Mark Scheme (Results)

Summer 2022

Pearson Edexcel International Advanced Level
In Pure Mathematics P2 (WMA12) Paper 01

Question	Scheme	Marks
1	$1024 + \ldots$	**B1**
	$\ldots + {}^{10}C_1 \times \ldots x + {}^{10}C_2 \times \ldots x^2 + {}^{10}C_3 \times \ldots x^3 + \ldots$	**M1**
	$\ldots + 10 \times 2^9 \times \dfrac{3}{8}x + 45 \times 2^8 \times \left(\dfrac{3}{8}x\right)^2 + 120 \times 2^7 \times \left(\dfrac{3}{8}x\right)^3$ Or two of $\ldots + 1920x + 1620x^2 + 810x^3$	**A1**
	$\ldots + 1920x + 1620x^2 + 810x^3$	**A1**
		(4)
		(4 marks)

Notes:

B1: Correct constant term of 1024 as an integer.

M1: Correct binomial coefficient multiplied by the correct powers of x for **at least 2 terms** in x, x^2 or x^3.

Allow e.g. ${}^{10}C_1$, ${}^{10}C_2$, ${}^{10}C_3$ or $\begin{pmatrix}10\\1\end{pmatrix}$, $\begin{pmatrix}10\\2\end{pmatrix}$, $\begin{pmatrix}10\\3\end{pmatrix}$ or evaluated coefficients and condone missing

brackets e.g. $\dfrac{3}{8}x^2$ for $\left(\dfrac{3}{8}x\right)^2$ or $\dfrac{3}{8}x^3$ for $\left(\dfrac{3}{8}x\right)^3$

May take out a common factor of 2^{10} first, but again look for correct binomial coefficients multiplied by the correct powers of x for **at least 2 terms** in x, x^2 or x^3.

E.g. $\left(2 + \dfrac{3}{8}x\right)^{10} = 2^{10}\left(1 + \dfrac{3}{16}x\right)^{10} = 2^{10}\left(1 + {}^{10}C_1 \ldots x + {}^{10}C_2 \ldots x^2 + {}^{10}C_3 \ldots x^3 + \ldots\right)$

A1: Allow for a fully correct unsimplified expression (ignoring the constant term) with **evaluated binomial coefficients**, must be expanded if a common factor of 2^{10} is taken out first **OR** two of the three terms in x, x^2 and x^3 correct and simplified.

E.g. $\ldots + 10 \times 2^9 \times \dfrac{3}{8}x + 45 \times 2^8 \times \left(\dfrac{3}{8}x\right)^2 + 120 \times 2^7 \times \left(\dfrac{3}{8}x\right)^3$

Or $\ldots + 10 \times 2^{10} \times \dfrac{3}{16}x + 45 \times 2^{10} \times \left(\dfrac{3}{16}x\right)^2 + 120 \times 2^{10} \times \left(\dfrac{3}{16}x\right)^3$

The brackets must be present unless they are implied by subsequent work.

OR two of $\ldots + 1920x + 1620x^2 + 810x^3$. Allow terms to be "listed".

A1: Final three terms fully correct and simplified. Allow terms to be "listed".
Once a correct expansion (or list of terms) is seen then isw. E.g. some candidates think they have to list the coefficients separately but apply isw.
Ignore any extra terms if found.

Question	Scheme						Marks
2(a)	x	0.5	1	1.5	2	2.5	3	**B1**
	y	1.319	**1.075**	1.001	**1.041**	1.223	1.850	**B1**
								(2)

(b)	$h = 0.5$ seen or implied	**B1**
	$\text{Area} \approx \left\{\dfrac{0.5}{2}\right\} \times \left(1.319 + 2("1.075" + 1.001 + "1.041" + 1.223) + 1.850\right)$	**M1**
	$= \text{awrt } 2.96$	**A1**
		(3)

(c)	$3 + \log_{10}\left(\sin(x)\right) = 4 - \left(1 - \log_{10}\left(\sin(x)\right)\right)$	**B1**
	$\displaystyle\int_{0.5}^{3} 3 + \log_{10}(\sin(x))\,\mathrm{d}x = \left[4x\right]_{0.5}^{3} - "2.96" = 12 - 2 - "2.96"$	**M1**
	$= 7.04$ follow through $10 -$ their 2.96	**A1ft**
		(3)

Alt (c)	$\displaystyle\int_{0.5}^{3} \log_{10}(\sin(x))\,\mathrm{d}x = \int_{0.5}^{3} 1\,\mathrm{d}x - "2.96" = 2.5 - "2.96"$	**B1**
	$\displaystyle\int_{0.5}^{3} 3 + \log_{10}(\sin(x))\,\mathrm{d}x = \left[3x\right]_{0.5}^{3} + \left(2.5 - "2.96"\right) = 9 - 1.5 + 2.5 - "2.96"$	**M1**
	$= 7.04$ follow through $10 -$ their 2.96	**A1ft**
		(3)

		(8 marks)

Notes:

(a)
B1: Either of awrt 1.075 or awrt.1.041 correct
B1: Both of awrt 1.075 and awrt.1.041 correct
Remember to check the body of the script for the values if the table has not been completed.
Note that the incorrect values of 2.758 and 2.457 come from the use of degrees.
(Which gives 4.51 in (b))

(b)
B1: Correct interval width seen or implied by working.
M1: Correct application of the trapezium rule. Must all be y values, and should be first + last + 2(sum of middle values), but if one middle value is omitted allow as a slip. A repeated value is M0. Follow through their values from (a). Allow obvious copying slips.

Note that $\left\{\dfrac{"0.5"}{2}\right\} \times 1.319 + 2("1.075" + 1.001 + "1.041" + 1.223) + 1.850$ scores M0 unless the

missing brackets are implied by subsequent work.

Allow this mark if they add the areas of individual trapezia e.g.

$$\left\{\frac{"0.5"}{2}\right\}\times\left(1.319+"1.075"\right)+\left\{\frac{"0.5"}{2}\right\}\times\left("1.075"+1.001\right)+...$$

A1: For awrt 2.96. Allow to come from answers in (a) incorrectly rounded or wrong accuracy.
FYI: Use of rounded figure gives 2.96225, use of calculated figures gives 2.9625726… Actual value is 2.89 to 2 decimal places.

(c) Condone all work in (c) written without the "10" in $\log_{10}\left(\sin(x)\right)$

B1: Writes $3+\log_{10}\left(\sin(x)\right)$ correctly in terms of $1-\log_{10}\left(\sin(x)\right)$. May be implied.

M1: Integrates the K in $K\pm\left(1-\log_{10}\left(\sin(x)\right)\right)$ to $Kx,\ \ K\neq3,$ substitutes in the limits 3 and 0.5 and subtracts oe e.g. $4\times(3-0.5)$ and uses the answer to (b).

A1ft: Correct answer awrt 7.04, follow through their 2.96 if given to 3sf or better.

Alt:

B1: Rearranges integral to find $\displaystyle\int_{0.5}^{3}\log_{10}(\sin(x))\,dx=2.5-"2.96"$. May be implied.

M1: Integrates the 3 and substitutes in given limits and subtracts oe e.g. $3\times(3-0.5)$ and adds their attempt at $\displaystyle\int_{0.5}^{3}\log_{10}(\sin(x))\,dx$ found using the answer to (b).

A1ft: Correct answer awrt 7.04, follow through their 2.96 if given to 3sf or better.

<u>**A common incorrect method in (c):**</u>

$$\int_{0.5}^{3}3+\log_{10}(\sin(x))\,dx=\left[3x\right]_{0.5}^{3}+"2.96"="10.46"$$

Scores no marks as they have not used the integral in part (b)

Attempts to use the trapezium rule again in (c) score no marks

Question	Scheme	Marks
3(i)	E.g. $n = 1 : 2^3 - 1^3 = 7$, $n = 2 : 3^3 - 2^3 = 19$, $n = 3 : 4^3 - 3^3 = ...$ Or identifies counterexample directly.	**M1**
	e.g. $6^3 - 5^3 = 91 = 7 \times 13$ so not true for $n = 5$, hence statement is not true.	**A1**
		(2)
	Notes for part (i) **M1:** Shows evidence of trying to find a counter example for a positive integer (at least one attempt). $2^3 - 1^3$ is prime is sufficient. **A1:** Gives a correct counter example with reason (shows factorisation) and concludes e.g. "which is not prime". Ignore any previous "incorrect" attempts e.g. $6^3 - 5^3 = 91$ which is prime. Note $n = 7$ ($169 = 13 \times 13$) and $n = 8$ ($217 = 7 \times 31$) and $n = 12$ ($469 = 7 \times 67$) are the next few counter examples. (Bigger examples are not likely to be seen!) Allow equivalent reasons for not being prime e.g. $169/13 = 13$ or 169 is divisible by 13 (condone "can be divided by 13") **Generally algebraic approaches score no marks unless they substitute numbers as indicated above.**	
3(ii)	**The majority of methods here will follow ways 1, 2 or 3 below** **In these cases the general guidance is as follows:** M1: Attempts to find • the gradient of any relevant line, e.g. *AC* or *BC* **or** • the length of any relevant line, e.g. AB/AB^2 or BC/BC^2 or AC/AC^2 **or** • the mid-point *M* of line *AB* A1: Correct **relevant** calculation of • gradients *AC* and *BC* • lengths of lines AB/AB^2, BC/BC^2 and AC/AC^2 • mid-point of line *AB* dM1: Full attempt at combining all **relevant** information required to solve the problem • attempts product of gradients or equivalent • attempts to show Pythagoras $AB^2 = AC^2 + BC^2$ • attempts to show $MA^2 = MC^2$ A1: Correct calculations or equivalent providing required evidence for the above A1: Provides correct reason and conclusion with all previous marks scored.	

Way 1	$m_{AC} = \dfrac{-6-0}{7-1} = \ldots$ or $m_{BC} = \dfrac{-6-(-10)}{7-3} = \ldots$	M1
	$m_{AC} = -1$ and $m_{BC} = 1$	A1
	So $m_{AC} \times m_{BC} = -1 \times 1 = -1$ or e.g. m_{AC} is negative reciprocal of m_{BC}	dM1A1
	So e.g. angle (at C) is a right angle hence AB is a diameter (Or equivalent)	A1
		(5)

M1: Attempts the gradients of AC or BC. Allow slips but score M0 if both attempts are clearly incorrect.
A1: Correct gradients from correct formulae
dM1: Applies perpendicular condition. May be seen as shown but allow equivalent work.
A1: Correct calculations or equivalent
A1: Suitable explanation and conclusion given with no errors and all previous marks awarded with no incorrect statements seen.

Way 2	$AB = \sqrt{(3-1)^2 + (-10-0)^2} = \ldots$ or $AC = \sqrt{(7-1)^2 + (-6-0)^2} = \ldots$ or $BC = \sqrt{(7-3)^2 + (-6+10)^2} = \ldots$	M1
	$AB = \sqrt{104}\left(2\sqrt{26}\right), AC = \sqrt{72}\left(6\sqrt{2}\right), BC = \sqrt{32}\left(4\sqrt{2}\right)$	A1
	$AB^2 = 104 = 72 + 32 = AC^2 + BC^2$	dM1A1
	Hence ABC is a right-angle triangle with hypotenuse AB hence AB is a diameter. (Or equivalent)	A1
		(5)

M1: Attempts length of AB or AC or BC or their squares. Allow slips but score M0 if attempts are clearly incorrect.
A1: Correct values for AB, AC and BC or their squares.
dM1: Applies Pythagoras' theorem with their values. (May see cosine rule used.)
A1: All calculations correct for this approach.
A1: Suitable explanation and conclusion given with no errors and all previous marks awarded with no incorrect statements seen.

Way 3	If AB is diameter centre must be midpoint of AB ie $M\left(\dfrac{1+3}{2},\dfrac{0-10}{2}\right)$	M1
	$=(2,-5)$	A1
	$MA=\sqrt{(2-1)^2+(-5-0)^2}=\sqrt{26},\ \ MC=\sqrt{(2-7)^2+(-5-(-6))^2}=\sqrt{26}$	dM1A1
	$MA=\sqrt{26},MC=\sqrt{26}$ so $MA{=}MC({=}MB)$ As the length from M to each of A and C is the same M is the centre of the circle hence AB is a diameter. (Or equivalent)	A1
		(5)

M1: Attempts midpoint of AB. If no method is shown accept one correct coordinate as evidence.
A1: Correct midpoint
dM1: Attempts length of MC and at least one of MA or MB, or AB. *As M is midpoint of AB there is no need to find both MA and MB, these may be assumed to be the same.* If they find AB then they must halve it to find the radius.
A1: All required calculations correct for this approach.
A1: Suitable explanation made which may be in a preamble and conclusion given with no errors and all previous marks awarded.

The following approach is less common and should be marked as shown:

Way 4	If AB is diameter centre must be midpoint of AB ie $M\left(\dfrac{1+3}{2},\dfrac{0-10}{2}\right)$	M1
	$=(2,-5)$	A1
	$MA=r=\sqrt{(2-1)^2+(-5-0)^2}=\sqrt{26}$ $\Rightarrow(x-2)^2+(y+5)^2=26$ $C(7,-6)\Rightarrow(7-2)^2+(-6+5)^2=5^2+1^2=26$	dM1A1
	As C also satisfies the equation of the circle then AB must be the diameter (or equivalent) There must be some further justification as above rather than just "AB is a diameter" which may be in a preamble e.g. If C lies on the circle…	A1
		(5)

M1: Attempts midpoint of AB. If no method is shown accept one correct coordinate as evidence.
A1: Correct midpoint
dM1: Attempts length of MA or MB to find r or r^2, forms equation of the circle and substitutes the coordinates of C.
A1: All required calculations correct for this approach.
A1: Suitable explanation and conclusion given with no errors and all previous marks awarded.

SEE END OF MARKSCHEME FOR SOME OTHER MORE UNUSUAL METHODS THAT MAY BE SEEN DURING MARKING WITH SUGGESTED MARKING GUIDELINES.

There may be other methods. Choose the way that best fits the overall response.
If you are in any doubt if a particular response deserves credit then use Review.

Question	Scheme	Marks
4	$\log_4 a + \log_4 b = \log_4 ab$ or $\log_4 "ab" = 3 \Rightarrow "ab" = 4^3$	M1
	$ab = K, a - b = 8 \Rightarrow a(a-8) = K$ or $(b+8)b = K$	dM1
	$a^2 - 8a - 64 = 0$ or $b^2 + 8b - 64 = 0$	A1
	$a = \dfrac{-(-8) + \sqrt{(-8)^2 - 4 \times 1 \times -64}}{2 \times 1} = ...$ or $b = \dfrac{-8 + \sqrt{8^2 - 4 \times 1 \times -64}}{2 \times 1} = ...$	M1
	$a = '4 + 4\sqrt{5}' \Rightarrow b = -4 + 4\sqrt{5}$	M1
	$a = 4 + 4\sqrt{5}$ and $b = -4 + 4\sqrt{5}$ and no other solutions.	A1
		(6)
		(6 marks)

Notes:

M1: Correct addition law applied (may be implied) **or** undoes a log equation correctly, **or**
replaces the 3 by $\log_4 4^3$ oe

The addition law may be seen after substitution from the first equation e.g.
$a - b = 8 \Rightarrow a = b + 8 \Rightarrow \log_4 a + \log_4 b = \log_4(b+8) + \log_4 b = \log_4 b(b+8)$

Condone $\log_4(b+8) + \log_4 b = \log_4 b^2 + 8b$

dM1: Removes logs correctly and proceeds from an equation of the form $ab = K$ and the
given equation $a - b = 8$ to a quadratic equation in either a or b.

A1: Correct quadratic equation. Brackets must be expanded but terms not necessarily all on one
side. The " $=0$" may be implied by their attempt to solve.

M1: Solves a 3TQ in a or b to obtain a positive solution. There is no need to see the
negative solution in their working.

M1: Uses their a to find b or vice versa. This depends on having solved a 3TQ earlier and obtained a
root of the form $p + q\sqrt{r}$, $p, q, r \neq 0$ to obtain another root of a similar form i.e. not a decimal.

The value of a or b does not necessarily have to be positive for this mark.
Note that having found a or b they may repeat the process above to find the other value which
is acceptable.

A1: Both of $a = 4 + 4\sqrt{5}$ and $b = -4 + 4\sqrt{5}$ simplified, with no other solutions.

Apply isw once correct answers are seen, e.g. if they subsequently go into decimals.

Question	Scheme	Marks
5	$3\tan(\theta+43°)=2\cos(\theta+43°) \Rightarrow 3\dfrac{\sin(\theta+43°)}{\cos(\theta+43°)}=2\cos(\theta+43°)$ $\Rightarrow 3\sin(\theta+43°)=2\cos^2(\theta+43°)$	M1
	$\Rightarrow 3\sin(\theta+43°)=2\left(1-\sin^2(\theta+43°)\right)$	M1
	$\Rightarrow 2\sin^2(\theta+43°)+3\sin(\theta+43°)-2=0$ $\Rightarrow \left(2\sin(\theta+43°)-1\right)\left(\sin(\theta+43°)+2\right)=0 \Rightarrow \sin(\theta+43°)=...$	M1
	$\sin(\theta+43°)=\dfrac{1}{2}$	A1
	$\theta=\arcsin"\dfrac{1}{2}"-43°$	M1
	$\theta=-13°,\ 107°$	A1
		(6)
		(6 marks)

Notes:

M1: Uses $\tan..=\dfrac{\sin..}{\cos..}$ and multiplies through to form an equation of the form $A\sin...=B\cos^2...$

Condone poor notation e.g.:

$3\tan(\theta+43°)=2\cos(\theta+43°)\Rightarrow 3\dfrac{\sin}{\cos}(\theta+43°)=2\cos(\theta+43°)$

$\Rightarrow 3\sin(\theta+43°)=2\cos^2(\theta+43°)$ (with or without brackets)

M1: Applies Pythagorean identity to obtain a 3 term quadratic equation in sin.

Allow use of $\cos^2...=\pm1\pm\sin^2...$

M1: Solves a 3 term quadratic in $\sin(\theta+43°)$ by any valid means.

This may be implied by at least one correct root for their quadratic.

Allow if they have $\sin(\theta+43°)=x$ or another variable or e.g. $\sin\alpha$ where $\alpha=\theta+43°$

A1: Correct value of $\sin(\theta+43°)$. If $\sin(\theta+43°)=x$ is used, it must be clear they mean

$\sin(\theta+43°)$ but this may be implied if they have e.g. $\sin\alpha=½$ where $\alpha=\theta+43°$

If $x=\dfrac{1}{2}$ is left as a final answer it is A0.

M1: Correct method for solving $\sin(\theta+43°)=k,|\,k\,|<1$, look for use of inverse sine followed by

subtraction of 43 from $\sin^{-1}$(their k). Implied by one correct solution for their k

Do **not** allow mixing of degrees and radians for this mark.

A1: Correct solutions and no others in the range.

Question	Scheme	Marks
6(a)	$u_n = ar^{n-1} \Rightarrow ar + ar^2 = 6$ and $ar^3 = 8$	M1
	$\Rightarrow \dfrac{ar + ar^2}{ar^3} = \dfrac{6}{8} \Rightarrow 1 + r = \dfrac{3}{4}r^2$	M1
	$\Rightarrow 3r^2 - 4r - 4 = 0 *$	A1*
		(3)
(a) Way 2	$u_4 = 8 \Rightarrow u_3 = \dfrac{8}{r}, \ u_2 = \dfrac{8}{r^2}$	M1
	$\dfrac{8}{r} + \dfrac{8}{r^2} = 6$	M1
	$\Rightarrow 3r^2 - 4r - 4 = 0 *$	A1*
		(3)
(b)	$r = -\dfrac{2}{3}$	B1
	$ar^3 = 8 \Rightarrow a = \dfrac{8}{\left(-\dfrac{2}{3}\right)^3} = \ldots$	M1
	$u_1 = -27$	A1ft
		(3)
(c)	$S_\infty = \dfrac{-27}{1 - \left(-\dfrac{2}{3}\right)} = \ldots$	M1
	$= -\dfrac{81}{5}$	A1
		(2)
		(8 marks)

Notes:

(a)   **Ignore labelling and mark (a), (b) and (c) together**

M1: Uses the correct *n*-th term formula for a GP to set up two equations in a and r

May also be in terms of u_1 and r or u_2 and r or u_3 and r e.g.

$u_1 r + u_1 r^2 = 6, \ u_1 r^3 = 8$ or $u_2 + u_2 r = 6, \ u_2 r^2 = 8$ or $\dfrac{u_3}{r} + u_3 = 6, \ ru_3 = 8$

Must be using the correct term formula so e.g. $ar^2 + ar^3 = 6$ and $ar^4 = 8$ is M0

Alternatively may use the correct sum formula for the first equation, $a\dfrac{1 - r^3}{1 - r} - a = 6$ oe.

M1: Attempts to solve their two equations to get an equation in *r*. Look for an attempt to divide the
two equations, or an attempt to find *a* in terms of *r* from one and substitute into the other.
Allow slips but the algebra should essentially be correct so do not allow use of e.g. $ar^3 = (ar)^3$

Alternatively, attempts to eliminate r from the equation, $r = \dfrac{2}{\sqrt[3]{a}} \Rightarrow \dfrac{2a}{\sqrt[3]{a}} + \dfrac{4a}{\sqrt[3]{a^2}} = 6$

$\Rightarrow 2a^{\frac{2}{3}} + 4a^{\frac{1}{3}} = 6$. Award when they reach a quadratic in $a^{\frac{1}{3}}$ in this case.

A1*: CSO Note that as we are marking (a), (b) and (c) together, allow the printed answer to appear anywhere as long as it follows correct work but the "=0" must be seen not implied.

Way 2:

M1: Uses $u_4 = 8$ to write u_2 and u_3 in terms of r

M1: Uses $u_2 + u_3 = 6$ to get an equation in r

A1*: CSO

(b)

B1: Correct value of r seen or used in their working even if subsequently rejected and ignore any other value offered e.g. $r = 2$

M1: Uses a value for r from solving the equation given in (a) where $|r| < 1$ in one of their equations from (a) to find a value for u_1 or a. Allow slips but the algebra should essentially be correct

A1ft: Correct value and no other values. Follow through on $\dfrac{8}{\left(\text{their } r\right)^3}$ for a value of r with $|r| < 1$

(c)

M1: Uses the correct sum formula with their a (u_1) obtained from $|r| < 1$ and r where $|r| < 1$, to find the sum to infinity.

A1: Correct answer and no other values. Allow equivalents e.g. -16.2

Question	Scheme	Marks
7	$(x+2)$ a factor $\Rightarrow f(-2) = 0 \Rightarrow -8A + 24 + 8 + B = 0$	M1A1
	$\int f(x)dx = \dfrac{A}{4}x^4 + 2x^3 - 2x^2 + Bx$	M1 A1
	$\int_3^5 f(x)dx = 176 \Rightarrow \left[\dfrac{A}{4}x^4 + 2x^3 - 2x^2 + Bx \right]_3^5 = 176$ $\Rightarrow \left(\dfrac{A}{4}5^4 + 2(5^3) - 2(5^2) + 5B \right) - \left(\dfrac{A}{4}3^4 + 2(3^3) - 2(3^2) + 3B \right) = 176$	dM1
	$\left. \begin{array}{l} 8A - B = 32 \\ 136A + 2B = 12 \end{array} \right\} \Rightarrow A = ..., B = ...$	dM1
	$A = \dfrac{1}{2}, \ B = -28$	A1
		(7)
		(7 marks)

Notes:

M1: Uses the factor theorem or long division (see below) to find one linear **equation** relating A and B. The "= 0" must be seen or implied by later work.

A1: Correct equation. Award once a correct equation is seen unsimplified or simplified.

So allow e.g. $A(-2)^3 + 6(-2)^2 - 4(-2) + B = 0$ but not e.g. $A \times -2^3 + 6 \times -2^2 - 4 \times -2 + B = 0$

unless the indices are processed correctly subsequently to e.g. $-8A + 24 + 8 + B = 0$

The "= 0" must be seen or implied by later work.

M1: Attempts to integrate the given expression, look for at least one power increased by 1.

This can also be evidenced by $B \rightarrow Bx$

A1: Correct integration. Allow unsimplified e.g. $\dfrac{6x^{2+1}}{2+1}$ for $2x^3$

Condone poor notation e.g. $\int f(x)dx = \int \dfrac{A}{4}x^4 + 2x^3 - 2x^2 + Bx$ and ignore any "+ c"

dM1: Applies limits 5 and 3 either way round and subtracts and equates to 176 to obtain an equation in A and B. **Depends on <u>previous</u> M.**

Condone poor use of brackets when subtracting e.g. allow

$\dfrac{A}{4}5^4 + 2(5^3) - 2(5^2) + 5B - \dfrac{A}{4}3^4 + 2(3^3) - 2(3^2) + 3B = 176$

dM1: Solves their two equations in A and B to find values for both constants.

Depends on <u>all previous</u> M marks <u>but allow this mark if the only error was to attempt f (2) = 0 at the beginning</u>.

A1: Correct values.

Note that some candidates eliminate A or B by rearranging the first equation and substituting into the integral. This essentially follows the main scheme:

$f(-2) = 0 \Rightarrow -8A + 24 + 8 + B = 0 \Rightarrow B = 8A - 32$	**M1A1**
$\displaystyle\int f(x)\,dx = \dfrac{A}{4}x^4 + 2x^3 - 2x^2 + 8Ax - 32x$	**M1A1**
$\Rightarrow \left(\dfrac{A}{4}5^4 + 2(5^3) - 2(5^2) + 40A - 160\right) - \left(\dfrac{A}{4}3^4 + 2(3^3) - 2(3^2) + 24A - 96\right) = 176$	**dM1**
$\Rightarrow A = ..., \quad B = ...$	**dM1**
$A = \dfrac{1}{2}, \; B = -28$	**A1**
<u>**Long Division for reference:**</u> $$\begin{array}{r} Ax^2 + (6-2A)x + 4A - 16 \\ x+2 \overline{\smash{\big)}\ Ax^3 + 6x^2 - 4x + B} \\ \underline{Ax^3 + 2Ax^2} \\ (6-2A)x^2 - 4x + B \\ \underline{(6-2A)x^2 + (12-4A)x} \\ (4A-16)x + B \\ \underline{(4A-16)x + 8A-32} \\ B - 8A + 32 \end{array}$$ Score the first M1 for a complete method to obtain a remainder in terms of A and B that is set $= 0$ and A1 for a correct equation e.g. $B - 8A + 32 = 0$	

Question	Scheme	Marks
8(a)	$$\left(\frac{dy}{dx}=\right)Ax^3+B+\frac{C}{x^3}$$	M1
	$$\left(\frac{dy}{dx}=\right)4\times256x^3-304-2\times27x^{-3} \text{ oe e.g. } 1024x^3-304-\frac{54}{x^3}$$	A1A1
		(3)
(b)	$$\frac{dy}{dx}=0 \text{ at SP} \Rightarrow 1024x^3-304-\frac{54}{x^3}=0 \Rightarrow 1024x^6-304x^3-54=0$$	M1
	$$\Rightarrow 512\left(x^3\right)^2-152x^3-27=0 \Rightarrow \left(8x^3+1\right)\left(64x^3-27\right)=0 \Rightarrow x^3=...$$	dM1
	$$x^3=-\frac{1}{8} \text{ or } x^3=\frac{27}{64} \text{ or } x=-\frac{1}{2} \text{ or } x=\frac{3}{4}$$	A1
	$$y=256\left("-\frac{1}{2}"\right)^4-304\left("-\frac{1}{2}"\right)-35+\frac{27}{\left("-\frac{1}{2}"\right)^2}=... \text{ or with } x=\frac{3}{4}$$	dM1
	Coordinates are $\left(-\frac{1}{2},241\right)$ and $\left(\frac{3}{4},-134\right)$	A1
		(5)
		(8 marks)

Notes:

(a)

M1: Attempts the derivative, with at least two terms of the correct form (ie $x^n \rightarrow ..x^{n-1}$ at least twice).

A1: At least two terms correct, need not be simplified.

A1: Fully correct derivative, need not be simplified. Isw after a correct (unsimplified) answer.

Note that there is no need to see the "$\frac{dy}{dx}=$" in (a) just look for the differentiation.

Ignore any spurious "= 0"

(b)

M1: Sets their derivative equal to zero and multiplies through by "x^3" to achieve a polynomial equation. If in doubt at least 2 terms must be multiplied.

Allow equivalent work e.g.

$$1024x^3-304-\frac{54}{x^3}=0 \Rightarrow \frac{1}{x^3}\left(1024x^6-304x^3-54\right)=0 \Rightarrow 1024x^6-304x^3-54=0$$

To score this mark, the derivative must have a negative power of x so allow for e.g.

$$\left(\frac{dy}{dx}=\right)1024x^3-304-54x^{-1}=0 \Rightarrow 1024x^4-304x-54=0$$

dM1: Solves a 3 term quadratic in x^3 by any valid means (including calculator).

Must come from an attempt at $\frac{dy}{dx}=0$ not $\frac{d^2y}{dx^2}=0$ or $\int y\,dx=0$

Condone use of inequalities rather than "=" as long as they solve an equation.

Note that here we accept e.g. $512\left(x^3\right)^2 - 152x^3 - 27 = 0 \Rightarrow (8y+1)(64y-27) = 0 \Rightarrow y = ...$

Or even $512\left(x^3\right)^2 - 152x^3 - 27 = 0 \Rightarrow (8x+1)(64x-27) = 0 \Rightarrow x = ...$

A1: Achieves at least one correct value for x or x^3. It must be clear that they are values for x^3 here so $y = ...$ is acceptable if $y = x^3$ is seen or implied. If they have $x = ...$ then they must have cube rooted. So e.g. $(8x+1)(64x-27) = 0 \Rightarrow x = -\dfrac{1}{8}$ unless they recover and recognise they have x^3 then this is A0

dM1: Proceeds to find the y coordinate for at least one value of x. Must **cube root** and not e.g. square root the solution from the quadratic first. The x must have come from a 2 or 3 term quadratic in x^3.
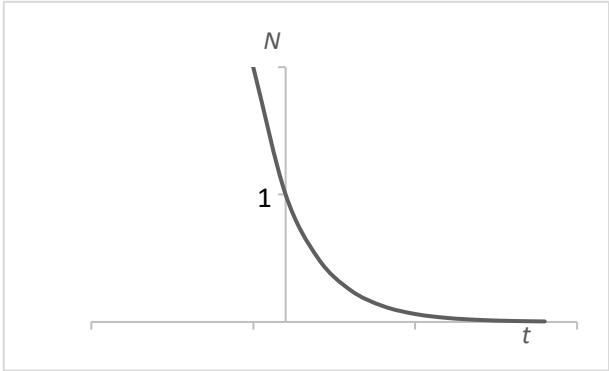If no method shown, accept any value for y having found an x value but score M0 if there is no evidence of cube rooting or clear evidence that they have substituted into something other than the curve equation but condone if they clearly just mis-copy the equation.
Depends on having scored at least 1 of the previous M marks in (b)

A1: Both correct pairs of coordinates. Accept if given as e.g. $x = -\dfrac{1}{2}$, $y = 241$ and $x = \dfrac{3}{4}$, $y = -134$

And no other values.

Allow equivalent fractions or values for the $-\dfrac{1}{2}$ and $\dfrac{3}{4}$

Question	Scheme	Marks
9(a)		
	Correct shape from or passing through a point on positive vertical axis. May extend to the left of the vertical axis and allow to pass into quadrant 4. There must be no (obvious) turning points. Labels not required on axes and ignore any that are given.	**M1**
	Shape and position correct, accept 1 or k as intercept on the positive vertical axis and allow to extend to the left of the vertical axis as shown. Condone $(1, 0)$ or $(k, 0)$ as long as it is in the correct position. The curve should approach a horizontal asymptote that is half-way between the horizontal axis and the intercept or below. Be tolerant with "wobbles" as it approaches the asymptote. May just "touch" the horizontal axis but not go below it. Labels not required on axes and ignore any that are given.	**A1**
		(2)
(b)	$\dfrac{1}{2}k = k\lambda^{5700} \Rightarrow \lambda^{5700} = \dfrac{1}{2}$ (see notes for method via substitution)	**M1**
	$\Rightarrow \lambda = \left(\dfrac{1}{2}\right)^{\frac{1}{5700}} = 0.999878$ to 6 d.p.*	**A1***
		(2)
(c)	When $t = 3250$, $N = 15 \times 0.999878^{3250} = ...$	**M1**
	$= \text{awrt } 10.1 \text{ (grams)}$	**A1**
		(2)
(d)	$18 = 25 \times 0.999878^{t}$	**B1**
	$\Rightarrow 0.999878^{t} = \dfrac{18}{25} \Rightarrow t = \dfrac{\log \dfrac{18}{25}}{\log 0.999878} = ...$	**M1**
	$t = 2692.49...$ so item is 2700 years old	**A1**
		(3)
		(9 marks)
Notes:		
(a) **M1:** See scheme. **A1:** See scheme.		

(b)

M1: Uses the information to set up a correct equation and rearranges to form $\lambda^a = b$ where a and b are constants. Allow $\lambda^{5700} = \dfrac{1}{2}$ to be written down directly.

A1*: Evidence of taking fractional root seen, leading to the given answer. Alternatively, may see logs used, $5700\log\lambda = \log\dfrac{1}{2} \Rightarrow \log\lambda = \dfrac{-\log 2}{5700} \Rightarrow \lambda = 10^{\frac{-\log 2}{5700}} = ...$ Look for at least one correct intermediate step (and no incorrect ones). Allow greater accuracy e.g. 0.9998784026…
Note that the use of 5699 instead of 5700 scores M1A0

Alt:

M1: Substitutes values 0.9998775 and 0.9998785 or a tighter range containing the root (e.g. 0.999878 and 0.9998785 will do) to calculate N or just λ^t at $t = 5700$.

A1*: Correct values, with suitable conclusion that as half value occurs between these $\lambda = 0.999878$ to 6 d.p.

(c)

M1: Substitutes given values (or more accurate λ) into the equation and evaluates. Implied by a correct answer, but an incorrect answer with no evidence is M0. Allow 3249 for 3250.
Condone the use of a less accurate λ e.g. 0.9999 if the intention is clear.

A1: Awrt 10.1. Units not required. Answer only can score both marks.

(d)

B1: Sets up the correct equation from the information given. It must be clear they are using the given λ or better accuracy so clear use of e.g. 0.9999 scores B0 but allow the M1 below.
This may be implied if they write e.g. $18 = 25 \times \lambda^t$ and then go on to use the given value of λ.
Allow this mark if a different letter is used for t as long as the equation is correct.

M1: Solves an equation of the form $a = b\lambda^t$ for t to obtain a value

Must be correct log work so look for e.g. $a = b\lambda^t \Rightarrow \dfrac{a}{b} = \lambda^t \Rightarrow t = \log_\lambda \dfrac{a}{b}$

Or e.g. $a = b\lambda^t \Rightarrow \dfrac{a}{b} = \lambda^t \Rightarrow t = \log\dfrac{a}{b} = \log\lambda^t = t\log\lambda \Rightarrow t = \dfrac{\log\dfrac{a}{b}}{\log\lambda}$

Allow a mis-copied λ or a less accurate λ for this mark.

A1: Correct answer of 2700 (not awrt). Accept "27 hundred" or even "2692.49… so 27"

FYI use of $\lambda = 2^{-\frac{1}{5700}}$ gives 2701.4 years.

Question	Scheme	Marks
10(a)	Equation of circle is $(x-3)^2+(y-5)^2=r^2$ and line is $y=2x+k$ So intersect when $(x-3)^2+(2x+k-5)^2=r^2$	M1
	$\Rightarrow x^2-6x+9+4x^2+4(k-5)x+(k-5)^2=r^2$ $\Rightarrow 5x^2+(-6+4k-20)x+9+k^2-10k+25-r^2=0$	dM1
	$\Rightarrow 5x^2+(4k-26)x+k^2-10k+34-r^2=0*$	A1*
		(3)
(b)	Tangent to $C \Rightarrow b^2-4ac=0 \Rightarrow (4k-26)^2-4\times 5\times(k^2-10k+34-r^2)=0$	M1
	$\Rightarrow 16k^2-208k+676-20k^2+200k-680+20r^2=0$ $\Rightarrow 5r^2=...$	M1
	$\Rightarrow 5r^2=k^2+2k+1=(k+1)^2$	A1
		(3)
(b) Way 2	Gradient of BX is $-\dfrac{1}{2}$ so equation of BX is $y-5=-\dfrac{1}{2}(x-3)$ $y-5=-\dfrac{1}{2}(x-3),\, y=2x+k \Rightarrow x=...,\, y=...\left(\dfrac{13-2k}{5},\dfrac{26+k}{5}\right)$	M1
	$\left(\dfrac{13-2k}{5}-3\right)^2+\left(\dfrac{26+k}{5}-5\right)^2=r^2$	dM1
	$\Rightarrow 5r^2=k^2+2k+1=(k+1)^2$	A1
		(3)
(c)	Triangle AXB is right angled so $AB^2+r^2=XA^2=(3-0)^2+(5-k)^2$	M1
	$AB^2=4r^2$ so $AB^2+r^2=5r^2$	M1
	$\Rightarrow 5r^2=9+(5-k)^2$	A1
	$\Rightarrow k^2+2k+1=9+25-10k+k^2$	M1
	$\Rightarrow 12k=33 \Rightarrow k=...$	dM1
	$k=\dfrac{11}{4}$	A1
		(6)
		(12 marks)

Notes:

(a)

M1: Forms equation of circle and substitutes in equation of line.

The circle equation must be of the form $(x\pm 3)^2+(y\pm 5)^2=r^2$

dM1: Expands both sets of brackets and collects terms in x^2 and x.

A1*: Reaches the answer given with no errors seen.

Note that some candidates expand the brackets first before substitution e.g.:

$(x-3)^2 + (y-5)^2 = r^2 \Rightarrow x^2 - 6x + 9 + y^2 - 10y + 25 = r^2 \Rightarrow x^2 - 6x + 9 + (2x+k)^2 - 10(2x+k) + 25 = r^2$

This implies the first M and then the second M will score when terms in x^2 and x are collected.

Note about poor squaring e.g. $(x-3)^2 = x^2 + 9$: The first M is available in both cases above but the second M requires at least two x^2 terms and at least two x terms from the expansions.

Note that it is acceptable to go from a completely correct full expansion to the printed answer e.g.

$(x-3)^2 + (y-5)^2 = r^2 \Rightarrow x^2 - 6x + 9 + y^2 - 10y + 25 = r^2 \Rightarrow x^2 - 6x + 9 + (2x+k)^2 - 10(2x+k) + 25 = r^2$

$\Rightarrow x^2 - 6x + 9 + 4x^2 + 4kx + k^2 - 20x - 10k + 25 = r^2 \Rightarrow 5x^2 + (4k-26)x + k^2 - 10k + 34 - r^2 = 0$

Scores full marks in (a)

(b)

M1: Uses the discriminant is zero to form an equation in k and r

dM1: Expands and rearranges to make αr^2 the subject

A1: Correct answer

Way 2

M1: Attempts the equation of *BX* and solves simultaneously with l to find the coordinates of *B*

Alternatively uses $x = -\dfrac{b}{2a} = \dfrac{13-2k}{5}$ at *B* and uses this to find y at *B*

dM1: Correct use of Pythagoras for *BX* and sets $= r^2$

A1: Correct answer

(c)

M1: Attempts XA^2 **correctly** in terms of k (the k may appear as y but must be replaced by k later) and uses it in Pythagoras theorem for triangle *AXB*.

May be implied.

M1: Applies $AB = 2r$ to get $AB^2 + r^2$ in terms of r. Condone with $AB^2 = 2r^2$ used.

A1: Correct equation.

M1: Substitutes the result from (b) and expands brackets.

dM1: Solves a linear equation in k. **Depends on the previous M.**

A1: Correct value.

Alt (c)		
	At B $x = -\dfrac{b}{2a} = \dfrac{13-2k}{5}$, $y = 2\left(\dfrac{13-2k}{5}\right) + k$	M1
	$AB^2 = \left(\dfrac{13-2k}{5} - 0\right)^2 + \left(\dfrac{26-4k}{5} + k - k\right)^2$	M1 A1
	$\Rightarrow 25 \times 4r^2 = (13-2k)^2 + (26-4k)^2 = ...$	M1
	$\Rightarrow 20 \times (k+1)^2 = (13-2k)^2 + (26-4k)^2 = 20k^2 - 260k + 845$ $\Rightarrow k^2 + 2k + 1 = k^2 - 13k + \dfrac{169}{4} \Rightarrow k = ...$	dM1
	$\Rightarrow k = \dfrac{11}{4}$	A1
		(6)

Notes:

(c)

M1: Uses the result in (a) to find the x coordinate where the line and circle meet and then finds y
    An alternative is to find the equation of BX as in (b) way 2 and solve with l to find x and y at B
    (May have already found the coordinates of B in (a) but must be re-stated or used in (c) to score
    this mark)

M1: Uses distance formula to find an expression in k for AB or AB^2

A1: Correct expression for AB or AB^2. Need not be simplified.

M1: Applies $AB = 2r$ to the equation. Condone with $AB^2 = 2r^2$ used.

dM1: Substitutes the result from (b) and solves a linear equation in k. **Depends on the previous M.**

A1: Correct value.

Some more unusual methods for 3ii

Via perpendicular bisectors:

Way 5	If AB is diameter centre must be midpoint of AB ie $M\left(\dfrac{1+3}{2},\dfrac{0-10}{2}\right)$	**M1**
	$=(2,-5)$	**A1**
	Attempts 2 of: $m_{BC}=\dfrac{-6+10}{7-3}=1$ and midpoint is $\left(\dfrac{7+3}{2},\dfrac{-6-10}{2}\right)=(5,-8)$ so perpendicular bisector is $y+"8"=-\dfrac{1}{"1"}(x-"5")$ or $m_{AC}=\dfrac{7-1}{-6-0}=-1$ and midpoint is $\left(\dfrac{7+1}{2},\dfrac{-6}{2}\right)=(4,-3)$ so perpendicular bisector is $y+"3"=-\dfrac{1}{"-1"}(x-"4")$ or $m_{AB}=\dfrac{-10-0}{3-1}=-5$ and midpoint is $\left(\dfrac{3+1}{2},\dfrac{-10}{2}\right)=(2,-5)$ so perpendicular bisector is $y+"5"=-\dfrac{1}{"-5"}(x-"2")$ $y+8=-(x-5)$ oe or $y+3=x-4$ oe or $y+5=\dfrac{1}{5}(x-2)$ oe **And solves simultaneously:** E.g. $y+3=x-4,\ y+8=5-x \Rightarrow 5-x-5=x-4 \Rightarrow x=2,\ y=-5$	**dM1**
	Hence centre of circle is $(2,-5)$	**A1**
	E.g. Midpoint of AB is the centre of the circle so AB is a diameter (or equivalent)	**A1**
		(5)

M1: Attempts midpoint of *AB*. If no method is shown accept one correct coordinate as evidence.
A1: Correct midpoint
dM1: Attempts 2 perpendicular bisectors, and solves simultaneously
A1: Obtains (2, −5)
A1: Suitable explanation and conclusion given with no errors and all previous marks awarded.

Via circle equation:

Way 6	Uses $(x-a)^2 + (y-b)^2 = r^2$ With (1, 0), (7, −6) and (3, −10) To find $(a, b) = \ldots$ or $r/r^2 = \ldots$	**M1**
	Centre (2, −5) or radius $\sqrt{26}$	**A1**
	E.g. Equation of *AB* is $y = -5(x-1)$ and $-5(2-1) = -5$ or $AB = \sqrt{(3-1)^2 + (-10-0)^2} = \sqrt{104} = 2\sqrt{26}$ or midpoint of *AB* is $\left(\dfrac{1+3}{2}, \dfrac{0-10}{2}\right) = (2, -5)$	**dM1A1**
	So centre is on *AB* or *AB* is twice the radius or midpoint is the centre hence *AB* is a diameter of the circle. (or equivalent)	**A1**
		(5)

M1: Uses all three points in circle equation to set up three equations in three unknowns to find centre or radius.
A1: Correct centre or correct radius
dM1: Finds e.g. equation of *AB*, distance *AB* or midpoint of *AB*
A1: Correct equation of *AB*, distance *AB* or midpoint of *AB*
A1: Suitable explanation and conclusion given with no errors and all previous marks awarded.

Via intersecting circles:

(ii) **Way 7**	If *AB* is diameter centre must be midpoint of *AB* ie $M\left(\dfrac{1+3}{2}, \dfrac{0-10}{2}\right)$	**M1**
	$= (2, -5)$	**A1**
	Circle centre *C* radius *r* is $(x-7)^2 + (y+6)^2 = r^2$ Circle centre *B* radius *r* is $(x-3)^2 + (y+10)^2 = r^2$ These intersect when $(x-7)^2 + (y+6)^2 = (x-3)^2 + (y+10)^2$ $\Rightarrow x + y = -3$ Circle centre *A* radius *r* is $(x-1)^2 + y^2 = r^2$ $(x-3)^2 + (y+10)^2 = (x-1)^2 + y^2 \Rightarrow x - 5y = 27$	**dM1A1**

		Solves simultaneously: $x + y = -3,\ x - 5y = 27 \Rightarrow x = 2,\ y = -5$	
		E.g. Midpoint of AB is the centre of the circle so AB is a diameter (or equivalent)	**A1**
			(5)

M1: Attempts midpoint of AB. If no method is shown accept one correct coordinate as evidence.

A1: Correct midpoint

dM1: Attempts equations of 2 circles with A, B or C as centre with radius r, repeats the process for 2 different circles and finds the intersection of both straight lines and solves simultaneously

A1: Correct coordinates of centre

A1: Suitable explanation and conclusion given with no errors and all previous marks awarded.