

# Theorem 2

In[1]:= (\* See Mathematica codes at <https://bit.ly/4bkvD03> \*)

(\* Define mu \*)

mu =

$$\begin{aligned} & (-8 p f p^3 (-1 + b l) b l (-1 + b r) b r + 2 p^4 (-1 + b l) b l (-1 + b r) b r - 2 p (-2 + b r + 3 p f b r - p f \\ & \quad b r^2 + p f b l^2 (-1 + 4 p f^2 (-1 + b r) b r) + b l (1 + p f (3 - 4 b r) - 4 p f^3 (-1 + b r) b r)) + \\ & \quad p f (-b r (-2 + p f + p f b r) + p f b l^2 (-1 + 2 p f^2 (-1 + b r) b r) - \\ & \quad b l (-2 + p f + 4 p f b r + 2 p f^3 (-1 + b r) b r)) + p^2 (-8 + 7 b r - b r^2 + \\ & \quad b l^2 (-1 + 12 p f^2 (-1 + b r) b r) + b l (7 + 4 (-1 + 3 p f^2) b r - 12 p f^2 b r^2))) / \\ & (1 + p f^4 (-1 + b l) b l (-1 + b r) b r - 4 p f p^3 (-1 + b l) b l (-1 + b r) b r + \\ & \quad p^4 (-1 + b l) b l (-1 + b r) b r - p f^2 (b l + b r + 2 b l b r) - \\ & \quad 2 p f p (b l + b r + 2 (-1 + p f^2) b l b r + 2 p f^2 b l^2 (-1 + b r) b r - 2 p f^2 b l b r^2) + \\ & \quad p^2 (-4 + 3 b r + 6 p f^2 b l^2 (-1 + b r) b r + b l (3 - 2 b r + 6 p f^2 b r - 6 p f^2 b r^2))) \end{aligned}$$

(\* Compute mu(B,0) - mu(B/2,B/2) \*)

compare = Simplify[mu /. {bl -> B, br -> 0}] - Simplify[mu /. {bl -> B/2, br -> B/2}]

(\* Specify range of parameters \*)

conditions = 0 < B < 1 && 0 ≤ p < pf ≤ 1/2

(\* Verify if it is possible to have mu(B,0) - mu(B/2,B/2) ≤ 0;

returns false if the difference > 0 for all parameters within the range \*)

Reduce[compare ≤ 0 && conditions, {B, pf, p}]

$$\begin{aligned} \text{Out[1]} = & (2 (-1 + b l) b l (-1 + b r) b r p^4 - 8 (-1 + b l) b l (-1 + b r) b r p^3 p f - \\ & 2 p (-2 + b r + 3 b r p f - b r^2 p f + b l^2 p f (-1 + 4 (-1 + b r) b r p f^2) + b l (1 + (3 - 4 b r) p f - 4 (-1 + b r) b r p f^3)) + \\ & p f (-b r (-2 + p f + b r p f) + b l^2 p f (-1 + 2 (-1 + b r) b r p f^2) - b l (-2 + p f + 4 b r p f + 2 (-1 + b r) b r p f^3)) + \\ & p^2 (-8 + 7 b r - b r^2 + b l^2 (-1 + 12 (-1 + b r) b r p f^2) + b l (7 - 12 b r^2 p f^2 + 4 b r (-1 + 3 p f^2))) / \\ & (1 + (-1 + b l) b l (-1 + b r) b r p^4 - 4 (-1 + b l) b l (-1 + b r) b r p^3 p f - (b l + b r + 2 b l b r) p f^2 + \\ & (-1 + b l) b l (-1 + b r) b r p f^4 - 2 p p f (b l + b r + 2 b l^2 (-1 + b r) b r p f^2 - 2 b l b r^2 p f^2 + 2 b l b r (-1 + p f^2)) + \\ & p^2 (-4 + 3 b r + 6 b l^2 (-1 + b r) b r p f^2 + b l (3 - 2 b r + 6 b r p f^2 - 6 b r^2 p f^2))) \end{aligned}$$

$$\begin{aligned} \text{Out[2]} = & - \frac{2 ((-2 + B) B p^2 + B p f (4 + (-2 + B) p f) - 2 (-2 + B) p (2 + B p f))}{4 + (-2 + B) B p^2 - 2 B (-2 + p f) p f + B^2 p f^2 - 2 (-2 + B) p (2 + B p f)} - \\ & \frac{(8 - 7 B + B^2) p^2 + B p f (-2 + p f + B p f) + p (-4 - 2 B^2 p f + B (2 + 6 p f))}{1 + (-4 + 3 B) p^2 - 2 B p p f - B p f^2} \end{aligned}$$

$$\text{Out[3]} = 0 < B < 1 \&\& 0 \leq p < p f \leq \frac{1}{2}$$

Out[4]= False