Unit 6: Introduction to linear regression

2. Prediction, outliers, and inference for regression

Sta 101 - Fall 2015

Duke University, Department of Statistical Science

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Slides posted at *http://bit.ly/sta101_f15*

Announcements

- PA 6 opens today, due Nov 22, Sun
- PS 6 due Nov 20, Fri
- RA 7 (last RA!) on Wednesday
- Project 2 questions?
  - If you want to see sample posters from previous years, stop by office hours
  - Most important advice: Sketch out a meeting / working plan with your team **TODAY**, keeping in mind Thanksgiving break

Clicker question

Suppose you want to predict annual murder count (per million) for a series of districts that were not included in the dataset. For which of the following districts would you be most comfortable with your prediction?

A district where % in poverty =

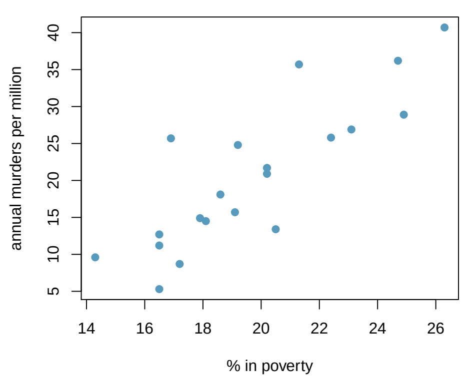
(a) 5%

(b) 15%

(c) 20%

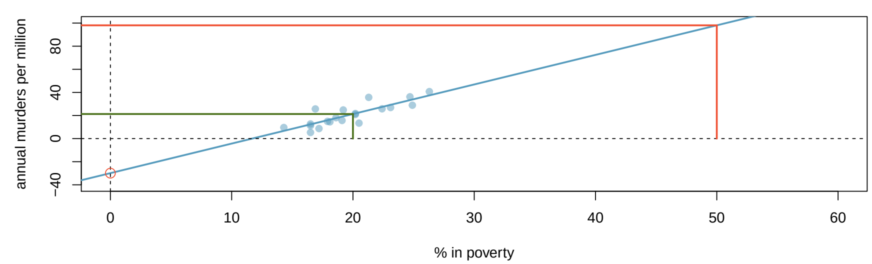
(d) 26%

(e) 40%



A note about the intercept

Sometimes the intercept might be an extrapolation: useful for adjusting the height of the line, but meaningless in the context of the data.



By hand: $\widehat{murder} = -29.91 + 2.56\ poverty$
The predicted number of murders per million per year for a county with 20% poverty rate is:

$$\widehat{murder} = -29.91 + 2.56 \times 20 = 21.29$$

In R:

```
# load data
murder <- read.csv("https://stat.duke.edu/~mc301/data/murder.csv")
# fit model
m_mur_pov <- lm(annual_murders_per_mil ~ perc_pov, data = murder)
# create new data
newdata <- data.frame(perc_pov = 20)
# predict
predict(m_mur_pov, newdata)
```

```
       1
21.28663
```


Calculating predicted values

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- Regression models are useful for making predictions for new observations not include in the original dataset.
- If the model is good, the predictions should be close to the true value of the response variable for this observation, however it may not be exact, i.e. $\hat{y}$ might be different than y.
- With any prediction we can (and should) also report a measure of uncertainty of the prediction.

A *prediction interval* for y for a given $x^\star$ is

$$\hat{y} \pm t^\star_{n-2} s \sqrt{1 + \frac{1}{n} + \frac{(x^\star - \bar{x})^2}{(n-1)s_x^2}}$$

where s is the standard deviation of the residuals, and $x^\star$ is a new observation.

- Interpretation: We are XX% confident that $\hat{y}$ for given $x^\star$ is within this interval.
- The width of the confidence interval for $\hat{y}$ increases as
  - $x^\star$ moves away from the center
  - s (the variability of residuals), i.e. the scatter, increases
- Prediction level: If we repeat the study of obtaining a regression data set many times, each time forming a XX% prediction interval at $x^\star$, and wait to see what the future value of y is at $x^\star$, then roughly XX% of the prediction intervals will contain the corresponding actual value of y.

Calculating the prediction interval

By hand:
Don't worry about it...

In R:

```
# predict
predict(m_mur_pov, newdata, interval = "prediction", level = 0.95)
```

```
       fit      lwr      upr
1 21.28663 9.418327 33.15493
```

We are 95% confident that the annual murders per million for a county with 20% poverty rate is between 9.52 and 33.15.

(1) R^2 assesses model fit -- higher the better

- R^2: percentage of variability in y explained by the model.
- For single predictor regression: R^2 is the square of the correlation coefficient, R.

```
murder %>%
    summarise(r_sq = cor(annual_murders_per_mil, perc_pov)^2)
```

```
       r_sq
1 0.7052275
```

- For all regression: $R^2 = \frac{SS_{reg}}{SS_{tot}}$

```
anova(m_mur_pov)
```

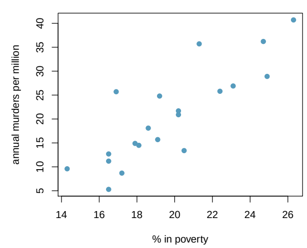
```
Analysis of Variance Table

Response: annual_murders_per_mil
          Df  Sum Sq Mean Sq F value    Pr(>F)
perc_pov   1 1308.34 1308.34  43.064 3.638e-06 ***
Residuals 18  546.86   30.38
```

$$R^2 = \frac{\textit{explained variabilty}}{\textit{total variability}} = \frac{SS_{reg}}{SS_{tot}} = \frac{1308.34}{1308.34 + 546.86} = \frac{1308.34}{1855.2} \approx 0.71$$

Clicker question

R^2 for the regression model for predicting annual murders per million based on percentage living in poverty is roughly 71%. Which of the following is the correct interpretation of this value?



(a) 71% of the variability in percentage living in poverty is explained by the model.

(b) 84% of the variability in the murder rates is explained by the model, i.e. percentage living in poverty.

(c) 71% of the variability in the murder rates is explained by the model, i.e. percentage living in poverty.

(d) 71% of the time percentage living in poverty predicts murder rates accurately.

Inference for regression uses the *t*-distribution

- Use a T distribution for inference on the slope, with degrees of freedom $n - 2$
  - Degrees of freedom for the slope(s) in regression is $df = n - p - 1$ where p is the number of predictors (explanatory variables) in the model.
- Hypothesis testing for a slope: $H_0 : \beta_1 = 0$; $H_A : \beta_1 \neq 0$
  - $T_{n-2} = \frac{b_1 - 0}{SE_{b_1}}$
  - p-value = P(observing a slope at least as different from 0 as the one observed if in fact there is no relationship between x and y
- Confidence intervals for a slope:
  - $b_1 \pm T^\star_{n-2} SE_{b_1}$

Conditions for regression

Important regardless of doing inference

- Linearity → randomly scattered residuals around 0 in the residuals plot – important regardless of doing inference

Important for inference

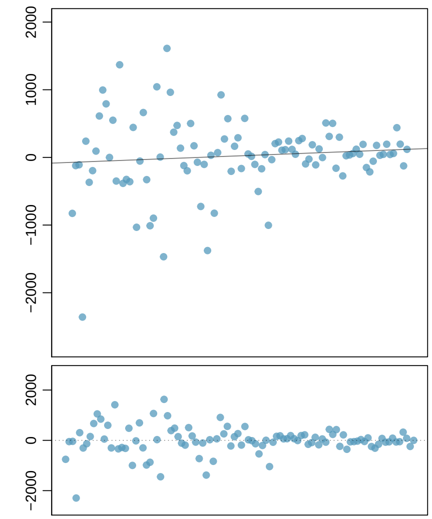
- Nearly normally distributed residuals → histogram or normal probability plot of residuals
- Constant variability of residuals (*homoscedasticity*) → no fan shape in the residuals plot
- Independence of residuals (and hence observations) → depends on data collection method, often violated for time-series data

Checking conditions

Clicker question

What condition is this linear model obviously and definitely violating?

(a) Linear relationship
(b) Non-normal residuals
(c) Constant variability
(d) Independence of observations



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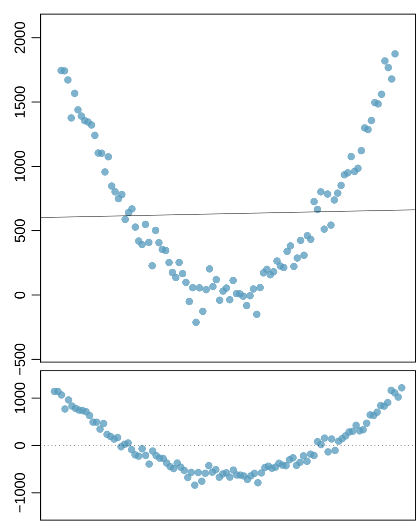
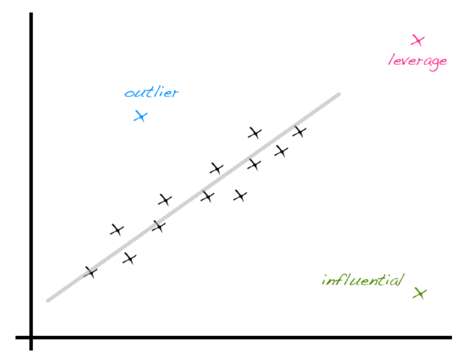


Type of outlier determines how it should be handled

- *Leverage* point is away from the cloud of points horizontally, does not necessarily change the slope
- *Influential* point changes the slope (most likely also has high leverage) – run the regression with and without that point to determine



- *Outlier* is an unusual point without these special characteristics (this one likely affects the intercept only)
- If clusters (groups of points) are apparent in the data, it might be worthwhile to model the groups separately.

Application exercise: 6.2 Linear regression

See course website for details

1. Predict, but don't extrapolate
2. Predicted values also have uncertainty around them
3. R^2 assesses model fit – higher the better
4. Inference for regression uses the t-distribution
5. Conditions for regression
6. Type of outlier determines how it should be handled