# CMPT 413
# Computational Linguistics

Anoop Sarkar
http://www.cs.sfu.ca/~anoop



# Formal Languages: Recap

- Symbols: a, b, c
- Alphabet : finite set of symbols $\Sigma = \{a, b\}$
- String: sequence of symbols bab
- Empty string: $\varepsilon$ Define: $\Sigma^{\varepsilon} = \Sigma \cup \{\varepsilon\}$
- Set of all strings: $\Sigma^*$ cf. *The Library of Babel*, Jorge Luis Borges
- (Formal) Language: a set of strings
  $\{ a^n b^n : n > 0 \}$

# Regular Languages

- The set of regular languages: each element is a regular language
- Each regular language is an example of a (formal) language, i.e. a set of strings

  e.g. { $a^m b^n$ : m, n are +ve integers }



# Regular Languages

- Defining the set of all regular languages:
  - The empty set and $\{a\}$ for all $a$ in $\Sigma^\varepsilon$ are regular languages
  - If $L_1$ and $L_2$ and L are regular languages, then:

    $L_1 \cdot L_2 = \{xy \mid x \in L_1 \text{ and } y \in L_2\}$ (concatenation)

    $L_1 \cup L_2$ (union)

    $L^* = \cup_{i=0}^{\infty} L^i$ (Kleene closure)

    are also regular languages
  - There are no other regular languages

# Formal Grammars

- A formal grammar is a concise description of a formal language
- A formal grammar uses a specialized syntax
- For example, a **regular expression** is a concise description of a regular language
  - *(a|b)*abb* : is the set of all strings over the alphabet {*a*, *b*} which end in *abb*



# Regular Expressions: Definition

- Every symbol of $\Sigma \cup \{ \varepsilon \}$ is a regular expression
- If $r_1$ and $r_2$ are regular expressions, so are
  - Concatenation: $r_1 r_2$
  - Alternation: $r_1|r_2$
  - Repetition: $r_1$*
- Nothing else is.
  - Grouping re's: e.g. aa|bc vs. ((aa)|b)c

# Regular Expressions: Examples

- Alphabet { V, C }  V: vowel  C: consonant
- A set of consonant-vowel sequences  (CV|CCV)*
- All strings that do not contain "VC" as a substring C*V*
- Need a decision procedure: does a particular regular expression (regexp) accept an input string
- Provided by: Finite State Automata

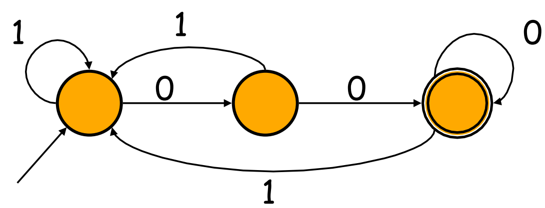


# Finite Automata: Recap

- A set of states S
  - One start state $q_0$, zero or more final states F
- An alphabet $\sum$ of input symbols
- A transition function:
  - $\delta$: S x $\Sigma \Rightarrow$ S
- Example: $\delta(1, a) = 2$

# Finite Automata: Example

- What regular expression does this automaton accept?




Answer: (0|1)*00

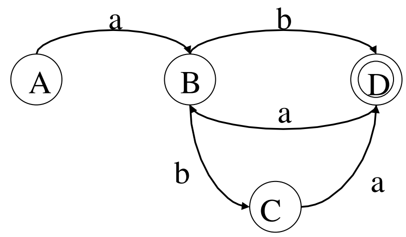


# NFAs

- NFA: like a DFA, except
  - A transition can lead to more than one state, that is, $\delta: S \times \Sigma \Rightarrow 2^S$
  - One state is chosen non-deterministically
  - Transitions can be labeled with $\varepsilon$, meaning states can be reached without reading any input, that is,

$$\delta: S \times \Sigma \cup \{ \varepsilon \} \Rightarrow 2^S$$

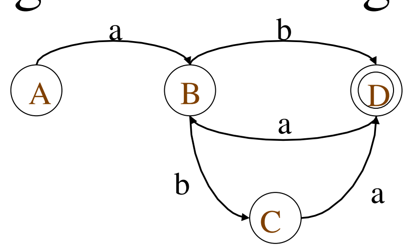
# Recognition of strings (NFAs)




- Input string: aba#
- Recognition problem: Is input string in the language generated by the NFA?
- Recognition (without conversion to DFA) is also called *simulation* of NFA



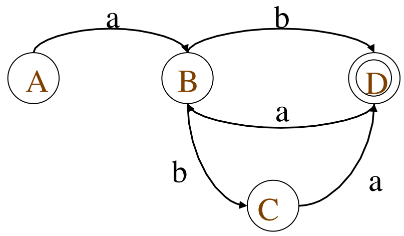
# Recognition of strings (NFAs)




q is the transition function for the NFA

- Input tape: $_0$ a $_1$ b $_2$ a $_3$ # $_4$
- Start State: A Agenda: { (A, 0) }
- Pop (A, 0) from Agenda
- q(A, a) = B, Agenda: { (B, 1) }
- Pop (B, 1) from Agenda
- q(B, b) = { D, C } Agenda: { (D, 2), (C, 2) }

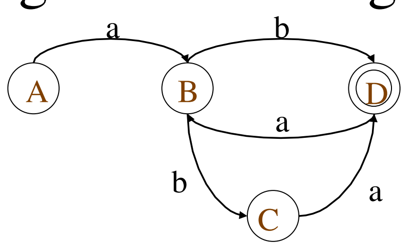
# Recognition of strings (NFAs)




- Input tape: $_0$ a $_1$ b $_2$ a $_3$ # $_4$
- Pop (D, 2) from Agenda
- q(D, a) = { B }   Agenda: { (B, 3), (C, 2) }
- Pop (B, 3) from Agenda:  B is not a final state
- Pop (C, 2) from Agenda: if Agenda empty, reject
- q(C, a) = { D }   Agenda: { (D, 3) }



# Recognition of strings (NFAs)




- Input tape: $_0$ a $_1$ b $_2$ a $_3$ # $_4$
- Pop (D, 3) from Agenda
- Is (D, 3) an accept item?
- Yes: D is a final state **and** 3 is index of the end-of-string marker #
- Return accept

# Recognition of strings (NFAs)

```
function NDRecognize (tape[], q):
    Agenda = { (start-state, 0) }
    Current = (state, index) = pop(Agenda)
    while (true) {
        if (Current is an accept item) return accept
        else Agenda = Agenda ∪ GenStates(q, state, tape[index])
        if (Agenda is empty) return reject
        else Current = (state, index) = pop(Agenda)
    }
function GenStates (q, state, index):
    return { (q', index) : for all q' = q(state, ε) } ∪
           { (q', index+1) : for all q' = q(state, tape[index+1]) }
```



*what if the input to this algorithm is a DFA?*



# Algorithms for FSMs

## (finite-state machines)

- Recognition of a string in a regular language: is a string accepted by an NFA?
- Conversion of regular expressions to NFAs
- Determinization: converting NFA to DFA
- Converting an NFA into a regular expression
- Other useful *closure* properties: union, concatenation, Kleene closure, intersection