CMPT 379
Compilers

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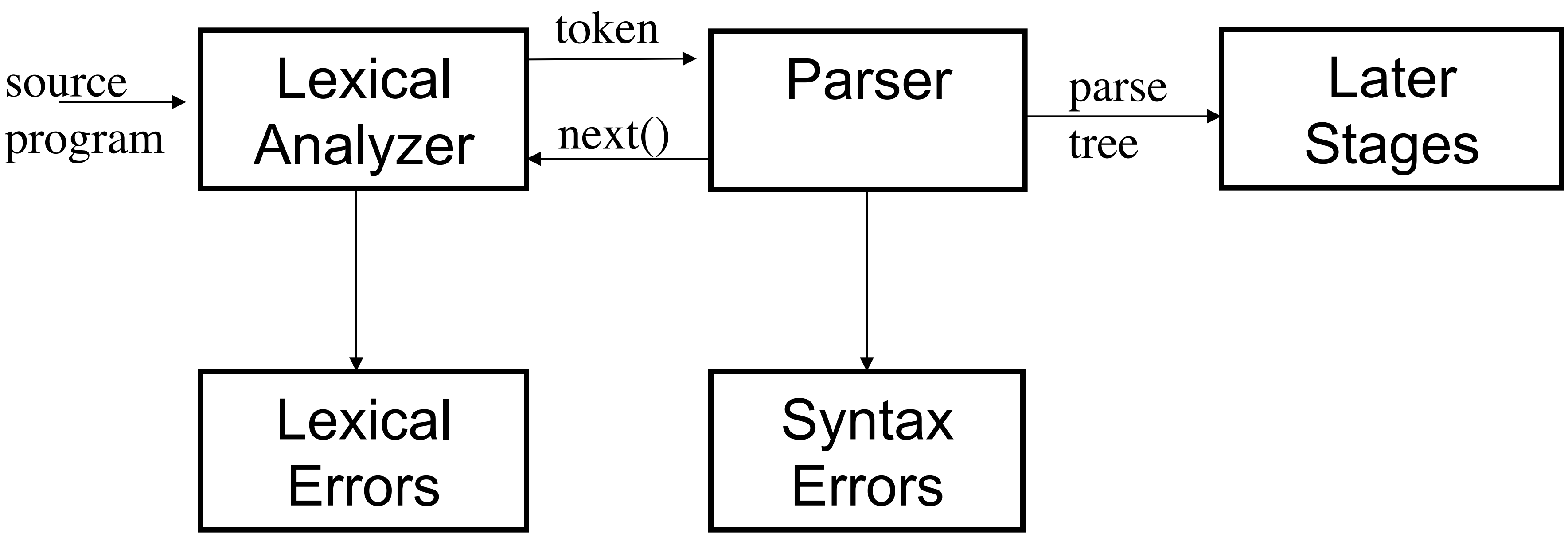
Parsing



source
program
Lexical
Analyzer
token
next()
Parser
parse
tree
Later
Stages
Lexical
Errors
Syntax
Errors

Context-free Grammars

- Set of rules by which valid sentences can be constructed.
- Example:

  Sentence → Noun Verb Object

  Noun → *trees* | *compilers*

  Verb → *are* | *grow*

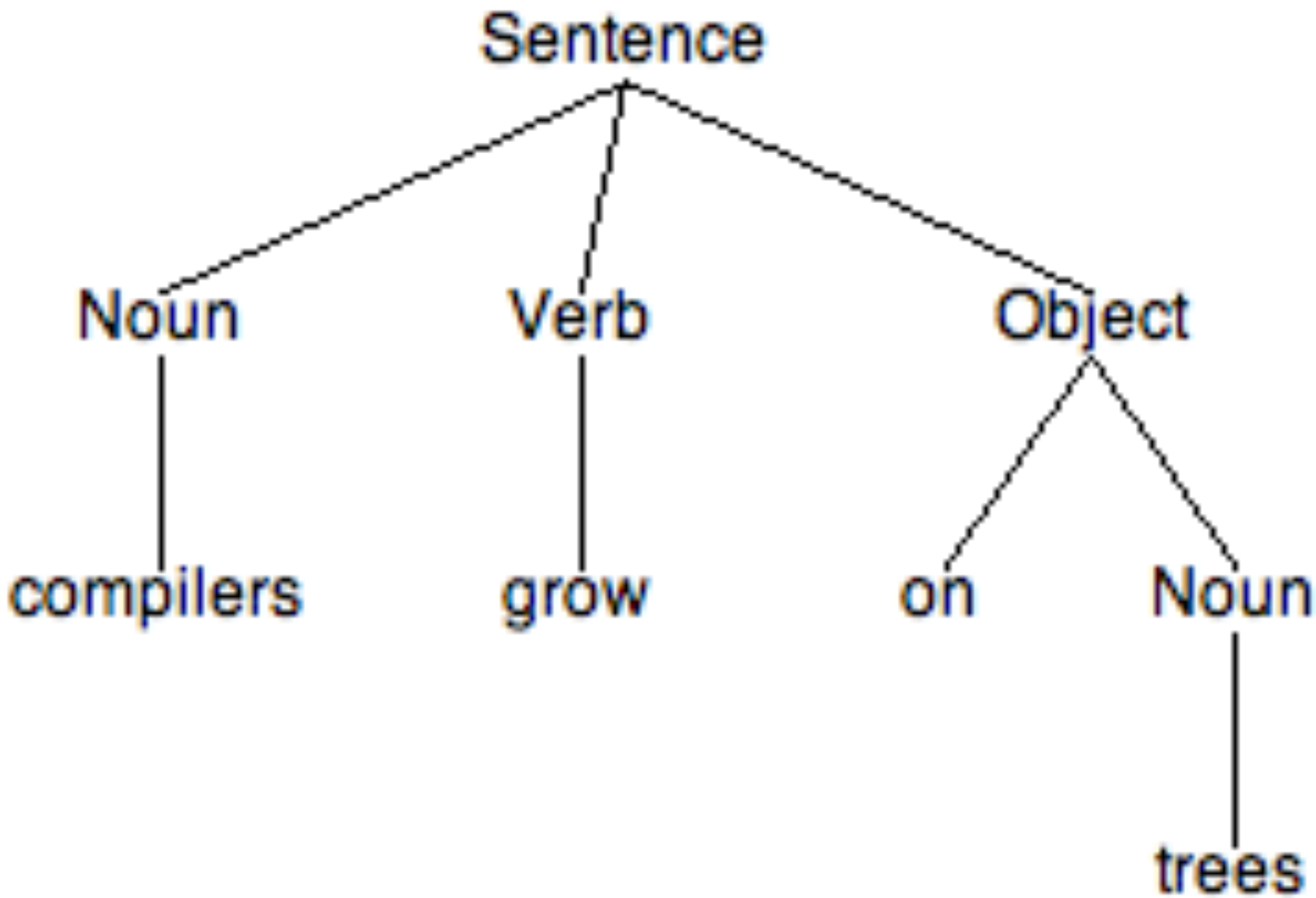
  Object → *on* Noun | Adjective

  Adjective → *slowly* | *interesting*
- What strings can Sentence *derive*?
- Syntax only – no semantic checking

Derivations of a CFG

- *compilers grow on trees*
- *compilers grow on* **Noun**
- *compilers grow* **Object**
- *compilers* **Verb Object**
- **Noun Verb Object**
- **Sentence**

Derivations and parse trees



Why use grammars for PL?

- Precise, yet easy-to-understand specification of language
- Construct parser automatically
  - Detect potential problems
- Structure and simplify remaining compiler phases
- Allow for evolution

CFG Notation

- A reference grammar is a concise description of a context-free grammar
- For example, a reference grammar can use regular expressions on the right hand sides of CFG rules
- Can even use ideas like comma-separated lists to simplify the reference language definition

Writing a CFG for a PL

- First write (or read) a reference grammar of what you want to be valid programs
- For now, we only worry about the structure, so the reference grammar might choose to over-generate in certain cases (e.g. `bool x = 20;`)
- Convert the reference grammar to a CFG
- Certain CFGs might be easier to work with than others (this is the **essence** of the study of CFGs and their parsing algorithms for compilers)

CFG Notation

- Normal CFG notation

  $E \rightarrow E * E$

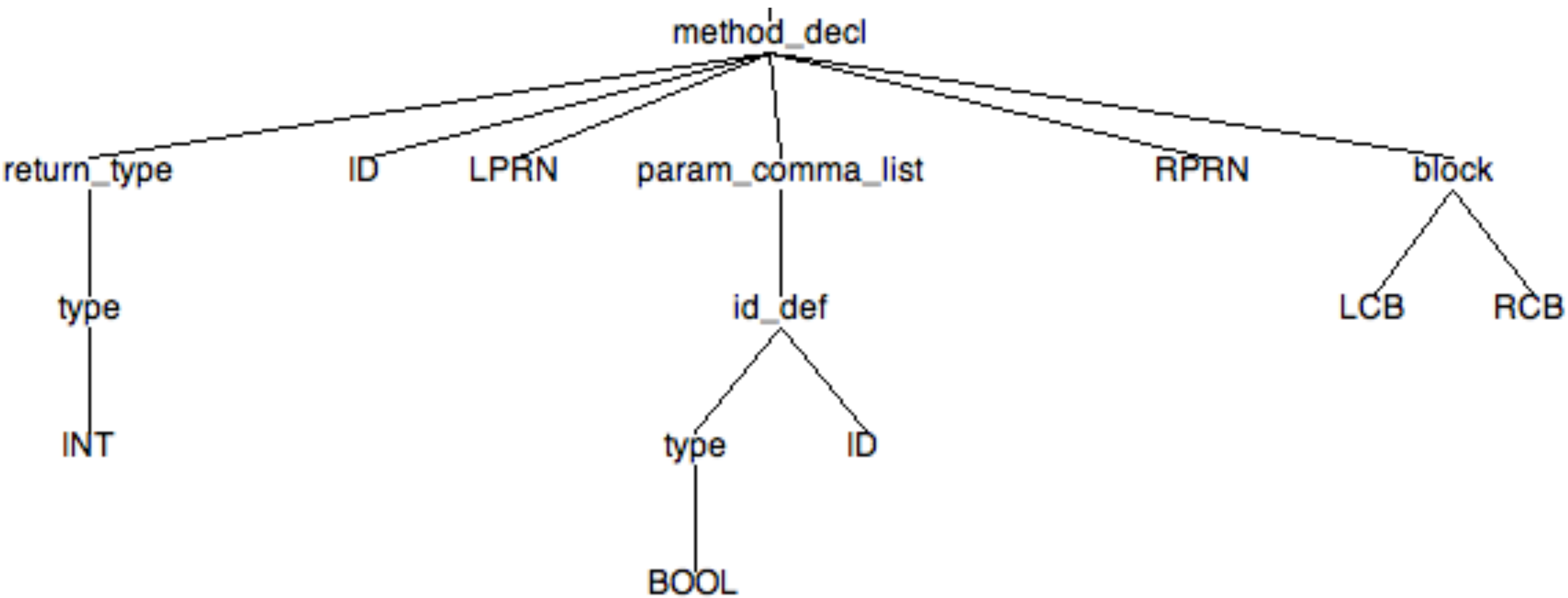
  $E \rightarrow E + E$

- Backus Naur notation

  $E ::= E * E \mid E + E$

  (an or-list of right hand sides)

Parse Trees for programs



Arithmetic Expressions

- $E \rightarrow E + E$
- $E \rightarrow E * E$
- $E \rightarrow (E)$
- $E \rightarrow - E$
- $E \rightarrow \textbf{id}$

Leftmost derivations for **id + id * id**

$E \rightarrow E + E$
$E \rightarrow E * E$
$E \rightarrow (E)$
$E \rightarrow - E$
$E \rightarrow \mathbf{id}$

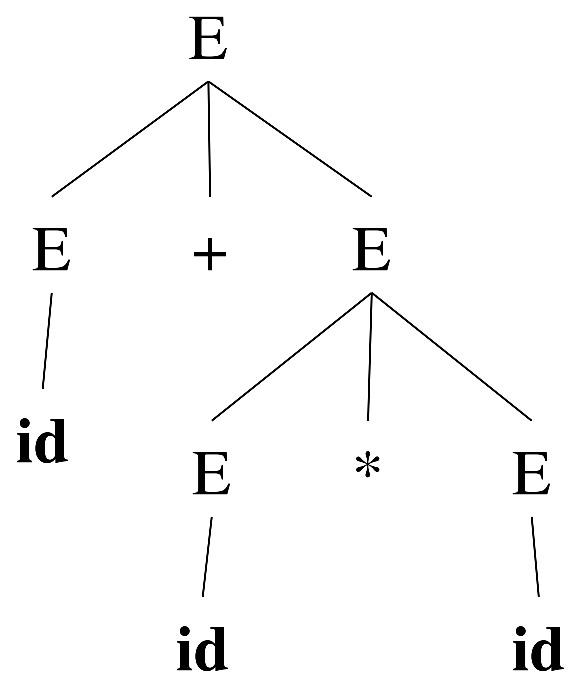
- $E \Rightarrow E + E$

$\Rightarrow \mathbf{id} + E$

$\Rightarrow \mathbf{id} + E * E$

$\Rightarrow \mathbf{id} + \mathbf{id} * E$

$\Rightarrow \mathbf{id} + \mathbf{id} * \mathbf{id}$



Leftmost derivations for **id + id * id**

$E \rightarrow E + E$
$E \rightarrow E * E$
$E \rightarrow (E)$
$E \rightarrow - E$
$E \rightarrow \mathbf{id}$

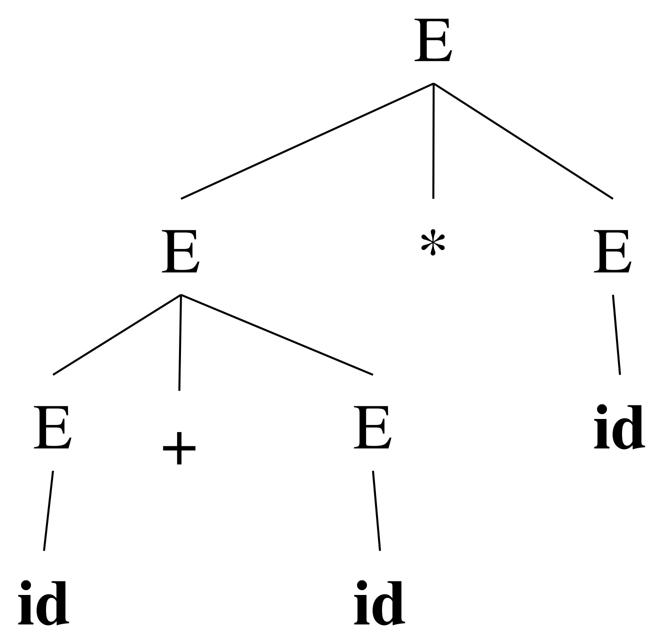
- $E \Rightarrow E * E$

$\Rightarrow E + E * E$

$\Rightarrow \mathbf{id} + E * E$

$\Rightarrow \mathbf{id} + \mathbf{id} * E$

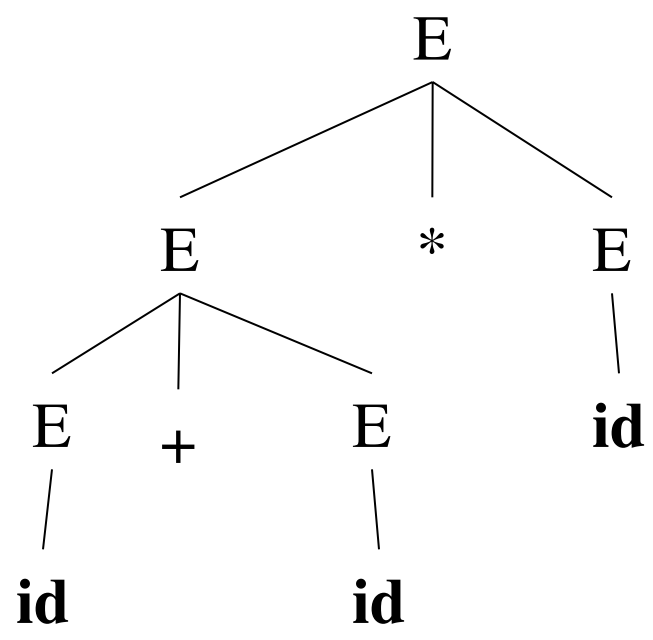
$\Rightarrow \mathbf{id} + \mathbf{id} * \mathbf{id}$



Rightmost derivation for **id + id * id**

$E \rightarrow E + E$
$E \rightarrow E * E$
$E \rightarrow (E)$
$E \rightarrow - E$
$E \rightarrow \mathbf{id}$

$$
\begin{aligned}
E &\Rightarrow E * E \\
&\Rightarrow E * \mathbf{id} \\
&\Rightarrow E + E * \mathbf{id} \\
&\Rightarrow E + \mathbf{id} * \mathbf{id} \\
&\Rightarrow \mathbf{id} + \mathbf{id} * \mathbf{id}
\end{aligned}
$$



Ambiguity

- Grammar is ambiguous if more than one parse tree is possible for some sentences
- Examples in English:
  - Two sisters reunited after 18 years in checkout counter
- Ambiguity is not acceptable in PL
  - Unfortunately, it's undecidable to check whether a given CFG is ambiguous
  - Some CFLs are inherently ambiguous (do not have an unambiguous CFG)

Ambiguity

- Alternatives
  - Massage grammar to make it unambiguous
  - Rely on "default" parser behavior
  - Augment parser
- Consider the original ambiguous grammar:

$E \rightarrow E + E \qquad E \rightarrow E * E$

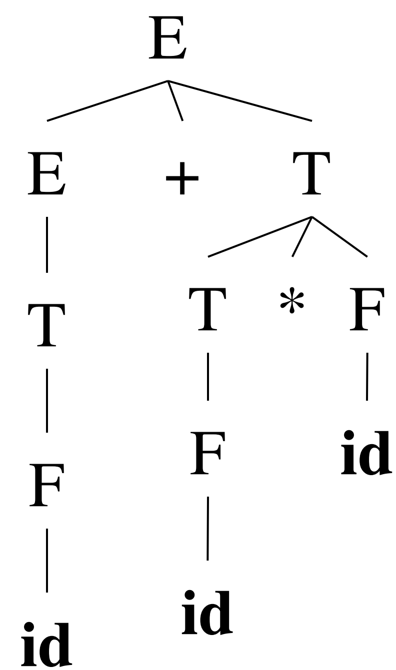
$E \rightarrow (E) \qquad E \rightarrow - E$

$E \rightarrow \mathbf{id}$

- How can we change the grammar to get only one tree for the input **id + id * id**

Ambiguity

- Original ambiguous grammar:
  - E → E + E   E → E * E
  - E → (E)   E → - E
  - E → id
- Unambiguous grammar:
  - E → E + T   T → T * F
  - E → T   T → F
  - F → (E) $\boxed{\text{F → - E}}$
  - F → id
- Input: id + id * id



Warning! Is this unambiguous?
Check derivations for – id + id

Compare with F → - F

Dangling else ambiguity

- Original Grammar (ambiguous)

  Stmt → **if** Expr **then** Stmt **else** Stmt

  Stmt → **if** Expr **then** Stmt

  Stmt → Other

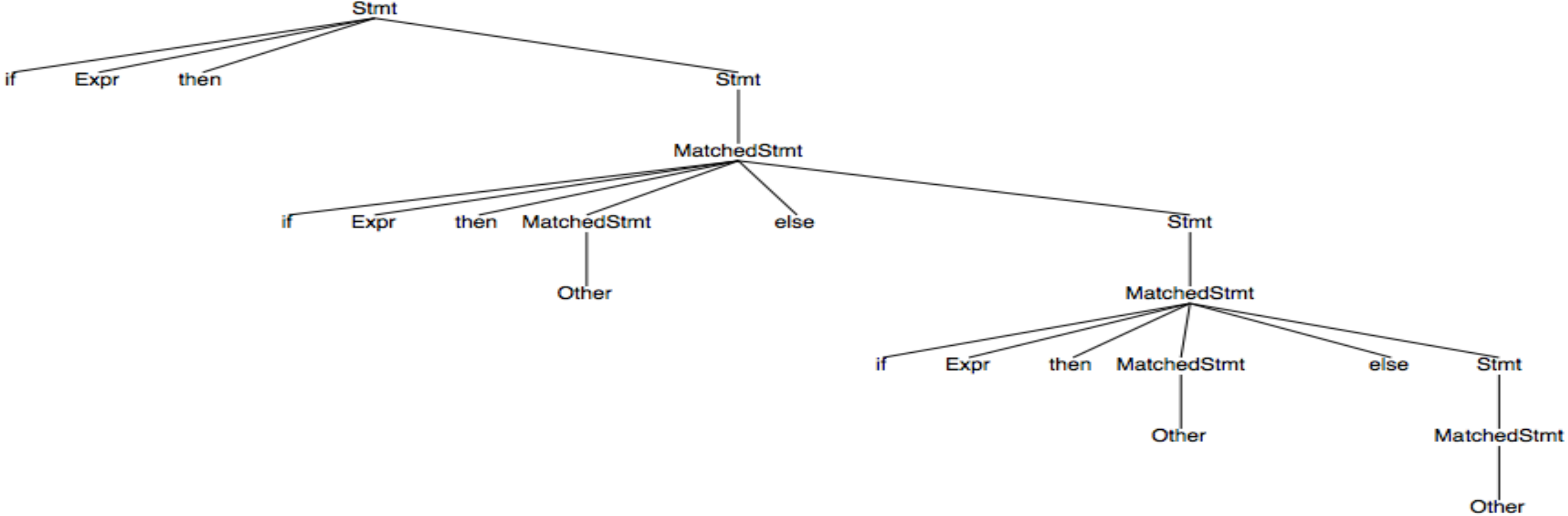
- Modified Grammar (unambiguous?)

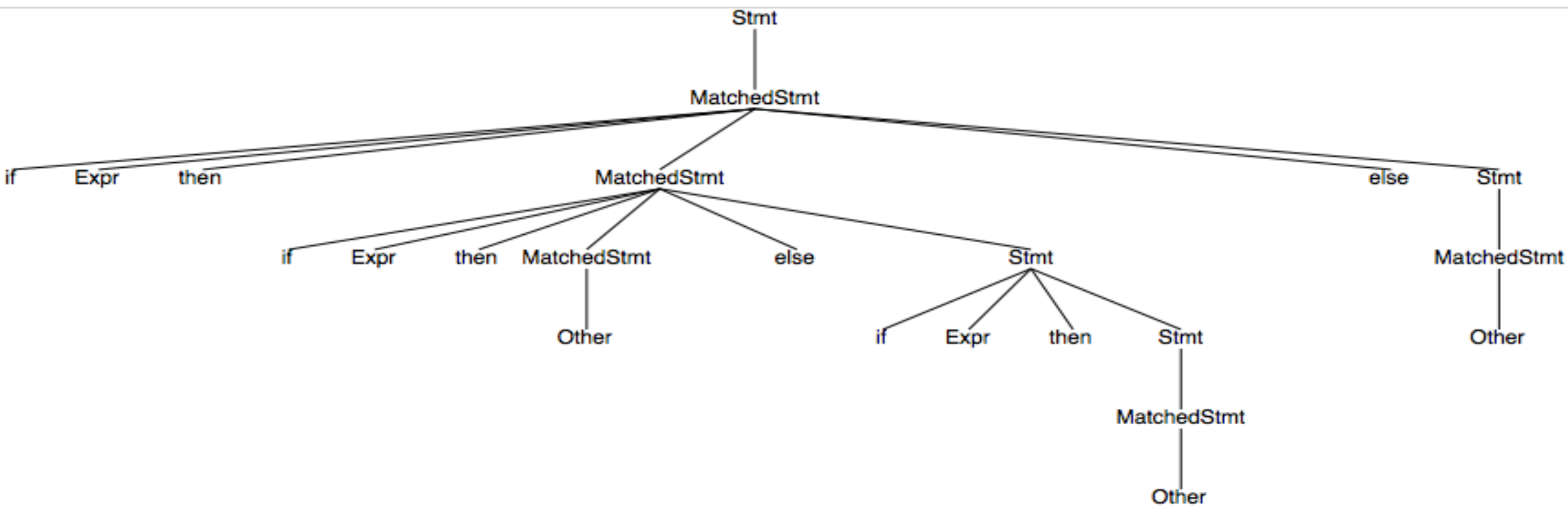
  Stmt → **if** Expr **then** Stmt

  Stmt → MatchedStmt

  MatchedStmt → **if** Expr **then** MatchedStmt **else** Stmt

  MatchedStmt → Other


Stmt
if Expr then Stmt
MatchedStmt
if Expr then MatchedStmt else Stmt
Other MatchedStmt
if Expr then MatchedStmt else Stmt
Other MatchedStmt
Other


Stmt
MatchedStmt
if Expr then MatchedStmt else Stmt
if Expr then MatchedStmt else Stmt MatchedStmt
Other if Expr then Stmt Other
MatchedStmt
Other

Dangling else ambiguity

- Original Grammar (ambiguous)

  Stmt → **if** Expr **then** Stmt **else** Stmt

  Stmt → **if** Expr **then** Stmt

  Stmt → Other

- Unambiguous grammar

  Stmt → MatchedStmt

  Stmt → UnmatchedStmt

  MatchedStmt → **if** Expr **then** MatchedStmt **else** MatchedStmt

  MatchedStmt → Other

  UnmatchedStmt → **if** Expr **then** Stmt

  UnmatchedStmt → **if** Expr **then** MatchedStmt **else** UnmatchedStmt

Dangling else ambiguity

- Check unambiguous dangling-else grammar with the following inputs:
  - if Expr then if Expr then Other else Other
  - if Expr then if Expr then Other else Other else Other
  - if Expr then if Expr then Other else if Expr then Other else Other

Other Ambiguous Grammars

- Consider the grammar

  R → R ‘|’ R | R R | R ‘*’ | ‘(‘ R ‘)’ | a | b
- What does this grammar generate?
- What’s the parse tree for *a|b*a*
- Is this grammar ambiguous?

Left Factoring

- Original Grammar (ambiguous)

  Stmt → **if** Expr **then** Stmt **else** Stmt

  Stmt → **if** Expr **then** Stmt

  Stmt → Other

- Left-factored Grammar (still ambiguous):

  Stmt → **if** Expr **then** Stmt OptElse

  Stmt → Other

  OptElse → **else** Stmt | ε

Left Factor:

A → XA
| XB
| X
| Y
| Z

Left Factoring

- In general, for rules

$$A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \ldots \mid \alpha\beta_n \mid \gamma$$

- Left factoring is achieved by the following grammar transformation:

$$A \rightarrow \alpha A' \mid \gamma$$
$$A' \rightarrow \beta_1 \mid \beta_2 \mid \ldots \mid \beta_n$$

Grammar Transformations

- G is converted to G' s.t. L(G') = L(G)
- Left Factoring
- Removing cycles: $A \Rightarrow^{+} A$
- Removing ε-rules of the form $A \rightarrow \varepsilon$
- Eliminating left recursion
- Conversion to normal forms:
  - Chomsky Normal Form, $A \rightarrow B\ C$ and $A \rightarrow a$
  - Greibach Normal Form, $A \rightarrow a\ \beta$

Eliminating Left Recursion

- Simple case, for left-recursive pair of rules: $A \to A\alpha \mid \beta$
- Replace with the following rules:

$$A \to \beta A'$$

$$A' \to \alpha A' \mid \epsilon$$

- Elimination of immediate left recursion

Eliminating Left Recursion

- Example:

  $E \rightarrow E + T, E \rightarrow T$

- Without left recursion:

  $E \rightarrow T\ E_1, E_1 \rightarrow +\ T\ E_1\ , E_1 \rightarrow \varepsilon$

- Simple algorithm doesn't work for 2-step recursion:

  $S \rightarrow A\ a\ , S \rightarrow b$

  $A \rightarrow A\ c\ , A \rightarrow S\ d\ , A \rightarrow \varepsilon$

Eliminating Left Recursion

- Problem CFG:

  $S \rightarrow A\, a$, $S \rightarrow b$

  $A \rightarrow A\, c$, $A \rightarrow S\, d$, $A \rightarrow \varepsilon$

- Expand possibly left-recursive rules:

  $S \rightarrow A\, a$, $S \rightarrow b$

  $A \rightarrow A\, c$, $A \rightarrow A\, a\, d$, $A \rightarrow b\, d$, $A \rightarrow \varepsilon$

- Eliminate immediate left-recursion

  $S \rightarrow A\, a$, $S \rightarrow b$

  $A \rightarrow b\, d\, A_1$, $A \rightarrow A_1$,

  $A_1 \rightarrow c\, A_1$, $A_1 \rightarrow a\, d\, A_1$, $A_1 \rightarrow \varepsilon$

Eliminating Left Recursion

- We cannot use the algorithm if the non-terminal also derives epsilon. Let's see why:

  $A \rightarrow AAa \mid b \mid \varepsilon$

- Using the standard lrec removal algorithm:

  $A \rightarrow bA_1 \mid A_1$

  $A_1 \rightarrow AaA_1 \mid \varepsilon$

Eliminating Left Recursion

- First we eliminate the epsilon rule:

  $A \rightarrow AAa \mid b \mid \varepsilon$

- Since A is the start symbol, create a new start symbol to generate the empty string:

  $A_1 \rightarrow A \mid \varepsilon \qquad A \rightarrow AAa \mid Aa \mid a \mid b$

- Now we can do the usual lrec algorithm:

  $A_1 \rightarrow A \mid \varepsilon \qquad A \rightarrow aA_2 \mid bA_2$

  $A_2 \rightarrow AaA_2 \mid aA_2 \mid \varepsilon$

Context-free languages and Pushdown Automata

- Recall that for each regular language there was an equivalent finite-state automaton
- The FSA was used as a recognizer of the regular language
- For each context-free language there is also an automaton that recognizes it: called a **pushdown automaton (pda)**

Context-free languages and Pushdown Automata

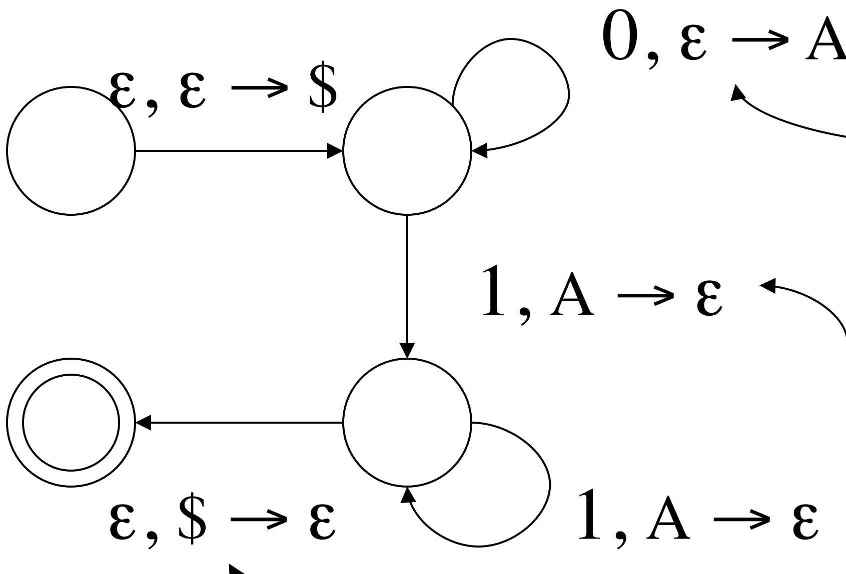
- Similar to FSAs there are non-deterministic pda and deterministic pda
- Unlike in the case of FSAs we cannot always convert a npda to a dpda
- Our goal in compiler design will be to choose grammars carefully so that we can always provide a dpda for it
- Similar to the FSA case, a DFA construction provides us with the algorithm for lexical analysis,
- In this case the construction of a dpda will provide us with the algorithm for parsing (take in strings and provide the parse tree)
- We will study later how to convert a given CFG into a parser by first converting into a PDA

Pushdown Automata

- PDA has
  - an alphabet (terminals) and
  - stack symbols (like non-terminals),
  - a finite-state automaton, and
  - stack

e.g. PDA for language
$L = \{ 0^n 1^n : n >= 0 \}$

→ implies a push/pop of stack symbol(s)



push stack symbol A

pop stack symbol A

check that stack is empty

Non-CF Languages

$$L_1 = \{wcw \mid w \in (a|b)*\}$$

$$L_2 = \{a^n b^m c^n d^m \mid n \geq 1, m \geq 1\}$$

$$L_3 = \{a^n b^n c^n \mid n \geq 0\}$$

CF Languages

$$L_4 = \{wcw^R \mid w \in (a|b)*\}$$

$$S \rightarrow aSa \mid bSb \mid c$$

$$L_5 = \{a^n b^m c^m d^n \mid n \geq 1, m \geq 1\}$$

$$S \rightarrow aSd \mid aAd$$

$$A \rightarrow bAc \mid bc$$

Summary

- CFGs can be used describe PL
- Derivations correspond to parse trees
- Parse trees represent structure of programs
- Ambiguous CFGs exist
- Some forms of ambiguity can be fixed by changing the grammar
- Grammars can be simplified by left-factoring
- Left recursion in a CFG can be eliminated
- CF languages can be recognized using Pushdown Automata

Extra Slides

Non-CF Languages

- The pumping lemma for CFLs [Bar-Hillel] is similar to the pumping lemma for RLs
- For a string $wuxvy$ in a CFL for $u,v \neq \varepsilon$ and the string is longer than p and $|xvy| \leq p$ then wu^nxv^ny is also in the CFL for $n \geq 0$
- Not strong enough to work for every non-CF language (cf. Ogden's Lemma)