ASEN 3802 Lab 3 Part 1

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1 Task 1: Creating Arifoils

Plots of Airfoils based on NACA 4 Number

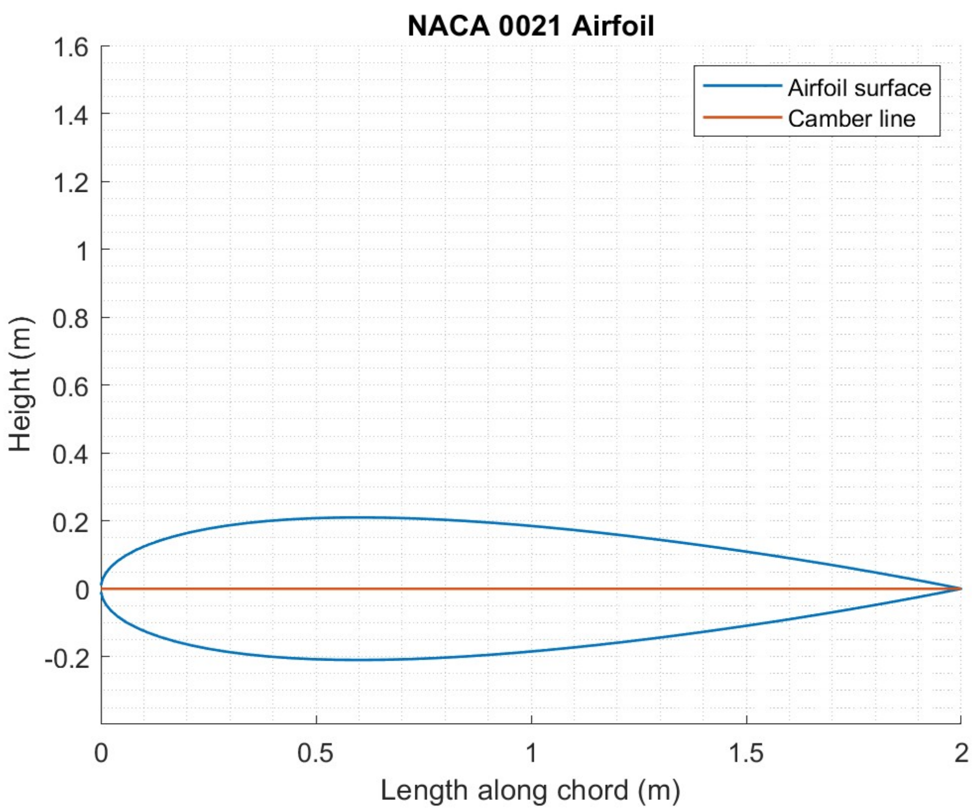

Figure 1: NACA 0021 Airfoil

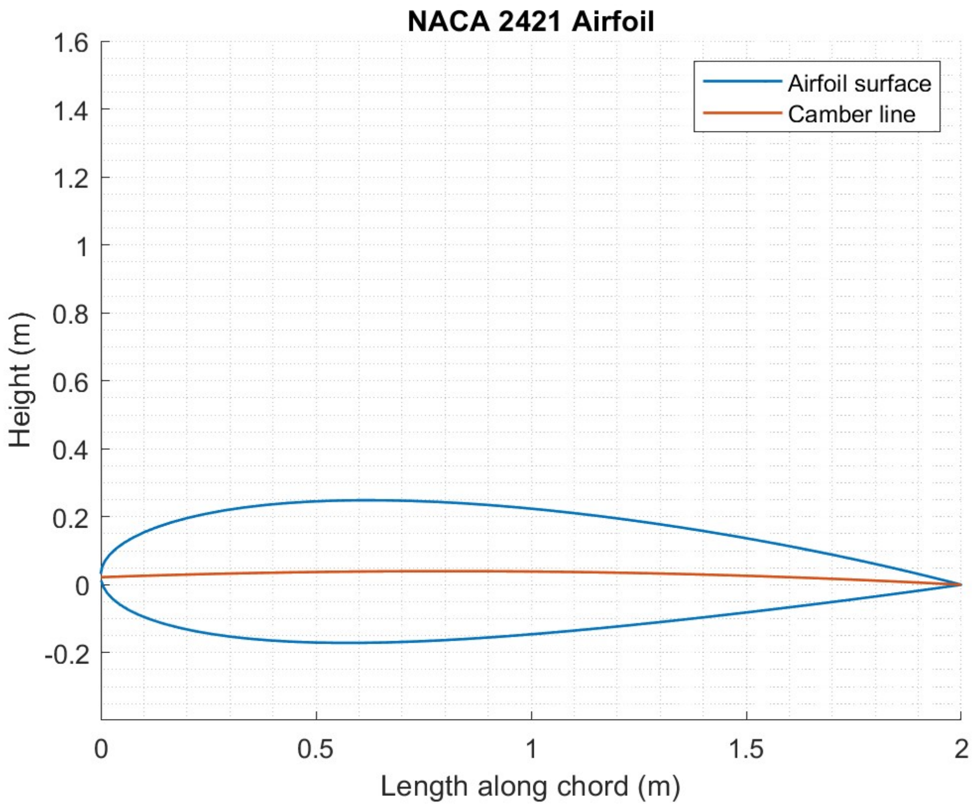

Figure 2: NACA 2421 Airfoil

In order to reduce the error at the leading edge, our function calculates the boundary points using Chebyshev nodes, which are calculated using the following formula:

$$x_k = \frac{c}{2}\cos\left(\frac{(2k-1)\pi}{2n}\right) + \frac{c}{2}, \quad k = 1, 2, ..., n$$

This formula just provides nodes that are closer together at the leading and trailing edges where changes in pressure are more prominent and a tighter clustering of points is more important.
These figures make sense. The first one depicts a symmetric airfoil, which is seen in both the NACA number and the graph. The second also makes sense, because the NACA number indicates the airfoil will have positive camber.

2 Task 2: Effect of Thickness

2.1 Plot of C_l versus number of Panels (upper and lower surfaces)

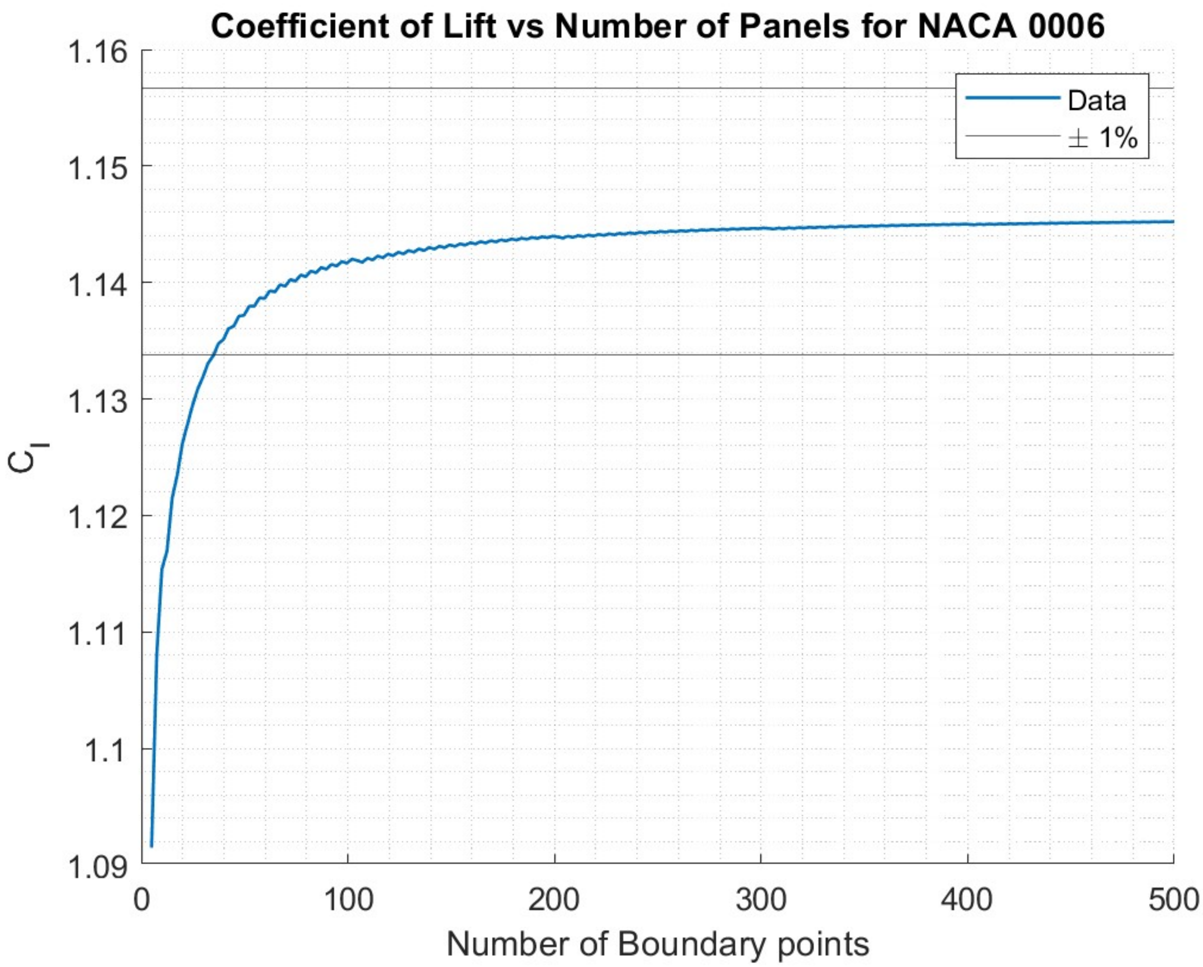

Figure 3: C_l convergence due to Number of Panels

As the number of panels increases, the accuracy of the estimation increases as well. By making n go very high ($n = 500$), we can see the asymptotic behavior. Assuming the final value is approximately the value of the asymptote, we can determine the first point that is within the 1% range, and the required number of panels to get that accuracy. With $n = 500$, $C_l = 1.1452$. The first value that is within $\pm 1\%$ of this is when $n = 14$, $C_l = 1.1348$. We can see the relative error is less than 1% by using the relative error formula:

$$\frac{|1.1452 - 1.1348|}{1.1452} = 0.00908 \Rightarrow 0.908\%$$

Therefore, to get within 1% relative error, we only need to use 14 panels on both the upper and lower surfaces.

2.1.1 Discussion:

Does the "exact" sectional lift coefficient make sense based on your understanding of airfoil theory? Is the number of panels you needed to achieve 1% relative error more or less than you expected? Do you believe it is worth using even more panels to further reduce the error? Why or why not?

The 'exact' sectional lift coefficient is certainly within reasonable bounds, however it doesn't quite match the experimental data in “Theory of Wing Sections”. Where we found that C_l approaches 1.1452 with 500 panels, the book seems to suggest a Cl somewhere closer to 0.9. Approximating within 1% in only 14 panels seems reasonable since numerical approximations usually approach asymptotes very quickly. By looking at the graph of C_l vs α, we can see a very quick approach to the solution so we can tell that this method is super-linearly convergent, meaning the required number of terms is low to form an accurate estimate. We know it has this convergence behavior because the vast majority of large discrepancies within the model disappear quite quickly. Using more panels would truly only be necessary for very in depth analysis. Working with less than 1% error is quite remarkable to begin with, and is extremely usable for most calculations. Additionally, this is also preferable because it keeps computation time down. Since we only need to compute 14 panels (28 overall), the programs can be optimized and shorter, whereas if we had to do 500 panels on the upper and lower surfaces, the programs would take much longer to run and would be more computationally expensive.

2.2 Deliverable 2: C_l vs α

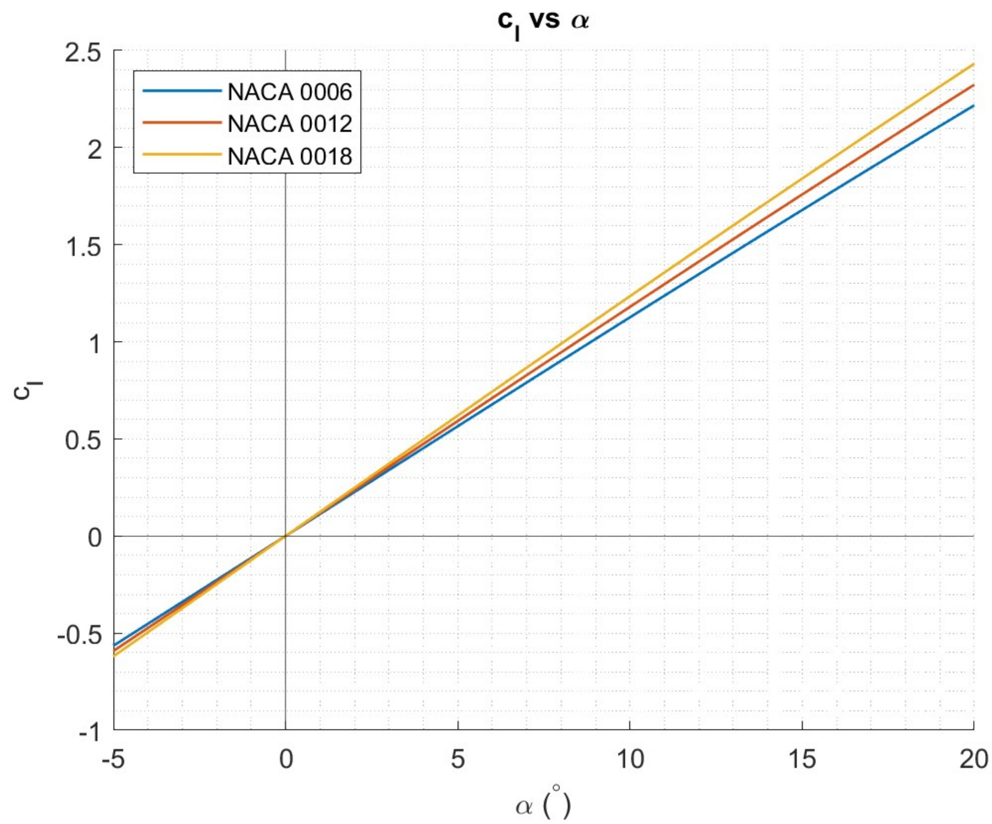

Figure 4: Plot of C_l vs α for Different Thickness Airfoils

Airfoil	$\alpha_{L=0}$ [°](Theory)	$\alpha_{L=0}$ [°](Model)	$\alpha_{L=0}$ [°](Exp)	α_0 [1/°] (Theory)	α_0 [1/°] (Model)	α_0 [1/°] (Exp)
NACA 0006	0	0	0	0.1097	0.1131	0.1
NACA 0012	0	0	0	0.1097	0.1185	0.1
NACA 0018	0	0	-	0.1097	0.1240	-

2.2.1 Finding $\alpha_{L=0}$

The angle for an airfoil at which the lift is zero is called the zero lift angle of attack. It can be calculated using the following equation:

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}\left(\cos\theta_0 - 1\right) d\theta_0 \tag{1}$$

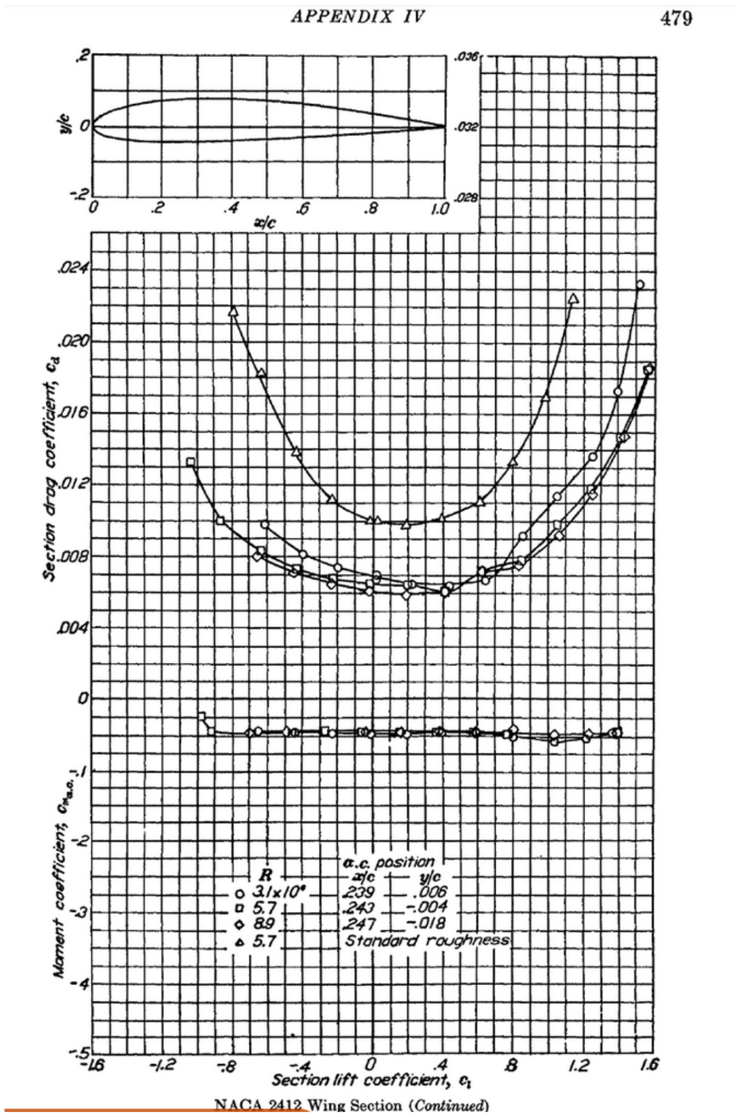

Figure 5: NACA 2412 Experimental cl vs cd

In order to get this equation, we use the following substitution: $x = \frac{c}{2}(1 - \cos\theta_0)$. Performing a variable transform we can rewrite equation 1 to

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{c/2} \frac{dz}{dx}\left(-\frac{2x}{c} + 1 - 1\right)\frac{1}{c}\sqrt{\frac{2}{x(c-x)}}dx \tag{2}$$

We use this in our code to compute the zero lift angle of attack, using numerical differentiators and integrators. Using this method, we get $\frac{dz}{dx} = 0$, leading to $\alpha_{L=0} = 0$ for all airfoils tested. This makes sense as the NACA 0006, 0012 and 0018 are all symmetric airfoils, and have a zero lift angle of attack of zero degrees/radians.

2.2.2 Discussion:

How do changes in the wing section thickness alter the sectional lift slope? How accurate is the assumption of thin airfoil theory for each wing section? How well do the two different theoretical methods (thin airfoil theory and vortex panel method) compare with the experimental data, and where do you think any disagreements might come from? The experimental data was collected at precise Reynolds numbers - what are the Reynolds numbers for the two theoretical methods?

In theory, wing section thickness shouldn't alter the sectional lift slope at all, however, in our model we found that increasing wing thickness also increased lift slopes. The thin airfoil assumption decreases in accuracy as the wing section becomes larger, but the error remains relatively small for the airfoils tested. Both methods seem to be a slight overestimation of the experimental data, but our Thin Airfoil results fall closer than the Vortex panel method. We believe that the discrepancies arise from assumptions made for each method as well as that Thin Airfoil theory does not take into account how the thickness of the different profiles changes. Both Thin Airfoil and Vortex sheet methods assume inviscid flow, meaning the Reynold's number is ignored or irrelevant. Since the Reynold's number is inversely proportional to dynamic viscosity ($Re \propto \frac{1}{\nu}$), if there is no viscosity as the model assumes, the Reynold's Number is theoretically infinite.

3 Task 3: Effect of Camber

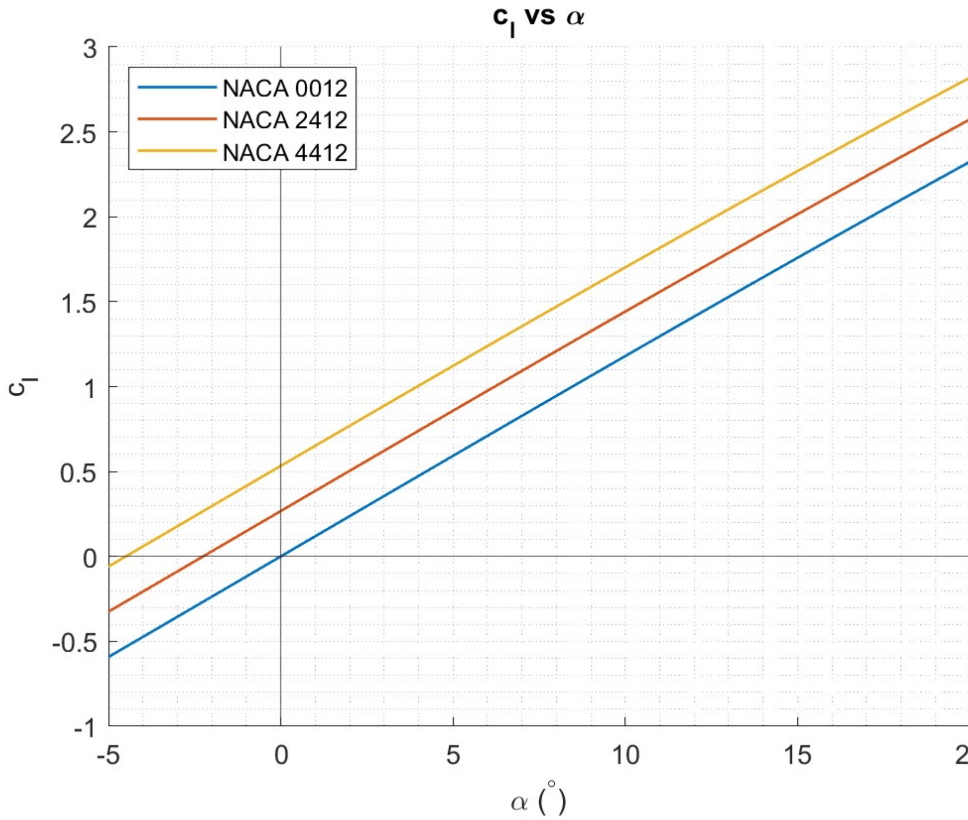

Figure 6: c_l vsα for Varying Camber

Airfoil	α_0 (Theory) [1/°]	α_0 (Model) [1/°]	α_0 (Exp) [1/°]	$\alpha_{L=0}$ (Theory)[°]	$\alpha_{L=0}$ (Model)[°]	$\alpha_{L=0}$ (Exp) [°]
NACA 0012	0.1185	0.1185	0.1	0	-5.5×10^{-6}	0
NACA 2412	0.1186	0.1186	0.0889	-2.3	-2.2538	-2
NACA 4412	0.1186	0.1186	0.1	-4.5	-4.4968	-4

3.1 Discussion:

How do changes in camber alter the sectional lift slope and the zero-lift angle of attack? How accurate is the assumption of thin airfoil theory for each wing section? How well do the two different theoretical methods (thin airfoil theory and vortex panel method) compare with the experimental data, and where do you think any disagreements might come from?

Increasing the camber lowers the zero-lift angle of attack from zero degrees, like it is for a non-cambered airfoil, while not changing the lift slope. Thin airfoil theory is quite accurate for the 0012 airfoil as well as 2412 and 4412 airfoils, but is less accurate for the latter airfoils as the camber increases. The Vortex Panel Method's fairs similarly with cambered airfoils, following the change in zero-lift angle of attack closely. The results for Thin-airfoil are exceedingly close to the Vortex panel results. The small differences (not shown in 4 decimal places) can be attributed to how the Vortex Panel method and Thin airfoil theory operate. Where the Vortex panel method models the flow around an airfoil, where changes in camber do alter the results, in Thin-airfoil theory the camber line becomes the airfoil's shape for aerodynamic analysis.

4 Appendix

4.1 Airfoil Function

```
    function coords = naca4series(digits,c,n)
    %interpreting the digits
    t = digits(3)/100; % Percent max thickness
    p = digits(2)/10; % 1/10 loc of max camber
    m = digits(1)/100; % Percent max camber

    % Creating panel nodes
    x = linspace(0,c,n)'; %Equispaced nodes

    % Chebyshev nodes
    for k = 1:n
        x(k) = c/2*cos((2*k-1)*pi/(2*n)) + c/2;
    end

    % Thickness Distribution
    yt = t/(0.2)*c.*(0.2969.*sqrt(x/c)-0.1260.*(x/c)-0.3516.*(x/c).^2+0.2843*(x/c).^3-0.1036.*(x/c
       ).^4);

    % Camber line distribution
    yc = zeros(length(x),1);
    discontinuity = find(x>p*c,1);
    for i = 1:discontinuity
        yc(i) = m*x(i)/p^2*(2*p-x(i)/c);
    end
    for i = discontinuity:length(x)
        yc(i) = m*(c-x(i))/(1-p)^2*(1+x(i)/c-2*p);
    end

    % Camber line derivative
    ycp = zeros(length(x),1);
    for i = 1:discontinuity
        ycp(i) = m*2*p/p^2 - -2*m*x(i)/(p^2*c);
    end
    for i = discontinuity:length(x)
        ycp(i) = m*(-x(i))/(1-p)^2*(1+x(i)/c-2*p) + m*(c-x(i))/(1-p)^2*(1/c);
    end

    % xi
    xi = atan(ycp);

    %Calculating  the upper and lower surfaces
    coords.xU = x - yt.*sin(xi);
    coords.xL = x + yt.*sin(xi);
    coords.yU = yc + yt.*cos(xi);
    coords.yL = yc - yt.*cos(xi);
    coords.yc = yc;
    coords.xc = x;
    coords.xb = cat(1,coords.xL,flip(coords.xU));
    coords.yb = cat(1,coords.yL,flip(coords.yU));

end
```

4.2 Task 1

```
    close all; clear; clc;

c = 2;%m
n = 50; %num panels + 1

%% NACA 0021
digits = [0,0,21];

coords = naca4series(digits,c,n);
coords0021.x = cat(1,flip(coords.xL),coords.xU);
coords0021.y = cat(1,flip(coords.yL),coords.yU);

figure()
hold on
plot(coords0021.x,coords0021.y,'Linewidth',1)
plot(coords.xc,coords.yc,'Linewidth',1)
legend("Airfoil surface","Camber line")
grid minor
xlabel("Length along chord (m)")
ylabel("Height (m)")
title("NACA 0021 Airfoil")
ymin = min(coords.xL)-0.4;
ylim([ymin,ymin+c ])
xlim([0,c])

%% NACA 2421
digits = [2,4,21];

coords = naca4series(digits,c,n);
coords2421.x = cat(1,flip(coords.xL),coords.xU);
coords2421.y = cat(1,flip(coords.yL),coords.yU);

figure()
hold on
plot(coords2421.x,coords2421.y,'Linewidth',1)
plot(coords.xc,coords.yc,'Linewidth',1)
legend("Airfoil surface","Camber line")
grid minor
xlabel("Length along chord (m)")
ylabel("Height (m)")
title("NACA 2421 Airfoil")
ymin = min(coords.xL)-0.4;
ylim([ymin,ymin+c ])
xlim([0,c])
```

4.3 Task 2

```
close all; clear; clc;

%% TASK 2: Convergence Study

% Properties
n = linspace(5,500,200);
c = 1;
digits = [0,0,6];
```

```matlab
alpha = 10; %deg

% Calculating the cl for every n value
for i = 1:length(n)
    coords0006 = naca4series(digits,c,n(i));
    cl0006(i) = Vortex_Panel(coords0006.xb,coords0006.yb,alpha);
end

% Calculating the relative error
error = zeros(length(n)-1,1);
for i = 1:length(n)-1
    error(i) = (cl0006(i)-cl0006(end))/cl0006(end);
end

% Plotting Results
figure()
hold on
plot(n,cl0006,"LineWidth",1)
yline(0.99*cl0006(end))
yline(1.01*cl0006(end))
legend("Data","\pm 1% ","")
grid minor
xlabel("Number of Boundary points")
ylabel("C_l")
title("Coefficient of Lift vs Number of Panels for NACA 0006")


index = find(cl0006>cl0006(end)*.99,1);

%% Plot cl/alpha for different thickness airfoils
clear; close all

n = 20; %Just more than we predicted for the sake of margin
c = 1; %m
alpha = linspace(-5,20,25);

% NACA 0006
digits = [0,0,6];
coords0006 = naca4series(digits,c,n);

% NACA 0012
digits = [0,0,12];
coords0012 = naca4series(digits,c,n);

% NACA 0018
digits = [0,0,18];
coords0018 = naca4series(digits,c,n);

% Finding the cl/alpha graphs
cl0006 = zeros(length(alpha),1);
cl0012 = zeros(length(alpha),1);
cl0018 = zeros(length(alpha),1);
for i = 1:length(alpha)
    cl0006(i) = Vortex_Panel(coords0006.xb,coords0006.yb,alpha(i));
    cl0012(i) = Vortex_Panel(coords0012.xb,coords0012.yb,alpha(i));
    cl0018(i) = Vortex_Panel(coords0018.xb,coords0018.yb,alpha(i));
end

```

```
% Plotting Results
figure()
hold on
plot(alpha,cl0006,'Linewidth',1)
plot(alpha,cl0012,'Linewidth',1)
plot(alpha,cl0018,'Linewidth',1)
xline(0)
yline(0)
grid minor
xlabel("\alpha (^\circ)")
ylabel("c_l")
title("c_l vs \alpha")
legend("NACA 0006", "NACA 0012", "NACA 0018",'Location','Northwest')

% Estimating the lift slope and 0 lift AoA

% For lift slop- mostly linear so can use simple slope equation
a0_NACA0006 = (cl0006(3)-cl0006(4))/(alpha(3)-alpha(4));
a0_NACA0012 = (cl0012(3)-cl0012(4))/(alpha(3)-alpha(4));
a0_NACA0018 = (cl0018(3)-cl0018(4))/(alpha(3)-alpha(4));


%% Estimating alphaL=0
cv = c*ones(length(coords0006.xc),1);
dzdx = finite_difference(coords0006.xc,coords0006.yc);
integrand = -1/pi*dzdx.*(2*coords0006.xc/c).*1/c.*sqrt(2/(coords0006.xc.*(cv-coords0006.xc)));
alphal0_0006 = trapz(integrand(:,1));
```

4.4 Task 3

```
close all; clear; clc;


%% Plot cl/alpha for different camber airfoils

n = 20; %Just more than we predicted for the sake of margin
c = 1; %m
alpha = linspace(-5,20,25);

% NACA 0012
digits = [0,0,12];
coords0012 = naca4series(digits,c,n);

% NACA 2412
digits = [2,4,12];
coords2412 = naca4series(digits,c,n);

% NACA 4418
digits = [4,4,12];
coords4412 = naca4series(digits,c,n);

% Finding the cl/alpha graphs
cl0012 = zeros(length(alpha),1);
cl2412 = zeros(length(alpha),1);
cl4412 = zeros(length(alpha),1);
for i = 1:length(alpha)
    cl0012(i) = Vortex_Panel(coords0012.xb,coords0012.yb,alpha(i));
```

```
    cl2412(i) = Vortex_Panel(coords2412.xb,coords2412.yb,alpha(i));
    cl4412(i) = Vortex_Panel(coords4412.xb,coords4412.yb,alpha(i));
end

% Plotting Results
figure()
hold on
plot(alpha,cl0012,'Linewidth',1)
plot(alpha,cl2412,'Linewidth',1)
plot(alpha,cl4412,'Linewidth',1)
xline(0)
yline(0)
grid minor
xlabel("\alpha (^\circ)")
ylabel("c_l")
title("c_l vs \alpha")
legend("NACA 0012", "NACA 2412", "NACA 4412",'Location','Northwest')

% Estimating the lift slope and 0 lift AoA

% For lift slop- mostly linear so can use simple slope equation
a0_NACA0012 = (cl0012(3)-cl0012(4))/(alpha(3)-alpha(4));
a0_NACA2412 = (cl2412(3)-cl2412(4))/(alpha(3)-alpha(4));
a0_NACA4412 = (cl4412(3)-cl4412(4))/(alpha(3)-alpha(4));

% Interpolating to find alpha L=0
index_neg = find(cl0012<0,1,"last");
index_pos = find(cl0012>0,1);
alphaL0_NACA0012 = interpinator(cl0012(index_neg),alpha(index_neg),cl0012(index_pos),alpha(index_pos),0);


index_neg = find(cl2412<0,1,"last");
index_pos = find(cl2412>0,1);
alphaL0_NACA2412 = interpinator(cl2412(index_neg),alpha(index_neg),cl2412(index_pos),alpha(index_pos),0);
% Using finite methods
cv = c*ones(length(coords2412.xc),1);
dzdx = finite_difference(coords2412.xc,coords2412.yc);
integrand = 1/pi*dzdx.*(2*coords2412.xc/c).*1/c.*sqrt(2/(coords2412.xc.*(cv-coords2412.xc)));
alphal0_2412 = trapz(integrand(:,10));


index_neg = find(cl4412<0,1,"last");
index_pos = find(cl4412>0,1);
alphaL0_NACA4412 = interpinator(cl4412(index_neg),alpha(index_neg),cl4412(index_pos),alpha(index_pos),0);
cv = c*ones(length(coords4412.xc),1);
dzdx = finite_difference(coords4412.xc,coords4412.yc);
integrand = 1/pi*dzdx.*(2*coords4412.xc/c).*1/c.*sqrt(2/(coords4412.xc.*(cv-coords4412.xc)));
alphal0_4412 = trapz(integrand(:,10));



function [yout] = interpinator(x1,y1,x2,y2,xq)
    m = (y2-y1)/(x2-x1);
    yout = m*(xq-x1)+y1;
end
```