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ELECTROMAGNETIC FOCUSING AND POLARIZATION OF NEUTRON BEAMS

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The application of methods used in the focusing and polarization of atomic beams is considered. It is found that in spite of the small value of the magnetic moment of the neutron, focusing and very high degree of polarization of slow neutrons with the aid of a magnetic sixpole lens is feasible.

The analogy between the motion of neutrons and atoms with spin $\frac{1}{2}$ is demonstrated by the Stern-Gerlach experiment (Sherwood et al.[1])) and the focusing of neutron beams raises the same type of problem as the focusing of beams of neutral atoms of spin $\frac{1}{2}$. The latter problem was analysed theoretically and the results were tested experimentally by Friedburg and Paul[2]), and the method has been used successfully by other experimenters also (e.g. Lemonick et al.[3])). In this method beam focusing is obtained by virtue of the translatory forces acting on the magnetic dipole moments of the particles in an inhomogeneous magnetic field of suitable design. The apparent difficulty of applying this approach to neutron beams is that the relevant magnetic moment in atomic beam work is of electronic origin and is, in practical cases, equal to 1 Bohr magneton, whereas the magnetic moment of the neutron is approximately 2 nuclear magnetons, three orders of magnitude lower than the previous quantity. However, the focusing properties of the fields depend on the square root of the magnetic moment of the particles concerned and if the neutrons are slow enough it seems possible to compensate the loss of "refractive power" of the focusing system by choosing its geometrical dimensions appropriately. As in the case of atomic beams focusing is obtained for one of the two spin states; particles in the other spin state are defocused and therefore a small aperture in a suitable position can select a highly polarized beam.

Given a beam of particles moving near the z-axis of a cylindrical coordinate system (r, θ, z), focusing will occur if the particles pass through a region where they are subject to a quasi-elastic radial force. In a magnetic field, $\boldsymbol{B}$, which can be derived from a scalar potential $U(r, \theta, z)$, the force acting on the dipole is $\boldsymbol{F} = \mathrm{grad}\,(\boldsymbol{\mu}\cdot\boldsymbol{B}) = \pm\mu\,\mathrm{grad}\,\mathrm{B}$. In order to obtain focusing it is required that $\mathrm{grad}\,B = -\alpha^2 r$; ($\alpha = \mathrm{constant}$).

Considering a $2n$-pole field characterized by the two dimensional potential distribution

$$U = \mathrm{const}\cdot r^n \cos n\theta$$

one finds that the force acting on the particles is:

$$\boldsymbol{F} = \pm(n-1)\mu(B_0/r_0)(r/r_0)^{n-2}\,\boldsymbol{r}$$

where $\boldsymbol{r}$ is the radial unit vector, and B_0 is the field at a radial distance r_0.

To obtain a quasi-elastic radial force, one has to choose a six-pole field: $n=3$, in which case the differential equation of the trajectories becomes

$$r'' = -q^2 r$$

with

$$q^2 = \pm\frac{\mu B_0}{kT}\left(\frac{v_T}{v}\right)^2\frac{1}{r_0^2}.$$

Here v is the axial component of the neutron velocity; $v_T = (2\,kT/m)^{\frac{1}{2}}$ is the most probable neutron velocity in a neutron gas in thermal equilibrium at an absolute temperature T; and k is Boltzmann's constant.

For particles of a given axial velocity q^2 is constant, and its sign depends on the spin state of the particle. The field acts as a lens and by well known methods of geometrical electron optics one finds that the distance of the principal planes from the middle of lens, Δ, in units of the length of the magnet, Z, is

$$\frac{\Delta}{Z} = \pm\frac{1}{2}\left(\frac{\tan\frac{1}{2}qZ}{\frac{1}{2}qZ} - 1\right).$$

The distance of the foci, in the same units, with respect to the median plane of the lens, is given by

$$\pm g/Z = \tfrac{1}{2} + 1/qZ\tan qZ.$$

and the focal length f of the system is given by

$$Z/f = qZ\sin qZ.$$

From these formulae a few general properties of the six-pole lens can be readily seen. 1) Particles are focused if $q^2 > 0$, i.e. in one spin state only. Those in the other spin state ($q^2 < 0$) are defocused. 2) For values about $qZ \simeq 2$ the focal length of a given lens passes through a rather flat minimum, but the positions of principal planes and foci vary significantly. 3) For values $qZ < \frac{1}{3}\pi$ the lens can be well approximated by a "thin" one, since the principal planes are located very close to the medium plane of the lens ($\Delta/Z < 0.1$).

To consider a numerical example take $B_0 \simeq 1$ Wb/m^2, $Z/r_0 = 200$, $T \simeq 300°$ K, $v/v_T \simeq \frac{1}{3}$ (the same applies to $v/v_T = 1$, $T \simeq 33°$ K). One finds $qZ \simeq 0.9$, yielding a practically thin lens with $Z/f \simeq 0.7$. Considering a magnet with a distance $2r_0 = 3$ cm between opposite pole tips, its length is $Z = 3$ m, and the foci are located at a distance $f \simeq 4.3$ m from its centre.

To obtain a highly polarized beam, the arrangement has to satisfy the following requirements. 1) It rejects all particles, irrespectively of their velocities, in one spin state. 2) All particles of a given velocity accepted by an entrance aperture can pass through an exit aperture, provided they are in the spin state for which the field is focusing. It cannot be avoided that the flux of particles even in the appropriate spin state is attenuated to some degree depending on the deviation of their velocity from the selected one.

These conditions are met if 1) the magnetic field produces a real image of the source, the exit aperture is situated in the image plane and its radius, w, is equal to or larger than the radius of the image $w \geqq Ms$ (s and M are the source radius and magnification respectively); 2) a ring aperture is used in the central plane of the lens and the geometrical shadow of the centre-stop completely covers the exit aperture.

Both these conditions are applied in the design of beta-ray lens spectrometers (e.g. Deutsch et al.[4]). A quantitative description of the performance of the present system is made cumbersome by two features usually avoided in charged particle spectrometers designed primarily for high momentum resolving power. (a) To obtain a practicable overall length in the present case, the parameter qZ is rather large and this makes the relationship between focal length and particle velocity somewhat complicated. (b) To increase the output flux density the centre-stop is chosen as small as compatible with condition (2) above, i.e. its radius is $r_i = (Ms + w)/(M + 1)$ and a very wide ring diaphragm is used. The condition under which calculations are greatly simplified is that $r_0 - r_i \gg r_0 + r_i$, where r_0 and r_i are the outside and inside radii of the ring respectively. In the present system approximations based on this condition are not applicable.

Suppose the magnetic field is set to produce an image of the source in the plane of the exit aperture if the neutrons have a velocity in the range v_0, $v_0 + \mathrm{d}v$, and the "luminosity" of the source in this range is $N(v_0)\mathrm{d}v$ neutrons/cm$^2\cdot$sec. Using an aperture diameter equal to the image diameter ($w = Ms$), one finds that the flux density of particles of velocity v_0 in the exit aperture is $\Phi(v_0) = \frac{1}{2}AN(v_0)$ with

$$A = \left(\frac{r_0}{f_0}\right)^2 \left(1 - \frac{f_0}{l_0}\right)^2 \left[1 - \left(\frac{r_i}{r_0}\right)^2\right],$$

where l_0 is the source distance ($l_0 \gg r_0$), and the factor $\frac{1}{2}$ represents the rejection of one of the spin states.

Taking numerical values as in the previous example, with $r_0/r_i = 3$, and using unit magnification to obtain minimum source-to-image distance, one finds $A \simeq 3 \times 10^{-6}$.

In order to determine the total flux density in the exit aperture one has to know the transmission of the system as a function of particle velocity $T(v)$. Subdividing the ring diaphragm into zones between radii R_k, $R_k + \delta R_k$, one can determine the flux density appearing in the exit apertures as the sum of the contributions of the single zones. For a ring diaphragm between radii R_k and $R_k + \delta R_k$ ($\delta R_k \ll R_k$), and $w = Ms$, $T(v)$ is given by an approximately triangular shaped curve with $T(v_0) = 1$ (i.e. full transmission at v_0); the velocity range transmitted depends on R_k. For the purpose of an order of magnitude estimate it is permissible to replace $T(v)$ by a square shaped transmission curve $t(v)$ such that $t(v) = 1$ if $T(v) \geqq \frac{1}{2}$ and $t(v) = 0$ if $T(v) < \frac{1}{2}$.

Denoting the total luminosity of the neutron source by $2N$ (neutrons/cm$^2\cdot$sec) and using the same numerical values of the relevant parameters as before, one finds that the totally polarized flux density in the exit aperture is $\Phi \simeq 4 \times 10^{-2}AN \simeq 10^{-7}N$. In the calculations spherical aberration was disregarded and it was assumed that the velocity distribution in the incident beam arises from a Maxwellian distribution when neutrons escape through a small aperture from a neutron gas at thermal equilibrium. Since the focused velocity v_0 is approximately $\frac{1}{3}v_T$ and $N(v)$ increases rapidly for $\frac{1}{3}v_T < v < v_T$, the approximation used by taking $t(v)$ rather than $T(v)$ as the transmission function gives an underestimate of the output flux. The high transmission is the consequence of a rather poor resolving power in velocity selection. It may be of interest to sacrifice transmission in order to improve the resolving power by reducing the source diameter and taking advantage of the formation of a ring focus.

It is rather difficult to compare the expected performance of the above system with that of any existing neutron beam filter and polarizer.

A comparison with a system where polarization and energy selection is obtained by diffraction on a ferromagnetic crystal is not appropriate, because the degree of monochromacy expected from the proposed system is necessarily much inferior. In a number of applications, however, a cruder energy selection is quite satisfactory, and then increased flux with high degree of polarization is valuable. Furthermore the performance of the proposed scheme is not limited towards low energies.

H. Maier-Leibnitz and Springer[5]) describe a bent copper tube used as neutron guide yielding an output flux density of about 6×10^5 n/cm$^2\cdot$sec for a thermal flux of 5×10^{12} n/cm$^2\cdot$sec close to the core end of the tube. This system does not polarize the beam at all. Møller, Passell and Stecher-Rasmussen[6]) describe a multi-channel collimator used as neutron beam filter and polarizer. From a neutron flux density of about 2.3×10^{13} n/cm$^2\cdot$sec measured at the interior end of the beam tube near the reactor core, an output beam is obtained with an estimated flux density of $3–5 \times 10^6$ n/cm$^2\cdot$sec. The overall polarization of the beam is 42 %.

The geometrical dimensions of the system proposed here, or rather the ratio Z/r_0, and/or magnetic flux density requirements can be considerably reduced, if the task is to carry out experiments with cold neutron beams.

Finally it should be remarked that the beam emerging from the system is polarized, but not aligned to any one direction defined in the laboratory. Alignment with respect to a direction in the laboratory can be achieved in the well known way if the neutrons, after leaving the six-pole field, pass through a homogeneous magnetic field. As the neutrons leave the six-pole field and move into the homogeneous field the spin orientation will change adiabatically, the neutrons conserving their spin states, provided that the change of the direction of the magnetic field as "seen" by the neutrons takes place slowly in comparison with the period of the Larmor precession.

The development of a six-pole magnet, based on the principles outlined above, is being considered at the Scottish Research Reactor Centre, East Kilbride.

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