Module 5 Small Sample Size

1. You are interested in the average emergency room (ER) wait time at your local hospital. You take a random sample of 50 patients who visit the ER over the past week. From this sample, the mean wait time was 30 minutes and the standard deviation was 20 minutes. Find a 95% confidence interval for the average ER wait time for the hospital.

Solution: Sample size $n = 50$, so df=50-1=49. Sample mean $\bar{x} = 30$ and sample standard deviation $s = 20$. Since the population standard deviation is unknown, we use the sample standard deviation.
The 95% Confidence Interval:

$$\bar{x} \pm t_{.025,49} \times \frac{s}{\sqrt{n}} = 30 \pm 2.009 \times \frac{20}{\sqrt{5}} = (24.32, 35.68)$$

Interpreting our interval: We are 95% confident that mean emergency room wait time at our local hospital is from 24.32 minutes to 35.68 minutes.

2. The mean length of the lumber is supposed to be 8.5 feet. A builder wants to check whether the shipment of lumber she receives has a mean length different from 8.5 feet. If the builder observes that the sample mean of 61 pieces of lumber is 8.3 feet with a sample standard deviation of 1.2 feet. What will she conclude? Conduct this test at a 1% level of significance.
Solution:

Step 1: Develop Hypotheses $H_0 : \mu = 8.5$ vs $H_a : \mu \neq 8.5$.

Step 2: Test Statistic $t = \frac{\bar{x}\mu_0}{s/\sqrt{n}} = \frac{8.3-8.5}{1.2/\sqrt{61}} = -1.30$, df=61-1=60.

Step 3: Determine P-value P-value $= P(t < -1.30) + P(t > 1.30) = 2P(t < -1.30) = 0.1986$.

Step 4: Make A Conclusion: With a p-value of 0.1986 which exceed our significance level, we fail to reject the null hypothesis at a 1% level of significance. We conclude that there is not enough statistical evidence that indicates that the mean length of lumber differs from 8.5 feet.

3. The administrator at your local hospital states that on weekends the average wait time for emergency room visits is 10 minutes.Based on discussions you have had with friends who have complained on how long they waited to be see in in the ER over a weekend, you dispute the administrator's claim. You decide to test you hypothesis. Over the course of a few weekends you record the wait time for 40 randomly selected patients. The average wait time for these 40 patients is 11 minutes with a standard deviation of 3 minutes.Do you have enough evidence to support your hypothesis that the average ER wait time exceeds 10 minutes? You opt to conduct the test at a 5% level of significance.

Solution:

Step 1: Develop Hypotheses $H_0 : \mu = 10$ vs $H_a : \mu > 10$.

Step 2: Test Statistic $t = \frac{\bar{x}\mu_0}{s/\sqrt{n}} = \frac{11-10}{3/\sqrt{40}} = 2.11$, df=40-1=39.

Step 3: Determine P-value P-value $= P(t > 2.11) = 0.021$.

Step 4: Make A Conclusion: P-value is 0.0021 which does not exceed our significance level, we can reject the null hypothesis. We have statistical evidence at the 5% level of significance to conclude that the average emergency wait time at the hospital is more than 10 minutes.