Pearson Edexcel

Mark Scheme (Results)

October 2020

Pearson Edexcel IAL Mathematics (WMA14)

Pure Mathematics P4

Question Number	Scheme	Marks
1	Assume that there exists a number *m* such that when m^3 is even, *m* is odd	B1
	If *m* is odd then $m = 2p+1$ (where *p* is an integer) and $m^3 = (2p+1)^3 = ...$	M1
	$= 8p^3 + 12p^2 + 6p + 1$	A1
	$2\times\left(4p^3+6p^2+3p\right)+1$ is odd and hence we have a contradiction so if n^3 is even, then n is even.	A1
		(4)
		(4 marks)

B1: For setting up the contradiction.

Eg Assume that there exists a number *m* such that when m^3 is even, *m* is odd

Condone a contra-positive statement here

"Assume that there exists a number *m* such that when m^3 is even, *m* is not even"

As a minimum accept "assume if m^3 is even then *m* is odd."

Condone the other way around " assume if *n* is odd then n^3 is even"

M1: Attempts to cube an odd number. Accept an attempt at $(2p+1)^3$, $(2p-1)^3$

Look for $(2p+1)^3 = ...p^3.....$

A1: $(2p+1)^3 = 8p^3 + 12p^2 + 6p + 1$ or simplified equivalent such as $2\times\left(4p^3+6p^2+3p\right)+1$.

For $(2p-1)^3 = 8p^3 - 12p^2 + 6p - 1$ or equivalent such as $2\times\left(4p^3-6p^2+3p-1\right)+1$

A1: For a fully correct proof. Requires correct calculations with reason and conclusion

E.g. 1 Correct calculations $(2p+1)^3 = 8p^3 + 12p^2 + 6p + 1 =$

Reason (even +1) = odd

Conclusion "hence we have a contradiction, so if n^3 is even, then n is even."

E.g. 2 Correct calculations $(2p+1)^3 = 8p^3 + 12p^2 + 6p + 1$

Reason $= 2\times\left(4p^3+6p^2+3p\right)+1$ = odd

Conclusion "this is contradiction, so proven."

E.g. 3 Correct calculations $(2p-1)^3 = 8p^3 - 12p^2 + 6p - 1$

Reason = $8p^3 - 12p^2 + 6p$ is even so $8p^3 - 12p^2 + 6p - 1$ is odd

Conclusion: So if n^3 is even then n must be even

Note that B0 M1 A1 A1 is possible

Question Number	Scheme	Marks
2(a)	$4^{-\frac{1}{2}}$ or $\dfrac{1}{4^{\frac{1}{2}}}$ or $\dfrac{1}{2}$	B1
	$(4-5x)^{-\frac{1}{2}} = \dfrac{1}{2}\left(1-\dfrac{5x}{4}\right)^{-\frac{1}{2}}$	
	$= \ldots\left(1+\left(-\dfrac{1}{2}\right)\times\left(-\dfrac{5x}{4}\right)+\dfrac{\left(-\dfrac{1}{2}\right)\times\left(-\dfrac{3}{2}\right)}{2!}\times\left(-\dfrac{5x}{4}\right)^2+\ldots\right)$	M1A1
	$= \dfrac{1}{2}+\dfrac{5}{16}x+\dfrac{75}{256}x^2+\ldots$	A1
		(4)
	$\dfrac{2+kx}{(2-3x)^3} = (2+kx)\left(\dfrac{1}{2}+\dfrac{5}{16}x+\dfrac{75}{256}x^2+\ldots\right)$	
(b)	Compares x terms leading to $k = \ldots$. E.g. $\dfrac{10}{16}+\dfrac{k}{2}=\dfrac{3}{10} \Rightarrow k = -\dfrac{13}{20}$	M1 A1
		(2)
(c)	Compares x^2 terms leading to $m = \ldots$ E.g. $m = \dfrac{75}{128}+\dfrac{5}{16}\times'-\dfrac{13}{20}' \Rightarrow m = \dfrac{49}{128}$	M1 A1
		(2)
		(8 marks)
Alt (a)	$4^{-\frac{1}{2}}$ or $\dfrac{1}{4^{\frac{1}{2}}}$	B1
	$(4-5x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}+\left(-\dfrac{1}{2}\right)4^{-\frac{3}{2}}(-5x)+\dfrac{-\dfrac{1}{2}\times-\dfrac{3}{2}}{2}4^{-\frac{5}{2}}(-5x)^2$	M1A1
	$= \dfrac{1}{2}+\dfrac{5}{16}x+\dfrac{75}{256}x^2+\ldots$	A1
		(4)

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(a)

B1: For taking out a factor of $4^{-\frac{1}{2}}$ or $\frac{1}{2}$

For a direct expansion look for $4^{-\frac{1}{2}}+....$ or equivalent.

M1: For the form of the binomial expansion $(1+ax)^{-\frac{1}{2}}$ where $a \neq 1$ or -5

To score M1 it is sufficient to see either term two or term three. Allow a slip on the sign.

So allow for either $\left(-\frac{1}{2}\right)(\pm ax)$ or $\dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}(\pm ax)^2$

In the alternative version look for $\left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(-5x)$ or $\dfrac{-\frac{1}{2}\times-\frac{3}{2}}{2}4^{-\frac{5}{2}}(-5x)^2$ condoning sign slips

A1: Any (unsimplified) but correct form of the binomial expansion for . $\left(1-\frac{5x}{4}\right)^{-\frac{1}{2}}$

Ignore the factor preceding the bracket for this mark

Score for $1+\left(-\frac{1}{2}\right)\times\left(-\frac{5x}{4}\right)+\dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2!}\times\left(-\frac{5x}{4}\right)^2$ o.e.

In the alternative version look for $(4-5x)^{-\frac{1}{2}}=4^{-\frac{1}{2}}+\left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(-5x)+\dfrac{-\frac{1}{2}\times-\frac{3}{2}}{2}4^{-\frac{5}{2}}(-5x)^2$

A1: cao $\frac{1}{2}+\frac{5}{16}x+\frac{75}{256}x^2+...$ This must be simplified

$$(2+kx)\left(P+Qx+Rx^2+...\right)=1+\frac{3}{10}x+mx^2$$

(b)

M1: For a correct equation in k formed by comparing the x terms. It must lead to a value for k

Follow through on their expansion. So look for $Pk+2Q=\frac{3}{10}\Rightarrow k=...$ Condone slips, i.e copying errors.

Condone $Pkx+2Qx=\frac{3}{10}x$ as long as it leads to a value for k

A1: $k=-\frac{13}{20}$

(c)

M1: Correctly compares the x^2 terms, following through on their expansion and their value for k leading to a value for m . Condone slips, i.e copying errors.

Look for $Qk+2R=m$ Condone $Qkx^2+2Rx^2=mx^2$ as long as it leads to a value for m

A1: $m=\dfrac{49}{128}$ oe Condone sight of $\dfrac{49}{128}x^2$ as evidence for a correct value for m.

Question Number	Scheme	Marks
3	Calculates the upper limit as $2\ln 2$ or $\ln 4$	B1
	$\int \left(e^{0.5x}-2\right)^2 dx = \int \left(e^{x}-4e^{0.5x}+4\right) dx = e^{x}-8e^{0.5x}+4x$	M1 A1 A1
	Volume $= \pi\left["e^{x}-8e^{0.5x}+4x"\right]_0^{"2\ln 2"} = ...$	dM1
	Volume $= 8\pi\ln 2-5\pi$	A1
		(6 marks)

B1: Calculates the upper limit as $2\ln 2$ or $\ln 4$ only. Do not accept 1.386, $\dfrac{\ln 2}{0.5}$ or anything else.

Recovery from $\dfrac{\ln 2}{0.5}$ is allowed so if the candidate progresses to $8\pi\ln 2-5\pi$ do not withhold this mark.

M1: Attempts to square $\left(e^{0.5x}-2\right)$ **with** $\int e^{kx}dx \to e^{kx}$ seen at least once. Ignore any factor of π

This may be awarded when candidates fails to square $e^{0.5x}$ correctly

So for example $\int \left(e^{0.5x}\right)^2 - 4e^{0.5x}+4\, dx \to ....\pm...e^{0.5x}$ is sufficient

A1: Two terms correct $e^{x}-8e^{0.5x}+4x$ Ignore any factor of π

A1: All three terms correct $e^{x}-8e^{0.5x}+4x$ Ignore any factor of π

dM1: A full and complete method of finding the volume of the solid generated.

Look for an attempt to find $\pi\int_0^{"a"}\left(e^{0.5x}-2\right)^2 dx$ with a being $2\ln 2$, $\ln 4$, 1.38.. or $\dfrac{\ln 2}{2}$

It is dependent upon the previous M so some correct integration must be seen as well as use of both limits. There must be some evidence for the 0. Those terms cannot just "disappear" or be set to 0.

A1: cao $8\pi\ln 2-5\pi$ or $-5\pi+8\pi\ln 2$ Condone $\pi\left(8\ln 2-5\right)$

(Note that a requirement of the question is to write in the form $a\ln 2+b$)

Question Number	Scheme	Marks
4(a)	A and B are where $t^3-4t=0 \Rightarrow t(t^2-4)=0 \Rightarrow t=2 \text{ or } -2$	M1
	Substitutes $t=2, x=2\times4-6\times2=-4$ Hence $A=(-4,0)$	A1
	When $t=-2, x=2\times4-6\times-2=20, (y=0)$ Hence $B=(20,0)$ *	B1*
		(3)
(b)	$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{3t^2-4}{4t-6}$	M1A1
	Sub $t=-2$ into $\frac{dy}{dx}=\frac{3t^2-4}{4t-6} \Rightarrow$ gradient $=\left(-\frac{4}{7}\right)$	M1
	Uses their $\left(-\frac{4}{7}\right)$ and $(20,0)$ to produce eqn of tangent $\Rightarrow 7y+4x-80=0$ *	M1 A1*
		(5)
(c)	Substitutes $x=2t^2-6t,\ y=t^3-4t,$ into $7y+4x-80=0$	
	$\Rightarrow 7(t^3-4t)+4(2t^2-6t)-80=0$	M1
	$\Rightarrow 7t^3+8t^2-52t-80=0$	A1
	$\Rightarrow (t+2)^2(7t-20)=0$	
	$t="\frac{20}{7}" \Rightarrow x=....$	dM1
	$x=-\frac{40}{49}$	A1
		(4)
		(12 marks)

(a)

M1: Sets $t^3-4t=0 \Rightarrow t=\ldots$ to reach $t=2$ or -2

A1: Substitutes $t=2 \Rightarrow x=2\times4-6\times2=-4$ and states coordinates of $A=(-4,0)$

Accept $t=2 \Rightarrow A=(-4,0)$ provided the M1 has been awarded

B1*: Show that the coordinates of $B=(20,0)$

Possible ways of this are

1) Solves $t^3-4t=0 \Rightarrow t=-2$ and substitutes into $2t^2-6t \Rightarrow x=20 \Rightarrow B=(20,0)$.

This is a show that and there must be some evidence for the "20", not just sight of $t=-2$

Look for $2\times(-2)^2-6\times(-2) \Rightarrow x=20$ or $8+12 \Rightarrow x=20$

2) Alternatively sets $2t^2-6t=20\Rightarrow t=-2$ and substitutes into $y=t^3-4t\rightarrow y=0\Rightarrow B=(20,0)$

This is a show that and there must be some evidence for the "0", not just sight of $t=-2$

(b)

M1: Attempts to differentiate $x(t)$ and $y(t)$ and calculates $\frac{\mathrm{d}y}{\mathrm{d}x}$ by using $\frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}$ in part (b)

A1: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3t^2-4}{4t-6}$

M1: Sub 'their' $t=-2$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to find the gradient of the tangent at B.

M1: Uses (20,0) and their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to find an equation of the tangent.

If they use $y=mx+c$ they need to proceed as far as $c=...$

A1*: Achieves given answer. cso $\Rightarrow 7y+4x-80=0$ The $=0$ must be seen

ISW after sight of this

(c) The demand here is to use algebra which is satisfied by the setting up of the cubic equation.

M1: Attempts to substitute $x=2t^2-6t,\ y=t^3-4t$ into $7y+4x-80=0$

A1: Correct simplified equation in t: $7t^3+8t^2-52t-80=0$ The $=0$ may be implied by further work

dM1: Uses a correct method to find x

- Using a calculator this can be scored for finding a value of $t\neq-2$ and substituting this in x (t).You may have to check this on a calculator if their equation is incorrect. An accuracy of 2 sf is required for non exact solutions. Note that the equation may be in different forms under this method so $7t^3+8t^2-52t=80\Rightarrow t=\frac{20}{7}$ is fine

  Note: $7(t^3-4t)+4(2t^2-6t)-80=0\Rightarrow t=\frac{20}{7}\Rightarrow x=-\frac{40}{49}$ would score M1 A0 dM1 A0

- Via factorisation look for factors where the ends work,

  E.g. $"7t^3+8t^2-52t-80"=(Pt+a)(Qt+b)(Rt+c)$ with $PQR="7"$ and $abc="\pm 80"$

  followed by substitution of a value of $t\neq-2$ in x (t)

A1: CSO $x=-\frac{40}{49}$ Do not accept decimals here but remember to isw following a correct answer.

Ignore any references to the y coordinate. Penalise extra solutions, incorrect factorisation etc on this mark.

Question Number	Scheme	Marks
5(a)	$\int \frac{\ln x}{x^2}\,dx = \int x^{-2}\ln x\,dx = -x^{-1}\ln x + \int x^{-1}\times\frac{1}{x}\,dx$	M1
	$= -x^{-1}\ln x - x^{-1}(+c)$	dM1 A1
		(3)
(b)	$\frac{3+2x+\ln x}{x^2} = 3x^{-2}+2x^{-1}+x^{-2}\ln x$ oe	B1
	Area $= \left[-\frac{3}{x}+2\ln x\text{"}-\frac{\ln x}{x}-\frac{1}{x}\text{"}\right]_2^4$	M1
	$= \left(-\frac{3}{4}+2\ln 4-\frac{\ln 4}{4}-\frac{1}{4}\right)-\left(-\frac{3}{2}+2\ln 2-\frac{\ln 2}{2}-\frac{1}{2}\right)=1+2\ln 2$	M1 A1
		(4)
		(7 marks)

(a)

M1: Attempts by parts to reach a form $\int x^{-2}\ln x\,dx = \pm ax^{-1}\ln x \pm b\int x^{-1}\times\frac{1}{x}\,dx$ where $a, b \neq 0$

If a formula is stated it must be correct.

dM1: Integrates again to reach a form $\pm ax^{-1}\ln x \pm bx^{-1}$

A1: $-x^{-1}\ln x - x^{-1}(+c)$ o.e such as $-\frac{\ln x}{x}-\frac{1}{x}+c$

Condone the omission of the constant of integration. Condone $-x^{-1}\ln x+\left(-x^{-1}\right)$

(b)

B1: $\frac{3+2x+\ln x}{x^2} = 3x^{-2}+2x^{-1}+x^{-2}\ln x$ or $\frac{3}{x^2}+\frac{2}{x}+\frac{\ln x}{x^2}$.

The $\frac{2}{x}$ may be implied by later work. That is if they state $\int \frac{2x}{x^2}\,dx = 2\ln x$ or $\ln x^2$

M1: Area $= \left[-\frac{3}{x}+2\ln x\text{"}-\frac{\ln x}{x}-\frac{1}{x}\text{"}\right]_2^4$ following through on their part (a)

M1: For substituting 2 and 4 **into an expression of the form** $\left[\frac{p}{x}+q\ln x+r\frac{\ln x}{x}+\frac{s}{x}\right]_2^4$ and attempts to simplify by using one log law correctly. (Note that the two terms in $\frac{1}{x}$ may be combined before substitution)

A1: Exact answer: Likely to be $1+2\ln 2$ or $1+\ln 4$

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For those who don't read the instruction in (b) to use (a) and do via parts score

B1: $\int \frac{3+2x+\ln x}{x^2}\,\mathrm{d}x = -x^{-1}(3+2x+\ln x) + \int \left(2+\frac{1}{x}\right)\times\frac{1}{x}\,\mathrm{d}x$

M1: Area $= \left[\pm x^{-1}(3+2x+\ln x)" \pm 2\ln x \pm x^{-1}\right]_2^4$

Question Number	Scheme	Marks
6 (a)	$y = x^{\sin x} \Rightarrow \ln y = \sin x \ln x$	B1
	Differentiates $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sin x}{x} + \ln x \cos x$	M1 M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y\sin x}{x} + y\ln x\cos x$ oe	A1
		(5)
(b)	Puts $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \frac{\sin x}{x} + \ln x\cos x = 0$	M1
	$\frac{\sin x}{\cos x} + x\ln x = 0 \Rightarrow \tan x + x\ln x = 0$*	A1*
		(2)
		(7 marks)

(a)

B1: $y = x^{\sin x} \Rightarrow \ln y = \sin x \ln x$ o.e. Condone this written $\log y = \sin x \log x$

This may have been found via $\log_x y = \sin x \to \frac{\ln y}{\ln x} = \sin x$

M1: For differentiation of $\ln y \to \frac{1}{y}\times\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\log_e y \to \frac{1}{y}\times\frac{\mathrm{d}y}{\mathrm{d}x}$

M1: Attempts the product rule on $\sin x \ln x$ If the rule is quoted it must be correct.
If candidates set $u = ..., u' = ..., v = ..., v' = ...$ it may be implied by their $vu' + uv'$

If it is neither quoted nor implied score for $\frac{\sin x}{x} \pm \ln x \cos x$

A1: Correct differentiation to $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sin x}{x} + \ln x\cos x$

A1: Achieves $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y\sin x}{x} + y\ln x\cos x$ or exact equivalent such as $\frac{\mathrm{d}y}{\mathrm{d}x} = y\left(\frac{\sin x + x\ln x\cos x}{x}\right)$

Accept this in terms of just x. For instance $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^{\sin x}\sin x}{x} + x^{\sin x}\ln x\cos x$

ISW after sight of a correct answer.

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Note that you will see candidates who write $\frac{1}{y}\,\mathrm{d}y = \frac{\sin x}{x}\,\mathrm{d}x + \ln x\cos x\,\mathrm{d}x$ which can be marked in the same way

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(b)

M1: Sets their $\frac{\mathrm{d}y}{\mathrm{d}x}=0$, divides by y or $x^{\sin x}$ to form a simplified equation in x.

This may be implied by $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y\sin x}{x}+y\ln x\cos x=0\Rightarrow \sin x+x\ln x\cos x=0$ o.e.

For this to be scored $\frac{\mathrm{d}y}{\mathrm{d}x}$ **must be in the form**

$$\frac{y\sin x}{...}\pm y\ln x\cos x$$

$$\frac{x^{\sin x}\sin x}{...}\pm x^{\sin x}\ln x\cos x \text{ where ... could be 1}$$

A1*: CSO Proceeds to the given answer of $\tan x+x\ln x=0$

showing at least the intermediate line $\frac{\sin x}{\cos x}+x\ln x=0$

It can be implied by $\sin x+x\ln x\cos x=0\quad(\div\cos x)\Rightarrow\tan x+x\ln x=0$

6 (a)	$y=x^{\sin x}\Rightarrow y=\mathrm{e}^{\sin x\ln x}$	B1
	Differentiates $y=\mathrm{e}^{\sin x\ln x}\rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{\sin x\ln x}\times...$	M1
	$\frac{\mathrm{d}}{\mathrm{d}x}(\sin x\ln x)=\frac{\sin x}{x}+\ln x\cos x$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{\sin x\ln x}\times\left(\frac{\sin x}{x}+\ln x\cos x\right)$	A2
		(5)

B1: $y=x^{\sin x}\Rightarrow y=\mathrm{e}^{\sin x\ln x}$

M1: Differentiates $\mathrm{e}^{\sin x\ln x}\rightarrow\mathrm{e}^{\sin x\ln x}\times...$ where ... could be 1

M1: Attempts the product rule on $\sin x\ln x$. **See main scheme on how to apply this mark**.

A2: Correct differentiation $=\frac{\sin x}{x}+\ln x\cos x$

Question	Scheme		Marks
7 (i)	Chooses a suitable substitution for $\int_1^5 \frac{3x}{\sqrt{2x-1}}\,\mathrm{d}x$		
	E.g. $u=\sqrt{2x-1}$	E.g. $u=2x-1$	B1
	$u=\sqrt{2x-1} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=u$	$u=2x-1 \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{2}$	
	$\int \frac{3x}{\sqrt{2x-1}}\,\mathrm{d}x$	$\int \frac{3x}{\sqrt{2x-1}}\,\mathrm{d}x$	
	$=\int \frac{3}{2}u^2+\frac{3}{2}\,\mathrm{d}u$	$=\int \frac{3}{4}u^{\frac{1}{2}}+\frac{3}{4}u^{-\frac{1}{2}}\,\mathrm{d}u$	M1
	$=\frac{1}{2}u^3+\frac{3}{2}u$	$=\frac{1}{2}u^{\frac{3}{2}}+\frac{3}{2}u^{\frac{1}{2}}$	dM1 A1
	Limits $=\left[\frac{1}{2}u^3+\frac{3}{2}u\right]_1^3=\ldots$	$=\left[\frac{1}{2}u^{\frac{3}{2}}+\frac{3}{2}u^{\frac{1}{2}}\right]_1^9=\ldots$	M1
	$\left(\frac{1}{2}\times 3^3+\frac{3}{2}\times 3\right)-\left(\frac{1}{2}\times 1^3+\frac{3}{2}\times 1\right)=16$	$\left(\frac{1}{2}\times 9^{\frac{3}{2}}+\frac{3}{2}\times 9^{\frac{1}{2}}\right)-\left(\frac{1}{2}\times 1+\frac{3}{2}\times 1\right)=16$	A1
			(6)
7 (ii)	$\frac{6x^2-16}{(x+1)(2x-3)}=3+\ldots..$		B1
	$\frac{6x^2-16}{(x+1)(2x-3)}=3+\frac{2}{x+1}-\frac{1}{2x-3}$		M1 A1
	$\int \frac{6x^2-16}{(x+1)(2x-3)}\,\mathrm{d}x=\int 3+\frac{2}{x+1}-\frac{1}{2x-3}\,\mathrm{d}x$		
	$=3x+2\ln\lvert x+1\rvert-\frac{1}{2}\ln\lvert 2x-3\rvert+c$		M1 A1ft A1
			(6)
			(12 marks)

(i)

B1: States or uses a suitable substitution. That is one that will work.

M1: Attempts to change the integral in x to an integral in 'u'

Expect to see all aspects changed including the dx

For $u=\sqrt{2x-1}$ expect to see $\int Au^2+B\,\mathrm{d}u$

For $u = 2x-1$ expect to see $\int Pu^{\frac{1}{2}} + Qu^{-\frac{1}{2}}\,du$

dM1: For correct method of integration. Allow for correct powers. Dependent upon previous M

A1: See scheme. Correct integration which may be left unsimplified

M1: Uses correct limits for their **valid** substitution.

A1: CSO 16. Look for 18 -2 or such before the 16.

16 with no working scores no marks. This is evidence of use of a calculator

(ii) Note that epen scoring is M1 M1 A1 M1 A1 A1

B1: For a quotient of 3.

This can be scored by setting $6x^2 - 16 = A(x+1)(2x-3) + B(x+1) + C(2x-3)$ and reaching $A = 3$

or by division with 3 in the "correct place"

M1: A full attempt to find A, B and C.

Either sets $6x^2 - 16 = A(x+1)(2x-3) + B(x+1) + C(2x-3)$ condoning slips leading to values of A, B and C

or divides and uses partial fractions on the remainder to form an expression of the correct form.

A1: Correct partial fractions seen or implied. $\frac{6x^2-16}{(x+1)(2x-3)} = 3 + \frac{2}{(x+1)} - \frac{1}{(2x-3)}$

M1: Two of $A \to ...x$, $\int \frac{B}{(x+1)}\,dx \to ...\ln(x+1)$ $\int \frac{C}{(2x-3)}\,dx \to ...\ln(2x-3)$

A1ft: Two correct of $A \to Ax$ $\int \frac{B}{(x+1)}\,dx \to B\ln(x+1)$ and $\int \frac{C}{(2x-3)}\,dx \to \frac{C}{2}\ln(2x-3)$

Please note that answers such as $\int \frac{1}{2(2x-3)}\,dx \to \frac{1}{2}\ln(4x-6)$ are correct

A1: $3x + 2\ln|x+1| - \frac{1}{2}\ln|2x-3| + c$ but condone $3x + 2\ln(x+1) - \frac{1}{2}\ln(2x-3) + c$ **The $+c$ is required.**

You should isw after a correct answer.

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Note that it is possible to do **part (i)** by parts. This does not satisfy the demand of the question but can score some marks

B1: $\int \frac{3x}{\sqrt{2x-1}}\,dx = 3x\sqrt{2x-1} - \int 3\sqrt{2x-1}\,dx$

M1: $\int \frac{3x}{\sqrt{2x-1}}\,dx = ax\sqrt{2x-1} - b(2x-1)^{\frac{3}{2}}$ FYI: Correct when $a = 3, b = 1$

third M1 (5th mark on epen): applies limits to a valid expression $\int_1^5 \frac{3x}{\sqrt{2x-1}}\,dx = \left[ax\sqrt{2x-1} - b(2x-1)^{\frac{3}{2}}\right]_1^5 = ...$

second A1 (6th mark on epen) $\int_1^5 \frac{3x}{\sqrt{2x-1}}\,dx = \left[3x\sqrt{2x-1} - (2x-1)^{\frac{3}{2}}\right]_1^5 = (45-27)-(3-1) = 16$ o.e.

to score on e-epen 110011

....

Note that in **part (ii)** some candidates will write

$$\frac{6x^2-16}{(x+1)(2x-3)}=\frac{B}{x+1}+\frac{C}{2x-3}$$ and via substitution will get "a correct" $$\frac{2}{x+1}-\frac{1}{2x-3}$$

$$\int\frac{6x^2-16}{(x+1)(2x-3)}\mathrm{d}x=\int\frac{"B"}{x+1}+\frac{"C"}{2x-3}\mathrm{d}x="B"\ln(x+1)+\frac{"C"}{2}\ln(2x-3)$$

Scores SC B0 M0 A0 M1 A1ft A0 for 2 out of 6 if ft integration is correct

Question Number	Scheme	Marks
8 (a)	$\begin{pmatrix}4\\-3\\2\end{pmatrix}+\lambda\begin{pmatrix}3\\-2\\-1\end{pmatrix}=\begin{pmatrix}2\\0\\-9\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\-3\end{pmatrix}\Rightarrow \begin{matrix}4+3\lambda=2+2\mu\\-3-2\lambda=0-1\mu\\2-1\lambda=-9-3\mu\end{matrix}$ **any two** of these	M1
	Full method to find either λ **or** μ eg (1) +3(3) $\Rightarrow \mu=...$	M1
	Either $\lambda=-4$ or $\mu=-5$	A1
	Position vector of intersection is $\begin{pmatrix}4\\-3\\2\end{pmatrix}-4\begin{pmatrix}3\\-2\\-1\end{pmatrix}=$ OR $\begin{pmatrix}2\\0\\-9\end{pmatrix}-5\begin{pmatrix}2\\-1\\-3\end{pmatrix}=$	dM1
	$=\begin{pmatrix}-8\\5\\6\end{pmatrix}$	A1
		(5)
(b)	Co-ordinates or position vector of point $Q=\begin{pmatrix}2+2\mu\\0-1\mu\\-9-3\mu\end{pmatrix}$	
	$\overrightarrow{PQ}=\begin{pmatrix}2+2\mu\\0-1\mu\\-9-3\mu\end{pmatrix}-\begin{pmatrix}10\\-7\\0\end{pmatrix}=\begin{pmatrix}2\mu-8\\7-1\mu\\-9-3\mu\end{pmatrix}$	M1
	Uses $\overrightarrow{PQ}.\begin{pmatrix}2\\-1\\-3\end{pmatrix}=0\Rightarrow 4\mu-16-7+\mu+27+9\mu=0\Rightarrow \mu=-\frac{2}{7}$	dM1 A1
	Substitutes their μ into $\begin{pmatrix}2+2\mu\\0-1\mu\\-9-3\mu\end{pmatrix}\Rightarrow Q=\left(\frac{10}{7},\frac{2}{7},-\frac{57}{7}\right)$	ddM1 A1
		(5)
		(10 marks)

(a)

M1: For writing down **any two equations** that give the coordinates of the point of intersection.

Accept two of $4+3\lambda=2+2\mu$, $-3-2\lambda=0-1\mu$, $2-1\lambda=-9-3\mu$

There must be an attempt to set the coordinates equal but condone one slip.

Setting the vectors equal to each other is insufficient for this but correct calculations will imply this.

M1: A full method to find **either** λ **or** μ. There are a few answers with limited or no working.

In such cases this M mark can be implied only if **either** λ **or** μ are correct for their equations.

A1: Either value correct $\mu=-5$ or $\lambda=-4$. Correct value(s) following correct equations implies M1 A1

dM1: Substitutes their value of λ into l_1 to find the coordinates or position vector of the point of intersection. It is dependent upon having scored second method mark. Alternatively substitutes their value of μ into l_2 to find the coordinates or position vector of the point of intersection. It can be implied in cases where there is limited working by two correct coordinates for their λ **or** μ.

A1: Either $\begin{pmatrix}-8\\5\\6\end{pmatrix}$ or $-8\mathbf{i}+5\mathbf{j}+6\mathbf{k}$ but not the coordinate $(-8, 5, 6)$

However ISW after sight of the correct **vector****

(b)

M1: States or uses a general point on $l_2=\begin{pmatrix}2+2\mu\\0-1\mu\\-9-3\mu\end{pmatrix}$ and attempt $\overrightarrow{PQ}=\begin{pmatrix}2+2\mu\\0-1\mu\\-9-3\mu\end{pmatrix}-\begin{pmatrix}10\\-7\\0\end{pmatrix}=\begin{pmatrix}2\mu-8\\7-1\mu\\-9-3\mu\end{pmatrix}$ either way around with their general point. Condone slips

dM1: Uses the scalar product with fact that $\overrightarrow{PQ}$ and $\begin{pmatrix}2\\-1\\-3\end{pmatrix}$ are perpendicular vectors to find μ

See scheme but again condone slips.

Alternatively uses the scalar product with fact that $\overrightarrow{PQ}$ and $\overrightarrow{XQ}$ are perpendicular vectors to find μ

$$\overrightarrow{PQ}.\overrightarrow{XQ}=0\Rightarrow(2\mu-8)(2\mu+10)+(7-\mu)(-\mu-5)+(-9-3\mu)(-15-3\mu)=0\Rightarrow\mu=\left(-\frac{2}{7}\right)$$

A1: $\mu=-\frac{2}{7}$

ddM1: Uses their μ to find the coordinates of Q.

If no method is seen imply by two correct coordinates for their μ

A1: $Q=\left(\frac{10}{7},\frac{2}{7},-\frac{57}{7}\right)$ but not (**) $\frac{10}{7}\mathbf{i}+\frac{2}{7}\mathbf{j}-\frac{57}{7}\mathbf{k}$ o.e

** Only penalise incorrect notation once, the first time that it occurs. **

Alt using distances

(b)	Co-ordinates or position vector of point $Q = \begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix}$	
	$\overrightarrow{PQ} = \begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix} - \begin{pmatrix} 10 \\ -7 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\mu-8 \\ 7-1\mu \\ -9-3\mu \end{pmatrix}$	M1
	Uses Pythagoras with $PQ^2 + QX^2 = PX^2 \Rightarrow 28\mu^2 + 148\mu + 544 = 504 \Rightarrow \mu = \cancel{-5}, -\frac{2}{7}$	dM1 A1
	Substitutes their μ into $\begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix} \Rightarrow Q = \left(\frac{10}{7}, \frac{2}{7}, -\frac{57}{7}\right)$	ddM1 A1

Alt using minimum distance and differentiation

(b)	Co-ordinates or position vector of point $Q = \begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix}$	
	$\overrightarrow{PQ} = \begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix} - \begin{pmatrix} 10 \\ -7 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\mu-8 \\ 7-1\mu \\ -9-3\mu \end{pmatrix}$	M1
	Uses Pythagoras with $PQ^2 = (2\mu-8)^2 + (7-\mu)^2 + (-9-3\mu)^2$ $\frac{\mathrm{d}}{\mathrm{d}\mu}PQ^2 = 0 \Rightarrow 4(2\mu-8) - 2(7-\mu) - 6(-9-3\mu) = 0 \Rightarrow \mu = -\frac{2}{7}$	dM1 A1
	Substitutes their μ into $\begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix} \Rightarrow Q = \left(\frac{10}{7}, \frac{2}{7}, -\frac{57}{7}\right)$	ddM1 A1

Question Number	Scheme	Marks
9 (a)	$\dfrac{dA}{dt}=\dfrac{A^{\frac{3}{2}}}{5t^2}\Rightarrow\int\dfrac{dA}{A^{\frac{3}{2}}}=\int\dfrac{dt}{5t^2}$ oe	M1
	$-2A^{-\frac{1}{2}}=-\dfrac{1}{5}t^{-1}(+c)$	M1 M1 A1
	Substitutes $t=3, A=2.25\Rightarrow c=...\left(-\dfrac{19}{15}\right)$	M1
	Uses their $\dfrac{2}{\sqrt{A}}=\dfrac{1}{5t}+\dfrac{19}{15}\Rightarrow A=\left(\dfrac{30t}{19t+3}\right)^2$	M1, A1
		(7)
(b)	As $t\to\infty, A\to\left(\dfrac{30}{19}\right)^2=\dfrac{900}{361}$ or awrt 2.49 cm^2	M1 A1
		(2) **(9 marks)**

(a)

M1: Attempts to separate the variables either $\int\dfrac{dA}{A^{\frac{3}{2}}}=\int...\dfrac{dt}{t^2}$ oe such as $\int...\dfrac{dA}{A^{\frac{3}{2}}}=\int\dfrac{dt}{t^2}$

Condone without the integral signs but the $A^{\frac{3}{2}}$, dA, t^2 and dt must be in the correct positions.
Don't be too concerned on the position of the "5"

M1: Integrates one side correctly. Look for $pA^{-\frac{1}{2}}$ or qt^{-1}

dM1: Integrates both sides correctly. Look for $pA^{-\frac{1}{2}}$ and qt^{-1}

A1: Correct intermediate stage (without $+c$).

Either $-2A^{-\frac{1}{2}}=-\dfrac{1}{5}t^{-1}(+c)$ or $-10A^{-\frac{1}{2}}=-t^{-1}(+c)$ or equivalent such as $-\dfrac{A^{-0.5}}{0.5}=-\dfrac{1}{5}t^{-1}(+c)$

M1: Must have a "$+c$ " now. It is for using $t=3, A=2.25\Rightarrow c=....$

There must be some evidence for this award which may be implied by their value for c if no working is seen.
One index must have started out correct
This may be awarded after an incorrect change of subject has occurred.

M1: It is for proceeding to $A=\left(\dfrac{pt}{qt+r}\right)^2$ from an equation of the form $\dfrac{p}{\sqrt{A}}=\dfrac{q}{t}+r$ with correct operations.

Candidates do not need to have found a <u>**value**</u> for r (their constant) to score this mark
The attempt must involve

- using a common factor on the rhs of $\dfrac{p}{\sqrt{A}}=\dfrac{q}{t}+r$
- "inverting" both sides not each term
- an attempt to square both sides and not each term to reach $A = ...$

A1: $A=\left(\dfrac{30t}{19t+3}\right)^2$ cso. This mark can be awarded independent of the first M1

Be aware that $A=\left(\frac{300t}{190t+30}\right)^2$ is also correct

(b)

M1: $t\to\infty, A\to\left(\frac{a}{b}\right)^2$ The form of part (a) must be $A=\left(\frac{at}{bt+c}\right)^2$ where a, b and c are all positive.

The reason for this is that if we had "$bt-c$", the area would be infinite at $t=\frac{c}{b}$ and a limit would not exist

A1: $\frac{900}{361}$ or awrt 2.49 cm^2 following correct work.

Condone for both marks $A<\frac{900}{361}$ following correct work.