Simplified Risk Aware CVaR-based POMDP With Performance Guarantees: a Risk Envelope Perspective Supplementary Material

Ido Nutov[1] and Vadim Indelman[2]

[1] Technion Autonomous Systems Program
[2] Department of Aerospace Engineering
Technion - Israel Institute of Technology, Haifa 32000, Israel
notov@campus.technion.ac.il, vadim.indelman@technion.ac.il

This document provides supplementary material to [1]. Therefore, it should not be considered a self-contained document, but instead regarded as an appendix of [1]. Throughout this report, all notations and definitions are with compliance to the ones presented in [1].

1 Proof Theorem 2

Denote by $\bar{\xi} \in \mathcal{U}_{cvar}(\alpha, \bar{\mathbb{P}}(z_{k+1}|b_k, a_k))$ and $\xi^* \in \mathcal{U}_{cvar}(\alpha, \mathbb{P}(z_{k+1}|b_k, a_k))$ the optimal risk ratios, considering the corresponding risk envelopes,

$$\overline{CVaR}_\alpha(\mathcal{H}(\hat{\bar{b}}_{k+1}) \mid \hat{b}_k, a_k) = \min_{\xi \in \mathcal{U}_{\mathrm{cvar}}(\alpha, \bar{\mathbb{P}}(z_{k+1}|\hat{b}_k, a_k))} \bar{\mathbb{E}}_{z_{k+1}}[\xi(z_{k+1})\mathcal{H}(\hat{\bar{b}}_{k+1})|\hat{b}_k, a_k] = \bar{\mathbb{E}}_{z_{k+1}}[\bar{\xi}(z_{k+1})\mathcal{H}(\hat{\bar{b}}_{k+1})|\hat{b}_k, a_k]$$

$$CVaR_\alpha(\mathcal{H}(\hat{b}_{k+1})|\hat{b}_k, a_k) = \min_{\xi \in \mathcal{U}_{\mathrm{cvar}}(\alpha, \mathbb{P}(z_{k+1}|\hat{b}_k, a_k))} \mathbb{E}_{z_{k+1}}[\xi(z_{k+1})\mathcal{H}(\hat{b}_{k+1})|\hat{b}_k, a_k] = \mathbb{E}_{z_{k+1}}[\xi^*(z_{k+1})\mathcal{H}(\hat{b}_{k+1})|\hat{b}_k, a_k].$$

Lower bound

We use the dual form and Lemma 2 to prove the lower bound,

$$\overline{CVaR}_\alpha(\mathcal{H}(\hat{\bar{b}}_{k+1}) \mid \hat{b}_k, a_k) - CVaR_\alpha(\mathcal{H}(\hat{b}_{k+1}) \mid \hat{b}_k, a_k) = \tag{1}$$

$$\bar{\mathbb{E}}_{z_{k+1}}(\bar{\xi}(z_{k+1})\mathcal{H}(\hat{\bar{\mathbb{P}}}(x_{k+1}|\hat{b}_k, a_k, z_{k+1})) - \mathop{\mathbb{E}}_{z_{k+1}}(\xi^*(z_{k+1})\mathcal{H}(\hat{\mathbb{P}}(x_{k+1}|\hat{b}_k, a_k, z_{k+1}))) \underbrace{\geq}_{\text{Lemma 2}} \tag{2}$$

$$\bar{\mathbb{E}}_{z_{k+1}}(\bar{\xi}(z_{k+1})\mathcal{H}(\hat{\bar{\mathbb{P}}}(x_{k+1}|\hat{b}_k, a_k, z_{k+1})) - \mathop{\mathbb{E}}_{z_{k+1}}(\bar{\xi}(z_{k+1})\mathcal{H}(\hat{\mathbb{P}}(x_{k+1}|\hat{b}_k, a_k, z_{k+1}))), \tag{3}$$

where we use $\hat{\mathbb{P}}(x_{k+1}|\hat{b}_k, a_k, z_{k+1})$ to explicitly denote the dependence of the particle belief $\hat{b}_{k+1}$ on $\hat{b}_k, a_k$ and z_{k+1} (and similarly for $\hat{\bar{\mathbb{P}}}(x_{k+1}|\hat{b}_k, a_k, z_{k+1})$ and $\hat{\bar{b}}_{k+1}$).

We now plug-in the entropy estimator ((34) in the main paper)

$$-\bar{\eta}_{k+1}\sum_{m=1}^{M}\bar{\xi}(z^m_{k+1})\sum_{i=1}^{N}\bar{Z}(z^m_{k+1}|x^i_{k+1})q^i_k\log(\frac{\bar{Z}(z^m_{k+1}|x^i_{k+1})\sum_{j=1}^{N}T(x^i_{k+1}|x^i_k,a_k)q^j_k}{\sum_{i'=1}^{N}\bar{Z}(z^m_{k+1}|x^{i'}_{k+1})q^{i'}_k})+ \tag{4}$$

$$\bar{\eta}_{k+1}\sum_{m=1}^{M}\bar{\xi}(z^m_{k+1})\sum_{i=1}^{N}Z(z^m_{k+1}|x^i_{k+1})q^i_k\log(\frac{Z(z^m_{k+1}|x^i_{k+1})\sum_{j=1}^{N}T(x^i_{k+1}|x^i_k,a_k)q^j_k}{\sum_{i'=1}^{N}Z(z^m_{k+1}|x^{i'}_{k+1})q^{i'}_k})=$$

$$-\bar{\eta}_{k+1}\sum_{c=1}^{C}\sum_{t=K\cdot(c-1)+1}^{K\cdot C}\bar{\xi}(z^t_{k+1})\sum_{i=1}^{N}Z(z^t_{k+1}|x^i_{k+1})q^i_k\cdot\log(\frac{\bar{Z}(z^t_{k+1}|x^i_{k+1})\sum_{i'=1}^{N}Z(z^t_{k+1}|x^{i'}_{k+1})q^{i'}_k}{Z(z^t_{k+1}|x^i_{k+1})\sum_{i'=1}^{N}\bar{Z}(z^t_{k+1}|x^{i'}_{k+1})q^{i'}_k}).$$

Using the inequality $\log(x)\leq x-1,\forall x>0$,

$$-\bar{\eta}_{k+1}\sum_{c=1}^{C}\sum_{t=K\cdot(c-1)+1}^{K\cdot C}\bar{\xi}(z^t_{k+1})\sum_{i=1}^{N}Z(z^t_{k+1}|x^i_{k+1})q^i_k\cdot\log(\frac{\bar{Z}(z^t_{k+1}|x^i_{k+1})\sum_{i'=1}^{N}Z(z^t_{k+1}|x^{i'}_t)q^{i'}_k}{Z(z^t_{k+1}|x^i_{k+1})\sum_{i'=1}^{N}\bar{Z}(z^t_{k+1}|x^{i'}_{k+1})q^{i'}_k})\geq \tag{5}$$

$$\bar{\eta}_{k+1}\sum_{c=1}^{C}\sum_{t=K\cdot(c-1)+1}^{K\cdot C}\bar{\xi}(z^t_{k+1})\sum_{i=1}^{N}Z(z^t_{k+1}|x^i_{k+1})q^i_k[1-\frac{\bar{Z}(z^t_{k+1}|x^i_{k+1})\sum_{i'=1}^{N}Z(z^t_{k+1}|x^{i'}_{k+1})q^{i'}_k}{Z(z^t_{k+1}|x^i_{k+1})\sum_{i'=1}^{N}\bar{Z}(z^t_{k+1}|x^{i'}_{k+1})q^{i'}_k}]=$$

$$\bar{\eta}_{k+1}\sum_{c=1}^{C}\sum_{t=K\cdot(c-1)+1}^{K\cdot C}\bar{\xi}(z^t_{k+1})\sum_{i=1}^{N}Z(z^t_{k+1}|x^i_{k+1})q^i_k-\bar{\eta}_{k+1}\sum_{c=1}^{C}\sum_{t=K\cdot(c-1)+1}^{K\cdot C}\bar{\xi}(z^t_{k+1})\sum_{i=1}^{N}Z(z^t_{k+1}|x^i_{k+1})q^i_k=0.$$

Upper bound

We now prove the upper bound:

$$\overline{CVaR}_\alpha(\mathcal{H}(\hat{\bar{b}}_{k+1})\mid\hat{b}_k,a_k)-CVaR_\alpha(\mathcal{H}(\hat{b}_{k+1})\mid\hat{b}_k,a_k)= \tag{6}$$

$$\overline{\mathbb{E}}_{z_{k+1}}(\bar{\xi}(z_{k+1})\mathcal{H}(\hat{\bar{\mathbb{P}}}(x_{k+1}|\hat{b}_k,a_k,z_{k+1}))-\mathop{\mathbb{E}}_{z_{k+1}}(\xi^*(z_{k+1})\mathcal{H}(\hat{\mathbb{P}}(x_{k+1}|\hat{b}_k,a_k,z_{k+1})))= \tag{7}$$

$$-\bar{\eta}_{k+1}\sum_{m=1}^{M}\bar{\xi}(z^m_{k+1})\sum_{i=1}^{N}\bar{Z}(z^m_{k+1}|x^i_{k+1})q^i_k\log(\frac{\bar{Z}(z^m_{k+1}|x^i_{k+1})\sum_{j=1}^{N}T(x^i_{k+1}|x^i_k,a_k)q^j_k}{\sum_{i'=1}^{N}\bar{Z}(z^m_{k+1}|x^{i'}_{k+1})q^{i'}_k})+ \tag{8}$$

$$\bar{\eta}_{k+1}\sum_{m=1}^{M}\xi^*(z^m_{k+1})\sum_{i=1}^{N}Z(z^m_{k+1}|x^i_{k+1})q^i_k\log(\frac{Z(z^m_{k+1}|x^i_{k+1})\sum_{j=1}^{N}T(x^i_{k+1}|x^i_k,a_k)q^j_k}{\sum_{i'=1}^{N}Z(z^m_{k+1}|x^{i'}_{k+1})q^{i'}_k}). \tag{9}$$

We select a general element g from the risk envelope $\mathcal{U}_{\text{cvar}}(\alpha,\bar{\mathbb{P}}(z^m_{k+1}|\hat{\bar{b}}_k,a_k))$. Since a risk measure involves solving a minimization problem over this envelope, considering a general element within the risk envelope makes the entire expression larger. Additionally, we define $h(m)=[\frac{m}{K}]+1$ for a general integer m. We define g as follows and then prove it indeed belongs to $\mathcal{U}_{\text{cvar}}(\alpha,\bar{\mathbb{P}}(z^m_{k+1}|\hat{\bar{b}}_k,a_k))$:

$$g(z^m_{k+1})\triangleq\frac{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)}\xi^*(z^t_{k+1})\mathbb{P}(z^t_{k+1}|\hat{b}_k,a_k)}{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)}\mathbb{P}(z^t_{k+1}|\hat{b}_k,a_k)}. \tag{10}$$

We now prove that $g(z^m_{k+1}) \in \mathcal{U}_{\text{cvar}}(\alpha, \bar{\mathbb{P}}(z^m_{k+1}|\hat{\bar{b}}_k, a_k))$:

$$\sum_{m=1}^{M} g(z^m_{k+1})\bar{\mathbb{P}}(z^m_{k+1}|\hat{\bar{b}}_k, a_k) = \sum_{m=1}^{M} \frac{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \xi^*(z^t_{k+1})\mathbb{P}(z^t_{k+1}|\hat{b}_k, a_k)}{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \mathbb{P}(z^t_{k+1}|\hat{b}_k, a_k)} \bar{\mathbb{P}}(z^m_{k+1}|\hat{\bar{b}}_k, a_k) = \tag{11}$$

$$\sum_{m=1}^{M} \frac{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \xi^*(z^t_{k+1})\mathbb{P}(z^t_{k+1}|\hat{b}_k, a_k)}{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \sum_{i=1}^{N} Z(z^t_{k+1}|x^i_{k+1}) \sum_{j=1}^{N} T(x^i_{k+1}|x^j_k, a_k)q^j_k} \sum_{i=1}^{N} \bar{Z}(z^m_{k+1}|x^i_{k+1}) \sum_{j=1}^{N} T(x^i_{k+1}|x^j_k, a_k)q^j_k =$$

$$\sum_{m=1}^{M} \frac{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \xi^*(z^t_{k+1})\mathbb{P}(z^t_{k+1}|\hat{b}_k, a_k)}{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \sum_{i=1}^{N} Z(z^t_{k+1}|x^i_{k+1}) \sum_{j=1}^{N} T(x^i_{k+1}|x^j_k, a_k)q^j_k} \cdot$$

$$\frac{1}{K} \sum_{i=1}^{N} \sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} Z(z^t_{k+1}|x^i_{k+1}) \sum_{j=1}^{N} T(x^i_{k+1}|x^j_k, a_k)q^j_k =$$

$$\sum_{m=1}^{M} \frac{1}{K} \sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \xi^*(z^t_{k+1})\mathbb{P}(z^t_{k+1}|\hat{b}_k, a_k) = 1,$$

and

$$g(z^m_{k+1}) = \frac{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \xi^*(z^t_{k+1})\mathbb{P}(z^t_{k+1}|\hat{b}_k, a_k)}{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \mathbb{P}(z^t_{k+1}|\hat{b}_k, a_k)} \leq \frac{1}{\alpha} \frac{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \mathbb{P}(z^t_{k+1}|\hat{b}_k, a_k)}{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \mathbb{P}(z^t_{k+1}|\hat{b}_k, a_k)} = \frac{1}{\alpha}. \tag{12}$$

To conclude, we showed that $\sum_{m=1}^{M} g(z^m_{k+1})\bar{\mathbb{P}}(z^m_{k+1}|\hat{\bar{b}}_k, a_k) = 1$ and $g(z^m_{k+1}) \leq \frac{1}{\alpha}$. Therefore, according to (11), $g(z^m_{k+1}) \in \mathcal{U}_{\text{cvar}}(\alpha, \bar{\mathbb{P}}(z^m_{k+1}|\hat{\bar{b}}_k, a_k))$.

Referring to (6), we now take notice that $\log(\frac{Z(z^m_{k+1}|x^i_{k+1}) \sum_{j=1}^{N} T(x^i_{k+1}|x^i_k, a_k)q^j_k}{\sum_{i'=1}^{N} Z(z^m_{k+1}|x^{i'}_{k+1})q^{i'}_k}) < 0$, as the log of a discrete probability distribution. By replacing the optimal risk ratio $\bar{\xi}(z^m_{k+1}) \in \mathcal{U}_{\text{cvar}}(\alpha, \bar{\mathbb{P}}(z^m_{k+1}|\hat{\bar{b}}_k, a_k))$ with the risk ratio $g(z^m_{k+1}) \in \mathcal{U}_{\text{cvar}}(\alpha, \bar{\mathbb{P}}(z^m_{k+1}|\hat{\bar{b}}_k, a_k))$, in (6), we get the bound

$$\overline{CVaR}_\alpha(\mathcal{H}(\hat{\bar{b}}_{k+1}) \mid \hat{b}_k, a_k) - CVaR_\alpha(\mathcal{H}(\hat{b}_{k+1}) \mid \hat{b}_k, a_k) \leq \tag{13}$$

$$-\bar{\eta}_{k+1} \sum_{m=1}^{M} g(z^m_{k+1}) \sum_{i=1}^{N} \bar{Z}(z^m_{k+1}|x^i_{k+1})q^i_k \log(\frac{\bar{Z}(z^m_{k+1}|x^i_{k+1}) \sum_{j=1}^{N} T(x^i_{k+1}|x^i_k, a_k)q^j_k}{\sum_{i'=1}^{N} \bar{Z}(z^m_{k+1}|x^{i'}_{k+1})q^{i'}_k}) + \tag{14}$$

$$\bar{\eta}_{k+1} \sum_{m=1}^{M} \xi^*(z^m_{k+1}) \sum_{i=1}^{N} Z(z^m_{k+1}|x^i_{k+1})q^i_k \log(\frac{Z(z^m_{k+1}|x^i_{k+1}) \sum_{j=1}^{N} T(x^i_{k+1}|x^i_k, a_k)q^j_k}{\sum_{i'=1}^{N} Z(z^m_{k+1}|x^{i'}_{k+1})q^{i'}_k}). \tag{15}$$

We now look at one of the clusters (without loss of generality, at the first one, i.e. $m = 1$), plug-in the definition of the abstraction observation model (29), and define a matrix notation of the cluster,

$$g(z^{m=1}_{k+1}) \sum_{i=1}^{N} \bar{Z}(z^t_{k+1}|x^i_{k+1})q^i_k = \frac{\sum_{t=1}^{K} \xi^*(z^t_{k+1})\mathbb{P}(z^t_{k+1}|\hat{b}_k, a_k)}{\sum_{t=1}^{K} \mathbb{P}(z^t_{k+1}|\hat{b}_k, a_k)} \sum_{i=1}^{N} \bar{Z}(z^t_{k+1}|x^i_{k+1})q^i_k \triangleq \frac{1}{K} \frac{\xi^T P \mathbf{1}^T Z_q}{\mathbf{1}^T P}, \tag{16}$$

where

$$\xi \triangleq \begin{bmatrix} \xi^*(z^1_{k+1}) \\ \cdot \\ \cdot \\ \xi^*(z^K_{k+1}) \end{bmatrix}, Z_q \triangleq \begin{bmatrix} \sum_{i=1}^N Z(z^1_{k+1}|x^i_{k+1})q^i_k \\ \cdot \\ \cdot \\ \sum_{i=1}^N Z(z^K_{k+1}|x^i_{k+1})q^i_k \end{bmatrix}, P \triangleq \begin{bmatrix} \mathbb{P}(z^1_{k+1}|\hat{b}_k, a_k) \\ \cdot \\ \cdot \\ \mathbb{P}(z^K_{k+1}|\hat{b}_k, a_k) \end{bmatrix}, \mathbf{1} \triangleq \begin{bmatrix} 1 \\ \cdot \\ \cdot \\ 1 \end{bmatrix}. \tag{17}$$

We apply

$$\frac{1}{K}\frac{\xi^T P \mathbf{1}^T Z_q}{\mathbf{1}^T P} = \frac{1}{K}\frac{tr(\xi^T P \mathbf{1}^T Z_q)}{tr(\mathbf{1}^T P)} \leq \frac{1}{K}\frac{tr(P\mathbf{1}^T)tr(Z_q \xi^T)}{tr(\mathbf{1}^T P)} = \frac{1}{K}tr(Z_q\xi^T) = \xi^T Z_q. \tag{18}$$

Hence, we re-write (14) as

$$-\bar{\eta}_{k+1}\sum_{m=1}^{M}\xi^*(z^m_{k+1})\sum_{i=1}^{N}Z(z^m_{k+1}|x^i_{k+1})q^i_k \log(\frac{\bar{Z}(z^m_{k+1}|x^i_{k+1})\sum_{j=1}^N T(x^i_{k+1}|x^i_k, a_k)q^j_k}{\sum_{i'=1}^N \bar{Z}(z^m_{k+1}|x^{i'}_{k+1})q^{i'}_k})+ \tag{19}$$

$$\bar{\eta}_{k+1}\sum_{m=1}^{M}\xi^*(z^m_{k+1})\sum_{i=1}^{N}Z(z^m_{k+1}|x^i_{k+1})q^i_k \log(\frac{Z(z^m_{k+1}|x^i_{k+1})\sum_{j=1}^N T(x^i_{k+1}|x^i_k, a_k)q^j_k}{\sum_{i'=1}^N Z(z^m_{k+1}|x^{i'}_{k+1})q^{i'}_k}) =$$

$$\underbrace{\bar{\eta}_{k+1}\sum_{m=1}^{M}\xi^*(z^m_{k+1})\sum_{i=1}^{N}Z(z^m_{k+1}|x^i_{k+1})q^i_k \log(\frac{Z(z^m_{k+1}|x^i_{k+1})}{\bar{Z}(z^m_{k+1}|x^i_{k+1})})}_{(a)}+$$

$$\underbrace{\bar{\eta}_{k+1}\sum_{m=1}^{M}\xi^*(z^m_{k+1})\sum_{i=1}^{N}Z(z^m_{k+1}|x^i_{k+1})q^i_k \log(\frac{\sum_{i=1}^N Z(z^m_{k+1}|x^i_{k+1})q^i_k}{\sum_{i=1}^N \bar{Z}(z^m_{k+1}|x^i_{k+1})q^i_k})}_{(b)}.$$

We now treat the terms (a) and (b) separately, starting with (a):

$$(a) = \bar{\eta}_{k+1}\sum_{m=1}^{M}\xi^*(z^m_{k+1})\sum_{i=1}^{N}Z(z^m_{k+1}|x^i_{k+1})q^i_k \log(K) + \bar{\eta}_{k+1}\sum_{m=1}^{M}\xi^*(z^m_{k+1})\sum_{i=1}^{N}Z(z^m_{k+1}|x^i_{k+1})q^i_k \log(\frac{Z(z^m_{k+1}|x^i_{k+1})}{\sum Z(z^m_{k+1}|x^i_{k+1})})$$

$$\leq \bar{\eta}_{k+1}\sum_{m=1}^{M}\xi^*(z^m_{k+1})\sum_{i=1}^{N}Z(z^m_{k+1}|x^i_{k+1})q^i_k \log(K) = \log(K).$$

For term (b), we utilize the Jensen's inequality,

$$(b) = \bar{\eta}_{k+1}\sum_{m=1}^{M}\xi^*(z^m_{k+1})\sum_{i=1}^{N}Z(z^m_{k+1}|x^i_{k+1})q^i_k \log(\frac{\sum_{i=1}^N \bar{Z}(z^m_{k+1}|x^i_{k+1})q^i_k}{\sum_{i=1}^N Z(z^m_{k+1}|x^i_{k+1})q^i_k})$$

$$\leq \log(\bar{\eta}_{k+1}\sum_{m=1}^{M}\xi^*(z^m_{k+1})\sum_{i=1}^{N}Z(z^m_{k+1}|x^i_{k+1})q^i_k(\frac{\sum_{i=1}^N \bar{Z}(z^m_{k+1}|x^i_{k+1})q^i_k}{\sum_{i=1}^N Z(z^m_{k+1}|x^i_{k+1})q^i_k})) = \log(\bar{\eta}_{k+1}\sum_{m=1}^{M}\xi^*(z^m_{k+1})\sum_{i=1}^{N}\bar{Z}(z^m_{k+1}|x^i_{k+1})q$$

We again look at one of the clusters in matrix notations (without loss of generality, at the first),

$$\frac{1}{K}tr((\mathbf{1}^T\xi)(\mathbf{z}^T\mathbf{1})) \leq \frac{1}{K}tr(\xi\mathbf{z}^T)tr(\mathbf{1}\mathbf{1}^T) = tr(\xi\mathbf{z}^T). \tag{20}$$

Hence, for the term (b) we get,

$$\log(\bar{\eta}_{k+1} \sum_{m=1}^{M} \xi^*(z_{k+1}^m) \sum_{i=1}^{N} \bar{Z}(z_{k+1}^m|x_{k+1}^i) q_k^i) \leq \log(\bar{\eta}_{k+1} \sum_{m=1}^{M} \xi^*(z_{k+1}^m) \sum_{i=1}^{N} Z(z_{k+1}^m|x_{k+1}^i) q_k^i) = 0. \tag{21}$$

Therefore, $0 \leq \overline{CVaR}_\alpha(\mathcal{H}(\hat{\bar{b}}_{k+1}) \mid \hat{b}_k, a_k) - CVaR_\alpha(\mathcal{H}(\hat{b}_{k+1}) \mid \hat{b}_k, a_k) \leq \log(K)$.

References

1. I. Nutov and V. Indelman. Simplified risk aware cvar-based pomdp with performance guarantees: a risk envelope perspective. In *Proc. of the Intl. Symp. of Robotics Research (ISRR)*, 2024. Submitted.