CMPT 379
Compilers

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Lexical Analysis

- Also called *scanning*, take input program *string* and convert into tokens
- Example:

```
double f = sqrt(-1);
```

```
T_DOUBLE       ("double")
T_IDENT        ("f")
T_OP           ("=")
T_IDENT        ("sqrt")
T_LPAREN       ("(")
T_OP           ("-")
T_INTCONSTANT  ("1")
T_RPAREN       (")")
T_SEP          (";")
```

Token Attributes

- Some tokens have attributes
  - T_IDENT "sqrt"
  - T_INTCONSTANT 1
- Other tokens do not
  - T_WHILE
- *Token*=T_IDENT, *Lexeme*="sqrt", *Pattern*
- Source code location for error reports

Lexical errors

- What if user omits the space in “doublef”?
  - No lexical error, single token T_IDENT(“doublef”) is produced instead of sequence T_DOUBLE, T_IDENT(“f”)!
- Typically few lexical error types
  - E.g., illegal chars, opened string constants or comments that are not closed

Lexical errors

- Lexical analysis should not disambiguate tokens,
  - e.g. unary op + versus binary op +
  - Use the same token T_PLUS for both
  - It’s the job of the parser to disambiguate based on the context
- Language definition should not permit crazy long distance effects (e.g. Fortran)

DO 5 I = 1,5	T_DO T_INT(5) T_ID(I)
DO 5 I = 1.5	T_ID(DO5I) T_EQ

Ad-hoc Scanners

Implementing Lexers: Loop and switch scanners

- Ad hoc scanners
- Big nested switch/case statements
- Lots of getc()/ungetc() calls
  - Buffering; Sentinels for push-backs; streams
- Can be error-prone, use only if
  - Your language's lexical structure is very simple
  - The tools do not provide what you need for your token definitions
- Changing or adding a keyword is problematic
- Have a look at an actual implementation of an ad-hoc scanner

Implementing Lexers: Loop and switch scanners

- Another problem: how to show that the implementation actually captures all tokens specified by the language definition?
- How can we show correctness
- Key idea: separate the definition of tokens from the implementation
- Problem: we need to reason about patterns and how they can be used to define tokens (recognize strings).

Specification of Patterns using Regular Expressions

Formal Languages: Recap

- Symbols: a, b, c
- Alphabet : finite set of symbols $\Sigma = \{a, b\}$
- String: sequence of symbols bab
- Empty string: ε Define: $\Sigma^{\varepsilon} = \Sigma \cup \{\varepsilon\}$
- Set of all strings: Σ^* cf. *The Library of Babel*, Jorge Luis Borges
- (Formal) Language: a set of strings

  $\{ a^n b^n : n > 0 \}$

Regular Languages

- The set of regular languages: each element is a regular language
- Each regular language is an example of a (formal) language, i.e. a set of strings

  e.g. { a^m b^n : m, n are +ve integers }

Regular Languages

- Defining the set of all regular languages:
  - The empty set and {a} for all a in Σ^{ε} are regular languages
  - If L_1 and L_2 and L are regular languages, then:

    $L_1 \cdot L_2 = \{xy \mid x \in L_1 \text{ and } y \in L_2\}$ (concatenation)

    $L_1 \cup L_2$ (union)

    $L^* = \cup_{i=0}^{\infty} L^i$ (Kleene closure)

    are also regular languages
  - There are no other regular languages

Formal Grammars

- A formal grammar is a concise description of a formal language
- A formal grammar uses a specialized syntax
- For example, a **regular expression** is a concise description of a regular language

  *(a|b)*abb* : is the set of all strings over the alphabet {*a*, *b*} which end in *abb*
- We will use regular expressions (regexps) in order to define tokens in our compiler,
  - e.g. lexemes for string tokens are \" (Σ\")* \"

Regular Expressions: Definition

- Every symbol of $\Sigma \cup \{ \varepsilon \}$ is a regular expression
  - E.g. if $\Sigma = \{a,b\}$ then 'a', 'b' are regexps
- If r_1 and r_2 are regular expressions, then the core operators to combine two regexps are
  - Concatenation: $r_1 r_2$, e.g. 'ab' or 'aba'
  - Alternation: $r_1 | r_2$, e.g. 'a|b'
  - Repetition: r_1*, e.g. 'a*' or 'b*'
- No other core operators are defined
  - But other operators can be defined using the basic operators (as in lex regular expressions) e.g. a+ = aa*

Lex regular expressions

Expression	Matches	Example	Using core operators
c	non-operator character c	`a`	
c	character c literally	`\*`	
"*s*"	string s literally	`"**"`	
.	any character but newline	`a.*b`	
^	beginning of line	`^abc`	used for matching
$	end of line	`abc$`	used for matching
[s]	any one of characters in string s	`[abc]`	(a|b|c)
[^s]	any one character not in string s	`[^a]`	(b|c) where Σ = {a,b,c}
*r**	zero or more strings matching r	`a*`	
r+	one or more strings matching r	`a+`	aa*
r?	zero or one r	`a?`	(a|ε)
r{m,n}	between m and n occurences of r	`a{2,3}`	(aa|aaa)
$r_1 r_2$	an r_1 followed by an r_2	`ab`	
$r_1 \mid r_2$	an r_1 or an r_2	`a|b`	
(r)	same as r	`(a|b)`	
r_1/r_2	r_1 when followed by an r_2	`abc/123`	used for matching

Regular Expressions are Trees

Regular Expressions: Definition

- Note that operators apply recursively and these applications can be ambiguous
  - E.g. is aa|bc equal to a(a|b)c or ((aa)|b)c?
- Avoid such cases of ambiguity - provide explicit arguments for each regexp operator
  - For convenience, for examples on this page, let us use the symbol '·' to denote the operator for concatenation
- Remove ambiguity with an explicit regexp tree

Regular Expressions: Definition

- Remove ambiguity with an explicit regexp tree

  a(a|b)c is written as

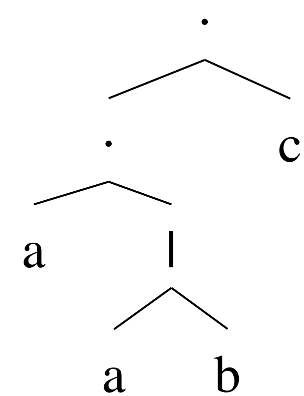
  (·(·a(|ab))c)

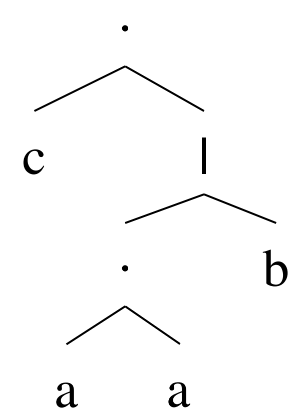
  or in postfix: aab|·c·

  ((aa)|b)c is written as

  (·(|(·aa)b)c)

  or in postfix: aa·b|c·
- Does the order of concatenation matter?





Equivalence of Regexps

- (R|S)|T == R|(S|T) == R|S|T
- (RS)T == R(ST)
- (R|S) == (S|R)
- R*R* == (R*)* == R* == RR*| ε
- R** == R*
- (R|S)T = RT|ST
- R(S|T) == RS | RT
- (R|S)* == (R*S*)* == (R*S)*R* == (R*|S*)*
- RR* == R*R
- (RS)*R == R(SR)*
- R = R|R = Rε

Equivalence of Regexps

- 0(10)*1|(01)*
- (01)(01)*|(01)*
- (01)(01)*|(01)(01)*|ε
- (01)(01)*|ε
- (01)*

- (RS)*R == R(SR)*
- RS == (RS)
- R* == RR*|ε
- R == R|R
- R* == RR*| ε

Implementing Regular Expressions with Finite-state Automata

Regular Expressions

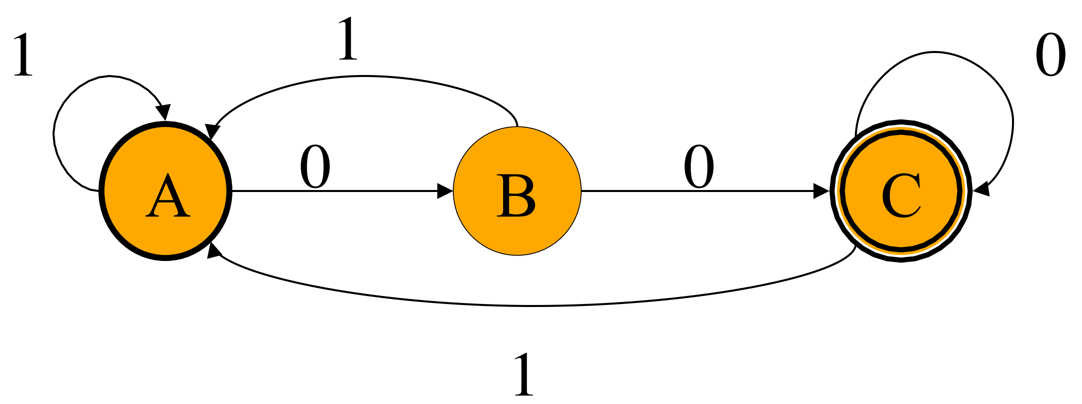
- To describe all lexemes that form a token as a *pattern*
  - (0|1|2|3|4|5|6|7|8|9)+
- Need decision procedure: to which token does a given sequence of characters belong (if any)?
  - Finite State Automata
  - Can be deterministic (DFA) or non-deterministic (NFA)

Deterministic Finite State Automata: DFA

- A set of states S
  - One start state q_0, zero or more final states F
- An alphabet $\sum$ of input symbols
- A transition function:
  - δ: S x $\Sigma \Rightarrow$ S
- Example: $\delta(1, a) = 2$

DFA: Example

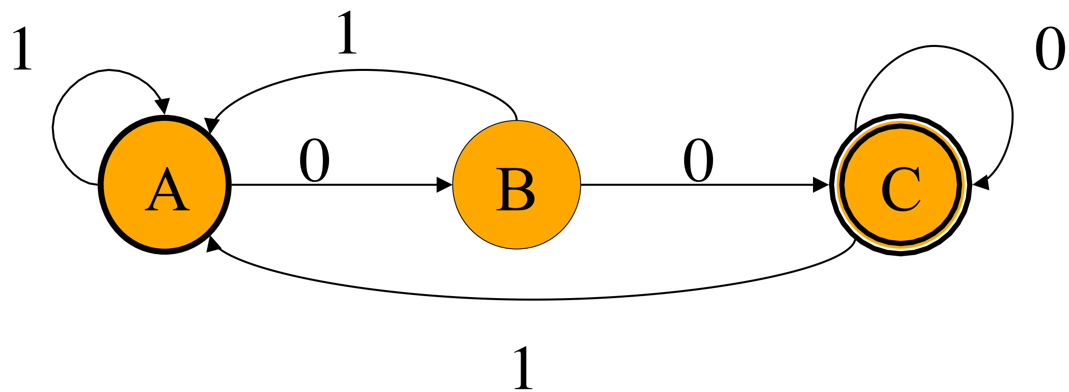
- What regular expression does this automaton accept?



A: start state
C: final state

Answer: (0|1)*00

DFA simulation



Input string: 00100

DFA simulation takes at most n steps for input of length n to return accept or reject

- Start state: A
  1. $\delta(A,0) = B$
  2. $\delta(B,0) = C$
  3. $\delta(C,1) = A$
  4. $\delta(A,0) = B$
  5. $\delta(B,0) = C$
- no more input and C is final state: **accept**

Building a Lexical Analyzer

- Token $\Rightarrow$ Pattern
- Pattern $\Rightarrow$ Regular Expression
- Regular Expression $\Rightarrow$ NFA
- NFA $\Rightarrow$ DFA
- DFAs or NFAs for all the tokens $\Rightarrow$ **Lexical Analyzer**
- Two basic rules to deal with multiple matching: **greedy match** + **regexp ordering**

Note that **greedy** means *longest leftmost match*

Lexical Analysis using `Lex`

```
%{
#include <stdio.h>
#define NUMBER     256
#define IDENTIFIER 257
%}

/* regexp definitions */
num [0-9]+

%%

{num}           { return NUMBER; }
[a-zA-Z0-9]+    { return IDENTIFIER; }

%%

int
main () {
  int token;
  while ((token = yylex())) {
    switch (token) {
      case NUMBER: printf("NUMBER: %s, LENGTH:%d\n", yytext, yyleng); break;
      case IDENTIFIER: printf("IDENTIFIER: %s, LENGTH:%d\n", yytext, yyleng); break;
      default: printf("Error: %s not recognized\n", yytext);
    }
  }
}
```

simpletok.lex

Converting Regular Expressions into Non-deterministic Automata

NFAs

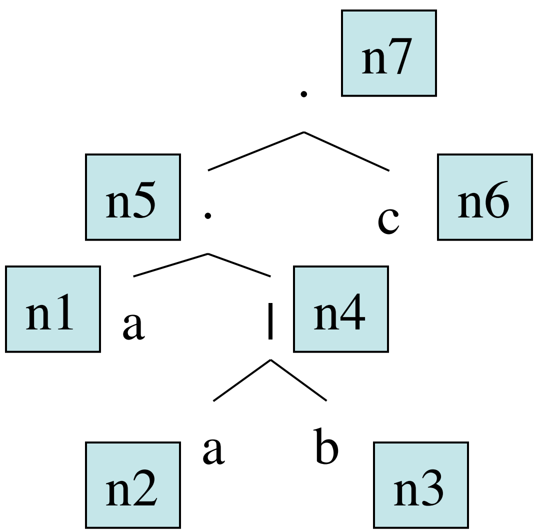
- NFA: like a DFA, except
  - A transition can lead to more than one state, that is, δ: S x $\Sigma \Rightarrow 2^S$
  - One state is chosen non-deterministically
  - Transitions can be labeled with ε, meaning states can be reached without reading any input, that is,

$$\delta: S \times \Sigma \cup \{ \varepsilon \} \Rightarrow 2^S$$

Thompson's construction

Converts regexps to NFA

Build NFA recursively from regexp tree	Build NFA with left-to-right parse of postfix string using a stack



Input = aab|·c·

- read a, push n1 = nfa(a)
- read a, push n2 = nfa(a)
- read b, push n3 = nfa(b)
- read |, n3=pop(); n2=pop(); push

n4 = nfa(or, n2, n3)

- read ·, n4 = pop(); n1 = pop(); push

n5 = nfa(cat, n1 , n4)

- read c, push n6 = nfa(c)
- read ·, n6 = pop(); n5 = pop(); push

n7 = nfa(cat, n5 , n6)

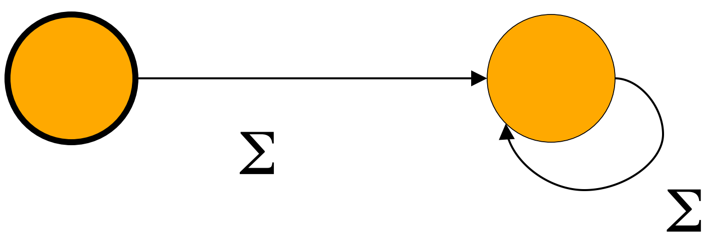
Thompson's construction

- Converts regexps to NFA
- Six simple rules
  - Empty language
  - Symbols
  - Empty String
  - Alternation (r_1 or r_2)
  - Concatenation (r_1 followed by r_2)
  - Repetition (r_1*)

Used by Ken Thompson for pattern-based search in text editor QED (1968)
To keep things simple our version is more verbose

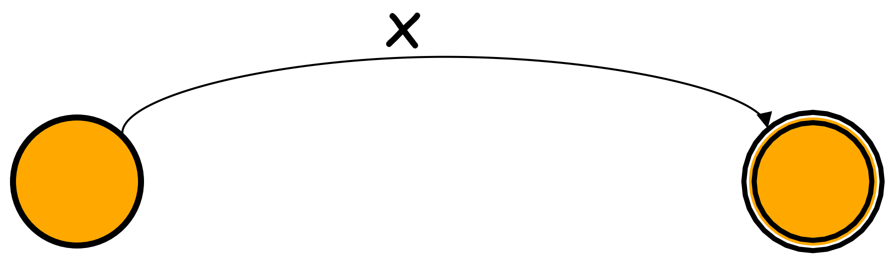
Thompson Rule 0

- For the empty language ϕ (optionally include a *sinkhole* state)



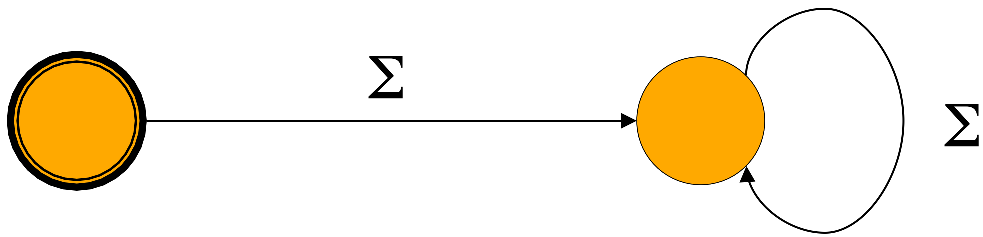
Thompson Rule 1

- For each symbol *x* of the alphabet, there is a NFA that accepts it (include a *sinkhole* state)



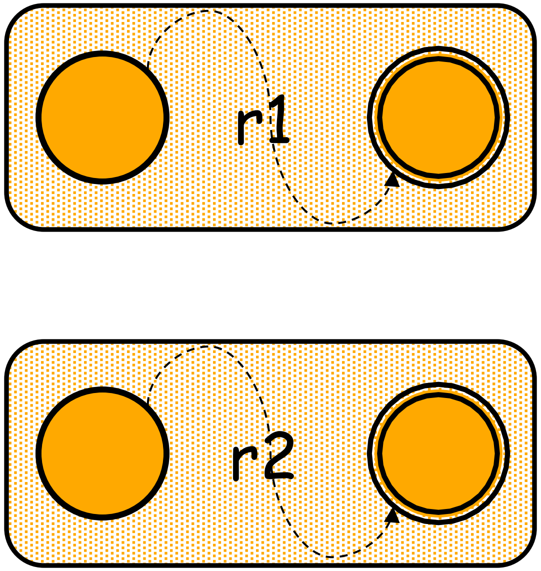
Thompson Rule 2

- There is an NFA that accepts only ε



Thompson Rule 3

- Given two NFAs for r_1, r_2, there is a NFA that accepts $r_1|r_2$



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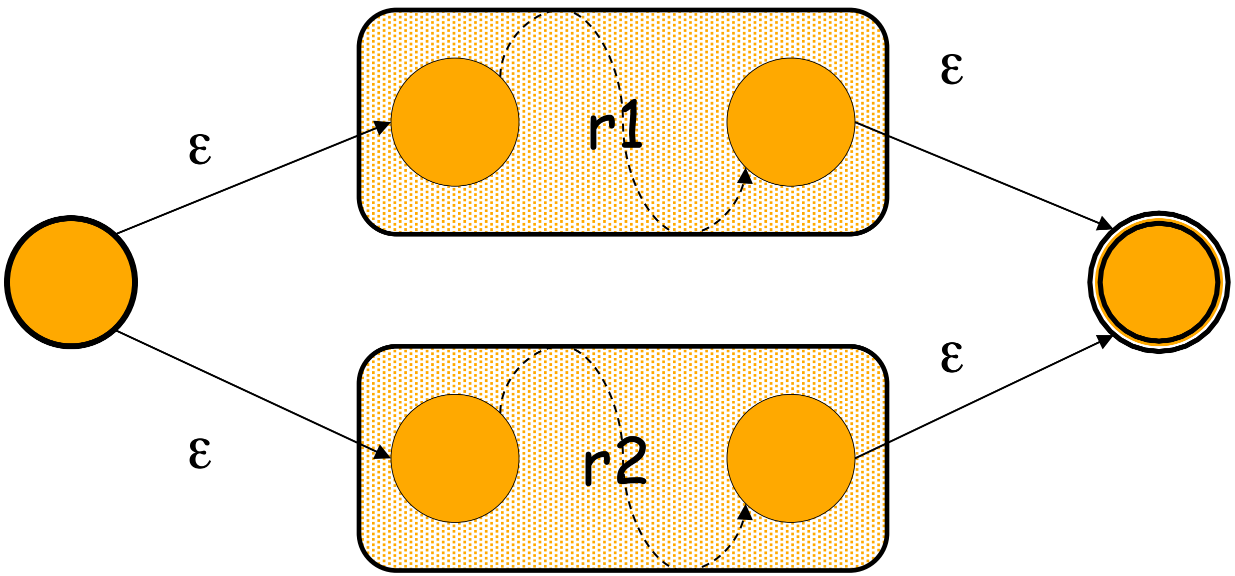
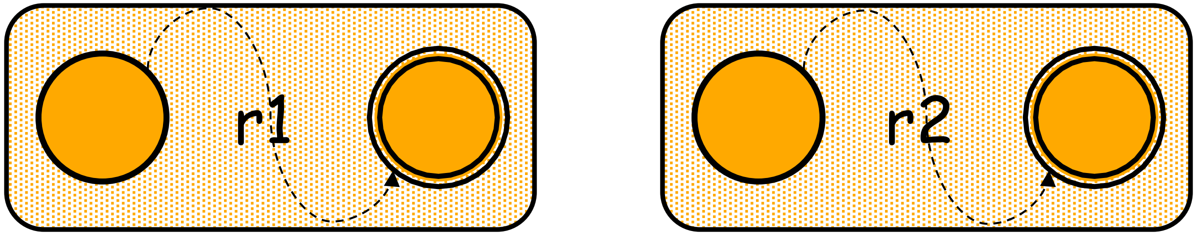


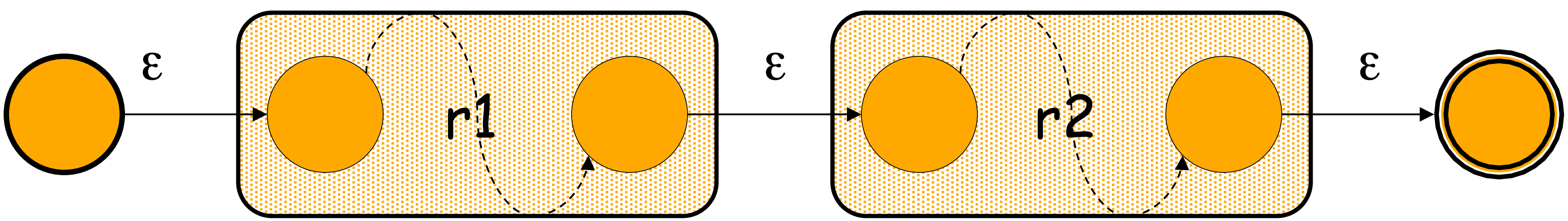
Thompson Rule 4

- Given two NFAs for r_1, r_2, there is a NFA that accepts $r_1 r_2$



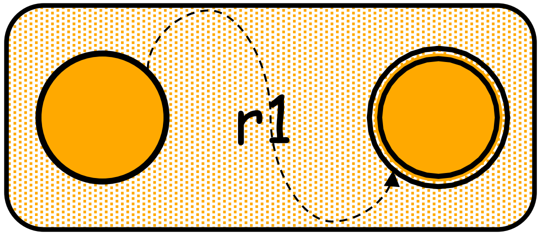
Thompson Rule 4

- Given two NFAs for r_1, r_2, there is a NFA that accepts r_1r_2



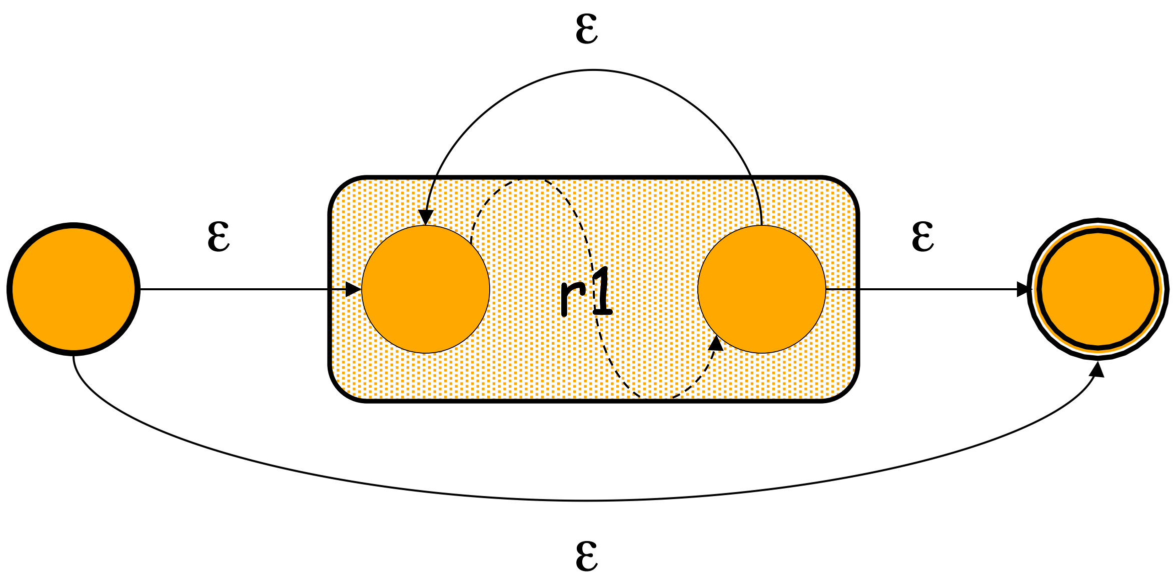
Thompson Rule 5

- Given a NFA for r_1, there is an NFA that accepts r_1*



Thompson Rule 5

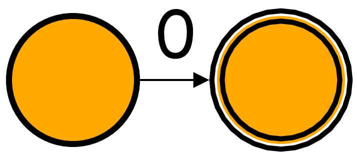
- Given a NFA for r_1, there is an NFA that accepts r_1*

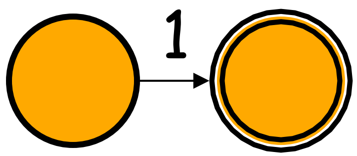


Example

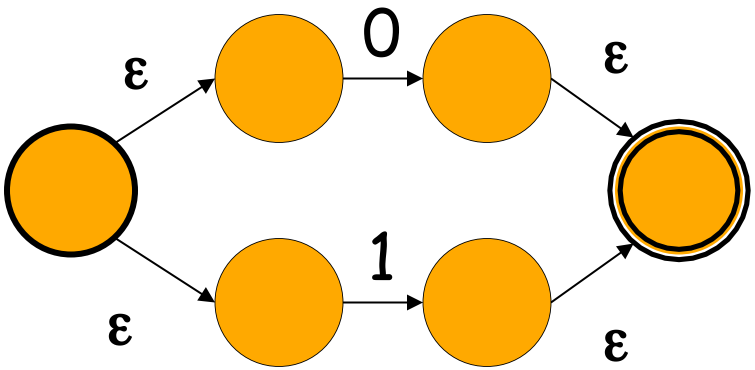
- Set of all binary strings that are divisible by four (include 0 in this set)
- Defined by the regexp: ((0|1)*00) | 0
- Apply Thompson's Rules to create an NFA

Basic Blocks 0 and 1

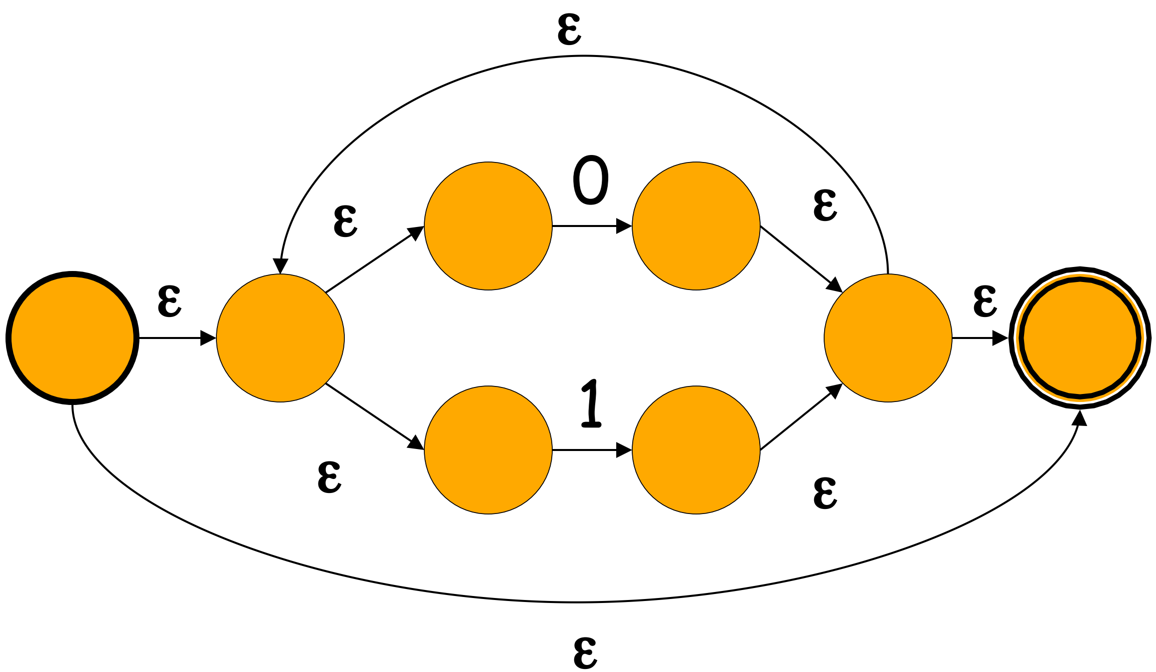
- 0 

- 1 

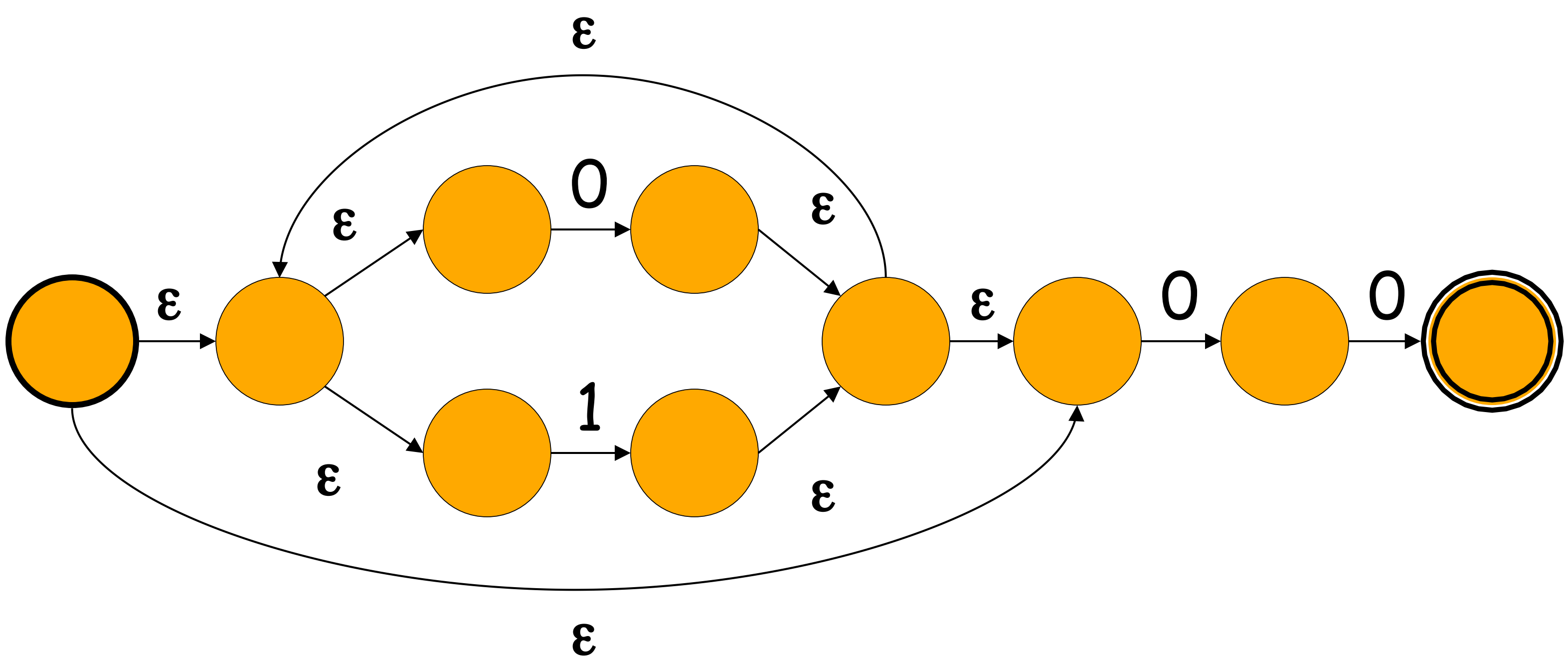
(this version does not report errors: no *sinkholes*)



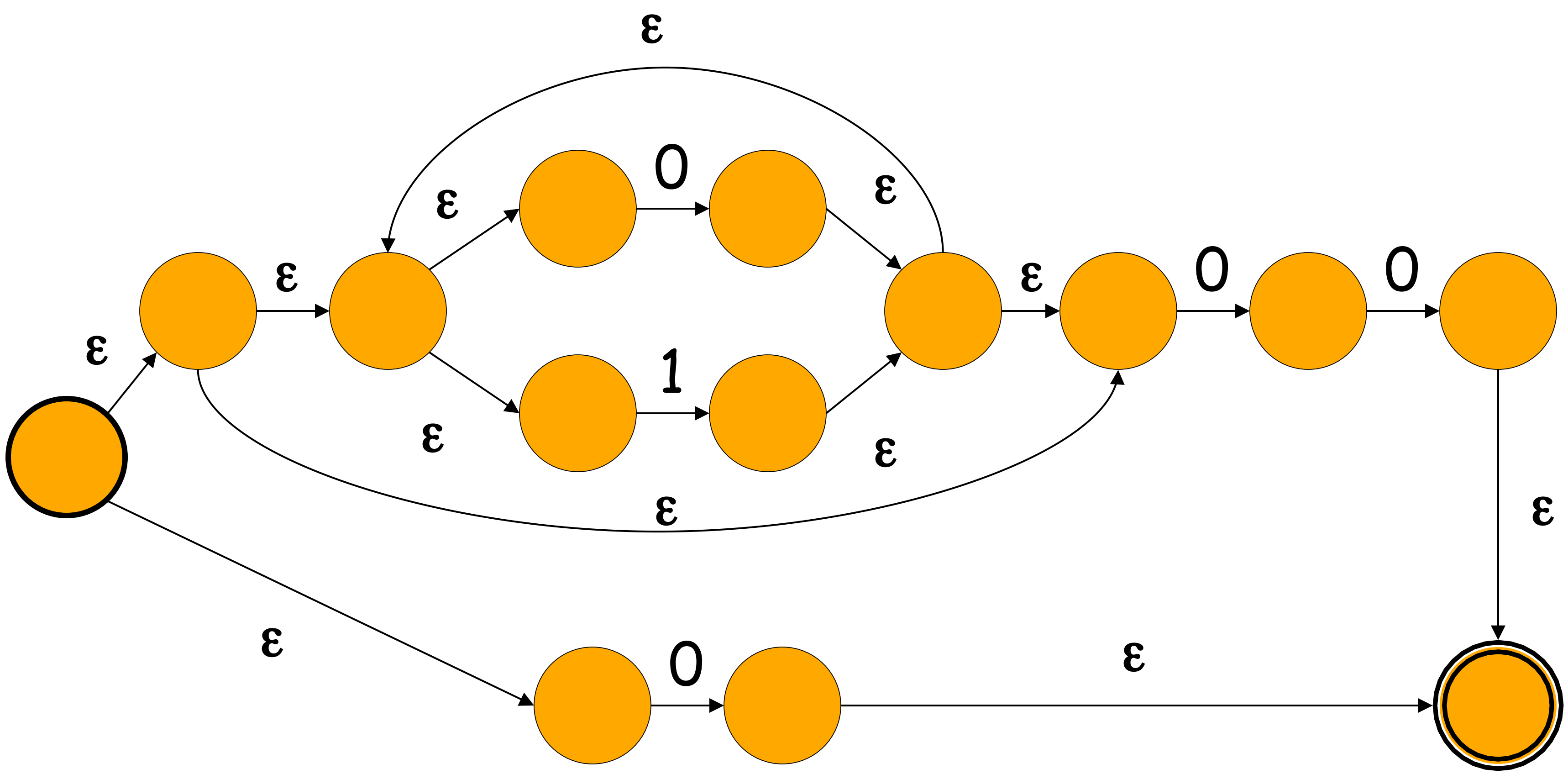
0|1



(0|1)*



(0|1)*00



((0|1)*00)|0

Matching Patterns using Non-deterministic Automata (conversion from NFA to DFA)

Simulating NFAs

- Simulation == Given a NFA and input string, does the string match the pattern?
- Similar to DFA simulation
- But have to deal with ε transitions and multiple transitions on the same input
- Instead of one state, we have to consider *sets* of states

NFA to DFA Conversion

- Simulation implicitly converts NFA -> DFA
- Subset construction
- Idea: subsets of set of all NFA states are *equivalent* and become one DFA state
- Algorithm simulates movement through NFA
- Key problem: how to treat ε-transitions?

ε-Closure

- Start state: q_0
- ε-closure(S): S is a set of states

$$\textbf{initialize: } S \leftarrow \{q_0\}$$
$$T \leftarrow S$$
$$\textbf{repeat } T' \leftarrow T$$
$$\quad T \leftarrow T' \cup [\cup_{s \in T'} \textbf{move}(s, \epsilon)]$$
$$\textbf{until } T = T'$$

ε-Closure (T: set of states)

```
push all states in T onto stack
 initialize ε-closure(T) to T
 while stack is not empty do begin
       pop t off stack
       for each state u with u ∈ move(t, ε) do
           if u ∉ ε-closure(T) do begin
               add u to ε-closure(T)
               push u onto stack
           end
 end
```

NFA Simulation

- After computing the ε-*closure* move, we get a set of states
- On some input extend all these states to get a new set of states

$$\mathbf{DFAedge}(T, c) = \epsilon\textbf{-closure}\left(\cup_{q \in T} \mathbf{move}(q, c)\right)$$

NFA Simulation

- $\mathbf{DFAedge}(T, c) = \epsilon\textbf{-closure}\left(\cup_{q \in T} \mathbf{move}(q, c)\right)$
- Start state: q_0
- Input: $c_1, \ldots, c_k$

$$T \leftarrow \epsilon\textbf{-closure}(\{q_0\})$$

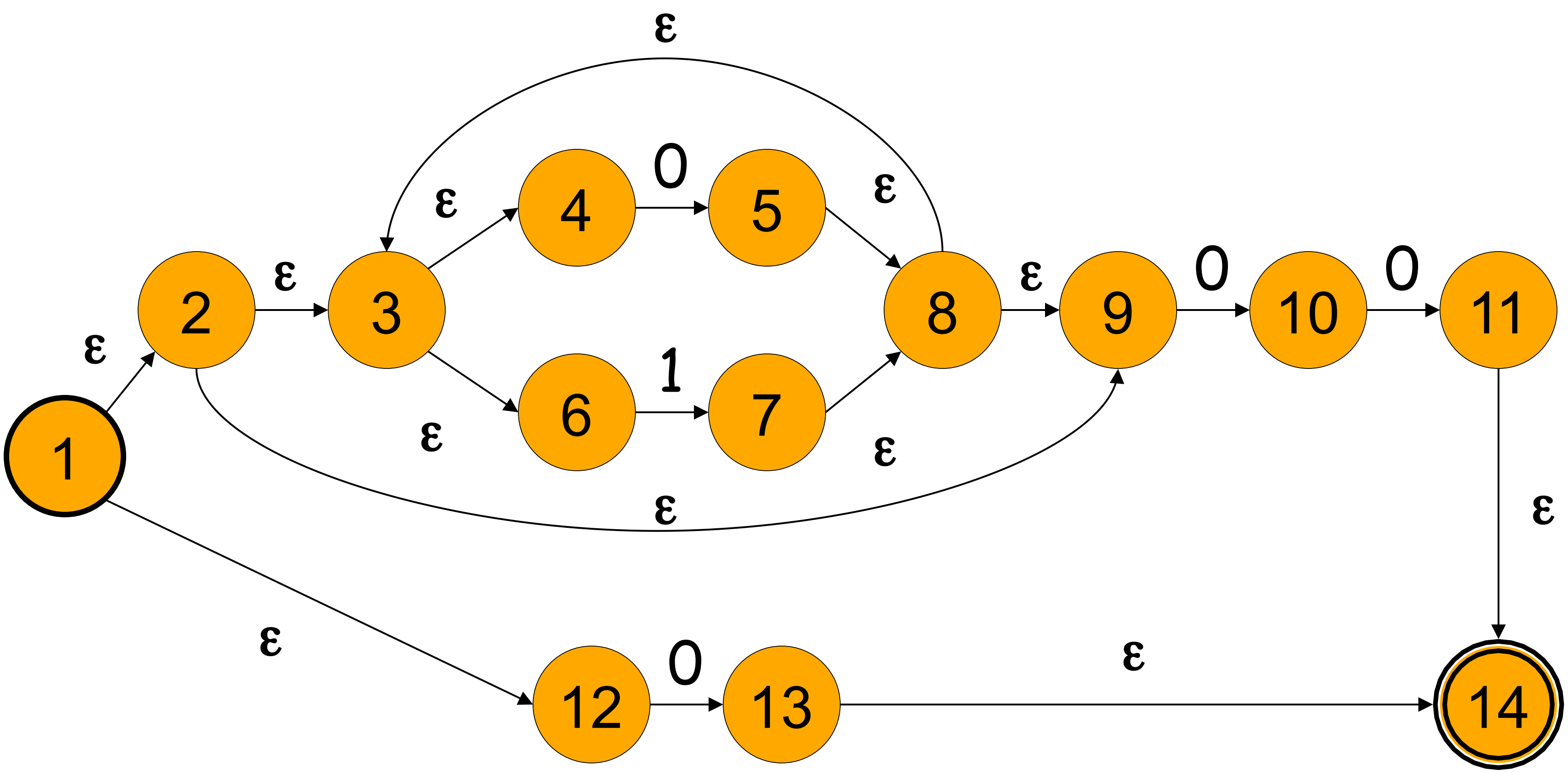
$$\textbf{for } i \leftarrow 1 \textbf{ to } k$$

$$\quad T \leftarrow \mathbf{DFAedge}(T, c_i)$$

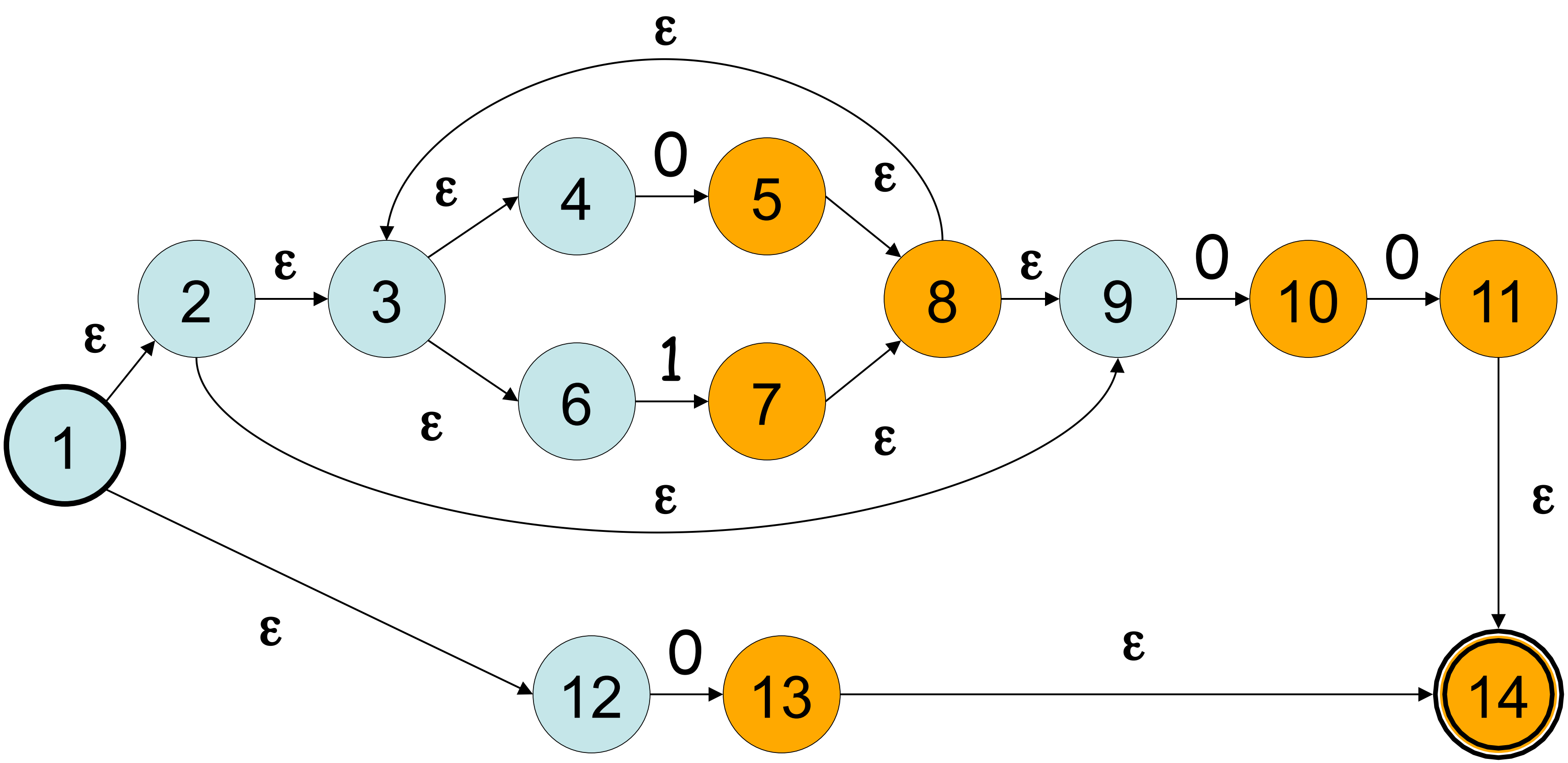
Conversion from NFA to DFA

- Conversion method closely follows the NFA simulation algorithm
- Instead of simulating, we can collect those NFA states that behave identically on the same input
- Group this set of states to form one state in the DFA

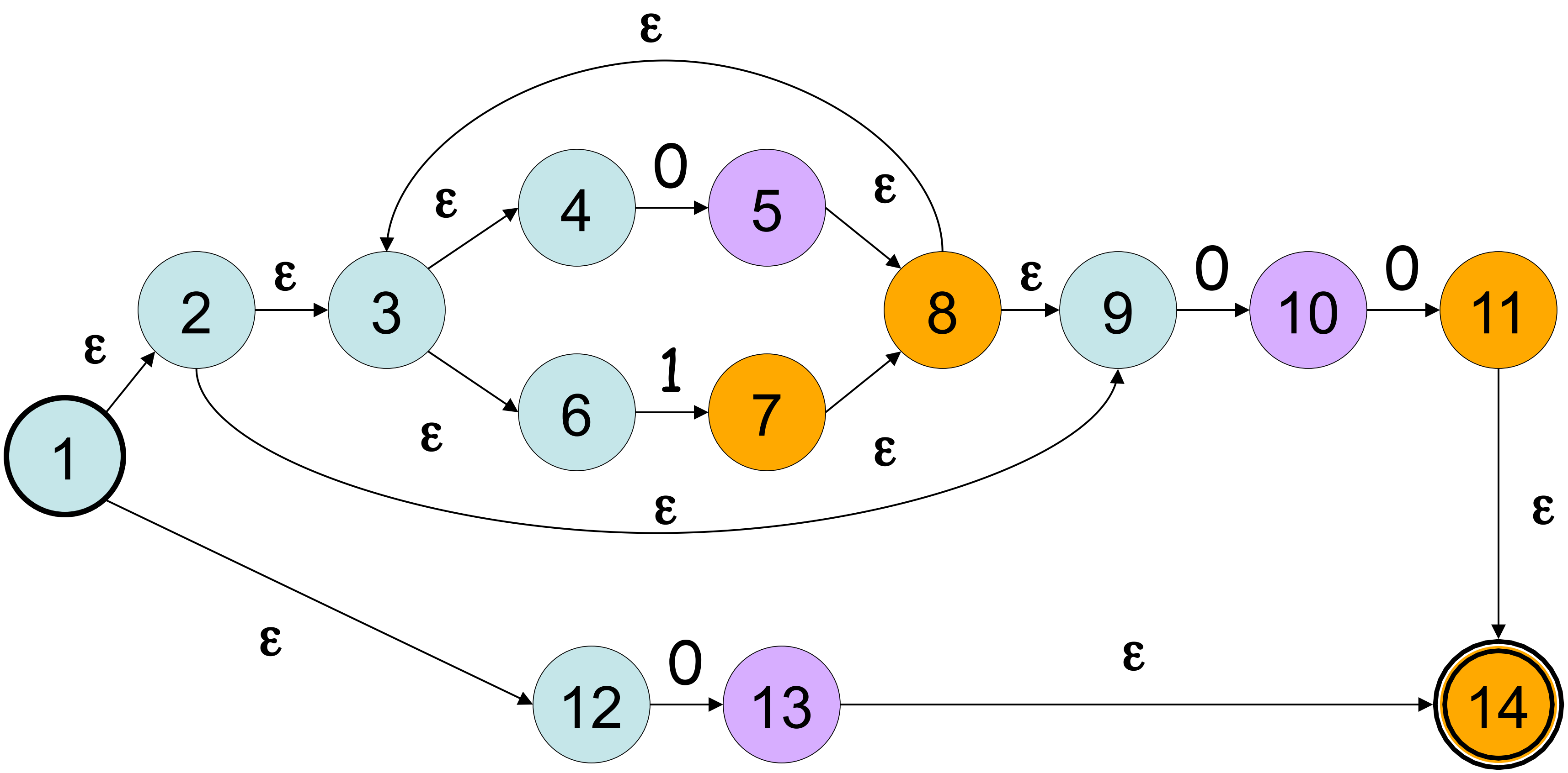
Example: subset construction



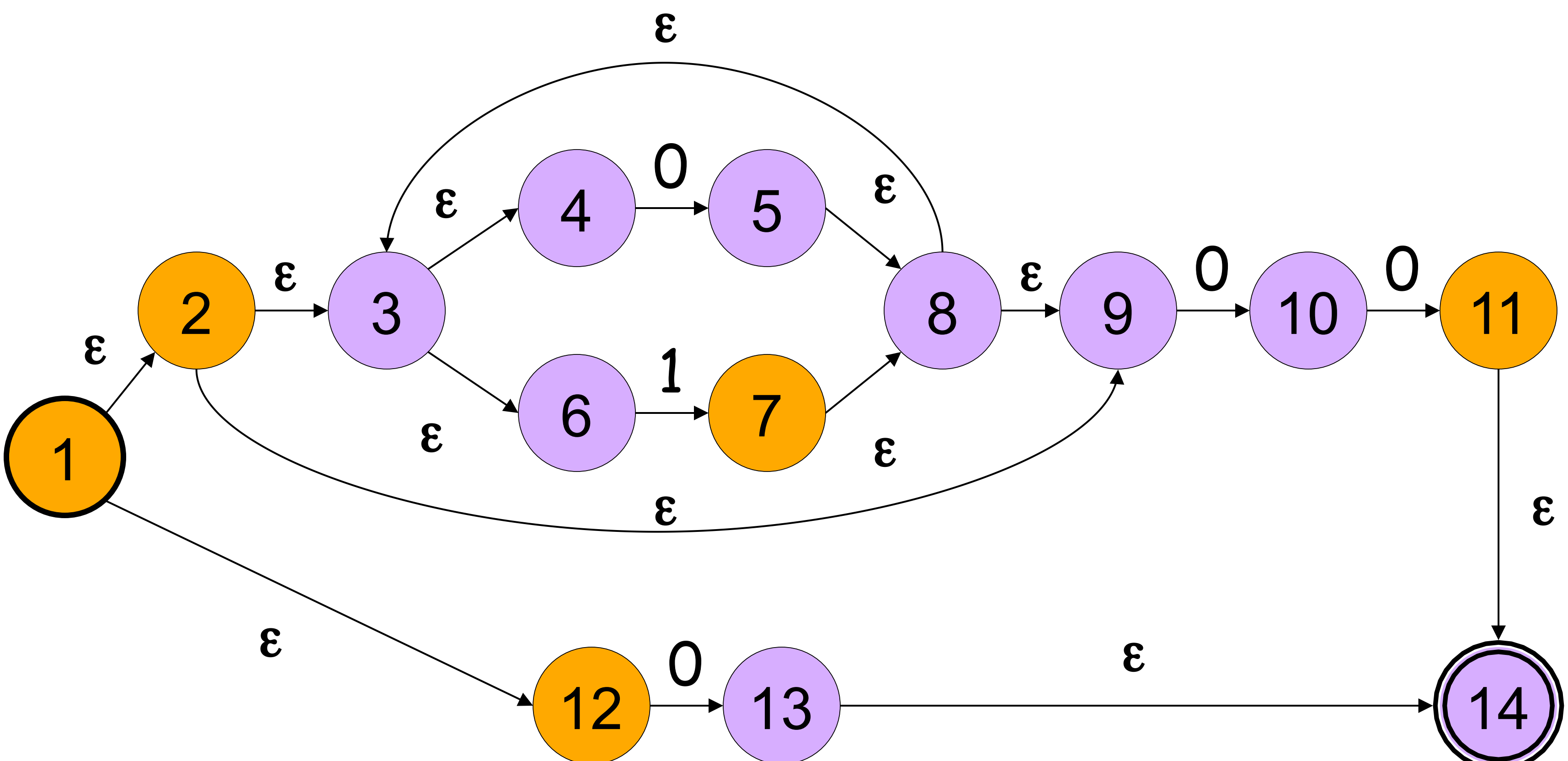
ε-*closure*(q_0)



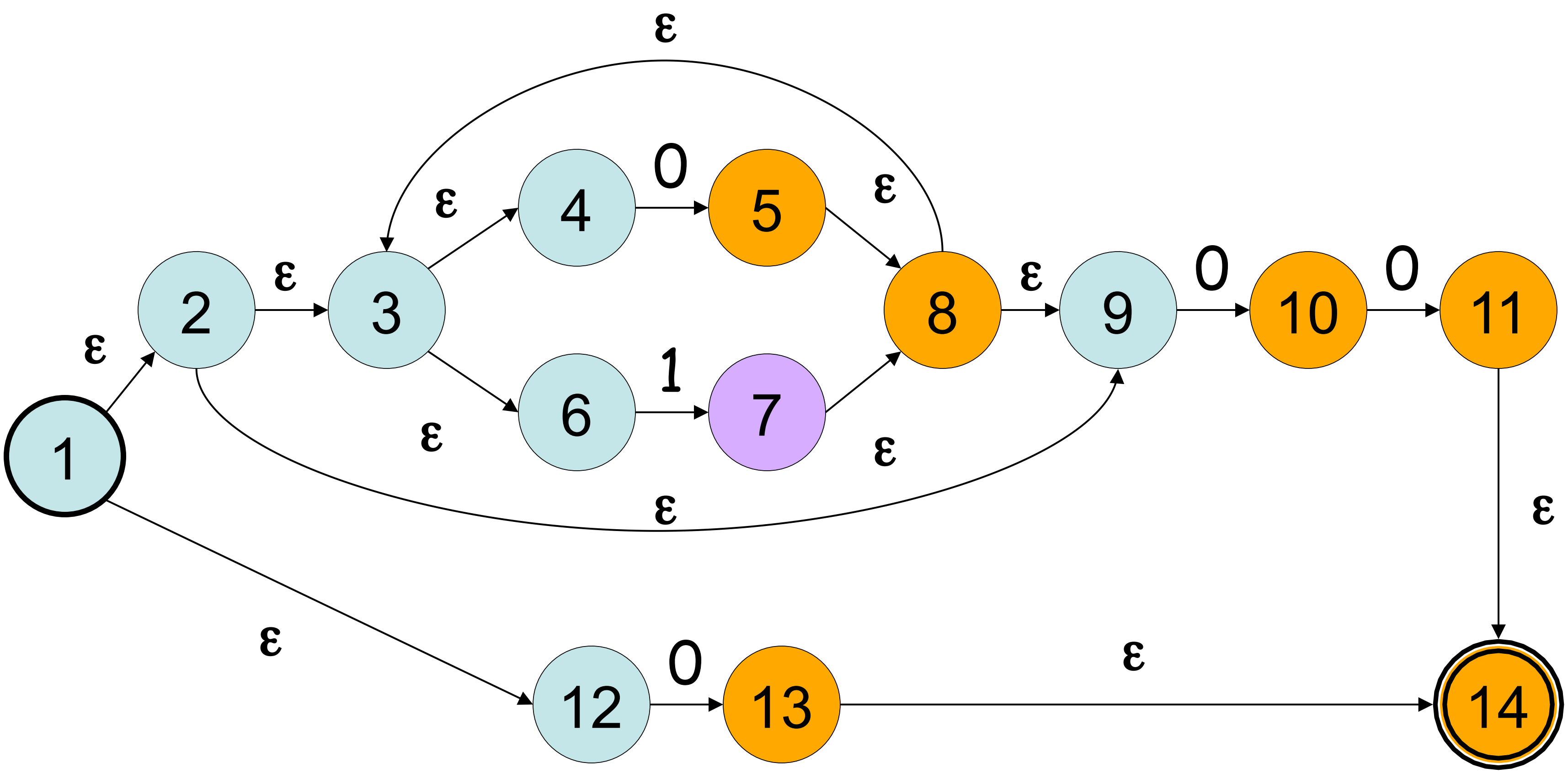
move(ε-*closure*(q_0), 0)



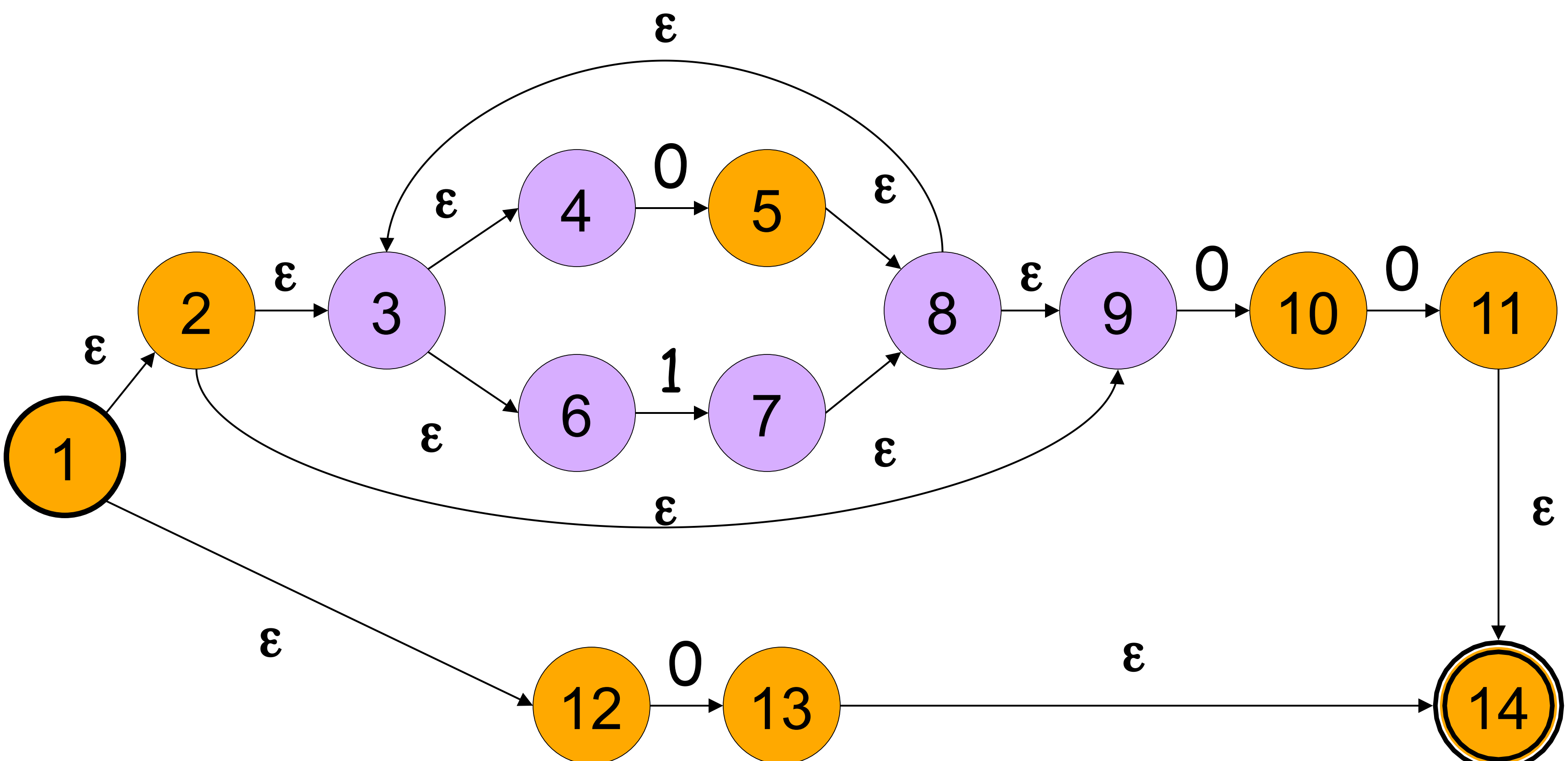
ε-*closure*(move(ε-*closure*(q_0), 0))



move(ε-*closure*(q_0), 1)



ε-*closure*(move(ε-*closure*(q_0), 1))



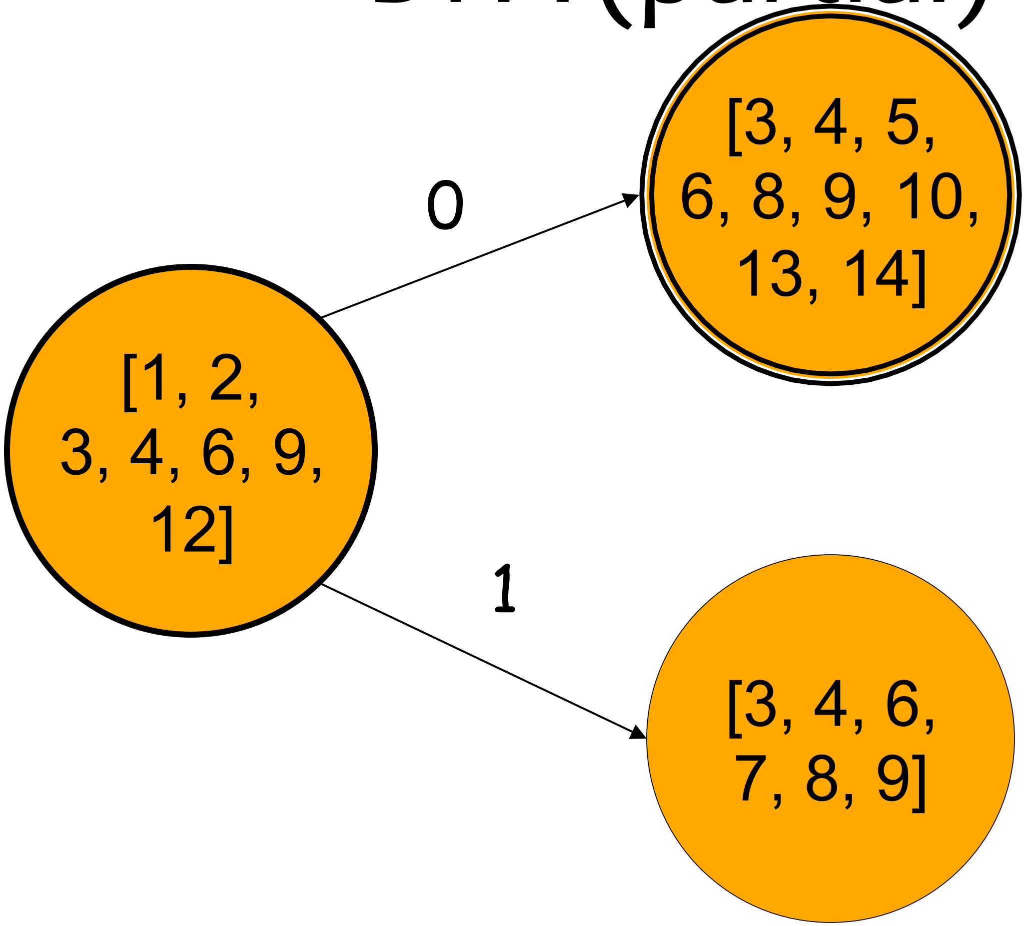
Subset Construction

```
add ε-closure(q0) to Dstates unmarked
while ∃ unmarked T ∈ Dstates do begin
    mark T;
    for each symbol c do begin
        U := ε-closure(move(T, c));
        if U ∉ Dstates then
            add U to Dstates unmarked
        Dtrans[d, c] := U;
    end
end
```

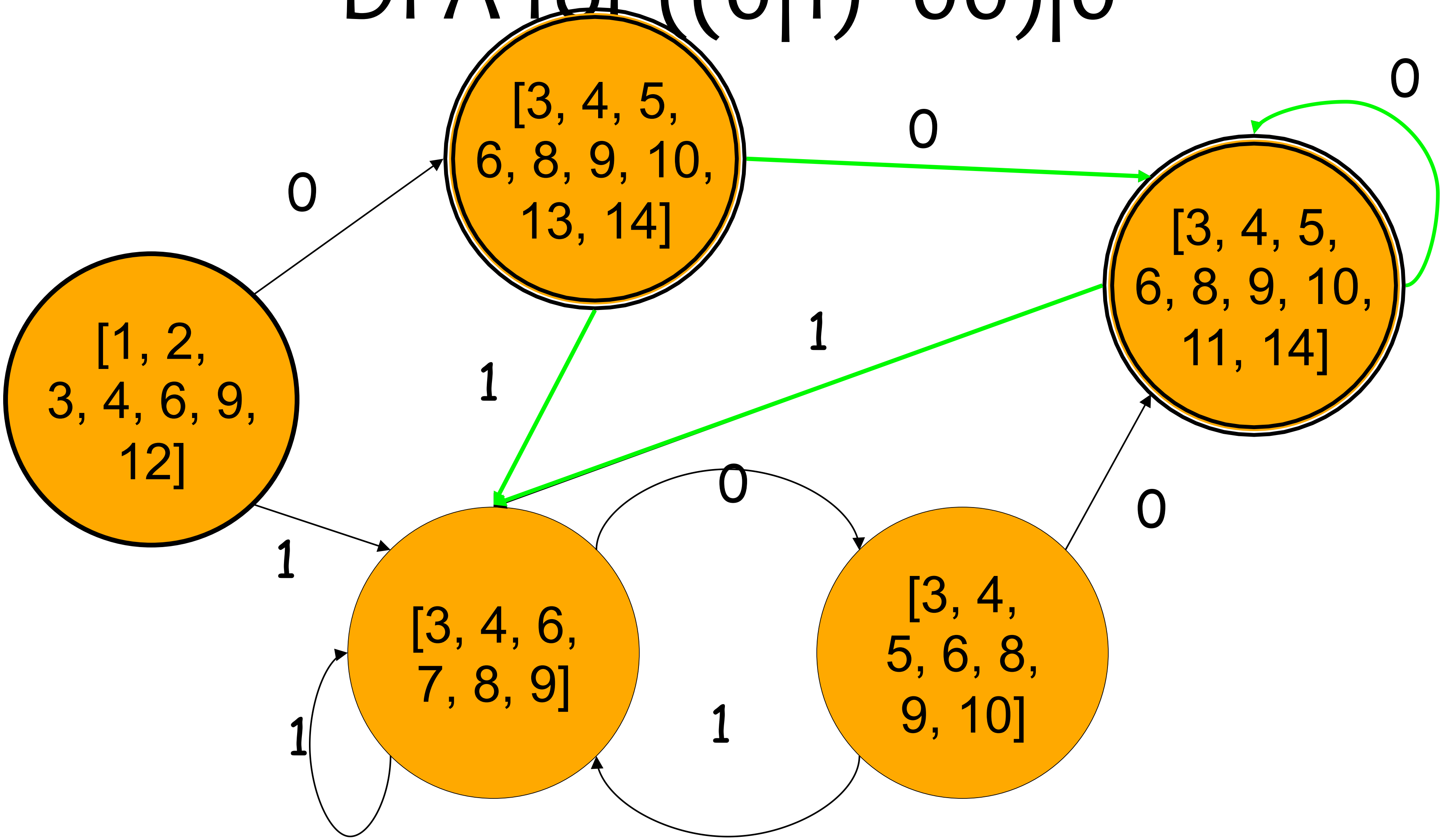
Subset Construction

```
states[0] = ε-closure({q0})
p = j = 0
while j ≤ p do begin
        for each symbol c do begin
                e = DFAedge(states[j], c)
                if e = states[i] for some i ≤ p
                then    Dtrans[j, c] = i
                else    p = p+1
                        states[p] = e
                        Dtrans[j, c] = p
        j = j + 1
        end
end
```

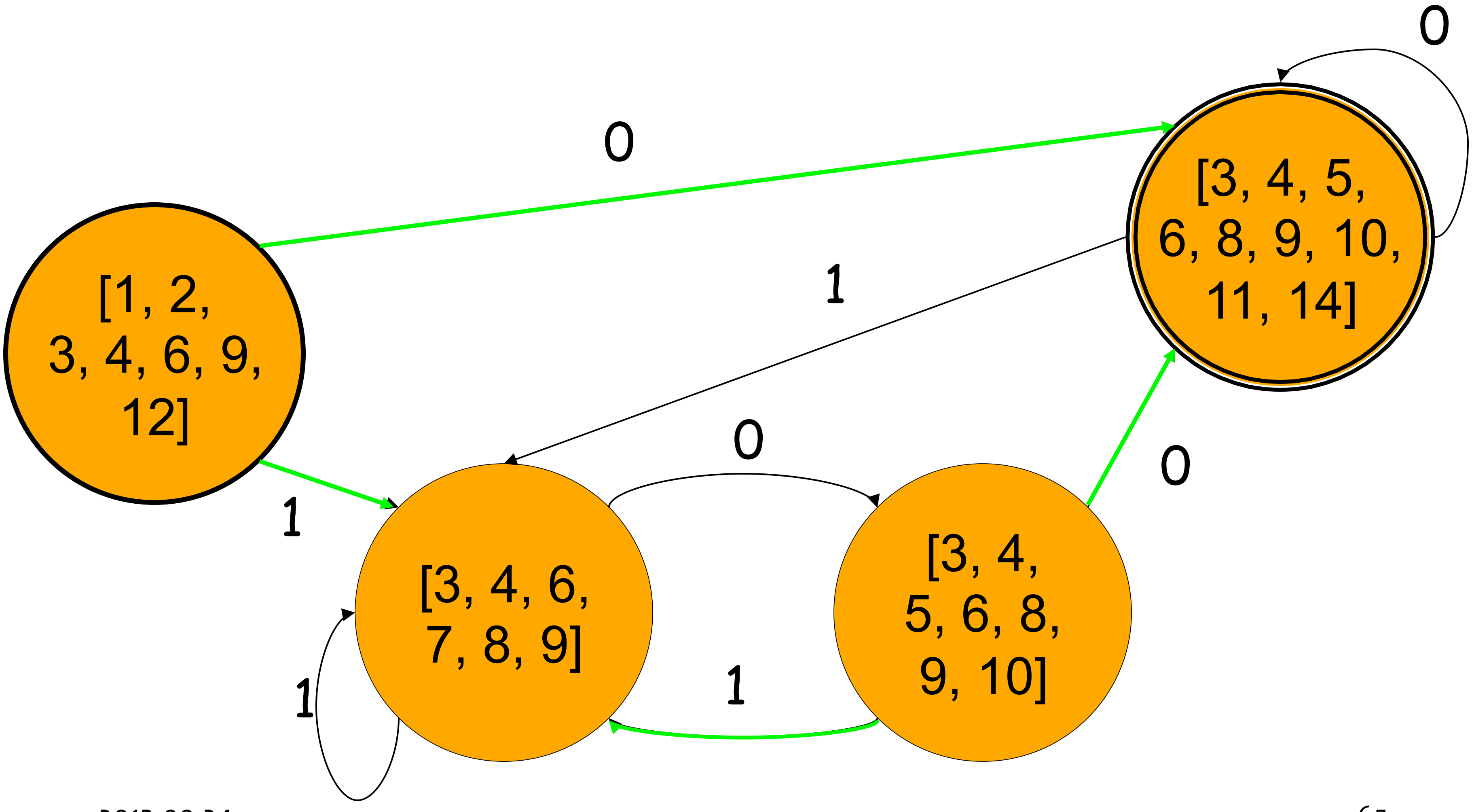
DFA (partial)



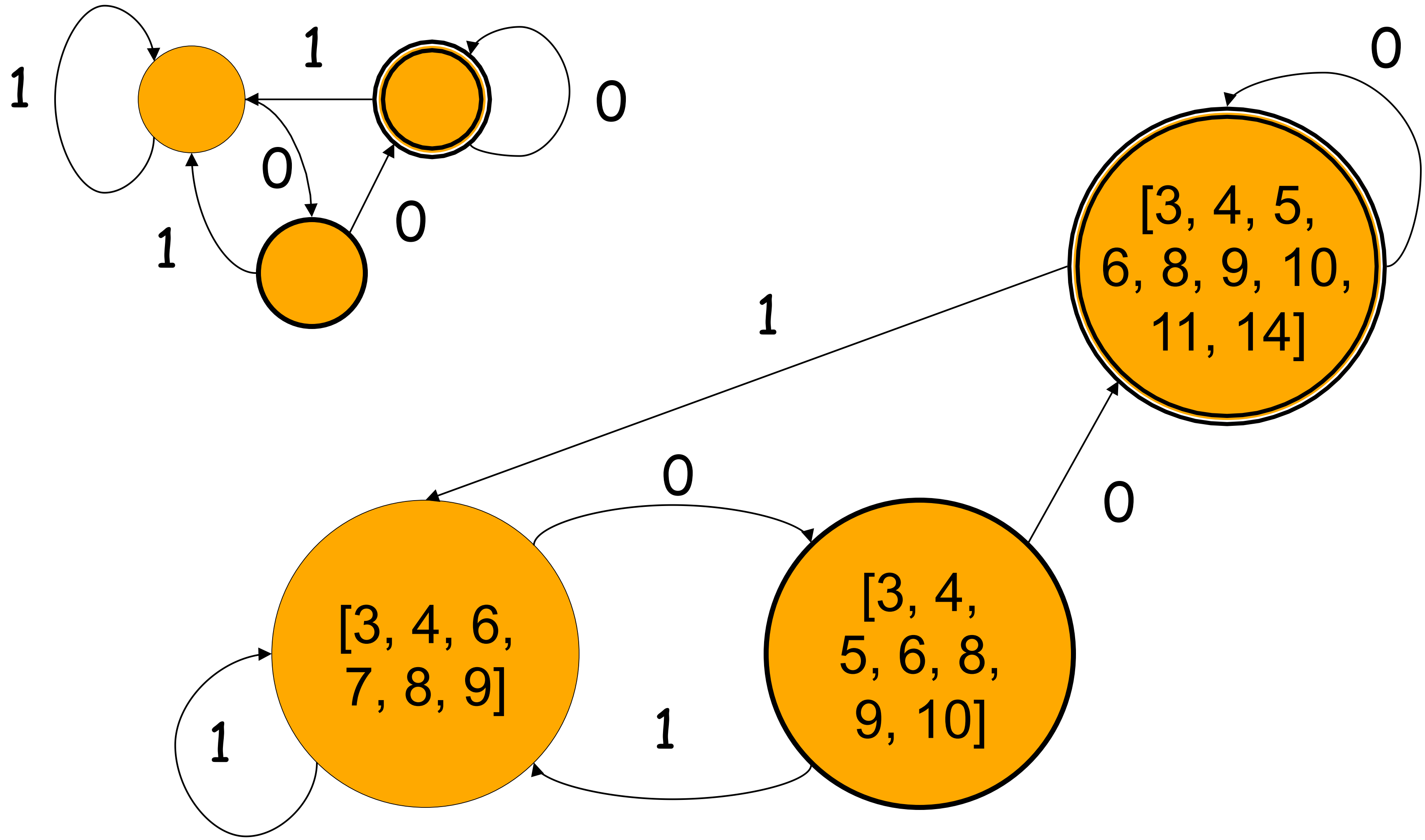
DFA for ((0|1)*00)|0



Minimization of DFAs



Minimization of DFAs



NFA to DFA Complexity Analysis

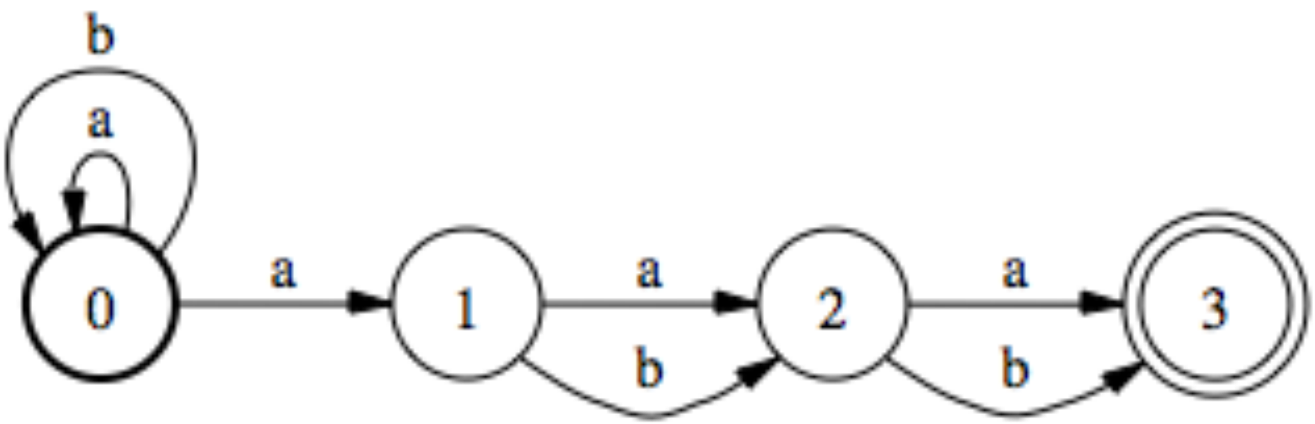
NFA to DFA

- Subset construction converts NFA to DFA
- Complexity:
  - For FSAs, we measure complexity in terms of initial cost (creating the automaton) and per string cost
  - Let *r* be the length of the regexp and *n* be the length of the input string
  - NFA, Initial cost: $O(r)$; Per string: $O(rn)$
  - DFA, Initial cost: $O(r^2s)$; Per string: $O(n)$
  - DFA, common case, $s = r$, but worst case $s = 2^r$

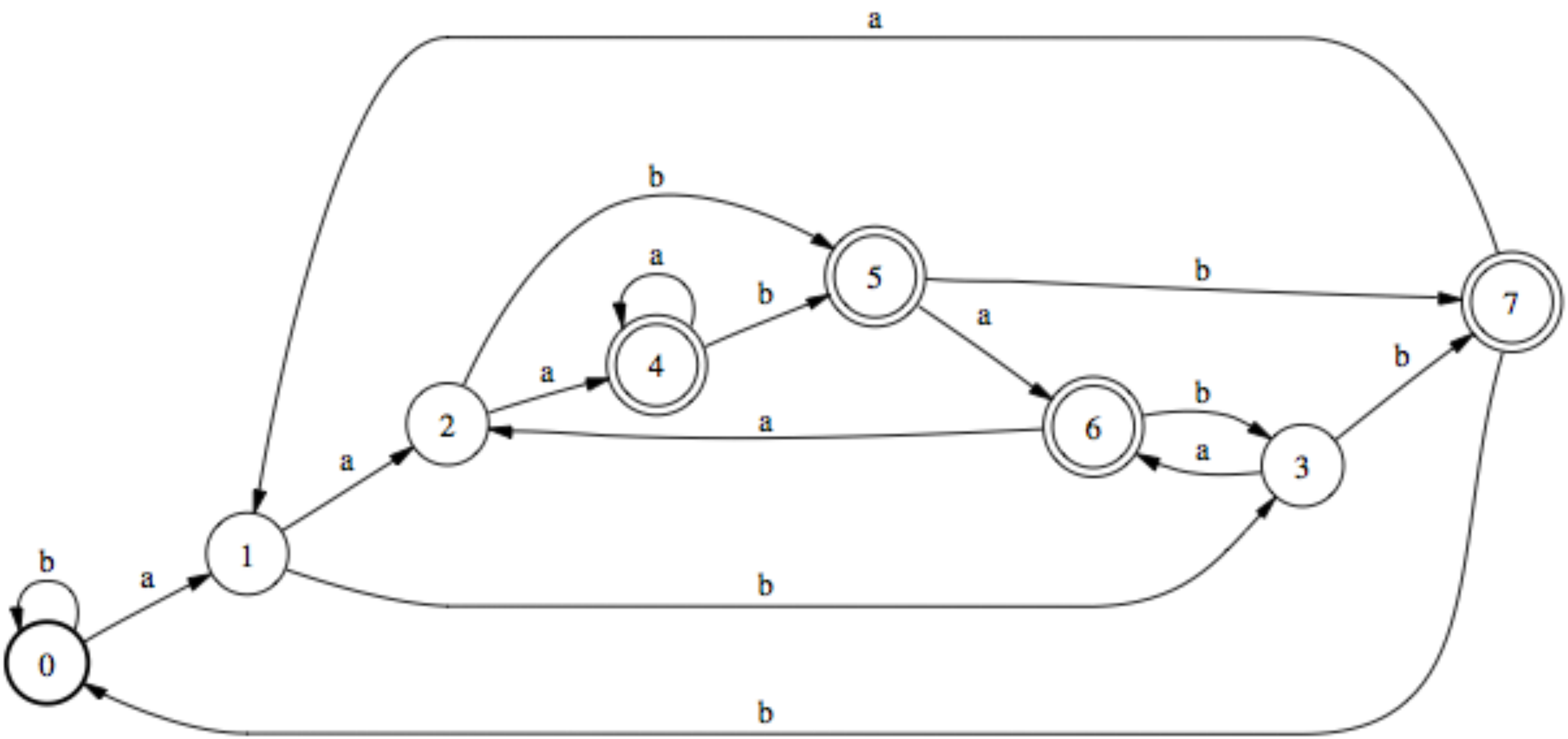
NFA to DFA

- A regexp of size *r* can become a 2^r state DFA, an exponential increase in complexity
  - Try the subset construction on NFA built for the regexp $\mathbf{A^*aA^{n-1}}$ where **A** is the regexp **(a|b)**
- Note that the NFA for regexp of size *r* will have *r* states
- Minimization can reduce the number of states
- But minimization requires determinization

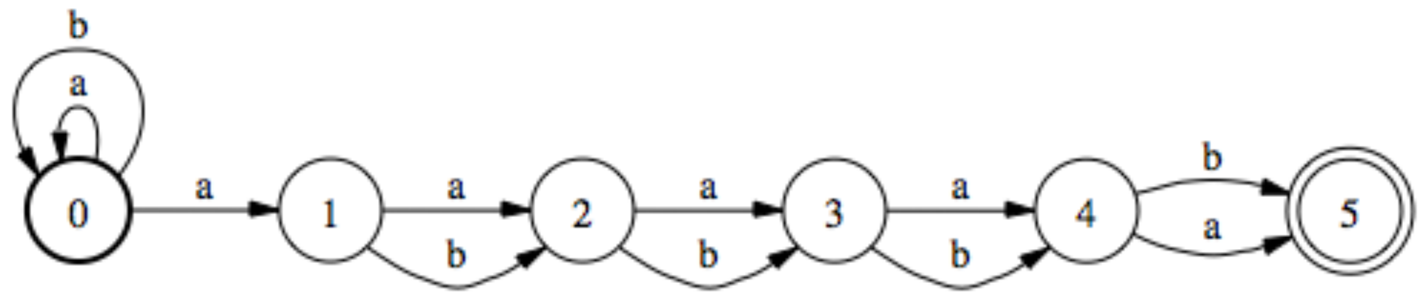
NFA to DFA

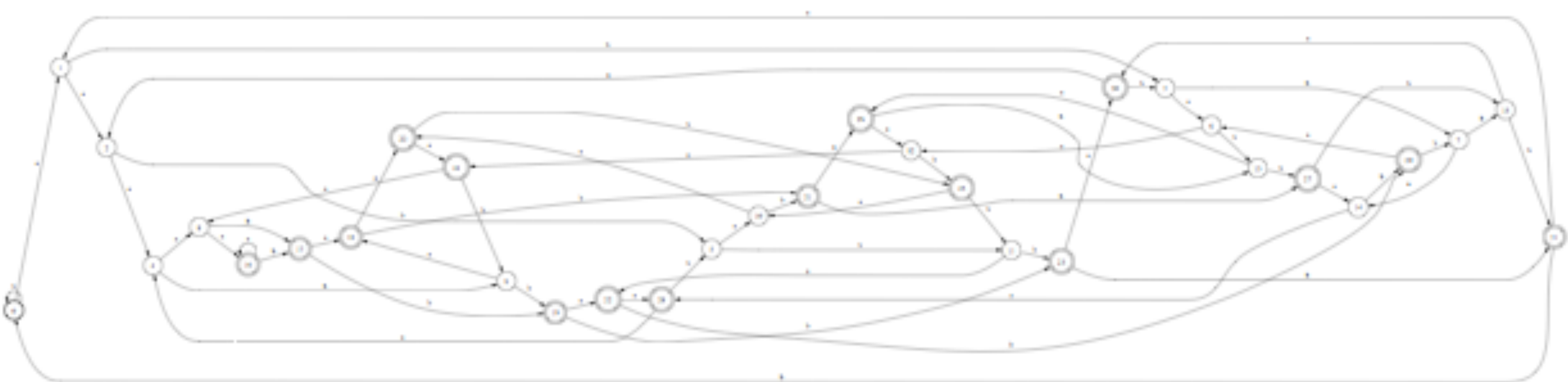


NFA to DFA



NFA to DFA





$2^5 = 32$ states

NFA vs. DFA in the wild

Engine Type	**Programs**
DFA	*awk* (most versions), *egrep* (most versions), *flex*, *lex*, MySQL, Procmail
Traditional NFA	GNU *Emacs*, Java, *grep* (most versions), *less*, *more*, .NET languages, PCRE library, Perl, PHP (pcre routines), Python, Ruby, *sed* (most versions), vi
POSIX NFA	*mawk*, MKS utilities, GNU *Emacs* (when requested)
Hybrid NFA/DFA	GNU *awk*, GNU *grep*/*egrep*, Tcl

Extensions to Regular Expressions

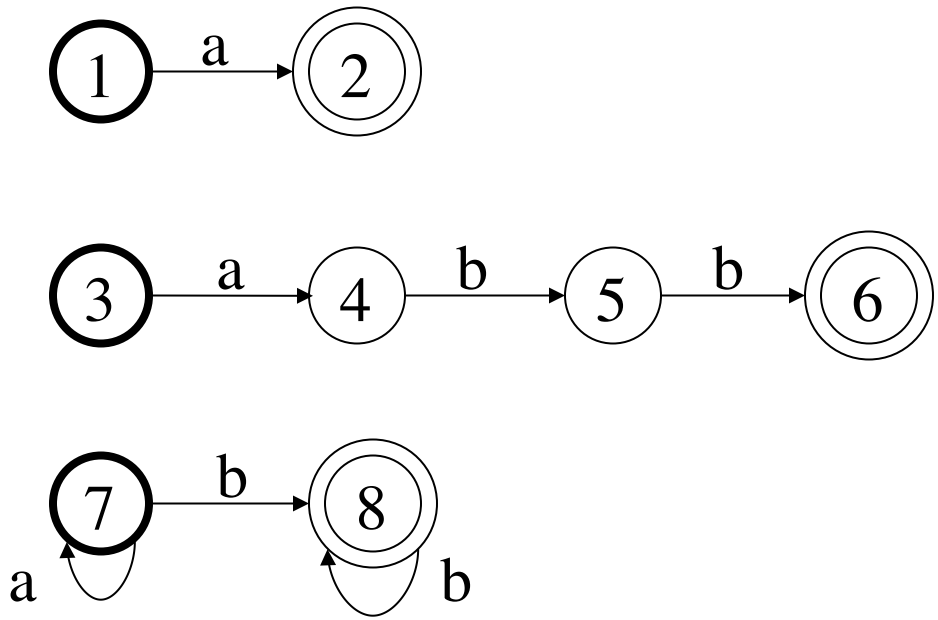
- Most modern regexp implementations provide extensions:
  - matching groups; \1 refers to the string matched by the first grouping (), \2 to the second match, etc.,
    - e.g. ([a-z]+)\1 which matches abab where \1=ab
  - match and replace operations,
    - e.g. s/([a-z]+)/\1\1/g which changes ab into abab where \1=ab
- These extensions are no longer "regular". In fact, extended regexp matching is NP-hard
  - Extended regular expressions (including POSIX and Perl) are called REGEX to distinguish from regexp (which *are* regular)
- In order to capture these difficult cases, the algorithms used even for simple regexp matching run in time exponential in the length of the input

Implementing a Lexical Analyzer

Lexical Analyzer using NFAs

- For each token convert its regexp into a DFA or NFA
- Create a new start state and create a transition on ε to the start state of the automaton for each token
- For input $i_1, i_2, \ldots, i_n$ run NFA simulation which returns some final states (each final state indicates a token)
- If no final state is reached then raise an error
- Pick the final state (token) that has the longest match in the input,
  - e.g. prefer DFA #8 over all others because it read the input until i_{30} and none of the other DFAs reached i_{30}
  - If two DFAs reach the same input character then pick the one that is listed first in the ordered list

Lexical Analysis using NFAs

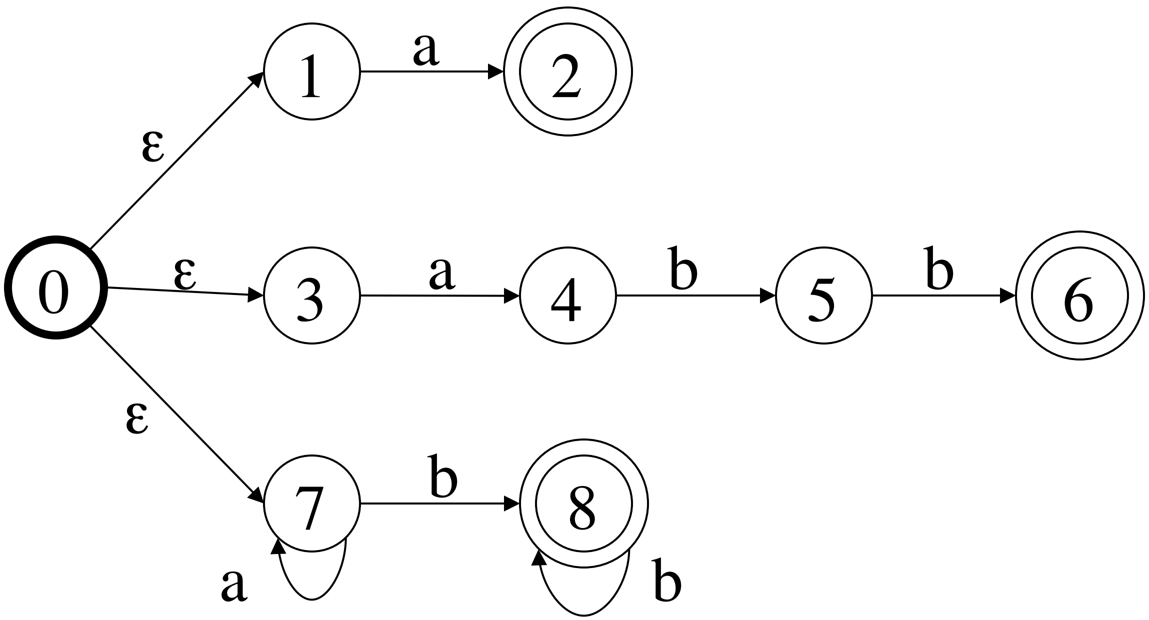


TOKEN_A = a

TOKEN_B = abb

TOKEN_C = a*b+

Lexical Analysis using NFAs



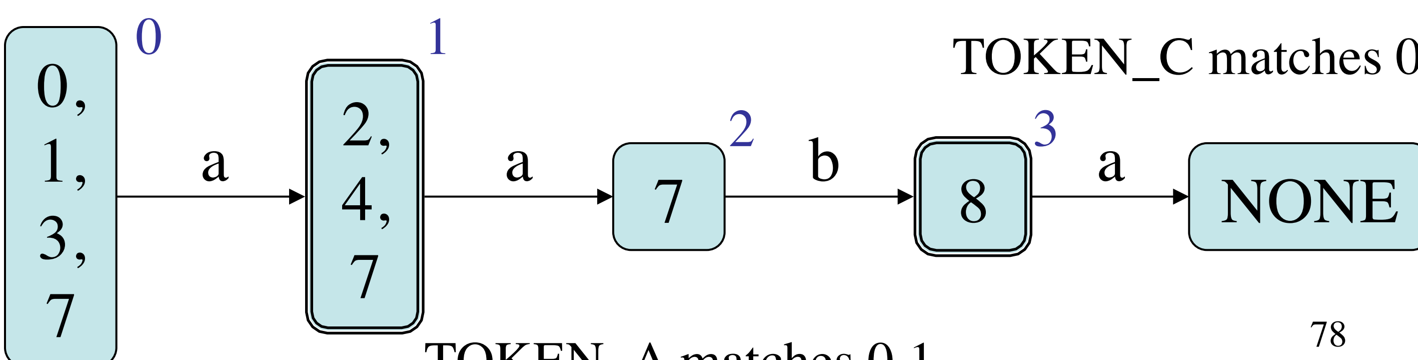
TOKEN_A = a

TOKEN_B = abb

TOKEN_C = a*b+

Input: aaba

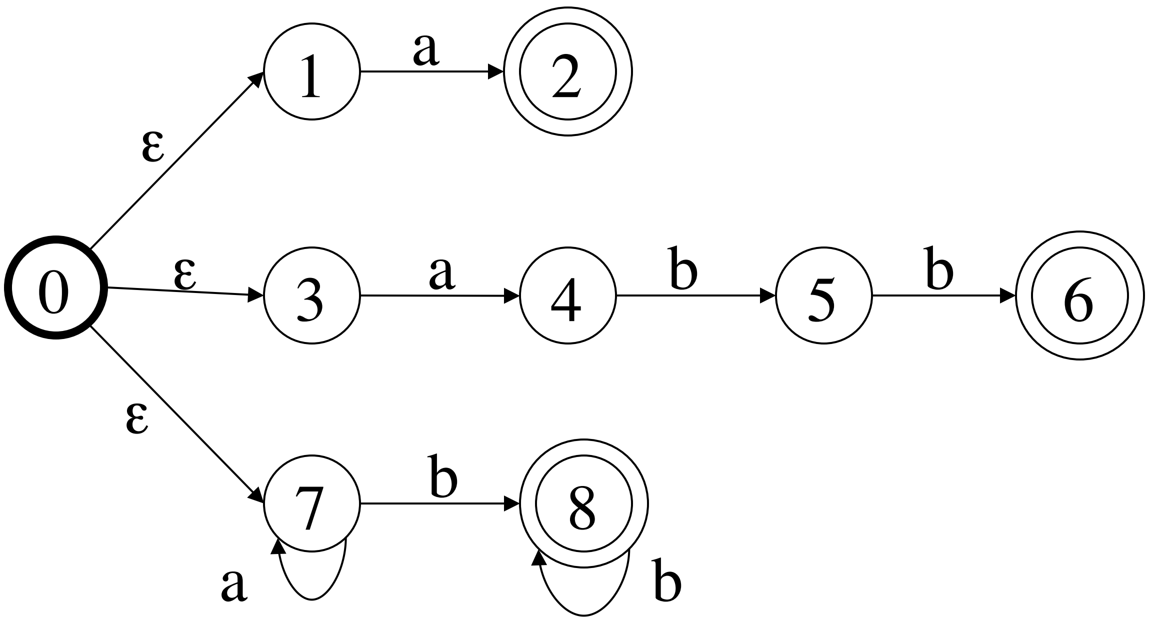
$_0a_1a_2b_3a_4$



TOKEN_C matches 0,3

TOKEN_A matches 0,1

Lexical Analysis using NFAs



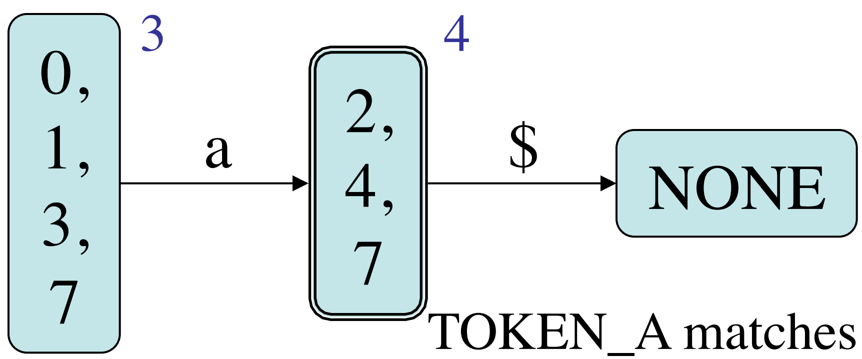
TOKEN_A = a

TOKEN_B = abb

TOKEN_C = a*b+

Input: aaba

$_0a_1a_2b_3a_4$



Output:
TOKEN_C aab [0,3]
TOKEN_A a [3,4]

TOKEN_A matches 3,4

Lexical Analyzer using DFAs

- Each token is defined using a regexp r_i
- Merge all regexps into one big regexp
  - $R = (r_1 \mid r_2 \mid \ldots \mid r_n)$
- Convert R to an NFA, then DFA, then minimize
  - remember orig NFA final states with each DFA state

Lexical Analyzer using DFAs

- The DFA recognizer has to find the *longest leftmost match* for a token
  - continue matching and report the last final state reached once DFA simulation cannot continue
  - e.g. longest match: <*print*> and not <*pr*>, <*int*>
  - e.g. leftmost match: for input string *aabaaaaab* the regexp a^+b will match *aab* and not *aaaaab*
- If two patterns match the same token, pick the one that was listed earlier in R
  - e.g. prefer final state (in the original NFA) of r_2 over r_3

Lookahead operator

- Implementing r_1/r_2 : match r_1 when followed by r_2
- e.g. *a*b+/a*c* accepts a string *bac* but not *abd*
- The lexical analyzer matches $r_1 \varepsilon r_2$ up to position *q* in the input
- But remembers the position *p* in the input where r_1 matched but not r_2
- Reset to start state and start from position *p*

Summary

- Token $\Rightarrow$ Pattern
- Pattern $\Rightarrow$ Regular Expression
- Regular Expression $\Rightarrow$ NFA
  - Thompson's Rules
- NFA $\Rightarrow$ DFA
  - Subset construction
- DFA $\Rightarrow$ minimal DFA
  - Minimization

$\Rightarrow$ **Lexical Analyzer (multiple patterns)**

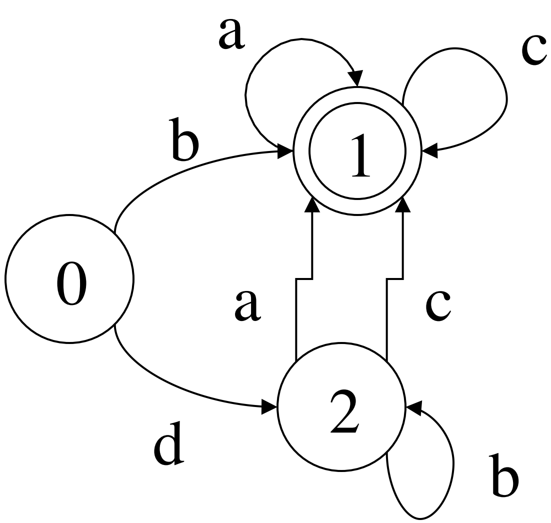
Extra Slides

Efficient data-structures for DFAs

Implementing DFAs

- 2D array storing the transition table
- Adjacency list, more space efficient but slower
- Merge two ideas: array structures used for sparse tables like DFA transition tables
  - base & next arrays: Tarjan and Yao, 1979
  - Dragon book (default+base & next+check)

Implementing DFAs



	a	b	c	d
0	-	1	-	2
1	1	-	1	-
2	1	2	1	-

Implementing DFAs

	a	b	c	d
0	-	1	-	2
1	1	-	1	-
2	1	2	1	-

		-	1	-	2			
				1	-	1	-	
1	2	1	-					
1	2	1	1	1	2	1	-	next
0	1	2	3	4	5	6	7	

base

0	2
1	4
2	0

2	2	2	0	1	0	1	-	check

$nextstate(s, x)$:

L := base[s] + x

return next[L] **if** check[L] **eq** s

Implementing DFAs

	a	b	c	d
0	-	1	-	2
1	1	-	1	-
2	1	2	1	-

		-	1	-	2		
				1	-	1	-
	-	2	-	-			
next	-	2	1	1	2	1	-
	0	1	2	3	4	5	6
check	-	2	0	1	0	1	-

base

0	1
1	3
2	0

-
-
1

default

$nextstate(s, x)$:

L := base[s] + x

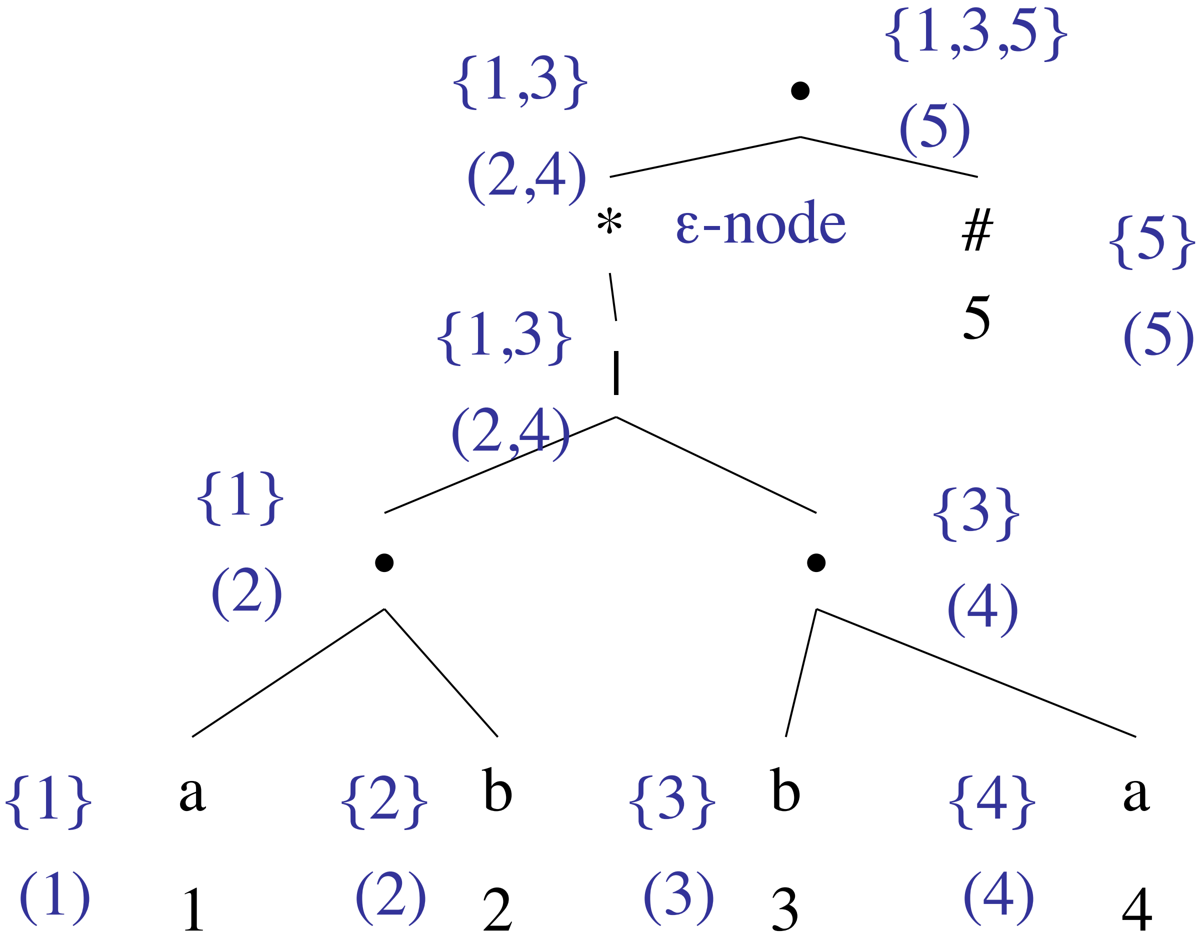
return next[L] **if** check[L] **eq** s

else return $nextstate$(default[s], x)

Converting Regular Expressions directly into DFAs

This algorithm was first used by Al Aho in `egrep`, and used in `awk`, `lex`, `flex`

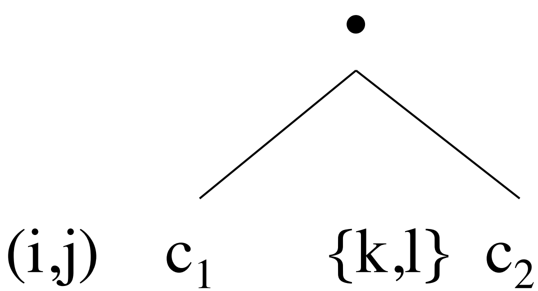
Regexp to DFA: `((ab)|(ba))*#`



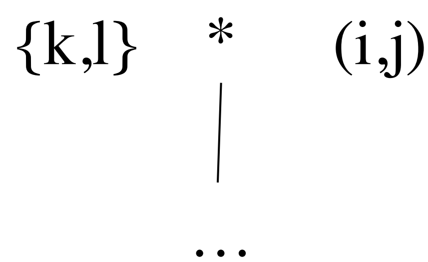
firstpos = {}
lastpos = ()

Regexp to DFA: *followpos*

- *followpos(p)* tells us which positions can follow a position *p*
- There are two rules that use the *firstpos* {} and *lastpos* () information

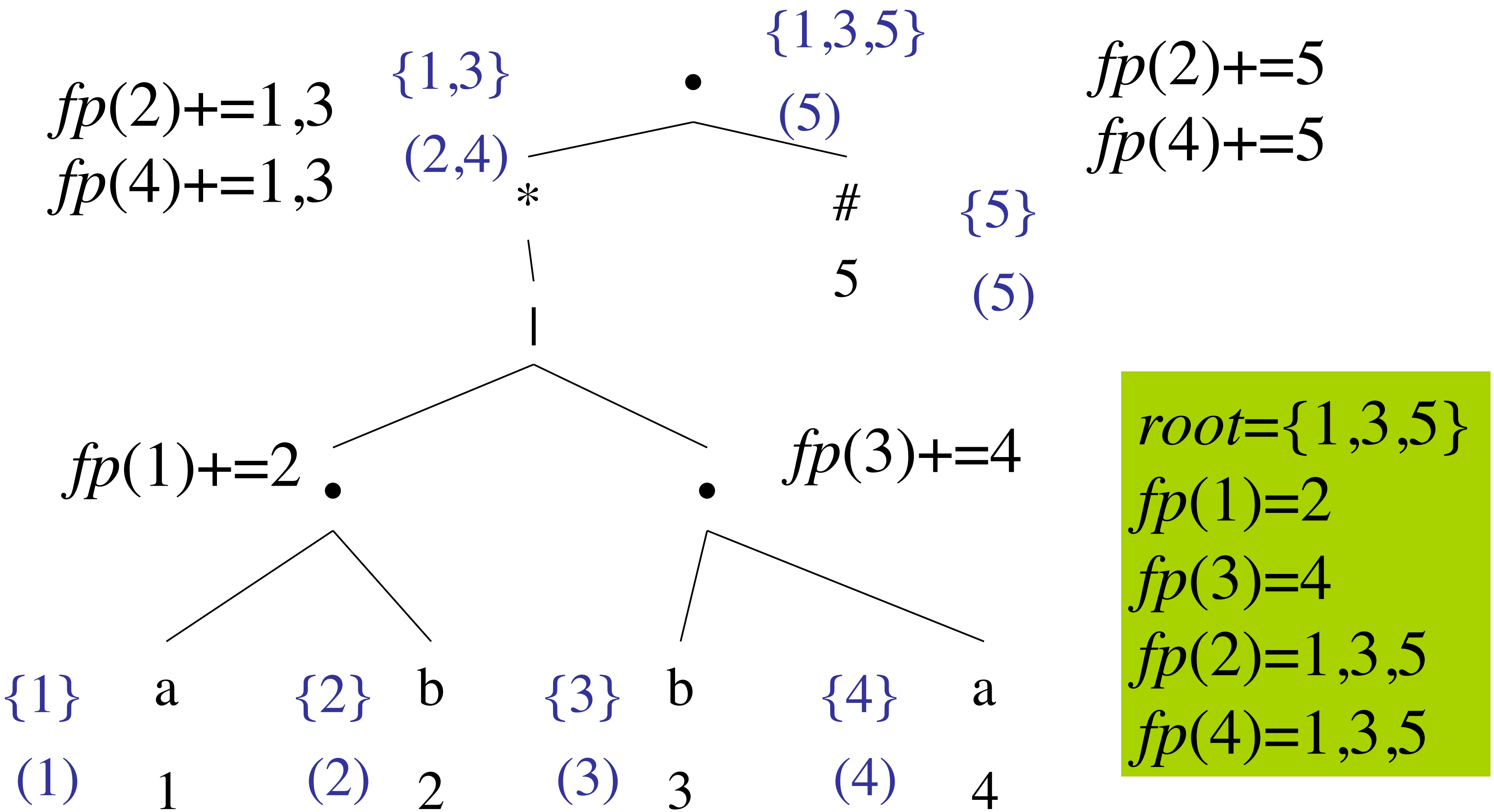


followpos(i)+=k,l
followpos(j)+=k,l



followpos(i)+=k,l
followpos(j)+=k,l

Regexp to DFA: `((ab)|(ba))*#`



Regexp to DFA: `((ab)|(ba))*#`

root={1,3,5}
fp(1)=2
fp(3)=4
fp(2)=1,3,5
fp(4)=1,3,5

1:a
2:b
3:b
4:a
5:#

{1,3,5} A

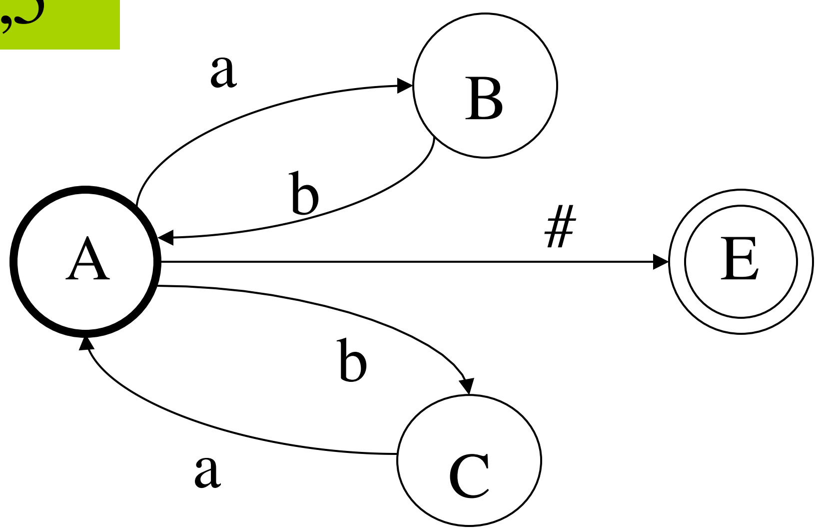
A: fp(1),a {2},a B,a

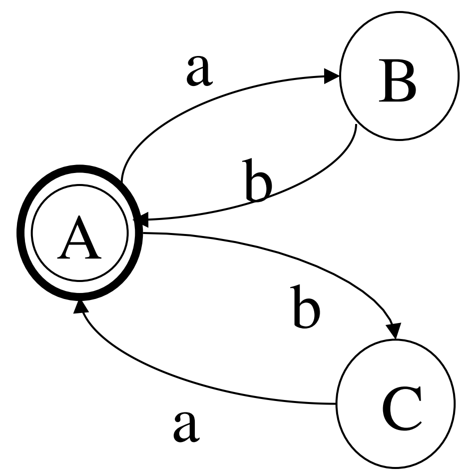
A: fp(3),b {4},b C,b

A: fp(5),# {},# E,#

B: fp(2),b {1,3,5},b A,b

C: fp(4),a {1,3,5},a A,a

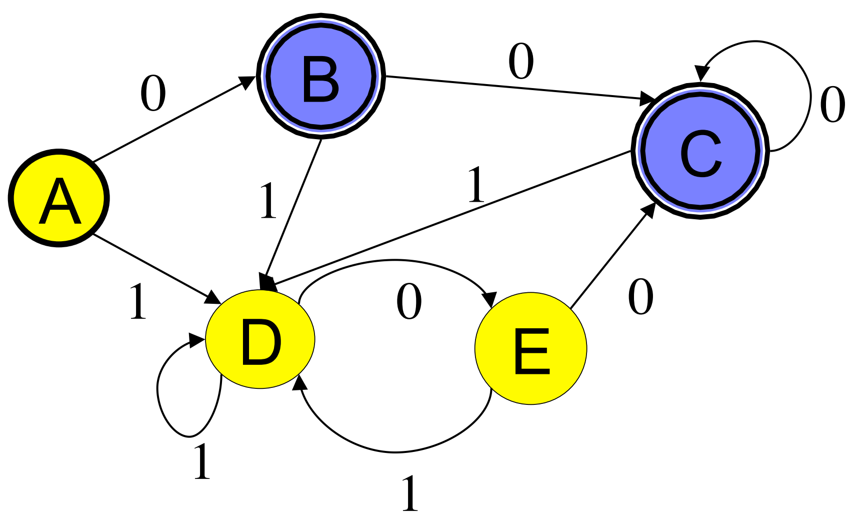




Minimization of DFAs

Minimization of DFAs

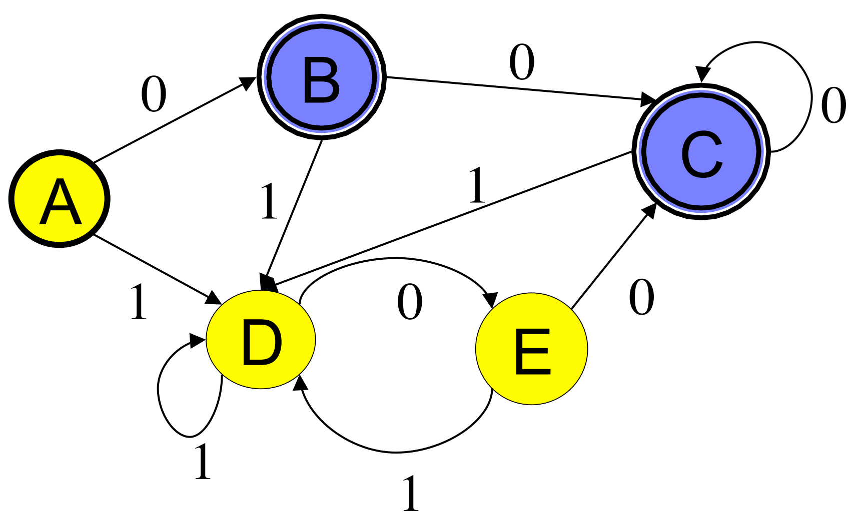
- Algorithm for minimizing the number of states in a DFA
- Step 1: partition states into 2 groups: accepting and non-accepting



Minimization of DFAs

- Step 2: in each group, find a sub-group of states having property P
- P: The states have transitions on each symbol (in the alphabet) to the *same* group

A, 0: blue
A, 1: yellow
E, 0: blue
E, 1: yellow
D, 0: yellow
D, 1: yellow

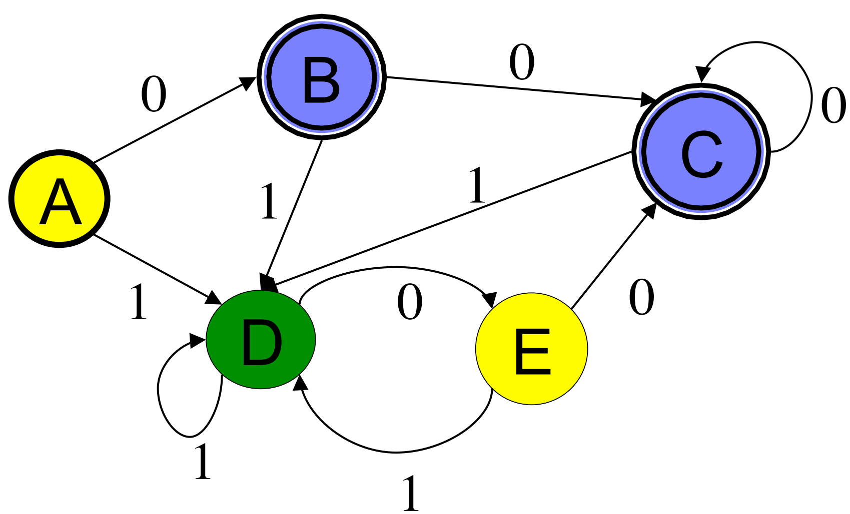


B, 0: blue
B, 1: yellow
C, 0: blue
C, 1: yellow

Minimization of DFAs

- Step 3: if a sub-group does not obey P split up the group into a separate group
- Go back to step 2. If no further sub-groups emerge then continue to step 4

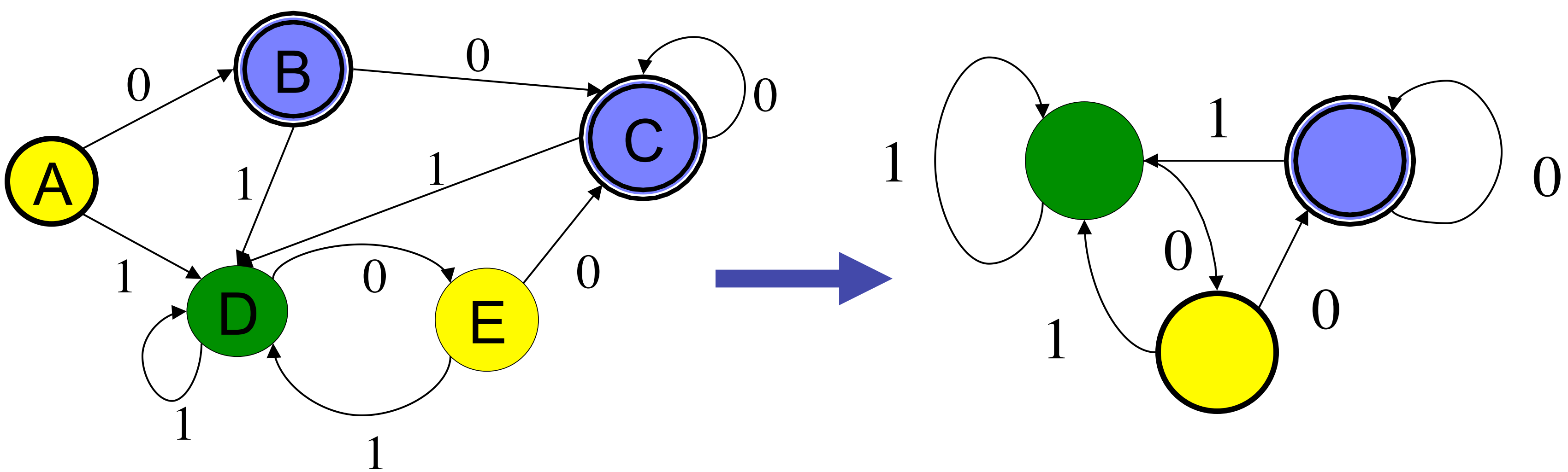
A, 0: blue
A, 1: green
E, 0: blue
E, 1: green
D, 0: yellow
D, 1: green



B, 0: blue
B, 1: green
C, 0: blue
C, 1: green

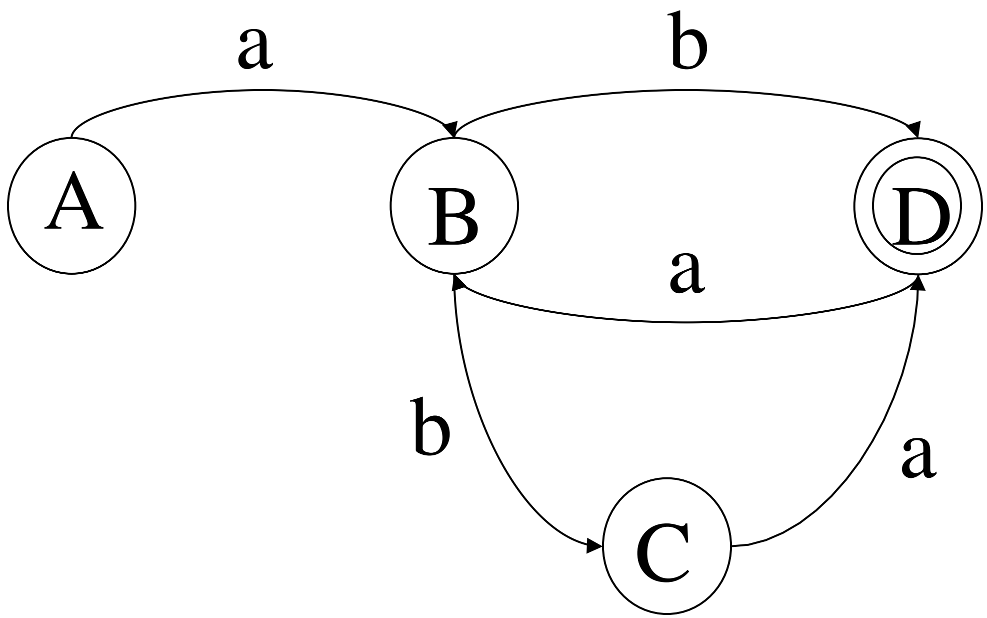
Minimization of DFAs

- Step 4: each group becomes a state in the minimized DFA
- Transitions to individual states are mapped to a single state representing the group of states



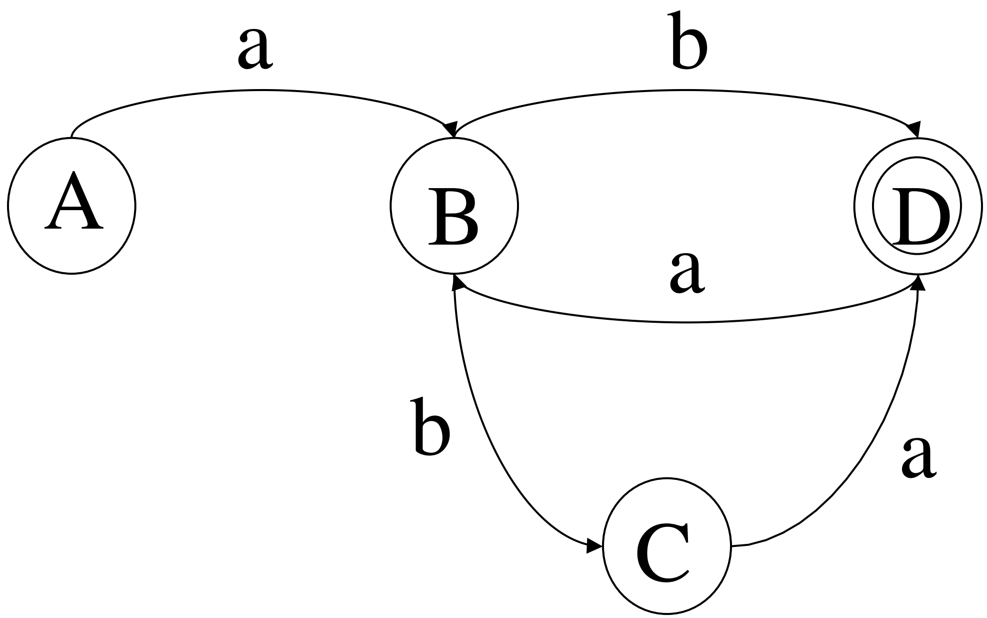
Converting an NFA into a Regular Expression

NFA to RegExp



What is the regular expression for this NFA?

NFA to RegExp



- A = a B
- B = b D | b C
- D = a B | ε
- C = a D

NFA to RegExp

- Three steps in the algorithm (apply in any order):

1. Substitution: for B = X pick every A = B | T and replace to get A = X | T
2. Factoring: (R S) | (R T) = R (S | T) and (R T) | (S T) = (R | S) T
3. Arden's Rule: For any set of strings S and T, the equation X = (S X) | T has X = (S*) T as a solution.

NFA to RegExp

- $A = a\ B$

  $B = b\ D \mid b\ C$

  $D = a\ B \mid \varepsilon$

  $C = a\ D$
- Substitute:

  $A = a\ B$

  $B = b\ D \mid b\ a\ D$

  $D = a\ B \mid \varepsilon$
- Factor:

  $A = a\ B$

  $B = (\ b \mid b\ a\ )\ D$

  $D = a\ B \mid \varepsilon$
- Substitute:

  $A = a\ (\ b \mid b\ a\ )\ D$

  $D = a\ (b \mid b\ a)\ D \mid \varepsilon$

NFA to RegExp

$A = a\,(\,b \mid b\,a\,)\,D$

$D = a\,(b \mid b\,a)\,D \mid \varepsilon$

- Factor:

$A = (a\,b \mid a\,b\,a\,)\,D$

$D = (a\,b \mid a\,b\,a)\,D \mid \varepsilon$

- Arden:

$A = (a\,b \mid a\,b\,a\,)\,D$

$D = (a\,b \mid a\,b\,a)^*\,\varepsilon$

- Remove epsilon:

$A = (a\,b \mid a\,b\,a\,)\,D$

$D = (a\,b \mid a\,b\,a)^*$

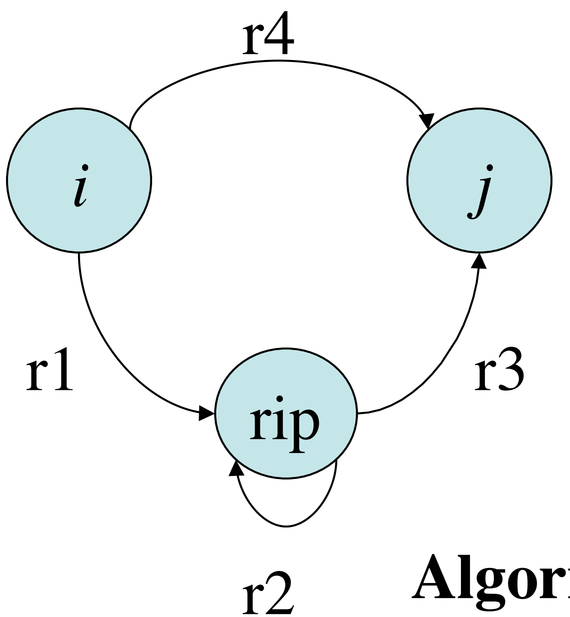
- Substitute:

$A = (a\,b \mid a\,b\,a\,)$
$\quad (a\,b \mid a\,b\,a)^*$

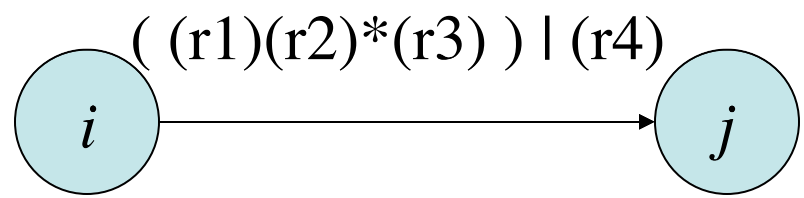
- Simplify:

$A = (a\,b \mid a\,b\,a)+$

NFA to Regexp using GNFAs



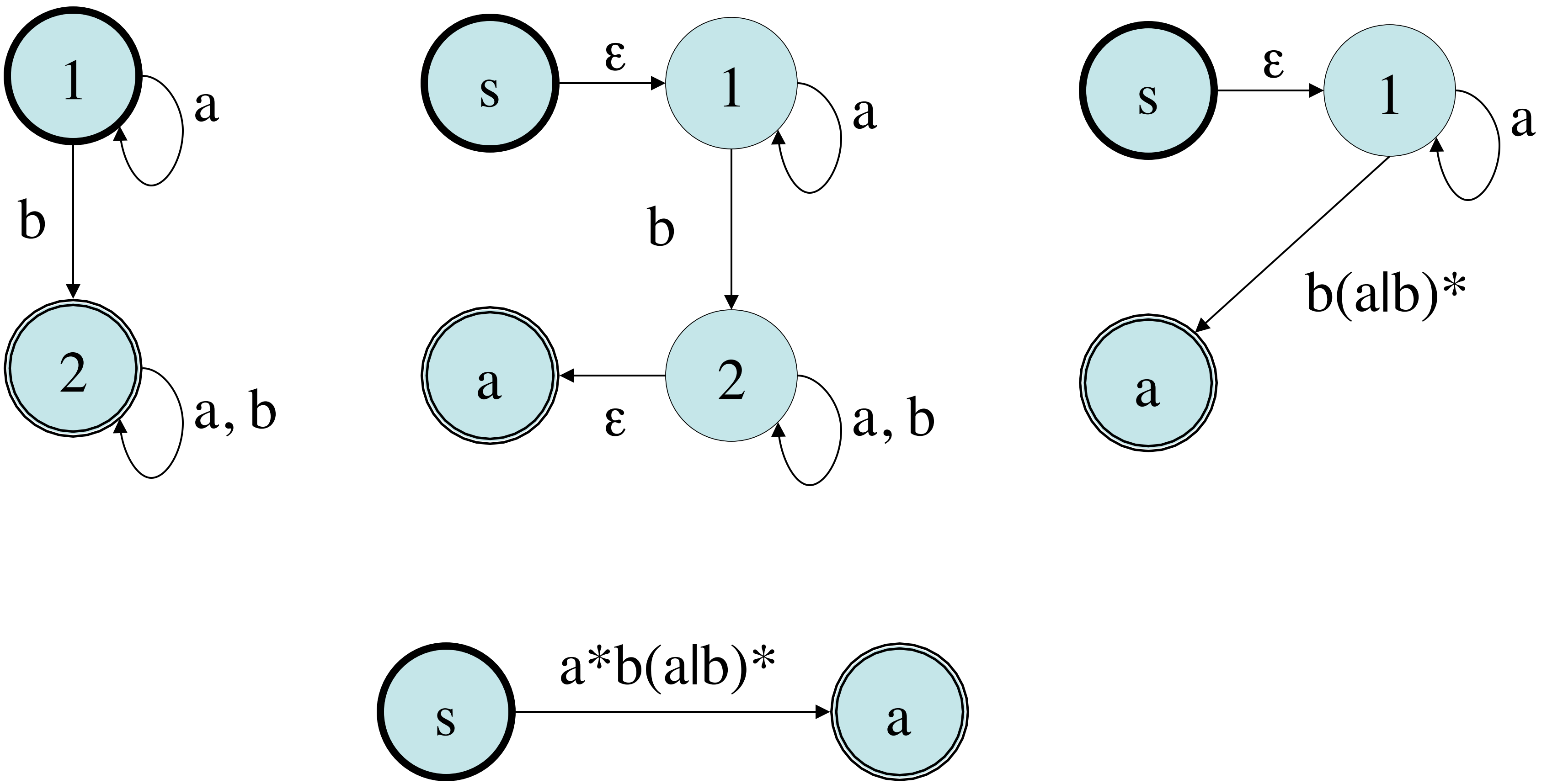
Generalized NFA: transition function takes state and regexp and returns a set of states



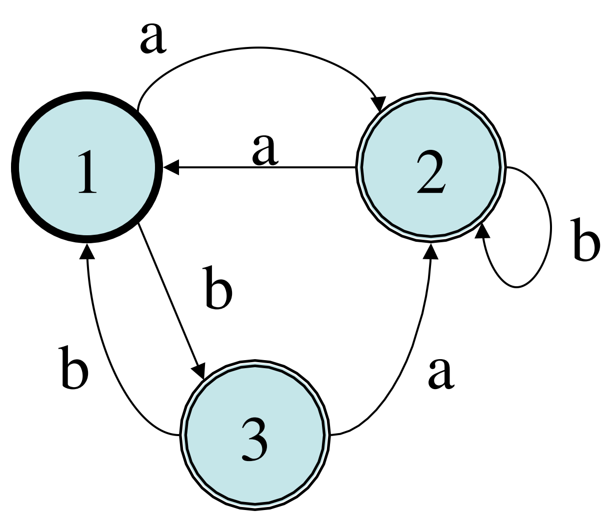
Algorithm:

1. Add new start & accept state
2. For each state *s*: rip state *s* creating GNFA, consider each state *i* and *j* adjacent to *s*
3. Return regexp from start to accept state

NFA to Regexp using GNFAs



NFA to Regexp using GNFAs



Rip states 1, 2, 3 in that order, and we get: (a(aa|b)*ab|b)((ba|a)(aa|b)*ab|bb)*((ba|a)(aa|b)*|ε)|a(aa|b)*