CMPT 379
Compilers

Anoop Sarkar

http://www.cs.sfu.ca/~anoop

Converting Regular Expressions into Non-deterministic Automata

NFAs

- NFA: like a DFA, except
  - A transition can lead to more than one state, that is, δ: S x $\Sigma \Rightarrow 2^S$
  - One state is chosen non-deterministically
  - Transitions can be labeled with ε, meaning states can be reached without reading any input, that is,

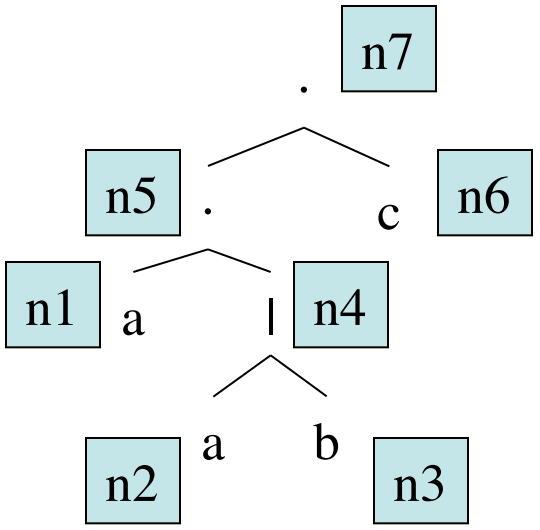
$$\delta: S \times \Sigma \cup \{ \varepsilon \} \Rightarrow 2^S$$

Thompson's construction

Converts regexps to NFA

Build NFA recursively from regexp tree

Build NFA with left-to-right parse of postfix string using a stack



Input = aab|·c·

- read a, push n1 = nfa(a)
- read a, push n2 = nfa(a)
- read b, push n3 = nfa(b)
- read |, n3=pop(); n2=pop(); push n4 = nfa(or, n2, n3)
- read ·, n4 = pop(); n1 = pop(); push n5 = nfa(cat, n1, n4)
- read c, push n6 = nfa(c)
- read ·, n6 = pop(); n5 = pop(); push n7 = nfa(cat, n5, n6)

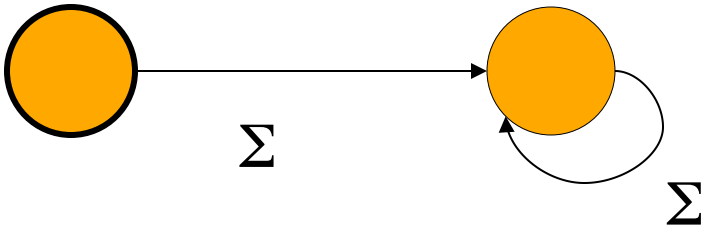
Thompson's construction

- Converts regexps to NFA
- Six simple rules
  - Empty language
  - Symbols
  - Empty String
  - Alternation (r_1 or r_2)
  - Concatenation (r_1 followed by r_2)
  - Repetition (r_1*)

Used by Ken Thompson for pattern-based search in text editor QED (1968)
To keep things simple our version is more verbose

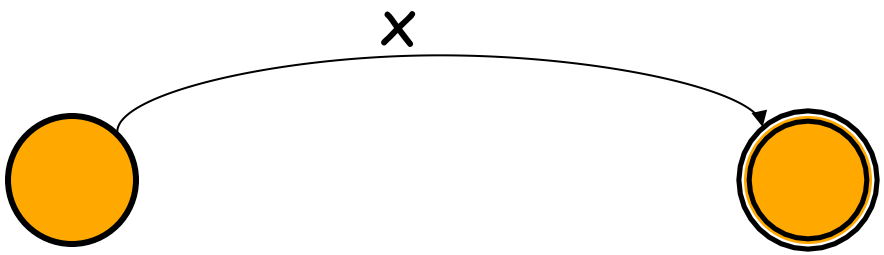
Thompson Rule 0

- For the empty language ϕ (optionally include a *sinkhole* state)



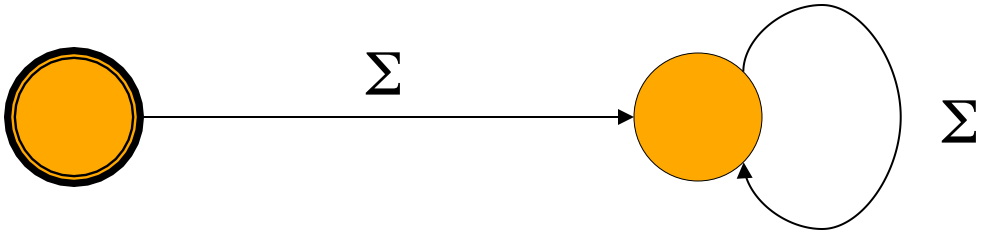
Thompson Rule 1

- For each symbol *x* of the alphabet, there is a NFA that accepts it (include a *sinkhole* state)



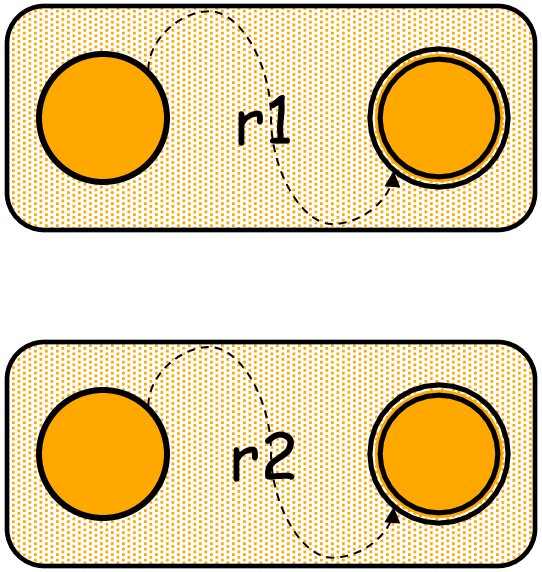
Thompson Rule 2

- There is an NFA that accepts only ε



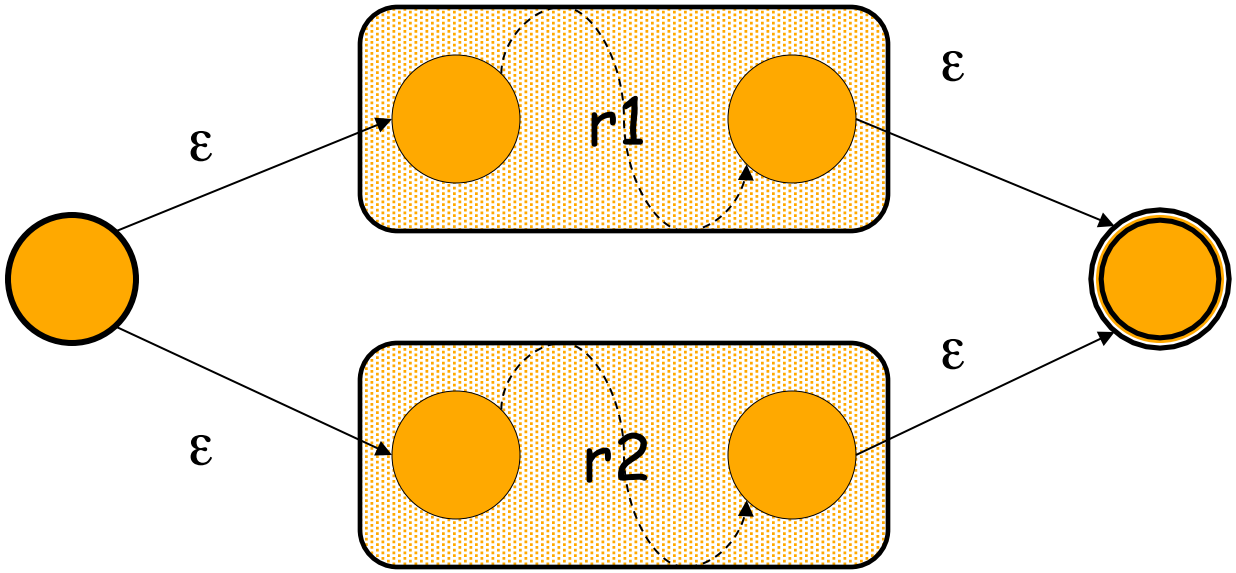
Thompson Rule 3

- Given two NFAs for r_1, r_2, there is a NFA that accepts $r_1|r_2$



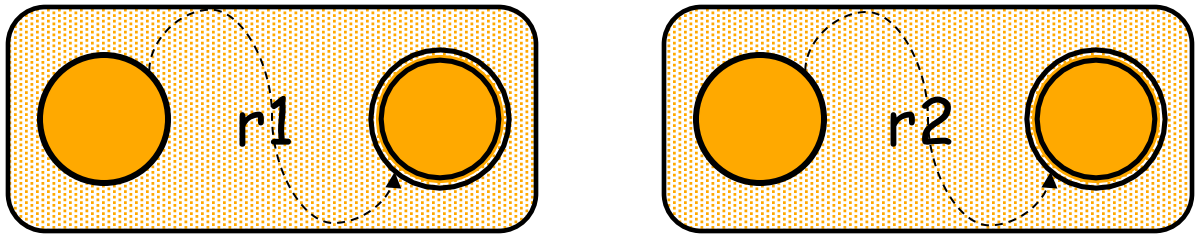
Thompson Rule 3

- Given two NFAs for r_1, r_2, there is a NFA that accepts $r_1|r_2$



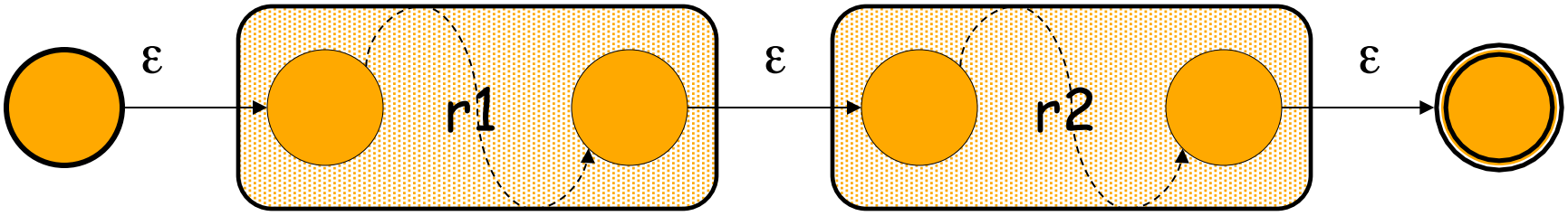
Thompson Rule 4

- Given two NFAs for r_1, r_2, there is a NFA that accepts $r_1 r_2$



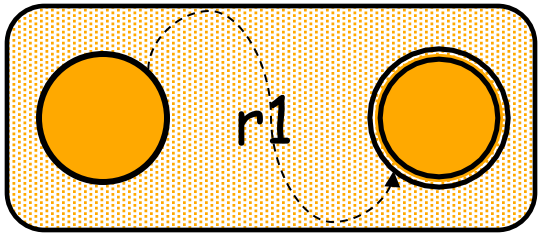
Thompson Rule 4

- Given two NFAs for r_1, r_2, there is a NFA that accepts $r_1 r_2$



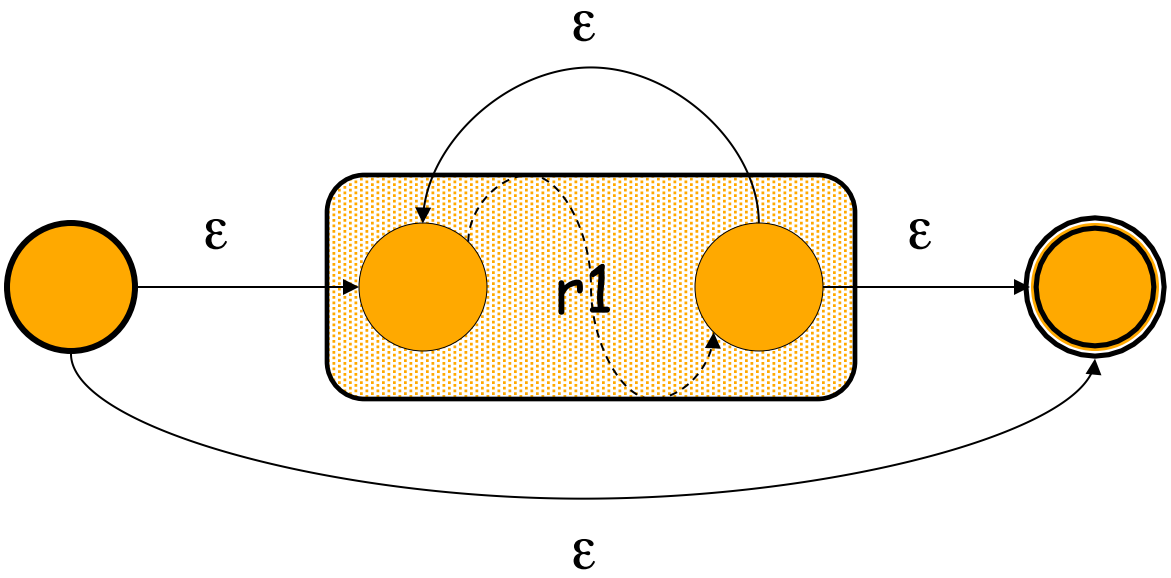
Thompson Rule 5

- Given a NFA for r_1, there is an NFA that accepts r_1*



Thompson Rule 5

- Given a NFA for r_1, there is an NFA that accepts r_1*

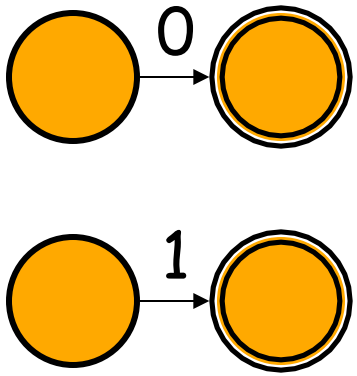


Example

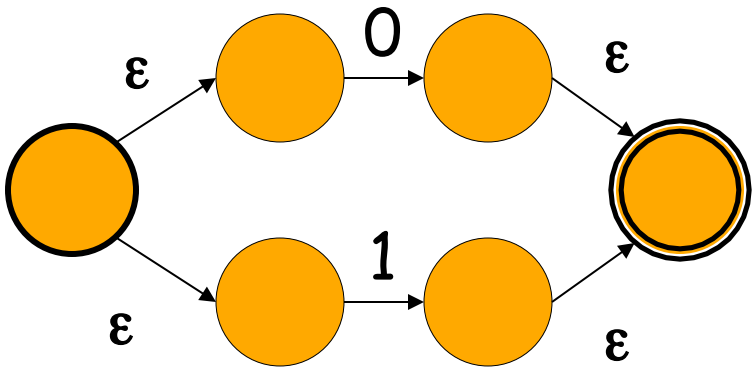
- Set of all binary strings that are divisible by four (include 0 in this set)
- Defined by the regexp: ((0|1)*00) | 0
- Apply Thompson's Rules to create an NFA

Basic Blocks 0 and 1

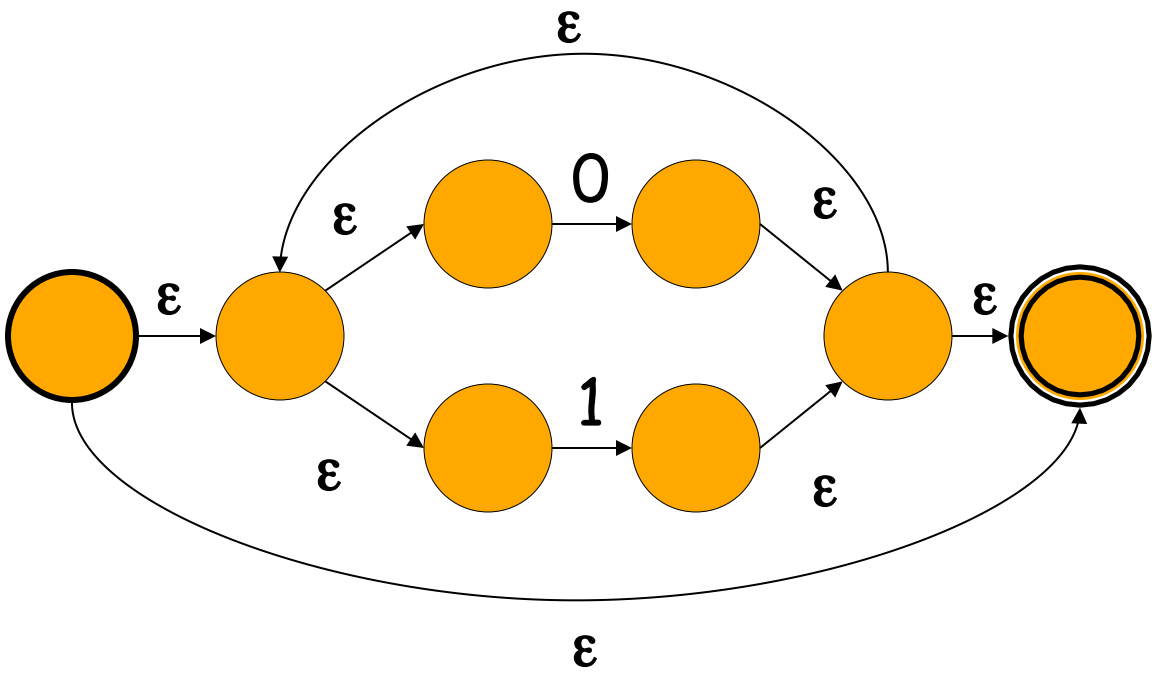
- 0
- 1



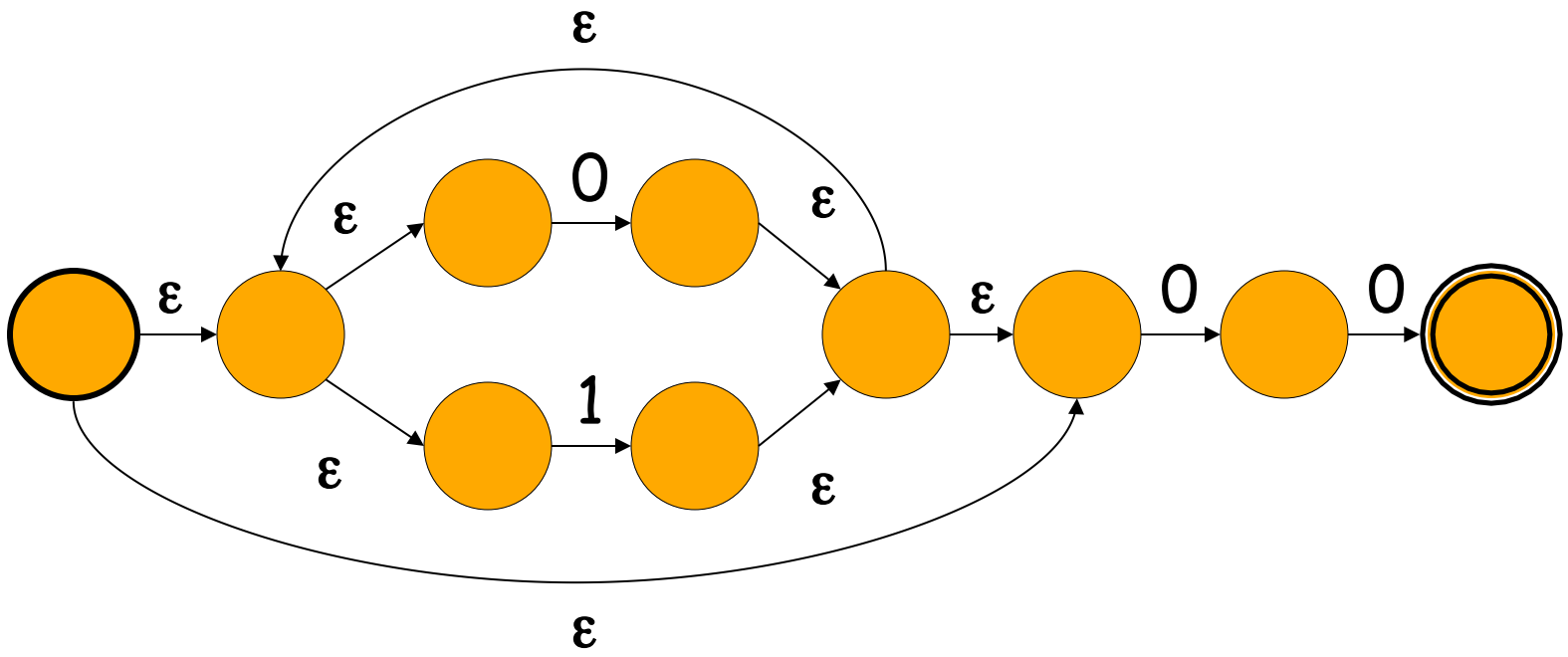
(this version does not report errors: no *sinkholes*)



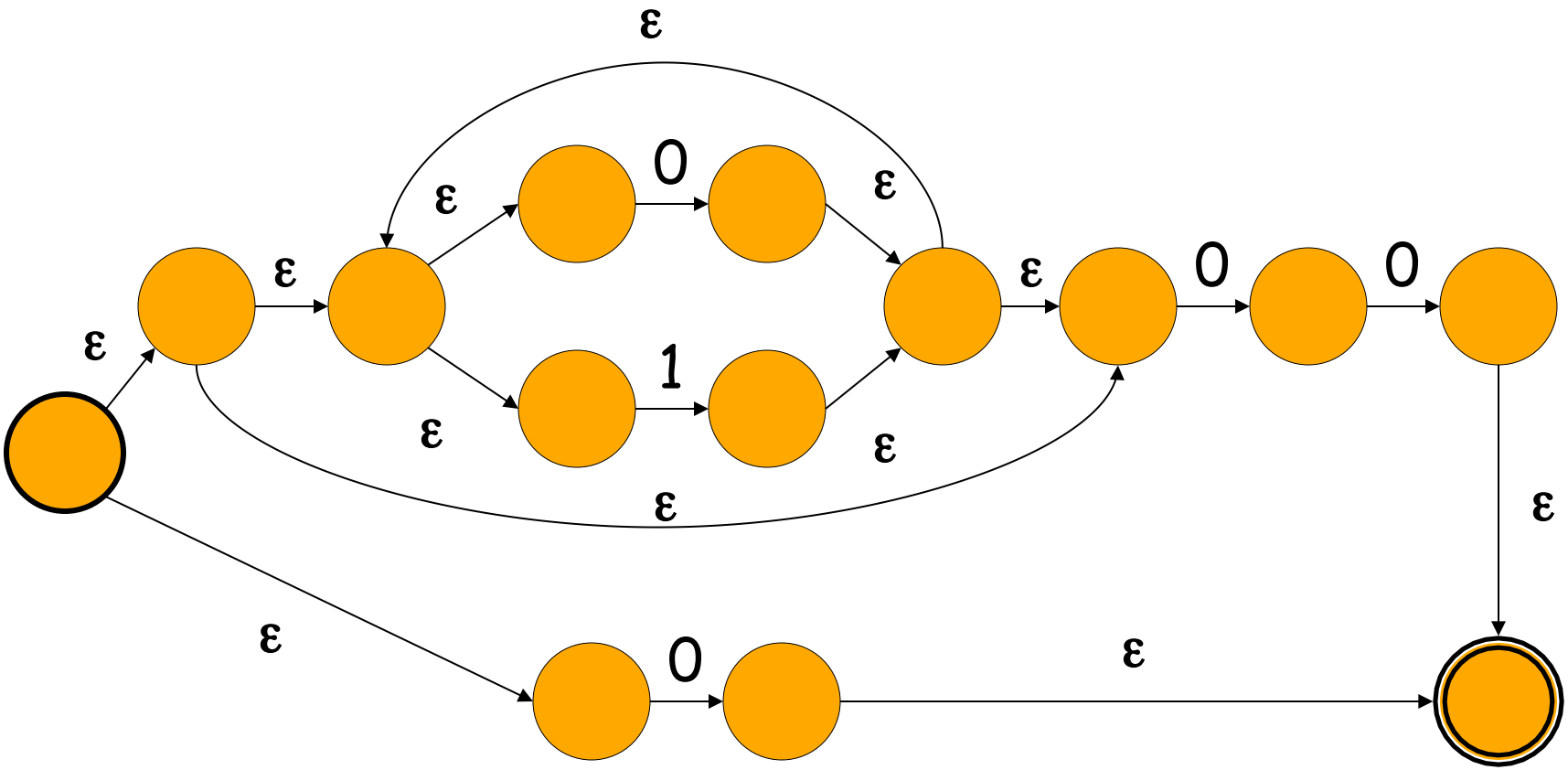
0|1



(0|1)*



(0|1)*00



((0|1)*00)|0