Representing multi-qubit states

1

→ Single bit has two possible states.
→ qubit state has two complex amplitude.
and bits have four possible states:

00 01 10 11

→ The state of two qubits requires four complex amplitudes. and store these amplitudes in a 4D-Vector

$$|a\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle = \begin{bmatrix} a_{00} \\ a_{01} \\ a_{10} \\ a_{11} \end{bmatrix}$$

→ $|00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $|01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

$|10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, $|11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$$|a\rangle = a_{00}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + a_{01}\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + a_{10}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + a_{11}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} a_{00} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ a_{01} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ a_{10} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ a_{11} \end{bmatrix}$$

$$= \begin{bmatrix} a_{00} + 0 + 0 + 0 \\ 0 + a_{01} + 0 + 0 \\ 0 + 0 + a_{10} + 0 \\ 0 + 0 + 0 + a_{11} \end{bmatrix} = \begin{bmatrix} a_{00} \\ a_{01} \\ a_{10} \\ a_{11} \end{bmatrix} \#$$

Scanned with CamScanner

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$\Rightarrow |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1\times 1 + 1\times 0 \\ 1\times 1 + (-1)\times 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1+0 \\ 1-0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \left[|0\rangle + |1\rangle \right]$$

$$\boxed{H|0\rangle = |+\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \left[|0\rangle + |1\rangle \right]}$$

$$\boxed{H|1\rangle = |-\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{\sqrt{2}} \left[|0\rangle - |1\rangle \right]}$$