

AA274 (Winter 2017-18): Problem Set 3

Anqi Fu

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1 Extended Kalman Filter (EKF)

- (i) See submitted code. Let the state variables be $\mathbf{x}(t) = (x(t), y(t), \theta(t))$ and the control variables be $\mathbf{u}(t) = (V(t), \omega(t))$. The differential drive model is

$$\begin{aligned}\dot{x}(t) &= V(t) \cos(\theta(t)) \\ \dot{y}(t) &= V(t) \sin(\theta(t)) \\ \dot{\theta}(t) &= \omega(t),\end{aligned}$$

which we can discretize by assuming a zero-order hold on the control, i.e. $\mathbf{u}(t)$ is constant over a time interval of length dt . Then we can integrate to get

$$\theta_t - \theta_{t-dt} = \int_{t-dt}^t \dot{\theta}(s) ds = \int_{t-dt}^t \omega(s) ds = \omega_t \int_{t-dt}^t ds = \omega_t dt.$$

If $\omega(t) = 0$ over the interval, $\theta(t)$ remains constant, so our coordinates can be found easily by integrating

$$\begin{aligned}x_t - x_{t-dt} &= \int_{t-dt}^t \dot{x}(s) ds = \int_{t-dt}^t V(s) \cos(\theta(s)) ds = V_t \cos(\theta_{t-dt}) dt \\ y_t - y_{t-dt} &= \int_{t-dt}^t \dot{y}(s) ds = \int_{t-dt}^t V(s) \sin(\theta(s)) ds = V_t \sin(\theta_{t-dt}) dt.\end{aligned}$$

With a slight abuse of notation, We may thus write

$$\mathbf{x}_t = \begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} = \begin{pmatrix} x_{t-1} + V_t \cos(\theta_{t-1}) dt \\ y_{t-1} + V_t \sin(\theta_{t-1}) dt \\ \theta_{t-1} + \omega_t dt \end{pmatrix} = g(\mathbf{x}_{t-1}, \mathbf{u}_t)$$

and the Jacobians of g with respect to \mathbf{x} and \mathbf{u} are, respectively,

$$G_x = \begin{pmatrix} 1 & 0 & -V_t \sin(\theta_{t-1}) dt \\ 0 & 1 & V_t \cos(\theta_{t-1}) dt \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad G_u = \begin{pmatrix} \cos(\theta_{t-1}) dt & 0 \\ \sin(\theta_{t-1}) dt & 0 \\ 0 & dt \end{pmatrix}.$$

If $\omega(t) \neq 0$, we exploit the fact that $\dot{\theta}(t) = \omega(t)$ to get

$$\begin{aligned} x_t - x_{t-dt} &= \int_{t-dt}^t V(s) \cos(\theta(s)) \frac{\dot{\theta}(s)}{\omega(s)} ds = \frac{V_t}{\omega_t} \int_{t-dt}^t \cos(\theta(s)) \dot{\theta}(s) ds \\ &= \frac{V_t}{\omega_t} (\sin(\theta_t) - \sin(\theta_{t-dt})) = \frac{V_t}{\omega_t} (\sin(\theta_{t-dt} + \omega_t dt) - \sin(\theta_{t-dt})) \\ y_t - y_{t-dt} &= \frac{V_t}{\omega_t} \int_{t-dt}^t \sin(\theta(s)) \dot{\theta}(s) ds = \frac{V_t}{\omega_t} (-\cos(\theta_{t-dt} + \omega_t dt) + \cos(\theta_{t-dt})) \end{aligned}$$

Our discrete model becomes

$$\mathbf{x}_t = \begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} = \begin{pmatrix} x_{t-1} + \frac{V_t}{\omega_t} (\sin(\theta_{t-1} + \omega_t dt) - \sin(\theta_{t-1})) \\ y_{t-1} - \frac{V_t}{\omega_t} (\cos(\theta_{t-1} + \omega_t dt) - \cos(\theta_{t-1})) \\ \theta_{t-1} + \omega_t dt \end{pmatrix} = g(\mathbf{x}_{t-1}, \mathbf{u}_t)$$

with Jacobians

$$\begin{aligned} G_x &= \begin{pmatrix} 1 & 0 & \frac{V_t}{\omega_t} (\cos(\theta_{t-1} + \omega_t dt) - \cos(\theta_{t-1})) \\ 0 & 1 & \frac{V_t}{\omega_t} (\sin(\theta_{t-1} + \omega_t dt) - \sin(\theta_{t-1})) \\ 0 & 0 & 1 \end{pmatrix} \\ G_u &= \frac{1}{\omega_t} \begin{pmatrix} \sin(\theta_t) - \sin(\theta_{t-1}) & -V_t (\frac{1}{\omega_t} (\sin(\theta_t) - \sin(\theta_{t-1})) - \cos(\theta_t) dt) \\ -\cos(\theta_t) + \cos(\theta_{t-1}) & V_t (\frac{1}{\omega_t} (\cos(\theta_t) - \cos(\theta_{t-1})) + \sin(\theta_t) dt) \\ 0 & \omega_t dt \end{pmatrix}. \end{aligned}$$

where $\theta_t := \theta_{t-1} + \omega_t dt$.

- (ii) See submitted code. Let the belief state at $t-1$ be distributed as $\mathcal{N}(\mathbf{x}_{t-1}, P_{t-1})$. We model the dynamic certainty with white noise $\nu \sim \mathcal{N}(\mathbf{0}, Q)$ applied to the control input. For a time step of dt , the EKF prediction step is

$$\begin{aligned} \bar{\mathbf{x}}_t &= g(\mathbf{x}_{t-1}, \mathbf{u}_t) \\ \bar{P}_t &= G_x P_{t-1} G_x^T + dt \cdot G_u Q G_u^T. \end{aligned}$$

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2 EKF Simultaneous Localization and Mapping (SLAM)

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(iii)

3 Turtlebot Localization

(i) See submitted code.

(ii) See submitted code.