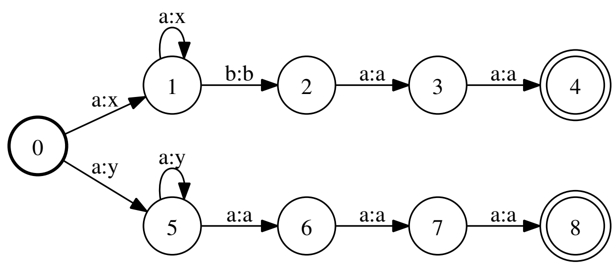
CMPT 413 - Spring 2013 - Midterm

There are three questions (25pts). Please write down "Midterm" on the top of the answer booklet.

When you have finished, return your answer booklet along with this question booklet.

(1) (5pts) **Finite-state transducers:**

For the following finite-state transducer provide exactly two rewrite rules that defines the same regular relation. Both the rewrite rules must be left to right obligatory rules. Use regular expression '^' to match the start of string and '$ to match the end of string.



Hint: It is nonsensical to write down a replacement rule like $a^* \to x^*$ or $a^n \to x^n$ regardless of the left or right context. However, it is correct to use x^* in the left or right context.

> *Answer:* The regular relation can be defined verbally as follows: for any $n \geq 1$ if the input is $a^n baa$ then the output is $x^n baa$, and if the input is a^{n+3} then the output is $y^n aaa$.
>
> $$a \to x / \hat{}\ \_\ a^* baa\$$$
> $$a \to y / \hat{}\ \_\ a^* aaa\$$$
>
> The following solution works for $n \geq 0$ (note that the above FST corresponds to the $n \geq 1$ case):
>
> $$a \to x / \_\ x^* baa\$$$
> $$a \to y / \_\ y^* aaa\$$$

(2) **Edit distance**: Assume insertion of a character has cost 1, deletion has cost 1, and substitution of one character for another has cost 2.

a. (2pts) What is the minimum edit distance value between target word *goal* and source word *hole*?

b. (3pts) The following is a visual display of one possible alignment between target word *goal* and source word *hole* using the usual notation.

```
g o a l _
  |   |
h o _ l e
```

Using the above notation for alignments, provide any other possible alignments that have the same edit distance as the alignment shown above.

Answer:

```
levenshtein distance = 4
alignment number 1 for [4,4]:
_ g o a l _
    |   |
h _ o _ l e

alignment number 2 for [4,4]:
g _ o a l _
    |   |
_ h o _ l e

alignment number 3 for [4,4]:
g o a l _
  |   |
h o _ l e

total of 3 alignments
```

(3) **Hidden Markov Models**:

The probability model $P(t_i \mid t_{i-2}, t_{i-1})$ is provided below where each t_i is a part of speech tag, e.g. the sixth row of the left table below corresponds to $P(\text{D} \mid \text{N}, \text{V}) = \frac{1}{3}$. Also provided is $P(w_i \mid t_i)$ that a word w_i has a part of speech tag t_i, e.g. the seventh line of the right table below corresponds to $P(\text{flies} \mid \text{V}) = \frac{1}{2}$.

$P(t_i \mid t_{i-2}, t_{i-1})$	t_{i-2}	t_{i-1}	t_i
1	`bos`	`bos`	N
$\frac{1}{2}$	`bos`	N	N
$\frac{1}{2}$	`bos`	N	V
$\frac{1}{2}$	N	N	V
$\frac{1}{2}$	N	N	P
$\frac{1}{3}$	N	V	D
$\frac{1}{3}$	N	V	V
$\frac{1}{3}$	N	V	P
1	V	D	N
1	V	V	D
1	N	P	D
1	V	P	D
1	P	D	N
1	D	N	`eos`

$P(w_i \mid t_i)$	t_i	w_i
1	D	an
$\frac{2}{5}$	N	time
$\frac{2}{5}$	N	arrow
$\frac{1}{5}$	N	flies
1	P	like
$\frac{1}{2}$	V	like
$\frac{1}{2}$	V	flies
1	`eos`	`eos`
1	`bos`	`bos`

The part of speech tag definitions are: `bos` (*begin sentence marker*), N (*noun*), V (*verb*), D (*determiner*), P (*preposition*), `eos` (*end of sentence marker*).

a. (10pts) Provide a Hidden Markov Model (*hmm*) that uses the trigram part of speech probability $P(t_i \mid t_{i-2}, t_{i-1})$ as the transition probability $P_{hmm}(s_j \mid s_k)$ and the probability $P(w_i \mid t_i)$ as the emission probability $P_{hmm}(w_j \mid s_j)$.

Important: Provide the *hmm* in the form of two tables as shown below. The first table contains transitions between states in the *hmm* and the transition probabilities and the second table contains the words emitted at each state and the emission probabilities. Do not provide entries with zero probability.

from-state s_k	to-state s_j	$P(s_j \mid s_k)$

state s_j	emission w	$P(w \mid s_j)$

Hint: In your *hmm* the state $\langle N, \texttt{eos} \rangle$ will have emission of word `eos` with probability 1 and will not have transitions to any other states.

Answer: Here are the two tables that define the HMM, the transition table on the left and the emission table on the right:

from-state s_k	to-state s_j	$P(s_j \mid s_k)$	
bos, bos	bos, N	$P(N \mid bos, bos)$	1
bos, N	N, N	$P(N \mid bos, N)$	$\frac{1}{2}$
bos, N	N, V	$P(V \mid bos, N)$	$\frac{1}{2}$
N, N	N, V	$P(V \mid N, N)$	$\frac{1}{2}$
N, N	N, P	$P(P \mid N, N)$	$\frac{1}{2}$
N, V	V, D	$P(D \mid N, V)$	$\frac{1}{3}$
N, V	V, V	$P(V \mid N, V)$	$\frac{1}{3}$
N, V	V, P	$P(P \mid N, V)$	$\frac{1}{3}$
V, D	D, N	$P(N \mid V, D)$	1
V, V	V, D	$P(D \mid V, V)$	1
N, P	P, D	$P(D \mid N, P)$	1
V, P	P, D	$P(D \mid V, P)$	1
P, D	D, N	$P(N \mid P, D)$	1
D, N	N, eos	$P(eos \mid D, N)$	1

state s_j	emission w	$P(w \mid s_j)$
bos, bos	bos	1
bos, N	time	$\frac{2}{5}$
bos, N	arrow	$\frac{2}{5}$
bos, N	flies	$\frac{1}{5}$
N, N	time	$\frac{2}{5}$
N, N	arrow	$\frac{2}{5}$
N, N	flies	$\frac{1}{5}$
N, V	like	$\frac{1}{2}$
N, V	flies	$\frac{1}{2}$
V, D	an	1
V, V	like	$\frac{1}{2}$
V, V	flies	$\frac{1}{2}$
N, P	like	1
V, P	like	1
P, D	an	1
D, N	time	$\frac{2}{5}$
D, N	arrow	$\frac{2}{5}$
D, N	flies	$\frac{1}{5}$

b. (5pts) Based on your *hmm* constructed in 3a. what is the state sequence with the highest probability for the following observation sequence:

`bos bos` time flies like an arrow `eos`

Answer:

bos	time	flies	like	an	arrow	eos	
(bos,bos) 1	(bos,N) $\times 1 \times \frac{2}{5}$	(N,V) $\times \frac{1}{2} \times \frac{1}{2}$	(V,P) $\times \frac{1}{3} \times 1$	(P,D) $\times 1 \times 1$	(D,N) $\times 1 \times \frac{2}{5}$	(N,eos) $\times 1 \times 1$	$= \frac{1}{75}*$
(bos,bos) 1	(bos,N) $\times 1 \times \frac{2}{5}$	(N,V) $\times \frac{1}{2} \times \frac{1}{2}$	(V,V) $\times \frac{1}{3} \times \frac{1}{2}$	(V,D) $\times 1 \times 1$	(D,N) $\times 1 \times \frac{2}{5}$	(N,eos) $\times 1 \times 1$	$= \frac{1}{150}$
(bos,bos) 1	(bos,N) $\times 1 \times \frac{2}{5}$	(N,N) $\times \frac{1}{2} \times \frac{1}{5}$	(N,P) $\times \frac{1}{2} \times 1$	(P,D) $\times 1 \times 1$	(D,N) $\times 1 \times \frac{2}{5}$	(N,eos) $\times 1 \times 1$	$= \frac{1}{125}$
(bos,bos) 1	(bos,N) $\times 1 \times \frac{2}{5}$	(N,N) $\times \frac{1}{2} \times \frac{1}{5}$	(N,V) $\times \frac{1}{2} \times \frac{1}{2}$	(V,D) $\times \frac{1}{3} \times 1$	(D,N) $\times 1 \times \frac{2}{5}$	(N,eos) $\times 1 \times 1$	$= \frac{1}{750}$