

Disciplined Convex Optimization with CVXR

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useR! Conference 2018

Convex Optimization

CVXR

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Convex Optimization

$$\begin{array}{ll}\text{minimize} & f_0(x) \\ \text{subject to} & f_i(x) \leq 0, \quad i = 1, \dots, M \\ & Ax = b\end{array}$$

with variable $x \in \mathbf{R}^n$

- ▶ Objective and inequality constraints f_0, \dots, f_M are convex
- ▶ Equality constraints are linear

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Why?

- ▶ We can solve convex optimization problems
- ▶ There are many applications in many fields, including machine learning and statistics

Convex Problems in Statistics

- ▶ Least squares, nonnegative least squares
- ▶ Ridge and lasso regression
- ▶ Isotonic regression
- ▶ Huber (robust) regression
- ▶ Logistic regression
- ▶ Support vector machine
- ▶ Sparse inverse covariance
- ▶ Maximum entropy and related problems
- ▶ ... and new methods being invented every year!

Domain Specific Languages for Convex Optimization

- ▶ Special languages/packages for general convex optimization
- ▶ CVX, CVXPY, YALMIP, Convex.jl
- ▶ Slower than custom code, but extremely flexible and enables fast prototyping

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```
from cvxpy import *  
beta = Variable(n)  
cost = norm(X * beta - y)  
prob = Problem(Minimize(cost))  
prob.solve()  
beta.value
```


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A modeling language in R for convex optimization

- ▶ Connects to many open source solvers
- ▶ Uses disciplined convex programming to verify convexity
- ▶ Mixes easily with general R code and other libraries

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Ordinary Least Squares (OLS)

- ▶ minimize $\|X\beta - y\|_2^2$
- ▶ $\beta \in \mathbf{R}^n$ is variable, $X \in \mathbf{R}^{m \times n}$ and $y \in \mathbf{R}^m$ are constants

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```
library(CVXR)
beta <- Variable(n)
obj <- sum_squares(y - X %*% beta)
prob <- Problem(Minimize(obj))
result <- solve(prob)
solution$value
solution$getValue(beta)
```

- ▶ X and y are constants; β , obj , and prob are S4 objects
- ▶ `solve` method returns a list that includes optimal β and objective value

Non-Negative Least Squares (NNLS)

- ▶ minimize $\|X\beta - y\|_2^2$ subject to $\beta \geq 0$

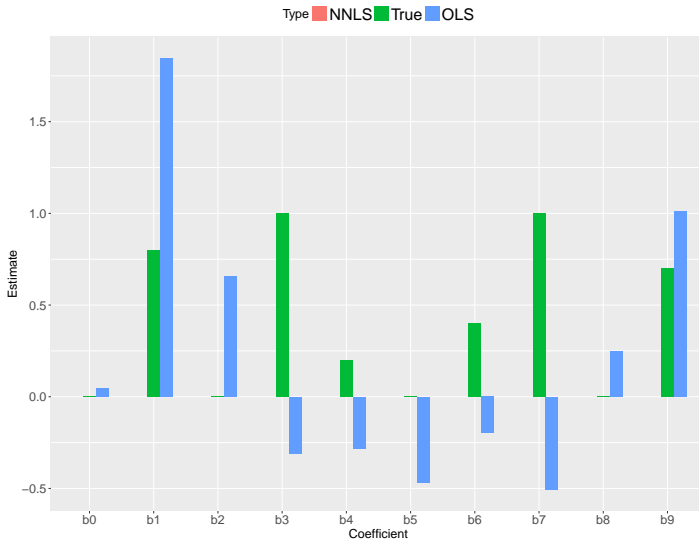
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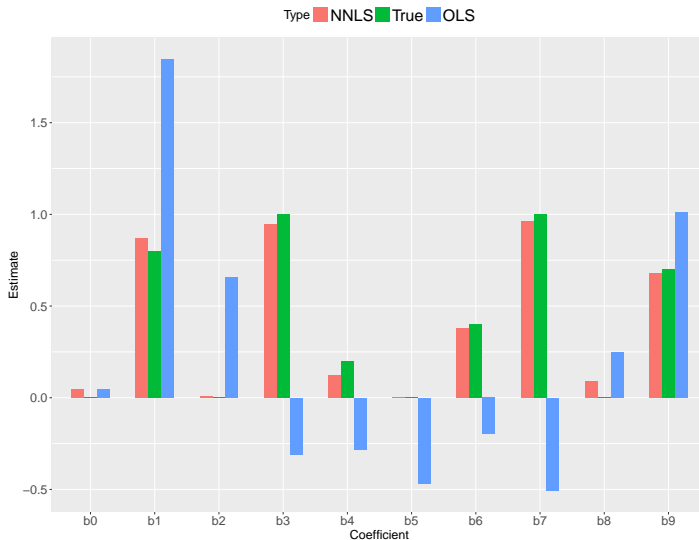
```
constr <- list(beta >= 0)
prob2 <- Problem(Minimize(obj), constr)
result2 <- solve(prob2)
result2$value
result2$getValue(beta)
```

- ▶ Construct new problem with list `constr` of constraints formed from constants and variables
- ▶ Variables, parameters, expressions, and constraints exist outside of any problem

True vs. Estimated Coefficients



True vs. Estimated Coefficients



Sparse Inverse Covariance Estimation

- ▶ Samples $x_i \in \mathbf{R}^n$ drawn i.i.d. from $N(0, \Sigma)$
- ▶ Know covariance $\Sigma \in \mathbf{S}_+^n$ has **sparse** inverse $S = \Sigma^{-1}$

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- ▶ Samples $x_i \in \mathbf{R}^n$ drawn i.i.d. from $N(0, \Sigma)$
- ▶ Know covariance $\Sigma \in \mathbf{S}_+^n$ has **sparse** inverse $S = \Sigma^{-1}$
- ▶ One way to estimate S is by maximizing the log-likelihood with a sparsity constraint:

$$\begin{aligned} & \underset{S}{\text{maximize}} && \log \det(S) - \text{tr}(SQ) \\ & \text{subject to} && S \in \mathbf{S}_+^n, \quad \sum_{i=1}^n \sum_{j=1}^n |S_{ij}| \leq \alpha. \end{aligned}$$

- ▶ $Q = \frac{1}{m-1} \sum_{i=1}^m (x_i - \bar{x})(x_i - \bar{x})^\top$ is sample covariance
- ▶ $\alpha \geq 0$ is a parameter controlling the degree of sparsity

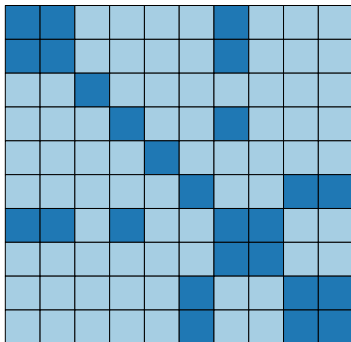
Sparse Inverse Covariance Estimation

```
S <- Semidef(n)
obj <- log_det(S) - matrix_trace(S %*% Q)
constr <- list(sum(abs(S)) <= alpha)
prob <- Problem(Maximize(obj), constr)
result <- solve(prob)
result$getValue(S)
```

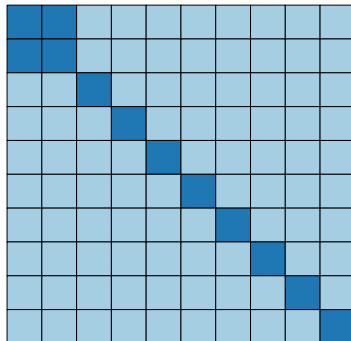
- ▶ Semidef restricts variable to positive semidefinite cone
- ▶ Must use `log_det(S)` instead of `log(det(S))` since `det` is not a supported atom
- ▶ `result$getValue(S)` returns an R matrix

True vs. Estimated Sparsity of Inverse

True Inverse

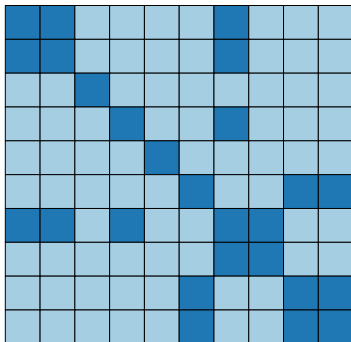


Estimate ($\alpha = 1$)

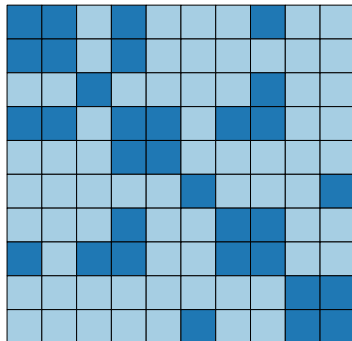


True vs. Estimated Sparsity of Inverse

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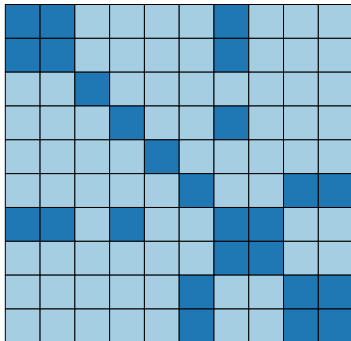


Estimate ($\alpha = 6$)

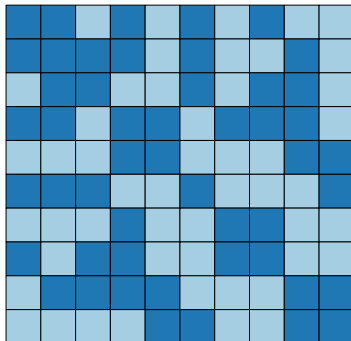


True vs. Estimated Sparsity of Inverse

True Inverse



Estimate ($\alpha = 10$)



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- ▶ Flesh out convex functions in library
- ▶ Develop more applications and examples
- ▶ Add warm start support
- ▶ Further speed improvements

Official site: `cvxr.rbind.io`

CRAN page: `CRAN.R-project.org/package=CVXR`