AA274 (Winter 2017-18): Problem Set 2

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1 Camera Calibration

(i) See submitted code. We are given a 7×9 chessboard grid with each square's side length of $d_{square} = 20.5$ mm. The axes are arranged in traditional fashion with the origin at the bottom left. To generate the world coordinates, we stack the rows of

$$G_X = \begin{pmatrix} 0 & d_{square} & \dots & 8d_{square} \\ 0 & d_{square} & \dots & 8d_{square} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & d_{square} & \dots & 8d_{square} \end{pmatrix} \quad \text{and} \quad G_Y = \begin{pmatrix} 6d_{square} & 6d_{square} & \dots & 6d_{square} \\ 5d_{square} & 5d_{square} & \dots & 5d_{square} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix},$$

producing $(X_k, Y_k) \in \mathbb{Z}_+^{2 \times 63}$ for each of the $k = 1, \dots, 23$ chessboards.

(ii) See submitted code. For a given chessboard i, form the row vector $\tilde{M} = (X_k, Y_k, \mathbf{1})^T \in \mathbb{Z}_+^{189}$ by stacking the world coordinates and a 63-length vector of ones. Define

$$L = \begin{pmatrix} \tilde{M}^T & \mathbf{0}^T & -u_{meas}\tilde{M}^T \\ \mathbf{0}^T & \tilde{M}^T & -v_{meas}\tilde{M}^T \end{pmatrix} \in \mathbf{R}^{378 \times 9}$$

and solve $Lx = 0$ by taking the SVD of L and setting x^* equal to the right singular vector associated with the smallest singular value. Then, $x^* = (h_1, h_2, h_3) \in \mathbf{R}^9$ is the stacked columns of our homography matrix H_k for chessboard k.

(iii) See submitted code. Let the i-th column vector of H_k be $h_i = (h_{i1}, h_{i2}, h_{i3})$, and define

$$v_{ij} = \begin{pmatrix} h_{i1}h_{j1} \\ h_{i1}h_{j2} + h_{i2}h_{j1} \\ h_{i2}h_{j2} \\ h_{i3}h_{j1} + h_{i1}h_{j3} \\ h_{i3}h_{j2} + h_{i2}h_{j3} \\ h_{i3}h_{j3} \end{pmatrix} \quad \text{and} \quad V_k = \begin{pmatrix} v_{12}^T \\ (v_{11} - v_{22})^T \end{pmatrix}$$

for a particular chessboard k. Form $V \in \mathbb{R}^{378\times 6}$ by stacking the V_k matrices. We can solve $Vb = 0$ by applying the SVD as in the previous part. Given the solution $b^* = (B_{11}, B_{12}, B_{22}, B_{13}, B_{23}, B_{33})$, we can compute

$$\begin{aligned} v_0 &= (B_{12}B_{13} - B_{11}B_{23})/(B_{11}B_{22} - B_{12}^2) \\ \lambda &= B_{33} - (B_{13}^2 + v_0(B_{12}B_{13} - B_{11}B_{23}))/B_{11} \\ \alpha &= \sqrt{\lambda/B_{11}} \\ \beta &= \sqrt{\lambda B_{11}/(B_{11}B_{22} - B_{12}^2)} \\ \gamma &= -B_{12}\alpha^2\beta/\lambda \\ u_0 &= \gamma v_0/\beta - B_{13}\alpha^2/\lambda \end{aligned}$$

and form the camera intrinsic matrix

$$A = \begin{pmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

(iv) See submitted code. Given A, we compute for each chessboard

$$q_1 = \lambda A^{-1}h_1, \quad q_2 = \lambda A^{-1}h_2 \quad q_3 = q_1 \times q_2$$

where h_j is column j of H_k, and our normalizing factor is $\lambda = 1/\|A^{-1}h_1\|$. Then, we form the matrix $Q = (q_1, q_2, q_3)$ by concatenating the column vectors and take its SVD to get $Q = USV^T$. The desired rotation matrix is $R = UV^T$ with translation $t = \lambda A^{-1}h_3$.

(v) See submitted code. Assuming $f = 1$, the desired transformation comes from the lecture notes:

$$\begin{pmatrix} X_C \\ Y_C \\ Z_C \end{pmatrix} = \begin{pmatrix} R & t \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ \mathbf{1} \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} X_C/Z_C \\ Y_C/Z_C \end{pmatrix}$$

$$\begin{pmatrix} x_h \\ y_h \\ z_h \end{pmatrix} = A \begin{pmatrix} R & t \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ \mathbf{1} \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} x_h/z_h \\ y_h/z_h \end{pmatrix}$$

(vi) See submitted code. Given $k = (0.15, 0.01)$, we can solve for

$$\begin{aligned} \breve{x} &= x + x(k_1(x^2 + y^2) + k_2(x^2 + y^2)^2) \\ \breve{y} &= y + y(k_1(x^2 + y^2) + k_2(x^2 + y^2)^2) \\ \breve{u} &= u_0 + \alpha\breve{x} + \gamma\breve{y} \\ \breve{v} &= v_0 + \beta\breve{y} \end{aligned}$$

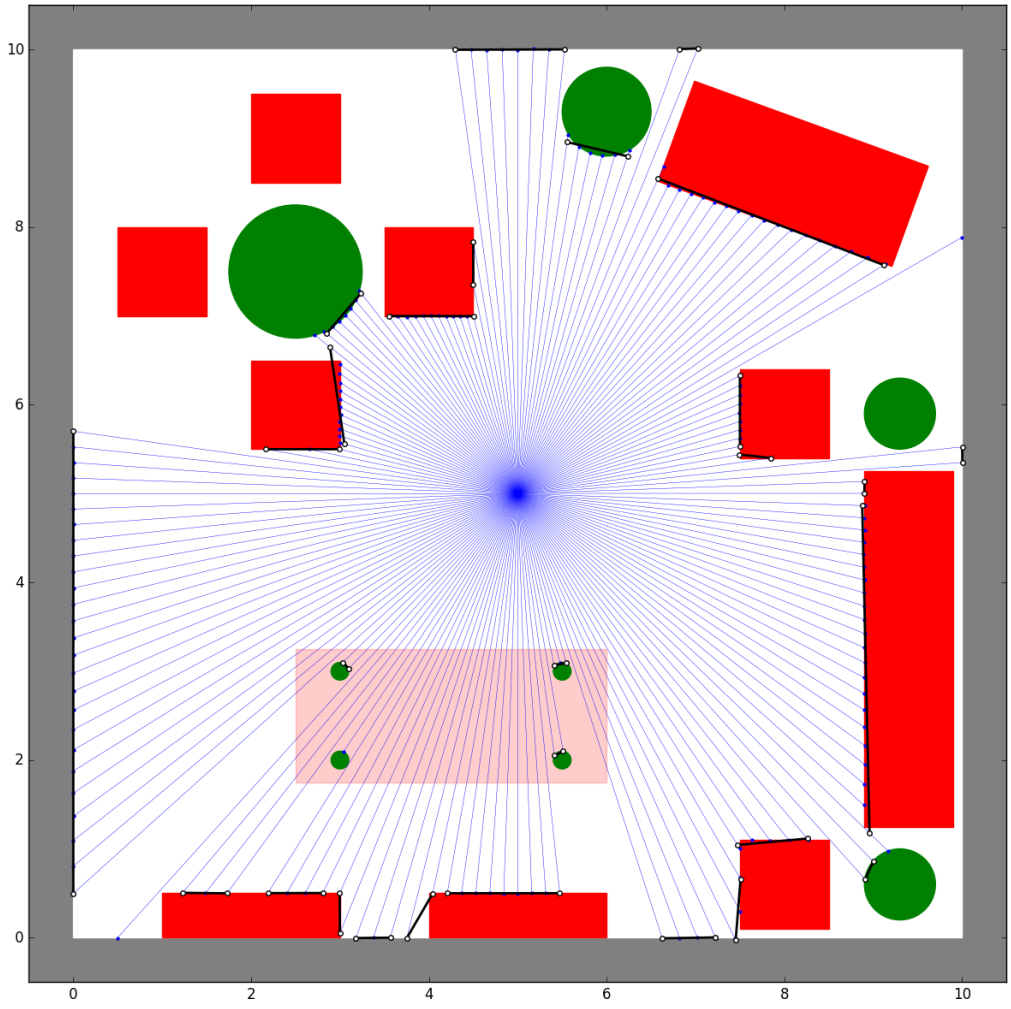
2 Line Fitting

(i) See submitted code.

(ii) Using the suggested algorithm, we extracted the following lines for each data set.

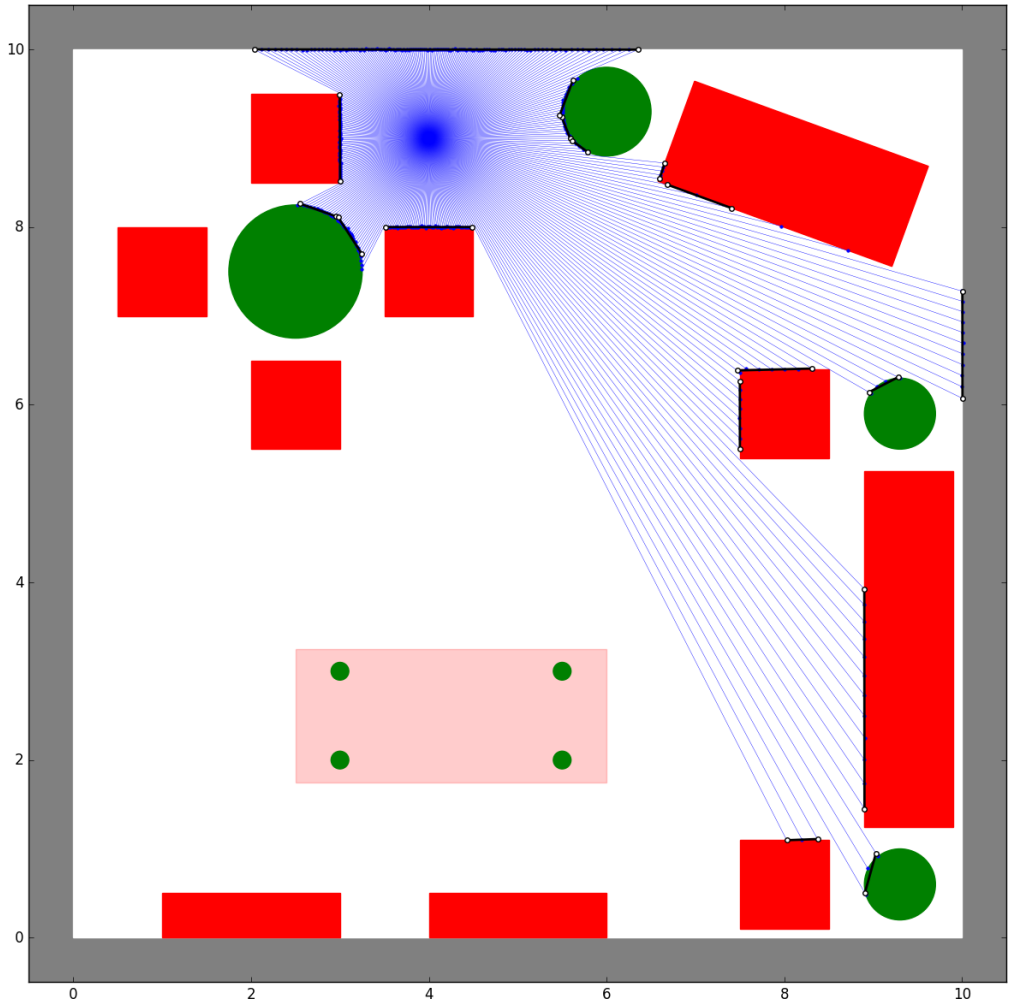
Range Data $(x_r, y_r, n_{pts}) = (5, 5, 180)$

```
LINE_POINT_DIST_THRESHOLD = 0.05
MIN_POINTS_PER_SEGMENT = 0.065
MIN_SEG_LENGTH = 2
MAX_P2P_DIST = 0.8
```



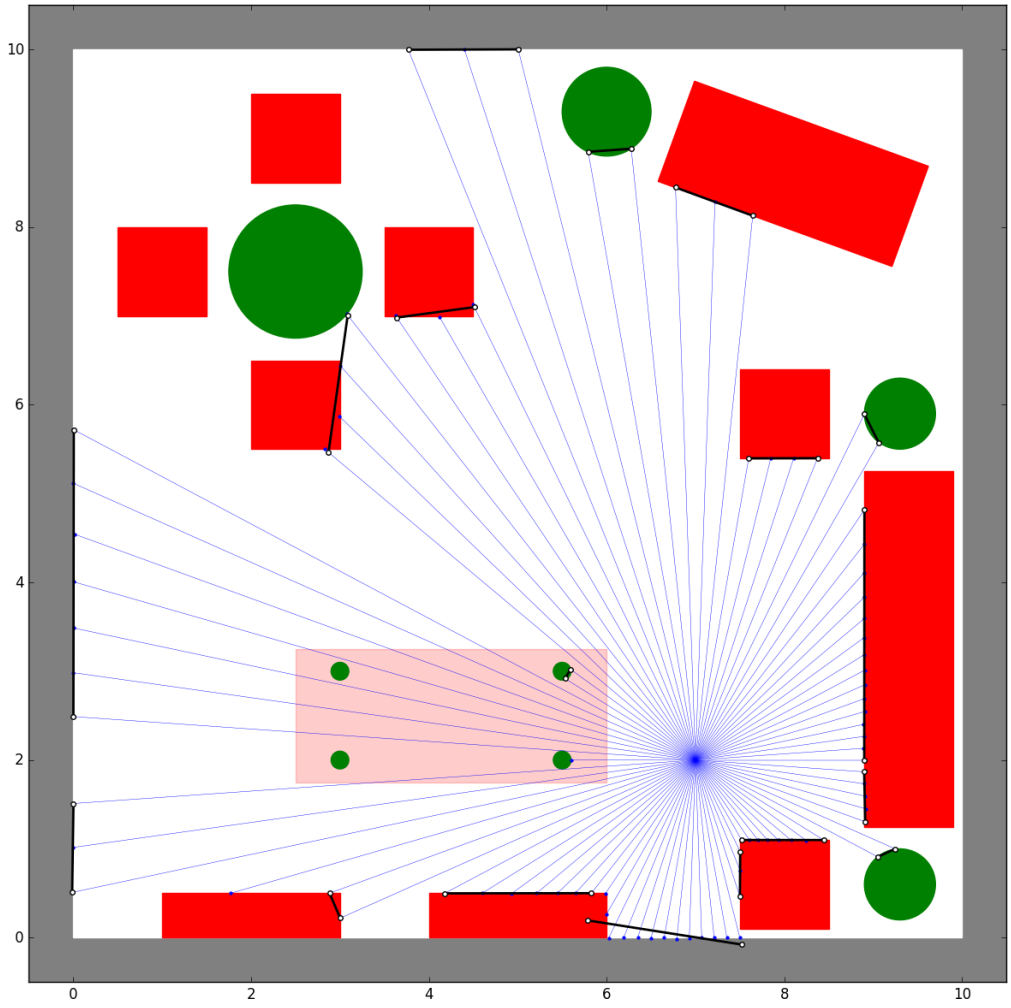
Range Data $(x_r, y_r, n_{pts}) = (4, 9, 360)$

```
LINE_POINT_DIST_THRESHOLD = 0.05
MIN_POINTS_PER_SEGMENT = 0.025
MIN_SEG_LENGTH = 3
MAX_P2P_DIST = 0.54
```



Range Data $(x_r, y_r, n_{pts}) = (7, 2, 90)$

```
LINE_POINT_DIST_THRESHOLD = 0.05
MIN_POINTS_PER_SEGMENT = 0.165
MIN_SEG_LENGTH = 2
MAX_P2P_DIST = 0.42
```



3 Tensorflow and HOG + SVM Pedestrian Detection

(i) Let $p, q \in \mathbb{R}^2$ be two points on the parallel hyperplanes, i.e. $w \cdot p - b = 1$ and $w \cdot q - b = -1$. The perpendicular distance between the planes is the projection of the vector $(p - q)$ onto the unit normal to the plane, $\frac{w}{\|w\|}$, which is simply

$$(p - q) \cdot \frac{w}{\|w\|} = \frac{w \cdot p - w \cdot q}{\|w\|} = \frac{(b + 1) - (b - 1)}{\|w\|} = \frac{2}{\|w\|}$$

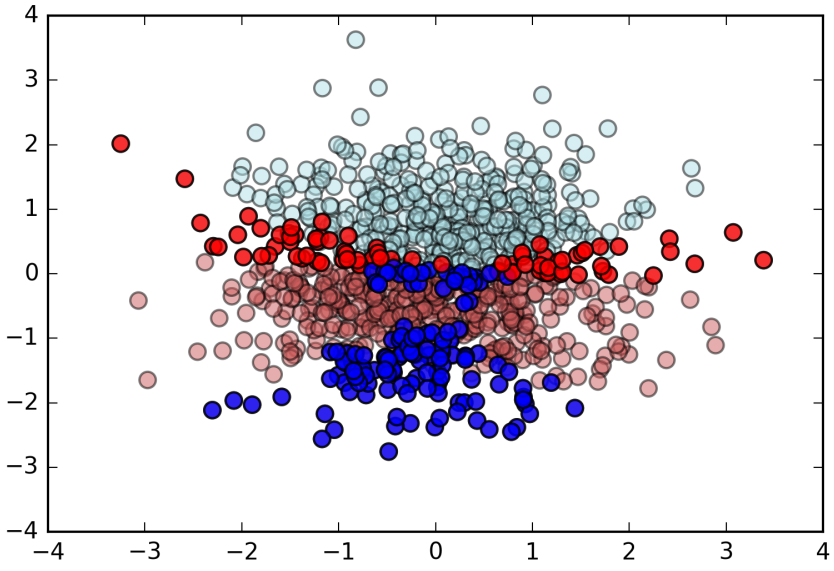
(ii) TensorFlow's workflow was created for modeling and executing distributed computati-

ons. The computation graph allows the user to see which mathematical operations will occur in parallel, making it easier to design and debug algorithms. Moreover, the actual data analysis is separated into another step to facilitate execution across multiple, distributed machines. Problems with the algorithm can thus be isolated from problems with the devices that carry out the computation, such as a breakdown in communication. This workflow differs from numpy, which immediately executes operations in the order they are entered. TensorFlow's paradigm makes sense for machine learning, because in this space, algorithms are complex and datasets are generally very large, requiring significant runtime on multiple machines. To avoid wasting resources, users will first test their computation graph locally before starting a cluster and feeding it the input data.

(iii) See submitted code.

(iv) With a learning rate of $\gamma = 0.1$ and $\lambda = 0.4$, I achieved a misclassification rate of 0.214 on the test dataset. This is rather high because the classes cannot be separated by a single hyperplane. The red class has curved boundaries on both top and bottom with the blue class. Thus, no matter where we draw the hyperplane, a large proportion of the classes will be misclassified. This model has placed it along the top boundary, so that all the blue points below it are labeled as red.

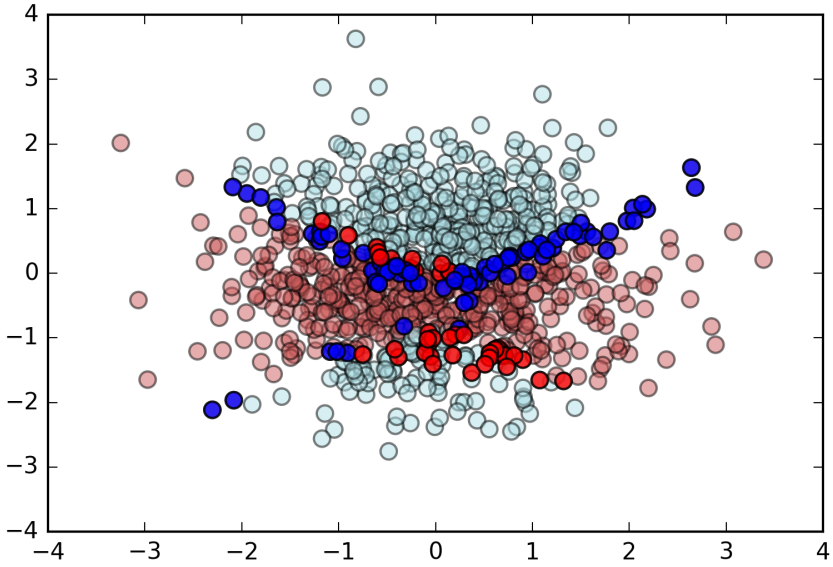
Misclassified Data with Original Features
$(\gamma = 0.1,\ \lambda = 0.4)$



(v) See submitted code.

(vi) I selected the features $(x_1, x_2, x_1^2, x_2^2, x_1x_2)$ because the boundaries between the two classes look like parabolas, so I expected there to be a quadratic dependency in the data. With a learning rate of $\gamma = 0.1$ and $\lambda = 1.1$, I achieved a misclassification rate of 0.120 on the test dataset.

Misclassified Data with Custom Features
$(\gamma = 0.1,\ \lambda = 1.1)$



(vii) See submitted code.

(viii) With a learning rate of $\gamma = 0.075$ and $\lambda = 0.825$, I achieved a misclassification rate of 0.130 on the test dataset.

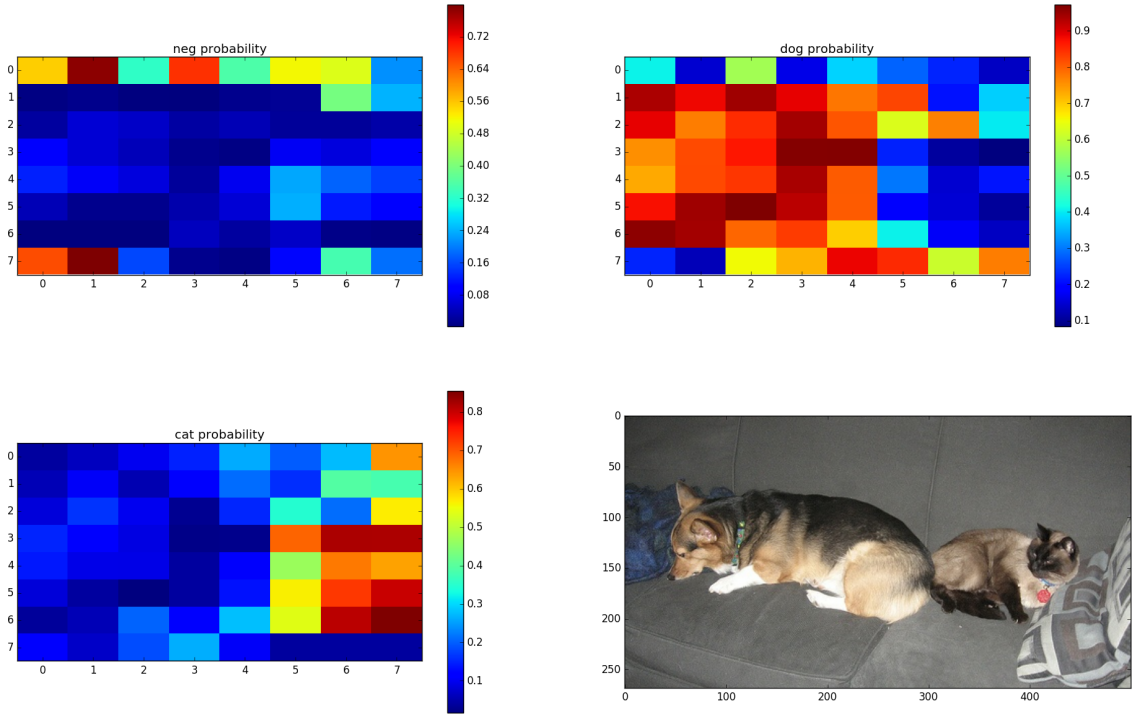
4 Classification and Sliding Window Detection

(i) By examining `bottleneck_input/BottleneckInputPlaceholder` in the `final_training_ops` node, we see that each bottleneck image summary has dimension 1×2048. There are 3 bias inputs and 2048×3 weights for a total of 6147 parameters. I achieved a misclassification rate of 0.10 (15/150) on the test dataset.

(ii) See submitted code.

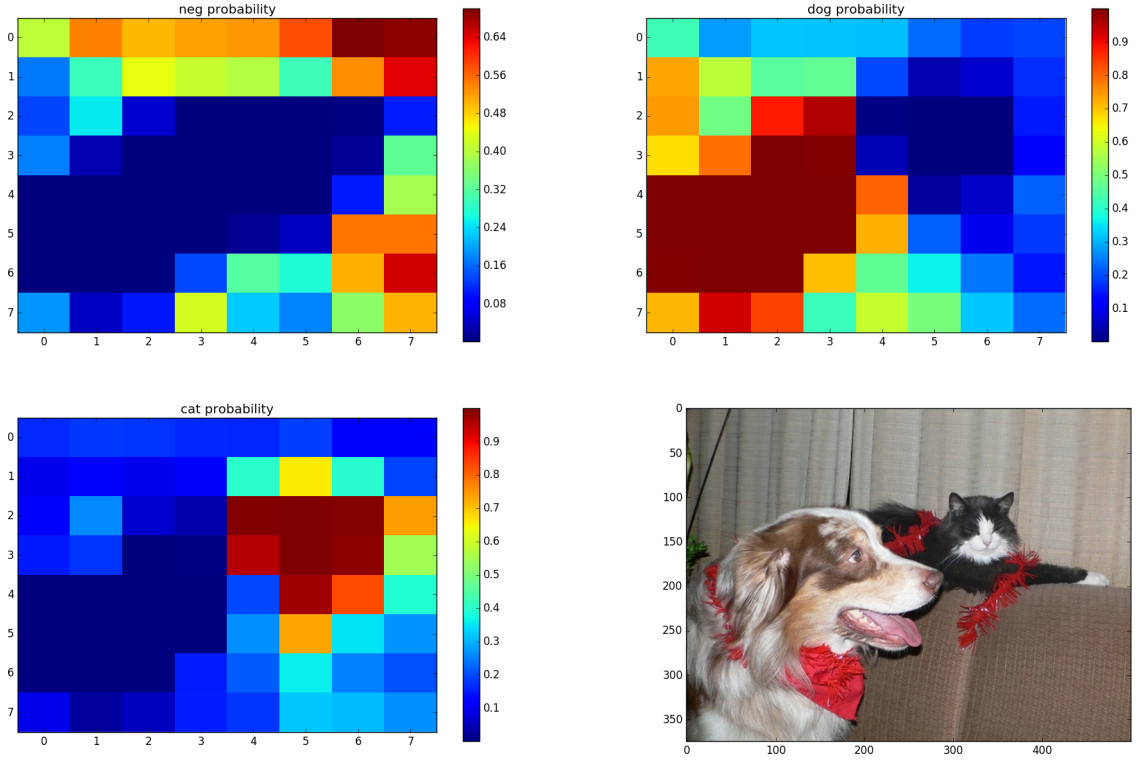
(iii) See submitted code. Below, I display the results on `catswithdogs/007417.jpg` with window padding on each side of about 15% the total height/width.

Brute Force Classification
Padding: 40 px (height), 75 px (width)



(iv) See submitted code.

Convolutional $K \times K$ Classification

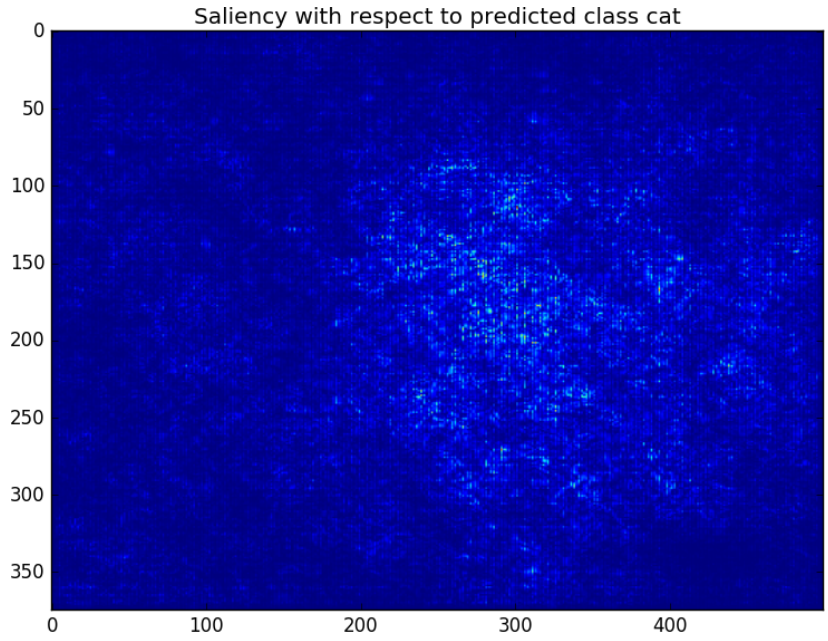


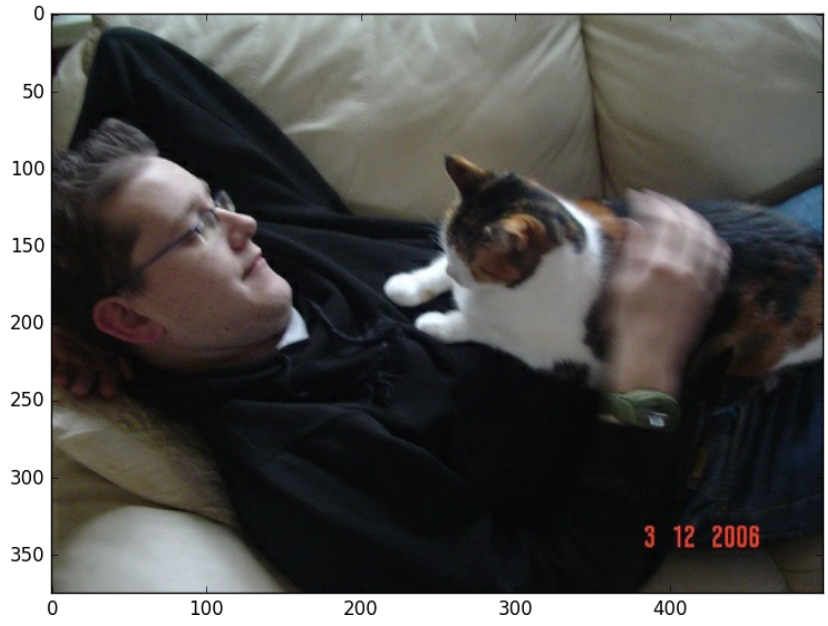
(v) The output of the final convolutional layer, `mixed_10/join:0`, feeds into the node `pool_3/(pool_3)`, which performs the AvgPool operation across the 8×8 windows. Thus, the feature vector for the image as a whole is the average of the feature vectors for each image subregion.

(vi) See submitted code. Given the raw gradient matrix $w \in \mathbb{R}^{m \times n \times p}$, we calculate the class saliency map to be $M \in \mathbb{R}^{m \times n}$ with entries $M_{ij} = \max_c |w_{ijc}|$, i.e. the maximum magnitude of w across all color channels.

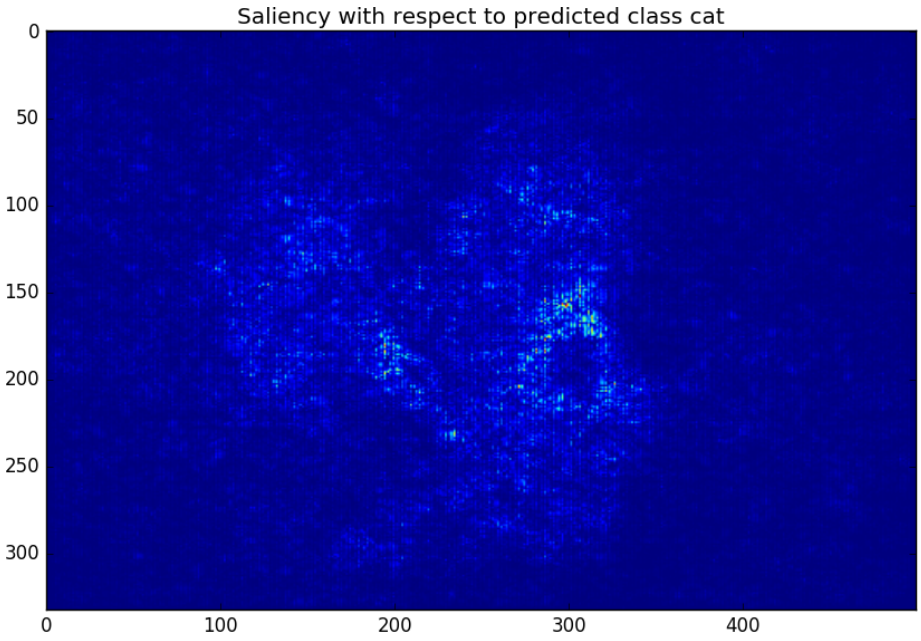
(vii) I compute and plot the saliency map for test image `000516.png`, which was correctly classified as a cat. In contrast, test image `004831.png` depicts neither a cat nor a dog, but was misclassified as a cat with class probabilities $(P_{neg}, P_{dog}, P_{cat}) = (0.23824486, 0.2742611, 0.48749402)$. From the saliency, it is clear that the classifier detected the baby as a cat, possibly due to the angle of the photo and light saturation along the arm.

Correctly Classified Image (Cat)





Misclassified Image (Neg)



5 Stop Sign Detection and FSM in ROS

(i) `supervisor.py` publishes the current desired pose, which is a message of type `Pose2D` that contains the goal position (x_g, y_g, θ_g).

(ii) See submitted code.

(iii) See submitted code.

(iv) See submitted code. From Figure 13, we can solve for

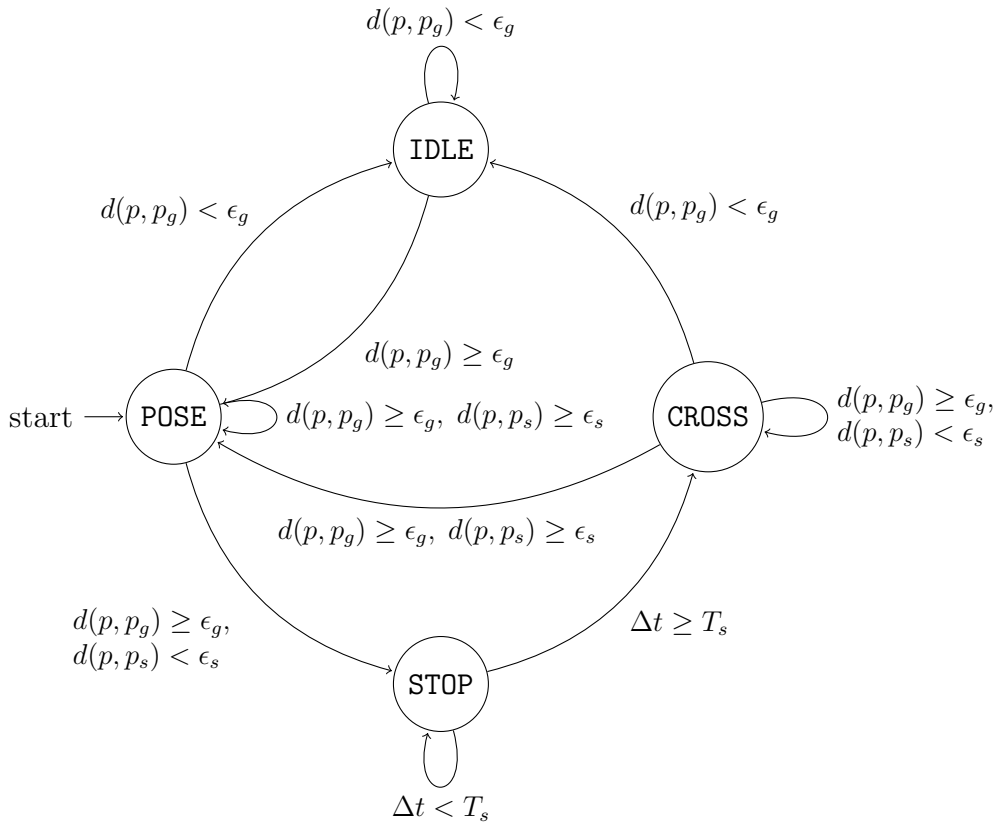
$$\frac{\tilde{x} - \tilde{x}_0}{f_x} = \frac{X_C}{Z_C} \quad \text{and} \quad \frac{\tilde{y} - \tilde{y}_0}{f_y} = \frac{Y_C}{Z_C} \tag{1}$$

Then, Z_C is determined by the unit vector constraint

$$\begin{aligned} 1 &= X_C^2 + Y_C^2 + Z_C^2 \\ \frac{1}{Z_C^2} &= \left(\frac{X_C}{Z_C}\right)^2 + \left(\frac{Y_C}{Z_C}\right)^2 + 1 \\ Z_C &= \left[\left(\frac{\tilde{x} - \tilde{x}_0}{f_x}\right)^2 + \left(\frac{\tilde{y} - \tilde{y}_0}{f_y}\right)^2 + 1\right]^{-\frac{1}{2}} \end{aligned}$$

which we can plug back into (1) to get X_C and Y_C.

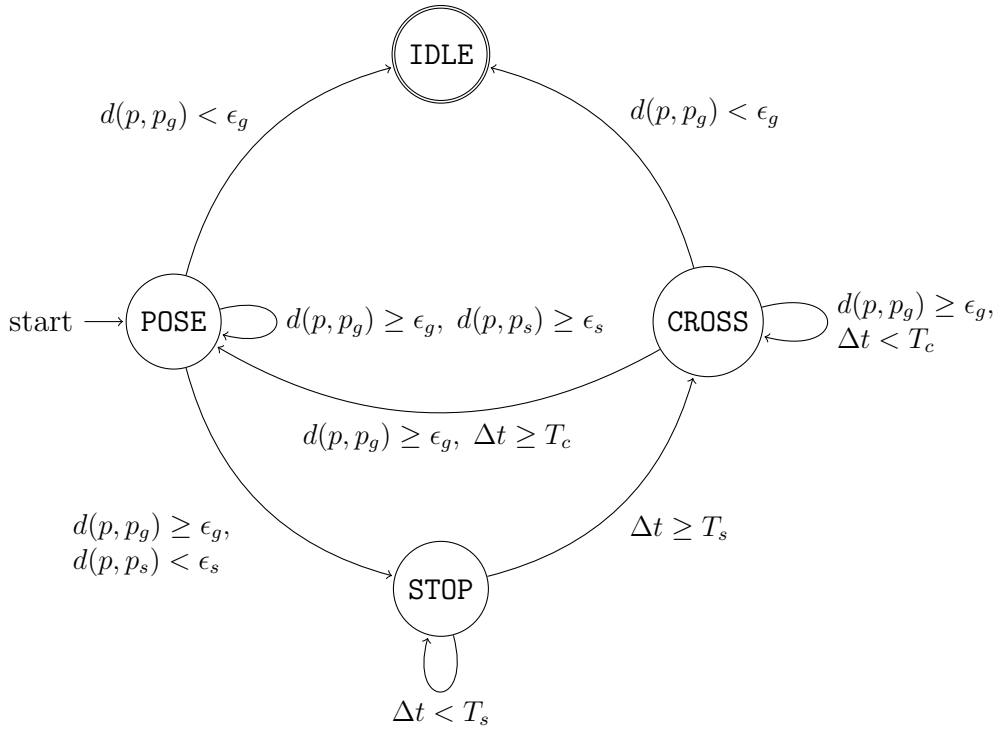
(v) Let $p = (x, y, \theta)$ be the current position, $p_g = (x_g, y_g, \theta_g)$ be the goal position, and $p_s = (x_s, y_s, \theta_s)$ be the location of the closest stop sign. Define $d : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ to be the standard distance function. If no stop sign is detected, we take $d(p, p_s) = \infty$. Furthermore, define the proximity thresholds $\epsilon_g = (0.1, 0.1, 0.15)$ and $\epsilon_s = 0.5$ m, and stopping time $T_s = 3$ seconds. We consider four states: `POSE`, `IDLE`, `STOP`, and `CROSS`. In `POSE` and `CROSS`, the current position p updates at each time step t (robot is navigating toward goal), while it stays the same in `STOP` and `IDLE` (robot is stationary). The robot starts in `POSE` and takes as input p_g and p_s, making the appropriate state changes according to the finite state machine (FSM),



In words, the transition rules are

- `POSE`: If the robot is at the goal, transition to `IDLE`. If the robot is not at the goal, but at a stop sign, transition to `CROSS`. If the robot is neither at the goal nor at a stop sign, remain in `POSE`.
- `IDLE`: If the robot is at the goal, remain in `IDLE`, otherwise transition to `POSE`.
- `STOP`: If 3 seconds have passed, transition to `CROSS`, otherwise remain in `STOP`.
- `CROSS`: If the robot is at the goal, transition to `IDLE`. If the robot is not at the goal, but at a stop sign, remain in `CROSS`. If the robot is neither at the goal nor at a stop sign, transition to `POSE`.

For the purposes of this implementation, we make two simplifying assumptions. First, we consider only a single goal, so once in `IDLE`, the robot remains there indefinitely. Second, we assume it takes $T_c = 3$ seconds to cross an intersection. Thus in `CROSS`, the robot will move toward the goal, ignoring all stop signs, for a fixed 3 seconds. If at any point it reaches the goal, it will transition to `IDLE`, otherwise it switches to `POSE` after the 3 seconds are over. The simplified FSM is shown below.



(vi) See submitted code.

(vii) The turtlebot started in `POSE`, transitioned to `STOP` at the stop sign, then continued to `CROSS` after 3 seconds. Once it passed the stop sign, it changed back to `POSE` until it reached its destination, where it remained in `IDLE`.

Turtlebot Path and Velocity Profile