CMPT 379
Compilers

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Parsing CFGs

- Consider the problem of parsing with arbitrary CFGs
- For any input string, the parser has to produce a parse tree
- The simpler problem: print **yes** if the input string is generated by the grammar, print **no** otherwise
- This problem is called *recognition*

CKY Recognition Algorithm

- The Cocke-Kasami-Younger algorithm
- As we shall see it runs in time that is polynomial in the size of the input
- It takes space polynomial in the size of the input
- **Remarkable fact:** it can find all possible parse trees (exponentially many) in polynomial time

Chomsky Normal Form

- Before we can see how CKY works, we need to convert the input CFG into Chomsky Normal Form
- CNF means that the input CFG G is converted to a new CFG G' in which all rules are of the form:

  $A \rightarrow B\ C$

  $A \rightarrow a$

Epsilon Removal

- First step, remove epsilon rules

  $A \rightarrow B\ C$

  $C \rightarrow \varepsilon \mid C\ D \mid a$

  $D \rightarrow b \quad B \rightarrow b$

- After ε-removal:

  $A \rightarrow B \mid B\ C\ D \mid B\ a$

  $C \rightarrow D \mid C\ D\ D \mid a\ D \mid C\ D \mid a$

  $D \rightarrow b \quad B \rightarrow b$

Removal of Chain Rules

- Second step, remove chain rules

  $A \rightarrow B\ C \mid C\ D\ C$

  $C \rightarrow D \mid a$

  $D \rightarrow d \quad B \rightarrow b$

- After removal of chain rules:

  $A \rightarrow B\ a \mid B\ D \mid a\ D\ a \mid a\ D\ D \mid D\ D\ a \mid D\ D\ D$

  $D \rightarrow d \quad B \rightarrow b$

Eliminate terminals from RHS

- Third step, remove terminals from the rhs of rules

  $A \rightarrow B\ a\ C\ d$

- After removal of terminals from the rhs:

  $A \rightarrow B\ N_1\ C\ N_2$

  $N_1 \rightarrow a$

  $N_2 \rightarrow d$

Binarize RHS with Nonterminals

- Fourth step, convert the rhs of each rule to have two non-terminals

  $A \rightarrow B\ N_1\ C\ N_2$

  $N_1 \rightarrow a$

  $N_2 \rightarrow d$

- After converting to binary form:

  $A \rightarrow B\ N_3 \qquad N_1 \rightarrow a$

  $N_3 \rightarrow N_1\ N_4 \qquad N_2 \rightarrow d$

  $N_4 \rightarrow C\ N_2$

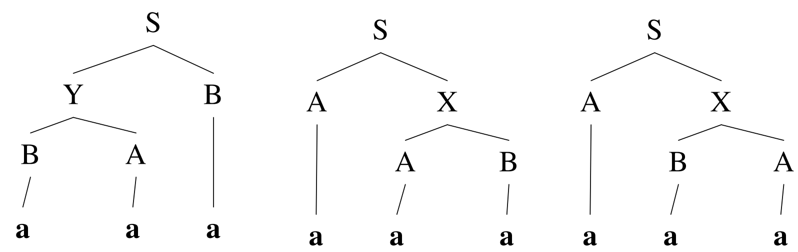
CKY algorithm

- We will consider the working of the algorithm on an example CFG and input string
- Example CFG:
  $S \rightarrow A\ X \mid Y\ B$
  $X \rightarrow A\ B \mid B\ A \qquad Y \rightarrow B\ A$
  $A \rightarrow a \quad B \rightarrow a$
- Example input string: *aaa*

CKY Algorithm

	0	1	2	3
0		A, B $A \rightarrow a$ $B \rightarrow a$	X, Y $X \rightarrow A\ B \mid B\ A$ $Y \rightarrow B\ A$	S $S \rightarrow A_{(0,1)}\ X_{(1,3)}$ $S \rightarrow Y_{(0,2)}\ B_{(2,3)}$
1			A, B $A \rightarrow a$ $B \rightarrow a$	X, Y $X \rightarrow A\ B \mid B\ A$ $Y \rightarrow B\ A$
2				A, B $A \rightarrow a$ $B \rightarrow a$
		a	a	a

Parse trees



CKY Algorithm

Input string **input** of size *n*
Create a 2D table **chart** of size n^2
for i=0 **to** n-1
  chart[i][i+1] = A **if** there is a rule $A \rightarrow a$ and **input**[i]=a
for j=2 **to** N
  for i=j-2 **downto** 0
    for k=i+1 **to** j-1
      chart[i][j] = A **if** there is a rule $A \rightarrow B\ C$ **and**
        chart[i][k] = B **and chart**[k][j] = C
return *yes* **if chart**[0][n] has the start symbol
else return *no*

CKY algorithm summary

- Parsing arbitrary CFGs
- For the CKY algorithm, the time complexity is $O(|G|^2 n^3)$
- The space requirement is $O(n^2)$
- The CKY algorithm handles arbitrary ambiguous CFGs
- All ambiguous choices are stored in the chart
- For compilers we consider parsing algorithms for CFGs that do not handle ambiguous grammars

GLR – Generalized LR Parsing

- Works for any CFG (just like CKY algorithm)
  - Masaru Tomita [1986]
- If you have shift/reduce conflict, just clone your stack and shift in one clone, reduce in the other clone
  - proceed in lockstep
  - parser that get into error states die
  - merge parsers that lead to identical reductions (graph structured stack)
- Careful implementation can provide $O(n^3)$ bound

Parsing - Summary

- Parsing arbitrary CFGs using the CKY algorithm: $O(n^3)$ time complexity
- Chomsky Normal Form (CNF) provides the n^3 time bound
- LR parsers can be extended to Generalized LR parsers to deal with arbitrary CFGs, complexity is still $O(n^3)$

Parsing - Additional Results

- $O(n^2)$ time complexity for linear grammars
  - All rules are of the form $S \rightarrow aSb$ or $S \rightarrow a$
  - Reason for $O(n^2)$ bound is the linear grammar normal form: $A \rightarrow aB$, $A \rightarrow Ba$, $A \rightarrow B$, $A \rightarrow a$
- Left corner parsers
  - extension of top-down parsing to arbitrary CFGs
- Earley's parsing algorithm
  - $O(n^3)$ worst case time for arbitrary CFGs just like CKY
  - $O(n^2)$ worst case time for unambiguous CFGs
  - $O(n)$ for specific unambiguous grammars (e.g. $S \rightarrow aSa \mid bSb \mid \varepsilon$)