Further Pure Mathematics FP3 Mark scheme

Question	Scheme		Marks
1	$y = 9\cosh x + 3\sinh x + 7x$		
	$\frac{dy}{dx} = 9\sinh x + 3\cosh x + 7$	Correct derivative	B1
	$9\frac{(e^x - e^{-x})}{2} + 3\frac{(e^x + e^{-x})}{2} + 7 = 0$	Replaces sinh$x$ and cosh$x$ by the correct exponential forms	M1
	Note that the first 2 marks can score the other way round: M1: $y = 9\frac{(e^x + e^{-x})}{2} + 3\frac{(e^x - e^{-x})}{2} + 7x$ B1: $\frac{dy}{dx} = 9\frac{(e^x - e^{-x})}{2} + 3\frac{(e^x + e^{-x})}{2} + 7$		
	$12e^{2x} + 14e^x - 6 = 0$ oe	M1: Obtains a quadratic in e^x A1: Correct quadratic	M1 A1
	$(3e^x - 1)(2e^x + 3) = 0 \Rightarrow e^x = ...$	Solves their quadratic as far as $e^x = ...$	M1
	$x = \ln\left(\frac{1}{3}\right)$	cso (Allow –ln3) $e^x = -\frac{3}{2}$ need not be seen. Extra answers, award A0	A1
	Alternative		
	$\frac{dy}{dx} = 9\sinh x + 3\cosh x + 7$	Correct derivative	B1
	$9\sinh x = -3\cosh x - 7 \Rightarrow 81\sinh^2 x = 9\cosh^2 x + 42\cosh x + 49$		
	$72\cosh^2 x - 42\cosh x - 130 = 0$	Squares and attempts quadratic in cosh$x$	M1
	$(3\cosh x - 5)(12\cosh x + 13) = 0 \Rightarrow \cosh x = \frac{5}{3}$	M1: Solves quadratic A1: Correct value	M1 A1
	$x = \ln\left(\frac{5}{3} \pm \sqrt{\left(\frac{5}{3}\right)^2 - 1}\right)$	Use of ln form of arcosh	M1
	$x = \ln\left(\frac{1}{3}\right)$	cso (Allow – ln3)	A1
	NB: Ignore any attempts to find the y coordinate		
			(6 marks)

Question	Scheme		Marks
2(a)	$\frac{x^2}{25}+\frac{y^2}{4}=1, \quad P(5\cos\theta, 2\sin\theta)$		
	$\frac{dx}{d\theta}=-5\sin\theta,\ \frac{dy}{d\theta}=2\cos\theta$ or $\frac{2x}{25}+\frac{2y}{4}\frac{dy}{dx}=0$	Correct derivatives or correct implicit differentiation	B1
	$\frac{dy}{dx}=\frac{2\cos\theta}{-5\sin\theta}$	Divides their derivatives correctly or substitutes and rearranges	M1
	$M_N=\frac{5\sin\theta}{2\cos\theta}$	Correct perpendicular gradient rule	M1
	$y-2\sin\theta=\frac{5\sin\theta}{2\cos\theta}(x-5\cos\theta)$	Correct straight line method (any complete method) **Must** use their gradient of the normal.	M1
	$5x\sin\theta-2y\cos\theta=21\sin\theta\cos\theta$*	cso	A1*
			(5)
(b)	At Q, $x=0 \Rightarrow y=-\frac{21}{2}\sin\theta$		B1
	M is $\left(\frac{0+5\cos\theta}{2}, \frac{2\sin\theta-\frac{21}{2}\sin\theta}{2}\right)$ $\left(=\left(\frac{5}{2}\cos\theta, -\frac{17}{4}\sin\theta\right)\right)$	Correct mid-point method for at least one coordinate Can be implied by a correct x coordinate	M1
	L cuts x-axis at $\frac{21}{5}\cos\theta$		B1
	Area $OPM = OLP + OLM$ $\frac{1}{2}.\frac{21}{5}\cos\theta.2\sin\theta+\frac{1}{2}.\frac{21}{5}\cos\theta.\frac{17}{4}\sin\theta$	M1: Correct triangle area method using their coordinates A1: Correct expression	M1 A1
	$=\frac{105}{16}\sin 2\theta$	Or $6.5625\sin 2\theta$ must be positive	A1
			(6)

Question	Scheme		Marks
2(b) *continued*	**Alternative 1: Using Area *OPM***		
	See above for B1M1		B1 M1
	Area $\Delta OPM = \frac{1}{2}\begin{vmatrix} 0 & 5\cos\theta & \frac{5}{2}\cos\theta & 0 \\ 0 & 2\sin\theta & -\frac{17}{4}\sin\theta & 0 \end{vmatrix}$	M1: Correct determinant with their coords, with 2 or 3 points. $\begin{matrix}0\\0\end{matrix}$ should be at both or neither end. A1: Correct determinant (There are more complicated determinants using the 3 points.)	M1 A1
	$=\frac{1}{2}\left(0+5\sin\theta\cos\theta+0-0+\frac{85}{4}\sin\theta\cos\theta-0\right)$	A1	A1
	$=\frac{105}{4}\sin\theta\cos\theta$		
	$=\frac{105}{16}\sin 2\theta$		A1
			(6)
	Alternative 2: Using Area *OPQ*		
	At Q, $x=0 \Rightarrow y=-\frac{21}{2}\sin\theta$		B1
	Area $\Delta OPQ = \frac{1}{2}\begin{vmatrix} 5\cos\theta & 0 \\ 2\sin\theta & -\frac{21}{2}\sin\theta \end{vmatrix}$	Can be implied by the following line	M1 A1
	$=\frac{1}{2}\times\frac{105}{2}\sin\theta\cos\theta$	OQ is base, x coord of P is height	A1
	$=\frac{105}{8}\sin 2\theta$		
	Area $OPM = \frac{1}{2}$ Area OPQ		M1
	$=\frac{105}{16}\sin 2\theta$		A1
			(6)

Question	Scheme		Marks
2(b) *continued*	**Alternative 3**		
	At Q, $x=0 \Rightarrow y=-\frac{21}{2}\sin\theta$		B1
	M is $\left(\frac{0+5\cos\theta}{2}, \frac{2\sin\theta-\frac{21}{2}\sin\theta}{2}\right)$ $\left(=\left(\frac{5}{2}\cos\theta, -\frac{17}{4}\sin\theta\right)\right)$		M1
	$OP=\sqrt{4\sin^2\theta+25\cos^2\theta}$ $\left(=\sqrt{4+21\cos^2\theta}\right)$		B1
	$d=\dfrac{\frac{5}{2}\cos\theta\times\frac{2\sin\theta}{5\cos\theta}+\frac{17}{4}\sin\theta}{\sqrt{\frac{4\sin^2\theta}{25\cos^2\theta}+1}}=\dfrac{\frac{21}{4}\sin\theta}{\sqrt{\frac{4+21\cos^2\theta}{25\cos^2\theta}}}$		
	Area $=\frac{1}{2}\times\dfrac{\frac{21}{4}\sin\theta}{\sqrt{\frac{4+21\cos^2\theta}{25\cos^2\theta}}}\times\sqrt{4+21\cos^2\theta}$		M1 A1
	$=\frac{105}{16}\sin 2\theta$		A1
			(6)
			(11 marks)

Question	Scheme		Marks
3(a)	$x^2+4x+13 \equiv (x+2)^2+9$		B1
	$\int \frac{1}{(x+2)^2+9}\,dx = \frac{1}{3}\arctan\left(\frac{x+2}{3}\right)$	M1: $k\arctan f(x)$. A1: Correct expression	M1 A1
	$\left[\frac{1}{3}\arctan\left(\frac{x+2}{3}\right)\right]_{-2}^{1} = \frac{1}{3}(\arctan 1 - \arctan 0)$	Correct use of limits arctan0 need not be shown	M1
	$\frac{\pi}{12}$	cao	A1
			(5)
	Alternative		
	Sub $x+2=3\tan t$		
	$x^2+4x+13 \equiv (x+2)^2+9$		B1
	$\frac{dx}{dt} = 3\sec^2 t \qquad x=-2, \tan t=0,\ t=0;\ x=1, \tan t=1,\ t=\frac{\pi}{4}$		
	$\int \frac{3\sec^2 t}{9\tan^2 t+9}\,dt = \frac{1}{3}\int dt = \frac{1}{3}t$	M1 sub and integrate inc use of $\tan^2+1=\sec^2$ A1 Correct expression Ignore limits	M1 A1
	$\left[\frac{\pi}{12}\right]_0^{\frac{\pi}{4}}$.	Either change limits and substitute Or reverse substitution and substitute original imits	M1
	$\frac{\pi}{12}$	cao	A1
			(5)

Question	Scheme		Marks
3(b)	$4x^2-12x+34=4\left(x-\frac{3}{2}\right)^2+25$ or $(2x-3)^2+25$	M1: $4(x\pm p)^2\pm q,\ (p,q\neq 0)$ A1: $4\left(x-\frac{3}{2}\right)^2+25$	M1 A1
	$\int\frac{1}{\sqrt{4\left(x-\frac{3}{2}\right)^2+25}}\,dx=\frac{1}{2}\int\frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2+\frac{25}{4}}}\,dx=\frac{1}{2}\text{arsinh}\left(\frac{x-\frac{3}{2}}{\frac{5}{2}}\right)$ M1: $k\text{arsinh}\,f(x)$. A1: Correct expression		M1 A1
	$\left[\frac{1}{2}\text{arsinh}\left(\frac{x-\frac{3}{2}}{\frac{5}{2}}\right)\right]_{-1}^{4}=\frac{1}{2}\left(\text{ar}\sinh(1)-\text{ar}\sinh(-1)\right)$	Correct use of limits	M1
	$=\frac{1}{2}\left(\ln\left(1+\sqrt{2}\right)-\ln\left(-1+\sqrt{2}\right)\right)$	Uses the logarithmic form of arsinh	M1
	$=\frac{1}{2}\ln\left(3+2\sqrt{2}\right)$ or $\ln\left(1+\sqrt{2}\right)$	cao	A1
			(7)
	Alternative: Second M1 A1		
	Sub $2x-3=u$ or $2x-3=5\sinh u$		
	$\int_{\text{arsinh}-1}^{\text{arsinh}1}\frac{1}{\sqrt{25\sinh^2 u+25}}5\cosh u\,du=\left[\frac{1}{2}\text{arsinh}\left(\frac{u}{5}\right)\right]_{-5}^{5}$		M1 A1
	$\int_{-5}^{5}\frac{1}{2\sqrt{u^2+25}}\,du=\left[\frac{1}{2}\text{arsinh}\left(\frac{u}{5}\right)\right]_{-5}^{5}$		
			(12 marks)

Question	Scheme		Marks
4(a)	$\mathbf{M}=\begin{pmatrix}1 & k & 0\\ -1 & 1 & 1\\ 1 & k & 3\end{pmatrix}$		
	$\lvert\mathbf{M}\rvert=3-k-k(-3-1)(=3k+3)$	Correct determinant in any form	B1
	$\mathbf{M}^{\mathrm{T}}=\begin{pmatrix}1 & -1 & 1\\ k & 1 & k\\ 0 & 1 & 3\end{pmatrix}$ or minors $\begin{pmatrix}3-k & -4 & -k-1\\ 3k & 3 & 0\\ k & 1 & 1+k\end{pmatrix}$ or cofactors $\begin{pmatrix}3-k & 4 & -k-1\\ -3k & 3 & 0\\ k & -1 & 1+k\end{pmatrix}$		B1
	$\mathbf{M}^{-1}=\dfrac{1}{3+3k}\begin{pmatrix}3-k & -3k & k\\ 4 & 3 & -1\\ -k-1 & 0 & 1+k\end{pmatrix}$	M1: Identifiable full attempt at inverse **including reciprocal of determinant**. Could be indicated by at least 6 correct elements. A1ft: Two rows or two columns correct (**follow through their determinant** but not incorrect entries in the matrices used) A1ft: Fully correct inverse (follow through as before)	M1 A1ft A1ft
	NB: If every element is the negative of the correct element, allow M1A1A0		
			(5)
(b)	$\mathbf{MN}=\begin{pmatrix}3 & 5 & 6\\ 4 & -1 & 1\\ 3 & 2 & -3\end{pmatrix}\Rightarrow\mathbf{N}=\mathbf{M}^{-1}\begin{pmatrix}3 & 5 & 6\\ 4 & -1 & 1\\ 3 & 2 & -3\end{pmatrix}$	Correct statement	B1
	$\mathbf{N}=\dfrac{1}{3}\begin{pmatrix}3 & 0 & 0\\ 4 & 3 & -1\\ -1 & 0 & 1\end{pmatrix}\begin{pmatrix}3 & 5 & 6\\ 4 & -1 & 1\\ 3 & 2 & -3\end{pmatrix}=\begin{pmatrix}3 & 5 & 6\\ 7 & 5 & 10\\ 0 & -1 & -3\end{pmatrix}$	M1: Multiplies the given matrix by their $\mathbf{M}^{-1}$ in the correct order Must include the "$\frac{1}{3}$" A2: Correct matrix (-1 each error). If left with $\frac{1}{3}$ outside the matrix award A0	M1 A(2, 1, 0)
			(4)
			(9 marks)

Question	Scheme		Marks
5(a)	$y = \text{artanh}(\cos x)$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{1-\cos^2 x} \times -\sin x$	Correct use of the chain rule	M1
	$= \frac{-\sin x}{\sin^2 x} = \frac{-1}{\sin x} = -\text{cosec}\, x$ *	A1: Correct completion with no errors	A1
			(2)
	Alternative 1		
	$\tanh y = \cos x \Rightarrow \text{sech}^2 y \frac{\mathrm{d}y}{\mathrm{d}x} = -\sin x$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-\sin x}{\text{sech}^2 y} = \frac{-\sin x}{1-\cos^2 x}$	Correct differentiation to obtain a function of x	M1
	$= \frac{-\sin x}{\sin^2 x} = \frac{-1}{\sin x} = -\text{cosec}\, x$ *	A1: Correct completion with no errors	A1
			(2)
	Alternative 2		
	$\text{artanh}(\cos x) = \frac{1}{2}\ln\left(\frac{1+\cos x}{1-\cos x}\right)$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2} \times \frac{1-\cos x}{1+\cos x} \times \frac{-\sin x(1-\cos x) - \sin x(1+\cos x)}{(1-\cos x)^2}$	Correct differentiation to obtain a function of x	M1
	$= \frac{-2\sin x}{2(1-\cos^2 x)} = -\text{cosec}\, x$ *	A1: Correct completion with no errors	A1
			(2)
(b)	$\int \cos x\, \text{artanh}(\cos x)\,\mathrm{d}x = \sin x\, \text{artanh}(\cos x) - \int \sin x \times -\text{cosec}x\,\mathrm{d}x$ M1: Parts in the correct direction A1: Correct expression		M1 A1
	$\left[\sin x\, \text{artanh}(\cos x) + x\right]_0^{\frac{\pi}{6}} = \frac{1}{2}\text{artanh}\left(\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}(-(0))$ M1: Correct use of limits on either part (provided both parts are integrated). Lower limit need not be shown		M1
	$= \frac{1}{4}\ln\left(\frac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}}\right) + \frac{\pi}{6}$	Use of the logarithmic form of artanh	M1
	$= \frac{1}{4}\ln\left(7+4\sqrt{3}\right) + \frac{\pi}{6}$ or $\frac{1}{2}\ln\left(2+\sqrt{3}\right) + \frac{\pi}{6}$	Cao (oe)	A1
	The last 2 M marks may be gained in reverse order.		**(5)**
			(7 marks)

Question	Scheme		Marks
6(a)	$\overrightarrow{AB}=\begin{pmatrix}-2\\1\\1\end{pmatrix},\ \overrightarrow{AC}=\begin{pmatrix}1\\-1\\3\end{pmatrix},\ \overrightarrow{BC}=\begin{pmatrix}3\\-2\\2\end{pmatrix}$	Two correct vectors in Π Can be negatives of those shown	B1
	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-2 & 1 & 1\\1 & -1 & 3\end{vmatrix}=\begin{pmatrix}4\\7\\1\end{pmatrix}$	M1: Attempt cross product of two vectors lying in Π (At least one no. to be correct.) A1: Correct normal vector	M1 A1
	$\begin{pmatrix}4\\7\\1\end{pmatrix}\bullet\begin{pmatrix}1\\2\\3\end{pmatrix}=4+14+3$	Attempt scalar product with their normal and a point in the plane	dM1
	$4x+7y+z=21$	Cao (oe)	A1
			(5)
	Alternative 1		
	$a+2b+3c=d$ $-a+3b+4c=d$ $2a+b+6c=d$	Correct equations	B1
	$a=\frac{4}{21}d,\ b=\frac{1}{3}d,\ c=\frac{1}{21}d$	M1: Solve for a, b and c in terms of d A1: Correct equations	M1 A1
	$d=21\Rightarrow a=...,\ b=...,\ c=...$	Obtains values for a, b, c and d	M1
	$4x+7y+z=21$	Cao (oe)	A1
			(5)
	Alternative 2: Using $\mathbf{r}=\mathbf{a}+s\mathbf{b}+t\mathbf{c}$ where $\mathbf{b}$ and $\mathbf{c}$ are vectors in Π		
	Two correct vectors in the plane	See main scheme	B1
	Eg $\mathbf{r}=\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}+s\begin{pmatrix}-2\\1\\1\end{pmatrix}+t\begin{pmatrix}1\\-1\\3\end{pmatrix}$		M1
	$x=1-2s+t$ $y=2+s-t$ $z=3+s+3t$	Deduce 3 correct equations	A1
	$4x+7y+z=21$	M1: Eliminate s, t A1: Cao	M1 A1
			(5)

Question	Scheme		Marks
6(b)	AD•AB×AC	Attempt suitable triple product	M1
	$=\begin{pmatrix}4\\7\\1\end{pmatrix}\bullet\begin{pmatrix}k-1\\2\\11\end{pmatrix}=4k-4+14+11$		
	$\therefore \frac{1}{6}(4k+21)=6$	M1: Set $\frac{1}{6}$(their triple product) = 6 A1: Correct equation	dM1 A1
	$k=\frac{15}{4}$	Cao (oe)	A1
			(4)
	Alternative		
	Area ABC $=\frac{1}{2}\left\|\overrightarrow{AB}\right\|\left\|\overrightarrow{AC}\right\|=\frac{1}{2}\sqrt{6}\sqrt{11}$ D to Π is $\frac{4k+28+14-21}{\sqrt{16+49+1}}$	Attempt area ABC and distance between DandΠ	M1
	$\frac{1}{6}\sqrt{6}\sqrt{11}\frac{4k+28+14-21}{\sqrt{16+49+1}}=6$	M1: Set $\frac{1}{3}$(their area x their distance) = 6 A1: Correct equation	dM1 A1
	$k=\frac{15}{4}$	Cao (oe)	A1
			(4)
			(9 marks)

Question	Scheme		Marks
7(a)	$x = 3t^4, \quad y = 4t^3$		
	$\dfrac{dx}{dt} = 12t^3, \quad \dfrac{dy}{dt} = 12t^2$	Correct derivatives	B1
	$S = (2\pi)\displaystyle\int y\left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right)^{\frac{1}{2}} dt = (2\pi)\int 4t^3\sqrt{(12t^3)^2 + (12t^2)^2}\, dt$ $\left(= (2\pi)\displaystyle\int 4t^3 (144t^6 + 144t^4)^{\frac{1}{2}} dt\right)$ M1: Substitutes their derivatives into a correct formula (2π not required)		M1
	$S = (2\pi)\displaystyle\int 4t^3 (144t^4)^{\frac{1}{2}} (t^2 + 1)^{\frac{1}{2}} dt$	Attempt to factor out at least t^4 - numerical factor may be left	M1
	$S = 96\pi\displaystyle\int_0^1 t^5 (t^2 + 1)^{\frac{1}{2}} dt$	Correct completion	A1
			(4)
(b)	$u^2 = t^2 + 1 \Rightarrow 2u\dfrac{du}{dt} = 2t$ or $2u = 2t\dfrac{dt}{du}$	Correct differentiation	B1
	$t = 0 \Rightarrow u = 1, \ t = 1 \Rightarrow u = \sqrt{2}$	Correct limits **Alternative:** Reverse the substitution later. (Treat as M1 in this case and award later when work seen)	B1
	$S = (96\pi)\displaystyle\int t^5 \times u \times \frac{u}{t} du$		
	$S = (96\pi)\displaystyle\int (u^2 - 1)^2 \times u^2 \, du$	M1: Complete substitution A1: Correct integral in terms of u. Ignore limits, need not be simplified	M1 A1
	$S = (96\pi)\displaystyle\int (u^6 - 2u^4 + u^2) du = (96\pi)\left[\frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3}\right]$ M1: Expands and attempts to integrate		dM1
	$S = 96\pi\left[\dfrac{u^7}{7} - \dfrac{2u^5}{5} + \dfrac{u^3}{3}\right]_1^{\sqrt{2}} = 96\pi\left\{\left(\dfrac{\sqrt{2}^7}{7} - \dfrac{2\sqrt{2}^5}{5} + \dfrac{\sqrt{2}^3}{3}\right) - \left(\dfrac{1}{7} - \dfrac{2}{5} + \dfrac{1}{3}\right)\right\}$ M1: Correct use of their changed limits (both to be changed) **Alternative:** If sub reversed, substitute the original limits		ddM1
	$S = \dfrac{192\pi}{105}\left(11\sqrt{2} - 4\right)$	Cao eg $\dfrac{64\pi}{35}$	A1
			(7)
			(11 marks)

Question	Scheme		Marks
8(a)	$$I_n = \int_0^{\ln 2} \tanh^{2n} x \, dx, \quad n \geq 0$$		
	$\tanh^{2n} x = \tanh^{2(n-1)} x \tanh^2 x$		B1
	$\tanh^{2n} x = \pm\tanh^{2(n-1)} x\left(1-\text{sech}^2 x\right)$		M1
	$I_n = \int_0^{\ln 2} \tanh^{2(n-1)} x \, dx - \int_0^{\ln 2} \tanh^{2(n-1)} x \, \text{sech}^2 x \, dx$		
	$I_n = I_{n-1} - \left[\frac{1}{2n-1}\tanh^{2n-1} x\right]_0^{\ln 2}$	M1: Correctly substitutes for I_{n-1} and obtains $\int \tanh^{2(n-1)} x \, \text{sech}^2 x \, dx = k\tanh^{2n-1} x$ A1: Correct expression	M1 A1
	$= I_{n-1} - \frac{1}{2n-1}\left(\frac{3}{5}\right)^{2n-1}$ *	Correct completion with no errors	A1*
			(5)
	Alternative		
	$I_n - I_{n-1} = \int_0^{\ln 2} \left(\tanh^{2n} x - \tanh^{2(n-1)} x\right) dx$		
	$= \int_0^{\ln 2} \tanh^{2(n-1)} x\left(\tanh^2 x - 1\right) dx$		B1
	$= \int_0^{\ln 2} \tanh^{2(n-1)} x\left(-\text{sech}^2 x\right) dx$	$= \int_0^{\ln 2} \tanh^{2(n-1)} x\left(\pm\text{sech}^2 x\right) dx$	M1
	$I_n - I_{n-1} = -\left[\frac{1}{2n-1}\tanh^{2n-1} x\right]_0^{\ln 2}$	M1: Obtains $\int \tanh^{2(n-1)} x \, \text{sech}^2 x \, dx = k\tanh^{2n-1} x$ A1: Correct expression	M1 A1
	$= I_{n-1} - \frac{1}{2n-1}\left(\frac{3}{5}\right)^{2n-1}$ *	Correct completion with no errors	A1*
			(5)

Question	Scheme		Marks
8(b)	$I_0 = \ln 2$	The integration must be seen.	B1
	$I_2 = I_1 - \frac{1}{3}\left(\frac{3}{5}\right)^3$	Applies the reduction formula once	M1
	$I_2 = I_0 - \frac{1}{1}\left(\frac{3}{5}\right)^1 - \frac{1}{3}\left(\frac{3}{5}\right)^3$	M1: Second application of the reduction formula A1: Correct expression	M1A1
	$I_2 = \ln 2 - \frac{84}{125}$	cao	A1
	Special Case: If I_4 is found award B1 for I_0 or I_1 and M1M0A0A0		
			(5)
	Alternative		
	$I_1 = \int_0^{\ln 2} \tanh^2 x \, dx = \int_0^{\ln 2} \left(1 - \operatorname{sech}^2 x\right) dx$		
	$I_1 = \left[x - \tanh x\right]_0^{\ln 2}$	Correct integration	B1
	$I_2 = I_1 - \frac{1}{3}\left(\frac{3}{5}\right)^3$	Applies the reduction formula once	M1
	$I_1 = \ln 2 - \tanh\left(\ln 2\right) = \ln 2 - \frac{3}{5}$	M1: Uses limits A1: Correct expression	M1A1
	$I_2 = \ln 2 - \frac{3}{5} - \frac{1}{3}\left(\frac{3}{5}\right)^3$		
	$= \ln 2 - \frac{84}{125}$		A1
			(5)
			(10 marks)