Theorem 1: One-sided Allocation > Balanced Allocation

In[1]:=
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(* Define mu *)
mu = -p^4 ((4 - 3 br)^2 + bl^2 (9 - 10 br + 2 br^2) - 2 bl (12 - 16 br + 5 br^2)) -
    pf (br (-2 + pf + pf br - 2 pf^2 br + pf^3 br) +
        bl (-2 + pf + 4 pf br + 6 pf^3 br^2 - 4 pf^2 br (1 + br)) +
        pf bl^2 (1 - 2 pf (1 + 2 br) + pf^2 (1 + 6 br + 2 br^2))) +
    2 p^3 ((-4 + 3 br) (-2 + br + 2 pf br) + bl^2 (3 - 2 br + 2 pf (3 - 6 br + 2 br^2)) -
        2 bl (5 - 5 br + br^2 + 2 pf (2 - 6 br + 3 br^2))) +
    2 p (2 + (-1 - 3 pf + 2 pf^2) br + (pf + pf^2 - 2 pf^3) br^2 +
        bl (-1 + 4 pf^3 (-2 + br) br + pf (-3 + 4 br) + pf^2 (2 + 6 br - 6 br^2)) +
        pf bl^2 (1 + pf - 6 pf br + pf^2 (-2 + 4 br + 4 br^2))) -
    p^2 (8 + (-7 - 16 pf + 8 pf^2) br + (1 + 10 pf - 2 pf^2) br^2 +
        bl (-7 + 4 br + pf^2 (8 - 12 br^2) - 4 pf (4 - 9 br + 3 br^2)) +
        bl^2 (1 - 2 pf (-5 + 6 br) + 2 pf^2 (-1 - 6 br + 6 br^2)));
(* Compute mu(B, 0) - mu(B/2, B/2) *)
compare = Simplify[mu /. {bl → B, br → 0}] - Simplify[mu /. {bl → B/2, br → B/2}];
(* Specify range of parameters *)
conditions = 0 < B ≤ 1 && 0 ≤ p < pf ≤ 1/2;
(* Verify if it is possible to have mu(B, 0) - mu(B/2, B/2) ≤ 0;
returns false if the difference > 0 for all parameters within the range *)
Reduce[compare ≤ 0 && conditions, {B, pf, p}]
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Out[4]=
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False
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