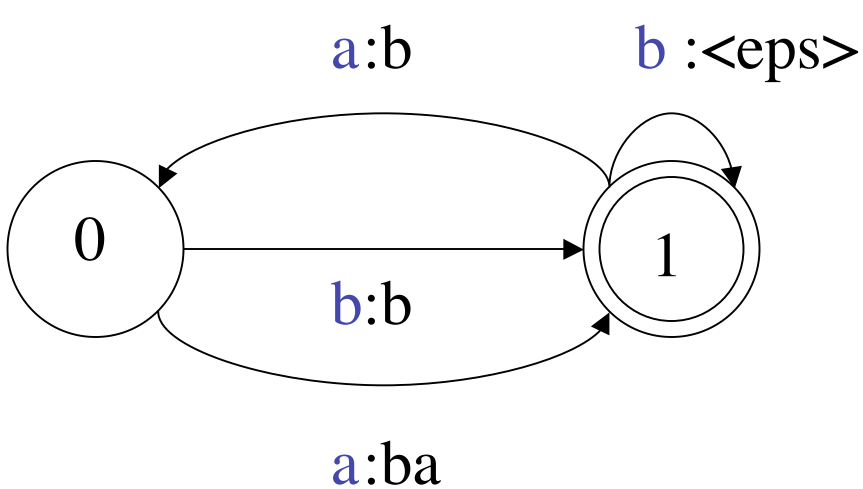
CMPT 825
Natural Language Processing

Anoop Sarkar

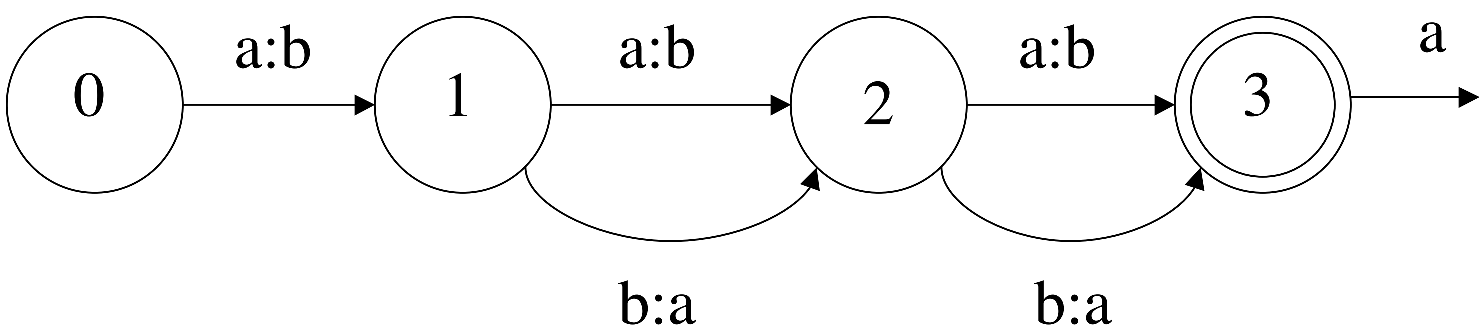
http://www.cs.sfu.ca/~anoop

Sequential transducers

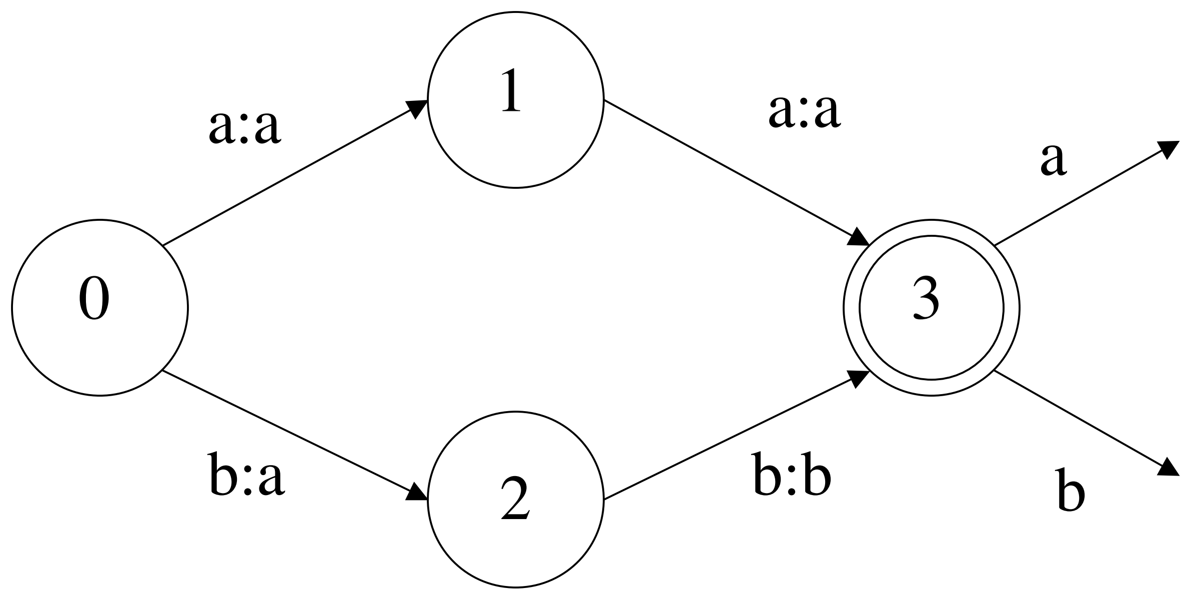


- determinization
- minimization

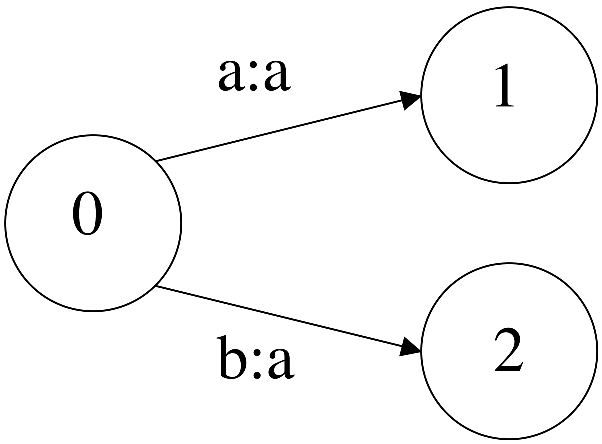
Subsequential transducers

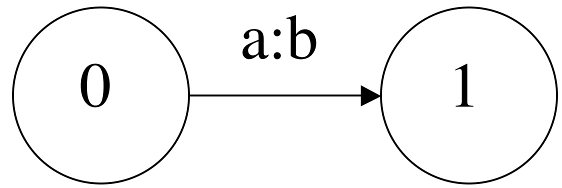


p-subsequential Transducers



Composition: T_1 o T_2



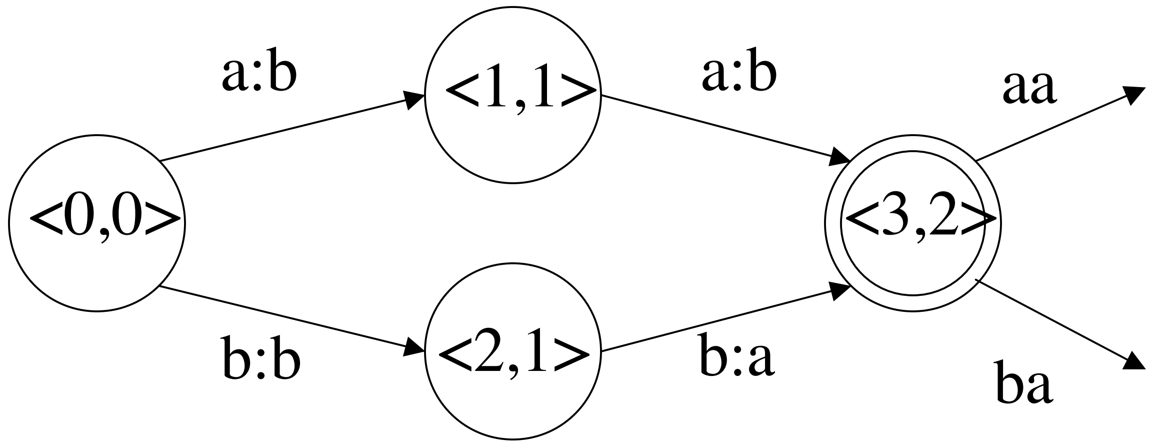


$\langle 0,0 \rangle$
$\langle 0,1 \rangle$
$\langle 1,0 \rangle$
$\langle 1,1 \rangle$
$\langle 2,0 \rangle$
$\langle 2,1 \rangle$

$\langle 0,0 \rangle \rightarrow \langle\, \mathbf{tf}_1(0,a)\ ,\ \mathbf{tf}_2(0,\ \mathbf{out}_1(0,a))\rangle \rightarrow \langle 1,1 \rangle$

$\langle 0,0 \rangle \rightarrow \langle\, \mathbf{tf}_1(0,b)\ ,\ \mathbf{tf}_2(0,\ \mathbf{out}_1(0,b))\rangle \rightarrow \langle 2,1 \rangle$

Composition: T_1 o T_2



$$\langle 0,0\rangle \to \langle\ \mathbf{tf}_1(0,a)\ ,\ \mathbf{tf}_2(0,\ \mathbf{out}_1(0,a))\rangle \to \langle 1,1\rangle$$

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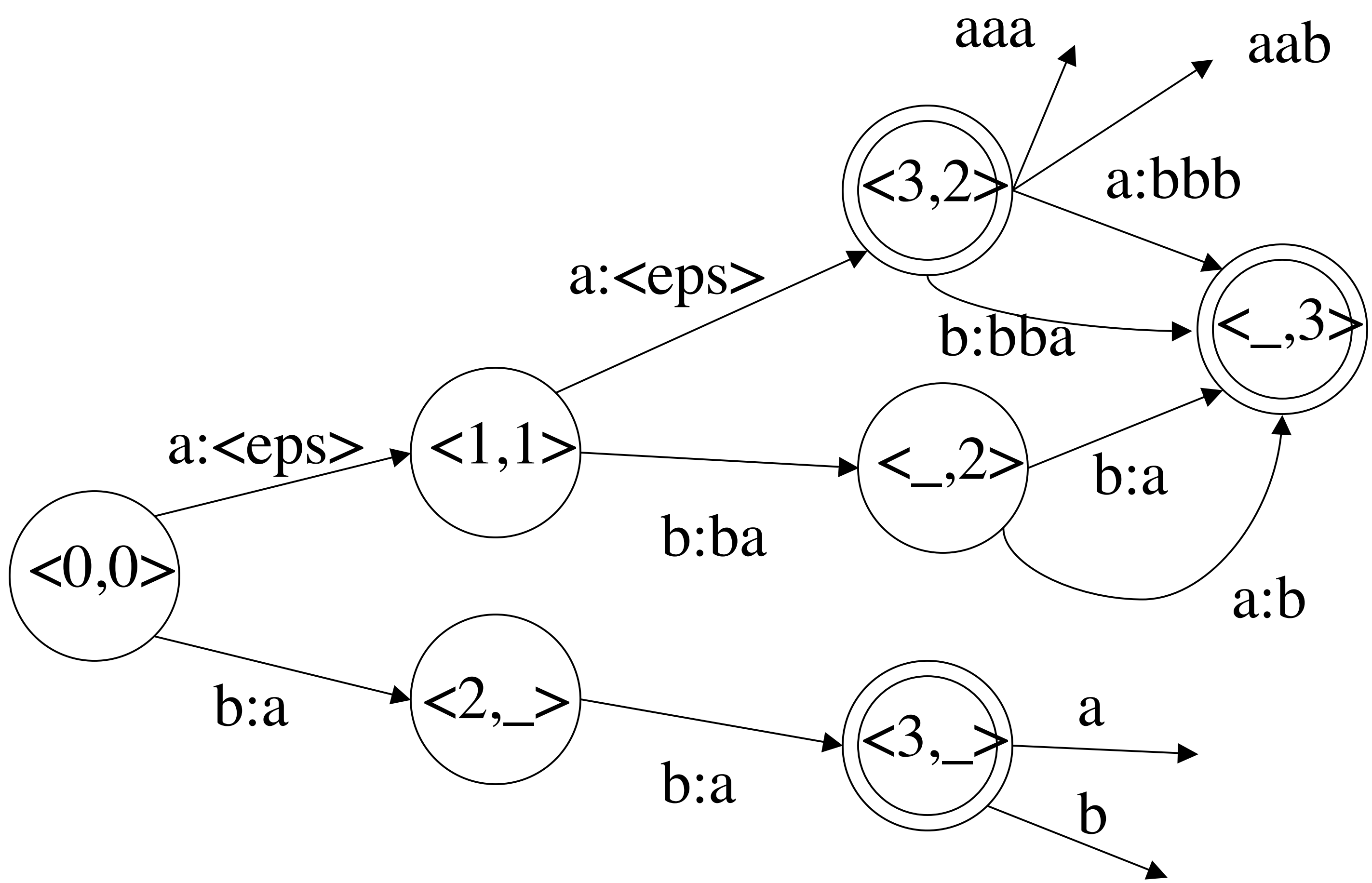
Union: $T_1 + T_2$



Arbitrary Transducers

- We've seen algorithms over subsequential transducers: but what about arbitrary transducers?
- There exist transducers with no sequential equivalent. e.g. $a^{|w|}$ if $|w|$ is even; else $b^{|w|}$
- If f is a transducer, then there is a left sequential l and a right sequential r such that $f = l \bullet r$
- It is decidable if a transducer is sequential
- See details in Mohri 1997.

Slides on Minimization taken from the presentation made by Jason Eisner at the following conference talk

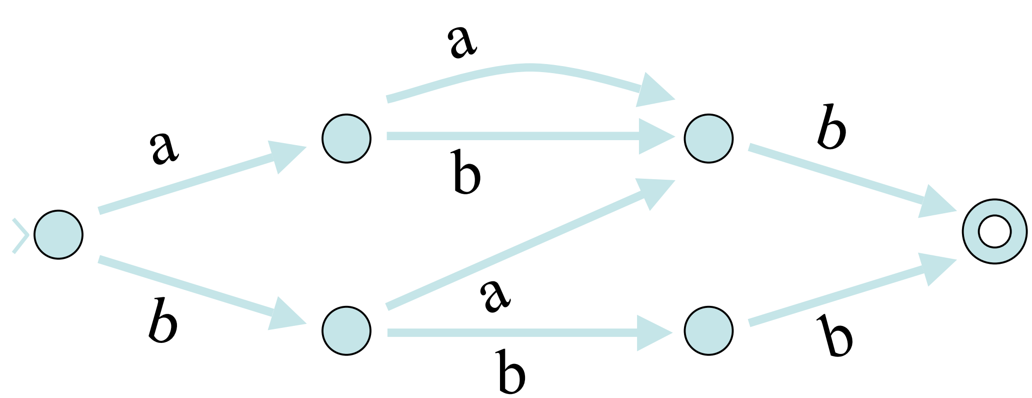
Simpler & More General Minimization for Weighted Finite-State Automata

Jason Eisner
Johns Hopkins University

May 28, 2003 — HLT-NAACL

The Minimization Problem

Input:	A DFA	(deterministic finite-state automaton)
Output:	An equiv. DFA with as few states as possible	
Complexity:	O(|arcs| log |states|)	(Hopcroft 1971)



Represents the language {aab, abb, bab, bbb}

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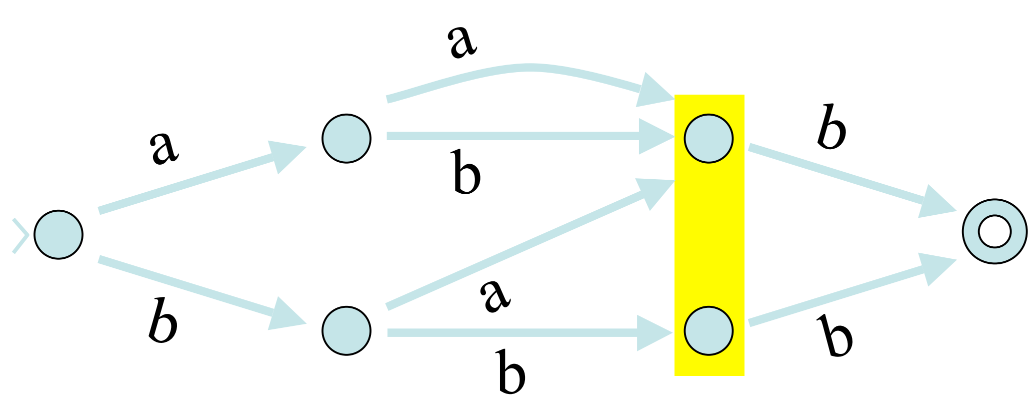

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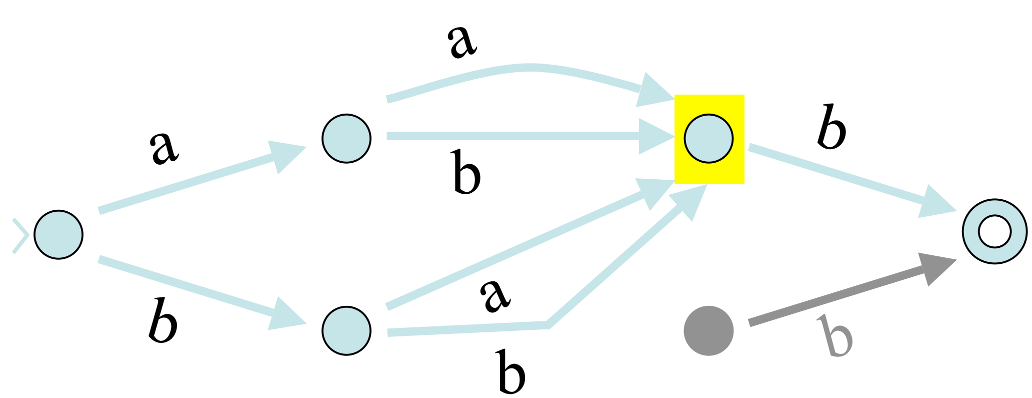

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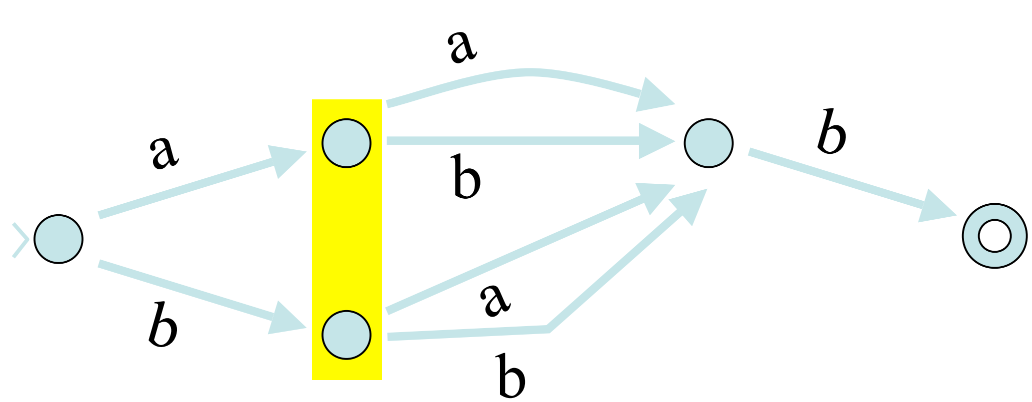

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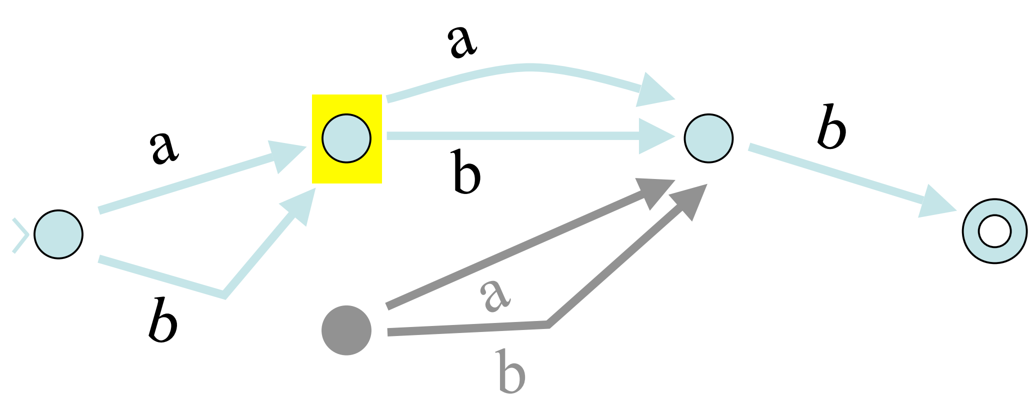

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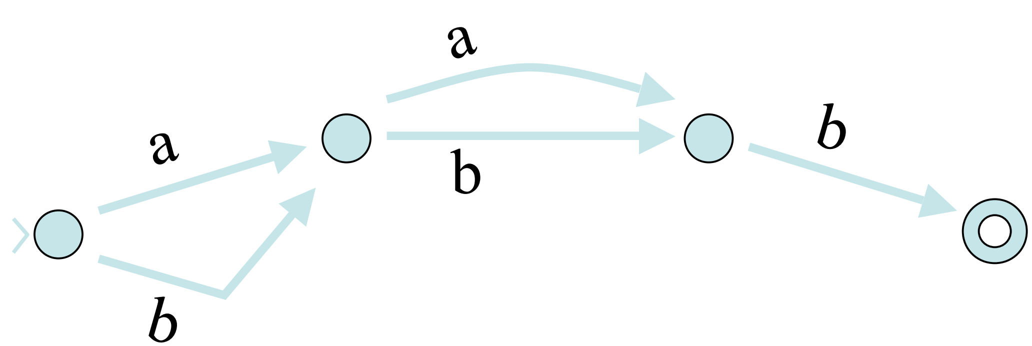


Can't always work backward from final state like this.
A bit more complicated because of cycles.

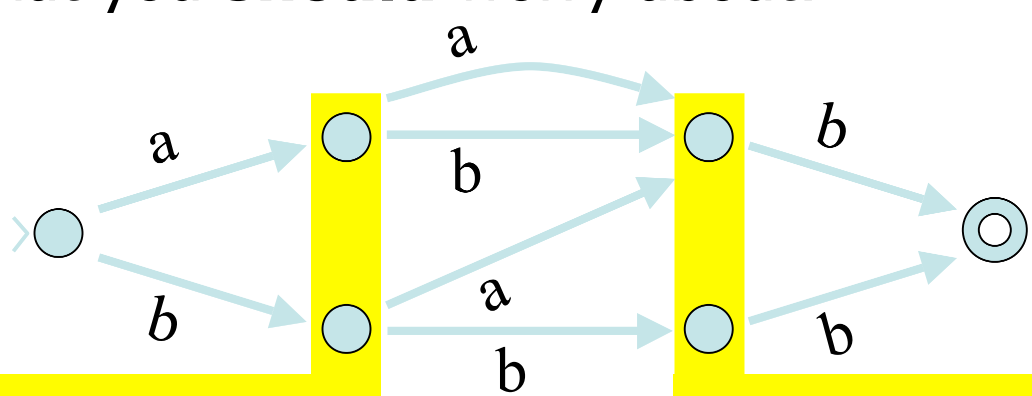
The Minimization Problem

Input: A DFA (deterministic finite-state automaton)

Output: An equiv. DFA with as few states as possible

Complexity: O(|arcs| log |states|) (Hopcroft 1971)

Here's what you **should** worry about:



Mergeable because they have the same **suffix language**: {ab,bb}

Mergeable because they have the same **suffix language**: {b}

An equivalence relation on states … merge the equivalence classes

The Minimization Problem

Input:	A DFA	(deterministic finite-state automaton)
Output:	An equiv. DFA with as few states as possible	
Complexity:	O(|arcs| log |states|)	(Hopcroft 1971)

Q: Why minimize # states, rather than # arcs?

A: Minimizing # states also minimizes # arcs!

Q: What if the input is an NDFA (nondeterministic)?

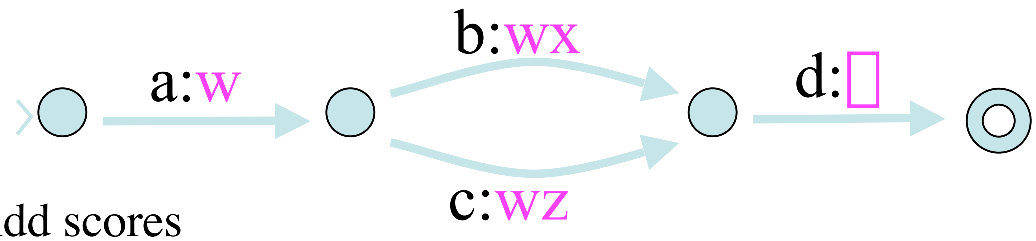
A: Determinize it first. (could yield exponential blowup ☹)

Q: How about minimizing an NDFA to an NDFA?

A: Yes, could be exponentially smaller ☺,
but problem is PSPACE-complete so we don't try. ☹

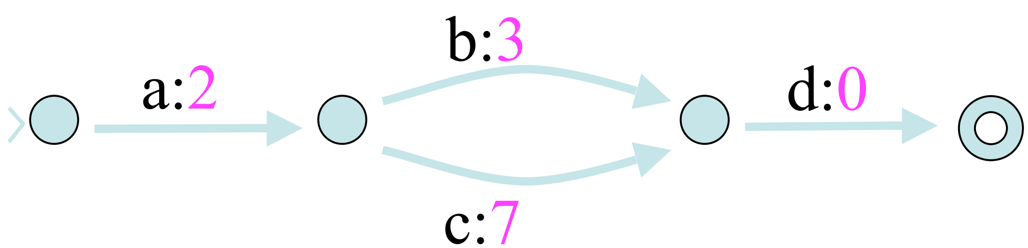
Real-World NLP:
Automata With Weights or Outputs

- Finite-state computation of functions
  - Concatenate strings



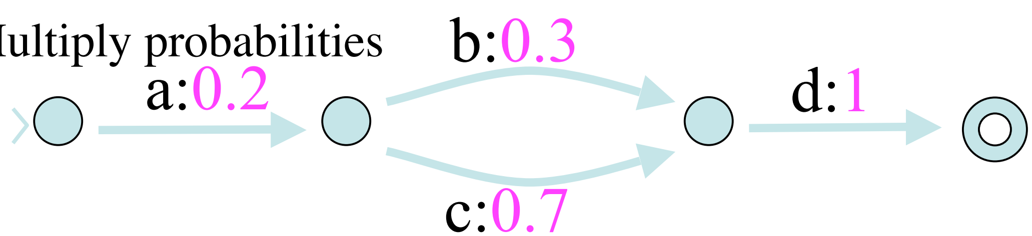
abd □ wwx
acd □ wwz

  - Add scores



abd □ 5
acd □ 9

  - Multiply probabilities

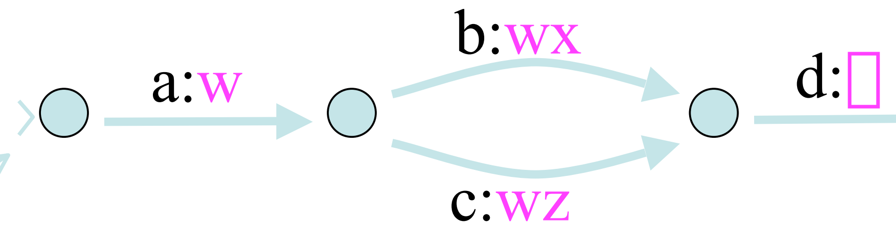


abd □ 0.06
acd □ 0.14

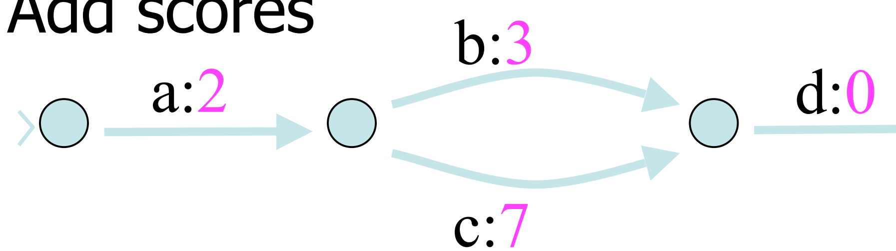
Weight Algebras

- Specify a weight algebra (K,   )
- Define DFAs over (K,   )
- Arcs have weights in set K
- A path's weight is also in K: multiply its arc weights with
- Examples:
  - (strings, concatenation)
  - (scores, addition)
  - (probabilities, multiplication)
  - (score vectors, addition)
  - (real weights, multiplication)
  - (objective func & gradient, product-rule multiplication)
  - (bit vectors, conjunction)

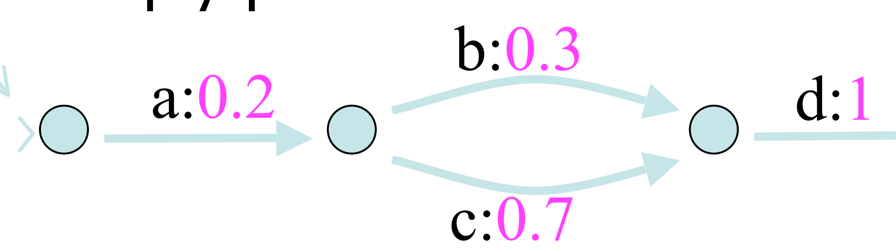
- Finite-state computation of fu
  - Concatenate strings



  - Add scores



  - Multiply probabilities

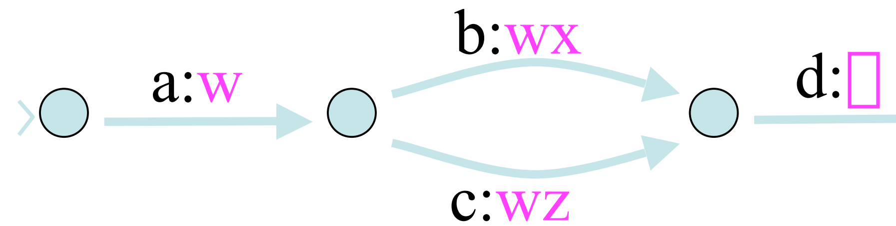


Weight Algebras

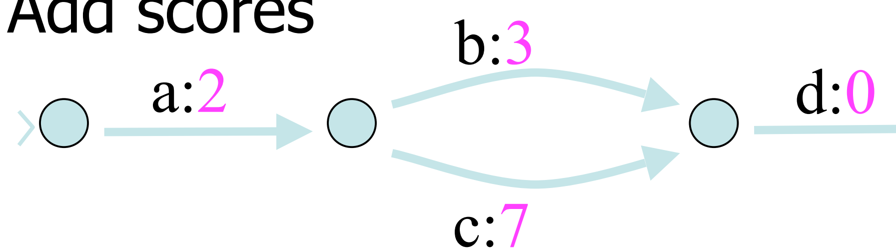
- Specify a weight algebra (K,)
- Define DFAs over (K,)
- Arcs have weights in set K
- A path's weight is also in K: multiply its arc weights with

- Q: Semiring is (K,⊕,). Why not talk about ⊕ too?
- A: Minimization is about DFAs.
- At most one path per input.
- So no need to ⊕ the weights of multiple accepting paths.

- Finite-state computation of fu
  - Concatenate strings



  - Add scores



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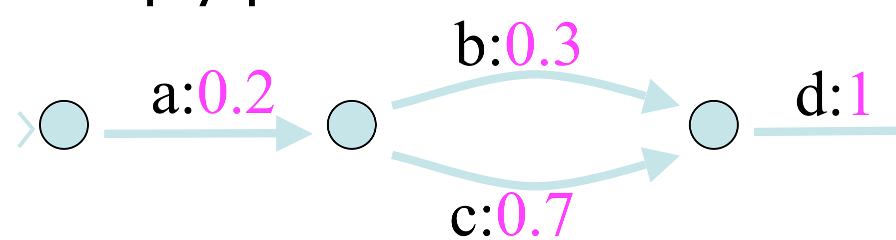
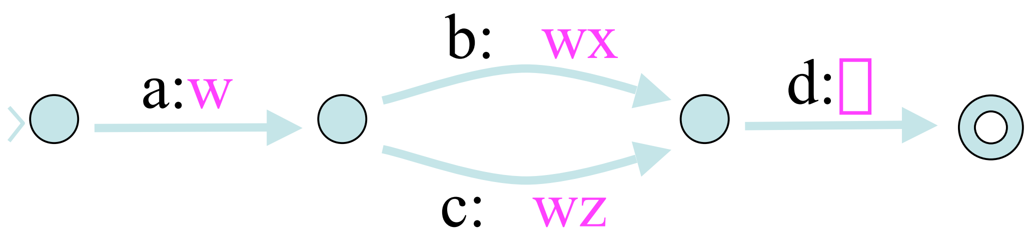


Shifting Outputs Along Paths

- Doesn't change the function computed:



abd □ wwx
acd □ wwz

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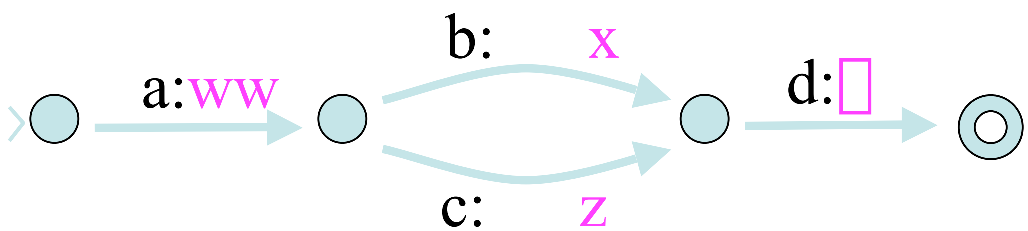

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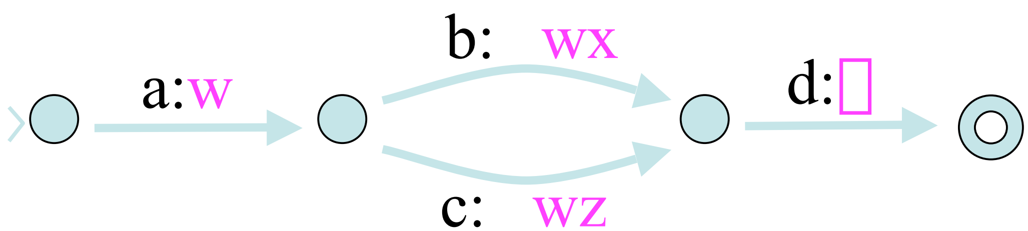

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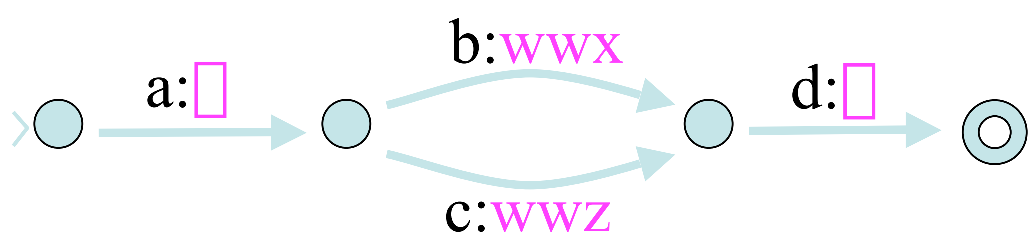

abd [illegible] wwx
acd [illegible] wwz

Shifting Outputs Along Paths

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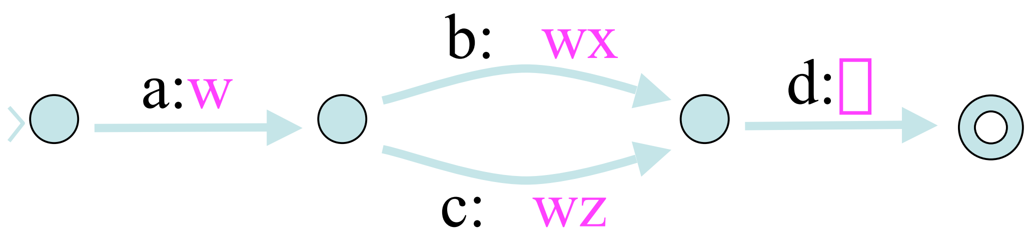

abd ▯ wwx
acd ▯ wwz

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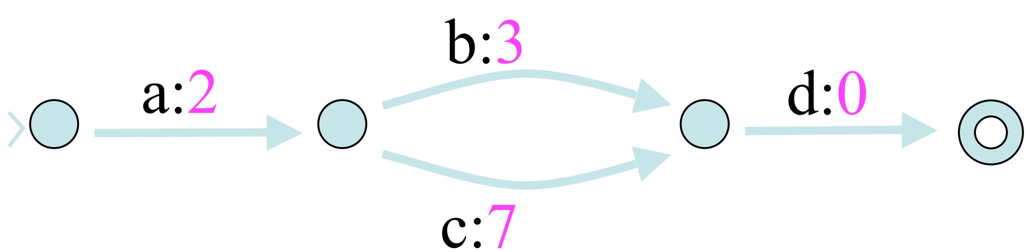

abd □ 5
acd □ 9

Shifting Outputs Along Paths

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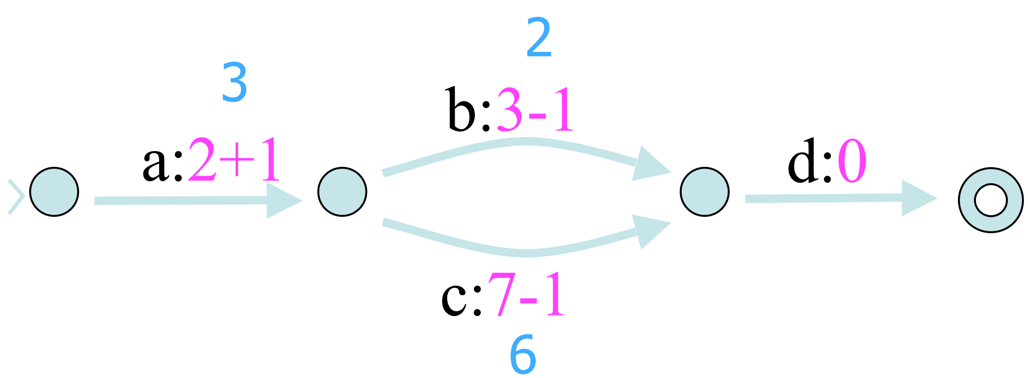

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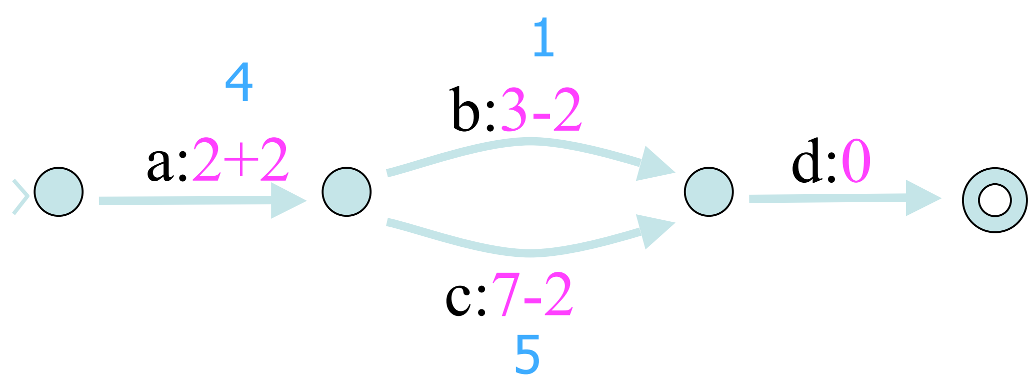

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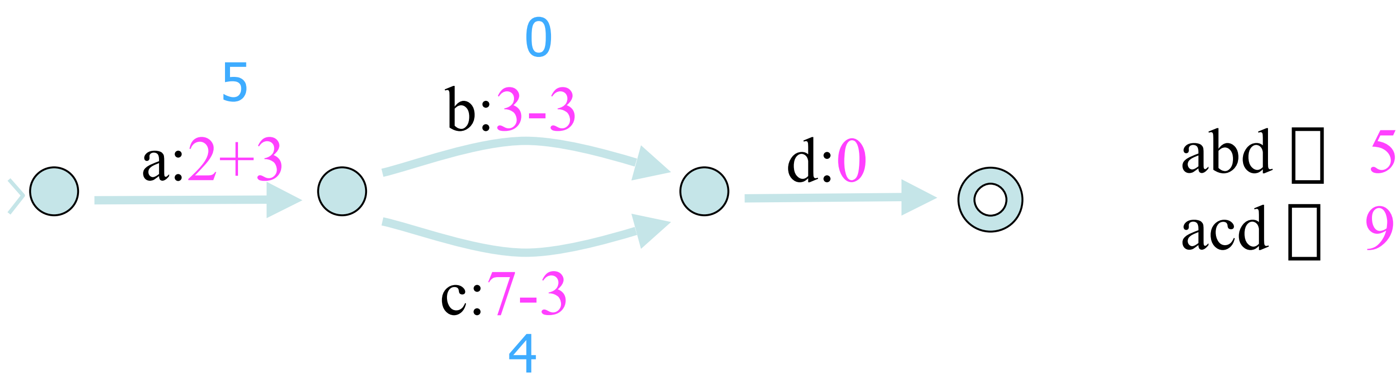

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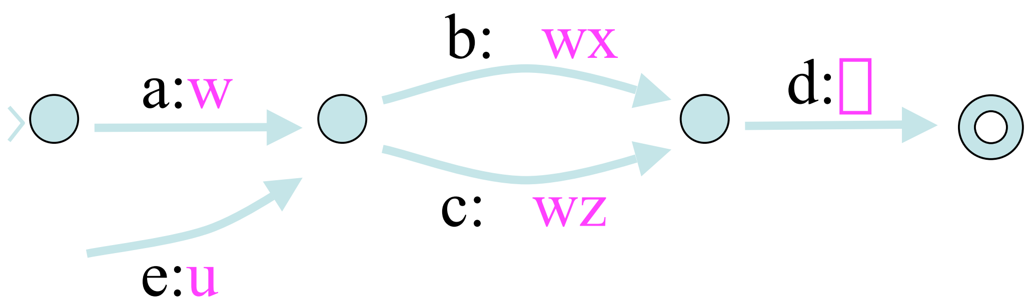
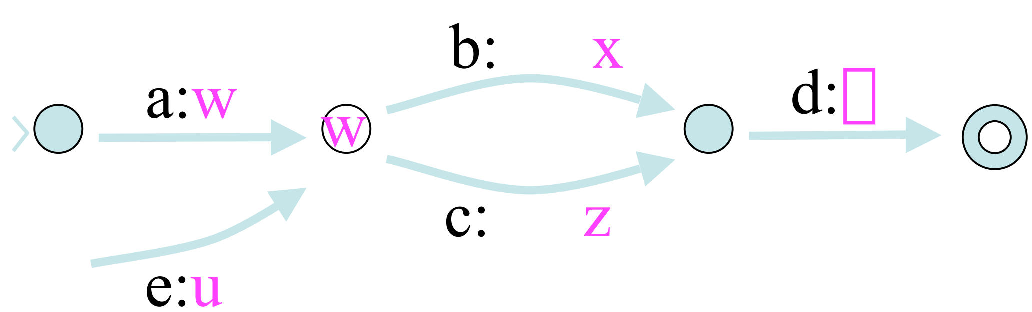


abd □ wwx
acd □ wwz
…ebd □ uwx
…ecd □ uwz

Shifting Outputs Along Paths

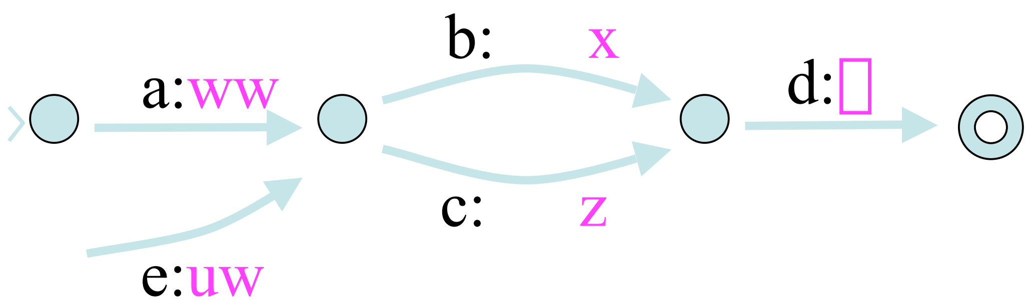
- State sucks back a prefix from its out-arcs



abd wwx
acd wwz
…ebd uwx
…ecd uwz

Shifting Outputs Along Paths

- State sucks back a prefix from its out-arcs and deposits it at end of its in-arcs.



abd □ wwx
acd □ wwz
…ebd □ uwx
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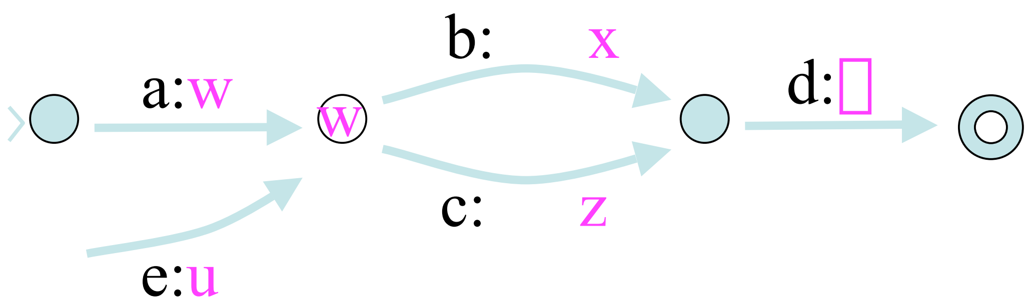
Shifting Outputs Along Paths



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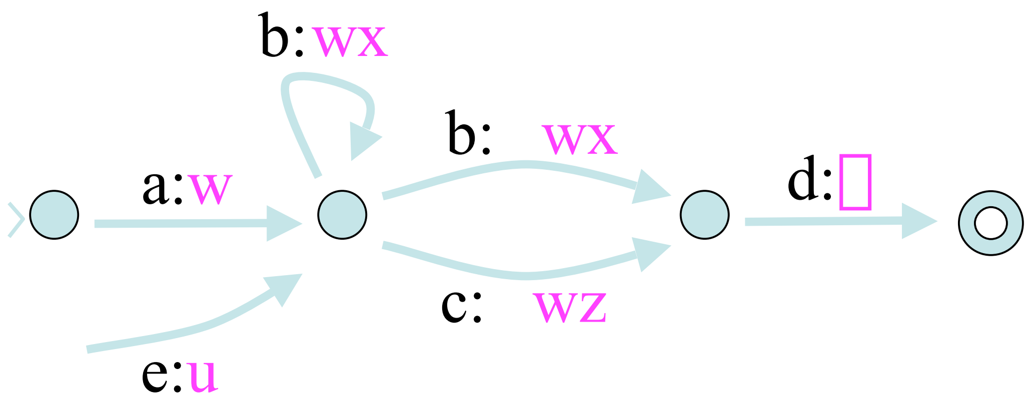
Shifting Outputs Along Paths



abd □ wwx

acd □ wwz

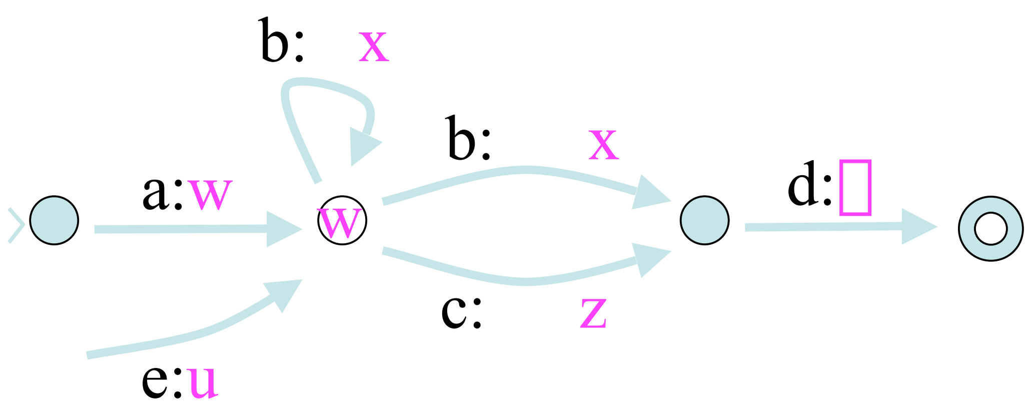
…ebd □ uwx

…ecd □ uwz

…ab^{n}bd □ u(wx)^{n}wx

…ab^{n}cd □ u(wx)^{n}wz

Shifting Outputs Along Paths



abd □ wwx

acd □ wwz

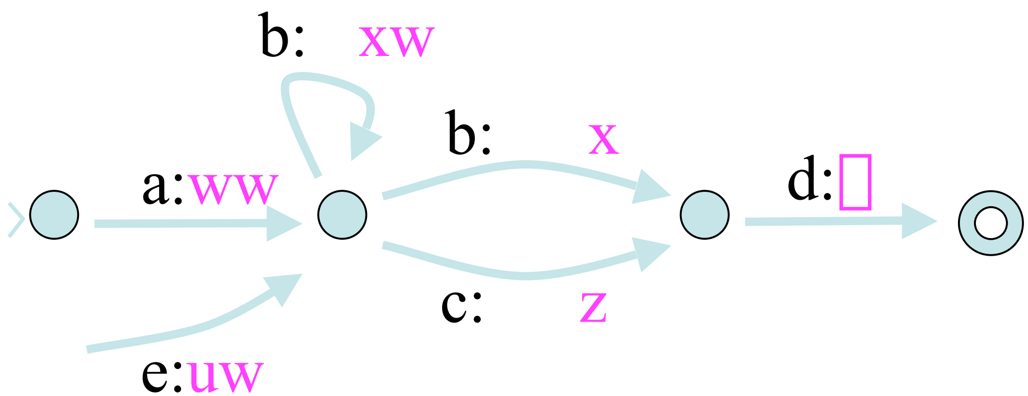
…ebd □ uwx

…ecd □ uwz

…$ab^n bd$ □ $u(wx)^n wx$

…$ab^n cd$ □ $u(wx)^n wz$

Shifting Outputs Along Paths

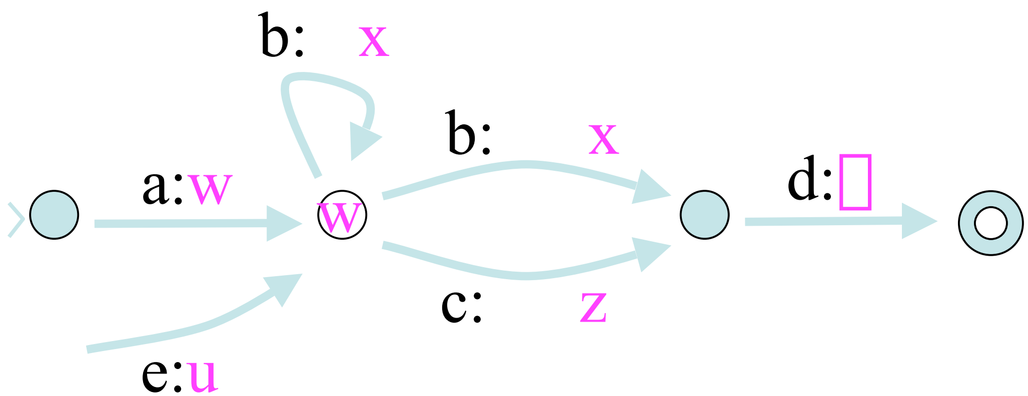


abd □ wwx
acd □ wwz
…ebd □ uwx
…ecd □ uwz

…ab^{n}bd □ u(wx)^{n}wx
…ab^{n}cd □ u(wx)^{n}wz

…ab^{n}bd □ uw(xw)^{n}x
…ab^{n}cd □ uw(xw)^{n}z

Shifting Outputs Along Paths



abd → wwx

acd → wwz

…ebd → uwx

…ecd → uwz

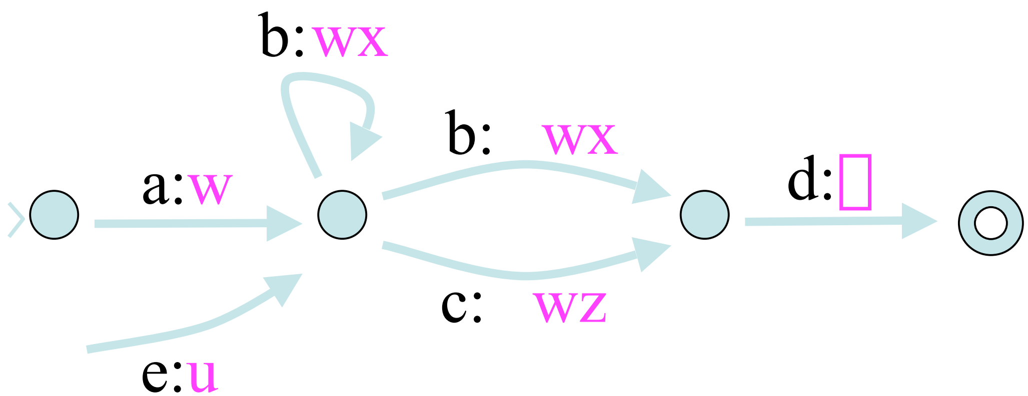
…ab^{n}bd → u(wx)^{n}wx

…ab^{n}cd → u(wx)^{n}wz

…ab^{n}bd → uw(xw)^{n}x

…ab^{n}cd → uw(xw)^{n}z

Shifting Outputs Along Paths



abd □ wwx

acd □ wwz

…ebd □ uwx

…ecd □ uwz

…ab^nbd □ $u(wx)^nwx$

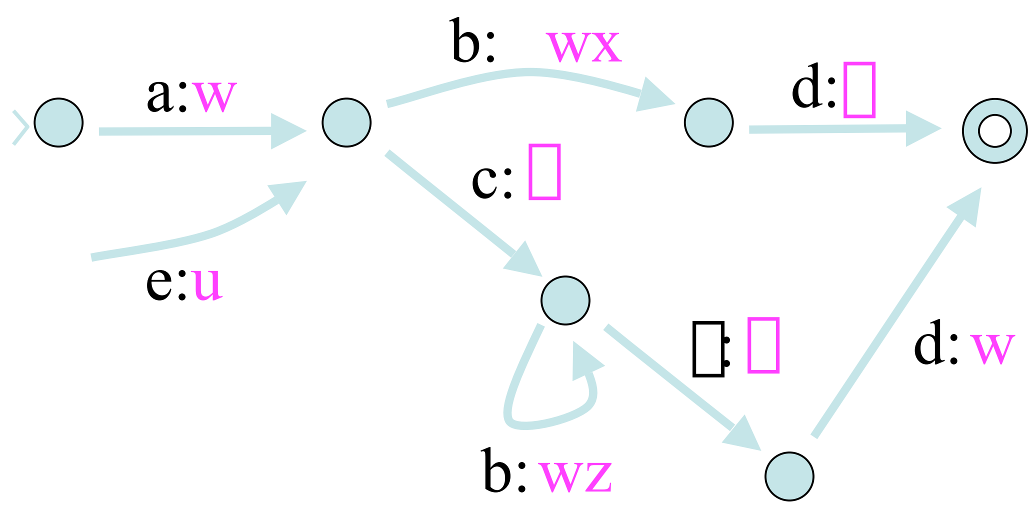
…ab^ncd □ $u(wx)^nwz$

…ab^nbd □ $uw(xw)^nx$

…ab^ncd □ $uw(xw)^nz$

Shifting Outputs Along Paths (Mohri)

- Here, not all the out-arcs start with w
- But all the out-**paths** start with w
- Do pushback at later states first:



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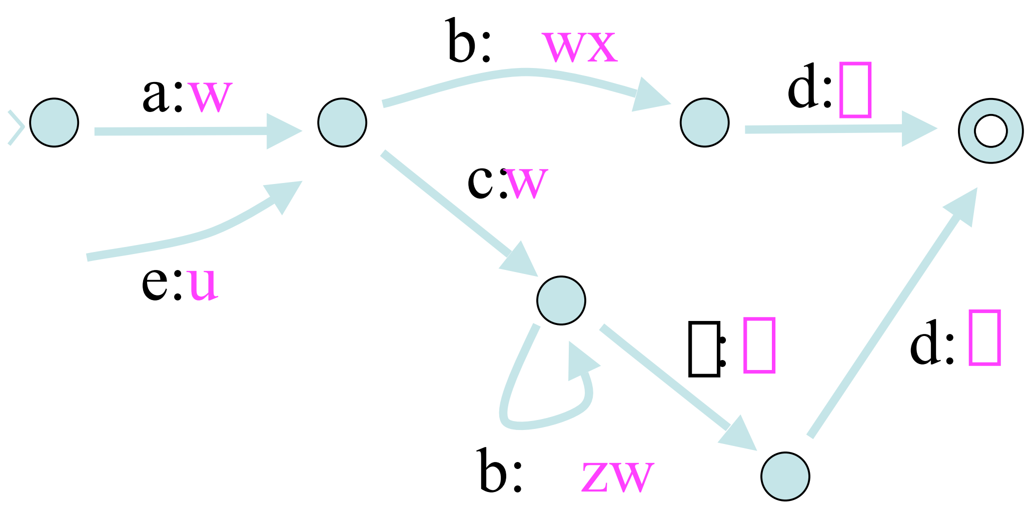

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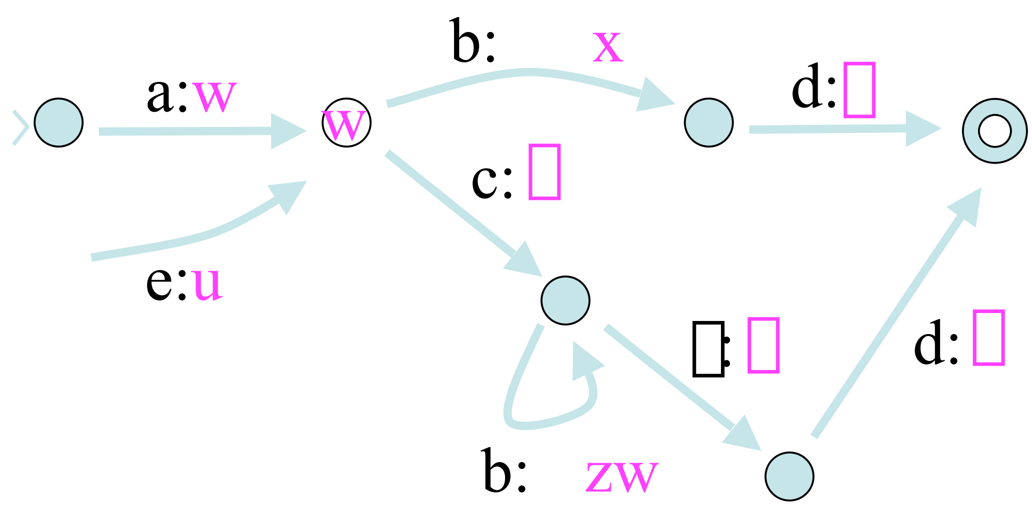

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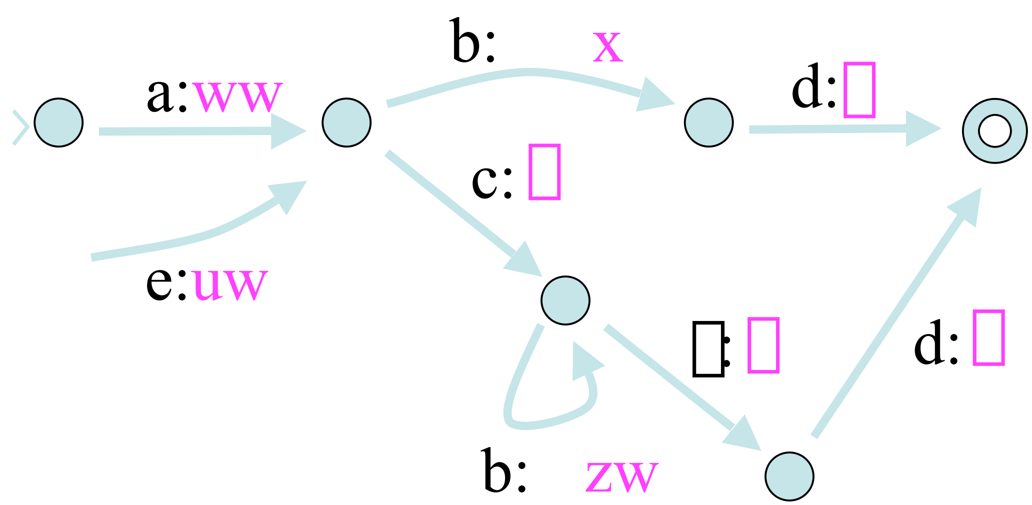
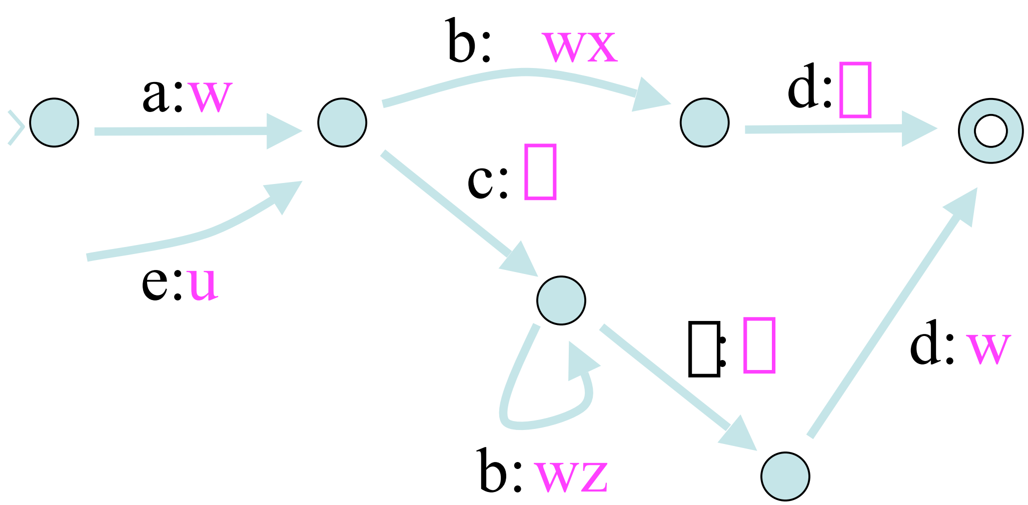


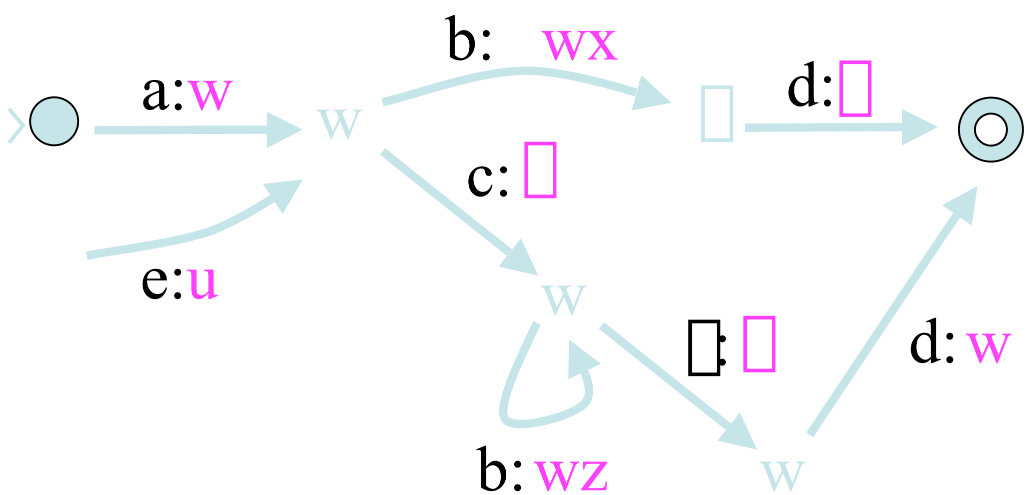
Shifting Outputs Along Paths (Mohri)

- Actually, push back at all states at once



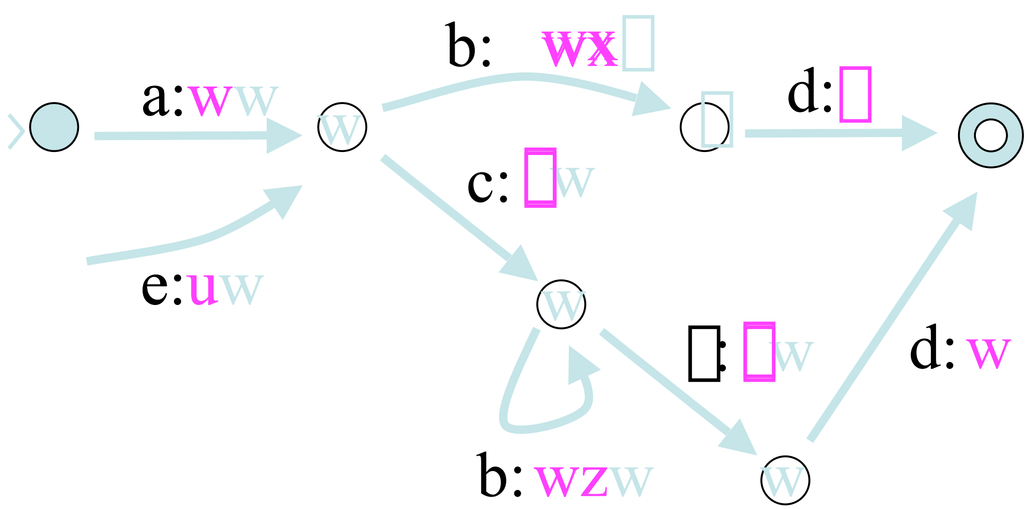
Shifting Outputs Along Paths (Mohri)

- Actually, push back at all states at once
- At every state q, compute some □(q)



Shifting Outputs Along Paths (Mohri)

- Actually, push back at all states at once
- Add □(q) to end of q's in-arcs



Shifting Outputs Along Paths (Mohri)

- Actually, push back at all states at once
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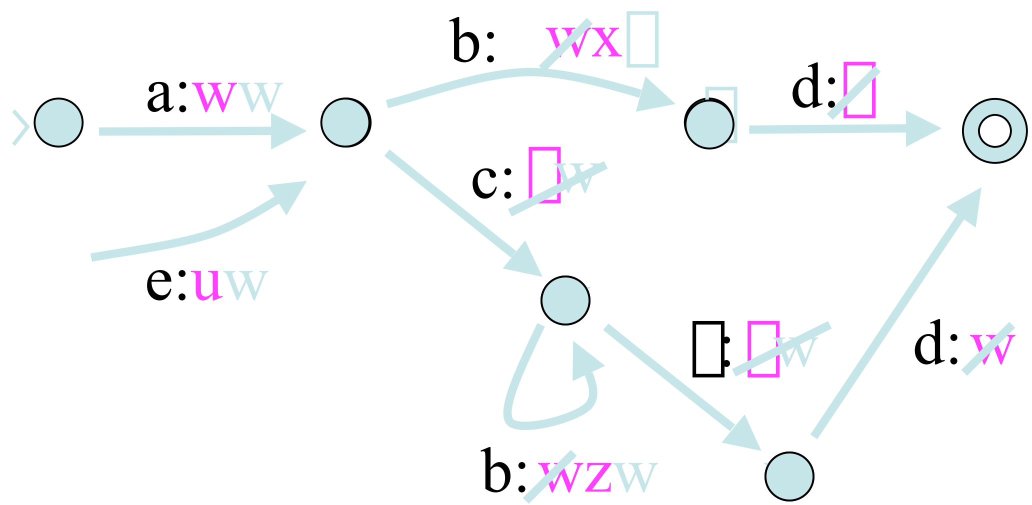
- Remove □(q) from start of q's out-arcs



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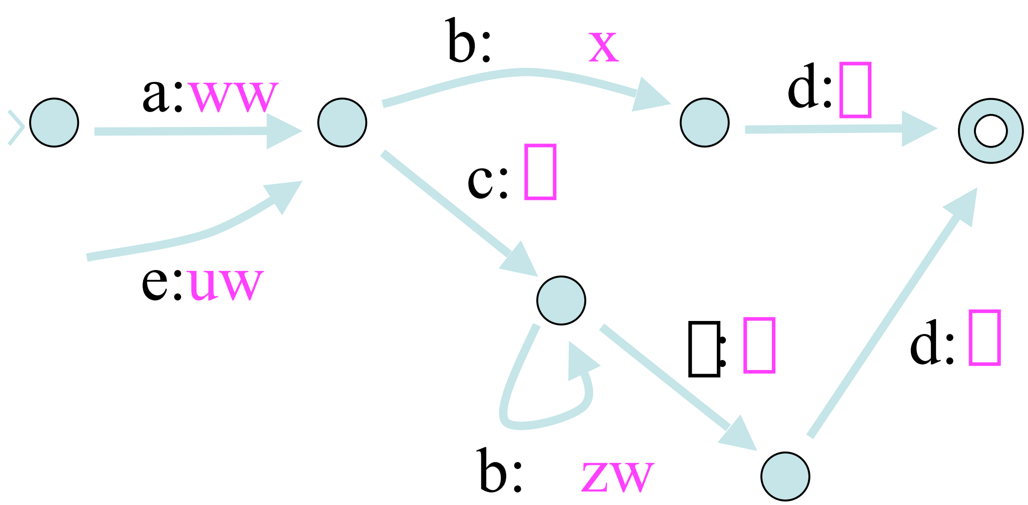
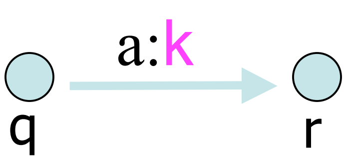
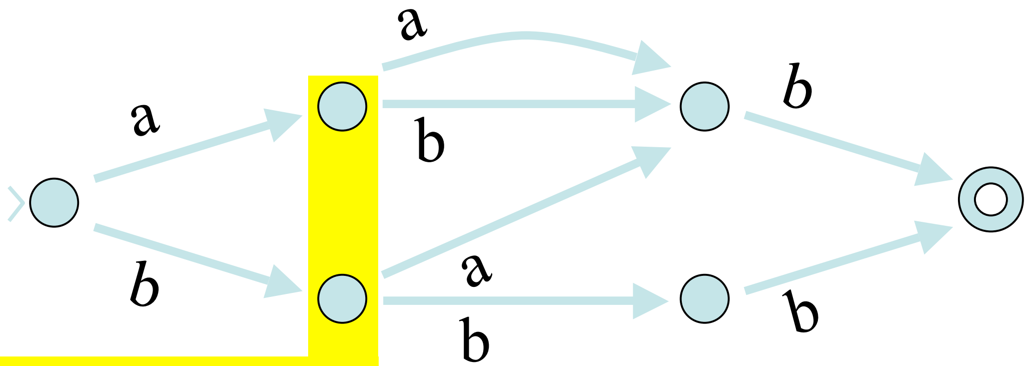




becomes

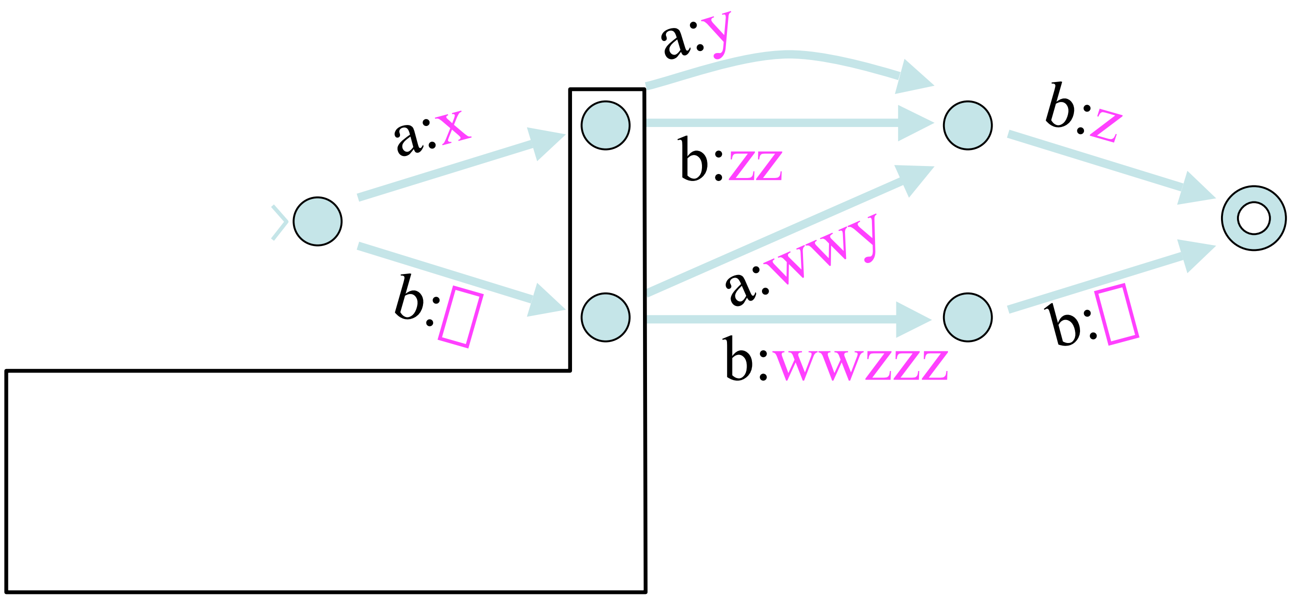
Minimizing Weighted DFAs (Mohri)



Mergeable because they accept the same **suffix language**: {ab,bb}

Minimizing Weighted DFAs (Mohri)

Still accept same suffix language,
  but produce different outputs on it



Minimizing Weighted DFAs (Mohri)

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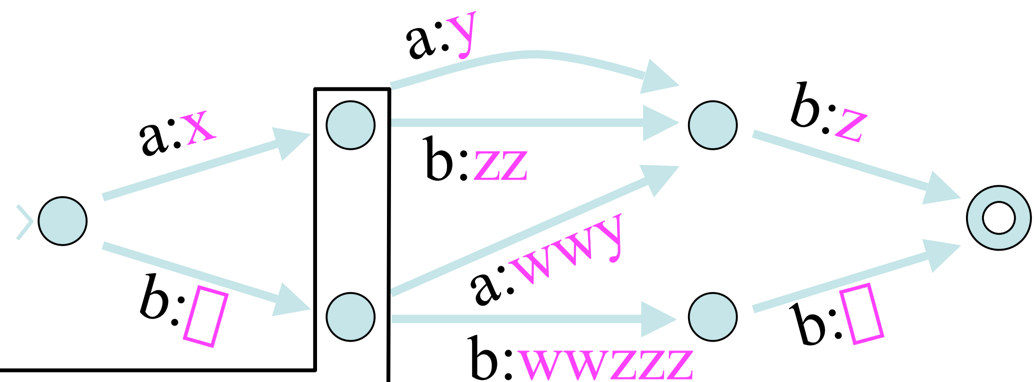


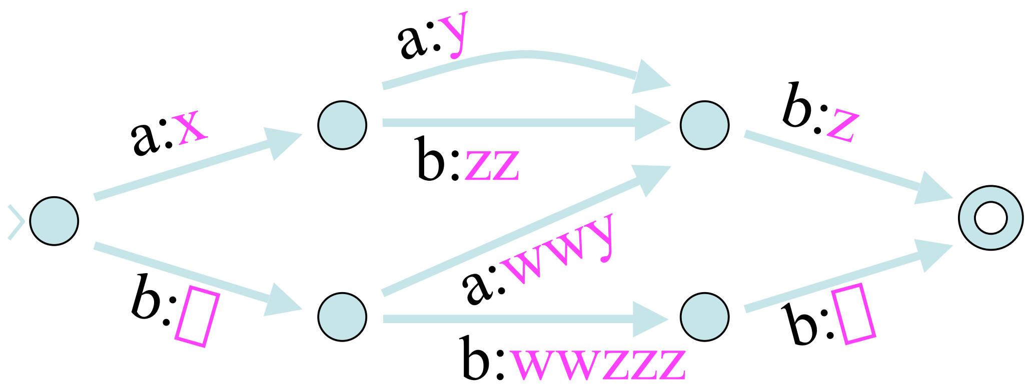
Not mergeable - compute different **suffix functions**:

ab □ yz or wwy

acd □ zzz or wwzzz

Minimizing Weighted DFAs (Mohri)

Fix by shifting outputs leftward …



Minimizing Weighted DFAs (Mohri)

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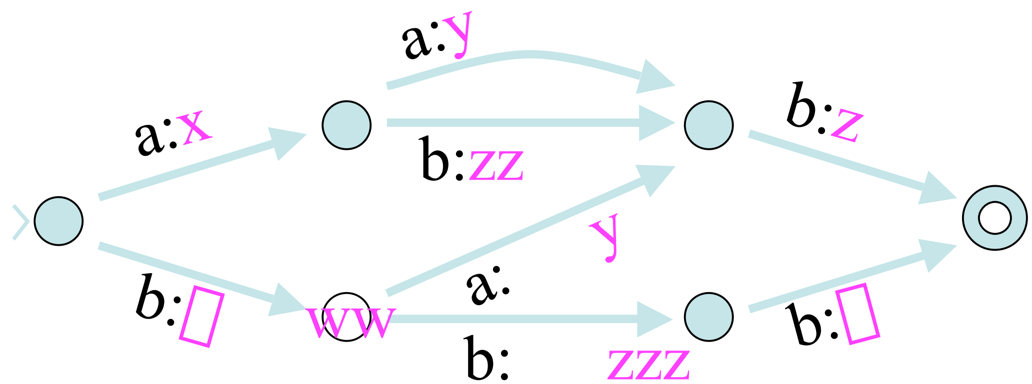

Minimizing Weighted DFAs (Mohri)

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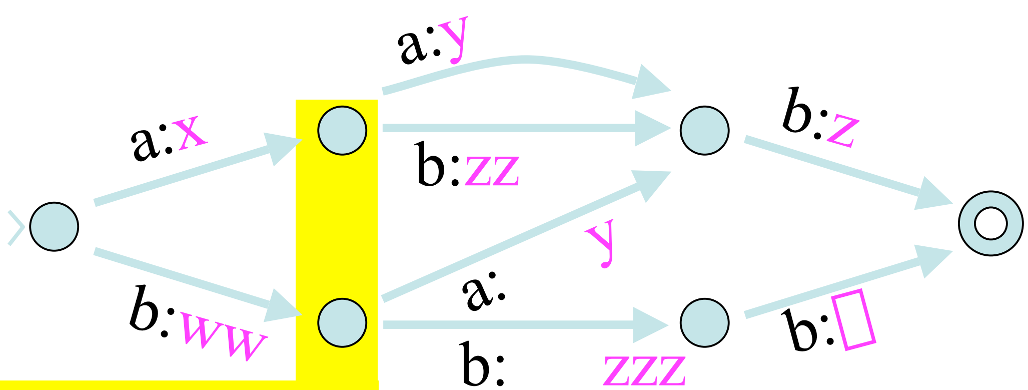


Now mergeable - they have the same **suffix function**:

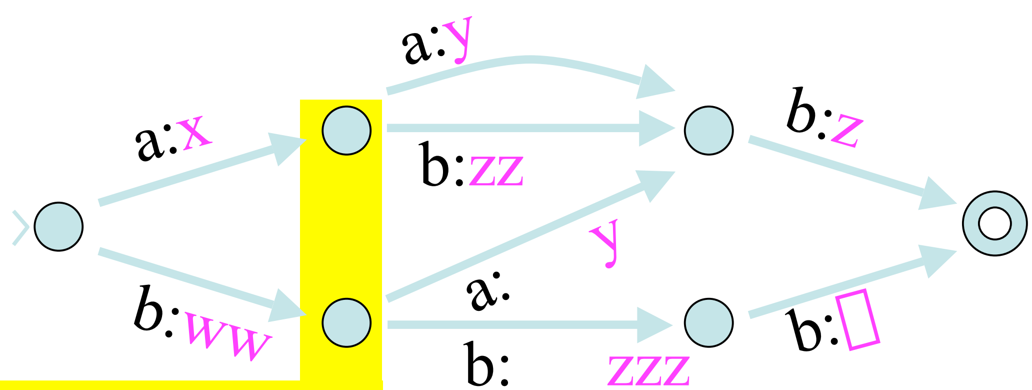
ab □ yz

acd □ zzz

But still no easy way to detect mergeability.

Minimizing Weighted DFAs (Mohri)

If we do this at all states as before …



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Minimizing Weighted DFAs (Mohri)

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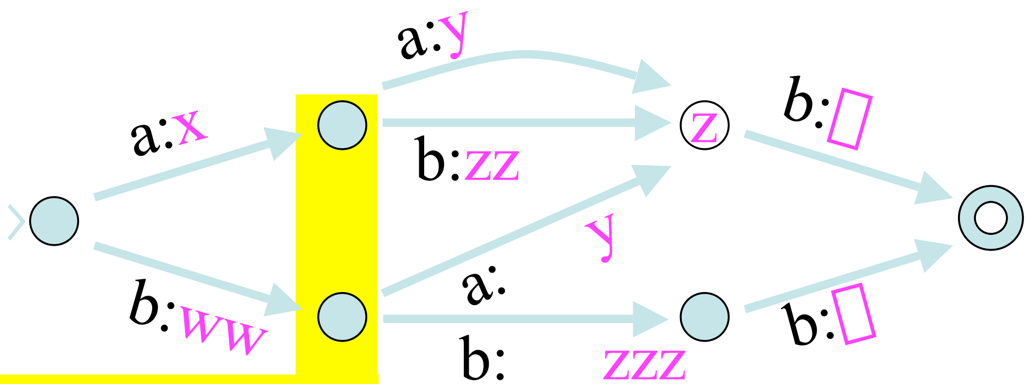

Now mergeable - they have the same **suffix function**:

ab → yz

acd → zzz

Minimizing Weighted DFAs (Mohri)

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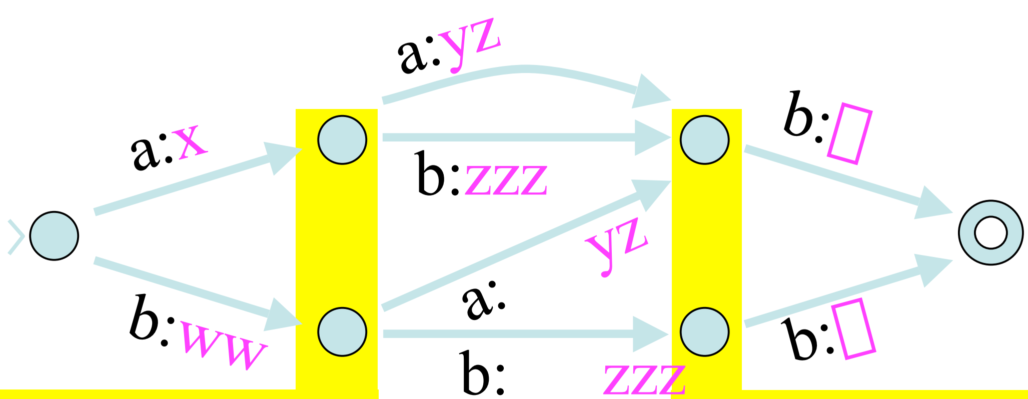
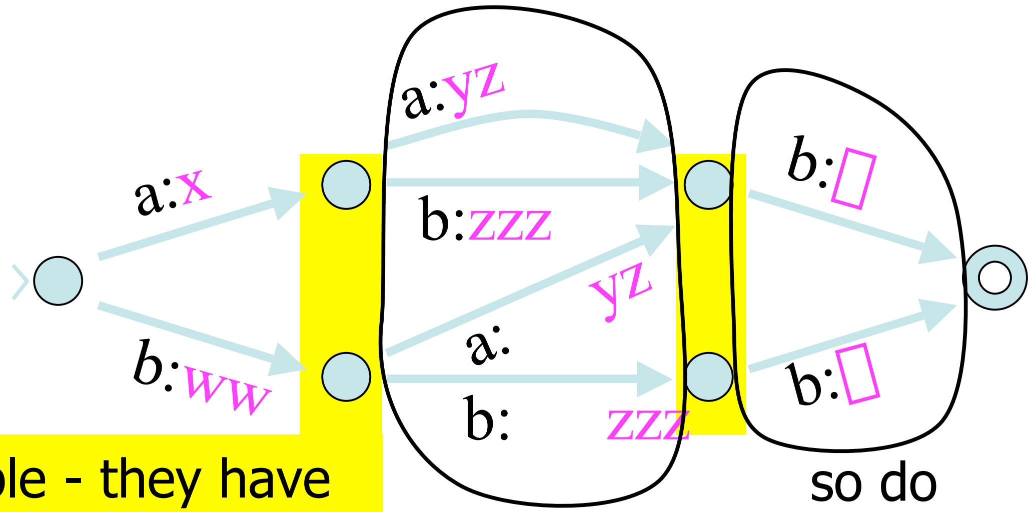


Now mergeable - they have the same **suffix function**:
ab □ yz
acd □ zzz

Now these have the same sufffix function too:
b □ □

Minimizing Weighted DFAs (Mohri)

Now we can discover & perform the merges:



Now mergeable - they have
the same **suffix function**:
ab □ yz
acd □ zzz

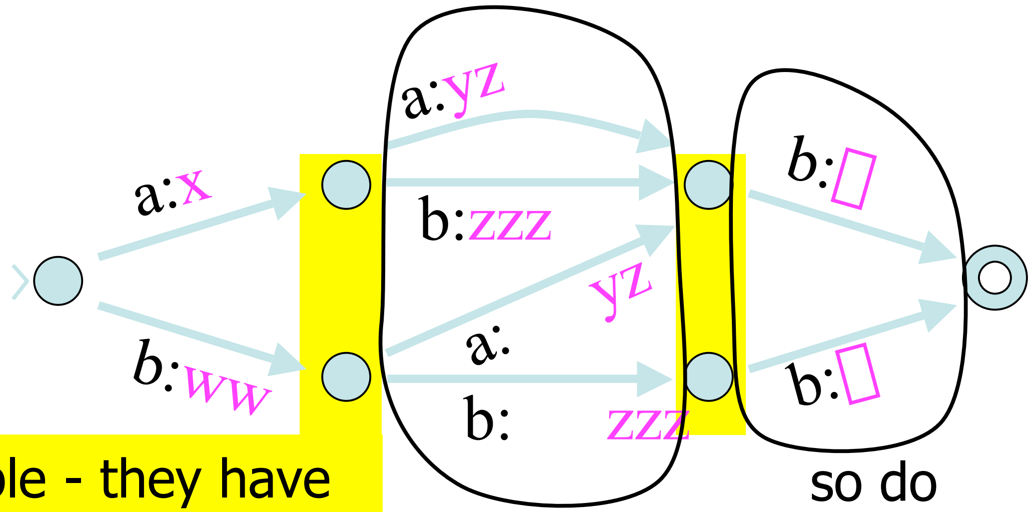
now these
have same
arc labels

so do
these

because we arranged for
a canonical placement of
outputs along paths

Minimizing Weighted DFAs (Mohri)

Treat each label "a:yz" as a single atomic symbol



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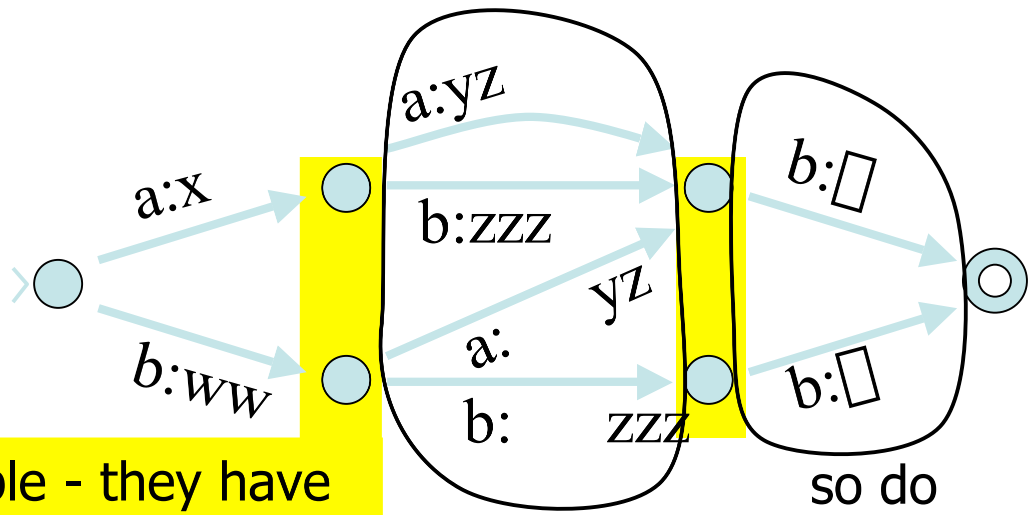

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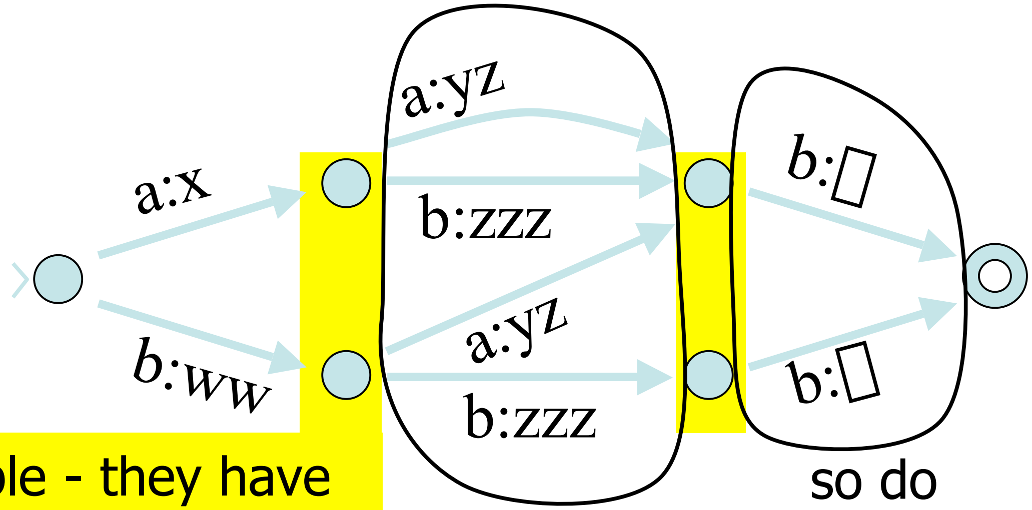

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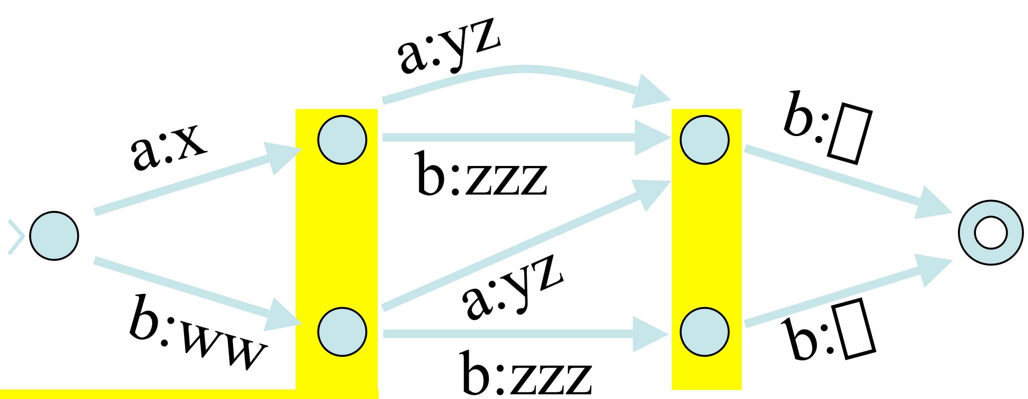
Minimizing Weighted DFAs (Mohri)

Treat each label "a:yz" as a single atomic symbol
Use *unweighted* minimization algorithm!



Now mergeable - they have the same **suffix function**:

ab → yz

acd → zzz

Minimizing Weighted DFAs (Mohri)

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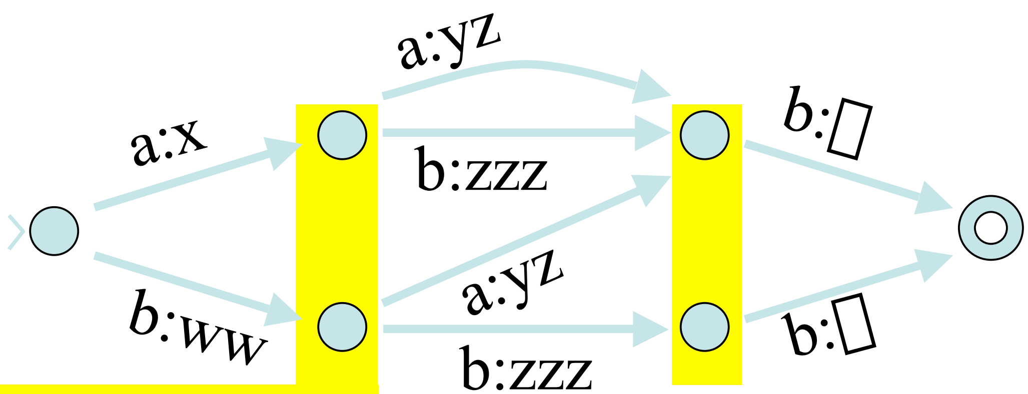

Now mergeable - they have the same **suffix language**:
{a:yz b:□,
b:zzz b:□}

Minimizing Weighted DFAs (Mohri)

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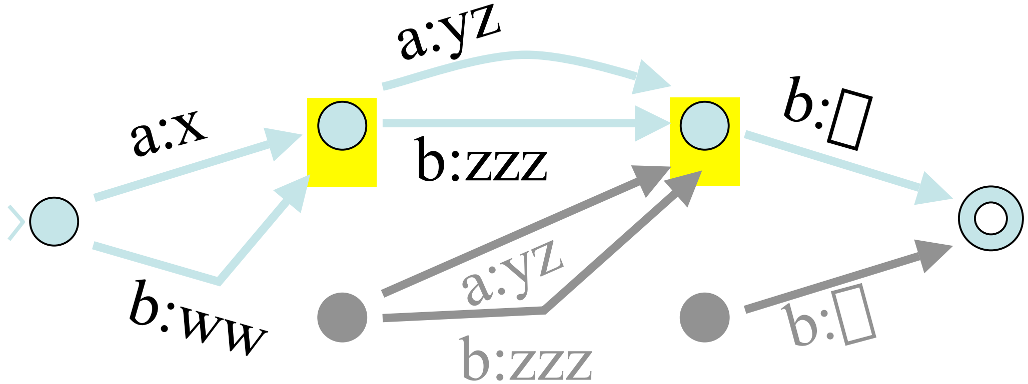
Use *unweighted* minimization algorithm!



Minimizing Weighted DFAs (Mohri)

Summary of weighted minimization algorithm:

1. **Compute** □(q) at each state q
2. **Push** each □(q) back through state q;
   this changes arc weights
3. **Merge** states via unweighted minimization

Step 3 merges states

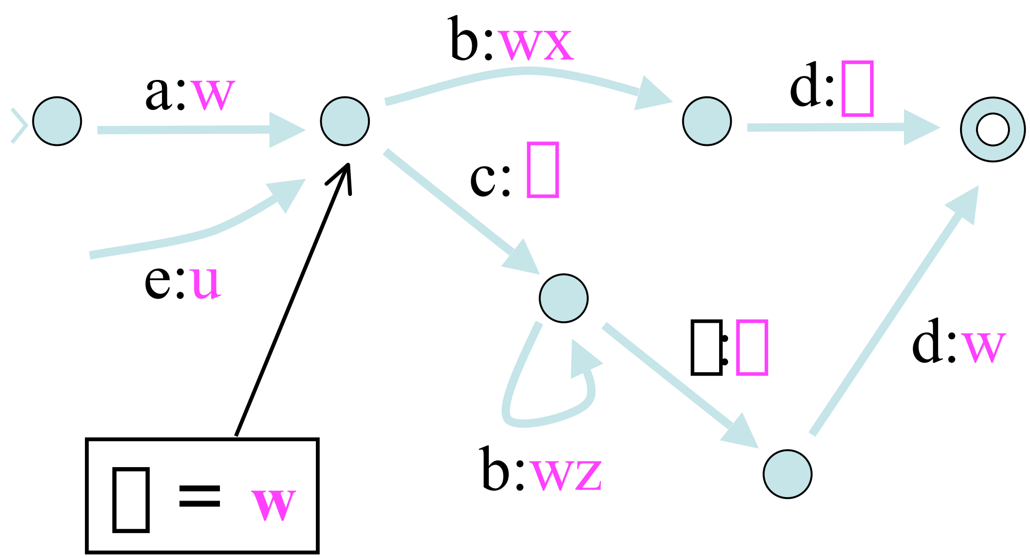
Step 2 allows more states to merge at step 3

Step 1 controls what step 2 does – preferably, to give states the same suffix function whenever possible

So define □(q) carefully at step 1!

Mohri's Algorithms (1997, 2000)

- Mohri treated two versions of (K,)
- (K,) = (strings, concatenation)
  - □(q) = longest common prefix of all paths from q
  - Rather tricky to find



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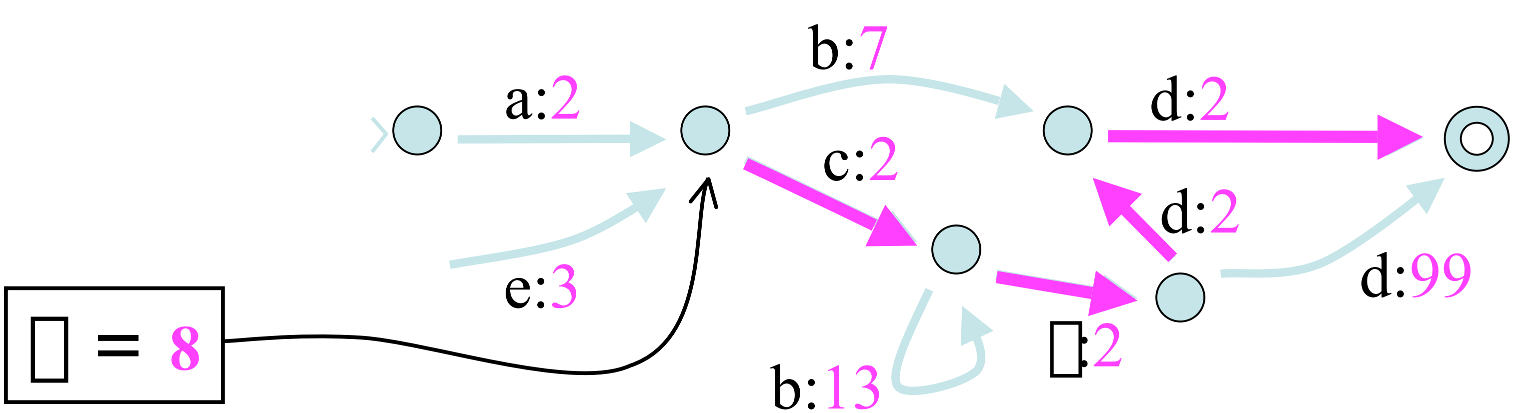
  - Find it by Dijkstra's shortest-path algorithm



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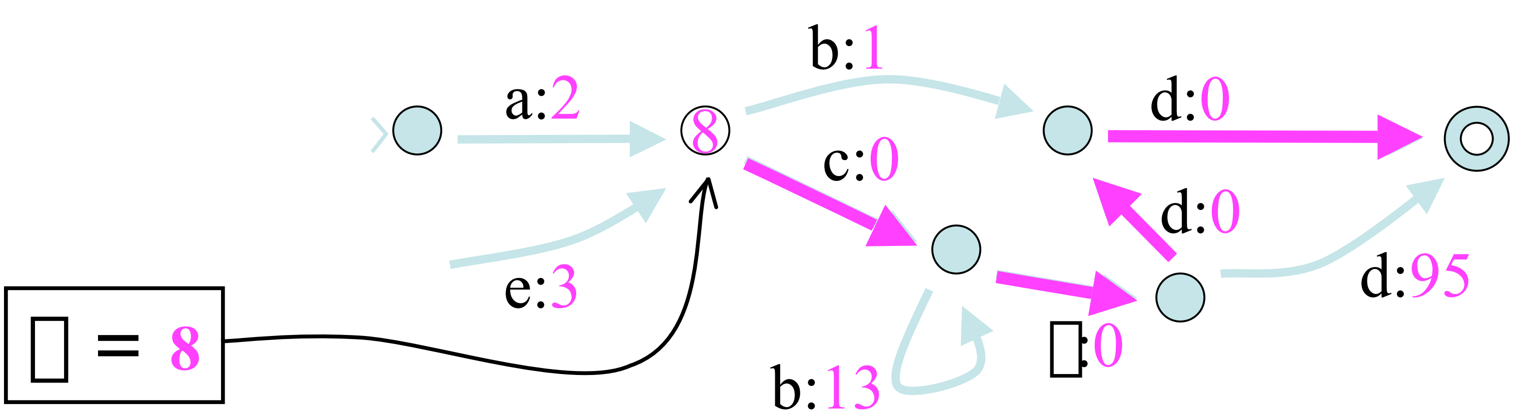

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