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Inverted Pendulum

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ABSTRACT TODO:Add abstract

INDEX TERMS TODO:Add keywords

I. INTRODUCTION

THE

A. EQUATION SYSTEM

$$(M+m)\ddot{x} + b\dot{x} + ml\theta\cos\theta - ml\dot{\theta}^2 \sin theta = F \quad (1)$$

$$(I+ml^2)\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta \quad (2)$$

isolating the variable $\ddot{x}$ in 1 we get,

$$\ddot{x} = \frac{F - b\dot{x} - ml\ddot{\theta}\cos\theta + ml\dot{\theta}^2\sin\theta}{M+m}$$

substituting in 2 and isolating $\ddot{\theta}$ we get

$$\ddot{\theta} = \frac{-ml(F - b\dot{x} + ml\dot{\theta}^2\sin\theta) - mgl(M+m)\sin\theta}{I(M+m) + Mml^2 + m^2l^2\sin^2\theta} \quad (3)$$

Similarly, isolating $\ddot{\theta}$ in 2 we get,

$$\ddot{\theta} = \frac{-ml(\ddot{x}\cos\theta + g\sin\theta)}{I+ml^2}$$

substituting and isolating $\ddot{x}$, we get

$$\ddot{x} = \frac{(I+ml^2)(F - b\dot{x} + ml\dot{\theta}^2\sin\theta) + m^2gl^2\sin\theta\cos\theta}{I(M+m) + Mml^2 + m^2l^2\sin^2\theta} \quad (4)$$

B. NON-LINEAR STATE SPACE MODEL

1) state space variables

$x_1 = x$	cart position
$x_2 = \theta$	pendulum angle
$x_3 = \dot{x}$	cart velocity
$x_4 = \dot{\theta}$	angular velocity
$u = F$	input force

C. STATE SPACE EQUATION SYSTEM

$$\begin{aligned}
\dot{x_1} &= x_3\\
\dot{x_2} &= x_4\\
\dot{x_3} &= \frac{(I+ml^2)(F - bx_3 + mlx_4^2\sin x_2) + m^2gl^2\sin x_2\cos x_2}{I(M+m) + Mml^2 + m^2l^2\sin^2 x_2}\\
\dot{x_4} &= \frac{-ml(F - bx_3 + mlx_4^2\sin x_2) - mgl(M+m)\sin x_2}{I(M+m) + Mml^2 + m^2l^2\sin^2 x_2}
\end{aligned} \quad (5)$$

D. LINEARIZATION

When $x_2 \to 0$,

- $\sin x_2 \to x_2$
- $\cos x_2 \to 1$
- $\sin^2 x_2 \to x_2^2 \approx 0$
- $x_4^2 \approx 0$

therefore, we can express 5 when $\theta \to 0$ as,

$$\begin{aligned}
\dot{x_1} &= x_3\\
\dot{x_2} &= x_4\\
\dot{x_3} &= \frac{(I+ml^2)(F - bx_3) + m^2gl^2x_2}{I(M+m) + Mml^2}\\
\dot{x_4} &= \frac{-ml(F - bx_3) - mgl(M+m)x_2}{I(M+m)}
\end{aligned} \quad (6)$$

or in matrix form

$$\begin{aligned}
\dot{x} &= Ax + Bu\\
y &= Cx + Du
\end{aligned} \quad (7)$$

where

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \in \mathbb{R}^{4\times 1}$$

$$u = F \in \mathbb{R}$$

$$y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in \mathbb{R}^{2\times 1}$$

$$A = \frac{1}{I(M+m)+Mml^2} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & m^2gl^2 & -(I+ml^2)b & 0 \\ 0 & -mgl(M+m) & mlb & 0 \end{pmatrix} \in \mathbb{R}^{4\times 4}$$

$$B = \frac{1}{I(M+m)+Mml^2} \begin{pmatrix} 0 \\ 0 \\ I+ml^2 \\ -ml \end{pmatrix} \in \mathbb{R}^{4\times 1}$$

$C =$ Andrea!!! we need to define our outputs $\in \mathbb{R}^{4\times 2}$

$D = 0 \in \mathbb{R}^{2\times 1}$