Unit 4: Inference for numerical data

1. Inference using the t-distribution

Sta 101 - Fall 2015

Duke University, Department of Statistical Science

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Slides posted at *http://bit.ly/sta101_f15*

1. Housekeeping

2. Main ideas
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    2. When comparing means of two groups, ask if paired or independent

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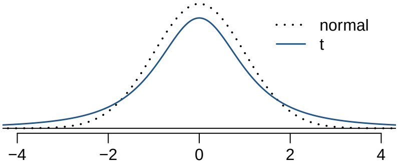
- Also has a bell shape, but its tails are *thicker* than the normal model’s
  - Observations are more likely to fall beyond two SDs from the mean than under the normal distribution.
- Extra thick tails are helpful for mitigating the effect of a less reliable estimate for the standard error of the sampling distribution.



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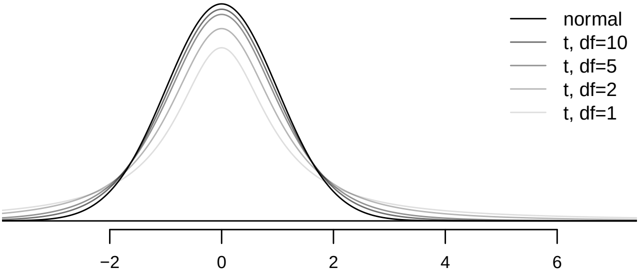

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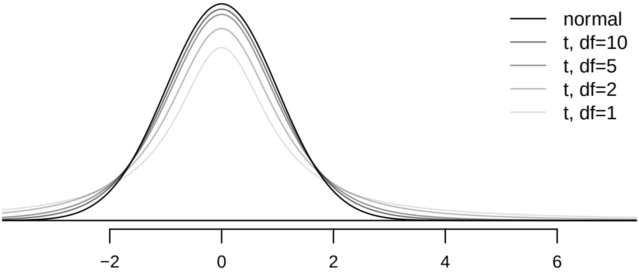

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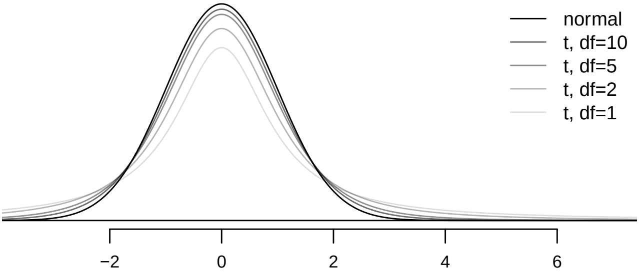

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Approaches normal.

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- dependent (paired) groups (e.g. pre/post weights of subjects in a weight loss study, twin studies, etc.)

$$SE_{\bar{x}_{diff}} = \frac{s_{diff}}{\sqrt{n_{diff}}}$$

- independent groups (e.g. grades of students across two sections)

$$SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Trace metals in drinking water affect the flavor and an unusually high concentration can pose a health hazard. Ten pairs of data were taken measuring zinc concentration in bottom water and surface water at 10 randomly sampled locations.

Location	bottom	surface
1	0.43	0.415
2	0.266	0.238
3	0.567	0.39
4	0.531	0.41
5	0.707	0.605
6	0.716	0.609
7	0.651	0.632
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Water samples collected at the same location, on the surface and in the bottom, cannot be assumed to be independent of each other, hence we need to use a *paired* analysis.

Source: *https://onlinecourses.science.psu.edu/stat500/node/51*

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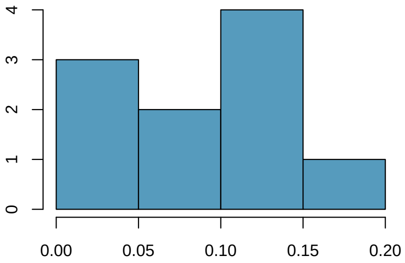
- It is important that we always subtract using a consistent order.

Location	bottom	surface	difference
1	0.43	0.415	0.015
2	0.266	0.238	0.028
3	0.567	0.39	0.177
4	0.531	0.41	0.121
5	0.707	0.605	0.102
6	0.716	0.609	0.107
7	0.651	0.632	0.019
8	0.589	0.523	0.066
9	0.469	0.411	0.058



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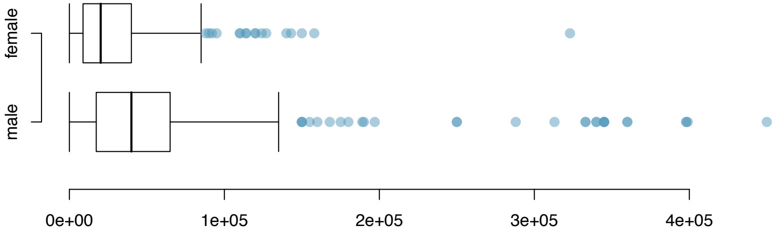
- *Point estimate:* Average difference between the bottom and surface zinc measurements of drinking water from the *sampled* locations.

$$\bar{x}_{diff}$$

Since 2005, the American Community Survey polls ~3.5 million households yearly. The following summarizes distribution of salaries of males and females from a random sample of individuals who responded to the 2012 ACS:

	$\bar{x}$	s	n
male	55,890	68,767.88	470
female	29,240	32,025.98	373



ACS: Surge of media attention in spring 2012 when the House of Representatives voted to eliminate the survey. Daniel Webster, Republican congressman from Florida: “in the end this is not a scientific survey. It’s a random survey.”

How are the two examples different from each other? How are they similar to each other?