LONGITUDINAL STERN–GERLACH EFFECT WITH SLOW NEUTRONS

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The spin-dependent retardation and acceleration of neutrons entering a magnetic field has been measured by means of the high resolution backscattering technique. Its relation to a neutron magnetic resonance system is discussed.

It is a well-known rule of quantum mechanics that for any physical system which can be described by a hamiltonian which does not explicitly depend on time, i.e. $\partial\mathcal{H}/\partial t = 0$, the total energy of the system is a constant of the motion since in that case

$$(\mathrm{d}/\mathrm{d}t)\langle\mathcal{H}\rangle = 0\,. \tag{1}$$

The hamiltonian of a neutron in a magnetic field $\boldsymbol{B}$ is

$$\mathcal{H} = p^2/2m - \boldsymbol{\mu}\boldsymbol{B}\,, \tag{2}$$

where $\boldsymbol{p} = \hbar\boldsymbol{k} = -i\hbar\,\nabla$ is the neutron momentum operator, m is the neutron mass and $\boldsymbol{\mu} = \mu\boldsymbol{\sigma}$ the neutron magnetic moment operator, where $\mu = -6.031 \times 10^{-8}$ eV/T is the magnetic moment of the neutron and $\boldsymbol{\sigma}$ is the Pauli spin operator. If the magnetic field is stationary, i.e. its time dependence in the moving neutron's reference frame described by $B(t) \equiv B(r(t))$ and hence $\partial B/\partial t = 0$, eq. (1) holds for the hamiltonian (2) and the energy of the neutron is conserved, whatever the spatial distribution of the field along the neutron trajectory may be.

Consequently, the change of the potential energy that is associated with each spatial variation of a static magnetic field inevitably implies an equivalent inverse change of the kinetic energy of the neutron for its total energy to remain constant. In a semiclassical approximation, where it is assumed that the field variations take place along distances much larger than the spatial extent of the wave packet representing the neutron, this momentum change is accomplished by a force

$$\boldsymbol{F} = \nabla(\boldsymbol{\mu}\boldsymbol{B}) = (\boldsymbol{\mu}\nabla)\boldsymbol{B}\,. \tag{3}$$

The second equality sign in eq. (3) holds since for a static magnetic field in vacuum $\nabla \times \boldsymbol{B} = 0$ and since $\boldsymbol{\mu}$ does not depend explicitly on the position, i.e. $\nabla_i\mu_i = 0$. In the "classical" Stern–Gerlach experiment, where the neutrons pass through an inhomogeneous magnetic field, whose gradient is oriented perpendicularly to their initial propagation direction, the resulting force leads to a transversal acceleration of the neutrons and thus to the well-known spatial separation of their two possible spin states [1,2]. Similar acceleration effects are responsible for the refraction of neutrons by magnetic fields as measured by Just et al. [3] and Badurek et al. [4]. Field gradients along the neutron-propagation direction, on the other hand, result in purely longitudinal momentum changes. Zeilinger and Shull [5] have recently demonstrated that the associated small wavelength modifications have a remarkable influence on the dynamical diffraction of neutrons in perfect cystals. In fact, use is made of the longitudinal Stern–Gerlach effect in polarizing ultra-cold neutrons by perpendicular transmission through the magnetic fields [6,7]. Funahashi [8] has proposed to use the longitudinal momentum separation of the two spin states to polarize highly monochromatic neutron beams by perfect crystal backreflection. In this proposal it was incorrectly stated, however, that the neutron energy is not conserved if the magnetic field changes its direc-

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tion so abruptly along the beam path that transitions between the two Zeeman energy levels are induced. In the literature such field configurations are usually referred to as "nonadiabatic" to distinguish them from "adiabatic" field distributions where no spin transitions are induced. But one should not be confused by the notations "nonadiabatic" and "adiabatic" in this case. They are only related to the (non-)conservation of potential and kinetic energy, respectively. The total neutron energy, however, is a constant of the motion for any static field configuration.

Here we report a direct measurement of the acceleration and retardation of neutrons entering a magnetic field by exploiting the very high momentum resolution of Bragg reflections at perfect crystals in backscattering geometry. The experiment was performed at the backscattering spectrometer [9] at the FRJ-2 reactor. The neutron wavelength was $\lambda = 6.28$ Å, corresponding to an energy of 2.1 meV. The experimental set up is sketched in fig. 1. The incident neutrons are monochromatized by means of backreflection at the perfect Si-crystal 1 mounted on a Doppler drive. The relative momentum resolution of such an arrangement is [10]:

$$\Delta k/k = \tfrac{1}{2}\Delta E_k/E_k = (\Delta d/d)^2 + (\Delta\alpha/2)^2 \approx 0.4 \times 10^{-4}, \tag{4}$$

where $E_k = \hbar^2 k^2/2m$ is the kinetic energy, $\Delta d/d \approx 2 \times 10^{-5}$ is the dynamic reflection width and $\Delta\alpha(\approx 0.9 \times 10^{-2}$ rad in our case) is the angular divergence of the beam. A semi-transparent detector, which is electronically gated synchronously with a mechanical disk chopper to discriminate against the incident flux, registered those neutrons that are backreflected at a second Si-crystal placed between the pole shoes of an electromagnet. If no field is applied there is no change of kinetic energy of the neutrons along their path between the two crystals and the detector intensity

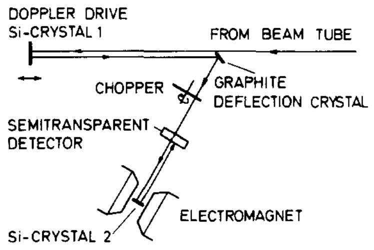

Fig. 1. Sketch of the experimental arrangement.

reaches its maximal value if the Doppler energy shift ΔE of the primary reflection is zero. The result of this measurement is shown at the top of fig. 2. The observed width of the lorentzian-type resolution function of ≈ 0.3 μeV agrees with the estimation according to eq. (4). If the magnetic field is switched on, energy conservation demands that the kinetic energy of the neutrons is modified from $E_k = E_{tot} = \hbar^2 k^2/2m$ ($k = 2\pi/\lambda$ is the neutron wave number outside the field region) into

$$E'_k = \hbar^2 k'^2/2m = \hbar^2 k^2/2m \pm \mu B ,$$

where the signs refer to the two spin states parallel (+) and antiparallel (−) to the magnetic induction. The kinetic energy spectrum therefore splits into two lines separated by $\Delta E_k = 2\mu B$. Fitting two lorentzians to the experimental data yields an experimental splitting of 0.26 ± 0.03 μeV as indicated in fig. 2. For the given field of $B = 1.964$ T (as measured with a proton reso-

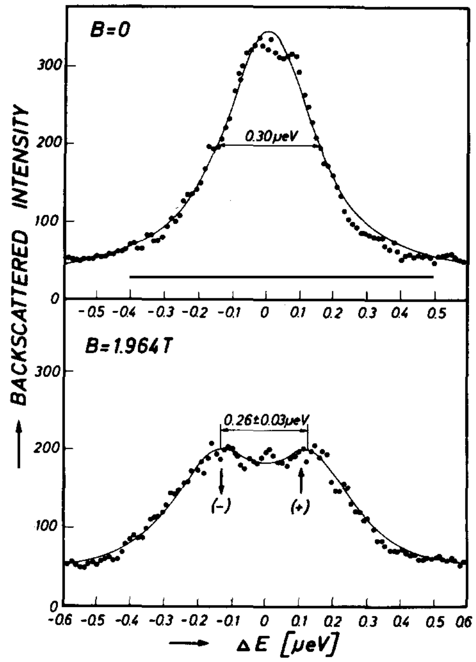

Fig. 2. Backscattered intensity versus Doppler energy shift of the incident neutrons without and with a magnetic field applied to the backscattering crystal. The arrows indicate the two spin states parallel (+) and antiparallel (−) to the magnetic induction. The solid lines correspond to least squares fits of lorentzians to the experimental points. The data collection time was ≈10 h for each of the two measurements.

nance probe) the theoretically expected energy separation is 0.237 μeV. The appearance of the spurious central peak should have no physical reasons but is likely due to an anomalous transmission of the spectrometer (see the curve for $B = 0$) or may possibly be a purely statistical fluctuation.

It is evident, that the initial wavelength of the neutrons is resorted after they have left the magnetic field again since the retardations and accelerations occurring at the entrance are exactly compensated by their respective inverse processes on the exit from the field region. However, if inside this static field $\boldsymbol{B}$ spin-flip transitions are induced between the Zeeman levels by means of a time-dependent perturbation, as e.g. a (small) magnetic field $\boldsymbol{B}_1(t)$ oscillating normally to the static field, the neutron energy is no longer a constant of the motion. The neutron magnetic moment couples significantly to the photon states of this field if the oscillation frequency ν of this field is equal to the Larmor frequency ($h\nu = 2\mu B$). For a suitably chosen interaction time one can achieve that an originally strictly monochromatic unpolarized neutron beam after passage through such an arrangement of crossed static and time-dependent magnetic fields will be composed of two polarized subbeams which differ in energy by an amount of $4\mu B$. By means of a technique similar to that used for the present paper we could recently verify experimentally this inelasticity feature of the interaction of neutrons with time-dependent magnetic fields [11,12], which is of basic importance for the so-called "dynamical polarization" of neutron beams [13].

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