Review
Bayesian vs. frequentist inference

Sta 101 - Spring 2015

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Slides posted at http://bitly.com/sta101sp15

- Poster sessions in Link Classroom 3 tomorrow, submission on Sakai due by your lab session time.
- Posttest due by Friday 5/1 at midnight
- Final exam review: Thursday, 4/30, 5:30 - 6:30pm

- We have a population of M&Ms. The percentage of yellow M&Ms is either 10% or 20%.
- You have been hired as a statistical consultant to decide whether the true percentage of yellow M&Ms is 10%. You are being asked to make a decision, and there are associated payoff/losses that you should consider.

	True state of the population	
Decision	% yellow = 10%	% yellow = 20%
% yellow = 10%	*Your boss gives you a bonus, and I'll bring you candy on Wednesday*	*You lose your job, and no candy for you*
%yellow = 20%	*You lose your job, and no candy for you*	*Your boss gives you a bonus, and I'll bring you candy on Wednesday*

- I will show you a random sample from the population, but you pay $200 for each M&M, and you must buy in $1000 increments.
- That is, you may buy 5, 10, 15, or 20 M&Ms.

- Hypotheses:
  - H_0: 10% yellow M&Ms
  - H_A: more than 10% yellow M&Ms
- Your test statistic is the number of yellow M&Ms you observe in the sample.
- The p-value will be the probability of observing this many or more yellow M&Ms given the null hypothesis is true.

Application exercise: Set up -- data: clicker

[CLICKER] How many M&Ms would you buy? Decide as a team and vote.

(a) 5 (b) 10 (c) 15 (d) 20

Application exercise: Set up -- significance level

[CLICKER] Discuss at what significance level you will reject the null hypothesis. Submit a value between 0 and 1.

Now we will take a sequence of M&Ms, and you record the number of yellows in the first n draws.

- $n = 5 \rightarrow$ RGYBO
- $n = 10 \rightarrow$ RGYBO BBGOY
- $n = 15 \rightarrow$ RGYBO BBGOY YRBRR
- $n = 20 \rightarrow$ RGYBO BBGOY YRBRR GORBY

Application exercise: FR.1 Frequentist inference

1. What is your sample size? This is your n.
2. How many yellows are in your sample? This is your k.
3. Calculate the p-value using the Binomial distribution: p-value = P(k or more yellows | n, %yellow is 10%)
4. Do you reject the null hypothesis based on the α you chose earlier?
5. What is the conclusion of your hypothesis test, i.e. what do you report to your boss?

Now we will start over, with 1:1 odds for the two competing hypotheses. These are our priors:

- H_1: 10% yellow M&Ms $\rightarrow P(H_1 : p = 0.10) = 0.5$
- H_2: 20% yellow M&Ms $\rightarrow P(H_2 : p = 0.10) = 0.5$

Application exercise: FR.2 Bayesian inference

Using the same data and Bayes' theorem to calculate the probability the percentage of yellow is 10% and 20% given the observed data in your sample, i.e.

1. $P(p = 0.10 \mid data)$
2. $P(p = 0.20 \mid data)$

Hint:

$$P(p = 0.10 \mid data) = \frac{P(data \mid 10\%yellow) \times P(10\%yellow)}{P(data)}$$

$$= \frac{P(data \mid 10\%yellow) \times P(10\%yellow)}{P(data \mid 10\%yellow) \times P(10\%yellow) + P(data \mid 20\%yellow) \times P(20\%yellow)}$$

$$= \frac{Binom(k \mid n, p = 0.10) \times P(H_1 : p = 0.10)}{Binom(k \mid n, p = 0.10) \times P(H_1 : p = 0.10) + Binom(k \mid n, p = 0.20) \times P(H_2 : p = 0.20)}$$

	Frequentist: p-value		Bayesian: Posterior	
# of yellow M&Ms in	P(K $\geq$ k | n, 10% yellow)	Decision	P(10% yellow | n, k)	P(20% yellow | n, k)
$n = 5 : k = 1$				
$n = 10 : k = 2$				
$n = 15 : k = 3$				
$n = 20 : k = 4$				

- The frequentist approach (using p-values) does not allow us to reject the null hypothesis of 10% yellow
- The Bayesian approach yields a higher posterior probability for 20% yellow
- The frequentist approach depends on the null hypothesis heavily (we would get different results if we had set $p = 0.20$ as the null hypothesis), but the Bayesian approach allows you to consider an array of hypotheses at once
- The Bayesian approach also gives you the actual probabilities you want, $P(hypothesis \mid data)$, and brings basic probability into the context of decision making scenarios more naturally than the frequentist p-value