Pearson Edexcel

Mark Scheme (Results)

January 2023

Pearson Edexcel International Advanced Level
In Statistics S2 (WST02) Paper 01

Question Number	Scheme	Marks
1 (a)	Po(isson) with $(\lambda =)4$	B1
		(1)
(b)	Pairs of **shoes** (are **sold**) singly/randomly/independently/at a **constant** (average) **rate**	B1
		(1)
(c) (i)	X = number of sales per hour $\Rightarrow X \sim$ Po(4)	
	$P(X > 4) = 1 - P(X \leqslant 4)$	M1
	= 0.3712 awrt 0.371	A1
(ii)	$('0.371...')^3$	M1
	= 0.051147… 0.05115 or awrt 0.0511	A1
		(4)
(d)	$H_0 : \lambda = '4'$ $H_1 : \lambda > '4'$	B1ft
	$P(X \geqslant 7) = 1 - P(X \leqslant 6)$ or $P(X \geqslant 9) = 1 - P(X \leqslant 8) = 0.0214$	M1
	= 0.1107 or CR $X \geqslant 9$ awrt 0.111	A1
	Not significant/Do not reject H_0 /Not in the critical region	M1
	There is insufficient evidence of an <u>increase</u> in <u>sales</u> following the appearance of the advert/<u>manager's</u> <u>belief</u> is not supported.	dA1
		(5)
	Notes	**Total 11**

(a)	**B1**	For Po or Poisson and 4 must be seen in part (a). Do not allow P(4)
(b)	**B1**	For one of the given assumptions in context (must have context of **shoes** or **sales**). Ignore extraneous non-contradictory comments.
(c) (i)	**M1**	For writing or using $P(X > 4) = 1 - P(X \leqslant 4)$
	A1	awrt 0.371
(ii)	**M1**	'part (i)'^{3}
	A1	0.05115 or awrt 0.0511 (Calculator gives 0.051132…)
(d)	**B1ft**	Both hypotheses correct. Must be in terms of λ or μ ft their λ from part (a) Must be attached to H_0 and H_1
	M1	For writing or using $P(X \geqslant 7) = 1 - P(X \leqslant 6)$ If a CR approach is taken then award M1 for $P(X \geqslant 9) = 1 - P(X \leqslant 8)$ written or used This mark may be implied by a correct *p*-value or CR
	A1	awrt 0.111 or CR $X \geqslant 9$
	M1	Any correct ft statement consistent with their *p*-value and 0.05 **or** their CR and 7 – no context needed but do not allow contradicting non contextual comments. The comparison of their *p*-value and the significance level is not counted as a non contextual statement. May be implied by a correct ft conclusion in context.
	dA1	Dependent on 1st M1 - Correct conclusion in context which must be **not rejecting** H_0. Must include the underlined words (oe).

Question Number	Scheme	Marks
2 (a)	20, 20, 20 20, 20, 50 (×3) 20, 50, 50 (×3) 50, 50, 50	B2
		(2)
(b)	$a = 30$ and $b = 40$	B1
		(1)
(c)	$p^3 = \frac{4913}{8000}$ or $q^3 = \frac{27}{8000}$	M1
	$p = \frac{17}{20}$ (0.85) and $q = \frac{3}{20}$ (0.15)	A1
		(2)
(d)	$[\text{P}(30)] = 3\times"p^2"\times"q"$ $[\text{P}(40)] = 3\times"p"\times"q^2"$	M1 M1
	$c = \frac{2601}{8000}$ $d = \frac{459}{8000}$	A1
		(3)
(e)	M: 20, 50 P($M = m$): $\frac{3757}{4000}$, $\frac{243}{4000}$	B1 M1 A1ft
		(3)
	Notes	**Total 11**

(a)	**B2**	For all 4 correct combinations (B1 for 3 correct combinations) Ignore extraneous repetitions of any of the given combinations
(b)	**B1**	For $a = 30$ and $b = 40$
(c)	**M1**	Either $p^3 = \frac{4913}{8000}$ or $q^3 = \frac{27}{8000}$
	A1	$p = 0.85$oe and $q = 0.15$oe
(d)	**M1**	$[\text{P}(30)] = 3\times(\text{their } p)^2\times(\text{their } q)$ or $[\text{P}(40)] = 3\times(\text{their } p)\times(\text{their } q)^2$ must see values substituted and must be using their values from part (c)
	M1	$[\text{P}(30)] = 3\times(\text{their } p)^2\times(\text{their } q)$ and $[\text{P}(40)] = 3\times(\text{their } p)\times(\text{their } q)^2$ or use of sum of probabilities = 1 i.e. $c + d = \frac{153}{400}$
	A1	For $c = \frac{2601}{8000} (= 0.325125)$ and $d = \frac{459}{8000} (= 0.057375)$
(e)	**B1**	For 20 and 50 only (ignore notation used for M)
	M1	Either $\frac{4913}{8000} + \text{their } c$ or $\frac{27}{8000} + \text{their } d$ for ft answers only values will need to be checked
	A1ft	For $\frac{3757}{4000}$ oe and $\frac{243}{4000}$ oe Follow through their values for c and d but P($M = 20$) + P($M = 50$) must sum to 1 (A table is not required).
	NB	If a and b are reversed then allow $a = 40$ and $b = 30$ – this will mean $p = 0.15$ and $q = 0.85$, $c = \frac{459}{8000}$ $d = \frac{2601}{8000}$

Question Number	Scheme	Marks
3 (a) (i)	$X \sim \mathrm{B}(10, 0.1)$	
	$\mathrm{P}(X \geqslant 4) = 1 - \mathrm{P}(X \leqslant 3) = 1 - 0.9872$	M1
	$= 0.0128$ awrt 0.0128	A1
(ii)	$\mathrm{P}(1 < X < 5) = \mathrm{P}(X \leqslant 4) - \mathrm{P}(X \leqslant 1) = 0.9984 - 0.7361$ or $\mathrm{P}(X = 2) + \mathrm{P}(X = 3) + \mathrm{P}(X = 4) = 0.1937 + 0.0574 + 0.0112$	M1
	$= 0.2623$ awrt 0.262	A1
		(4)
(b)	$\mathrm{H}_0 : p = 0.1 \quad \mathrm{H}_1 : p < 0.1$	B1
	$X \sim \mathrm{B}(50, 0.1)$	
	$\mathrm{P}(X \leqslant 2) = 0.1117$ or CR $X \leqslant 1$	B1
	Do not reject H_0/Not in the critical region	M1
	There is insufficient evidence to suggest that this result supports the managing director's claim/not enough evidence to suggest a reduction in the probability of a tennis ball failing the bounce test	A1
		(4)
(c)	$X \sim \mathrm{B}(n, 0.1)$ and we reject H_0 if $\mathrm{P}(X = 0) < 0.01$	
	$\mathrm{P}(X = 0) = \left[{}^{n}C_0 \times 0.1^0 \right] \times 0.9^n [< 0.01]$	M1
	$0.9^{44} = 0.00969... [< 0.01]$ | $n > \dfrac{\ln 0.01}{\ln 0.9} \Rightarrow n > 43.7$	M1
	$n = 44$	A1
		(3)

		Notes	Total 11
(a) (i)	**M1**	for writing or using $\mathrm{P}(X \geqslant 4) = 1 - \mathrm{P}(X \leqslant 3)$	
	A1	awrt 0.0128	
(ii)	**M1**	for writing or using $\mathrm{P}(X \leqslant 4) - \mathrm{P}(X \leqslant 1)$ or for writing or using $\mathrm{P}(X = 2) + \mathrm{P}(X = 3) + \mathrm{P}(X = 4)$	
	A1	awrt 0.262	
(b)	**B1**	Both hypotheses correct. Must be in terms of p or π Must be attached to H_0 and H_1	
	B1	awrt 0.112 or CR $\leqslant 1$	
	M1	A correct ft statement consistent with their p –value and 0.05 **or** their CR and 2– no context needed but do not allow contradicting non contextual comments. The comparison of their p-value and the significance level is not counted as a non contextual statement. May be implied by a correct ft conclusion in context. Must have a p-value or CR to access this mark.	
	A1	Correct conclusion in context which must be **not rejecting** H_0. Must use underlined words (oe). No hypotheses then A0	
(c)	**M1**	For recognising $\mathrm{P}(X=0) = 0.9^n$	
	M1	For $0.9^{44} (= 0.00969...)$ or $0.9^{43} (= 0.01077...)$ or rearranging to $n > \dfrac{\ln 0.01}{\ln 0.9}$ … (Allow =) $n >$ awrt 43.7 implies M1M1 (Allow n = awrt 43.7 for M1M1)	
	A1	Cao	
	SC	Use of tables only, $n = 40$, $p = 0.0148$ **and** $n = 50$, $p = 0.0052$ scores M1M0A0	

Question Number	Scheme	Marks
4 (a)	$\frac{9}{20}$	B1
		(1)
(b)	$(21k-k)\times\frac{\pi}{20}=1$	M1
	$k=\frac{1}{\pi}$ *	A1*
		(2)
(c) (i)	$\left[\mathrm{E}(X)=\frac{1}{2}(k+21k)\right]=\frac{11}{\pi}$	B1
(ii)	$\mathrm{Var}(X)=\frac{1}{12}(21k-k)^2$ or $\mathrm{Var}(X)=\int_{\frac{1}{\pi}}^{\frac{21}{\pi}}\frac{\pi}{20}x^2\,\mathrm{d}x-\left(\frac{11}{\pi}\right)^2$	M1
	$=\frac{100}{3\pi^2}$	A1
		(3)
(d)	$\mathrm{E}(A)=\pi\mathrm{E}(X^2)+4\mathrm{E}(X)+\frac{4}{\pi}$ $\mathrm{E}(A)=\int_{k}^{21k}\mathrm{f}(x)(A)\,\mathrm{d}x=\int_{k}^{21k}\frac{\pi}{20}(\pi)\left(x^2+\frac{4}{\pi}x+\frac{4}{\pi^2}\right)\mathrm{d}x$	M1
	$\mathrm{E}(X^2)=\frac{100}{3\pi^2}+\left(\frac{11}{\pi}\right)^2=\frac{463}{3\pi^2}$ $\mathrm{E}(A)=\frac{\pi}{20}(\pi)\left(\frac{x^3}{3}+(\frac{4}{\pi})\frac{x^2}{2}+\frac{4}{\pi^2}x\right)$	M1
	$\mathrm{E}(A)=\frac{463}{3\pi}+\frac{44}{\pi}+\frac{4}{\pi}$ sub limits $\frac{21}{\pi}$ and $\frac{1}{\pi}$	M1
	$=\frac{607}{3\pi}$ = awrt 64.4	A1
		(4)

	Notes	Total 10
(a)	**B1**	0.45oe cao
(b)	**M1**	use of the area of the rectangle = 1 Any equivalent rearrangement, allow $20k$ instead of $(21k - k)$
	A1*	answer is given so a fully correct solution must be seen
(c)(i)	**B1**	oe must be in terms of π (isw after correct answer seen)
(ii)	**M1**	use of $\frac{(b-a)^2}{12}$ or $\mathrm{Var}(X)=\int_{\frac{1}{\pi}}^{\frac{21}{\pi}}\frac{\pi}{20}x^2\,\mathrm{d}x-\left(\frac{11}{\pi}\right)^2$
	A1	for $\frac{100}{3\pi^2}$ oe must be in terms of π (isw after correct answer seen)
	SC	If both final answers are given in terms of k, score B1M1A0 for (c)(i) $11k$ and (c)(ii) $\frac{100}{3}k^2$
(d)	**M1**	for expanding $\mathrm{E}(A)=\mathrm{E}\left(\pi X^2+4X+\frac{4}{\pi}\right)$ **or** for setting up correct integral (ignore limits)
	M1	Valid method for finding $\mathrm{E}(X^2)$ i.e. use of $\mathrm{Var}(X)+\mathrm{E}(X)^2$ or integration of $x^2\mathrm{f}(x)$ **or** for integration of their $\mathrm{f}(x)A$ with at least one $x^n \to x^{n+1}$
	M1	substitution of their $\mathrm{E}(X)$ and their $\mathrm{E}(X^2)$ into their $\mathrm{E}(A)$ **or** for use of correct limits
	A1	for $\frac{607}{3\pi}$ or awrt 64.4

Question Number	Scheme	Marks
5 (a)	$X \sim \text{Po}(5)$	
	$\text{P}(X \leqslant 5) = 0.6160$ awrt 0.616	M1 A1
		(2)
(b)	$X \sim \text{B}(4,"0.616")$	B1ft
	$\text{P}(X < 2) = \text{P}(X \leqslant 1)$	M1
	$= 0.384^4 + 4 \times 0.616 \times 0.384^3$	M1
	$= 0.16126...$ awrt 0.161	A1
		(4)
(c)	X = The number of defects per x meters	
	$X \sim \text{N}\left(\frac{x}{16}, \frac{x}{16}\right)$	B1
	$\text{P}(X < 26) = \text{P}\left(Z < \frac{25.5 - \frac{x}{16}}{\sqrt{\frac{x}{16}}}\right) = 0.5398$	M1
	$\frac{25.5 - \frac{x}{16}}{\frac{1}{4}\sqrt{x}} = 0.1$	B1 M1 A1ft
	$\frac{1}{16}x + \frac{1}{40}\sqrt{x} - 25.5 = 0 \rightarrow \sqrt{x} = 20$ (or $\sqrt{x} = -20.4$)	M1
	$(\sqrt{x})^2 = 20^2$	M1
	$x = 400$	A1
		(8)

	Notes	Total 14
(a)	**M1**	For writing or using $\text{P}(X \leqslant 5)$
	A1	awrt 0.616
(b)	**B1ft**	For $X \sim \text{B}(4,0.616)$ Follow through their part (a). May be implied by a correct ft expression for the 2nd M1
	M1	For writing or using $\text{P}(X \leqslant 1)$ (May be implied by 2nd M1)
	M1	For $= [{}^4C_0](1-p)^4 + {}^4C_1 \times p \times (1-p)^3 \quad 0 < p < 1$
	A1	awrt 0.161 correct answer on its own scores 4 out of 4
(c)	**B1**	For $X \sim \text{N}\left(\frac{x}{16}, \frac{x}{16}\right)$ May be implied by values in standardisation.
	M1	For use of a continuity correction either 25.5 or 26.5 (Allow 24.5)
	B1	$z = \pm 0.1$ Allow calculator value if seen $\pm 0.0999(2986\ldots)$
	M1	Standardising using either 24.5 or 25 or 25.5 or 26 or 26.5 and equate to a z value. Follow through their mean and variance
	A1ft	A correct equation with compatible signs ft their mean and variance provided mean = variance
	M1	For solving their 3 term equation by factorising, completing the square or use of formula. May be implied by –20.4 , otherwise if answer is incorrect working must be shown.
	M1	For correct squaring of both sides. May be implied by 416[.16] from correct equation This mark may be scored prior to solving a 3TQ, e.g. $\left(25.5 - \frac{x}{16}\right)^2 = \left(\frac{1}{40}\sqrt{x}\right)^2$. Do not award if squaring each individual term
	A1	$x = 400$ only. This is dependent upon all previous marks in (c).
	SC	Use of $X \sim \text{N}\left(\frac{x}{16}, \frac{15x}{256}\right)$ leading to $x = 400$ scores max B0M1B1M1A0M1M1A0

Question Number	Scheme	Marks
6 (a)	$[\mathrm{F}(k)=1\Rightarrow]ak+bk^2=1\Rightarrow ak=1-bk^2$ *	B1*
		(1)
(b)	$\mathrm{f}(x)=a+2bx$	B1
	$\mathrm{E}(X)=\int_0^k\left(ax+2bx^2\right)\,\mathrm{d}x\left[=\frac{6}{5}\right]\Rightarrow\left[\frac{ax^2}{2}+\frac{2bx^3}{3}\right]_0^k\left[=\frac{6}{5}\right]$	M1
	$\frac{ak^2}{2}+\frac{2bk^3}{3}=\frac{6}{5}$	dM1, A1
	$15ak^2+20bk^3=36$	
	$15k\left(1-bk^2\right)+20bk^3=36$	M1
	$5bk^3=36-15k$ *	A1*
		(6)
(c)	$\mathrm{E}(X^2)=\int_0^k\left(ax^2+2bx^3\right)\,\mathrm{d}x\Rightarrow\left[\frac{ax^3}{3}+\frac{bx^4}{2}\right]_0^k$	M1
	$\mathrm{Var}(X)=\frac{ak^3}{3}+\frac{bk^4}{2}-\frac{36}{25}=\frac{22}{75}$	dM1 A1
	$10ak^3+15bk^4=52$	
	$10k^2\left(1-bk^2\right)+15bk^4=52$	M1
	$5bk^4=52-10k^2$ *	A1*
		(5)
(d)	$\frac{1}{k}=\frac{36-15k}{52-10k^2}$	M1
	$5k^2-36k+52=0$	A1
	$(k-2)(5k-26)=0$	M1
	$k=2$	A1
		(4)
(e)	$'40'b=36-'30'\Rightarrow b=\frac{3}{20}$ or $'80'b=52-'40'\Rightarrow b=\frac{3}{20}$	B1ft
	$2a+\frac{3}{5}=1\Rightarrow a=\frac{1}{5}$	B1ft
		(2)
	Notes	**Total 18**

(a)	**B1***	Answer is given so no incorrect working can be seen
(b)	**B1**	For a correct expression for f(x) (may be implied by a correct expression for E(X)
	M1	For an attempt to integrate x f(x) (Ignore limits) at least one $(x^n\rightarrow x^{n+1})$. F.t. their f(x) f(x) must be a changed expression from F(x) so integrating xF(x) is M0
	dM1	Dependent on the previous M mark. For equating to $\frac{6}{5}$ and substitution of k (no need to see substitution of lower limit 0).
	A1	For a correct equation any form
	M1	For substitution of $ak=1-bk^2$ oe into their equation
	A1*	Answer is given so no incorrect working can be seen

(c)	**M1**	For an attempt to integrate x^2 f(x) (Ignore limits) at least one $(x^n \rightarrow x^{n+1})$ F.t. their f(x) x^2 F(x) is M0
	dM1	Dependent on previous M mark. For substitution of correct limits and subtraction of $\frac{36}{25} = \frac{22}{75}$
	A1	For a correct equation any form
	M1	For substitution of $ak = 1 - bk^2$ oe into their equation
	A1*	Answer is given so no incorrect working can be seen
(d)	**M1**	For solving simultaneously to set up an equation in k only
	A1	For a correct 3 term quadratic
	M1	For solving their 3 term quadratic by factorising, completing the square or using formula. $k = 5.2$ implies M1A1M1
	A1	2 only cao. Correct answer on its own scores 4 out of 4
(e)	**B1ft**	For $b = \frac{3}{20}$ ft their k $\quad b = \frac{36-15k}{5k^3}$ Common ft answer is $b = \frac{-525}{8788} = \text{awrt} -0.0597$ coming from choosing $k = 5.2$
	B1ft	For $a = \frac{1}{5}$ ft their k and their b $\quad a = \frac{1-bk^2}{k}$ Common ft answer is $a = \frac{85}{169} = \text{awrt } 0.503$ coming from choosing $k = 5.2$