CMPT 379
Compilers

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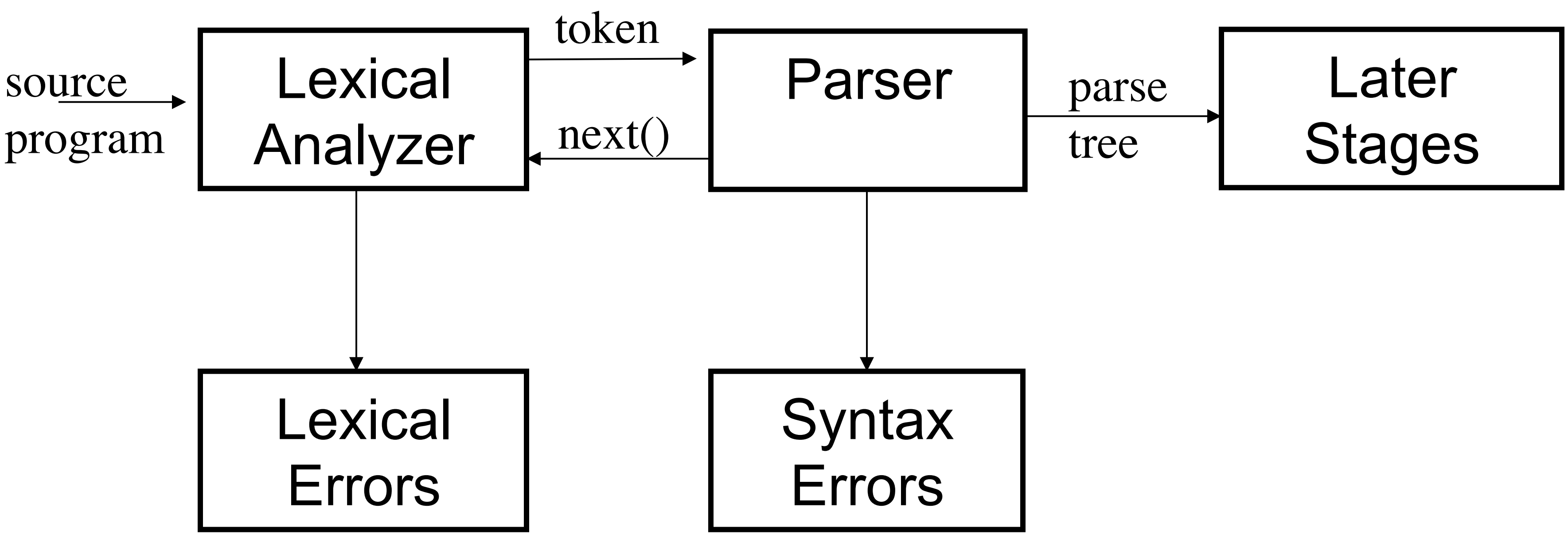
Parsing



token
source program
Lexical Analyzer
next()
Parser
parse tree
Later Stages
Lexical Errors
Syntax Errors

Context-free Grammars

- Set of rules by which valid sentences can be constructed.
- Example:

  Sentence → Noun Verb Object

  Noun → *trees* | *compilers*

  Verb → *are* | *grow*

  Object → *on* Noun | Adjective

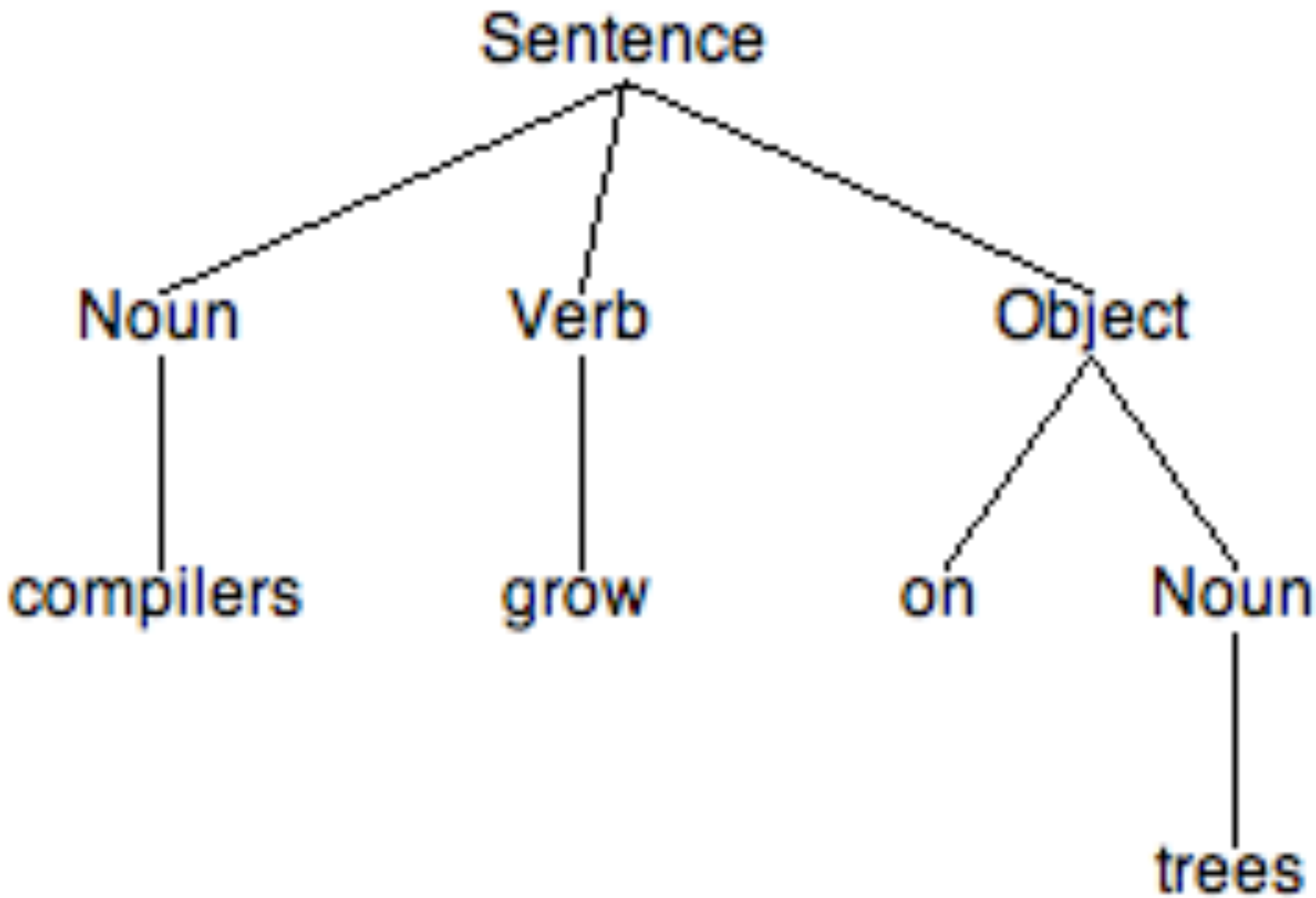
  Adjective → *slowly* | *interesting*

- What strings can Sentence *derive*?
- Syntax only – no semantic checking

Derivations of a CFG

- *compilers grow on trees*
- *compilers grow on* **Noun**
- *compilers grow* **Object**
- *compilers* **Verb Object**
- **Noun Verb Object**
- **Sentence**

Derivations and parse trees



Why use grammars for PL?

- Precise, yet easy-to-understand specification of language
- Construct parser automatically
  - Detect potential problems
- Structure and simplify remaining compiler phases
- Allow for evolution

CFG Notation

- A reference grammar is a concise description of a context-free grammar
- For example, a reference grammar can use regular expressions on the right hand sides of CFG rules
- Can even use ideas like comma-separated lists to simplify the reference language definition

Writing a CFG for a PL

- First write (or read) a reference grammar of what you want to be valid programs
- For now, we only worry about the structure, so the reference grammar might choose to over-generate in certain cases (e.g. `bool x = 20;`)
- Convert the reference grammar to a CFG
- Certain CFGs might be easier to work with than others (this is the **essence** of the study of CFGs and their parsing algorithms for compilers)

CFG Notation

- Normal CFG notation

  $E \rightarrow E * E$

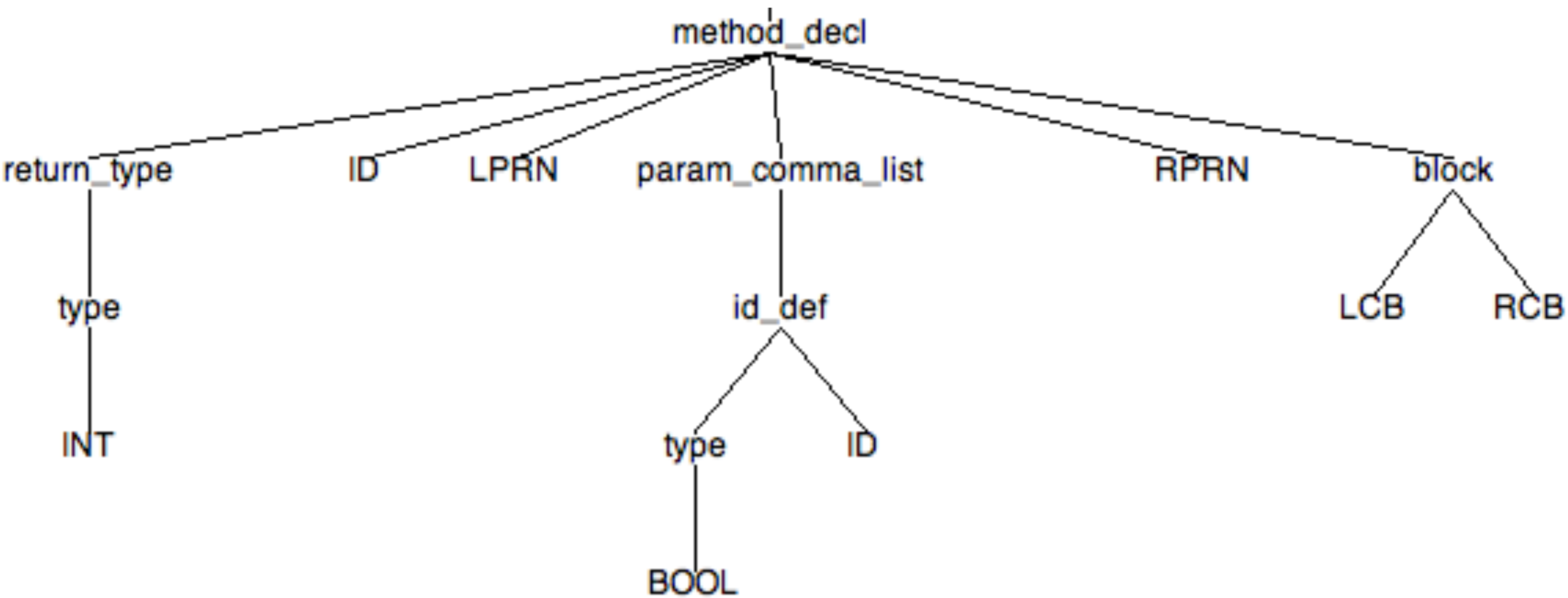
  $E \rightarrow E + E$

- Backus Naur notation

  E ::= E * E | E + E

  (an or-list of right hand sides)

Parse Trees for programs



Arithmetic Expressions

- $E \rightarrow E + E$
- $E \rightarrow E * E$
- $E \rightarrow (E)$
- $E \rightarrow - E$
- $E \rightarrow \mathbf{id}$

Leftmost derivations for
id + id * id

$E \rightarrow E + E$
$E \rightarrow E * E$
$E \rightarrow (E)$
$E \rightarrow - E$
$E \rightarrow \mathbf{id}$

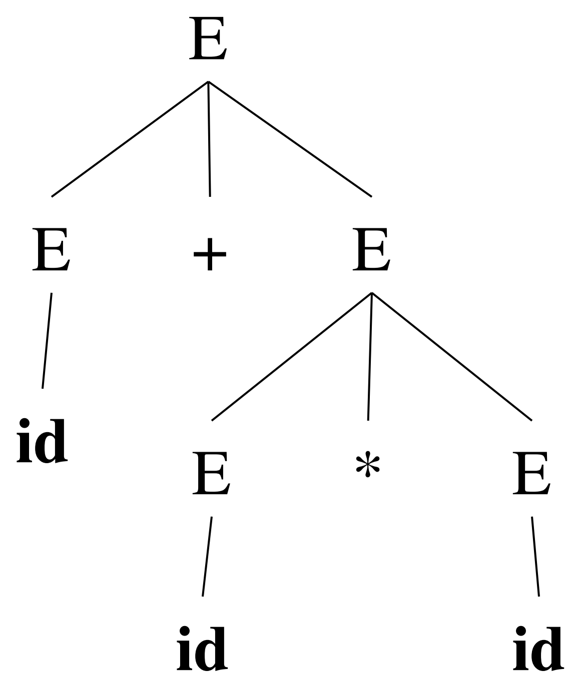
- $E \Rightarrow E + E$

$\Rightarrow \mathbf{id} + E$

$\Rightarrow \mathbf{id} + E * E$

$\Rightarrow \mathbf{id} + \mathbf{id} * E$

$\Rightarrow \mathbf{id} + \mathbf{id} * \mathbf{id}$



Leftmost derivations for **id + id * id**

$E \rightarrow E + E$
$E \rightarrow E * E$
$E \rightarrow (E)$
$E \rightarrow - E$
$E \rightarrow \mathbf{id}$

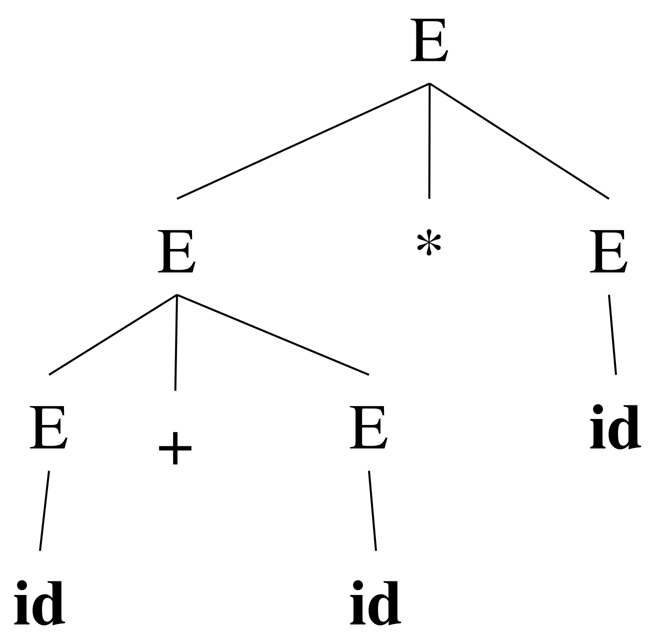
- $E \Rightarrow E * E$

$\Rightarrow E + E * E$

$\Rightarrow \mathbf{id} + E * E$

$\Rightarrow \mathbf{id} + \mathbf{id} * E$

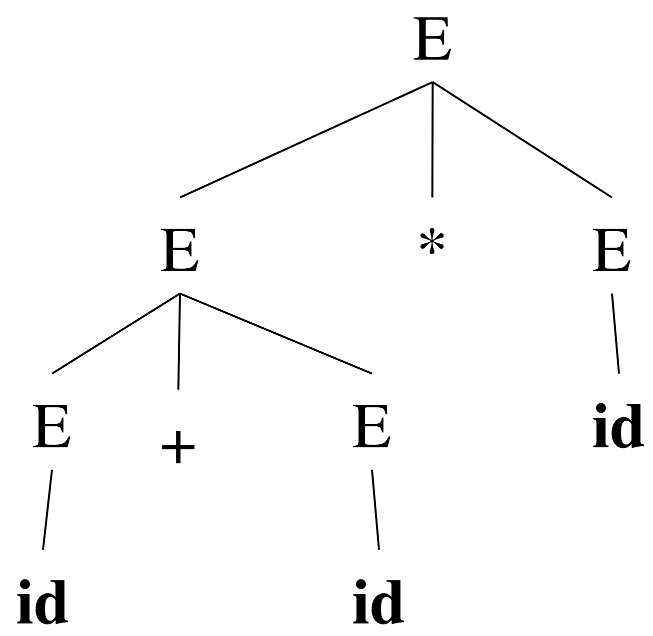
$\Rightarrow \mathbf{id} + \mathbf{id} * \mathbf{id}$



Rightmost derivation for **id + id * id**

$E \rightarrow E + E$
$E \rightarrow E * E$
$E \rightarrow (E)$
$E \rightarrow - E$
$E \rightarrow \mathbf{id}$

$E \Rightarrow E * E$
$\Rightarrow E * \mathbf{id}$
$\Rightarrow E + E * \mathbf{id}$
$\Rightarrow E + \mathbf{id} * \mathbf{id}$
$\Rightarrow \mathbf{id} + \mathbf{id} * \mathbf{id}$



Ambiguity

- Grammar is ambiguous if more than one parse tree is possible for some sentences
- Examples in English:
  - Two sisters reunited after 18 years in checkout counter
- Ambiguity is not acceptable in PL
  - Unfortunately, it's undecidable to check whether a given CFG is ambiguous
  - Some CFLs are inherently ambiguous (do not have an unambiguous CFG)

Ambiguity

- Alternatives
  - Massage grammar to make it unambiguous
  - Rely on "default" parser behavior
  - Augment parser
- Consider the original ambiguous grammar:

$E \rightarrow E + E \qquad E \rightarrow E * E$

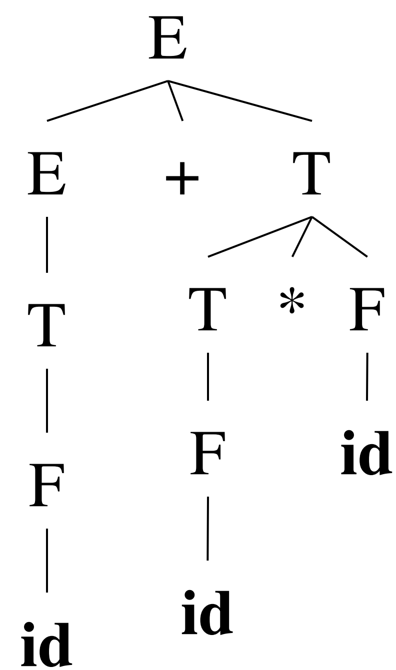
$E \rightarrow (E) \qquad E \rightarrow - E$

$E \rightarrow \mathbf{id}$

- How can we change the grammar to get only one tree for the input **id + id * id**

Ambiguity

- Original ambiguous grammar:
  - E → E + E E → E * E
  - E → (E) E → - E
  - E → id
- Unambiguous grammar:
  - E → E + T T → T * F
  - E → T T → F
  - F → (E) $\boxed{\text{F} \rightarrow \text{- E}}$
  - F → id
- Input: id + id * id



Warning! Is this unambiguous?
Check derivations for – id + id

Compare with F → - F

Dangling else ambiguity

- Original Grammar (ambiguous)

  Stmt → **if** Expr **then** Stmt **else** Stmt

  Stmt → **if** Expr **then** Stmt

  Stmt → Other

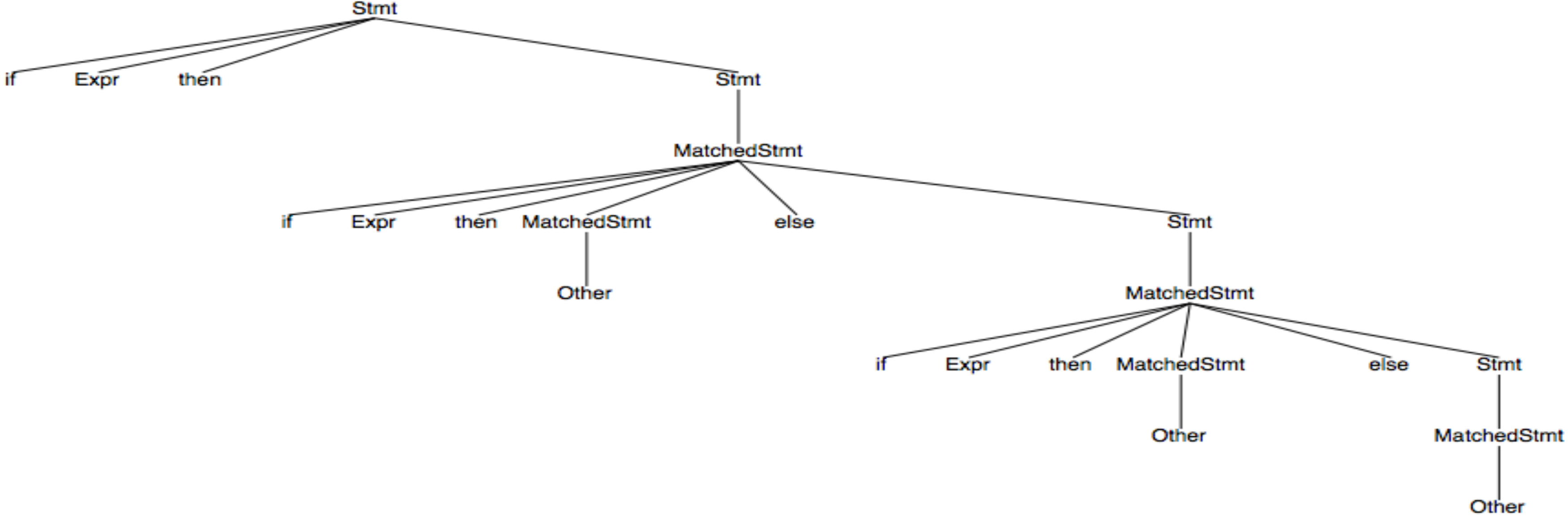
- Modified Grammar (unambiguous?)

  Stmt → **if** Expr **then** Stmt

  Stmt → MatchedStmt

  MatchedStmt → **if** Expr **then** MatchedStmt **else** Stmt

  MatchedStmt → Other


Stmt
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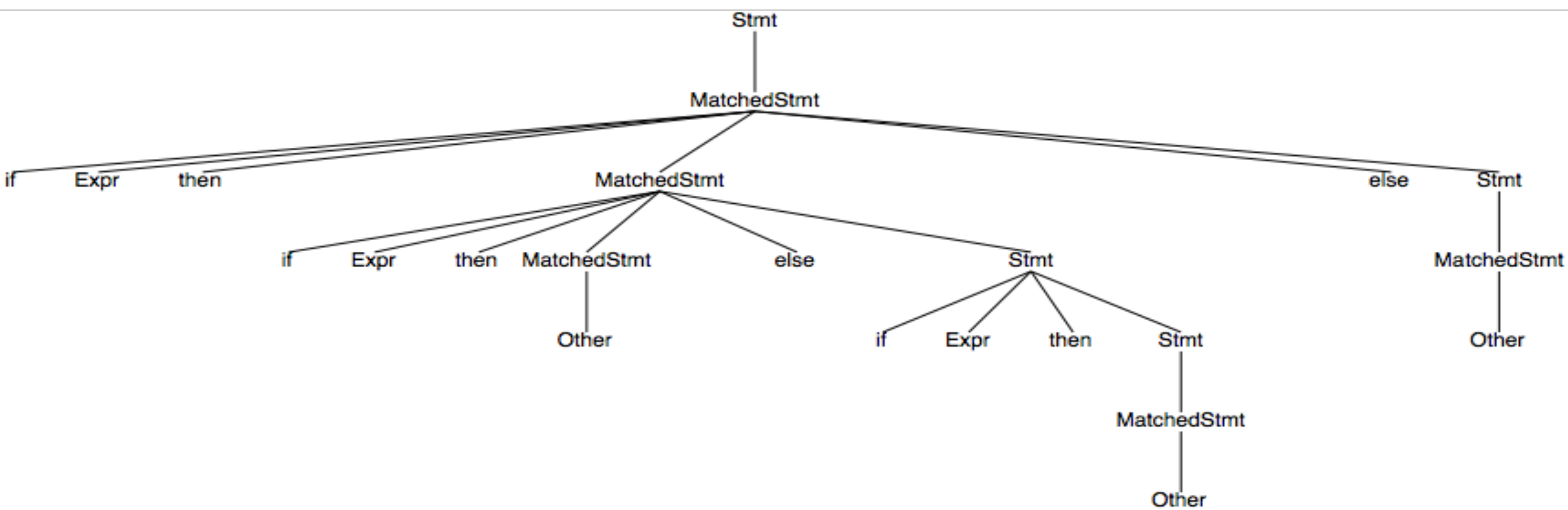
Stmt
Other
MatchedStmt
Other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Other
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then
Stmt
Other
MatchedStmt
Other

Dangling else ambiguity

- Original Grammar (ambiguous)

  Stmt → **if** Expr **then** Stmt **else** Stmt

  Stmt → **if** Expr **then** Stmt

  Stmt → Other

- Unambiguous grammar

  Stmt → MatchedStmt

  Stmt → UnmatchedStmt

  MatchedStmt → **if** Expr **then** MatchedStmt **else** MatchedStmt

  MatchedStmt → Other

  UnmatchedStmt → **if** Expr **then** Stmt

  UnmatchedStmt → **if** Expr **then** MatchedStmt **else** UnmatchedStmt

Dangling else ambiguity

- Check unambiguous dangling-else grammar with the following inputs:
  - if Expr then if Expr then Other else Other
  - if Expr then if Expr then Other else Other else Other
  - if Expr then if Expr then Other else if Expr then Other else Other

Other Ambiguous Grammars

- Consider the grammar

  $R \rightarrow R$ ‘|’ R | R R | R ‘*’ | ‘(‘ R ‘)’ | a | b
- What does this grammar generate?
- What’s the parse tree for *a|b*a*
- Is this grammar ambiguous?

Left Factoring

- Original Grammar (ambiguous)

  Stmt → **if** Expr **then** Stmt **else** Stmt

  Stmt → **if** Expr **then** Stmt

  Stmt → Other

- Left-factored Grammar (still ambiguous):

  Stmt → **if** Expr **then** Stmt OptElse

  Stmt → Other

  OptElse → **else** Stmt | ε

Left Factoring

Left Factor:

$$
\begin{aligned}
A \rightarrow\ & XA \\
\mid\ & XB \\
\mid\ & X \\
\mid\ & Y \\
\mid\ & Z
\end{aligned}
$$

- In general, for rules

$$A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \ldots \mid \alpha\beta_n \mid \gamma$$

- Left factoring is achieved by the following grammar transformation:

$$A \rightarrow \alpha A' \mid \gamma$$

$$A' \rightarrow \beta_1 \mid \beta_2 \mid \ldots \mid \beta_n$$

Grammar Transformations

- G is converted to G' s.t. L(G') = L(G)
- Left Factoring
- Removing cycles: $A \Rightarrow^{+} A$
- Removing ε-rules of the form $A \rightarrow \varepsilon$
- Eliminating left recursion
- Conversion to normal forms:
  - Chomsky Normal Form, $A \rightarrow B\ C$ and $A \rightarrow a$
  - Greibach Normal Form, $A \rightarrow a\ \beta$

Eliminating Left Recursion

- Simple case, for left-recursive pair of rules: $$A \rightarrow A\alpha \mid \beta$$

- Replace with the following rules:

$$A \rightarrow \beta A'$$

$$A' \rightarrow \alpha A' \mid \epsilon$$

- Elimination of immediate left recursion

Eliminating Left Recursion

- Example:

  $E \rightarrow E + T,\ E \rightarrow T$

- Without left recursion:

  $E \rightarrow T\ E_1,\ E_1 \rightarrow +\ T\ E_1\ ,\ E_1 \rightarrow \varepsilon$

- Simple algorithm doesn't work for 2-step recursion:

  $S \rightarrow A\ a\ ,\ S \rightarrow b$

  $A \rightarrow A\ c\ ,\ A \rightarrow S\ d\ ,\ A \rightarrow \varepsilon$

Eliminating Left Recursion

- Problem CFG:

  $S \rightarrow A\ a\ ,\ S \rightarrow b$

  $A \rightarrow A\ c\ ,\ A \rightarrow S\ d\ ,\ A \rightarrow \varepsilon$

- Expand possibly left-recursive rules:

  $S \rightarrow A\ a\ ,\ S \rightarrow b$

  $A \rightarrow A\ c\ ,\ A \rightarrow A\ a\ d\ ,\ A \rightarrow b\ d\ ,\ A \rightarrow \varepsilon$

- Eliminate immediate left-recursion

  $S \rightarrow A\ a\ ,\ S \rightarrow b$

  $A \rightarrow b\ d\ A_1\ ,\ A \rightarrow A_1\ ,$

  $A_1 \rightarrow c\ A_1\ ,\ A_1 \rightarrow a\ d\ A_1\ ,\ A_1 \rightarrow \varepsilon$

Eliminating Left Recursion

- We cannot use the algorithm if the non-terminal also derives epsilon. Let's see why:

  $A \rightarrow AAa \mid b \mid \varepsilon$

- Using the standard lrec removal algorithm:

  $A \rightarrow bA_1 \mid A_1$

  $A_1 \rightarrow AaA_1 \mid \varepsilon$

Eliminating Left Recursion

- First we eliminate the epsilon rule:

  $A \rightarrow AAa \mid b \mid \varepsilon$

- Since A is the start symbol, create a new start symbol to generate the empty string:

  $A_1 \rightarrow A \mid \varepsilon \qquad A \rightarrow AAa \mid Aa \mid a \mid b$

- Now we can do the usual lrec algorithm:

  $A_1 \rightarrow A \mid \varepsilon \qquad A \rightarrow aA_2 \mid bA_2$

  $A_2 \rightarrow AaA_2 \mid aA_2 \mid \varepsilon$

Context-free languages and Pushdown Automata

- Recall that for each regular language there was an equivalent finite-state automaton
- The FSA was used as a recognizer of the regular language
- For each context-free language there is also an automaton that recognizes it: called a **pushdown automaton (pda)**

Context-free languages and Pushdown Automata

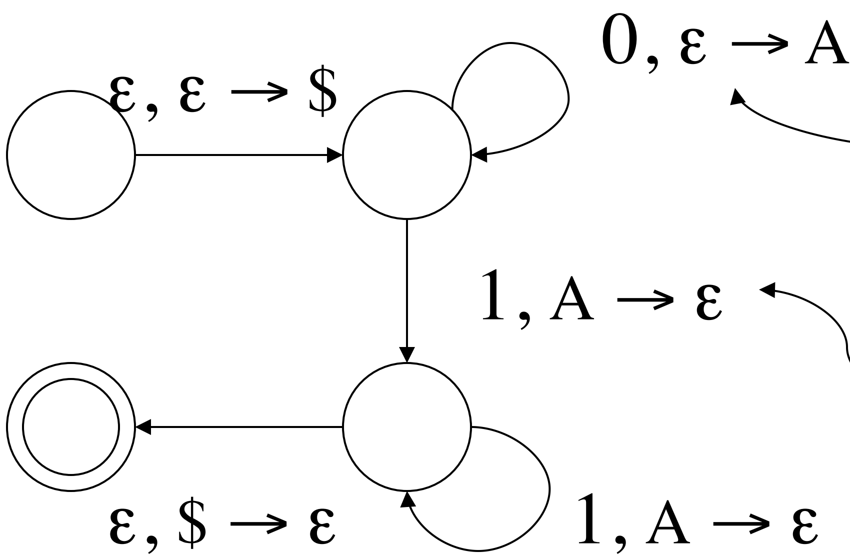
- Similar to FSAs there are non-deterministic pda and deterministic pda
- Unlike in the case of FSAs we cannot always convert a npda to a dpda
- Our goal in compiler design will be to choose grammars carefully so that we can always provide a dpda for it
- Similar to the FSA case, a DFA construction provides us with the algorithm for lexical analysis,
- In this case the construction of a dpda will provide us with the algorithm for parsing (take in strings and provide the parse tree)
- We will study later how to convert a given CFG into a parser by first converting into a PDA

Pushdown Automata

- PDA has
  - an alphabet (terminals) and
  - stack symbols (like non-terminals),
  - a finite-state automaton, and
  - stack

e.g. PDA for language
$L = \{ 0^n 1^n : n >= 0 \}$

$\rightarrow$ implies a push/pop of stack symbol(s)



push stack symbol A

pop stack symbol A

check that stack is empty

Non-CF Languages

$$L_1 = \{wcw \mid w \in (a|b)*\}$$

$$L_2 = \{a^n b^m c^n d^m \mid n \geq 1, m \geq 1\}$$

$$L_3 = \{a^n b^n c^n \mid n \geq 0\}$$

CF Languages

$$L_4 = \{wcw^R \mid w \in (a|b)*\}$$

$$S \rightarrow aSa \mid bSb \mid c$$

$$L_5 = \{a^n b^m c^m d^n \mid n \geq 1, m \geq 1\}$$

$$S \rightarrow aSd \mid aAd$$

$$A \rightarrow bAc \mid bc$$

Summary

- CFGs can be used describe PL
- Derivations correspond to parse trees
- Parse trees represent structure of programs
- Ambiguous CFGs exist
- Some forms of ambiguity can be fixed by changing the grammar
- Grammars can be simplified by left-factoring
- Left recursion in a CFG can be eliminated
- CF languages can be recognized using Pushdown Automata

Extra Slides

Non-CF Languages

- The pumping lemma for CFLs [Bar-Hillel] is similar to the pumping lemma for RLs
- For a string $wuxvy$ in a CFL for $u,v \neq \varepsilon$ and the string is longer than p and $|xvy| \leq p$ then wu^nxv^ny is also in the CFL for $n \geq 0$
- Not strong enough to work for every non-CF language (cf. Ogden's Lemma)