appear next to questions or on the diagram.
If there is a contradiction between the answer by the question/on the diagram and the main body of the work then the answer in the main body of the work takes precedence.

(a)

B1: $a=2$ which may be stated or seen in the expression $2(x\pm...)^2$.

M1: $...(x\pm3)^2...$ may be implied by $b=\pm3$. Accept unsimplified versions e.g. $...\left(x\pm\dfrac{12}{4}\right)^2...$

A1: $2(x-3)^2-4$ isw following a correct expression (e.g. if they proceed to state incorrect values for *a*, *b* and/or *c*). If they just state the values then withhold this mark. Constants must be integers.

SC110 $2(x-3)-4$ (slip writing missing the squared on the bracket)

(b)

B1ft: $(3,\ -4)$ or e.g. $x=3,\ y=-4$. Condone invisible brackets. Follow through from their part (a) so score for $(-b,\ c)$

(c)

M1: Attempts to find the gradient between their minimum point and $(-1, 28)$. Score for the expression $\frac{28-"-4"}{-1-"3"}$ or equivalent. They must be subtracting the correct way round. Do not condone sign slips for this mark. If a value appears with no method (nor implied using their minimum point and $(-1, 28)$), then this mark cannot be scored (M0dM0A0)

May be part of the equation for l e.g. $\frac{28-"-4"}{y-28}=\frac{-1-"3"}{x+1}$ or $\frac{28-"-4"}{y+4}=\frac{-1-"3"}{x-3}$

dM1: Attempts to find the equation using their gradient. |It is dependent on the previous method mark. Score for $y-28="-8"(x+1)$ or $y+4="-8"(x-3)$ but condone one sign slip substituting in the coordinates for x and y.

If they use $y=mx+c$ they must proceed as far as $c=\ldots$.

A1: $y=-8x+20$

(d) Allow consistent use of either $\leqslant$ $\geqslant$ or $<$ $>$ Ignore use of and/or or equivalent notation for this question.

Note that $2x^2-12x+14$ may be expressed as $2(x-3)^2-4$

Allow the use of f(x) instead of y but the use of R instead of y is not acceptable.

B1ft: One of

- $y \leqslant "-8x+20"$ follow through on their part (c)
- $y \geqslant 2x^2-12x+14$ follow through on their part (b) so $"2"(x-"3")^2-"4"$

May be part of an inconsistent set of inequalities e.g. $2x^2-12x+14 < y \leqslant "-8x+20"$

B1ft: Both

- $y \leqslant "-8x+20"$ follow through on their part (c)
- $y \geqslant 2x^2-12x+14$ follow through on their part (b) so $"2"(x-"3")^2-"4"$

May also be seen as $2x^2-12x+14 \leqslant y \leqslant "-8x+20"$

B1cso: $y \leqslant -8x+20$ $\quad y \geqslant 2x^2-12x+14$ $\quad y \geqslant 0,\ x \geqslant 0$ Do not isw for this mark.

Also accept $2x^2-12x+14 \leqslant y \leqslant -8x+20$ $\quad y \geqslant 0,\ x \geqslant 0$

Accept inequalities $0 \leqslant x \leqslant p$ provided $p \geqslant \frac{5}{2}$

$0 \leqslant y \leqslant q$ provided $q \geqslant 20$

Note: The equations may be written in alternative forms e.g. $8x+y \leqslant 20$ which is also acceptable