

## Module 7 Chi-Squared Test

1. For a fair six-sided die, the probability of any given outcome on a single roll would be  $1/6$ . The data in Table were obtained by rolling a six-sided die 36 times. Are these data consistent with the hypothesis that the die is a fair die?

Outcome	Frequency
1	8
2	5
3	9
4	2
5	7
6	5

### Solution

$H_0$ : the probability of any given outcome on a single roll would be  $1/6$ .  $H_a$ : Not all the probability of any given outcome on a single roll are  $1/6$ . The expected frequency of each outcome is  $36/6=6$ .

$$\chi^2 = \frac{(8-6)^2}{6} + \frac{(5-6)^2}{6} + \frac{(9-6)^2}{6} + \frac{(2-6)^2}{6} + \frac{(7-6)^2}{6} + \frac{(5-6)^2}{6} = 5.33$$

df=6-1=5 and P-value= $P(\chi^2 > 5.33)=0.3769 > 0.05$ .

Therefore, the null hypothesis that the die is fair cannot be rejected at 0.05 significance level.

2. A safari park in Africa is divided into 8 zones, each containing a known population of elephants. A sample is taken of the number of elephants found in each zone to determine whether the distribution of elephants is significantly different from what would be expected based on the known population in each zone.

Zone	Sample	Population
1	7	24
2	11	27
3	5	20
4	6	22
5	9	35
6	4	17
7	5	32
8	8	28
Total	55	205

### Solution

$H_0$ : The distribution of elephants in the sample among the zones is the same as the known population.  $H_a$ : The distribution of elephants in the sample among the zones is different from the known population. Thus, we have

Zone	Sample	Population	Expected	$\frac{(O-E)^2}{E}$
1	7	24	$\frac{24}{205} \times 55 = 6.4390$	$\frac{(7-6.4390)^2}{6.4390} = 0.0489$
2	11	27	$\frac{27}{205} \times 55 = 7.2439$	$\frac{(11-7.2439)^2}{7.2439} = 1.9476$
3	5	20	$\frac{20}{205} \times 55 = 5.3658$	$\frac{(5-5.3658)^2}{5.3658} = 0.0249$
4	6	22	$\frac{22}{205} \times 55 = 5.9024$	$\frac{(6-5.9024)^2}{5.9024} = 0.0016$
5	9	35	$\frac{35}{205} \times 55 = 9.3902$	$\frac{(9-9.3902)^2}{9.3902} = 0.0162$
6	4	17	$\frac{17}{205} \times 55 = 4.5610$	$\frac{(4-4.5610)^2}{4.5610} = 0.0690$
7	5	32	$\frac{32}{205} \times 55 = 8.5854$	$\frac{(5-8.5854)^2}{8.5854} = 1.4973$
8	8	28	$\frac{28}{205} \times 55 = 7.5122$	$\frac{(8-7.5122)^2}{7.5122} = 0.0317$
Total	55	205		

$$\chi^2 = 0.0489 + 1.9476 + 0.0249 + 0.0016 + 0.0162 + 0.0690 + 1.4973 + 0.0317 = 3.6372$$

df=8-1=7 and P-value= $P(\chi^2 > 3.6372) = 0.82 > 0.05$ .

Therefore, we don't reject the null hypothesis, and conclude there is no significant difference between the distribution of elephants in the sample among the zones compared with the known population.