

Disciplined Convex Optimization with CVXR

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useR! Conference 2016

Convex Optimization

CVXR

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Outline

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Convex Optimization

$$\begin{array}{ll}\text{minimize} & f_0(x) \\ \text{subject to} & f_i(x) \leq 0, \quad i = 1, \dots, m \\ & Ax = b\end{array}$$

with variable $x \in \mathbf{R}^n$

- ▶ Objective and inequality constraints f_0, \dots, f_m are convex
- ▶ Equality constraints are linear

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Why?

- ▶ We can solve convex optimization problems
- ▶ There are many applications in many fields, including machine learning and statistics

Convex Problems in Statistics

- ▶ Least squares, ridge and lasso regression, non-negative least squares
- ▶ Isotonic regression
- ▶ Logistic regression
- ▶ Huber (robust) regression
- ▶ Maximum entropy and related problems
- ▶ Support vector machines
- ▶ Sparse inverse covariance
- ▶ ... and new methods being invented every year!

Domain Specific Languages for Convex Optimization

- ▶ CVX, CVXPY, YALMIP, Convex.jl
- ▶ Slower than custom code, but extremely flexible, fast prototyping, etc
- ▶ TODO: CVXPY snippet

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CVXR

A modeling language in R for convex optimization

- ▶ Connects to many open source solvers
- ▶ Supports parameters, multiple constraints
- ▶ Mixes easily with general R code and other libraries

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Ordinary Least Squares (OLS)

- ▶ minimize $\|X\beta - y\|_2^2$
- ▶ $\beta \in \mathbf{R}^m$ is variable, X and y are constants

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```
library(cvxr)
beta <- Variable(m)
obj <- SumSquares(y - X %*% beta)
prob <- Problem(Minimize(obj))
solution <- solve(prob)
solution$opt_val
solution$beta
```

- ▶ X and y are constants; β , obj , and prob are S4 objects
- ▶ `solve` method returns a list that includes optimal β

Non-Negative Least Squares

- ▶ minimize $\|X\beta - y\|_2^2$ subject to $\beta \geq 0$

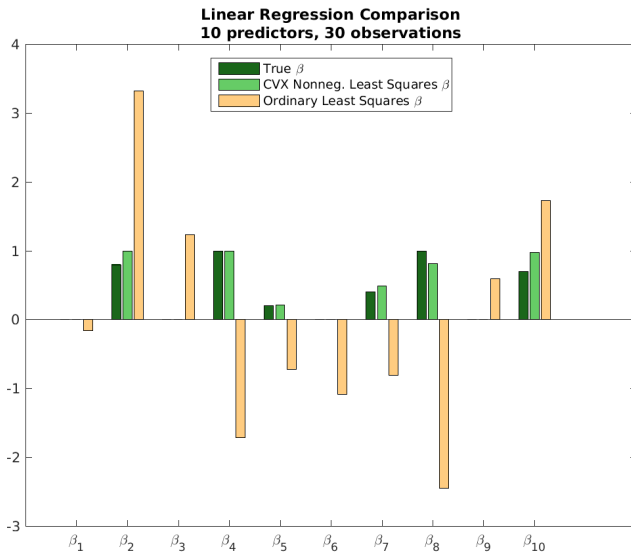
Non-Negative Least Squares

- ▶ minimize $\|X\beta - y\|_2^2$ subject to $\beta \geq 0$

```
constr <- list(beta >= 0)
prob2 <- Problem(Minimize(obj), constr)
solution2 <- solve(prob2)
solution2$opt_val
solution2$beta
```

- ▶ Construct new problem with list `constr` of constraints formed from constants and variables
- ▶ Variables, parameters, expressions, and constraints exist outside of any problem

Non-Negative Least Squares



Direct Standardization

- ▶ Samples (X, y) drawn **non-uniformly** from a distribution
- ▶ Expectations of features in X are a known quantity $b \in \mathbf{R}^m$
- ▶ Want to predict probability p_i for each sample
- ▶ Choose p_i to match known expectations, while maximizing total entropy

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$$\begin{array}{ll}\text{maximize} & \sum_i^n \text{entr}(p_i) \\ \text{subject to} & p \geq 0 \quad \mathbf{1}^T p = 1 \quad X^T p = b\end{array}$$

- ▶ $\text{entr}(p) = -p_i \log p_i$
- ▶ (p_i, y_i) is a guess of true distribution

Example: Direct Standardization

```
probs <- Variable(n)
cost <- SumEntries(Entr(probs))
constr <- list(probs >= 0, SumEntries(probs) == 1,
  t(X) %*% probs == b)
prob <- Problem(Maximize(cost), constr)
solution <- solve(prob)
solution$probs
```

- TODO: Details about code, constraint on fraction of women

Results: Direct Standardization

TODO: Plot of CDF from original and skewed

Results: Direct Standardization

TODO: Plot of CDF from original, skewed, and estimated distributions

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- ▶ More solvers - SCS, CVXOPT, etc
- ▶ More convex functions and constraints
- ▶ Warm start to speed up convergence