Flows

* Conservation Constraint

$\sum_s f(s \to w) = \sum_t f(w \to t)$

flow in = flow out

* $|f|$ net flow out of source vertex

$$|f| := \sum_w f(s \to w) - \sum_u f(u \to s)$$

$\partial f(v)$ — net flow out of any vertex

$$\partial f(v) := \sum_u f(u \to v) - \sum_w f(v \to w)$$

since $\partial f(v) = 0 \ \forall v \notin s, t$

$$\sum_v \partial f(v) = \partial f(s) + \partial f(t)$$

$c : E \to \mathbb{R}_{\geq 0}$

flow f is feasible wrt to c

if $0 \leq f(e) \leq c(e) \ \forall$ edges

$f(e) = c(e) \to$ saturated edge

$f(e) = 0 \to$ Avoids edge

ESPPRC

$G = N(\cup\{v_s, v_e\}, A)$

N – customers, (v_e, v_s) – start & end depots

- (v_i, v_j) is an arc that reps travel bet customers & their distances (d_{ij}) & times (t_{ij})
- Reduced cost for every arc is calc as $r_{ij} = d_{ij} - \lambda_{ij}$ ← dual multiplier
- ESPPRC is a subproblem in master problem of column generation. (routes are columns)
  - Vehicle capacity is Q & q_i total demand should not exceed
  - Time window $[a_i, b_i]$

Subproblem

$v_s \to v_e$

min r_{ij} considering q_{ij} & t_{ij}

Proof

TP: $|f| = \|S, T\|$

$|f| = \partial f(v)$ $\Big|\ \sum_{v \in S} \partial f(v)$

$= \cancel{\sum_{v \in S} \partial f(v \to w)} - \cancel{\sum_{u \in S} f(u \to v)}$

$= \sum_{v \in S} \sum_{w} f(v \to w) - \sum_{v \in S} \sum_{u} f(u \to v)$

$= \sum_{v \in S} \sum_{w \notin S} f(v \to w) - \sum_{v \in S} \sum_{u \notin S} f(u \to v)$

$= \sum_{v \in S} \sum_{w \in T} f(v \to w)$

$\quad - \sum_{v \in S} \sum_{u \in T} f(u \to v)$

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