CMPT-413
Computational Linguistics

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Why are parsing algorithms important?

- A linguistic theory is implemented in a formal system to generate the set of grammatical strings and rule out ungrammatical strings.
- Such a formal system has computational properties.
- One such property is a simple decision problem: given a string, can it be generated by the formal system *(recognition)*.
- If it is generated, what were the steps taken to recognize the string *(parsing)*.

Why are parsing algorithms important?

- Consider the recognition problem: find algorithms for this problem for a particular formal system.
- The algorithm must be decidable.
- Preferably the algorithm should be polynomial: enables computational implementations of linguistic theories.
- Elegant, polynomial-time algorithms exist for formalisms like CFG

Top-down, depth-first, left to right parsing

$$
\begin{array}{rcl}
S & \rightarrow & NP\ VP \\
NP & \rightarrow & Det\ N \\
NP & \rightarrow & Det\ N\ PP \\
VP & \rightarrow & V \\
VP & \rightarrow & V\ NP \\
VP & \rightarrow & V\ NP\ PP \\
PP & \rightarrow & P\ NP \\
NP & \rightarrow & I \\
Det & \rightarrow & a \mid the \\
V & \rightarrow & saw \\
N & \rightarrow & park \mid dog \mid man \mid telescope \\
P & \rightarrow & in \mid with
\end{array}
$$

Top-down, depth-first, left to right parsing

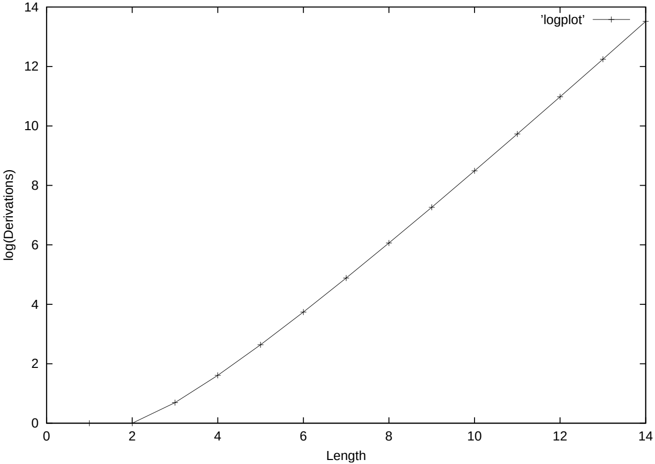
- Consider the input string: *the dog saw a man in the park*
- S . . . (S (NP VP)) . . . (S (NP Det N) VP) . . . (S (NP (Det the) N) VP) . . . (S (NP (Det the) (N dog)) VP) . . .
- (S (NP (Det the) (N dog)) VP) . . . (S (NP (Det the) (N dog)) (VP V NP PP)) . . . (S (NP (Det the) (N dog)) (VP (V saw) NP PP)) . . .
- (S (NP (Det the) (N dog)) (VP (V saw) (NP Det N) PP)) . . .
- (S (NP (Det the) (N dog)) (VP (V saw) (NP (Det a) (N man)) (PP (P in) (NP (Det the) (N park)))))

Number of derivations

CFG rules { S → S S , S → a }

n : a^n	number of parses
1	1
2	1
3	2
4	5
5	14
6	42
7	132
8	429
9	1430
10	4862
11	16796

Number of derivations grows exponentially



$L(G) = a+$ using CFG rules $\{$ `S` $\rightarrow$ `S S` , `S` $\rightarrow$ `a` $\}$

Syntactic Ambiguity: (Church and Patil 1982)

- Algebraic character of parse derivations
- Power Series for grammar for coordination type of grammars (more general than PPs):

  `N → natural | language | processing | course`

  `N → N N`
- We write an equation for algebraic expansion starting from `N`
- The equation represents generation of each string in the language as the terms, and the number of different ways of generating the string as the coefficients:

```
N = nat. + lang. + proc. + course +
  + nat. lang. + nat. proc. + ...
  + 2(nat. lang. proc.) + 2(lang. proc. course) + ...
  + 5(nat. lang. proc. course) + ...
  + 14 ...
```

CFG Ambiguity

- Coefficients in previous equation equal the number of parses for each string derived from E
- These ambiguity coefficients are Catalan numbers:

$$Cat(n) = \frac{1}{n+1} \left(\begin{array}{c} 2n \\ n \end{array} \right)$$

- $\left(\begin{array}{c} a \\ b \end{array} \right)$ is the *binomial coefficient*

$$\left(\begin{array}{c} a \\ b \end{array} \right) = \frac{a!}{(b!(a-b)!)}$$

Catalan numbers

- Why Catalan numbers? Cat(n) is the number of ways to parenthesize an expression of length n with two conditions:
  1. there must be equal numbers of open and close parens
  2. they must be properly nested so that an open precedes a close

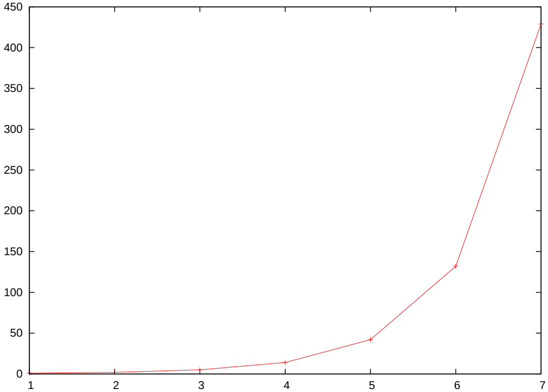
Catalan numbers

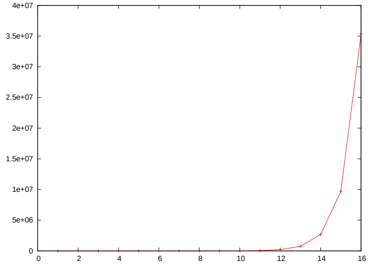
- For an expression of with n ways to form constituents there are a total of $2n$ choose n parenthesis pairs, e.g. for $n = 2$, $\binom{4}{2} = 6$:
  `a(bc)`, `a)bc(`, `)a(bc`, `(ab)c`, `)ab(c`, `ab)c(`
- But for each valid parenthesis pair, additional n pairs are created that have the right parenthesis to the left of its matching left parenthesis, from e.g. above: `a)bc(`, `)a(bc`, `)ab(c`, `ab)c(`
- So we divide $2n$ choose n by $n + 1$:

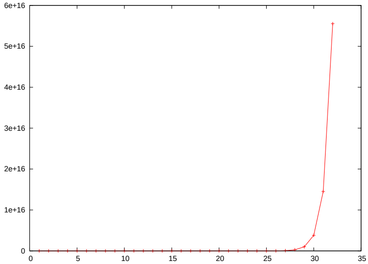
$$Cat(n) = \frac{\binom{2n}{n}}{n+1}$$

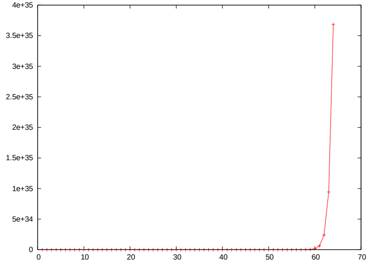
Catalan numbers

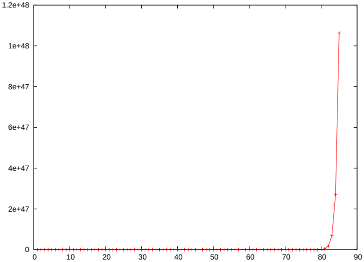
n	catalan(n)
1	1
2	2
3	5
4	14
5	42
6	132
7	429
8	1430
9	4862
10	16796




4e+07
3.5e+07
3e+07
2.5e+07
2e+07
1.5e+07
1e+07
5e+06
0
0 2 4 6 8 10 12 14 16


6e+16
5e+16
4e+16
3e+16
2e+16
1e+16
0
0 5 10 15 20 25 30 35


4e+35
3.5e+35
3e+35
2.5e+35
2e+35
1.5e+35
1e+35
5e+34
0
0 10 20 30 40 50 60 70


1.2e+48
1e+48
8e+47
6e+47
4e+47
2e+47
0
0 10 20 30 40 50 60 70 80 90

Syntactic Ambiguity

- $Cat(n)$ also provides exactly the number of parses for the sentence: *John saw the man on the hill with the telescope* (generated by the grammar given below, a different grammar will have different number of parses)

$$
\begin{array}{rcl}
S & \rightarrow & NP\ VP \\
NP & \rightarrow & John \mid Det\ N \\
N & \rightarrow & man \mid hill \mid telescope \\
VP & \rightarrow & V\ NP \\
Det & \rightarrow & the
\end{array}
\qquad
\begin{array}{rcl}
VP & \rightarrow & VP\ PP \\
NP & \rightarrow & NP\ PP \\
PP & \rightarrow & P\ NP \\
V & \rightarrow & saw \\
P & \rightarrow & on \mid with
\end{array}
$$

number of parse trees $= \mathrm{Cat}(2+1) = 5$.
With 8 PPs: $\mathrm{Cat}(9) = 4862$ parse trees

Syntactic Ambiguity

- For grammar on previous page,
  number of parse trees $= \text{Cat}(2 + 1) = 5$.
- Why $\text{Cat}(2 + 1)$?
  - For 2 PPs, there are 4 things involved: VP, NP, PP-1, PP-2
  - We want the items over which the grammar imposes **all possible parentheses**
  - The grammar is structured in such a way that each combination with a VP or an NP reduces the set of items over which we obtain all possible parentheses to 3
  - This can be viewed schematically as VP * NP * PP-1 * PP-2
    1. (VP (NP (PP-1 PP-2)))
    2. (VP ((NP PP-1) PP-2))
    3. ((VP NP) (PP-1 PP-2))
    4. ((VP (NP PP-1)) PP-2)
    5. (((VP NP) PP-1) PP-2)
  - Note that combining PP-1 and PP-2 is valid because PP-1 has an NP inside it.

Syntactic Ambiguity

- Other sub-grammars are simpler. For chains of adjectives: *cross-eyed pot-bellied ugly hairy professor* We can write the following grammar, and compute the power series:

$$
\begin{aligned}
ADJP &\rightarrow adj\ ADJP \mid \epsilon \\
ADJP &= 1 + adj + adj^2 + adj^3 + \ldots
\end{aligned}
$$

Syntactic Ambiguity

- Now consider power series of combinations of sub-grammars:
  $\texttt{S} = \texttt{NP} \cdot \texttt{VP}$

  `( The number of products over sales ... )`
  `( is near the number of sales ... )`

- Both the `NP` subgrammar and the `VP` subgrammar power series have Catalan coefficients

Syntactic Ambiguity

- The power series for the `S` $\rightarrow$ `NP VP` grammar is the multiplication:

$$(\ N \sum_i Cat_i \ (\ P \ N \ )^i) \ \cdot \ (\ is \sum_j Cat_j (\ P \ N \ )^j)$$

- In a parser for this grammar, this leads to a cross-product:

$$L \ \times \ R = \{ (\ l, \ r \ ) \mid l \in L \ \& \ r \in R \ \}$$

Syntactic Ambiguity

- A simple change:

  `Is ( The number of products over sales ... )`
  `   ( near the number of sales ... )`

$$
\begin{aligned}
&= \texttt{Is}\ N \sum_i Cat_i\ (\ P\ N\ )^i) \cdot (\sum_j Cat_j(\ P\ N\ )^j) \\
&= \texttt{Is}\ N \sum_i \sum_j Cat_i\ Cat_j(\ P\ N\ )^{i+j} \\
&= \texttt{Is}\ N \sum_{i+j} Cat_{i+j+1}\ (\ P\ N\ )^{i+j}
\end{aligned}
$$

Dealing with Ambiguity

- A CFG for natural language can end up providing exponentially many analyses, approx $n!$, for an input sentence of length n
- Much worse than the worst case in the part of speech tagging case, which was n^m for m distinct part of speech tags
- If we actually have to process all the analyses, then our parser might as well be exponential
- Typically, we can directly use the compact description (in the case of CKY, the chart or 2D array, also called a *forest*)

Dealing with Ambiguity

- Solutions to this problem:
  - CKY algorithm: computes all parses in $\mathcal{O}(n^3)$ time. Problem is that worst-case and average-case time is the same.
  - Earley algorithm: computes all parses in $\mathcal{O}(n^3)$ time for arbitrary CFGs,
    $\mathcal{O}(n^2)$ for unambiguous CFGs, and $\mathcal{O}(n)$ for so-called bounded-state CFGs (e.g. $S \rightarrow aSa \mid bSb \mid aa \mid bb$ which generates palindromes over the alphabet a, b).
    Also, average case performance of Earley is better than CKY.
  - Deterministic parsing: only report one parse. Two options: top-down (LL parsing) or bottom-up (LR or shift-reduce) parsing

Shift-Reduce Parsing

- Every CFG has an equivalent pushdown automata: a finite state machine which has additional memory in the form of a stack
- Consider the grammar: $NP \rightarrow Det\ N$, $Det \rightarrow the$, $N \rightarrow dogs$
- Consider the input: *the dogs*
- shift the first word *the* into the stack, check if the top n symbols in the stack matches the right hand side of a rule in which case you can **reduce** that rule, or optionally you can shift another word into the stack

Shift-Reduce Parsing

- reduce using the rule $Det \rightarrow the$, and push Det onto the stack
- shift *dogs*, and then reduce using $N \rightarrow dogs$ and push N onto the stack
- the stack now contains Det, N which matches the rhs of the rule $NP \rightarrow Det\ N$ which means we can reduce using this rule, pushing NP onto the stack
- If NP is the start symbol and since there is no more input left to shift, we can accept the string
- Can this grammar get stuck (that is, there is no shift or reduce possible at some stage while parsing) on a valid string?
- What happens if we add the rule $NP \rightarrow dogs$ to the grammar?

Shift-Reduce Parsing

- Sometimes humans can be "led down the garden-path" when processing a sentence (from left to right)
- Such garden-path sentences lead to a situation where one is forced to backtrack because of a commitment to only one out of many possible derivations
- Consider the sentence:
  The emergency crews hate most is domestic violence.
- Consider the sentence:
  The horse raced past the barn fell

Shift-Reduce Parsing

- Once you process the word *fell* you are forced to reanalyze the previous word *raced* as being a verb inside a *relative clause*: *raced past the barn*, meaning *the horse that was raced past the barn*
- Notice however that other examples with the same structure but different words do not behave the same way.
- For example:
  the flowers delivered to the patient arrived

Earley Parsing

- Earley Parsing is a more advanced form of CKY parsing with two novel ideas:
  - A *dotted rule* as a way to get around the explicit conversion of a CFG to Chomsky Normal Form
  - Do not explore every single element in the CKY parse chart. Instead use goal-directed search
- Since natural language grammars are quite large, and are often modified to be able to parse more data, avoiding the explicit conversion to CNF is an advantage
- A dotted rule denotes that the right hand side of a CF rule has been partially recognized/parsed
- By avoiding the explicit n^3 loop of CKY, we can parse some grammars more efficiently, in time n^2 or n.
- Goal-directed search can be done in any order including left to right (more psychologically plausible)

Earley Parsing

- $S \rightarrow \bullet NP\ VP$ indicates that once we find an NP and a VP we have recognized an S
- $S \rightarrow NP\ \bullet\ VP$ indicates that we've recognized an NP and we need a VP
- $S \rightarrow NP\ VP\ \bullet$ indicates that we have a complete S
- Consider the dotted rule $S \rightarrow \bullet NP\ VP$ and assume our CFG contains a rule $NP \rightarrow John$
  Because we have such an NP rule we can **predict** a new dotted rule $NP \rightarrow \bullet\ John$

Earley Parsing

- If we have the dotted rule: $NP \rightarrow \bullet\ John$ and the next input symbol on our *input tape* is the word *John* we can **scan** the input and create a new dotted rule $NP \rightarrow John\ \bullet$
- Consider the dotted rule $S \rightarrow \bullet NP\ VP$ and $NP \rightarrow John\ \bullet$ Since *NP* has been completely recognized we can **complete** $S \rightarrow NP\ \bullet\ VP$
- These three steps: *predictor*, *scanner* and *completer* form the *Earley parsing algorithm* and can be used to parse using any CFG without conversion to CNF
  Note that we have not accounted for ϵ in the *scanner*

Earley Parsing

- A *state* is a dotted rule plus a span over the input string, e.g. $(S \rightarrow NP \bullet VP, [4, 8])$ implies that we have recognized an NP
- We store all the states in a *chart* – in *chart[j]* we store all states of the form: $(A \rightarrow \alpha \bullet \beta, [i, j])$, where $\alpha, \beta \in (N \cup T)^*$

Earley Parsing

- Note that $(S \rightarrow NP\ \bullet\ VP, [0,8])$ implies that in the chart there are two states $(NP \rightarrow \alpha\ \bullet, [0,8])$ and $(S \rightarrow \bullet\ NP\ VP, [0,0])$ — this is the *completer* rule, the heart of the Earley parser
- Also if we have state $(S \rightarrow \bullet\ NP\ VP, [0,0])$ in the chart, then we always *predict* the state $(NP \rightarrow \bullet\ \alpha, [0,0])$ for all rules $NP \rightarrow \alpha$ in the grammar

Earley Parsing

$$
\begin{array}{rcl}
S & \rightarrow & NP\ VP \\
NP & \rightarrow & Det\ N \mid NP\ PP \mid John \\
Det & \rightarrow & the \\
N & \rightarrow & cookie \mid table \\
VP & \rightarrow & VP\ PP \mid V\ NP \mid V \\
V & \rightarrow & ate \\
PP & \rightarrow & P\ NP \\
P & \rightarrow & on
\end{array}
$$

Consider the input: *0 John 1 ate 2 on 3 the 4 table 5*
What can we predict from the state $(S \rightarrow \bullet\ NP\ VP, [0, 0])$?
What can we complete from the state $(V \rightarrow ate\ \bullet, [1, 2])$?

Earley Parsing

- enqueue(state, j):

  input: state $= (A \rightarrow \alpha \bullet \beta, [i, j])$
  input: j (insert state into chart[j])
  if state not in chart[j] **then**
  chart[j].add(state)
  end if

- predictor(state):

  input: state $= (A \rightarrow B \bullet C, [i, j])$
  for all rules $C \rightarrow \alpha$ in the grammar **do**
  newstate $= (C \rightarrow \bullet \alpha, [j, j])$
  enqueue(newstate, j)
  end for

Earley Parsing

- scanner(state, tokens):

  input: state $= (A \rightarrow B \bullet a\ C, [i, j])$
  input: tokens (list of input tokens to the parser)
  if tokens[j] $==$ a **then**
  newstate $= (A \rightarrow B\ a \bullet C, [i, j+1])$
  enqueue(newstate, j+1)
  end if

- completer(state):

  input: state $= (A \rightarrow B\ C \bullet, [j, k])$
  for all rules $X \rightarrow Y \bullet A\ Z, [i, j]$ in chart[j] **do**
  newstate $= (X \rightarrow Y\ A \bullet Z, [i, k])$
  enqueue(newstate, k)
  end for

Earley Parsing

- earley(tokens[0 ... N], grammar):

```
for each rule S → α where S is the start symbol do
    add (S → • α, [0, 0]) to chart[0]
end for
for 0 ≤ j ≤ N + 1 do
    for state in chart[j] that has not been marked do
        mark state
        if state = (A → α • B β, [i, j]) then
            predictor(state)
        else if state = (A → α • b β, [i, j]), j < N + 1 then
            scanner(state, tokens)
        else
            completer(state)
        end if
    end for
end for
return yes if chart[N + 1] has a final state
```

Earley Parsing

- isIncomplete(state):

  if state is of type $(A \rightarrow \alpha \bullet, [i,j])$ **then**

  return False

  end if

  return True

- nextCategory(state):

  if state $== (A \rightarrow B \ \bullet \ \nu \ C, [i,j])$ **then**

  return ν (ν can be terminal or non-terminal)

  else

  raise error

  end if

Earley Parsing

- isFinal(state):

  input: state $= (A \rightarrow \alpha \bullet, [i, j])$
  cond1 $=$ A is a start symbol
  cond2 $=$ isIncomplete(state) is False
  cond3 $=$ j is equal to length(tokens)
  if cond1 and cond2 and cond3 **then**
      return True
  end if
  return False

- isToken(category):

  if category is terminal symbol **then**
      return True
  end if
  return False