# Mark Scheme (Results)

January 2023

Pearson Edexcel International Advanced Level
In Pure Mathematics P4 (WMA14) Paper 01

| Question Number | Scheme | Marks |
|---|---|---|
| **1 (a)** | $\dfrac{5x+10}{(1-x)(2+3x)} \equiv \dfrac{A}{1-x}+\dfrac{B}{2+3x} \Rightarrow$ Value for $A$ or $B$ | **M1** |
| | One correct value, either $A=3$ or $B=4$ | **A1** |
| | Correct PF form $\dfrac{3}{1-x}+\dfrac{4}{2+3x}$ | **A1** |
| | | **(3)** |
| **(b)(i)** | $\dfrac{A}{1-x}=A(1-x)^{-1}=A\left(1+x+x^2+\ldots\right)$ | **B1** |
| | $\left\{\dfrac{B}{2}\right\}\left(1+\dfrac{3x}{2}\right)^{-1}=\left\{\dfrac{B}{2}\right\}\left(1+(-1)\dfrac{3x}{2}+\dfrac{(-1)(-2)}{2}\left(\dfrac{3x}{2}\right)^2+\ldots\right);=\dfrac{B}{2}\left(1-\dfrac{3x}{2}+\dfrac{9x^2}{4}+\ldots\right)$ | **M1; A1** |
| | $\text{f}(x)=\ 3\times\left(1+x+x^2+\ldots\right)+\ \dfrac{4}{2}\left(1-\dfrac{3x}{2}+\dfrac{9x^2}{4}+\ldots\right)$ | **M1** |
| | $=5+\dfrac{15}{2}x^2+\ldots$ | **A1** |
| | | **(5)** |
| **(b)(ii)** | $\lvert x\rvert<\dfrac{2}{3}$ | **B1** |
| | | **(1)** |
| | | **(9 marks)** |
| **(b)(i) Alt 1** | $(1-x)^{-1}=1+x+x^2+\ldots$ | **B1** |
| | $\left\{\dfrac{1}{2}\right\}\left(1+\dfrac{3x}{2}\right)^{-1}=\left\{\dfrac{1}{2}\right\}\left(1+(-1)\dfrac{3x}{2}+\dfrac{(-1)(-2)}{2}\left(\dfrac{3x}{2}\right)^2+\ldots\right);=\dfrac{1}{2}\left(1-\dfrac{3x}{2}+\dfrac{9}{4}x^2+\ldots\right)$ | **M1; A1** |
| | $\dfrac{5x+10}{(1-x)(2+3x)}=(5x+10)\left(1+x+x^2+\ldots\right)\times\dfrac{1}{2}\left(1-\dfrac{3x}{2}+\dfrac{9x^2}{4}+\ldots\right)=5+\ldots x+\ldots x^2$ | **M1** |
| | $=5+\dfrac{15}{2}x^2+\ldots$ | **A1** |
| | | **(5)** |
| **(b)(i) Alt 2** | $\dfrac{5x+10}{(1-x)(2+3x)}=(5x+10)\left(2+\left(x-3x^2\right)\right)^{-1}=\dfrac{1}{2}(5x+10)\left(1+\dfrac{1}{2}\left(x-3x^2\right)\right)^{-1}$ | **B1** |
| | $(1+p(x))^{-1}=\left(1\pm p(x)+\dfrac{(-1)(-2)}{2}\left(p(x)\right)^2+\ldots\right);\ \dfrac{1}{2}\left(1-\dfrac{1}{2}\left(x-3x^2\right)+\dfrac{1}{4}\left(x-3x^2\right)^2+\ldots\right)$ | **M1; A1** |
| | $(10+5x)\left(\dfrac{1}{2}-\dfrac{1}{4}x+\dfrac{3}{4}x^2+\dfrac{1}{8}x^2+\ldots\right)=5-\dfrac{5}{2}x+\dfrac{35}{4}x^2+\dfrac{5}{2}x-\dfrac{5}{4}x^2+\ldots$ | **M1** |

| | | |
|---|---|---|
| | $= 5 + \frac{15}{2}x^2 + \ldots$ | **A1** |
| | | **(5)** |

**Notes:**

a)
M1: Attempts at correct PF. Correct form identified (may be implicit) and achieves a value for at least one of the constants.
A1: One correct value or term.
A1: Correct PF form $\frac{3}{1-x} + \frac{4}{2+3x}$. This may be awarded if seen in (b) but the correct final form (not just values) must be seen somewhere in the question. Accept at $3(1-x)^{-1} + 4(2+3x)^{-1}$

(b)(i)
B1: $\frac{A}{1-x} = A(1-x)^{-1} = A\left(1 + x + x^2 + \ldots\right)$ which may be unsimplified. Allow with their $A$ or with $A = 1$.

M1: Attempts to expand $\frac{1}{2+3x} = (2+3x)^{-1}$ binomially either by taking out the factor 2 first, or directly. Look for $(1+kx)^{-1} = \ldots\left(1 \pm kx + \frac{(-1)(-2)}{2}(kx)^2 + \ldots\right)$ where $k \neq 1$ following an attempt at taking out a factor 2, or $\frac{1}{2+3x} = (2+3x)^{-1} = \left(2^{-1} \pm 2^{-2}kx + \frac{(-1)(-2)}{2}2^{-3}(kx)^2 + \right)$ by direct expansion. Allow missing brackets on $kx^2$ in either case.

A1: $\frac{B}{2+3x} = \frac{B}{2}\left(1 + \frac{3x}{2}\right)^{-1} = \frac{B}{2}\left(1 - \frac{3x}{2} + \frac{9}{4}x^2 + \right)$ oe with their $B$ from (a) or with $B = 1$

M1: Uses their coefficients and attempts to add both series.

A1cao: $5 + \frac{15}{2}x^2 + \ldots$ Condone additional higher order terms. Terms may be either order.

(b)(ii)
B1: $|x| < \frac{2}{3}$ or exact equivalent. This must be clearly identified as the answer. B0 if both ranges are given with no choice of which is correct. (But B1 if formal set notation with $\cap$ used.)

**(b)(i) Alt 1:**

B1: $(1-x)^{-1} = 1 + x + x^2 + \ldots$ which may be unsimplified.

M1: Same as main scheme.
A1: Correct expansion (see main scheme, $B = 1$ allowed).
M1: Attempts to expand all three brackets, achieving the correct constant term at least.
A1cso: $5 + \frac{15}{2}x^2 + \ldots$ Condone additional higher order terms. Terms may be either order.

**(b)(i) Alt 2**

B1: Writes f($x$) as $(5x+10)\left(2 + \left(x - 3x^2\right)\right)^{-1}$ or with the 2 extracted, with the $\left(x - 3x^2\right)$ clear.

M1: Attempts the binomial expansion on $\left(1+p(x)\right)^{-1}$ or $\left(2+p(x)\right)^{-1}$ for $p(x)$ of form $ax+bx^2$.

Same conditions as for main scheme.

A1: Correct expansion. For direct expansion $\left(\frac{1}{2}-\frac{1}{4}\left(x-3x^2\right)+\frac{1}{8}\left(x-3x^2\right)^2+...\right)$

M1: Expands the brackets achieving at least the correct constant term.

A1cao: $5+\frac{15}{2}x^2+...$ Condone additional higher order terms. Terms may be either order.

| Question Number | Scheme | Marks |
|---|---|---|
| **2 (a)** | E.g. $x=\dfrac{t-1}{2t+1}\Rightarrow t=\dfrac{x+1}{1-2x}$ or $y=\dfrac{6}{2t+1}\Rightarrow t=\dfrac{6-y}{2y}$ | **M1** |
| | E.g. $y=\dfrac{6}{2t+1}\Rightarrow y=\dfrac{6}{2\times\left(\dfrac{x+1}{1-2x}\right)+1}$ or $t=\dfrac{6-y}{2y}\Rightarrow x=\dfrac{\dfrac{6-y}{2y}-1}{2\times\dfrac{6-y}{2y}+1}$ | **A1** |
| | E.g. $y=\dfrac{6}{2\times\left(\dfrac{x+1}{1-2x}\right)+1}\Rightarrow y=\dfrac{6(1-2x)}{2\times(x+1)+1(1-2x)}=ax+b$ | **dM1** |
| | E.g. $y=\dfrac{6(1-2x)}{3}, y=2(1-2x)$ oe so linear * | **A1*** |
| | | **(4)** |
| | | |
| **(b)** | $y=2(1-2x)$ and $y=x+12\Rightarrow\ 2(1-2x)=x+12\Rightarrow x=...$ | **M1** |
| | $x=-2$ | **A1cao** |
| | | **(2)** |
| **Alt (b)** | $\dfrac{6}{2t+1}=\dfrac{t-1}{2t+1}+12\Rightarrow t=\left(-\dfrac{1}{5}\right)$ | **M1** |
| | $x=\dfrac{-\frac{1}{5}-1}{2\times-\frac{1}{5}+1}=-2$ | **A1** |
| | | **(2)** |
| | | **(6 marks)** |

**Notes:**

(a) Do not recover marks for part (a) from part (b) if there is an attempt at part (a). If there is no labelling mark as a whole.

M1: For an attempt to get $t$ in terms of $x$ or $y$ **or** $x$ and $y$ $\dfrac{x}{y}=\dfrac{t-1}{6}\Rightarrow t=\dfrac{6x+y}{y}$ or full method to eliminate $t$ from the equations.

A1: Forms a correct equation linking $x$ and $y$ only. Other forms are possible using $t$ in terms of $x$ and $y$ in either equation for $x$ or $y$ etc.

dM1: Depends on first M. Attempts to simplify the fraction reaching a linear form in $x$ and $y$. Allow if there are slips but an unsimplified equation of form $ax + by = c$ must be achieved.

A1: Achieves $y=2(1-2x)$ o.e. (and isw after a correct linear equation) and states linear or hence on line etc. There must be a reference to linearity in some form (similarly for the Alts).

(b)

M1: Solves their "$y=2(1-2x)$" (may not be linear) with $y=x+12$, E.g. $2(1-2x)=x+12\Rightarrow x=...$

| |
|---|
| A1: cao $x=-2$ (ignore any references to the *y* coordinate). Do not accept $-\frac{10}{5}$<br>Alt (b)<br>M1: Solves the parametric equations simultaneously with the line equation to find a value for *t*<br>A1: cao Deduces correct value for *x*. |

| | | |
|---|---|---|
| **2 (a) Alt 1** | $\frac{dx}{dt}=\frac{(2t+1)-2(t-1)}{(2t+1)^2}$ and $\frac{dy}{dt}=\frac{-12}{(2t+1)^2}$ o.e. | **M1**<br>**A1** |
| | $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$ | **dM1** |
| | $\frac{dy}{dx}=\frac{-12}{(2t+1)^2}\div\frac{3}{(2t+1)^2}=-4$ (which is a constant,) hence linear | **A1*** **(4)** |
| Alt 1 (a) via differentiation **Notes:** | | |
| M1: Attempts $\frac{dx}{dt}$ and $\frac{dy}{dt}$ using appropriate rule for at least one.<br>A1: Both correct<br>dM1: Depends on first M. Attempts $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$ leading to constant.<br>A1: Achieves $\frac{dy}{dx}=-4$ and makes suitable conclusion e.g. “hence linear” * | | |
| **2 (a) Alt 2** | $x=\frac{t-1}{2t+1}\Rightarrow x=A-\frac{B}{2t+1}$; $x=\frac{1}{2}-\frac{3}{2(2t+1)}$ | **M1;**<br>**A1** |
| | $x=\frac{1}{2}-\frac{3}{2\left(6/y\right)}$ | **dM1** |
| | $x=\frac{1}{2}-\frac{1}{4}y$ hence linear * | **A1*** **(4)** |
| Alt 2 (a) via division **Notes:** | | |
| M1: Attempts to write *x* in terms of just 2*t*+1. E.g $x=\frac{t-1}{2t+1}\Rightarrow x=A-\frac{B}{2t+1}$.<br>A1: $x=\frac{1}{2}-\frac{3}{2(2t+1)}$<br>dM1: Uses $y=\frac{6}{2t+1}$ to form an equation linking *x* and *y*<br>A1: $x=\frac{1}{2}-\frac{1}{4}y$ and states linear* | | |
| **2 (a) Alt 3** | $ax+by=\frac{at-a+6b}{2t+1}=\frac{k\left(2t+1\right)}{2t+1}\Rightarrow a=2k, 6b-a=k$ | **M1** |
| | $a=12b-2a\Rightarrow a=4b$ | **A1** |
| | E.g. $4x+y=\frac{2\left(2t+1\right)}{2t+1}=\ldots$ | **dM1** |
| | $4x+y=2$ (oe) hence linear * | **A1*** **(4)** |

| Alt 3 (a) via elimination **Notes:** |
|---|
| M1: Writes $ax+by=...$ as a single fraction and attempts to compare coefficients of numerator with the denominator.<br>A1: Correct ratio between $a$ and $b$ deduced.<br>dM1: Uses their ratio to eliminate $t$ from the equation.<br>A1: $4x+y=2$ (oe) and states linear*<br>Note: It is possible to spot the correct values for $a$ and $b$ directly so the following would gain full marks: $2x+\frac{1}{2}y=\frac{2t-2+3}{2t+1}=\frac{2t+1}{2t+1}=1$ hence linear. * |

| Question Number | Scheme | Marks |
|---|---|---|
| **3.** | States or implies Volume = $\int_{\sqrt{5}}^{5} \pi \left(\sqrt{\dfrac{3x}{3x^2+5}}\right)^2 \mathrm{d}x$ | **B1** |
| | $\int \left(\sqrt{\dfrac{3x}{3x^2+5}}\right)^2 \mathrm{d}x = \int \dfrac{3x}{(3x^2+5)} \mathrm{d}x = \dfrac{1}{2}\ln(3x^2+5)$ | **M1A1** |
| | Volume = $\{\pi\}\left(\dfrac{1}{2}\ln(3\times 25+5) - \dfrac{1}{2}\ln(3\times 5+5)\right)$ | **M1** |
| | $= \pi \ln 2$ | **A1** |
| | | |
| | | **(5 marks)** |

**Notes:**

B1: States or implies Volume = $\int_{\sqrt{5}}^{5} \pi \left(\sqrt{\dfrac{3x}{3x^2+5}}\right)^2 \mathrm{d}x$ o.e. The limits may be implied by subsequent work, and the d$x$ may be missing. This is for knowing the correct formula rather than for notation. The $\pi$ may be implied by later work.

M1: Attempts $\int \left(\sqrt{\dfrac{3x}{3x^2+5}}\right)^2 \mathrm{d}x$ to achieve $k\ln(3x^2+5)$ (oe). May use substitution, either $u$ = $3x^2$ or $u = 3x^2+5$, in which case they should achieve $k\ln(u+5)$ or $k\ln(u)$ (oe) respectively. Allow if the brackets are missing.

A1: Correct result of integration, which may be left unsimplified. May be in terms of $u$ if a substitution has been used. Allow if missing brackets are recovered, but A0 if never recovered.

M1: Having achieved an integral of the form $p(x)\ln(3x^2+5)$ (allowing for missing brackets) where $p(x)$ is constant or a polynomial in $x$ (oe in terms of $u$ for substitution), uses the limits within their integral - substitutes **correct limits for their variable** and subtracts, allowing either way round. The $\pi$ may be missing for this mark.

A1: $\pi \ln 2$ cao. Note $\dfrac{\pi}{2}\ln 4$ is A0 as form is not as specified.

| Question Number | Scheme | Marks |
|---|---|---|
| **4 (a)** | $a=3, b=5$ | **B1** |
| | E.g $u=\sqrt{2x+1} \Rightarrow \dfrac{dx}{du}=u$ or $\dfrac{du}{dx}=(2x+1)^{-\frac{1}{2}}$ o.e. | **B1** |
| | $\int \sqrt{8x+4}\ e^{\sqrt{2x+1}}\ dx = \int 2u\ e^{u}\ u\ du$ | **M1** |
| | $= \int_3^5 2u^2\ e^u\ du$ | **A1** |
| | | **(4)** |
| **(b)** | $\int \sqrt{8x+4}\ e^{2x+1}\ dx = \int 2u^2\ e^u\ du$ | |
| | $= 2u^2e^u - \int 4u\ e^u\ du$ | **M1** |
| | $= \ = 2u^2e^u - \left(4ue^u - 4e^u\right) = 2u^2e^u - 4ue^u + 4e^u$ | **dM1 A1ft** |
| | $\int_4^{12} \sqrt{8x+4}\ e^{\sqrt{2x+1}}\ dx = \left[2u^2e^u - 4ue^u + 4e^u\right]_3^5 = \left(50e^5 - 20e^5 + 4e^5\right) - \left(18e^3 - 12e^3 + 4e^3\right)$ | **ddM1** |
| | $= 34e^5 - 10e^3$ | **A1** |
| | | **(5)** |
| | | **(9 marks)** |

**Notes:**

(a)

B1: For both $a=3, b=5$ seen in their solution. Allow if these are recovered in (b).

B1: For a correct expression involving $\dfrac{du}{dx}$ or $\dfrac{dx}{du}$ or d$u$ and d$x$ separately. May be unsimplified

M1: Attempts to fully change $\int \sqrt{8x+4}\ e^{\sqrt{2x+1}}\ dx$ into an integral with respect to $u$. Must include an attempt at replacing d$x$ to get d$u$ so M0 if there are no d terms present or d$x$ becomes d$u$ without an attempt at connecting them first (ie there must have been an attempt at $\dfrac{du}{dx}$ oe).

A1: Complete method to show I = $\int_3^5 2u^2\ e^u\ du$. Must include the correct limits and the d$u$.

(b) Note: you may see different ways of presenting the application of parts e.g D/I method.

M1: Use of integration by parts once to obtain $pu^2e^u - \int qu\ e^u\ du$, where $p, q > 0$ (if $k$ is positive, otherwise signs will be opposite) and may be in terms of $k$ (as can the dM mark).

dM1: Completely integrates by parts twice to a form $pu^2e^u - que^u \pm re^u$ where $p, q > 0$ (if $k > 0$ as before). Note they may evaluate in stages, but look for the complete integration overall.

A1ft: $\int_a^b ku^2\ e^u\ du = ku^2e^u - 2kue^u + 2ke^u$ (oe) accepted with $k$ or their value for $k$. May have the last two terms bracketed but must be seen as a complete answer in their work.

ddM1: Substitutes their $a$ and $b$ into a form $pu^2e^u - que^u \pm re^u$ and subtracts (either way), but must be using a value for $k$ at this stage. May be done in stages.

A1: $34e^{5}-10e^{3}$ or exact equivalent in a simplified form such as $2e^{3}\left(17e^{2}-5\right)$

| Question Number | Scheme | Marks |
|---|---|---|
| **5 (a)** | $y^2 = 2x^2 + 15x + 10y \Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 4x + 15 + 10\frac{\mathrm{d}y}{\mathrm{d}x}$ | **M1 A1** |
| | $(2y-10)\frac{\mathrm{d}y}{\mathrm{d}x} = 4x + 15 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4x+15}{2y-10}$ oe | **M1, A1** |
| | | **(4)** |
| | | |
| **(b)** | Deduces that $2y - 10 = 0 \Rightarrow y = 5$ | **B1ft** |
| | Substitutes $y = 5$ into $y^2 = 2x^2 + 15x + 10y \Rightarrow 2x^2 + 15x + 25 = 0$ and solves for $x$ | **M1** |
| | $(p =) -5, (q =) -\frac{5}{2}$ | **A1** |
| | | **(3)** |
| | | **(7 marks)** |

**Notes:**

(a)

M1: Correct differentiation of one of the $y$ terms, ie $y^2 \to 2y\frac{\mathrm{d}y}{\mathrm{d}x}$ or $10y \to 10\frac{\mathrm{d}y}{\mathrm{d}x}$.

A1: Fully correct differentiation $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 4x + 15 + 10\frac{\mathrm{d}y}{\mathrm{d}x}$ o.e.

M1: Rearranges to make $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject. The differentiated expression must contain exactly two $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms - one from each $y$ term, not an extra $\frac{\mathrm{d}y}{\mathrm{d}x} = \ldots$.

A1: $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4x+15}{2y-10}$ oe

(b)

B1ft: Deduces that $2y - 10 = 0 \Rightarrow y = 5$. Follow through on a denominator of form $ay + b$, $a, b \neq 0$. This deduction may arise from use of the symmetry of a hyperbola.

E.g. $x = 0 \Rightarrow y^2 - 10y = 0 \Rightarrow y = 0, 10$ so $p, q$ when $y = 5$

M1: Substitutes their $y = 5$ into $y^2 = 2x^2 + 15x + 10y\left(\Rightarrow 2x^2 + 15x + 25 = 0\right)$ and solves for $x$ (usual rules, if no working shown (by calculator) they must give correct values for their quadratic).

A1: $(p =) -5, (q =) -\frac{5}{2}$ Correct values, do not be concerned about the labels and accept if they give as the end points of the interval (ie accept if they give $\left(-5, -\frac{5}{2}\right)$ as their answer).

(b) Alt method 1

$$y^2 = 2x^2 + 15x + 10y \Rightarrow y^2 - 10y - \left(2x^2 + 15x\right) = 0$$

B1: Deduces that roots for $x$ don't exist when $100 + 4\times\left(2x^2 + 15x\right) < 0$

M1: Solves their $2x^2 + 15x + 25 < 0$

A1: $(p =) - 5, (q =) - \frac{5}{2}$ Correct values, see note on main scheme.

(b) Alt method 2

B1: $y = 0 \Rightarrow 2x^2 + 15x = 0 \Rightarrow x = 0, -\frac{15}{2}$ Correct values found for $x$ when $y = 0$

M1: Full method to use symmetry to deduce the required values of $x$. E.g. by symmetry, values required are one third and two thirds way between these $\Rightarrow x = \frac{1}{3}\times -\frac{15}{2}, \frac{2}{3}\times -\frac{15}{2}$

A1: $(p =) - 5, (q =) - \frac{5}{2}$ Correct values, see note on main scheme.

| Question Number | Scheme | Marks |
|---|---|---|
| **6 (a)(i)** | $\overrightarrow{AB}=(\pm)\left[(8\mathbf{i}+3\mathbf{j}-7\mathbf{k})-(2\mathbf{i}-3\mathbf{j}+5\mathbf{k})\right]=...$ | **M1** |
| | $\overrightarrow{AB}=6\mathbf{i}+\mathbf{6j}-12\mathbf{k}$ | **A1** |
| **(ii)** | $\mathbf{r}=\begin{pmatrix}2\\-3\\5\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\-2\end{pmatrix}$ o.e. such as $\mathbf{r}=\begin{pmatrix}8\\3\\-7\end{pmatrix}+\lambda\begin{pmatrix}6\\6\\-12\end{pmatrix}$ | **B1ft** |
| | | **(3)** |
| **(b)** | Attempts $\pm\overrightarrow{CP}=\pm\begin{pmatrix}2+\lambda-3\\-3+\lambda-5\\5-2\lambda-2\end{pmatrix}$ | **M1** |
| | $\overrightarrow{CP}\bullet k\begin{pmatrix}1\\1\\-2\end{pmatrix}=0\Rightarrow\begin{pmatrix}\lambda-1\\\lambda-8\\-2\lambda+3\end{pmatrix}\bullet k\begin{pmatrix}1\\1\\-2\end{pmatrix}=0\Rightarrow 1(\lambda-1)+1(\lambda-8)-2(-2\lambda+3)=0$<br>Alt: $(\lambda-1)^2+(\lambda-8)^2+(-2\lambda+3)^2=6\lambda^2-30\lambda+74=6\left(\lambda-\frac{5}{2}\right)^2+\frac{73}{2}$ | **dM1** |
| | $\Rightarrow\lambda=\frac{5}{2}$ $\left[\text{use of }\overrightarrow{AB}\text{ in }\overrightarrow{CP}\text{ gives }\lambda=\frac{5}{12},\text{use of }\overrightarrow{OB}\ \lambda=\frac{-7}{2}\text{ or }\frac{-7}{12}\right]$ | **A1** |
| | $\overrightarrow{OP}=\begin{pmatrix}2\\-3\\5\end{pmatrix}+\frac{5}{2}\begin{pmatrix}1\\1\\-2\end{pmatrix}=\frac{9}{2}\mathbf{i}-\frac{1}{2}\mathbf{j}$ | **ddM1, A1**<br>**(5)** |
| | | **(8 marks)** |

**Notes:**

Accept either form of vector notation throughout. Accept with **i**,**j** and **k** in their column vectors.

(a)(i)

M1: Attempts to subtract vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ either way around. May be implied by two correct components.

A1: $\overrightarrow{AB}=6\mathbf{i}+\mathbf{6j}-12\mathbf{k}$ o.e.

(a)(ii)

B1ft: Any correct equation for the line, may use a correct or follow through multiple of $\overrightarrow{AB}$ for direction and with any point on the line. Must start **r** = … or accept $x\mathbf{i}+y\mathbf{j}+z\mathbf{k}=...$ ($l$ = … is B0).

(b)

M1: Attempts $\pm\overrightarrow{CP}$ using point $C$ and a general point on their $l$

dM1: Sets the scalar product of their $\overrightarrow{CP}$ (either direction) and their direction of $l$ (or $\overrightarrow{AB}$) to 0 and proceeds to an equation in $\lambda$. Condone sign slips in components if the intention is clear.

Alternatively attempts to minimise the distance $CP$ (by completing square as shown, or by differentiation) to obtain a linear equation in $\lambda$.

A1: Finds a correct value of $\lambda$ for their $l$. Note if they use $\overrightarrow{AB}$ the correct value is $\frac{5}{12}$

ddM1: Substitutes their $\lambda$ (from a correct method) into their $l$

A1: $\overrightarrow{OP} = \frac{9}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}$ Accept as coordinates, and accept $P = \ldots$ instead of $\overrightarrow{OP}$.

| Question Number | Scheme | Marks |
|---|---|---|
| **7 (a)** | $\frac{dV}{dr} = 4\pi r^2$ | **B1** **(1)** |
| **(b) (i)** | $\frac{dV}{dt} = \frac{900}{(2t+3)^2} \Rightarrow V = -\frac{450}{2t+3} + c$ (oe) | **M1 A1** |
| | $t = 0, V = 0 \Rightarrow 0 = -\frac{450}{3} + c \Rightarrow c = ...$ | **M1** |
| | $V = 150 - \frac{450}{2t+3} = \frac{300t + 450 - 450}{2t+3} = \frac{300t}{2t+3}$ * | **A1 *** |
| **(ii)** | 150 cm$^3$ | **B1** |
| | | **(5)** |
| **(c)** | $t = 3 \Rightarrow V = \frac{300 \times 3}{2 \times 3 + 3} = (100)$ | **M1** |
| | $100 = \frac{4}{3}\pi r^3 \Rightarrow r = 2.88\text{ cm}$ | **dM1 A1cao** **(3)** |
| **(d)** | $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow \frac{900}{(2t+3)^2} = 4\pi r^2 \times \frac{dr}{dt}$ | **M1** |
| | $t = 3, r = "2.88" \Rightarrow \frac{900}{81} = 4\pi \times 2.88^2 \times \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = ...$ | **dM1** |
| | $\Rightarrow \frac{dr}{dt} = \text{awrt } 0.11\text{ cm s}^{-1}$ | **A1** **(3)** |
| | | **(12 marks)** |

**Notes:** Mark the question as a whole. Penalise only once for missing/incorrect units in the question.

(a)
B1: cao See scheme.
(b)(i)
M1: Integrates to a form $V = \frac{k}{2t+3}$ (oe) with or without + $c$. Condone a sign error in $2t - 3$.
A1: $V = -\frac{450}{2t+3}(+c)$ (oe). There is no need for + $c$
M1: Substitutes $V = 0, t = 0$ and proceeds to find a value for $c$. There must have been an attempt at integrating to achieve a function in $V$ and $t$ with a constant of integration.
A1*: Correct integration and value for $c$ with at least one intermediate step with $c$ substituted back in the equation before proceeding to the given answer.
(b)(ii)
B1: 150 cm$^3$. Must include units.
(c)
M1: Attempts to substitute t = 3 into the equation for $V$. Allow if there is a slip in substitution.
dM1: Uses their $V$ in $V = \frac{4}{3}\pi r^3$ to find a value for $r$
A1: cao $r = 2.88\text{ cm}$. Must be to 3 s.f.. Must include units unless already penalised in (b)(ii).
(d)
M1: Attempts to use $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ (oe) with the given formula for $\frac{dV}{dt}$ and an attempt at substituting their $\frac{dV}{dr}$ (allow if this substitution is not in the correct place if a correct chain rule has been stated.)

dM1: Substitutes both $t = 3$ and their value for $r$ and proceeds to find a value for $\frac{dr}{dt}$. If no substitution shown, the answer must be correct for their $r$ to imply the method (may need to check).
A1: awrt $0.11$ cm s$^{-1}$. Must include units unless already penalised in (b)(ii) or (c).

| Question Number | Scheme | Marks |
|---|---|---|
| **8 (a)** | At $t=\frac{\pi}{4}$ $P=\left(\frac{1}{2},2\right)$ | **B1** |
| | $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}=\frac{2\sec^2 t}{2\sin t\cos t}=4$ when $t=\frac{\pi}{4}$ | **M1 A1** |
| | Equation of $l$: $y-2=-\frac{1}{4}\left(x-\frac{1}{2}\right)\Rightarrow 8y-16=-2x+1\Rightarrow 8y+2x=17*$ | **dM1 A1 * cso** |
| | | **(5)** |
| **(b)** | $\int y\frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t=\int 2\tan t\times 2\sin t\cos t\,\mathrm{d}t$ | **M1** |
| | $=\int 4\sin^2 t\,\mathrm{d}t$ | **A1** |
| | $=\int 2-2\cos 2t\,\mathrm{d}t = 2t-\sin 2t$ | **dM1 A1** |
| | Total area of $S=\left[2t-\sin 2t\right]_0^{\frac{\pi}{4}}+\frac{1}{2}\times 8\times 2=\frac{\pi}{2}-1+8=\frac{\pi}{2}+7$ | **M1 A1** |
| | | **(6)** |
| | | **(11 marks)** |

**Notes:**

(a)

B1: Correct coordinates for $P$ stated or implied by working.

M1: Attempts to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ using $\frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}$ at $t=\frac{\pi}{4}$. Condone poor differentiation. Substitution of the $\frac{\pi}{4}$ is sufficient for the method. Alternatively, may attempt $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $-\frac{\mathrm{d}x}{\mathrm{d}y}$. Accept a value following finding $\frac{\mathrm{d}y}{\mathrm{d}x}$ (or its reciprocal etc) as an attempt to evaluate at $t=\frac{\pi}{4}$ if no contrary working is shown **but check carefully** as the correct answer may arise from incorrect working.

A1: Correct $\frac{\mathrm{d}y}{\mathrm{d}x}=4$ (oe equation) following correct differentiation. May be implied.

dM1: Attempts to find the equation of the normal at $t=\frac{\pi}{4}$. It is dependent upon the previous M and use of their $P$. The value of the gradient used must be correct for their differential.

A1*: **cso** Correct proof leading to $8y+2x=17$

(b)

M1: Attempts $\int y\frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t=\int 2\tan t\times "2\sin t\cos t"\,\mathrm{d}t$ with their $\frac{\mathrm{d}x}{\mathrm{d}t}$ condoning slips on coefficients.

A1: $\int 4\sin^2 t\,\mathrm{d}t$

| |
|---|
| dM1: Uses $\cos 2t = \pm 1 \pm 2\sin^2 t$ and integrates $\int \pm p \pm q\cos 2t\,\mathrm{d}t$ to a form $\pm at \pm b\sin 2t$<br>See note below.<br>A1: $\int y\dfrac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t = 2t - \sin 2t$ See note below.<br>M1: Full method to find area of region $S$. Finds the sum of their values for $\int_0^{\frac{\pi}{4}} y\dfrac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t$ and $\dfrac{1}{2}\left(\dfrac{17}{2} - P_x\right)\times P_y$ . Condone poor integration for this mark as long as they are attempting to apply the correct limits to their result. They may attempt the area under the line by integration: $\int_{P_x}^{\frac{17}{2}} -\dfrac{1}{4}x + \dfrac{17}{8}\,\mathrm{d}x$ In such a method condone minor slips, but must be attempting correct limits.<br>A1: $\dfrac{\pi}{2} + 7$<br>**Note:** If the $t$'s becomes $x$'s during the integration, then allow the M's and the A's if recovered but if $2x - \sin 2x$ or with mixed variables is found and $x$ values substituted then it is M1A0 for the integral and M0 for the method for area. |

| | | |
|---|---|---|
| **8 (a) Alt** | At $t = \dfrac{\pi}{4}$ $P = \left(\dfrac{1}{2}, 2\right)$ | **B1** |
| | $y = \dfrac{2\sin t}{\cos t} = \dfrac{2\sqrt{x}}{\sqrt{1-x}}$ $y^2 = \dfrac{4x}{1-x}$<br>$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x^{-\frac{1}{2}}(1-x)^{\frac{1}{2}} - 2\sqrt{x}\times -\frac{1}{2}(1-x)^{-\frac{1}{2}}}{1-x} \Rightarrow \left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=\frac{1}{2}} = 4$ or<br>$2y\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{4(1-x) - 4x\times -1}{(1-x)^2} \Rightarrow \left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=\frac{1}{2}, y=2} = 4$ oe | **M1 A1** |
| | Equation of $l$:<br>$y - 2 = -\dfrac{1}{4}\left(x - \dfrac{1}{2}\right) \Rightarrow 8y - 16 = -2x + 1 \Rightarrow 8y + 2x = 17$* | **dM1 A1 * cso** |
| | | **(5)** |

| |
|---|
| (a)<br>B1: Correct coordinate for $P$ $\left(\dfrac{1}{2}, 2\right)$ stated or implied by working.<br>M1: Attempts to find Cartesian equation for $C$, any form, and attempts $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ (or an equivalent as main scheme) with appropriate differentiation methods for their Cartesian form, allowing for slips and finds $x$ and/or $y$ using $t = \dfrac{\pi}{4}$ and evaluate the derivative with these values. |

A1: Correct $\frac{\mathrm{d}y}{\mathrm{d}x} = 4$ (oe equation) following correct differentiation and from correct work.

dM1: Attempts to find the equation of the normal at their $x$ and $y$ values.
It is dependent upon the previous M and use of their $P$.

A1*: **cso** Correct proof leading to $8y + 2x = 17$

| Question Number | Scheme | Marks |
|---|---|---|
| **9 (a)** | Let $p=3k+2$ then $(3k+2)^3=27k^3+54k^2+36k+8$ | **M1** |
| | $=3\times\left(9k^3+18k^2+12k+3\right)-1$ not a multiple of 3 | **A1** |
| | So *p* cannot be of form 3*k* + 1 or 3*k* + 2, since $p^3$ is a multiple of 3. Hence *p* must be a multiple of 3, a contradiction of our assumption, hence for all integers *p*, when $p^3$ is a multiple of 3, then *p* is a multiple of 3 | **A1** |
| | | **(3)** |
| **(b)** | Assumption: there exist (integers) *p* and *q* such that $\sqrt[3]{3}=\frac{p}{q}$ (where *p* and *q* have no (non-trivial) common factors.) | **B1** |
| | Then $\sqrt[3]{3}=\frac{p}{q}\Rightarrow p^3=3q^3$ | **M1** |
| | So $p^3$ is a multiple of 3 and (so) *p* is a multiple of 3 | **A1** |
| | But $p=3k\Rightarrow 27k^3=3q^3\Rightarrow q^3=9k^3$ | **dM1** |
| | Hence $q^3$ is a multiple of 3 so *q* is a multiple of 3, but as *p* and *q* have no (non-trivial) common factors, this is a contradiction. Hence $\sqrt[3]{3}$ is an irrational number.* | **A1*** |
| | | **(5)** |
| | | **(8 marks)** |

**Notes:**

(a)

M1: Attempts to expand $(3k+2)^3$ or $(3k-1)^3$.

Look for a cubic expression with **4 terms** with at least **two correct** (allowing for incorrect signs).

A1: Achieves a correct $3\times(...)+r,|r|<10$ form for the expansion and states not a multiple of 3.

Suitable forms include $(3k+2)^3=3\left(9k^3+18k^2+12k+2\right)+2=3\left(9k^3+18k^2+12k+3\right)-1$ or

$3\left(9k^3+18k^2+12k\right)+8$ or $(3k-1)^3=3\left(9k^3-9k^2+3k\right)-1$ etc.

Alternatively, achieves correct $(3k+2)^3=27k^3+54k^2+36k+8$ or $(3k-1)^3=27k^3-27k^2+9k-1$ with a **reason** why it is not a multiple of 3 e.g 3 divides 27, 54 and 36, but not 8 hence not divisible by 3.

A1: Completes the proof. Must have scored both previous marks and a reference to **both cases** (in some form) leading to a "**contradiction"** and some indication that **proof is complete**. It is unlikely to be as complete as that shown in the scheme, but all three bold points must be conveyed. E.g. as a minimum after satisfying the first A "both cases give a contradiction hence the original statement is true".

(b)

May use different letters throughout.

B1: Sets up algebraically the initial statement to be contradicted. Essentially for showing they know what a rational number is algebraically. There is no requirement for this mark to state that $p$ and $q$ are integers with no (non-trivial) common factors (this may be implied for this mark).
M1: Cubes correctly and multiplies through by $q^3$
A1: Deduces that both $p^3$ is a multiple of 3 and hence $p$ is a multiple of 3. Jumping directly to $p$ is a multiple of 3 is A0.
dM1: Sets $p = 3k$ and proceeds to find $q^3$ in terms of $k$. (May use a different letter.)
A1: Completes the proof. This requires

- Correct algebraic statements
- Correct deductions in correct order. E.g. $p^3$ is a multiple of 3 so $p$ is a multiple of 3
- initial statement must have included that $p$ and $q$ are integers (or accept natural numbers or a $\dfrac{p}{q}$ is a fraction) and have no common factors (or are co-prime, or in simplest form)
- correct reason for contradiction and acceptable conclusion
- There must have been no contrary statements during the proof (e.g. that $p$ and $q$ are prime)