Disciplined Convex Optimization with CVXR

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useR! Conference 2018

Convex Optimization

CVXR

Examples

Future Work

Outline

Convex Optimization

CVXR

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Convex Optimization

$$
\begin{array}{ll}
\text{minimize} & f_0(x) \\
\text{subject to} & f_i(x) \leq 0, \quad i = 1, \ldots, M \\
& Ax = b
\end{array}
$$

with variable $x \in \mathbf{R}^n$

- Objective and inequality constraints $f_0, \ldots, f_M$ are convex
- Equality constraints are linear

Convex Optimization

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Why?

- We can solve convex optimization problems
- There are many applications in many fields, including machine learning and statistics

Convex Problems in Statistics

- ▶ Least squares, nonnegative least squares
- ▶ Ridge and lasso regression
- ▶ Isotonic regression
- ▶ Huber (robust) regression
- ▶ Logistic regression
- ▶ Support vector machine
- ▶ Sparse inverse covariance
- ▶ Maximum entropy and related problems
- ▶ . . . and new methods being invented every year!

Domain Specific Languages for Convex Optimization

- Special languages/packages for general convex optimization
- CVX, CVXPY, YALMIP, Convex.jl
- Slower than custom code, but extremely flexible and enables fast prototyping

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```
from cvxpy import *
beta = Variable(n)
cost = norm(X * beta - y)
prob = Problem(Minimize(cost))
prob.solve()
beta.value
```

Outline

CVXR

A modeling language in R for convex optimization

- ▶ Connects to many open source solvers
- ▶ Uses disciplined convex programming to verify convexity
- ▶ Mixes easily with general R code and other libraries

Outline

Ordinary Least Squares (OLS)

- minimize $||X\beta - y||_2^2$
- $\beta \in \mathbf{R}^n$ is variable, $X \in \mathbf{R}^{m \times n}$ and $y \in \mathbf{R}^m$ are constants

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```
library(CVXR)
beta <- Variable(n)
obj <- sum_squares(y - X %*% beta)
prob <- Problem(Minimize(obj))
result <- solve(prob)
solution$value
solution$getValue(beta)
```

- X and y are constants; beta, obj, and prob are S4 objects
- solve method returns a list that includes optimal beta and objective value

Non-Negative Least Squares (NNLS)

- minimize $||X\beta - y||_2^2$ subject to $\beta \geq 0$

Non-Negative Least Squares (NNLS)

▶ minimize $||X\beta - y||_2^2$ subject to $\beta \geq 0$

```
constr <- list(beta >= 0)
prob2 <- Problem(Minimize(obj), constr)
result2 <- solve(prob2)
result2$value
result2$getValue(beta)
```
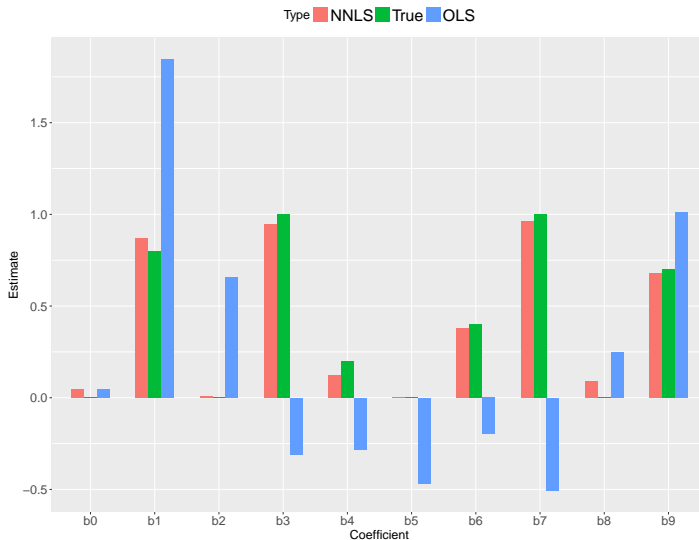
▶ Construct new problem with list `constr` of constraints formed from constants and variables
▶ Variables, parameters, expressions, and constraints exist outside of any problem

True vs. Estimated Coefficients
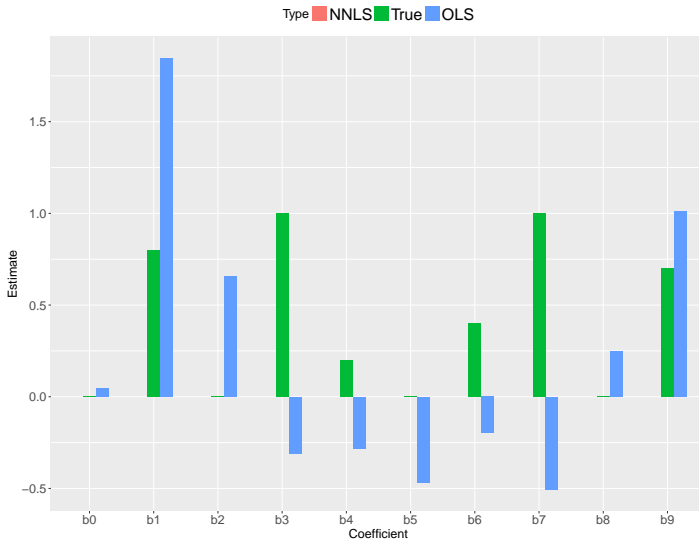
True vs. Estimated Coefficients

Fastest Mixing Markov Chain

- ▶ TODO: Set up FMMC problem

Fastest Mixing Markov Chain

- TODO: Code for FMMC problem

Outline

Future Work

- ▶ Flesh out convex functions in library
- ▶ Develop more applications and examples
- ▶ Add warm start support
- ▶ Further speed improvements

Official site: `cvxr.rbind.io`
CRAN page: `CRAN.R-project.org/package=CVXR`