Pearson Edexcel

Mark Scheme (Results)

January 2022

Pearson Edexcel International A Level
In Further Pure Mathematics F3 (WFM03)
Paper 01

Question Number	Scheme	Notes	Marks
1(a)	$8\cosh^4 x = 8\left(\dfrac{e^x + e^{-x}}{2}\right)^4 = \dfrac{8}{16}\left(e^{4x} + 4e^{2x} + 6 + 4e^{-2x} + e^{-4x}\right)$ Applies $\cosh x = \dfrac{e^x + e^{-x}}{2}$ and attempts to expand the bracket to at least 4 different and no more than 5 different terms of the correct form but they may be "uncollected" depending on how they do the expansion. Allow unsimplified terms e.g. $\left(e^x\right)^3 e^{-x}$. May see $8\left(\dfrac{e^x + e^{-x}}{2}\right)^2\left(\dfrac{e^x + e^{-x}}{2}\right)^2$ but must attempt to expand as above		M1
	$= \dfrac{1}{2}\left(e^{4x} + e^{-4x}\right) + 4\left(\dfrac{e^{2x} + e^{-2x}}{2}\right) + 3 = ...$	Collects appropriate terms and reaches the form $\cosh 4x + p\cosh 2x + q$ or obtains values of p and q.	M1
	$= \cosh 4x + 4\cosh 2x + 3$	Correct expression or values e.g. $p = 4$ and $q = 3$	A1
	No marks are available in (a) if exponentials are not used but note that they may appear in combination with the use of hyperbolic identities e.g.: $8\cosh^4 x = 8\left(\cosh^2 x\right)^2 = 8\left(\dfrac{\cosh 2x + 1}{2}\right)^2 = 2\left(\dfrac{e^{2x} + e^{-2x}}{2} + 1\right)^2$ $= 2\left(\dfrac{e^{4x} + 2 + e^{-4x}}{4} + e^{2x} + e^{-2x} + 1\right) = \dfrac{e^{4x} + e^{-4x}}{2} + 4\left(\dfrac{e^{2x} + e^{-2x}}{2}\right) + 2$ $= \cosh 4x + 4\cosh 2x + 3$ **Allow to "meet in the middle" e.g. expands as above and compares with** $\dfrac{1}{2}\left(e^{4x} + e^{-4x}\right) + p\left(\dfrac{e^{2x} + e^{-2x}}{2}\right) + q \Rightarrow p = ..., q = ...$ **but to score any marks the expansion must be attempted.**		
			(3)

(b) Way 1	$\cosh 4x - 17\cosh 2x + 9 = 0 \Rightarrow 8\cosh^4 x - 4\cosh 2x - 3 - 17\cosh 2x + 9 = 0$ $\Rightarrow 8\cosh^4 x - 21\cosh 2x + 6 = 0 \Rightarrow 8\cosh^4 x - 21\left(2\cosh^2 x - 1\right) + 6 = 0$ Uses **their** result from part (a) and $\cosh 2x = \pm 2\cosh^2 x \pm 1$ to obtain a quadratic equation in $\cosh^2 x$ **or** $\cosh 4x - 17\cosh 2x + 9 = 0 \Rightarrow 2\left(2\cosh^2 x - 1\right)^2 - 1 - 17\left(2\cosh^2 x - 1\right) + 9 = 0$ Uses $\cosh 4x = \pm 2\cosh^2 2x \pm 1$ and $\cosh 2x = \pm 2\cosh^2 x \pm 1$ to obtain a quadratic equation in $\cosh^2 x$		M1
	$\Rightarrow 8\cosh^4 x - 42\cosh^2 x + 27 = 0$	Correct 3TQ in $\cosh^2 x$	A1
	$\Rightarrow 8\cosh^4 x - 42\cosh^2 x + 27 = 0$ $\Rightarrow \cosh^2 x = \frac{9}{2}\left(, \frac{3}{4}\right)$	Solves 3TQ in $\cosh^2 x$ (apply usual rules if necessary) to obtain $\cosh^2 x = k$ $(k \in \mathbb{R}$ and $> 1)$. May be implied by their values – check if necessary.	M1
	$\cosh^2 x = \frac{9}{2} \Rightarrow \cosh x = \frac{3}{\sqrt{2}} \Rightarrow x = \pm\ln\left(\frac{3}{\sqrt{2}} + \sqrt{\frac{9}{2} - 1}\right)$ or $\cosh x = \frac{3}{\sqrt{2}} \Rightarrow \frac{e^x + e^{-x}}{2} = \frac{3}{\sqrt{2}} \Rightarrow \sqrt{2}e^{2x} - 6e^x + \sqrt{2} = 0 \Rightarrow e^x = \ldots \Rightarrow x = \ldots$ or $\cosh^2 x = \frac{9}{2} \Rightarrow \left(\frac{e^x + e^{-x}}{2}\right)^2 = \frac{9}{2} \Rightarrow e^{4x} - 16e^{2x} + 1 = 0 \Rightarrow e^{2x} = \ldots \Rightarrow x = \ldots$ Takes square root to obtain $\cosh x = k$ $(k > 1)$ and applies the correct logarithmic form for arcosh or uses the correct exponential form for cosh x to obtain at least one value for x The root(s) must be real to score this mark.		M1
	$x = \pm\ln\left(\frac{3\sqrt{2}}{2} + \frac{\sqrt{14}}{2}\right)$ Both correct and exact including brackets. Accept simplified equivalents e.g. $x = \ln\left(\frac{3}{\sqrt{2}} \pm \frac{\sqrt{7}}{\sqrt{2}}\right)$ but withhold this mark if additional answers are given unless they are the same e.g. allow $x = \pm\ln\left(\frac{3\sqrt{2}}{2} \pm \frac{\sqrt{14}}{2}\right)$		A1
			(5)

(b) Way 2	$\cosh 4x - 17\cosh 2x + 9 = 0 \Rightarrow 2\cosh^2 2x - 1 - 17\cosh 2x + 9 = 0$ Applies $\cosh 4x = \pm 2\cosh^2 2x \pm 1$ to obtain a quadratic equation in cosh $2x$		M1
	$2\cosh^2 2x - 17\cosh 2x + 8 = 0$	Correct 3TQ in cosh $2x$	A1
	$2\cosh^2 2x - 17\cosh 2x + 8 = 0$ $\Rightarrow \cosh 2x = 8\left(, \frac{1}{2}\right)$	Solves 3TQ in cosh $2x$ (apply usual rules if necessary) to obtain $\cosh 2x = k \ \left(k \in \mathbb{R} \text{ and } > 1\right)$	M1
	$\cosh 2x = 8 \Rightarrow 2x = \pm\ln\left(8 + \sqrt{8^2 - 1}\right)$ or $\cosh 2x = 8 \Rightarrow \frac{e^{2x} + e^{-2x}}{2} = 8 \Rightarrow e^{4x} - 16e^{2x} + 1 = 0 \Rightarrow e^{2x} = ... \Rightarrow 2x = ...$ Applies the correct logarithmic form for arcosh from $\cosh 2x = k \ \left(k > 1\right)$ or uses the correct exponential form for cosh $2x$ to obtain at least one value for $2x$ The root(s) must be real to score this mark.		M1
	$x = \pm\frac{1}{2}\ln\left(8 + 3\sqrt{7}\right)$ or e.g. $x = \pm\ln\left(8 + 3\sqrt{7}\right)^{\frac{1}{2}}$	Both correct and exact with brackets. Accept simplified equivalents e.g. $x = \frac{1}{2}\ln\left(8 \pm \sqrt{63}\right)$ but withhold this mark if additional answers are given unless they are the same as above.	A1
(b) Way 3	$\cosh 4x - 17\cosh 2x + 9 = 0 \Rightarrow \frac{e^{4x} + e^{-4x}}{2} - \frac{17}{2}\left(e^{2x} + e^{-2x}\right) + 9 = 0$ $\Rightarrow e^{8x} - 17e^{6x} + 18e^{4x} - 17e^{2x} + 1 = 0$ M1: Applies the correct exponential forms and attempts a quartic equation in e^{2x} A1: Correct equation		M1A1
	$e^{8x} - 17e^{6x} + 18e^{4x} - 17e^{2x} + 1 = 0$ $\Rightarrow e^{2x} = 8 \pm 3\sqrt{7}, ...$	Solves and proceeds to a value for e^{2x} where $e^{2x} > 1$ and real.	M1
	$\Rightarrow e^{2x} = 8 \pm 3\sqrt{7} \Rightarrow 2x = \ln\left(8 \pm 3\sqrt{7}\right)$	Takes ln's to obtain at least one value for $2x$ The root(s) must be real to score this mark.	M1
	$x = \frac{1}{2}\ln\left(8 \pm 3\sqrt{7}\right)$ or e.g. $x = \ln\left(8 \pm 3\sqrt{7}\right)^{\frac{1}{2}}$	Both correct and exact with brackets. Accept simplified equivalents e.g. $x = \pm\frac{1}{2}\ln\left(8 + 3\sqrt{7}\right)$ but withhold this mark if additional answers are given unless they are the same as above.	A1
			Total 8

Question Number	Scheme	Notes	Marks
2	$\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=\dfrac{\sec\theta\tan\theta+\sec^2\theta}{\sec\theta+\tan\theta}-\cos\theta$ Correct derivative. Do not condone missing brackets e.g. $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=\dfrac{1}{\sec\theta+\tan\theta}\times\sec\theta\tan\theta+\sec^2\theta-\cos\theta$ unless a correct expression is implied by subsequent work. Award when a correct expression is seen but note that other forms are possible e.g. $\sec\theta-\cos\theta,\ \tan\theta\sin\theta$		B1
	$\left(\dfrac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2+\left(\dfrac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2=\left(\dfrac{\sec\theta\tan\theta+\sec^2\theta}{\sec\theta+\tan\theta}-\cos\theta\right)^2+(-\sin\theta)^2$ Attempts $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}$ and then $\left(\dfrac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2+\left(\dfrac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2$		M1
	$S=(2\pi)\displaystyle\int\cos\theta\sqrt{\left(\dfrac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2+\left(\dfrac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta$ $=(2\pi)\displaystyle\int\cos\theta\sqrt{\left(\dfrac{\sec\theta\tan\theta+\sec^2\theta}{\sec\theta+\tan\theta}-\cos\theta\right)^2+(-\sin\theta)^2}\,\mathrm{d}\theta$ Applies a correct surface area formula using their $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}$ and their $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}$ with or without the 2π For reference: $\sqrt{\left(\dfrac{\sec\theta\tan\theta+\sec^2\theta}{\sec\theta+\tan\theta}-\cos\theta\right)^2+(-\sin\theta)^2}=\tan\theta$ Allow π in front of the integral but must be an integral		M1
	$(2\pi)\displaystyle\int\sin\theta\ \mathrm{d}\theta$	Fully correct simplified integral with or without the 2π	A1
	$=(2\pi)[-\cos\theta](+c)$	Correct integration with or without the 2π	A1
	$(2\pi)[-\cos\theta]_0^{\frac{\pi}{4}}=(2\pi)\left(-\dfrac{1}{\sqrt{2}}+1\right)$ Applies the limits 0 and $\dfrac{\pi}{4}$. Must see evidence of both limits if necessary but condone e.g. $(2\pi)\left(-\dfrac{1}{\sqrt{2}}-1\right)$ **Depends on both previous method marks.**		dM1
	TSA = $2\pi\left(-\dfrac{1}{\sqrt{2}}+1\right)+\pi\times1^2+\pi\times\left(\dfrac{1}{\sqrt{2}}\right)^2$	**Correct** expressions for the 2 "ends" and adds these to their curved surface area. **Depends on the previous method mark.**	dM1
	$=\dfrac{\pi}{2}\left(7-2\sqrt{2}\right)$	Correct answer in the required form or correct values for p and q.	A1
	Note: **The final answer should follow correct work. The final mark should be withheld following e.g. $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}$ clearly seen as +sin θ or $\int\sin\theta\ \mathrm{d}\theta=+\cos\theta$** **Note:** **Without the "ends" the answer is $\dfrac{\pi}{2}\left(4-2\sqrt{2}\right)$ (usually scores 6/8)**		
			(8)
			Total 8

Alternative for first 4 marks:

	$\frac{dx}{d\theta}=\frac{\sec\theta\tan\theta+\sec^2\theta}{\sec\theta+\tan\theta}-\cos\theta$ Correct derivative. Do not condone missing brackets e.g. $\frac{dx}{d\theta}=\frac{1}{\sec\theta+\tan\theta}\times\sec\theta\tan\theta+\sec^2\theta-\cos\theta$ unless a correct expression is implied by subsequent work. Award when a correct expression is seen but note that other forms are possible e.g. $\sec\theta-\cos\theta,\ \tan\theta\sin\theta$		B1
	$1+\left(\frac{dy}{dx}\right)^2=1+\left(\frac{-\sin\theta}{\sec\theta-\cos\theta}\right)^2$ Attempts $1+\left(\frac{dy}{dx}\right)^2$ with $\frac{dy}{dx}=\frac{dy}{d\theta}\times\frac{d\theta}{dx}$		M1
	$S=(2\pi)\int\cos\theta\sqrt{1+\left(\frac{dy}{dx}\right)^2}\frac{dx}{d\theta}d\theta$ $=(2\pi)\int\cos\theta\sqrt{1+\left(\frac{-\sin\theta}{\sec\theta-\cos\theta}\right)^2}(\sec\theta-\cos\theta)d\theta$ Applies a correct surface area formula using their $\frac{dx}{d\theta}$ and their $\frac{dy}{dx}$ with or without the 2π For reference: $\sqrt{1+\left(\frac{-\sin\theta}{\sec\theta-\cos\theta}\right)^2}(\sec\theta-\cos\theta)=\tan\theta$ Allow π in front of the integral but must be an integral		M1
	$(2\pi)\int\sin\theta\ d\theta$	Fully correct simplified integral with or without the 2π	A1

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td>3(a)</td><td>$y = \text{arsech}\left(\frac{x}{2}\right) \Rightarrow \text{sech}\, y = \frac{x}{2}$
$\Rightarrow \frac{dx}{dy} = -2\,\text{sech}\, y \tanh y$</td><td>Takes "sech" of both sides and differentiates to obtain $\frac{dx}{dy} = k\,\text{sech}\, y \tanh y$ or equivalent.</td><td>M1</td></tr>
<tr><td></td><td colspan="2">$\Rightarrow \frac{dx}{dy} = -2\left(\frac{x}{2}\right)\sqrt{1-\left(\frac{x}{2}\right)^2}$
M1: Replaces sech y with $\frac{x}{2}$ and tanh y with $\sqrt{1-\left(\frac{x}{2}\right)^2}$
A1: Correct equation involving $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in any form in terms of x only.</td><td>M1A1</td></tr>
<tr><td></td><td>$\Rightarrow \frac{dy}{dx} = \frac{-2}{x\sqrt{4-x^2}}$</td><td>Correct derivative in the required form or correct values for p and q.</td><td>A1</td></tr>
<tr><td></td><td></td><td></td><td>(4)</td></tr>
<tr><td>(a)
Way 2</td><td>$y = \text{arsech}\left(\frac{x}{2}\right) \Rightarrow \text{sech}\, y = \frac{x}{2}$
$\Rightarrow \cosh y = \frac{2}{x} \Rightarrow \sinh y \frac{dy}{dx} = -\frac{2}{x^2}$</td><td>Takes "sech" of both sides, changes to "cosh" and differentiates to obtain $\sinh y \frac{dy}{dx} = \frac{k}{x^2}$ or equivalent.</td><td>M1</td></tr>
<tr><td></td><td colspan="2">$\Rightarrow \frac{dy}{dx} = -\frac{2}{x^2 \sinh y} = -\frac{2}{x^2\sqrt{\left(\frac{2}{x}\right)^2 - 1}}$
M1: Replaces sinh y with $\sqrt{\left(\frac{2}{x}\right)^2 - 1}$
A1: Correct equation involving $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in any form in terms of x only.</td><td>M1A1</td></tr>
<tr><td></td><td>$\Rightarrow \frac{dy}{dx} = \frac{-2}{x\sqrt{4-x^2}}$</td><td>Correct derivative in the required form or correct values for p and q.</td><td>A1</td></tr>
<tr><td>(a)
Way 3</td><td colspan="2">$y = \text{arsech}\left(\frac{x}{2}\right) \Rightarrow y = \text{arcosh}\left(\frac{2}{x}\right)$
Changes to "arcosh" correctly. Score this as the second M mark on EPEN.</td><td>M1</td></tr>
<tr><td></td><td colspan="2">$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{\left(\frac{2}{x}\right)^2 - 1}} \times -\frac{2}{x^2}$
M1: Differentiates to the form $\frac{k}{x^2\sqrt{\left(\frac{2}{x}\right)^2 - 1}}$ oe
A1: Correct equation involving $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in any form in terms of x only.
Score this as the first M mark and first A mark on EPEN.</td><td>M1A1</td></tr>
<tr><td></td><td>$\Rightarrow \frac{dy}{dx} = \frac{-2}{x\sqrt{4-x^2}}$</td><td>Correct derivative in the required form or correct values for p and q.</td><td>A1</td></tr>
</table>

<table>
<tr><td>(a)
Way 4</td><td colspan="2">$$y=\text{arsech}\left(\frac{x}{2}\right)\Rightarrow \text{sech } y=\frac{x}{2}\Rightarrow\left(\frac{x}{2}\right)^2=\text{sech}^2 y\Rightarrow \tanh y=\sqrt{1-\left(\frac{x}{2}\right)^2}$$ $$\Rightarrow \text{sech}^2 y\frac{\mathrm{d}y}{\mathrm{d}x}=-x\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}$$ Differentiates to $\text{sech}^2 y\dfrac{\mathrm{d}y}{\mathrm{d}x}=kx\left(1-\dfrac{x^2}{4}\right)^{-\frac{1}{2}}$ or equivalent</td><td>M1</td></tr>
<tr><td></td><td colspan="2">$$\Rightarrow \text{sech}^2 y\frac{\mathrm{d}y}{\mathrm{d}x}=-x\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}\Rightarrow\frac{x^2}{4}\frac{\mathrm{d}y}{\mathrm{d}x}=-x\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{4}{x}\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}$$ M1: Replaces $\text{sech}^2 y$ with $\left(\dfrac{2}{x}\right)^2$
A1: Correct equation involving $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ or $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in any form in terms of x only.</td><td>M1A1</td></tr>
<tr><td></td><td>$$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-2}{x\sqrt{4-x^2}}$$</td><td>Correct derivative in the required form or correct values for p and q.</td><td>A1</td></tr>
<tr><td>(a)
Way 5</td><td colspan="2">$$y=\text{arsech}\left(\frac{x}{2}\right)\Rightarrow \text{sech } y=\frac{x}{2}\Rightarrow y=\text{artanh}\left(\sqrt{1-\left(\frac{x}{2}\right)^2}\right)$$ Changes to “artanh” correctly. Score this as the second M mark on EPEN.</td><td>M1</td></tr>
<tr><td></td><td colspan="2">$$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{1}{2}\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}}{1-\left(1-\frac{x^2}{4}\right)}\times-\frac{x}{2}$$ M1: Differentiates to the form $\dfrac{kx\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}}{1-\left(1-\frac{x^2}{4}\right)}$ oe
A1: Correct equation involving $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ or $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in any form in terms of x only.
Score this as the first M mark and first A mark on EPEN.</td><td>M1A1</td></tr>
<tr><td></td><td>$$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-2}{x\sqrt{4-x^2}}$$</td><td>Correct derivative in the required form or correct values for p and q.</td><td>A1</td></tr>
</table>

There may be other methods used.
If you are in any doubt if the method deserves any marks use Review.

(b)	$\mathrm{f}(x)=\tanh^{-1}(x)+\operatorname{sech}^{-1}\left(\frac{x}{2}\right)\Rightarrow \mathrm{f}'(x)=\frac{1}{1-x^2}-\frac{2}{x\sqrt{4-x^2}}$ Correct f ′(x) following through their (a) of the form $\frac{p}{x\sqrt{q-x^2}}$ Also allow with "made up" p and q or the letters p and q.		B1ft
	$\frac{1}{1-x^2}-\frac{2}{x\sqrt{4-x^2}}=0\Rightarrow 2\left(1-x^2\right)=x\sqrt{4-x^2}\Rightarrow 4\left(1-x^2\right)^2=x^2\left(4-x^2\right)$ Sets $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ with their (a) of the form $\frac{p}{x\sqrt{q-x^2}}$ and squares both sides to reach a quartic equation		M1
	$5x^4-12x^2+4=0$	Correct quartic	A1
	$5x^4-12x^2+4=0\Rightarrow x^2=2,\ 0.4$ $\Rightarrow x=\ldots$	Solves their quartic equation to obtain a value for x^2 and proceeds to a value for x. Apply usual rules for solving and check if necessary. Allow complex roots.	M1
	$x=\sqrt{\frac{2}{5}}$	Correct exact answer (allow equivalents e.g. $\frac{\sqrt{10}}{5}$). If any extra answers given score A0 e.g. $x=\pm\sqrt{\frac{2}{5}}$	A1
			(5)
			Total 9

<u>Special case:</u>

It is possible for a correct solution in (b) following a sign error in (a) e.g.

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{x\sqrt{4-x^2}}$$

$$\mathrm{f}(x)=\tanh^{-1}(x)+\operatorname{sech}^{-1}\left(\frac{x}{2}\right)\Rightarrow \mathrm{f}'(x)=\frac{1}{1-x^2}+\frac{2}{x\sqrt{4-x^2}}$$

$$\frac{1}{1-x^2}+\frac{2}{x\sqrt{4-x^2}}=0\Rightarrow 2\left(1-x^2\right)=-x\sqrt{4-x^2}\Rightarrow 4\left(1-x^2\right)^2=x^2\left(4-x^2\right)\text{ etc.}$$

This is likely to score M1M1A0A0 in (a) but allow full recovery in (b) if it leads to the correct answer.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td>4(a)</td><td colspan="2">$$\lambda = 3 \Rightarrow \left|\mathbf{M} - 3\mathbf{I}\right| = \begin{vmatrix} 3 & k & 2 \\ k & 2 & 0 \\ 2 & 0 & 4 \end{vmatrix} = 0 \Rightarrow 3(8) - k(4k) + 2(-4) = 0$$
or e.g.
$$\left|\mathbf{M} - \lambda\mathbf{I}\right| = \begin{vmatrix} 6-\lambda & k & 2 \\ k & 5-\lambda & 0 \\ 2 & 0 & 7-\lambda \end{vmatrix} = 0$$
$$\Rightarrow (6-\lambda)(5-\lambda)(7-\lambda) - k\left(k(7-\lambda)\right) + 2\left(0 - 2(5-\lambda)\right) = 0 \Rightarrow 24 - k(4k) - 8 = 0$$
Correct interpretation of 3 being an eigenvalue leading to the formation of a quadratic equation in k only.
If the method for forming the determinant is not clear then look for at least 2 correct “components”.
NB rule of Sarrus gives 24 – 8 – 4k^2 = 0</td><td>M1</td></tr>
<tr><td></td><td>$\Rightarrow 4k^2 = 16 \Rightarrow k = ...$</td><td>Solves quadratic.
Depends on the first M.</td><td>dM1</td></tr>
<tr><td></td><td>$k = \pm 2$</td><td>Correct values</td><td>A1</td></tr>
<tr><td></td><td></td><td></td><td>(3)</td></tr>
<tr><td>(a)
Way 2</td><td colspan="2">$$\begin{pmatrix} 6 & k & 2 \\ k & 5 & 0 \\ 2 & 0 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3\begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{aligned} 6x + ky + 2z &= 3x \\ kx + 5y &= 3y \\ 2x + 7z &= 3z \end{aligned}$$
$$z = -\frac{1}{2}x,\ \ y = -\frac{1}{2}kx \Rightarrow 6x - \frac{k^2 x}{2} - x = 3x \Rightarrow \frac{k^2}{2} = 2$$
Eliminates z and y and reaches a quadratic equation in k only</td><td>M1</td></tr>
<tr><td></td><td>$\frac{k^2}{2} = 2 \Rightarrow k = ...$</td><td>Solves quadratic.
Depends on the first M.</td><td>dM1</td></tr>
<tr><td></td><td>$k = \pm 2$</td><td>Correct values</td><td>A1</td></tr>
<tr><td>(b)</td><td colspan="2">$$k = -2 \Rightarrow \left|\mathbf{M} - \lambda\mathbf{I}\right| = \begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 5-\lambda & 0 \\ 2 & 0 & 7-\lambda \end{vmatrix}$$
$$\Rightarrow (6-\lambda)(7-\lambda)(5-\lambda) + 2(2\lambda - 14) + 2(2\lambda - 10) = 0$$
Applies a value of k from (a) and a recognisable attempt at the characteristic equation (the “= 0” is not needed here).
If the method is not clear then look for at least 2 correct “components”.</td><td>M1</td></tr>
<tr><td></td><td>$\Rightarrow \lambda^3 - 18\lambda^2 + 99\lambda - 162 = 0 \Rightarrow \lambda = ...$</td><td>Solves cubic. May use λ = 3 as a factor or calculator to solve. Depends on the first mark. Allow complex roots.</td><td>dM1</td></tr>
<tr><td></td><td>$\lambda = 6,\ 9\ (,3)$</td><td>Correct values.
Allow to come from k = 2</td><td>A1</td></tr>
<tr><td></td><td colspan="2"></td><td>(3)</td></tr>
</table>

(c)	$$\begin{pmatrix}6 & -2 & 2\\ -2 & 5 & 0\\ 2 & 0 & 7\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=3\begin{pmatrix}x\\ y\\ z\end{pmatrix}\Rightarrow \begin{matrix}6x-2y+2z=3x\\ -2x+5y=3y\\ 2x+7z=3z\end{matrix}\Rightarrow\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\ldots$$ or $$\begin{pmatrix}3 & -2 & 2\\ -2 & 2 & 0\\ 2 & 0 & 4\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}\Rightarrow \begin{matrix}6x-2y+2z=0\\ -2x+5y=0\\ 2x+7z=0\end{matrix}\Rightarrow\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\ldots$$ Correct strategy for finding the eigenvector using a value of *k* from (a) Note that the cross product of any 2 rows or columns of **M** – 3**I** gives an eigenvector		M1
	$p\begin{pmatrix}2\\ 2\\ -1\end{pmatrix}$	Any correct eigenvector	A1
	$\frac{1}{3}\begin{pmatrix}2\\ 2\\ -1\end{pmatrix}$	Any correct normalised eigenvector	A1
			(3)
			Total 9

Question Number	Scheme	Notes	Marks
5(i)	$x^2-3x+5=\left(x-\frac{3}{2}\right)^2+\frac{11}{4}$	Correct completion of the square	B1
	$\int\frac{1}{\sqrt{x^2-3x+5}}\,\mathrm{d}x=\int\frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2+\frac{11}{4}}}\,\mathrm{d}x=\sinh^{-1}\frac{2x-3}{\sqrt{11}}(+c)$ M1: Use of $\sinh^{-1}$ A1: Fully correct expression (condone omission of $+c$) Allow equivalent correct expressions e.g. $\sinh^{-1}\frac{x-\frac{3}{2}}{\sqrt{\frac{11}{4}}}(+c)$, $\sinh^{-1}\frac{x-\frac{3}{2}}{\frac{\sqrt{11}}{2}}(+c)$ Allow equivalents for $\sinh^{-1}$ e.g. arsinh, arcsinh but **not** arsin or arcsin		M1A1
	You may see logarithmic forms for the answer: e.g. $\ln\left(\frac{2x-3}{\sqrt{11}}+\sqrt{\left(\frac{2x-3}{\sqrt{11}}\right)^2+1}\right)$, $\ln\left(x-\frac{3}{2}+\sqrt{\left(x-\frac{3}{2}\right)^2+\frac{11}{4}}\right)$ but apply isw once a correct answer is seen.		
			(3)
(ii)	$63+4x-4x^2=-4\left(x^2-x-\frac{63}{4}\right)$ $=-4\left(\left(x-\frac{1}{2}\right)^2-\frac{64}{4}\right)$	Obtains $-4\left(\left(x-\frac{1}{2}\right)^2\pm\ldots\right)$ or $-4\left(x-\frac{1}{2}\right)^2\pm\ldots$ or $\ldots-(2x-1)^2$	M1
	$-4\left(\left(x-\frac{1}{2}\right)^2-16\right)$ or $64-4\left(x-\frac{1}{2}\right)^2$ or $64-(2x-1)^2$	Correct completion of the square	A1
	$\int\frac{1}{\sqrt{63+4x-4x^2}}\,\mathrm{d}x=\frac{1}{2}\sin^{-1}\left(\frac{2x-1}{8}\right)(+c)$ M1: Use of $\sin^{-1}$ A1: Fully correct expression (condone omission of $+c$) Allow equivalent correct expressions e.g. $\frac{1}{2}\sin^{-1}\frac{x-\frac{1}{2}}{4}(+c)$, $-\frac{1}{2}\sin^{-1}\frac{\frac{1}{2}-x}{4}(+c)$ Allow equivalents for $\sin^{-1}$ e.g. arsin, arcsin but not arsinh or arcsinh		M1A1
			(4)
	In (ii) there are no marks for using $\int\frac{1}{\sqrt{63+4x-4x^2}}\,\mathrm{d}x=-\int\frac{1}{\sqrt{4x^2-63-4x}}\mathrm{d}x$ But if completion of square attempted first allow M1A1 e.g. for $\int\frac{1}{\sqrt{63+4x-4x^2}}\mathrm{d}x=\int\frac{1}{\sqrt{64-(2x-1)^2}}\mathrm{d}x$ but then M0 for $=\int\frac{-1}{\sqrt{(2x-1)^2-64}}\mathrm{d}x$		
			Total 7

Question Number	Scheme	Notes	Marks
6(a)	$\int \mathrm{e}^x \sin^n x\,\mathrm{d}x = \mathrm{e}^x \sin^n x - n\int \mathrm{e}^x \sin^{n-1} x\cos x\,\mathrm{d}x$ Applies integration by parts to obtain $\pm\mathrm{e}^x \sin^n x \pm \alpha\int \mathrm{e}^x \sin^{n-1} x\cos x\,\mathrm{d}x$		M1
	$= \mathrm{e}^x \sin^n x - n\left\{\mathrm{e}^x \sin^{n-1} x\cos x - \int \mathrm{e}^x\left((n-1)\sin^{n-2} x\cos^2 x - \sin^n x\right)\mathrm{d}x\right\}$ M1: Applies integration by parts to $\pm\alpha\int \mathrm{e}^x \sin^{n-1} x\cos x\,\mathrm{d}x$ to obtain $\pm\mathrm{e}^x \sin^{n-1} x\cos x \pm \int \mathrm{e}^x\left(\alpha\sin^{n-2} x\cos^2 x - \beta\sin^n x\right)\mathrm{d}x$ Or equivalent e.g. $\pm\mathrm{e}^x \sin^{n-1} x\cos x \pm \int \mathrm{e}^x\left(\alpha\sin^{n-2} x - \beta\sin^n x\right)\mathrm{d}x$ (if Pythagoras applied first) A1: Fully correct expression for I_n from parts applied twice.		**d**M1A1
	$= \mathrm{e}^x \sin^n x - n\left\{\mathrm{e}^x \sin^{n-1} x\cos x - \int \mathrm{e}^x\left((n-1)\sin^{n-2} x\left(1-\sin^2 x\right) - \sin^n x\right)\mathrm{d}x\right\}$ Applies $\cos^2 x = 1 - \sin^2 x$		**d**M1
	$= \mathrm{e}^x \sin^n x - n\left\{\mathrm{e}^x \sin^{n-1} x\cos x - \int \mathrm{e}^x\left((n-1)\sin^{n-2} x - (n-1)\sin^n x - \sin^n x\right)\mathrm{d}x\right\}$ $= \mathrm{e}^x \sin^n x - n\left\{\mathrm{e}^x \sin^{n-1} x\cos x - \int \mathrm{e}^x\left((n-1)\sin^{n-2} x - n\sin^n x\right)\mathrm{d}x\right\}$ $= \mathrm{e}^x \sin^n x - n\mathrm{e}^x \sin^{n-1} x\cos x + n(n-1)I_{n-2} - n^2 I_n \Rightarrow I_n = \ldots$ Completes by introducing I_{n-2} and I_n and makes I_n the subject		**d**M1
	$I_n = \dfrac{\mathrm{e}^x \sin^{n-1} x}{n^2+1}(\sin x - n\cos x) + \dfrac{n(n-1)}{n^2+1}I_{n-2}$ * Fully correct proof with no errors but allow e.g. the occasional missing "dx" but any clear errors must be recovered before final answer e.g. missing brackets.		A1*
			(6)

(b)	$$I_4 = \frac{e^x \sin^3 x}{17}(\sin x - 4\cos x) + \frac{12}{17}I_2$$ or $$I_2 = \frac{e^x \sin x}{5}(\sin x - 2\cos x) + \frac{2}{5}I_0$$ Applies the reduction formula once	M1
	$$= \frac{e^x \sin^3 x}{17}(\sin x - 4\cos x) + \frac{12}{17}\left(\frac{e^x \sin x}{5}(\sin x - 2\cos x) + \frac{2}{5}I_0\right)$$ $$= \frac{e^x \sin^3 x}{17}(\sin x - 4\cos x) + \frac{12e^x \sin x}{85}(\sin x - 2\cos x) + \frac{24}{85}e^x$$ Applies the reduction formula again and uses $I_0 = \int e^x \, dx = e^x$ to obtain an expression in terms of x	M1
	$$\int_0^{\frac{\pi}{2}} e^x \sin^4 x \, dx = \left[\frac{e^x \sin^3 x}{17}(\sin x - 4\cos x) + \frac{12e^x \sin x}{85}(\sin x - 2\cos x) + \frac{24}{85}e^x\right]_0^{\frac{\pi}{2}}$$ $$= \frac{e^{\frac{\pi}{2}}}{17} + \frac{12e^{\frac{\pi}{2}}}{85} + \frac{24e^{\frac{\pi}{2}}}{85} - \frac{24}{85}$$ Uses the limits 0 and $\frac{\pi}{2}$ and subtracts. **Depends on both previous marks.**	**d**M1
	$$= \frac{41e^{\frac{\pi}{2}}}{85} - \frac{24}{85}$$ Correct expression or correct values e.g. $A = \ldots$, $B = \ldots$	A1
		(4)
	Note that the limits may be applied as they go e.g.: $$\text{M1: } I_4 = \frac{e^{\frac{\pi}{2}}}{17}(1-0) + \frac{12}{17}I_2$$ $$I_2 = \frac{e^{\frac{\pi}{2}}}{5}(1-0) + \frac{2}{5}I_0$$ $$I_0 = e^{\frac{\pi}{2}} - 1$$ $$\text{M1M1: } I_4 = \frac{e^{\frac{\pi}{2}}}{17} + \frac{12}{17}\left(\frac{e^{\frac{\pi}{2}}}{5} + \frac{2}{5}\left(e^{\frac{\pi}{2}} - 1\right)\right)$$ $$\text{A1: } = \frac{41e^{\frac{\pi}{2}}}{85} - \frac{24}{85}$$	
		Total 10

Question Number	Scheme	Notes	Marks
7(a)	$\frac{x-3}{4}=\frac{y-5}{-2}=\frac{z-4}{7}\Rightarrow \mathbf{r}=\begin{pmatrix}3\\5\\4\end{pmatrix}\pm\lambda\begin{pmatrix}4\\-2\\7\end{pmatrix}$	Converts to parametric form. "**r** =" is not required	M1
	$2x+4y-z=1$ $\Rightarrow 2(3+4\lambda)+4(5-2\lambda)-4-7\lambda=1$ $\Rightarrow \lambda=...(3)\Rightarrow P$ is ...	Correct strategy for finding *P*. Condone the use of $2x+4y-z=0$ for the plane equation.	M1
	$(15,\ -1,\ 25)$	Correct coordinates. Condone if given as a vector.	A1
			(3)
(a) Way 2	$\frac{x-3}{4}=\frac{y-5}{-2}\Rightarrow x=13-2y$	Uses the Cartesian equation to find *x* in terms of *y*	M1
	$2x+4y-z=1\Rightarrow 26-4y+4y-z=1$ $\Rightarrow z=...,\ x=...,\ y=...$	Correct strategy for finding *P*. Condone the use of $2x+4y-z=0$ for the plane equation.	M1
	$(15,\ -1,\ 25)$	Correct coordinates. Condone if given as a vector.	A1
(b)	$\begin{pmatrix}4\\-2\\7\end{pmatrix}\bullet\begin{pmatrix}2\\4\\-1\end{pmatrix}=8-8-7=-7$	Applies the scalar product between the direction of l_1 and the normal to the plane	M1
	Examples: $\phi=\cos^{-1}\frac{\pm 7}{\sqrt{69}\sqrt{21}}=...\quad \phi=\sin^{-1}\frac{\pm 7}{\sqrt{69}\sqrt{21}}=...$ Attempts to find a relevant angle in degrees or radians. **Depends on the first method mark.**		**d**M1
	$\theta=10.6°$	Allow awrt 10.6 but do **not** isw and mark the final answer. For reference $\theta=10.5965654°$	A1
			(3)
(b) Way 2	$\begin{pmatrix}4\\-2\\7\end{pmatrix}\times\begin{pmatrix}2\\4\\-1\end{pmatrix}=\begin{pmatrix}26\\-18\\-20\end{pmatrix}$	Attempts vector product of normal to Π and direction of l_1	M1
	$\sqrt{26^2+18^2+20^2}=\sqrt{21}\sqrt{69}\sin\alpha$ $\sin\alpha=\frac{10\sqrt{46}}{69}\Rightarrow\alpha=...$	Attempts to find a relevant angle. **Depends on the first method mark.**	**d**M1
	$\theta=10.6°$	Allow awrt 10.6 but do **not** isw and mark the final answer. For reference $\theta=10.5965654°$	A1

(c)	$\mathbf{a}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & 4 & -1\\ 4 & -2 & 7\end{vmatrix}=\begin{pmatrix}26\\-18\\-20\end{pmatrix}$	Attempts vector product of normal to Π and direction of l_1. If no method is seen expect at least 2 correct components.	M1
	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 13 & -9 & -10\\ 2 & 4 & -1\end{vmatrix}=\begin{pmatrix}49\\-7\\70\end{pmatrix}$	Attempts vector product of "**a**" with normal to Π to find direction of l_2	M1
		Correct direction for l_2	A1
	$\mathbf{r}=\begin{pmatrix}15\\-1\\25\end{pmatrix}+\mu\begin{pmatrix}7\\-1\\10\end{pmatrix}$	Depends on both previous M marks Attempts vector equation using their direction vector and their P	ddM1
		Correct equation or any equivalent correct vector equation	A1
			(5)
(c) **Way 2**	$\lambda=1\Rightarrow(7,\ 3,\ 11)$ lies on l_1 $\mathbf{r}=\begin{pmatrix}7\\3\\11\end{pmatrix}+t\begin{pmatrix}2\\4\\-1\end{pmatrix}$ $\Rightarrow 2(7+2t)+4(3+4t)-11+t=1$ $t=-\frac{2}{3}\Rightarrow\left(\frac{17}{3},\ \frac{1}{3},\ \frac{35}{3}\right)$ is on l_2	Complete method to find a point on l_2	M1
	Direction of l_2 is $\begin{pmatrix}15\\-1\\25\end{pmatrix}-\frac{1}{3}\begin{pmatrix}17\\1\\35\end{pmatrix}=\frac{1}{3}\begin{pmatrix}28\\-4\\40\end{pmatrix}$	Uses their point and their P to find direction of l_2	M1
		Correct direction for l_2	A1
	$\mathbf{r}=\begin{pmatrix}15\\-1\\25\end{pmatrix}+\mu\begin{pmatrix}7\\-1\\10\end{pmatrix}$	Attempts vector equation using their direction vector and their point on l_2	ddM1
		Correct equation or any equivalent correct vector equation. Must have $\mathbf{r}=$ and not e.g. $l_2=\ldots$	A1
(c) **Way 3**	Normal to plane from l_1 $\mathbf{r}=\begin{pmatrix}3\\5\\4\end{pmatrix}+t\begin{pmatrix}2\\4\\-1\end{pmatrix}$ $\Rightarrow 2(3+2t)+4(5+4t)-(4-t)=1$ $t=-1\Rightarrow(1,\ 1,\ 5)$ is on l_2	Complete method to find a point on l_2	M1
	Direction of l_2 is $\begin{pmatrix}15\\-1\\25\end{pmatrix}-\begin{pmatrix}1\\1\\5\end{pmatrix}=\begin{pmatrix}14\\-2\\20\end{pmatrix}$	Uses their point and their P to find direction of l_2	M1
		Correct direction for l_2	A1
	$\mathbf{r}=\begin{pmatrix}1\\1\\5\end{pmatrix}+\mu\begin{pmatrix}7\\-1\\10\end{pmatrix}$	Attempts vector equation using their direction vector and their point on l_2	ddM1
		Correct equation or any equivalent correct vector equation. Must have $\mathbf{r}=$ and not e.g. $l_2=\ldots$	A1
			Total 11

Question Number	Scheme	Notes	Marks
8(a)	$b^2 = a^2\left(1-e^2\right) \Rightarrow 4 = 9\left(1-e^2\right) \Rightarrow e = \ldots$ or e.g. $e = \sqrt{1-\dfrac{b^2}{a^2}} \Rightarrow e = \ldots$	Uses a correct formula with a and b correctly placed to find a value for e	M1
	$e = \dfrac{\sqrt{5}}{3}$	Correct value (or equivalent) $e = \pm\dfrac{\sqrt{5}}{3}$ scores A0	A1
			(2)
(b)(i)	$(\pm ae,\ 0) = \left(\pm\sqrt{5},\ 0\right)$ or $\left(\pm 3\dfrac{\sqrt{5}}{3},\ 0\right)$ Correct foci. Must be coordinates but allow unsimplified and isw if necessary. Follow through their e so allow for ($\pm$ 3$\times$their e, 0)		B1ft
(ii)	$x = \pm\dfrac{a}{e} = \pm\dfrac{9}{\sqrt{5}}$ or $x = \pm\dfrac{3}{\frac{\sqrt{5}}{3}}$ Correct directrices. Must be equations but allow unsimplified and isw if necessary. Follow through their e so allow for $x = \pm 3/\text{their } e$		B1ft
			(2)
	Special case: Use of a^2 for a and b^2 for b **consistently** scores M0A0 in (a) and B1ft B1ft in (b). This gives $e = \dfrac{\sqrt{65}}{9}$, $\left(\pm\sqrt{65},\ 0\right)$, $x = \pm\dfrac{81}{\sqrt{65}}$		
(c)	$\dfrac{dx}{d\theta} = -3\sin\theta,\ \dfrac{dy}{d\theta} = 2\cos\theta$ or $\dfrac{2x}{9} + \dfrac{2y}{4}\dfrac{dy}{dx} = 0$ or $y = \left(4 - \dfrac{4x^2}{9}\right)^{\frac{1}{2}} \Rightarrow \dfrac{dy}{dx} = -\dfrac{4x}{9}\left(4 - \dfrac{4x^2}{9}\right)^{-\frac{1}{2}}$ $\Rightarrow \dfrac{dy}{dx} = \ldots\left(= \dfrac{2\cos\theta}{-3\sin\theta}\right)$	Correct strategy for the gradient of l in terms of θ. Allow $\dfrac{dy}{dx} = \dfrac{2\cos\theta}{-3\sin\theta}$ to be stated.	M1
	$y - 2\sin\theta = \dfrac{2\cos\theta}{-3\sin\theta}\left(x - 3\cos\theta\right)$	Correct straight line method (any complete method). Finding the equation of the normal is M0.	M1
	$-3y\sin\theta + 6\sin^2\theta = 2x\cos\theta - 6\cos^2\theta$ $2x\cos\theta + 3y\sin\theta = 6$*	Cso with at least one intermediate line of working	A1*
			(3)

(d)	$l_2: \ \ y=\dfrac{3\sin\theta}{2\cos\theta}x$	Correct equation for l_2	B1
	$2x\cos\theta+3y\sin\theta=6,\ y=\dfrac{3\sin\theta}{2\cos\theta}x$ $\Rightarrow x=\ldots,\ y=\ldots$	Complete method for Q	M1
	$Q:\left(\dfrac{12\cos\theta}{4\cos^2\theta+9\sin^2\theta},\ \dfrac{18\sin\theta}{4\cos^2\theta+9\sin^2\theta}\right)$ Correct coordinates. Allow as $x=\ldots,\ y=\ldots$ and allow equivalent correct expressions as long as they are single fractions e.g. $x=\dfrac{12\cos\theta}{4+5\sin^2\theta}\quad y=\dfrac{18\sin\theta}{4+5\sin^2\theta},\quad x=\dfrac{12\cos\theta}{9-5\cos^2\theta}\quad y=\dfrac{18\sin\theta}{9-5\cos^2\theta}$		A1
			(3)

(e)	At Q, $\dfrac{y}{x}=\dfrac{3}{2}\tan\theta$	Uses their coordinates of Q to attempt an equation connecting x, y and θ or states or uses the equation found in (d)	M1
	$x=\dfrac{12\cos\theta}{4\cos^2\theta+9\sin^2\theta}=\dfrac{12\sec\theta}{4+9\tan^2\theta}\Rightarrow x^2=\dfrac{144\sec^2\theta}{\left(4+9\tan^2\theta\right)^2}=\dfrac{144\left(1+\dfrac{4y^2}{9x^2}\right)}{\left(4+9\times\dfrac{4y^2}{9x^2}\right)^2}$ or $y=\dfrac{18\sin\theta}{4\cos^2\theta+9\sin^2\theta}=\dfrac{12\sec\theta\tan\theta}{4+9\tan^2\theta}$ $\Rightarrow y^2=\dfrac{324\sec^2\theta\tan^2\theta}{\left(4+9\tan^2\theta\right)^2}=\dfrac{324\left(1+\dfrac{4y^2}{9x^2}\right)\dfrac{4y^2}{9x^2}}{\left(4+9\times\dfrac{4y^2}{9x^2}\right)^2}$ Eliminates θ **Depends on the first mark.**		**d**M1
	$\Rightarrow x^2=\dfrac{x^2\left(9x^2+4y^2\right)}{\left(x^2+y^2\right)^2}\Rightarrow\left(x^2+y^2\right)^2=9x^2+4y^2$ or $\Rightarrow 9\times16x^2y^2\left(1+\dfrac{y^2}{x^2}\right)^2=4\times18^2\left(1+\dfrac{4y^2}{9x^2}\right)\Rightarrow\left(x^2+y^2\right)^2=9x^2+4y^2$ Correct equation or correct values for α and β.		A1
			(3)
(e) **Way 2**	$x=\dfrac{12\cos\theta}{4+5\sin^2\theta}\quad y=\dfrac{18\sin\theta}{4+5\sin^2\theta}\Rightarrow\left(x^2+y^2\right)^2=\left(\dfrac{144\cos^2\theta+324\sin^2\theta}{\left(4+5\sin^2\theta\right)^2}\right)^2$ Uses their Q to obtain an expression for $\left(x^2+y^2\right)^2$ in terms of θ		M1
	$\left(\dfrac{144\cos^2\theta+324\sin^2\theta}{\left(4+5\sin^2\theta\right)^2}\right)^2=\left(\dfrac{144+180\sin^2\theta}{\left(4+5\sin^2\theta\right)^2}\right)^2=\left(\dfrac{36\left(4+5\sin^2\theta\right)}{\left(4+5\sin^2\theta\right)^2}\right)^2=\dfrac{1296}{\left(4+5\sin^2\theta\right)^2}$ $\dfrac{1296}{\left(4+5\sin^2\theta\right)^2}=\alpha x^2+\beta y^2=\alpha\dfrac{144\cos^2\theta}{\left(4+5\sin^2\theta\right)^2}+\beta\dfrac{324\sin^2\theta}{\left(4+5\sin^2\theta\right)^2}\Rightarrow\alpha=...,\beta=...$ Substitutes into the given answer and solves for α and β **Depends on the first mark.**		**d**M1
	$\left(x^2+y^2\right)^2=9x^2+4y^2$	Correct expression or correct values for α and β.	A1
			Total 13