

# **Self Attention**

**Advanced NLP: Summer 2023**

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# Preliminaries

# LayerNorm

<https://arxiv.org/abs/1607.06450>

also see: <https://arxiv.org/abs/1911.07013>

$$\mathbf{x} = (x_1, x_2, \dots, x_H)$$

$$\mu = \frac{1}{H} \sum_{i=1}^H x_i \quad \sigma^2 = \frac{1}{H} \sum_{i=1}^H (x_i - \mu)^2$$

$$N(\mathbf{x}) = \frac{\mathbf{x} - \mu}{\sigma + \epsilon} \quad \epsilon \text{ avoids div by zero}$$

$$\mathbf{h} = \mathbf{g} \cdot N(\mathbf{x}) + \mathbf{b}$$

**g** and **b** are hyperparameters with dimension H

In PyTorch

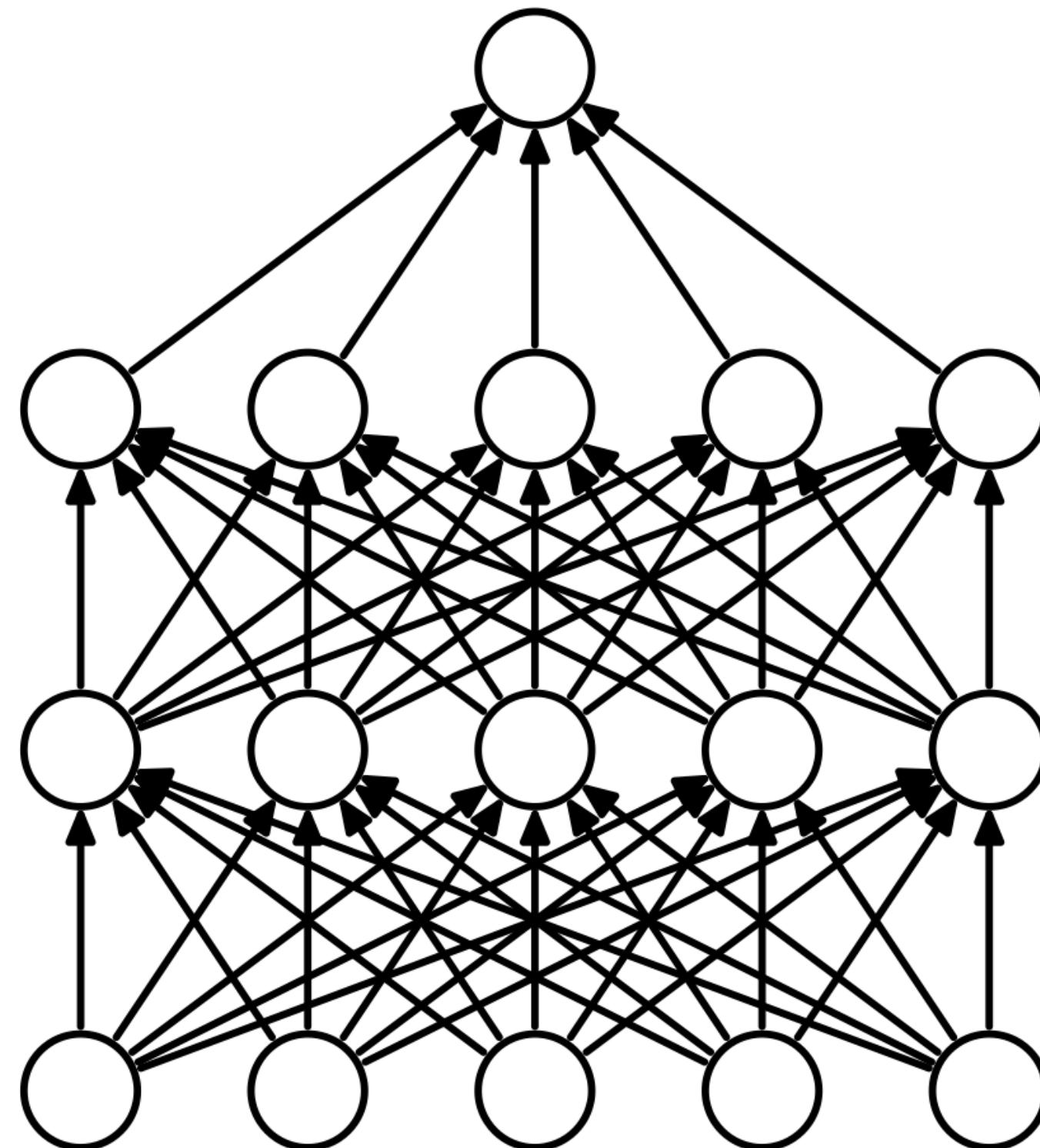
```
>>> # NLP Example
>>> batch, sentence_length, embedding_dim = 20, 5, 10
>>> embedding = torch.randn(batch, sentence_length, embedding_dim)
>>> layer_norm = nn.LayerNorm(embedding_dim)
>>> # Activate module
>>> layer_norm(embedding)
```

# Dropout

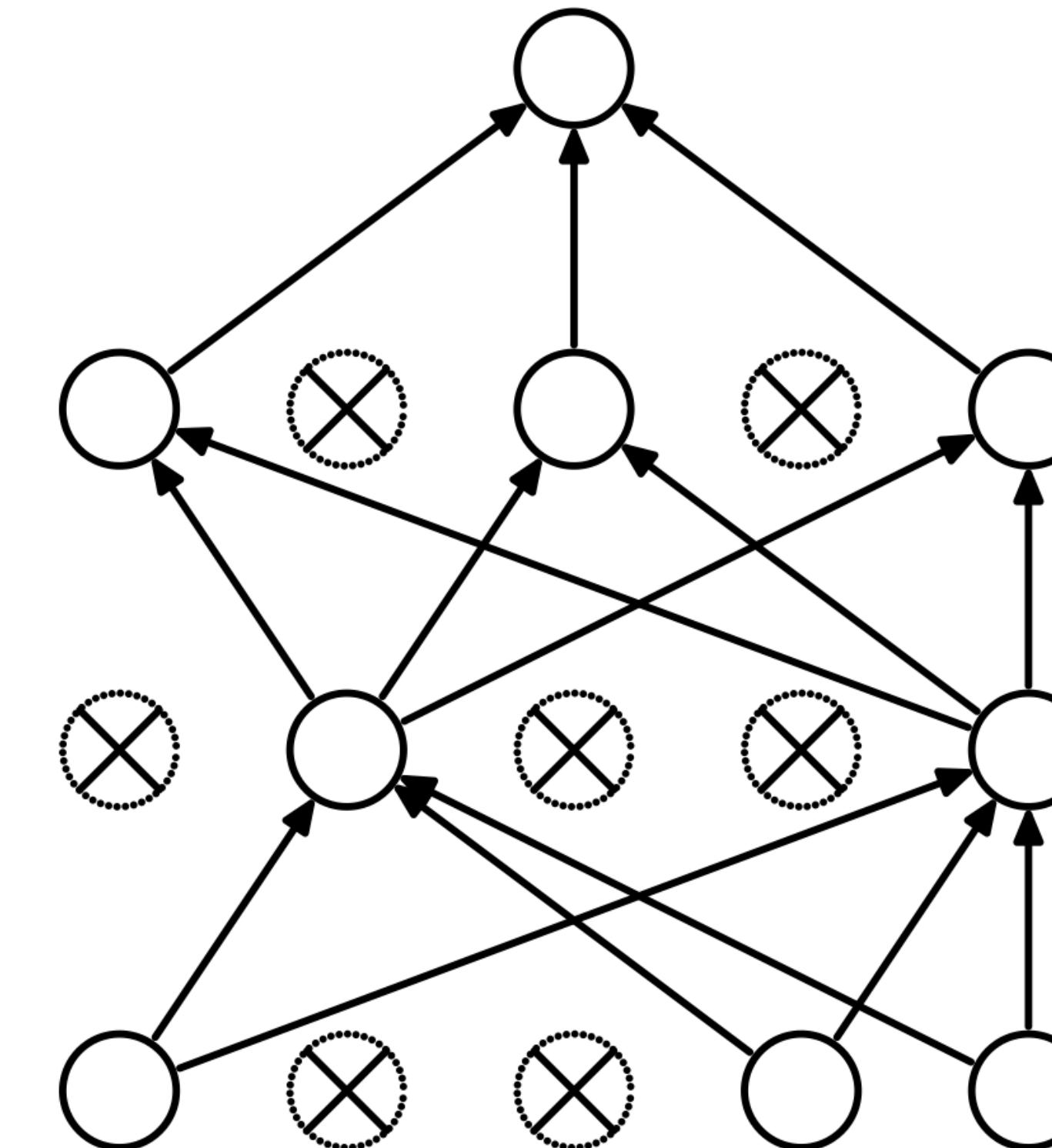
<https://jmlr.org/papers/v15/srivastava14a.html>

<https://arxiv.org/abs/1207.0580>

aka how to train  $2^n$  neural networks when it has  $n$  units



(a) Standard Neural Net



(b) After applying dropout.

## Before dropout

$$\begin{aligned} z_i^{(l+1)} &= \mathbf{w}_i^{(l+1)} \mathbf{y}^l + b_i^{(l+1)}, \\ y_i^{(l+1)} &= f(z_i^{(l+1)}), \end{aligned}$$

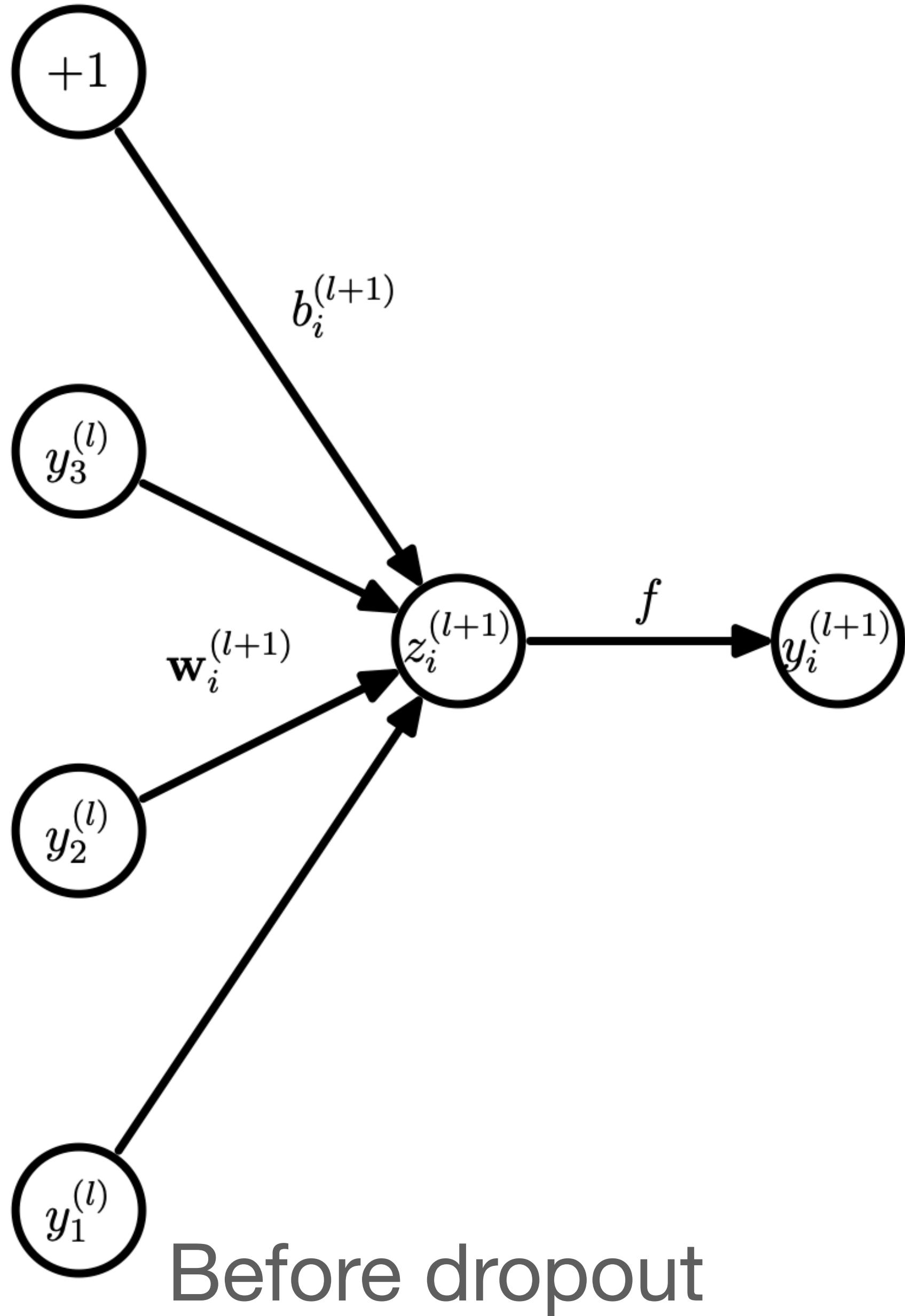
## After dropout

$$\begin{aligned} r_j^{(l)} &\sim \text{Bernoulli}(p), \\ \tilde{\mathbf{y}}^{(l)} &= \mathbf{r}^{(l)} * \mathbf{y}^{(l)}, \\ z_i^{(l+1)} &= \mathbf{w}_i^{(l+1)} \tilde{\mathbf{y}}^l + b_i^{(l+1)}, \\ y_i^{(l+1)} &= f(z_i^{(l+1)}). \end{aligned}$$

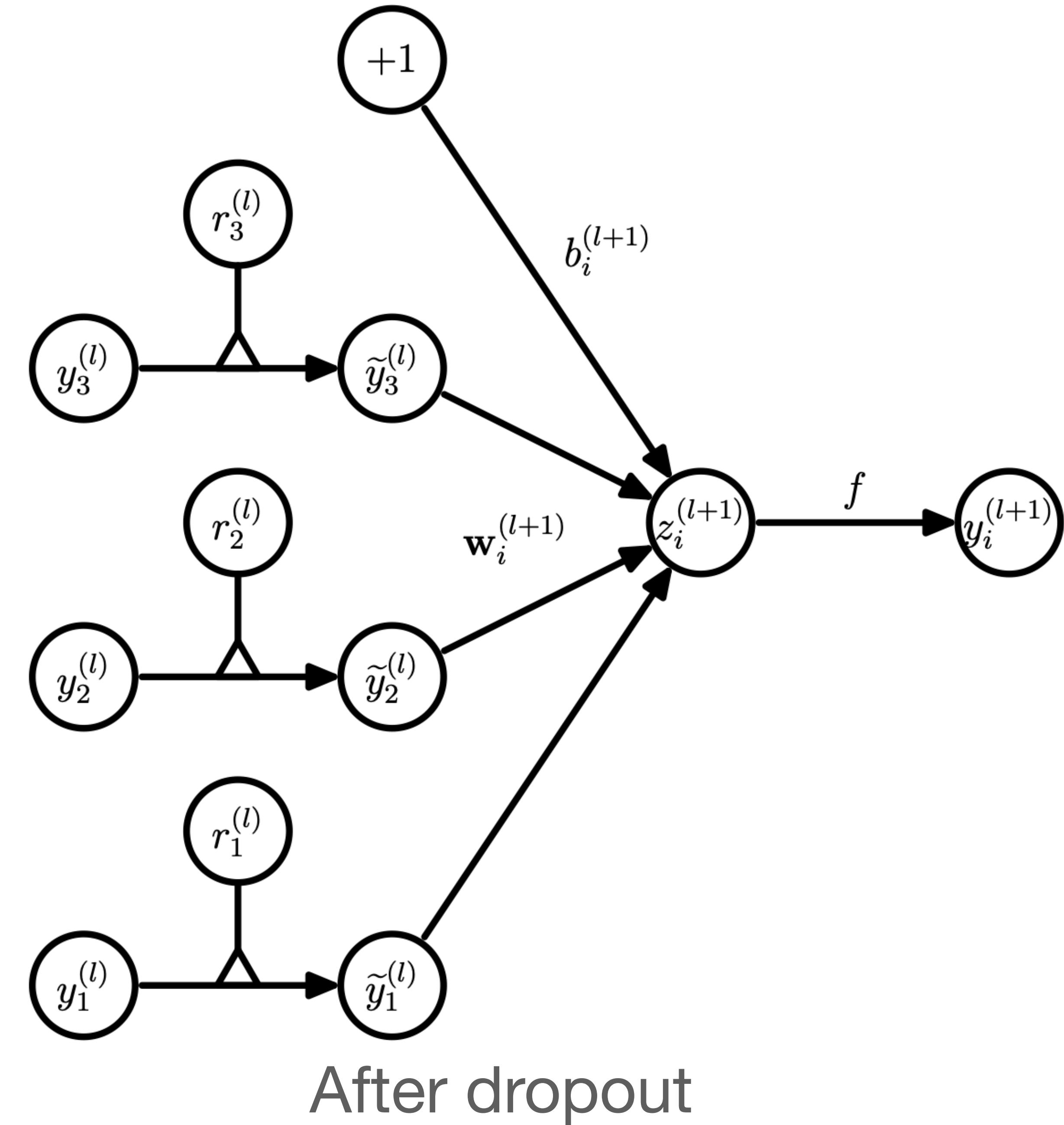
## In PyTorch

```
>>> m = nn.Dropout(p=0.2)
>>> input = torch.randn(20, 16)
>>> output = m(input)
```

default: 0.5



Before dropout



After dropout

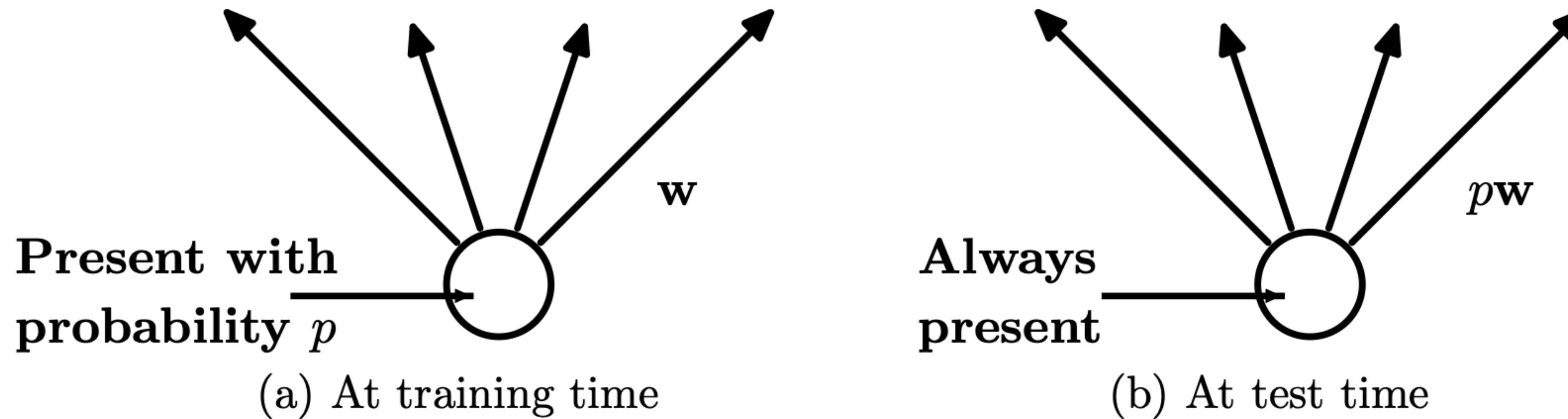


Figure 2: **Left:** A unit at training time that is present with probability  $p$  and is connected to units in the next layer with weights  $w$ . **Right:** At test time, the unit is always present and the weights are multiplied by  $p$ . The output at test time is same as the expected output at training time.

In Pytorch the outputs are scaled by a factor of  $\frac{1}{1-p}$  during training so at inference/test/evaluation time the dropout function simply computes the identity function

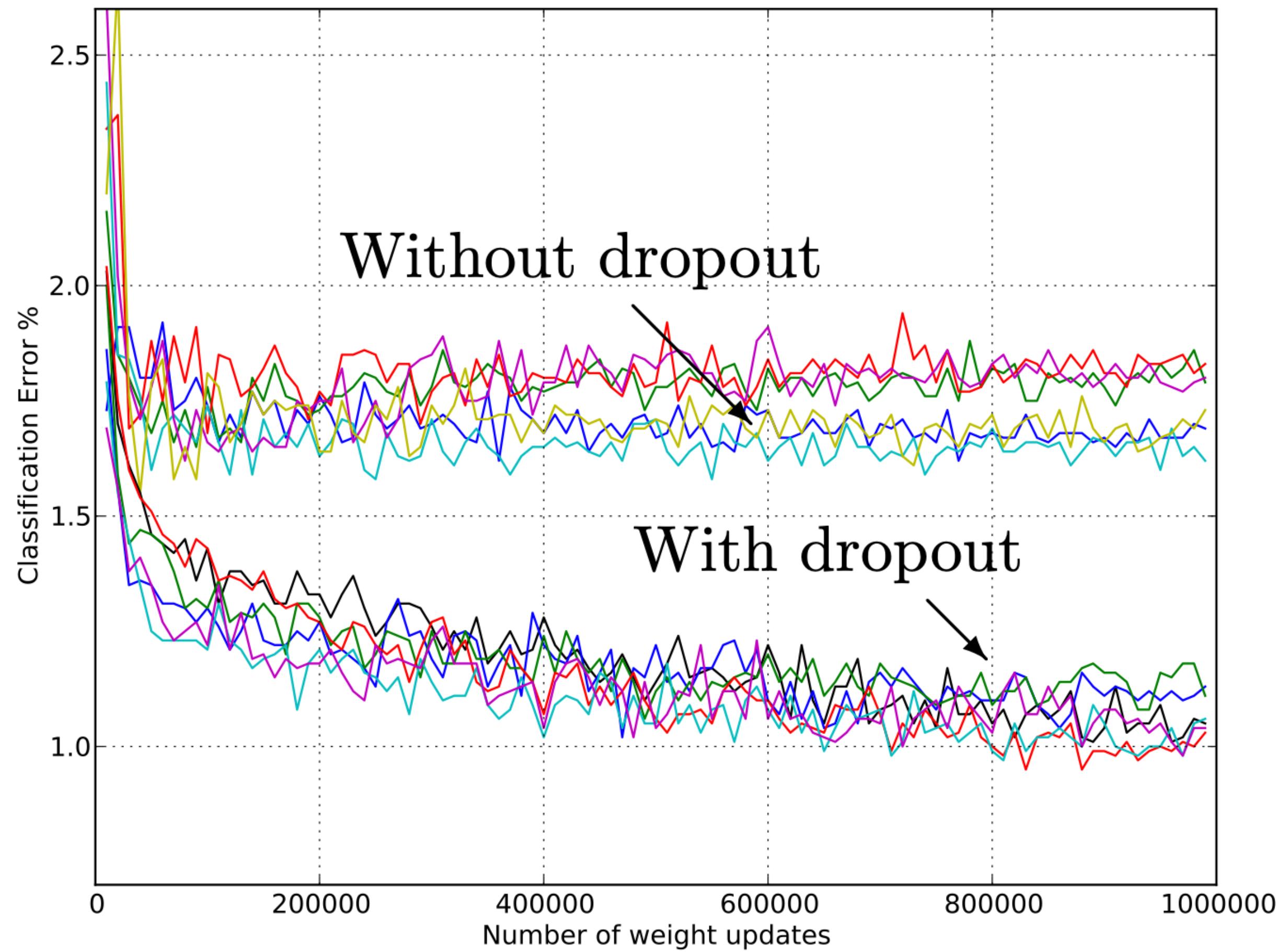
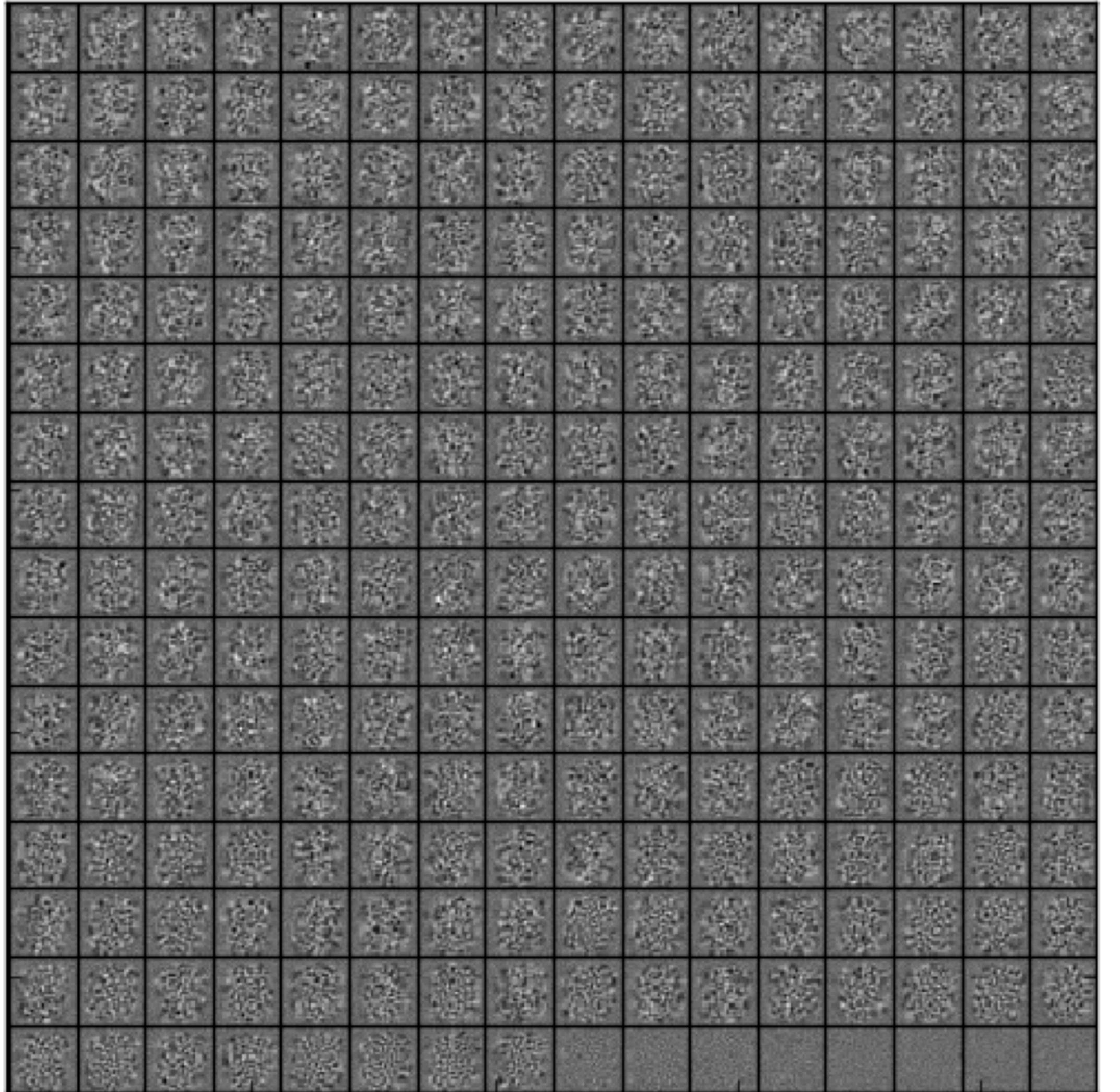
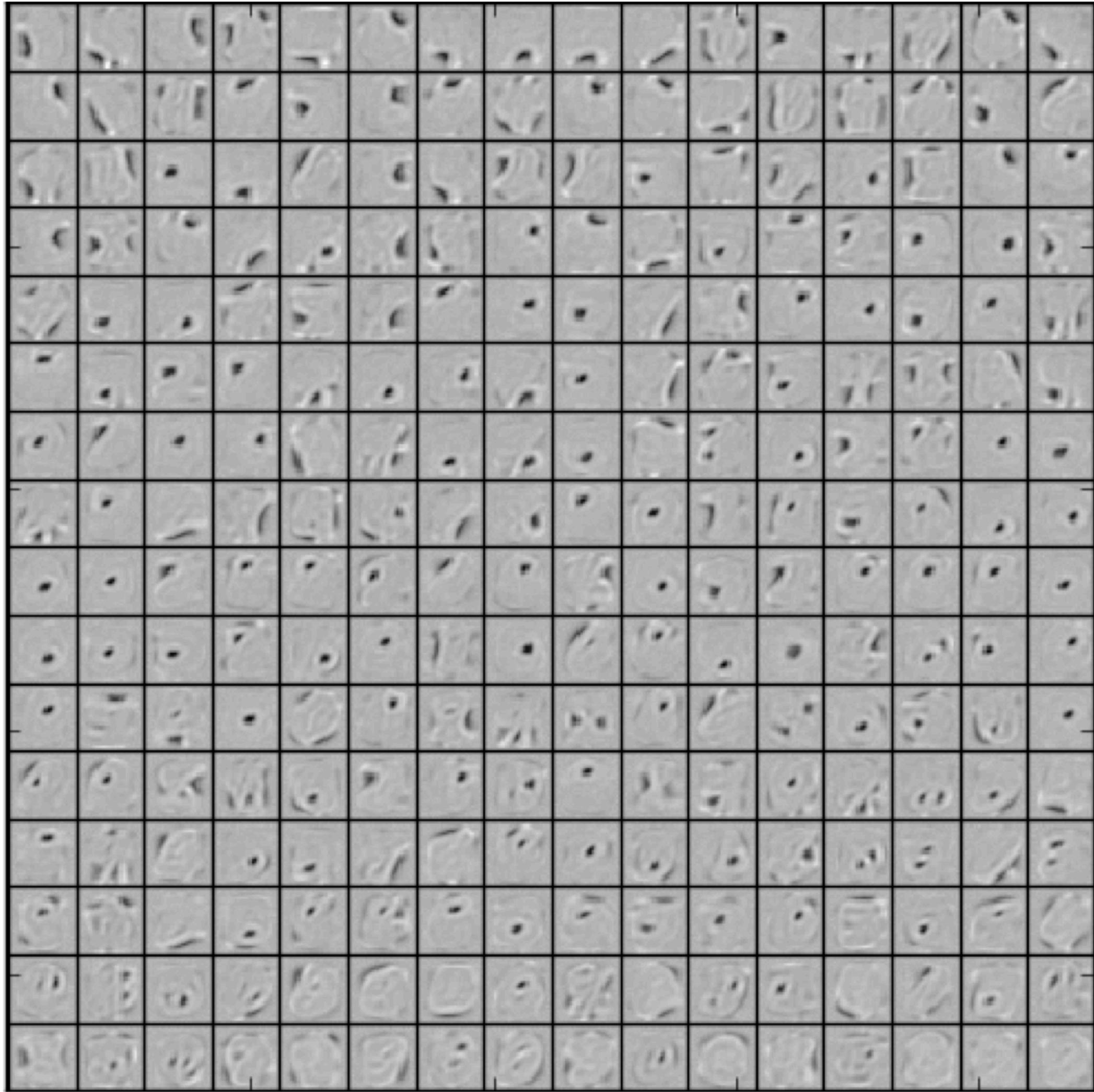


Figure 4: Test error for different architectures with and without dropout. The networks have 2 to 4 hidden layers each with 1024 to 2048 units.

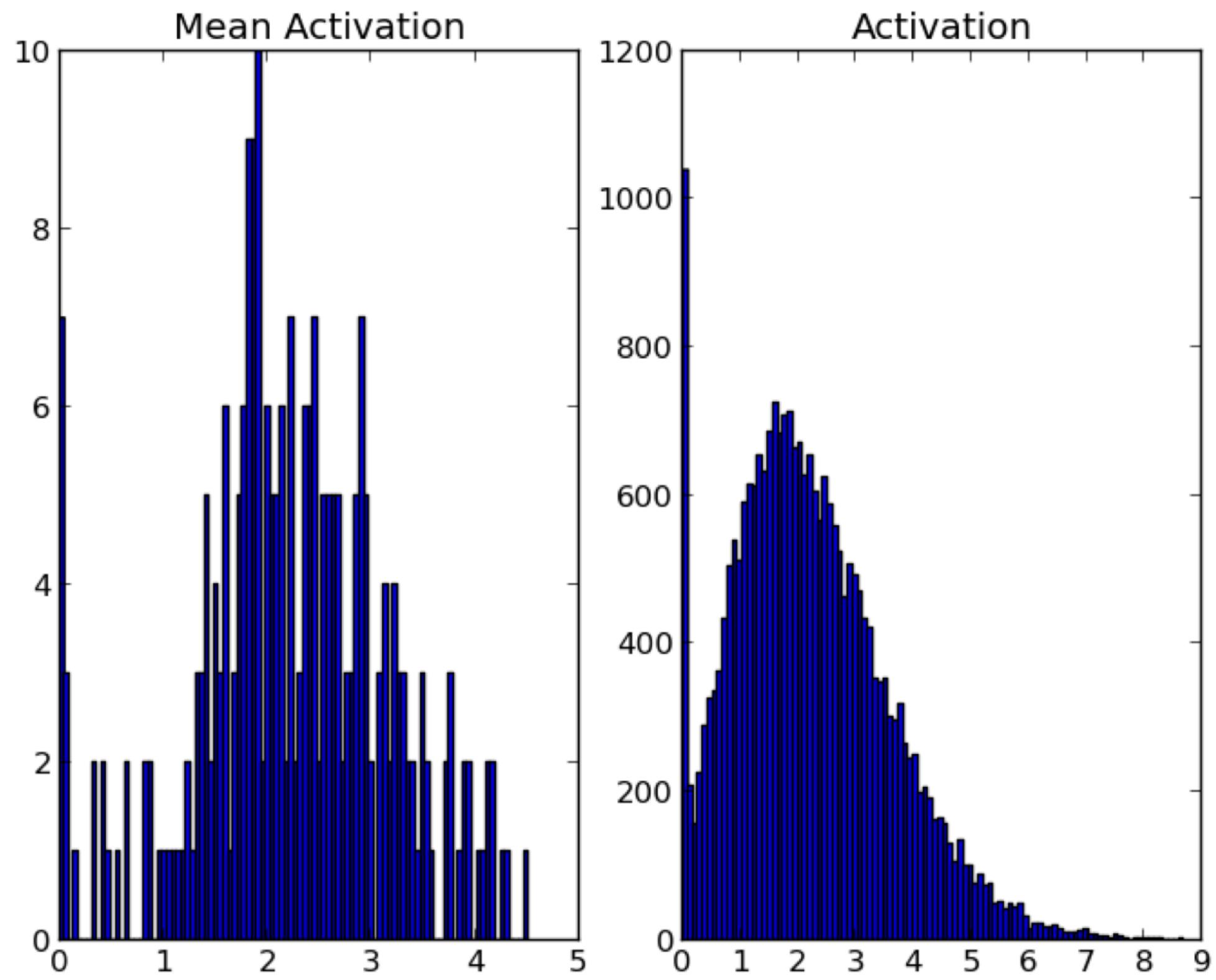


(a) Without dropout

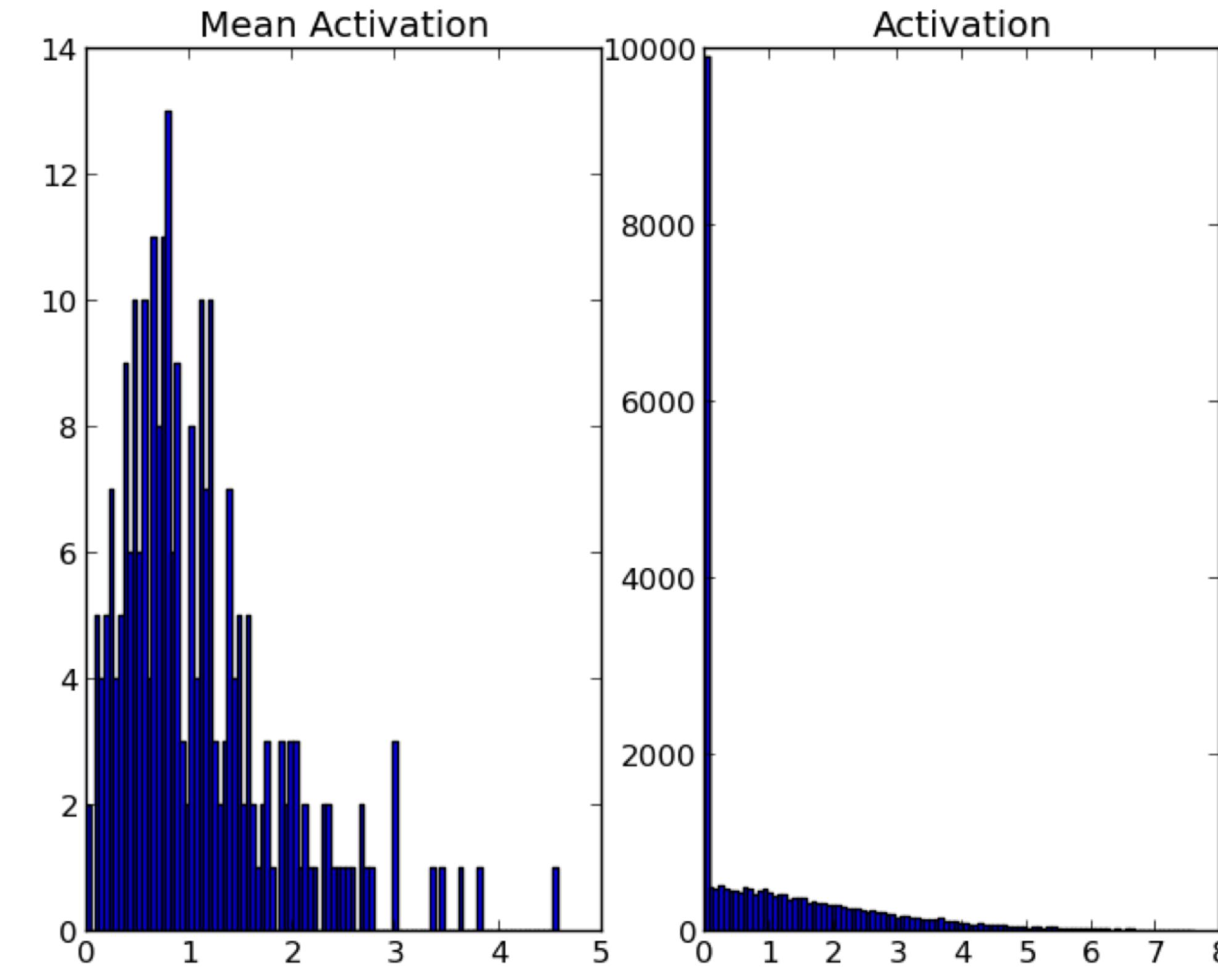


(b) Dropout with  $p = 0.5$ .

Figure 7: Features learned on MNIST with one hidden layer autoencoders having 256 rectified linear units.



(a) Without dropout



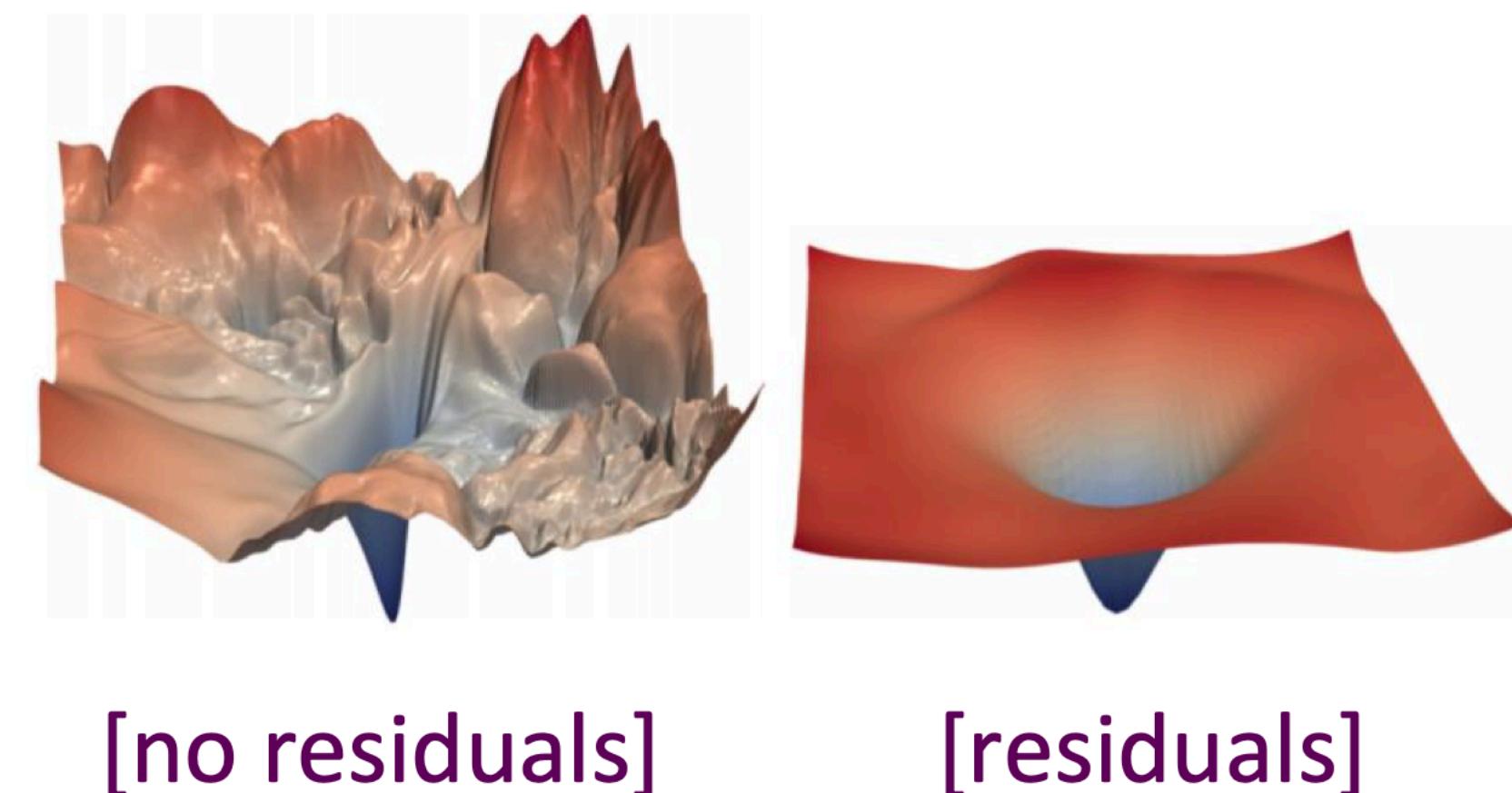
(b) Dropout with  $p = 0.5$ .

Figure 8: Effect of dropout on sparsity. ReLUs were used for both models. **Left:** The histogram of mean activations shows that most units have a mean activation of about 2.0. The histogram of activations shows a huge mode away from zero. Clearly, a large fraction of units have high activation. **Right:** The histogram of mean activations shows that most units have a smaller mean activation of about 0.7. The histogram of activations shows a sharp peak at zero. Very few units have high activation.

# Residual Connections

Add input of a layer to output of that layer

- $\mathbf{z}^{\ell+1} = f(\mathbf{z}^\ell) + \mathbf{z}^\ell$
- Local gradient is 1 for the identity function
- Easier to learn the difference from the identity function than to learn the function from scratch.



<https://arxiv.org/pdf/1712.09913.pdf>

# **Transformer Encoder-Decoder**

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# Attention Is All You Need

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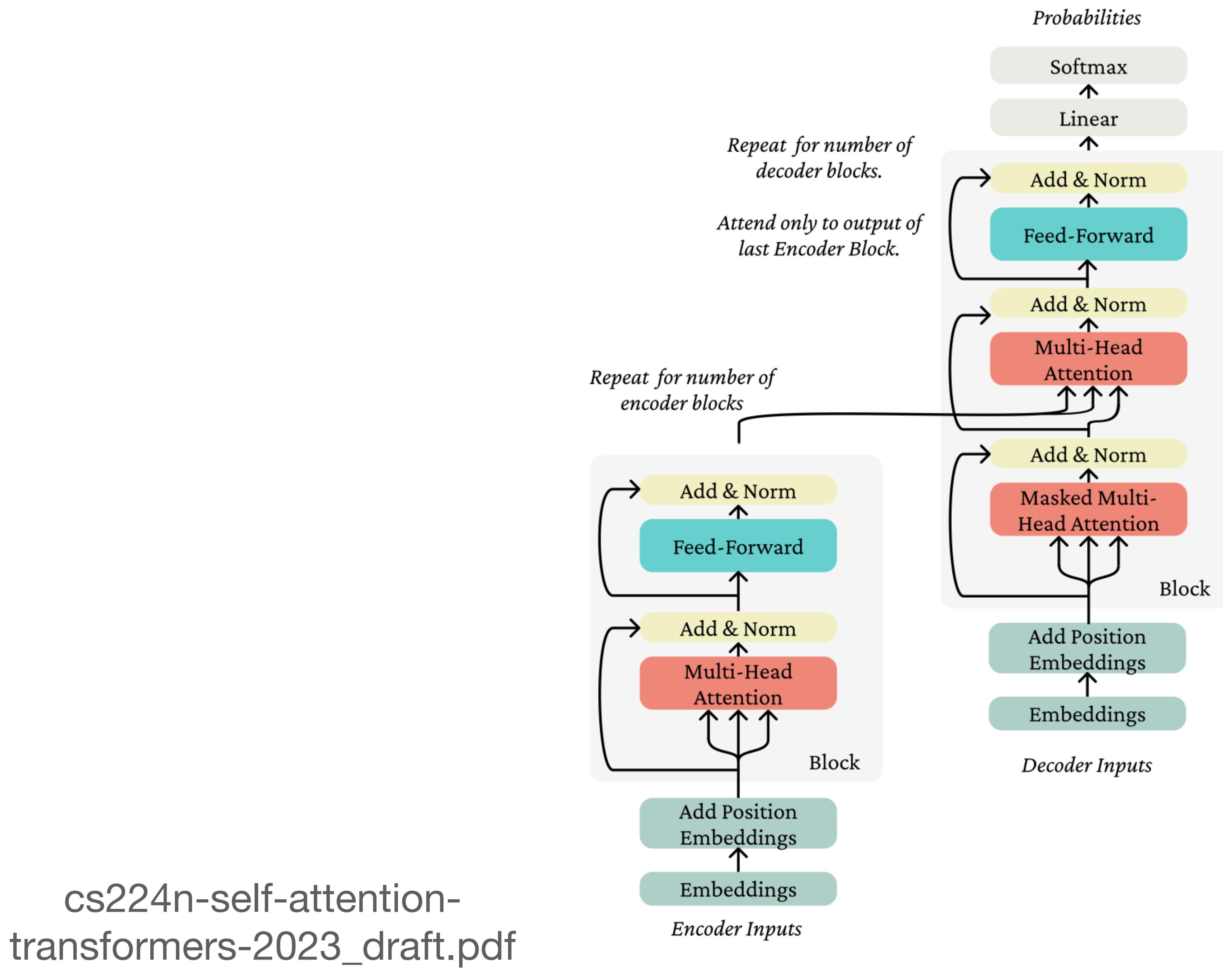
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NIPS (2017)

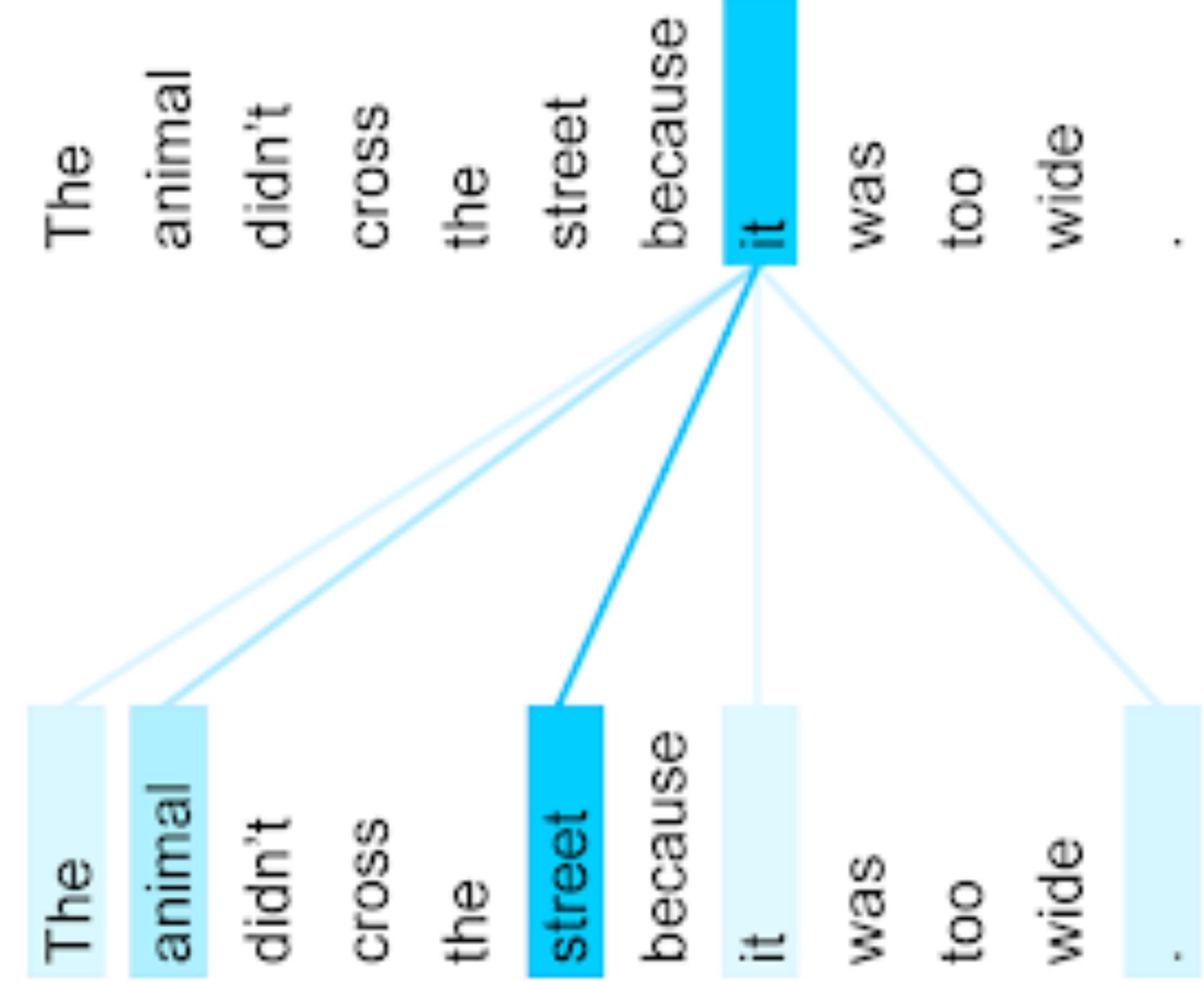
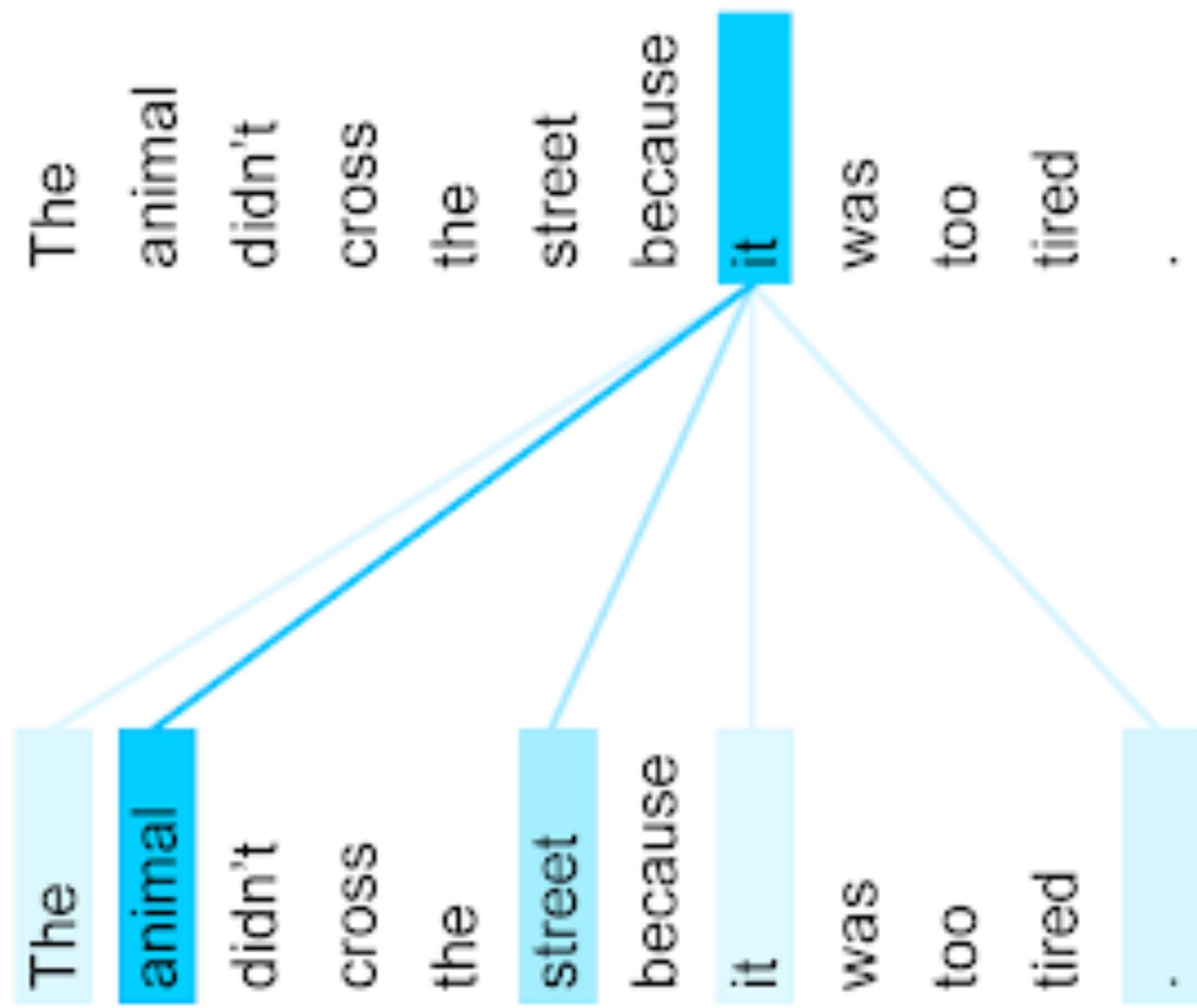


<https://ai.googleblog.com/2017/08/transformer-novel-neural-network.html>

*The animal didn't cross the street because it was too tired.  
L'animal n'a pas traversé la rue parce qu'il était trop fatigué.*

*The animal didn't cross the street because it was too wide.  
L'animal n'a pas traversé la rue parce qu'elle était trop large.*

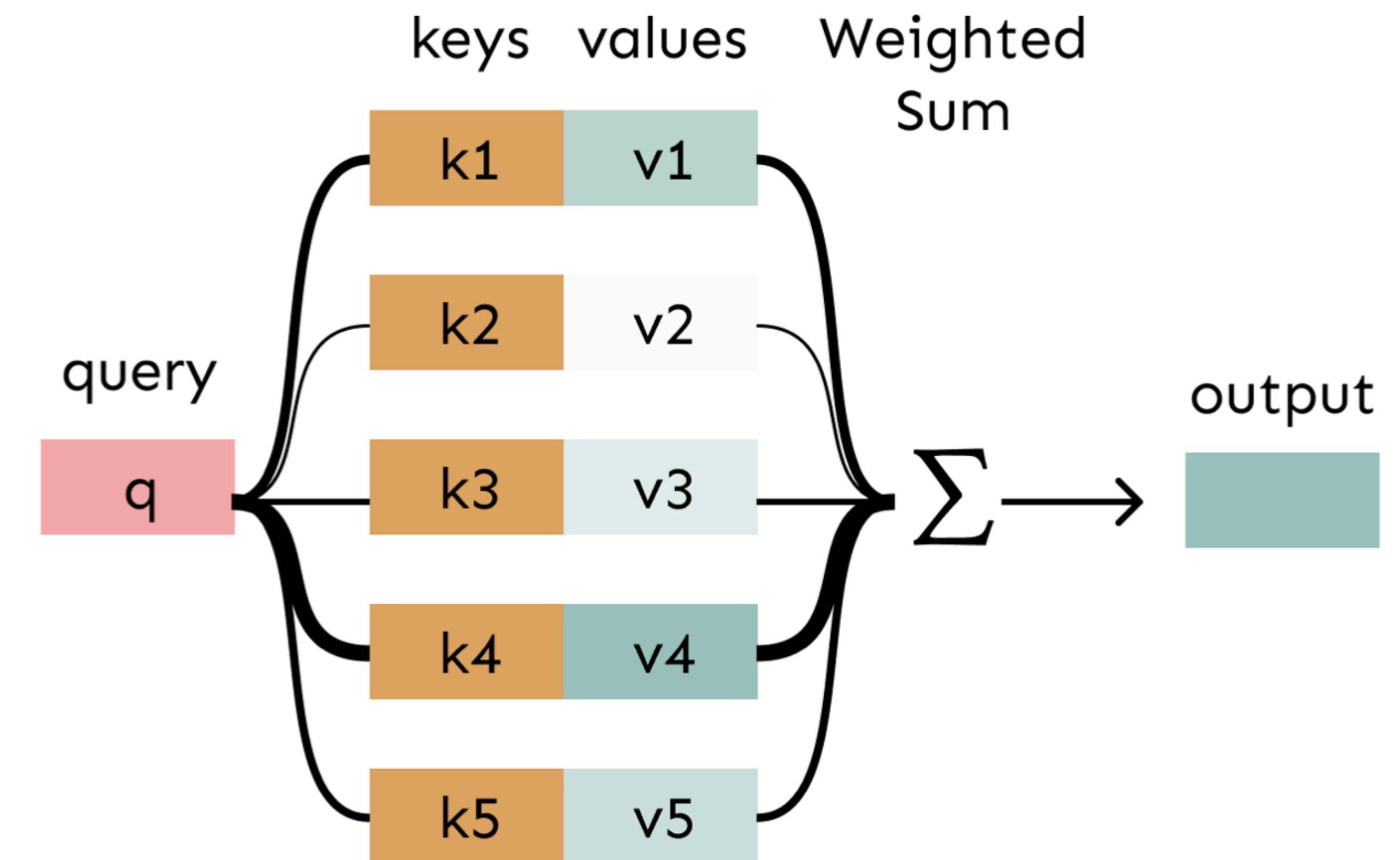
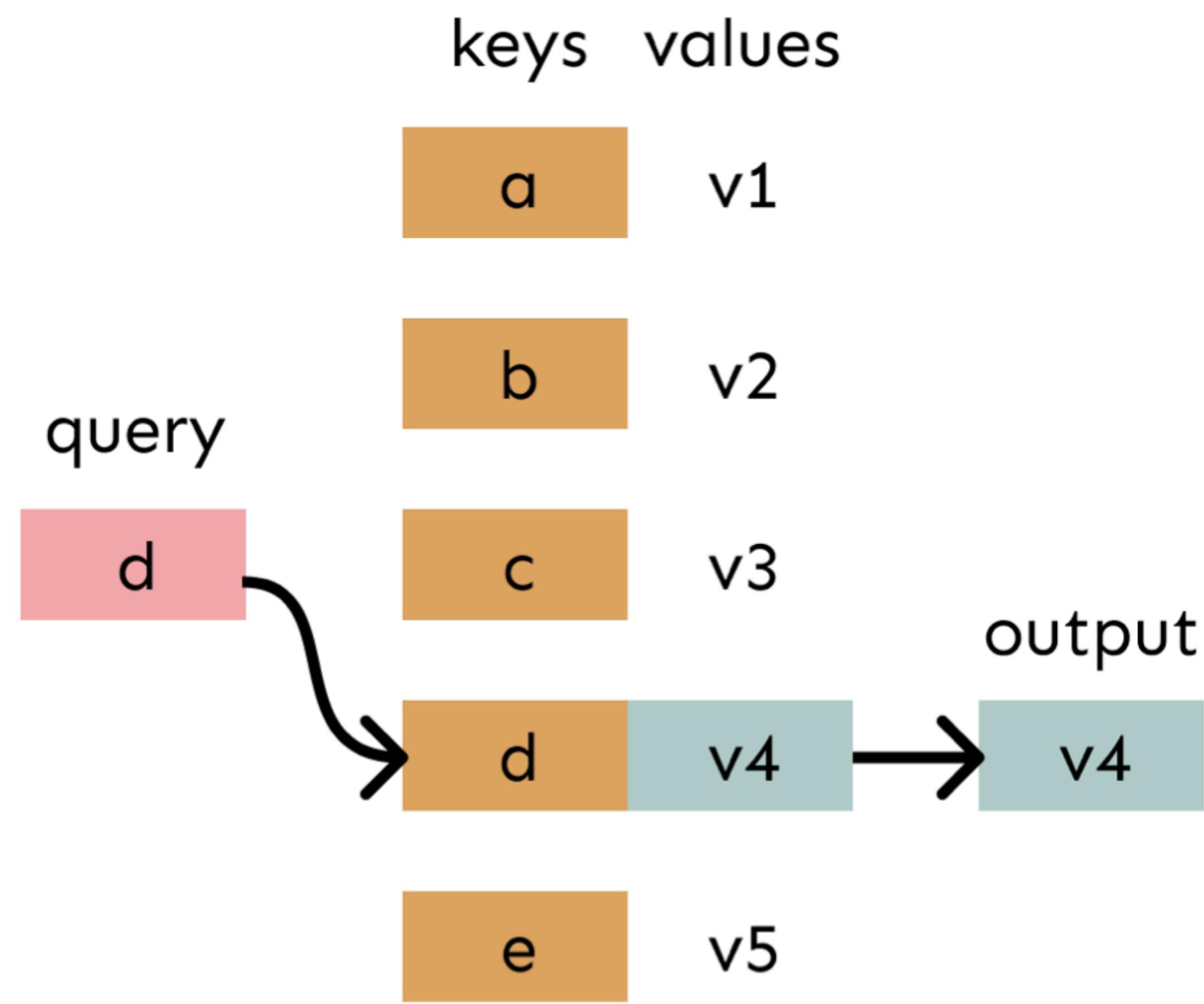
the translation for “it” depends on the gender of the noun it refers to - and in French “animal” and “street” have different genders



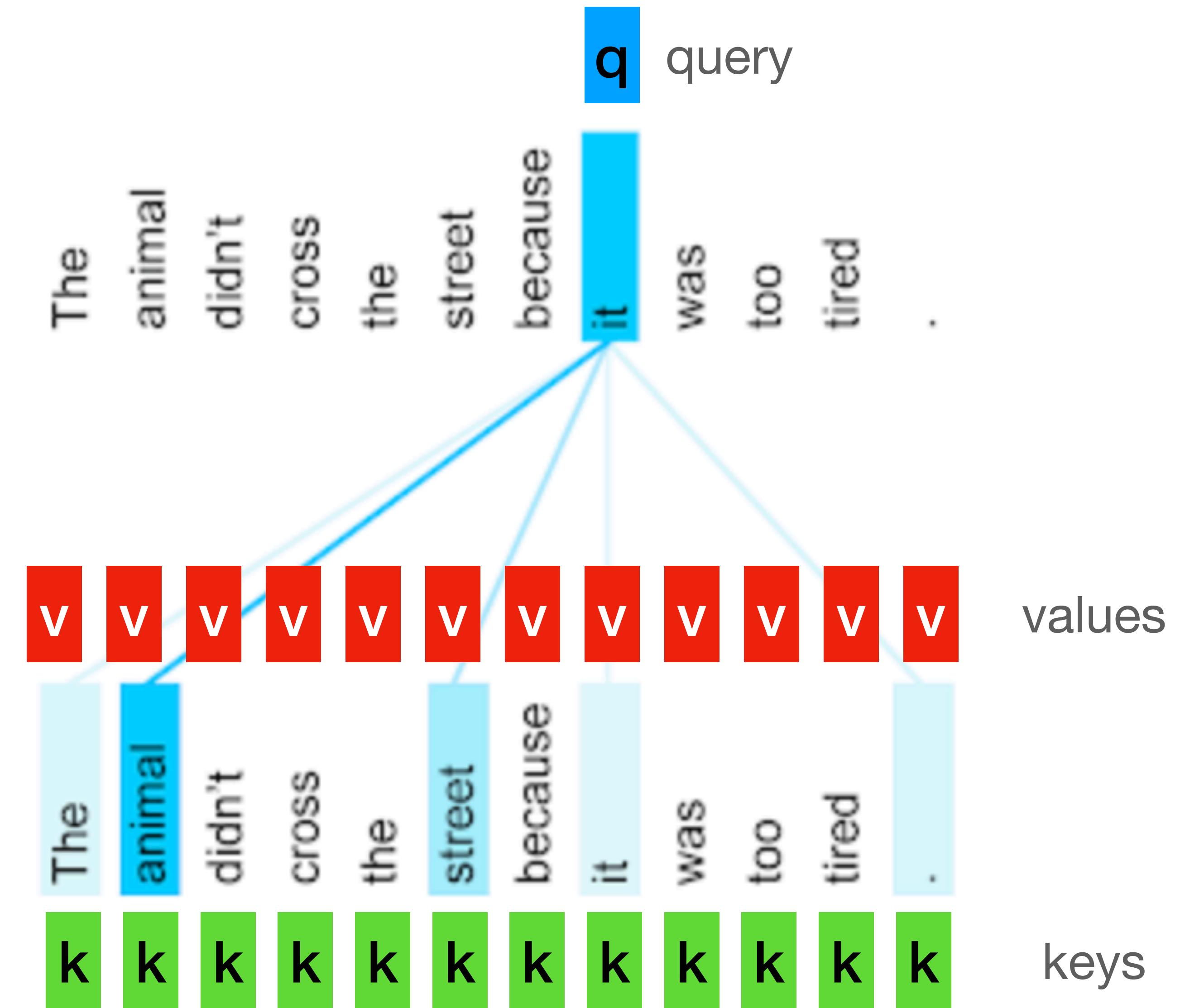
The encoder self-attention distribution for the word “it” from the 5th to the 6th layer of a Transformer trained on English to French translation (one of eight attention heads).

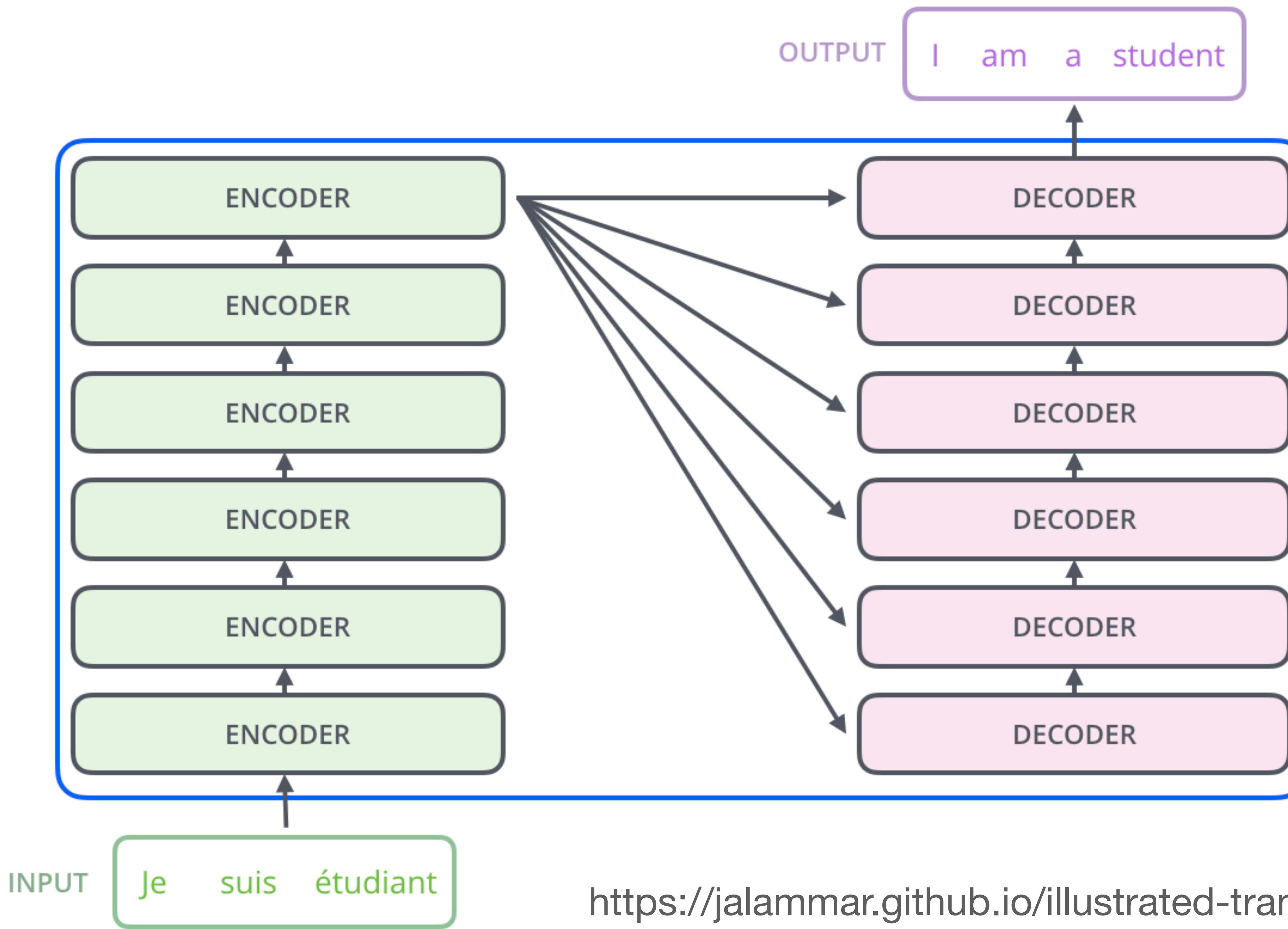
# Self Attention

- Take a query vector (based on one token)
- Do a "**soft lookup**" in a key-value store; **pick up** the key **most like** the query and return the value vector
- "**pick up**" = return the average value based on a probability distribution
- "**most like**" = higher probability for a key means it is **more like** the query
- "**more like**" = dot product e.g.
- In *self attention* we use the same tokens for queries, keys and values



# Self Attention





<https://jalammar.github.io/illustrated-transformer/>

# Self-attention

## keys, queries and values from the same sequence

- Let  $\mathbf{w} = (w_1, \dots, w_n)$  be a sequence of tokens, like "Havana is the capital of"
  - For each  $w_i$  let  $\mathbf{x}_i = E\mathbf{w}_i$  where  $E \in \mathbb{R}^{d \times |V|}$  is an embedding matrix.  $V$  is the vocabulary.
  - Let  $Q, K, V$  be matrices in  $\mathbb{R}^{d \times d}$ 
    - $\mathbf{q}_i = Q\mathbf{x}_i$
    - $\mathbf{k}_i = K\mathbf{x}_i$
    - $\mathbf{v}_i = V\mathbf{x}_i$
- Output for each word is a weighted sum of values:
- $$\mathbf{o}_i = \sum_j \text{softmax}_j(\mathbf{q}_i^T \mathbf{k}_j) \cdot \mathbf{v}_i$$

# Self Attention: Three Problems

| Problem   | Solution  |
|---|---|
| Encoder and decoder has no inherent notion of ordering. It's just a bag of words. | Add position representations to each token                |
| Just a weighted average of a vector. No non-linearities.                          | Apply feedforward network to each self attention output   |
| Decoder should not look into the future while training the predictor.             | Mask out the future by setting attention weights to zero. |

# Self-attention

## Fixing the sequence order problem

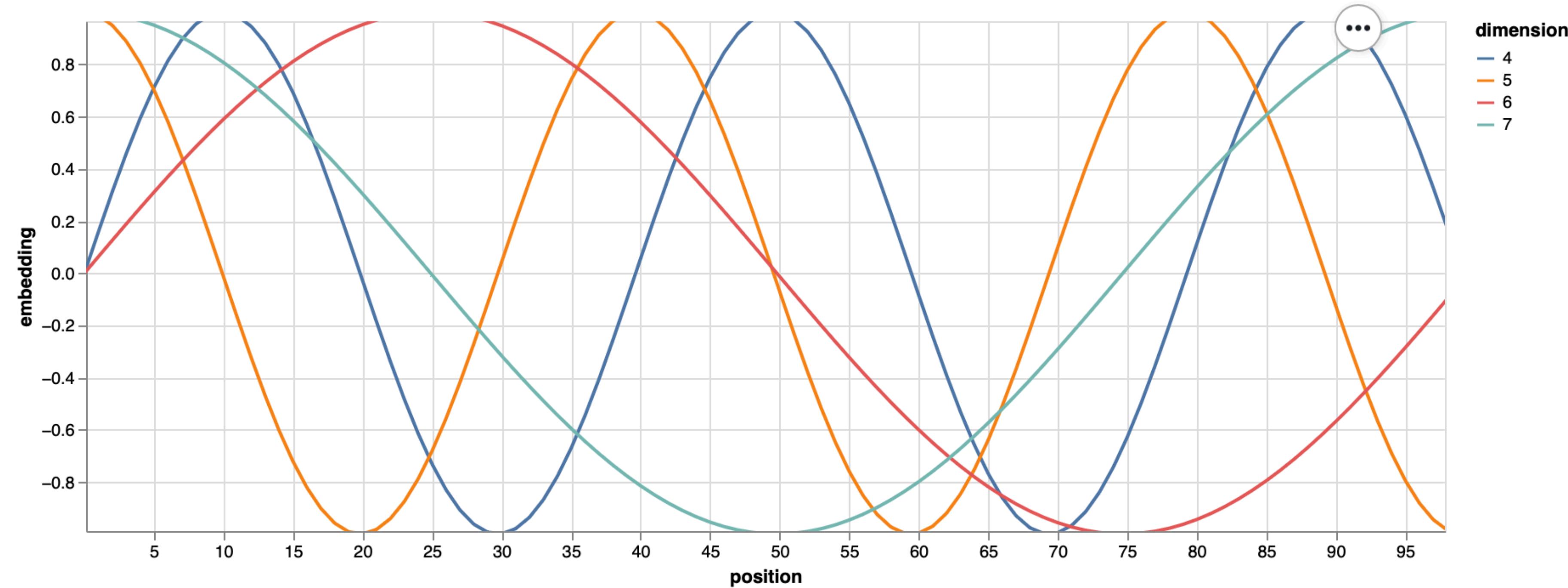
- We need to encode the order of the tokens in a sentence in the keys, values and queries
- We want a position embedding (similar to a word embedding)
- Let  $\mathbf{p}_i \in \mathbb{R}^d$  for  $i \in 1, \dots, n$  be the position embeddings
- If  $\mathbf{x}_i$  is the embedding for the word  $w_i$  then the combined word plus position embedding is  $\tilde{\mathbf{x}}_i = \mathbf{x}_i + \mathbf{p}_i$
- Either concatenate  $\mathbf{x}_i$  and  $\mathbf{p}_i$  or just add them. Adding is more common.

# Position embeddings without learning

cs224n-self-attention-transformers-2023\_draft.pdf

Use a periodic function like sine and cosine with different periods to get an embedding vector without any parameter updates.

$$\mathbf{p}_i = \begin{pmatrix} \sin(i/10000^{2*1/d}) \\ \cos(i/10000^{2*1/d}) \\ \vdots \\ \sin(i/10000^{2*\frac{d}{2}/d}) \\ \cos(i/10000^{2*\frac{d}{2}/d}) \end{pmatrix}$$



Pros:

- \* Periodicity means absolute position is not important
- \* Can extrapolate to longer sequences as periods restart

Cons:

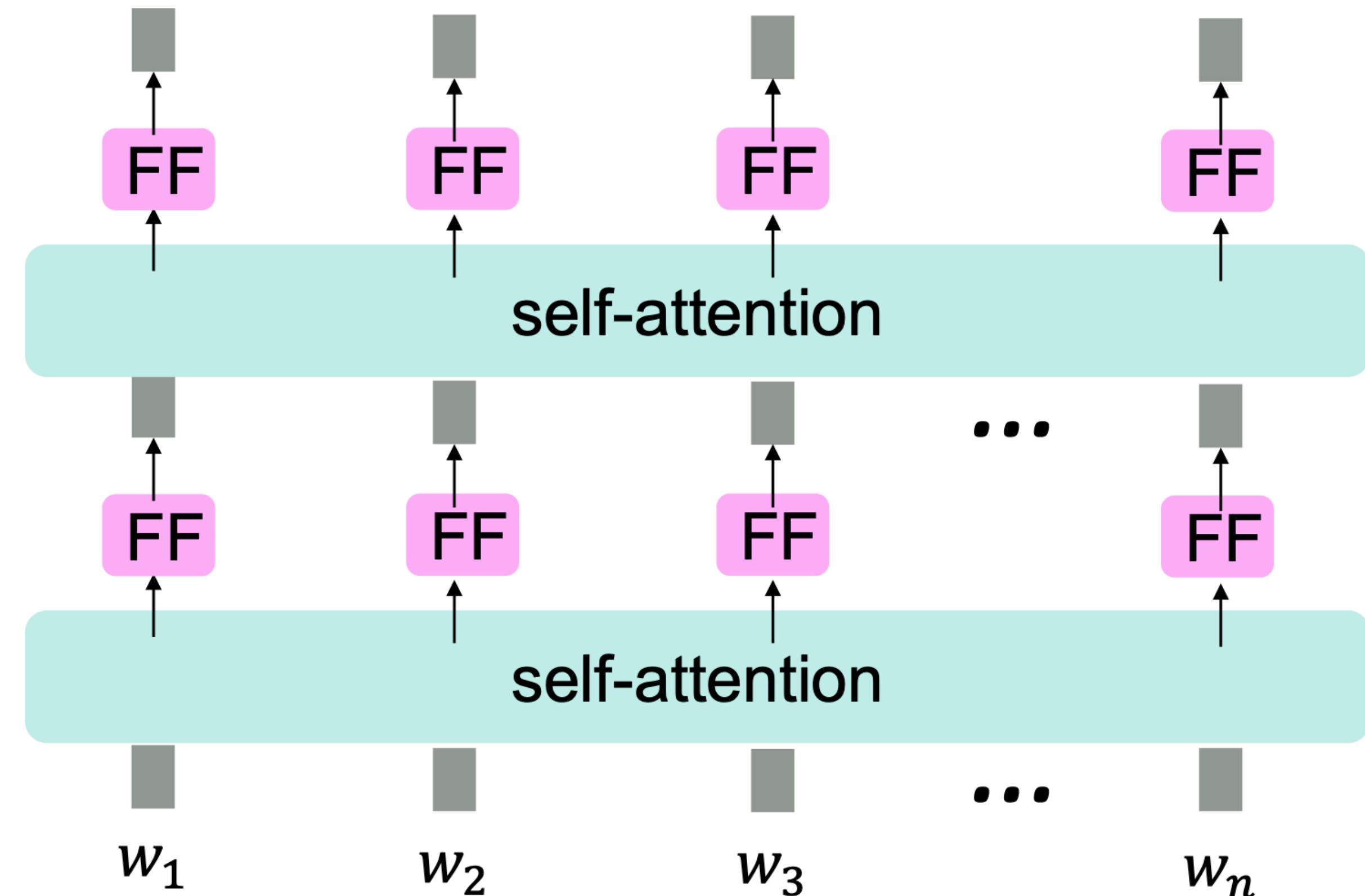
- \* Not learnable
- \* Extrapolation does not work that well for some applications

<http://nlp.seas.harvard.edu/annotated-transformer/#positional-encoding>

# Self Attention Encoder using a Feed-forward Network

$$\begin{aligned} m_i &= \text{MLP}(\text{output}_i) \\ &= W_2 * \text{ReLU}(W_1 \text{ output}_i + b_1) + b_2 \end{aligned}$$

Intuition: the feed-forward (FF) network processes the attention vector and makes it usable by the next layer



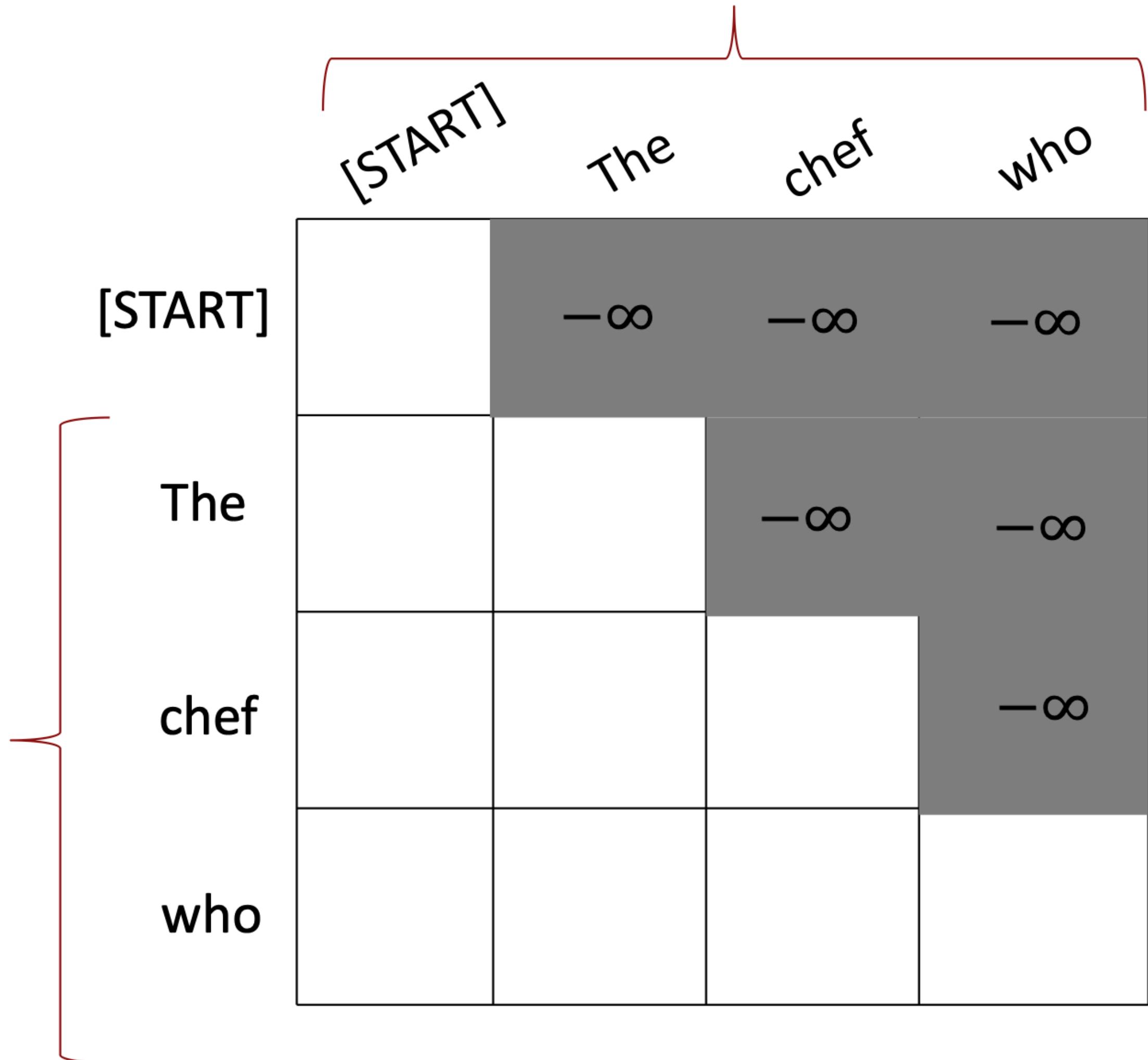
# Decoders should **not** see into the future

- \* During training we mask the attention vector by setting attention scores to  $-\infty$
- \* During inference, we decode from left to right and use the output from previous time-step as input to the next

$$e_{ij} = \begin{cases} q_i^T k_j, & j \leq i \\ -\infty, & j > i \end{cases}$$

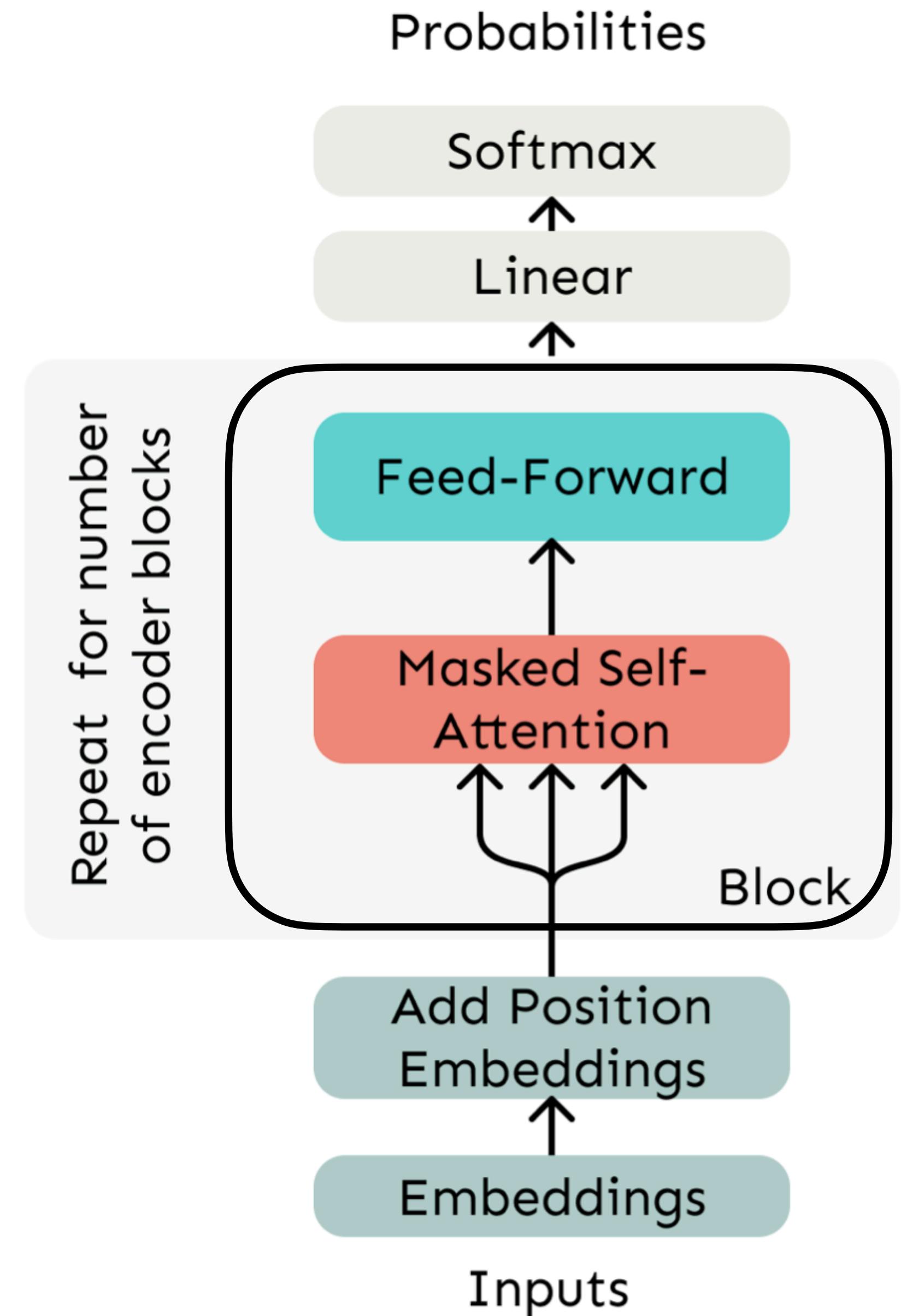
For encoding  
these words

We can only look at the non-greyed out words in the attention vector



# Self-attention building block

- \* **Self attention**
  - \* need this!
- \* **Position embeddings**
  - \* since self-attention is unordered
- \* **Nonlinearities**
  - \* For the output of attention block
  - \* Simple feed-forward network that is easy to train
- \* **Masking**
  - \* To parallelize operations while not looking at the future (during training)
  - \* Enforces training to behave like inference



# From Single Attention Head to Multiple Attention Heads

The animal didn't cross the street because it was too tired .

The diagram illustrates a single attention head. A central blue rectangle labeled 'it' has three light blue lines extending from it to three other blue rectangles: 'animal' (the first word), 'street' (the fourth word), and 'tired' (the eighth word). All other words in the sentence are in black.

The animal didn't cross the street because it was too wide .

The diagram illustrates multiple attention heads. A central blue rectangle labeled 'it' has four light blue lines extending from it to four other blue rectangles: 'animal' (the first word), 'street' (the fourth word), 'wide' (the ninth word), and 'too' (the tenth word). All other words in the sentence are in black.

# Each Layer has Multi-head Self-Attention

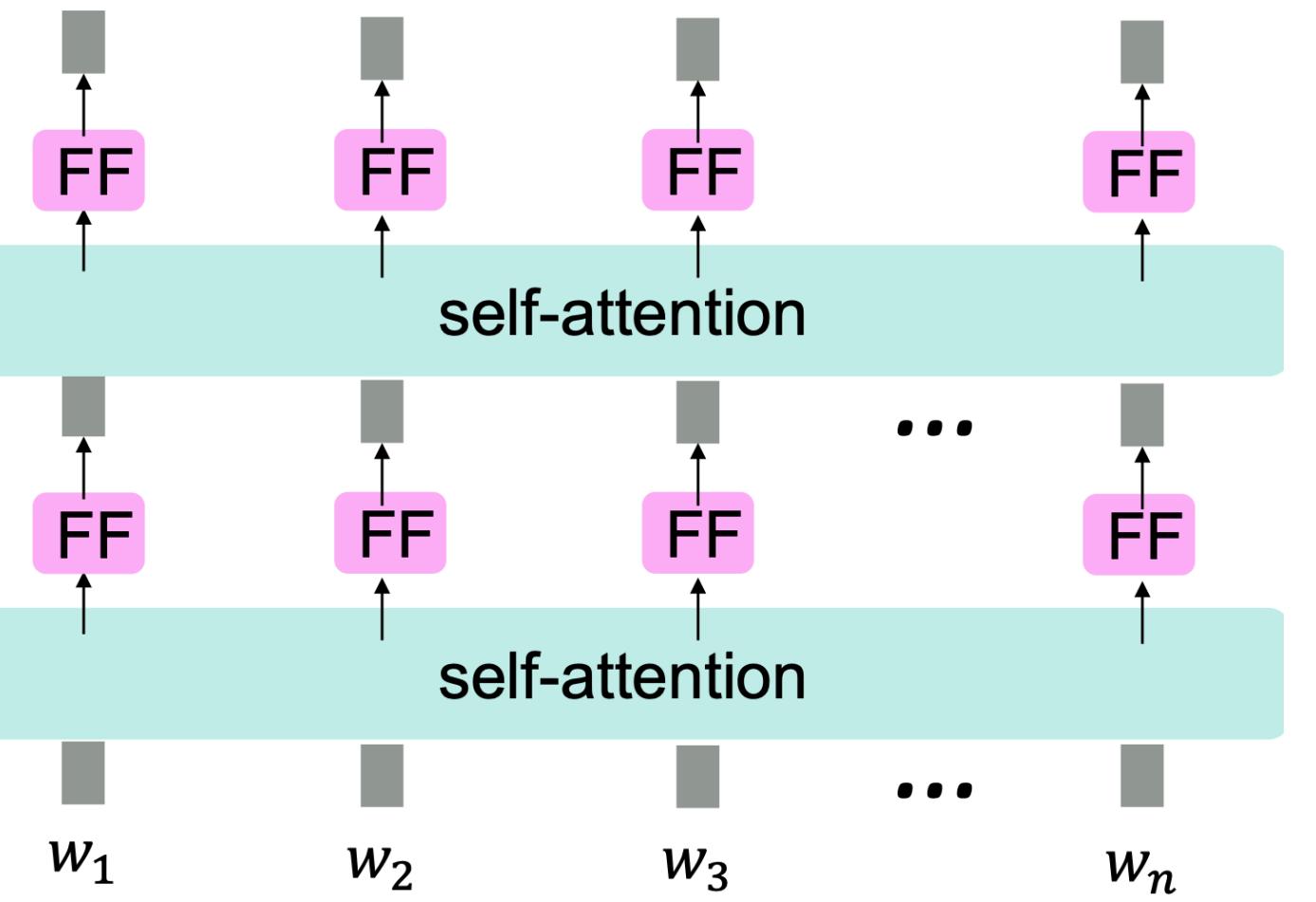
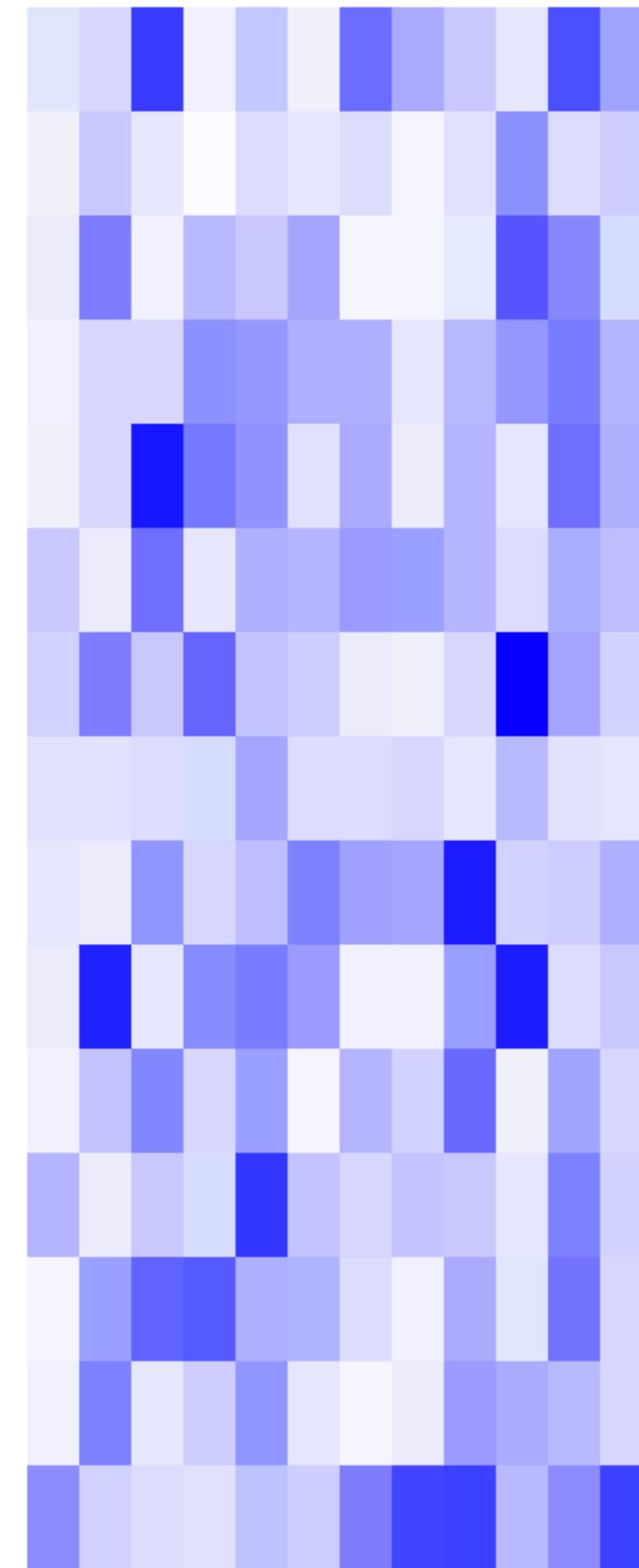
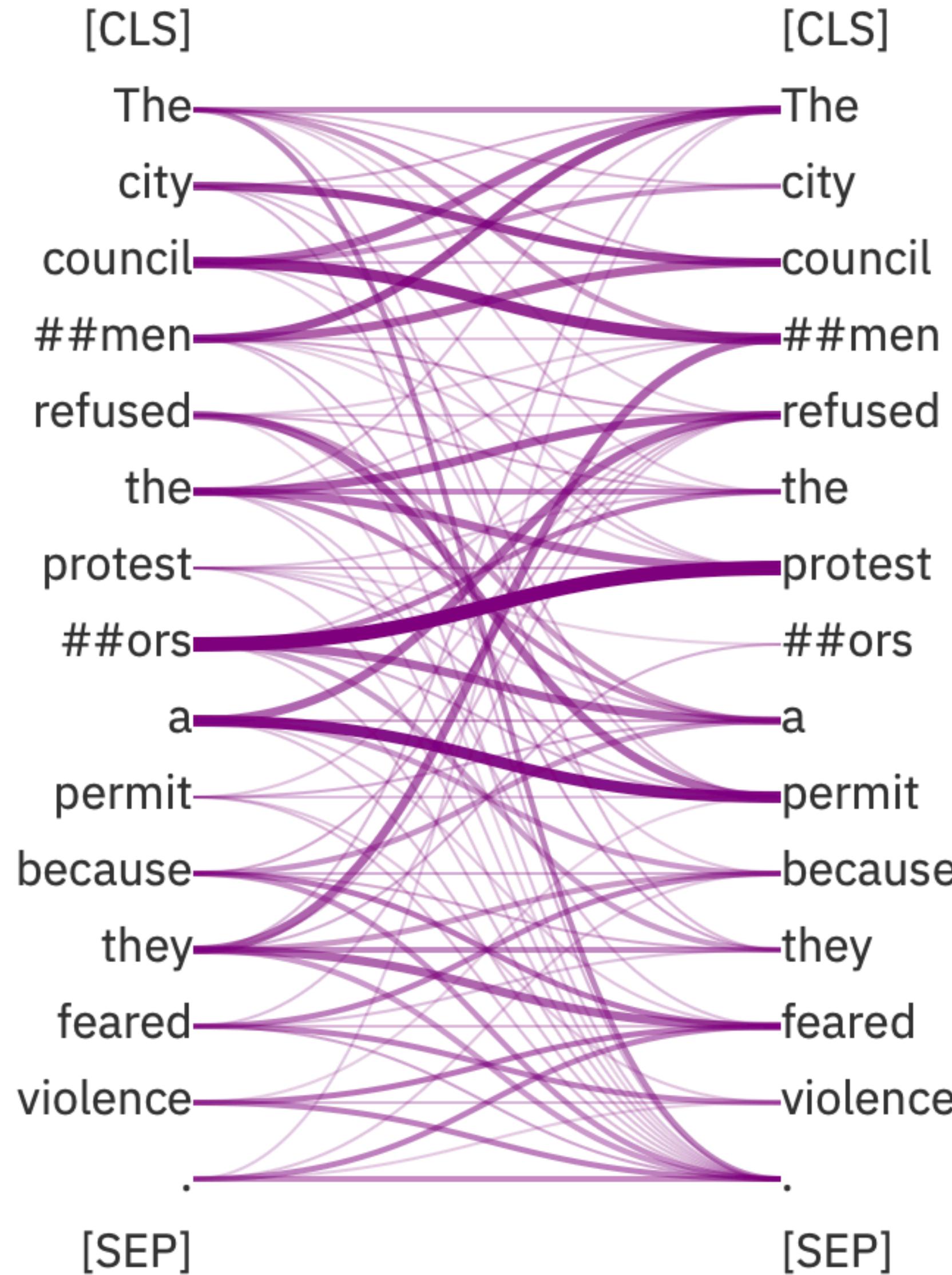
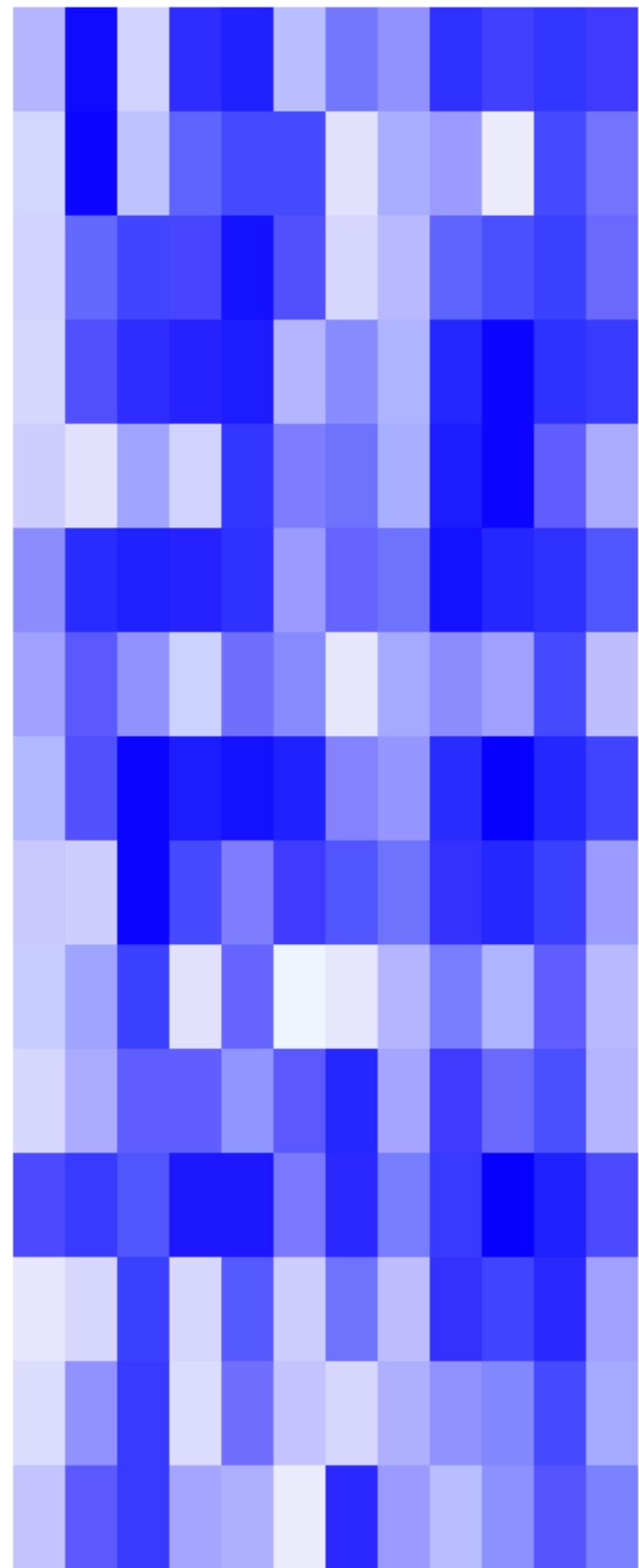


Image shows Layer 5 of  
a 12 Layer Transformer

12 attention heads for  
each layer

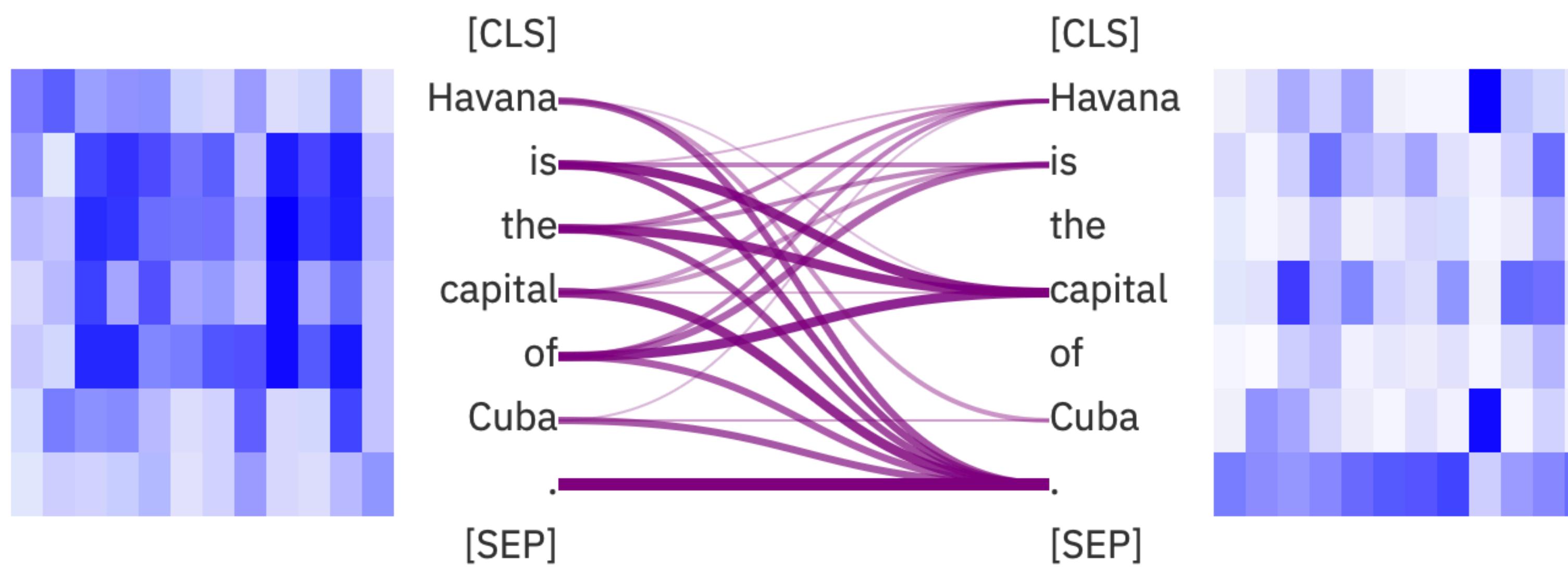
[https://huggingface.co/  
spaces/exbert-project/exbert](https://huggingface.co/spaces/exbert-project/exbert)



**Layer**

1 2 3 4 5 6 7 8 9 10 11 12

**Selected heads:** 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12



# Self-attention

## Matrix form

- Let  $\mathbf{w} = (w_1, \dots, w_n)$  be a sequence of tokens, like "Cuba is the capital of"
- For each  $w_i$  let  $\mathbf{x}_i = E\mathbf{w}_i$  where  $E \in \mathbb{R}^{d \times |V|}$  is an embedding matrix.  $V$  is the vocabulary.
- Let  $\mathbf{X} = [\mathbf{x}_1; \dots; \mathbf{x}_n] \in \mathbb{R}^{n \times d}$  be the concatenation of the input word vectors
- Let  $Q, K, V$  be matrices in  $\mathbb{R}^{d \times d}$  then  $XQ \in \mathbb{R}^{n \times d}, XK \in \mathbb{R}^{n \times d}, XV \in \mathbb{R}^{n \times d}$

# Self-attention

## Matrix form

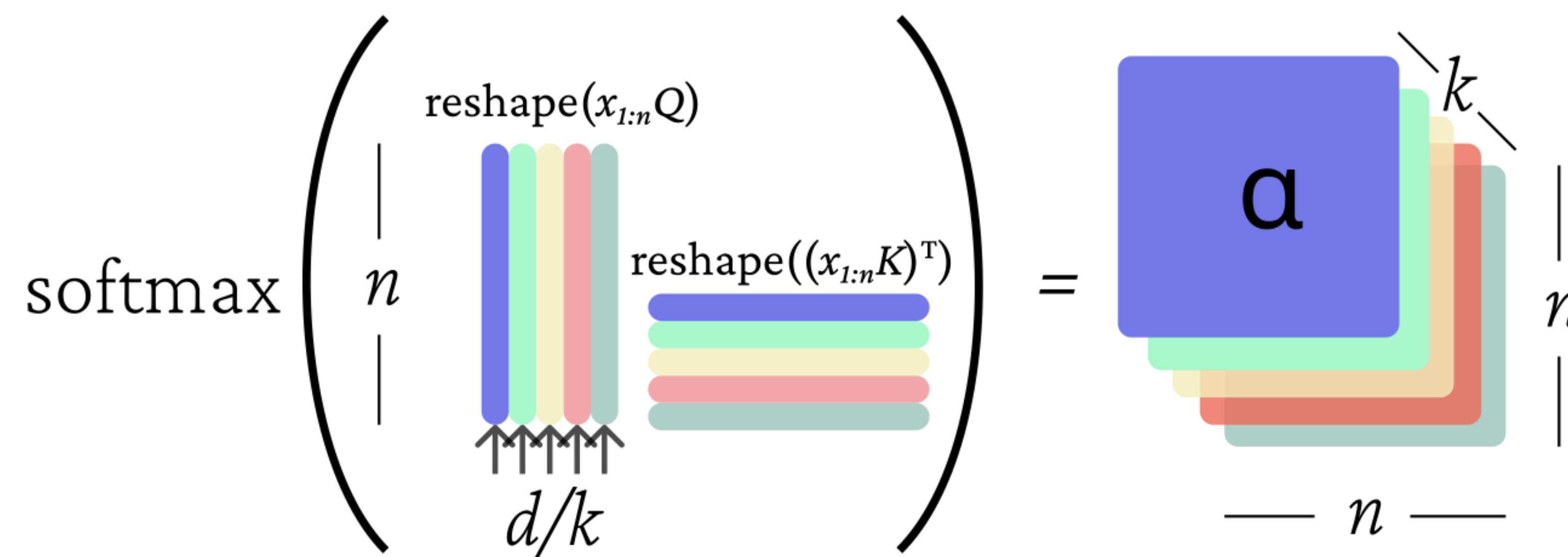
- First take the query-key dot products in matrix form:  $XQ(XK)^T$
- Next softmax and compute the weighted average:  $\text{softmax}(XQ(XK)^T)$ 
  - $XV \in \mathbb{R}^{n \times d}$
- Output is the context vector for each  $w_i$  but in matrix form:  $\mathbb{R}^{n \times d}$

$$\text{softmax} \left( \begin{array}{c|c} n & x_{1:n}Q \\ \hline & (x_{1:n}K)^T \\ -d- & \end{array} \right) = \begin{array}{c|c} a & n \\ \hline & \end{array}$$

# Multi-head Self-attention

## Matrix form

- Let  $h$  range from  $1 \dots k$  for  $k$  total attention heads.
- $Q_h, K_h, V_h \in \mathbb{R}^{d \times \frac{d}{k}}$  so the output  $O_h = \text{softmax}(XQ_h(XK_h)^T) \cdot XV_h \in \mathbb{R}^{\frac{d}{k}}$
- Combine all the heads:  $O = [O_1, \dots, O_k]$



# Add & Norm

## Residual Connections and Layer Norm

- Combine residual connection and layer norm into a single "Add & Norm" component
- Two choices:
  - Pre-norm:  $\mathbf{z}^{\ell+1} = f(\text{LN}(\mathbf{z}^\ell)) + \mathbf{z}^\ell$
  - Post-norm:  $\mathbf{z}^{\ell+1} = \text{LN}(f(\mathbf{z}^\ell) + \mathbf{z}^\ell)$
- Pre-norm leads to faster training. <https://arxiv.org/abs/2002.04745>

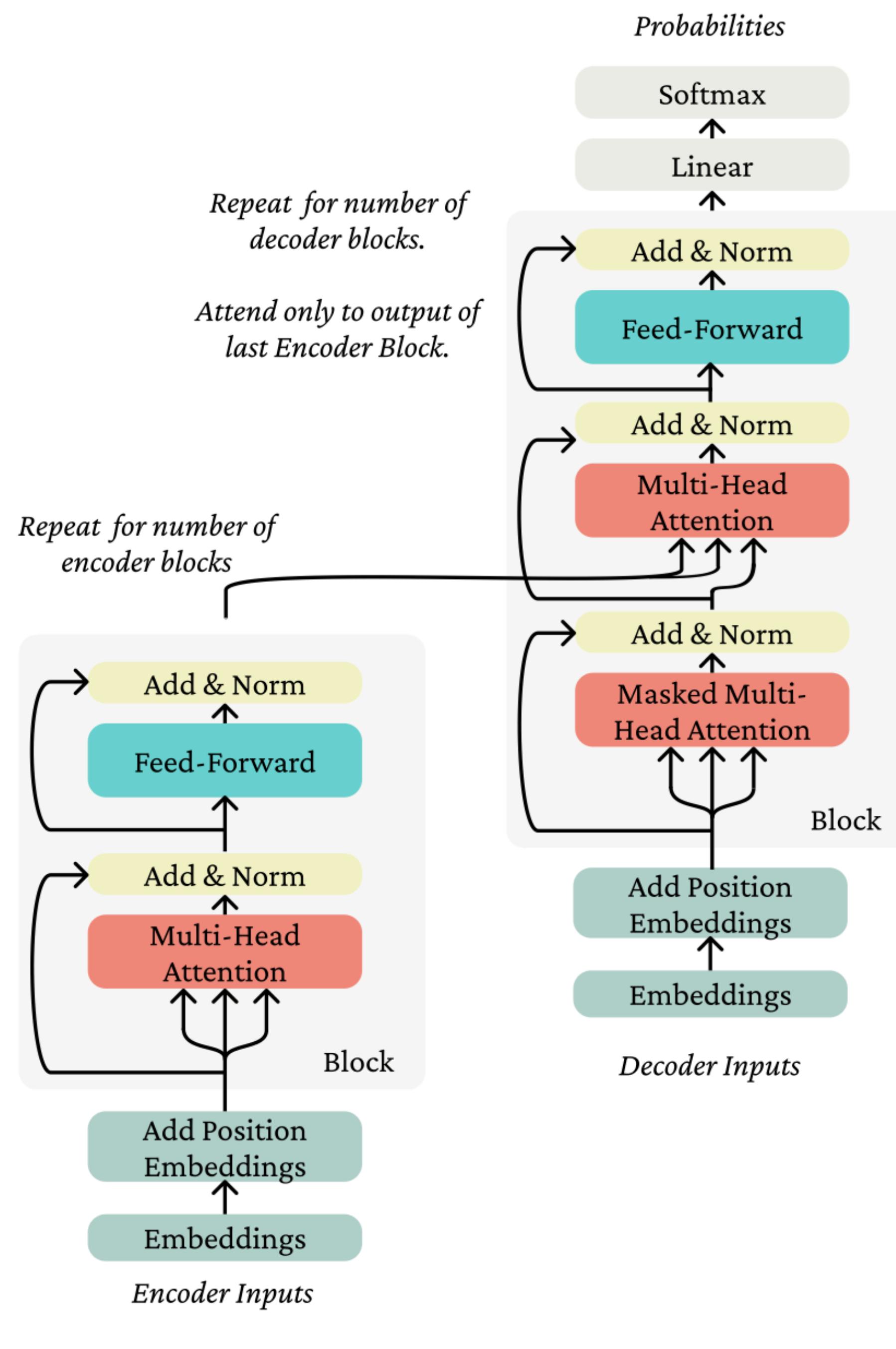
# Scaled dot product attention

## Attention with logit scaling

- Scaling to large dimension vectors  $d$
- Dot product of random vectors (at initialization) grows proportional to  $\sqrt{d}$
- Normalize the dot products by  $\sqrt{d}$  to stop this iterative scaling upwards

$$\text{softmax} \left( \frac{\begin{matrix} n \\ | \\ x_{1:n}Q \\ | \\ -d- \end{matrix}}{\sqrt{d}} \begin{matrix} (x_{1:n}K)^T \end{matrix} \right) = \begin{matrix} a \\ | \\ n \\ | \\ ---n--- \end{matrix}$$

The diagram illustrates the computation of scaled dot product attention. On the left, a softmax function is applied to a matrix divided by  $\sqrt{d}$ . The matrix has dimensions  $n \times d$ , indicated by the vertical bar and the label  $-d-$  below it. The matrix is partitioned into two parts:  $x_{1:n}Q$  (the first  $n$  columns of  $x$ ) and  $(x_{1:n}K)^T$  (the transpose of the first  $n$  columns of  $K$ ). The result is a vector  $a$  of length  $n$ .



# Machine Translation Results

Table 2: The Transformer achieves better BLEU scores than previous state-of-the-art models on the English-to-German and English-to-French newstest2014 tests at a fraction of the training cost.

| Model                           | BLEU        |              | Training Cost (FLOPs) |                                       |
|---------------------------------|-------------|--------------|-----------------------|---------------------------------------|
|                                 | EN-DE       | EN-FR        | EN-DE                 | EN-FR                                 |
| ByteNet [18]                    |             | 23.75        |                       |                                       |
| Deep-Att + PosUnk [39]          |             | 39.2         |                       | $1.0 \cdot 10^{20}$                   |
| GNMT + RL [38]                  | 24.6        | 39.92        | $2.3 \cdot 10^{19}$   | $1.4 \cdot 10^{20}$                   |
| ConvS2S [9]                     | 25.16       | 40.46        | $9.6 \cdot 10^{18}$   | $1.5 \cdot 10^{20}$                   |
| MoE [32]                        | 26.03       | 40.56        | $2.0 \cdot 10^{19}$   | $1.2 \cdot 10^{20}$                   |
| Deep-Att + PosUnk Ensemble [39] |             | 40.4         |                       | $8.0 \cdot 10^{20}$                   |
| GNMT + RL Ensemble [38]         | 26.30       | 41.16        | $1.8 \cdot 10^{20}$   | $1.1 \cdot 10^{21}$                   |
| ConvS2S Ensemble [9]            | 26.36       | <b>41.29</b> | $7.7 \cdot 10^{19}$   | $1.2 \cdot 10^{21}$                   |
| Transformer (base model)        | 27.3        | 38.1         |                       | <b><math>3.3 \cdot 10^{18}</math></b> |
| Transformer (big)               | <b>28.4</b> | <b>41.8</b>  |                       | $2.3 \cdot 10^{19}$                   |

# Same Transformer model applied to constituency parsing

Table 4: The Transformer generalizes well to English constituency parsing (Results are on Section 23 of WSJ)

| Parser                              | Training                 | WSJ 23 F1 |
|-------------------------------------|--------------------------|-----------|
| Vinyals & Kaiser el al. (2014) [37] | WSJ only, discriminative | 88.3      |
| Petrov et al. (2006) [29]           | WSJ only, discriminative | 90.4      |
| Zhu et al. (2013) [40]              | WSJ only, discriminative | 90.4      |
| Dyer et al. (2016) [8]              | WSJ only, discriminative | 91.7      |
| Transformer (4 layers)              | WSJ only, discriminative | 91.3      |
| Zhu et al. (2013) [40]              | semi-supervised          | 91.3      |
| Huang & Harper (2009) [14]          | semi-supervised          | 91.3      |
| McClosky et al. (2006) [26]         | semi-supervised          | 92.1      |
| Vinyals & Kaiser el al. (2014) [37] | semi-supervised          | 92.1      |
| Transformer (4 layers)              | semi-supervised          | 92.7      |
| Luong et al. (2015) [23]            | multi-task               | 93.0      |
| Dyer et al. (2016) [8]              | generative               | 93.3      |

Table 3: Variations on the Transformer architecture. Unlisted values are identical to those of the base model. All metrics are on the English-to-German translation development set, newstest2013. Listed perplexities are per-wordpiece, according to our byte-pair encoding, and should not be compared to per-word perplexities.

|      | $N$ | $d_{\text{model}}$ | $d_{\text{ff}}$ | $h$  | $d_k$ | $d_v$ | $P_{\text{drop}}$ | $\epsilon_{ls}$ | train steps | PPL (dev)   | BLEU (dev)  | params $\times 10^6$ |
|------|-----|--------------------|-----------------|------|-------|-------|-------------------|-----------------|-------------|-------------|-------------|----------------------|
| base | 6   | 512                | 2048            | 8    | 64    | 64    | 0.1               | 0.1             | 100K        | 4.92        | 25.8        | 65                   |
| (A)  |     |                    |                 | 1    | 512   | 512   |                   |                 |             | 5.29        | 24.9        |                      |
|      |     |                    |                 | 4    | 128   | 128   |                   |                 |             | 5.00        | 25.5        |                      |
|      |     |                    |                 | 16   | 32    | 32    |                   |                 |             | 4.91        | 25.8        |                      |
|      |     |                    |                 | 32   | 16    | 16    |                   |                 |             | 5.01        | 25.4        |                      |
| (B)  |     |                    |                 |      |       | 16    |                   |                 |             | 5.16        | 25.1        | 58                   |
|      |     |                    |                 |      |       | 32    |                   |                 |             | 5.01        | 25.4        | 60                   |
| (C)  |     |                    |                 | 2    |       |       |                   |                 |             | 6.11        | 23.7        | 36                   |
|      |     |                    |                 | 4    |       |       |                   |                 |             | 5.19        | 25.3        | 50                   |
|      |     |                    |                 | 8    |       |       |                   |                 |             | 4.88        | 25.5        | 80                   |
|      |     |                    | 256             |      | 32    | 32    |                   |                 |             | 5.75        | 24.5        | 28                   |
|      |     |                    | 1024            |      | 128   | 128   |                   |                 |             | 4.66        | 26.0        | 168                  |
|      |     |                    |                 | 1024 |       |       |                   |                 |             | 5.12        | 25.4        | 53                   |
|      |     |                    |                 | 4096 |       |       |                   |                 |             | 4.75        | 26.2        | 90                   |
|      |     |                    |                 |      |       |       | 0.0               |                 |             | 5.77        | 24.6        |                      |
| (D)  |     |                    |                 |      |       |       | 0.2               |                 |             | 4.95        | 25.5        |                      |
|      |     |                    |                 |      |       |       |                   | 0.0             |             | 4.67        | 25.3        |                      |
|      |     |                    |                 |      |       |       |                   | 0.2             |             | 5.47        | 25.7        |                      |
|      | (E) |                    |                 |      |       |       |                   |                 |             | 4.92        | 25.7        |                      |
| big  | 6   | 1024               | 4096            | 16   |       |       | 0.3               |                 | 300K        | <b>4.33</b> | <b>26.4</b> | 213                  |

# Problems with Transformers

## What needs fixing?

- Quadratic compute cost
  - In prior models like RNNs attention grew linearly since it only paid attention to the previous time step
  - In Transformers, attention takes  $O(n^2d)$  time to compute for input of length  $n$  and dimensionality  $d$
- Positional representations
  - Simple indices the best we can do?
  - In the decoder, attention at time  $t$  is independent of previous time steps.