Mathematical Formulation

1 Data :

number of employee : I

number of events : J

K : a large enough integer number

First we will transform time windows data that represent both the events start (begin) and end (end) times and also the employees shifts start(startshift) and end times(endshift), into a suitable format(seconds) that is compatible with the resolutions approach steps.

in addition to that we will generate a J×J Conflict matrix with values of 1 or 2, were the value 2 in $Conflict_{ij}$ indicates that their is no overlap between the i-eith and j-eith events time windows and it would be 1 if theirs is an overlap.

2 Indices :

i : to refer to an employee, with i = 1,...I

j : to refer to an event, with j = 1,...J

3 Mathematical Model :

3.1 Decision Variables :

We consider one type of Boolean variables as follows :

$$x_{i,j} = \begin{cases} 1 & \text{if the employee i take delivery j} \\ 0 & otherwise \end{cases}$$

C_{max} : continues variable that represent the maximum time span.

3.2 Constraints :

3.2.1 Time Span Constraints :

We would like to assign events as fairly as possible their for we will introduce this set of constraints.The time span for each employee i upper bounded with the objectif to minimize as follows: $\forall i = 1, ...I$

$$\sum_{j=1}^{n}(end_j - begin_j) \times x_{ij} \leq C_{max} \qquad (1.1)$$

3.2.2 Demand Satisfaction Constraints:

Each costumer j is served by exactly one employee i : $\forall j = 1, ..., J$

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad (1.1)$$

3.2.3 No overlap constraints :

For each employee we forbid the assignment of two distinct pair of deliveries with time windows that overlap:

$\forall i = 1, ..., I,$ $\quad \forall$ j, p = 1,...,J with j < p

$$\mathrm{x}_{ij} + x_{ip} \leq \mathrm{conflict}_{jp} \qquad (1.1)$$

3.2.4 Time Shifts Constraints:

For each employee, we assign tasks within his/her daily time shift :

After the start of his shift : $\forall i = 1, ..., I,$ $\forall$ j = 1,...,J

$$K \times (1 - x_{ij}) + begin_j \geq shiftstart_i$$

The upper bound of his last task does not exceeds the end of his time shift :

$\forall i = 1, ..., I,$ $\forall$ j = 1,...,J

$$end_j - K \times (1 - x_{ij}) \leq shiftend_i$$

3.3 Objectif Function :

the objectif is to minimize the maximum time span :

$$Z(min) = C_{max}$$

3.4

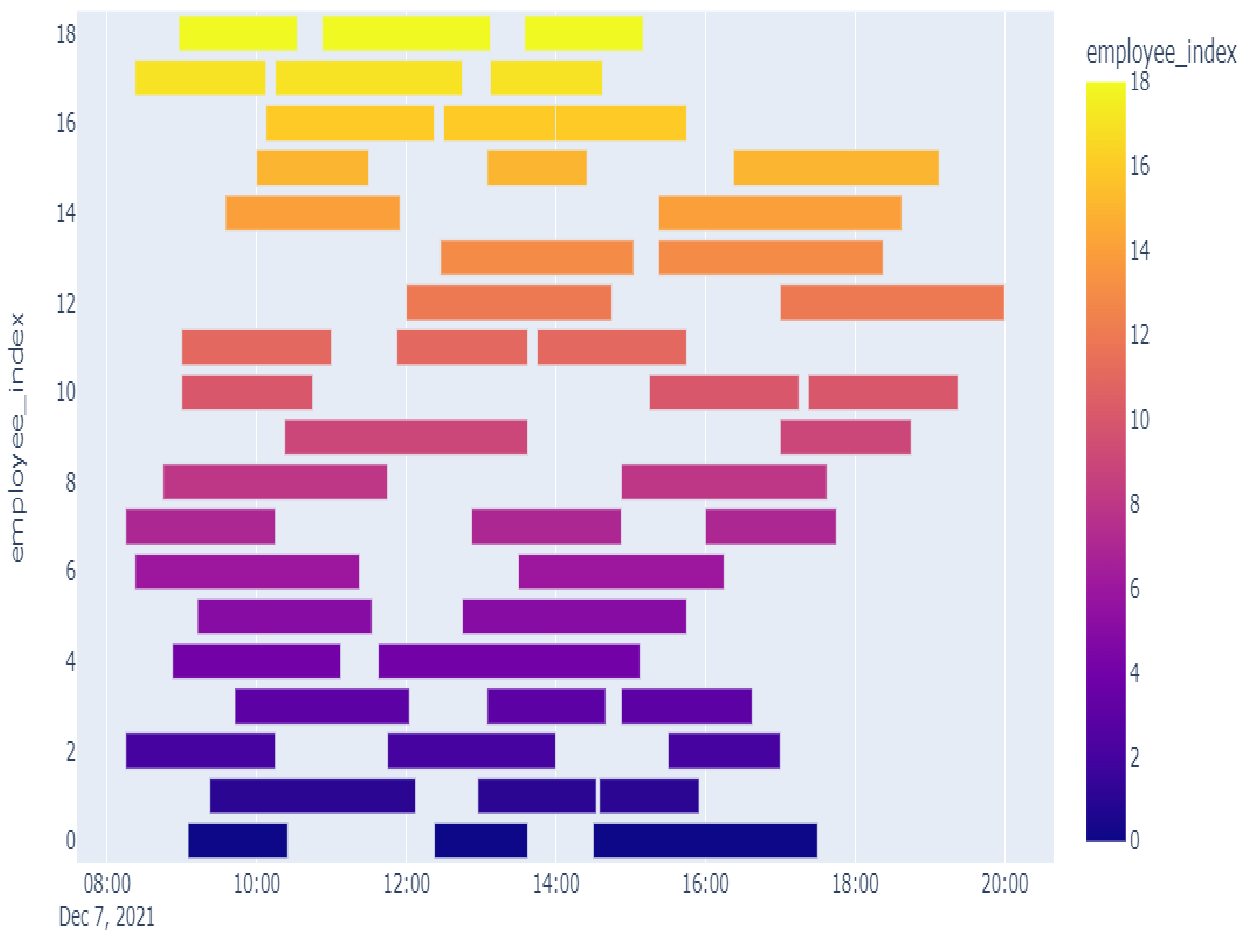

Figure 1: time table of the exmple