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An Analysis of Projectile Motion Using Numerical Approximation

I. Introduction

We investigated and compared the motion of a projectile object in numerical simulations of four different air density models: constant air density, zero air density, the adiabatic air density profile, and the isothermal air density profile. For each model, we calculated the x velocity, y velocity, x position, and y position of the projectile object at user-designated time intervals until a user-designated end time. In this study, we focus on times that are 0.0005 seconds apart on the domain of [0,50], but the step size and range may be altered in order to determine the patterns of each model.

We then analyzed the position of the projectile object in each density model using graphs; we changed the muzzle velocity and firing angle in order to observe the effects those constants have on the plots. The chosen values for those constants are in the table below. Furthermore, we studied the impact of time interval size and air density at sea level; the values we used are in the table below. When changing the constants, we held every other constant at its default value and only changed the one we were focusing on. We assumed that the object – except in special cases specified later in the paper – began its motion at the origin with a 45-degree firing angle, and we used the default values suggested by the projectile motion document on this course's Athena page.

Finally, we graphed all of the models on one plot in order to discover when the models began to diverge from each other. The results, and the divergence criteria, are explicated further in the final section of this report.

	Smaller Value	**Default Value**	**Larger Value**
Muzzle Velocity	343 m/s	827 m/s	1000 m/s
Firing Angle	20 degrees	45 degrees	70 degrees
Time Interval Size	0.00005 seconds	0.0005 seconds	0.005 seconds
Air Density at Sea Level	0.1 kg/m3	1.225 kg/m3	3 kg/m3

Table 1. Table that details the constants that were changed in this analysis, their default values in the code, and the smaller and larger values that were used in the following analysis.

II. Individual Analysis of Models

A. Constant Air Density Model

In the constant air density model, with all of the default constants, the graph of the projectile object's position is as follows:

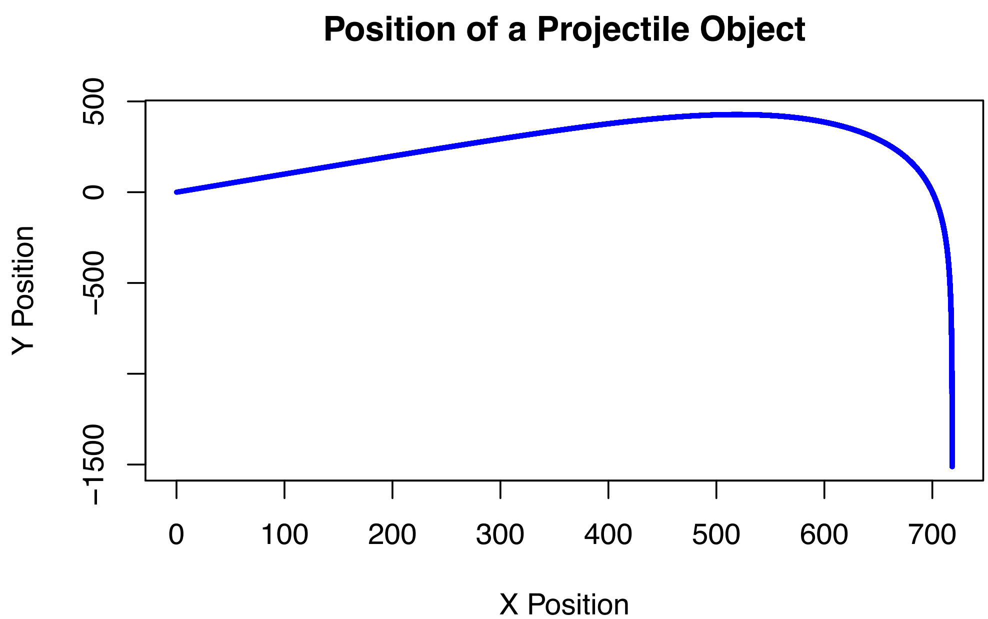

Figure 1. Graph of projectile object's position in a simulation of constant air density. All of the constants are at their default values.

Decreasing the muzzle velocity preserves the overall shape of the graph, but the object does not attain even half of the maximum y-position found earlier, and it travels about 200 fewer meters in the x-direction. Additionally, increasing the muzzle velocity also produces a similar curve, but the descent of the object occurs at a faster rate, and the maximum x and y positions are greater.

Altering the firing angle changes the initial path of the object' ascent: a smaller angle results in a flatter ascent, while a larger angle results in a steeper ascent. However, no matter the starting angle, the descent remains similar to the original graph – almost vertical.

While decreasing the time step has little effect on the graph – as the time steps are already very small – increasing the time step dramatically alters the accuracy of the model. Even though the y-position of the projectile object remains similar to the original graph, the maximum x-position decreases by about 30 meters. This may be due to the model's approximations being spaced too far apart; as a result, the increases or decreases in velocity become overestimates and the model is inaccurate. Furthermore, the details of the object's ascent become less clear because the points are spaced further apart.

Due to the simple definition and implementation of a constant density model, changing the air density at sea level has a direct, predictable effect. Decreasing its value makes the object's ascent last longer, its descent less steep, and nearly doubles the distance it travels in the x-direction while increasing the maximum y-position it reaches. In contrast, increasing air density flattens the object's ascent, steepens the descent, and more than halves the maximum x and y positions the object achieves.

B. Zero Density Model

In comparison to the constant density model, the zero density model creates the following graph with the default constants:

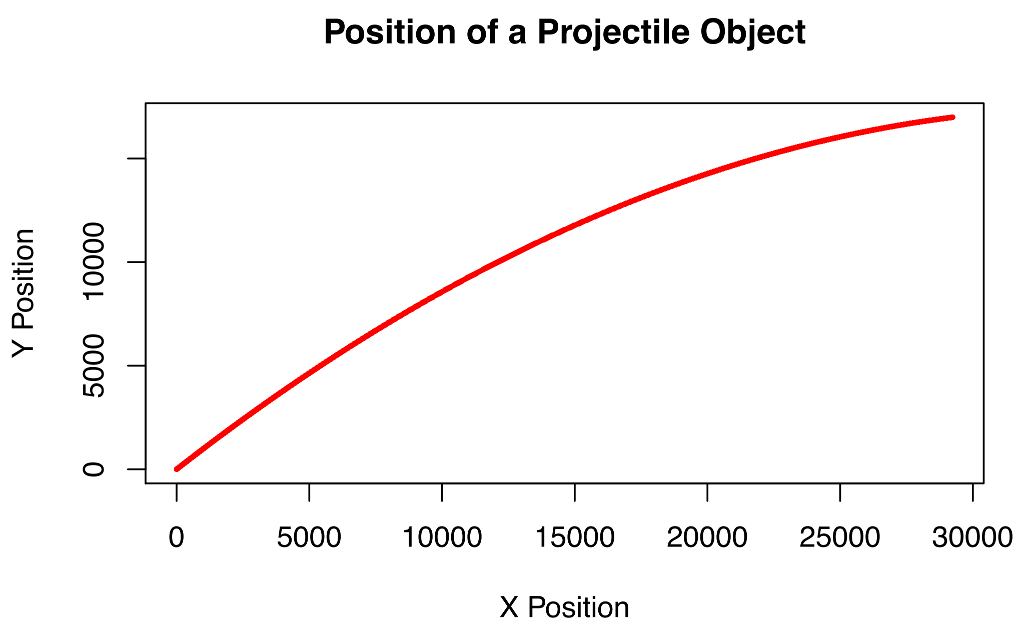

Figure 2. Graph of projectile object's position in a simulation of zero air density. All of the constants are at their default values.

One distinct different between the previous density model and this one is that there is no air density controlling the motion of the projectile object; as a result, the maximum x and y positions the object achieves in zero air density are much greater. The constant and zero air density models diverge quickly as a result; however, if one decreases the starting velocity of the object traveling in zero density, the graphs begin to look more similar.

Because the zero air density model follows the same equation as that used for constant air density – just replacing the rho constant with zero – the effects of the constants detailed above cause nearly identical changes to the position graph. The most significant, and perhaps obvious, difference is that changing the air density at sea level will not alter the position values in this graph because that constant is not used in calculations. Observing the exact effects of the other constants on this object's ascent and descent become more pronounced when the end time is doubled.

C. Adiabatic Air Density Profile

When using the default constants, the position graph of a projectile object in a simulation of the adiabatic air density profile appears as follows:

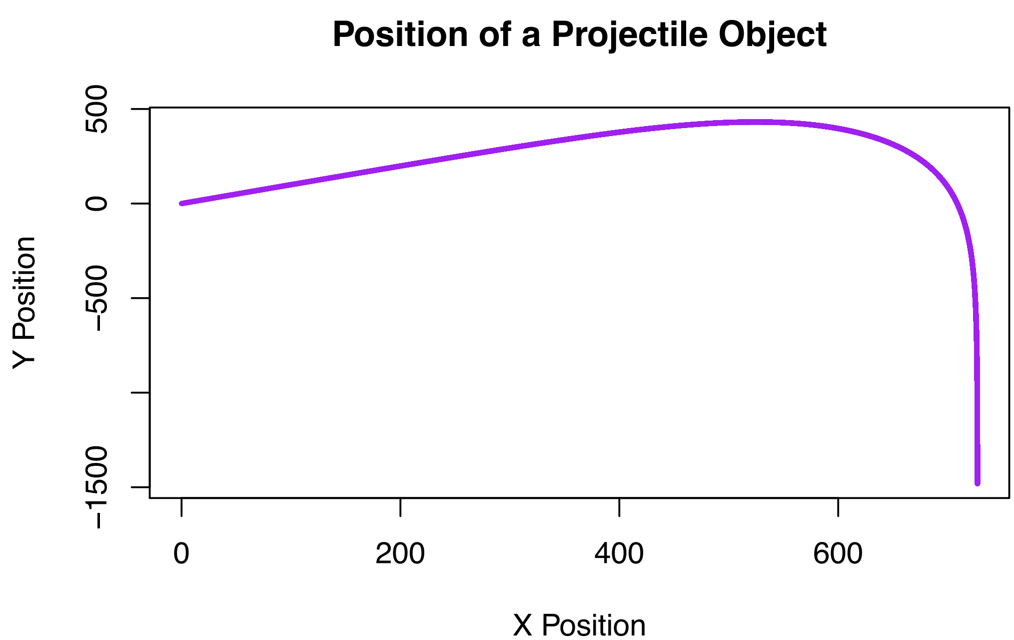

Figure 3. Graph of projectile object's position in a simulation of the adiabatic air density profile. All of the constants are at their default values.

Altering the firing angle of the projectile object has a similar, albeit more pronounced effect on its motion than it did in the previous two air density models. Decreasing the firing angle flattens the ascent and increases the x distance that the object travels by about 50 meters, while increasing the firing angle has the opposite effect.

When the muzzle velocity of the projectile object decreases, the "elbow" of the graph becomes more visible: the ascent flattens and the descent occurs more rapidly; in addition, the maximum x-position decreases by about 100 m. Even though increasing the initial velocity increases the maximum x-position, it doesn't have as dramatic of an effect on the shape of the graph: its shape is almost identical to the graph with all of the default constants.

Decreasing the time interval has a minimal effect: the maximum x-position is marginally larger. In contrast, increasing the time interval results in a lower density of points during the object's ascent, causing a higher error in approximations between those points. However, the rest of the graph remains practically the same; only the maximum x-position experiences a change and decreases.

Finally, decreasing the air density at sea level has a similar effect to the one it had in the constant density model: the ascent becomes steeper, both the maximum x and y-positions nearly double, and descent is not as vertical as in the control graph. Increasing air density results in a more pronounced "elbow" – the ascent is nearly horizontal and the descent is nearly vertical – and the x and y distances travelled decrease in comparison to the control.

D. Isothermal Air Density Profile

The final air density profile we studied was the isothermal air density profile. Unlike the previous models, simulating this air density model required a starting y-position that was greater than zero, as the density is a function of altitude. Thus, with a starting y-position of 10^4 meters and the other constants at their default values, the projectile object's position produced the following graph:

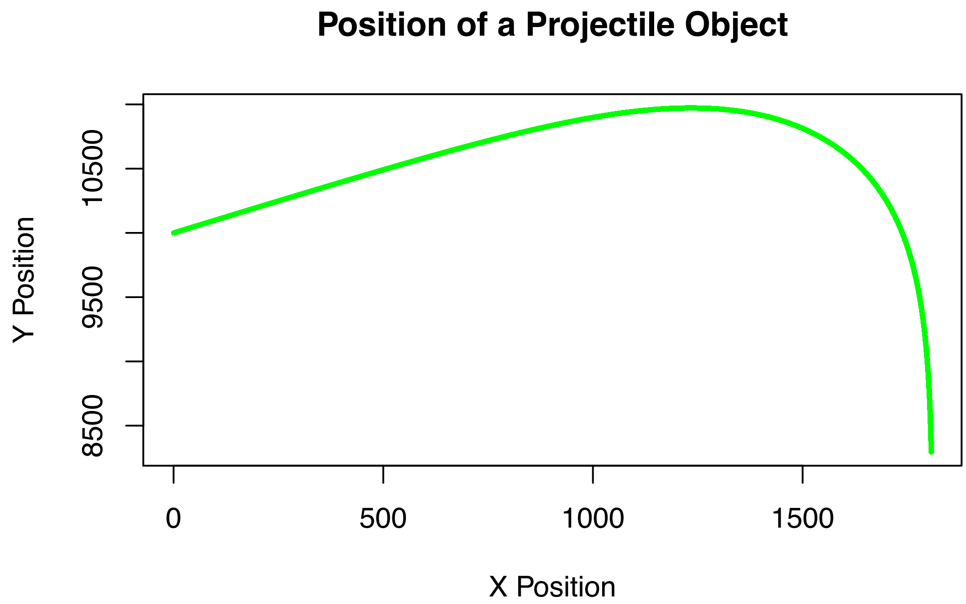

Figure 4. Graph of projectile object's position in a simulation of the isothermal air density profile. All of the constants are at their default values.

Decreasing the firing angle of the projectile doesn't alter the descent or final x-position as significantly as it does in the other air density models; however, the ascent flattens. Increasing the firing angle decreases the final x-position by a much larger amount than in the other models: about 600 meters. However, this model experiences many of the same changes the other models do: the ascent of the projectile steepens, and the descent is not as sudden as it is in the control graph.

While decreasing the muzzle velocity produced a graph similar to the one displayed above, the ascent of the object occurred at a slower rate, and the object traveled about 100 m less in the x-direction. However, increasing the muzzle velocity produced one of the least dramatic changes in the simulation of this model. The ascent steepened slightly, but the x-distance traveled marginally increased, and the shape of the graph with an increased velocity was nearly identical to that with the default velocity.

In addition, changing the step size had minimal effect on the position graph. The only visible change was the decreased density of the points at the beginning of the object's motion when the step size was increased.

Lastly, we studied the impact of air density at sea level on the position graph of the projectile object in this simulation. Decreasing air density had the most pronounced effect in this model. The shape of the graph changed drastically – it appeared to take on the appearance of a downwards-opening parabola, and the maximum x and y positions increased by a factor of 1.5. In contrast, the graph of the object's position with an increased air density was more similar to the control graph. However, the final x-position almost halved, and the maximum y-position decreased by about 500 meters.

III. Divergence

Our final aim was to study where the position of the projectile object diverged in each simulation of the four air density models. In order to investigate this divergence, we needed to plot each simulation on the same graph and compare where the position of the object differed. However, because the isothermal air density profile requires a high magnitude, positive starting y-position, we altered the default constants of the other models so that each simulation began at the coordinates $(0, 10^4)$. Doing so produced the following graph:

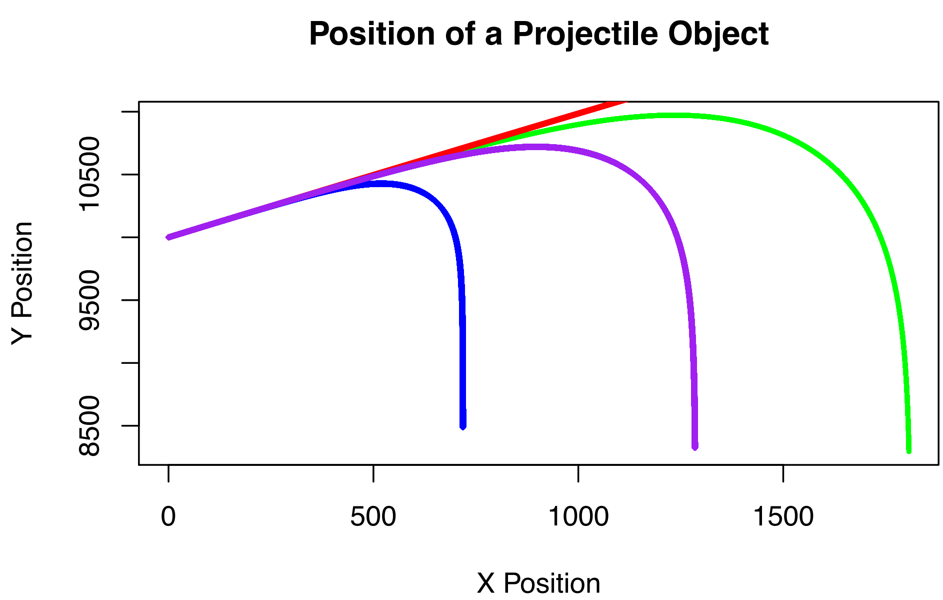

Figure 5. Graph of projectile object's position in the simulations of each air density model mentioned earlier in the report. Except for the initial position, which is now $(0, 10^4)$, all of the constants are at their default values. The graph for the simulation of the constant air density model is in blue, the graph for the zero air density model is in red, the graph for the adiabatic air density profile is in purple, and the graph for the isothermal air density profile is in green.

Our criterion for divergence was whether or not the graphs of the projectile object's position in different simulations overlapped. Following this standard, we found that the different air density models caused a divergence when the object reached an x-position of about 500 meters. However, while the object simulated in the constant air density model diverged first, the remaining models – adiabatic, isothermal, and zero – don't diverge until the object reaches an x-position of about 750 meters.

IV. Conclusion

Although the calculations that approximated velocity and position remained constant for each model, changing the calculation for air density dramatically altered the path each projectile object took in each simulation. Due to these mathematical differences, changing the values of constants like muzzle velocity and firing angle also had different impacts on each simulation. For example, decreasing the air density altered the graph of the projectile's position in an isothermal air density profile the greatest. Ultimately, this analysis report serves as a guide for the user of our projectile motion code: he or she can use it to predict the position graph a certain set of constants will produce, or to choose the air density model that best suits his or her needs.