CMPT 379
Compilers

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Top-Down vs. Bottom Up

Grammar: $S \rightarrow A\ B$
$A \rightarrow c \mid \varepsilon$
$B \rightarrow cbB \mid ca$

Input String: ccbca

Top-Down/leftmost		Bottom-Up/rightmost	
$S \Rightarrow AB$	$S \rightarrow AB$	$ccbca \Leftarrow Acbca$	$A \rightarrow c$
$\Rightarrow cB$	$A \rightarrow c$	$\Leftarrow AcbB$	$B \rightarrow ca$
$\Rightarrow ccbB$	$B \rightarrow cbB$	$\Leftarrow AB$	$B \rightarrow cbB$
$\Rightarrow ccbca$	$B \rightarrow ca$	$\Leftarrow S$	$S \rightarrow AB$

Parsing - Roadmap

- Parser:
  - decision procedure: builds a parse tree
- Top-down vs. bottom-up
- LL(1) – Deterministic Parsing
  - recursive-descent
  - table-driven
- LR(k) – Deterministic Parsing
  - LR(0), SLR(1), LR(1), LALR(1)
- Parsing arbitrary CFGs – Polynomial time parsing

Top-Down: Backtracking

$S \rightarrow A\ B$
$A \rightarrow c \mid \varepsilon$
$B \rightarrow cbB \mid ca$

True/False
$S \Rightarrow^* cbca?$

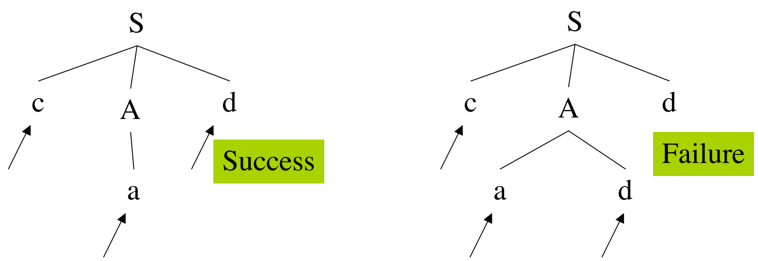
S	cbca	try $S \rightarrow AB$
AB	cbca	try $A \rightarrow c$
cB	cbca	match c
B	bca	dead-end, try $A \rightarrow \varepsilon$
εB	cbca	try $B \rightarrow cbB$
cbB	cbca	match c
bB	bca	match b
B	ca	try $B \rightarrow cbB$
cbB	ca	match c
bB	a	dead-end, try $B \rightarrow ca$
ca	ca	match c
a	a	match a, Done!

Backtracking

$S \rightarrow cAd \mid c$
$A \rightarrow a \mid ad$

Input: cad

$S \rightarrow cAd \mid c$
$A \rightarrow ad \mid a$



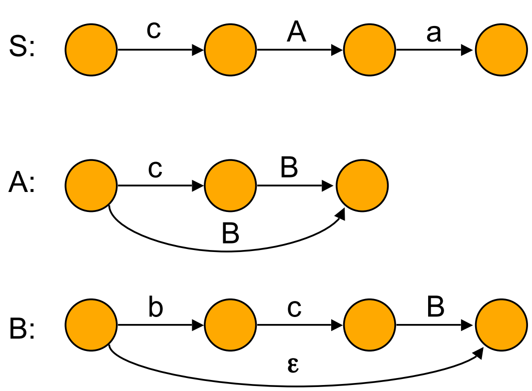
For some grammars, rule ordering is crucial for backtracking parsers, e.g $S \rightarrow aSa$, $S \rightarrow aa$

Transition Diagram

$S \rightarrow cAa$

$A \rightarrow cB \mid B$

$B \rightarrow bcB \mid \varepsilon$



Predictive Top-Down Parser

- Knows which production to choose based on single lookahead symbol
- Need LL(1) grammars
  - First L: reads input Left to right
  - Second L: produce Leftmost derivation
  - 1: one symbol of lookahead
- Can't have left-recursion
- Must be left-factored (no left-factors)
- Not all grammars can be made LL(1)

Leftmost derivation for **id + id * id**

$E \rightarrow E + E$
$E \rightarrow E * E$
$E \rightarrow (E)$
$E \rightarrow - E$
$E \rightarrow \textbf{id}$

$E \Rightarrow E + E$
$\Rightarrow \textbf{id} + E$
$\Rightarrow \textbf{id} + E * E$
$\Rightarrow \textbf{id} + \textbf{id} * E$
$\Rightarrow \textbf{id} + \textbf{id} * \textbf{id}$

$E \Rightarrow^*_{lm}$ id + E * E

Predictive Parsing Table

	Productions
1	**T → F T’**
2	**T’ → ε**
3	**T’ → * F T’**
4	**F → id**
5	**F → (T)**

	*	(	)	id	$
T		**T → F T’**		**T → F T’**	
T’	**T’ → * F T’**		**T’ → ε**		**T’ → ε**
F		**F → (T)**		**F → id**	

Trace “(id)*id”

	*	(	)	id	$
T		T → FT’		T → FT’	
T’	T’ → *FT’		T’ → ε		T’ → ε
F		F → (T)		F → id	

Stack	Input	Output
$T	(id)*id$	
$T’F	(id)*id$	**T → F T’**
$T’)T(	(id)*id$	**F → (T)**
$T’)T	id)*id$	
$T’)T’F	id)*id$	**T → F T’**
$T’)T’id	id)*id$	**F → id**
$T’)T’	)*id$	
$T’)	)*id$	**T’ → ε**

Trace “(id)*id”

	*	(	)	id	$
T		T → FT’		T → FT’	
T’	T’ → *FT’		T’ → ε		T’ → ε
F		F → (T)		F → id	

Stack	Input	Output
$T’	*id$	
$T’F*	*id$	**T’ → * F T’**
$T’F	id$	
$T’id	id$	**F → id**
$T’	$	
$	$	**T’ → ε**

Table-Driven Parsing

stack.push($); stack.push(S);
a = input.read();
forever do begin
  X = stack.peek();
  if X = a **and** a = $ **then** return SUCCESS;
  elsif X = a **and** a != $ **then**
    pop X; a = input.read();
  elsif X != a **and** X ∈ **N and** M[X,a] **then**
    pop X; push right-hand side of M[X,a];
  else ERROR!
end

Predictive Parsing table

- Given a grammar produce the predictive parsing table
- We need to to know for all rules $A \rightarrow \alpha \mid \beta$ the lookahead symbol
- Based on the lookahead symbol the table can be used to pick which rule to push onto the stack
- This can be done using two sets: FIRST and FOLLOW

FIRST and FOLLOW

$$a \in \mathrm{FIRST}(\alpha) \text{ if } \alpha \Rightarrow^* a\beta$$

$$\text{if } \alpha \Rightarrow^* \epsilon \text{ then } \epsilon \in \mathrm{FIRST}(\alpha)$$

$$a \in \mathrm{FOLLOW}(A) \text{ if } S \Rightarrow^* \alpha A a \beta$$

$$a \in \mathrm{FOLLOW}(A) \text{ if } S \Rightarrow^* \alpha A \gamma a \beta$$

$$\text{and } \gamma \Rightarrow^* \epsilon$$

Conditions for LL(1)

- Necessary conditions:
  - no ambiguity
  - no left recursion
  - Left factored grammar
- A grammar G is LL(1) iff - whenever $A \rightarrow \alpha \mid \beta$
  1. $\mathrm{First}(\alpha) \cap \mathrm{First}(\beta) = \varnothing$
  2. $\alpha \Rightarrow^* \varepsilon$ implies **!**$(\beta \Rightarrow^* \varepsilon)$
  3. $\alpha \Rightarrow^* \varepsilon$ implies $\mathrm{First}(\beta) \cap \mathrm{Follow}(A) = \varnothing$

ComputeFirst(α: string of symbols)

// assume $\alpha = X_1\ X_2\ X_3 \ldots X_n$
if $X_1 \in$ **T then** First[α] := $\{X_1\}$
else begin
  i:=1; First[α] := ComputeFirst(X_1)\{**ε**};
  while $X_i \Rightarrow^* \varepsilon$ **do begin**
    if i < n **then**
      First[α] := First[α] ∪ ComputeFirst(X_{i+1})\{**ε**};
    else
      First[α] := First[α] ∪ {**ε**};
    i := i + 1;
  end
end

ComputeFirst(α: string of symbols)

// assume $\alpha = X_1 X_2 X_3 \ldots X_n$
if $X_1 \in \mathbf{T}$ **then** $\text{First}[\alpha] := \{X_1\}$
else begin
  $i{:=}1$; $\text{First}[\alpha] := \text{ComputeFirst}(X_1) \setminus \{\boldsymbol{\varepsilon}\}$;
  while $X_i \Rightarrow^* \boldsymbol{\varepsilon}$ **do begin**
    if $i < n$ **then**
      $\text{First}[\alpha] := \text{First}[\alpha] \cup \text{ComputeFirst}(X_{i+1}) \setminus \{\boldsymbol{\varepsilon}\}$;
    else
      $\text{First}[\alpha] := \text{First}[\alpha] \cup \{\boldsymbol{\varepsilon}\}$;
    $i := i + 1$;
  end
end

Recursion in computing FIRST causes problems when faced with left-recursive grammars

ComputeFirst; modified

foreach $X \in \mathbf{T}$ **do** $\text{First}[X] := X$;
foreach $p \in \mathbf{P} : X \to \boldsymbol{\varepsilon}$ **do** $\text{First}[X] := \{\boldsymbol{\varepsilon}\}$;
repeat foreach $X \in \mathbf{N}$, $p : X \to Y_1\ Y_2\ Y_3 \ldots Y_n$ **do**
  begin $i{:=}1$;
   while $Y_i \Rightarrow^* \boldsymbol{\varepsilon}$ and $i <= n$ **do begin**
     $\text{First}[X] := \text{First}[X] \cup \text{First}[Y_i] \setminus \{\boldsymbol{\varepsilon}\}$;
     $i := i{+}1$;
   end
  if $i = n{+}1$ **then** $\text{First}[X] := \text{First}[X] \cup \{\boldsymbol{\varepsilon}\}$;
  else $\text{First}[X] := \text{First}[X] \cup \text{First}[Y_i]$;
until no change in First[X] for any X;

ComputeFirst; modified

foreach $X \in \mathbf{T}$ **do** $\text{First}[X] := X$;
foreach $p \in \mathbf{P} : X \to \boldsymbol{\varepsilon}$ **do** $\text{First}[X] := \{\boldsymbol{\varepsilon}\}$;
repeat foreach $X \in \mathbf{N}$, $p : X \to Y_1\ Y_2\ Y_3 \ldots Y_n$ **do**
  begin $i{:=}1$;
   while $Y_i \Rightarrow^*$
     $\text{First}[X] :=$
     $i := i{+}1$;
   end
  if $i = n{+}1$ **then** $\text{First}[X] := \text{First}[X] \cup \{\boldsymbol{\varepsilon}\}$;
  else $\text{First}[X] := \text{First}[X] \cup \text{First}[Y_i]$;
until no change in First[X] for any X;

Non-recursive FIRST computation works with left-recursive grammars. Computes a fixed point for FIRST[X] for all non-terminals X in the grammar. But this algorithm is very inefficient.

ComputeFollow

$\text{Follow}(S) := \{\$\}$;
repeat
 foreach $p \in \mathbf{P}$ **do**
   case $p = A \to \alpha B \beta$ **begin**
     $\text{Follow}[B] := \text{Follow}[B] \cup \text{ComputeFirst}(\beta) \setminus \{\boldsymbol{\varepsilon}\}$;
     if $\varepsilon \in \text{First}(\beta)$ **then**
      $\text{Follow}[B] := \text{Follow}[B] \cup \text{Follow}[A]$;
   end
   case $p = A \to \alpha B$
     $\text{Follow}[B] := \text{Follow}[B] \cup \text{Follow}[A]$;
until no change in any Follow[N]

Example First/Follow

$S \rightarrow AB$

$A \rightarrow c \mid \varepsilon$ Not an LL(1) grammar

$B \rightarrow cbB \mid ca$

First(A) = {c, ε} Follow(A) = {c}

First(B) = {c} Follow(A) ∩

First(cbB) = First(c) = {c}

First(ca) = {c} Follow(B) = {$}

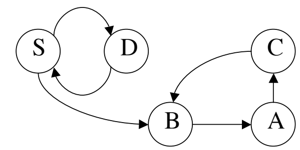
First(S) = {c} Follow(S) = {$}

ComputeFirst on Left-recursive Grammars

- ComputeFirst as defined earlier loops on left-recursive grammars
- Here is an alternative algorithm for ComputeFirst
  1. Compute non-recursive cases of FIRST
  2. Create a graph of recursive cases where FIRST of a non-terminal depends on another non-terminal
  3. Compute Strongly Connected Components (SCC)
  4. Compute FIRST starting from root of SCC to avoid cycles
- Unlike top-down LL parsing, bottom-up LR parsing allows left-recursive grammars, so this algorithm is useful for LR parsing

ComputeFirst on Left-recursive Grammars

- $S \rightarrow BD$
- $D \rightarrow d \mid Sd$
- $A \rightarrow CB \mid a$
- $C \rightarrow Bb \mid \varepsilon$
- $B \rightarrow Ab \mid b$



Compute Strongly Connected Components

$FIRST_0[A] := \{a, b\}$

$FIRST_0[C] := \{\}$

$FIRST_0[B] := \{b\}$

$FIRST_0[S] := \{\}$

$FIRST_0[D] := \{d\}$

2 SCCs: e.g. consider B-A-C

$FIRST[B] := FIRST_0[B] + FIRST[A]$

$FIRST[A] := FIRST_0[A] + FIRST[C]$

$FIRST[C] := FIRST_0[C] + FIRST_0[B]$

Converting to LL(1)

$S \rightarrow AB$

$A \rightarrow c \mid \varepsilon$

$B \rightarrow cbB \mid ca$

Note that grammar is regular: c? (cb)* ca

c (c b c b … c b) c a	→	c c (b c b … c b c) a
(c b c b … c b) c a		c (b c b … c b c) a

same as:

c c? (bc)* a

$S \rightarrow cAa$

$A \rightarrow cB \mid B$

$B \rightarrow bcB \mid \varepsilon$

Verifying LL(1) using F/F sets

$S \rightarrow cAa$

$A \rightarrow cB \mid B$

$B \rightarrow bcB \mid \varepsilon$

First(A) = $\{b, c, \varepsilon\}$	Follow(A) = $\{a\}$
First(B) = $\{b, \varepsilon\}$	Follow(B) = $\{a\}$
First(S) = $\{c\}$	Follow(S) = $\{\$\}$

Building the Parse Table

- Compute First and Follow sets
- For each production $A \rightarrow \alpha$
  - foreach $a \in$ First(α) add $A \rightarrow \alpha$ to M[A,a]
  - If $\boldsymbol{\varepsilon} \in$ First(α) add $A \rightarrow \alpha$ to M[A,b] for each b in Follow(A)
  - If $\boldsymbol{\varepsilon} \in$ First(α) add $A \rightarrow \alpha$ to M[A,\$] if $\$ \in$ Follow(α)
  - All undefined entries are errors

Revisit conditions for LL(1)

- A grammar G is LL(1) iff - whenever $A \rightarrow \alpha \mid \beta$
  1. First(α) $\cap$ First(β) = $\varnothing$
  2. $\alpha \Rightarrow^* \varepsilon$ implies $!(\beta \Rightarrow^* \varepsilon)$
  3. $\alpha \Rightarrow^* \varepsilon$ implies First(β) $\cap$ Follow(A) = $\varnothing$
- No more than one entry per table field

Error Handling

- Reporting & Recovery
  - Report as soon as possible
  - Suitable error messages
  - Resume after error
  - Avoid cascading errors
- Phrase-level vs. Panic-mode recovery

Panic-Mode Recovery

- Skip tokens until *synchronizing set* is seen
  - Follow(A)
    - garbage or missing things after
  - Higher-level start symbols
  - First(A)
    - garbage before
  - Epsilon
    - if nullable
  - Pop/Insert terminal
    - "auto-insert"
- Add "synch" actions to table

Summary so far

- LL(1) grammars
  - necessary conditions
    - No left recursion
    - Left-factored
- Not all languages can be generated by LL(1) grammar
- LL(1) grammars can be parsed by simple predictive recursive-descent parser
  - Alternative: table-driven top-down parser

Bottom-up parsing overview

- Start from terminal symbols, search for a path to the start symbol
- Apply shift and reduce actions: postpone decisions
- LR parsing:
  - L: left to right parsing
  - R: rightmost derivation (in reverse or bottom-up)
- LR(0) → SLR(1) → LR(1) → LALR(1)
  - 0 or 1 or *k* lookahead symbols

Actions in Shift-Reduce Parsing

- Shift
  - add terminal to parse stack, advance input
- Reduce
  - If αw on stack, and $A \rightarrow w$, and there is a $\beta \in T^*$ such that $S \Rightarrow^*_{rm} \alpha A \beta \Rightarrow_{rm} \alpha w \beta$ then we can *prune the handle* w; we reduce αw to αA on the stack
  - αw is a *viable prefix*
- Error
- Accept

Questions

- When to shift/reduce?
  - What are valid handles?
  - Ambiguity: Shift/reduce conflict
- If reducing, using which production?
  - Ambiguity: Reduce/reduce conflict

Rightmost derivation for **id + id * id**

Grammar	Derivation	
$E \rightarrow E + E$	$E \Rightarrow E * E$	
$E \rightarrow E * E$	$\Rightarrow E * \mathbf{id}$	
$E \rightarrow (E)$	$\Rightarrow E + E * \mathbf{id}$	
$E \rightarrow - E$	$\Rightarrow E + \mathbf{id} * \mathbf{id}$	reduce with $E \rightarrow \mathbf{id}$
$E \rightarrow \mathbf{id}$	$\Rightarrow \mathbf{id} + \mathbf{id} * \mathbf{id}$	shift

$E \Rightarrow^{*}_{rm} E + E$ * id

LR Parsing

- Table-based parser
  - Creates rightmost derivation (in reverse)
  - For "less massaged" grammars than LL(1)
- Data structures:
  - Stack of states/symbols {s}
  - Action table: **action**[s, a]; a ∈ **T**
  - Goto table: **goto**[s, X]; X ∈ **N**

Action/Goto Table

Productions	
1	**T → F**
2	**T → T*F**
3	**F → id**
4	**F → (T)**

	*	(	)	id	$	T	F
0		S5		S8		2	1
1	R1	R1	R1	R1	R1		
2	S3				Acc!		
3		S5		S8			4
4	R2	R2	R2	R2	R2		
5		S5		S8		6	1
6	S3		S7				
7	R4	R4	R4	R4	R4		
8	R3	R3	R3	R3	R3		

Trace “(id)*id”

Stack	Input	Action
0	(id) * id $	Shift S5
0 5	id) * id $	Shift S8
0 5 8	) * id $	Reduce 3 F→id, pop 8, goto [5,F]=1
0 5 1	) * id $	Reduce 1 T→ F, pop 1, goto [5,T]=6
0 5 6	) * id $	Shift S7
0 5 6 7	* id $	Reduce 4 F→ (T), pop 7 6 5, goto [0,F]=1
0 1	* id $	Reduce 1 T → F pop 1, goto [0,T]=2

Productions	
1	T → F
2	T → T*F
3	F → id
4	F → (T)

“(id)*id”

	*	(	)	id	$	T	F
0		S5		S8		2	1
1	R1	R1	R1	R1	R1		
2	S3				A		
3		S5		S8			4
4	R2	R2	R2	R2	R2		
5		S5		S8		6	1
6	S3		S7				
7	R4	R4	R4	R4	R4		
8	R3	R3	R3	R3	R3		

	Input	A
0	(id) * id $	Sh
0 5	id) * id $	Sh
0 5 8	) * id $	Re po
0 5 1	) * id $	Reduce 1 T→ F, pop 1, goto [5,T]=6
0 5 6	) * id $	Shift S7
0 5 6 7	* id $	Reduce 4 F→ (T), pop 7 6 5, goto [0,F]=1
0 1	* id $	Reduce 1 T → F pop 1, goto [0,T]=2

Trace “(id)*id”

Stack	Input	Action
0 1	* id $	Reduce 1 T→F, pop 1, goto [0,T]=2
0 2	* id $	Shift S3
0 2 3	id $	Shift S8
0 2 3 8	$	Reduce 3 F→id, pop 8, goto [3,F]=4
0 2 3 4	$	Reduce 2 T→T * F pop 4 3 2, goto [0,T]=2
0 2	$	Accept

Productions	
1	T → F
2	T → T*F
3	F → id
4	F → (T)

“(id)*id”

	*	(	)	id	$	T	F
0		S5		S8		2	1
1	R1	R1	R1	R1	R1		
2	S3				A		
3		S5		S8			4
4	R2	R2	R2	R2	R2		
5		S5		S8		6	1
6	S3		S7				
7	R4	R4	R4	R4	R4		
8	R3	R3	R3	R3	R3		

Stack	Input	Actio
0 1	* id $	Reduc pop 1,
0 2	* id $	Shift S
0 2 3	id $	Shift S8
0 2 3 8	$	Reduce 3 F→id, pop 8, goto [3,F]=4
0 2 3 4	$	Reduce 2 T→T * F pop 4 3 2, goto [0,T]=2
0 2	$	Accept

Tracing LR: **action**[*s*, *a*]

- case **shift** *u*:
  - push state *u*
  - read new *a*
- case **reduce** *r*:
  - lookup production *r*: $X \rightarrow Y_1..Y_k$;
  - pop *k* states, find state *u*
  - push **goto**[*u*, *X*]
- case **accept**: done
- no entry in action table: **error**

Configuration set

- Each set is a parser state
- Consider

$$\mathbf{T \rightarrow T * \bullet F}$$
$$\mathbf{F \rightarrow \bullet (T)}$$
$$\mathbf{F \rightarrow \bullet id}$$

- Like NFA-to-DFA conversion

Closure

Closure property:

- If $\mathbf{T \rightarrow X_1 \ldots X_i \bullet X_{i+1} \ldots X_n}$ is in set, and $\mathbf{X_{i+1}}$ is a nonterminal, then $\mathbf{X_{i+1} \rightarrow \bullet Y_1 \ldots Y_m}$ is in the set as well for all productions $\mathbf{X_{i+1} \rightarrow Y_1 \ldots Y_m}$
- Compute as fixed point

Starting Configuration

- Augment Grammar with S’
- Add production S’ → S
- Initial configuration set is

closure(S’ → • S)

Example: I = closure($S' \to \bullet T$)

$S' \to \bullet T$

$T \to \bullet T * F$

$T \to \bullet F$

$F \to \bullet \text{id}$

$F \to \bullet (T)$

$S' \to T$
$T \to F \mid T * F$
$F \to \text{id} \mid (T)$

Successor(I, X)

Informally: “move by symbol X”

1. move dot to the right in all items where dot is before X
2. remove all other items

   (viable prefixes only!)
3. compute closure

Successor Example

I = {$S' \to \bullet T$,

$T \to \bullet F$,

$T \to \bullet T * F$,

$F \to \bullet \text{id}$,

$F \to \bullet (T)$ }

$S' \to T$
$T \to F \mid T * F$
$F \to \text{id} \mid (T)$

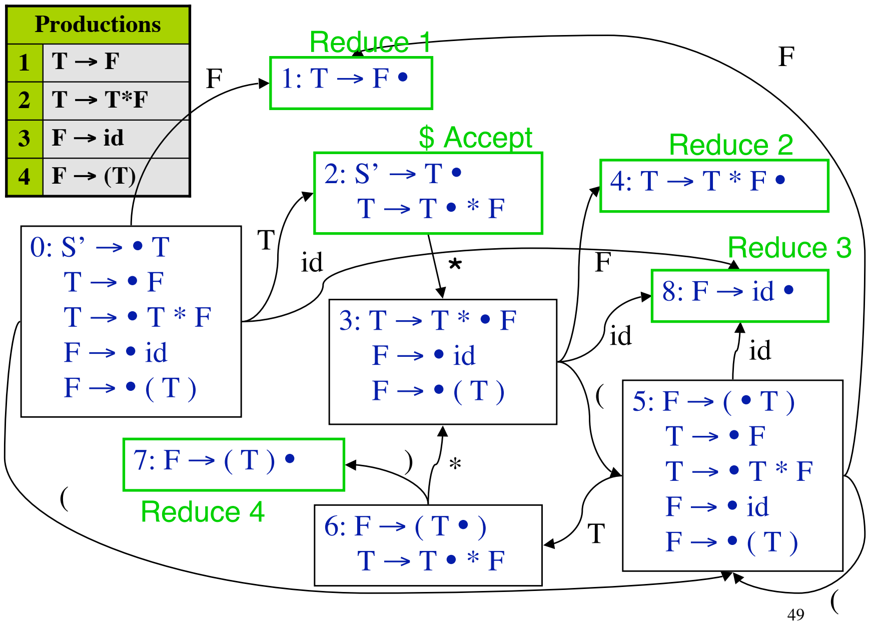
Compute **Successor**(I, “(“)

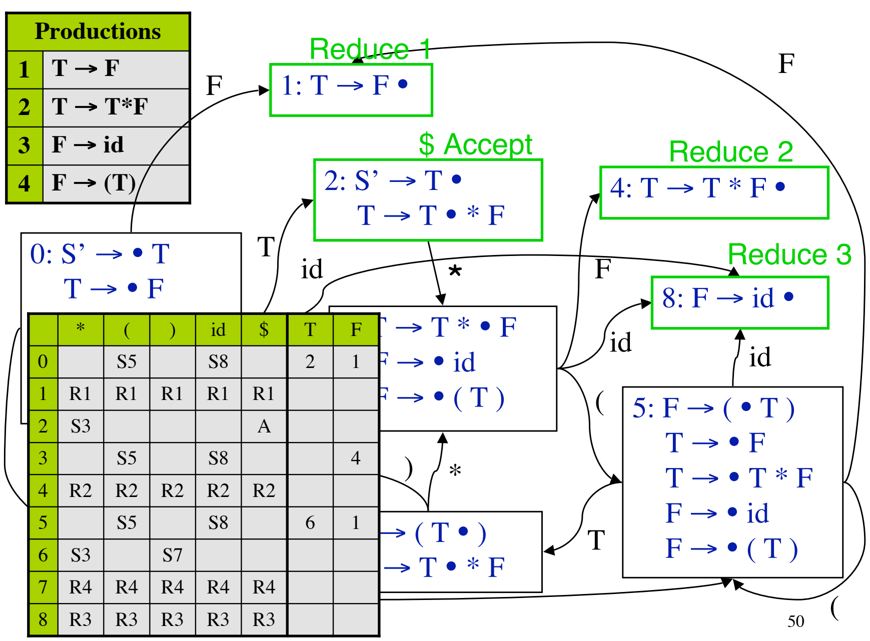
{ $F \to (\bullet T)$, $T \to \bullet F$, $T \to \bullet T * F$, $F \to \bullet \text{id}$, $F \to \bullet (T)$ }

Sets-of-Items Construction

Family of configuration sets

function items(G’)
    C = { closure({$S' \to \bullet S$}) };
    do foreach I $\in$ C **do**
        foreach X $\in$ **(N** $\cup$ **T) do**
            C = C $\cup$ { **Successor**(I, X) };
    while C changes;





	*	(	)	id	$	T	F
0		S5		S8		2	1
1	R1	R1	R1	R1	R1		
2	S3				A		
3		S5		S8			4
4	R2	R2	R2	R2	R2		
5		S5		S8		6	1
6	S3		S7				
7	R4	R4	R4	R4	R4		
8	R3	R3	R3	R3	R3		

LR(0) Construction

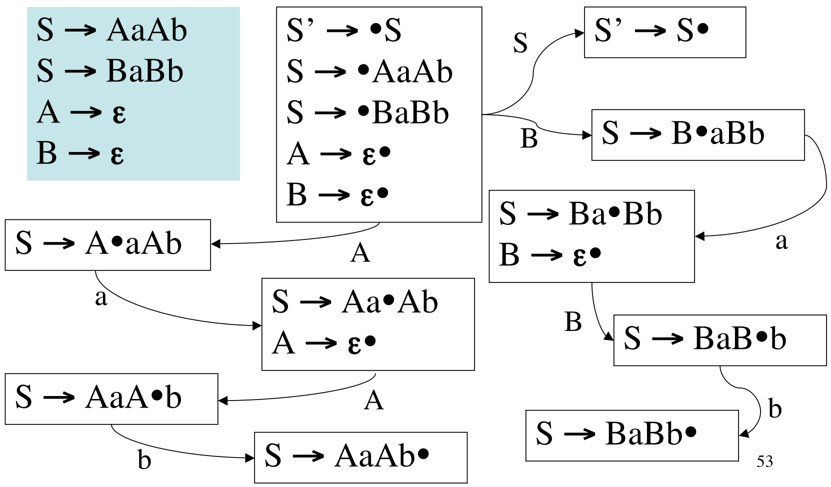
1. Construct F = $\{I_0, I_1, \ldots I_n\}$
2. a) if $\{A \rightarrow \alpha\bullet\} \in I_i$ and A != S
   then action[i, _] := reduce $A \rightarrow \alpha$
   b) if $\{S' \rightarrow S\bullet\} \in I_i$
   then action[i,$] := accept
   c) if $\{A \rightarrow \alpha\bullet a\beta\} \in I_i$ and Successor(I_i,a) = I_j
   then action[i,a] := shift j
3. if Successor(I_i,A) = I_j then goto[i,A] := j

LR(0) Construction (cont’d)

4. All entries not defined are errors
5. Make sure I_0 is the initial state

- Note: LR(0) always reduces if $\{A \rightarrow \alpha\bullet\} \in I_i$, no lookahead
- Shift and reduce items can’t be in the same configuration set
  - Accepting state doesn’t count as reduce item
- At most one reduce item per set

Set-of-items with Epsilon rules



SLR(1) : Simple LR(1) Parsing

0: S' → • T
T → • F
T → • T * F
T → • C (T)
F → • id
F → • id ++
F → • (T)
C → • id

id

S' → T
T → F | T * F | C (T)
F → id | id ++ | (T)
C → id

1: F → id •
F → id • ++
C → id •

Follow(F) = { *,), $ }

Follow(C) = { (}

action[1,*]= action[1,)] = action[1,$] = Reduce F → id
action[1,(] = Reduce C → id
action[1,++] = Shift

LR(0) conflicts:

S' → F
F → id | (T)
F → id = T ;
T → T * F
T → id

5: F → id •
F → id • = T

Shift/reduce conflict

2: F → id •
T → id •

Reduce/Reduce conflict

Need more lookahead: SLR(1)

SLR(1) Construction

1. Construct F = $\{I_0, I_1, \ldots I_n\}$
2. a) if $\{A \rightarrow \alpha\bullet\} \in I_i$ and A != S'
   then action[i, b] := reduce $A \rightarrow \alpha$
   for all $b \in$ Follow(A)

   b) if $\{S' \rightarrow S\bullet\} \in I_i$
   then action[i, $] := accept

   c) if $\{A \rightarrow \alpha\bullet a\beta\} \in I_i$ and Successor(I_i, a) = I_j
   then action[i, a] := shift j
3. if Successor(I_i, A) = I_j then goto[i, A] := j

SLR(1) Construction (cont'd)

4. All entries not defined are errors
5. Make sure I_0 is the initial state

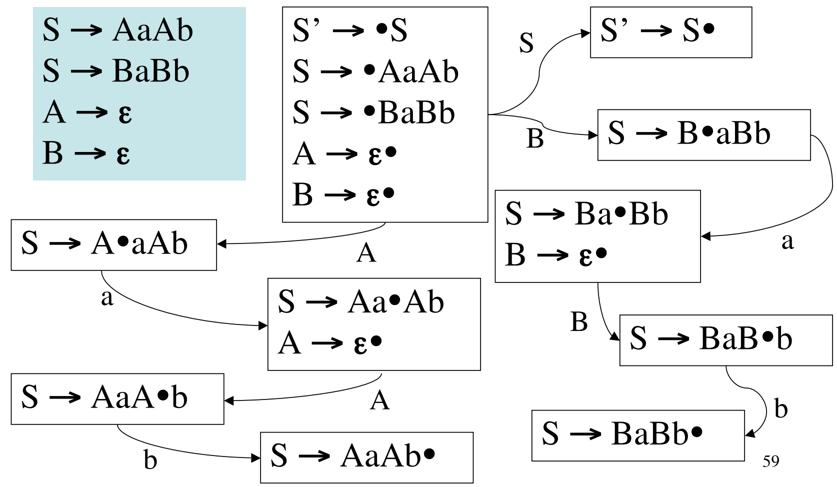
- Note: SLR(1) only reduces $\{A \rightarrow \alpha\bullet\}$ if lookahead in Follow(A)
- Shift and reduce items or more than one reduce item can be in the same configuration set as long as lookaheads are disjoint

SLR(1) Conditions

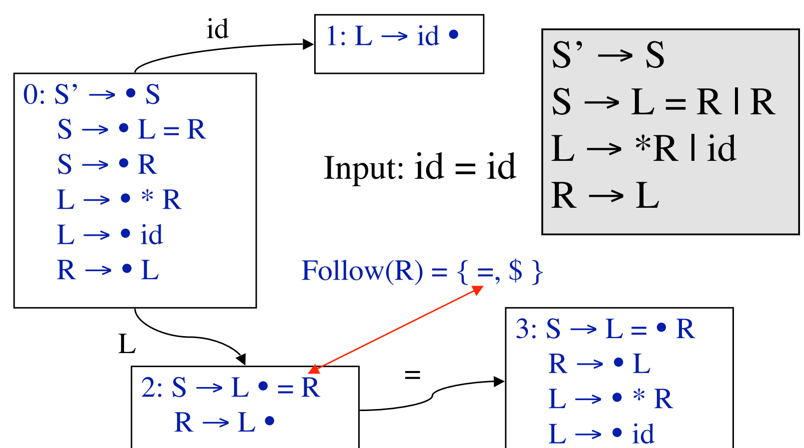
- A grammar is SLR(1) if for each configuration set:
  - For any item $\{A \rightarrow \alpha\bullet x\beta: x \in T\}$ there is no $\{B \rightarrow \gamma\bullet: x \in \text{Follow}(B)\}$
  - For any two items $\{A \rightarrow \alpha\bullet\}$ and $\{B \rightarrow \beta\bullet\}$ $\text{Follow}(A) \cap \text{Follow}(B) = \varnothing$

LR(0) Grammars $\subset$ SLR(1) Grammars

Is this grammar SLR(1)?



SLR limitation: lack of context



Solution: Canonical LR(1)

- Extend definition of configuration
  - Remember lookahead
- New closure method
- Extend definition of Successor

LR(1) Configurations

- $[A \rightarrow \alpha\bullet\beta, a]$ for $a \in T$ is valid for a viable prefix $\delta\alpha$ if there is a rightmost derivation
  $S \Rightarrow^* \delta A\eta \Rightarrow^* \delta\alpha\beta\eta$ and
  $(\eta = a\gamma)$ or $(\eta = \varepsilon$ and $a = \$)$
- Notation: $[A \rightarrow \alpha\bullet\beta, a/b/c]$
  - if $[A \rightarrow \alpha\bullet\beta, a]$, $[A \rightarrow \alpha\bullet\beta, b]$, $[A \rightarrow \alpha\bullet\beta, c]$ are valid configurations

LR(1) Configurations

$S \rightarrow B\ B$
$B \rightarrow a\ B \mid b$

- $S \Rightarrow^*_{rm} aaBab \Rightarrow_{rm} aaaBab$
- Item $[B \rightarrow a \bullet B, a]$ is valid for viable prefix *aaa*
- $S \Rightarrow^*_{rm} BaB \Rightarrow_{rm} BaaB$
- Also, item $[B \rightarrow a \bullet B, \$]$ is valid for viable prefix *Baa*

LR(1) Closure

Closure property:

- If $[A \rightarrow \alpha \bullet B\beta, a]$ is in set, then $[B \rightarrow \bullet\ \gamma, b]$ is in set if $b \in \text{First}(\beta a)$
- Compute as fixed point
- Only include contextually valid lookaheads to guide reducing to B

Starting Configuration

- Augment Grammar with S’ just like for LR(0), SLR(1)
- Initial configuration set is

  I = closure([S’ → • S, $])

Example: closure([S’ → • S, $])

[S’ → • S, $]
[S → • L = R, $]
[S → • R, $]
[L → • * R, =]
[L → • id, =]
[R → • L, $]
[L → • *R, $]
[L → • id, $]

S’ → S
S → L = R | R
L → *R | id
R → L

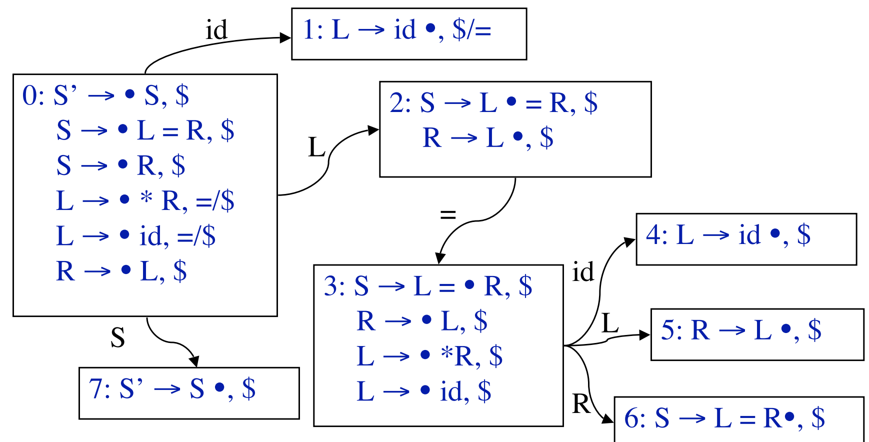
LR(1) Successor(C, X)

- Let I = [A → α•Bβ, a] **or** [A → α•bβ, a]
- Successor(I, B)

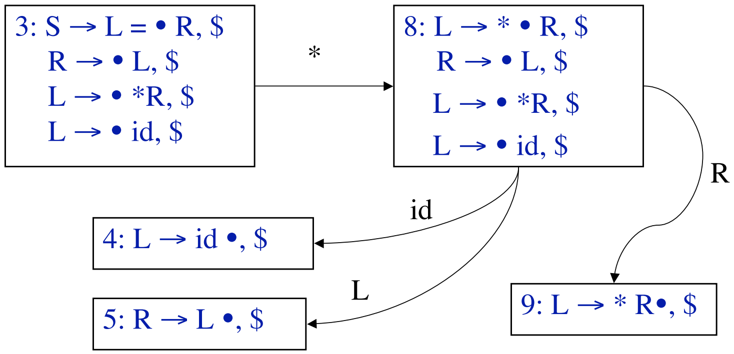
  = closure([A → αB • β, a])
- Successor(I, b)

  = closure([A → αb • β, a])

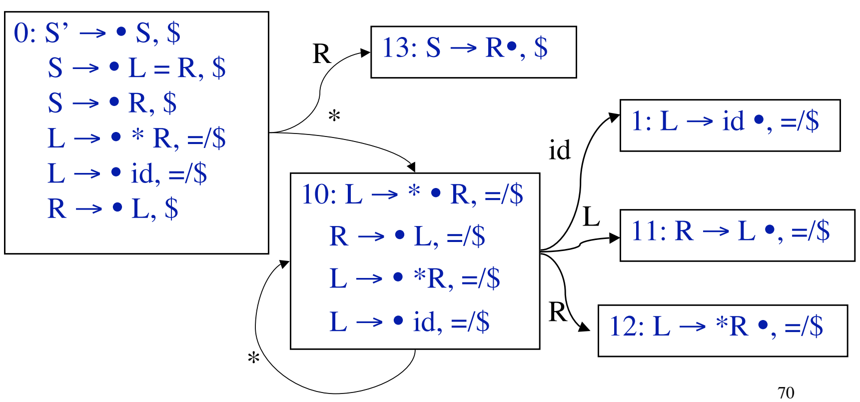
LR(1) Example



LR(1) Example (contd)



LR(1) Example (contd)



Productions	
1	**S → L = R**
2	**S → R**
3	**L → * R**
4	**L → id**
5	**R → L**

	id	=	*	$	S	L	R
0	S1		S10		7	2	13
1		R4		R4			
2		S3		R5			
3	S4		S8			5	6
4				R4			
5				R5			
6				R1			
7				Acc			
8	S4					5	9
9				R3			
10	S1		S10			11	12
11		R5		R5			
12		R3		R3			
13				R2			

LR(1) Construction

1. Construct $F = \{I_0, I_1, \ldots I_n\}$
2. a) if $[A \to \alpha\bullet, a] \in I_i$ and A != S'
   then action[i, a] := reduce $A \to \alpha$
   b) if $[S' \to S\bullet, \$] \in I_i$
   then action[i, \$] := accept
   c) if $[A \to \alpha\bullet a\beta, b] \in I_i$ and Successor(I_i, a)=I_j
   then action[i, a] := shift j
3. if Successor(I_i, A) = I_j then goto[i, A] := j

LR(1) Construction (cont'd)

4. All entries not defined are errors
5. Make sure I_0 is the initial state

- Note: LR(1) only reduces using $A \rightarrow \alpha$ for $[A \rightarrow \alpha\bullet, a]$ if a follows
- LR(1) states remember context by virtue of lookahead
- Possibly many states!
  - LALR(1) combines some states

LR(1) Conditions

- A grammar is LR(1) if for each configuration set holds:
  - For any item $[A \rightarrow \alpha\bullet x\beta, a]$ with $x \in T$ there is no $[B \rightarrow \gamma\bullet, x]$
  - For any two complete items $[A \rightarrow \gamma\bullet, a]$ and $[B \rightarrow \beta\bullet, b]$ it follows a and a != b.
- Grammars:
  - $LR(0) \subset SLR(1) \subset LR(1) \subset LR(k)$
- Languages expressible by grammars:
  - $LR(0) \subset SLR(1) \subset LR(1) = LR(k)$

Canonical LR(1) Recap

- LR(1) uses left context, current handle and lookahead to decide when to reduce or shift
- Most powerful parser so far
- LALR(1) is practical simplification with fewer states

Merging States in LALR(1)

- $S' \rightarrow S$
  $S \rightarrow XX$
  $X \rightarrow aX$
  $X \rightarrow b$
- Same **Core Set**
- Different lookaheads

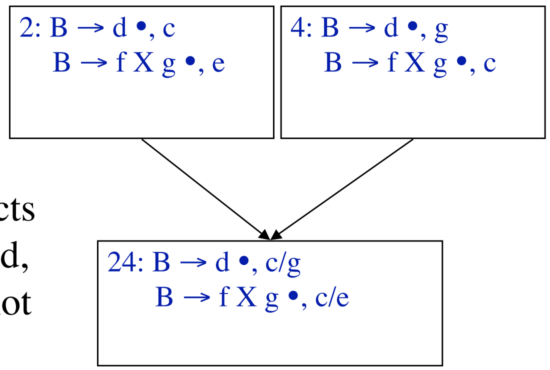
3: $X \rightarrow a \bullet X$, a/b
$X \rightarrow \bullet\, a X$, a/b
$X \rightarrow \bullet\, b$, a/b

6: $X \rightarrow a \bullet X$, \$
$X \rightarrow \bullet\, a X$, \$
$X \rightarrow \bullet\, b$, \$

36: $X \rightarrow a \bullet X$, a/b/\$
$X \rightarrow \bullet\, a X$, a/b/\$
$X \rightarrow \bullet\, b$, a/b/\$

R/R conflicts when merging

- B → d
  B → f X g
  X → …



- If R/R conflicts are introduced, grammar is not LALR(1)!

LALR(1)

- LALR(1) Condition:
  – Merging in this way does not introduce reduce/reduce conflicts
  – Shift/reduce can't be introduced
- Merging brute force or step-by-step
- More compact than canonical LR, like SLR(1)
- More powerful than SLR(1)
  – Not always merge to full Follow Set

S/R & ambiguous grammars

- Lx(k) Grammar vs. Language
  – Grammar is Lx(k) if it can be parsed by Lx(k) method – according to criteria that is specific to the method.
  – A Lx(k) grammar may or may not exist for a language.
- Even if a given grammar is not LR(k), shift/reduce parser can *sometimes* handle them by accounting for ambiguities
  – Example: 'dangling' else
    - Preferring shift to reduce means matching inner 'if'

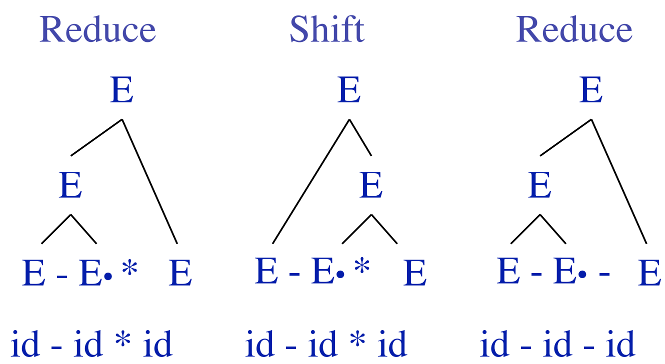
Dangling 'else'

1. S → if E then S
2. S → if E then S else S

- Viable prefix "if E then if E then S"
  – Then read else
- Shift "else" (means go for 2)
- Reduce (reduce using production #1)
- NB: dangling else as written above is ambiguous
  – NB: Ambiguity can be resolved, but there's still no LR(k) grammar

Precedence & Associativity

- Consider $\boxed{E \rightarrow E - E \mid E * E \mid id}$



Precedence Relations

- Let $A \rightarrow w$ be a rule in the grammar
- And b is a terminal
- In some state q of the LR(1) parser there is a shift-reduce conflict:
  - either reduce with $A \rightarrow w$ or shift on b
- Write down a rule, either:

  $A \rightarrow w, < b$ or $A \rightarrow w, > b$

Precedence Relations

- $A \rightarrow w, < b$ means rule has less precedence and so we shift if we see b in the lookahead
- $A \rightarrow w, > b$ means rule has higher precedence and so we reduce if we see b in the lookahead
- If there are multiple terminals with shift-reduce conflicts, then we list them all:

  $A \rightarrow w, > b, < c, > d$

Precedence Relations

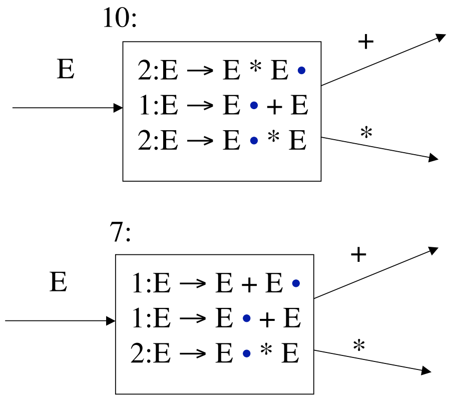
- Consider the grammar

  $E \rightarrow E + E \mid E * E \mid (E) \mid a$
- Assume left-association so that E+E+E is interpreted as (E+E)+E
- Assume multiplication has higher precedence than addition
- Then we can write precedence rules/relns:

  $E \rightarrow E + E, > +, < *$

  $E \rightarrow E * E, > +, > *$

Precedence & Associativity



$E \rightarrow E + E, > +, < *$
$E \rightarrow E * E, > +, > *$

	+	*
7	R1	Shift
10	R2	R2

Handling S/R & R/R Conflicts

- Have a conflict?
  - No? – Done, grammar is compliant.
- Already using most powerful parser available?
  - No? – Upgrade and goto 1
- Can the grammar be rearranged so that the conflict disappears?
  - While preserving the language!

Conflicts revisited (cont'd)

- Can the grammar be rearranged so that the conflict disappears?
  - No?
    - Is the conflict S/R and does shift-to-reduce preference yield desired result?
      - Yes: Done. (Example: dangling else)
    - Else: Bad luck
  - Yes: Is it worth it?
    - Yes, resolve conflict.
    - No: live with default or specified conflict resolution (precedence, associativity)

Compiler (parser) compilers

- Rather than build a parser for a particular grammar (e.g. recursive descent), write down a grammar as a text file
- Run through a compiler compiler which produces a parser for that grammar
- The parser is a program that can be compiled and accepts input strings and produces user-defined output

Compiler (parser) compilers

- For LR parsing, all it needs to do is produce action/goto table
  - Yacc (yet another compiler compiler) was distributed with Unix, the most popular tool. Uses LALR(1).
  - Many variants of yacc exist for many languages
- As we will see later, translation of the parse tree into machine code (or anything else) can also be written down with the grammar
- Handling errors and interaction with the lexical analyzer have to be precisely defined

Parsing CFGs

- Consider the problem of parsing with arbitrary CFGs
- For any input string, the parser has to produce a parse tree
- The simpler problem: print **yes** if the input string is generated by the grammar, print **no** otherwise
- This problem is called *recognition*

CKY Recognition Algorithm

- The Cocke-Kasami-Younger algorithm
- As we shall see it runs in time that is polynomial in the size of the input
- It takes space polynomial in the size of the input
- **Remarkable fact:** it can find all possible parse trees (exponentially many) in polynomial time

Chomsky Normal Form

- Before we can see how CKY works, we need to convert the input CFG into Chomsky Normal Form
- CNF means that the input CFG G is converted to a new CFG G' in which all rules are of the form:

  $A \rightarrow B\ C$

  $A \rightarrow a$

Epsilon Removal

- First step, remove epsilon rules

  $A \rightarrow B\ C$

  $C \rightarrow \varepsilon \mid C\ D \mid a$

  $D \rightarrow b \quad B \rightarrow b$

- After ε-removal:

  $A \rightarrow B \mid B\ C\ D \mid B\ a$

  $C \rightarrow D \mid C\ D\ D \mid a\ D \mid C\ D \mid a$

  $D \rightarrow b \quad B \rightarrow b$

Removal of Chain Rules

- Second step, remove chain rules

  $A \rightarrow B\ C \mid C\ D\ C$

  $C \rightarrow D \mid a$

  $D \rightarrow d \quad B \rightarrow b$

- After removal of chain rules:

  $A \rightarrow B\ a \mid B\ D \mid a\ D\ a \mid a\ D\ D \mid D\ D\ a \mid D\ D\ D$

  $D \rightarrow d \quad B \rightarrow b$

Eliminate terminals from RHS

- Third step, remove terminals from the rhs of rules

  $A \rightarrow B\ a\ C\ d$

- After removal of terminals from the rhs:

  $A \rightarrow B\ N_1\ C\ N_2$

  $N_1 \rightarrow a$

  $N_2 \rightarrow d$

Binarize RHS with Nonterminals

- Fourth step, convert the rhs of each rule to have two non-terminals

  $A \rightarrow B\ N_1\ C\ N_2$

  $N_1 \rightarrow a$

  $N_2 \rightarrow d$

- After converting to binary form:

  $A \rightarrow B\ N_3 \qquad N_1 \rightarrow a$

  $N_3 \rightarrow N_1\ N_4 \qquad N_2 \rightarrow d$

  $N_4 \rightarrow C\ N_2$

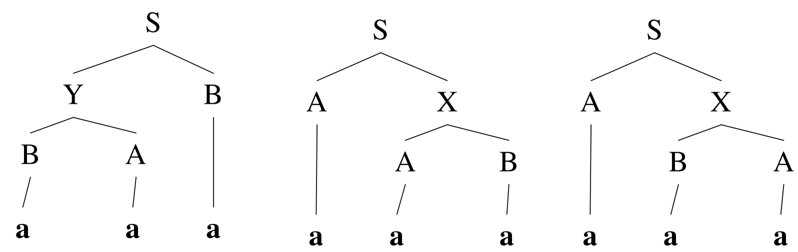
CKY algorithm

- We will consider the working of the algorithm on an example CFG and input string
- Example CFG:
  S → A X | Y B
  X → A B | B A   Y → B A
  A → a   B → a
- Example input string: *aaa*

CKY Algorithm

	0	1	2	3
0		A, B A → a B → a	X, Y X → A B | B A Y → B A	S S → $A_{(0,1)}$ $X_{(1,3)}$ S → $Y_{(0,2)}$ $B_{(2,3)}$
1			A, B A → a B → a	X, Y X → A B | B A Y → B A
2				A, B A → a B → a
		a	a	a

Parse trees



CKY Algorithm

Input string **input** of size *n*
Create a 2D table **chart** of size n^2
for i=0 **to** n-1
    chart[i][i+1] = A **if** there is a rule A → a and **input**[i]=a
for j=2 **to** N
    for i=j-2 **downto** 0
        for k=i+1 **to** j-1
            chart[i][j] = A **if** there is a rule A → B C **and**
              chart[i][k] = B **and chart**[k][j] = C
return *yes* **if chart**[0][n] has the start symbol
else return *no*

CKY algorithm summary

- Parsing arbitrary CFGs
- For the CKY algorithm, the time complexity is $O(|G|^2 n^3)$
- The space requirement is $O(n^2)$
- The CKY algorithm handles arbitrary ambiguous CFGs
- All ambiguous choices are stored in the chart
- For compilers we consider parsing algorithms for CFGs that do not handle ambiguous grammars

GLR – Generalized LR Parsing

- Works for any CFG (just like CKY algorithm)
  - Masaru Tomita [1986]
- If you have shift/reduce conflict, just clone your stack and shift in one clone, reduce in the other clone
  - proceed in lockstep
  - parser that get into error states die
  - merge parsers that lead to identical reductions (graph structured stack)

Parsing - Summary

- Parsing arbitrary CFGs: $O(n^3)$ time complexity
- Top-down vs. bottom-up
- Lookahead: FIRST and FOLLOW sets
- LL(1) – Parsing: $O(n)$ time complexity
  - recursive-descent and table-driven predictive parsing
- LR(k) – Parsing : $O(n)$ time complexity
  - LR(0), SLR(1), LR(1), LALR(1)
- Resolving shift/reduce conflicts
  - using precedence, associativity