

ASEN 3802 Lab 2 Part 2

Andrew Patella, Lauren Lajoie, Skylar Harris, Yaseen Mustapha

March 17, 2025

1 Analytical Transient Solution Derivation

1.1 Derivation

Derivation of b_n , the Fourier coefficients.

$$b_n = -\frac{2H}{L} \int_0^L x \sin(\lambda_n x) dx, \quad \lambda_n = \frac{(2n-1)\pi}{2L}$$

Using integration by parts,

$$\begin{aligned} b_n &= -\frac{2H}{L} \left[-x \frac{1}{\lambda_n} \cos(\lambda_n x) \Big|_0^L + \int_0^L \frac{1}{\lambda_n} \cos(\lambda_n x) dx \right] \\ b_n &= -\frac{2H}{L} \left[-x \frac{1}{\lambda_n} \cos(\lambda_n x) + \frac{1}{\lambda_n^2} \sin(\lambda_n x) \right]_0^L \\ b_n &= -\frac{2H}{L\lambda_n} \left[-x \cos(\lambda_n x) + \frac{1}{\lambda_n} \sin(\lambda_n x) \right]_0^L \end{aligned}$$

Plugging in λ_n ,

$$b_n = -\frac{2H}{L \frac{(2n-1)\pi}{2L}} \left[-x \cos\left(\frac{(2n-1)\pi}{2L} x\right) + \frac{2L}{(2n-1)\pi} \sin\left(\frac{(2n-1)\pi}{2L} x\right) \right]_0^L$$

$$b_n = -\frac{4H}{(2n-1)\pi} \left[-L \cos\left(\frac{(2n-1)\pi}{2L} L\right) + \frac{2L}{(2n-1)\pi} \sin\left(\frac{(2n-1)\pi}{2L} L\right) - \left(-0 \cos\left(\frac{(2n-1)\pi}{2L} 0\right) + \frac{2L}{(2n-1)\pi} \sin\left(\frac{(2n-1)\pi}{2L} 0\right) \right) \right]$$

$$b_n = -\frac{4H}{(2n-1)\pi} \left[-L \cos\left(\frac{(2n-1)\pi}{2L} L\right) + \frac{2L}{(2n-1)\pi} \sin\left(\frac{(2n-1)\pi}{2L} L\right) \right]$$

$$b_n = -\frac{4H}{(2n-1)\pi} \left[-L \cos\left(\frac{(2n-1)\pi}{2}\right) + \frac{2L}{(2n-1)\pi} \sin\left(\frac{(2n-1)\pi}{2}\right) \right]$$

$$b_n = -\frac{4H}{(2n-1)\pi} \left[\frac{2L}{(2n-1)\pi} \sin\left(\frac{(2n-1)\pi}{2}\right) \right]$$

$$b_n = -\frac{8HL(-1)^{n+1}}{(2n-1)^2\pi^2} = \frac{8HL(-1)^n}{(2n-1)^2\pi^2}$$

$$u(x, t) = T_0 + Hx + \sum_{n=1}^{\infty} b_n \sin(\lambda_n x) e^{-\lambda_n^2 \alpha t}$$

Using the values found from Part 1 for aluminum at 26 V, we have $T_0 = 12^\circ\text{C}$ and $H_{an} = 98.6 \frac{^\circ\text{C}}{\text{m}}$. We were given its thermal conductivity, specific heat, density, and we were able to calculate the length of the bar.

1.2 Plot

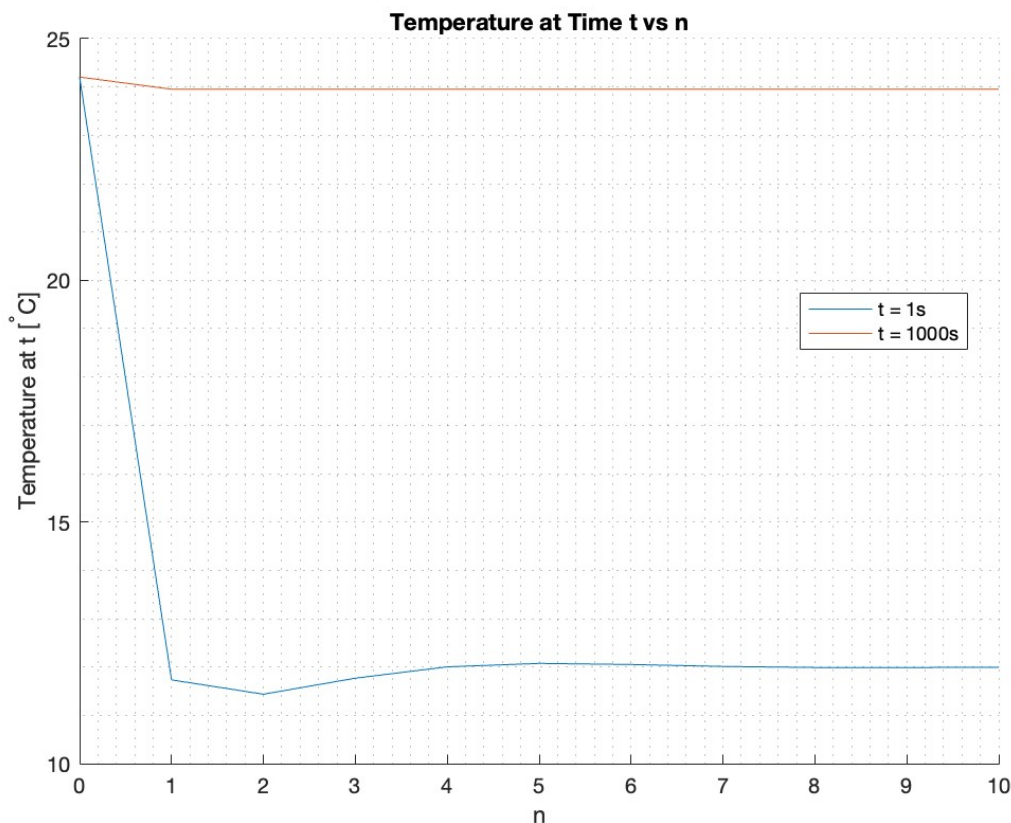


Figure 1: Convergence of the temperature of aluminum at $t = 1\text{s}$ and $t = 1000\text{s}$

Using our derivation and the values provided, we plotted the temperature at 2 different times from $n = 0$ to $n = 10$. This plot shows the convergence of the temperature to the desired steady state as more and more terms are added. As discussed in section 1.3: as time increases, fewer terms are necessary to reach convergence to the solution. At $t = 1000$ seconds, only 1 term is necessary to reach convergence, but for $t = 1$ seconds, about 5 terms are necessary to reach convergence. From just looking at Figure 1 we can see this result, the plot for $t = 1000$ seconds oscillates less and settles within fewer iterations than when $t = 1$ seconds.

1.3 Discussion

For this experiment one term is not sufficient. Using the Fourier number equation, $F_0 = \frac{\alpha t}{L^2}$, we found that the Fourier numbers at 1 second and 1000 seconds are 0.0016 and 1.5808, respectively. For aluminum at 1 second, the Fourier number is significantly lower than 0.2, the cutoff for one term being sufficient. At 1000 seconds, the Fourier number is above 0.2. We noted that as t increases, the Fourier number increases, and it is sufficient to have fewer terms. However, in order to have a model that is accurate at all time inputs, we need to use more than one term. In general, although one term is 98% accurate for models with Fourier numbers above 0.2, more terms lead to better accuracy. Since it took about 5 terms for the series to converge at 1 second, we chose to use $n = 10$ for the models to ensure accuracy at earlier times.

2 Task 2: Time-Dependent Temperature Profiles - Model IA

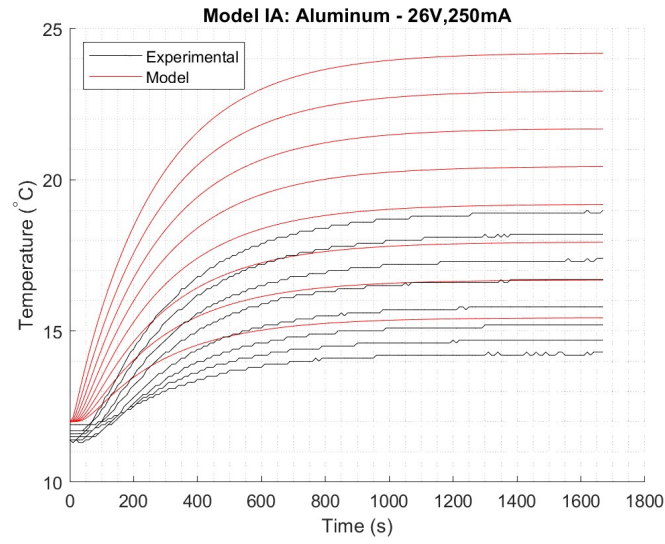


Figure 2: Aluminum at 26V: Model vs Experiment

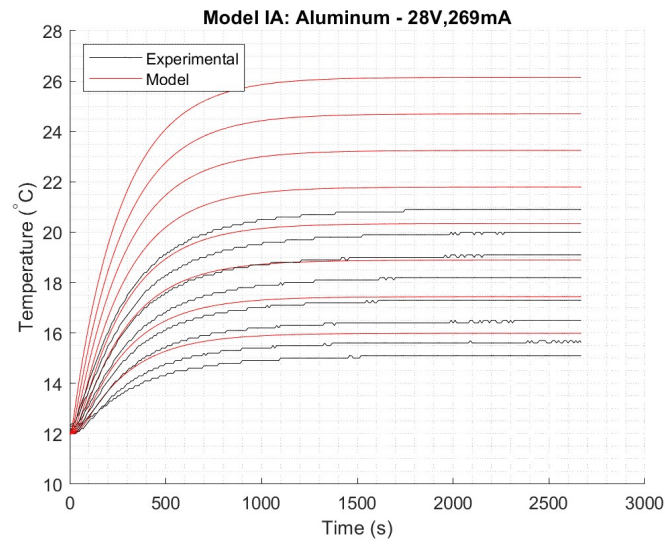


Figure 3: Aluminum at 28V: Model vs Experiment

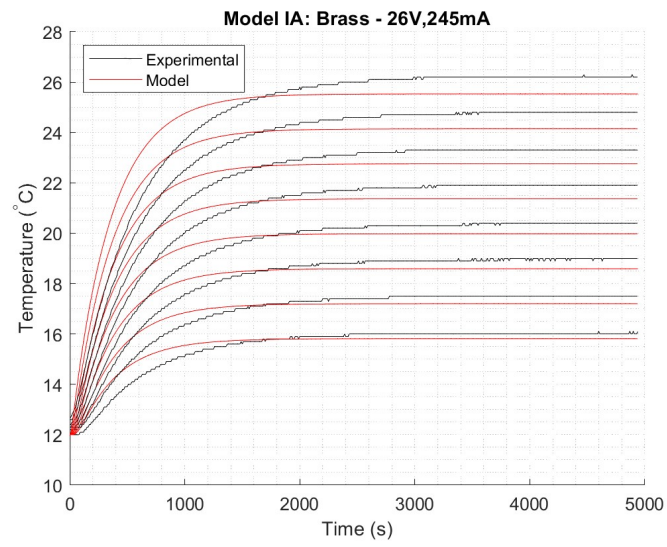


Figure 4: Brass at 26V: Model vs Experiment

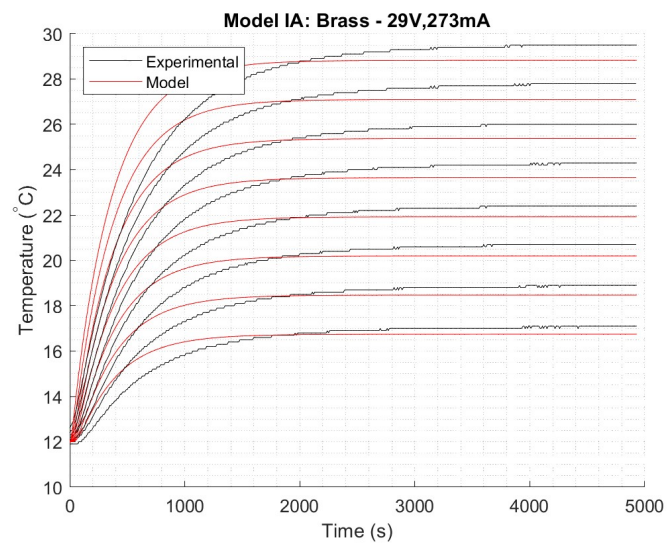


Figure 5: Brass at 29V: Model vs Experiment

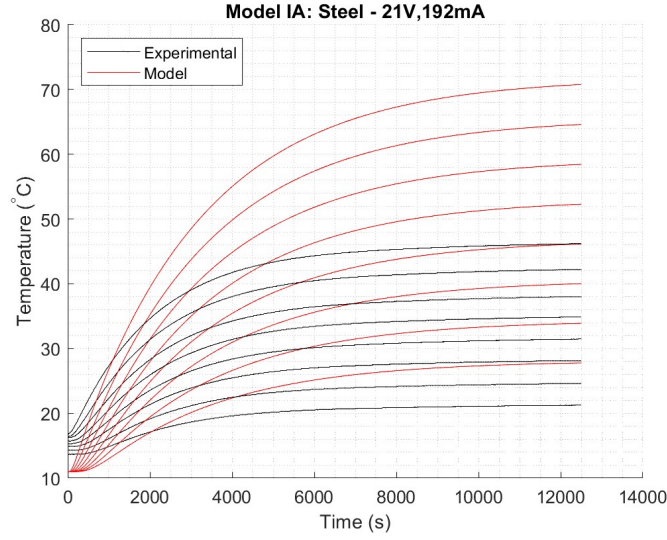


Figure 6: Steel at 21V: Model vs Experiment

Model IA is a good approximation for the shape of the temperature vs. time curve. However, the model does not work well for aluminum or steel. Looking at Figures 2 and 3, the final temperature that the model reaches is higher than the experimental temperature by about 5°C . This result makes sense because our calculated analytical slope for aluminum was about 80% more than the experimental steady-state slope. We see a similar result in Figure 6 where the model predicts a final temperature almost 30 degrees higher than the measured temperature. For steel, it looks as if the model never reaches a steady-state temperature in the same time frame as the experiment. This result is expected because the analytical slope we calculated for Steel in Part 1 was about 50% more than the experimental steady-state slope, resulting in a large discrepancy between the model and the experiment. Model IA best predicted the temperature for brass because our calculated analytical slope was about 3% less than the experimental steady-state slope. For brass, the model slightly underestimated the temperature, as seen in Figures 4 and 5. There is some error in the steady-state temperature distributions of the bar, i.e., the final temperatures are different between the model and the experiment. For brass, the discrepancy is within 1°C , however, for aluminum and steel the difference is much greater than 1°C . We account for this error as a result of the vast difference between the analytical and experimental steady-state slopes for those materials.

3 Task 3: Time-Dependent Temperature Profiles - Model IB

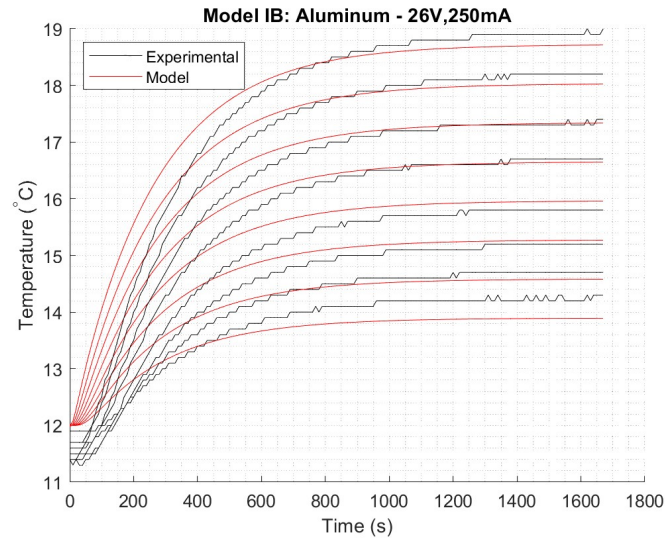


Figure 7: Aluminum at 26V: Model vs Experiment

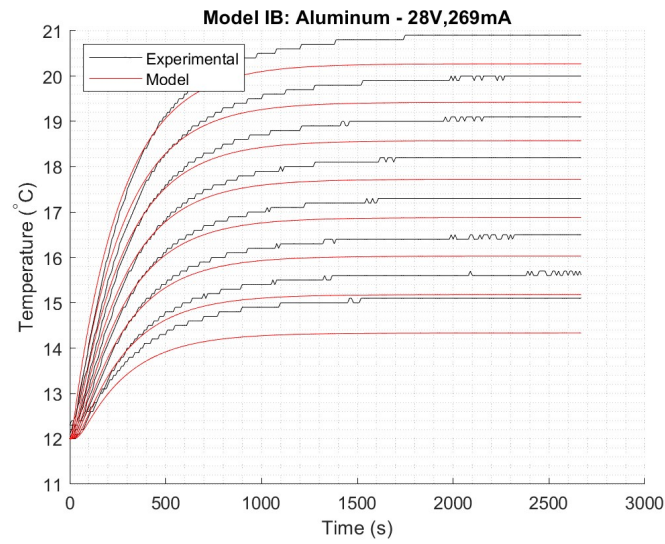


Figure 8: Aluminum at 28V: Model vs Experiment

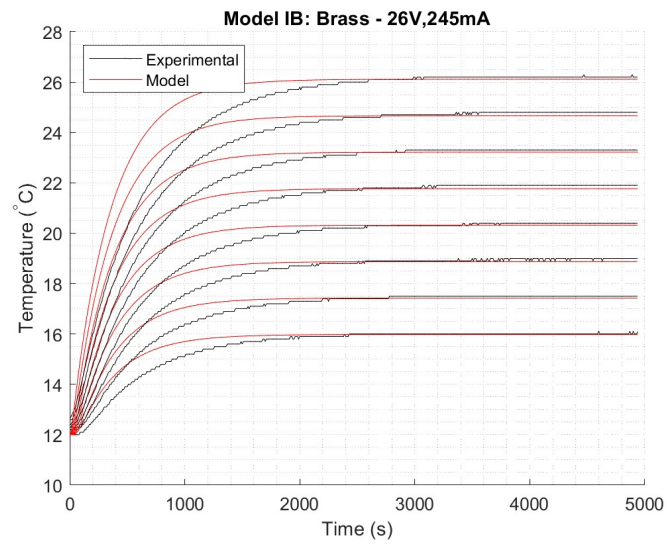


Figure 9: Brass at 26V: Model vs Experiment

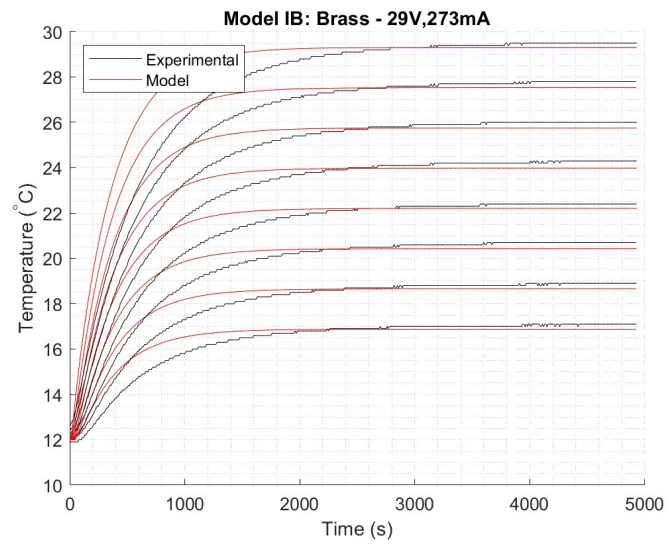


Figure 10: Brass at 29V: Model vs Experiment

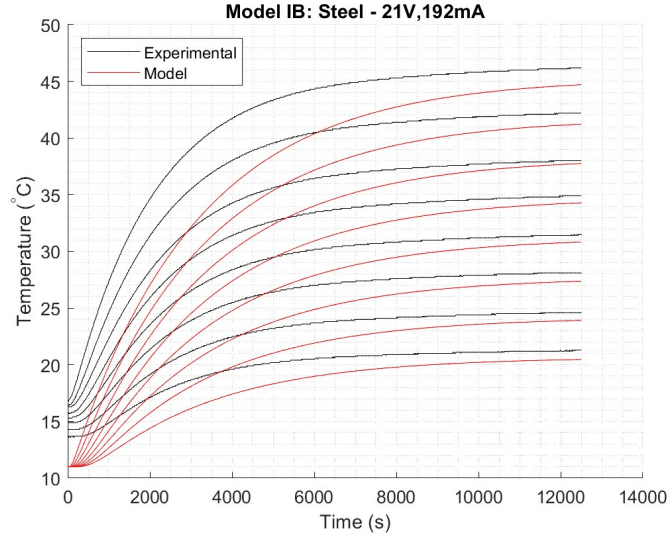


Figure 11: Steel at 21V: Model vs Experiment

As explained in Task 2, there is significant error in the steady-state temperatures for the aluminum and steel bars for Model IA. This error was due to the vast difference between the analytical and experimental steady-state slopes for those materials. We would expect Model IB to be a better approximation of the steady-state temperatures due to the slope being used was calculated from the experiment. As shown in Figures 7-11, Model IB is a better approximation for the experiment than Model IA because the model now uses the experimental steady-state slopes calculated in Part 1. Using the experimental slope led to a good approximation for both brass and aluminum, where the final temperatures are within 1°C , as shown in Figures 7-10. As shown in Figure 11, the model does not accurately depict the temperature for steel. The final temperatures of the experimental results and the model are within 1°C , however, the initial temperature distribution is not within that bound. The model starts from slightly different initial temperatures than the experiment. Overall, using the experimental slopes for Model IB gave us a better model for the thermocouple. This model performs better than Model IA for the steady-state distribution, but it does not accurately model the initial temperature distribution.

4 Task 4: Time-Dependent Temperature Profiles - Model II

When developing our analytical model, we assumed that the bar was initially at a constant T_0 , found by taking the intercept of an interpolated final state. However, after comparing the model to the experimental data, it is clear that this may not be a very accurate assumption. In order to improve the model, we used new $g(x)$ and $f(x)$ functions when deriving the Fourier Coefficients, b_n . We used $f(x) = Mx + T_0$ and $g(x) = (M - H)x$ to more accurately match the initial temperature distribution of the bar. The variable M , is the experimental steady-state slope of the initial condition of the data. This value was found for each material in Part 1 of the lab. Due to the error in the thermocouples, the values of M ranged drastically since each thermocouple has a certainty of $\pm 2^{\circ}\text{C}$.

By changing the functions $g(x)$ and $f(x)$, we are able to produce a more accurate model because it more closely matches the data at $t = 0$.

$$b_n = \frac{2}{L} \int_0^L g(x) \sin(\lambda_n x) dx, \quad g(x) = (M - H)x$$

$$b_n = \frac{2(M - H)}{L} \int_0^L x \sin(\lambda_n x) dx$$

By modifying the result from Section 1.1, we can find the equation for the Fourier Coefficients:

$$b_n = \frac{8(M - H)L(-1)^{n+1}}{(2n - 1)^2\pi^2}$$

Using the new values for b_n , we can plot a new model which should more accurately represent the experimental results at $t = 0$. The results for this new model are shown below.

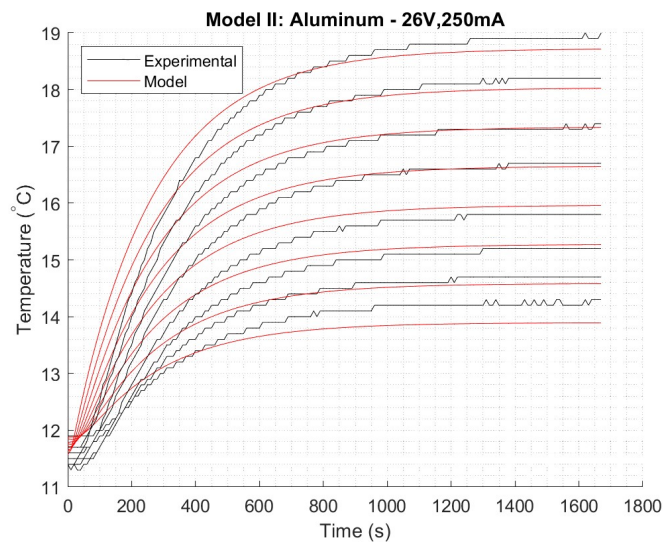


Figure 12: Aluminum at 26V: Model vs Experiment

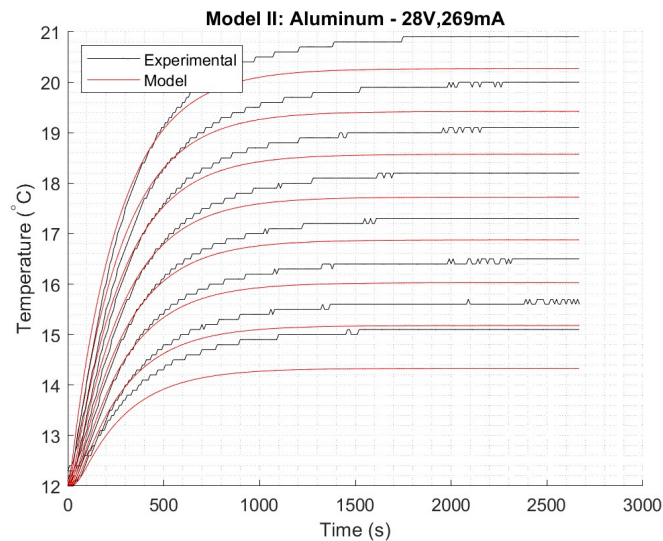


Figure 13: Aluminum at 28V: Model vs Experiment

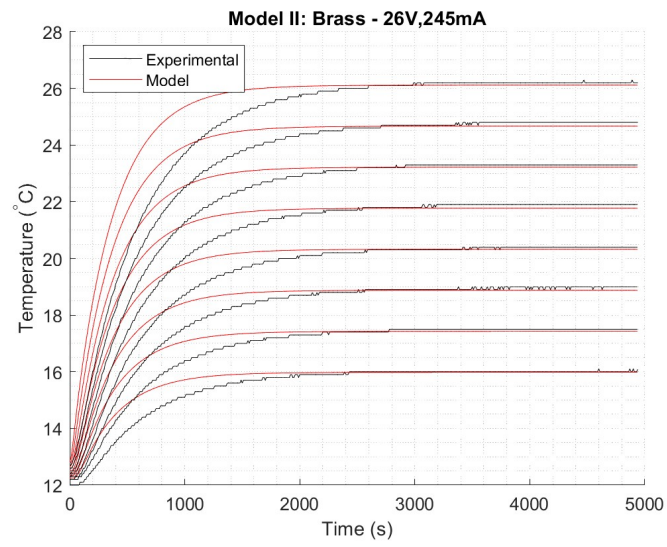


Figure 14: Brass at 26V: Model vs Experiment

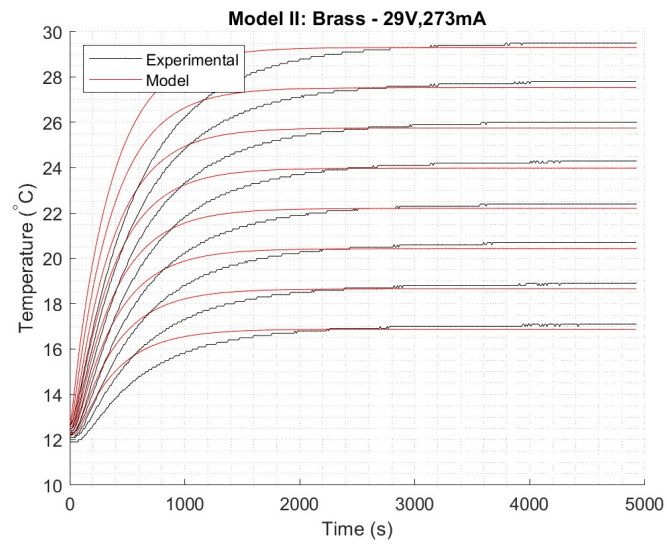


Figure 15: Brass at 29V: Model vs Experiment

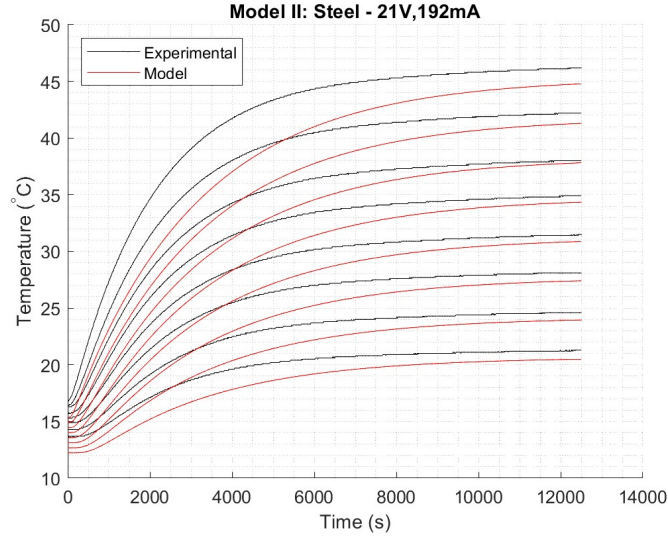


Figure 16: Steel at 21V: Model vs Experiment

As shown in Figures 12-16, we can see that Model II improves the estimate of the initial temperature distribution. The experimental results and Model II are closer at $t = 0$, more so than the previous models. Model II is the most accurate at predicting the initial temperatures of the rods, while continuing to accurately represent the steady-state temperatures. We were able to see this improvement visually in our plots because the initial temperatures are much closer and all seem within $\pm 2[^\circ C]$ degrees of the experimental data.

5 Appendix

5.1 Task 1

```
1 %% ASEN 3802 Lab 2 Part 2 Task 1
2
3 clc;clear;close all;
4
5 const.T0 = 12; % [C]
6 const.H_an = 98.6; % [C/m]
7 const.k = 130; % [W/mK]
8 const.rho = 2810; % [kg/m^3]
9 const.c_p = 960; % [J/kgK]
10 const.alpha = const.k/(const.rho*const.c_p); % [m^2/s]
11 const.x = 0.1238; % [m]
12 const.L = const.x+0.0508; % [m]
13
14 n = 10;
15 t1 = 1; % [s]
16 t2 = 1000; % [s]
17
18 u_1s = heatdistribution(t1,n,const);
19 u_1000s = heatdistribution(t2,n,const);
20
21 Fo_1s = (const.alpha*t1)/const.L^2;
22 Fo_1000s = (const.alpha*t2)/const.L^2;
23
24 figure
25 hold on
26 plot(0:n,u_1s)
27 plot(0:n,u_1000s)
28 title('Temperature at time t vs n')
29 xlabel('n')
30 ylabel('Temperature at t [C]')
31 legend('t=1s','t=1000s')
32
33
34 function u = heatdistribution(t,n,const)
35     sum = 0;
36     u(1) = const.T0+const.H_an*const.x;
37     for i = 1:n
38         b_n = (8*const.H_an*const.L*(-1)^i)/((2*i-1)^2*pi^2);
39         lambda_n = pi*(2*i-1)/(2*const.L);
40         sum = sum + b_n*sin(lambda_n*const.x)*exp(-1*lambda_n^2*const.alpha*t);
41         u(i+1) = const.T0+const.H_an*const.x + sum;
42     end
43 end
```

5.2 Task 2

```
1 %% ASEN 3802 Lab 2 Part 2 Task 2
2
3 clc;clear;close all;
4
5 x1 = 0.0349; % [m]
6 x2 = 0.0476; % [m]
```

```

7 x3 = 0.0603; % [m]
8 x4 = 0.073; % [m]
9 x5 = 0.0857; % [m]
10 x6 = 0.0984; % [m]
11 x7 = 0.1111; % [m]
12 x8 = 0.1238; % [m]
13 x = [x1,x2,x3,x4,x5,x6,x7,x8];
14
15 const_st.H_an = 491.2; % [C/m]
16 const_st.T0 = 11; % [C]
17 const_st.k = 16.2; % [W/mK]
18 const_st.cp = 500; %J/kgK
19 const_st.rho = 8000; % [kg/m^3]
20 const_st.alpha = const_st.k/(const_st.rho*const_st.cp); % [m^2/s]
21 const_st.L = x8+0.0508; % [m]
22
23 const_br26.H_an = 109.3; % [C/m]
24 const_br26.T0 = 12; % [C]
25 const_br26.k = 115; % [W/mK]
26 const_br26.cp = 380; %J/kgK
27 const_br26.rho = 8500; % [kg/m^3]
28 const_br26.alpha = const_br26.k/(const_br26.rho*const_br26.cp); % [m^2/s]
29 const_br26.L = x8+0.0508; % [m]
30
31 const_br29.H_an = 135.9; % [C/m]
32 const_br29.T0 = 12; % [C]
33 const_br29.k = 115; % [W/mK]
34 const_br29.cp = 380; % [J/kgK]
35 const_br29.rho = 8500; % [kg/m^3]
36 const_br29.alpha = const_br29.k/(const_br29.rho*const_br29.cp); % [m^2/s]
37 const_br29.L = x8+0.0508; % [m]
38
39 const_al26.H_an = 98.6; % [C/m]
40 const_al26.T0 = 12; % [C]
41 const_al26.k = 130; % [W/mK]
42 const_al26.cp = 960; % [J/kgK]
43 const_al26.rho = 2810; % [kg/m^3]
44 const_al26.alpha = const_al26.k/(const_al26.rho*const_al26.cp); % [m^2/s]
45 const_al26.L = x8+0.0508; % [m]
46
47 const_al28.H_an = 114.3; % [C/m]
48 const_al28.T0 = 12; % [C]
49 const_al28.k = 130; % [W/mK]
50 const_al28.cp = 960; % [J/kgK]
51 const_al28.rho = 2810; % [kg/m^3]
52 const_al28.alpha = const_al28.k/(const_al28.rho*const_al28.cp); % [m^2/s]
53 const_al28.L = x8+0.0508; % [m]
54
55 t_st = 0:10:12510; % [s]
56 t_al26 = 0:10:1670; % [s]
57 t_al28 = 0:10:2670; % [s]
58 t_br26 = 0:10:4940; % [s]
59 t_br29 = 0:10:4930; % [s]
60
61
62 for i = 1:length(t_st)
63     u_st(i,:) = heatdistribution(t_st(i),x,const_st);
64 end

```

```

65 for i = 1:length(t_al26)
66     u_al26(i,:) = heatdistribution(t_al26(i),x,const_al26);
67 end
68 for i = 1:length(t_al28)
69     u_al28(i,:) = heatdistribution(t_al28(i),x,const_al28);
70 end
71 for i = 1:length(t_br26)
72     u_br26(i,:) = heatdistribution(t_br26(i),x,const_br26);
73 end
74 for i = 1:length(t_br29)
75     u_br29(i,:) = heatdistribution(t_br29(i),x,const_br29);
76 end
77
78 al26 = readmatrix("Aluminum_26V_250mA");
79 al28 = readmatrix("Aluminum_28V_269mA");
80 br26 = readmatrix("Brass_26V_245mA");
81 br29 = readmatrix("Brass_29V_273mA");
82 st = readmatrix("Steel_21V_192mA");
83
84 dx = 0.0127; %m
85 x0 = 0.034925;
86 xpos = x0:dx:x0 + 0.1016;
87 % breaking data into channels
88 % cell row is which thermocouple
89 al26_c = cell(8,1);
90 al28_c = cell(8,1);
91 br26_c = cell(8,1);
92 br29_c = cell(8,1);
93 st_c = cell(8,1);
94 al26_t = al26(:,1);
95 al28_t = al28(:,1);
96 br26_t = br26(:,1);
97 br29_t = br29(:,1);
98 st_t = st(:,1);
99 for i = 1:8
100     al26_c{i} = al26(:,i+1);
101     al28_c{i} = al28(:,i+1);
102     br26_c{i} = br26(:,i+1);
103     br29_c{i} = br29(:,i+1);
104     st_c{i} = st(:,i+1);
105 end
106
107 figure()
108 hold on
109 for i = 1:8
110     plot(al26_t,al26_c{i},'k')
111     plot(t_al26,u_al26(:,i),'r')
112 end
113 grid minor
114 xlabel("Time (s)")
115 ylabel('Temperature_␣(^∘C)')
116 legend('Experimental','Model','','','','','','','','','','','','','Location','northwest')
117 title("Model IA: Aluminum - 26V,250mA")
118
119 figure()
120 hold on
121 for i = 1:8
122     plot(al28_t,al28_c{i},'k')

```

```

123 plot(t_al28,u_al28(:,i),'r')
124 end
125 grid minor
126 xlabel("Time (s)")
127 ylabel('Temperature_\(^{\circ}\text{C}\)')
128 legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
129 title("Model IA: Aluminum - 28V,269mA")
130
131 figure()
132 hold on
133 for i = 1:8
134 plot(br26_t,br26_c{i},'k')
135 plot(t_br26,u_br26(:,i),'r')
136 end
137 grid minor
138 xlabel("Time (s)")
139 ylabel('Temperature_\(^{\circ}\text{C}\)')
140 legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
141 title("Model IA: Brass - 26V,245mA")
142
143 figure()
144 hold on
145 for i = 1:8
146 plot(br29_t,br29_c{i},'k')
147 plot(t_br29,u_br29(:,i),'r')
148 end
149 grid minor
150 xlabel("Time (s)")
151 ylabel('Temperature_\(^{\circ}\text{C}\)')
152 legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
153 title("Model IA: Brass - 29V,273mA")
154
155 figure()
156 hold on
157 for i = 1:8
158 plot(st_t,st_c{i},'k')
159 plot(t_st,u_st(:,i),'r')
160 end
161 grid minor
162 xlabel("Time (s)")
163 ylabel('Temperature_\(^{\circ}\text{C}\)')
164 legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
165 title("Model IA: Steel - 21V,192mA")
166
167
168 function u = heatdistribution(t,x,const)
169     sum = [0,0,0,0,0,0,0,0];
170     for n = 1:10
171         b_n = (8*const.H_an*const.L*(-1)^n)/((2*n-1)^2*pi^2);
172         lambda_n = pi*(2*n-1)/(2*const.L);
173         sum = sum + b_n.*sin(lambda_n.*x).*exp(-1*lambda_n^2*const.alpha*t);
174     end
175     u = const.T0+const.H_an.*x + sum;
176 end

```

5.3 Task 3


```

1 %% ASEN 3802 Lab 2 Part 2 Task 3
2
3 clc;clear;close all;
4
5 x1 = 0.0349; % [m]
6 x2 = 0.0476; % [m]
7 x3 = 0.0603; % [m]
8 x4 = 0.073; % [m]
9 x5 = 0.0857; % [m]
10 x6 = 0.0984; % [m]
11 x7 = 0.1111; % [m]
12 x8 = 0.1238; % [m]
13 x = [x1,x2,x3,x4,x5,x6,x7,x8];
14
15 const_st.H_exp = 277; % [C/m]
16 const_st.T0 = 11; % [C]
17 const_st.k = 16.2; % [W/mK]
18 const_st.cp = 500; % [J/kgK]
19 const_st.rho = 8000; % [kg/m^3]
20 const_st.alpha = const_st.k/(const_st.rho*const_st.cp); % [m^2/s]
21 const_st.L = x8+0.0508; % [m]
22
23 const_br26.H_exp = 114; % [C/m]
24 const_br26.T0 = 12; % [C]
25 const_br26.k = 115; % [W/mK]
26 const_br26.cp = 380; % [J/kgK]
27 const_br26.rho = 8500; % [kg/m^3]
28 const_br26.alpha = const_br26.k/(const_br26.rho*const_br26.cp); % [m^2/s]
29 const_br26.L = x8+0.0508; % [m]
30
31 const_br29.H_exp = 139.8; % [C/m]
32 const_br29.T0 = 12; % [C]
33 const_br29.k = 115; % [W/mK]
34 const_br29.cp = 380; % [J/kgK]
35 const_br29.rho = 8500; % [kg/m^3]
36 const_br29.alpha = const_br29.k/(const_br29.rho*const_br29.cp); % [m^2/s]
37 const_br29.L = x8+0.0508; % [m]
38
39 const_al26.H_exp = 54.3; % [C/m]
40 const_al26.T0 = 12; % [C]
41 const_al26.k = 130; % [W/mK]
42 const_al26.cp = 960; % [J/kgK]
43 const_al26.rho = 2810; % [kg/m^3]
44 const_al26.alpha = const_al26.k/(const_al26.rho*const_al26.cp); % [m^2/s]
45 const_al26.L = x8+0.0508; % [m]
46
47 const_al28.H_exp = 66.8; % [C/m]
48 const_al28.T0 = 12; % [C]
49 const_al28.k = 130; % [W/mK]
50 const_al28.cp = 960; % [J/kgK]
51 const_al28.rho = 2810; % [kg/m^3]
52 const_al28.alpha = const_al28.k/(const_al28.rho*const_al28.cp); % [m^2/s]
53 const_al28.L = x8+0.0508; % [m]
54
55 t_st = 0:10:12510; % [s]
56 t_al26 = 0:10:1670; % [s]
57 t_al28 = 0:10:2670; % [s]

```

```

58 t_br26 = 0:10:4940; % [s]
59 t_br29 = 0:10:4930; % [s]
60
61 for i = 1:length(t_st)
62     u_st(i,:) = heatdistribution(t_st(i),x,const_st);
63 end
64 for i = 1:length(t_al26)
65     u_al26(i,:) = heatdistribution(t_al26(i),x,const_al26);
66 end
67 for i = 1:length(t_al28)
68     u_al28(i,:) = heatdistribution(t_al28(i),x,const_al28);
69 end
70 for i = 1:length(t_br26)
71     u_br26(i,:) = heatdistribution(t_br26(i),x,const_br26);
72 end
73 for i = 1:length(t_br29)
74     u_br29(i,:) = heatdistribution(t_br29(i),x,const_br29);
75 end
76
77 al26 = readmatrix("Aluminum_26V_250mA");
78 al28 = readmatrix("Aluminum_28V_269mA");
79 br26 = readmatrix("Brass_26V_245mA");
80 br29 = readmatrix("Brass_29V_273mA");
81 st = readmatrix("Steel_21V_192mA");
82
83 dx = 0.0127; %m
84 x0 = 0.034925;
85 xpos = x0:dx:x0 + 0.1016;
86 % breaking data into channels
87 % cell row is which thermocouple
88 al26_c = cell(8,1);
89 al28_c = cell(8,1);
90 br26_c = cell(8,1);
91 br29_c = cell(8,1);
92 st_c = cell(8,1);
93 al26_t = al26(:,1);
94 al28_t = al28(:,1);
95 br26_t = br26(:,1);
96 br29_t = br29(:,1);
97 st_t = st(:,1);
98 for i = 1:8
99     al26_c{i} = al26(:,i+1);
100    al28_c{i} = al28(:,i+1);
101    br26_c{i} = br26(:,i+1);
102    br29_c{i} = br29(:,i+1);
103    st_c{i} = st(:,i+1);
104 end
105
106 figure()
107 hold on
108 for i = 1:8
109     plot(al26_t,al26_c{i},'k')
110     plot(t_al26,u_al26(:,i),'r')
111 end
112 grid minor
113 xlabel("Time (s)")
114 ylabel('Temperature_␣(^␣circC)')
115 legend('Experimental','Model','','','','','','','','','','','','','Location','northwest')

```

```

116 title("Model IB: Aluminum - 26V,250mA")
117
118 figure()
119 hold on
120 for i = 1:8
121 plot(al28_t,al28_c{i},'k')
122 plot(t_al28,u_al28(:,i),'r')
123 end
124 grid minor
125 xlabel("Time (s)")
126 ylabel('Temperature_□(^{\circ}C)')
127 legend('Experimental','Model','','','','','','','','','','','','','Location','northwest')
128 title("Model IB: Aluminum - 28V,269mA")
129
130 figure()
131 hold on
132 for i = 1:8
133 plot(br26_t,br26_c{i},'k')
134 plot(t_br26,u_br26(:,i),'r')
135 end
136 grid minor
137 xlabel("Time (s)")
138 ylabel('Temperature_□(^{\circ}C)')
139 legend('Experimental','Model','','','','','','','','','','','','','Location','northwest')
140 title("Model IB: Brass - 26V,245mA")
141
142 figure()
143 hold on
144 for i = 1:8
145 plot(br29_t,br29_c{i},'k')
146 plot(t_br29,u_br29(:,i),'r')
147 end
148 grid minor
149 xlabel("Time (s)")
150 ylabel('Temperature_□(^{\circ}C)')
151 legend('Experimental','Model','','','','','','','','','','','','','Location','northwest')
152 title("Model IB: Brass - 29V,273mA")
153
154 figure()
155 hold on
156 for i = 1:8
157 plot(st_t,st_c{i},'k')
158 plot(t_st,u_st(:,i),'r')
159 end
160 grid minor
161 xlabel("Time (s)")
162 ylabel('Temperature_□(^{\circ}C)')
163 legend('Experimental','Model','','','','','','','','','','','','','Location','northwest')
164 title("Model IB: Steel - 21V,192mA")
165
166
167 function u = heatdistribution(t,x,const)
168     sum = [0,0,0,0,0,0,0,0];
169     for n = 1:10
170         b_n = (8*const.H_exp*const.L*(-1)^n)/((2*n-1)^2*pi^2);
171         lambda_n = pi*(2*n-1)/(2*const.L);
172         sum = sum + b_n.*sin(lambda_n.*x).*exp(-1*lambda_n^2*const.alpha*t);
173     end

```

```

174     u = const.T0+const.H_exp.*x + sum;
175 end

```

5.4 Task 4

```

1 %% ASEN 3802 Lab 2 Part 2 Task 4
2
3 clc;clear;close all;
4
5 x1 = 0.0349; % [m]
6 x2 = 0.0476; % [m]
7 x3 = 0.0603; % [m]
8 x4 = 0.073; % [m]
9 x5 = 0.0857; % [m]
10 x6 = 0.0984; % [m]
11 x7 = 0.1111; % [m]
12 x8 = 0.1238; % [m]
13 x = [x1,x2,x3,x4,x5,x6,x7,x8];
14
15 const_st.H_exp = 277; % [C/m]
16 const_st.M_exp = 35.2; % [C/m]
17 const_st.T0 = 11; % [C]
18 const_st.k = 16.2; % [W/mK]
19 const_st.cp = 500; % [J/kgK]
20 const_st.rho = 8000; % [kg/m^3]
21 const_st.alpha = const_st.k/(const_st.rho*const_st.cp); % [m^2/s]
22 const_st.L = x8+0.0508; % [m]
23
24 const_br26.H_exp = 114; % [C/m]
25 const_br26.M_exp = 7; % [C/m]
26 const_br26.T0 = 12; % [C]
27 const_br26.k = 115; % [W/mK]
28 const_br26.cp = 380; % [J/kgK]
29 const_br26.rho = 8500; % [kg/m^3]
30 const_br26.alpha = const_br26.k/(const_br26.rho*const_br26.cp); % [m^2/s]
31 const_br26.L = x8+0.0508; % [m]
32
33 const_br29.H_exp = 139.8; % [C/m]
34 const_br29.M_exp = 6.6; % [C/m]
35 const_br29.T0 = 12; % [C]
36 const_br29.k = 115; % [W/mK]
37 const_br29.cp = 380; % [J/kgK]
38 const_br29.rho = 8500; % [kg/m^3]
39 const_br29.alpha = const_br29.k/(const_br29.rho*const_br29.cp); % [m^2/s]
40 const_br29.L = x8+0.0508; % [m]
41
42 const_al26.H_exp = 54.3; % [C/m]
43 const_al26.M_exp = -3.4; % [C/m]
44 const_al26.T0 = 12; % [C]
45 const_al26.k = 130; % [W/mK]
46 const_al26.cp = 960; % [J/kgK]
47 const_al26.rho = 2810; % [kg/m^3]
48 const_al26.alpha = const_al26.k/(const_al26.rho*const_al26.cp); % [m^2/s]
49 const_al26.L = x8+0.0508; % [m]
50
51 const_al28.H_exp = 66.8; % [C/m]
52 const_al28.M_exp = 0.3; % [C/m]

```

```

53 const_al28.T0 = 12; % [C]
54 const_al28.k = 130; % [W/mK]
55 const_al28.cp = 960; % [J/kgK]
56 const_al28.rho = 2810; % [kg/m^3]
57 const_al28.alpha = const_al28.k/(const_al28.rho*const_al28.cp); % [m^2/s]
58 const_al28.L = x8+0.0508; % [m]
59
60 t_st = 0:10:12510; % [s]
61 t_al26 = 0:10:1670; % [s]
62 t_al28 = 0:10:2670; % [s]
63 t_br26 = 0:10:4940; % [s]
64 t_br29 = 0:10:4930; % [s]
65
66 for i = 1:length(t_st)
67     u_st(i,:) = heatdistribution(t_st(i),x,const_st);
68 end
69 for i = 1:length(t_al26)
70     u_al26(i,:) = heatdistribution(t_al26(i),x,const_al26);
71 end
72 for i = 1:length(t_al28)
73     u_al28(i,:) = heatdistribution(t_al28(i),x,const_al28);
74 end
75 for i = 1:length(t_br26)
76     u_br26(i,:) = heatdistribution(t_br26(i),x,const_br26);
77 end
78 for i = 1:length(t_br29)
79     u_br29(i,:) = heatdistribution(t_br29(i),x,const_br29);
80 end
81
82 al26 = readmatrix("Aluminum_26V_250mA");
83 al28 = readmatrix("Aluminum_28V_269mA");
84 br26 = readmatrix("Brass_26V_245mA");
85 br29 = readmatrix("Brass_29V_273mA");
86 st = readmatrix("Steel_21V_192mA");
87
88 dx = 0.0127; %m
89 x0 = 0.034925;
90 xpos = x0:dx:x0 + 0.1016;
91 % breaking data into channels
92 % cell row is which thermocouple
93 al26_c = cell(8,1);
94 al28_c = cell(8,1);
95 br26_c = cell(8,1);
96 br29_c = cell(8,1);
97 st_c = cell(8,1);
98 al26_t = al26(:,1);
99 al28_t = al28(:,1);
100 br26_t = br26(:,1);
101 br29_t = br29(:,1);
102 st_t = st(:,1);
103 for i = 1:8
104     al26_c{i} = al26(:,i+1);
105     al28_c{i} = al28(:,i+1);
106     br26_c{i} = br26(:,i+1);
107     br29_c{i} = br29(:,i+1);
108     st_c{i} = st(:,i+1);
109 end
110

```

```

111 figure()
112 hold on
113 for i = 1:8
114 plot(al26_t,al26_c{i},'k')
115 plot(t_al26,u_al26(:,i),'r')
116 end
117 grid minor
118 xlabel("Time (s)")
119 ylabel('Temperature_\(\circ C\)')
120 legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
121 title("Model II: Aluminum - 26V,250mA")
122
123 figure()
124 hold on
125 for i = 1:8
126 plot(al28_t,al28_c{i},'k')
127 plot(t_al28,u_al28(:,i),'r')
128 end
129 grid minor
130 xlabel("Time (s)")
131 ylabel('Temperature_\(\circ C\)')
132 legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
133 title("Model II: Aluminum - 28V,269mA")
134
135 figure()
136 hold on
137 for i = 1:8
138 plot(br26_t,br26_c{i},'k')
139 plot(t_br26,u_br26(:,i),'r')
140 end
141 grid minor
142 xlabel("Time (s)")
143 ylabel('Temperature_\(\circ C\)')
144 legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
145 title("Model II: Brass - 26V,245mA")
146
147 figure()
148 hold on
149 for i = 1:8
150 plot(br29_t,br29_c{i},'k')
151 plot(t_br29,u_br29(:,i),'r')
152 end
153 grid minor
154 xlabel("Time (s)")
155 ylabel('Temperature_\(\circ C\)')
156 legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')
157 title("Model II: Brass - 29V,273mA")
158
159 figure()
160 hold on
161 for i = 1:8
162 plot(st_t,st_c{i},'k')
163 plot(t_st,u_st(:,i),'r')
164 end
165 grid minor
166 xlabel("Time (s)")
167 ylabel('Temperature_\(\circ C\)')
168 legend('Experimental','Model','','','','','','','','','','','','','','','Location','northwest')

```

```

169 title("Model II: Steel - 21V,192mA")
170
171
172 function u = heatdistribution(t,x,const)
173     sum = [0,0,0,0,0,0,0,0];
174     for n = 1:10
175         b_n = (8*(const.M_exp-const.H_exp)*const.L*(-1)^(n+1))/((2*n-1)^2*pi^2);
176         lambda_n = pi*(2*n-1)/(2*const.L);
177         sum = sum + b_n.*sin(lambda_n.*x).*exp(-1*lambda_n^2*const.alpha*t);
178     end
179     u = const.T0+const.H_exp.*x + sum;
180 end

```