Topological semantics of modal logic

$x \models \Box\varphi \overset{\text{def}}{\iff} \exists U_x \, \cancel{\forall} \, \forall y \in U_x \, (x \in U_x \wedge y \models \varphi)$

$x \models \Diamond\varphi \overset{\text{def}}{\iff} \forall U_x \, \cancel{\forall} \, \exists y \in U_x \, (x \in U_x \wedge y \models \varphi)$

Kuratowski's axioms resemble the axioms of S4

$[\![\varphi]\!] = \{x \in X \mid x \models \varphi\}$

interior

$x \models \Box\varphi$ <u>iff</u> ~~iff~~ $x \in i[\![\varphi]\!]$

$x \models \Diamond\varphi$ <u>iff</u> $x \in c[\![\varphi]\!]$

closure.

(McKinsey-Tarski, 1944) : S4 is the logic of an arbitrary (nonempty) crowded/dense-in-itself metric space

↳ every point is a limit point

Rasiowa - Sikorski book ← reference for Mckinsey - Tarski thm

~~How to prove~~

→ ~~Modal logic S4 has~~

Building a topological model from a finite frame

How to prove

→ S4 has finite-model property

→ use ~~bisimulat~~ functional bisimulation, ~~or~~ or p-morphism ?

→ We ~~w~~ will ~~define~~ pull-back along

$f : X \to \mathcal{F}$

X: Top space

$\mathcal{F}$: frame

Where ~~do~~ did we use finiteness ?

Infinite depth will cancel out density

Grzegorczyk logic

↳ closed w.r.t. to ?

[Look at the recording

Euclidean Hierarchy

↳ Van Bentham - Gehrke 2003
Guram Bezhanishvili

Canton Bendixon theorem

Shetman (1999)

Connectedness of a top. sp

⇕ 'clopen'-ness

$\forall p (\Diamond p \Rightarrow \Box p) \Rightarrow (\forall p \vee \forall \neg p)$

$\forall$ ← universal modality