#### **Key Deliverables:**

- 5 "Check your answer" questions in Canvas (10 pts)
  - You only get 3 attempts (correct answers carry over to next attempt) at the quiz.
  - There is a 5 minute "cool-down" between attempts. Use this time to debug and reevaluate your code. Once you compute a new answer, make sure to critically think about it before submitting again.
  - No additional attempts will be provided for any reason.
- 1 .zip file containing the following (naming convention lastname\_firstname\_CC1.zip) (15pts):
  - Functioning Matlab code (.m file(s) if using multiple files, please name driver code main.m)
  - Published code (.pdf file)
  - Flowchart sketch (.pdf file)

## Background

We have continued the study of statistical concepts by considering multiple measurements of a singular quantity, defining weighted average and standard deviation of the weighted average. We have also considered how multiple uncertain measurements can be combined through the general method of error propagation to compute error of a more complex equation. Lastly, we talked about the stochastic method known as Monte Carlo.

In this coding challenge you will begin by analyzing two sets of data taken in the static-fire rocket test stand. The goal will be to compute the weighted average (Eqn. 1) and associated uncertainty (Eqn. 2) of the thrust after the start-up phase. The experimental data was simultaneously collected from two different sensors mounted in the load cell. You will then explore the Tsiolkovsky equation (a different form of the ideal rocket equation),  $\Delta v = I_{sp}g_0 \ln \frac{m_0}{m_f}$ , where  $\Delta v$  is the change in velocity (m/s),  $I_{sp}$  is the specific impulse (s),  $g_0$  is the gravitational acceleration constant (m/s<sup>2</sup>), and  $m_0$  is the total mass of the rocket (structure + propellant) where  $m_0 = m_s + m_p$  (kg) and  $m_f = m_s$  is the mass of the rocket with no propellant (kg). You will use the general method of error propagation to determine the best estimate of the change in velocity along with its associated uncertainty Eqn. 3. Lastly, a Monte Carlo simulation will be performed to compare to the results of the general method.

$$\overline{x}_{W,ave} = \frac{\sum (W_i \overline{x}_i)}{\sum W_i}$$
, where  $W_i = \frac{1}{\sigma_i^2}$  (1)

$$\sigma_{W,ave} = \frac{1}{\sqrt{\sum W_i}} \tag{2}$$

For some 
$$q = q(x, ..., y)$$
,  $\delta q = \sqrt{\left(\frac{\partial q}{\partial x} \delta x\right)^2 + ... + \left(\frac{\partial q}{\partial z} \delta z\right)^2}$  (3)

#### Step 1: Read in and clean the data set

Download the data set from Coding Challenge 2 Assignment in Canvas, named **Static\_Thrust\_Data.xlsx**. Use any Matlab function of your choosing to read in the data set (you may **not** use the Import Data Tool). As previewed in class, clean the data set of any missing (or NaN) values and plot to take note of any trends. We are looking for the average thrust *during* the times when thrust is being produced. Consider what values should be included in the weighted average and remove the values that should not be included. Isolate the time vector  $(N \times 1)$  from the two thrusts  $(N \times 2)$ .

In Canvas quiz, enter the number of rows (N) in your data set for time/thrust.

# Step 2: Calculate the weighted average and associated uncertainty in a function

Calculate the weighted average and standard deviation using a **single** function that you write. Then, create a figure with both thrust profiles and the weighted average thrust (solid black line) and confidence interval (i.e.,  $\pm 1\sigma$  black dashed lines).

**HINT:** the thrust data is  $(N \times 2)$ , what happens when you use the mean() and std() functions on a matrix?

In Canvas quiz, enter the weighted average thrust and standard deviation. (2 separate questions)

## Step 3: The General Method

You are given the following quantities and uncertainties:

$$g_0 = 9.81 \text{ m/s}^2$$
  
 $I_{sp} = 459 \pm 11 \text{ s}$   
 $m_s = 13050 \pm 60 \text{ kg}$   
 $m_p = 71800 \pm 300 \text{ kg}$ 

Compute the best estimate of  $\Delta v$  and associated uncertainty by the general method. You will need to manually derive the formula in terms of only the provided quantities and then input into Matlab to compute the value. Note that  $\delta m_0 \neq 0$  and  $\delta m_f \neq 0$ .

**HINT:** Some math identities that might prove helpful:

$$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$$
$$\frac{\partial}{\partial x} \ln(x) = \frac{1}{x}$$

**HINT:** In Matlab ln() as a mathematical operator is the log() function

**HINT:** The Matlab function norm() is helpful in computing root mean square (i.e.  $\sqrt{(\cdot)^2 + ... + (\cdot)^2}$ )

In Canvas quiz, enter the uncertainty associated with  $\Delta v$ , in m/s, for the General Method.

#### Step 4: Perform a Monte Carlo Simulation

Use the same given quantities as the previous section. Create normal distributions of each uncertain quantity using randn() function as discussed in class. The size random samples is up to you and your best engineering judgement (i.e, start low and increase until you are happy with the resulting distribution). Plot this using the histogram function with a sufficient number of bins.

In Canvas quiz, enter the uncertainty associated with  $\Delta v$ , in m/s, for the Monte Carlo Simulation.

#### Step 5: Compare the General Method and Monte Carlo Simulations

Compare the histogram (you may want to limit the number of bins as N gets very large) and PDF on one plot (you may find that the histogram function options are helpful). Include lines indicating the mean (solid black line) and  $\pm 1$  standard deviation from the mean (using dashed black lines) from the general method. Be sure to include axis labels and a legend.

# Reflection Questions

Do the general method and Monte Carlo simulation mean and uncertainty agree? What would happen to the value of  $\delta(\Delta v)$  if you performed these analyses with errors associated with  $2\sigma$  of each uncertain quantity?

Is the weighted average and associated uncertainty of thrust the **same** as taking the average of the two thrust profiles and then taking the standard deviation? Explain your reasoning.

Please write out the answers to these questions in the comments of your Matlab script. This should be about 1 paragraph in length.

# Coding Rubric

	Excellent (100%)	Above Average (80%)	Average (70%)	Below Average (50%)
Requirements and Delivery (2pts)	<ul> <li>Completed 90-100% of the requirements</li> <li>Delivered on time, and in correct format.</li> </ul>	<ul> <li>Completed 80-90% of the requirements</li> <li>Delivered on time, and in correct format.</li> </ul>	<ul> <li>Completed 70-80% of the requirements</li> <li>Delivered on time, and in correct format.</li> </ul>	<ul> <li>Completed &lt;70% of the requirements</li> <li>Delivered on time, but not in correct format.</li> </ul>
Coding Standards (2pts)	<ul> <li>Includes full header*</li> <li>Excellent use of variables (no global or unambiguous variable naming)</li> </ul>	<ul> <li>Includes full header*</li> <li>Good use of variables (1-2 global or ambiguous variable naming)</li> </ul>	<ul> <li>Includes incomp. header*</li> <li>Fine use of variables (3-5 global or ambiguous variable naming)</li> </ul>	<ul> <li>No header</li> <li>Poor use of variables (many global or ambiguous variable naming)</li> </ul>
Documentation (2pts) Comment your code	<ul> <li>Clearly documented</li> <li>Specific purpose noted for each function and/or section</li> </ul>	Well documented     Specific purpose noted for each function and/or section	Some documentation     Purpose noted for each function and/or section	• Limited to no documentation
Runtime (1pt)	• Executes quickly, without errors	• Executes without errors, over 1 min runtime	• Executes with warnings/errors	• Does not execute
Efficiency (2pts)	• Easy to understand, and maintain	Logical, without sac- rificing readability and understanding	Difficult to follow	• Difficult to follow, huge and appears patched together
Figure Quality (2pts)	• Easy to understand, labels and legend present	• Easy to understand, lacks labels and legend	• Difficult to understand, labels and legend present	• Difficult to understand, lacks labels and legend
Reflection Questions (4pts)	• Easy to understand, fully thought out	• Easy to understand, mostly thought out	• Hard to understand, effort made	Hard to understand, lacking effort

<sup>\*</sup> Header includes author name(s), assignment title, purpose, creation date, revisions (applicable to **group** projects only)