Constructive Proof of Boolean algebras by a distributive lattice

L ← ~~Distributive~~ lattice

$\mathrm{Con}L$ ← Lattice of congruences of L

↳ $a \,\theta\, b \Rightarrow (a \vee x)\,\theta\,(b \vee x)$ and $(a \wedge x)\,\theta\,(b \wedge x)$ (the congruence)

★ $\mathrm{Con}L$ is distributive

Take $a \in L$

$\theta_a :=$ $x, \theta_a y$ if $(x \vee a) = (y \vee a)$

$\theta^a :=$ $x\,\theta^a\,y$ if $(x \wedge a) = (y \wedge a)$

★ If L is ~~con~~ distributive then θ_a and θ^a are congruences

$\theta_a \vee \theta^a = 1 = L \times L$

$\theta_a \wedge \theta^a = 0 = \{ (x,x) \mid x \in L \}$