ASEN 3802 Lab 2 Part 2

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1 Analytical Transient Solution Derivation

1.1 Derivation

Derivation of b_n, the Fourier coefficients.

$$b_n = -\frac{2H}{L}\int_0^L x\sin(\lambda_n x)dx, \qquad \lambda_n = \frac{(2n-1)\pi}{2L}$$

Using integration by parts,

$$b_n = -\frac{2H}{L}\left[-x\frac{1}{\lambda_n}\cos(\lambda_n x)\Big|_0^L + \int_0^L \frac{1}{\lambda_n}\cos(\lambda_n x)dx\right]$$

$$b_n = -\frac{2H}{L}\left[-x\frac{1}{\lambda_n}\cos(\lambda_n x) + \frac{1}{\lambda_n^2}\sin(\lambda_n x)\right]_0^L$$

$$b_n = -\frac{2H}{L\lambda_n}\left[-x\cos(\lambda_n x) + \frac{1}{\lambda_n}\sin(\lambda_n x)\right]_0^L$$

Plugging in λ_n,

$$b_n = -\frac{2H}{L\frac{(2n-1)\pi}{2L}}\left[-x\cos\left(\frac{(2n-1)\pi}{2L}x\right) + \frac{2L}{(2n-1)\pi}\sin\left(\frac{(2n-1)\pi}{2L}x\right)\right]_0^L$$

$$b_n = -\frac{4H}{(2n-1)\pi}\left[-L\cos\left(\frac{(2n-1)\pi}{2L}L\right) + \frac{2L}{(2n-1)\pi}\sin\left(\frac{(2n-1)\pi}{2L}L\right) - \left(\cancel{-0\cos\left(\frac{(2n-1)\pi}{2L}0\right)} + \cancel{\frac{2L}{(2n-1)\pi}\sin\left(\frac{(2n-1)\pi}{2L}0\right)}\right)\right]$$

$$b_n = -\frac{4H}{(2n-1)\pi}\left[-L\cos\left(\frac{(2n-1)\pi}{2\cancel{L}}\cancel{L}\right) + \frac{2L}{(2n-1)\pi}\sin\left(\frac{(2n-1)\pi}{2\cancel{L}}\cancel{L}\right)\right]$$

$$b_n = -\frac{4H}{(2n-1)\pi}\left[\cancel{-L\cos\left(\frac{(2n-1)\pi}{2}\right)} + \frac{2L}{(2n-1)\pi}\sin\left(\frac{(2n-1)\pi}{2}\right)\right]$$

$$b_n = -\frac{4H}{(2n-1)\pi}\left[\frac{2L}{(2n-1)\pi}\sin\left(\frac{(2n-1)\pi}{2}\right)\right]$$

$$\boxed{b_n = -\frac{8HL(-1)^{n+1}}{(2n-1)^2\pi^2} = \frac{8HL(-1)^n}{(2n-1)^2\pi^2}}$$

$$u(x,t) = T_0 + Hx + \sum_{n=1}^{\infty} b_n \sin(\lambda_n x)e^{-\lambda_n^2\alpha t}$$

Using the values found from Part 1 for aluminum at 26 V, we have $T_0 = 12°C$ and $H_{an} = 98.6\frac{°C}{m}$. We were given its thermal conductivity, specific heat, density, and we were able to calculate the length of the bar.

1.2 Plot

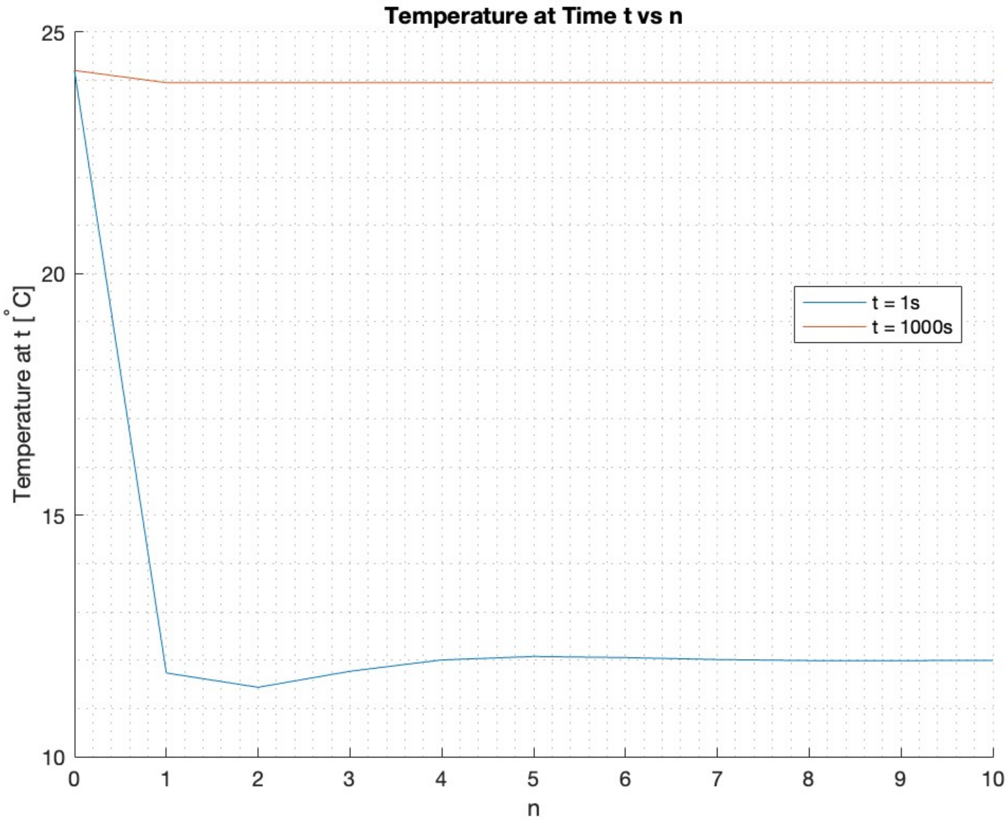

Figure 1: Convergence of the temperature of aluminum at $t = 1$s and $t = 1000$s

Using our derivation and the values provided, we plotted the temperature at 2 different times from $n = 0$ to $n = 10$. This plot shows the convergence of the temperature to the desired steady state as more and more terms are added. As discussed in section 1.3: as time increases, fewer terms are necessary to reach convergence to the solution. At $t = 1000$ seconds, only 1 term is necessary to reach convergence, but for $t = 1$ seconds, about 5 terms are necessary to reach convergence. From just looking at Figure 1 we can see this result, the plot for $t = 1000$ seconds oscillates less and settles within fewer iterations than when $t = 1$ seconds.

1.3 Discussion

For this experiment one term is not sufficient. Using the Fourier number equation, $F_0 = \frac{\alpha t}{L^2}$, we found that the Fourier numbers at 1 second and 1000 seconds are 0.0016 and 1.5808, respectively. For aluminum at 1 second, the Fourier number is significantly lower than than 0.2, the cutoff for one term being sufficient. At 1000 seconds, the Fourier number is above 0.2. We noted that as t increases, the Fourier number increases, and it is sufficient to have fewer terms. However, in order to have a model that is accurate at all time inputs, we need to use more than one term. In general, although one term is 98% accurate for models with Fourier numbers above 0.2, more terms lead to better accuracy. Since it took about 5 terms for the series to converge at 1 second, we chose to use $n = 10$ for the models to ensure accuracy at earlier times.

2 Task 2: Time-Dependent Temperature Profiles - Model IA

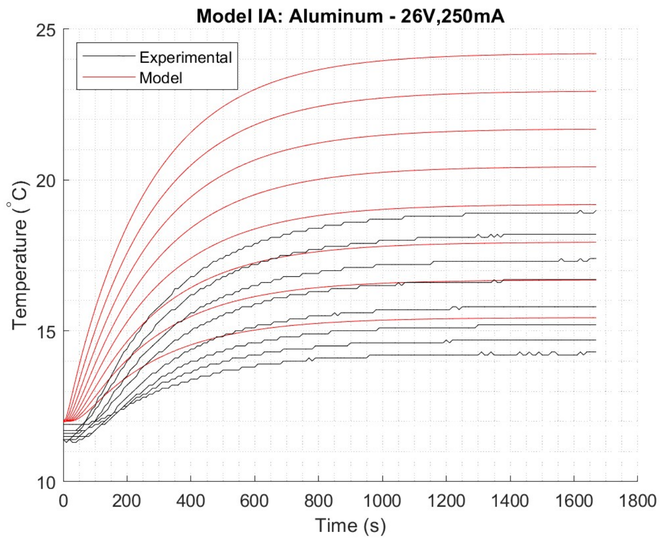

Figure 2: Aluminum at 26V: Model vs Experiment

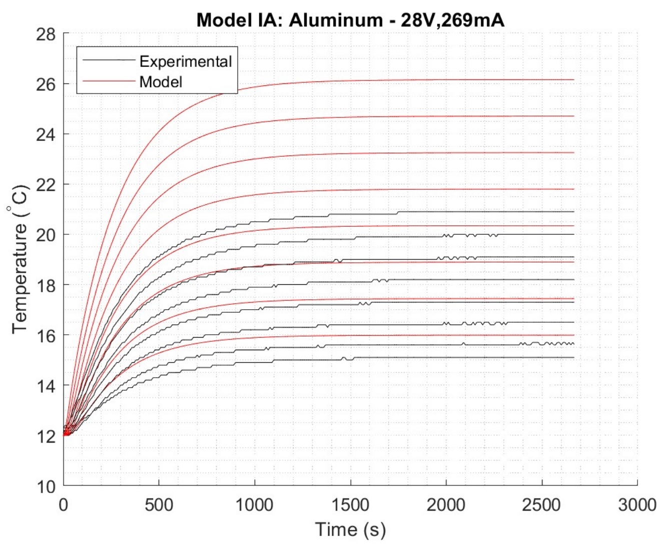

Figure 3: Aluminum at 28V: Model vs Experiment

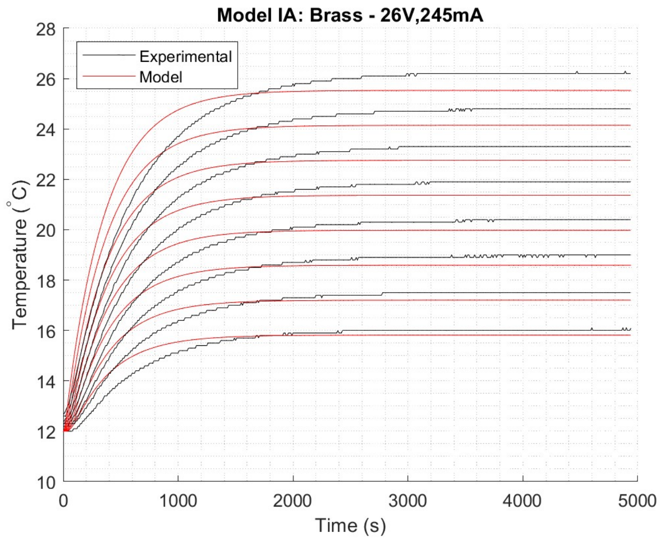

Figure 4: Brass at 26V: Model vs Experiment

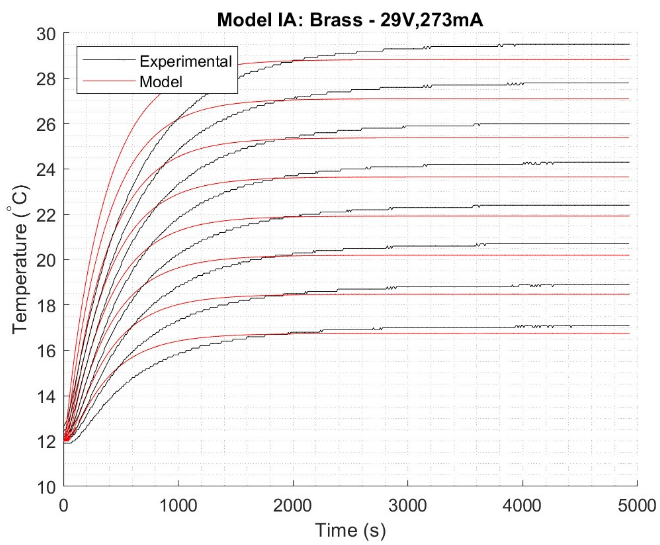

Figure 5: Brass at 29V: Model vs Experiment

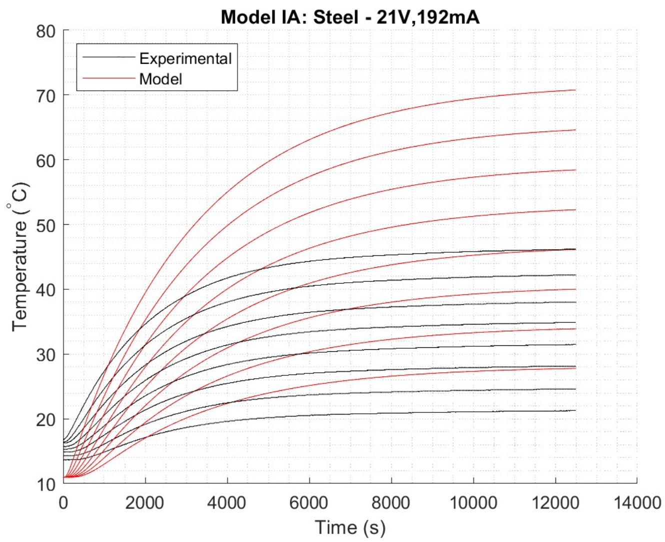

Figure 6: Steel at 21V: Model vs Experiment

Model IA is a good approximation for the shape of the temperature vs. time curve. However, the model does not work well for aluminum or steel. Looking at Figures 2 and 3, the final temperature that the model reaches is higher than the experimental temperature by about $5°C$. This result makes sense because our calculated analytical slope for aluminum was about 80% more than the experimental steady-state slope. We see a similar result in Figure 6 where the model predicts a final temperature almost 30 degrees higher than the measured temperature. For steel, it looks as if the model never reaches a steady-state temperature in the same time frame as the experiment. This result is expected because the analytical slope we calculated for Steel in Part 1 was about 50% more than the experimental steady-state slope, resulting in a large discrepancy between the model and the experiment. Model IA best predicted the temperature for brass because our calculated analytical slope was about 3% less than the experimental steady-state slope. For brass, the model slightly underestimated the temperature, as seen in Figures 4 and 5. There is some error in the steady-state temperature distributions of the bar, i.e., the final temperatures are different between the model and the experiment. For brass, the discrepancy is within $1°C$, however, for aluminum and steel the difference is much greater than $1°C$. We account for this error as a result of the vast difference between the analytical and experimental steady-state slopes for those materials.

3 Task 3: Time-Dependent Temperature Profiles - Model IB

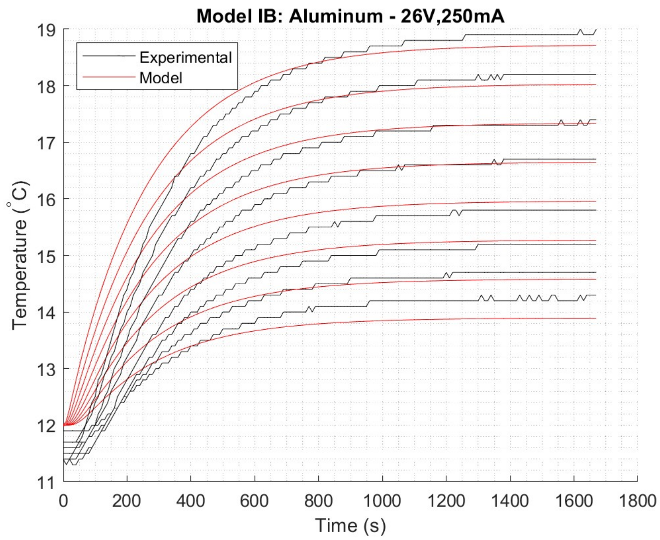

Figure 7: Aluminum at 26V: Model vs Experiment

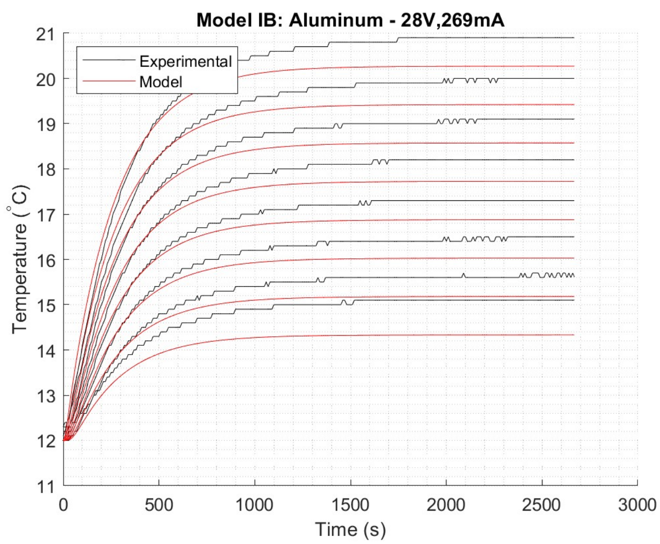

Figure 8: Aluminum at 28V: Model vs Experiment

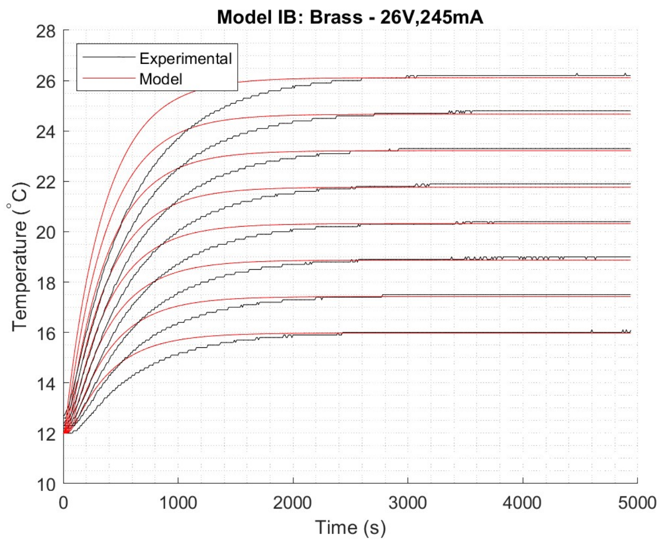

Figure 9: Brass at 26V: Model vs Experiment

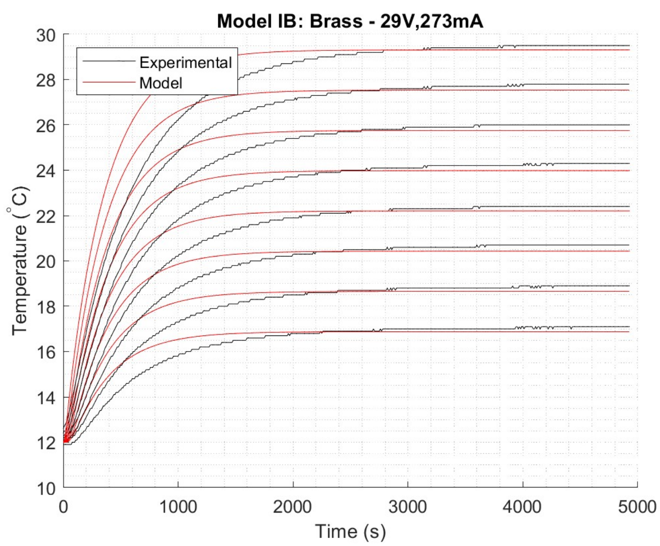

Figure 10: Brass at 29V: Model vs Experiment

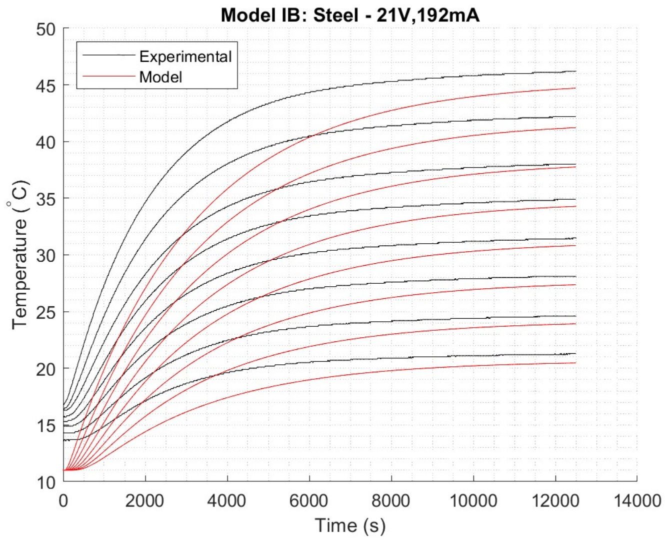

Figure 11: Steel at 21V: Model vs Experiment

As explained in Task 2, there is significant error in the steady-state temperatures for the aluminum and steel bars for Model IA. This error was due to the vast difference between the analytical and experimental steady-state slopes for those materials. We would expect Model IB to be a better approximation of the steady-state temperatures due to the slope being used was calculated from the experiment. As shown in Figures 7-11, Model IB is a better approximation for the experiment than Model IA because the model now uses the experimental steady-state slopes calculated in Part 1. Using the experimental slope led to a good approximation for both brass and aluminum, where the final temperatures are within $1°C$, as shown in Figures 7-10. As shown in Figure 11, the model does not accurately depict the temperature for steel. The final temperatures of the experimental results and the model are within $1°C$, however, the initial temperature distribution is not within that bound. The model starts from slightly different initial temperatures than the experiment. Overall, using the experimental slopes for Model IB gave us a better model for the thermocouple. This model performs better than Model IA for the steady-state distribution, but it does not accurately model the initial temperature distribution.

4 Task 4: Time-Dependent Temperature Profiles - Model II

When developing our analytical model, we assumed that the bar was initially at a constant T_0, found by taking the intercept of an interpolated final state. However, after comparing the model to the experimental data, it is clear that this may not be a very accurate assumption. In order to improve the model, we used new $g(x)$ and $f(x)$ functions when deriving the Fourier Coefficients, b_n. We used $f(x) = Mx + T_0$ and $g(x) = (M - H)x$ to more accurately match the initial temperature distribution of the bar. The variable M, is the experimental steady-state slope of the initial condition of the data. This value was found for each material in Part 1 of the lab. Due to the error in the thermocouples, the values of M ranged drastically since each thermocouple has a certainty of $\pm 2°C$.

By changing the functions $g(x)$ and $f(x)$, we are able to produce a more accurate model because it more closely matches the data at $t = 0$.

$$b_n = \frac{2}{L}\int_0^L g(x)\sin(\lambda_n x)dx, \quad g(x) = (M - H)x$$

$$b_n = \frac{2(M - H)}{L}\int_0^L x\sin(\lambda_n x)dx$$

By modifying the result from Section 1.1, we can find the equation for the Fourier Coefficients:

$$\boxed{b_n = \frac{8(M-H)L(-1)^{n+1}}{(2n-1)^2\pi^2}}$$

Using the new values for b_n, we can plot a new model which should more accurately represent the experimental results at $t = 0$. The results for this new model are shown below.

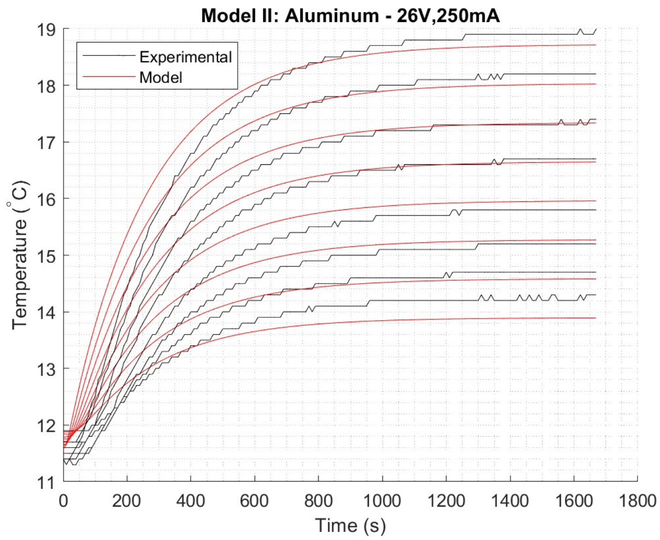

Figure 12: Aluminum at 26V: Model vs Experiment

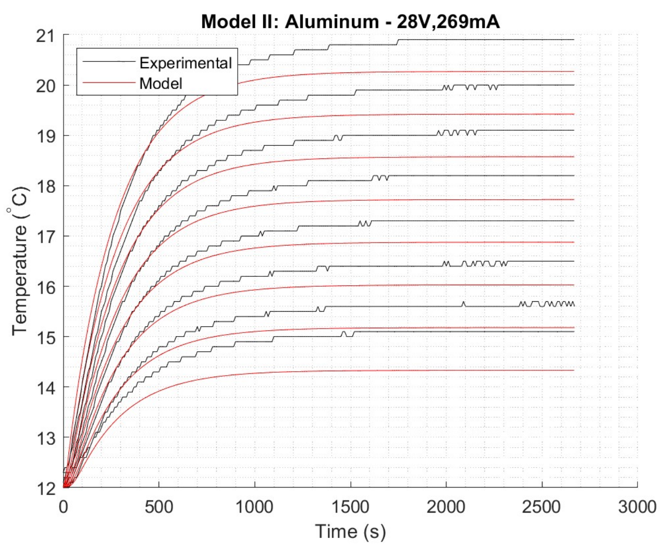

Figure 13: Aluminum at 28V: Model vs Experiment

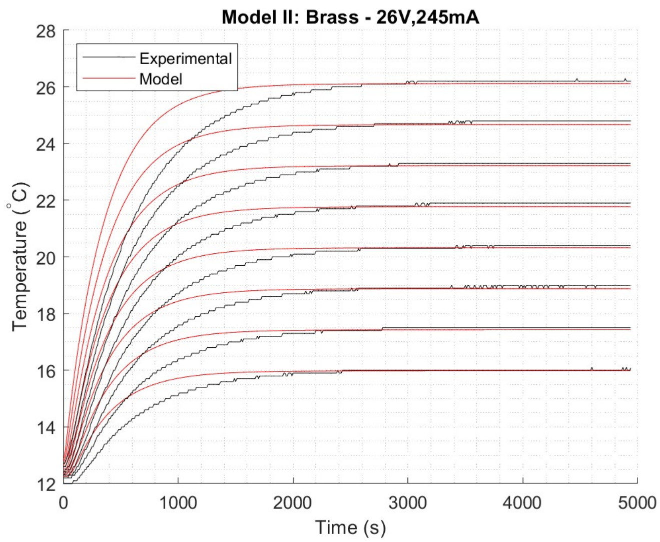

Figure 14: Brass at 26V: Model vs Experiment

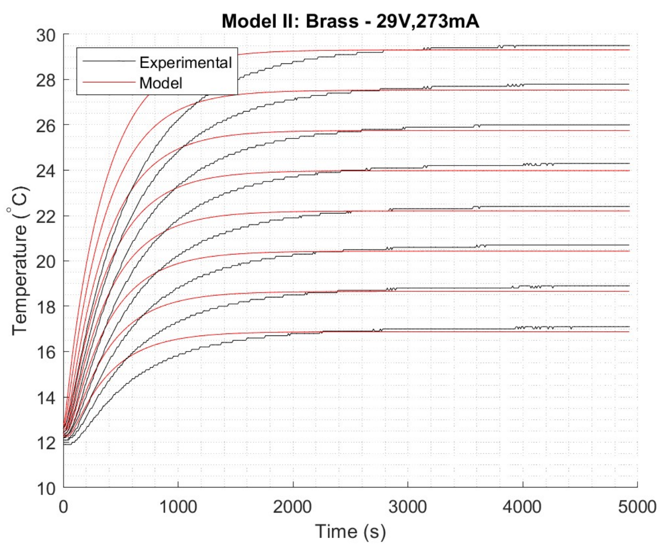

Figure 15: Brass at 29V: Model vs Experiment

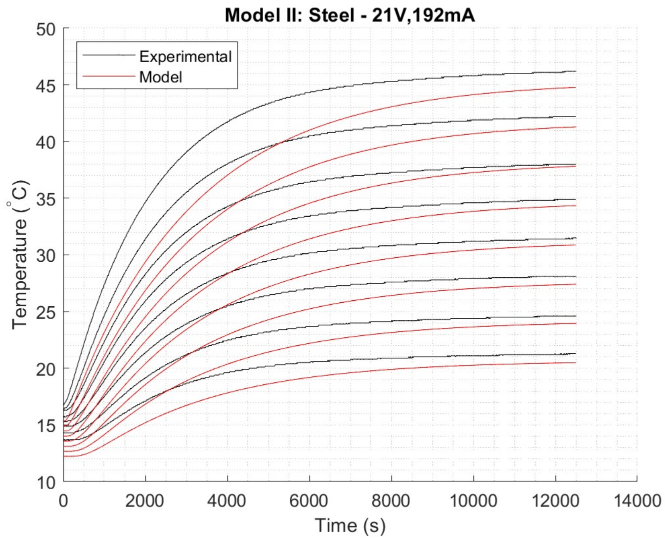

Figure 16: Steel at 21V: Model vs Experiment

As shown in Figures 12-16, we can see that Model II improves the estimate of the initial temperature distribution. The experimental results and Model II are closer at $t = 0$, more so than the previous models. Model II is the most accurate at predicting the initial temperatures of the rods, while continuing to accurately represent the steady-state temperatures. We were able to see this improvement visually in our plots because the initial temperatures are much closer and all seem within $\pm 2[^{\circ}C]$ degrees of the experimental data.

5 Appendix

5.1 Task 1

```
%% ASEN 3802 Lab 2 Part 2 Task 1

clc;clear;close all;

const.T0 = 12; % [C]
const.H_an = 98.6;  % [C/m]
const.k = 130; % [W/mK]
const.rho = 2810; % [kg/m^3]
const.c_p = 960; % [J/kgK]
const.alpha = const.k/(const.rho*const.c_p); % [m^2/s]
const.x = 0.1238; % [m]
const.L = const.x+0.0508; % [m]

n = 10;
t1 = 1; % [s]
t2 = 1000; % [s]

u_1s = heatdistribution(t1,n,const);
u_1000s = heatdistribution(t2,n,const);

Fo_1s = (const.alpha*t1)/const.L^2;
Fo_1000s = (const.alpha*t2)/const.L^2;

figure
hold on
plot(0:n,u_1s)
plot(0:n,u_1000s)
title('Temperature at time t vs n')
xlabel('n')
ylabel('Temperature at t [C]')
legend('t = 1s','t = 1000s')


function u = heatdistribution(t,n,const)
    sum = 0;
    u(1) = const.T0+const.H_an*const.x;
    for i = 1:n
        b_n = (8*const.H_an*const.L*(-1)^i)/((2*i-1)^2*pi^2);
        lambda_n = pi*(2*i-1)/(2*const.L);
        sum = sum + b_n*sin(lambda_n*const.x)*exp(-1*lambda_n^2*const.alpha*t);
        u(i+1) = const.T0+const.H_an*const.x + sum;
    end
end
```

5.2 Task 2

```
%% ASEN 3802 Lab 2 Part 2 Task 2

clc;clear;close all;

x1 = 0.0349; % [m]
x2 = 0.0476; % [m]
```

```
x3 = 0.0603; % [m]
x4 = 0.073; % [m]
x5 = 0.0857; % [m]
x6 = 0.0984; % [m]
x7 = 0.1111; % [m]
x8 = 0.1238; % [m]
x = [x1,x2,x3,x4,x5,x6,x7,x8];

const_st.H_an = 491.2; % [C/m]
const_st.T0 = 11; % [C]
const_st.k = 16.2; % [W/mK]
const_st.cp = 500; %J/kgK
const_st.rho = 8000; % [kg/m^3]
const_st.alpha = const_st.k/(const_st.rho*const_st.cp); % [m^2/s]
const_st.L = x8+0.0508; % [m]

const_br26.H_an = 109.3; % [C/m]
const_br26.T0 = 12; % [C]
const_br26.k = 115; % [W/mK]
const_br26.cp = 380; %J/kgK
const_br26.rho = 8500; % [kg/m^3]
const_br26.alpha = const_br26.k/(const_br26.rho*const_br26.cp); % [m^2/s]
const_br26.L = x8+0.0508; % [m]

const_br29.H_an = 135.9; % [C/m]
const_br29.T0 = 12; % [C]
const_br29.k = 115; % [W/mK]
const_br29.cp = 380; % [J/kgK]
const_br29.rho = 8500; % [kg/m^3]
const_br29.alpha = const_br29.k/(const_br29.rho*const_br29.cp); % [m^2/s]
const_br29.L = x8+0.0508; % [m]

const_al26.H_an = 98.6; % [C/m]
const_al26.T0 = 12; % [C]
const_al26.k = 130; % [W/mK]
const_al26.cp = 960; % [J/kgK]
const_al26.rho = 2810; % [kg/m^3]
const_al26.alpha = const_al26.k/(const_al26.rho*const_al26.cp); % [m^2/s]
const_al26.L = x8+0.0508; % [m]

const_al28.H_an = 114.3; % [C/m]
const_al28.T0 = 12; % [C]
const_al28.k = 130; % [W/mK]
const_al28.cp = 960; % [J/kgK]
const_al28.rho = 2810; % [kg/m^3]
const_al28.alpha = const_al28.k/(const_al28.rho*const_al28.cp); % [m^2/s]
const_al28.L = x8+0.0508; % [m]

t_st = 0:10:12510; % [s]
t_al26 = 0:10:1670; % [s]
t_al28 = 0:10:2670; % [s]
t_br26 = 0:10:4940; % [s]
t_br29 = 0:10:4930; % [s]


for i = 1:length(t_st)
    u_st(i,:) = heatdistribution(t_st(i),x,const_st);
end
```

```
for i = 1:length(t_al26)
    u_al26(i,:) = heatdistribution(t_al26(i),x,const_al26);
end
for i = 1:length(t_al28)
    u_al28(i,:) = heatdistribution(t_al28(i),x,const_al28);
end
for i = 1:length(t_br26)
    u_br26(i,:) = heatdistribution(t_br26(i),x,const_br26);
end
for i = 1:length(t_br29)
    u_br29(i,:) = heatdistribution(t_br29(i),x,const_br29);
end

al26 = readmatrix("Aluminum_26V_250mA");
al28 = readmatrix("Aluminum_28V_269mA");
br26 = readmatrix("Brass_26V_245mA");
br29 = readmatrix("Brass_29V_273mA");
st = readmatrix("Steel_21V_192mA");

dx = 0.0127; %m
x0 = 0.034925;
xpos = x0:dx:x0 + 0.1016;
% breaking data into channels
% cell row is which thermocouple
al26_c = cell(8,1);
al28_c = cell(8,1);
br26_c = cell(8,1);
br29_c = cell(8,1);
st_c = cell(8,1);
al26_t = al26(:,1);
al28_t = al28(:,1);
br26_t = br26(:,1);
br29_t = br29(:,1);
st_t = st(:,1);
for i = 1:8
al26_c{i} = al26(:,i+1);
al28_c{i} = al28(:,i+1);
br26_c{i} = br26(:,i+1);
br29_c{i} = br29(:,i+1);
st_c{i} = st(:,i+1);
end

figure()
hold on
for i = 1:8
plot(al26_t,al26_c{i},'k')
plot(t_al26,u_al26(:,i),'r')
end
grid minor
xlabel("Time (s)")
ylabel('Temperature (^\circC)')
legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
title("Model IA: Aluminum - 26V,250mA")

figure()
hold on
for i = 1:8
plot(al28_t,al28_c{i},'k')
```

```
plot(t_al28,u_al28(:,i),'r')
end
grid minor
xlabel("Time (s)")
ylabel('Temperature␣(^\circC)')
legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
title("Model IA: Aluminum - 28V,269mA")

figure()
hold on
for i = 1:8
plot(br26_t,br26_c{i},'k')
plot(t_br26,u_br26(:,i),'r')
end
grid minor
xlabel("Time (s)")
ylabel('Temperature␣(^\circC)')
legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
title("Model IA: Brass - 26V,245mA")

figure()
hold on
for i = 1:8
plot(br29_t,br29_c{i},'k')
plot(t_br29,u_br29(:,i),'r')
end
grid minor
xlabel("Time (s)")
ylabel('Temperature␣(^\circC)')
legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
title("Model IA: Brass - 29V,273mA")

figure()
hold on
for i = 1:8
plot(st_t,st_c{i},'k')
plot(t_st,u_st(:,i),'r')
end
grid minor
xlabel("Time (s)")
ylabel('Temperature␣(^\circC)')
legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
title("Model IA: Steel - 21V,192mA")


function u = heatdistribution(t,x,const)
    sum = [0,0,0,0,0,0,0,0];
    for n = 1:10
        b_n = (8*const.H_an*const.L*(-1)^n)/((2*n-1)^2*pi^2);
        lambda_n = pi*(2*n-1)/(2*const.L);
        sum = sum + b_n.*sin(lambda_n.*x).*exp(-1*lambda_n^2*const.alpha*t);
    end
    u = const.T0+const.H_an.*x + sum;
end
```

5.3 Task 3

```matlab
%% ASEN 3802 Lab 2 Part 2 Task 3

clc;clear;close all;

x1 = 0.0349; % [m]
x2 = 0.0476; % [m]
x3 = 0.0603; % [m]
x4 = 0.073; % [m]
x5 = 0.0857; % [m]
x6 = 0.0984; % [m]
x7 = 0.1111; % [m]
x8 = 0.1238; % [m]
x = [x1,x2,x3,x4,x5,x6,x7,x8];

const_st.H_exp = 277; % [C/m]
const_st.T0 = 11; % [C]
const_st.k = 16.2; % [W/mK]
const_st.cp = 500; % [J/kgK]
const_st.rho = 8000; % [kg/m^3]
const_st.alpha = const_st.k/(const_st.rho*const_st.cp); % [m^2/s]
const_st.L = x8+0.0508; % [m]

const_br26.H_exp = 114; % [C/m]
const_br26.T0 = 12; % [C]
const_br26.k = 115; % [W/mK]
const_br26.cp = 380; % [J/kgK]
const_br26.rho = 8500; % [kg/m^3]
const_br26.alpha = const_br26.k/(const_br26.rho*const_br26.cp); % [m^2/s]
const_br26.L = x8+0.0508; % [m]

const_br29.H_exp = 139.8; % [C/m]
const_br29.T0 = 12; % [C]
const_br29.k = 115; % [W/mK]
const_br29.cp = 380; % [J/kgK]
const_br29.rho = 8500; % [kg/m^3]
const_br29.alpha = const_br29.k/(const_br29.rho*const_br29.cp); % [m^2/s]
const_br29.L = x8+0.0508; % [m]

const_al26.H_exp = 54.3; % [C/m]
const_al26.T0 = 12; % [C]
const_al26.k = 130; % [W/mK]
const_al26.cp = 960; % [J/kgK]
const_al26.rho = 2810; % [kg/m^3]
const_al26.alpha = const_al26.k/(const_al26.rho*const_al26.cp); % [m^2/s]
const_al26.L = x8+0.0508; % [m]

const_al28.H_exp = 66.8; % [C/m]
const_al28.T0 = 12; % [C]
const_al28.k = 130; % [W/mK]
const_al28.cp = 960; % [J/kgK]
const_al28.rho = 2810; % [kg/m^3]
const_al28.alpha = const_al28.k/(const_al28.rho*const_al28.cp); % [m^2/s]
const_al28.L = x8+0.0508; % [m]

t_st = 0:10:12510; % [s]
t_al26 = 0:10:1670; % [s]
t_al28 = 0:10:2670; % [s]
```

```
t_br26 = 0:10:4940; % [s]
t_br29 = 0:10:4930; % [s]

for i = 1:length(t_st)
    u_st(i,:) = heatdistribution(t_st(i),x,const_st);
end
for i = 1:length(t_al26)
    u_al26(i,:) = heatdistribution(t_al26(i),x,const_al26);
end
for i = 1:length(t_al28)
    u_al28(i,:) = heatdistribution(t_al28(i),x,const_al28);
end
for i = 1:length(t_br26)
    u_br26(i,:) = heatdistribution(t_br26(i),x,const_br26);
end
for i = 1:length(t_br29)
    u_br29(i,:) = heatdistribution(t_br29(i),x,const_br29);
end

al26 = readmatrix("Aluminum_26V_250mA");
al28 = readmatrix("Aluminum_28V_269mA");
br26 = readmatrix("Brass_26V_245mA");
br29 = readmatrix("Brass_29V_273mA");
st = readmatrix("Steel_21V_192mA");

dx = 0.0127; %m
x0 = 0.034925;
xpos = x0:dx:x0 + 0.1016;
% breaking data into channels
% cell row is which thermocouple
al26_c = cell(8,1);
al28_c = cell(8,1);
br26_c = cell(8,1);
br29_c = cell(8,1);
st_c = cell(8,1);
al26_t = al26(:,1);
al28_t = al28(:,1);
br26_t = br26(:,1);
br29_t = br29(:,1);
st_t = st(:,1);
for i = 1:8
al26_c{i} = al26(:,i+1);
al28_c{i} = al28(:,i+1);
br26_c{i} = br26(:,i+1);
br29_c{i} = br29(:,i+1);
st_c{i} = st(:,i+1);
end

figure()
hold on
for i = 1:8
plot(al26_t,al26_c{i},'k')
plot(t_al26,u_al26(:,i),'r')
end
grid minor
xlabel("Time (s)")
ylabel('Temperature␣(^\circC)')
legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
```

```
title("Model IB: Aluminum - 26V,250mA")

figure()
hold on
for i = 1:8
plot(al28_t,al28_c{i},'k')
plot(t_al28,u_al28(:,i),'r')
end
grid minor
xlabel("Time (s)")
ylabel('Temperature␣(^\circC)')
legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
title("Model IB: Aluminum - 28V,269mA")

figure()
hold on
for i = 1:8
plot(br26_t,br26_c{i},'k')
plot(t_br26,u_br26(:,i),'r')
end
grid minor
xlabel("Time (s)")
ylabel('Temperature␣(^\circC)')
legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
title("Model IB: Brass - 26V,245mA")

figure()
hold on
for i = 1:8
plot(br29_t,br29_c{i},'k')
plot(t_br29,u_br29(:,i),'r')
end
grid minor
xlabel("Time (s)")
ylabel('Temperature␣(^\circC)')
legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
title("Model IB: Brass - 29V,273mA")

figure()
hold on
for i = 1:8
plot(st_t,st_c{i},'k')
plot(t_st,u_st(:,i),'r')
end
grid minor
xlabel("Time (s)")
ylabel('Temperature␣(^\circC)')
legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
title("Model IB: Steel - 21V,192mA")


function u = heatdistribution(t,x,const)
    sum = [0,0,0,0,0,0,0,0];
    for n = 1:10
        b_n = (8*const.H_exp*const.L*(-1)^n)/((2*n-1)^2*pi^2);
        lambda_n = pi*(2*n-1)/(2*const.L);
        sum = sum + b_n.*sin(lambda_n.*x).*exp(-1*lambda_n^2*const.alpha*t);
    end
```

```
    u = const.T0+const.H_exp.*x + sum;
end
```

5.4 Task 4

```
%% ASEN 3802 Lab 2 Part 2 Task 4

clc;clear;close all;

x1 = 0.0349; % [m]
x2 = 0.0476; % [m]
x3 = 0.0603; % [m]
x4 = 0.073; % [m]
x5 = 0.0857; % [m]
x6 = 0.0984; % [m]
x7 = 0.1111; % [m]
x8 = 0.1238; % [m]
x = [x1,x2,x3,x4,x5,x6,x7,x8];

const_st.H_exp = 277; % [C/m]
const_st.M_exp = 35.2; % [C/m]
const_st.T0 = 11; % [C]
const_st.k = 16.2; % [W/mK]
const_st.cp = 500; % [J/kgK]
const_st.rho = 8000; % [kg/m^3]
const_st.alpha = const_st.k/(const_st.rho*const_st.cp); % [m^2/s]
const_st.L = x8+0.0508; % [m]

const_br26.H_exp = 114; % [C/m]
const_br26.M_exp = 7; % [C/m]
const_br26.T0 = 12; % [C]
const_br26.k = 115; % [W/mK]
const_br26.cp = 380; % [J/kgK]
const_br26.rho = 8500; % [kg/m^3]
const_br26.alpha = const_br26.k/(const_br26.rho*const_br26.cp); % [m^2/s]
const_br26.L = x8+0.0508; % [m]

const_br29.H_exp = 139.8; % [C/m]
const_br29.M_exp = 6.6; % [C/m]
const_br29.T0 = 12; % [C]
const_br29.k = 115; % [W/mK]
const_br29.cp = 380; % [J/kgK]
const_br29.rho = 8500; % [kg/m^3]
const_br29.alpha = const_br29.k/(const_br29.rho*const_br29.cp); % [m^2/s]
const_br29.L = x8+0.0508; % [m]

const_al26.H_exp = 54.3; % [C/m]
const_al26.M_exp = -3.4; % [C/m]
const_al26.T0 = 12; % [C]
const_al26.k = 130; % [W/mK]
const_al26.cp = 960; % [J/kgK]
const_al26.rho = 2810; % [kg/m^3]
const_al26.alpha = const_al26.k/(const_al26.rho*const_al26.cp); % [m^2/s]
const_al26.L = x8+0.0508; % [m]

const_al28.H_exp = 66.8; % [C/m]
const_al28.M_exp = 0.3; % [C/m]
```

```
const_al28.T0 = 12; % [C]
const_al28.k = 130; % [W/mK]
const_al28.cp = 960; % [J/kgK]
const_al28.rho = 2810; % [kg/m^3]
const_al28.alpha = const_al28.k/(const_al28.rho*const_al28.cp); % [m^2/s]
const_al28.L = x8+0.0508; % [m]

t_st = 0:10:12510; % [s]
t_al26 = 0:10:1670; % [s]
t_al28 = 0:10:2670; % [s]
t_br26 = 0:10:4940; % [s]
t_br29 = 0:10:4930; % [s]

for i = 1:length(t_st)
    u_st(i,:) = heatdistribution(t_st(i),x,const_st);
end
for i = 1:length(t_al26)
    u_al26(i,:) = heatdistribution(t_al26(i),x,const_al26);
end
for i = 1:length(t_al28)
    u_al28(i,:) = heatdistribution(t_al28(i),x,const_al28);
end
for i = 1:length(t_br26)
    u_br26(i,:) = heatdistribution(t_br26(i),x,const_br26);
end
for i = 1:length(t_br29)
    u_br29(i,:) = heatdistribution(t_br29(i),x,const_br29);
end

al26 = readmatrix("Aluminum_26V_250mA");
al28 = readmatrix("Aluminum_28V_269mA");
br26 = readmatrix("Brass_26V_245mA");
br29 = readmatrix("Brass_29V_273mA");
st = readmatrix("Steel_21V_192mA");

dx = 0.0127; %m
x0 = 0.034925;
xpos = x0:dx:x0 + 0.1016;
% breaking data into channels
% cell row is which thermocouple
al26_c = cell(8,1);
al28_c = cell(8,1);
br26_c = cell(8,1);
br29_c = cell(8,1);
st_c = cell(8,1);
al26_t = al26(:,1);
al28_t = al28(:,1);
br26_t = br26(:,1);
br29_t = br29(:,1);
st_t = st(:,1);
for i = 1:8
al26_c{i} = al26(:,i+1);
al28_c{i} = al28(:,i+1);
br26_c{i} = br26(:,i+1);
br29_c{i} = br29(:,i+1);
st_c{i} = st(:,i+1);
end

```

```
figure()
hold on
for i = 1:8
plot(al26_t,al26_c{i},'k')
plot(t_al26,u_al26(:,i),'r')
end
grid minor
xlabel("Time (s)")
ylabel('Temperature (^\circC)')
legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
title("Model II: Aluminum - 26V,250mA")

figure()
hold on
for i = 1:8
plot(al28_t,al28_c{i},'k')
plot(t_al28,u_al28(:,i),'r')
end
grid minor
xlabel("Time (s)")
ylabel('Temperature (^\circC)')
legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
title("Model II: Aluminum - 28V,269mA")

figure()
hold on
for i = 1:8
plot(br26_t,br26_c{i},'k')
plot(t_br26,u_br26(:,i),'r')
end
grid minor
xlabel("Time (s)")
ylabel('Temperature (^\circC)')
legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
title("Model II: Brass - 26V,245mA")

figure()
hold on
for i = 1:8
plot(br29_t,br29_c{i},'k')
plot(t_br29,u_br29(:,i),'r')
end
grid minor
xlabel("Time (s)")
ylabel('Temperature (^\circC)')
legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
title("Model II: Brass - 29V,273mA")

figure()
hold on
for i = 1:8
plot(st_t,st_c{i},'k')
plot(t_st,u_st(:,i),'r')
end
grid minor
xlabel("Time (s)")
ylabel('Temperature (^\circC)')
legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
```

```
title("Model II: Steel - 21V,192mA")


function u = heatdistribution(t,x,const)
    sum = [0,0,0,0,0,0,0,0,0];
    for n = 1:10
        b_n = (8*(const.M_exp-const.H_exp)*const.L*(-1)^(n+1))/((2*n-1)^2*pi^2);
        lambda_n = pi*(2*n-1)/(2*const.L);
        sum = sum + b_n.*sin(lambda_n.*x).*exp(-1*lambda_n^2*const.alpha*t);
    end
    u = const.T0+const.H_exp.*x + sum;
end
```