Computer Vision AI – Assignment 2
Epipolar Geometry and Chaining

April 25, 2014

The analysis, the conclusions and the source code must be included in the final delivery. Students are supposed to work on this assignment for two weeks.

Important: Starting from this week you are going to implement partially (not all the steps) a simplified version of the method described in "3D Object Modeling and Recognition Using Local Affine-Invariant Image Descriptors and Multi-View Spatial Constraints".

1 Fundamental Matrix Estimation

In this assignment you will write a function that takes two images as input and computes fundamental matrix by Normalized Eight-point Algorithm with Ransac 1.3. You will work with supplied teddy bear and house images. The overall scheme can be summarized as follows:

1. Detect interest points in each image.
2. Characterize the local appearance of the regions around interest points.
3. Get a set of supposed matches between region descriptors in each image.
4. Estimate the fundamental matrix for the given two images.

The first three steps can be performed using vl_feat functions (http://www.vlfeat.org/download.html):

Note: Eliminate detected interest points on background.

In the next stage, for given $n \geq 8$ known corresponding points' pairs in stereo images, we can formulate a homogenous linear equation as follows:

$$\begin{bmatrix} x_i' y_i' 1 \end{bmatrix} \underbrace{\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}}_{F} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = 0 \; , \tag{1}$$

where x_i and y_i denote x and y coordinates of the i^{th} point p_i, respectively. Equation 1 can also be written as

$$\underbrace{\begin{bmatrix} x_1x_1{}' & x_1y_1{}' & x_1 & y_1x_1{}' & y_1y_1{}' & y_1 & x_1{}' & y_1{}' & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_nx_n{}' & x_ny_n{}' & x_n & y_nx_n{}' & y_ny_n{}' & y_n & x_n{}' & y_n{}' & 1 \end{bmatrix}}_{A} \begin{bmatrix} f_{11} \\ f_{21} \\ f_{31} \\ f_{12} \\ f_{22} \\ f_{32} \\ f_{13} \\ f_{23} \\ f_{33} \end{bmatrix} = 0 \ , \qquad (2)$$

where F denotes the fundamental matrix.

1.1 Eight-point Algorithm

- Construct the $n \times 9$ matrix A
- Find the SVD of A: $A = UDV^T$
- The entries of F are the components of the column of V corresponding to the smallest singular value.

Estimated fundamental matrix F is almost always non-singular (read pages 215 – 221 from Forsyth & Ponce book for further details). The singularity of fundamental matrix is enforced by adjusting the entries of estimated F:

- Find the SVD of F: $F = U_f D_f {V_f}^T$
- Set the smallest singular value in the diagonal matrix D_f to zero in order to obtain the corrected matrix $D'{}_f$
- Recompute F: $F = U_f D'_f {V_f}^T$

1.2 Normalized Eight-point Algorithm

It turns out that a careful normalization of the input data (the point correspondences) leads to an enormous improvement in the conditioning of the problem, and hence in the stability of the result. The added complexity necessary for this transformation is insignificant.

1.2.1 Normalization:

We want to apply a similarity transformation to the set of points $\{p_i\}$ so that their mean is 0 and the average distance to the mean is $\sqrt{2}$.

Let $m_x = \frac{1}{n}\sum_{i=1}^{n} x_i$, $m_y = \frac{1}{n}\sum_{i=1}^{n} y_i$, $d = \frac{1}{n}\sum_{i=1}^{n}\sqrt{(x_i - m_x)^2) + (y_i - m_y)^2)}$, and $T = \begin{bmatrix} \sqrt{2}/d & 0 & -m_x\sqrt{2}/d \\ 0 & \sqrt{2}/d & -m_y\sqrt{2}/d \\ 0 & 0 & 1 \end{bmatrix}$, where x_i and y_i denote x and y coordinates of a point p_i, respectively.

Then $\widehat{p_i} = Tp_i$. Check and show that the set of points $\{\widehat{p_i}\}$ satisfies our criteria. Similarly, define a transformation T' using the set $\left\{\widehat{p_i}'\right\}$, and let $\widehat{p_i}' = T'p_i'$.

1.2.2 Find a fundamental matrix:

- Construct a matrix A from the matches $\widehat{p_i} \leftrightarrow \widehat{p_i}'$ and get $\widehat{F}$ from the last column of V in the SVD of A.
- Find the SVD of $\widehat{F}$: $\widehat{F} = U_{\widehat{f}} D_{\widehat{f}} V_{\widehat{f}}^T$
- Set the smallest singular value in the diagonal matrix $D_{\widehat{f}}$ to zero in order to obtain the corrected matrix $D'_{\widehat{f}}$
- Recompute $\widehat{F}$: $\widehat{F} = U_{\widehat{f}} D'_{\widehat{f}} V_{\widehat{f}}^T$

1.2.3 Denormalization:

Finally, let $F = T'^T \widehat{F}' T$.

1.3 Normalized Eight-point Algorithm with Ransac

Fundamental matrix estimation step given in Section 1.2.2 can also be performed via a RANSAC-based approach. First pick 8 point correspondences randomly from the set $\left\{\widehat{p_i} \leftrightarrow \widehat{p_i}'\right\}$, then, calculate a fundamental matrix $\widehat{F}'$, and count the number of inliers (the other correspondences that agree with this fundamental matrix). Repeat this process many times, pick the largest set of inliers obtained, and apply fundamental matrix estimation step given in Section 1.2.2 to the set of all inliers.

In order to determine whether a match $p_i \leftrightarrow p_i{}'$ agrees with a fundamental matrix F, we typically use the Sampson distance as follows:

$$d_i = \frac{(p_i'^T F p_i)^2}{(Fp_i)_1^2 + (Fp_i)_2^2 + (F^T p_i')_1^2 + (F^T p_i')_2^2}, \tag{3}$$

where $(Fp)_j^2$ is the square of the j^{th} entry of the vector Fp. If d_i is smaller than some threshold, the match is said to be an inlier.

2 Chaining

Construct point-view matrix with the matches found in last step for all consecutive teddy-bear images (1-2, 2-3, 3-4, ..., 15-16, 16-1). Rows of the point-view matrix will be representing your images while columns will be points.

1. Start from any two consecutive image matches. Add a new column to point-view matrix for each newly introduced point.

2. If a point which is already introduced in the point-view matrix and another image contains that point, mark this matching on your point-view matrix using the previously defined point column. Do not introduce a new column.