# New California Export Report

on Steel, Truck and Equipment Production

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## Honor Code

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# 1 Description of the Problem

The New California has traditionally exported steel, trucks, and equipment. Our goal here is to **maximize the net dollar value of the country's exports for the coming year** (the net dollar value of exports is defined as exports less the cost of all materials imported by the country). The existing world market prices are \$900 per unit for steel, \$3,000 per unit for trucks and \$2,500 per unit of equipment.

However, there are technological constraints when deciding the maximized net dollar value of exports:

1. One unit of steel requires 0.08 units of trucks, 0.05 units of equipment, and 0.5 man-years of labor. It also needs 2 units of ore purchased on the world market for \$100/unit and other imported materials costing \$100, which adds up to a total of \$300. In addition, the steel mills have maximum capacity of 300,000 units/year.

2. One unit of trucks requires 1 unit of steel, 0.10 units of equipment and 3 man-years of labor. Total of \$500 of imported materials is needed for one unit of trucks, and the truck capacity is 550,000 units/year.

3. One unit of equipment requires 0.75 units of steel, 0.12 units of trucks, and 5 man-years of labor. Total of \$150 of materials must be imported for each unit of equipment produced, and the capacity of the country's equipment plants is 50,000 units/year.

4. The total manpower available for production of steel, trucks and equipment is 1,200,000 men/year.

# 2 Selecting Model

## 2.1 Model Formulation

Denote $a$, $b$ and $c$ as the units of steel, trucks and equipment **exported** for the coming year respectively, and let $x$, $y$ and $z$ be be the unit of steel, trucks and equipment **produced** for the coming year respectively.

Since one unit of steel can be sold at \$900, one unit of truck can be sold at \$3,000 and one unit of equipment can be sold at \$2,500, the total exported value for the next year is $900a + 3000b + 2500c$. However, producing one unit of steel requires \$300 of imported materials, producing one unit of trucks requires \$500 imported materials, and producing one unit of equipment requires \$150 imported materials, so **the net dollar value of exports on steel** is $900a + 3000b + 2500c - 300x - 500y - 150z$.

To satisfy the constraints we mentioned in the description, we first need to make the total number of steel production less than 300,000 units, so $x \leq 300000$. Similarly, total number of truck production should be less than 550,000 units so $y \leq 550000$, and total number of equipment production should be less than 50,000 units so $z \leq 50000$. In addition, the total manpower should be less than 1,200,000, which means $0.5x + 3y + 5z \leq 1200000$.

Also note that producing 1 unit of trucks requires 1 unit of steel and producing 1 units of equipment requires 0.75 unit of steel. Therefore, producing $y$ units of truck and $z$ units of equipment will cost $y + 0.75z$ units of steel. Then the total unit of steel remained after producing trucks and equipment is $x - y - 0.75z$. Since the amount of remained steel produced should be greater or equal to the amount of steel exported(because we assume we cannot import this type of good), we have the constraint $x - y - 0.75z - a \geq 0$.

Similarly, since producing 1 unit of steel requires 0.08 units of truck and 1 unit of equipment requires 0.12 units of truck, producing $x$ units of steel and $z$ units of equipment will cost $0.08x + 0.12z$ units of truck. Then the total unit of trucks remained after producing steel and equipment is $-0.08x + y - 0.12z$. Since the amount of remained trucks produced should be greater or equal to the amount of steel exported, we have the constraint $-0.08x + y - 0.12z - b \geq 0$.

Producing 1 unit of steel requires 0.05 units of equipment and 1 unit of truck requires 0.1 units of equipment, so producing $x$ units of steel and $y$ units of truck will cost $0.05x + 0.1y$ units of equipment. Then the total unit of equipment remained after producing steel and trucks is $-0.05x - 0.1y + z$. Since the amount of remained trucks produced should be greater or equal to the amount of steel exported, we have the constraint $-0.05x - 0.1y + z - c \geq 0$.

Also, the number of each unit of steel, truck and equipment exported and produced should all be nonnegative, because we are not assuming the liquidity that we can convert produced goods into the materials used to produce it.

Therefore, combining all the results above, we formulate the mathematical model as:

$$
\begin{aligned}
\max \quad & 900a + 3000b + 2500c - 300x - 500y - 150z, \\
\text{subject to} \quad & x \leq 300000 \\
& y \leq 550000 \\
& z \leq 50000 \\
& 0.5x + 3y + 5z \leq 1200000 \\
& x - y - 0.75z - a \geq 0 \\
& y - 0.08x - 0.12z - b \geq 0 \\
& z - 0.05x - 0.1y - c \geq 0 \\
& a, b, c, x, y, z \geq 0.
\end{aligned} \tag{1}
$$

## 2.2 Checking Linear Programming Criteria

Since we formulated a linear programming model, we need to justify that our model is appropriate. There are four assumptions to check if a model is appropriate for linear programming, Proportionality, Additivity, Certainty and Divisibility. If the four assumptions are all satisfied, then a linear programming model is appropriate.

Firstly, we know that any extra unit of steel, truck or equipment exported will exactly add \$900, \$3,000 or \$2,500 in our total net dollar value of export, and any extra unit of steel, truck or equipment produced will decrease in our total net dollar value of export by exactly \$300, \$500 or \$1500. Also, any extra unit of steel, truck or equipment produced will increase 0.5, 3 or 5 units in manpower; 1, -1 or -0.75 unit of steel sold; -0.08, 1 or -0.12 unit of truck sold; -0.05, -.01 or 1 unit of equipment sold. This means that our model satisfies the Proportionality criteria.

Secondly, the total net dollar value of export is calculated by the sum of the exported value of units of steel, truck and equipment and cost of imported materials for units of steel, truck and equipment produced. Also, the total number of units of manpower is the sum of manpower required for producing steel, truck and equipment separately. Also, the total units of steel, truck and equipment sold are also sums of steel, truck and equipment produced in different proportion. In this case, our model satisfies Additivity criteria.

Thirdly, since the net dollar value of export, manpower, and the total number of units steel, truck and equipment sold are all certain and fixed, so the Certainty criteria is met.

Finally, the number of units can be fraction, so the Divisibility criteria is satisfied.

Therefore, **we can conclude that it is reasonable to say the assumptions of a Linear**

**Programming model is satisfied.**

## 3 Solution of the Model

Using Lingo, we get the result in Figure 1.

```
Global optimal solution found.
Objective value:                              0.4906250E+09
Infeasibilities:                              0.000000
Total solver iterations:                             4
Elapsed runtime seconds:                          0.07

Model Class:                                        LP

Total variables:                      6
Nonlinear variables:                  0
Integer variables:                    0

Total constraints:                    8
Nonlinear constraints:                0

Total nonzeros:                      24
Nonlinear nonzeros:                   0


                              Variable           Value        Reduced Cost
                                     A        0.000000            1350.000
                                     B        232500.0            0.000000
                                     C        8750.000            0.000000
                                     X        300000.0            0.000000
                                     Y        262500.0            0.000000
                                     Z        50000.00            0.000000

                                   Row    Slack or Surplus      Dual Price
                                     1       0.4906250E+09        1.000000
                             STEEL_CAP        0.000000            1585.000
                             TRUCK_CAP        287500.0            0.000000
                             EQUIP_CAP        0.000000            302.5000
                              MANPOWER        12500.00            0.000000
                             STEEL_REM        0.000000           -2250.000
                             TRUCK_REM        0.000000           -3000.000
                             EQUIP_REM        0.000000           -2500.000
```

Figure 1: Model Solution

Note that not all of the information contained in this program output is essential for decision making, and we will only focus on the most important ones.

The "Objective value" row shows the calculated maximized net dollar value of export for the coming year, which is \$490,625,000 here. The "Variable" column tells the names of each variable, here "A", "B", "C" represents units of steel, trucks and equipment exported respectively, and "X", "Y", "Z" represents units of steel, trucks and equipment produced respectively.

The “Value” column shows the number of units of steel, trucks and equipment to produce and export for the coming year according to the specific variable to maximize net dollar value of export. Here we can see 300,000 units of steel, 262,500 units of trucks and 50,000 units of equipment are required to be produced for the coming year, and accordingly we need to export 0 units of steel, 232,500 units of trucks and 8,750 units of equipment for the coming year.

**Therefore, by satisfying all the constraints mentioned, our final result is that, for the coming year, if we produce 300,000 units of steel, 262,500 units of trucks and 50,000 units of equipment and, accordingly, export 232,500 units of truck and 8,750 units of equipment, we will reach our maximized net dollar value of export, which is $490,625,000.**

## 4 Interpretation of the Solution

By the result we got, for the maximized net dollar value of export, we will produce 300,000 units of steel, 262,500 units of trucks and 50,000 units of equipment. However, we only export 232,500 units of truck and 8,750 units of equipment accordingly.

This means that all the 300,000 units of steel are used to produce truck or equipment; $262,500 - 232,500 = 30,000$ units of truck are used to produce steel or equipment; and $50,000 - 8,750 = 41,250$ units of equipment are used to produce steel or truck.

By looking at the “Slack or Surplus” column in Figure 1, for “TRUCK_CAP” constraint, it shows $287500 > 0$, and this means that the units of truck produced is less the maximum capacity of storing the truck by 287,500 units. However, for “STEEL_CAP” and “EQUIP_CAP” constraints, the “Slack or Surplus” are all 0, meaning the production of units of steel and equipment reached the maximum capacity.

## 5 Other Considerations

The following analysis are based on the Sensitivity Report in Figure 2. To understand what this report is saying, we give the columns named “Allowable” the two sections in the report.

In the “Objective Coefficient Ranges” section, the “Allowable Increase” and “Allowable Decrease” shows that how large the coefficient of units of steel, truck and equipment to export or produce can increase/decrease without changing the original optimal production strategy for the maximized net dollar value of export(even though the maximized profit value may change).

In the “Righthand Side Ranges” section, the “Allowable Increase” and “Allowable Decrease” shows the range that the right-hand side of each constraints can change so that the “basic variables”

```
Ranges in which the basis is unchanged:

                                  Objective Coefficient Ranges:

                        Current         Allowable        Allowable
    Variable          Coefficient        Increase         Decrease
           A           900.0000          1350.000         INFINITY
           B           3000.000          347.7011         1350.000
           C           2500.000          10566.67         281.3953
           X          -300.0000          INFINITY         1585.000
           Y          -500.0000          403.3333         1350.000
           Z          -150.0000          INFINITY         302.5000

                                     Righthand Side Ranges:

                        Current         Allowable        Allowable
         Row              RHS            Increase         Decrease
   STEEL_CAP           300000.0          3571.429         252717.4
   TRUCK_CAP           550000.0          INFINITY         287500.0
   EQUIP_CAP           50000.00          4545.455         8139.535
    MANPOWER           1200000.          INFINITY         12500.00
   STEEL_REM           0.000000          232500.0         4166.667
   TRUCK_REM           0.000000          232500.0         INFINITY
   EQUIP_REM           0.000000          8750.000         INFINITY
```

Figure 2: LINGO Sensitivity Report

selected in the current solution would stay the same, and most importantly, our changed maximized net dollar value is optimal under the same set of "basic variables"(but both the optimal strategy as well as the optimal net profit value would change in general).

## 5.1 Price of Steel Increased to $1,250/unit and Export One Unit of Steel

If the world market price of steel increased from $900/unit to $1,250/unit, and the world market price of trucks and equipment doesn't change, the coefficient of $a$ in our objective function will increase by 350.

By looking at the sensitivity report of our original model in Figure 2, in this scenario we will need to increase the coefficient for $a$ by 350, which is inside the range 1,350 "Allowable Increase" of $x$. Since the change is inside the allowable range, our optimal production strategy won't change. We will still export 0 units of steel, and produce 300,000 units of steel.

Although the price of Steel Increased to $1,250/unit, we don't export steel, and with other coefficient in the function for net value of export won't change staying the same, the net value of export won't change.

Hence, **if the world market price of steel increased to $1250/unit, the export and production on steel, trunks and equipment remains the same, and the total net dollar value of exports stays the same, being $490,625,000.**

Now we consider the case that the country chose to export one unit of steel(without the marker price change). Since we are forced to export one unit, the constraint on total units of steels exported now becomes $a \geq 1$. Then according to Figure 1, since the "Reduced Cost" column for A, units of steel exported, is 1,350, we know one unit increase in A will decrease the original total net dollar value of exports by $1,350. Therefore, **if the country chose to export one unit of steel, the total net dollar value of exports will decrease.**

Now combining the two situations together, since the increase in world market price of steel neither changes the optimal production strategy nor changes the total net dollar value of exports, we would have export zero unit of steel as part of the optimal strategy. Now exporting zero unit is already optimal, forcing the export of steel to be one unit could not increase the profit, so the only possibly would be stay unchanged or decrease for the maximized net profit. Therefore, **if the world market price of steel increased to $1,250/unit and the country chose to export one unit of steel, the net value of export could only decrease or stay unchanged**.
(The result is verified by building the new LP model. More detailed result refer to Figure 4, as the maximized profit is decreased by 1,000)

## 5.2 The Last Preferable Resource to be Used by New Products

Note that the "Dual Price" in Figure 1 means how many value we will lose if we divert resources away from our optimal production mix, which is generally the opportunity cost.

Referring to Figure 1, the opportunity cost for both truck capacity and manpower are 0, which means that diverting either any unit of truck capacity or manpower will not lose values in the net value of export.

Also, the opportunity cost for steel capacity is 1585, meaning diverting 1 unit of steel capacity to our new product will lose us $1585. Similarly, the opportunity cost for equipment capacity is 302.5, meaning diverting 1 unit of equipment capacity to our new product will lose us $302.5.

Therefore, by comparing the opportunity costs for the four possible resources, we will lose most money by diverting steel capacity to new production. Hence, **units of steel production is the last preferable to be used by new products, and should be used as sparingly as possible.**

## 5.3 Spend $500,000 to Expand Capacity

Suppose that we will spend $500, 000 on buying 300 units of steel capacity, 500 units of truck capacity or 1000 units of equipment capacity. Here we want to choose which capacity to expand will lead to a larger net value of export.

By looking at the Sensitivity Report in Figure 2, we can see that the "Allowable Increase" for capacity constraints on the three resources "STEEL_CAP", "TRUCK_CAP" and "EQUIP_CAP" are 3571.429, INFINITY and 4545.455 respectively. All the increases in capacity are within this range, so, in this case, the net profit will still be maximized under the same set of "basic variable" before.

Now looking at the "Dual Price" in Figure 1, which shows how much the maximized value of export will change if the Right-hand side of each constraints increases by 1 unit. We can see that for capacity constraints on the three resources "STEEL_CAP", "TRUCK_CAP" and "EQUIP_CAP", the "Dual Price" for "TRUCK_CAP" is 0, so we shouldn't consider expanding 500 units of truck capacity.

"STEEL_CAP" has the highest "Dual Price" of 1,585, so expanding 300 units of steel capacity will generate $1585 \cdot 300 = 475,500$ for net value of export. The "Dual Price" for units of equipment produced is 302.5, so expanding 1,000 units of steel capacity will generate $302.5 \cdot 1000 = 302,500$ for net value of export.

Although $\$475,500 > \$302,500$, $\$475,500 < \$500,000$, so **the best investment is not choosing to spend $500,000 on buying any units of capacity.**

(The result is verified by building the new LP model. More detailed result refer to Figure 5 and 6, and the model result shows that the net value of export increased by buying 300 units of steel capacity is $491,100,500, subtracting $500,000, we get $490,600,500 < $490,625,000).

## 5.4 World Market Price of the Imported Materials for Trucks Increase by $400

Originally, the imported materials needed to produce one unit of trucks was $500. If the world market price of the imported materials needed to produce one unit of trucks were to increase by $400, now it becomes $900. Therefore, the coefficient for $y$ decreases from -500 to -900.

By looking at the sensitivity report of our original model in Figure 2, in this scenario the "Allowable Decrease " for units of trucks produced (Y) is 1350, so our decrease is in the allowable range for holding the optimal strategy the same. Now we still export 232,500 units of truck and 8,750 units of equipment, and produce 300,000 units of steel, 262,500 units of trucks and 50,000 units of equipment as we reasoned at the very beginning of Section 5.

Now our total net dollar value of export is $900 \cdot 0 + 3000 \cdot 232500 + 2500 \cdot 8750 - 300 \cdot 300000 - 900 \cdot 262500 - 150 \cdot 50000 = 385,625,000$

In conclusion, **if the world market price of the imported materials needed to produce one unit of trucks were to increase by $400, we still produce 300,000 units of steel, 262,500 units of trucks and 50,000 units of equipment and the total net dollar value of export is $385,625,000.**

(This result is verified by building the new LP model. More detailed result refer to Figure 7.)

### 5.5 Minister of Defense Stockpile an Additional 10,000 Units of Steel

If minister would like to stockpile an additional 10,000 units of steel during the coming year, we need to change the steel remained constraint ("STEEL_REM"), and all the other constraints remain the same. We need to stockpile 10,000 units of steel during the coming year, which means the total amount of steel that we are going to produce minus the amount that we are going to export should be 10,000. Therefore, the right hand side of the steel sold constraint should becomes $x - a - y - 0.75z \geq 10,000$ such that we could be able to stockpile 10,000 units of steel. In this case, from Figure 2, the change 10,000 is within the allowable range of 287,500 to keep the original solution ("basic variable") optimal. However, it is still possible that the optimal production strategy and value of the maximized net profit would change. From Figure 1, the dual price for "STEEL_REM" is -2,250, which means that any one unit increase of steel remained will cause a decrease of $2,250 in our total net value of export. Therefore, here we will decrease $\$2,250 \cdot 1000 = \$2,250,000$ in our total net value of export.

**In conclusion, if minister would like to stockpile an additional 10,000 units of steel during the coming year, it will cause a decrease of $2,250,000 in total net value of export.**

(This is verified by Lingo result in Figure 8. The result of total net value of export for our new model is $488,375,000, which is exactly $2,250,000 less than our original model of $490,625,000.)

### 5.6 Inventory Requirement Model Formulation

Suppose we start the year with $S_s$ units of steel in the inventory, $M_s$ units of equipment, and $T_s$ units of trucks, instead of zero units for each of them. At the end of the year, we want to have at least $S_e$ units of steel in the inventory, $M_e$ units of equipment, and $T_e$ units of trucks, instead of zero units for each of them.

Then the lower bound for the difference in the amount produced and the amount sold for steel, instead of zero, would instead to be modified into $S_e - S_s$, and similarly, such lower bound for trucks is $T_e - T_s$ and for equipment is $M_e - M_s$. To see why this is the case, we take the example of steel and recognize that the total amount of steel left over from production during the year would be $S_s + x - y - 0.75z$ units, and we demand that we leave at least $S_e$ units after selling steel on the market, so the constraint on the inventory balance of steel would be $S_s + x - y - 0.75z - a \geq S_e$. Now moving the $S_s$ to the other side of the inequality, and doing the same thing for the other products, we obtain the new set of constraints: (if "want to have" is interpreted to have a more strict meaning that we want exact $S_s$ units of steel at the end of year, then the inequality should be modified into an equality)

$$\begin{aligned} &x - y - 0.75z - a \geq S_e - S_s \\ &y - 0.08x - 0.12z - b \geq T_e - T_s \\ &z - 0.05x - 0.1y - c \geq M_e - M_s \end{aligned} \tag{2}$$

So our model after this change would become:

$$\begin{aligned} \max \quad & 900a + 3000b + 2500c - 300x - 500y - 150z, \\ \text{subject to} \quad & x \leq 300000 \\ & y \leq 550000 \\ & z \leq 50000 \\ & 0.5x + 3y + 5z \leq 1200000 \\ & x - y - 0.75z - a \geq S_e - S_s \\ & y - 0.08x - 0.12z - b \geq T_e - T_s \\ & z - 0.05x - 0.1y - c \geq M_e - M_s \\ & a, b, c, x, y, z \geq 0. \end{aligned} \tag{3}$$

## 5.7 Adding New Product, Product X

A government R&D group has recently come to Michael with Product X to be exported produced by 1.5 man-years of labor and 0.3 units of machinery for each unit produced. In order to make profit from producing and exporting Product X, we need to calculate the price for Product X.

| Variable | Value | Reduced Cost |
|---|---|---|
| A | 0.000000 | 1350.000 |
| B | 232500.0 | 0.000000 |
| C | 8750.000 | 0.000000 |
| X | 300000.0 | 0.000000 |
| Y | 262500.0 | 0.000000 |
| Z | 50000.00 | 0.000000 |

| Row | Slack or Surplus | Dual Price |
|---|---|---|
| 1 | 0.4906250E+09 | 1.000000 |
| STEEL_CAP | 0.000000 | 1585.000 |
| TRUCK_CAP | 287500.0 | 0.000000 |
| EQUIP_CAP | 0.000000 | 302.5000 |
| MANPOWER | 12500.00 | 0.000000 |
| STEEL_REM | 0.000000 | -2250.000 |
| TRUCK_REM | 0.000000 | -3000.000 |
| EQUIP_REM | 0.000000 | -2500.000 |

Figure 3: LINGO Dual Price Report

From the plot above we know the dual price of one unit of machinery(equipment) is 302.5, and that of manpower is 0. Since we need 1.5 man-years of labor and 0.3 units of machinery for each unit of Product X, the opportunity cost of each unit of Product X can be computed as the sum of the the units required multiplied by the dual price of each kind of resource:

$$1.5 \cdot 0 + 0.3 \cdot 302.5 = 90.75 \tag{4}$$

Therefore, we conclude **in order to attract the production of Product X, it must be sold greater or equal than $ 90.75.**

## 5.8 Strike influence

If there is a possibility of strikes during the year that will impair Machinery production such that the strike would reduce the machinery capacity from 50000 to 40000. Our interest is to estimate the maximized net dollar exports after this reduction in capacity.

Since in the previous section we see that the dual price of one unit of machinery(equipment) is 302.5, we would give the estimate as the current maximized net dollar exports, subtracted by the units of reduction(10000) multiplied by this dual price:

$$490625000 - 10000 \cdot 302.5 = 487600000 \tag{5}$$

The value computed above is an estimate because from the sensitivity analysis report, the allowable range for the machinery capacity to decrease is 8139.535 which is less than the amount of decrease here(10000), which means the "basic variable" that produces the maximized net dollar exports would be different than the current one, and as a result our interpretation of the dual price as the "value" of one unit of machinery capacity would no hold appropriately.

From the Weak duality property, we know that this estimate is an upper bound to the real net dollar exports had we solved the new LP problem after the change of constraint. To see why this is true, we first note the Weak duality property states that, for an LP problem in Canonical form with $c$ storing the coefficient of the objective function and $b$ storing the right hand sides of the constraints, if $x_0$ is a feasible solution to the primal problem and $y_0$ is a feasible solution to the dual problem, then

$$c^T x_0 \leq b^T y_0 \tag{6}$$

In our case of the original problem, we have an optimal solution $x^*$ with its corresponding solution in the dual problem $y^*$. By the Fundamental Theorem of Duality, we have $b^T y_* = 490625000$ as the optimal value in the original problem. We also know that the third element in $y^*$ is the same as the dual price column in our model solution excluding the first row. Note the third element in $y^*$ is 302.5.

Then new problem after the reduction of machinery capacity would have the right hand side vector of constraints being $b - (0, 0, 10000, 0, 0, 0, 0)^T$, and the coefficients of the objective function remains unchanged as $c$. Now we notice that since the feasible region of the dual problem only depends on the matrix storing the left hand side coefficients in the constraints, and the coefficients of the objective function, we have both of these unaffected by the reduction of machinery capacity, so the feasible region of the dual problem is the same regardless whether the strike occurs or not! Therefore, since $y^*$ is a feasible point of the dual problem before the reduction, it would also be an feasible point of the dual problem after the reduction.

Now let $x^{**}$ be the optimal solution to the problem after the reduction in machinery capacity, and we directly apply the Weak duality property to conclude:

$$c^T x^{**} \leq (b - (0, 0, 10000, 0, 0, 0, 0)^T)^T y^* = by^* - 10000 \cdot y_3^* = 490625000 - 10000 \cdot 302.5 = 487600000 \tag{7}$$

where the left hand side of inequality above is by definition the real net value of export of the new model after the reduction. We therefore have proved that the estimate we have is an upper bound of the real net dollar exports value.

Therefore, we conclude **the estimated value of the maximized net value of export $ 487,600,000 is only an estimate, and we know further that it is an upper bound of the true value from Weak duality property.**

(This is verified by Lingo result in Figure 9. The result of total net value of export for our new model is \$466,466,700, which is exactly \$2,250,000 less than our estimate \$487,600,000, so our estimate is indeed an upper bound.)

## 6 Recommendations

In the recommendation, we are going to give the recommended solution in order to maximize the net dollar value of exports, and summary of different scenarios based on the sensitivity report of the original LP model.

- The maximized net dollar value will be reached by producing 300,000 units of steel, 262,500 units of trucks and 50,000 units of equipment and exporting 0 unit of steel, 232,500 units of truck and 8,750 units of equipment. And the maximized net dollar value of export \$490,625,000. After we have done all the other considerations and varied one or more constraints, the new net dollar value are all less or equal than the original net dollar value in this solution.

- If the price of steel increased from \$900 to \$1,250 per unit, the coefficient of a in objective function will increased by 350 which is inside the range of allowable increase of $x$ from the sensitivity report. But the net dollar value will decrease if we choose to export steel. So the net dollar value of exports remains \$490,625,000 or decrease if any steel is exported.

- Spending \$500,000 to expand capacity will not increase the net dollar value of exports. We calculate the units we can expand for each material, and the increases are all within the allowable range, so the optimal mix will remain the same. And then, we found out the return by exporting the extra material are all lower than \$ 500,000. Therefore, it is not profitable to spend \$500,000 to expand capacity.

- If the price of producing one unit of trucks increases by \$400, the optimal mix will not change because the change is within the allowable range in the sensitivity report. However, the net dollar value will decreases to \$385,625,000 correspondingly.

- If we would like to stockpile an additional 10,000 units of steel, there will be one constraint need to be changed which is the steel remained constraint. If we stockpile 10,000 unit of steel, the net dollar value will decrease to \$ 488,375,000. Hence, this is not a beneficial change for maximizing net dollar value.

- Product X is recently introduced by R&D Group, and each unit of Product X is produced by 1.5 man-years of labor and 0.3 units of 1.5 man-years of labor and 0.3 units. From the dual price, we calculate the opportunity coast for each unit of Product X is \$ 90.75. So, in order to make profit from Product X, it must be sold at a price greater than \$ 90.75.

- Under the strike, the machinery capacity reduced from 50000 to 40000. In this case, we will have a new net dollar exports, and we have proved that this net dollar exports is only an estimated value, and this value, \$ 487,600,00, is the upper bound of the exact real value.

# A Models and Solution Reports in Alternative Settings

## A.1 Price of Steel Increased to \$1250/unit and Export 1 Unit of Steel

### A.1.1 Model

$$
\begin{aligned}
\max \quad & 1250a + 3000b + 2500c - 300x - 500y - 150z, \\
\text{subject to} \quad & x \leq 300000 \\
& y \leq 550000 \\
& z \leq 50000 \\
& 0.5x + 3y + 5z \leq 1200000 \\
& x - a - y - 0.75z \geq 0 \\
& y - b - 0.08x - 0.12z \geq 0 \\
& z - c - 0.05x - 0.1y \geq 0 \\
& a = 1 \\
& a, b, c, x, y, z \geq 0.
\end{aligned}
\tag{8}
$$

### A.1.2 Solution Report

```
MAX 1250A+3000B+2500C-300X-500Y-150Z
SUBJECT TO
STEEL_CAP)X <= 300000
TRUCK_CAP)Y <= 550000
EQUIP_CAP)Z <= 50000
MANPOWER) 0.5X+3Y+5Z <= 1200000
STEEL_REM)X-A-Y-0.75Z >= 0
TRUCK_REM)Y-B-0.08X-0.12Z >= 0
EQUIP_REM)Z-C-0.05X-0.1Y >= 0
STEEL_EXP) A = 1
END
```

```
Global optimal solution found.
Objective value:                              0.4906240E+09
Infeasibilities:                               0.000000
Total solver iterations:                              1
Elapsed runtime seconds:                           0.10

Model Class:                                         LP

Total variables:                  5
Nonlinear variables:              0
Integer variables:                0

Total constraints:                8
Nonlinear constraints:            0

Total nonzeros:                  22
Nonlinear nonzeros:               0


                        Variable           Value        Reduced Cost
                               A        1.000000            0.000000
                               B        232499.0            0.000000
                               C        8750.100            0.000000
                               X        300000.0            0.000000
                               Y        262499.0            0.000000
                               Z        50000.00            0.000000

                             Row    Slack or Surplus      Dual Price
                               1      0.4906240E+09         1.000000
                       STEEL_CAP        0.000000            1585.000
                       TRUCK_CAP        287501.0            0.000000
                       EQUIP_CAP        0.000000            302.5000
                        MANPOWER        12503.00            0.000000
                       STEEL_REM        0.000000           -2250.000
                       TRUCK_REM        0.000000           -3000.000
                       EQUIP_REM        0.000000           -2500.000
                       STEEL_EXP        0.000000           -1000.000
```

Figure 4: LINGO Solution Report

## A.2 Expand Steel Capacity by 300

### A.2.1 Model

$$
\begin{aligned}
\max \quad & 900a + 3000b + 2500c - 300x - 500y - 150z, \\
\text{subject to} \quad & x \leq 300300 \\
& y \leq 550000 \\
& z \leq 50000 \\
& 0.5x + 3y + 5z \leq 1200000 \\
& x - a - y - 0.75z \geq 0 \\
& y - b - 0.08x - 0.12z \geq 0 \\
& z - c - 0.05x - 0.1y \geq 0 \\
& a, b, c, x, y, z \geq 0.
\end{aligned} \tag{9}
$$

### A.2.2 Solution Report

```
MAX 900A+3000B+2500C-300X-500Y-150Z
SUBJECT TO
STEEL_CAP)X <= 300300
TRUCK_CAP)Y <= 550000
EQUIP_CAP)Z <= 50000
MANPOWER) 0.5X+3Y+5Z <= 1200000
STEEL_REM)X-A-Y-0.75Z >= 0
TRUCK_REM)Y-B-0.08X-0.12Z >= 0
EQUIP_REM)Z-C-0.05X-0.1Y >= 0
END
```

```
Global optimal solution found.
Objective value:                              0.4911005E+09
Infeasibilities:                              0.000000
Total solver iterations:                             4
Elapsed runtime seconds:                          0.08

Model Class:                                        LP

Total variables:                      6
Nonlinear variables:                  0
Integer variables:                    0

Total constraints:                    8
Nonlinear constraints:                0

Total nonzeros:                      24
Nonlinear nonzeros:                   0


                                Variable           Value        Reduced Cost
                                       A        0.000000            1350.000
                                       B        232776.0            0.000000
                                       C        8705.000            0.000000
                                       X        300300.0            0.000000
                                       Y        262800.0            0.000000
                                       Z        50000.00            0.000000

                                     Row    Slack or Surplus      Dual Price
                                       1      0.4911005E+09         1.000000
                               STEEL_CAP        0.000000            1585.000
                               TRUCK_CAP        287200.0            0.000000
                               EQUIP_CAP        0.000000            302.5000
                                MANPOWER        11450.00            0.000000
                               STEEL_REM        0.000000           -2250.000
                               TRUCK_REM        0.000000           -3000.000
                               EQUIP_REM        0.000000           -2500.000
```

Figure 5: LINGO Solution Report

## A.3 Expand Equipment Capacity by 1000

### A.3.1 Model

$$
\begin{aligned}
\max \quad & 900a + 3000b + 2500c - 300x - 500y - 150z, \\
\text{subject to} \quad & x \leq 300000 \\
& y \leq 550000 \\
& z \leq 51000 \\
& 0.5x + 3y + 5z \leq 1200000 \\
& x - a - y - 0.75z \geq 0 \\
& y - b - 0.08x - 0.12z \geq 0 \\
& z - c - 0.05x - 0.1y \geq 0 \\
& a, b, c, x, y, z \geq 0.
\end{aligned} \tag{10}
$$

### A.3.2 Solution Report

```
MAX 900A+3000B+2500C-300X-500Y-150Z
SUBJECT TO
STEEL_CAP)X <= 300000
TRUCK_CAP)Y <= 550000
EQUIP_CAP)Z <= 51000
MANPOWER) 0.5X+3Y+5Z <= 1200000
STEEL_REM)X-A-Y-0.75Z >= 0
TRUCK_REM)Y-B-0.08X-0.12Z >= 0
EQUIP_REM)Z-C-0.05X-0.1Y >= 0
END
```

```
Global optimal solution found.
Objective value:                              0.4909275E+09
Infeasibilities:                              0.000000
Total solver iterations:                             4
Elapsed runtime seconds:                          0.09

Model Class:                                        LP

Total variables:                      6
Nonlinear variables:                  0
Integer variables:                    0

Total constraints:                    8
Nonlinear constraints:                0

Total nonzeros:                      24
Nonlinear nonzeros:                   0


                       Variable           Value        Reduced Cost
                              A        0.000000            1350.000
                              B        231630.0            0.000000
                              C        9825.000            0.000000
                              X        300000.0            0.000000
                              Y        261750.0            0.000000
                              Z        51000.00            0.000000

                            Row    Slack or Surplus      Dual Price
                              1       0.4909275E+09        1.000000
                      STEEL_CAP        0.000000            1585.000
                      TRUCK_CAP        288250.0            0.000000
                      EQUIP_CAP        0.000000            302.5000
                       MANPOWER        9750.000            0.000000
                      STEEL_REM        0.000000           -2250.000
                      TRUCK_REM        0.000000           -3000.000
                      EQUIP_REM        0.000000           -2500.000
```

Figure 6: LINGO Solution Report

## A.4 World Market Price of the Imported Materials for Trucks Increase by \$400

### A.4.1 Model

$$
\begin{aligned}
\max \quad & 900a + 3000b + 2500c - 300x - 900y - 150z, \\
\text{subject to} \quad & x \leq 300000 \\
& y \leq 550000 \\
& z \leq 50000 \\
& 0.5x + 3y + 5z \leq 1200000 \\
& x - a - y - 0.75z \geq 0 \\
& y - b - 0.08x - 0.12z \geq 0 \\
& z - c - 0.05x - 0.1y \geq 0 \\
& a, b, c, x, y, z \geq 0.
\end{aligned} \tag{11}
$$

### A.4.2 Solution Report

```
MAX 900A+3000B+2500C-300X-900Y-150Z
SUBJECT TO
STEEL_CAP)X <= 300000
TRUCK_CAP)Y <= 550000
EQUIP_CAP)Z <= 50000
MANPOWER) 0.5X+3Y+5Z <= 1200000
STEEL_REM)X-A-Y-0.75Z >= 0
TRUCK_REM)Y-B-0.08X-0.12Z >= 0
EQUIP_REM)Z-C-0.05X-0.1Y >= 0
END
```

```
Global optimal solution found.
Objective value:                              0.3856250E+09
Infeasibilities:                               0.000000
Total solver iterations:                              4
Elapsed runtime seconds:                           0.08

Model Class:                                         LP

Total variables:                   6
Nonlinear variables:               0
Integer variables:                 0

Total constraints:                 8
Nonlinear constraints:             0

Total nonzeros:                   24
Nonlinear nonzeros:                0


                        Variable           Value        Reduced Cost
                               A        0.000000            950.0000
                               B        232500.0            0.000000
                               C        8750.000            0.000000
                               X        300000.0            0.000000
                               Y        262500.0            0.000000
                               Z        50000.00            0.000000

                             Row    Slack or Surplus      Dual Price
                               1       0.3856250E+09        1.000000
                       STEEL_CAP        0.000000            1185.000
                       TRUCK_CAP        287500.0            0.000000
                       EQUIP_CAP        0.000000            602.5000
                        MANPOWER        12500.00            0.000000
                       STEEL_REM        0.000000           -1850.000
                       TRUCK_REM        0.000000           -3000.000
                       EQUIP_REM        0.000000           -2500.000
```

Figure 7: LINGO Solution Report

## A.5 Minister of Defense Stockpile an Additional 10,000 Units of Steel

### A.5.1 Model

$$
\begin{aligned}
\max \quad & 900a + 3000b + 2500c - 300x - 500y - 150z, \\
\text{subject to} \quad & x \leq 300000 \\
& y \leq 550000 \\
& z \leq 50000 \\
& 0.5x + 3y + 5z \leq 1200000 \\
& x - a - y - 0.75z \geq 10,000 \\
& y - b - 0.08x - 0.12z \geq 0 \\
& z - c - 0.05x - 0.1y \geq 0 \\
& a, b, c, x, y, z \geq 0.
\end{aligned} \tag{12}
$$

### A.5.2 Solution Report

```
MAX 900A+3000B+2500C-300X-500Y-150Z
SUBJECT TO
STEEL_CAP)X <= 300000
TRUCK_CAP)Y <= 550000
EQUIP_CAP)Z <= 50000
MANPOWER) 0.5X+3Y+5Z <= 1200000
STEEL_REM)X-A-Y-0.75Z >= 1000
TRUCK_REM)Y-B-0.08X-0.12Z >= 0
EQUIP_REM)Z-C-0.05X-0.1Y >= 0
END
```

```
Global optimal solution found.
Objective value:                              0.4883750E+09
Infeasibilities:                              0.000000
Total solver iterations:                             4
Elapsed runtime seconds:                          0.08

Model Class:                                        LP

Total variables:                  6
Nonlinear variables:              0
Integer variables:                0

Total constraints:                8
Nonlinear constraints:            0

Total nonzeros:                  24
Nonlinear nonzeros:               0


                             Variable           Value        Reduced Cost
                                    A        0.000000            1350.000
                                    B        231500.0            0.000000
                                    C        8850.000            0.000000
                                    X        300000.0            0.000000
                                    Y        261500.0            0.000000
                                    Z        50000.00            0.000000

                                  Row    Slack or Surplus      Dual Price
                                    1       0.4883750E+09        1.000000
                            STEEL_CAP        0.000000            1585.000
                            TRUCK_CAP        288500.0            0.000000
                            EQUIP_CAP        0.000000            302.5000
                             MANPOWER        15500.00            0.000000
                            STEEL_REM        0.000000           -2250.000
                            TRUCK_REM        0.000000           -3000.000
                            EQUIP_REM        0.000000           -2500.000
```

Figure 8: LINGO Solution Report

## A.6 Strike influence

### A.6.1 Model

$$
\begin{aligned}
\max \quad & 900a + 3000b + 2500c - 300x - 500y - 150z, \\
\text{subject to} \quad & x \leq 300000 \\
& y \leq 550000 \\
& z \leq 40000 \\
& 0.5x + 3y + 5z \leq 1200000 \\
& x - a - y - 0.75z \geq 0 \\
& y - b - 0.08x - 0.12z \geq 0 \\
& z - c - 0.05x - 0.1y \geq 0 \\
& a, b, c, x, y, z \geq 0.
\end{aligned} \tag{13}
$$

### A.6.2 Solution Report

```
MAX 900A+3000B+2500C-300X-500Y-150Z
SUBJECT TO
STEEL_CAP) X <= 300000
TRUCK_CAP) Y <= 550000
EQUIP_CAP) Z <= 40000
MANPOWER) 0.5X+3Y+5Z <1200000
STEEL_SOLD) X-Y-0.75Z-A >= 0
TRUCK_SOLD) Y-0.08X-0.12Z-B >= 0
EQUIP_SOLD) Z-0.05X-0.1Y-C >= 0
END
```

```
Global optimal solution found.
Objective value:                              0.4664667E+09
Infeasibilities:                               0.000000
Total solver iterations:                              5
Elapsed runtime seconds:                           0.03

Model Class:                                         LP

Total variables:                      6
Nonlinear variables:                  0
Integer variables:                    0

Total constraints:                    8
Nonlinear constraints:                0

Total nonzeros:                      24
Nonlinear nonzeros:                   0


                      Variable           Value        Reduced Cost
                             A        0.000000            293.3333
                             B        228933.3            0.000000
                             C        0.000000            10566.67
                             X        286666.7            0.000000
                             Y        256666.7            0.000000
                             Z        40000.00            0.000000

                           Row    Slack or Surplus      Dual Price
                             1      0.4664667E+09        1.000000
                     STEEL_CAP        13333.33            0.000000
                     TRUCK_CAP        293333.3            0.000000
                     EQUIP_CAP        0.000000            11661.67
                      MANPOWER        86666.67            0.000000
                    STEEL_SOLD        0.000000           -1193.333
                    TRUCK_SOLD        0.000000           -3000.000
                    EQUIP_SOLD        0.000000           -13066.67
```

Figure 9: LINGO Solution Report