CMPT 379
Compilers

Anoop Sarkar
http://www.cs.sfu.ca/~anoop

Lexical Analysis

- Also called *scanning*, take input program *string* and convert into tokens
- Example:

```
double f = sqrt(-1);
```

```
T_DOUBLE      ("double")
T_IDENT       ("f")
T_OP          ("=")
T_IDENT       ("sqrt")
T_LPAREN      ("(")
T_OP          ("-")
T_INTCONSTANT ("1")
T_RPAREN      (")")
T_SEP         (";")
```


Token Attributes

- Some tokens have attributes
  - T_IDENT "sqrt"
  - T_INTCONSTANT 1
- Other tokens do not
  - T_WHILE
- *Token*=T_IDENT, *Lexeme*="sqrt", *Pattern*
- Source code location for error reports

Lexical errors

- What if user omits the space in "doublef"?
  - No lexical error, single token T_IDENT("doublef") is produced instead of sequence T_DOUBLE, T_IDENT("f")!
- Typically few lexical error types
  - E.g., illegal chars, opened string constants or comments that are not closed

Implementing Lexers: Loop and switch scanners

- Ad hoc scanners
- Big nested switch/case statements
- Lots of getc()/ungetc() calls
  - Buffering
- Can be error-prone, use only if
  - Your language's lexical structure is simple
  - Tools don't do what you want
- Changing or adding a keyword is problematic
- Key idea: separate the defn from the implementation
- Problem: we need to reason about patterns and how they can be used to define tokens (recognize strings).

Formal Languages: Recap

- Symbols: a, b, c
- Alphabet : finite set of symbols $\Sigma = \{a, b\}$
- String: sequence of symbols bab
- Empty string: ε Define: $\Sigma^{\varepsilon} = \Sigma \cup \{\varepsilon\}$
- Set of all strings: Σ^* cf. *The Library of Babel*, Jorge Luis Borges
- (Formal) Language: a set of strings

  $\{ a^n b^n : n > 0 \}$

Regular Languages

- The set of regular languages: each element is a regular language
- Each regular language is an example of a (formal) language, i.e. a set of strings

  e.g. $\{ a^m b^n : m, n \text{ are +ve integers} \}$

Regular Languages

- Defining the set of all regular languages:
  - The empty set and $\{a\}$ for all a in Σ^{ε} are regular languages
  - If L_1 and L_2 and L are regular languages, then:

    $L_1 \cdot L_2 = \{xy \mid x \in L_1 \text{ and } y \in L_2\}$ (concatenation)

    $L_1 \cup L_2$ (union)

    $L^* = \cup_{i=0}^{\infty} L^i$ (Kleene closure)

    are also regular languages
  - There are no other regular languages

Formal Grammars

- A formal grammar is a concise description of a formal language
- A formal grammar uses a specialized syntax
- For example, a **regular expression** is a concise description of a regular language

*(a|b)*abb* : is the set of all strings over the alphabet {*a*, *b*} which end in *abb*

Regular Expressions: Definition

- Every symbol of $\Sigma \cup \{ \varepsilon \}$ is a regular expression
- If r_1 and r_2 are regular expressions, so are
  - Concatenation: $r_1 \, r_2$
  - Alternation: $r_1 | r_2$
  - Repetition: r_1*
- Nothing else is.
  - Grouping re's: e.g. aa|bc vs. ((aa)|b)c

Regular Expressions: Examples

- Alphabet { 0, 1 }
- All strings that represent binary numbers divisible by 4 (but accept 0) ((0|1)*00)|0
- All strings that do not contain "01" as a substring 1*0*

Regular Expressions

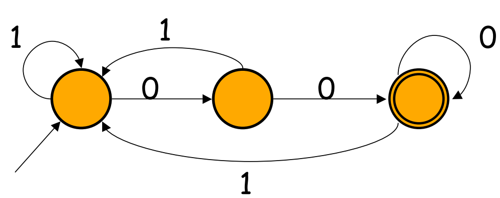
- To describe all lexemes that form a token as a *pattern*
  - (0|1|2|3|4|5|6|7|8|9)+
- Need decision procedure: to which token does a given sequence of characters belong (if any)?
  - Finite State Automata

Finite Automata: Recap

- A set of states S
  - One start state q_0, zero or more final states F
- An alphabet $\sum$ of input symbols
- A transition function:
  - δ: S x $\Sigma \Rightarrow$ S
- Example: $\delta(1, a) = 2$

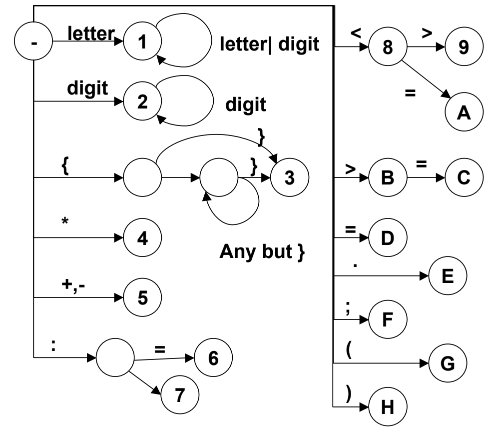
Finite Automata: Example

- What regular expression does this automaton accept?



Answer: (0|1)*00

FA: Pascal Example



Building a Lexical Analyzer

- Token $\Rightarrow$ Pattern
- Pattern $\Rightarrow$ Regular Expression
- Regular Expression $\Rightarrow$ NFA
- NFA $\Rightarrow$ DFA
- DFA $\Rightarrow$ Lexical Analyzer

NFAs

- NFA: like a DFA, except
  - A transition can lead to more than one state, that is, $\delta: S \times \Sigma \Rightarrow 2^S$
  - One state is chosen non-deterministically
  - Transitions can be labeled with ε, meaning states can be reached without reading any input, that is,

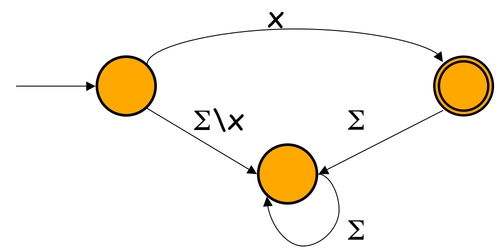
$$\delta: S \times \Sigma \cup \{ \varepsilon \} \Rightarrow 2^S$$

Thompson's construction

- Converts regexps to NFA
- Five simple rules
  - Symbols
  - Empty String
  - Alternation (r_1 or r_2)
  - Concatenation (r_1 followed by r_2)
  - Repetition (r_1*)

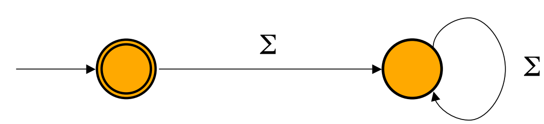
Thompson Rule 1

- For each symbol *x* of the alphabet, there is a NFA that accepts it (include a *sinkhole* state)



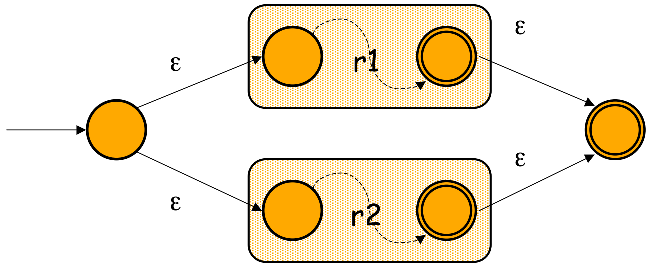
Thompson Rule 2

- There is an NFA that accepts only ε



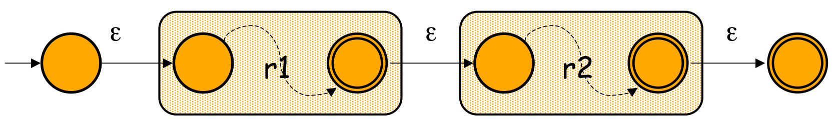
Thompson Rule 3

- Given two NFAs for r_1, r_2, there is a NFA that accepts $r_1|r_2$



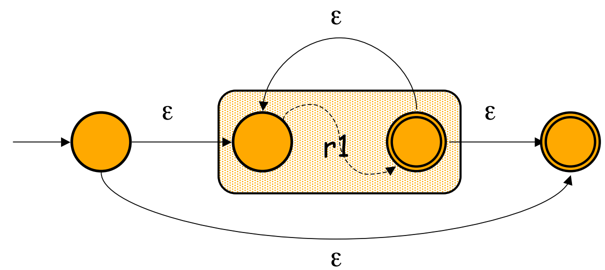
Thompson Rule 4

- Given two NFAs for r_1, r_2, there is a NFA that accepts r_1r_2



Thompson Rule 5

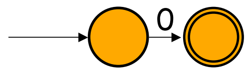
- Given a NFA for r_1, there is an NFA that accepts r_1*

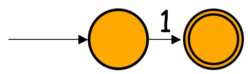


Example

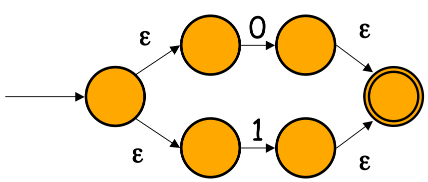
- Set of all binary strings that are divisible by four (include 0 in this set)
- Defined by the regexp: ((0|1)*00) | 0
- Apply Thompson's Rules to create an NFA

Basic Blocks 0 and 1

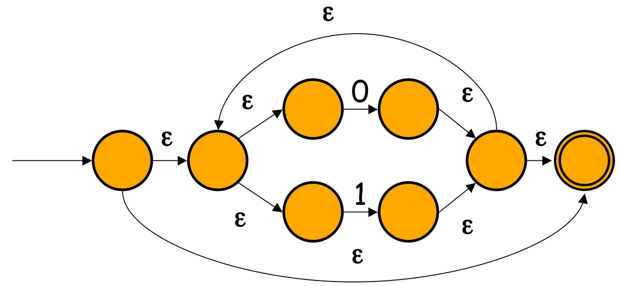
- 0 

- 1 

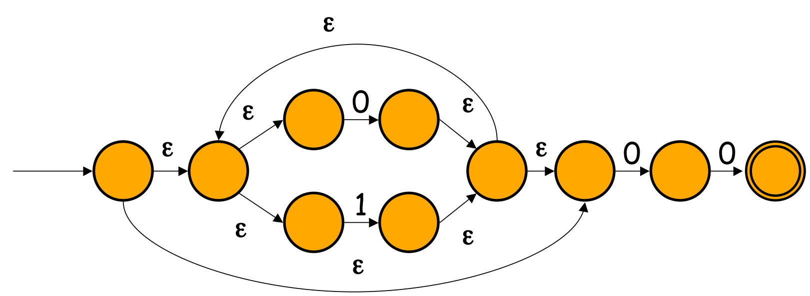
(this version does not report errors: no *sinkholes*)



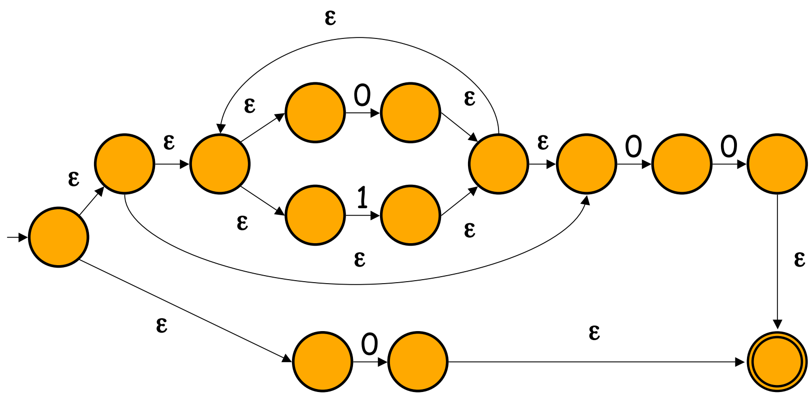
0|1



(0|1)*



(0|1)*00



((0|1)*00)|0

Simulating NFAs

- Similar to DFA simulation
- But have to deal with ε transitions and multiple transitions on the same input
- Instead of one state, we have to consider *sets* of states
- Simulating NFAs is a problem that is closely linked to converting a given NFA to a DFA

NFA to DFA Conversion

- Subset construction
- Idea: subsets of set of all NFA states are *equivalent* and become one DFA state
- Algorithm simulates movement through NFA
- Key problem: how to treat ε-transitions?

ε-Closure

- Start state: q_0
- ε-closure(S): S is a set of states

$$\begin{array}{l} \textbf{initialize: } S \leftarrow \{q_0\} \\ T \leftarrow S \\ \textbf{repeat } T' \leftarrow T \\ \quad T \leftarrow T' \cup [\cup_{s \in T'} \textbf{move}(s, \epsilon)] \\ \textbf{until } T = T' \end{array}$$

ε-Closure (T: set of states)

```
push all states in T onto stack
initialize ε-closure(T) to T
while stack is not empty do begin
    pop t off stack
    for each state u with u ∈ move(t, ε) do
        if u ∉ ε-closure(T) do begin
            add u to ε-closure(T)
            push u onto stack
        end
end
```


NFA Simulation

- After computing the ε-*closure* move, we get a set of states
- On some input extend all these states to get a new set of states

$$\mathbf{DFAedge}(T, c) = \epsilon\textbf{-closure}\left(\cup_{q \in T} \mathbf{move}(q, c)\right)$$

NFA Simulation

- Start state: q_0
- Input: $c_1, \ldots, c_k$

$$T \leftarrow \epsilon\textbf{-closure}(\{q_0\})$$
$$\textbf{for } i \leftarrow 1 \textbf{ to } k$$
$$\quad T \leftarrow \mathbf{DFAedge}(T, c_i)$$

Conversion from NFA to DFA

- Conversion method closely follows the NFA simulation algorithm
- Instead of simulating, we can collect those NFA states that behave identically on the same input
- Group this set of states to form one state in the DFA

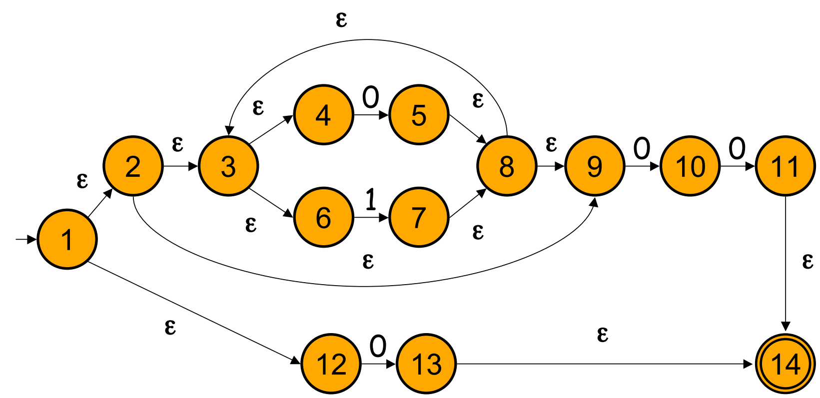
Subset Construction

add ε-*closure*(q_0) to *Dstates* unmarked
while ∃ unmarked T ∈ *Dstates* **do begin**
    mark T;
    for each symbol *c* **do begin**
        U := **ε**-*closure*(**move**(T, *c*));
        if U ∉ *Dstates* **then**
            add U to *Dstates* unmarked
        Dtrans[d, *c*] := U;
    end
end

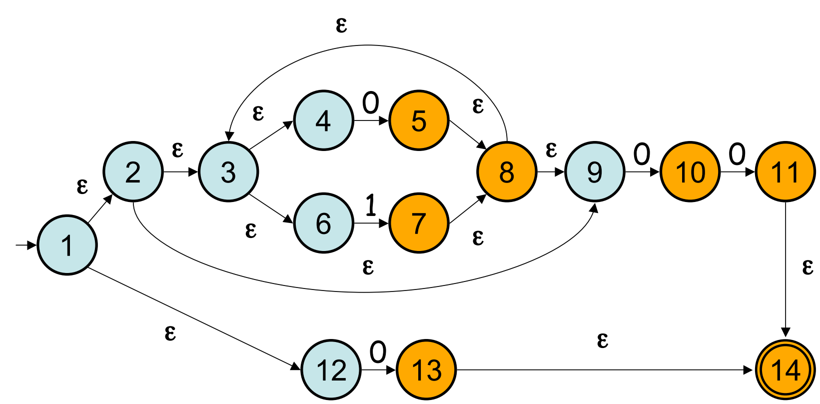
Subset Construction

states[0] = ε-**closure**({q_0})
p = j = 0
while j ≤ p **do begin**
    for each symbol *c* **do begin**
        e = **DFAedge**(states[j], c)
        if e = states[i] for some i ≤ p
        then Dtrans[j, c] = i
        else p = p+1
            states[p] = e
            Dtrans[j, c] = p
    j = j + 1
    end
end

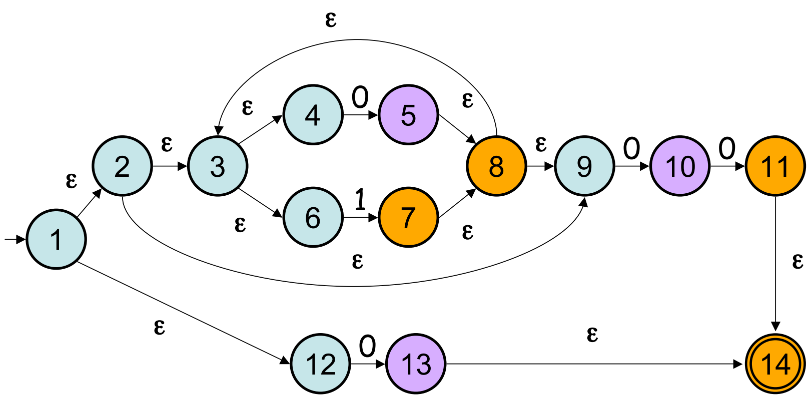
Example: subset construction



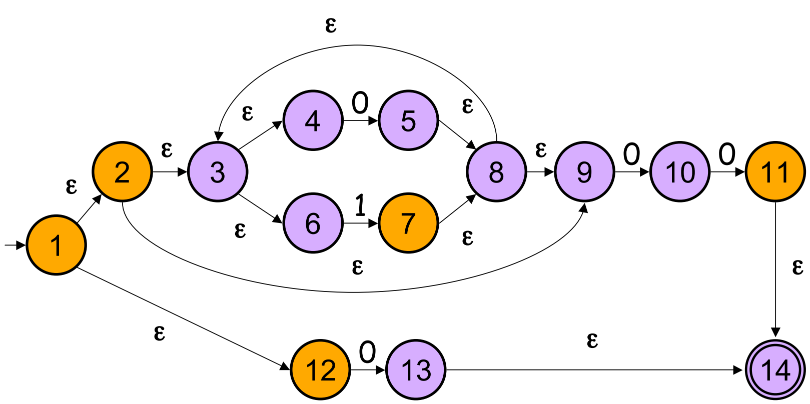
ε-*closure*(q_0)



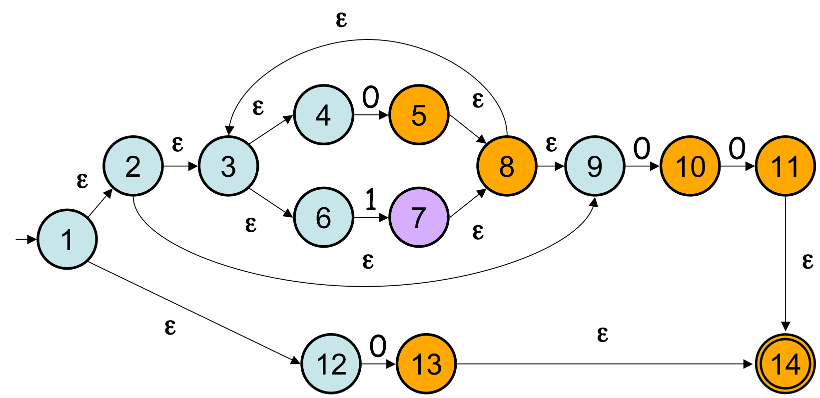
move(ε-*closure*(q_0), 0)



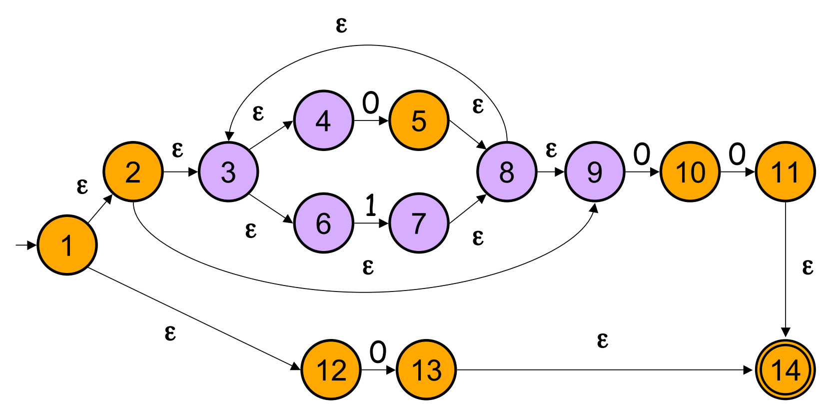
ε-*closure*(move(ε-*closure*(q_0), 0))



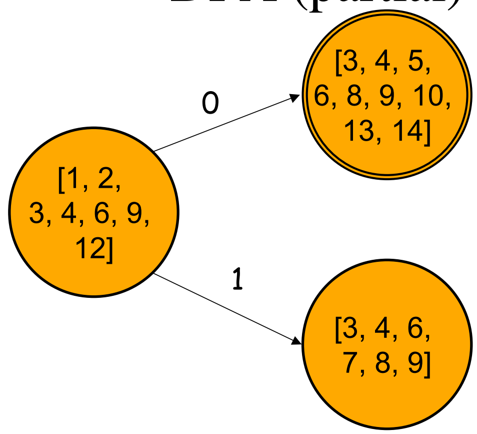
move(ε-*closure*(q_0), 1)



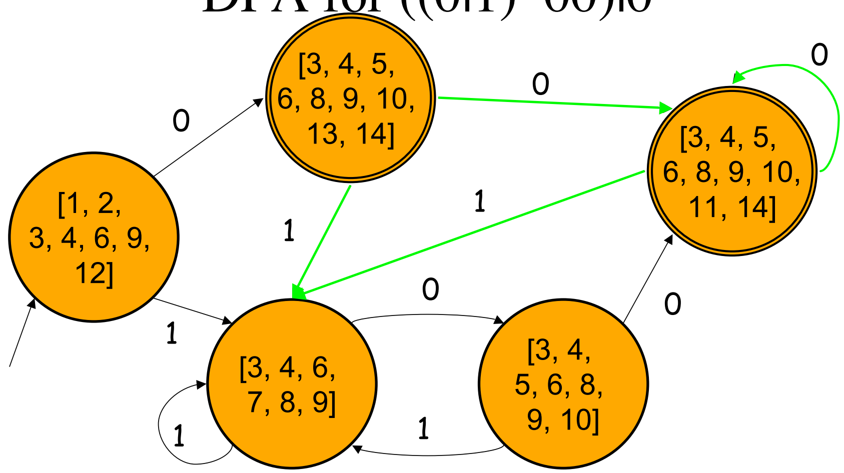
ε-*closure*(move(ε-*closure*(q_0), 1))



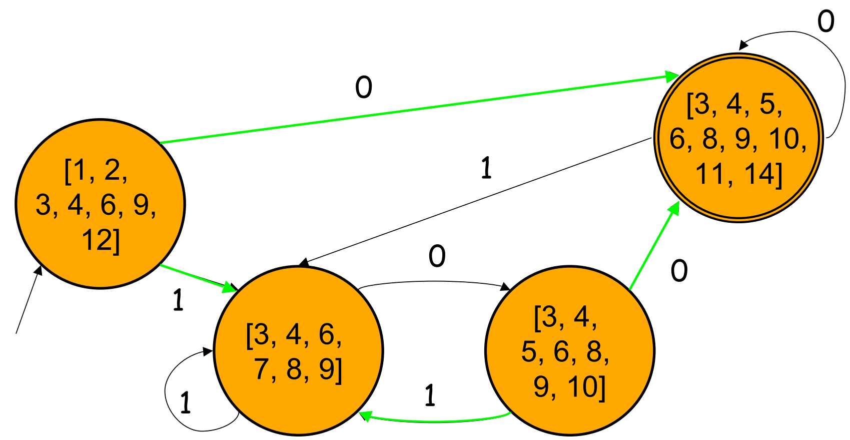
DFA (partial)



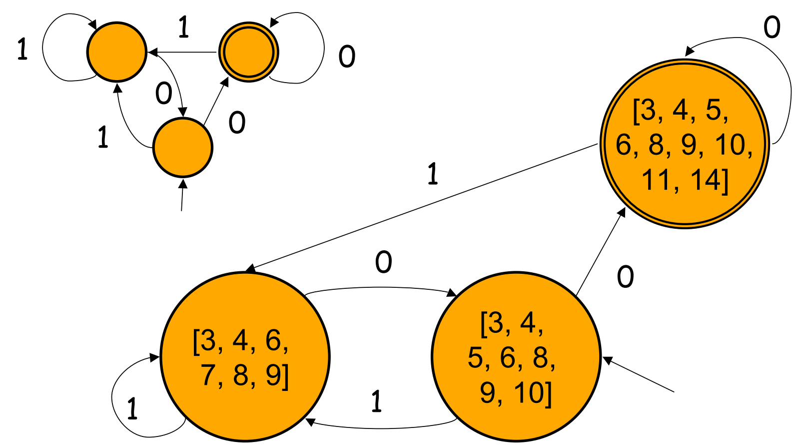
DFA for ((0|1)*00)|0



Minimization (I)

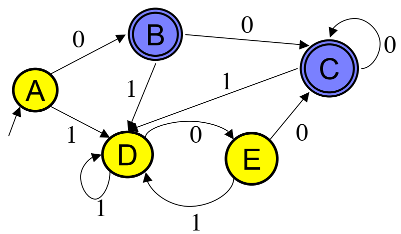


Minimization (II)



Minimization of DFAs

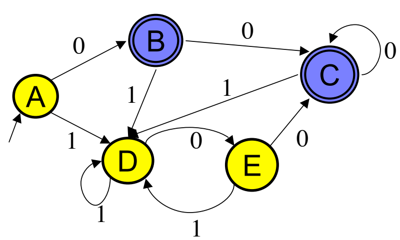
- Algorithm for minimizing the number of states in a DFA
- Step 1: partition states into 2 groups: accepting and non-accepting



Minimization of DFAs

- Step 2: in each group, find a sub-group of states having property P
- P: The states have transitions on each symbol (in the alphabet) to the *same* group

A, 0: blue
A, 1: yellow
E, 0: blue
E, 1: yellow
D, 0: yellow
D, 1: yellow

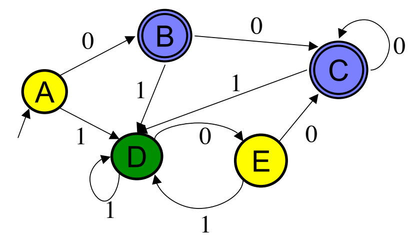


B, 0: blue
B, 1: yellow
C, 0: blue
C, 1: yellow

Minimization of DFAs

- Step 3: if a sub-group does not obey P split up the group into a separate group
- Go back to step 2. If no further sub-groups emerge then continue to step 4

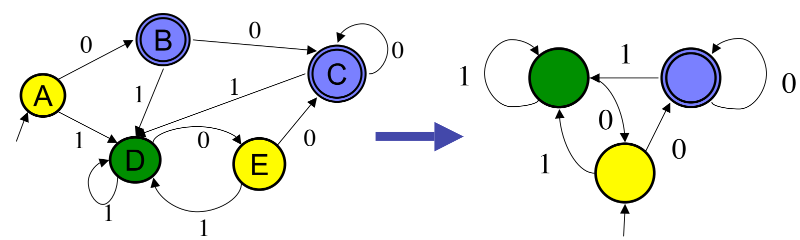
A, 0: blue
A, 1: green
E, 0: blue
E, 1: green
D, 0: yellow
D, 1: green



B, 0: blue
B, 1: green
C, 0: blue
C, 1: green

Minimization of DFAs

- Step 4: each group becomes a state in the minimized DFA
- Transitions to individual states are mapped to a single state representing the group of states



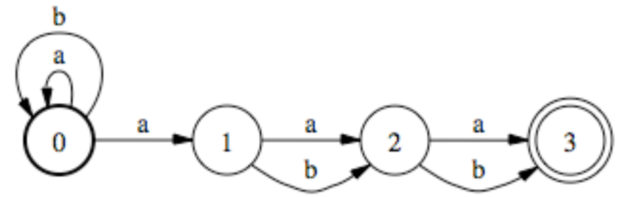
NFA to DFA

- Subset construction converts NFA to DFA
- Complexity:
  - in programs we measure time complexity in number of steps
  - For FSAs, we measure complexity in terms of the number of states

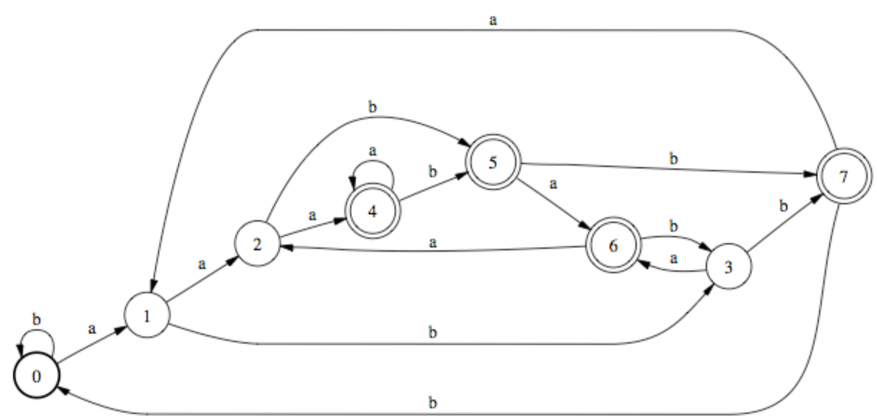
NFA to DFA

- Problem: An *n* state NFA can sometimes become a 2^n state DFA, an exponential increase in complexity
  - Try the subset construction on NFA built for the regexp $\mathbf{A^*aA^{n-1}}$ where **A** is the regexp **(a|b)**
- Minimization can reduce the number of states
- But minimization requires determinization

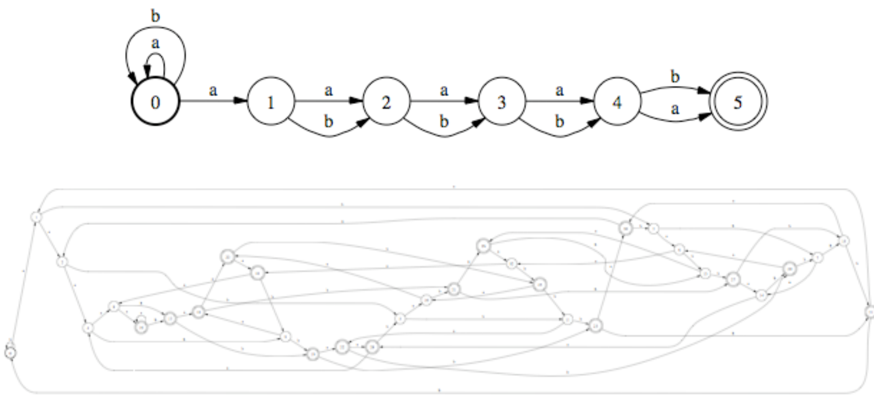
NFA to DFA



NFA to DFA



NFA to DFA

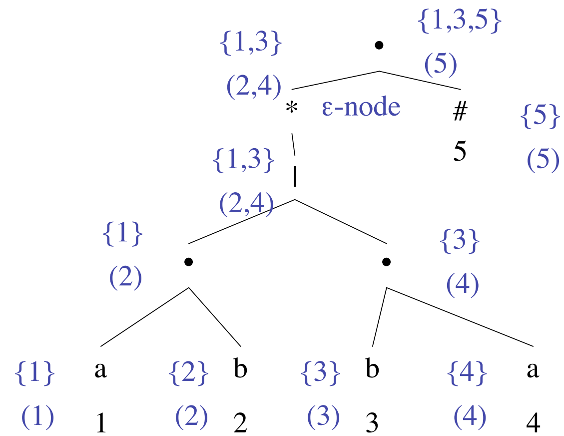


$2^5 = 32$ states

NFA vs. DFA in the wild

Engine Type	**Programs**
DFA	*awk* (most versions), *egrep* (most versions), *flex*, *lex*, MySQL, Procmail
Traditional NFA	GNU *Emacs*, Java, *grep* (most versions), *less*, *more*, .NET languages, PCRE library, Perl, PHP (pcre routines), Python, Ruby, *sed* (most versions), vi
POSIX NFA	*mawk*, MKS utilities, GNU *Emacs* (when requested)
Hybrid NFA/DFA	GNU *awk*, GNU *grep*/*egrep*, Tcl

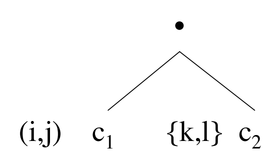
Regexp to DFA: `(ab|ba)*#`



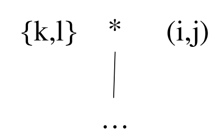
firstpos = {}
lastpos = ()

Regexp to DFA: *followpos*

- *followpos(p)* tells us which positions can follow a position *p*
- There are two rules that use the *firstpos* {} and *lastpos* () information

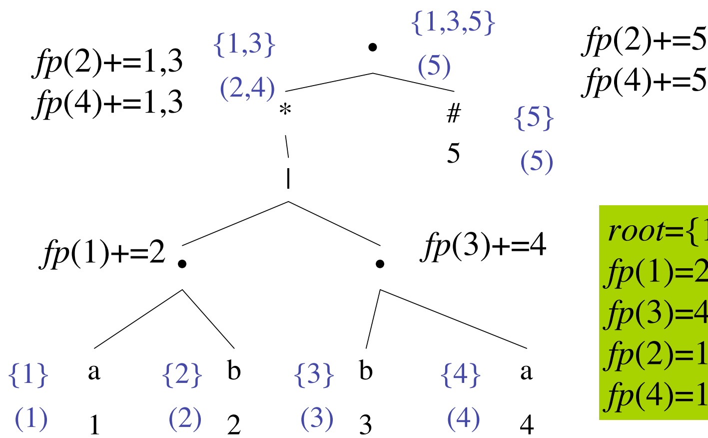


followpos(i)+=k,l
followpos(j)+=k,l



followpos(i)+=k,l
followpos(j)+=k,l

Regexp to DFA: `(ab|ba)*#`

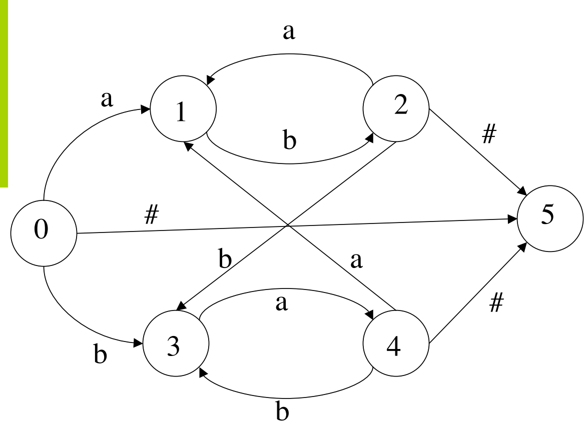


root={1,3,5}
fp(1)=2
fp(3)=4
fp(2)=1,3,5
fp(4)=1,3,5

Regexp to DFA: `(ab|ba)*#`

root={1,3,5}
fp(1)=2
fp(3)=4
fp(2)=1,3,5
fp(4)=1,3,5

1:a
2:b
3:b
4:a
5:#



Regexp to DFA: `(ab|ba)*#`

root={1,3,5}
fp(1)=2
fp(3)=4
fp(2)=1,3,5
fp(4)=1,3,5

{1,3,5} A

A: fp(1),a {2},a B,a

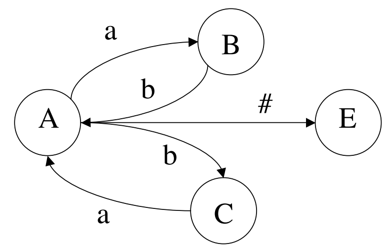
A: fp(3),b {4},b C,b

A: fp(5),# {},# E,#

B: fp(2),b {1,3,5},b A,b

C: fp(4),a {1,3,5},a A,a

1:a
2:b
3:b
4:a
5:#



Equivalence of Regexps

- (R|S)|T == R|(S|T) == R|S|T
- (RS)T == R(ST)
- (R|S) == (S|R)
- R*R* == (R*)* == R* == RR*| ε
- R** == R*
- (R|S)T = RT|ST
- R(S|T) == RS | RT
- (R|S)* == (R*S*)* == (R*S)*R* == (R*|S*)*
- RR* == R*R
- (RS)*R == R(SR)*
- R = R|R = R|ε

Equivalence of Regexps

- 0(10)*1|(01)*
- (01)(01)*|(01)*
- (01)(01)*|(01)(01)*|ε
- (01)(01)*|ε
- (01)*

- (RS)*R == R(SR)*
- RS == (RS)
- R* == RR*|ε
- R == R|R
- R* == RR*| ε

Lexical Analyzer using DFAs

- Each token is defined using a regexp r_i
- Merge all regexps into one big regexp
  - $R = (r_1 \mid r_2 \mid \ldots \mid r_n)$
- Convert *R* to an NFA, then DFA, then minimize
  - remember orig NFA final states with each DFA state

Lexical Analyzer using DFAs

- The DFA recognizer has to find the *longest match* for a token
  - e.g. *<print>* and not *<pr>, <int>*
- If two patterns match the same token, pick the one that was listed earlier in R
  - e.g. prefer final state (in the original NFA) of r_2 over r_3

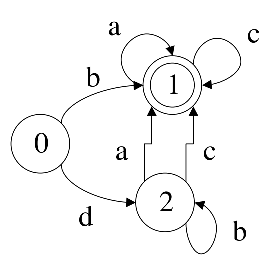
Lexical Analyzer using DFAs

- Alternative method:
  - Organize all the DFAs for each token in an ordered list
  - For input $i_1, i_2, \ldots, i_n$ run all DFAs until some reach a final state (pick the longest match for each DFA)
  - Pick the token for which some DFA could read the longest match in the input,
    - e.g. prefer DFA #8 over all others because it read the input until i_{30} and none of the other DFAs reached i_{30}
  - If two DFAs reach the same input character then pick the one that is listed first in the ordered list

Implementing DFAs

- 2D array storing the transition table
- Adjacency list, more space efficient but slower
- Merge two ideas: array structures used for sparse tables like DFA transition tables
  - base & next arrays: Tarjan and Yao, 1979
  - Dragon book (default+base & next+check)

Implementing DFAs



	a	b	c	d
0	-	1	-	2
1	1	-	1	-
2	1	2	1	-

Implementing DFAs

	a	b	c	d
0	-	1	-	2
1	1	-	1	-
2	1	2	1	-

		-	1	-	2		
				1	-	1	-
1	2	1	-				
1	2	1	1	1	2	1	-
0	1	2	3	4	5	6	7

next

2	2	2	0	1	0	1	-

check

base

0	2
1	4
2	0

nextstate(*s*, *x*) :
L := base[*s*] + *x*
return next[L] **if** check[L] **eq** *s*

Implementing DFAs

	a	b	c	d
0	-	1	-	2
1	1	-	1	-
2	1	2	1	-

	-	1	-	2		
			1	-	1	-
-	2	-	-			
-	2	1	1	2	1	-
0	1	2	3	4	5	6

next

-	2	0	1	0	1	-

check

base

0	2	-
1	3	-
2	0	1

default

nextstate(*s*, *x*) :
L := base[*s*] + *x*
return next[L] **if** check[L] **eq** *s*
else return *nextstate*(default[*s*], *x*)

Summary

- Token $\Rightarrow$ Pattern
- Pattern $\Rightarrow$ Regular Expression
- Regular Expression $\Rightarrow$ NFA
  - Thompson's Rules
- NFA $\Rightarrow$ DFA
  - Subset construction
- DFA $\Rightarrow$ minimal DFA
  - Minimization

$\Rightarrow$ Lexical Analyzer (multiple patterns)