On the Supply of Autonomous Vehicles in Platforms

The likely arrival of autonomous vehicle (AV) technology in the near future has the potential to fundamentally change the transportation landscape. Due to the high cost of AV hardware, the most likely path to widespread use of AVs is via platforms that can sustain high-utilization, outsource the high capital burden, and complement the network with human drivers joining as individual contractors (ICs). In this paper, we study a supply chain game between a platform, an outside AV supplier, and ICs. We show that such a setting is subject to a risk of AV underutilization because of the need to maintain the ICs' utilization sufficiently high to ensure ICs remain engaged. We show that in a decentralized supply chain, this can have a very significant negative effect on the supply chain efficiency, with an unbounded profit loss. We then study potential contracting solutions and argue that if demand scenarios are contractible, usage contracts can align the supply chain. If demand scenarios are not contractible, prioritization contracts can still guarantee that at least half of the optimal supply chain profit is attained.

Key words: Supply Chain Contracting, Ride-hailing, Ridesharing, Autonomous Vehicles, Gig Economy, Sharing Economy

---

1. Introduction

The development of commercially viable autonomous vehicles (AVs) will, if successful, transform transportation. As of August 2023 AVs have been deployed, either operationally or in testing phases, in at least 18 U.S. cities (Muller 2023). This includes commercial deployments, with no safety driver, in multiple major cities including San Francisco, Phoenix, Los Angeles and Las Vegas (Liedtke 2023, Mickle et al. 2023a, Capoot and Piazza 2023, Hawkins 2022, Reuters 2022). The strong momentum is expected to continue: some estimate the autonomous vehicle market size to reach 2,161.79 Billion US Dollars by 2030, achieving a Compound Annual Growth Rate of 40.1% (Allied Market Research 2023).

One important obstacle in the path of widespread AV adoption is the high cost of the hardware that AVs need. John Krafcik, the CEO of Waymo, one of the firms leading the race to develop AV technology, is famously bearish on the question of whether AVs will be adopted in mass for private use any time soon due to the technology's high cost (Moreno 2022). Similarly, analysts widely expect that Level 5 (Full Driving Automation) AVs will not reach individual households before 2035 (Lowery 2023a). Instead, AVs are widely expected to first build a major presence in settings where they could be highly utilized, such as in ride-hailing and delivery, settings where they are already commercially deployed (Bellan 2022b, Giacomin 2023, Lowery 2023b).

In line with such expectations, our work focuses on four potential operational models to commercially deploy AVs in the ride-hailing industry. We first consider (1) a model where AVs are adopted within the context of open platforms (Lyft 2021), which are platforms that allow third-parties such as human drivers and AV suppliers to bring their vehicles into the system. This is akin to the operational model currently adopted by ridehailing platforms like Uber and Lyft and AV technology companies such as Waymo (owned by Google parent company Alphabet Inc.) and Motional in Phoenix, Las Vegas and Los Angeles (Capoot and Piazza 2023, Bellan 2023, Hawkins 2022, Reuters 2022). As this is the prevailing model of AV deployment nowadays we focus on this model for most of this paper. However, we also analyze (2) a platform that sources AVs from a supplier through leasing contracts (either short- or long-term). This model more closely follows the payment structure in a traditional supply chain, though it is subject to the well-studied risk of double marginalization and has worse theoretical guarantees than the open platform model (see Section 5.2). Moreover, we also consider (3) an AV-only platform that is operated independently by the AV supplier. This is the operational model piloted by Waymo and Cruise in San Francisco (Liedtke 2023, Mickle et al. 2023a), though we show that it can face significant inefficiencies due to its inability to access flexible human drivers to handle stochastic demand (see Section 5.1). Finally, we consider (4) a benchmark model where AVs and human drivers are integrated on a single ride-hailing platform. This operational model is akin to the "Waymo buys Lyft" model, and can in theory lead to an efficient supply chain design. However, due to legal and financial complications AV technology companies such as Motional and Aurora Innovation, Inc. have openly expressed that they do not intend to build ride-hailing or carrier platforms themselves, but will instead build the driving technology to power those businesses (Bellan 2022a, Soper 2022).Likewise, in an effort to reign in costs, platforms like Uber and Lyft have mostly abandoned their plans to develop/build their own AV fleets (Mullaney 2023), striving instead to become "a smart hub for Cruise, Waymo, Tesla and AV players to come." In summary, our results mostly focus on the open platform operational model for several reasons:

- it is an active operating model today and expected to be more widely adopted in the future;
- we demonstrate that it has significant efficiency benefits when compared to the alternatives;
- it involves interesting dynamics that add a new dimension to the literature.

Operationally, most of our deployment models (except for AV-only platforms) enable hybrid fleets that rely on a combination of human drivers joining as independent contractors (ICs) and AVs (Taqla 2022, Shetty 2022). Intuitively, AVs, having lower variable costs, could serve base demand, while ICs, having lower fixed costs, could help cover peak demand. Thus, on a superficial level, the respective use of AVs and ICs to serve base and peak demand resembles classical ideas in operations management like dual-sourcing (Allon and Van Mieghem 2010), where a firm sources inventory from two locations, one that has long lead-times but is cheaper, and a second that has shorter lead-times but is more expensive. A standard solution in dual sourcing is to use the cheap source for base demand while the flexible source handles random fluctuations. In our setting, the supply of AVs is similarly unresponsive and has a lower marginal operating cost whereas ICs react more quickly but at a larger marginal operating cost; thus, one might expect the same outcome. However, this analogy is imperfect. Using ICs to cover only peak demand is likely to be infeasible as it would require ICs to be left idle for large periods of time. An additional constraint that is not present in dual sourcing is that ICs need to be incentivized to participate, and this requires offering them a sufficient workload. Consider the following stylized example:

Example 1. *Suppose a platform pays ICs \$15 per request served, and ICs have a reservation earnings level of \$15, i.e., they only join the platform if they are guaranteed to serve an order (with a probability of 100%). Suppose further that the platform has access to 20 AVs that operate at a marginal cost of \$1 per unit of demand served. Demand is assumed to be equal to 20 or 30, each with probability of* $1/2$.

The platform would like to serve 20 units (base demand) via AVs and, when demand is high, 10 units via ICs. This however, is not feasible because it would leave ICs with a utilization of 50%. In order to have 10 ICs present in the system, 10 units must also be served via ICs when demand is low to maintain sufficient IC utilization. AVs end up unutilized some of the time despite operating at a very low marginal cost.[1]

The distinction with the dual sourcing setting is due to the ICs' utilization constraints, i.e., ICs may only be available to serve high demand if they also get to serve some demand even when demand is low. We refer to this phenomenon, in which AVs are available and have a lower operating cost but it is nonetheless optimal for the platform to not fully utilize them, as the *AV underutilization effect*. When supply decisions are made centrally this effect may be a mere curiosity. However, we will show that in the case of open platforms that do not own the AVs, but rather pay an independent outside supplier (e.g., a car rental company, an AV manufacturer, or an investment fund) for AV usage, the AV underutilization can propagate backwards in the supply chain and reduce the supplier's incentive to invest in AV capacity.

In this work, we study a supply chain game that involves an open platform, an AV supplier and ICs. We describe the AV underutilization effect and show that it can have a significant impact on the supply chain profits. We then study the power of supply chain contracts to resolve this incentive misalignment.

[1] In this example we assume for simplicity that the IC pay is set exogenously. If it were endogenous, the platform could set the IC pay higher, and have ICs operate at lower utilization in this example; however, one can easily compute that the most cost-effective way for the platform to attract 10 ICs to join is by setting their pay to \$15 and fully utilizing them rather than increasing their pay and decreasing their utilization. In particular, this means that even with endogenous IC pay, AVs would end up unutilized some of the time.

1.1. Contributions

Our work is, to the best of our knowledge, the first study of the strategic interaction between a platform and an AV supplier, and the implications of this interaction on the availability of AVs within open platforms. As Uber and Lyft are currently working to set up partnerships with AV suppliers, guidance on how these partnerships should be structured is critically needed, especially as platforms plan to more widely operate with a combination of ICs and AVs for the foreseeable future. In Section 2, we define a sequential game with the following events: at first the platform commits to how much it will pay the AV supplier for AV usage, then the AV supplier determines the AV capacity it provides, then a demand scenario (but not the actual demand) realizes, followed by a number of ICs deciding whether to work in the realized scenario, and finally the actual demand realizes and the platform decides how to fulfill it given the ICs and the AVs at its disposal. We discuss some of the underlying model assumptions in Section 2.2.

Supply Chain (SC) Inefficiency. In Section 3, we investigate the subgame perfect equilibrium of the above-described game. We compare the combined supply chain profit, i.e., that of platform and supplier combined, in equilibrium with that of an integrated supply chain that controls both the capacity decision (*how many AVs to order*) and the dispatch decision (*how much demand to fulfill through AVs and ICs, respectively*). We find that the subgame perfect equilibrium can have an arbitrarily large efficiency loss, i.e., the Price of Anarchy (PoA) (Papadimitriou 2001) is unbounded.[2] This occurs because the AV underutilization effect may lead the supplier to supply no AVs at all, even when it would be near-optimal to operate with AVs only. Though we show the unbounded PoA through a sequence of pathological instances, we find in our numerical study (see Section 6) that the efficiency loss averages 5.1% across a plausible set of parameters, with a tenth of the instances involving an efficiency loss of more than 12%.

Our findings unveil a potential pitfall of the open platform model by demonstrating that the platform's optimal dispatch decision, via AV underutilization, may lead to market failures. This is a surprising outcome as AVs have lower operating costs, and one may thus expect them to be highly utilized. However, we show that to operate efficiently, the platform needs to provide the supplier with contractual utilization guarantees.

Utilization-based SC Contracts. The possibility of a significant SC misalignment begs the question of whether supply chain contracts could help realign the incentives of platform and supplier. In Section 4, we explore that possibility under a range of different permissible contracts. Our main

[2] The *Price of Anarchy* (Koutsoupias and Papadimitriou 1999, Papadimitriou 2001) compares the value of the worst equilibrium relative to the welfare-maximizing solution assuming a worst-case instance; a related concept, the *Price of Stability* (Schulz and Stier Moses 2002, Anshelevich et al. 2008) compares the best equilibrium relative to the welfare-maximizing solution under a worst-case instance; our results show that both are unbounded in our setting.

focus is on usage contracts, as these are closest to the platforms' current modus operandi. We find that when the platform can contractually commit to an AV utilization level *under each demand realization in each scenario*, then the AV underutilization effect cannot propagate backwards, and the supply chain can be as profitable as if it were integrated. However, when the scenario is not contractible, then alignment is (in general) only possible via integration. Nonetheless, we find that, even in this case, where perfect alignment without integration is not possible, the open platform model can potentially become much more efficient with well-designed contracts that make dispatch commitments. As a proof of concept, we show a guarantee of the following flavor for a simple prioritization contract: either the supply chain profit under the contract is at least half of that of an integrated supply chain or the supply chain can achieve half the profit of an integrated supply chain even without a contract. Despite the inability to fully align the supply chain, this contract demonstrates the value of supply chain contracts when compared to the potentially unbounded efficiency loss in the absence of a contract. Indeed, in our numerical results we find significantly improved outcomes with an average loss of just 0.2% across our instances.

1.2. Related Work

Our work lies at the intersection of supply chain contracting and platform operations, especially with a focus on (i) different worker types, (ii) self-scheduling workers, and (iii) the integration of autonomous vehicles. We highlight the most closely important related papers in this section.

Supply chain management. Though our work has some similarities with dual sourcing (Allon and Van Mieghem 2010), it is quite far from the significant literature on the optimal implementation of dual sourcing inventory systems (Song et al. 2017, Xin and Goldberg 2018). Rather, the main focus of our work is on the strategic interaction in a supply chain. In doing so our work centers on the hold-up problem (Williamson 1985), where a party under-invests in relation-specific investment due to an uncertainty in future transactions. In particular, our model presents a novel hold-up problem that arises from both demand uncertainty and the platform's access to a more flexible but utilization-driven supply source (the ICs). This novel effect goes hand-in-hand with a rather surprising optimal strategy: in traditional supply chain models with demand uncertainty the risk of low demand drives the inefficiency, as the supplier's excess capacity is unused when demand is low (Tomlin 2003, Taylor and Plambeck 2007). In contrast, our model presents a case where high demand generates the risk for AV underutilization. This is due to the dual-sourcing that occurs downstream, wherein the platform may need to allocate higher utilization to ICs during high demand scenarios to keep them engaged, which in turn risks lower usage of AVs. We illustrate this point through a family of instances in Appendix E.2, in which the demand is monotone increasing (in a stochastic dominance sense), but the equilibrium capacity decisions can be inverted, with the higher demand leading to lower capacity.

In addressing AV underutilization, our work connects to classical solutions to the hold-up problem, which rely on full contractibility of future states (Rogerson 1992, Maskin and Tirole 1999). While a similar solution applies to our setting when demand scenarios are contractible, we strengthen the robustness of usage contracts by showing that they can vastly improve the efficiency guarantee even when demand is not fully contractible. Our work also relates to double marginalization (Spengler 1950) and the contracting literature that aims to align supply chains in the face of it (Lariviere and Porteus 2001, Cachon and Lariviere 2005). While that literature stream proposes revenue-sharing contracts to align inventory decisions, we propose dispatch prioritization contracts to align AV capacity decisions.

The issue of managing multiple sources of capacity with different degrees of flexibility also appears in other domains, such as in power systems. Renewable energy such as solar and wind power often has low marginal cost but is fairly unpredictable, while other energy sources have higher marginal cost but are more reliable. There is a literature on how to manage such systems (Severance 2011, Costello and Hemphill 2014) and the risk of a capacity "death spiral." But this connection is fairly loose: in transportation, the cost structure is different and the less flexible resource (AVs) is the one with lower marginal cost. More closely related to our work is the notion of supply retention in repeated split-reward auctions: Chaturvedi et al. (2014) study the phenomenon that, rather than sourcing from only the cheapest supplier, a seller may buy some of its supply from other suppliers to maintain them as suppliers for the future. This notion has also been corroborated experimentally by Chaturvedi et al. (2019) and variants thereof were studied by Huang et al. (2018). This line of works shares the interesting feature that a decision maker would take the costlier option to ensure (future) participation, with the notable distinction that in a traditional supply chain underutilized capacity can be managed as inventory, whereas in service systems such as ride-hailing, underutilized capacity is lost due to its time-sensitive nature.

Platform operations. Our work is part of a growing stream of work on platform operations, especially for platforms with self-scheduling capacity (Cachon et al. 2017, Gurvich et al. 2019, Banerjee et al. 2015). Within this stream of work, there are relatively few papers that consider different worker types, usually drawing the distinction between employees and ICs — our work is similar to these, but with AVs instead of employees (Dong and Ibrahim 2020, Lobel et al. 2021, Castro et al. 2022, Hu et al. 2022). Usually, these papers assume that the platform can hire some number of employees before the ICs join due to some equilibrium condition (often the same as ours, as motivated by Hall et al. 2021 and Chandar et al. 2019). The key distinction that arises in our setting is that the AV supply is set by an outside supplier as opposed to the platform itself, i.e., we assume that both types of supply join the platform based on strategic considerations. We also remark that the prioritization scheme we consider is more general than most in the literature, except for Castro et al. (2022), allowing for arbitrary prioritization of either supply type.

Our work also naturally relates to some very recent studies on AV integration within platforms, though none of them consider the interaction between an AV supplier and the platform. Lian and van Ryzin (2020) study four different possible market designs and investigate the prices that result in each of them for customers; their setting that is closest to ours is that of a common platform with monopoly AV, though they assume equal prioritization of ICs and AVs. Siddiq and Taylor (2022) similarly investigate how platform profit and IC welfare changes when a platform gains access to AVs, and in particular how that is affected by the AV cost. Along this line, Benjaafar et al. (2021) further examine a fluid model consisting of two locations with asymmetric demands and strategic drivers who can reposition, demonstrating the displacement and incentive effects of automation on driver welfare. In our study of the platform's dispatch problem, our work relates to Castro and Frazelle (2023), in which the platform has access to an AV fleet of known size (exogenous), and needs to decide how many of these to deploy before demand realizes. Before that decision, the platform announces an IC wage (endogenous) and the ICs make a decision whether to join the platform; once demand realizes, it is assumed that the platform's prioritization is so that AVs are always *fully prioritized* (see Section 3). Roughly speaking, they find that the platform should not deploy too many AVs, as a large AV deployment causes ICs to only join at a high wage, i.e., the platform may price itself out of the market for ICs. In contrast, we consider the IC wage as exogenously given, while allowing for arbitrary prioritization. Crucially, our focus is on the strategic interaction between an AV supplier and the platform, which has not been previously considered.

Finally, we note that our model focuses on the strategic interaction of firms in the supply chain, not on details of operational implementation. As such, our model does not focus on platform features such as pricing (Bimpikis et al. 2019, Banerjee et al. 2022), repositioning (Braverman et al. 2019), or routing (Arslan et al. 2019). All of these have been studied extensively in the literature, but are abstracted away in our model.

2. The Model

We model the interaction between a platform and an AV supplier as a sequential game. At a high level, the following sequence of events takes place: (1) the platform determines the payment to the AV supplier per unit of demand served; (2) the AV supplier chooses its fleet size; (3) the demand scenario (but not the actual demand) is realized; (4) the platform decides its *dispatch prioritization*, i.e., how to allocate demand to AVs and ICs; (5) ICs make participation decisions; (6) the demand realizes and AVs/ICs are both dispatched according to the chosen prioritization. Fig. 1 below provides an illustration of the sequential game and indicates the key decisions. This timeline is based on the assumption that the AV fleet size is negotiated far in advance of its deployment, and the AV per-use payment is agreed upon upfront to avoid an unrealistic bad equilibrium where the AV fleet size is zero because the platform can choose to pay nothing for its use once it is deployed.

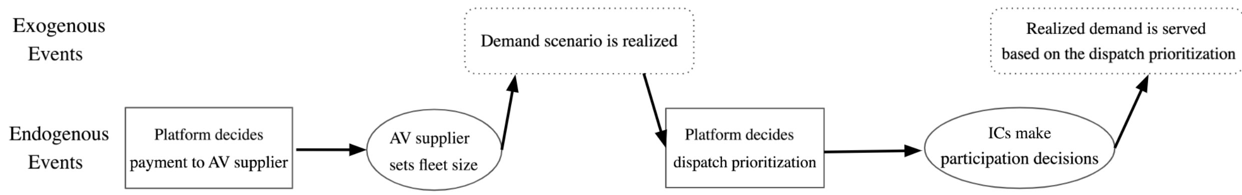


Figure 1 Illustration of stages of the sequential game.

Demand. Demand in our model is characterized by two levels of stochasticity. The first level of stochasticity comes from the realization of n potential states of the world, which we call *scenarios*, each occurring with probability α_i, $i \in \{1, ..., n\}$. Then, in a second level of stochasticity — in each scenario i — the demand realizes as a non-negative random variable D_i with cumulative distribution function F_i. The distinction between demand scenario and demand is meant to capture the fact that ICs have some information about the demand when they choose whether to participate, but they do not know the precise demand level. In contrast, the platform and the AV supplier know much less about demand when setting c_P and the AV fleet size.

AV Supply. To secure its AV supply, the platform guarantees upfront the supplier a payment of $c_{AV} + c_P$ per unit of demand served. The term c_{AV} represents the AV supplier's exogenous variable cost incurred whenever AVs serve a unit of demand, while c_P represents the AV supplier's profit per unit of demand served.[3] Since c_{AV} is exogenous, we will use c_P to represent the platform's first-period decision of how much to pay its AV supplier per unit of AV use. The AV supplier also faces an upfront fixed cost of c_F per unit of AV capacity it provides. This fixed cost is incurred regardless of whether the AVs are eventually used to serve demand. As the supplier makes its AV investment in the second stage after observing c_P, we denote the AV capacity by $K(c_P)$. The amount of demand the platform serves through AVs in the third stage is given by the function $\mathcal{A}_i\left(\cdot\,|c_P, K(c_P)\right)$ in each scenario i, a choice of notation that highlights the dependence on both c_P and $K(c_P)$. That is, the amount of demand the platform fulfills through AVs in scenario i is a random variable $\mathcal{A}_i\left(D_i|c_P, K(c_P)\right)$. We assume that each unit of supply capacity can serve up to one unit of demand, and therefore we must have $\mathcal{A}_i\left(D_i|c_P, K(c_P)\right) \leq \min\left\{D_i, K(c_P)\right\}$ for all c_p, i and D_i.

[3] We assume that c_P is set by the platform for simplicity, but the choice of decision-maker for c_P—be it the platform, the AV supplier, or a social planner—does not alter the main result of the model (see Theorem 1).

IC Supply. We follow the common assumption that ICs self-schedule their work on the platform depending on their expected earnings on the platform and their own reservation earnings level. Denoting by $\mathcal{H}_i\left(\,\cdot\,|c_P, K\left(c_P\right)\right)$ the amount of demand the platform serves through ICs in scenario i and by c_I the platform pay per demand fulfilled, the ICs' combined earnings evaluate to $\mathcal{H}_i\left(D_i|c_P, K\left(c_P\right)\right) \cdot c_I$. In the main body we focus on a setting where all ICs have a common reservation earnings level $t > 0$.[4] This case reflects a perfectly elastic labor market, which is motivated by the empirical findings of Hall et al. (2021) and Chandar et al. (2019) who show that Uber's supply pool is highly elastic. In such a setting, denoting by $y_i\left(c_P, K\left(c_P\right)\right)$ the number of ICs that join in scenario i, the IC supply is based on the following equilibrium condition

$$ty_i\left(c_P, K\left(c_P\right)\right) = c_I \mathbb{E}_{D_i \sim F_i}\left[\mathcal{H}_i\left(D_i|c_P, K\left(c_P\right)\right)\right]. \tag{1}$$

Similar to AVs, the IC supply bounds the amount of demand served through ICs, i.e., $\mathcal{H}_i\left(D_i|c_P, K\left(c_P\right)\right) \leq \min\left\{D_i, y_i\left(c_P, K\left(c_P\right)\right)\right\}$. We then refer to $\left(\mathcal{A}\left(\cdot\right), \mathcal{H}\left(\cdot\right)\right)$ as the platform's *dispatch policy.* Whereas some of our results in the main body assume that c_I is an exogenous parameter with $c_I \geq t$ (with $c_I < t$ no ICs would ever join), we show in Appendix A.2 that most of our main insights continue to hold when c_I is an endogenous decision variable for the platform.

Revenue. The platform earns a fixed revenue, normalized to 1, per unit of fulfilled demand, where we assume without loss of generality that $c_{AV} < 1$ and $c_I < 1$, i.e., fulfilling demand, whether through AVs or ICs, is profitable. Finally, we define an *instance* of our model in Definition 1 as a collection of all of the exogenous parameters.

Definition 1. An *instance* $I = \left(\vec{\alpha}, \vec{F}, c_I, t, c_{AV}, c_F\right)$ of our model is specified by the exogenous parameters $\vec{\alpha}, \vec{F}, c_I, t, c_{AV}$, and c_F. We denote by $\mathcal{I}$ the set of all instances.

The sequential game. We formalize our model through a three-stage game. At first, the platform sets c_P knowing that (i) the variable profit of serving demand with AVs and ICs are $1 - c_{AV} - c_P$ and $1 - c_I$, respectively; (ii) the resulting AV and IC dispatches depend on c_P and $K\left(c_P\right)$:

$$\max_{c_P} \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[\left(1 - c_{AV} - c_P\right)\mathcal{A}_i\left(D_i|c_P, K\left(c_P\right)\right) + \left(1 - c_I\right)\mathcal{H}_i\left(D_i|c_P, K\left(c_P\right)\right)\right] \tag{2}$$

In the second stage, given c_P, the AV supplier optimizes K knowing that the resulting AV dispatch would be $\mathcal{A}_i\left(\,\cdot\,|c_P, K\right)\ \forall i:$

$$\max_K \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[c_P \mathcal{A}_i\left(D_i|c_P, K\right)\right] - c_F K \tag{3}$$

[4] In contrast, in Appendix A.3 we consider a model wherein the number of ICs that join in scenario i follows some general function $S\left(\mathbb{E}_{D_i \sim F_i}\left[\mathcal{H}_i\left(D_i|c_P, K\left(c_P\right)\right)\right] \cdot c_I\right)$.

In the final decision stage, given c_P and $K(c_P)$, the platform decides its dispatch policy $(\mathcal{A}(\cdot), \mathcal{H}(\cdot))$, which also gives rise to $\vec{\mathbf{y}}$:

$$\max_{\vec{\mathbf{y}},\mathcal{A},\mathcal{H}} \sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[(1-c_{AV}-c_P)\mathcal{A}_i(D_i)+(1-c_I)\mathcal{H}_i(D_i)\right] \tag{4}$$

$$\text{s.t. } 0 \le \mathcal{A}_i(D_i) \le \min\left\{D_i, K(c_P)\right\}, \forall i \tag{5}$$

$$0 \le \mathcal{H}_i(D_i) \le \min\{D_i, y_i\}, \forall i \tag{6}$$

$$\mathcal{A}_i(D_i)+\mathcal{H}_i(D_i) \le \min\left\{D_i, K(c_P)+y_i\right\}, \forall i \tag{7}$$

$$ty_i = c_I \mathbb{E}_{D_i\sim F_i}\left[\mathcal{H}_i(D_i)\right] \ \forall i \tag{8}$$

Here Eq. (5), Eq. (6) and Eq. (7) represent the capacity constraints of available AVs and ICs, and Eq. (8) is the equilibrium condition for the participation of ICs.

Equilibrium outcome. The standard solution concept in a dynamic game without private information is the pure-strategies *subgame perfect equilibrium (SPE)*, which is the solution concept we consider. An SPE s requires that the action taken by a player in any stage of the game tree maximizes their forward-looking expected utility. An SPE s defines an equilibrium outcome $(K^s, \vec{\mathbf{y}}^s, \mathcal{A}^s, \mathcal{H}^s, c_P^s)$, which we will sometimes also refer to as the SPE of the game in a slight abuse of notation. For any $I \in \mathcal{I}$ and an SPE s, we denote the resulting equilibrium profits of the platform and the AV supplier respectively by

$$V_P^s(I) := \sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[(1-c_{AV}-c_P^s)\mathcal{A}_i^s(D_i)+(1-c_I)\mathcal{H}_i^s(D_i)\right],$$
$$V_A^s(I) := \sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[c_P^s\mathcal{A}_i^s(D_i)\right] - c_F K^s.$$

Then, the aggregate supply chain profit for I in the equilibrium is given by

$$V^s(I) := V_P^s(I) + V_A^s(I) = \sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[(1-c_{AV})\mathcal{A}_i^s(D_i)+(1-c_I)\mathcal{H}_i^s(D_i)\right] - c_F K^s.$$

2.1. Other Deployment Models

We benchmark the performance of an SPE, and later of a contracted SPE, against a vertically integrated supply chain in which the platform and the supplier are just one entity.[5] This serves as a natural upper bound for the profit in any deployment model. We then consider three alternatives to the above-described "open platform" model: an AV supplier operating an AV-only platform with no ICs or a platform that sources its AVs from a supplier through either short-term or long-term leasing contracts. We present the integrated benchmark here and defer the presentation of the other models and the corresponding results to Section 5.

[5] As we surveyed in the introduction, due to the platforms' reluctance in substantial AV investment and the AV suppliers' inclination to leverage existing ride-sharing networks, this operational model is currently not prevalent.

Centralized benchmark. In the centralized benchmark, the platform owns the AVs; thus, it does not need to pay c_P to an outside supplier for the use of the AVs but instead incurs the fixed cost of c_F per unit of AV capacity. This centralized problem and our model are two modeling variants known in the contracting literature respectively as the first-best and second-best solutions (Lipsey and Lancaster 1956). Mathematically, the goal of the platform is to optimize the AV fleet size K, IC quantity $\vec{\mathbf{y}}$ and dispatch policy $\mathcal{A}$ and $\mathcal{H}$ for an instance $I \in \mathcal{I}$.

$$\max_{K,\vec{\mathbf{y}},\mathcal{A},\mathcal{H}} \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i} \left[(1-c_{AV})\mathcal{A}_i\left(D_i\right) + (1-c_I)\mathcal{H}_i\left(D_i\right)\right] - c_F K \tag{9}$$

$$\text{s.t. } (6), (8), 0 \le \mathcal{A}_i\left(D_i\right) \le \min\left\{D_i, K\right\}, \mathcal{A}_i\left(D_i\right) + \mathcal{H}_i\left(D_i\right) \le \min\left\{D_i, K + y_i\right\}, \forall i. \tag{10}$$

We denote a centralized solution of instance $I \in \mathcal{I}$ by $(K^\star, \vec{\mathbf{y}}^\star, \mathcal{A}^\star, \mathcal{H}^\star)$ and write

$$V^\star(I) := \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i} \left[(1-c_{AV})\mathcal{A}_i^\star(D_i) + (1-c_I)\mathcal{H}_i^\star(D_i)\right] - c_F K^\star$$

for its supply chain profits. Throughout our work we are interested in the following questions: (1) How does the equilibrium supply chain profit compare to that of an integrated supply chain? (2) If the gap between the two is large, are there practically implementable contracts to limit the efficiency loss? We provide answers to questions (1) and (2) in Sections 3 and 4, respectively.

2.2. Discussion of Model Assumptions

The goal of our model is to capture key features of the interaction between an AV supplier and an open platform. In this section we explain some of the modeling choices we make toward this goal.

Sequence of events. The platform and the AV supplier both face significant demand uncertainty when the former decides on the price to pay for AV usage and the latter makes their investment; this is captured by the uncertainty over different possible demand scenarios at the time of the capacity decision, which we assume occurs many months or perhaps even a year or two in advance of a prioritization decision. We follow the framework of Lobel et al. (2021) in assuming there is still some residual demand uncertainty when ICs make participation decisions. We assume the dispatch policy is decided before ICs choose to participate to represent in a reduced-form way that participation decisions are daily or weekly decisions, without having to formally model a repeated game. That is, there is no possibility of ICs being there as a sunk-cost decision despite a dispatch policy that is not in their favor (as could happen with AVs), as ICs would quickly leave the platform. The assumption that AVs are exclusively deployed through an open platform can be relaxed to accommodate the existence of outside options. In Appendix A.1 we show that, regardless of whether the outside option is offered to the supplier before the demand scenario realizes, in-between it realizing and the demand realizing, or after the demand realizing, the result in Theorem 1 continues to hold.

Demand scenarios. The scenario definitions can be used to capture both observable and unobservable market conditions, while allowing for demand to remain stochastic even when conditioning on a given scenario. Concretely, one may interpret this as follows: different scenarios encode different time windows (e.g., workday mornings during the summer, weekend evenings during the winter, etc.) as well as general demand conditions (e.g., construction sites, concerts, weather, or strikes that affect demand). Even with full knowledge of these conditions demand remains stochastic. We assume that all market participants (platform, AV supplier, and ICs) have accurate distributional information, i.e., the probability of a given scenario realizing (or, viewed differently, the frequency with which each scenario occurs) and the probability that demand of a given magnitude realizes conditioned on the scenario are common knowledge.

As we model the AV investment as a long-term decision, we assume that the investment is made before the realization of the scenario. In other words, the supplier cannot redeploy its AVs based on short-term market conditions (e.g., add vehicles for one evening to serve demand going to a concert, but then not incur the fixed cost for owning them between 3-6AM when they would not be used).[6] ICs, on the other hand, know of the market conditions and can adapt accordingly. Specifically, ICs (i) observe the market conditions (e.g., they know of the concert in advance and can decide to drive during that time, without needing to incur a fixed cost between 3-6AM when they are sound asleep), and (ii) make rational decisions based on the conditions and the platform's anticipated dispatch prioritization. Finally, we also assume that the platform (i) adapts quickly to the market conditions, i.e., it has the ability to prioritize ICs in one scenario, and prioritize AVs in another, and (ii) does so in a way that ICs can anticipate, i.e., the ICs are able (potentially thanks to platform communication) to form rational expectations of the platform's dispatch prioritization in a given scenario. Of course, with demand remaining stochastic, residual uncertainty remains for the ICs, i.e., though full knowledge of the platform's dispatch decisions and the distribution of demand allows them to accurately predict their expected utilization, it does not guarantee perfect utilization (e.g., if too many drivers show up to serve demand coming from the concert, a driver may end up idle).

Lack of friction. In the interest of tractability our model abstracts away the spatial friction that exists in many service systems in which AVs would be deployed. In most service systems, the service quality improves when more supply is available, e.g., because spatial density speeds up pick-up times (Besbes et al. 2022, Nikzad 2017, Freund and van Ryzin 2021); similarly, in spatial settings, prioritizing one type of supply over another would worsen the service quality. Nonetheless, as such prioritization already occurs in practice (Krishnan et al. 2021) we view this as a useful abstraction

[6]Though, in Appendix A.1, we consider an extension where the supplier can redeploy AVs at a cost.

that allows us to focus on the key strategic interactions in the supply chain. We highlight that our model assumes that the platform is able to prioritize at will between AVs and ICs; this can be viewed as a generalization of existing notions in the literature that either consider a platform that does not actively prioritize either supply type (Lian and van Ryzin 2020) or a platform that fully prioritizes the supply type with lower marginal cost (Castro and Frazelle 2023).

3. The AV Underutilization Effect

At first sight, it might appear that the incentives of the platform and the AV supplier are fairly well-aligned. After all, we already assumed that c_P is chosen upfront in order to encourage the AV supplier to build capacity. However, this is not the case, as there exists a more subtle source of inefficiency in this supply chain, which we call the *AV underutilization effect*. In this section, we explore this effect, its causes, and its consequences.

We first define *AV prioritization*: for given c_P and K the ratio $\mathbb{E}_{D_i\sim F_i}\left[\mathcal{A}_i\left(D_i|c_P,K\right)\right]/\mathbb{E}_{D_i\sim F_i}\left[\min\left\{D_i,K\right\}\right]$ measures to what extent AVs are prioritized in scenario i. We say that the platform *fully prioritizes* AVs in scenario i if $\mathcal{A}_i\left(D_i|c_P,K\right)=\min\left\{D_i,K\right\}$, i.e., when AVs fulfill as much demand as possible. The supplier always prefers full AV prioritization, as their revenue is proportional to $\mathbb{E}_{D_i\sim F_i}\left[\mathcal{A}_i\left(D_i|c_P,K\right)\right]$. Intuitively, with AVs having lower marginal cost, i.e., $c_{AV}+c_P<c_I$, one might expect that it is also in the platform's interest to fully prioritize them. We show below that this is not the case. Indeed, the platform may even want to prioritize ICs fully, given y_i in scenario i set $\mathcal{H}_i\left(D_i|c_P,K\right)=\min\left\{D_i,y_i\right\}$. We refer to such a deviation from full AV prioritization as AV underutilization.

Definition 2. Given K,c_{AV},c_I and c_P, we say that AVs are *underutilized* in scenario i if $c_{AV}+c_P<c_I$ but $\mathcal{A}_i\left(D_i|c_P,K\right)<\min\left\{D_i,K\right\}$ for some realization of D_i. Furthermore, the magnitude of *AV underutilization* in scenario i is measured by $1-\mathbb{E}_{D_i\sim F_i}\left[\mathcal{A}_i\left(D_i|c_P,K\right)\right]/\mathbb{E}_{D_i\sim F_i}\left[\min\left\{D_i,K\right\}\right]$.

The possibility of AV underutilization raises two questions: (1) if AV capacity investment is set at a fixed K, how severe can AV underutilization be? (2) Given a supplier's rational expectation of AV underutilization, how should they make their investment decision? In other words, can AV underutilization propagate backward to earlier stages of the game, and discourage the AV supplier from making the socially optimal AV investment, thus resulting in a source of supply chain inefficiency? In the rest of this section we aim to answer these questions.

3.1. Dispatch Prioritization

In this section we show that the platform may prefer a non-trivial dispatch prioritization with ICs sometimes having higher priority than AVs, and that this can occur both in an SPE and in an integrated supply chain. This is a consequence of the greater flexibility of ICs: while the AV capacity must be secured in advance of when the platform makes dispatch decisions, the number of ICs varies across scenarios based on how much demand they are assigned in expectation in that scenario. Thus, in peak demand scenarios it may even be in the platform's interest to fully prioritize ICs in order to ensure sufficient IC capacity is available to meet demand.[7] In Proposition 1, we constructively show the occurrence of such a phenomenon through an instance of our model.

Proposition 1. *There exists an instance* $I \in \mathcal{I}$ *with unique centralized solution* $(K^\star, \vec{\mathbf{y}}^\star, \mathcal{A}^\star, \mathcal{H}^\star)$ *and SPE* $(K^s, \vec{\mathbf{y}}^s, \mathcal{A}^s, \mathcal{H}^s, c_P^s)$ *such that* $c_{AV} < c_{AV} + c_P^s < c_I$ *holds and*

(i) $y_i^\star > 0, \mathcal{H}_i^\star(D_i) = \min\{D_i, y_i^\star\}$ *and* $\mathcal{A}_i^\star(D_i) < \min\{D_i, K^\star\}$ *for scenario* i *and realization* D_i*;*

(ii) $y_i^s > 0, \mathcal{H}_i^s(D_i) = \min\{D_i, y_i^s\}$ *and* $\mathcal{A}_i^s(D_i) < \min\{D_i, K^s\}$ *for scenario* i *and realization* D_i*.*

The proposition shows that the AV underutilization effect occurs in both an integrated supply chain (part ii) and in equilibrium (part iii). The AV underutilization effect arises from ICs being (i) more flexible, and (ii) constrained by their equilibrium participation condition holding *in expectation.* It occurs because, even though AVs might be cheaper to operate on the margin, they are less flexible and the need to incentivize ICs to participate can trump the marginal cost differences.[8]

The magnitude of the AV underutilization effect depends on c_P. The higher the value of c_P, the smaller the upside of using AVs and therefore the more likely the platform will be to prioritize ICs. As the integrated supply chain pays only c_{AV} per unit of use, it corresponds to $c_P = 0$. Thus, the AV underutilization effect becomes more problematic for a distributed supply chain than for an integrated one. We formalize this result in Proposition 2 and illustrate it in Fig. 2 through an instance of our model. Proposition 2 also states that, even with $c_P < c_I - c_{AV}$, AV underutilization can be arbitrarily bad, i.e., AV prioritization can be arbitrarily close to 0.

Proposition 2. *For any* I, K, c_P *and* c_P' *where* $c_P \le c_P'$*, there exist equilibrium solutions with*

$$\mathbb{E}_{D_i \sim F_i}\left[\mathcal{A}_i^s(D_i | c_P, K)\right] \ge \mathbb{E}_{D_i \sim F_i}\left[\mathcal{A}_i^s(D_i | c_P', K)\right], \forall i, \tag{11}$$

$$\mathbb{E}_{D_i \sim F_i}\left[\mathcal{H}_i^s(D_i | c_P, K)\right] \le \mathbb{E}_{D_i \sim F_i}\left[\mathcal{H}_i^s(D_i | c_P', K)\right], \forall i. \tag{12}$$

Moreover, for any $\epsilon > 0$*, there exist* $I \in \mathcal{I}$*,* c_P *and* K *where* $c_{AV} + c_P < c_I$ *and*

$$\frac{\sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[\mathcal{A}_i^s(D_i | c_P, K)\right]}{\sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, K\}\right]} < \epsilon.$$

[7]With exogenous IC wages, prioritization is the only way to ensure IC participation; however, we show in Appendix A.2 that the results in this subsection continue to hold even when the platform sets IC wages endogenously.

[8]However, AV underutilization does not occur without the second stage of stochasticity; if demand is deterministic within each scenario, the "necessary" number of ICs show up even with full AV prioritization (see Appendix B).

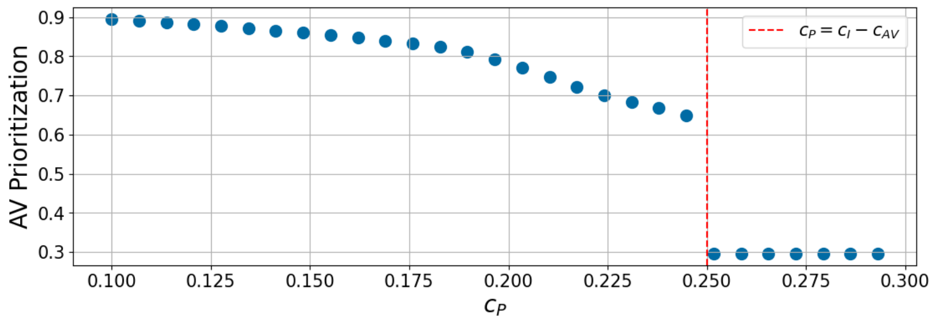


Figure 2 Optimal AV prioritization for an instance with $c_I = 0.75, c_{AV} = 0.5, c_F = 0.1$ and $t = 0.75 \cdot c_I$; the demand scenarios $D_1 \sim U(0,20)$ and $D_2 \sim U(0,40)$ occur with probability 0.7 and 0.3, and we take $K = K^\star = 17$. As c_P increases, AV prioritization decreases, and when $c_P > c_I - c_{AV}$ the platform fully prioritizes ICs.

3.2. The Profit Ratio

A key question in our paper is whether the AV underutilization effect is a source of economic inefficiency and, if yes, how big a source is it. If the extent of AV underutilization is the same under first-best (an integrated supply chain) and second-best (a decentralized supply chain), perhaps we do not need to worry about its occurrence since, in this case, the AV utilization would be set at a level chosen by a social planner. Proposition 2 suggests that this is not the case: the AV underutilization effect is stronger under a decentralized supply chain than under an integrated one. This implies that the AV underutilization effect is likely a source of supply chain inefficiency. In this section, we explore the degree of inefficiency introduced by the AV underutilization effect in a decentralized supply chain. One of our key results is to prove that this inefficiency, as measured by the profit ratio we are defining next, can be arbitrarily large.

Definition 3. For any $I \in \mathcal{I}$ and SPE s, we define its profit ratio by $\mathrm{PR}^s(I) := V^\star(I)/V^s(I)$.

We know that $\mathrm{PR}^s(I) \geq 1, \forall I \in \mathcal{I}$ as a centralized solution always leads to higher supply chain profits than any SPE. When $\mathrm{PR}^s(I) = 1$, we say that the supply chain is *aligned* for I. This opens the question of how large the profit ratio can be. We show below that there is no constant bound on the profit ratio. Our result is actually a stronger one, demonstrating that the profit ratio can be arbitrarily large for any given c_P in the first stage (not just the c_P from an SPE). For this purpose, we enhance the definition of profit ratio in Definition 4 to take into account an arbitrary $c_P \geq 0$. That is, we remove the first stage of the sequential game and only solve the second and third stages of the game with a given c_P. We denote the corresponding supply chain profit by $V^s(I|c_P)$, which we then use to compare with $V^\star(I)$.

Definition 4. For any $I \in \mathcal{I}, c_P \geq 0$ we define the profit ratio given c_P as $\mathrm{PR}^s(I|c_P) := \frac{V^\star(I)}{V^s(I|c_P)}$.

Equipped with this new definition, we are ready to show in Theorem 1 that there are instances with arbitrarily large profit ratio regardless of the choice of $c_P \geq 0$.

Theorem 1. *$\forall M \in \mathbb{R} : \exists I \in \mathcal{I}$ such that for any SPE s and any $c_P \geq 0$, $PR^s(I|c_P) \geq M$.*

Intuitively, we construct an instance where, for any c_P, the AV underutilization effect is so strong that the AV supplier responds by not investing in capacity at all. The platform then uses only ICs, whose variable cost is set so close to the revenue that the platform's profit margin vanishes. We remark that inefficient SPEs can also occur when suppliers have outside options for their AVs (Appendix A.1) or the platform sets c_I endogenously (Appendix A.2).

Moreover, while such efficiency loss is also partially driven by double marginalization — a supply chain distortion that results from successive markups by independent firms at different vertical levels of the supply chain (Spengler 1950) — we note that the key factor leading to the unbounded profit ratio is the AV underutilization effect. The difference between the AV underutilization effect and the double marginalization phenomenon is exhibited by Theorem 1: even if c_P is chosen by a social planner (and thus there is no efficiency loss from pricing), the profit ratio can still be arbitrarily large. Finally, we observe an interesting contrast to many traditional supply chain models where inefficiency is driven by the risk of low demand. Due to the nature of the dual-sourcing that occurs downstream (wherein the platform may utilize ICs highly in a high demand scenario to keep them engaged), it may actually be high demand scenarios that generate the risk of low AV utilization. We illustrate this point through families of instances in Appendix E.2 where the equilibrium capacity decision decreases while expected demand monotonically increases.

In addition to our theoretical results showing potentially unbounded inefficiency for an equilibrium outcome, we examine in Section 6 the inefficiency of the AV underutilization effect under plausible parameter choices. Our instances show that misalignment of equilibrium outcomes are commonly observed among a wide range of instances.

4. Supply Chain Coordination Contracts

We have argued so far that the AV underutilization effect creates an incentive misalignment that can lead to an underprovision of AV capacity. A natural follow-up question is whether the use of supply chain contracts might be able to ameliorate or resolve this supply chain misalignment. The model considered so far can be viewed as a simple contract model where the platform promises a profit per use c_P, and our previous section proves that this kind of contract is not powerful enough to resolve the misalignment. Indeed, it follows from the proof of Theorem 1 that a contract that stipulates the AV fleet size K as well as c_P does not suffice to guarantee that the supply chain achieves a constant fraction of the first-best objective. To address AV underutilization, the root cause of supply chain inefficiency, one needs to contract on usage rather than just price c_P and quantity K. In this section we study a broader set of contracts which assumes that conditions on the platform's dispatch policy $\mathcal{A}$ are also contractible.

We now propose a broader set of contracts. These contracts specify the value of c_P, K and impose a lower bound on the AV dispatch $\mathcal{A}$. In addition to the decision variables in the sequential game, the contract specifies an additional parameter c_S, which is a side payment made by the platform to the AV supplier as an additional incentive, independent of the AV usage, for the AV supplier to invest in autonomous technologies. We formalize the set of allowed contracts as follows.

Definition 5. The set Π contains all contracts π that specify the parameters c_S^π, c_P^π, K^π and $\underline{\mathcal{A}}^\pi$ in the following way:

(i) The platform pays the AV supplier c_P^π for each unit of demand served by AVs and a side payment of c_S^π;

(ii) The platform serves at least $\underline{\mathcal{A}}^\pi$ units of demand through AVs;

(iii) The AV supplier provides K^π units of AV capacity.

To allow for contracts in our formulation, we expand the game to add a stage-zero contracting phase. At stage 0, some contract $\pi \in \Pi$ is offered to both the platform and the AV supplier. We do not focus on who chooses which contract is under consideration, which could be chosen by one of the two parties or by a third-party moderator. Both parties have the option to accept or reject the contract. If they both accept the contract, they play a subgame perfect equilibrium according to the contract rules. If either party rejects the contract, they play an SPE as defined in Section 2. In this case, we say they are playing a no-contract equilibrium s. Therefore, a contract π is accepted if $V_P^\pi(I) \geq V_P^s(I)$ and $V_A^\pi(I) \geq V_A^s(I)$. Moreover, we denote the supply chain profit for I given π by $V^\pi(I)$, so we have $V^\pi(I) = V_P^\pi(I) + V_A^\pi(I)$ when the contract is accepted and $V^\pi(I) = V^s(I)$ otherwise. For a given contract π, we next describe how we measure the supply chain's profit ratio.

Definition 6. The profit ratio for I given π is measured by $\mathrm{PR}^\pi(I) := V^\star(I)/V^\pi(I)$. In particular, we say that π *aligns* the supply chain for I if $\mathrm{PR}^\pi(I) = 1$.

In general, for any $I \in \mathcal{I}$ with a centralized solution $(K^\star, \vec{\mathbf{y}}^\star, \mathcal{A}^\star, \mathcal{H}^\star)$ we can always vertically integrate the supply chain through a contract π that sets $c_S^\pi = c_F K^\star + V_A^s(I), c_P^\pi = 0, K^\pi = K^\star$ and $\underline{\mathcal{A}}_i^\pi(\cdot) = 0\; \forall i$. We refer to such a contract as an *integration contract* for I, denoted by $\pi^\star(I)$, and use superscript $\star$ as a shorthand notation for the contract. We show in Proposition 3 that $\pi^\star(I)$ aligns the supply chain for I, a result that is well-understood in the supply chain literature.

Proposition 3. *For any $I \in \mathcal{I}$, the integration contract $\pi^\star(I)$ ensures that the platform and the AV supplier adopt a centralized solution.*

A natural question is whether traditional revenue-sharing contracts, like integration contracts, can also align the platform's and the supplier's incentives and thus maximize the supply chain profit. Superficially, the parameter c_P may suggest that the platform and the supplier, even in the absence of a contract, already implement revenue-sharing for the demand served through AVs. However, for revenue-sharing to align the supply chain, the platform needs to also pay the supplier for demand served through ICs, which is analogous to vertical integration (see Appendix C.1).

Given that such full integration seems unlikely, we focus on the power of *usage contracts* to align the supply chain in different situations. We say a contract is a usage one if it pays AVs per unit of AV use, disallowing integration-style side payments as well as paying the AV supplier for demand served by ICs. Formally, we denote the set of usage contracts by $\Pi^U \subseteq \Pi$, with Π^U containing all contracts $\pi \in \Pi$ with $c_S^\pi = 0$ (to be clear, the contract still stipulates c_P^π, $\underline{\mathcal{A}}^\pi$ and K^π). In particular, we first look at the most general class of usage contracts that allow the utilization of the AVs to be contractible in each scenario, i.e., the contract specifies $\underline{\mathcal{A}}_i^\pi(\cdot)$ for each scenario i; in Section 4.1 we find that this suffices, without any side payment, to align the supply chain. This solution echos classical approaches to tackle hold-up problems, but requires (i) the ability to accurately predict the probability of each possible state of nature and (ii) the ability to contract based on the realized state of nature (Rogerson 1992, Maskin and Tirole 1999). As we discussed in Section 2.2, the realization of scenarios may not be contractible in practice; thus, we consider in Section 4.2 a subset of Π where $\underline{\mathcal{A}}_i^\pi(\cdot) = \underline{\mathcal{A}}_j^\pi(\cdot)\ \forall i, j$. We first show that whenever such a contract includes even a small amount of usage pay ($c_P^\pi > 0$), it cannot align the supply chain regardless of the side payment c_S^π. Nonetheless, usage contracts can be valuable, even with this restriction: we prove that, when committing the platform to fully prioritize AVs, usage contracts can guarantee to achieve half of the first-best supply chain profit, whenever an SPE fails to do so. Though not guaranteeing alignment, this result demonstrates the practical value of utilization commitments in the AV supply chain.

4.1. Setting with Contractible Scenarios

We start by allowing contracts that specify different values $\underline{\mathcal{A}}_i^\pi(\cdot)$ for different scenarios i. As the most general set of contracts Π includes the integration contract, we know that the supply chain can always be aligned through a contract from Π. However, our goal here is to identify a usage contract to align the supply chain, i.e., whether there exists a $\pi \in \Pi^U$ that aligns the supply chain. Our next result affirms that contracts from Π^U are sufficiently powerful to do so.

Theorem 2. *For every instance $I \in \mathcal{I}$, there exists some $\pi \in \Pi^U$ that leads to $PR^\pi(I) = 1$.*

For an outcome to obtain the first-best objective, it must be the case that the AV fleet size K, the number of ICs $\vec{y}$, and their respective utilization in each scenario are all equal to their respective values in a first-best solution. The contract constructed in the proof of Theorem 2 specifies K and the utilization for each realized demand accordingly, and thus allows the platform to attract $\vec{\mathbf{y}}^\star$ ICs to complement the AVs. This almost fully specifies an outcome with a supply chain profit that is greater than under the second-best; what remains open is how that larger profit is apportioned between the supplier and the platform. Given these fixed values, c_P fully determines the share of the profit the supplier and the platform each receive; by setting it appropriately one can guarantee that both the platform and the supplier agree to the contract. In Appendix C.2 we show (Corollary 1) that the same outcome can be achieved when the platform's dispatch commitment to the supplier is based on the IC dispatch, instead of on the realized demand, *as long as it may be scenario-dependent*; this may be of interest when demand served by ICs is more easily observed than total latent demand.[9] Due to the additional notations needed to define such dispatch policies and contracts, we defer the formalization of this result to Appendix C.2.

4.2. Setting where Scenario is Not Contractible

Though the contracts covered by Theorem 2 align with classical solutions to the hold-up problem in the supply chain literature, they may not be a practical solution to AV underutilization. Given the high dimensionality of a ridehailing marketplace, the realization of a scenario may be an unobservable, or at least unverifiable, event (see Section 2.2). We now study the efficacy of usage contracts in a more challenging setting where we assume that scenarios are not contractible. Specifically, we consider contracts from the following set.

DEFINITION 7. The set of contracts $\Pi^D \subseteq \Pi$ contains all contracts π with $\underline{\mathcal{A}}_i^\pi(\cdot) = \underline{\mathcal{A}}_j^\pi(\cdot)\ \forall j \neq i$.

We observe that, without the help of any usage payments, the vertical integration contract $\pi^\star(I)$ is a trivial solution in Π^D that aligns the supply chain. But does there exist a contract $\pi \in \Pi^D$ that involves usage pay, i.e., with $c_P^\pi > 0$, and aligns the supply chain? We start with an impossibility result that gives a negative answer to this question: no matter how small $c_P^\pi > 0$ is, the AV underutilization effect can arise and misalign the supply chain.

THEOREM 3. *For any $\epsilon > 0$, there exist $c_P \in (0, \epsilon)$ and $I \in \mathcal{I}$ with c_{AV} and c_I such that we have (i) $c_{AV} + c_P < c_I$ and (ii) $PR^\pi(I) > 1$ for any $\pi \in \Pi^D$ with $c_P^\pi \geq c_P$.*

Intuitively, Theorem 3 shows that no matter how small a value of c_P is chosen, and despite AVs having lower variable cost (i.e., $c_{AV} + c_P < c_I$), it may still be in the interest of the platform to underutilize AVs in order to incentivize the ICs to participate in the high demand scenarios. Thus, the only contracting solution that guarantees supply chain alignment in this setting is to set $c_P = 0$ and incentivize the supplier solely through side payments, i.e., full integration. The proof of the theorem is based on constructing an instance where the AV platform cannot tell whether AVs are being underutilized on purpose or because the scenario is one where less capacity is needed. The inability to contract on scenarios therefore causes a misalignment.

Nonetheless, contracting on commitment to AVs can have significant value: there exists a simple class of contracts that does not contract on scenarios which guarantees that the supply chain obtains at least half of the profit of a first-best solution when an SPE fails to provide the same guarantee.

[9] This only requires public ridership data which many cities provide (e.g., NYC at `https://data.cityofnewyork.us/Transportation/uber-Data/gre9-vvjv` and Chicago at `https://data.cityofchicago.org/Transportation/Transportation-Network-Providers-Trips/m6dm-c72p`)

We refer to these contracts as *full prioritization contracts*. Intuitively, full prioritization contracts allow the platform to commit to fully prioritize AVs, which prevents the AV underutilization effect, and thus ensures the AV supplier's incentive to invest.

Definition 8. For any $I \in \mathcal{I}$, the set of full prioritization contracts, denoted by $\Pi^F(I)$, contains all contracts π with $\underline{\mathcal{A}}_i^\pi(D_i) = \min\{D_i, K^\pi\}, \forall i$.

We establish in Theorem 4 that the better of "always prioritizing AVs" and an SPE (without resorting to contracts) leads to a profit ratio of 2. The full prioritization contract $\pi \in \Pi^F$ that we design in the proof of the theorem lies in the intersection of Π^U and Π^D, i.e., it is a usage contract and does not rely on contracting on scenarios.

Theorem 4. *For any $I \in \mathcal{I}$ with SPE s at least one of the following two statements is true:*

(i) The profit ratio of the SPE s satisfies $PR^s(I) \leq 2$;

(ii) Both the platform and the supplier agree to a $\pi \in \Pi^F$ that guarantees a profit ratio $PR^\pi(I) \leq 2$.

The result is based on constructing two solutions, one that relies only on ICs and one that relies only on AVs, and showing that one of them must earn at least half the profit from an integrated supply chain. Since the solution that relies only on ICs can be enforced by the platform without resorting to the AV supplier, we know that any no-contract equilibrium s must do at least as good as the IC-only solution. Then, we argue that when the IC-only solution is bad (i.e., when it obtains less than a half of the integrated supply chain profit), we can find a contract from Π^F that ensures both the platform and the supplier have incentives to adopt the AV-only solution, which then yields at least a half of the integrated supply chain profit. As in the case of Theorem 2, the reliance of Theorem 4 on the contractibility of the realized demand can be replaced with the contractibility of the dispatch of ICs (see Appendix C.2).

Theorem 4 highlights the importance of usage commitments when demand scenarios are not contractible. Even a simple prioritization contract can substantially enhance supply chain profit ratio guarantee by providing the platform and the supplier with the option to commit to full AV prioritization. In Section 6 we further numerically explore the performances of the full prioritization contracts across various instances of our model, finding that the better of an SPE and the full prioritization contracts yields the most robust performances out of all models/contracts considered.

5. Alternative Deployment Models

In this section we consider the supply chain performance of three alternative deployment models: the AV-only model, the short-term leasing model and the long-term leasing model. Specifically, we identify lower bounds on the worst-case profit ratios in these models to compare them with a contracted open platform model.

5.1. AV-only Model

We start with a model of an AV supplier that builds a ride-hailing platform using only AVs; such a platform solves (9)-(10) subject to $\vec{\mathbf{y}} = \mathbf{0}$, i.e.,

$$V^{AV}(I) := \max_{K,\mathcal{A}} \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[(1-c_{AV})\mathcal{A}_i(D_i)\right] - c_F K \quad \text{s.t.} \quad 0 \le \mathcal{A}_i(D_i) \le \min\{D_i, K\}, \forall i. \tag{13}$$

We define the profit ratio for I in an AV-only platform as $\text{PR}^{AV}(I) := \frac{V^\star(I)}{V^{AV}(I)}$. Since AVs are the only source of supply, the AV-only platform avoids the risk of AV underutilization. However, the inability to dynamically adjust its fleet size in different demand scenarios can lead to significant inefficiency relative to an integrated platform. In particular, we prove that the profit of an AV-only platform can be arbitrarily worse than $V^\star(I)$.[10]

PROPOSITION 4 **(Inefficiency of AV-only)**. *For any $M > 0$, there exists I with $PR^{AV}(I) \ge M$.*

5.2. Leasing Models

In this section, we study two models in which a platform secures its AV supply through leasing contracts as opposed to making per-trip payments. We distinguish between short- and long-term leases: for short-term leases, the leased AV quantities are scenario-dependent, for long-term leases they are not. Such leasing contracts require the platform to commit to fixed leasing quantities over a longer time span and thereby alleviate the risk of AV underutilization (similar to the AV-only model). However, they also carry a risk of double marginalization, as the AV supplier applies a markup on c_F to ensure a profit. In this section we show that both short- and long-term leasing contracts can have profit ratios worse than 2, i.e., they may be more inefficient than a well-contracted open platform.

5.2.1. Short-term Leasing In our short-term leasing model the platform leases $\mathcal{K}_i$ AVs in scenario i and pays c_P per AV leased; thereafter, it only incurs the usage cost c_{AV} for serving demand through AVs, not an additional payment to the supplier. The model evolves as follows:

(1) A social planner sets c_P to maximize supply chain profit, i.e., the platform's profit from fulfilling demand minus the fixed cost of AVs. The social planner knows that for given c_P the supplier responds with capacity investment $K(c_P)$ and the platform responds with leasing quantity $\mathcal{K}_i(c_P, K(c_P))$, dispatch policy $\mathcal{A}_i\left(D_i|c_P, K(c_P)\right)$ and $\mathcal{H}_i\left(D_i|c_P, K(c_P)\right)$ and solves:

$$\max_{c_P} \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[(1-c_{AV})\,\mathcal{A}_i\left(D_i|c_P, K(c_P)\right) + (1-c_I)\,\mathcal{H}_i\left(D_i|c_P, K(c_P)\right)\right] - c_F K(c_P)$$

[10] This inefficiency is similar to the one that arises from a platform which has only employees and no contractors, as identified by (Lobel et al. 2021).

(2) Anticipating, for given c_P and K, the AV leasing quantities $\mathcal{K}_i(c_P, K), \forall i$, the supplier solves:

$$\max_K \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i} \left[c_P \mathcal{K}_i(c_P, K) \right] - c_F K;$$

(3) Given c_P and $K(c_P)$, the platform sets $\mathcal{K}, \vec{\mathbf{y}}, \mathcal{A}$ and $\mathcal{H}$ as follows:

$$\max_{\mathcal{K}, \vec{\mathbf{y}}, \mathcal{A}, \mathcal{H}} \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i} \left[(1 - c_{AV}) \mathcal{A}_i(D_i) + (1 - c_I) \mathcal{H}_i(D_i) - c_P \mathcal{K}_i \right]$$
$$\text{s.t. } (6), (8), 0 \le \mathcal{A}_i(D_i) \le \min\{D_i, \mathcal{K}_i\}, \mathcal{A}_i(D_i) + \mathcal{H}_i(D_i) \le \min\{D_i, \mathcal{K}_i + y_i\}, \mathcal{K}_i \le K(c_P), \forall i.$$

We denote the supply chain profit and profit ratio of an SPE in an instance I of the above sequential game by $V^{SL}(I)$ and $\mathrm{PR}^{SL}(I) := \frac{V^\star(I)}{V^{SL}(I)}$. The following bound is our main result on the short-term leasing model.

Proposition 5 **(Inefficiency of Short-term Leasing)**. *There exists I with $PR^{SL}(I) \ge 100$.*

We prove this proposition by constructing an instance in which it is beneficial for the platform to have access to AVs across different demand scenarios, though it is not in the platform's interest to deploy AVs in every demand scenario. Consequently, the expected supplier's expected revenue from these intermittent leasing quantities does not justify the investment in AVs. The bound of 100 shows that a short-term leasing model can be much worse than a well-contracted open platform.

5.2.2. Long-term Leasing In the long-term leasing model the quantity of AVs leased by the platform is scenario-independent. In this model the payment c_P is made for each unit AV that the platform leases from the supplier before a scenario realizes, regardless of the actual AV usage. For given c_P, it is trivially optimal for the supplier to invest in whatever AV quantity the platform decides to lease, i.e., K is effectively chosen by the platform. Here, we assume that (1) the AV supplier sets $c_P = \arg\max (c_P - c_F) K(c_P)$ where $K(c_P)$ denotes the resulting AV leasing quantity and (2) given c_P, the platform sets $K, \vec{\mathbf{y}}, \mathcal{A}$ and $\mathcal{H}$ so as to

$$\max_{\mathcal{K}, \vec{\mathbf{y}}, \mathcal{A}, \mathcal{H}} \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i} \left[(1 - c_{AV}) \mathcal{A}_i(D_i) + (1 - c_I) \mathcal{H}_i(D_i) \right] - c_P K \quad \text{subject to} \quad (10).$$

We denote the supply chain profit and profit ratio of an SPE in an instance I of the above sequential game $V^{LL}(I)$ and $\mathrm{PR}^{LL}(I) := \frac{V^\star(I)}{V^{LL}(I)}$. We show the following bound for the profit ratio.

Proposition 6 **(Inefficiency of Long-term Leasing)**. *There exists I with $PR^{LL}(I) \ge 2.8$.*

The long-term leasing model largely mitigates the risk of AV underutilization by requiring the platform to commit to a fixed leasing quantity before the scenario realizes. Post-commitment, the AV supplier is indifferent to the usage of the leased vehicles. However, the supply chain suffers under double marginalization — the AV supplier adds a profit-maximizing markup to the fixed cost c_F, which drives the AV leasing quantity below the socially-optimal level. Proposition 6 states that this effect can lead to greater inefficiency than in a well-contracted open platform.

6. Numerical Results

In this section, we complement our theoretical bounds through a numerical investigation of the AV underutilization effect and the supply chain inefficiencies caused by it. Specifically, we examine (i) $\mathrm{PR}^{AV}(I)$ (ii) $\mathrm{PR}^{LL}(I)$, (iii) $\mathrm{PR}^{SL}(I)$, (iv) $\mathrm{PR}^{s}(I)$ as well as (v) the profit ratio of a full prioritization contract $\pi \in \Pi^F$ in the open platform model, with $K^\pi = K^\star, \underline{\mathcal{A}}_i^\pi(D_i) = \min\{D_i, K^\pi\}\ \forall i$ and

$$c_P^\pi = \frac{V_A^s(I) + c_F K^\star}{\sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}[\min\{D_i, K^\star\}]} \qquad \text{(set as in the proof of Theorem 4).}$$

For the full prioritization contract, we use $\vec{\mathbf{y}}^\pi$ and $\mathrm{PR}^f(I) := V^\star(I) / \left(V_P^\pi(I) + V_A^\pi(I)\right)$ to respectively denote the number of ICs and the profit ratio assuming that the contract π is accepted. Observe that we know from $V_A^\pi(I) = V_A^s(I)$ that π is always accepted by the supplier, and it is accepted by the platform if and only if $V_P^\pi(I) \geq V_P^s(I)$, i.e., when $\mathrm{PR}^f(I) \leq \mathrm{PR}^s(I)$. Finally, we define the profit ratio for the better of (iv) and (v) in the open platform model by $\mathrm{PR}^{B2}(I) = \min\left\{\mathrm{PR}^s(I), \mathrm{PR}^f(I)\right\}$. Notice that Theorem 4 upper bounds $\mathrm{PR}^{B2}(I)$ by 2. We omit $\mathrm{PR}^{B2}(I)$ whenever it can be inferred from $\mathrm{PR}^s(I)$ and $\mathrm{PR}^f(I)$.

We structure this section as follows. We first conduct a stylized calibration to derive plausible parameters across which we evaluate each profit ratio. Next (Figs. 3 & 4), we aggregate the results to illustrate the inefficiency faced by each of (i)-(iv); Finally (Figs. 5–**??**), we explore in a more granular way how the inefficiency of each deployment mode depends on particular parameters.

Parameters. Our results are based on a setting where the demand is governed by some uniform distribution $U(a_i, b_i)$ in one of three scenarios: a low-demand scenario with $D_1 \sim U(10, 20)$, a high-variance scenario with $D_2 \sim U(10, 40)$, and a high-demand scenario with $D_3 \sim U(30, 40)$. We vary the scenario probability, $\vec{\alpha}$, to examine the profit ratios under different demand distributions.

We begin by giving some back-of-the-envelope motivation for the range of parameters we consider. For ICs, we rely on current platforms' business practices. Uber and Lyft charge commissions that are typically in the $20-25\%$ range but can go up to 40% (Parrott and Reich 2018); on the other extreme, Juno, aimed to charge only 10% (Solomon 2016). This motivates 0.6 to 0.9 as a natural range for c_I; for t, we follow empirically observed utilization values of around 60% Parrott and Reich (2018) and set $t = 0.60 \cdot c_I$. Though the AV technology is still in development, with cost parameters changing as the business scales, AVs are generally expected to have higher fixed cost and lower marginal cost than ICs (indeed, given the reduced need of labor and the additional equipment necessary, one could view this as a defining distinction). In particular, current sources suggest that an AV operated by Cruise costs \$150,000 to \$200,000 (Mickle et al. 2023b), which is much higher than the approximate cost of \$25,000 of an economy car operated through Uber's platform (Tucker 2023). Thus, for c_F, if an AV costs \$150,000 and serves 60,000 trips in its lifetime

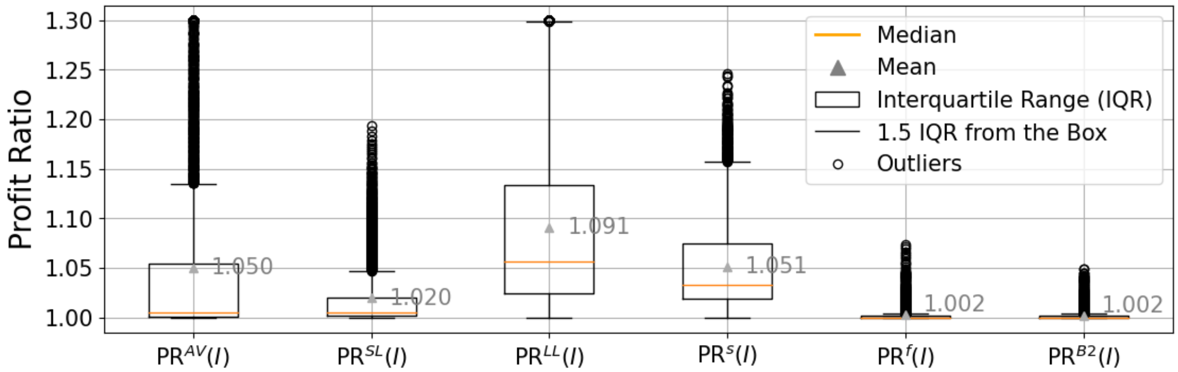


Figure 3 **Profit ratio of different deployment modes across all parameters.**

at an average revenue of \$25 per ride, then the normalized fixed cost would be 0.1 (we do not try to incorporate the higher cost of capital here). In contrast, the marginal cost of AVs is expected to be lower than that of a conventional vehicle due to the savings in labor costs. To estimate a reasonable cost for c_{AV}, we need to include the costs of fuel, wear and tear, insurance, and cleaning, i.e., all of the costs ICs currently face except for driver costs (which is replaced by compute cost). We consider a range for c_{AV} between 0.1 and 0.5, i.e., between a tenth and a half of the revenue of a trip. Though these numbers are generally consistent with those reported in the media, e.g., by Lau (2021) and Fannin (2022), we emphasize that they should only be interpreted as educated guesses; by numerically exploring a wide range with $c_F \in [0.025, 0.4]$ and $c_{AV} \in [0.1, 0.5]$ we aim to include realistic numbers. Across our experiments, we solve K to a precision level of 0.5 and c_P to a precision level of $(1 - c_{AV} - c_F)/100$.

Aggregate results. Fig. 3 shows the range of profit ratios for our different deployment modes. Whereas PR^{AV}, PR^{SL}, PR^{LL}, and PR^{s} can face significant inefficiencies, PR^{f} displays a small profit ratio for almost all parameters. In the rare instances where PR^{f} is large, Fig. 4 shows that PR^{s} is small, which reflects the spirit of Theorem 4 (at most one of them can be > 2). As our parameters are not chosen adversarially, we do not expect to see the unboundedly large inefficiencies we observed in our theoretical results; nonetheless we find the most significant inefficiencies in those modes (PR^{AV}, PR^{SL}, PR^{LL}, and PR^{s}) for which our theoretical results suggest that they can occur. Our numerical results thus reflect our theoretical finding that a well-contracted open platform more effectively hedges against significant inefficiencies and provides a superior deployment model.

Varying costs for AVs. We next examine how profit ratios change as we vary the cost parameters of AVs. Fixing c_I, we respectively vary c_{AV} and c_F in Fig. 5 (a) and (b), finding that the AV-only platform is most sensitive to changes in c_{AV} and c_F.[11] While PR^{AV} is close to 1 when c_{AV} and c_F

[11]We find that the patterns with respect to the cost parameters hold for varying demand distributions. Each number reported in Fig. 5-7 captures the average profit ratio over 12 demand distributions with $D_1 \sim U(10, 20), D_2 \sim U(10, 40), D_3 \sim U(30, 40)$ and varying scenario probabilities.

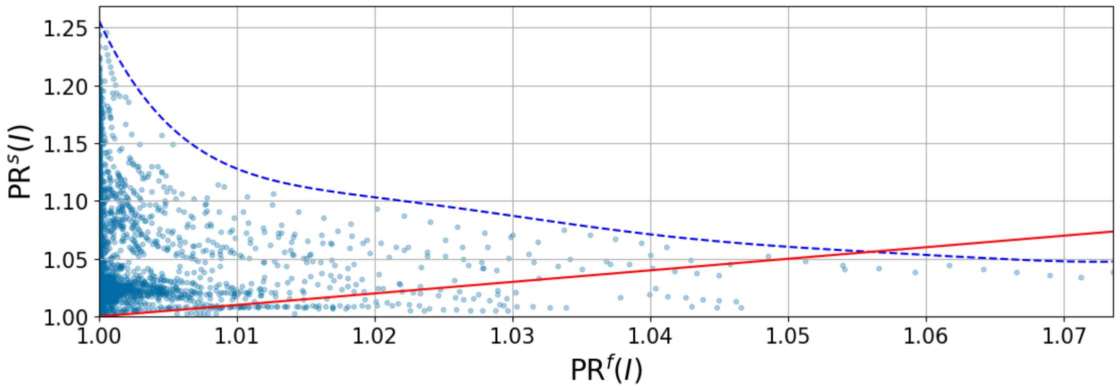
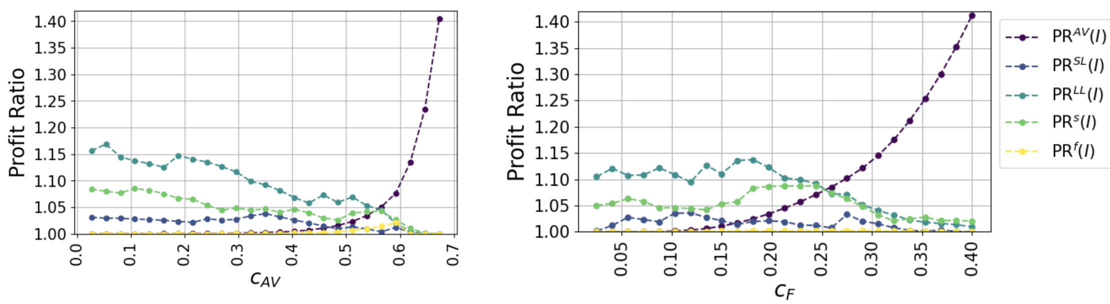


Figure 4 Scatter plot for $\mathrm{PR}^s(I)$ and $\mathrm{PR}^f(I)$ across the experiments. The red line indicates the boundary where $\mathrm{PR}^s(I) = \mathrm{PR}^f(I)$, above which the full prioritization contract is accepted. We observe that $\mathrm{PR}^f(I)$ and $\mathrm{PR}^s(I)$ are never both very large, and the dashed blue line highlights a trade-off between the two profit ratios.



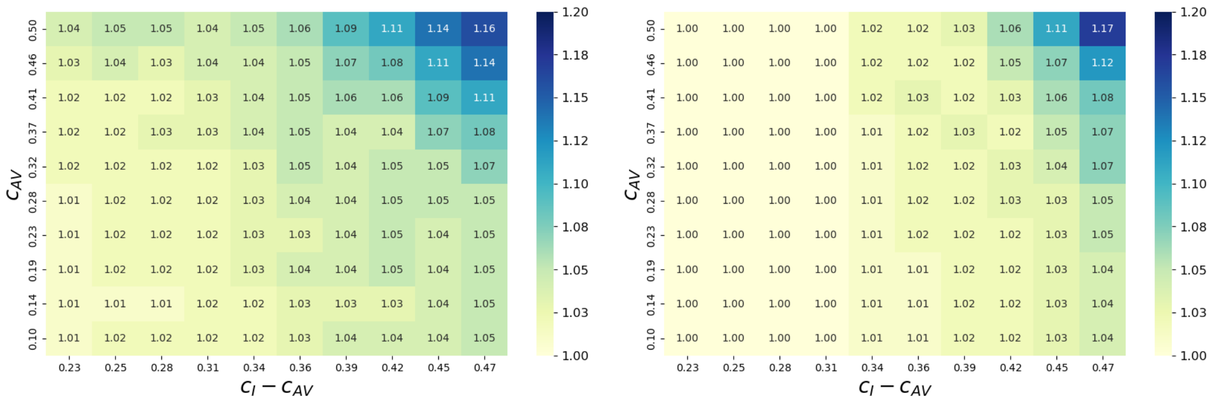
(a) Scale up c_{AV} for $c_F = 0.1$

(b) Scale up c_F for $c_{AV} = 0.3$

Figure 5 Experiments that respectively scale up c_{AV} and c_F for fixed $c_I = 0.75$.

are small, it quickly deteriorates as the cost parameters of AVs grow larger. In contrast, $\mathrm{PR}^s(I)$ and $\mathrm{PR}^{LL}(I)$ tend to decrease in c_{AV} and c_F, ultimately achieving a profit ratio of almost 1. Intuitively, this follows because AVs have a larger advantage relative to ICs when c_{AV} and c_F are smaller, and thus any misalignment due to AV underutilization is more costly. In contrast, for sufficiently large c_{AV} and c_F all operational models (except for AV-only), including the integrated benchmark, rely only on ICs and AV underutilization no longer disrupts the supply chain. In particular, we observe that an AV-only platform suffers greatly from the lack of access to ICs for large c_{AV} and c_F, whereas the the full prioritization contract, though also requiring commitment to AVs, leads to $\mathrm{PR}^f(I) \le 1.03$ for all instances. This occurs because the full prioritization contract sets its AV investment to be $K^\star$, which is decreasing in c_{AV} and c_F .

Inefficient SPEs. We then focus on $\mathrm{PR}^s(I)$ to investigate its dependence on c_I and c_{AV}. We find that the misalignment tends to be worse when $c_I - c_{AV}$ is large and particularly when c_{AV} is also large. Intuitively, this occurs because when $c_I - c_{AV}$ is small, a platform may perform well by relying only on ICs, which bounds the effect of a misalignment with the supplier; in contrast, when $c_I - c_{AV}$ is large, the benefit of AVs is significant, yet the supplier may undersupply to avoid underutilization. When c_{AV} and c_I are both large $\text{PR}^s(I)$ is particularly bad, as we examine next.



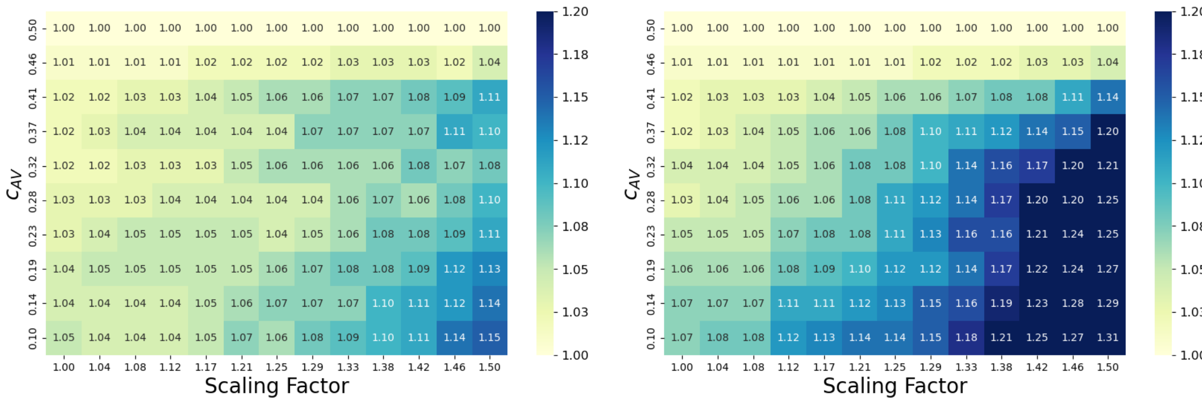
(a) $c_F = 0.1$ (b) $c_F = 0.3$

Figure 6 Heatmap for profit ratio of SPE in open platform across different c_{AV} and $c_I - c_{AV}$.

Scaling Up Costs. In our next set of numerical results we consider a setting where costs scale up proportionally, so profit margins shrink for both AVs and ICs. Specifically, we set initial values of c_I, c_{AV}, and c_F, and then proportionately scale these three cost parameters by factors between 1 and 1.5. Fig. 7 presents the outcomes of these experiments for different initial values of c_{AV} and scaling factors. Fig. 7 (a) shows that, for almost all initial values of c_{AV}, the resulting misalignment of the SPE in the open platform model worsens as the cost parameters scale up. Intuitively, as costs scale large the profit margin of any solution is small, which magnifies the absolute loss in profit relative to the integrated benchmark. We find similar patterns for the long-term leasing contract in Fig. 7 (b), with performances deteriorating even faster as the cost parameters grow large. Though we do not include the plots here, we find that the full prioritization contract, short-term leasing contract, and the solution for the AV-only platform are more robust (typically $< 5\%$ change in efficiency loss) as we simultaneously increase the different cost parameters. In particular, $\text{PR}^f(I) < 1.02$ for all instances in this experiment.

7. Conclusion

Our paper explores a new kind of supply chain misalignment that is likely to emerge as AVs move from being prototypes to being deployed technology. The misalignment is due to the fact that AVs will likely have to be supplemented with human-driven vehicles for managing stochastic demand, and since human drivers need to be engaged to ensure participation, the platform might need to reduce the prioritization of AVs. This is an unusual misalignment where the platform deviates from low-cost to high-cost to ensure sufficient capacity remains available for peak moments of demand.



(a) $\mathrm{PR}^s(I)$ (b) $\mathrm{PR}^{LL}(I)$

Figure 7 Heatmap for profit ratio across different initial values of c_{AV} and scaling factors. The initial value of c_I and c_F are respectively fixed at 0.6 and 0.1.

The supply chain management literature has solutions in place for aligning incentives, but they all have their challenges in a platform setting like this one. As usual, vertical integration is an option, though it requires merging two very different businesses, one of them being a capital-intensive one. A revenue-sharing contract requires sharing profits from ICs and is thus akin to a vertical merger. We show that usage commitments are an effective alternative to align the incentives in AV supply chain: in the unlikely setting that demand scenarios are contractible, they fully align the supply chain; and even when that is not the case, the option to commit to full AV prioritization offers a strong profit ratio guarantee.

References

Allied Market Research. Autonomous vehicle market size to generate an estimate of $2,161.79 billion in 2030, at a compound annual growth rate (cagr) of 40.1%. *Yahoo Finance*, 2023. URL `https://finance.yahoo.com/news/autonomous-vehicle-market-size-generate-124300044.html?guccounter=1`. Wed, November 1, 2023 at 8:43 AM EDT·4 min read.

Gad Allon and Jan A Van Mieghem. Global dual sourcing: Tailored base-surge allocation to near-and offshore production. *Management Science*, 56(1):110–124, 2010.

Elliot Anshelevich, Anirban Dasgupta, Jon Kleinberg, Éva Tardos, Tom Wexler, and Tim Roughgarden. The price of stability for network design with fair cost allocation. *SIAM Journal on Computing*, 38(4): 1602–1623, 2008.

Alp M Arslan, Niels Agatz, Leo Kroon, and Rob Zuidwijk. Crowdsourced delivery—a dynamic pickup and delivery problem with ad hoc drivers. *Transportation Science*, 53(1):222–235, 2019.

Siddhartha Banerjee, Carlos Riquelme, and Ramesh Johari. Pricing in ride-share platforms: A queueing-theoretic approach. *Working Paper, Cornell University*, 2015.

Siddhartha Banerjee, Daniel Freund, and Thodoris Lykouris. Pricing and optimization in shared vehicle systems: An approximation framework. *Operations Research*, 70(3):1783–1805, 2022.

Rebecca Bellan. Inside motional's strategy to bring robotaxis to market. https://techcrunch.com/2022/10/13/inside-motionals-strategy-to-bring-robotaxis-to-market/?guccounter=1, 2022a. Accessed: October 13, 2022.

Rebecca Bellan. Uber eats pilots autonomous delivery with serve robotics, motional. *Tech Crunch*, 2022b. URL https://techcrunch.com/2022/05/15/uber-eats-pilots-autonomous-delivery-with-serve-robotics-motional/.

Rebecca Bellan. Waymo's self-driving cars will be available on uber's app starting in phoenix. *TechCrunch*, 2023. URL https://techcrunch.com/2023/05/23/waymos-self-driving-cars-will-be-available-on-ubers-app-starting-in-phoenix/.

Saif Benjaafar, Zicheng Wang, and Xiaotang Yang. Autonomous vehicles for ride-hailing. *Available at SSRN 3919411*, 2021.

Omar Besbes, Francisco Castro, and Ilan Lobel. Spatial capacity planning. *Operations Research*, 70(2):1271–1291, 2022.

Kostas Bimpikis, Ozan Candogan, and Daniela Saban. Spatial pricing in ride-sharing networks. *Operations Research*, 67(3):744–769, 2019.

Anton Braverman, Jim G Dai, Xin Liu, and Lei Ying. Empty-car routing in ridesharing systems. *Operations Research*, 67(5):1437–1452, 2019.

Gérard P Cachon and Martin A Lariviere. Supply chain coordination with revenue-sharing contracts: strengths and limitations. *Management Science*, 51(1):30–44, 2005.

Gerard P Cachon, Kaitlin M Daniels, and Ruben Lobel. The role of surge pricing on a service platform with self-scheduling capacity. *Manufacturing & Service Operations Management*, 19(3):368–384, 2017.

Ashley Capoot and Jake Piazza. Uber begins offering rides in self-driving waymo cars. CNBC, October 2023. URL https://www.cnbc.com/2023/10/26/uber-begins-offering-rides-in-self-driving-waymo-cars.html. Published Thu, Oct 26 2023 9:58 AM EDT; Updated Thu, Oct 26 2023 4:00 PM EDT.

Francisco Castro and Andrew Frazelle. Getting out of your own way: Introducing autonomous vehicles on a ride-hailing platform. *Available at SSRN 3912137*, 2023.

Francisco Castro, Jian Gao, and Sebastien Martin. Supply prioritization in hybrid marketplaces. *Working Paper, UCLA*, 2022.

Bharat K Chandar, Ali Hortaçsu, John A List, Ian Muir, and Jeffrey M Wooldridge. Design and analysis of cluster-randomized field experiments in panel data settings. Technical report, National Bureau of Economic Research, 2019.

Aadhaar Chaturvedi, Damian R Beil, and Victor Martínez-de Albéniz. Split-award auctions for supplier retention. *Management Science*, 60(7):1719–1737, 2014.

Aadhaar Chaturvedi, Elena Katok, and Damian R Beil. Split-award auctions: Insights from theory and experiments. *Management Science*, 65(1):71–89, 2019.

Kenneth W Costello and Ross C Hemphill. Electric utilities''death spiral': Hyperbole or reality? *The Electricity Journal*, 27(10):7–26, 2014.

Jing Dong and Rouba Ibrahim. Managing supply in the on-demand economy: flexible workers, full-time employees, or both? *Operations Research*, 68(4):1238–1264, 2020.

Rebecca Fannin. Where the billions spent on autonomous vehicles by u.s. and chinese giants is heading. *CNBC*, 2022. URL `https://www.cnbc.com/2022/05/21/why-the-first-autonomous-vehicles-winners-wont-be-in-your-driveway.html`.

Daniel Freund and Garrett van Ryzin. Pricing fast and slow: Limitations of dynamic pricing mechanisms in ride-hailing. *Working Paper, MIT*, 2021.

Joseph Giacomin. How much will an autonomous car cost? a lot. *Automotive News Europe*, 2023. URL `https://europe.autonews.com/guest-columnist/how-much-will-autonomous-car-cost-lot`.

Itai Gurvich, Martin Lariviere, and Antonio Moreno. Operations in the on-demand economy: Staffing services with self-scheduling capacity. In *Sharing economy*, pages 249–278. Springer, 2019.

Jonathan V Hall, John J Horton, and Daniel T Knoepfle. Pricing in designed markets: The case of ride-sharing, 2021.

Andrew J. Hawkins. Uber and motional's robotaxi service expands to las vegas. *The Verge*, 2022. URL `https://www.theverge.com/2022/12/7/23496383/uber-motional-av-robotaxi-las-vegas-ridehail`.

Ming Hu, Jianfu Wang, and Zhoupeng Jack Zhang. Implications of worker classification in on-demand economy. *Working Paper, University of Toronto*, 2022.

He Huang, Zhipeng Li, and Hongyan Xu. Wholesale price auctions for dual sourcing under supply risk. *Decision Sciences*, 49(4):754–780, 2018.

Elias Koutsoupias and Christos Papadimitriou. Worst-case equilibria. In *Annual Symposium on Theoretical Aspects of Computer Science (STOC)*, pages 404–413. Springer, 1999.

Varun Krishnan, Ramon Iglesias, Sebastien Martin, Varun Pattabhiraman, Su Wang, and Garrett van Ryzin. Solving the ride-sharing productivity paradox: Priority dispatch and optimal priority sets. *Working Paper, Northwestern University*, 2021.

Martin A Lariviere and Evan L Porteus. Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing & Service Operations Management*, 3(4):293–305, 2001.

Alan Lau. Self-driving cars faq: How far away is far away? *Motor Trend*, 2021. URL `https://www.motortrend.com/news/self-driving-cars-faq-can-i-buy-one/`.

Zhen Lian and Garrett van Ryzin. Autonomous vehicle market design. *Working Paper, Cornell University*, 2020.

Michael Liedtke. Two rival robotaxi services win approval to operate throughout san francisco despite safety concerns. *Associated Press*, 2023. URL https://apnews.com/article/robotaxis-cruise-waymo-san-francisco-artificial-intelligence-98f0dab03984bd71685b92c2ac7aad0c.

Richard G Lipsey and Kelvin Lancaster. The general theory of second best. *The Review of Economic Studies*, 24(1):11–32, 1956.

Ilan Lobel, Sebastien Martin, and Haotian Song. Employees, contractors, or hybrid: An operational perspective. *Working Paper, NYU*, 2021.

Lurah Lowery. S&p predicts avs need at least 10 more years to hit widespread adoption. *Repairer Driven News*, September 2023a. URL https://www.repairerdrivennews.com/2023/09/26/sp-predicts-avs-need-at-least-10-more-years-to-hit-widespread-adoption/. Market Trends — Technology.

Lurah Lowery. S&p predicts avs need at least 10 more years to hit widespread adoption. *Repairer Driven News*, 2023b. URL https://www.repairerdrivennews.com/2023/09/26/sp-predicts-avs-need-at-least-10-more-years-to-hit-widespread-adoption/.

Lyft. The lyft network: Autonomous fact sheet. https://s27.q4cdn.com/263799617/files/doc_presentations/2021.04.26-Lyft-Conference-Call-Fact-Sheet.pdf, 2021. Accessed: 2022-07-30.

Eric Maskin and Jean Tirole. Unforeseen contingencies and incomplete contracts. *The Review of Economic Studies*, 66(1):83–114, 1999.

Tripp Mickle, Yiwen Lu, and Mike Isaac. Waymo makes progress in driverless cars in san francisco. *The New York Times*, 2023a. URL https://www.nytimes.com/2023/08/21/technology/waymo-driverless-cars-san-francisco.html.

Tripp Mickle, Cade Metz, and Yiwen Lu. G.m.'s cruise moved fast in the driverless race. it got ugly. *The New York Times*, November 2023b. URL https://www.nytimes.com/2023/11/03/technology/cruise-general-motors-self-driving-cars.html.

Johan Moreno. Waymo ceo says tesla is not a competitor, gives estimated cost of autonomous vehicles. *Forbes*, 2022. URL https://www.forbes.com/sites/johanmoreno/2021/01/22/waymo-ceo-says-tesla-is-not-a-competitor-gives-estimated-cost-of-autonomous-vehicles/?sh=11b77eee541b.

Tim Mullaney. Where uber's climate and autonomous driving goals will meet in the future. CNBC, August 2023. URL https://www.cnbc.com/2023/08/24/where-ubers-climate-and-autonomous-driving-goals-meet-in-the-future.html. Published Thu, Aug 24 2023 11:42 AM EDT; Updated Fri, Aug 25 2023 10:28 AM EDT.

Joann Muller. Robotaxis hit the accelerator in growing list of cities nationwide. *Axios*, 2023. URL `https://www.axios.com/2023/08/29/cities-testing-self-driving-driverless-taxis-robotaxi-waymo`.

Afshin Nikzad. Thickness and competition in ride-sharing markets. *Working Paper, USC*, 2017.

Christos Papadimitriou. Algorithms, games, and the internet. In *Proceedings of the Annual ACM Symposium on Theory of Computing (STOC)*, pages 749–753, 2001.

James A Parrott and Michael Reich. An earnings standard for new york city's app-based drivers. *New York: The New School: Center for New York City Affairs*, 2018.

Reuters. Lyft and motional launch robotaxi service in los angeles. *Reuters*, 2022. URL `https://www.reuters.com/business/autos-transportation/lyft-motional-launch-robotaxi-service-los-angeles-2022-11-17/`.

William P Rogerson. Contractual solutions to the hold-up problem. *The Review of Economic Studies*, 59 (4):777–793, 1992.

Andreas S Schulz and Nicols E Stier Moses. On the performance of user equilibrium in traffic networks. *Proceedings of SODA*, 2002.

Craig A Severance. A practical, affordable (and least business risk) plan to achieve "80% clean electricity" by 2035. *The Electricity Journal*, 24(6):8–26, 2011.

Sameepa Shetty. Uber's self-driving cars are a key to its path to profitability. *CNBC*, 2022. URL `https://www.cnbc.com/2020/01/28/ubers-self-driving-cars-are-a-key-to-its-path-to-profitability.html`.

Auyon Siddiq and Terry A Taylor. Ride-hailing platforms: Competition and autonomous vehicles. *Manufacturing & Service Operations Management*, 2022.

Brian Solomon. Stealth startup juno will take on uber by treating drivers better. *Forbes*, 2016. Accessed: 2022-08-31.

Jing-Sheng Song, Li Xiao, Hanqin Zhang, and Paul Zipkin. Optimal policies for a dual-sourcing inventory problem with endogenous stochastic lead times. *Operations Research*, 65(2):379–395, 2017.

Taylor Soper. The road ahead for autonomous vehicles: Aurora ceo chris urmson on the future of cars and trucks. *GeekWire*, 2022. URL `https://www.geekwire.com/2022/the-road-ahead-for-autonomous-vehicles-aurora-ceo-chris-urmson-on-the-future-of-cars-and-trucks/`.

Joseph J Spengler. Vertical integration and antitrust policy. *Journal of Political Economy*, 58(4):347–352, 1950.

Joaquin Taqla. Lyft's future is autonomous and driverless vehicles, according to co-founder. *Tech Times*, 2022. URL `https://www.techtimes.com/articles/276467/20220608/lyfts-future-autonomous-driverless-vehicles-according-co-founder.htm`.

Terry A. Taylor and Erica L. Plambeck. Simple relational contracts to motivate capacity investment: Price only vs. price and quantity. *Manufacturing & Service Operations Management*, 9(1):94–113, 2007.

Brian Tomlin. Capacity investments in supply chains: Sharing the gain rather than sharing the pain. *Manufacturing & Service Operations Management*, 5(4):317–333, 2003.

Sean Tucker. Average used car price held steady in june. Kelley Blue Book, July 2023. URL `https://www.kbb.com/car-news/average-used-car-price-held-steady-in-june/`. Accessed: 2023-11-03.

Oliver E. Williamson. *The Economic Institutions of Capitalism.* Free Press, 1985.

Linwei Xin and David A Goldberg. Asymptotic optimality of tailored base-surge policies in dual-sourcing inventory systems. *Management Science*, 64(1):437–452, 2018.

Appendix A: Modeling Extensions

A.1. Outside Option

In this section we consider a setting where the AV supplier has an option to divest its AV investment. Depending on whether the AV supplier is able to divest (1) after demand realizes, (2) after demand scenario realizes but before demand realizes, or (3) before demand scenario realizes, we analyze three cases under this setting. We refer to these cases by demand-dependent outside option, scenario-dependent outside option, and scenario-independent outside option, respectively. By extending Theorem 1, we show that in either of the cases AV underutilization could still lead to arbitrarily bad supply chain consequences.

A.1.1. Demand-dependent Outside Option We start with the most flexible outside option through which the supplier divests after demand realizes. Specifically, after demand realizes and the platform executes its dispatch decisions, the supplier can divest at a revenue of $c_O > 0$ each unit of AV that is unused by the platform. Since the supplier only acts on its divestment after the platform makes its decision, this outside option solely impacts the AV supplier and does not impact the decisions made by the platform. Mathematically, this means replacing (3) by

$$\max_K \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i} \left[c_P \mathcal{A}_i \left(D_i | c_P, K \right) + c_O \Big(K - \mathcal{A}_i \left(D_i | c_P, K \right) \Big) \right] - c_F K. \tag{14}$$

in the sequential game, with all else being the same.

Proposition 7 **(With Demand-dependent Outside Option Outcome Can Be Arbitrarily Bad).** *For any $M \in \mathbb{R}^+$, there exists $I \in \mathcal{I}$ with demand-dependent outside option $c_O > 0$ such that for any SPE s and any $c_P \geq 0$, $PR^s(I|c_P) \geq M$.*

A.1.2. Scenario-dependent Outside Option We next examine an outside option through which the supplier divests after demand scenario realizes but before demand realizes. Specifically, after demand scenario realizes, the supplier can divest its vehicles at a revenue of $c_O > 0$ per unit. These divested vehicles would then no longer be used by the platform. We show that there exists $c_O > 0$ such that the existence of a scenario-dependent outside option does not alter any decisions made by the supplier and the platform in $I(\delta_1)$ constructed for Theorem 1, thereby extending the theorem to this setting.

Proposition 8 **(With Scenario-dependent Outside Option Outcome Can Be Arbitrarily Bad).** *For any $M \in \mathbb{R}^+$, there exists $I \in \mathcal{I}$ with scenario-dependent outside option $c_O > 0$ such that for any SPE s and any $c_P \geq 0$, $PR^s(I|c_P) \geq M$.*

A.1.3. Scenario-independent Outside Option We conclude by briefly commenting on the setting with scenario-independent outside option, in which the supplier obtains $c_P > 0$ per unit of AV it divests before demand scenario realizes. Since in this setting the AV supplier divests without obtaining any information on demand scenarios, the profit of divestment is trivially $c_O - c_F$, and the supplier would not divest given any $c_O \in (0, c_F)$. Thus, by taking any $c_O \in (0, c_F)$, a scenario-independent outside option would not alter any decisions made by the supplier and the platform in $I(\delta_1)$ constructed for Theorem 1, thereby extending the theorem to this setting.

A.2. Endogenous IC Wages

In this section we consider an extension of the model that allows for one additional degree of freedom for the platform in optimizing IC payment c_I. Specifically, to model to dynamic pricing that is often in place in ride-hailing market today, we assume that the platform is free to choose c_I^i in each demand scenario i, with potentially different values of c_I^i being chosen in different scenarios. Mathematically, it is equivalent to replacing stage 3 of the sequential game by

$$\begin{aligned} \max_{\vec{\mathbf{y}},\mathcal{A},\mathcal{H},c_I} & \sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[(1-c_{AV}-c_P)\mathcal{A}_i\left(D_i\right)+(1-c_I^i)\mathcal{H}_i\left(D_i\right)\right] \\ \text{s.t. } & (5),(6),(7), ty_i = c_I^i \mathbb{E}_{D_i\sim F_i}\left[\mathcal{H}_i\left(D_i\right)\right], c_I^i \geq 0, \forall i. \end{aligned} \tag{15}$$

with all else being the same. As in the main body of the paper, we consider the solution concept of SPE, and with a slight abuse of notation we denote the outcome of the endogenous IC wage model by the superscript s within this subsection. The supply chain profit of the endogenous IC wage model is denoted by $V^{ED}(I)$ for an instance I. We start by providing the optimality condition for $\vec{\mathbf{y}}^s$ in this model.

Lemma 1. *For any $I \in \mathcal{I}$, given any $c_P \in [0, 1-c_{AV})$ and $K \geq 0$, the smallest optimal solution for $\vec{\mathbf{y}}$ in (15) satisfies*

$$y_i^s(c_P, K) := \min\left\{y_i \middle| \mathbb{P}\left[D_i > K + y_i\right] + \mathbb{P}\left[y_i < D_i \leq K + y_i\right]\cdot(c_{AV}+c_P) \leq t\right\}, \forall i.$$

Moreover, $\mathcal{A}_i^s(D_i|c_P, K) = \min\left\{(D_i - y_i^s(c_P, K))^+, K\right\}, \forall i.$

For any $c_P \in [0, 1-c_{AV})$ and $K \geq 0$, Lemma 1 guarantees that $y_i^s(c_P, K)$ is non-decreasing as c_P increases, and consequently $\mathcal{A}_i^s(D_i|c_P, K)$ is non-increasing as c_P increases. Thus, Lemma 1 implies that in the endogenous IC wage model, in line with the spirit of Proposition 2, AV underutilization is strengthened as c_P increases. In particular, since $c_P = 0$ in a centralized solution, an SPE with $c_P > 0$ is subject to the risk of more significant AV underutilization.

We next present an analogue of Proposition 1 that shows indeed AV underutilization may occur in both a centralized solution and an SPE even with endogenous IC wages.

Proposition 9. *(i) There exists $I \in \mathcal{I}$ with centralized solution $(K^\star, \vec{\mathbf{y}}^\star, \mathcal{A}^\star, \mathcal{H}^\star)$ such that*

$$y_i^\star > 0, \mathcal{H}_i^\star\left(D_i\right) = \min\left\{D_i, y_i^\star\right\} \text{ and } \mathcal{A}_i^\star\left(D_i\right) < \min\left\{D_i, K^\star\right\} \text{ for some } i \text{ and } D_i.$$

(ii) There exists $I \in \mathcal{I}$ with SPE $(K^s, \vec{\mathbf{y}}^s, \mathcal{A}^s, \mathcal{H}^s, c_P^s)$, where $c_{AV} < c_{AV} + c_P^s < c_I$, such that

$$y_i^s > 0, \mathcal{H}_i^s\left(D_i\right) = \min\left\{D_i, y_i^s\right\} \text{ and } \mathcal{A}_i^s\left(D_i\right) < \min\left\{D_i, K^s\right\} \text{ for some } i \text{ and } D_i.$$

Given the lack of closed-form solution to this model, proving that the profit ratio can be arbitrarily large, as we have shown for the exogenous IC wage model in Theorem 1, can be challenging. However, we show that AV underutilization can still lead to significant supply chain efficiency loss in this model by constructing an explicit example. In particular, we prove a result that shows, no matter how c_P is set in the first stage, the resulting supply chain profit $V^{ED}\left(I|c_P\right)$ can be bad relative to $V^\star\left(I\right)$.

Proposition 10 **(Outcome of the Endogenous IC Wage Model Can Be Bad)**. *There exists $I \in \mathcal{I}$ such that for any $c_P \geq 0$, $\frac{V^\star(I)}{V^{ED}(I|c_P)} \geq 1.49$.*

A.3. Heterogeneous IC Reservation Earnings

In this section we consider an extension of the model that allow ICs to be heterogeneous in their reservation earnings. Specifically, we assume that there exists a function $S(c_e)$ that maps $c_e \geq 0$, the expected total earning for all ICs in the market, to a scaled monetary value in $\mathbb{R}_{\geq 0}$. Then, we replace the IC participation constraint in (8) by

$$S\left(\mathbb{E}_{D_i \sim F_i}\left[\mathcal{H}_i\left(D_i\right)\right] \cdot c_I\right)/t = y_i \,\forall i. \tag{16}$$

We require that (1) $S(c_e)$ is monotonically increasing with respect to $c_e \geq 0$, and that (2) $S(0) = 0$ since ICs should not drive for free.

The function $S(c_e)$ captures the distribution of the heterogeneous reservation earnings required by ICs: when the average reservation earning required by potential supply of ICs exceeds t, we have $S(c_e) < c_e$; when the average reservation earning required is below t, we have $S(c_e) > c_e$. In the special case where $S(c_e) = c_e, \forall c_e \geq 0$, we recover (8), i.e., the IC utilization constraint in the homogeneous model.

With the homogeneous setting being a special case to the heterogeneous setting, we immediately know that Theorem 1 and Theorem 3 still hold as the worst-case performances under this more general setting. To see that Theorem 2 also holds in this more general setting, we show that given $K^\pi = K^\star, \underline{\mathcal{A}}^\pi = \mathcal{A}^\star$ and any $c_P^\pi > 0$ it is not in the interest of the platform to deviate from $\mathcal{H}^\star$ and $\mathcal{A}^\star$. We prove by contradiction and assume that there exists some feasible $\mathcal{H}_i\left(D_i\right) \neq \mathcal{H}_i^\star(D_i)$ and $\mathcal{A}_i\left(D_i\right) \geq \mathcal{A}_i^\star(D_i)$ for some i such that

$$\mathbb{E}_{D_i \sim F_i}\left[(1 - c_{AV} - c_P^\pi)\mathcal{A}_i\left(D_i\right) + (1 - c_I)\mathcal{H}_i\left(D_i\right)\right] > \mathbb{E}_{D_i \sim F_i}\left[(1 - c_{AV} - c_P^\pi)\mathcal{A}_i^\star(D_i) + (1 - c_I)\mathcal{H}_i^\star(D_i)\right].$$

Then, from $\mathcal{A}_i\left(D_i\right) \geq \mathcal{A}_i^\star(D_i)$ we must have

$$\mathbb{E}_{D_i \sim F_i}\left[(1 - c_{AV})\mathcal{A}_i\left(D_i\right) + (1 - c_I)\mathcal{H}_i\left(D_i\right)\right] > \mathbb{E}_{D_i \sim F_i}\left[(1 - c_{AV})\mathcal{A}_i^\star(D_i) + (1 - c_I)\mathcal{H}_i^\star(D_i)\right],$$

which contracts the optimality of $\mathcal{A}_i^\star(D_i)$ and $\mathcal{H}_i^\star(D_i)$ in scenario i of a centralized solution. The rest of the arguments follow from those in the proof of Theorem 2.

Finally, Theorem 4 trivially holds because, if a centralized solution $(K^\star, \vec{\mathbf{y}}^\star, \mathcal{A}^\star)$ is feasible, then (1) the "AV only" solution $K = K^\star, \vec{\mathbf{y}} = \vec{0}, \mathcal{A}\left(D_i\right) = \min\left\{D_i, K^\star\right\}, \forall i$, and (2) the "IC only" solution $K = 0, \vec{\mathbf{y}} = \vec{\mathbf{y}}^\star, \mathcal{A}\left(D_i\right) = 0, \forall i$ are both still feasible, with at least one of the solution leading to supply chain profit $\geq \frac{1}{2}V^\star(I)$. The rest of the arguments follow from the original proof of Theorem 4. Thus, all main results in the paper hold in the setting with heterogeneous IC reservation earnings.

Appendix B: The Advanced-Stochasticity-Only Model

In this appendix, we analyze a special case of our model that only has the *advanced stochasticity*, i.e., stochasticity over the different scenarios. In particular, in such a model the demand is deterministic within each scenario.

Definition 9. An instance I is *advanced-stochasticity-only* if its demand is deterministic in all scenarios, i.e., $\mathbb{P}[D_i = d_i] = 1$ for some value $d_i \geq 0, \forall i$. We denote the set of advanced-stochasticity-only instances by $\mathcal{I}_P$.

In the advanced-stochasticity-only model, ICs know the precise realized demand when making participation decisions. This implies that the platform can contract the exact number of ICs it needs without having to worry about prioritizing them. In this case, the platform uses as many ICs as needed and it prioritizes its supply use solely based on the variable costs, i.e., the platform fully prioritizes AVs when their variable cost is lower, and vice versa. The following proposition formalizes this intuition.

PROPOSITION 11. *For any instance $I \in \mathcal{I}_P$, any given $c_P \geq 0$ and K, the following statements hold:*

(i) It is optimal for the platform to fully prioritize AVs if $c_{AV} + c_P \leq c_I$;

(ii) It is optimal for the platform to fully prioritize ICs if $c_{AV} + c_P > c_I$.

Appendix C: Alternative Contracts

C.1. The Revenue-Sharing Contract

In this appendix, we consider a revenue-sharing contract which enables the platform to pay the supplier for the demand served by both AVs and ICs. Through such a contract the AV supplier becomes indifferent to the underutilization of AVs and the supply chain misalignment can thus be overcome. Such a contract is unlikely to be adopted in practice, however, as it requires the platform to pay the AV supplier for demand that is not actually fulfilled by AVs.

DEFINITION 10. The set of revenue-sharing contracts, Π^S, contains all contracts π that specify the parameters c_P^π, c_R^π and K^π in the following way:

(i) The platform must pay the AV supplier c_P^π for each unit of demand served by AVs and c_R^π for each unit of demand served by ICs;

(ii) The AV supplier must invest in K^π units of AVs.

We show in Proposition 12 the ability of the revenue-sharing contract to align the supply chain. Notice, in particular, that the revenue-sharing contract does not need to specify the dispatch of AVs in order to align the supply chain. Thus, its enforcement relies on observing the platform's dispatch of AVs and ICs, but not the scenarios or the realized demand.

PROPOSITION 12. *For every instance $I \in \mathcal{I}$, there exists some contract $\pi \in \Pi^S$ that leads to $PR^\pi(I) = 1$.*

C.2. Dispatch-Based Contracts

In this section we provide corollaries to Theorem 2 and Theorem 4 that show that the original results still apply if we rely on the contractibility of IC dispatch rather than contractibility on realized demand. Contracting on realized rides (such as AV and IC dispatches) is likely far more feasible in practice than contracting on realized demand, which might involve ride requests that were not fulfilled. Specifically, we start by providing a corollary to Theorem 2 that relies on specifying the dispatch of AVs based on the dispatch of ICs (rather than realized demand), and then provide a corollary to Theorem 4 that only relies on specifying the dispatch of ICs.

To establish the corollary to Theorem 2, we denote by the function $\widetilde{\mathcal{A}}_i(\mathcal{H}_i)$ the units of demand served by AVs in scenario i given $\mathcal{H}_i$ units of demand served by ICs. Then in Definition 11 we build a new set of contracts Π^C that specify $\widetilde{\mathcal{A}}^\pi$ rather than $\underline{\mathcal{A}}^\pi$ but are otherwise equivalent to their counterparts in Π^U.

Definition 11. *The set Π^C contains all contracts π that specify the parameters c_P^π, K^π and $\widetilde{\mathcal{A}}^\pi$ in the following way:*

(i) *The platform must pay the AV supplier c_P^π for each unit of demand served by AVs and must dispatch at least $\widetilde{\mathcal{A}}^\pi$ units of AVs;*

(ii) *The AV supplier must invest in K^π units of AVs.*

Definition 11 allows us to establish a corollary to Theorem 2 that relies on the contractibility of the IC dispatch rather than the realized demand:

Corollary 1. *For every instance $I \in \mathcal{I}$, there exists some contract $\pi \in \Pi^C$ that leads to $PR^\pi(I) = 1$.*

For the corollary to Theorem 4, we similarly consider an alternative definition of full prioritization contracts that imposes restrictions only on the IC dispatch:

Definition 12. *The set Π^H contains all contracts π that specify the parameters c_S^π, c_P^π, K^π and $\mathcal{H}^\pi$ in the following way:*

(i) *The platform must pay the AV supplier c_S^π upfront, c_P^π for each unit of demand served by AVs, and must dispatch no ICs, i.e., $\mathcal{H}^\pi = 0$;*

(ii) *The AV supplier must invest in K^π units of AVs.*

Definition 12 allows us to establish a corollary to Theorem 4 that relies on the contractibility of the dispatch of ICs rather than the realized demand:

Corollary 2. *For any $I \in \mathcal{I}$, at least one of the following two statements is true:*

- *There exists a $\pi \in \Pi^H$ that guarantees a profit ratio $PR^\pi(I) \le 2$;*
- *For any no-contract equilibrium s, the profit ratio satisfies $PR^s(I) \le 2$.*

Appendix D: Proofs

D.1. Proofs of Results in Section 3

The proofs in this section rely on specifying the number of ICs that the platform uses given c_P and K. We provide this solution in Lemma 2 when $0 \le c_P \le c_I - c_{AV}$ and in Lemma 3 when $c_P > c_I - c_{AV}$. The proofs of these results are deferred to Appendix D.3.

We use $\bar{F}_i$ to denote the tail distribution of D_i and $\bar{F}_i^{-1}$ for the inverse of the tail distribution.

Lemma 2. *For any $I \in \mathcal{I}$, given any $c_P \in [0, c_I - c_{AV}]$ and $K \ge 0$, we define*

$$\bar{y}_i := \bar{F}_i^{-1}\left(\frac{t(c_I - c_{AV} - c_P)}{c_I(1 - c_{AV} - c_P)}\right) - K, \forall i. \tag{17}$$

$$y_i^{ub} := \max\left\{ y_i \middle| c_I \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, y_i\}\right] = t y_i \right\}, \forall i \tag{18}$$

$$y_i^{lb} := \max\left\{ y_i \middle| c_I \mathbb{E}_{D_i \sim F_i}\left[\min\left\{(D_i - K)^+, y_i\right\}\right] = t y_i \right\}, \forall i. \tag{19}$$

Then, given c_P and K, the smallest optimal solution for y_i in Eq. (4) is

$$y_i^s(c_P, K) = \begin{cases} y_i^{lb} & \text{if } \bar{y}_i < y_i^{lb} \\ \bar{y}_i & \text{if } y_i^{lb} \le \bar{y}_i \le y_i^{ub} \\ y_i^{ub} & \text{if } \bar{y}_i > y_i^{ub}. \end{cases} \tag{20}$$

Moreover, the platform's maximum achievable profit in Eq. (4) *given* c_P *and* K *is non-decreasing in* y_i *when* $y_i^{lb} \le y_i \le y_i^s(c_P, K)$ *and non-increasing in* y_i *when* $y_i^s(c_P, K) \le y_i \le y_i^{ub}$.

Lemma 3. *For any* $I \in \mathcal{I}$, *given* $c_P > c_I - c_{AV}$ *and* K, *the optimal solution for* $\vec{\mathbf{y}}$ *in Eq.* (4) *is*

$$y_i^s(c_P, K) = y_i^{ub} := \max\left\{ y_i \middle| c_I \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, y_i\}\right] = t y_i \right\}, \forall i. \tag{21}$$

D.1.1. Proof of Proposition 1

Proof. We prove Proposition 1 constructively using an instance I with $\vec{\alpha} = \left[3/4, 1/4\right], c_I = 1/4, t = 1/4, c_{AV} = 1/8$ and $c_F = 1/30$. Moreover, the demand distributions in scenario 1 and 2 are respectively given by $D_1 = 10$ and

$$D_2 = \begin{cases} 10 & \text{w.p. } 3/4 \\ 20 & \text{w.p. } 1/4. \end{cases}$$

Step 1: find the SPE

We start by showing that in the unique subgame perfect equilibrium the platform sets $c_P^s = 8/195$, the AV supplier sets $K^s = 10$, and the platform set $\vec{\mathbf{y}}^s = [0, 10]$. Moreover, the platform's dispatch policies for AVs are $\mathcal{A}_1^s(D_1) = 10$ and

$$\mathcal{A}_2^s(D_2) = \begin{cases} 0 & \text{when } D_2 = 10 \\ 10 & \text{when } D_2 = 20. \end{cases}$$

Observe, in particular, that the platform dispatches no AVs in scenario 2 when $D_2 = 10$ despite $c_{AV} + c_P^s < c_I$ and $K^s > 0$. In contrast, the platform's dispatch policies for ICs are $\mathcal{H}_1^s(D_1) = 0$ and

$$\mathcal{H}_2^s(D_2) = \begin{cases} 10 & \text{when } D_2 = 10 \\ 10 & \text{when } D_2 = 20. \end{cases}$$

That is, $y_2^s > 0$ and $\mathcal{H}_2^s(D_2) = \min\{D_2, y_2^s\}$, so ICs are fully prioritized in scenario 2. Thus, this construction satisfies Proposition 1 (i) and (iii).

To verify the subgame perfect equilibrium solution above, we consider c_P within four sets of possible values, and prove that $c_P = 8/195$ is indeed the equilibrium outcome. We start by observing that when $c_P < c_F = 1/30$ the AV supplier cannot break even with any $K > 0$. Thus, $K^s(c_P) = 0$ in this case and the platform can only serve the demand with ICs, which leads to

$$y_1^s\left(c_P, K^s(c_P)\right) = \min\left\{y_1^{ub}, \bar{y}_1\right\} = 10 \text{ and } y_2^s\left(c_P, K^s(c_P)\right) = \min\left\{y_1^{ub}, \bar{y}_1\right\} = 10$$

by Lemma 2. Thus, the profit for the platform in this case is $(1 - c_I)10 = 7.5$.

Next, when $c_F \le c_P < 8/195$, we find

$$\bar{F}_1^{-1}\left(\frac{t(c_I - c_{AV} - c_P)}{c_I(1 - c_{AV} - c_P)}\right) = 10 \text{ and } \bar{F}_2^{-1}\left(\frac{t(c_I - c_{AV} - c_P)}{c_I(1 - c_{AV} - c_P)}\right) = 20.$$

Now, for $0 < K \le 10$ we have $\bar{y}_2 = 20 - K \ge 10$, so that $y_2^s(c_P, K) = y_2^{ub} = 10$ by Eq. (20) and $\mathcal{H}_2^s\left(D_2 | c_P, K\right) = 10$ by the equilibrium condition in Eq. (8). Then, the marginal profit for the AV supplier to own a unit of AV is at most

$$c_P\left(\alpha_1 + \alpha_2 \mathbb{P}\left[D_2 = 20\right]\right) - c_F < 0$$

because AVs are not used when $D_2 = 10$. On the other hand, if $K > 10$ we find that the marginal profit for the AV supplier to own more than 10 units of AV is at most $c_P\alpha_2 - c_F < 0$ because the highest possible demand in scenario 1 is 10. Thus, with $c_F \le c_P < 8/195$ we again find $K^s(c_P) = 0$ and the profit for the platform is 7.5.

When $8/195 \le c_P \le c_I - c_{AV}$, we again find that when $K > 10$ the marginal profit for the AV supplier to own more than 10 units of AV is at most $c_P\alpha_2 - c_F < 0$. Then, when $0 < K \le 10$ we solve

$$y_1^s(c_P, K) = y_1^{lb} = \bar{y}_1 = 10 - K \text{ and } y_2^s(c_P, K) = y_2^{ub} = 10 < \bar{y}_2 = 20 - K,$$

which implies that $\mathcal{A}_1^s\left(D_1|c_P, K\right) = K$ and $\mathcal{A}_2^s\left(D_2|c_P, K\right) = K$ when $D_2 = 20$. Thus, with $c_P \ge 8/195$, the marginal profit for the AV supplier to own a unit of AV becomes

$$c_P\left(\alpha_1 + \alpha_2\mathbb{P}\left[D_2 = 20\right]\right) - c_F \ge 0, \tag{22}$$

and the supplier responds by setting $K^s(c_P) = 10$. Since by construction 8/195 is the minimum c_P that leads to an equality in Eq. (22) (and consequently the minimum c_P that leads to $K^s(c_P) = 10$), it is optimal for the platform to set $c_P = 8/195$ and $\vec{\mathbf{y}}^s\left(c_P, K^s(c_P)\right) = [0, 10]$. By Eq. (2) and Eq. (8), this solution yields a profit of

$$(1 - c_{AV} - c_P)\left(\alpha_1 + \alpha_2\mathbb{P}\left[D_2 = 20\right]\right)K^s(c_P) + (1 - c_I)\,\alpha_2 y_2^s\left(c_P, K^s(c_P)\right) \approx 8.65$$

for the platform. In particular, this profit is higher than the solution that involves only ICs.

Finally, when $c_P > c_I - c_{AV}$, the variable cost of AVs becomes higher than that of the ICs, so the platform uses as many ICs as possible by setting

$$y_1^s(c_P, K) = y_1^{ub} = 10 \text{ and } y_2^s(c_P, K) = y_2^{ub} = 10$$

for any K. Since AVs are only used when ICs are insufficient for covering all demand, i.e., when $D_2 = 20$, the marginal profit for owning a unit of AV is at most $\alpha_2\mathbb{P}\left[D_2 = 20\right] - c_F < 0$. Since the supplier again cannot break even with any $K > 0$, the platform adopts the IC-only solution $\vec{\mathbf{y}}^s\left(c_P, K^s(c_P) = 0\right) = [10, 10]$, which we know is not optimal. We thus conclude that $c_P = 8/195$ is indeed the equilibrium outcome.

Step 2: find the centralized solution

We next argue that the unique centralized solution is to set $K^\star = 10$: for $0 \le K \le 10$ the marginal profit for owning a unit of AV is at least $(c_I - c_{AV})\alpha_1 - c_F > 0$, which is the increase in profit for the platform to use AVs to serve demand that could otherwise be served with ICs. On the other hand, for $K > 10$, since $y_2^{ub} = 10$ and $y_2^{lb} = 0$ we have $y_2^s(c_P = 0, K) = \bar{y}_2 = 20 - K$. Then, the marginal profit for owning an additional unit of AV is at most $(c_I - c_{AV})\alpha_2 - c_F < 0$ since the demand could otherwise be served with ICs. Given $K^\star = 10$, in the centralized problem the platform incurs no cost of c_P, so by plugging $c_P = 0$ and $K^\star$ into Lemma 2 we find

$$\bar{y}_1 = \bar{F}_1^{-1}\left(\frac{t(c_I - c_{AV})}{c_I(1 - c_{AV})}\right) - 10 = 0 \text{ and } \bar{y}_2 = \bar{F}_2^{-1}\left(\frac{t(c_I - c_{AV})}{c_I(1 - c_{AV})}\right) - 10 = 10.$$

By construction of the demand distributions we also have $y_i^{ub} = 10$ and $y_i^{lb} = 10 - K^\star = 0\;\forall i$, so $\vec{\mathbf{y}}^\star = [0, 10]$. Finally, the solutions for $K^\star$ and $\vec{\mathbf{y}}^\star$ uniquely lead to $\mathcal{H}_1^\star(D_1) = 0$ and

$$\mathcal{H}_2^\star(D_2) = \begin{cases} 10 & \text{when } D_2 = 10 \\ 10 & \text{when } D_2 = 20. \end{cases}$$

That is, $y_2^\star > 0$ and $\mathcal{H}_2^\star(D_i) = \min\{D_i, y_2^\star\}$. This also implies that $\mathcal{A}_2^\star(10) \le 10 - \mathcal{H}_2^\star(10) = 0 < 10 = \min\{D_2, K^\star\}$ when $D_2 = 10$, so Proposition 1 (ii) also holds.

□

D.1.2. Proof of Proposition 2

Proof. We consider an equilibrium s that always breaks ties in favor of the AV supplier. For any instance $I \in \mathcal{I}$ and fixed K, we consider the cases where $0 < c_P' \le c_I - c_{AV}$ and $c_P' > c_I - c_{AV}$ separately. When $0 < c_P \le c_P' < c_I - c_{AV}$, we claim that the smallest optimal solution for y_i constructed in Lemma 2 is also the optimal solution that is most favored by the AV supplier.

Claim 1. *Given any $c_P \in (0, c_I - c_{AV}]$ and K, among the optimal solutions for $(\vec{\mathbf{y}}, \mathcal{A}, \mathcal{H})$ in Eq. (4), the ones that maximize Eq. (3) contain the smallest optimal y_i in each scenario i.*

Then, we observe that

$$\bar{F}_i^{-1}\left(\frac{t(c_I - c_{AV} - c_P)}{c_I(1 - c_{AV} - c_P)}\right) - K \le \bar{F}_i^{-1}\left(\frac{t(c_I - c_{AV} - c_P')}{c_I(1 - c_{AV} - c_P')}\right) - K.$$

Since we also know that the constructions of y_i^{ub} and y_i^{lb} depend only on I and K, by Lemma 2 we obtain $y_i^s(c_P, K) \le y_i^s(c_P', K), \forall i$.

When $c_P' > c_I - c_{AV}$, by Lemma 3 we have the unique optimal solution $y_i^s(c_P', K) = y_i^{ub}, \forall i$. Thus, for any c_P we have $y_i^s(c_P, K) \le y_i^{ub} = y_i^s(c_P', K), \forall i$. Since the above holds in both cases, by the equality constraint in Eq. (8) we obtain

$$\mathbb{E}_{D_i \sim F_i}\left[\mathcal{H}_i^s(D_i | c_P, K)\right] \le \mathbb{E}_{D_i \sim F_i}\left[\mathcal{H}_i^s(D_i | c_P', K)\right], \forall i.$$

For the analyses of $\mathcal{A}_i^s(D_i | c_P', K)$, notice that we may assume without loss of generality that $c_P' < 1 - c_{AV}$, because otherwise the platform uses no AVs given c_P' and we trivially have

$$\mathbb{E}_{D_i \sim F_i}\left[\mathcal{A}_i^s(D_i | c_P, K)\right] \ge \mathbb{E}_{D_i \sim F_i}\left[\mathcal{A}_i^s(D_i | c_P', K)\right] = 0, \forall i.$$

When $c_P' < 1 - c_{AV}$, given $\mathcal{H}_i^s(D_i | c_P', K)$ we observe that we must have

$$\mathcal{A}_i^s(D_i | c_P', K) = \min\left\{K, D_i - \mathcal{H}_i^s(D_i | c_P', K)\right\}, \forall i, D_i \tag{23}$$

because when $\mathcal{A}_i^s(D_i | c_P', K)$ is smaller we can increase objective in (4) without violating any feasibility constraint, and $\mathcal{A}_i^s(D_i | c_P', K)$ also cannot be larger by constraint (5) and (7). Similarly, we find that

$$\mathcal{A}_i^s(D_i | c_P, K) = \min\left\{K, D_i - \mathcal{H}_i^s(D_i | c_P, K)\right\}, \forall i, D_i. \tag{24}$$

Since $\mathbb{E}_{D_i \sim F_i}\left[\mathcal{H}_i^s(D_i | c_P, K)\right] \le \mathbb{E}_{D_i \sim F_i}\left[\mathcal{H}_i^s(D_i | c_P', K)\right]$ $\forall i$, we can always fix $\mathcal{H}_i^s(D_i | c_P', K)$ and find $\mathcal{H}_i^s(D_i | c_P, K)$ such that

$$\mathcal{H}_i^s(D_i | c_P, K) \le \mathcal{H}_i^s(D_i | c_P', K), \forall i, D_i.$$

Thus, taking expectation over D_i in (23) and (24) we find

$$\begin{aligned}\mathbb{E}_{D_i \sim F_i}\left[\mathcal{A}_i^s(D_i | c_P, K)\right] &= \mathbb{E}_{D_i \sim F_i}\left[\min\left\{K, D_i - \mathcal{H}_i^s(D_i | c_P, K)\right\}\right] \\ &\ge \mathbb{E}_{D_i \sim F_i}\left[\min\left\{K, D_i - \mathcal{H}_i^s(D_i | c_P', K)\right\}\right] = \mathbb{E}_{D_i \sim F_i}\left[\mathcal{A}_i^s(D_i | c_P', K)\right], \forall i.\end{aligned}$$

Finally, we provide a constructive proof that AV underutilization can be arbitrarily strong. Notice that we may assume without loss of generality that $\epsilon \in (0, 1/4)$ because the construction for a small ϵ value immediately implies the result for a larger choice of ϵ. Fix $\epsilon \in (0, 1/4)$, we know that we can find ϵ_1, ϵ_2 such that $0 < \epsilon_1 < \epsilon_2 < \epsilon$. Then, we construct an instance I with $\vec{\alpha} = [1], c_I = t = 1/4$ and $c_{AV} = \frac{1/4-\epsilon_1}{1-\epsilon_1}$. We take $c_P = 0$ and $K = 10$. Notice that this construction ensures $c_{AV} + c_P = c_{AV} < c_I, \forall \epsilon \in (0, 1/4)$. We assume that the demand distribution in scenario 1, which is also the only scenario, is

$$D_1 = \begin{cases} 10 & \text{w.p. } 1-\epsilon_2 \\ 20 & \text{w.p. } \epsilon_2. \end{cases}$$

By plugging $c_P = 0$ and $K = 10$ into Lemma 2 we find

$$\bar{y}_1 = \bar{F}_1^{-1}\left(\frac{t(c_I - c_{AV})}{c_I(1-c_{AV})}\right) - 10 = \bar{F}_1^{-1}(\epsilon_1) - 10 = 10.$$

By construction of the demand distribution we also have $y_1^{ub} = 10$ and $y_1^{lb} = 10 - K = 0$, so $y_1^s(c_P, K) = 10$. Since $t/c_I = 1$, ICs need to be fully utilized and $\mathcal{H}_1^s(D_1|c_P, K) = 10, \forall D_1$. Thus, $\mathcal{A}_1^s\left(10|c_P, K\right) \leq 10 - \mathcal{H}_1^s(10|c_P, K) = 0$ and $\mathcal{A}_1^s\left(20|c_P, K\right) \leq 20 - \mathcal{H}_1^s(20|c_P, K) = 10$. This implies that

$$\frac{\sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[\mathcal{A}_i^s(D_i|c_P, K)\right]}{\sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, K\}\right]} \leq \frac{10 \cdot \epsilon_2}{10} = \epsilon_2 < \epsilon.$$

This completes the proof.

□

D.1.3. Proof of Theorem 1

Proof. To show that the profit ratio is unbounded, we construct a sequence of instances that depend on the parameter $\delta_1 \in (0, 0.01]$. With a slight abuse of notation we use $I(\delta_1)$ to highlight the dependency of the instance on δ_1. Then, we show that

$$\lim_{\delta_1 \to 0^+} \sup_{c_P} \mathrm{PR}^s(I(\delta_1)|c_P) = \infty,$$

which immediately implies the theorem statement.

For any $\delta_1 \in (0, 0.01]$, let $\delta_2 = \sqrt{0.049875\delta_1 + 0.7375\delta_1^2} - \frac{1}{2}\delta_1$. Then, we construct an instance I with

$$\vec{\alpha} = [0.65, 0.35], c_I = 1 - \delta_1, c_{AV} = 0.81, c_F = 0.19 - \delta_2 \text{ and } t = \frac{2(1-\delta_1)}{5 - 3(\delta_2 - \delta_1)/\delta_2}.$$

Moreover, the demand distributions in scenario 1 and 2 are respectively given by $D_1 = 10$ and

$$D_2 = \begin{cases} 10 & \text{w.p. } 1-p \\ 25 & \text{w.p. } p \end{cases}$$

where $p = \frac{t}{1-\delta_1}\left(1 - \frac{\delta_1}{\delta_2}\right)$. In particular, we verify that the instances are well-defined by first observing that $\delta_1 < \delta_2$ holds when $\delta_1 \in (0, 0.01]$. Then, $1 - c_{AV} - c_F = \delta_2 > \delta_1 = 1 - c_I$, i.e., $c_{AV} < c_I$. Moreover,

$$t = \frac{2(1-\delta_1)}{5 - 3(\delta_2 - \delta_1)/\delta_2} = (1-\delta_1)\frac{2}{5 - 3\left(1 - \frac{\delta_1}{\delta_2}\right)} \leq 1 - \delta_1 = c_I$$

and $p = \frac{t}{1-\delta_1}\left(1 - \frac{\delta_1}{\delta_2}\right) < \frac{t}{c_I} < 1$.

In order to show that $\lim_{\delta_1 \to 0^+} \sup_{c_P} \mathrm{PR}^s(I(\delta_1)|c_P) = \infty$, we first provide a lower bound on the profit of a centralized solution, i.e., a lower bound on $V^\star\left(I(\delta_1)\right)$. Then, we show an upper bound on the profit of an SPE given any c_P, i.e., $\sup_{c_P} V^s\left(I(\delta_1)|c_P\right)$. Finally, we show that the profit ratio is unbounded as δ_1 approaches 0 from the right.

Step 1: construct a lower bound on $V^\star\left(I\left(\delta_1\right)\right)$

Notice that a feasible solution to the centralized problem is given by $K = 10, \vec{\mathbf{y}} = [0,0]$, and this leads to a supply chain profit of $(1-c_{AV})\,10 - c_F 10$. Since this is a lower bound on the profit achievable by a centralized solution, we have

$$V^\star(I(\delta_1)) \geq (1-c_{AV})\,10 - c_F 10 = (0.19 - c_F)\,10 = \delta_2 10. \tag{25}$$

Step 2: find $\sup_{c_P} V^s\left(I\left(\delta_1\right)|c_P\right)$

To find c_P that leads to the highest possible $V^s\left(I\left(\delta_1\right)|c_P\right)$, we consider 3 cases: (i) $c_P < c_F$; (ii) $c_F \leq c_P \leq c_I - c_{AV}$; and (iii) $c_P > c_I - c_{AV}$.

In case (i), the marginal profit for the AV supplier to own a unit of AV is at most $c_P - c_F < 0$, so $K^s\left(c_P\right) = 0$. For the platform, then, from Lemma 2 we find

$$y_1^s\left(c_P, K^s(c_P)\right) = y_1^s\left(c_P, 0\right) = y_1^{ub} = y_1^{lb} = 10 \text{ and } y_2^s\left(c_P, K^s(c_P)\right) = y_2^s\left(c_P, 0\right) = y_2^{ub} = y_2^{lb} = 25$$

by the construction of t. Thus, the profit for the platform in this case is $(1-c_I)\left(10+\alpha_2 p 15\right) = \delta_1\left(10+5.25p\right)$. This is also the profit of the supply chain since the AV supplier makes no profit. That is,

$$V^s\left(I\left(\delta_1\right)|c_P\right) = \delta_1\left(10+5.25p\right), \forall c_P < c_F.$$

In case (ii), we find

$$\bar{F}_2^{-1}\left(\frac{t(c_I - c_{AV} - c_P)}{c_I(1-c_{AV}-c_P)}\right) \geq \bar{F}_2^{-1}\left(\frac{t(\delta_2-\delta_1)}{c_I(\delta_2)}\right) = 25$$

for any $c_P \geq c_F$ by the construction of p. Since the highest possible realized demand is 25, we trivially have $K^s(c_P) \in [0,25]$. Then, by Lemma 2 we find, for any $K \in [0,25]$, $y_2^{ub} = 25, \bar{y}_2 = 25 - K$. We also claim that $y_2^{lb} \leq 25 - K$ and defer the proof of Claim 2 to Appendix D.3.5.

Claim 2. *Given $I\left(\delta_1\right)$ constructed above, we have $y_2^{lb} \leq 25 - K$ for any $K \in [0,25]$.*

Thus, we know that $y_2^s\left(c_P, K\right) = \bar{y}_2 = 25 - K, \forall c_P \in \left[c_F, c_I - c_{AV}\right]$. We also make the following claim on the platform's dispatch policy given c_P and K and defer its proof to Appendix D.3.6.

Claim 3. *Given $I\left(\delta_1\right)$ constructed above, for any $c_P \in \left[c_F, c_I - c_{AV}\right]$ and $K \in [0,25]$, the platform's dispatch policy satisfies*

$$\mathbb{E}_{D_2 \sim F_2}\left[\mathcal{A}_2^s\left(D_2|c_P, K\right)\right] = \mathbb{E}_{D_2 \sim F_2}\left[\min\left\{D_2, K + y_2^s\left(c_P, K\right)\right\}\right] - \frac{t}{c_I} y_2^s\left(c_P, K\right).$$

In particular, by plugging $y_2^s\left(c_P, K\right) = 25 - K$ into Claim 3 we find

$$\mathbb{E}_{D_2 \sim F_2}\left[\mathcal{A}_2^s\left(D_2|c_P, K\right)\right] = \mathbb{E}\left[D_2\right] - \frac{t}{c_I}(25-K) = \mathbb{E}\left[D_2\right] + \frac{t}{c_I}(K-25).$$

Thus, by the supplier's objective function in Eq. (3), the marginal profit for the supplier to own a unit of AV is at most

$$\begin{aligned}
c_P\left(\alpha_1 + \alpha_2\frac{t}{c_I}\right) - c_F \leq &(c_I - c_{AV})\left(\alpha_1 + \alpha_2\frac{t}{c_I}\right) - c_F \\
&= (0.19 - \delta_1)\left(0.35 + 0.65\frac{2}{5 - 3\left(\delta_2 - \delta_1\right)/\delta_2}\right) - 0.19 + \delta_2 \\
&= -2\delta_2 + \delta_2 < 0
\end{aligned}$$

by the construction of δ_2. Thus, $K^s(c_P)=0$ and we again find $\vec{\mathbf{y}}^s\left(c_P,K^s(c_P)\right)=[10,25]$ for any $c_P\in[c_F,c_I-c_{AV}]$, which leads to $V^s\left(I(\delta_1)\,|c_P\right)=\delta_1(10+5.25p)$.

Finally, we argue that case (iii) is not optimal for the platform because the platform can fulfill all demand with ICs alone by setting $\vec{\mathbf{y}}=[10,25]$, and the marginal cost of using a unit of AV to serve the demand is $c_{AV}+c_P>c_I$. Thus, we conclude that

$$\sup_{c_P} V^s\left(I(\delta_1)\,|c_P\right)=\delta_1(10+5.25p). \tag{26}$$

Step 3: bound the profit ratio

Putting Eq. (25) and Eq. (26) together, we obtain

$$\sup_{c_P}\mathrm{PR}^s(I(\delta_1)\,|c_P)=\sup_{c_P}\frac{V^\star(I(\delta_1))}{V^s\left(I(\delta_1)\,|c_P\right)}\geq\frac{\delta_2 10}{\delta_1(10+5.25p)}=\frac{10}{\frac{\delta_1}{\delta_2}\left(10+5.25\frac{2}{5-3\left(1-\frac{\delta_1}{\delta_2}\right)}\left(1-\frac{\delta_1}{\delta_2}\right)\right)},$$

where

$$\frac{\delta_1}{\delta_2}=\frac{\delta_1}{\sqrt{0.049875\delta_1+0.7375\delta_1^2}-\frac{1}{2}\delta_1}=\frac{1}{\sqrt{0.049875/\delta_1+0.7375}-\frac{1}{2}}.$$

By plugging in the value of $\frac{\delta_1}{\delta_2}$ and taking the limit with respect to δ_1, we conclude that

$$\begin{aligned}&\lim_{\delta_1\to0^+}\sup_{c_P}\mathrm{PR}^s(I(\delta_1)\,|c_P)\\&\geq\lim_{\delta_1\to0^+}\frac{10\left(\sqrt{0.049875/\delta_1+0.7375}-\frac{1}{2}\right)}{10+10.5\left(1-\frac{1}{\sqrt{0.049875/\delta_1+0.7375}-\frac{1}{2}}\right)\Big/\left(5-3\left(1-\frac{1}{\sqrt{0.049875/\delta_1+0.7375}-\frac{1}{2}}\right)\right)}=\infty.\end{aligned}$$

□

Observe that in our sequence of instances, as the profit ratio becomes arbitrarily large, the profit of even the integrated supply chain becomes infinitesimally small. This aligns with our numerical observations in Fig. 5, wherein the profit ratio of an SPE grows under thinner margins.

D.2. Proofs of Results in Section 4

D.2.1. Proof of Proposition 3

Proof. We start by finding the platform's solution of $\vec{\mathbf{y}},\mathcal{A}$ and $\mathcal{H}$ if $\pi^\star(I)$ is accepted by both the platform and the supplier, and then show that it is indeed in the interest of both sides to accept $\pi^\star(I)$. Because c_P^π and $K^\pi=K^\star$ are specified by the contract, the platform only needs to solve the third stage of the sequential game. Given $c_P^\pi=0$ and a fixed cost of $c_FK^\star+V_A^s(I)$ for AVs, the platform solves the following optimization problem:

$$\begin{aligned}\max_{\vec{\mathbf{y}},\mathcal{A},\mathcal{H}}\ &\sum_i\alpha_i\mathbb{E}_{D_i\sim F_i}\left[(1-c_{AV})\mathcal{A}_i(D_i)+(1-c_I)\mathcal{H}_i(D_i)\right]-c_FK^\star-V_A^s(I)\\ \text{s.t. }&0\leq\mathcal{A}_i(D_i)\leq\min\{D_i,K^\star\},\forall i\\ &0\leq\mathcal{H}_i(D_i)\leq\min\{D_i,y_i\},\forall i\\ &\mathcal{A}_i(D_i)+\mathcal{H}_i(D_i)\leq\min\{D_i,K^\star+y_i\},\forall i\\ &ty_i=c_I\mathbb{E}_{D_i\sim F_i}\left[\mathcal{H}_i(D_i)\right],\forall i\end{aligned}$$

Since the fixed cost is independent of the choice of $\vec{\mathbf{y}}, \mathcal{A}$ and $\mathcal{H}$, the platform equivalently solves

$$\max_{\vec{\mathbf{y}},\mathcal{A},\mathcal{H}} \sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[(1-c_{AV})\mathcal{A}_i(D_i)+(1-c_I)\mathcal{H}_i(D_i)\right] \tag{27}$$

subject to the same constraints above. Notice that (27) is exactly the inner problem of Eq. (9) when the optimal solution of $K^\star$ is given, so $\pi^\star(I)$ leads the platform to adopt the centralized solution $\vec{\mathbf{y}}^\star, \mathcal{A}^\star$ and $\mathcal{H}^\star$ if it is accepted.

Then, to show that it is in the interest of both sides to accept the contract, we make use of Lemma 4 below and defer its proof to Appendix D.3.3.

Lemma 4. *For an instance I with a centralized solution $(K^\star, \vec{\mathbf{y}}^\star, \mathcal{A}^\star, \mathcal{H}^\star)$, both the platform and the supplier accept a contract π if the following holds true:*

(i) $K^\pi = K^\star$ *and it is optimal for the platform to set* $\vec{\mathbf{y}} = \vec{\mathbf{y}}^\star, \mathcal{A} = \mathcal{A}^\star$ *when π is accepted;*

(ii) $V_A^\pi(I) = V_A^s(I)$.

Notice that $\pi^\star(I)$ ensures $K^\pi = K^\star$ and it is optimal for the platform to adopt $\vec{\mathbf{y}} = \vec{\mathbf{y}}^\star$, so Lemma 4 (*i*) is satisfied. Moreover, the supplier obtains a profit of $c_F K^\star + V_A^s(I) - c_F K^\star = V_A^s(I)$ if $\pi^\star(I)$ is accepted, so Lemma 4 (*ii*) is also satisfied. Thus, it is optimal for both sides to accept $\pi^\star(I)$ and adopt the centralized solution.

□

D.2.2. Proof of Theorem 2

Proof. Given a centralized solution $(K^\star, \vec{\mathbf{y}}^\star, \mathcal{A}^\star, \mathcal{H}^\star)$, we construct

$$c_P^\pi = \frac{V_A^s(I) + c_F K^\star}{\sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[\mathcal{A}_i^\star(D_i)\right]} \geq 0.$$

Moreover, we find $\pi \in \Pi^U$ with $c_S^\pi = 0, K^\pi = K^\star, \underline{\mathcal{A}}^\pi = \mathcal{A}^\star$ and c_P^π as above, and then argue that such contract π guarantees $\mathrm{PR}^\pi(I) = 1$. Specifically, we first argue that it is optimal for the platform to set $\vec{\mathbf{y}} = \vec{\mathbf{y}}^\star$ and $\mathcal{A} = \mathcal{A}^\star$ when π is accepted, and then show that it is in the interest of both sides to accept π.

To find an optimal solution for $\vec{\mathbf{y}}$, we first simplify the centralized problem by plugging in the constraint Eq. (8). Then, given $K^\star$ we find that the centralized problem in scenario i is equivalent to

$$\max_{y_i,\mathcal{A}_i,\mathcal{H}_i} \mathbb{E}_{D_i\sim F_i}\left[(1-c_{AV})\mathcal{A}_i(D_i)\right] + (1-c_I)\frac{t}{c_I} y_i \tag{28}$$

subject to the constraints in Eq. (10). Since $\underline{\mathcal{A}}_i^\pi = \mathcal{A}_i^\star$, we already know that the platform must use $\mathcal{A}_i \geq \mathcal{A}_i^\star$. Now, given $K^\star$, if there exist some $y_i > y_i^\star, \mathcal{A}_i \geq \mathcal{A}_i^\star$ and $\mathcal{H}_i$ that are feasible for Eq. (10), then the resulting objective in Eq. (28) is strictly greater than that of $(K^\star, \vec{\mathbf{y}}^\star, \mathcal{A}^\star, \mathcal{H}^\star)$, which contradicts the optimality of the centralized solution. Thus, we know that the platform must set $y_i \leq y_i^\star\ \forall i$.

For the other direction, notice that we have $y_i^\star = y_i^s(c_P = 0, K^\star)$ in any scenario i. Now, since $c_P^\pi \geq 0$, given $K^\star$ we have

$$\bar{F}_i^{-1}\left(\frac{t(c_I - c_{AV} - c_P^\pi)}{c_I(1 - c_{AV} - c_P^\pi)}\right) - K^\star \geq \bar{F}_i^{-1}\left(\frac{t(c_I - c_{AV})}{c_I(1 - c_{AV})}\right) - K^\star$$

in any scenario i. Then, by Lemma 2, since the lower and upper bounds constructed for $y_i^s(c_P^\pi, K^\star)$ and $y_i^s(c_P = 0, K^\star)$ are exactly the same, we know that $y_i^s(c_P^\pi, K^\star) \geq y_i^s(c_P = 0, K^\star) = y_i^\star, \forall i$. Moreover, since the platform's maximum achievable profit given c_P^π and $K^\star$ is non-decreasing in y_i when $y_i \le y_i^\star \le y_i^s\left(c_P^\pi, K^\star\right)$, it is optimal for the platform to set $y_i = y_i^\star\ \forall i$,

Given $K^\star$ and $y_i = y_i^\star\ \forall i$, if there exist $\mathcal{A}_i > \mathcal{A}_i^\star$ and $\mathcal{H}_i$ that are feasible for Eq. (10), we again obtain a higher objective in Eq. (28) and contradict the optimality of the centralized solution. Thus, it is optimal for the platform to set $\vec{\mathbf{y}} = \vec{\mathbf{y}}^\star$ and $\mathcal{A} = \mathcal{A}^\star$ when π is accepted.

For the supplier, then, by the construction of c_P^π and $\mathcal{A} = \mathcal{A}^\star$ we have

$$V_A^\pi\left(I\right) = c_P^\pi \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[\mathcal{A}_i^\star(D_i)\right] - c_F K^\star = V_A^s\left(I\right).$$

Thus, both conditions in Lemma 4 are satisfied and we know that π is accepted. In particular, since π sets $K^\pi = K^\star$ and leads the platform to set $\vec{\mathbf{y}}^\star, \mathcal{A}^\star$ and $\mathcal{H}^\star$, the centralized solution is adopted and $\mathrm{PR}^\pi\left(I\right) = 1$.

□

D.2.3. Proof of Theorem 3

Proof. For $\epsilon > 0$, we pick any $c_P \in \left(0, \min\left\{\epsilon, \frac{1}{20}\right\}\right)$. Then, we construct an instance I with $\vec{\alpha} = [0.98, 0.01, 0.01]$, $c_I = 1/4, t = 1/4, c_{AV} = 1/5$ and $c_F = 1/32$. Moreover, the demand distributions in scenario $1, 2$ and 3 are respectively given by $D_1 = 10$,

$$D_2 = \begin{cases} 10 & \text{w.p. } 7/8 \\ 20 & \text{w.p. } 1/8 \end{cases}$$

and

$$D_3 = \begin{cases} 10 & \text{w.p. } 1-p \\ 20 & \text{w.p. } p \end{cases}$$

where $p = \frac{t(c_I - c_{AV} - c_P)}{c_I(1 - c_{AV} - c_P)}$. Then, Theorem 3 (i) is trivially satisfied with $c_P < \frac{1}{20} = c_I - c_{AV}$. Notice that the probability p depends on c_P and satisfies $0 < p < \frac{1}{16}$ by construction. To show that I satisfies Theorem 3 (ii) as well, we start by providing the unique centralized solution, and then show that no contract $\pi \in \Pi^D$ with $c_P^\pi \ge c_P$ leads the platform and the supplier to adopt such a solution.

Step 1: find the centralized solution

By solving the centralized problem for I we find, for $0 < p < \frac{1}{16}$, the unique centralized solution $K^\star = 10$. To see this, observe that for $0 \le K \le 10$ the marginal profit for owning a unit of AV is at least $(c_I - c_{AV})\alpha_1 - c_F > 0$, while for $K > 10$ the marginal profit for owning an additional unit of AV is at most $r\left(\alpha_2 + \alpha_3\right) - c_F < 0$. Moreover, in the centralized problem the platform incurs no cost of c_P, so by plugging $c_P = 0$ and $K^\star = 10$ into Lemma 2 we find

$$\bar{y}_1 = \bar{F}_1^{-1}\left(\frac{t(c_I - c_{AV})}{c_I(1 - c_{AV})}\right) - 10 = 0,$$

and similarly $\bar{y}_2 = 10, \bar{y}_3 = 0$. By construction of the demand distributions we also have $y_i^{ub} = 10$ and $y_i^{lb} = 10 - K^\star = 0\ \forall i$, so $\vec{\mathbf{y}}^\star = [0, 10, 0]$. Finally, the solutions for $K^\star$ and $\vec{\mathbf{y}}^\star$ uniquely lead to $\mathcal{A}_1^\star(D_1) = 10$,

$$\mathcal{A}_2^\star(D_2) = \begin{cases} 0 & \text{when } D_2 = 10 \\ 10 & \text{when } D_2 = 20 \end{cases}$$

and

$$\mathcal{A}_3^\star(D_3) = \begin{cases} 10 & \text{when } D_3 = 10 \\ 10 & \text{when } D_3 = 20. \end{cases}$$

That is, AVs are fully prioritized in scenario 3, but not in scenario 2.

Step 2: show $\mathbf{PR}^\pi(I) > 1$ for $\pi \in \Pi^D$ with $c_P^\pi \geq c_P$

For any contract $\pi \in \Pi^D$ with $c_P^\pi \geq c_P$, we have $\mathrm{PR}^\pi(I) = 1$ if and only if the contract makes the supplier and the platform adopt the unique centralized solution. Thus, $K^\pi = 10$. Moreover, since $\pi \in \Pi^D$, given that $\mathcal{A}_2^s(D_2) = 0$ when $D_2 = 10$ the contract must set the lower bound $\underline{\mathcal{A}}_i^\pi(10) = 0, \forall i$.

With $c_P^\pi \geq c_P$, we find

$$\bar{F}_2^{-1}\left(\frac{t(c_I - c_{AV} - c_P^\pi)}{c_I(1 - c_{AV} - c_P^\pi)}\right) = 20$$

by the construction of p. Then, by Lemma 2 we have $y_3^s(c_P^\pi, K^\pi) = \bar{y}_3 = y_3^{ub} = 10$ when $K^\pi = 10$. Since using y_3^{ub} units of ICs requires the platform to fully prioritize ICs, we obtain

$$\mathcal{A}_3^s(D_3|c_P^\pi, K^\pi) = \begin{cases} 0 & \text{when } D_3 = 10 \\ 10 & \text{when } D_3 = 20. \end{cases}$$

Given that $\underline{\mathcal{A}}_i^\pi(10) = 0, \forall i$, the above dispatch policy is allowed by π and the platform adopts

$$\mathcal{A}_3(D_3|c_P^\pi, K^\pi) = 0 < 10 = \mathcal{A}_3^\star(D_3)$$

in scenario 3 when $D_3 = 10$. Thus, $\mathrm{PR}^\pi(I) > 1$ and Theorem 3 (*ii*) holds.

□

D.2.4. Proof of Theorem 4

Proof. For any $I \in \mathcal{I}$ with a centralized solution $(K^\star, \vec{\mathbf{y}}^\star, \mathcal{A}^\star, \mathcal{H}^\star)$, we define the "AV only" and "IC only" solution as follows:

(i) The "AV only" solution sets $K = K^\star, \vec{\mathbf{y}} = \vec{0}, \mathcal{A}(D_i) = \min\{D_i, K^\star\}$ and $\mathcal{H}(D_i) = 0\ \forall i$;

(ii) The "IC only" solution sets $K = 0, \vec{\mathbf{y}} = \vec{\mathbf{y}}^\star, \mathcal{A}(D_i) = 0$ and $\mathcal{H}(D_i) = \min\left\{y_i^\star, D_i - \mathcal{A}_i^\star(D_i)\right\}\ \forall i$.

Notice that the above solutions trivially satisfy the constraints in Eq. (10). We start by arguing that at least one of the above solutions leads to a supply chain profit $\geq \frac{1}{2}V^\star(I)$, and then show how this result implies that $\mathrm{PR}^\pi(I) \leq 2$ for some $\pi \in \Pi^F(I)$.

For the "AV only" solution, the profit of the supply chain is

$$\begin{aligned} &\sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[(1 - c_{AV})\mathcal{A}(D_i) + (1 - c_I)\mathcal{H}(D_i)\right] - c_F K \\ =&\sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[(1 - c_{AV})\min\{D_i, K^\star\}\right] - c_F K^\star. \end{aligned} \tag{29}$$

Then, for the "IC only" solution, the profit of the supply chain is

$$\sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[(1 - c_I)\min\left\{y_i^\star, D_i - \mathcal{A}_i^\star(D_i)\right\}\right]. \tag{30}$$

Since the supply chain profit given the centralized solution is

$$V^\star(I) = \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[(1 - c_{AV})\mathcal{A}_i^\star(D_i) + (1 - c_I)\mathcal{H}_i^\star(D_i)\right] - c_F K^\star,$$

we find that

$$\begin{aligned} (29) + (30) &= \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[(1 - c_{AV})\min\{D_i, K^\star\} + (1 - c_I)\min\left\{y_i^\star, D_i - \mathcal{A}_i^\star(D_i)\right\}\right] - c_F K^\star \\ &\geq \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[(1 - c_{AV})\mathcal{A}_i^\star(D_i) + (1 - c_I)\mathcal{H}_i^\star(D_i)\right] - c_F K^\star = V^\star(I), \end{aligned}$$

where the inequality comes from the facts that

$$\mathcal{A}_i^\star(D_i) \le \min\{D_i, K^\star\} \text{ and } \mathcal{H}_i^\star(D_i) \le \min\left\{y_i^\star, D_i - \mathcal{A}_i^\star(D_i)\right\}, \forall i.$$

Thus, at least one of (29) $\ge \frac{1}{2}V^\star(I)$ and (30) $\ge \frac{1}{2}V^\star(I)$ must hold.

Notice that we may assume that $V^\star(I) > 2V^s(I)$ because otherwise Theorem 4 (*ii*) holds true. Similarly, we know that the platform can always adopt the "IC only" solution in the subgame by setting $c_P = 0$; thus, (30) $\le V_P^s(I) \le V^s(I)$. Therefore, if (30) $\ge \frac{1}{2}V^\star(I)$, then $V^s(I) \ge \frac{1}{2}V^\star(I)$, and Theorem 4 (*ii*) again holds true. Thus, we can focus on the case where (29) $\ge \frac{1}{2}V^\star(I) > V^s(I)$ holds true. We construct $\pi \in \Pi^F$ with $K^\pi = K^\star, \underline{\mathcal{A}}^\pi(D_i) = \min\{D_i, K^\star\}$ $\forall i$ and

$$c_P^\pi = \frac{V_A^s(I) + c_F K^\star}{\sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[\min\{D_i, K^\star\}\right]}.$$

so then (29) $\ge \frac{1}{2}V^\star(I)$ must hold.

Since the constraint in Eq. (5) guarantees that $\mathcal{A}_i(D_i) \le \min\{D_i, K^\star\}$ $\forall i$, the requirement on $\underline{\mathcal{A}}^\pi$ means that the platform has to set $\mathcal{A}_i(D_i) = \min\{D_i, K^\star\}$ $\forall i$. Then, from the non-negativity of $\mathcal{H}$ we know that the supply chain profit when π is accepted is

$$\begin{aligned} V_A^\pi(I) + V_P^\pi(I) &= \sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[(1-c_{AV})\min\{D_i, K^\star\} + (1-c_I)\mathcal{H}_i(D_i)\right] - c_F K^\star \\ &\ge \sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[(1-c_{AV})\min\{D_i, K^\star\}\right] - c_F K^\star = (29). \end{aligned}$$

Now, the profit for the AV supplier when π is accepted is

$$V_A^\pi(I) = c_P^\pi \sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[\min\{D_i, K^\star\}\right] - c_F K^\star = V_A^s(I),$$

so the AV supplier is willing to accept π. Moreover, since (29) $\ge \frac{1}{2}V^\star(I)$ the platform receives

$$V_P^\pi(I) \ge (29) - V_A^\pi(I) \ge \frac{1}{2}V^\star(I) - V_A^s(I) > V^s(I) - V_A^s(I) = V_P^s(I),$$

so the platform also accepts π. Since the supply chain profit is lower bounded by (29) when π is accepted, we conclude that $\mathrm{PR}^\pi(I) \le 2$.

□

D.3. Proofs of Auxiliary Results

D.3.1. Proof of Lemma 2

Proof. Since the optimal number of ICs can be solved independently in each scenario, we show that Eq. (20) holds for any given scenario i. Specifically, we start by simplifying the optimization problem in Eq. (4) and providing the smallest unconstrained first-order solution $\bar{y}_i$. Then we delineate the upper and lower bound y_i^{ub} and y_i^{lb}, respectively, and finally we summarize the solution and monotonicity results for $y_i^s(c_P, K)$, which depend on the relationship among $\bar{y}_i, y_i^{ub}$ and y_i^{lb}.

Step 1: find the unconstrained first-order solution $\bar{y}_i$

We start by observing that at optimality the constraint in Eq. (7) is tight. That is, at optimality we must have

$$\mathcal{A}_i^s\left(D_i|c_P, K\right) + \mathcal{H}_i^s\left(D_i|c_P, K\right) = \min\left\{D_i, K + y_i^s(c_P, K)\right\}. \tag{31}$$

We prove Eq. (31) by contradiction. When the above is not true, we know that at least one of the following holds:

$$(i)\, \mathcal{A}_i^s\left(D_i|c_P,K\right) < \min\left\{D_i - \mathcal{H}_i^s\left(D_i|c_P,K\right),K\right\};$$
$$(ii)\, \mathcal{H}_i^s\left(D_i|c_P,K\right) < \min\left\{D_i - \mathcal{A}_i^s\left(D_i|c_P,K\right),y_i^s\left(c_P,K\right)\right\}.$$

because otherwise

$$\begin{aligned}
&\mathcal{A}_i^s\left(D_i|c_P,\right) + \mathcal{H}_i^s\left(D_i|c_P,K\right)\\
\geq &\min\left\{D_i - \mathcal{H}_i^s\left(D_i|c_P,K\right),K\right\} + \min\left\{D_i - \mathcal{A}_i^s\left(D_i|c_P,K\right),y_i^s\left(c_P,K\right)\right\}\\
\geq &\min\left\{D_i, K + y_i^s\left(c_P,K\right)\right\}.
\end{aligned}$$

When (i) is true, from $c_{AV} + c_P \leq c_I < r$ we know that we can increase the platform's profit by increasing $\mathcal{A}_i^s\left(D_i|c_P,K\right)$ to $\min\left\{D_i - \mathcal{H}_i^s\left(D_i|c_P,K\right),K\right\}$, which contradicts the optimality assumption. Similarly, when (ii) is true, from $c_I < r$ we know that we can increase the platform's profit by increasing $\mathcal{H}_i^s\left(D_i|c_P,K\right)$ (and simultaneously increasing y_i through Eq. (8)) until

$$\mathcal{H}_i^s\left(D_i|c_P,K\right) = \min\left\{D_i - \mathcal{A}_i^s\left(D_i|c_P,K\right),y_i^s\left(c_P,K\right)\right\},$$

which again contradicts the optimality assumption. Thus, we can add

$$\mathcal{A}_i\left(D_i\right) + \mathcal{H}_i\left(D_i\right) = \min\left\{D_i, K + y_i\right\}. \tag{32}$$

as a constraint into the optimization problem in Eq. (4) without affecting its optimal solution(s).

Since we can solve $y_i^s\left(c_P,K\right)$ independently in each scenario, we plug Eq. (32) into the objective function in Eq. (4) and find that the optimization problem that the platform faces in scenario i is

$$\max_{y_i,\mathcal{A}_i,\mathcal{H}_i} \mathbb{E}_{D_i\sim F_i}\left[(1-c_{AV}-c_P)\left(\min\left\{D_i,K+y_i\right\} - \mathcal{H}_i\left(D_i\right)\right) + (1-c_I)\mathcal{H}_i\left(D_i\right)\right]$$

subject to Eq. (5) - Eq. (8) and Eq. (32). Then, we substitute Eq. (8) into the objective of the optimization problem, which simplifies the problem to

$$\max_{y_i,\mathcal{A}_i,\mathcal{H}_i} \mathbb{E}_{D_i\sim F_i}\left[(1-c_{AV}-c_P)\min\left\{D_i,K+y_i\right\}\right] - \left(c_I - c_{AV} - c_P\right)\frac{t}{c_I}y_i \tag{33}$$

subject to Eq. (5) - Eq. (8) and Eq. (32). In particular, Eq. (33) is now an optimization problem for y_i alone, and its derivative with respect to y_i is given by

$$(1-c_{AV}-c_P)\mathbb{P}\left[D > K + y_i\right] - \left(c_I - c_{AV} - c_P\right)\frac{t}{c_I}. \tag{34}$$

Thus, by the first-order condition we know that $\bar{y}_i := \bar{F}_i^{-1}\left(\frac{t(c_I-c_{AV}-c_P)}{c_I(1-c_{AV}-c_P)}\right) - K$ is optimal to the unconstrained optimization problem in Eq. (33). Specifically, for a discrete distribution $\bar{y}_i$ is the smallest optimal solution for y_i in the unconstrained problem, and for a continuous distribution this optimal solution is unique.

Step 2: construct y_i^{ub} and y_i^{lb}

We start by constructing the upper and lower bounds y_i^{ub} and y_i^{lb} using the constraints in Eq. (5) - Eq. (8) and Eq. (32), which implies the necessity of y_i^{ub} and y_i^{lb} for an optimal solution. Then we show that the bounds are also sufficient in the sense that any $y_i \in \left[y_i^{lb}, y_i^{ub}\right]$ leads to a feasible $\mathcal{H}_i$ that satisfies Eq. (6) and Eq. (8).

For the upper bound y_i^{ub}, from Eq. (6) we know that the number of ICs must fulfill

$$c_I \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, y_i\}\right] \geq t y_i,$$

i.e., the expected earning of ICs must be greater-equal to their reservation earning when the platform fully prioritizes them. Thus, the maximum number of ICs that can be hired by the platform is given by

$$\begin{aligned} y_i^{ub} &:= \max\left\{ y_i \middle| c_I \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, y_i\}\right] = t y_i \right\} \\ &= \max\left\{ y_i \middle| \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, K + y_i\}\right] - \frac{t}{c_I} y_i = \mathbb{E}_{D_i \sim F_i}\left[\min\left\{(D_i - y_i)^+, K\right\}\right] \right\}. \end{aligned} \tag{35}$$

Let $f(y_i) := c_I \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, y_i\}\right]$. Since the derivative $f'(y_i) = c_I \mathbb{P}\left[D_i > y_i\right]$ is non-increasing in y_i, $f(y_i)$ is concave. Now, since y_i^{ub} is the largest solution to $f(y_i) = t y_i$, by the concavity of $f(y_i)$ and the linearity of $t y_i$ we know that $f(y_i) < t y_i$ for any $y_i > y_i^{ub}$. Thus, no $y_i > y_i^{ub}$ leads to a feasible solution $\mathcal{H}_i$ that satisfies Eq. (6).

Similarly, for the lower bound y_i^{lb}, observe that the platform cannot use so few ICs that

$$c_I \mathbb{E}_{D_i \sim F_i}\left[\min\left\{(D_i - K)^+, y_i\right\}\right] > t y_i,$$

i.e., the expected earning of ICs exceeds their reservation earning even if the platform fully prioritizes AVs. In particular, by taking $\mathcal{H}_i(D_i) < \min\left\{(D_i - K)^+, y_i\right\}$ when $\mathcal{A}_i(D_i) \leq \min\{D_i, K\}$ one violates Eq. (32). Thus, the minimum number of ICs that must be hired by the platform is given by

$$\begin{aligned} y_i^{lb} &:= \max\left\{ y_i \middle| c_I \mathbb{E}_{D_i \sim F_i}\left[\min\left\{(D_i - K)^+, y_i\right\}\right] = t y_i \right\} \\ &= \max\left\{ y_i \middle| \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, K + y_i\}\right] - \frac{t}{c_I} y_i = \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, K\}\right] \right\}, \end{aligned}$$

where the maximum is taken because $y_i = 0$ trivially satisfies the equality condition in the bracket. Now, let $g(y_i) := \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, K + y_i\}\right] - \frac{t}{c_I} y_i$. Since the derivative

$$g'(y_i) = \mathbb{P}\left[D_i > K + y_i\right] - \frac{t}{c_I}$$

is non-increasing in y_i, $g(y_i)$ is concave. Then, there are at most two solutions [12] to

$$g(y_i) = \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, K\}\right].$$

Since $y_i = 0$ is trivially one of the solutions, y_i^{lb} must be the larger (or equal) solution, which means that $g(y_i) > \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, K\}\right]$ for any $y_i \in (0, y_i^{lb})$. Thus, no $y_i \in (0, y_i^{lb})$ leads to a feasible solution $\mathcal{H}_i$ that satisfies Eq. (32).

[12]Notice that we assume without loss of generality that there is no degeneracy in any non-zero solution because in the degenerate case it is strictly better to take the largest degenerate solution: given $\mathcal{A}_i(D_i) = \min\{D_i, K\}\ \forall i$, the platform's objective becomes

$$\sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[(1 - c_{AV} - c_P)\min\{D_i, K\} + (1 - c_I)\frac{t}{c_I} y_i\right],$$

which is increasing in y_i.

Finally, we show that for any y_i such that $y_i^{lb} \le y_i \le y_i^{ub}$ we can find $\mathcal{H}_i$ that satisfies Eq. (6) and Eq. (8). Since y_i^{lb} is the largest solution to $g(y_i) = \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, K\}\right]$ and $g(y_i)$ is concave, we know that $g(y_i)$ is non-increasing in $y \ge y_i^{lb}$. Moreover, since

$$\mathbb{E}_{D_i \sim F_i}\left[\min\left\{(D_i - y_i)^+, K\right\}\right] \le \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, K\}\right]$$

and y_i^{ub} is the largest solution to $g(y_i) = \mathbb{E}_{D_i \sim F_i}\left[\min\left\{(D_i - y_i)^+, K\right\}\right]$, we have $y_i^{lb} \le y_i^{ub}$. In particular, any $y_i \in \left[y_i^{lb}, y_i^{ub}\right]$ satisfies

$$\mathbb{E}_{D_i \sim F_i}\left[\min\left\{(D_i - y_i)^+, K\right\}\right] \le g(y_i) \le \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, K\}\right].$$

That is, any $y_i \in \left[y_i^{lb}, y_i^{ub}\right]$ satisfies

$$\mathbb{E}_{D_i \sim F_i}\left[\min\left\{(D_i - K)^+, y_i\right\}\right] \le \frac{t}{c_I} y_i \le \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, y_i\}\right].$$

Since $\mathcal{H}_i(D_i)$ is feasible whenever

$$\min\left\{(D_i - K)^+, y_i\right\} \le \mathcal{H}_i(D_i) \le \min\{D_i, y_i\},$$

for any y_i such that $y_i^{lb} \le y_i \le y_i^{ub}$ we can find $\mathcal{H}_i$ that satisfies Eq. (6) and Eq. (8).

Step 3: find $y_i^s(c_P, K)$

When $y_i^{lb} \le \bar{y}_i \le y_i^{ub}$, by construction of the upper and lower bounds we know that there exists a feasible solution for $\mathcal{H}_i$ that satisfies Eq. (6) and Eq. (8). By taking $\mathcal{A}_i(D_i) = \min\{D_i, K + \bar{y}_i\} - \mathcal{H}_i(D_i)$ we also find a feasible solution for $\mathcal{A}_i$ that satisfies Eq. (5), Eq. (7) and Eq. (32). Since such $\bar{y}_i, \mathcal{H}_i$ and $\mathcal{A}_i$ simultaneously satisfy the first-order condition in Eq. (34) and the constraints in Eq. (5) - Eq. (8) and Eq. (32), such $\bar{y}_i$ is optimal. In particular, by the first-order condition in Eq. (34) we know that the objective value in Eq. (33) is non-decreasing in y_i for $y_i \le \bar{y}_i$ and non-increasing in y_i for $y_i \ge \bar{y}_i$.

When $\bar{y}_i > y_i^{ub}$, by the first-order condition we find that for all $y_i \le y_i^{ub}$ the objective value in Eq. (33) is non-decreasing in y_i. Since by construction y_i^{ub} is the largest number of ICs that leads to a feasible $\mathcal{H}_i$ that satisfies Eq. (6), we have $y_i^s(c_P, K) = y_i^{ub}$. Similarly, when $\bar{y}_i < y_i^{lb}$, we find that for all $y_i \ge y_i^{lb}$ the objective value in Eq. (33) is non-increasing in y_i. Since by construction y_i^{lb} is the smallest number of ICs that leads to a feasible $\mathcal{H}_i$ while satisfying Eq. (32), we have $y_i^s(c_P, K) = y_i^{lb}$.

□

D.3.2. Proof of Lemma 3

Proof. We separately consider the cases where $c_P > 1 - c_{AV}$ and $c_I - c_{AV} < c_P \le 1 - c_{AV}$. We begin by observing that when $c_P > 1 - c_{AV}$ the marginal profit of using AVs to serve demand is negative, so the platform trivially sets $\mathcal{A}_i^s\left(D_i | c_P, K\right) = 0\ \forall i$. Then, the platform solves:

$$\begin{aligned} \max_{\vec{y}, \mathcal{H}} \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[(1 - c_I)\mathcal{H}_i(D_i)\right] \\ \text{s.t. } 0 \le \mathcal{H}_i(D_i) \le \min\{D_i, y_i\}, \forall i \\ t y_i = c_I \mathbb{E}_{D_i \sim F_i}\left[\mathcal{H}_i(D_i)\right]\ \forall i. \end{aligned}$$

We find the objective equivalent to $\sum_i \alpha_i(1-c_I)\frac{t}{c_I}y_i$, which monotonically increases as each y_i increases because $c_I < 1$. Thus, it is optimal for the platform to use the maximum possible number of ICs, which we know from Eq. (35) is uniquely given by

$$y_i^s(c_P, K) = y_i^{ub} := \max\left\{y_i \middle| c_I \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, y_i\}\right] = t y_i\right\}, \forall i.$$

When $c_P > c_I - c_{AV}$, from Eq. (34) we know that in each scenario i the unconstrained first-order condition for solving Eq. (4) is

$$(1 - c_{AV} - c_P)\mathbb{P}[D > K + y_i] + (c_{AV} + c_P - c_I)\frac{t}{c_I} > 0.$$

Therefore, it is again optimal for the platform to use the maximum possible number of ICs and we have $y_i^s(c_P, K) = y_i^{ub}, \forall i.$

□

D.3.3. Proof of Lemma 4

Proof. By Lemma 4 (*ii*) we immediately know that the supplier is willing to accept π. Now, the profit of the supply chain is given by

$$\sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[(1 - c_{AV})\mathcal{A}_i(D_i) + (1 - c_I)\mathcal{H}_i(D_i)\right] - c_F K \tag{36}$$

for any feasible solutions of $(K, \vec{\mathbf{y}}, \mathcal{A}, \mathcal{H})$ since the payments between the supplier and the platform cancel out. By plugging the constraint in Eq. (8) into Eq. (36) we equivalently find

$$(36) = \sum_i \alpha_i \left(\mathbb{E}_{D_i \sim F_i}\left[(1 - c_{AV})\mathcal{A}_i(D_i)\right] + (1 - c_I)\frac{t}{c_I}y_i\right) - c_F K,$$

which now only depends on $K, \vec{\mathbf{y}}$ and $\mathcal{A}$. Thus, Lemma 4 (*i*) implies that the profit of the supply chain is $V^\star(I)$ when π is accepted. Then, the platform obtains a profit of

$$V_P^\pi(I) = V^\star(I) - V_A^s(I) \geq V^s(I) - V_A^s(I) = V_P^s(I)$$

when π is accepted. Thus, the platform also accepts π.

□

D.3.4. Proof of Claim 1

Proof. With the optimality condition in Eq. (32) and the constraint in Eq. (8) we find that the supplier's optimization problem in Eq. (3) is equivalent to

$$\max_K c_P \sum_i \alpha_i \left(\mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, K + y_i(c_P, K)\}\right] - \frac{t}{c_I}y_i(c_P, K)\right) - c_F K.$$

Then, when K is fixed, the derivative of the objective function with respect to $y_i(c_P, K)$ is

$$c_P \cdot \alpha_i \left(\mathbb{P}\left[D > K + y_i(c_P, K)\right] - \frac{t}{c_I}\right). \tag{37}$$

By Eq. (34) we know that there can be multiple optimal solutions for y_i in Eq. (4) only when there is some $y_i > \bar{F}_i^{-1}\left(\frac{t(c_I - c_{AV} - c_P)}{c_I(1 - c_{AV} - c_P)}\right) - K$ that is also optimal. By plugging $\bar{F}_i^{-1}\left(\frac{t(c_I - c_{AV} - c_P)}{c_I(1 - c_{AV} - c_P)}\right) - K$ into Eq. (37) we find

$$\begin{aligned} c_P \cdot \alpha_i \left(\mathbb{P}[D > K + y_i] - \frac{t}{c_I}\right) \leq & c_P \cdot \alpha_i \left(\frac{t(c_I - c_{AV} - c_P)}{c_I(1 - c_{AV} - c_P)} - \frac{t}{c_I}\right) \\ = & c_P \cdot \alpha_i \left(\frac{t}{c_I}\frac{c_I - 1}{1 - c_{AV} - c_P}\right) < 0, \forall y_i \geq \bar{F}_i^{-1}\left(\frac{t(c_I - c_{AV} - c_P)}{c_I(1 - c_{AV} - c_P)}\right) - K. \end{aligned}$$

That is, for fixed K the objective value in Eq. (3) is decreasing in y_i for $y_i \geq \bar{F}_i^{-1}\left(\frac{t(c_I-c_{AV}-c_P)}{c_I(1-c_{AV}-c_P)}\right)-K$. Since the argument holds in each scenario i, we conclude that among the optimal solutions for $(\vec{\mathbf{y}},\mathcal{A},\mathcal{H})$ in Eq. (4), the ones that maximize Eq. (3) contain the smallest optimal y_i in each scenario i.

□

D.3.5. Proof of Claim 2

Proof. By definition,

$$\begin{aligned} y_2^{lb} &= \max\left\{y_2 \middle| c_I \mathbb{E}_{D_2\sim F_i}\left[\min\left\{(D_2-K)^+, y_2\right\}\right] = t y_2\right\} \\ &= \max\left\{y_2 \middle| \mathbb{E}_{D_2\sim F_2}\left[\min\{D_2, K+y_2\}\right] - \frac{t}{c_I} y_2 = \mathbb{E}_{D_2\sim F_2}\left[\min\{D_2,K\}\right]\right\}. \end{aligned} \tag{38}$$

Let $g(y_2) := \mathbb{E}_{D_2\sim F_2}\left[\min\{D_2, K+y_2\}\right] - \frac{t}{c_I}y_2$. By construction of t and p we know

$$\frac{t}{c_I} = \frac{(1-p)10+p25}{25} = \frac{\mathbb{E}[D_2]}{25}.$$

Thus, for $0 \leq K \leq 10$ we have

$$g(y_2) < \mathbb{E}[D_2] - \frac{t}{c_I}(25-K) = \frac{t}{c_I}K \leq K = \mathbb{E}_{D_2\sim F_2}\left[\min\{D_2,K\}\right] \ \forall y_2 > 25-K.$$

Similarly, for $10 < K \leq 25$,

$$g(25-K) < \frac{t}{c_I}K \leq (1-p)10 + pK = \mathbb{E}_{D_2\sim F_2}\left[\min\{D_2,K\}\right] \ \forall y_2 > 25-K,$$

so by Eq. (38) we know that $y_2^{lb} \leq 25-K$ for $K \in [0,25]$. □

D.3.6. Proof of Claim 3

Proof. For any $c_P \in [c_F, c_I - c_{AV}]$ and $K \in [0,25]$, we know by the optimality condition in Eq. (32) that

$$\mathcal{A}_2^s\left(D_2|c_P,K\right) = \min\left\{D_2, K + y_2^s(c_P,K)\right\} - \mathcal{H}_2^s\left(D_2|c_P,K\right).$$

Taking expectation on both sides, we obtain

$$\mathbb{E}_{D_2\sim F_2}\left[\mathcal{A}_2^s\left(D_2|c_P,K\right)\right] = \mathbb{E}_{D_2\sim F_2}\left[\min\left\{D_2, K+y_2^s(c_P,K)\right\} - \mathcal{H}_2^s\left(D_2|c_P,K\right)\right].$$

Then, from the equilibrium condition in Eq. (8) we substitute $\mathbb{E}_{D_2\sim F_2}\left[\mathcal{H}_2^s\left(D_2|c_P,K\right)\right]$ with $\frac{t}{c_I}y_2^s(c_P,K)$ and conclude that

$$\mathbb{E}_{D_2\sim F_2}\left[\mathcal{A}_2^s\left(D_2|c_P,K\right)\right] = \mathbb{E}_{D_2\sim F_2}\left[\min\left\{D_2, K+y_2^s(c_P,K)\right\}\right] - \frac{t}{c_I}y_2^s(c_P,K).$$

□

D.4. Proofs of Results in Section 5

D.4.1. Proof of Proposition 4

Proof. We begin by characterizing the profits of AV and IC-only platforms. Since the objective in (13) monotonically increases with respect to $\mathcal{A}_i(D_i)$ for any D_i and i, (13) becomes

$$V^{AV}(I) = \max_K \sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[(1-c_{AV})\min\{D_i,K\}\right] - c_F K.$$

We find that this is equivalent to

$$\Pi^{\text{emp}} := \max_{x \geq 0} \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i} \left[p \min\{D_i, x\} \right] - cx$$

in Lobel et al. (2021) when $p = 1, c_{AV} = 0$ and $c_F = c$.

Similarly, an IC-only platform solves (9)-(10) subject to $K = 0$. That is,

$$\begin{aligned} V^{IC}(I) := \max_{\vec{\mathbf{y}}, \mathcal{H}} & \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i} \left[(1 - c_I) \mathcal{H}_i(D_i) \right] \\ \text{s.t. } & 0 \leq \mathcal{H}_i(D_i) \leq \min\{D_i, y_i\}, \forall i \\ & t y_i = c_I \mathbb{E}_{D_i \sim F_i} \left[\mathcal{H}_i(D_i) \right], \forall i. \end{aligned}$$

Since the objective monotonically increases with respect to $\mathcal{H}_i(D_i)$ for any D_i and i, the above becomes

$$\begin{aligned} V^{IC}(I) = \max_{\vec{\mathbf{y}}, \mathcal{H}} & \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i} \left[(1 - c_I) \min\{D_i, y_i\} \right] \\ \text{s.t. } & t y_i = c_I \mathbb{E}_{D_i \sim F_i} \left[\min\{D_i, y_i\} \right], \forall i. \end{aligned}$$

We find that this is equivalent to

$$\begin{aligned} \Pi^{\text{cont}} := \max_{\vec{\mathbf{s}}, w} & \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i} \left[(p - w) \min\{D_i, s_i\} \right] \\ \text{s.t. } & r s_i = w \mathbb{E}_{D_i \sim F_i} \left[\min\{D_i, s_i\} \right], \forall i \end{aligned} \tag{39}$$

when $p = 1, t = r$ and $c_I = w^\star$ returned as the optimal solution to w in (39).

Theorem 2 in Lobel et al. (2021) states that, for any $p > 0, r \in (0, p), c \in (0, p)$ and $\epsilon > 0$, there exists an instance with 2 demand scenarios such that $\frac{\Pi^{\text{cont}}}{\Pi^{\text{emp}}} \geq \frac{1}{\epsilon}$. Now, let $p = 1, t = r$ and $c_F = c$. Then, we know that for any $M > 0$, we can construct $\epsilon = 1/M > 0$ and there exists an instance $I \in \mathcal{I}$ such that

$$\frac{V^\star(I)}{V^{AV}(I)} \geq \frac{V^{IC}(I)}{V^{AV}(I)} = \frac{\Pi^{\text{cont}}}{\Pi^{\text{emp}}} \geq \frac{1}{\epsilon} = M.$$

□

D.4.2. Proof of Proposition 5

Proof. We construct an instance I with two scenarios D_1 and D_2, which respectively occurs with probability $\alpha_1 = 1 - 10^{-6}$ and $\alpha_2 = 10^{-6}$. $D_1 = 10$ deterministically, and

$$D_2 = \begin{cases} 10 & \text{w.p. } 2 \cdot 10^{-6} \\ 2000 & \text{w.p. } 1 - 2 \cdot 10^{-6}. \end{cases}$$

Let $c_{AV} = 0.001, c_I = 1 - 10^{-8}, t = \mathbb{E}[D_2] \cdot c_I$ and $c_F = (c_I - c_{AV}) \cdot \max\left(\alpha_1, \mathbb{E}[D_2]/2000\right) + 10^{-9}$. With a slight abuse of notation we denote an SPE in the short-term leasing model by superscript s in this proof. We show that the centralized solution sets $K^\star = 10$ and $\vec{\mathbf{y}}^\star = [0, 1990]$, whereas an SPE yields $K^s = \mathcal{K}_1^s = \mathcal{K}_2^s = 0, \vec{\mathbf{y}}^s = [10, 2000]$.

By construction, $\bar{F}_2^{-1}(\frac{t(c_I - c_{AV})}{c_I(1 - c_{AV})}) = 2000$ and $t/c_I = \mathbb{E}[D_2]/2000$. Thus, by applying Lemma 2 we find that in a centralized solution, for any given K, it is optimal to set $y_2^s(K) = 2000 - K$. Since ICs have an expected utilization of t/c_I, in scenario 2 each additional unit of AV reduces the expected IC usage by t/c_I. Thus, in a centralized solution the marginal profit of investing in AV when $K \in [0, 10)$ is $(c_I - c_{AV})\left[\alpha_1 + t/c_I \alpha_2\right] - c_F >$

0, and the marginal profit of investing in AV when $K \geq 10$ is upper bounded by $(c_I - c_{AV})\alpha_2 - c_F < 0$. Thus, $K^\star = 10$ and the resulting $\vec{\mathbf{y}}^\star = [0, 1990]$.

In the short-term leasing SPE, in contrast, we shall show that the platform has no incentive to lease AV in scenario 2 and the AV supplier responds by making no AV investment at all. Notice that the supplier trivially makes no investment when $c_P < c_F$. Similarly, since the platform could cover all demand with ICs alone it has no incentive to use AVs when $c_P > c_I - c_{AV}$. We thus focus on the case where $c_P \in [c_F, c_I - c_{AV}]$. In scenario 2, by the same analyses as above we find that the marginal profit for the platform to lease AV when $K \in [0, 10)$ is $(c_I - c_{AV})t/c_I - c_P \leq (c_I - c_{AV})t/c_I - c_F < 0$. Thus, given any $c_P \geq c_F$ and K, the platform would respond by setting $\mathcal{K}_2^s(c_P, K) = 0$. Then, for the AV supplier, the marginal profit of investing in AV becomes at most $c_P\alpha_1 - c_F \leq (c_I - c_{AV})\alpha_1 - c_F < 0$. Thus, in an SPE we have $K^s = \mathcal{K}_1^s = \mathcal{K}_2^s = 0$ and $\vec{\mathbf{y}}^s = [10, 2000]$. We thus find that

$$\frac{V^\star(I)}{V^{SL}(I)} \geq \frac{(1 - c_{AV}) \cdot 10\left[\alpha_1 + \alpha_2 t c_I\right] + (1 - c_I) \cdot 1990t/c_I\alpha_2 - 10c_F}{(1 - c_I) \cdot \left[\alpha_1 10 + \alpha_2 \mathbb{E}\left[D_2\right]\right]} > 100.$$

□

D.4.3. Proof of Proposition 6

Proof. We construct an instance I with two scenarios D_1 and D_2, which respectively occurs with probability $\alpha_1 = 0.2$ and $\alpha_2 = 0.8$. $D_1 \sim U(10, 12)$, and $D_2 \sim U(10, 5000)$. Let $c_{AV} = 0.001, c_I = 0.999, c_F = 0.763$ and $t = 7.99/8c_I$. With a slight abuse of notation we denote an SPE in the long-term leasing model by superscript s in this proof. We show that the centralized solution sets $K^\star \approx 236.02$ and $\vec{\mathbf{y}}^\star = [0, 0]$, whereas an SPE yields $K^s = 10, \vec{\mathbf{y}}^s = [0.005, 12.475]$.

In a centralized solution we have $\bar{F}_2^{-1}(\frac{t(c_I - c_{AV})}{c_I(1 - c_{AV})}) \approx 21.226$. Thus, unless $K < 22$ we should always have $\vec{\mathbf{y}}^s(c_P = 0, K) = [0, 0]$. When $K \geq 22$, the marginal profit of AV is $\mathbb{P}\left[D \geq K\right](1 - c_{AV}) - c_F$, which equals 0 when $K \approx 236.02$. It is easy to check that a solution of $K \geq 236.02, \vec{\mathbf{y}} = [0, 0]$ dominates any solution with $K < 22$, even if all remaining demand can be filled by ICs in the latter case. Thus, in a centralized solution we find that $K^\star \approx 236.02$ and the resulting $\vec{\mathbf{y}}^\star = [0, 0]$. This generates a supply chain profit greater than 6.6508.

In the long-term leasing SPE, we find that by taking $c_P = 0.9972$ the platform sets $K^s(c_P) = 10$ and the resulting $\vec{\mathbf{y}}^s(c_P, K^s(c_P)) = [0.005, 12.475]$. This generates at least 2.3419 for the AV supplier. We further find that by taking $c_P = 0.9972$ the platform sets $K^s(c_P) < 10$. We now iterate over values of c_P and apply a continuity argument to show that no c_P such that $K^s(c_P) \neq 10$ can be optimal for the AV supplier. Since the AV supplier will not take any $c_P < c_F$ and the platform will not take $c_P > 1$, we iterate over all choices of $c_P \in [c_F, 1 - c_{AV}]$ at a precision level of 0.0001 and solve the resulting $K^s(c_P)$ to the closest integral. Results yield no other c_P value such that (1) $K^s(c_P) \neq 10$, and (2) it generates more than a profit of 2.3 for the AV supplier. For any $c_P \in [c_F, 1 - c_{AV}]$ and $c'_P \in [c_P, c_P + 0.0001)$, $K^s(c'_P) \leq K^s(c_P)$. Thus, the supplier profit given c'_P is upper bounded by the supplier profit given c_P plus $0.0001 \cdot K^s(c_P)$. Here $K^s(c_P)$ is trivially upper bounded by $K^\star \approx 236.02$ for any $c_P \in [c_F, 1 - c_{AV}]$. Thus, for any c_P that yields profit no longer than 2.3, we know that the supplier profit given $c'_P \in [c_P, c_P + 0.0001)$ is upper bounded by $2.3 + 0.0001 \cdot 237 < 2.33$. Since this applies to all c_P such that $K^s(c_P) \neq 10$, we must have $K^s(c_P^s) = 10$. It is thus optimal for the AV supplier to set the largest c_P such that $K^s(c_P) = 10$, based on which we conclude $c_P^s \in [0.9972, 0.9973), K^s = 10$ and the resulting $\vec{\mathbf{y}}^s = [0.005, 12.475]$. This generates a supply chain profit smaller than 2.37. We therefore find that $\frac{V^\star(I)}{V^{LL}(I)} \geq \frac{6.6508}{2.37} > 2.806$. □

D.5. Proofs of Results in Appendices

D.5.1. Proof of Proposition 7

Proof. We observe that Eq. (14) is equivalent to

$$\max_K \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i} \left[(c_P - c_O) \mathcal{A}_i \left(D_i | c_P, K \right) \right] - (c_F - c_O) K.$$

Thus, by replacing the argument on c_P and c_F by $c_P - c_O$ and $c_F - c_O$, respectively, the construction in Theorem 1 directly extends. Specifically, for any $c_O > 0$ and $\delta_1 \in (0, 0.01]$, take $\delta_2 = \sqrt{0.049875\delta_1 + 0.7375\delta_1^2} - 0.5\delta_1$ and $c_F = c_O + 0.19 - \delta_2$. Then, construct $\vec{\alpha}, c_I, c_{AV}, t, p$ and D the same way as in Theorem 1, we recover instance $I(\delta_1)$ in Theorem 1, where any choice of c_P would lead to no AV investment on the supplier side. Thus, the bound on profit ratio extends and we have $\lim_{\delta_1 \to 0^+} \sup_{c_P} \mathrm{PR}^s(I(\delta_1) | c_P) = \infty$. □

D.5.2. Proof of Proposition 8

Proof. Using the same construction of $I(\delta_1)$ as in Theorem 1, we take any $c_O \in \left(0, \min\left(c_F, \frac{c_F - (c_I - c_{AV})\alpha_1}{\alpha_2}\right)\right)$. Since $(c_I - c_{AV})\alpha_1 < c_F$ by construction, the interval is guaranteed to be non-empty. Now, we consider three cases for c_P. When $c_P < c_F$, we have $\max(c_P, c_O) < c_F$ and thus $K^s(c_P) = 0$. When $c_P > c_I - c_{AV}$, platform trivially resorts to only ICs and thus $K^s(c_P) = 0$. When $c_P \in [c_F, c_I - c_{AV}]$, by $c_P \geq c_F > c_O$ we know that the AV supplier does not divest in scenario 1, when AVs are fully utilized. If the supplier divests in scenario 2, the marginal profit of investing in AVs is $c_P\alpha_1 + \alpha_2 c_O - c_F \leq (c_I - c_{AV})\alpha_1 + \alpha_2 c_O - c_F < 0$. If the supplier does not divest in scenario 2, the marginal profit remains $c_P\left[\alpha_1 + \alpha_2 t / c_I\right] - c_F < 0$ by construction. Thus, in either case the supplier would not invest in any AV and $K^s = 0$. The rest of the proof follows from Theorem 1. □

D.5.3. Proof of Lemma 1

Proof. Given any c_P, K, feasible $\vec{\mathbf{y}}$ and $\mathcal{A}_i(D_i)$, we start by observing that taking $\mathcal{H}_i(D_i) = \min\{D_i, K + y_i\} - \mathcal{A}_i(D_i), \forall i$, is always feasible to Eq. (15) and the objective is non-decreasing as $\mathcal{H}_i(D_i)$ increases. Thus, the third constraint in Eq. (15) is tight at optimality, and we can plug $\mathcal{H}_i(D_i) = \min\{D_i, K + y_i\} - \mathcal{A}_i(D_i) = \min\left\{D_i - \mathcal{A}_i(D_i), y_i\right\}, \forall i$, into Eq. (15). We then obtain

$$\begin{aligned} \max_{\vec{\mathbf{y}}, \mathcal{A}, \mathcal{H}, c_I} & \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i} \left[(1 - c_{AV} - c_P)\mathcal{A}_i(D_i) + (1 - c_I^i) \min\left\{D_i - \mathcal{A}_i(D_i), y_i\right\} \right] \\ \text{s.t. } & (5), (6), (7), t y_i = c_I^i \mathbb{E}_{D_i \sim F_i} \left[\mathcal{H}_i(D_i)\right], c_I^i \geq 0, \forall i. \end{aligned}$$

By plugging the last equality constraint into the above, we find that the optimization problem can be reformulated as:

$$\begin{aligned} \max_{\vec{\mathbf{y}}, \mathcal{A}} & \sum_i \alpha_i \left(\mathbb{E}_{D_i \sim F_i} \left[\min\left\{D_i, \mathcal{A}_i(D_i) + y_i\right\} - (c_{AV} + c_P)\mathcal{A}_i(D_i) \right] - t y_i \right) \\ \text{s.t. } & 0 \leq \mathcal{A}_i(D_i) \leq \min\{D_i, K\}, \forall i. \end{aligned}$$

Notice that in each scenario i, for given K and y_i the objective above is non-decreasing in $\mathcal{A}_i(D_i) \in \left[0,(D_i-y_i)^+\right]$ and non-increasing in $\mathcal{A}_i(D_i) \geq (D_i-y_i)^+$. Combined with the constraint that $\mathcal{A}_i(D_i) \leq \min\{D_i, K\}$, we conclude that at optimality we must have

$$\mathcal{A}_i(D_i) = \min\left\{(D_i-y_i)^+, K\right\}, \forall i. \tag{40}$$

Plugging this into the objective, we equivalently solve:

$$\max_{\vec{\mathbf{y}}} \sum_i \alpha_i \left(\mathbb{E}_{D_i\sim F_i}\left[\min\{D_i, K+y_i\} - (c_{AV}+c_P)\min\left\{(D_i-y_i)^+, K\right\}\right] - ty_i\right).$$

Since the optimization problem now contains a single variable, from the first-order condition on y_i we conclude that the objective increases with respect to y_i when $\mathbb{P}[y_i < D_i \leq K+y_i](c_{AV}+c_P) + \mathbb{P}[D_i > K+y_i] > t$ and the objective decreases with respect to y_i when $\mathbb{P}[y_i < D_i \leq K+y_i](c_{AV}+c_P) + \mathbb{P}[D_i > K+y_i] < t$. Thus, the smallest optimal solution for $\vec{\mathbf{y}}$ satisfies

$$y_i^s(c_P, K) := \min\left\{y_i \middle| \mathbb{P}[D_i > K+y_i] + \mathbb{P}[y_i < D_i \leq K+y_i]\cdot(c_{AV}+c_P) \leq t\right\}, \forall i.$$

Moreover, from Eq. (40) we obtain $\mathcal{A}_i^s(D_i|c_P, K) = \min\left\{(D_i - y_i^s(c_P,K))^+, K\right\}, \forall i.$ □

D.5.4. Proof of Proposition 10

Proof. We construct an instance I with two scenarios D_1 and D_2, which respectively occurs with probability $\alpha_1 = 0.001$ and $\alpha_2 = 0.999$. $D_1 = 10$ deterministically, and

$$D_2 = \begin{cases} 10 & \text{w.p. } 0.002 \\ 20 & \text{w.p. } 0.998. \end{cases}$$

Let $c_{AV} = 0.001, t = 0.999\cdot\mathbb{E}[D_2]/20$ and $c_F = 0.996005$. We show that the centralized solution sets $K^\star = 10$ and $\vec{\mathbf{y}}^\star = [0,10]$, whereas an equilibrium solution given any $c_P \geq 0$ yields $K^s(c_P) = 0, \vec{\mathbf{y}}^s(c_P, K^s(c_P)) = [10,10]$.

In a centralized solution, $c_P = 0$. If $K = 0$ we find from Lemma 1 that the resulting $\vec{\mathbf{y}}^s(c_P = 0, K = 0) = [10,10]$. This means that ICs are fully utilized and $c_I^i = t\,\forall i$, leading to a profit of $10(1-t)$. On the other hand, for any $K > 0$ we find that $\mathbb{P}[D_i > K+y_i] + \mathbb{P}[y_i < D_i \leq K+y_i]\cdot c_{AV} = 0.998 + 0.002c_{AV} > t$ for any $y \in [0,10)$ and thus the resulting $y_2^s(c_P = 0, K) = 10$. This means that AVs would only be used in scenario 2 when $D_2 = 20$. We also find that $y_1^s(c_P = 0, K) = 10 - K$. Thus, when $K \in [0,10)$ the marginal profit of using AV is $\alpha_1(t - c_{AV}) + \alpha_2 0.998(1-c_{AV}) - c_F > 0$, and when $K \geq 10$ the marginal profit of using AV is at most $\alpha_2 0.998(1-c_{AV}) - c_F < 0$. Therefore, $K^\star = 10$ and $\vec{\mathbf{y}}^\star = [0,10]$. The supply chain profit in a centralized solution is then given by $\alpha_1 10(1-c_{AV}) + \alpha_2\left[10(1-t) + 0.998\cdot 10\cdot(1-c_{AV})\right] - 10c_F$.

In an SPE, if $c_P < c_F$, the supplier trivially responds by setting $K^s(c_P) = 0$, and thus the platform sets $\vec{\mathbf{y}}^s(c_P, K^s(c_P)) = [10,10]$. We then focus on the case where $c_P \geq c_F$. From Lemma 1 we find that for any $K > 0$ and $c_P \geq 0$ we would have $y_2^s(c_P, K) = 10$. When $c_P \in [c_F, t - c_{AV}]$, the marginal profit for the AV supplier to invest in AV is thus upper bounded by $c_P(\alpha_1 + \alpha_2\cdot 0.998) - c_F < 0$ by construction of I, through which we know that $K^s(c_P) = 0$. When $c_P \in (t - c_{AV}, 1 - c_{AV}]$, platform resorts to only ICs in scenario 1. Because in this case the marginal profit of AV investment is upper bounded by $(1-c_{AV})\alpha_2\cdot 0.998 - c_F < 0$, we again have $K^s(c_P) = 0$. Finally, when $c_P > 1 - c_{AV}$, the platform trivially uses no AVs and the supplier sets $K^s(c_P) = 0$. Thus, for all c_P we find that the resulting $K^s(c_P) = 0$ and $\vec{\mathbf{y}}^s(c_P, K^s(c_P)) = [10,10]$. This implies a supply chain profit of $10(1-t)$. We thus find that

$$\frac{V^\star(I)}{V^{ED}(I|c_P)} = \frac{\alpha_1 10(1-c_{AV}) + \alpha_2\left[10(1-t) + 0.998\cdot 10\cdot(1-c_{AV})\right] - 10c_F}{10(1-t)} \geq 1.49, \forall c_P \geq 0.$$

This completes the proof. □

D.5.5. Proof of Proposition 9

Proof. We present the proof of Proposition 9 after that of Proposition 10 as it can be shown as a direct corollary of the latter. In particular, since a centralized solution for the instance I constructed in Proposition 10 satisfies $y_2^\star > 0, \mathcal{H}_2^\star(10) = 10$ and $\mathcal{A}_2^\star(10) = 0 < \min\{10, K^\star\} = 10$, we immediately obtain Proposition 9 (i).

To prove Proposition 9 (ii), it suffices to modify instance I to I' by taking $c_F = 0.996$. This construction ensures that all analyses for $c_P < c_F, c_P \in [c_F, t - c_{AV}]$ and $c_P > 1 - c_{AV}$ still hold, but when $c_P \in (t - c_{AV}, 1 - c_{AV}]$ the marginal profit of AV investment now becomes upper bounded by $(1 - c_{AV})\alpha_2 \cdot 0.998 - c_F > 0$ for $K \in [0, 10]$. By taking $c_P = c_F / (\alpha_2 \cdot 0.998) < 1 - c_{AV}$, the platform thus ensures that $K^s(c_P) = 10$. We verify that this strictly increases profit of the platform than taking $K = 0$ and $\vec{\mathbf{y}} = [10, 10]$. Thus, for I' we have $K^s = 10$ and $\vec{\mathbf{y}}^s = [10, 10]$. This instance satisfies $y_2^s > 0, \mathcal{H}_2^s(10) = 10$ and $\mathcal{A}_2^s(10) = 0 < \min\{10, K^s\} = 10$, which proves Proposition 9 (ii). □

D.5.6. Proof of Proposition 11

Proof. To show that (i) is true for any $I \in \mathcal{I}_P$, we argue by contradiction: when $c_{AV} + c_P \le c_I$, if it is not optimal to fully prioritize AVs, i.e., we always have $\mathcal{A}_i^s\left(D_i | c_P, K\right) < \min\left\{D_i, K\right\}$ in some scenario i, we show that we can construct another feasible solution that leads to at least the same profit for the platform as an optimal solution. In particular, we take $\mathcal{A}_i'(D_i) = \min\left\{D_i, K\right\}$,

$$\mathcal{H}_i'(D_i) = \left(\mathcal{H}_i^s\left(D_i | c_P, K\right) - \left(\mathcal{A}_i'(D_i) - \mathcal{A}_i^s\left(D_i | c_P, K\right)\right)\right)^+,$$
$$y_i' = y_i^s\left(c_P, K\right) - \frac{c_I}{t}\min\left\{\mathcal{H}_i^s\left(D_i | c_P, K\right), \mathcal{A}_i'(D_i) - \mathcal{A}_i^s\left(D_i | c_P, K\right)\right\}.$$

We start by showing that this construction is feasible, and then show that it is also optimal.

We first observe that the equilibrium condition for ICs, i.e., Eq. (8), becomes $ty_i = c_I \mathcal{H}_i\left(D_i\right)$ for $I \in \mathcal{I}_P$ as D_i becomes deterministic. Thus, $ty_i^s\left(c_P, K\right) = c_I \mathcal{H}_i^s\left(D_i | c_P, K\right)$ holds. Then, by construction of y_i' we have

$$\begin{aligned}
y_i' &= y_i^s\left(c_P, K\right) - \frac{c_I}{t}\min\left\{\mathcal{H}_i^s\left(D_i | c_P, K\right), \mathcal{A}_i'(D_i) - \mathcal{A}_i^s\left(D_i | c_P, K\right)\right\} \\
&= y_i^s\left(c_P, K\right) - \frac{c_I}{t}\left(\mathcal{H}_i^s\left(D_i | c_P, K\right) - \mathcal{H}_i'(D_i)\right) \\
&= y_i^s\left(c_P, K\right) - y_i^s\left(c_P, K\right) + \frac{c_I}{t}\mathcal{H}_i'(D_i) = \frac{c_I}{t}\mathcal{H}_i'(D_i),
\end{aligned}$$

so that y_i' satisfies Eq. (8). In particular, notice that $\mathcal{H}_i'(D_i) \le y_i'$ is still guaranteed. Moreover, since $\mathcal{A}_i^s\left(D_i | c_P, K\right)$ and $\mathcal{H}_i^s\left(D_i | c_P, K\right)$ are feasible for Eq. (5), Eq. (6) and Eq. (7), by reallocating

$$\min\left\{\mathcal{H}_i^s\left(D_i | c_P, K\right), \mathcal{A}_i'(D_i) - \mathcal{A}_i^s\left(D_i | c_P, K\right)\right\}$$

units of demand served by ICs to be served by AVs in scenario i, the solution $\mathcal{A}'$ and $\mathcal{H}'$ remain feasible to these three constraints.

Finally, observe that $\mathcal{A}'$ and $\mathcal{H}'$ lead to a change in profit of at least

$$\left((1 - c_{AV} - c_P) - (1 - c_I)\right) \cdot \min\left\{\mathcal{H}_i^s\left(D_i | c_P, K\right), \mathcal{A}_i'(D_i) - \mathcal{A}_i^s\left(D_i | c_P, K\right)\right\} \ge 0,$$

which is a contradiction.

We omit the proof of (ii) as it is exactly symmetrical to the construction above, i.e., we reallocate demand served by AVs to be served by ICs and show that the change in platform's profit is positive. □

D.5.7. Proof of Proposition 12

Proof. For $I \in \mathcal{I}$, we construct

$$\lambda := \frac{V_A^s(I) + c_F K^\star}{\sum_i \alpha_i \left((1-c_{AV}) \mathbb{E}_{D_i \sim F_i}\left[\mathcal{A}_i^\star(D_i)\right] + (1-c_I)\frac{t}{c_I} y_i^\star \right)} \geq 0$$

and propose a contract $\pi \in \Pi^S$ with $K^\pi = K^\star, c_P^\pi = \lambda(1-c_{AV})$ and $c_R^\pi = \lambda(1-c_I)$. Now we start by showing that the platform adopts $\vec{\mathbf{y}} = \vec{\mathbf{y}}^\star$ and $\mathcal{A} = \mathcal{A}^\star$ when π is accepted, and then show that π is indeed accepted.

Given π, the optimization problem that the platform faces is

$$\max_{\vec{\mathbf{y}}, \mathcal{A}, \mathcal{H}} \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[(1-c_{AV}-c_P^\pi)\mathcal{A}_i(D_i) + (1-c_I-c_R^\pi)\mathcal{H}_i(D_i)\right],$$

subject to Eq. (5) - Eq. (8) and $K = K^\star$. By plugging in the values of c_P^π and c_R^π, we find that the platform equivalently solves

$$\max_{\vec{\mathbf{y}}, \mathcal{A}, \mathcal{H}} (1-\lambda) \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[(1-c_{AV})\mathcal{A}_i(D_i) + (1-c_I)\mathcal{H}_i(D_i)\right] \tag{41}$$

subject to Eq. (5) - Eq. (8) and $K = K^\star$. Since the objective of Eq. (41) is a constant multiple of that in Eq. (4) when $c_P = 0$ and $K = K^\star$ and the two optimization problems have the same constraints, by Lemma 2 we know that an optimal solution for $\vec{\mathbf{y}}$ in Eq. (41) is $y_i^s(c_P = 0, K^\star) = y_i^\star, \forall i$.

Moreover, we have

$$\begin{aligned}\sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[\mathcal{A}_i^s(D_i | c_P = 0, K^\star)\right] &= \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, K^\star + y_i^\star\} - \mathcal{H}_i^s(D_i | c_P = 0, K^\star)\right] \\ &= \sum_i \alpha_i \left(\mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, K^\star + y_i^\star\}\right] - \frac{t}{c_I} y_i^\star \right),\end{aligned} \tag{42}$$

where the first equality comes from the optimality condition in Eq. (32) and the second equality comes from the constraint in Eq. (8). Given that it is optimal for the platform to use $y_i^\star$ $\forall i$, the objective value in Eq. (41) becomes deterministic and any solution for $\mathcal{A}$ that satisfies Eq. (42) is optimal for Eq. (41). In particular, this implies that it is optimal to set $y_i = y_i^\star, \mathcal{A}_i = \mathcal{A}_i^\star$ $\forall i$.

For the supplier, then, by the construction of λ and π we have

$$\begin{aligned}V_A^\pi(I) &= \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[\lambda(1-c_{AV})\mathcal{A}_i^\star(D_i) + \lambda(1-c_I)\mathcal{H}_i(D_i)\right] - c_F K^\star \\ &= \lambda \sum_i \alpha_i \left((1-c_{AV}) \mathbb{E}_{D_i \sim F_i}\left[\mathcal{A}_i^\star(D_i)\right] + (1-c_I)\frac{t}{c_I} y_i^\star \right) - c_F K^\star = V_A^s(I),\end{aligned}$$

where the second equality comes from the constraint in Eq. (8). Thus, both conditions in Lemma 4 are satisfied and we know that π is accepted. In particular, since π sets $K^\pi = K^\star$ and leads the platform to set $\vec{\mathbf{y}}^\star$ and $\mathcal{A}^\star$, the centralized solution is adopted and $\mathrm{PR}^\pi(I) = 1$.

□

D.5.8. Proof of Corollary 1

Proof. Similar to the proof of Theorem 2, where we provided a construction for $\pi' \in \Pi^U$ that aligns the supply chain, we now construct $\pi \in \Pi^C$ with $K^\pi = K^\star, \widetilde{\mathcal{A}}^\pi$ and

$$c_P^\pi = \frac{V_A^s(I) + c_F K^\star}{\sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[\mathcal{A}_i^\star(D_i)\right]} \geq 0.$$

Notice that π is the same as π' except that it sets $\widetilde{\mathcal{A}}^{\pi}$ instead of $\mathcal{A}^{\pi'} = \mathcal{A}_i^\star\ \forall i$. Thus, it suffices to construct some $\widetilde{\mathcal{A}}^{\pi}$ that ensures that the platform adopts $\mathcal{A}_i = \mathcal{A}_i^\star\ \forall i$, so that the rest of the proof follows exactly the arguments for Theorem 2.

We start by explicitly constructing a pair of dispatch policies $\mathcal{A}$ and $\mathcal{H}$ that induce $\vec{\mathbf{y}}^\star$. By Eq. (33) we know that, for given $K^\star$, $\vec{\mathbf{y}}^\star$ uniquely determines the optimal supply chain profit, and thus any $\mathcal{A}$ and $\mathcal{H}$ that fulfill Eq. (10) when $\vec{\mathbf{y}} = \vec{\mathbf{y}}^\star$ must be optimal to the centralized problem. We specifically construct, in any scenario i, an optimal pair of solutions $\mathcal{A}_i^\star(D_i)$ and $\mathcal{H}_i^\star(D_i)$ that are non-decreasing in D_i. In particular, we define

$$\beta_i := \min\left\{\beta \geq 0 \middle| \mathbb{E}_{D_i\sim F_i}\left[\min\left\{(D_i - K^\star)^+ + \beta, y_i^\star, D_i\right\}\right] = \frac{t}{c_I} y_i^\star\right\}.$$

Notice that when $\beta = y_i^\star$ we have

$$\mathbb{E}_{D_i\sim F_i}\left[\min\left\{(D_i - K^\star)^+ + \beta, y_i^\star, D_i\right\}\right] = \mathbb{E}_{D_i\sim F_i}\left[\min\left\{y_i^\star, D_i\right\}\right] \geq \frac{t}{c_I} y_i^\star$$

by the fact that $y_i^\star \leq y_i^{ub}$. Then, since $\mathbb{E}_{D_i\sim F_i}\left[\min\left\{(D_i - K^\star)^+ + \beta, y_i^\star, D_i\right\}\right]$ is non-decreasing in β, we know that $\beta_i \in [0, y_i^\star]$. Now we construct

$$\begin{aligned}
\mathcal{H}_i^\star(D_i) &= \min\left\{(D_i - K^\star)^+ + \beta_i, y_i^\star, D_i\right\},\\
\mathcal{A}_i^\star(D_i) &= \min\left\{D_i, K^\star + y_i^\star\right\} - \mathcal{H}_i^\star(D_i) = \max\left\{\min\left\{D_i, K^\star\right\} - \beta_i, \min\left\{D_i - y_i^\star, K^\star\right\}, 0\right\},
\end{aligned}$$

both of which are non-decreasing in D_i and fulfill Eq. (10).

We then find a mapping from $\mathcal{H}_i$ to $\widetilde{\mathcal{A}}^{\pi}$ and show that such $\widetilde{\mathcal{A}}^{\pi}$ ensures that the platform adopts $\mathcal{A}_i = \mathcal{A}_i^\star\ \forall i$. We start by defining

$$\mathcal{D}_i\left(\mathcal{H}_i\right) := \min\left\{d \middle| \mathcal{H}_i^\star(d) = \mathcal{H}_i\right\},$$

a function that allows the supplier to map $\mathcal{H}_i$ back to a lower bound on D_i, and then construct

$$\widetilde{\mathcal{A}}_i^{\pi}\left(\mathcal{H}_i\right) := \mathcal{A}_i^\star(\mathcal{D}_i\left(\mathcal{H}_i\right)).$$

In any scenario i, by the same argument as in the proof of Theorem 2 we know that given $c_P^\pi \geq 0$ the platform has no incentive to use $y_i < y_i^\star$. We next show that the platform also cannot use $y_i > y_i^\star$ given the requirement on $\widetilde{\mathcal{A}}^{\pi}$, so that $y_i = y_i^\star$ is guaranteed. To see this, we argue by contradiction: in order for the platform to use $y_i > y_i^\star$, by the constraint in Eq. (8) the platform must adopt some dispatch policy

$$\mathcal{H}_i(D_i) > \mathcal{H}_i^\star(D_i) = \min\left\{(D_i - K^\star)^+ + \beta_i, y_i^\star, D_i\right\}.$$

Then, from the fact that $\mathcal{H}_i^\star(D_i)$ is non-decreasing in D_i we know that $\mathcal{D}_i\left(\mathcal{H}_i\left(D_i\right)\right) \geq D_i$. Thus, from the fact that $\mathcal{A}_i^\star(D_i)$ is also non-decreasing in D_i we know that π requires the platform to use at least

$$\widetilde{\mathcal{A}}_i^{\pi}\left(\mathcal{H}_i\left(D_i\right)\right) = \mathcal{A}_i^\star(\mathcal{D}_i\left(\mathcal{H}_i\left(D_i\right)\right)) \geq \mathcal{A}_i^\star(D_i)$$

units of AVs. Since

$$\widetilde{\mathcal{A}}_i^{\pi}\left(\mathcal{H}_i\left(D_i\right)\right) + \mathcal{H}_i(D_i) > \mathcal{A}_i^\star\left(D_i\right) + \mathcal{H}_i^\star(D_i) = \min\left\{D_i, K^\star + y_i^\star\right\},$$

such dispatch policy $\mathcal{H}_i\left(D_i\right)$ is not feasible.

In any scenario i, for given $K^\star$ we know by Eq. (33) that $y_i = y_i^\star$ uniquely determines the optimal profit for the platform. Since $\mathcal{A}_i^\star$ is feasible for Eq. (5) - Eq. (8) when $y_i = y_i^\star$, it is optimal for the platform to set $\mathcal{A}_i = \mathcal{A}_i^\star$, which completes the proof.

□

D.5.9. Proof of Corollary 2

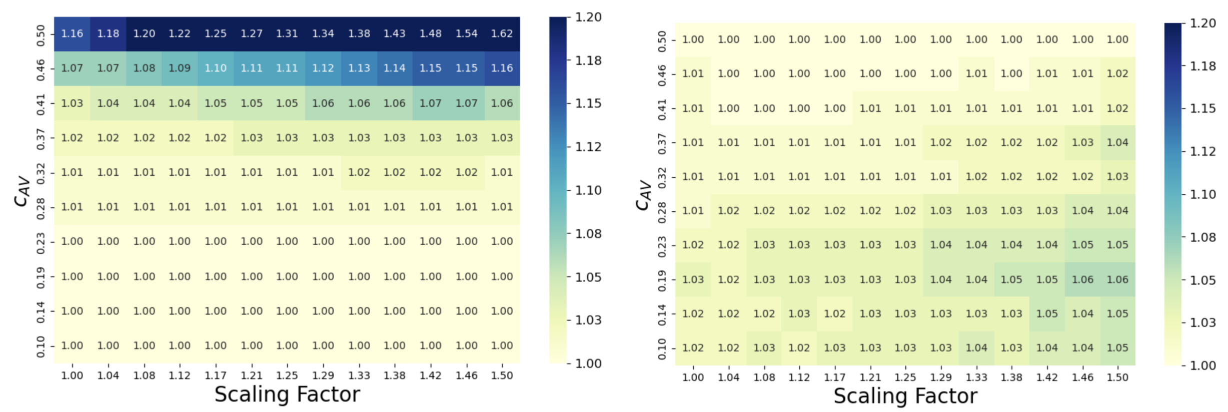
Proof. Since in the proof of Theorem 4 we have provided a construction for $\pi' \in \Pi^F$ that aligns the supply chain, we similarly construct $\pi \in \Pi^H$ with $K^\pi = K^\star, \mathcal{H}^\pi = 0$ and

$$c_P^\pi = \frac{V_A^s(I) + c_F K^\star}{\sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, K^\star\}\right]}.$$

Notice that π is the same as π' except that it sets $\mathcal{H}^\pi = 0$ instead of $\mathcal{A}^{\pi'}(D_i) = \min\{D_i, K^\star\}\ \forall i$. Thus, it suffices to show that $\mathcal{H}^\pi = 0$ implies that the platform adopts $\mathcal{A}_i(D_i) = \min\{D_i, K^\star\}\ \forall i$, so that the rest of the proof follows from Theorem 4. Indeed, from $\mathcal{H}^\pi = 0$ we know that the platform has to adopt $\vec{\mathbf{y}} = \vec{0}$ by the constraint in Eq. (8). Then, by the optimality condition in Eq. (32) we know the platform adopts $\mathcal{A}_i(D_i) = \min\{D_i, K^\star + 0\} - 0 = \min\{D_i, K^\star\}\ \forall i$. Then, π is equivalent to π' and we conclude the proof. $\square$

Appendix E: Additional Simulation Results

E.1. Profit Ratio as Costs Scale



(a) $\mathrm{PR}^{AV}(I)$ (b) $\mathrm{PR}^{SL}(I)$

Figure 8 Heatmap for profit ratio across different initial values of c_{AV} and scaling factors. The initial value of c_I and c_F are respectively fixed at 0.6 and 0.1. Each number in the plot reports the average profit ratio over 12 different demand distribution.

E.2. Capacity Decisions with Respect to Expected Value and Variance of Demand

In this subsection we design a sequence of instances that demonstrate interesting patterns of K^s with respect to expected value and variance of demand. Specifically, we take $c_{AV} = 0.81, c_I = 1 - 10^{-7}, c_F = 0.18991$ and $t = \frac{0.5(40 \cdot 0.0025 + 100 \cdot 0.9975) + 0.5 \cdot 75}{100} \cdot c_I$. We construct two demand scenarios that each occurs with probability $0.5:$

$$D_1 = \begin{cases} 40 & \text{w.p. } 1-\alpha \\ 100 & \text{w.p. } \alpha. \end{cases}$$

and $D_2 = 75$ deterministically. As we increase α from 0.995 to 1, expected demand increases and variance of demand decreases. In Fig. 9 we plot $K^\star$ and K^s with respect to the expected value and variance of demand.[13] We find in Fig. 9a that, as demand monotonically increases (in a stochastic dominance sense), the equilibrium capacity decision K^s can either increase or decrease, with higher demand potentially leading to lower AV investments. Similarly, as demonstrated in Fig. 9b, K^s can also either decrease or increase as variance of demand increases.

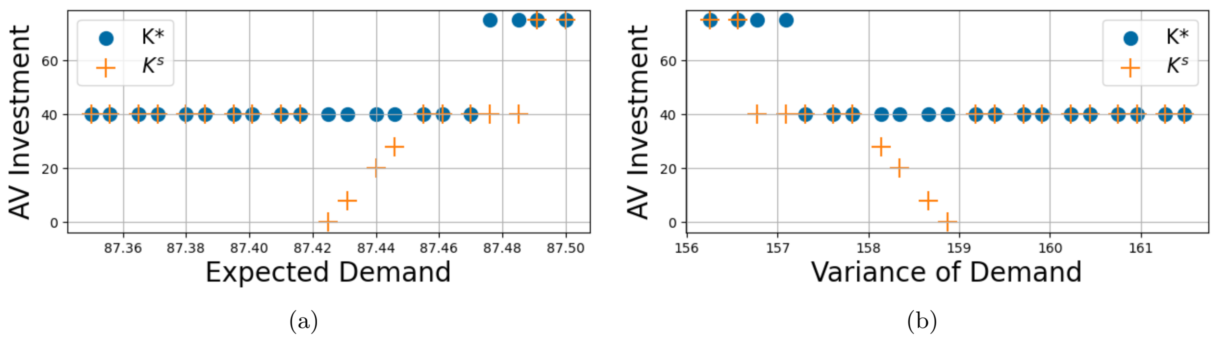


Figure 9 Instances where K^s demonstrates non-monotonic patterns with respect to expected value and variance of demand.

These findings are in contrast to the conventional wisdom in supply chain management that the risk of under-investment arises from low demand and high variance. Intuitively, when $\alpha \in [0.995, 0.9975)$, the incentives of the platform and the AV supplier are well-aligned, with $K^\star = K^s = 40$. However, when $\alpha = 0.9975$ the platform's optimal dispatch decision has a regime shift, moving from covering D_1 using only available AVs to sourcing additional ICs to cover $D_1 = 100$. In doing so, the platform underutilizes AVs, and the supplier thus responds by setting $K^s < K^\star$. This is the parameter setting where supply chain misalignment is the most prominent. Finally, as α increases to 1, the supply chain becomes re-aligned as the AV supplier benefits from increasing demand. Given sufficiently high expected demand, the AV supplier starts to invest again, leading to $K^\star = K^s = 75$.

[13] We numerically solve optimal values of K to closet integer values and c_P to a precision level of 10^{-7}.