Mark Scheme (Results)

January 2023

Pearson Edexcel International Advanced Level
In Mechanics M3 (WME03) Paper 01

Question Number	Scheme	Marks
1(a)	$\pi\int_0^1 (1+\sqrt{x})^2 \mathrm{d}x$	M1
	$=\pi\left[x+\frac{4}{3}x^{\frac{3}{2}}+\frac{1}{2}x^2\right]_0^1$	A1
	$=\frac{17\pi}{6}\text{ m}^3$ * including units	A1*
		(3)
(b)	$\pi\int_0^1 x(1+\sqrt{x})^2 \mathrm{d}x$	M1
	$=\pi\left[\frac{1}{2}x^2+\frac{4}{5}x^{\frac{5}{2}}+\frac{1}{3}x^3\right]_0^1$	A1
	$=\frac{49\pi}{30}$	A1
	$\bar{x}=\frac{\frac{49\pi}{30}}{\frac{17\pi}{6}}$	dM1
	$=\frac{49}{85}\text{ m}$ * including units	A1 *
		(5)
		(8)

Notes	
NB: Penalise missing units maximum of once per question.	
(a)	
M1	Use of $\pi\int_0^1 (1+\sqrt{x})^2 \mathrm{d}x$. Limits not needed. π is required.
A1	Correct integration – limits not needed
A1*	Correct given answer correctly obtained. Must include units. Limits must be seen (sight of substitution is not required). Accept $\frac{17}{6}\pi\text{ m}^3$
(b)	
M1	Use of $\pi\int_0^1 x(1+\sqrt{x})^2 \mathrm{d}x$. Limits not needed (π 's will cancel so it may not be seen)
A1	Correct integration – limits not needed
A1	Correct unsimplified with or without π (may see $\frac{1}{2}+\frac{4}{5}+\frac{1}{3}-0$)
dM1	Correct expression with their numerator (consistent π - seen in neither or both)
A1*	Correct given answer correctly obtained. Must include units.

Question Number	Scheme	Marks	
2.	$F\cos\alpha = mg$	M1	A1
	$F\sin\alpha = T$		A1
	$T = \dfrac{2mgx}{l}$ or $T = \dfrac{2mg(AB-l)}{l}$	M1	
	$\dfrac{3}{4}mg = \dfrac{2mgx}{l}$	dM1	
	$AB = \dfrac{11l}{8}$	A1	
		(6)	

Notes

M1	Resolve vertically or horizontally, correct no. of terms, condone sign errors and sin/cos confusion (or use trig on a right-angled triangle of forces)
A1	Correct vertical equation
A1	Correct horizontal equation (A2 for $T = mg\tan\alpha$ from triangle of forces)
M1	Hooke's Law. Must clearly be an extension and not *AB*. Since *x* is not defined in the question, other extensions may be used including ($AB - l$) or xl where x is found to be the constant $\dfrac{3}{8}$.
dM1	Substitute trig (not necessarily correctly) to produce an equation in '*x*' (and *l*) only, dependent on previous M's and on having two equations.
A1	Cao Accept $1.375l$, $1.4l$, $1.38l$

Question Number	Scheme	Marks
3(a)	Slant height, $l = \sqrt{\left(\frac{7a}{4}\right)^2 + (6a)^2}\ \left(=\frac{25a}{4}\right)$	M1
	Masses Square $16a^2$	B1 square
	Circle $\pi\left(\frac{7a}{4}\right)^2$	B1 circle
	Conical shell $\pi\times\frac{7a}{4}\times\frac{25a}{4}$ Total $\left[16a^2 - \pi\left(\frac{7a}{4}\right)^2 + \pi\times\frac{7a}{4}\times\frac{25a}{4}\right]$	B1ft (shell and total)
	Distances Square 0, Circle 0, Conical shell $2a$: Total $\bar{x}$	B1
	$\pi\times\frac{7a}{4}\times\frac{25a}{4}\times 2a = \left[16a^2 - \pi\left(\frac{7a}{4}\right)^2 + \pi\times\frac{7a}{4}\times\frac{25a}{4}\right]\bar{x}$	M1 A1
	$\bar{x} = \frac{175\pi a}{(63\pi+128)}$ *	A1*
		(8)
3(b)	$\tan\alpha = \frac{2a}{\left(\frac{175\pi a}{(63\pi+128)}\right)}$	M1
	$\tan\alpha = \frac{126\pi+256}{175\pi}$ (or $\frac{2(63\pi+128)}{175\pi}$)	A1
		(2)
		(10)

Notes	
(a)	
M1	Use of Pythagoras (unsimplified). May be seen on the diagram.
B1	Mass/area of square
B1	Mass/area of circle
B1 ft	Mass/area of conical shell and total. A common error is to use 6*a* as slant height, only ft on their calculated slant height. May derive conical shell formula from area of a sector.
B1	All distances correct
M1	Dimensionally correct moments equation. Must have correct number of terms including an attempt to subtract the circle. Condone a slip with an ‘*a*’ in one term.
A1	Correct equation (no ft)
A1*	Given answer correctly obtained. Condone missing brackets from denominator and terms reversed.
(b)	
M1	Allow reciprocal. Must use $2a$ and given $\bar{x}$.
A1	Cao Exact fraction required.

Question Number	Scheme	Marks
4(a)	$a = v\dfrac{\mathrm{d}v}{\mathrm{d}x}$	M1
	$= \dfrac{3}{2}(2x+1)^{\frac{1}{2}} \times 2 \times (2x+1)^{\frac{3}{2}} = 3(2x+1)^2$	A1
	$3(2x+1)^2 = 243$	M1
	$x = 4$	A1
		(4)
4(b)	$(2x+1)^{\frac{3}{2}} = \dfrac{\mathrm{d}x}{\mathrm{d}t}$ **OR** $a = 3v^{\frac{4}{3}} = \dfrac{\mathrm{d}v}{\mathrm{d}t}$	M1 A1
	$\int \mathrm{d}t = \int (2x+1)^{-\frac{3}{2}}\mathrm{d}x$ $\quad$ $\int 3\mathrm{d}t = \int v^{-\frac{4}{3}}\mathrm{d}v$	M1
	$t = -(2x+1)^{-\frac{1}{2}}(+C)$ $\quad$ $3t + (C) = -3v^{-\frac{1}{3}}$	A1
	$t = 0,\ x = 0 \Rightarrow C = 1$ $\quad$ $t = 0,\ x = 0 \Rightarrow v = 1 \Rightarrow C = -3$ and obtain an equation in v and t only.	M1
	$v = \dfrac{1}{(1-t)^3}$	A1
		(6)
		(10)

Notes

(a)	
M1	Use of $a = v\dfrac{\mathrm{d}v}{\mathrm{d}x}$ or $a = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$. Evidence of differentiation, power decreasing by 1. Should see a product of terms to imply 'use of'.
A1	Correct differentiation
M1	Independent. Use their result from differentiation and put a = 243 then solve for x
A1	Cao If -5 is seen then it must be rejected or 4 must be clearly identified.
(b)	
M1	Use of $v = \dfrac{\mathrm{d}x}{\mathrm{d}t}$ to obtain DE in x and t **OR** Use of $a = \dfrac{\mathrm{d}v}{\mathrm{d}t}$ to obtain DE in v and t
A1	Correct equation
M1	Separate and integrate (evidence of integration, power increasing by 1)
A1	Correct integration, condone missing C
M1	Use t = 0, x = 0 to obtain a value of C and obtain an equation in v and t *only*.
A1	Cao Accept $v = (1-t)^{-3}$ or $v = \dfrac{-1}{(t-1)^3}$ or $v = -(t-1)^{-3}$
	Note: No marks in (b) for use of a = 243

Question Number	Scheme	Marks
5(a)	Use of cosine rule on triangle *APB* **OR** trig. on 'half' of the triangle *APB* to find one relevant angle.	M1
	Given answers correctly obtained.*	A1*
		(2)
5(b)	$T_A \cos 30^o + T_B \cos 60^o = mg$	M1 A1
	$T_A \sin 30^o + T_B \sin 60^o = mr\omega^2$	M1A1A1
	$r = a\sin 60^o$ (or $r = a\sqrt{3}\cos 30$ or $r = a\frac{\sqrt{3}}{2}$)	B1
	Solve for T_A	dM1
	$T_A = \frac{1}{2}m\sqrt{3}(2g - a\omega^2)$ *	A1*
		(8)
5(c)	Attempt to obtain one inequality on ω^2	M1
	Correct inequality	A1
	Attempt to obtain another inequality on ω^2 and use both to obtain answer	M1
	$\frac{2g}{3a} < \omega^2 < \frac{2g}{a}$ *	A1 *
		(4)
		(14)

Notes

(a)	
M1	Either complete method to obtain one relevant angle.
A1*	Correct GIVEN angles correctly obtained. Sufficient annotation/justification leading to both given answers eg Stating $\angle OBP = 2 \times \angle OAP$ alone is not sufficient – additional annotation or justification is required. Use of triangles to verify is acceptable.
(b)	
M1	Resolve vertically, dimensionally correct equation with correct no. of terms, condone sign errors and sin/cos confusion.
A1	Correct equation
M1	Equation of motion horizontally: dimensionally correct equation with correct no. of terms, condone sign errors and sin/cos confusion.
A1	Correct equation, with at most one error. If $r\omega^2$ is never seen, this is an A error.
A1	Correct equation
B1	Cao If this is seen in (a) it must be used in (b) for this mark.
dM1	Solve for T_A in terms of *m*, *a*, *g* and ω
A1*	Given answer correctly obtained. Must see exactly.
(c)	
M1	Correct use of either $T_A > 0$ or their $T_B > 0$ oe to obtain one inequality on ω^2. Could be their expression for either Tension > 0.
A1	Correct inequality
M1	Use both $T_A > 0$ and their $T_B > 0$ to form inequalities in attempt to obtain answer. Could be their expression for either Tension > 0. Note: $T_B = \frac{3}{2}ma\omega^2 - mg$
A1*	Given answer correctly obtained

Question Number	Scheme	Marks
6(a)	$\frac{1}{2}mv^2 - mgl$ or $mgl - \frac{1}{2}mv^2$ seen or implied	B1
	Use of EPE	M1
	$\frac{mg}{2l}l^2$	A1
	$\frac{mg}{2l}(l\sqrt{2}-l)^2$	A1
	$\frac{1}{2}mv^2 + \frac{mg}{2l}(l\sqrt{2}-l)^2 = mgl + \frac{mg}{2l}l^2$	M1
	Solve for v^2	dM1
	$v^2 = 2gl\sqrt{2}$ *	A1*
		(7)
(b)	$T = \frac{mg(l\sqrt{2}-l)}{l} = mg(\sqrt{2}-1)$	M1 A1
	$\pm N + T\cos 45^o = \frac{mv^2}{l}$	M1A1A1
	$\pm N + mg(\sqrt{2}-1)\times\frac{\sqrt{2}}{2} = \frac{m}{l}\times 2gl\sqrt{2}$	dM1
	$N = \frac{1}{2}mg(5\sqrt{2}-2)$ *	A1*
		(7)
		(14)

	Notes
(a)	
B1	Difference between KE and GPE, seen either way round.
M1	Use of EPE formula at top or at *B*
A1	Correct EPE at top
A1	Correct EPE at B
M1	Use of conservation of energy, with 1 GPE, 1 KE and 2 EPE terms, condone sign errors
dM1	Solve for v^2, dependent on previous M
A1*	Exact given answer correctly obtained
(b)	
M1	Use of Hooke's Law at *B* – this may appear in an attempted equation of motion
A1	Correct unsimplified tension at *B*
M1	Equation of motion at *B* horizontally with correct terms, condone sign errors
A1	Correct equation with at most one error
A1	Correct equation
dM1	Sub for *T* and v^2. Dependent on both previous M marks
A1*	Given answer correctly obtained (exactly). If $N = -\frac{1}{2}mg\left(5\sqrt{2}-2\right)$ then clear justification is required to reach the given answer eg use of 'magnitude' or modulus signs.

Question Number	Scheme	Marks
7(a)	$T_A - T_B = m\ddot{x}$	M1
	$\frac{2mg}{l}\left(\frac{2l}{3}-x\right)-\frac{mg}{l}\left(\frac{4l}{3}+x\right)=m\ddot{x}$ or $\frac{mg}{l}\left(\frac{4l}{3}-x\right)-\frac{2mg}{l}\left(\frac{2l}{3}+x\right)=m\ddot{x}$.	dM1A1
	$-\frac{3g}{l}x=\ddot{x}$, so SHM	A1
	$T=\frac{2\pi}{\sqrt{\frac{3g}{l}}}=2\pi\sqrt{\frac{l}{3g}}$ *	M1 A1*
		(6)
7(b)	$\frac{1}{2}l\times\sqrt{\frac{3g}{l}}$ or $\frac{1}{2}\sqrt{3gl}$ or $\sqrt{\frac{3gl}{4}}$ oe	B1
		(1)
7(c)	$\frac{3g}{2}$ or $1.5g$	B1
		(1)
7(d)	$x=a\cos\omega t \Rightarrow v=-a\omega\sin\omega t$	M1
	$-\frac{3}{4}\sqrt{gl}=-a\omega\sin\omega t$ to find t	M1A1
	Solve for t	M1
	$t=\frac{\pi}{3}\sqrt{\frac{l}{3g}}$ oe	A1
		(5)
		(13)
Notes		
(a)		
M1	Equation of motion in a *general* position, allow a for acceleration, correct no. of terms, condone sign errors.	
dM1	Use Hooke's Law to sub for the two tensions, allow a for acceleration. Extensions must be different and of the form $(d \pm x)$ where d is a multiple of l.	
A1	Correct unsimplified equation, allow a for acceleration.	
A1	Correct equation using $\ddot{x}$ for acceleration.	
M1	Use of $\frac{2\pi}{\omega}$ Their ω from their equation of motion, which must be in terms of x.	
A1*cso	Given answer correctly obtained – this includes proof of SHM **with conclusion** and correct expression for the period.	
(b)		
B1	Cao Speed at O so must be positive. Unsimplified, ignore errors from subsequent 'simplifying' of surds.	
(c)		
B1	Cao Max acceleration so must be positive.	

(d)	
Main	
M1	Use of $x = a\cos\omega t$ to obtain $v = -a\omega\sin\omega t$ Substitution for a and ω is not required.
M1	Use $v = -a\omega\sin\omega t$ with $a = \frac{l}{2}$ and $\omega = \sqrt{\frac{3g}{l}}$ to obtain equation in t only, $-\frac{3}{4}\sqrt{gl} = -a\omega\sin\omega t$
A1	Correct equation in t only
M1	Solve to find the required time, t
A1	Cao for required time.
ALT 1	
M1	Use of $x = a\sin\omega t$ to obtain $v = a\omega\cos\omega t$ Substitution for a and ω is not required.
M1	Use $v = a\omega\cos\omega t$ with $a = \frac{l}{2}$ and $\omega = \sqrt{\frac{3g}{l}}$ to obtain equation in t only, $\frac{3}{4}\sqrt{gl} = a\omega\cos\omega t$
A1	Correct equation in t only
M1	Solve to find t and then subtract from $\frac{1}{4}$ period to find the required time. Eg $t = \frac{\pi}{6}\sqrt{\frac{l}{3g}}$ $\Rightarrow$ required time = $\frac{1}{4}\left(2\pi\sqrt{\frac{l}{3g}}\right) - \frac{\pi}{6}\sqrt{\frac{l}{3g}} = \frac{\pi}{3}\sqrt{\frac{l}{3g}}$
A1	Cao for required time, $t = \frac{\pi}{3}\sqrt{\frac{l}{3g}}$ oe
ALT2	
M1	Use of $x = a\cos\omega t$ or use of $x = a\sin\omega t$. Substitution for a and ω is not required.
M1	Using $v^2 = \omega^2(a^2 - x^2)$ with $a = \frac{l}{2}$ and $\omega = \sqrt{\frac{3g}{l}}$ to obtain equation in x only. $\left(-\frac{3}{4}\sqrt{gl}\right)^2 = \omega^2(a^2 - x^2)$
A1	Correct equation in x only. (Solution leads onto the first M mark in (d))
M1	Solves for t and then completes the method to find the required time. e.g. $\frac{l}{4} = \frac{l}{2}\cos\left(\sqrt{\frac{3g}{l}}t\right)$ or quarter period with sin method.
A1	Cao for required time, $t = \frac{\pi}{3}\sqrt{\frac{l}{3g}}$ oe
SPECIAL CASE where *a* = ½ is clearly stated as amplitude and consistently used in (b) (c) & (d)	
(b)	B1 $\frac{1}{2}\sqrt{\frac{3g}{l}}$
(c)	B1 $\frac{3g}{2l}$
(d)	Maximum M1 M1 A0 M0 A0