CMPT 413
Computational Linguistics

Anoop Sarkar

http://www.cs.sfu.ca/~anoop

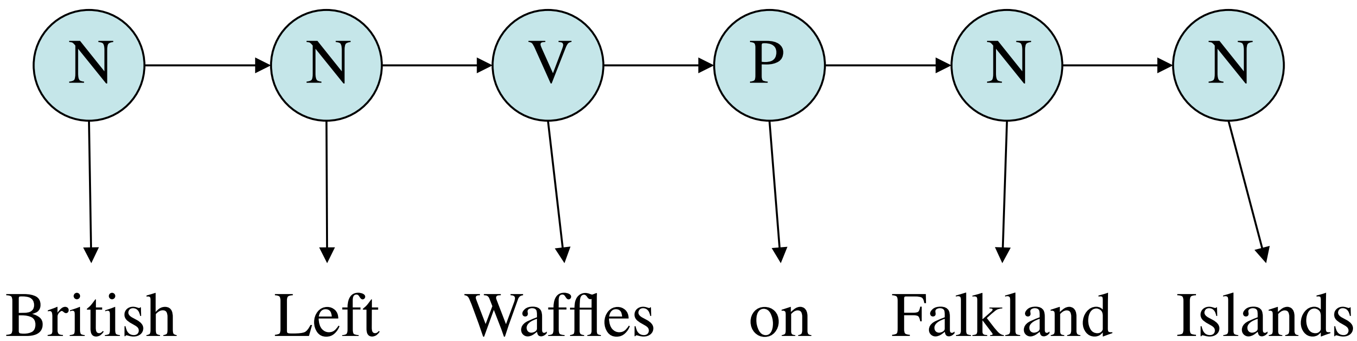
Sequence Learning

- British Left Waffles on Falkland Islands
  - (N, N, V, P, N, N)
  - (N, V, N, P, N, N)
- Segmentation 中国十四个边境开放城市经济建设成就显著
  - (b, i, b, i, b, b, i, b, i, b, i, b, i, b, i, b, i, b, i)

中国 十四 个 边境 开放 城市 经济 建设 成就 显著

China 's 14 open border cities marked economic achievements

Sequence Learning

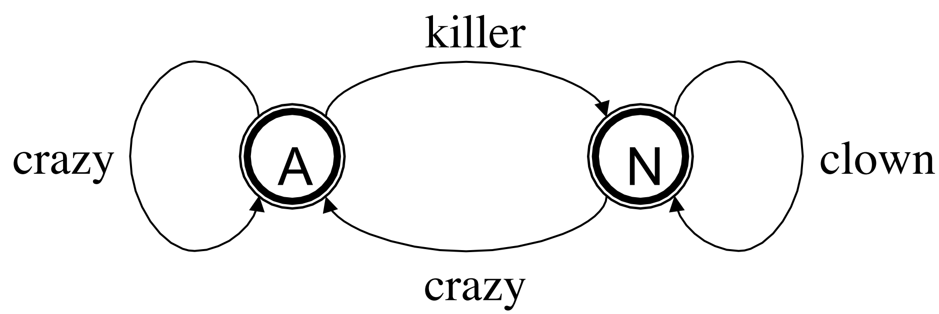


3 states: N, V, P

Observation sequence: $(o_1, \ldots o_6)$

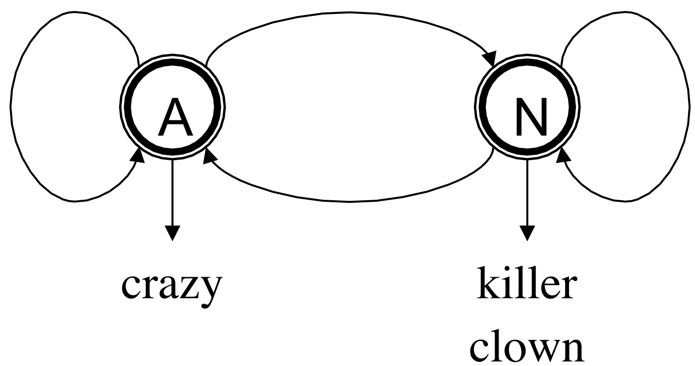
State sequence (6+1): *(Start, N, N, V, P, N, N)*

Finite State Machines



Mealy Machine

Finite State Machines

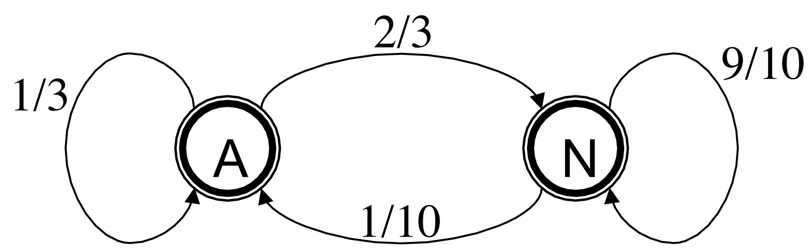


Moore Machine

Probabilistic FSMs

- Start at a state *i* with a *start state probability*: π_i
- Transition from state *i* to state *j* is associated with a *transition probability*: a_{ij}
- Emission of symbol *o* from state *i* is associated with an *emission probability*: $b_i(o)$
- Two conditions:
  - All outgoing transition arcs from a state must sum to 1
  - All symbol emissions from a state must sum to 1

Probabilistic FSMs



0 killer	1/3 killer
1.0 crazy	0 crazy
0 clown	1/3 clown
0 problem	1/3 problem

Probabilistic FSMs

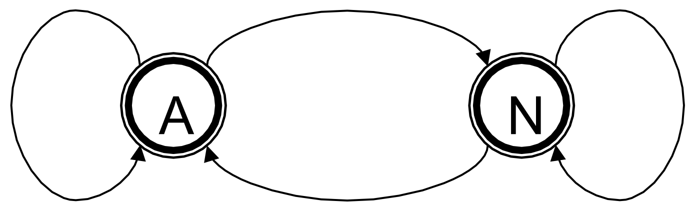
Emission

$b_A(\text{killer}) = 0$

$b_A(\text{crazy}) = 1$

$b_A(\text{clown}) = 0$

$b_A(\text{problem}) = 0$



killer	killer
crazy	crazy
clown	clown
problem	problem

Emission

$b_N(\text{killer}) = 1/3$

$b_N(\text{crazy}) = 0$

$b_N(\text{clown}) = 1/3$

$b_N(\text{problem}) = 1/3$

$$\sum_{o \in V} b_i(o) = 1$$

Start state

$\pi_A = 1/2$

$\pi_N = 1/2$

$$\sum_i \pi_i = 1$$

Transition

$a_{A,A} = 1/3$

$a_{A,N} = 2/3$

$a_{N,N} = 9/10$

$a_{N,A} = 1/10$

$$\sum_j a_{i,j} = 1$$

Hidden Markov Models

- There are *n* states $s_1, \ldots, s_i, \ldots, s_n$
- The emissions are observed (input data)
- Observation sequence $\mathbf{O}=(o_1, \ldots, o_t, \ldots, o_T)$
- The states are not directly observed (hidden)
- Data does not directly tell us which state X_t is linked with observation o_t

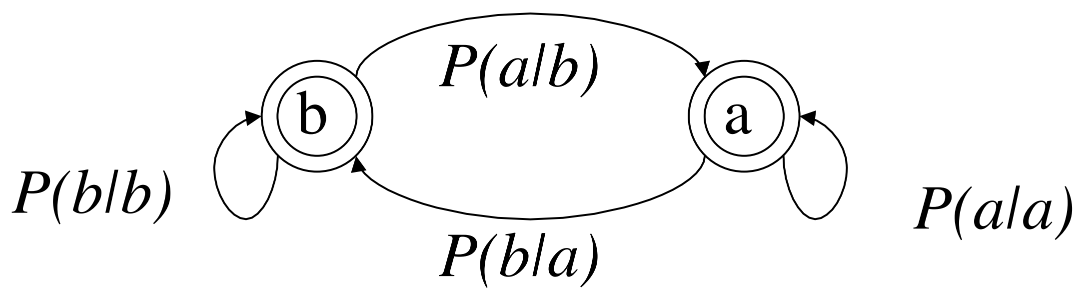
  $X_t \in \{s_1, \ldots, s_n\}$

Markov Chains vs. HMMs

- For observation sequence *babaa*

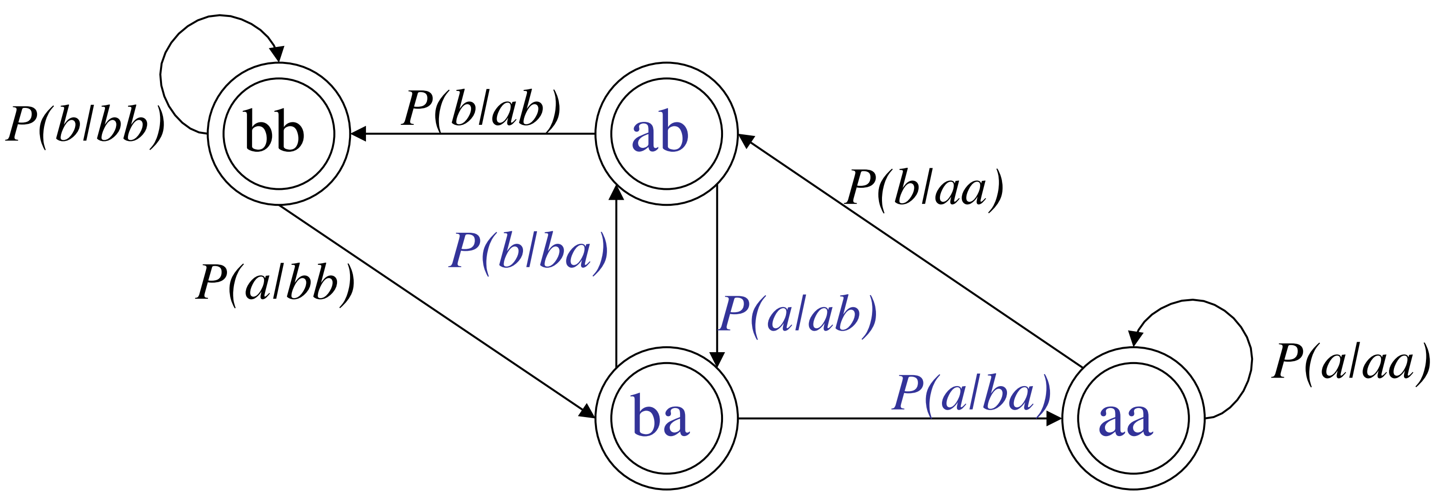
  $i.e{:}\ o_1=b,\ o_2=a,\ \ldots,\ o_5=a$
- Compute *P(babaa)* using a bigram model

  $P(b)*P(a|b)*P(b|a)*P(a|b)*P(a|a)$
- Equivalent Markov chain:



Markov Chains vs. HMMs

- For observation sequence *babaa*
  i.e: $o_1=b, o_2=a, \ldots, o_5=a$
- Compute *P(babaa)* using a trigram model
  *P(ba)*P(b|ba)*P(a|ab)*P(a|ba)*
- Equivalent Markov chain:

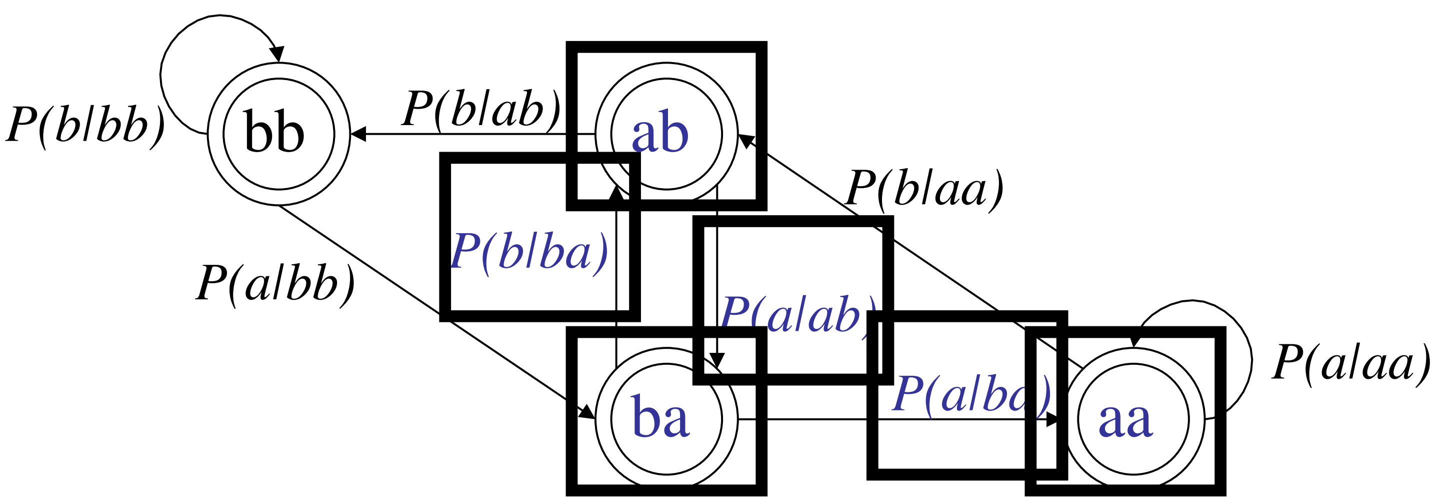


Markov Chains vs. HMMs

- For observation sequence *babaa*

  $i.e{:}\ o_1=b,\ o_2=a,\ \ldots,\ o_5=a$
- Compute *P(babaa)* using a trigram model

  *P(ba)*P(b|ba)*P(a|ab)*P(a|ba)*
- Equivalent Markov chain:



Markov Chains vs. HMMs

- Given an observation sequence

  O=$(o_1, \ldots, o_t, \ldots, o_T)$
- An *n*th order Markov Chain or *n*-gram model computes the probability

  $P(o_1, \ldots, o_t, \ldots, o_T)$
- An HMM computes the probability

  $P(X_1, \ldots, X_{T+1}, o_1, \ldots, o_T)$ where the state sequence is *hidden*

Properties of HMMs

- Markov assumption

$$P(X_t = s_i \mid \ldots, X_{t-1} = s_j)$$

- Stationary distribution

$$P(X_t = s_i \mid X_{t-1} = s_j) = P(X_{t+l} = s_i \mid X_{t+l-1} = s_j)$$

HMM Algorithms

- HMM as language model: compute probability of given observation sequence
- HMM as parser: compute the best sequence of states for a given observation sequence
- HMM as learner: given a set of observation sequences, learn its distribution, i.e. learn the transition and emission probabilities

HMM Algorithms

- HMM as language model: compute probability of given observation sequence
- Compute $P(o_1, \ldots, o_T)$ from the probability $P(X_1, \ldots, X_{T+1}, o_1, \ldots, o_T)$

$$= \prod_{t=1}^{T} P(X_{t+1} = s_j \mid X_t = s_i) \times P(o_t = k \mid X_{t+1} = s_j)$$

$$P(o_1, \ldots, o_T) = \sum_{X_1, \ldots, X_{T+1}} P(X_1, \ldots, X_{T+1}, o_1, \ldots, o_T)$$

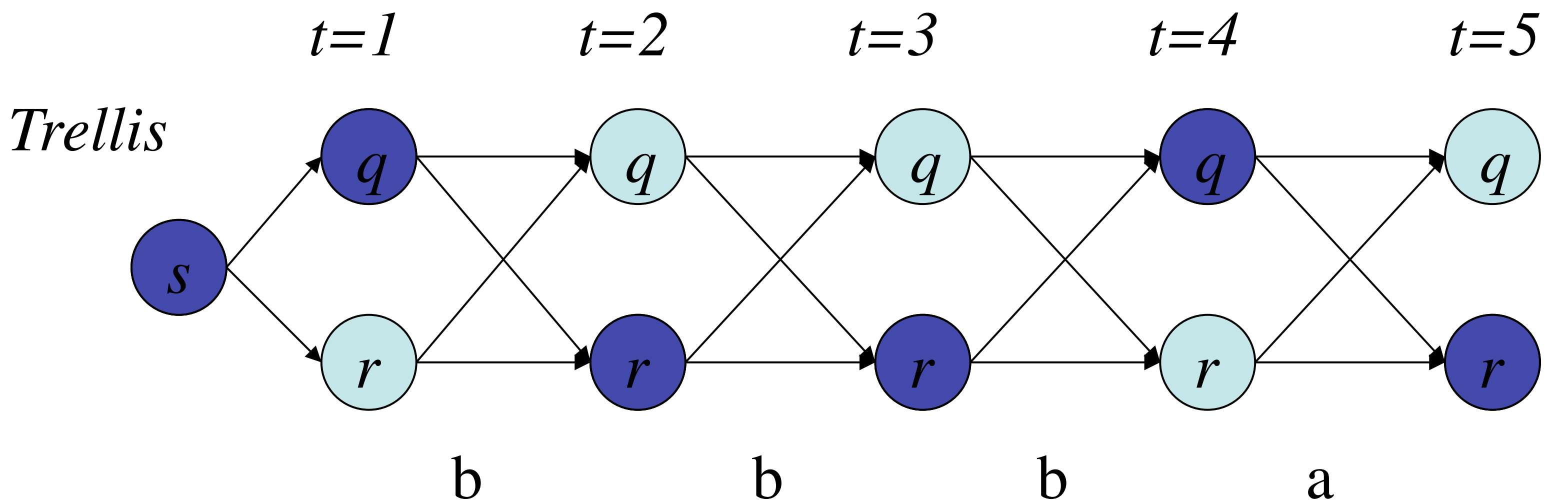
HMM Algorithms

- HMM as parser: compute the best sequence of states for a given observation sequence
- Compute best path $X_1, \ldots, X_{T+1}$ from the probability $P(X_1, \ldots, X_{T+1}, o_1, \ldots, o_T)$

Best state sequence $X^*_1, \ldots, X^*_{T+1}$

$$= \operatorname*{argmax}_{X_1,\ldots,X_{T+1}} P(X_1, \ldots, X_{T+1}, o_1, \ldots, o_T)$$

Best Path (Viterbi) Algorithm



- Key Idea 1: storing just the best path doesn’t work
- Key Idea 2: store the best path upto *each* state

Viterbi Algorithm

```
function viterbi (edges, input, obs): returns best path
edges = transition probability
input = emission probability
T = length of obs, the observation sequence
num-states = number of states in the HMM
Create a path-matrix: viterbi[num-states+1, T+1] # init to all 0s
for each state s: viterbi[s, 0] = π[s]
for each time step t from 0 to T:
   for each state s from 0 to num-states:
      for each s’ where edges[s,s’] is a transition probability:
         new-score = viterbi[s,t] * edges[s,s’] * input[s’,obs[t]]
         if (viterbi[s’,t+1] == 0) or (new-score > viterbi[s’, t+1]):
            viterbi[s’, t+1] = new-score
            back-pointer[s’,t+1] = s
```

Viterbi Algorithm

```
# finding the best path
best-final-score = best-final-state = 0
for each state s from 0 to num-states:
    if (viterbi[s,T+1] > best-final-score):
        best-final-state = s
        best-final-score = viterbi[s,T+1]
# start with the last state in the sequence
x = best-final-state
state-sequence.push(x)
for t from T+1 downto 0:
    state-sequence.push(back-pointer[x,t])
    x = back-pointer[x,t]
return state-sequence
```