ProgTeam Spring Week 12

Fast Fourier Transform

Fast Fourier Transform: Convolutions

- "Convolutional" sum: multiplying two polynomials

$$(4 + 2x + x^2)(3 + 2x + 5x^2)$$
$$= (12 + 14x + 27x^2 + 12x^3 + 5x^4)$$

PDF Viewers: ^This was convolution.gif

Example With Probabilities

You buy two boxes of chocolates, without knowing how many is in either one. However, you know the distribution of the amount of chocolates in either box. What is the distribution of the final amount of chocolates:

	0	1	2	3	4
Box A = {	.00,	.20,	.25,	.40,	.15}
Box B = {	.05,	.15,	.35,	.35,	.10}

The answer to the final distribution will be the *convolution* of these two probabilities

P(Total = 3) = P(Box A = 0) * P(Box B = 3) + P(Box A = 1) * P(Box B = 2)....

FFT: Solving Convolutions in N log N

- The very short version: using divide and conquer and complex numbers, we can compute the Discrete Fourier Transform, or compute the original function from a DFT
- Conveniently, for polynomials A and B, DFT(A * B) = DFT(A) + DFT(B)
- We can compute the DFT in N log N time, so we can compute the convolution in N log N time