Appendix E: Proofs of the Remaining Results

E.1. Proofs of the Remaining Results in Section 3 and Section 4

E.1.1. Proof of Proposition 1

Proof. We prove Proposition 1 constructively using an instance I with $\vec{\alpha} = \left[3/4, 1/4\right], c_I = 1/4, t = 1/4, c_{AV} = 1/8$ and $c_F = 1/30$. Moreover, the demand distributions in scenario 1 and 2 are respectively given by $D_1 = 10$ and

$$D_2 = \begin{cases} 10 & \text{w.p. } 3/4 \\ 20 & \text{w.p. } 1/4. \end{cases}$$

Step 1: find the SPE

We start by showing that in the unique subgame perfect equilibrium the platform sets $c_P^s = 8/195$, the AV supplier sets $K^s = 10$, and the platform set $\vec{\mathbf{y}}^s = [0, 10]$. Moreover, the platform's dispatch policies for AVs are $A_1^s(D_1) = 10$ and

$$A_2^s(D_2) = \begin{cases} 0 & \text{when } D_2 = 10 \\ 10 & \text{when } D_2 = 20. \end{cases}$$

Observe, in particular, that the platform dispatches no AVs in scenario 2 when $D_2 = 10$ despite $c_{AV} + c_P^s < c_I$ and $K^s > 0$. In contrast, the platform's dispatch policies for ICs are $H_1^s(D_1) = 0$ and

$$H_2^s(D_2) = \begin{cases} 10 & \text{when } D_2 = 10 \\ 10 & \text{when } D_2 = 20. \end{cases}$$

That is, $y_2^s > 0$ and $H_2^s\left(D_2\right) = \min\left\{D_2, y_2^s\right\}$, so ICs are fully prioritized in scenario 2. Thus, this construction satisfies Proposition 1 (i) and (iii).

To verify the subgame perfect equilibrium solution above, we consider c_P within four sets of possible values, and prove that $c_P = 8/195$ is indeed the equilibrium outcome. We start by observing that when $c_P < c_F = 1/30$ the AV supplier cannot break even with any $K > 0$. Thus, $K^s(c_P) = 0$ in this case and the platform can only serve the demand with ICs, which leads to

$$y_1^s\left(c_P, K^s(c_P)\right) = \min\left\{y_1^{ub}, \bar{y}_1\right\} = 10 \text{ and } y_2^s\left(c_P, K^s(c_P)\right) = \min\left\{y_1^{ub}, \bar{y}_1\right\} = 10$$

by Lemma 3. Thus, the profit for the platform in this case is $(1 - c_I)10 = 7.5$.

Next, when $c_F \le c_P < 8/195$, we find

$$\bar{F}_1^{-1}\left(\frac{t(c_I - c_{AV} - c_P)}{c_I(1 - c_{AV} - c_P)}\right) = 10 \text{ and } \bar{F}_2^{-1}\left(\frac{t(c_I - c_{AV} - c_P)}{c_I(1 - c_{AV} - c_P)}\right) = 20.$$

Now, for $0 < K \le 10$ we have $\bar{y}_2 = 20 - K \ge 10$, so that $y_2^s\left(c_P, K\right) = y_2^{ub} = 10$ by Eq. (23) and $H_2^s\left(D_2 | c_P, K\right) = 10$ by the equilibrium condition in Eq. (8). Then, the marginal profit for the AV supplier to own a unit of AV is at most

$$c_P\left(\alpha_1 + \alpha_2 \mathbb{P}\left[D_2 = 20\right]\right) - c_F < 0$$

because AVs are not used when $D_2 = 10$. On the other hand, if $K > 10$ we find that the marginal profit for the AV supplier to own more than 10 units of AV is at most $c_P\alpha_2 - c_F < 0$ because the highest possible demand in scenario 1 is 10. Thus, with $c_F \le c_P < 8/195$ we again find $K^s\left(c_P\right) = 0$ and the profit for the platform is 7.5.

When $8/195 \le c_P \le c_I - c_{AV}$, we again find that when $K > 10$ the marginal profit for the AV supplier to own more than 10 units of AV is at most $c_P\alpha_2 - c_F < 0$. Then, when $0 < K \le 10$ we solve

$$y_1^s(c_P, K) = y_1^{lb} = \bar{y}_1 = 10 - K \text{ and } y_2^s(c_P, K) = y_2^{ub} = 10 < \bar{y}_2 = 20 - K,$$

which implies that $A_1^s\left(D_1|c_P, K\right) = K$ and $A_2^s\left(D_2|c_P, K\right) = K$ when $D_2 = 20$. Thus, with $c_P \ge 8/195$, the marginal profit for the AV supplier to own a unit of AV becomes

$$c_P\left(\alpha_1 + \alpha_2\mathbb{P}\left[D_2 = 20\right]\right) - c_F \ge 0, \tag{30}$$

and the supplier responds by setting $K^s(c_P) = 10$. Since by construction $8/195$ is the minimum c_P that leads to an equality in Eq. (30) (and consequently the minimum c_P that leads to $K^s(c_P) = 10$), it is optimal for the platform to set $c_P = 8/195$ and $\vec{\mathbf{y}}^s\left(c_P, K^s(c_P)\right) = [0, 10]$. By Eq. (2) and Eq. (8), this solution yields a profit of

$$\left(1 - c_{AV} - c_P\right)\left(\alpha_1 + \alpha_2\mathbb{P}\left[D_2 = 20\right]\right)K^s(c_P) + (1 - c_I)\,\alpha_2 y_2^s\left(c_P, K^s(c_P)\right) \approx 8.65$$

for the platform. In particular, this profit is higher than the solution that involves only ICs.

Finally, when $c_P > c_I - c_{AV}$, the variable cost of AVs becomes higher than that of the ICs, so the platform uses as many ICs as possible by setting

$$y_1^s(c_P, K) = y_1^{ub} = 10 \text{ and } y_2^s(c_P, K) = y_2^{ub} = 10$$

for any K. Since AVs are only used when ICs are insufficient for covering all demand, i.e., when $D_2 = 20$, the marginal profit for owning a unit of AV is at most $\alpha_2\mathbb{P}\left[D_2 = 20\right] - c_F < 0$. Since the supplier again cannot break even with any $K > 0$, the platform adopts the IC-only solution $\vec{\mathbf{y}}^s\left(c_P, K^s(c_P) = 0\right) = [10, 10]$, which we know is not optimal. We thus conclude that $c_P = 8/195$ is indeed the equilibrium outcome.

Step 2: find the centralized solution

We next argue that the unique centralized solution is to set $K^\star = 10$: for $0 \le K \le 10$ the marginal profit for owning a unit of AV is at least $(c_I - c_{AV})\alpha_1 - c_F > 0$, which is the increase in profit for the platform to use AVs to serve demand that could otherwise be served with ICs. On the other hand, for $K > 10$, since $y_2^{ub} = 10$ and $y_2^{lb} = 0$ we have $y_2^s(c_P = 0, K) = \bar{y}_2 = 20 - K$. Then, the marginal profit for owning an additional unit of AV is at most $(c_I - c_{AV})\alpha_2 - c_F < 0$ since the demand could otherwise be served with ICs. Given $K^\star = 10$, in the centralized problem the platform incurs no cost of c_P, so by plugging $c_P = 0$ and $K^\star$ into Lemma 3 we find

$$\bar{y}_1 = \bar{F}_1^{-1}\left(\frac{t(c_I - c_{AV})}{c_I(1 - c_{AV})}\right) - 10 = 0 \text{ and } \bar{y}_2 = \bar{F}_2^{-1}\left(\frac{t(c_I - c_{AV})}{c_I(1 - c_{AV})}\right) - 10 = 10.$$

By construction of the demand distributions we also have $y_i^{ub} = 10$ and $y_i^{lb} = 10 - K^\star = 0\,\forall i$, so $\vec{\mathbf{y}}^\star = [0, 10]$. Finally, the solutions for $K^\star$ and $\vec{\mathbf{y}}^\star$ uniquely lead to $H_1^\star(D_1) = 0$ and

$$H_2^\star(D_2) = \begin{cases} 10 & \text{when } D_2 = 10 \\ 10 & \text{when } D_2 = 20. \end{cases}$$

That is, $y_2^\star > 0$ and $H_2^\star(D_i) = \min\{D_i, y_2^\star\}$. This also implies that $A_2^\star(10) \le 10 - H_2^\star(10) = 0 < 10 = \min\{D_2, K^\star\}$ when $D_2 = 10$, so Proposition 1 (ii) also holds.

□

E.1.2. Proof of Proposition 2

Proof. We consider an equilibrium s that always breaks ties in favor of the AV supplier. For any instance $I \in \mathcal{I}$ and fixed K, we consider the cases where $0 < c'_P \leq c_I - c_{AV}$ and $c'_P > c_I - c_{AV}$ separately. When $0 < c_P \leq c'_P < c_I - c_{AV}$, we claim that the smallest optimal solution for y_i constructed in Lemma 3 is also the optimal solution that is most favored by the AV supplier.

Claim 3. *Given any $c_P \in (0, c_I - c_{AV}]$ and K, among the optimal solutions for $(\vec{\mathbf{y}}, A, H)$ in Eq. (4), the ones that maximize Eq. (3) contain the smallest optimal y_i in each scenario i.*

Then, we observe that

$$\bar{F}_i^{-1}\left(\frac{t(c_I - c_{AV} - c_P)}{c_I(1 - c_{AV} - c_P)}\right) - K \leq \bar{F}_i^{-1}\left(\frac{t(c_I - c_{AV} - c'_P)}{c_I(1 - c_{AV} - c'_P)}\right) - K.$$

Since we also know that the constructions of y_i^{ub} and y_i^{lb} depend only on I and K, by Lemma 3 we obtain $y_i^s\left(c_P, K\right) \leq y_i^s\left(c'_P, K\right), \forall i$.

When $c'_P > c_I - c_{AV}$, by Lemma 4 we have the unique optimal solution $y_i^s\left(c'_P, K\right) = y_i^{ub}, \forall i$. Thus, for any c_P we have $y_i^s\left(c_P, K\right) \leq y_i^{ub} = y_i^s\left(c'_P, K\right), \forall i$. Since the above holds in both cases, by the equality constraint in Eq. (8) we obtain

$$\mathbb{E}_{D_i \sim F_i}\left[H_i^s\left(D_i | c_P, K\right)\right] \leq \mathbb{E}_{D_i \sim F_i}\left[H_i^s\left(D_i | c'_P, K\right)\right], \forall i.$$

For the analyses of $A_i^s\left(D_i | c'_P, K\right)$, notice that we may assume without loss of generality that $c'_P < 1 - c_{AV}$, because otherwise the platform uses no AVs given c'_P and we trivially have

$$\mathbb{E}_{D_i \sim F_i}\left[A_i^s\left(D_i | c_P, K\right)\right] \geq \mathbb{E}_{D_i \sim F_i}\left[A_i^s\left(D_i | c'_P, K\right)\right] = 0, \forall i.$$

When $c'_P < 1 - c_{AV}$, given $H_i^s\left(D_i | c'_P, K\right)$ we observe that we must have

$$A_i^s\left(D_i | c'_P, K\right) = \min\left\{K, D_i - H_i^s\left(D_i | c'_P, K\right)\right\}, \forall i, D_i \tag{31}$$

because when $A_i^s\left(D_i | c'_P, K\right)$ is smaller we can increase objective in (4) without violating any feasibility constraint, and $A_i^s\left(D_i | c'_P, K\right)$ also cannot be larger by constraint (5) and (7). Similarly, we find that

$$A_i^s\left(D_i | c_P, K\right) = \min\left\{K, D_i - H_i^s\left(D_i | c_P, K\right)\right\}, \forall i, D_i. \tag{32}$$

Since $\mathbb{E}_{D_i \sim F_i}\left[H_i^s\left(D_i | c_P, K\right)\right] \leq \mathbb{E}_{D_i \sim F_i}\left[H_i^s\left(D_i | c'_P, K\right)\right]$ $\forall i$, we can always fix $H_i^s\left(D_i | c'_P, K\right)$ and find $H_i^s\left(D_i | c_P, K\right)$ such that

$$H_i^s\left(D_i | c_P, K\right) \leq H_i^s\left(D_i | c'_P, K\right), \forall i, D_i.$$

Thus, taking expectation over D_i in (31) and (32) we find

$$\begin{aligned}\mathbb{E}_{D_i \sim F_i}\left[A_i^s\left(D_i | c_P, K\right)\right] &= \mathbb{E}_{D_i \sim F_i}\left[\min\left\{K, D_i - H_i^s\left(D_i | c_P, K\right)\right\}\right] \\ &\geq \mathbb{E}_{D_i \sim F_i}\left[\min\left\{K, D_i - H_i^s\left(D_i | c'_P, K\right)\right\}\right] = \mathbb{E}_{D_i \sim F_i}\left[A_i^s\left(D_i | c'_P, K\right)\right], \forall i.\end{aligned}$$

Finally, we provide a constructive proof that AV underutilization can be arbitrarily strong. Notice that we may assume without loss of generality that $\epsilon \in (0, 1/4)$ because the construction for a small ϵ value immediately implies the result for a larger choice of ϵ. Fix $\epsilon \in (0, 1/4)$, we know that we can find ϵ_1, ϵ_2 such that $0<\epsilon_1<\epsilon_2<\epsilon$. Then, we construct an instance I with $\vec{\alpha}=[1], c_I=t=1/4$ and $c_{AV}=\frac{1/4-\epsilon_1}{1-\epsilon_1}$. We take $c_P=0$ and $K=10$. Notice that this construction ensures $c_{AV}+c_P=c_{AV}<c_I, \forall \epsilon\in(0,1/4)$. We assume that the demand distribution in scenario 1, which is also the only scenario, is

$$D_1=\begin{cases}10 & \text{w.p. } 1-\epsilon_2\\ 20 & \text{w.p. } \epsilon_2.\end{cases}$$

By plugging $c_P=0$ and $K=10$ into Lemma 3 we find

$$\bar{y}_1=\bar{F}_1^{-1}\left(\frac{t(c_I-c_{AV})}{c_I(1-c_{AV})}\right)-10=\bar{F}_1^{-1}(\epsilon_1)-10=10.$$

By construction of the demand distribution we also have $y_1^{ub}=10$ and $y_1^{lb}=10-K=0$, so $y_1^s(c_P,K)=10$. Since $t/c_I=1$, ICs need to be fully utilized and $H_1^s(D_1|c_P,K)=10, \forall D_1$. Thus, $A_1^s\left(10|c_P,K\right)\le 10-H_1^s(10|c_P,K)=0$ and $A_1^s\left(20|c_P,K\right)\le 20-H_1^s(20|c_P,K)=10$. This implies that

$$\frac{\sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[A_i^s(D_i|c_P,K)\right]}{\sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[\min\{D_i,K\}\right]}\le \frac{10\cdot\epsilon_2}{10}=\epsilon_2<\epsilon.$$

This completes the proof.

□

E.1.3. Proof of Proposition 3

Proof. We start by finding the platform's solution of $\vec{\mathbf{y}}, A$ and H if $\pi^\star(I)$ is accepted by both the platform and the supplier, and then show that it is indeed in the interest of both sides to accept $\pi^\star(I)$. Because c_P^π and $K^\pi=K^\star$ are specified by the contract, the platform only needs to solve the third stage of the sequential game. Given $c_P^\pi=0$ and a fixed cost of $c_F K^\star+V_A^s(I)$ for AVs, the platform solves the following optimization problem:

$$\begin{aligned}\max_{\vec{\mathbf{y}},A,H}&\sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[(1-c_{AV})A_i(D_i)+(1-c_I)H_i(D_i)\right]-c_F K^\star-V_A^s(I)\\ \text{s.t. }& 0\le A_i(D_i)\le \min\{D_i,K^\star\},\forall i\\ & 0\le H_i(D_i)\le\min\{D_i,y_i\},\forall i\\ & A_i(D_i)+H_i(D_i)\le \min\{D_i,K^\star+y_i\},\forall i\\ & ty_i=c_I\mathbb{E}_{D_i\sim F_i}\left[H_i(D_i)\right],\forall i\end{aligned}$$

Since the fixed cost is independent of the choice of $\vec{\mathbf{y}}, A$ and H, the platform equivalently solves

$$\max_{\vec{\mathbf{y}},A,H}\sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[(1-c_{AV})A_i(D_i)+(1-c_I)H_i(D_i)\right] \tag{33}$$

subject to the same constraints above. Notice that (33) is exactly the inner problem of Eq. (9) when the optimal solution of $K^\star$ is given, so $\pi^\star(I)$ leads the platform to adopt the centralized solution $\vec{\mathbf{y}}^\star, A^\star$ and $H^\star$ if it is accepted.

Then, to show that it is in the interest of both sides to accept the contract, we make use of Lemma 5 below:

Lemma 5. *For an instance I with a centralized solution $(K^\star,\vec{\mathbf{y}}^\star,A^\star,H^\star)$, both the platform and the supplier accept a contract π if the following holds true:*

(i) $K^\pi = K^\star$ *and it is optimal for the platform to set* $\vec{\mathbf{y}} = \vec{\mathbf{y}}^\star, A = A^\star$ *when* π *is accepted;*

(ii) $V_A^\pi(I) = V_A^s(I)$.

Notice that $\pi^\star(I)$ ensures $K^\pi = K^\star$ and it is optimal for the platform to adopt $\vec{\mathbf{y}} = \vec{\mathbf{y}}^\star$, so Lemma 5 (i) is satisfied. Moreover, the supplier obtains a profit of $c_F K^\star + V_A^s(I) - c_F K^\star = V_A^s(I)$ if $\pi^\star(I)$ is accepted, so Lemma 5 (ii) is also satisfied. Thus, it is optimal for both sides to accept $\pi^\star(I)$ and adopt the centralized solution.

□

E.2. Proofs of the Auxiliary Results

E.2.1. Proof of Lemma 3

Proof. Since the optimal number of ICs can be solved independently in each scenario, we show that Eq. (23) holds for any given scenario i. Specifically, we start by simplifying the optimization problem in Eq. (4) and providing the smallest unconstrained first-order solution $\bar{y}_i$. Then we delineate the upper and lower bound y_i^{ub} and y_i^{lb}, respectively, and finally we summarize the solution and monotonicity results for $y_i^s(c_P, K)$, which depend on the relationship among $\bar{y}_i, y_i^{ub}$ and y_i^{lb}.

Step 1: find the unconstrained first-order solution $\bar{y}_i$

We start by observing that at optimality the constraint in Eq. (7) is tight. That is, at optimality we must have

$$A_i^s\left(D_i|c_P, K\right) + H_i^s\left(D_i|c_P, K\right) = \min\left\{D_i, K + y_i^s\left(c_P, K\right)\right\}. \tag{34}$$

We prove Eq. (34) by contradiction. When the above is not true, we know that at least one of the following holds:

$$\begin{aligned}
&(i)\; A_i^s\left(D_i|c_P, K\right) < \min\left\{D_i - H_i^s\left(D_i|c_P, K\right), K\right\};\\
&(ii)\; H_i^s\left(D_i|c_P, K\right) < \min\left\{D_i - A_i^s\left(D_i|c_P, K\right), y_i^s\left(c_P, K\right)\right\}.
\end{aligned}$$

because otherwise

$$\begin{aligned}
&A_i^s\left(D_i|c_P,\right) + H_i^s\left(D_i|c_P, K\right)\\
&\geq \min\left\{D_i - H_i^s\left(D_i|c_P, K\right), K\right\} + \min\left\{D_i - A_i^s\left(D_i|c_P, K\right), y_i^s\left(c_P, K\right)\right\}\\
&\geq \min\left\{D_i, K + y_i^s\left(c_P, K\right)\right\}.
\end{aligned}$$

When (i) is true, from $c_{AV} + c_P \leq c_I < r$ we know that we can increase the platform's profit by increasing $A_i^s\left(D_i|c_P, K\right)$ to $\min\left\{D_i - H_i^s\left(D_i|c_P, K\right), K\right\}$, which contradicts the optimality assumption. Similarly, when (ii) is true, from $c_I < r$ we know that we can increase the platform's profit by increasing $H_i^s\left(D_i|c_P, K\right)$ (and simultaneously increasing y_i through Eq. (8)) until

$$H_i^s\left(D_i|c_P, K\right) = \min\left\{D_i - A_i^s\left(D_i|c_P, K\right), y_i^s\left(c_P, K\right)\right\},$$

which again contradicts the optimality assumption. Thus, we can add

$$A_i\left(D_i\right) + H_i\left(D_i\right) = \min\left\{D_i, K + y_i\right\}. \tag{35}$$

as a constraint into the optimization problem in Eq. (4) without affecting its optimal solution(s).

Since we can solve $y_i^s(c_P, K)$ independently in each scenario, we plug Eq. (35) into the objective function in Eq. (4) and find that the optimization problem that the platform faces in scenario i is

$$\max_{y_i, A_i, H_i} \mathbb{E}_{D_i \sim F_i}\left[(1-c_{AV}-c_P)\left(\min\{D_i, K+y_i\} - H_i(D_i)\right) + (1-c_I)H_i(D_i)\right]$$

subject to Eq. (5) - Eq. (8) and Eq. (35). Then, we substitute Eq. (8) into the objective of the optimization problem, which simplifies the problem to

$$\max_{y_i, A_i, H_i} \mathbb{E}_{D_i \sim F_i}\left[(1-c_{AV}-c_P)\min\{D_i, K+y_i\}\right] - (c_I - c_{AV} - c_P)\frac{t}{c_I}y_i \tag{36}$$

subject to Eq. (5) - Eq. (8) and Eq. (35). In particular, Eq. (36) is now an optimization problem for y_i alone, and its derivative with respect to y_i is given by

$$(1-c_{AV}-c_P)\mathbb{P}[D > K + y_i] - (c_I - c_{AV} - c_P)\frac{t}{c_I}. \tag{37}$$

Thus, by the first-order condition we know that $\bar{y}_i := \bar{F}_i^{-1}\left(\frac{t(c_I - c_{AV} - c_P)}{c_I(1-c_{AV}-c_P)}\right) - K$ is optimal to the unconstrained optimization problem in Eq. (36). Specifically, for a discrete distribution $\bar{y}_i$ is the smallest optimal solution for y_i in the unconstrained problem, and for a continuous distribution this optimal solution is unique.

Step 2: construct y_i^{ub} and y_i^{lb}

We start by constructing the upper and lower bounds y_i^{ub} and y_i^{lb} using the constraints in Eq. (5) - Eq. (8) and Eq. (35), which implies the necessity of y_i^{ub} and y_i^{lb} for an optimal solution. Then we show that the bounds are also sufficient in the sense that any $y_i \in \left[y_i^{lb}, y_i^{ub}\right]$ leads to a feasible H_i that satisfies Eq. (6) and Eq. (8).

For the upper bound y_i^{ub}, from Eq. (6) we know that the number of ICs must fulfill

$$c_I \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, y_i\}\right] \geq t y_i,$$

i.e., the expected earning of ICs must be greater-equal to their reservation earning when the platform fully prioritizes them. Thus, the maximum number of ICs that can be hired by the platform is given by

$$\begin{aligned} y_i^{ub} &:= \max\left\{y_i \middle| c_I \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, y_i\}\right] = t y_i\right\} \\ &= \max\left\{y_i \middle| \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, K + y_i\}\right] - \frac{t}{c_I}y_i = \mathbb{E}_{D_i \sim F_i}\left[\min\left\{(D_i - y_i)^+, K\right\}\right]\right\}. \end{aligned} \tag{38}$$

Let $f(y_i) := c_I \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, y_i\}\right]$. Since the derivative $f'(y_i) = c_I \mathbb{P}[D_i > y_i]$ is non-increasing in y_i, $f(y_i)$ is concave. Now, since y_i^{ub} is the largest solution to $f(y_i) = t y_i$, by the concavity of $f(y_i)$ and the linearity of $t y_i$ we know that $f(y_i) < t y_i$ for any $y_i > y_i^{ub}$. Thus, no $y_i > y_i^{ub}$ leads to a feasible solution H_i that satisfies Eq. (6).

Similarly, for the lower bound y_i^{lb}, observe that the platform cannot use so few ICs that

$$c_I \mathbb{E}_{D_i \sim F_i}\left[\min\left\{(D_i - K)^+, y_i\right\}\right] > t y_i,$$

i.e., the expected earning of ICs exceeds their reservation earning even if the platform fully prioritizes AVs. In particular, by taking $H_i(D_i) < \min\left\{(D_i-K)^+, y_i\right\}$ when $A_i(D_i) \le \min\{D_i, K\}$ one violates Eq. (35). Thus, the minimum number of ICs that must be hired by the platform is given by

$$\begin{aligned} y_i^{lb} &:= \max\left\{ y_i \middle| c_I \mathbb{E}_{D_i\sim F_i}\left[\min\left\{(D_i-K)^+, y_i\right\}\right] = t y_i \right\} \\ &= \max\left\{ y_i \middle| \mathbb{E}_{D_i\sim F_i}\left[\min\{D_i, K+y_i\}\right] - \frac{t}{c_I} y_i = \mathbb{E}_{D_i\sim F_i}\left[\min\{D_i, K\}\right] \right\}, \end{aligned}$$

where the maximum is taken because $y_i = 0$ trivially satisfies the equality condition in the bracket. Now, let $g(y_i) := \mathbb{E}_{D_i\sim F_i}\left[\min\{D_i, K+y_i\}\right] - \frac{t}{c_I} y_i$. Since the derivative

$$g'(y_i) = \mathbb{P}\left[D_i > K + y_i\right] - \frac{t}{c_I}$$

is non-increasing in y_i, $g(y_i)$ is concave. Then, there are at most two solutions [14] to

$$g(y_i) = \mathbb{E}_{D_i\sim F_i}\left[\min\{D_i, K\}\right].$$

Since $y_i = 0$ is trivially one of the solutions, y_i^{lb} must be the larger (or equal) solution, which means that $g(y_i) > \mathbb{E}_{D_i\sim F_i}\left[\min\{D_i, K\}\right]$ for any $y_i \in (0, y_i^{lb})$. Thus, no $y_i \in (0, y_i^{lb})$ leads to a feasible solution H_i that satisfies Eq. (35).

Finally, we show that for any y_i such that $y_i^{lb} \le y_i \le y_i^{ub}$ we can find H_i that satisfies Eq. (6) and Eq. (8). Since y_i^{lb} is the largest solution to $g(y_i) = \mathbb{E}_{D_i\sim F_i}\left[\min\{D_i, K\}\right]$ and $g(y_i)$ is concave, we know that $g(y_i)$ is non-increasing in $y \ge y_i^{lb}$. Moreover, since

$$\mathbb{E}_{D_i\sim F_i}\left[\min\left\{(D_i-y_i)^+, K\right\}\right] \le \mathbb{E}_{D_i\sim F_i}\left[\min\{D_i, K\}\right]$$

and y_i^{ub} is the largest solution to $g(y_i) = \mathbb{E}_{D_i\sim F_i}\left[\min\left\{(D_i-y_i)^+, K\right\}\right]$, we have $y_i^{lb} \le y_i^{ub}$. In particular, any $y_i \in \left[y_i^{lb}, y_i^{ub}\right]$ satisfies

$$\mathbb{E}_{D_i\sim F_i}\left[\min\left\{(D_i-y_i)^+, K\right\}\right] \le g(y_i) \le \mathbb{E}_{D_i\sim F_i}\left[\min\{D_i, K\}\right].$$

That is, any $y_i \in \left[y_i^{lb}, y_i^{ub}\right]$ satisfies

$$\mathbb{E}_{D_i\sim F_i}\left[\min\left\{(D_i-K)^+, y_i\right\}\right] \le \frac{t}{c_I} y_i \le \mathbb{E}_{D_i\sim F_i}\left[\min\{D_i, y_i\}\right].$$

Since $H_i(D_i)$ is feasible whenever

$$\min\left\{(D_i-K)^+, y_i\right\} \le H_i(D_i) \le \min\{D_i, y_i\},$$

for any y_i such that $y_i^{lb} \le y_i \le y_i^{ub}$ we can find H_i that satisfies Eq. (6) and Eq. (8).

[14] Notice that we assume without loss of generality that there is no degeneracy in any non-zero solution because in the degenerate case it is strictly better to take the largest degenerate solution: given $A_i(D_i) = \min\{D_i, K\}\ \forall i$, the platform's objective becomes

$$\sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[(1 - c_{AV} - c_P)\min\{D_i, K\} + (1 - c_I)\frac{t}{c_I} y_i\right],$$

which is increasing in y_i.

Step 3: find $y_i^s(c_P, K)$

When $y_i^{lb} \le \bar{y}_i \le y_i^{ub}$, by construction of the upper and lower bounds we know that there exists a feasible solution for H_i that satisfies Eq. (6) and Eq. (8). By taking $A_i(D_i) = \min\{D_i, K + \bar{y}_i\} - H_i(D_i)$ we also find a feasible solution for A_i that satisfies Eq. (5), Eq. (7) and Eq. (35). Since such $\bar{y}_i, H_i$ and A_i simultaneously satisfy the first-order condition in Eq. (37) and the constraints in Eq. (5) - Eq. (8) and Eq. (35), such $\bar{y}_i$ is optimal. In particular, by the first-order condition in Eq. (37) we know that the objective value in Eq. (36) is non-decreasing in y_i for $y_i \le \bar{y}_i$ and non-increasing in y_i for $y_i \ge \bar{y}_i$.

When $\bar{y}_i > y_i^{ub}$, by the first-order condition we find that for all $y_i \le y_i^{ub}$ the objective value in Eq. (36) is non-decreasing in y_i. Since by construction y_i^{ub} is the largest number of ICs that leads to a feasible H_i that satisfies Eq. (6), we have $y_i^s(c_P, K) = y_i^{ub}$. Similarly, when $\bar{y}_i < y_i^{lb}$, we find that for all $y_i \ge y_i^{lb}$ the objective value in Eq. (36) is non-increasing in y_i. Since by construction y_i^{lb} is the smallest number of ICs that leads to a feasible H_i while satisfying Eq. (35), we have $y_i^s(c_P, K) = y_i^{lb}$.

□

E.2.2. Proof of Lemma 4

Proof. We separately consider the cases where $c_P > 1 - c_{AV}$ and $c_I - c_{AV} < c_P \le 1 - c_{AV}$. We begin by observing that when $c_P > 1 - c_{AV}$ the marginal profit of using AVs to serve demand is negative, so the platform trivially sets $A_i^s\left(D_i | c_P, K\right) = 0\, \forall i$. Then, the platform solves:

$$\begin{aligned} \max_{\vec{y}, H} \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i} & \left[(1 - c_I) H_i(D_i)\right] \\ \text{s.t. } 0 \le H_i(D_i) & \le \min\{D_i, y_i\}, \forall i \\ t y_i = c_I \mathbb{E}_{D_i \sim F_i} & \left[H_i(D_i)\right] \ \forall i. \end{aligned}$$

We find the objective equivalent to $\sum_i \alpha_i (1 - c_I) \frac{t}{c_I} y_i$, which monotonically increases as each y_i increases because $c_I < 1$. Thus, it is optimal for the platform to use the maximum possible number of ICs, which we know from Eq. (38) is uniquely given by

$$y_i^s(c_P, K) = y_i^{ub} := \max\left\{ y_i \middle| c_I \mathbb{E}_{D_i \sim F_i}\left[\min\{D_i, y_i\}\right] = t y_i \right\}, \forall i.$$

When $c_P > c_I - c_{AV}$, from Eq. (37) we know that in each scenario i the unconstrained first-order condition for solving Eq. (4) is

$$(1 - c_{AV} - c_P)\mathbb{P}[D > K + y_i] + (c_{AV} + c_P - c_I)\frac{t}{c_I} > 0.$$

Therefore, it is again optimal for the platform to use the maximum possible number of ICs and we have $y_i^s(c_P, K) = y_i^{ub}, \forall i$.

□

E.2.3. Proof of Lemma 5

Proof. By Lemma 5 *(ii)* we immediately know that the supplier is willing to accept π. Now, the profit of the supply chain is given by

$$\sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[(1 - c_{AV}) A_i(D_i) + (1 - c_I) H_i(D_i)\right] - c_F K \tag{39}$$

for any feasible solutions of $(K,\vec{\mathbf{y}},A,H)$ since the payments between the supplier and the platform cancel out. By plugging the constraint in Eq. (8) into Eq. (39) we equivalently find

$$(39) = \sum_i \alpha_i \left(\mathbb{E}_{D_i \sim F_i}\left[(1-c_{AV})A_i(D_i)\right] + (1-c_I)\frac{t}{c_I}y_i \right) - c_F K,$$

which now only depends on $K,\vec{\mathbf{y}}$ and A. Thus, Lemma 5 (i) implies that the profit of the supply chain is $V^\star(I)$ when π is accepted. Then, the platform obtains a profit of

$$V_P^\pi(I) = V^\star(I) - V_A^s(I) \geq V^s(I) - V_A^s(I) = V_P^s(I)$$

when π is accepted. Thus, the platform also accepts π.

□

E.2.4. Proof of Claim 3

Proof. With the optimality condition in Eq. (35) and the constraint in Eq. (8) we find that the supplier's optimization problem in Eq. (3) is equivalent to

$$\max_K c_P \sum_i \alpha_i \left(\mathbb{E}_{D_i \sim F_i}\left[\min\left\{D_i, K + y_i(c_P,K)\right\}\right] - \frac{t}{c_I}y_i(c_P,K) \right) - c_F K.$$

Then, when K is fixed, the derivative of the objective function with respect to $y_i(c_P,K)$ is

$$c_P \cdot \alpha_i \left(\mathbb{P}\left[D > K + y_i(c_P,K)\right] - \frac{t}{c_I} \right). \tag{40}$$

By Eq. (37) we know that there can be multiple optimal solutions for y_i in Eq. (4) only when there is some $y_i > \bar{F}_i^{-1}\left(\frac{t(c_I - c_{AV} - c_P)}{c_I(1-c_{AV}-c_P)}\right) - K$ that is also optimal. By plugging $\bar{F}_i^{-1}\left(\frac{t(c_I - c_{AV} - c_P)}{c_I(1-c_{AV}-c_P)}\right) - K$ into Eq. (40) we find

$$\begin{aligned} c_P \cdot \alpha_i \left(\mathbb{P}\left[D > K + y_i\right] - \frac{t}{c_I} \right) \leq & c_P \cdot \alpha_i \left(\frac{t(c_I - c_{AV} - c_P)}{c_I(1-c_{AV}-c_P)} - \frac{t}{c_I} \right) \\ = & c_P \cdot \alpha_i \left(\frac{t}{c_I} \frac{c_I - 1}{1 - c_{AV} - c_P} \right) < 0, \forall y_i \geq \bar{F}_i^{-1}\left(\frac{t(c_I - c_{AV} - c_P)}{c_I(1-c_{AV}-c_P)} \right) - K. \end{aligned}$$

That is, for fixed K the objective value in Eq. (3) is decreasing in y_i for $y_i \geq \bar{F}_i^{-1}\left(\frac{t(c_I - c_{AV} - c_P)}{c_I(1-c_{AV}-c_P)}\right) - K$. Since the argument holds in each scenario i, we conclude that among the optimal solutions for $(\vec{\mathbf{y}},A,H)$ in Eq. (4), the ones that maximize Eq. (3) contain the smallest optimal y_i in each scenario i.

□

E.2.5. Proof of Claim 1

Proof. By definition,

$$\begin{aligned} y_2^{lb} &= \max\left\{ y_2 \middle| c_I \mathbb{E}_{D_2 \sim F_i}\left[\min\left\{(D_2 - K)^+, y_2\right\}\right] = t y_2 \right\} \\ &= \max\left\{ y_2 \middle| \mathbb{E}_{D_2 \sim F_2}\left[\min\left\{D_2, K + y_2\right\}\right] - \frac{t}{c_I}y_2 = \mathbb{E}_{D_2 \sim F_2}\left[\min\left\{D_2, K\right\}\right] \right\}. \end{aligned} \tag{41}$$

Let $g(y_2) := \mathbb{E}_{D_2 \sim F_2}\left[\min\left\{D_2, K + y_2\right\}\right] - \frac{t}{c_I}y_2$. By construction of t and p we know

$$\frac{t}{c_I} = \frac{(1-p)10 + p25}{25} = \frac{\mathbb{E}\left[D_2\right]}{25}.$$

Thus, for $0 \leq K \leq 10$ we have

$$g(y_2) < \mathbb{E}[D_2] - \frac{t}{c_I}(25-K) = \frac{t}{c_I}K \leq K = \mathbb{E}_{D_2\sim F_2}\left[\min\{D_2, K\}\right] \ \forall y_2 > 25-K.$$

Similarly, for $10 < K \leq 25$,

$$g(25-K) < \frac{t}{c_I}K \leq (1-p)10 + pK = \mathbb{E}_{D_2\sim F_2}\left[\min\{D_2, K\}\right] \ \forall y_2 > 25-K,$$

so by Eq. (41) we know that $y_2^{lb} \leq 25-K$ for $K \in [0,25]$. □

E.2.6. Proof of Claim 2

Proof. For any $c_P \in [c_F, c_I - c_{AV}]$ and $K \in [0,25]$, we know by the optimality condition in Eq. (35) that

$$A_2^s(D_2|c_P, K) = \min\{D_2, K + y_2^s(c_P, K)\} - H_2^s(D_2|c_P, K).$$

Taking expectation on both sides, we obtain

$$\mathbb{E}_{D_2\sim F_2}\left[A_2^s(D_2|c_P, K)\right] = \mathbb{E}_{D_2\sim F_2}\left[\min\{D_2, K + y_2^s(c_P, K)\} - H_2^s(D_2|c_P, K)\right].$$

Then, from the equilibrium condition in Eq. (8) we substitute $\mathbb{E}_{D_2\sim F_2}\left[H_2^s(D_2|c_P, K)\right]$ with $\frac{t}{c_I}y_2^s(c_P, K)$ and conclude that

$$\mathbb{E}_{D_2\sim F_2}\left[A_2^s(D_2|c_P, K)\right] = \mathbb{E}_{D_2\sim F_2}\left[\min\{D_2, K + y_2^s(c_P, K)\}\right] - \frac{t}{c_I}y_2^s(c_P, K).$$

□

E.3. Proofs of Results in Section 5

E.3.1. Proof of Proposition 4

Proof. We begin by characterizing the profits of AV and IC-only platforms. Since the objective in (13) monotonically increases with respect to $A_i(D_i)$ for any D_i and i, (13) becomes

$$V^{AV}(I) = \max_K \sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[(1-c_{AV})\min\{D_i, K\}\right] - c_F K.$$

We find that this is equivalent to

$$\Pi^{\text{emp}} := \max_{x\geq 0} \sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[p\min\{D_i, x\}\right] - cx$$

in Lobel et al. (2021) when $p = 1, c_{AV} = 0$ and $c_F = c$.

Similarly, an IC-only platform solves (9)-(10) subject to $K = 0$. That is,

$$\begin{aligned} V^{IC}(I) := \max_{\vec{\mathbf{y}}, H} &\sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[(1-c_I)H_i(D_i)\right] \\ \text{s.t. } & 0 \leq H_i(D_i) \leq \min\{D_i, y_i\}, \forall i \\ & ty_i = c_I \mathbb{E}_{D_i\sim F_i}\left[H_i(D_i)\right], \forall i. \end{aligned}$$

Since the objective monotonically increases with respect to $H_i(D_i)$ for any D_i and i, the above becomes

$$\begin{aligned} V^{IC}(I) = \max_{\vec{\mathbf{y}}, H} &\sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[(1-c_I)\min\{D_i, y_i\}\right] \\ \text{s.t. } & ty_i = c_I \mathbb{E}_{D_i\sim F_i}\left[\min\{D_i, y_i\}\right], \forall i. \end{aligned}$$

We find that this is equivalent to

$$\Pi^{\text{cont}} := \max_{\vec{s},w} \sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[(p-w)\min\{D_i,s_i\}\right] \tag{42}$$
$$\text{s.t. } rs_i = w\mathbb{E}_{D_i\sim F_i}\left[\min\{D_i,s_i\}\right], \forall i$$

when $p=1, t=r$ and $c_I = w^\star$ returned as the optimal solution to w in (42).

Theorem 2 in Lobel et al. (2021) states that, for any $p>0, r\in(0,p), c\in(0,p)$ and $\epsilon>0$, there exists an instance with 2 demand scenarios such that $\frac{\Pi^{\text{cont}}}{\Pi^{\text{emp}}} \geq \frac{1}{\epsilon}$. Now, let $p=1, t=r$ and $c_F=c$. Then, we know that for any $M>0$, we can construct $\epsilon=1/M>0$ and there exists an instance $I\in\mathcal{I}$ such that

$$\frac{V^\star(I)}{V^{AV}(I)} \geq \frac{V^{IC}(I)}{V^{AV}(I)} = \frac{\Pi^{\text{cont}}}{\Pi^{\text{emp}}} \geq \frac{1}{\epsilon} = M.$$

□

E.3.2. Proof of Proposition 5

Proof. We construct an instance I with two scenarios D_1 and D_2, which respectively occurs with probability $\alpha_1 = 1-10^{-6}$ and $\alpha_2 = 10^{-6}$. $D_1 = 10$ deterministically, and

$$D_2 = \begin{cases} 10 & \text{w.p. } 2\cdot 10^{-6} \\ 2000 & \text{w.p. } 1-2\cdot 10^{-6}. \end{cases}$$

Let $c_{AV} = 0.001, c_I = 1-10^{-8}, t=\mathbb{E}[D_2]\cdot c_I$ and $c_F = (c_I - c_{AV})\cdot\max\left(\alpha_1, \mathbb{E}[D_2]/2000\right) + 10^{-9}$. With a slight abuse of notation we denote an SPE in the short-term leasing model by superscript s in this proof. We show that the centralized solution sets $K^\star = 10$ and $\vec{\mathbf{y}}^\star = [0,1990]$, whereas an SPE yields $K^s = \kappa_1^s = \kappa_2^s = 0, \vec{\mathbf{y}}^s = [10,2000]$.

By construction, $\bar{F}_2^{-1}(\frac{t(c_I-c_{AV})}{c_I(1-c_{AV})}) = 2000$ and $t/c_I = \mathbb{E}[D_2]/2000$. Thus, by applying Lemma 3 we find that in a centralized solution, for any given K, it is optimal to set $y_2^s(K) = 2000-K$. Since ICs have an expected utilization of t/c_I, in scenario 2 each additional unit of AV reduces the expected IC usage by t/c_I. Thus, in a centralized solution the marginal profit of investing in AV when $K\in[0,10)$ is $(c_I-c_{AV})\left[\alpha_1 + t/c_I\alpha_2\right] - c_F > 0$, and the marginal profit of investing in AV when $K\geq 10$ is upper bounded by $(c_I-c_{AV})\alpha_2 - c_F < 0$. Thus, $K^\star = 10$ and the resulting $\vec{\mathbf{y}}^\star = [0,1990]$.

In the short-term leasing SPE, in contrast, we shall show that the platform has no incentive to lease AV in scenario 2 and the AV supplier responds by making no AV investment at all. Notice that the supplier trivially makes no investment when $c_P < c_F$. Similarly, since the platform could cover all demand with ICs alone it has no incentive to use AVs when $c_P > c_I - c_{AV}$. We thus focus on the case where $c_P\in[c_F, c_I-c_{AV}]$. In scenario 2, by the same analyses as above we find that the marginal profit for the platform to lease AV when $K\in[0,10)$ is $(c_I-c_{AV})t/c_I - c_P \leq (c_I-c_{AV})t/c_I - c_F < 0$. Thus, given any $c_P\geq c_F$ and K, the platform would respond by setting $\kappa_2^s(c_P,K) = 0$. Then, for the AV supplier, the marginal profit of investing in AV becomes at most $c_P\alpha_1 - c_F \leq (c_I-c_{AV})\alpha_1 - c_F < 0$. Thus, in an SPE we have $K^s = \kappa_1^s = \kappa_2^s = 0$ and $\vec{\mathbf{y}}^s = [10,2000]$. We thus find that

$$\frac{V^\star(I)}{V^{SL}(I)} \geq \frac{(1-c_{AV})\cdot 10\left[\alpha_1 + \alpha_2 tc_I\right] + (1-c_I)\cdot 1990t/c_I\alpha_2 - 10c_F}{(1-c_I)\cdot\left[\alpha_1 10 + \alpha_2\mathbb{E}[D_2]\right]} > 100.$$

□

E.3.3. Proof of Proposition 6

Proof. We construct an instance I with two scenarios D_1 and D_2, which respectively occurs with probability $\alpha_1 = 0.2$ and $\alpha_2 = 0.8$. $D_1 \sim U(10,12)$, and $D_2 \sim U(10,5000)$. Let $c_{AV} = 0.001, c_I = 0.999, c_F = 0.763$ and $t = 7.99/8c_I$. With a slight abuse of notation we denote an SPE in the long-term leasing model by superscript s in this proof. We show that the centralized solution sets $K^\star \approx 236.02$ and $\vec{\mathbf{y}}^\star = [0,0]$, whereas an SPE yields $K^s = 10, \vec{\mathbf{y}}^s = [0.005, 12.475]$.

In a centralized solution we have $\bar{F}_2^{-1}(\frac{t(c_I - c_{AV})}{c_I(1-c_{AV})}) \approx 21.226$. Thus, unless $K < 22$ we should always have $\vec{\mathbf{y}}^s(c_P = 0, K) = [0,0]$. When $K \geq 22$, the marginal profit of AV is $\mathbb{P}[D \geq K](1 - c_{AV}) - c_F$, which equals 0 when $K \approx 236.02$. It is easy to check that a solution of $K \geq 236.02, \vec{\mathbf{y}} = [0,0]$ dominates any solution with $K < 22$, even if all remaining demand can be filled by ICs in the latter case. Thus, in a centralized solution we find that $K^\star \approx 236.02$ and the resulting $\vec{\mathbf{y}}^\star = [0,0]$. This generates a supply chain profit greater than 6.6508.

In the long-term leasing SPE, we find that by taking $c_P = 0.9972$ the platform sets $K^s(c_P) = 10$ and the resulting $\vec{\mathbf{y}}^s(c_P, K^s(c_P)) = [0.005, 12.475]$. This generates at least 2.3419 for the AV supplier. We further find that by taking $c_P = 0.9972$ the platform sets $K^s(c_P) < 10$. We now iterate over values of c_P and apply a continuity argument to show that no c_P such that $K^s(c_P) \neq 10$ can be optimal for the AV supplier. Since the AV supplier will not take any $c_P < c_F$ and the platform will not take $c_P > 1$, we iterate over all choices of $c_P \in [c_F, 1 - c_{AV}]$ at a precision level of 0.0001 and solve the resulting $K^s(c_P)$ to the closest integral. Results yield no other c_P value such that (1) $K^s(c_P) \neq 10$, and (2) it generates more than a profit of 2.3 for the AV supplier. For any $c_P \in [c_F, 1 - c_{AV}]$ and $c'_P \in [c_P, c_P + 0.0001)$, $K^s(c'_P) \leq K^s(c_P)$. Thus, the supplier profit given c'_P is upper bounded by the supplier profit given c_P plus $0.0001 \cdot K^s(c_P)$. Here $K^s(c_P)$ is trivially upper bounded by $K^\star \approx 236.02$ for any $c_P \in [c_F, 1 - c_{AV}]$. Thus, for any c_P that yields profit no longer than 2.3, we know that the supplier profit given $c'_P \in [c_P, c_P + 0.0001)$ is upper bounded by $2.3 + 0.0001 \cdot 237 < 2.33$. Since this applies to all c_P such that $K^s(c_P) \neq 10$, we must have $K^s(c_P^s) = 10$. It is thus optimal for the AV supplier to set the largest c_P such that $K^s(c_P) = 10$, based on which we conclude $c_P^s \in [0.9972, 0.9973), K^s = 10$ and the resulting $\vec{\mathbf{y}}^s = [0.005, 12.475]$. This generates a supply chain profit smaller than 2.37. We therefore find that $\frac{V^\star(I)}{V^{LL}(I)} \geq \frac{6.6508}{2.37} > 2.806$. $\square$

E.4. Proofs of Results in Appendices

E.4.1. Proof of Proposition 7

Proof. We observe that Eq. (14) is equivalent to

$$\max_K \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i} \left[(c_P - c_O) A_i \left(D_i | c_P, K \right) \right] - (c_F - c_O) K.$$

Thus, by replacing the argument on c_P and c_F by $c_P - c_O$ and $c_F - c_O$, respectively, the construction in Theorem 1 directly extends. Specifically, for any $c_O > 0$ and $\delta_1 \in (0, 0.01]$, take $\delta_2 = \sqrt{0.049875\delta_1 + 0.7375\delta_1^2} - 0.5\delta_1$ and $c_F = c_O + 0.19 - \delta_2$. Then, construct $\vec{\alpha}, c_I, c_{AV}, t, p$ and D the same way as in Theorem 1, we recover instance $I(\delta_1)$ in Theorem 1, where any choice of c_P would lead to no AV investment on the supplier side. Thus, the bound on profit ratio extends and we have $\lim_{\delta_1 \to 0^+} \sup_{c_P} \mathrm{PR}^s(I(\delta_1) | c_P) = \infty$. $\square$

E.4.2. Proof of Proposition 8

Proof. Using the same construction of $I(\delta_1)$ as in Theorem 1, we take any $c_O \in \left(0, \min\left(c_F, \frac{c_F-(c_I-c_{AV})\alpha_1}{\alpha_2}\right)\right)$. Since $(c_I - c_{AV})\alpha_1 < c_F$ by construction, the interval is guaranteed to be non-empty. Now, we consider three cases for c_P. When $c_P < c_F$, we have $\max(c_P, c_O) < c_F$ and thus $K^s(c_P) = 0$. When $c_P > c_I - c_{AV}$, platform trivially resorts to only ICs and thus $K^s(c_P) = 0$. When $c_P \in [c_F, c_I - c_{AV}]$, by $c_P \geq c_F > c_O$ we know that the AV supplier does not divest in scenario 1, when AVs are fully utilized. If the supplier divests in scenario 2, the marginal profit of investing in AVs is $c_P\alpha_1 + \alpha_2 c_O - c_F \leq (c_I - c_{AV})\alpha_1 + \alpha_2 c_O - c_F < 0$. If the supplier does not divest in scenario 2, the marginal profit remains $c_P\left[\alpha_1 + \alpha_2 t/c_I\right] - c_F < 0$ by construction. Thus, in either case the supplier would not invest in any AV and $K^s = 0$. The rest of the proof follows from Theorem 1. $\square$

E.4.3. Proof of Lemma 1

Proof. Given any c_P, K, feasible $\vec{\mathbf{y}}$ and $A_i(D_i)$, we start by plugging the last equality constraint into Eq. (16), which yields:

$$\begin{aligned}
&\max_{\vec{\mathbf{y}},A} \sum_i \alpha_i \left(\mathbb{E}_{D_i\sim F_i}\left[(1-c_{AV}-c_P)A_i(D_i) + H_i(D_i)\right] - ty_i\right)\\
&\text{s.t. } (5), (6), (7), ty_i = c_I^i \mathbb{E}_{D_i\sim F_i}\left[H_i(D_i)\right], c_I^i \geq 0, \forall i.
\end{aligned}$$

We then observe that taking $H_i(D_i) = \min\{D_i, K+y_i\} - A_i(D_i), \forall i$, is always feasible to Eq. (16) and the objective is non-decreasing as $H_i(D_i)$ increases. Thus, the third constraint in Eq. (16) is tight at optimality, and we can plug $H_i(D_i) = \min\{D_i, K+y_i\} - A_i(D_i) = \min\left\{D_i - A_i(D_i), y_i\right\}, \forall i$, into the above. We then obtain

$$\begin{aligned}
&\max_{\vec{\mathbf{y}},A} \sum_i \alpha_i \left(\mathbb{E}_{D_i\sim F_i}\left[(1-c_{AV}-c_P)A_i(D_i) + \min\left\{D_i - A_i(D_i), y_i\right\}\right] - ty_i\right)\\
&\text{s.t. } (5), (6), (7), ty_i = c_I^i \mathbb{E}_{D_i\sim F_i}\left[H_i(D_i)\right], c_I^i \geq 0, \forall i.
\end{aligned}$$

This is equivalent to:

$$\begin{aligned}
&\max_{\vec{\mathbf{y}},A} \sum_i \alpha_i \left(\mathbb{E}_{D_i\sim F_i}\left[\min\left\{D_i, A_i(D_i) + y_i\right\} - (c_{AV}+c_P)A_i(D_i)\right] - ty_i\right)\\
&\text{s.t. } 0 \leq A_i(D_i) \leq \min\{D_i, K\}, \forall i.
\end{aligned}$$

Notice that in each scenario i, for given K and y_i the objective above is non-decreasing in $A_i(D_i) \in \left[0, (D_i - y_i)^+\right]$ and non-increasing in $A_i(D_i) \geq (D_i - y_i)^+$. Combined with the constraint that $A_i(D_i) \leq \min\{D_i, K\}$, we conclude that at optimality we must have

$$A_i(D_i) = \min\left\{(D_i - y_i)^+, K\right\}, \forall i. \tag{43}$$

Plugging this into the objective, we equivalently solve:

$$\max_{\vec{\mathbf{y}}} \sum_i \alpha_i \left(\mathbb{E}_{D_i\sim F_i}\left[\min\{D_i, K+y_i\} - (c_{AV}+c_P)\min\left\{(D_i-y_i)^+, K\right\}\right] - ty_i\right). \tag{44}$$

Since the optimization problem now contains a single variable, from the first-order condition on y_i we conclude that the objective increases with respect to y_i when $\mathbb{P}[y_i < D_i \leq K + y_i](c_{AV}+c_P) + \mathbb{P}[D_i > K + y_i] > t$ and the objective decreases with respect to y_i when $\mathbb{P}[y_i < D_i \le K+y_i](c_{AV}+c_P)+\mathbb{P}[D_i > K+y_i] < t$. Thus, the smallest optimal solution for $\vec{\mathbf{y}}$ satisfies

$$y_i^s(c_P,K) := \min\left\{ y_i \middle| \mathbb{P}[D_i > K+y_i] + \mathbb{P}[y_i < D_i \le K+y_i]\cdot(c_{AV}+c_P) \le t \right\}, \forall i.$$

Moreover, from Eq. (43) we obtain $A_i^s(D_i|c_P,K) = \min\left\{(D_i - y_i^s(c_P,K))^+, K\right\}, \forall i.$ □

E.4.4. Proof of Proposition 10

Proof. We construct an instance I with two scenarios D_1 and D_2, which respectively occurs with probability $\alpha_1 = 0.001$ and $\alpha_2 = 0.999$. $D_1 = 10$ deterministically, and

$$D_2 = \begin{cases} 10 & \text{w.p. } 0.002 \\ 20 & \text{w.p. } 0.998. \end{cases}$$

Let $c_{AV} = 0.001, t = 0.999\cdot\mathbb{E}[D_2]/20$ and $c_F = 0.996005$. We show that the centralized solution sets $K^\star = 10$ and $\vec{\mathbf{y}}^\star = [0,10]$, whereas an equilibrium solution given any $c_P \ge 0$ yields $K^s(c_P) = 0, \vec{\mathbf{y}}^s(c_P, K^s(c_P)) = [10,10]$.

In a centralized solution, $c_P = 0$. If $K = 0$ we find from Lemma 1 that the resulting $\vec{\mathbf{y}}^s(c_P = 0, K = 0) = [10,10]$. This means that ICs are fully utilized and $c_I^i = t\ \forall i$, leading to a profit of $10(1-t)$. On the other hand, for any $K > 0$ we find that $\mathbb{P}[D_i > K+y_i] + \mathbb{P}[y_i < D_i \le K+y_i]\cdot c_{AV} = 0.998 + 0.002c_{AV} > t$ for any $y \in [0,10)$ and thus the resulting $y_2^s(c_P = 0, K) = 10$. This means that AVs would only be used in scenario 2 when $D_2 = 20$. We also find that $y_1^s(c_P = 0, K) = 10 - K$. Thus, when $K \in [0,10)$ the marginal profit of using AV is $\alpha_1(t - c_{AV}) + \alpha_2 0.998(1-c_{AV}) - c_F > 0$, and when $K \ge 10$ the marginal profit of using AV is at most $\alpha_2 0.998(1-c_{AV}) - c_F < 0$. Therefore, $K^\star = 10$ and $\vec{\mathbf{y}}^\star = [0,10]$. The supply chain profit in a centralized solution is then given by $\alpha_1 10(1-c_{AV}) + \alpha_2\left[10(1-t) + 0.998\cdot 10\cdot(1-c_{AV})\right] - 10c_F$.

In an SPE, if $c_P < c_F$, the supplier trivially responds by setting $K^s(c_P) = 0$, and thus the platform sets $\vec{\mathbf{y}}^s(c_P, K^s(c_P)) = [10,10]$. We then focus on the case where $c_P \ge c_F$. From Lemma 1 we find that for any $K > 0$ and $c_P \ge 0$ we would have $y_2^s(c_P,K) = 10$. When $c_P \in [c_F, t - c_{AV}]$, the marginal profit for the AV supplier to invest in AV is thus upper bounded by $c_P(\alpha_1 + \alpha_2\cdot 0.998) - c_F < 0$ by construction of I, through which we know that $K^s(c_P) = 0$. When $c_P \in (t - c_{AV}, 1 - c_{AV}]$, platform resorts to only ICs in scenario 1. Because in this case the marginal profit of AV investment is upper bounded by $(1-c_{AV})\alpha_2\cdot 0.998 - c_F < 0$, we again have $K^s(c_P) = 0$. Finally, when $c_P > 1 - c_{AV}$, the platform trivially uses no AVs and the supplier sets $K^s(c_P) = 0$. Thus, for all c_P we find that the resulting $K^s(c_P) = 0$ and $\vec{\mathbf{y}}^s(c_P, K^s(c_P)) = [10,10]$. This implies a supply chain profit of $10(1-t)$. We thus find that

$$\frac{V^\star(I)}{V^{ED}(I|c_P)} = \frac{\alpha_1 10(1-c_{AV}) + \alpha_2\left[10(1-t) + 0.998\cdot 10\cdot(1-c_{AV})\right] - 10c_F}{10(1-t)} \ge 1.49, \forall c_P \ge 0.$$

This completes the proof. □

E.4.5. Proof of Proposition 9

Proof. We present the proof of Proposition 9 after that of Proposition 10 as it can be shown as a direct corollary of the latter. In particular, since a centralized solution for the instance I constructed in Proposition 10 satisfies $y_2^\star > 0, H_2^\star(10) = 10$ and $A_2^\star(10) = 0 < \min\{10, K^\star\} = 10$, we immediately obtain Proposition 9 (i).

To prove Proposition 9 (ii), it suffices to modify instance I to I' by taking $c_F = 0.996$. This construction ensures that all analyses for $c_P < c_F, c_P \in [c_F, t - c_{AV}]$ and $c_P > 1 - c_{AV}$ still hold, but when $c_P \in (t - c_{AV}, 1 - c_{AV}]$ the marginal profit of AV investment now becomes upper bounded by $(1 - c_{AV})\alpha_2 \cdot 0.998 - c_F > 0$ for $K \in [0, 10]$. By taking $c_P = c_F/(\alpha_2 \cdot 0.998) < 1 - c_{AV}$, the platform thus ensures that $K^s(c_P) = 10$. We verify that this strictly increases profit of the platform than taking $K = 0$ and $\vec{\mathbf{y}} = [10, 10]$. Thus, for I' we have $K^s = 10$ and $\vec{\mathbf{y}}^s = [10, 10]$. This instance satisfies $y_2^s > 0, H_2^s(10) = 10$ and $A_2^s(10) = 0 < \min\{10, K^s\} = 10$, which proves Proposition 9 (ii). $\square$

E.4.6. Proof of Lemma 2

Proof. The proof of Lemma 2 generalizes the derivation of (44) in Lemma 1 to allow dynamic pricing, finding that the optimization problem in (19) is equivalent to:

$$\max_{\vec{\mathbf{y}},\mathbf{r}} \sum_i \alpha_i \left(\mathbb{E}_{D_i \sim F_i} \left[r_i(D_i) \cdot \min\left\{ \mathscr{D}_i\left(D_i, r_i(D_i)\right), K + y_i \right\} - (c_{AV} + c_P) \min\left\{ \left(\mathscr{D}_i\left(D_i, r_i(D_i)\right) - y_i\right)^+, K \right\} \right] - t y_i \right). \tag{45}$$

When $\vec{\mathbf{y}}$ is given, it suffices to consider the following three cases to find the optimal $r_i(D_i)$:

(1) $r_i(D_i) \le 2 - (K + y_i)/D_i$: in this case $\mathscr{D}_i\left(D_i, r_i(D_i)\right) \ge K + y_i$. Thus, (45) becomes $r_i(D_i) \cdot (K + y_i) - (c_{AV} + c_P) \cdot K$, which is trivially maximized at $r_i(D_i) = 2 - (K + y_i)/D_i$.

(2) $r_i(D_i) \in (2 - (K + y_i)/D_i, 2 - y_i/D_i]$: in this case $\mathscr{D}_i\left(D_i, r_i(D_i)\right) \in [y_i, K + y_i)$. Thus, (45) becomes $r_i(D_i) \cdot \mathscr{D}_i\left(D_i, r_i(D_i)\right) - (c_{AV} + c_P) \cdot \Big(\mathscr{D}_i\left(D_i, r_i(D_i)\right) - y_i \Big)$, which is optimized at $r_i(D_i) = 1 + (c_{AV} + c_P)/2$.

(3) $r_i(D_i) > 2 - y_i/D_i$: in this case $\mathscr{D}_i\left(D_i, r_i(D_i)\right) < y_i$. Thus, (45) becomes $r_i(D_i) \cdot \mathscr{D}_i\left(D_i, r_i(D_i)\right)$, which is maximized at $r_i(D_i) = 1$.

Combining the three cases above, we find that given any $c_P, K, \vec{\mathbf{y}} \ge 0$, in (19) it is optimal for the platform to set $r_i(D_i) = \max\Big\{ \min\Big\{ \max\left\{ 1 + (c_{AV} + c_P)/2, 2 - (y_i + K)/D_i \right\}, 2 - y/D_i \Big\}, 1 \Big\}, \forall i$. Once $r_i(D_i)$ is determined, from (43) we know that at optimality $H_i(D_i) = \min\Big\{ \mathscr{D}_i\left(D_i, r_i(D_i)\right), y_i \Big\}$ and thus $c_I^i(D_i) = t \cdot y_i / \min\Big\{ \mathscr{D}_i\left(D_i, r_i(D_i)\right), y_i \Big\}$ if $y_i > 0$ and 0 otherwise, $\forall i$. $\square$

E.4.7. Proof of Proposition 11

Proof. To show that (i) is true for any $I \in \mathcal{I}_P$, we argue by contradiction: when $c_{AV} + c_P \le c_I$, if it is not optimal to fully prioritize AVs, i.e., we always have $A_i^s\left(D_i | c_P, K\right) < \min\left\{D_i, K\right\}$ in some scenario i, we show that we can construct another feasible solution that leads to at least the same profit for the platform as an optimal solution. In particular, we take $A_i'(D_i) = \min\left\{D_i, K\right\}$,

$$H_i'(D_i) = \Big(H_i^s\left(D_i | c_P, K\right) - \Big(A_i'(D_i) - A_i^s\left(D_i | c_P, K\right) \Big) \Big)^+,$$

$$y_i' = y_i^s\left(c_P, K\right) - \frac{c_I}{t} \min\Big\{ H_i^s\left(D_i | c_P, K\right), A_i'(D_i) - A_i^s\left(D_i | c_P, K\right) \Big\}.$$

We start by showing that this construction is feasible, and then show that it is also optimal.

We first observe that the equilibrium condition for ICs, i.e., Eq. (8), becomes $t y_i = c_I H_i\left(D_i\right)$ for $I \in \mathcal{I}_P$ as D_i becomes deterministic. Thus, $t y_i^s\left(c_P, K\right) = c_I H_i^s\left(D_i | c_P, K\right)$ holds. Then, by construction of y_i' we have

$$y_i' = y_i^s\left(c_P, K\right) - \frac{c_I}{t} \min\Big\{ H_i^s\left(D_i | c_P, K\right), A_i'(D_i) - A_i^s\left(D_i | c_P, K\right) \Big\}$$

$$
\begin{aligned}
&= y_i^s\left(c_P, K\right) - \frac{c_I}{t}\left(H_i^s\left(D_i|c_P, K\right) - H_i'(D_i)\right) \\
&= y_i^s\left(c_P, K\right) - y_i^s\left(c_P, K\right) + \frac{c_I}{t}H_i'(D_i) = \frac{c_I}{t}H_i'(D_i),
\end{aligned}
$$

so that y_i' satisfies Eq. (8). In particular, notice that $H_i'(D_i) \leq y_i'$ is still guaranteed. Moreover, since $A_i^s\left(D_i|c_P, K\right)$ and $H_i^s\left(D_i|c_P, K\right)$ are feasible for Eq. (5), Eq. (6) and Eq. (7), by reallocating

$$
\min\left\{H_i^s\left(D_i|c_P, K\right), A_i'(D_i) - A_i^s\left(D_i|c_P, K\right)\right\}
$$

units of demand served by ICs to be served by AVs in scenario i, the solution A' and H' remain feasible to these three constraints.

Finally, observe that A' and H' lead to a change in profit of at least

$$
\left((1 - c_{AV} - c_P) - (1 - c_I)\right) \cdot \min\left\{H_i^s\left(D_i|c_P, K\right), A_i'(D_i) - A_i^s\left(D_i|c_P, K\right)\right\} \geq 0,
$$

which is a contradiction.

We omit the proof of (ii) as it is exactly symmetrical to the construction above, i.e., we reallocate demand served by AVs to be served by ICs and show that the change in platform's profit is positive. □

E.4.8. Proof of Proposition 12

Proof. For $I \in \mathcal{I}$, we construct

$$
\lambda := \frac{V_A^s(I) + c_F K^\star}{\sum_i \alpha_i \left((1 - c_{AV})\mathbb{E}_{D_i \sim F_i}\left[A_i^\star(D_i)\right] + (1 - c_I)\frac{t}{c_I}y_i^\star\right)} \geq 0
$$

and propose a contract $\pi \in \Pi^S$ with $K^\pi = K^\star, c_P^\pi = \lambda(1 - c_{AV})$ and $c_R^\pi = \lambda(1 - c_I)$. Now we start by showing that the platform adopts $\vec{\mathbf{y}} = \vec{\mathbf{y}}^\star$ and $A = A^\star$ when π is accepted, and then show that π is indeed accepted.

Given π, the optimization problem that the platform faces is

$$
\max_{\vec{\mathbf{y}}, A, H} \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[(1 - c_{AV} - c_P^\pi)A_i(D_i) + (1 - c_I - c_R^\pi)H_i(D_i)\right],
$$

subject to Eq. (5) - Eq. (8) and $K = K^\star$. By plugging in the values of c_P^π and c_R^π, we find that the platform equivalently solves

$$
\max_{\vec{\mathbf{y}}, A, H}(1 - \lambda)\sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[(1 - c_{AV})A_i(D_i) + (1 - c_I)H_i(D_i)\right] \tag{46}
$$

subject to Eq. (5) - Eq. (8) and $K = K^\star$. Since the objective of Eq. (46) is a constant multiple of that in Eq. (4) when $c_P = 0$ and $K = K^\star$ and the two optimization problems have the same constraints, by Lemma 3 we know that an optimal solution for $\vec{\mathbf{y}}$ in Eq. (46) is $y_i^s(c_P = 0, K^\star) = y_i^\star, \forall i$.

Moreover, we have

$$
\begin{aligned}
\sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[A_i^s\left(D_i|c_P = 0, K^\star\right)\right] &= \sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[\min\left\{D_i, K^\star + y_i^\star\right\} - H_i^s\left(D_i|c_P = 0, K^\star\right)\right] \\
&= \sum_i \alpha_i \left(\mathbb{E}_{D_i \sim F_i}\left[\min\left\{D_i, K^\star + y_i^\star\right\}\right] - \frac{t}{c_I}y_i^\star\right),
\end{aligned} \tag{47}
$$

where the first equality comes from the optimality condition in Eq. (35) and the second equality comes from the constraint in Eq. (8). Given that it is optimal for the platform to use $y_i^\star$ $\forall i$, the objective value in Eq. (46)

becomes deterministic and any solution for A that satisfies Eq. (47) is optimal for Eq. (46). In particular, this implies that it is optimal to set $y_i = y_i^\star, A_i = A_i^\star\ \forall i$.

For the supplier, then, by the construction of λ and π we have

$$\begin{aligned} V_A^\pi(I) =& \sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[\lambda(1-c_{AV})A_i^\star(D_i) + \lambda(1-c_I)H_i(D_i)\right] - c_F K^\star \\ =& \lambda \sum_i \alpha_i \left((1-c_{AV})\mathbb{E}_{D_i\sim F_i}\left[A_i^\star(D_i)\right] + (1-c_I)\frac{t}{c_I} y_i^\star \right) - c_F K^\star = V_A^s(I), \end{aligned}$$

where the second equality comes from the constraint in Eq. (8). Thus, both conditions in Lemma 5 are satisfied and we know that π is accepted. In particular, since π sets $K^\pi = K^\star$ and leads the platform to set $\vec{\mathbf{y}}^\star$ and $A^\star$, the centralized solution is adopted and $\mathrm{PR}^\pi(I) = 1$.

□

E.4.9. Proof of Corollary 1

Proof. Similar to the proof of Theorem 2, where we provided a construction for $\pi' \in \Pi^U$ that aligns the supply chain, we now construct $\pi \in \Pi^C$ with $K^\pi = K^\star, \widetilde{A}^\pi$ and

$$c_P^\pi = \frac{V_A^s(I) + c_F K^\star}{\sum_i \alpha_i \mathbb{E}_{D_i\sim F_i}\left[A_i^\star(D_i)\right]} \geq 0.$$

Notice that π is the same as π' except that it sets $\widetilde{A}^\pi$ instead of $A^{\pi'} = A_i^\star\ \forall i$. Thus, it suffices to construct some $\widetilde{A}^\pi$ that ensures that the platform adopts $A_i = A_i^\star\ \forall i$, so that the rest of the proof follows exactly the arguments for Theorem 2.

We start by explicitly constructing a pair of dispatch policies A and H that induce $\vec{\mathbf{y}}^\star$. By Eq. (36) we know that, for given $K^\star$, $\vec{\mathbf{y}}^\star$ uniquely determines the optimal supply chain profit, and thus any A and H that fulfill Eq. (10) when $\vec{\mathbf{y}} = \vec{\mathbf{y}}^\star$ must be optimal to the centralized problem. We specifically construct, in any scenario i, an optimal pair of solutions $A_i^\star(D_i)$ and $H_i^\star(D_i)$ that are non-decreasing in D_i. In particular, we define

$$\beta_i := \min\left\{\beta \geq 0 \middle| \mathbb{E}_{D_i\sim F_i}\left[\min\left\{(D_i - K^\star)^+ + \beta, y_i^\star, D_i\right\}\right] = \frac{t}{c_I} y_i^\star\right\}.$$

Notice that when $\beta = y_i^\star$ we have

$$\mathbb{E}_{D_i\sim F_i}\left[\min\left\{(D_i - K^\star)^+ + \beta, y_i^\star, D_i\right\}\right] = \mathbb{E}_{D_i\sim F_i}\left[\min\left\{y_i^\star, D_i\right\}\right] \geq \frac{t}{c_I} y_i^\star$$

by the fact that $y_i^\star \leq y_i^{ub}$. Then, since $\mathbb{E}_{D_i\sim F_i}\left[\min\left\{(D_i - K^\star)^+ + \beta, y_i^\star, D_i\right\}\right]$ is non-decreasing in β, we know that $\beta_i \in [0, y_i^\star]$. Now we construct

$$\begin{aligned} H_i^\star(D_i) &= \min\left\{(D_i - K^\star)^+ + \beta_i, y_i^\star, D_i\right\}, \\ A_i^\star(D_i) &= \min\left\{D_i, K^\star + y_i^\star\right\} - H_i^\star(D_i) = \max\left\{\min\left\{D_i, K^\star\right\} - \beta_i, \min\left\{D_i - y_i^\star, K^\star\right\}, 0\right\}, \end{aligned}$$

both of which are non-decreasing in D_i and fulfill Eq. (10).

We then find a mapping from H_i to $\widetilde{A}^\pi$ and show that such $\widetilde{A}^\pi$ ensures that the platform adopts $A_i = A_i^\star\ \forall i$. We start by defining

$$\mathcal{D}_i(H_i) := \min\left\{d \middle| H_i^\star(d) = H_i\right\},$$

a function that allows the supplier to map H_i back to a lower bound on D_i, and then construct

$$\widetilde{A}_i^{\pi}\left(H_i\right) := A_i^{\star}(\mathcal{D}_i\left(H_i\right)).$$

In any scenario i, by the same argument as in the proof of Theorem 2 we know that given $c_P^{\pi} \geq 0$ the platform has no incentive to use $y_i < y_i^{\star}$. We next show that the platform also cannot use $y_i > y_i^{\star}$ given the requirement on $\widetilde{A}^{\pi}$, so that $y_i = y_i^{\star}$ is guaranteed. To see this, we argue by contradiction: in order for the platform to use $y_i > y_i^{\star}$, by the constraint in Eq. (8) the platform must adopt some dispatch policy

$$H_i(D_i) > H_i^{\star}(D_i) = \min\left\{(D_i - K^{\star})^+ + \beta_i, y_i^{\star}, D_i\right\}.$$

Then, from the fact that $H_i^{\star}(D_i)$ is non-decreasing in D_i we know that $\mathcal{D}_i\left(H_i\left(D_i\right)\right) \geq D_i$. Thus, from the fact that $A_i^{\star}(D_i)$ is also non-decreasing in D_i we know that π requires the platform to use at least

$$\widetilde{A}_i^{\pi}\left(H_i\left(D_i\right)\right) = A_i^{\star}(\mathcal{D}_i\left(H_i\left(D_i\right)\right)) \geq A_i^{\star}(D_i)$$

units of AVs. Since

$$\widetilde{A}_i^{\pi}\left(H_i\left(D_i\right)\right) + H_i(D_i) > A_i^{\star}\left(D_i\right) + H_i^{\star}(D_i) = \min\left\{D_i, K^{\star} + y_i^{\star}\right\},$$

such dispatch policy $H_i\left(D_i\right)$ is not feasible.

In any scenario i, for given $K^{\star}$ we know by Eq. (36) that $y_i = y_i^{\star}$ uniquely determines the optimal profit for the platform. Since $A_i^{\star}$ is feasible for Eq. (5) - Eq. (8) when $y_i = y_i^{\star}$, it is optimal for the platform to set $A_i = A_i^{\star}$, which completes the proof.

□

E.4.10. Proof of Corollary 2

Proof. Since in the proof of Theorem 4 we have provided a construction for $\pi' \in \Pi^F$ that aligns the supply chain, we similarly construct $\pi \in \Pi^H$ with $K^{\pi} = K^{\star}, H^{\pi} = 0$ and

$$c_P^{\pi} = \frac{V_A^s\left(I\right) + c_F K^{\star}}{\sum_i \alpha_i \mathbb{E}_{D_i \sim F_i}\left[\min\left\{D_i, K^{\star}\right\}\right]}.$$

Notice that π is the same as π' except that it sets $H^{\pi} = 0$ instead of $A^{\pi'}(D_i) = \min\left\{D_i, K^{\star}\right\}$ $\forall i$. Thus, it suffices to show that $H^{\pi} = 0$ implies that the platform adopts $A_i(D_i) = \min\left\{D_i, K^{\star}\right\}$ $\forall i$, so that the rest of the proof follows from Theorem 4. Indeed, from $H^{\pi} = 0$ we know that the platform has to adopt $\vec{\mathbf{y}} = \vec{0}$ by the constraint in Eq. (8). Then, by the optimality condition in Eq. (35) we know the platform adopts $A_i(D_i) = \min\left\{D_i, K^{\star} + 0\right\} - 0 = \min\left\{D_i, K^{\star}\right\}$ $\forall i$. Then, π is equivalent to π' and we conclude the proof.

□