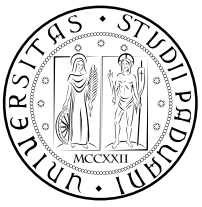

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Batch Size Estimate

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Ai miei genitori e allo zio Toni...

“ [...] sempre nella mia camicia [...]
Ho ancora la forza di chiedere anche scusa
o di incazzarmi ancora con la coscienza offesa,
di dirvi che comunque la mia parte
ve la posso garantire . . . ”
Francesco Guccini

“ E quando pensi che sia finita,
è proprio allora che comincia la salita.
Che fantastica storia è la vita. ”
Antonello Venditti

Abstract

In this work we analyze the main *batch resolution algorithms*. We particularly focus on the *tree-based class* to underline how their efficiency depends on the batch size. In fact, batch size is a critical parameter when using smart resolution strategies that take advantage this information to improve resolution efficiency. The dissertation will continue with the analysis of noteworthy techniques available in literature for the *batch size estimate*: in fact, original papers pay attention on the resolution process and leave the estimate problem in the background. Finally we propose and analyze *GEGA*, an estimate algorithm particularly good in terms of estimate accuracy over time taken by the estimate process.

Sommario

In questo lavoro sono stati analizzati i principali algoritmi per la risoluzione di insiemi di conflitto. In particolare ci si è concentrati sugli algoritmi ad albero evidenziando come la loro efficacia dipenda dalla taglia del problema. La cardinalità dell'insieme di collisione è infatti un parametro critico se si desidera utilizzare strategie ottimizzate per risolvere più efficientemente i nodi. A tal fine sono state analizzate, in primo luogo, le tecniche note in letteratura per la stima della cardinalità di insiemi di conflitto: infatti i paper originali non presentavano un'analisi adeguata del loro comportamento poiché l'attenzione era rivolta al processo di risoluzione dei nodi e non direttamente alla fase di stima. Nella parte finale del lavoro, al fine di ottenere una stima sufficientemente accurata per finalità operative, viene proposto *GEGA*. L'algoritmo presentato è particolarmente valido in termini di accuratezza della stima rispetto al tempo impiegato e permette di ottenere sempre una stima finita del numero di nodi sufficientemente vicina alla reale cardinalità dell'insieme esaminato.

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Chapter 1

Introduction

Generally speaking a set of actors contending for a common resource defines a *conflicting set*. As always, limited resources require access policies to provide efficient and, hopefully, fair use. When the system is distributed, resource access can be assimilated to a coordination problem. In the scenario considered in this thesis the contended resource is the physical transmission medium that is shared among several stations.
At the beginning of wired computer networks, multiple access control (MAC) was a big issue for efficient communication. The introduction of buffered switches in LANs reduced the conflicting set to only few stations simplifying the original problem. Switched networks, in fact, split large collision domains into smaller pieces thus realizing ring, star or mesh structures.
In a wireless context the problem can not be easily overcome, due to the broadcast nature of the wireless medium.

Nowadays wireless connectivity in pervasive computing has ephemeral character and can be used for creating ad-hoc networks, sensor networks, connections with RFID (Radio Frequency Identification) tags etc. The communication tasks in such wireless networks often involve an inquiry over a shared channel, which can be invoked for discovery of neighboring devices in ad-hoc networks, counting the number of RFID tags that have a certain property, estimating the mean value contained in a group of sensors, etc. Such an inquiry solicits replies from a possibly large number of terminals.

In particular we analyze the scenario where a reader broadcasts a query to the in-range nodes. Once the request is received, devices with data of interest are all concerned in transmitting the information back to the inquirer as soon as possible and, due to the shared nature of the communication medium, collision problems come in: only one successful transmission at time can be accomplished, concurrent transmissions result in destructive interference with waste of

energy/time. This data traffic shows a bursty nature which is the worst case for all shared medium scenarios.

In the literature this problem is referred to with different names: *Batch/Conflict Resolution Problem*, *Reader Collision Problem*, *Object Identification Problem.*
Algorithms trying to solve this problem are called *Batch Resolution Algorithms* (BRA) or *Collision Resolution Algorithms* (CRA).

Batch Resolution Problem is implicitly present in many practical applications over wireless networks such as:

- *Neighbor Discovery.* After being deployed, nodes generally need to discover their neighbors, which is an information required by almost all routing protocols, medium-access control protocols and several other topology-control algorithms, such as construction of minimum spanning trees. Ideally, nodes should discover their neighbors as quickly as possible since rapid discovery of neighbors often translates into energy efficiency and allows for other tasks to quickly start their execution on the system.

- *Batch Polling.* It consists in collecting a possibly very large number of messages from different devices in response to *time-driven* or *event-driven* events. *Time-driven* batch polling takes place when an inquirer periodically broadcasts a request to the nodes. *Event-driven* batch polling takes place when nodes send packets because triggered by the occurrence of events of interest. The problem is not properly a *Batch Resolution Problem* when the aim is to obtain only one message out of n as rapidly as possible. This case was studied in [4].

- *Object identification.* Physical objects are bridged to virtual ones by attaching to each object a sensor or an RFID tag. This allows asset tracking (e.g. libraries, animals), automated inventory and stock-keeping, toll collecting and similar tasks. Wireless connection allows unobtrusive management and monitoring of resources.

In these applications:

- Communications show spatially and timely correlated contention for channel access;

- In general, nodes multiplicity is time-varying. When a node wakes up it has no knowledge of the environment around it. In particular this holds when nodes sleep for most of time and seldom wake up to transmit.

To adapt to any scenario, BRAs can be oblivious of the batch multiplicity n: in this case, however, the expected batch resolution time (often referred to as *batch resolution interval,*

BRI) in not optimal. In fact, the knowledge of the conflict multiplicity n is the most critical factor to optimize the batch resolution and to allow the usage of advanced resolutions schemes characterized by higher resolution efficiency. For this reason the *Batch Size Estimate* is pivotal.

1.1 System Model

We consider the following standard model of a multiple access channel. A large number of geographically distributed nodes communicate through a common radio channel. Any node generates a single packet to be transmitted over the channel. The set of nodes with a packet to transmit constitutes the *batch* whose size is unknown. When not otherwise stated, we consider a pure-slotted system where time is partitioned in units of the same length, called *slots*.

In *pure-slotted* systems nodes are synchronized at slot boundaries. Nodes can start a transmission only at the beginning of the slot, otherwise they will stay quiet until the next slot to come. Each transmission lasts a single slot.

A different model refers to *carrier-sense multiple-access (CSMA)* networks, where each node is able to determine the beginning of a new slot by sensing the energy on the channel: when the channel is idle a device can start transmitting its message. In our scenario we assume that all the transmitted messages have a fixed duration. Once a node has started transmitting it can not sense the channel so that it can not be aware of the result of its transmission until it receives feedback. For this reason we have that a transmission always takes the same time, whether it results in a success or a collision. On the other hand, empty slots take less time than transmissions. Usually the duration of transmissions and idle slots are identified respectively by T_p and T_s. An important parameter in the CSMA channel is the *Carrier Sense Factor* $\beta = \frac{T_s}{T_p} < 1$.

We assume that there is no external source of interference and that a transmission fails only in case of collision. In short, saying k nodes transmit simultaneously in a slot, the following events may occur:

- $k = 0$: there are no transmissions in the slot, which is said to be *empty* or *idle*;
- $k = 1$: a single node transmits in the slot, which is said to be *successful*;
- $k \geq 2$: more than one node transmits, so that a collision occurs and the slot is said to be

collided.

Furthermore, throughout this work we assume that no new message is generated by the system while it is running an *estimate* or *resolution algorithm*. In other words, newly generated packets are inhibited from being transmitted while an algorithm is in progress and they will eventually be considered only in the subsequent estimate or resolution process. This way to manage the channel access of the nodes in the system is known as *obvious-access scheme*.

1.2 Goals

In this work, we will study and characterize different estimate techniques considering the relation between the estimate accuracy and the time required to provide it.
Most works concerning estimate techniques (e.g.: [7, 8]) define the estimate algorithm but do not analyze the "quality" of the estimate nor the time taken by the process. In fact, after proposing an estimate scheme, the authors focus on the definition of an optimized resolution scheme, considering perfect knowledge of the batch size n and ignoring the fact that the estimate of n is actually error prone, unless the batch is completely resolved.
In this work we will focus on the estimate phase preferring analytical error study when possible and using computer based simulations otherwise.
Finally, we will propose improvements to a technique in order to achieve better estimate quality and we will compare all the considered estimate algorithms to provide a comprehensive overview of pros and cons of each solution.

1.3 Content organization

This thesis is organized as follows.

- In Chapter 2 we will introduce the *Batch Resolution Problem* that motivates the study of *Batch Size Estimate Problem*. We will describe in details the fundamental schemes proposed in the literature and provide an overview of the most recent and advanced schemes. In particular we will concentrate on *binary tree algorithms*.
- Chapter 3 introduces the *Batch Size Estimate Problem*. We describe and analyze some noteworthy batch size estimate algorithm, both for slotted and framed scenarios.
- Chapter 4 is the core of the work. We will analytically and numerically analyze the algorithms described in Chapter 3 to provide a comprehensive evaluation of the tecniques.

Hence, we will propose and analyze a new algorithm, namely *GEGA*, with the goal to provide a good estimate as quickly as possible. Finally, we will compare the described algorithms.

- The last Chapter draws conclusions.

Chapter 2

Batch Resolution

Pure-ALOHA was the very first random-access protocol ever developed. It is as simple as possible:

- If you have data to send, send the data immediatly
- If the message collides with another transmission, try resending "later"

Slotted-ALOHA is an improvement over Pure-ALOHA in which time is *slotted* and transmissions can start only at the beginning of a slot boundary. Slotted-ALOHA assumes there is feedback from the receiver at the end of each slot so that all nodes learn whether or not a collision occurred.
ALOHA protocol was studied under the assumption that a large number of identical sources transmit on the channel such that the number of new packets generated during any slot is a Poisson random variable with mean λ (packets/slot).
Slotted ALOHA has equilibrium rate (maximum throughput) of $1/e \approx 0.368$ packets/slot but it has proven maximum stable throughput 0 (hence it is unstable).
The attempt to obtain stable throughput random-access protocols brought to the discovery of CRAs.

A *Collision Resolution Algorithm* (CRA) can be defined as a random-access protocol such that, whenever a collision occurs, then at some later time all senders will simultaneously learn from the feedback information that all packets involved in that collision have now been successfully transmitted. The crux of collision resolution is the exploitation of the feedback information to control the “random” retransmission process.

CRAs are interesting since their are able to solve conflicts of unknown multiplicities. Furthermore they are not tailored to solve only conflicts among packets arrivals characterized by

Poisson's distributions but they are robust since they work for any arrival process characterized by an average arrival rate λ.

CRAs can also be used to solve collisions among a batch of nodes which have a message to deliver. In this case CRAs are commonly called *Batch Resolution Algorithms* (BRAs).

In particular, the scenario we consider is the following: the reader probes a set of nodes. In-range devices try to reply as soon as possible transmitting in the wireless medium. If two ore more devices reply at the same time we get a collision and the delivery of the messages fails. Consequently we require each node to run a distributed algorithm which implements anti-collision schemes in order to resolve all the nodes.
There are many algorithms that enable batch resolution, and these, according to [3], can be classified into two categories: (a) *probabilistic*, and (b) *deterministic* [1].

In *probabilistic algorithms*, a framed ALOHA[2] scheme is used where the reader communicates the frame length, and the nodes pick a particular slot in the frame to transmit. The reader repeats this process until all nodes have transmitted at least once successfully in a slot without collisions.

Deterministic algorithms typically use a slotted ALOHA model tries to reduce the contending batch in the next slot based on the transmission result in the previous one. These algorithms fall into the class of tree-based algorithms with the nodes classified on a binary tree and the reader moving down the tree at each step to identify all nodes.

Deterministic algorithms are typically faster than probabilistic schemes in terms of actual node response slots used, however, they suffer from reader overhead since the reader has to specify address ranges to isolate subsets of contending nodes using a probe at the beginning of each slot.

Deterministic schemes assume that each node can understand and respond to complex commands from the reader, such as responding only if the *id* is within an address range specified

[1]Both classes of algorithms use a probabilistic approach to solve the problem. As aforementioned, CRA were initially thought to solve Poisson-like packets arrivals. Hence, each packet is characterized by an arrival time. CRAs are able so solve collisions respecting the *first-come first-served* (FCFS) policy. In *Batch Resolution Problem* packets can be associated to virtual arrival times. Once the arrival time (alias node ID) is fixed each run of the same *deterministic* algorithm behaves in the same way and nodes always get resolved in the same order and time amount. On the other hand, *probabilistic* algorithms are not able to maintain the relative order among resolved nodes in different runs of the algorithms even if they globally solve the same initial collision.

[2]framed ALOHA is a modification of slotted ALOHA where consecutive slots are grouped. Each node is allowed to choose only one slot per group. Its transmission is allowed to take place only in the choosed slot.

by the reader. Consequently not every device is able to support this class of algorithms. For example passive tags, which are the most dummy devices, cannot understand this kind of requests and they will continue to transmit in every resolution cycle. This lengthens the total time needed for the resolution process to complete. Wireless sensors, semi-active and active tags should allow to implement tree-based algorithms: the reader can acknowledge nodes that have succeeded at the end of each slot (immediate feedback), and hence those nodes can stay silent in subsequent slots, reducing the probability of collisions thereby shortening the overall identification time. Usually a node that successfully transmits its message is said *resolved* and stays silent until the end of the algorithm.
Furthermore, since tree algorithms require explicit feedback about channel status, they force devices to be alway active and listening to the channel in each step of the algorithm. Moreover, reader feedback in each slot adds overhead to the resolution.
On the other hand windows based algorithms are more energy saving since a device can sleep for most of time in the transmission window and only wake up in the slot it has decided to transmit on. In a windows of w slots a node will be up only for $1/w$ of time and wait for feedback at the end of the window.

Most of the batch resolution algorithms were originally developed for slotted scenarios. These algorithms can be flawlessly ported to the CSMA scheme.

2.1 Binary Tree Algorithms

In '70s, concern over the instability of most ALOHA-like protocols led some researchers to look for random-access schemes that were provably stable. The breakthrough in these efforts was reached in 1977 by J. Capetanakis [12], then a MIT doctoral student working with Prof. R. Gallager[3], and independently achieved shortly thereafter by two Soviet researchers, B. Tsybakov and V. Mihhailov [11]. Basic Binary Tree is the result of their studies.
In the following sections we will describe and analyze the *Basic Binary Tree Algorithm* and some of its variants applied to the *Batch Resolution Problem*.

We notice that *tree algorthims require for the nodes in the batch to be unique*. This is a necessity since otherwise it would be impossible to force two nodes to behave in a different way. In this Chapter we will refer to this node uniqueness property as *id*, *address*, *token*. Throughout the following sections, we will consider *ids*, *addresses* and *tokens* equivalent.

We also assume to operate in a *slotted*-ALOHA-like scenario, unless otherwise specified.

[3]FCFS splitting algorithm for Poisson's arrivals is one of his famous contributions.

2.1.1 Basic Binary Tree

Let's consider a batch $\mathcal{B}$ of size n.
Initially all the nodes try to transmit and we can have, according to what expressed in Section 1.1, three different events: *idle*, *success*, *collision*.
The supervisor broadcasts the result of the transmission to all the nodes.
If we get *idle* or *success* events, the resolution process stops meaning respectively that there were no nodes to resolve or there was only one node that was successfully resolved. That node delivered its message and will no longer take part in the current batch resolution.
In case of *collision* we know that at least 2 nodes are present and we have to solve the collision to obtain their messages. In this case all the n nodes run the same algorithm.
Each node chooses to transmit with probability p and not to transmit with probability $1-p$. Nodes that transmit are said to belong to set $\mathcal{R}$ while the others to set $\mathcal{S}$. Of course $\mathcal{R} \cap \mathcal{S} = \emptyset$ and $\mathcal{B} = \mathcal{R} \cup \mathcal{S}$. The two sets are then resolved recursively starting from $\mathcal{R}$.
Nodes in $\mathcal{S}$ wait until all terminals in $\mathcal{R}$ transmit successfully their packets, then they transmit.

Algorithm 1 BINARY TREE ($\mathcal{B}$)

```
  // current slot status can be idle, success, collision
Input: B batch with |B| = n
  each node transmits its message
  if (idle or success) then
      return
  else
      each node flips a fair coin
      R ← { nodes that flipped head}
      S ← { nodes that flipped tail}
      BINARY TREE (R)
      BINARY TREE (S)
  end if
```

Pseudo-code in Alg. 1 provides a very high level description of the splitting technique. The role of the supervisor is implicit in the code.

Intuitively setting $p = 1/2$ can be a good choice since the algorithm is in some sense "symmetric": best performance are achived when $\mathcal{R}$ and $\mathcal{S}$ are balanced.

Let L_n^{BT} be the expected running time in slots required to resolve a conflict among n nodes using the *Basic Binary Tree Algorithm* (BT). Let $Q_i(n) = \binom{n}{i} p^i (1-p)^{n-i}$ be the probability that i among n nodes decide to transmit in a slot (probability that $|\mathcal{R}| = i$). If i nodes transmit

we have first to solve a conflict of size $|\mathcal{R}| = i$ with expected time L_i^{BT} and later a conflict of size $|\mathcal{S}| = n - i$ with expected time L_{n-i}^{BT}. L_n^{BT} is given by the cost of the current slot plus the expected time to solve all the possible decompositions of the current set.
L_n^{BT} can be recursively computed (considering the factorial in $Q_i(n)$) collecting L_n^{BT} in the following equation:

$$L_n^{BT} = 1 + \sum_{i=0}^{n} Q_i(n)(L_i^{BT} + L_{n-i}^{BT}), \tag{2.1}$$

with

$$L_0^{BT} = L_1^{BT} = 1.$$

To obtain an upper bound on the expected time as $n \to \infty$ further analysis techniques have to be used.
Here we want simply focus on how the algorithm behaves when n grows.

The behaviour of L_n^{BT} is presented in Figure 2.1 and was obtained evaluating equation (2.1) for $n = 0, 1, \ldots, 19$. L_n^{BT} grows almost linearly after the initial step from 1 to 2 which dramatically impacts on the performance ($L_2^{BT} = 5$).
Figure 2.1 also reports the results for MBT Alg. that will be introduced in following Section 2.1.2. We chose to anticipate the results for MBT to avoid inserting too many figures and, at the same time, provide a useful performance comparison between BT and MBT.

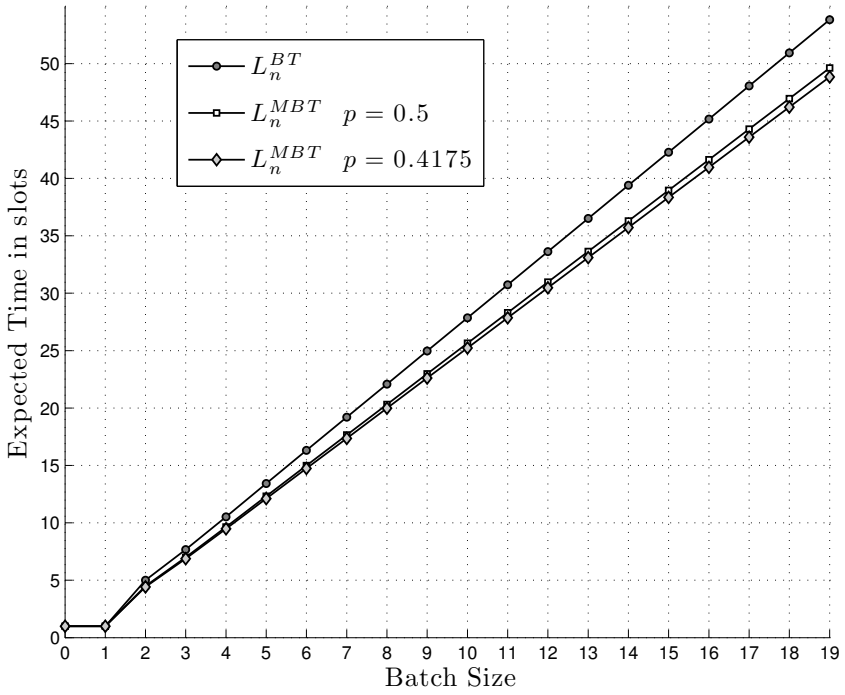


Figure 2.1: The plot illustrates the expected cost in slots to solve batches of size $n = 0, 1, \ldots, 19$ in a *slotted* Aloha-like scenario using all the basic variants of the tree based algorithms: BT, MBT with sub-optimal $p = 0.5$, MBT with optimal $p = 0.4175$. All the algorithms show the same behavior: almost linear grow for large n. Best performance are provided by MBT with optimal p.

Considering the efficiency $\eta_n = n/L_n^{BT}$ (number of resolved nodes over slots) we have a decreasing series $\eta_1 = 1$, $\eta_2 = 0.40$, $\eta_3 = 0.3913$, ..., $\eta_{16} = 0.3542$, ..., $\eta_{31} = 0.3505$. It can be shown [10] that $\eta_\infty \approx 0.347$.

Since the algorithm is much more efficient in solving small rather than large batches we would prefer to have (ideally) n batches of size 1 rather than 1 batch of size n.
Hence, knowing exactly the cardinality n of the initial batch $\mathcal{B}$, we can split the nodes into small groups, of approximately 1 node each, and resolve them faster.
This is the idea behind many improvements over the basic BT and it reveals the importance of having an accurate estimate of n to efficiently solve a batch.

Example

In Figure 2.2 we provide an example to further investigate the behavior of the algorithm. We notice that the instance starts with a collision in slot 1. Then nodes n_1, n_2, n_3 decide to proceed with a retransmission while n_4, n_5 remain idle. In slot 2 we see another collision, after it n_1 transmits again while n_2 and n_3 to stay quiet. In slot 3 we have the first resolution, n_1

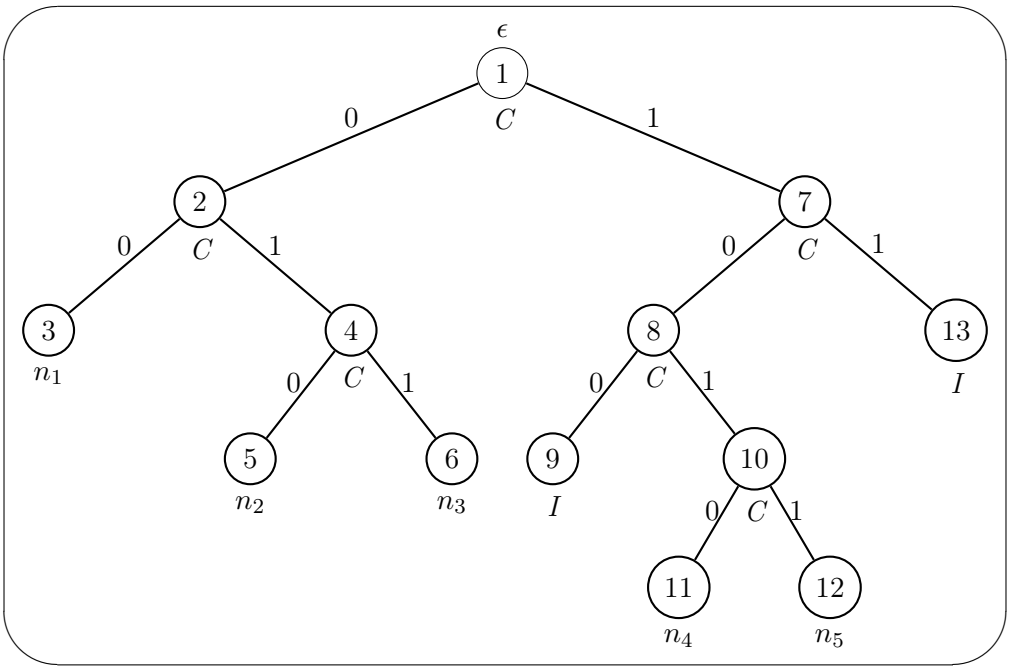


Figure 2.2: An istance of BT algorithm for $n = 5$ nodes. The number inside each circle identifies the slot number. The label below identifies the event occurring: I for *idle*, C for *collision*, n_i for resolution of node i. 0/1 branches is analogous to head/tail.

.

successfully send its message and leaves the collision resolution algorithm.

We notice that we can know the cardinality of a collision only after it has been fully resolved. For example we know only after slot 6 that the collision in slot 2 involved 3 nodes.

Nodes *id* interpretation

BRAs require each node to have a *unique id* to solve the batch. Usually the nodes *id*s are random generated at each algorithm run and can be used to identify a node inside the algorithm. There are multiple ways to generate that the *id*, such as:

- flipping a coin on demand after each collision (step-by-step *id* generation),
- generating a 'long enough' random binary string at the beginning of the algorithm.

We do not want to enter in the details of these choices since there is no reason to prefer one method to the others but device technical limitations.

Now we want to introduce an interesting interpretation of the *id* that will be used later in algorithms such as EBT (Section 2.1.5) and Cidon (Section 3.2).

In general any infinite length binary string $\mathbf{b_i} = (b_{i1}b_{i2}b_{i3}\ldots)$, with $b_{ij} \in \{0,1\}$, can be associated to a real number $r_i \in [0,1)$ by a bijective map r defined as follows:

$$r_i = r(\mathbf{b_i}) = \sum_{j=1}^{\infty} \frac{b_{ij}}{2^j} \tag{2.2}$$

Each node n_i can be associated to a point r_i within the real interval [0,1) as well as to the string $\mathbf{b_i}$.

For a finite length bit-string $\mathbf{a} = (a_1 a_2 \ldots a_L)$ with length $L = l(\mathbf{a})$ we adapt the definition of r as follows:

$$r(\mathbf{a}) = \sum_{j=1}^{L} \frac{a_j}{2^j} \tag{2.3}$$

In this case we have to carry L as auxiliary information to allow the map to remain bijective.
Following standard conventions, the empty string ϵ is prefix of any other string. It has length 0 and $r(\epsilon) = 0$.

Tree traversal rules

The duality in the interpretation of nodes' *id*s (bit-strings or real numbers) reflects on the duality of the tree traversal rules.
According to the adopted approach, enabled nodes can be specified by:

- a finite length string $\mathbf{a}$ which matches the path from the root to the first node in the sub-tree they belong to. In this case $\mathbf{a}$ is a prefix of the enabled nodes *id*s.
- the couple $\big(r(\mathbf{a}), l(\mathbf{a})\big)$ which enables the sub-interval $r(\mathbf{a}) \leq x < r(\mathbf{a}) + \dfrac{1}{2^{l(\mathbf{a})}}$

To complete the overview of the algorithm we now intuitively describe the tree visit. The following description uses the bit-string approach since a binary string can be immediately mapped to a path in the tree starting from the root. We assume, following the standard approach, to visit the tree in pre-order, giving precedence to left sub-trees, conventionally associated to 0 branches.
The visit starts from the root which has address ϵ.
Let $\mathbf{a}$ be the current enabled string, then the following rules apply:

- If $\mathbf{a} = \epsilon$ and last the event is a success or an idle slot then the whole conflict has been resolved;
- If the last event is a collision then we visit the left child of the current node ($\mathbf{a}0$);
- If the last event is a success or a collision and $\mathbf{a}_L = 0$ then we visit the right sibling of the current enabled node ($\mathbf{a}1$);

- If the last event is a success or a idle slot and $\mathbf{a}_L = 1$ then we look in the path back to the root for the first node whose sibling has not yet been visited. Since we visit the tree in pre-order the next enabled string will be in the form $\mathbf{a}_k 1$ with $k < L$.

A detailed pseudo-code of an algorithm that implements the rules above (and more) can be found in [2]. The *Modified Binary Tree* algorithm presented in next section gets a remarkable performance improvement over BT by adding just one new rule.

2.1.2 Modified Binary Tree

Modified binary tree is a simple way to improve the BT algorithm.
To keep the notation simple, we will explain the idea illustrating what happens the first time it is applied. In this case node τ is visited in slot τ. This does not holds in general, but explanation would have required to use two different indexed for slots and nodes, and Figure 2.3 would have been less immediate to understand.

The observation is that, during the tree traversal, sometimes we know in advance if the next slot will be collided. This happens when, after a collided slot τ, we get an idle slot $(\tau+1)$ in the left branch of the binary tree. In this case, visiting the right branch $(\tau + 2)$, we will certainly get a collision .
In fact, after sensing slot τ is collided, we know that there are at least 2 nodes in the last visited sub-tree. None of them belongs to the left-branch of that sub-tree since slot $(\tau + 1)$ is idle. Consequently they must be in the right branch of the sub-tree, whose enabling will hence result into a collision. This collision can be avoided by skipping node $(\tau + 2)$ and visiting its left-child node in slot $(\tau + 2)$.

Expected time analysis is analogous to Section 2.1.1. The only difference is that after a collision, if we get an idle slot, we will skip the "next one" (saving a slot that would certainly be wasted otherwise). Consequently the expected slot cost is $\big[1 \cdot \big(1 - Q_0(n)\big) + 0 \cdot Q_0(n)\big]$. Let L_n^{MBT} be the expected cost in slots to solve a batch of size n using the MBT Alg., then

$$L_n^{MBT} = \big(1 - Q_0(n)\big) + \sum_{i=0}^{n} Q_i(n)(L_i^{MBT} + L_{n-i}^{MBT}), \tag{2.4}$$

with

$$L_0^{MBT} = L_1^{MBT} = 1.$$

Intuitively in this case, since a higher probability to stay silent reduces the expected slot cost,

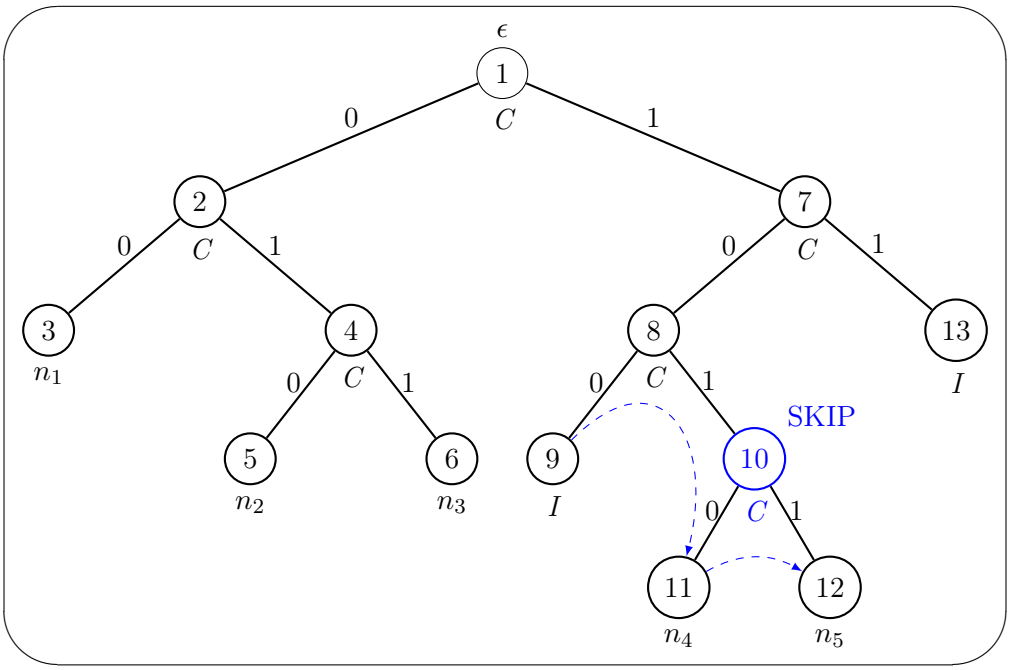


Figure 2.3: Same example as in Figure 2.2 but using MBT: tree structure do not change but node 10 is skipped in the traversal.

optimal transmit probability will be no longer equal to one half. At the same time, reducing the transmit probability will increase the number of (wasted) idle slots. Thus the new optimal probability p will be somewhere in the interval (0,0.5).

It can be shown [9] that the best result is achieved for $p = 0.4175$, for which the efficiency is asymptotically equal to $\eta \approx 0.381$. This is +10% higher than basic BT.

In general we have

$$L_n^{MBT} \leq C \cdot n + 1, \qquad \text{where} \qquad C = 2.623. \tag{2.5}$$

Using probability $p = \frac{1}{2}$ results, for large n, in about 1.6% peak performance loss ($C = 2.667$), which is a moderate decrease. It is important to notice that $p = 0.4175$ is very close to the optimal bias for small n as well.

2.1.3 Clipped Modified Binary Tree

In this section we will show that, in some cases, partial resolution algorithms can achieve higher efficiency than complete resolution algorithms.

The *Clipped Modified Binary Tree (CMBT)* is an adaptation of the *CBT* (see Section 3.1) algorithm for complete batch resolution. CBT is like the MBT Alg. but it ends up after two consecutive successful transmissions. Each CBT execution resolves at least two nodes but

eventually the last run. Hence, since the multiplicity of the batch to resolve decreases of at least two units after each run, the algorithm terminates for sure.

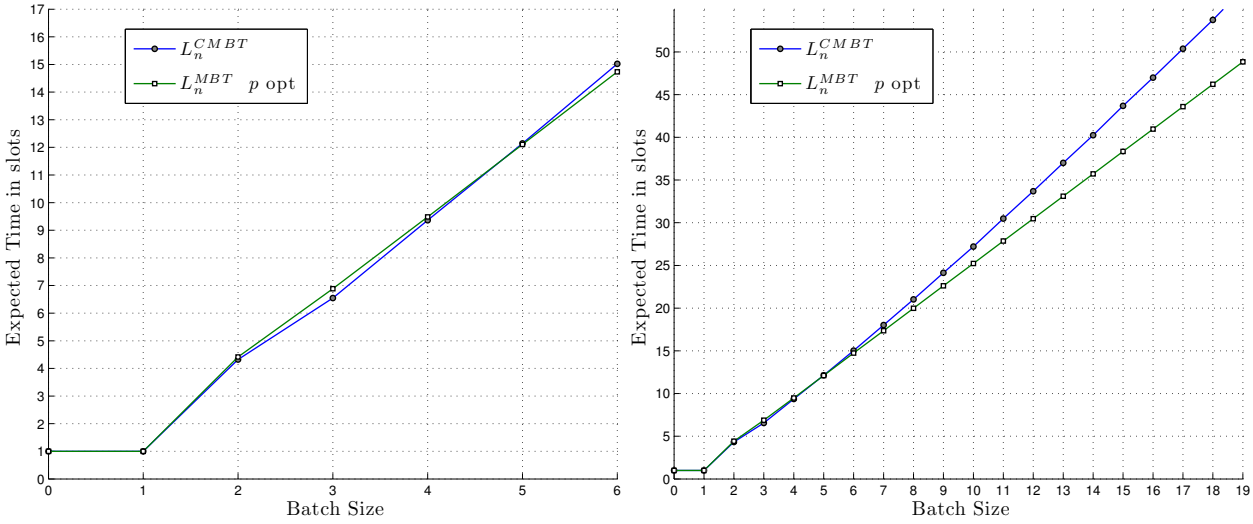
Collisions tree visit is defined by the following rules:

- If the last run of the CBT does not resolves completely the left sub-tree of the current root, then the next CBT starts from the same root;
- If the last run of the CBT resolves completely the left sub-tree of the current root, next CBT run is applied to the right child of the current root, that will be considered the new root;
- While current root left child is *idle*, set the root to the current root right child.

Applying the CBT starting each time from the current root makes the algorithm memoryless: its behavior is not affected by previous algorithm runs but the trivial root update. In fact, the root update allows to skip certain collisions and empty slots associated to already resolved sub-sets.

Efficiency in solving very small batches is CMBT most interesting property. This is a consequence of being memoryless. Figures 2.4 shows how in the average case, the CMBT performs a few better than the MBT for batch sizes smaller than 5. Expected maximum performance improvement is about 5% when batch size is 3. Nonetheless, for bigger batch sizes, MBT performs better than CMBT and the gap between the two algorithms gets bigger and bigger as the batch size increases.

Consequently, using CMBT is a good idea when the cardinality of the batch to solve is less than or equal to 5 with very high probability. For this reason CMBT is used by the EBT algorithm (following Section 2.1.5) to speed up the resolution.



(a) *CMBT vs MBT*: very small sizes

(b) *CMBT vs MBT*: larger sizes

Figure 2.4: Expected resolution time in slots for the *CMBT* and *MBT* algorithms for small batch sizes.

2.1.4 m Groups Tree Resolution

More advanced algorithms for batch resolution, such as those presented in [7] and [8], share a common idea: divide the initial batch into m groups. We will not deal here about the details of the algorithms and the different approaches they use to choose m. Instead we will concentrate on the common part of these algorithms: divide a batch of size n into m groups and apply a BRA to each group.

Given m groups, the probability to have exactly i among n nodes in a group is given by

$$P_m(i|n) = \binom{n}{i} \left(\frac{1}{m}\right)^i \left(1 - \frac{1}{m}\right)^{n-i}. \tag{2.6}$$

Let L_k be the expected cost to solve a batch of size k with a chosen BRA. Then the expected cost to solve a batch of size n dividing it into m groups is given by

$$L'_n(\rho) = m \sum_{i=0}^{n} L_i \binom{n}{i} \left(\frac{1}{m}\right)^i \left(1 - \frac{1}{m}\right)^{n-i}, \tag{2.7}$$

where $\rho = \dfrac{n}{m}$ is the expected number of nodes in a group.

Figure 2.5 plots $L'_n(\rho)$, the average time to solve a batch of size n using group splitting, over L_n, the average time without splitting. Results were obtained computing equation (2.7) for three different batch sizes varying ρ. We plotted the results for the MBT algorithm with optimal probability p in a slotted ALOHA scenario but considerations hold true for any *tree-based BRA*.

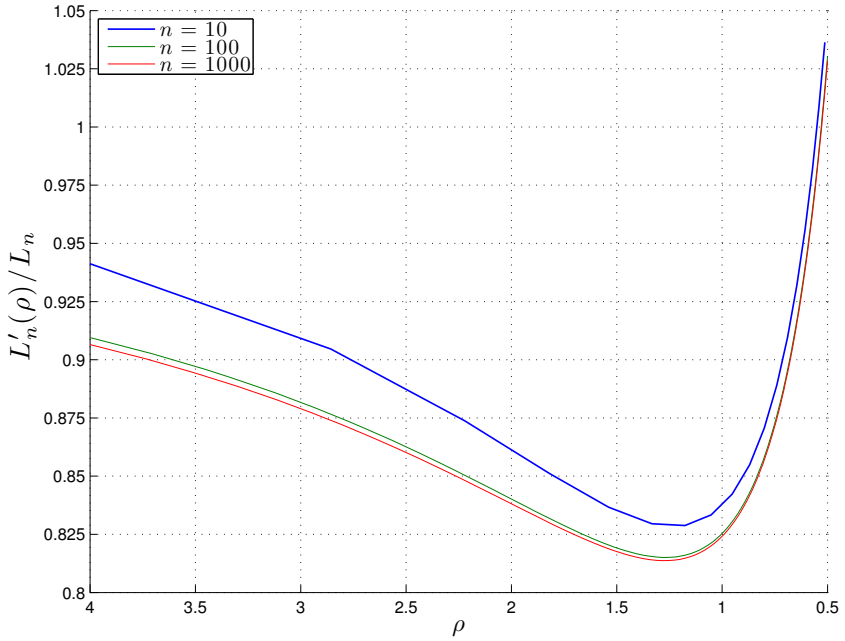


Figure 2.5: *m Groups Split: ALOHA scenario.* Expected performance of group splitting over trivial batch resolution using optimum MBT for different ρ. Lower means better.

Splitting the batch into groups achieves lower average resolution time than applying the BRA to the original batch for a wide range of ρ (when $L'_n(\rho)/L_n < 1$).
We note that $\rho \approx 1.26$ provides the best achievable performance. Furthermore we notice that the knowledge of the batch size is critical: m can be set to the optimal value only knowing exactly the batch size n but often we only have an estimate of n. If the estimate is smaller than n, performances smoothly degrade but still remain better than the trivial application of the BRA. On the other hand, overestimating n can lead to performance loss when $\rho \approx 0.5$ or smaller.

In a CSMA scenario the performances depend on β and on the ratio between the nodes packet size (S_n) and the supervisor feedback size (S_f). Figure 2.6 was obtained considering $\frac{S_n}{S_f} \approx 4$. We notice that in CSMA increasing idle slots probability (ρ small) can lower the expected cost. CSMA is also much more robust to an overestimate of n than *slotted*-ALOHA.

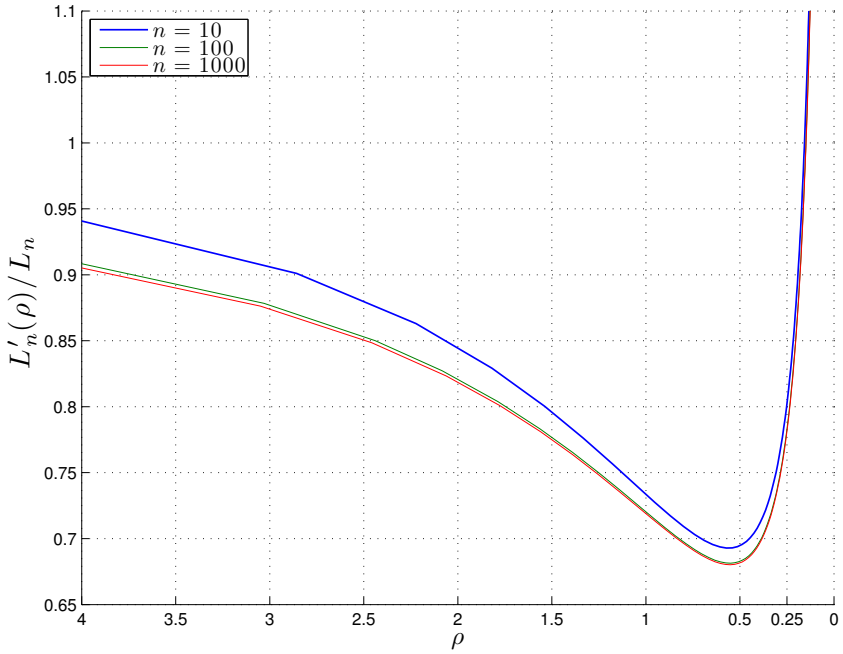


Figure 2.6: *m Groups Split: CSMA scenario.* Expected performance of group splitting over trivial batch resolution using optimum MBT for different ρ. Lower means better.

2.1.5 Estimating Binary Tree

Estimating Binary Tree (EBT) has been recently proposed in [2]. It does not work on parameters optimization but tries to use simple heuristics to skip tree nodes that will result in collisions with high probability.
Given a batch of size n, the keys to understand EBT are the following:

- in the luckiest scenario, running BT results in a balanced binary tree where all the leaves are at the same level,
- consequently it seems to be a good idea to assume that nodes at levels in the tree less than $\lfloor \log_2 n \rfloor$ will likely result in collided slots.

EBT tries to skip inner nodes of the tree by visiting only nodes at levels equal or deeper than $\lfloor \log_2 n \rfloor$. Since n is not *apriori* known, a dynamic estimating technique is adopted. To effectively use this estimate tecnique each node must be able to generate values from the standard uniform distribution on the interval [0,1) and to use that value as its unique *id*.
Assume that there are k nodes whose *id*s are in the sub-interval $[0,p_\epsilon)$ and that they have been previously resolved: all and only the nodes with *id* less than p_ϵ successfully transmitted their messages. Let $\hat{n}$ express the estimate of n. When p_ϵ is greater than 0, setting $\hat{n}$ to $\frac{k}{p_\epsilon}$ provides a good estimate of n that becomes more and more accurate as the algorithm goes on. EBT uses

$\hat{n}$ (as soon as it is available) to choose the right level in the tree to analyze.

The most straightforward interpretation of the EBT algorithm can be:

Algorithm 2 ESTIMATING BINARY TREE

```
Run the CBT algorithm
while batch is not resolved do
    Update the estimate n̂
    Start a new CMBT at the next node from level ⌊log2 n̂⌋
end while
```

Here we gave only a short insight of the algorithm and the estimate technique. In particular this estimate technique is a dynamic adaptation of that described later in Section 3.2 and originally proposed in [7].

2.2 Others

In this section we will shortly describe non *tree-based* algorithms for batch resolution.

2.2.1 IERC

The *Interval Estimation Conflict Resolution (IERC)* [2] is an adaptation of the FCFS [5] algorithm for Poisson's arrivals of packets to the batch resolution.
FCFS is the fastest known algorithm to solve Poisson's arrivals and it achieves efficiency $\eta = 0.4871$. It assumes *apriori* knowledge of the arrival rate λ of the packets and works by splitting the time in epochs of length τ.
Now we briefly describe FCFS Alg. and then we will show how it can be translated to the batch resolution problem.

At any time k, let $T(k)$ be the start time of the current enabled allocation interval. Packets generated in the enabled window $(T(k), T(k) + \tau]$ are allowed to be transmitted. When an *idle* or *successful* event occurs enabled time window gets incremented (shifted towards current time) by τ. On the other hand, if a *collision* takes place, a *Clipped Binary Tree* (see Section 3.1) algorithm is started. Given that CBT resolves the sub-interval $(T(k), T(k) + \alpha(k)]$, then next FCFS enabled window at time k' will be $(T(k'), T(k') + \min(\tau, k' - T(k'))]$ where $T(k') = T(k) + \alpha(k)$.

It can be shown [5] that setting $\tau = \frac{1.26}{\lambda}$ leads to the maximum efficiency. This means that, on average, there are 1.26 nodes in an allocation interval of length τ.
The batch resolution problem can be adapted to be solved with an FCFS-like strategy in the following way:

- nodes *tokens* can be interpreted as arrival times;
- nodes must be splitted into $m = \frac{n}{1.26}$ groups, so that in each group we expect to have 1.26 nodes;
- apply FCFS to solve the problem.

Following these ideas, IERC achieves the same efficiency $\eta = 0.4871$ of FCFS for Poisson's arrivals in solving the *Batch Resolution Problem*: it is the asymptotically fastest know algorithm allowed by our scenario model.

2.2.2 Window Based Approaches

Another possible approach to the batch resolution is to consider a *framed*-ALOHA scenario.
In *framed*-ALOHA, a frame (or *window*) is a sequence of consecutive slots. When a node in the collision batch decides to transmit, it uniformly picks one and only one slot in the window and transmits in that slot.
In windows based scenarios, usually we can work on the optimization of two parameters: m, the length of the window in slots, and p, the probability that a node transmits in the current window. Innovative approaches to the problem uses hybrid transmission scheme working by cycles that tries to optimize operational parameters after each transmission windows.

Chapter 3

Batch Size Estimate Techniques

Here we present some noteworthy techniques for batch size estimate that can be found in literature. If a technique was not already identified by a name or associated to an acronym, we used the name of one the authors as reference.

In general, we assume to have no *a priori* statistical knowledge about the conflict multiplicity. Estimation techniques are thus required to provide accurate estimates for the general zero-knowledge scenario.

3.1 Clipped Binary Tree

A simple way to obtain an estimate of the batch size is to solve a certain number of nodes and then infer the batch size according to the time taken to solve these nodes. This can be done, for example, by using deterministic algorithms such as the *Clipped Binary Tree (CBT) Algorithm.* CBT is a partial resolution algorithm since only a fraction of the packets of the batch are successfully transmitted. The algorithm is identical to the MBT, with $p = \frac{1}{2}$ since we require nodes to be uniformly distributed in the interval [0,1), except that it stops (the tree is clipped) whenever two consecutive successful transmissions follow a conflict.
When the algorithm stops, after i collisions, we know than the last two resolved nodes belong to the same level i of the tree (root is assumed to be at level 0). Therefore, an estimate of the initial batch size is given by:

$$\hat{n} \leftarrow 2^i \tag{3.1}$$

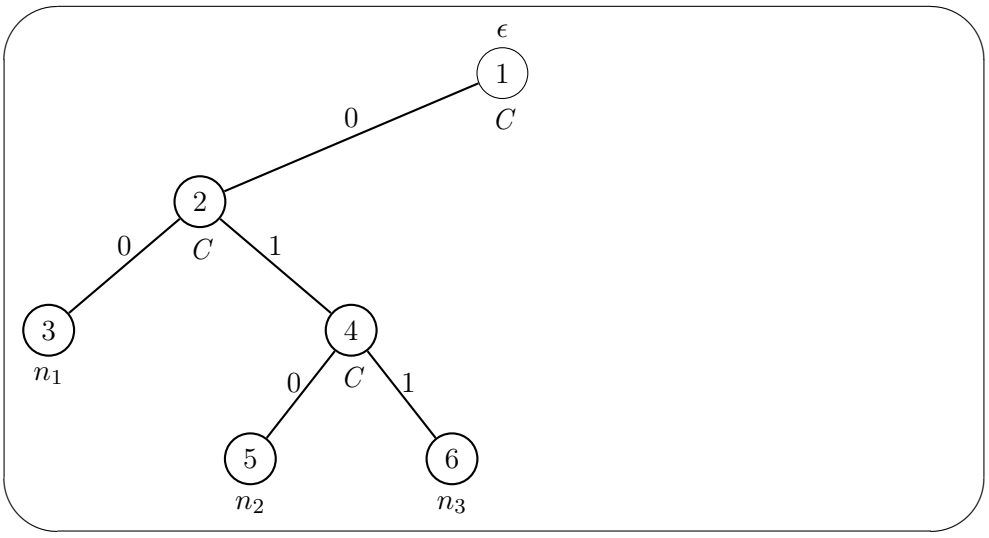


Figure 3.1: Same example as in Figure 2.2 but resolution using CBT ends up after two consecutive successful transmissions.

Experimental results show that the variance of this estimate is extremely high and the resulting accuracy is rather poor. This is due to the fact that the batch we use for the estimate becomes, at each level, smaller and smaller: in the average case we expect all the intervals to be balanced in the number of nodes. Using this algorithm we don't have knowledge of a sufficiently large number of intervals but we limit to analyze. the estimate, even for huge sizes, depends only on very few (3-5) nodes (those at the most left in the tree). Consequently estimate is quite inacurate.
From the tables A.5 reported in Appendix you can notice that the distribution probability decreases rather slowly.

3.2 Cidon

In [7] Cidon and Sidi proposed a complete resolution algorithm based on two phases:

1. Estimate the initial batch size using a partial deterministic resolution scheme.

2. Perform an optimized complete deterministic resolution based on the results of phase 1.

Phase 1 consists in resolving a small part of the initial batch, counting the number of successful transmissions. A node takes part either to the estimate phase or to the following resolution phase, not both. The probability p_ϵ, which is an algorithm input parameter, determines this choice. We called it p_ϵ to underlined that this initial choice reflects the expected accuracy of the resulting estimate, as it will be discussed in more detail later on.

As usual we consider a batch $\mathcal{B}$ of unknown size n. At the beginning of the algorithm each node chooses to transmit with probability p_ϵ. Thus the n nodes are partitioned into two sets, $\mathcal{E}$ and $\mathcal{D}$, where $\mathcal{E}$ consists of those that transmitted and $\mathcal{D}$ the rest. Clearly, $|\mathcal{E}| + |\mathcal{D}| = n$. If the resulting slot is empty or contains a successful transmission, we conclude that $|\mathcal{E}| = 0$ or $|\mathcal{E}| = 1$, respectively. When a conflict occurs ($|\mathcal{E}| \geq 2$), nodes in $\mathcal{E}$ use a complete BRA to resolve the conflict among them. Let j be the number of resolved nodes at the end of phase 1, i.e. $|\mathcal{E}| = j$. Then the estimate $\hat{n}$ is given by

$$\hat{n} \leftarrow \frac{j}{p_\epsilon}. \tag{3.2}$$

When the nodes are uniformly distributed in the real interval [0,1), $\frac{j}{p_\epsilon}$ identifies also the expected nodes density in the interval [0, 1) and the nodes in $\mathcal{E}$ are those whose *id* can be mapped to a value belonging to the sub-interval [0, p_ϵ).
Figure 3.2 illustrates the concept with a simple example.

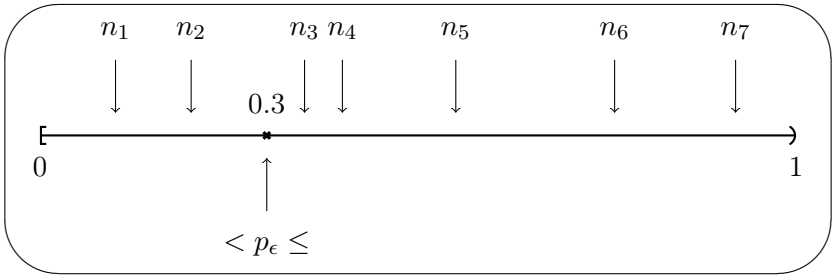


Figure 3.2: In this example $p_\epsilon = 0.3$. At the beginning of the algorithm each node generates its own *id* from the standard uniform distribution in the interval [0,1). Nodes whose *id* is less than p_ϵ belongs to $\mathcal{E}$. Nodes whose *id* is greater or equal to p_ϵ belongs to $\mathcal{D}$. Estimate of the batch returns $\lceil 2/0.3 \rceil = 7$ which, in this case, is the exact size of the batch.

After the first phase, nodes in $\mathcal{E}$ are resolved, whereas those in $\mathcal{D}$ has still to be counted. An estimate $\hat{k}$ of $|\mathcal{D}|$ can be obtained as

$$\hat{k} \leftarrow \frac{j}{p_\epsilon}(1 - p_\epsilon). \tag{3.3}$$

This estimate is used as starting point for the second phase of the Cidon algorithm, whose high level peudo-code is presented in Algorithm 3.
COMPLETE COLLISION RESOLUTION ($\mathcal{E}$) identifies any procedure able to resolve all the nodes in $\mathcal{E}$ allowing them to successfully transmit their messages.
OPTIMIZED COMPLETE COLLISION RESOLUTION ($\mathcal{D}$,$\hat{k}$, p_ϵ) identifies an optimized way to resolve the batch $\mathcal{D}$ that takes into account the expected multiplicity of $\mathcal{D}$.

Algorithm 3 CIDON($\mathcal{B}$, p_ϵ)

Input: $\mathcal{B}$ batch with $|\mathcal{B}| = n$
Input: p_ϵ, fraction of the whole batch to solve
1: // Phase 1
2: each node flips a coin getting 0 with probability p_ϵ, 1 otherwise
3: $\mathcal{E} \leftarrow$ { nodes that flipped 0}
4: $\mathcal{D} \leftarrow$ { nodes that flipped 1}
5: COMPLETE COLLISION RESOLUTION ($\mathcal{E}$)
6: $\hat{k} \leftarrow |\mathcal{E}|/p_\epsilon$
7: // Phase 2
8: OPTIMIZED COMPLETE COLLISION RESOLUTION ($\mathcal{D}$,$\hat{k}$, p_ϵ)

The original paper proposes to use an *m groups split* (Section 2.1.4) approach to resolve $\mathcal{D}$, where

$$m = \max(1, \lceil \alpha\hat{k} - \beta \rceil), \tag{3.4}$$

and each group is resolved by applying the MBT algorithm.
The parameter α determines the number of groups, and therefore the efficiency of the resolution process, when n is large, while β reduces the number of groups when n is small in order to reduce the risk of empty groups.

$\alpha = 0.786$ determines $\rho \approx 1.27$ which is the *unique* optimum nodes per group density. β and p_ϵ depend on operational requirements: in [7] is showed that setting $\beta = 8$ and $p_\epsilon = 0.1$ is a good compromise to get efficient resolution for a wide range of batch sizes.

The average cost of the estimate phase (phase 1) depends on the BRA used but in general, for tree-based BRAs, can be considered $O(p_\epsilon n)$.

3.3 Greenberg

Basic Greenberg algorithm searches for *a power of 2* that is close to n with high probability:

$$\hat{n} \geq 2^i \approx n \tag{3.5}$$

Let each of the n conflicting stations either or not transmit in accordance with the outcome of a biased binary coin. The coin is biased to turn up 0 (transmit) with probability 2^{-i} and 1 (do not transmit) with complementary probability $1 - 2^{-i}$. Since the expected number of transmitters in the slot is $2^{-i}n$, a conflict supports the hypothesis that $n \geq 2^i$.
Using this test repeatedly with $i = 1, 2, 3, \ldots$, leads to the Greenberg *base 2 estimation algorithm.* Each of the conflicting stations executes Algorithm (4), resulting in a series of collisions whose

length determines $\hat{n}$. The probability that at most one node transmits in a slot, monotonically grows with the slot number and approaches 1 very rapidly as i increases beyond $\log_2 n$. Consequently, we expect that the collision series stops at slot i that is close to $\log_2 n$.

Algorithm 4 BASIC GREENBERG ($\mathcal{B}$)

```
// Each node performs these operations
i ← 0
repeat
    i ← i + 1
    choose to transmit with probability p = 2^{-i}
until no collision occurs
n̂ ← 2^i
```

The idea behind Algorithm 4 appears to be quite simple: as the algorithm goes on, the initial unknown batch (of size n) is progressively sliced into smaller pieces. Only the nodes virtually inside the enabled slice are allowed to transmit. Slices get thinner and thinner until at most one node is contained in a slice. Figure 3.3 illustrates the idea with an example: visually, nodes can be thought to be uniformly distributed on a circumference. Using Greenberg algorithm we will analyze at each slot a smaller sector (in this case the half of the previous one) of the circle and discover when a sector contains at most 1 node. Note that no overlapping sectors are drawn to maintain the image simple. However, in general, enabled nodes gets redistributed at each transmission test performed by the algorithm.

Expected running time is $O(log_2 n)$. In particular, since in the *slotted*-ALOHA model feedback is supposed to be transmitted at the end of each transmission slot, the expected running time in slots can be expressed as $\approx 1 + \log_2 n$.

An important note is that the algorithm always involves all the nodes in the batch: at each slot, each node decides whatever or not transmit. Each choice is independent of the past. This is of great importance and allows $\hat{n}$ to have bounded statistical moments: it can be shown that, for large n, it holds:

$$E[\hat{n}] \approx n\phi, \tag{3.6}$$

$$E[\hat{n}^2] \approx n\Phi, \tag{3.7}$$

where

$$\phi = \frac{1}{\log 2} \int_0^\infty e^{-x}(1+x) \prod_{k=1}^{\infty} \left(1 - e^{-2^k x}(1+2^k x)\right) x^{-2}\, dx = 0.91422 \tag{3.8}$$

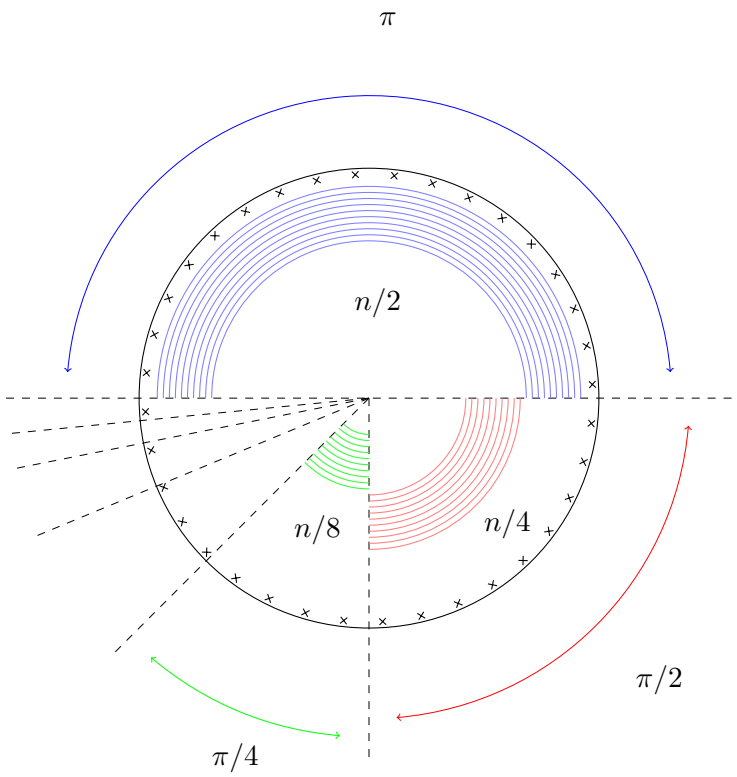


Figure 3.3: *Basic Greenberg*: batch split idea

$$\Phi = \frac{1}{\log 2} \int_0^\infty e^{-x}(1+x) \prod_{k=1}^{\infty} \left(1 - e^{-2^k x}(1+2^k x)\right) x^{-3}\, dx = 1.23278 \tag{3.9}$$

ϕ and Φ are obtained in [8] using advanced mathematical analysis supported by Mellin integral transform[1]. In general ϕ and Φ depend on the size of the problem. Table 3.1 shows the behavior of the expected estimate (and, therefore of ϕ) as a function of n. We note that the ratio $E[\hat{n}|n]/n$ monotonically decreases and gets stable at 0.9142. This shows that this estimate technique provide biased results.

The fact that, for large n, $E[\hat{n}] \approx n\phi$ suggests a way for correcting the estimate bias as $\hat{n}_+ = \dfrac{\hat{n}}{\phi}$. Due to the contribution of periodic functions, $\hat{n}_+$ is not an asymptotically unbiased estimate of n, in the sense that $E[\hat{n}_+]/n$ does not tend to 1 as n gets large. Fortunately, the amplitude of the periodic functions turns out to be less than $2 \cdot 10^{-5}$, so this bias is negligible for all practical purposes.

Interestingly, a simple variant of the estimation algorithm shows really poor performance.

[1] In this work we only report the final results. Please, refer to the original paper for details.

Table 3.1: *Basic Greenberg*: Expected estimate and bias

n	$E[\hat{n}\|n]$	$E[\hat{n}\|n]/n$
1	2.00	2.0000
2	2.56	1.2822
4	4.21	1.0533
8	7.89	0.9863
16	15.20	0.9498
32	29.82	0.9320
64	59.08	0.9231
128	117.59	0.9186
256	234.60	0.9164
512	468.64	0.9153
1024	936.71	0.9148
2048	1872.86	0.9145
4096	3745.14	0.9143
8192	7489.72	0.9143
16384	14978.86	0.9142
32768	29957.16	0.9142
65536	59913.74	0.9142

Consider the algorithm in which each station involved in the initial collision transmits to the channel with probability 1/2. If this causes another collision, then those that just transmitted, transmit again with probability 1/2. The others drop out. This process continues until there are no collisions. Let 2^i be the estimate of the conflict multiplicity, where i is the slot at which there is no collision. It can be shown [8] that the second and all higher moments of this estimate are infinite.

3.3.1 Base *b* variant

Using basic Algorithm 4, the expected value of $\hat{n}_+$ is likely different from n by a factor 2. In the original work, a small generalization of the base 2 algorithm is proposed to overcome this limitation, by simply using a base b instead of 2, with $1 < b \leq 2$.

Algorithm 5 BASE b GREENBERG ($\mathcal{B}$)

$i \leftarrow 0$
repeat
  $i \leftarrow i + 1$
  transmit with probability b^{-i}
until no collision occurs
$\hat{n}(b) \leftarrow 2^i$
$\hat{n}_+(b) \leftarrow \hat{n}(b)/\phi(b)$

In Algorithm 5, the term $\phi(b)$ corrects the bias of the estimator. Table 3.2 shows how ϕ and the expected cost in slots vary for different b. Expected cost ($\lesssim \log_b n$) is expressed as a multiplicative factor for the basic Greenberg algorithm cost ($\lesssim \log_2 n$). We can notice that smaller b results in smaller $\phi(b)$. This means that b deeply biases the estimate: if $b^{'} < b^{''}$ then $E[\hat{n}(b^{'})] < E[\hat{n}(b^{''})]$. This follows form the fact that, decreasing b, the stop probability "shift" towards "early slots" associated to lower estimates.

Table 3.2: *Base b Greenberg*: Bias and expected cost summary

b	$\phi(b)$	Expected cost in slots-1
2	≈ 0.9142	$\lesssim 1 \times \log_2 n$
1.1	≈ 0.3484	$\lesssim 7.27$
1.01	≈ 0.1960	$\lesssim 69.66$
1.001	≈ 0.1348	$\lesssim 693.49$
1.0001	≈ 0.1027	$\lesssim 6931.81$

In[8], it is shown that, *for all n greater than some constant* $n_0(b)$*, it holds*

$$\left| \frac{E[\hat{n}_+(b)]}{n} - 1 \right| < \epsilon(b), \tag{3.10}$$

$$\frac{\sigma\big(\hat{n}_+(b)\big)}{n} < \epsilon(b), \tag{3.11}$$

where $\epsilon(b) \to 0$ as $b \to 1$.
In other words when $b \to 1$ and n is large the estimate becomes unbiased and its variance goes to zero. However, our experimental results showed that the performance of base b estimator is not ideal in practical scenarios.

3.4 Window Based

Window based approaches use a *framed*-ALOHA transmission scheme, where the reader provides feedback to the nodes after each frame. *Framed*-ALOHA is characterized by the frame length L, whose choice is critical for the performance of the resolution process as well as the estimation phase. Setting L large compared to the batch size increases the probability to solve all the nodes in the current window but, on the other hand, it leads to a waste of slots and consequently sub-optimal running time. At the same time, a large value of L allows for a very accurate estimate of the batch size.
Setting L small increases the probability of collided slots. Since the number of nodes involved in a collision is unknown, collisions provide poor information about the batch multiplicity. The more collisions we have, the less accurate the estimate.

An interesting approach to the problem is presented in [3]. When the scenario allows to use a *probabilistic-framed-ALOHA*[2] scheme we can act on the parameter p, i.e. the probability for a node to take part in the current transmission window. Small p value allows to keep L small too and hence, when n is large, provides an accurate estimate of the whole batch considering only a sub-set of it. This results in shorter estimate time.

Since when $p = 1$ *probabilistic-framed-ALOHA* reduces to simple *framed-ALOHA* and it is reasonable to start any estimate algorithm with $p = 1$, we will show a possible approach to the problem in a *framed-ALOHA* scenario.

Let n be the batch size and L the window length. The number of nodes k out of n that choose the same slot is binomially distributed with parameters $B(n, 1/L)$. Then, the probability to get an *idle*, *successful* or *collision* slot is respectively given by:

$$p_0(n) = \left(1 - \frac{1}{L}\right)^n, \tag{3.12}$$

$$p_1(n) = \frac{n}{L}\left(1 - \frac{1}{L}\right)^{n-1}, \tag{3.13}$$

$$p_{2+}(n) = 1 - p_i(n) - p_s(n). \tag{3.14}$$

[2] *Probabilistic-framed-ALOHA* is an extension of the *framed-ALOHA* model where a node takes part to the current contention window with probabily p or waits for the next one with probability $1 - p$. Nodes that decide to transmit behave like in the standard *framed-ALOHA*.

Considering the whole window, the outcome consists in a tuple (i, s, c), denoting the number of idle, success, and collision slots, which satisfies $i + s + c = L$. Hence, the probability of (i,s,c) is given by:

$$P(i, s, c) = \frac{L!}{i!\, s!\, c!}\ p_0(n)^i\ p_1(n)^s\ p_{2+}(n)^c = \frac{L!}{i!\, s!\, c!}\ f_L(i, s, c, n). \tag{3.15}$$

We note that (3.15), does not take into account the fact that a node is allowed to transmit only once in a window: equation (3.15) is only an approximation of the distribution of the nodes in the slots when both n and L are large. A Poisson distribution of parameter $\lambda = n\frac{1}{L}$ would model better the scenario but (3.15) allows simpler numerical computation.

Once we have tried to resolve the batch using a window approach we know how many idle, successful or collided slots there were.

Then we find the estimate $\hat{n}$ as the batch size that maximizes the probability to see the tuple (i, s, c) in a L length window:

$$\hat{n} = \arg\max_n f_L(i, s, c, n), \tag{3.16}$$

where we discarded the factorial terms because they not contribute to identify the maximum since L, i, s, c are fixed. Furthermore, since L, i, s, c are fixed, $f_L(i, s, c, n)$ becomes a one-variable function which results well behaved in n, being initially monotonically increasing and then monotonically decreasing. Therefore $f_L(i, s, c, n)$ has only one maximum.

By setting

$$f'_L(i, s, c, n) = 0, \tag{3.17}$$

and numerically solving (3.17) we can obtain the batch size that maximizes our function. In general the solution will not be integer-valued and a rounding operation is necessary to achieve a real world batch estimate.

Table 3.3: Estimate given (i, c, s) when $L = i + c + s = 10$.

c \ s	0	1	2	3	4	5	6	7	8	9
1	3	3	4	5	6	7	8	9	10	11
2	5	6	7	8	8	10	11	12	13	-
3	7	8	9	10	11	12	13	14	-	-
4	10	11	12	13	14	15	16	-	-	-
5	12	14	15	16	17	19	-	-	-	-
6	16	17	19	20	22	-	-	-	-	-
7	20	22	23	25	-	-	-	-	-	-
8	26	28	30	-	-	-	-	-	-	-
9	35	38	-	-	-	-	-	-	-	-
10	∞	-	-	-	-	-	-	-	-	-

Table 3.3 shows the estimate provided by the explained technique when $L = 10$. Cells,

identified by (c, s), where $c + s > 10$ cannot be associated to any estimate since their events are impossible. The case $c = 0$ is trivial since the estimate is exact and is given by the number of successful transmission.

We note also that when we see only collisions, the estimator is not able to provide a finite estimate. The event $c = 10$ in the table is in fact associated to $\hat{n} = \infty$. In general, when $c = L$ we have that (3.15) reduces to:

$$P(0, 0, L) = \ p_{2+}(n)^L = \left[1 - \left(1 - \frac{1}{L}\right)^n - \frac{n}{L}\left(1 - \frac{1}{L}\right)^{n-1}\right]^L, \tag{3.18}$$

which is maximized ($P(0, 0, L) = 1$) by $n = \infty$ for any L. Hence we would not to have only collisions since they do not provide any information about the cardinality of the batch.

A larger window has to be used to get a finite estimate but its optimal length remains unknown.

Chapter 4

Estimate Performance Analysis

In this Chapter, we present a performance analysis of some of the already mentioned noteworthy *batch size estimation (BSE) algorithms*. The overview will obviously consider different batch sizes, algorithm parameters, time required and accuracy of the estimate. We want to support reader in choosing the best estimate technique suitable for its needs.
First we will provide a detailed analytical analysis of *Cidon* algorithm and then, supported by numerical computation, we will derive its features in term of time and accuracy.
We will then discuss the behavior of *Greenberg* with its pros and cons and try to overcome its limitations by introducing a slight modification of the original algorithm to achieve better estimate. *Enhanced Greenberg Algorithm (EGA)* is the middle-step in the development of our proposed estimate algorithm: it improves the estimate in terms of "choosing the best power of 2", maintaining the same target estimates allowed by basic Greenberg. Then, we will describe and analyze the *Generalized Enhanced Greenberg Algorithm (GEGA)*, which combines Greenberg with a further test and performs a *maximum likelihood estimation* to determine the "best" estimate. Finally, the comparison between *GEGA* and *Cidon* will lead to the conclusions.

4.1 Cidon BSE

The original paper [7] describes the estimate technique but does not characterize it in details. The interest is focused on other aspects and, therefore, no detailed analysis of the behavior of the estimate algorithm is presented: it is only mentioned that as n grows the estimator becomes more accurate. Hence, in this section we will provide a complete analysis of the estimate algorithm.

Following from Section 3.2, let j denote the number of nodes in $\mathcal{E}$ and p_ϵ be the expected fraction of nodes to be resolved in Cidon algorithm Phase 1. Parameter p_ϵ can be considered

fixed *a priori* since it is an input of the algorithm and, therefore, given a batch of size n, j is a binomially distributed random variable with parameters n and p_ϵ. Hence, we have the following:

$$E[j|n, p_\epsilon] = np_\epsilon, \tag{4.1}$$

$$\text{var}(j|n, p_\epsilon) = np_\epsilon(1 - p_\epsilon). \tag{4.2}$$

By applying Chebychev's Inequality (A.1), we have for any $\epsilon > 0$

$$P\Big(|j - np_\epsilon| \geq \epsilon n \,|\, n, p_\epsilon\Big) \leq \frac{p_\epsilon(1 - p_\epsilon)}{\epsilon^2 n}. \tag{4.3}$$

We remember that the Cidon estimate is given by $\hat{n} = \dfrac{j}{p_\epsilon}$. Then, from the aforementioned equations (4.1)-(4.3), it follows that:

$$E[\hat{n}|n, p_\epsilon] = \frac{1}{p_\epsilon} E\left[j|n, p_\epsilon\right] = n, \qquad \forall\, p_\epsilon \tag{4.4}$$

and

$$P\left(\left|\frac{\hat{n}}{n} - 1\right| \geq \epsilon \,\middle|\, n, p_\epsilon\right) \leq \frac{1 - p_\epsilon}{\epsilon^2 n p_\epsilon}. \tag{4.5}$$

which shows that using this estimation method we can trade off the accuracy with the consumption of resources in terms of time or messages. Furthermore, Cidon estimate is unbiased independently of p_ϵ, which influences the variance of the estimator. In fact, we have

$$\text{var}(\hat{n}|n) = \frac{n}{p_\epsilon} - n \tag{4.6}$$

which reveals that as the number of resolved terminals (and therefore p_ϵ) increases the estimate $\hat{n}$ becomes more accurate.

4.1.1 Performance

Figure 4.1 shows that the variance is strict monotonically decreasing in p_ϵ. Furthermore, you can notice that, for p_ϵ smaller than 0.1, a small increment in p_ϵ produces a large decrease in the normalized variance which reflects in a large estimate accuracy improvement. Hence, considering the tread-off between estimate overhead and resolution efficiency, solving up to $\frac{1}{10}$ of the initial batch in the estimate phase is a good practical choice.

However, it is difficult to establish in what measure an estimate can be considered accurate. The idea we adopt is to require the estimate to be "near" the real batch size with very high probability. Given the real batch size n, let $k \geq 1$ identify an interval surrounding n in the

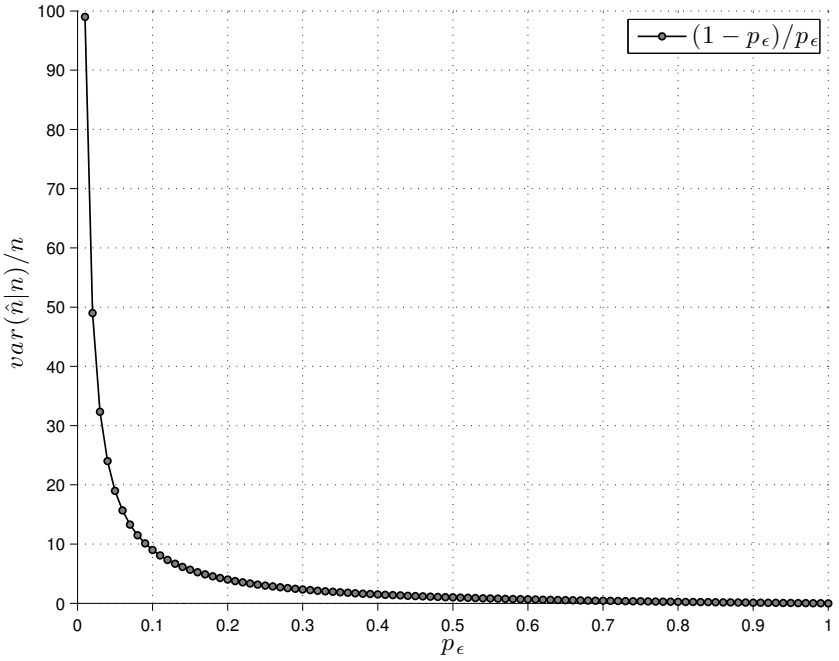


Figure 4.1: *Cidon*: estimate normalized variance as function of p_ϵ.

following way:

$$\frac{n}{k} \leq \hat{n} \leq kn \tag{4.7}$$

We can study the algorithm behavior by considering as index for the estimate accuracy

$$\theta_k = P\left(\frac{n}{k} \leq \hat{n} \leq kn\right) \tag{4.8}$$

Let θ be the minimum allowed value for θ_k: in other words θ is the probability we require for constrains (4.7) to be satisfied.
If we set $\theta = 0.99$, we can find the minimum p_ϵ, ensuring the estimate to be within interval (4.7), by solving the following problem.

$$\begin{aligned} P\left(\frac{n}{k} \leq \hat{n} \leq kn \middle| k, n\right) =& P\left(\frac{n}{k} \leq j/p_\epsilon \leq kn \middle| k, n, p_\epsilon\right) \\ =& P\left(\frac{np_\epsilon}{k} \leq j \leq knp_\epsilon \middle| k, n, p_\epsilon\right) \end{aligned} \tag{4.9}$$

Since j assumes positive integer values, we introduce rounding operations. In particular, rounding effect is non neglectible when $n \leq 200$.

$$f(k, n, p_\epsilon) = P\left(\left\lceil \frac{np_\epsilon}{k} \right\rceil \leq j \leq \lfloor knp_\epsilon \rfloor \,\middle|\, k, n, p_\epsilon\right) \geq \theta \tag{4.10}$$

Fixed k, n and θ, p_ϵ can be found by numerically solving the following equation:

$$f(k, n, p_\epsilon) = \theta \tag{4.11}$$

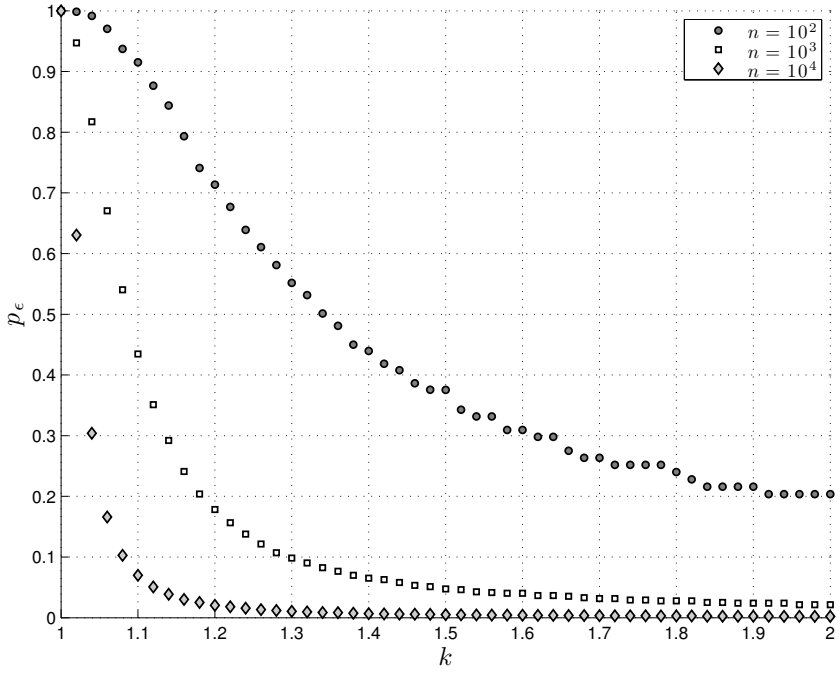


Figure 4.2: *Cidon*: Minimum p_ϵ as function of k when $\theta = 0.99$.

Figure 4.2 plots the minimum fraction p_ϵ of the batch to be resolved as function of k when $\theta = 0.99$. As you could expect, once the batch size is fixed, the minimum p_ϵ is monotonically decreasing in k. In the same manner, when k is fixed, the minimum fraction of the batch to resolve to achieve the desired accuracy is lower for larger batch sizes. In Figure 4.3 we report the expected time (in slots) required by *Cidon* to achieve the desired accuracy in the estimate. We notice that initially, when k is larger than 1.5, the time required to achieve a given accuracy is very similar for each considered batch size. For tighter accuracies the time required depends largely on the size of the problem.

Furthermore, the time required by the largest considered batch size to achieve a given accuracy provides an upper bound for smaller batch sizes: in the same amount of time smaller batch sizes are expected to achieve higher accuracy.

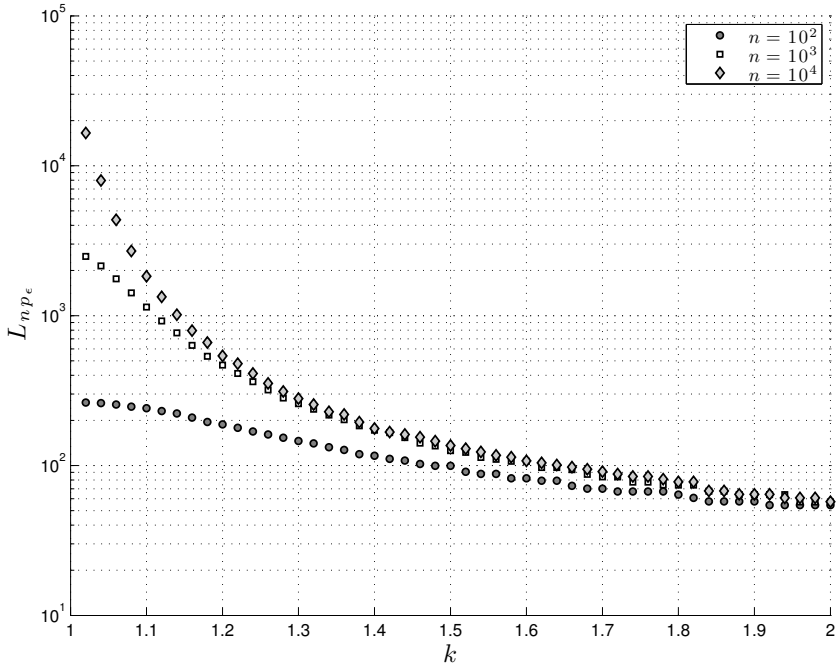


Figure 4.3: *Cidon*: Expected time as function of the required accuracy when $\theta = 0.99$.

4.2 Greenberg BSE

Given a current slot transmission probability p and a batch of size n we define respectively:

1. the probability to get an empty slot (no transmissions)

$$q_0(p, n) = (1 - p)^n \tag{4.12}$$

2. the probability to get a successful slot (exactly one transmission)

$$q_1(p, n) = np(1 - p)^{n-1} \tag{4.13}$$

3. the probability to get a collision (two or more transmissions)

$$q_{2+}(p, n) = 1 - q_0(p, n) - q_1(p, n) \tag{4.14}$$

In basic Greenberg (*algorithm* 4) each slot is associated with a different probability p. Numbering slots i from $1, 2$, and so on we hence have

$$p_i = p(i) = 2^i. \tag{4.15}$$

Given n nodes, the probability to terminate Greenberg algorithm in slot i is given by:

$$f_G(n,i) = \prod_{k=1}^{i-1} q_{2+}(p_k, n) \cdot \big(q_0(p_i, n) + q_1(p_i, n)\big). \tag{4.16}$$

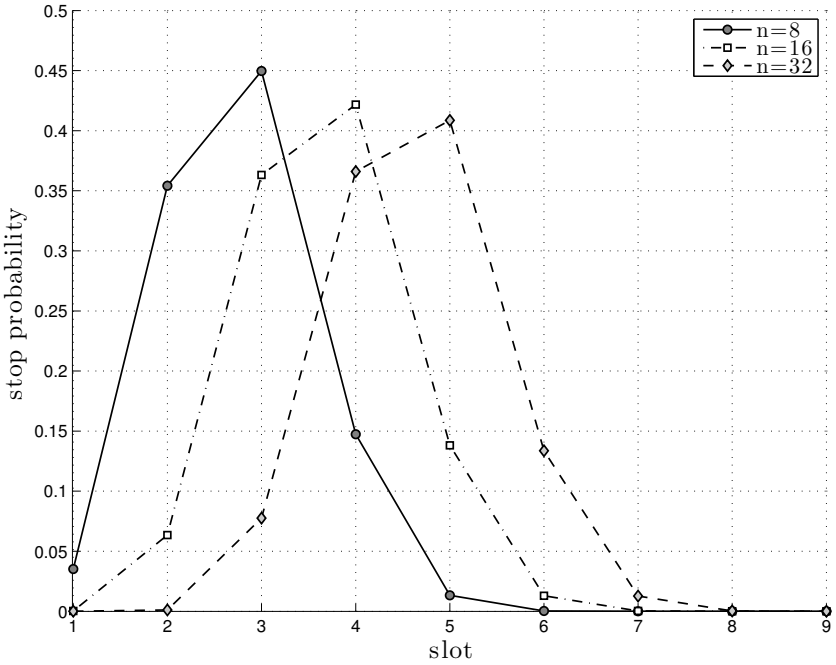


Figure 4.4: *Basic Greenberg*: Small 2^k sizes distribution.

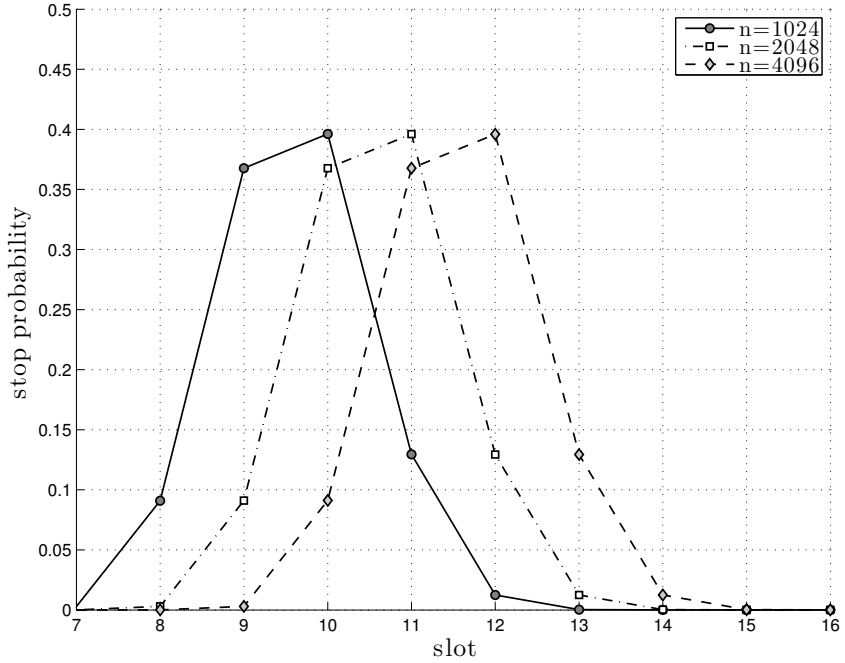


Figure 4.5: *Basic Greenberg*: Large 2^k sizes distribution.

An overview of the behavior of $f_G(n, i)$ is presented in Appendix in Table A.6 on page 63. Equation (4.16) defines the probability for biased estimate $\hat{n}$ to be equal to 2^i when the batch size is n.

$$\Pr\left(\hat{n} = 2^i | n\right) = f_G(n, i) \tag{4.17}$$

Figures 4.4 and 4.5 show how the distribution behaves respectively for small and large sizes. We note that, for any fixed n, the distribution is well behaved with a bell-like shape. It turns out that, for batch sizes larger than 128, the distribution is "stable" in the sense that doubling the number of nodes produces only a shift of the stop probability of one slot to the right (see Figure 4.5).

This is actually even for batch sizes that are not power of 2. Figure 4.6 shows the case.

4.2.1 Considerations

Greenberg method is really good in terms of running time cost, which is $O\left(\log_2 n\right)$. However it has some non negligible drawbacks:

a) The estimation phase results in a sequence of colliding messages. These provides information about the cardinality of the batch, but do not help to solve an even small portion of the batch and can not carry auxiliary information. An algorithm that allows to get an estimate

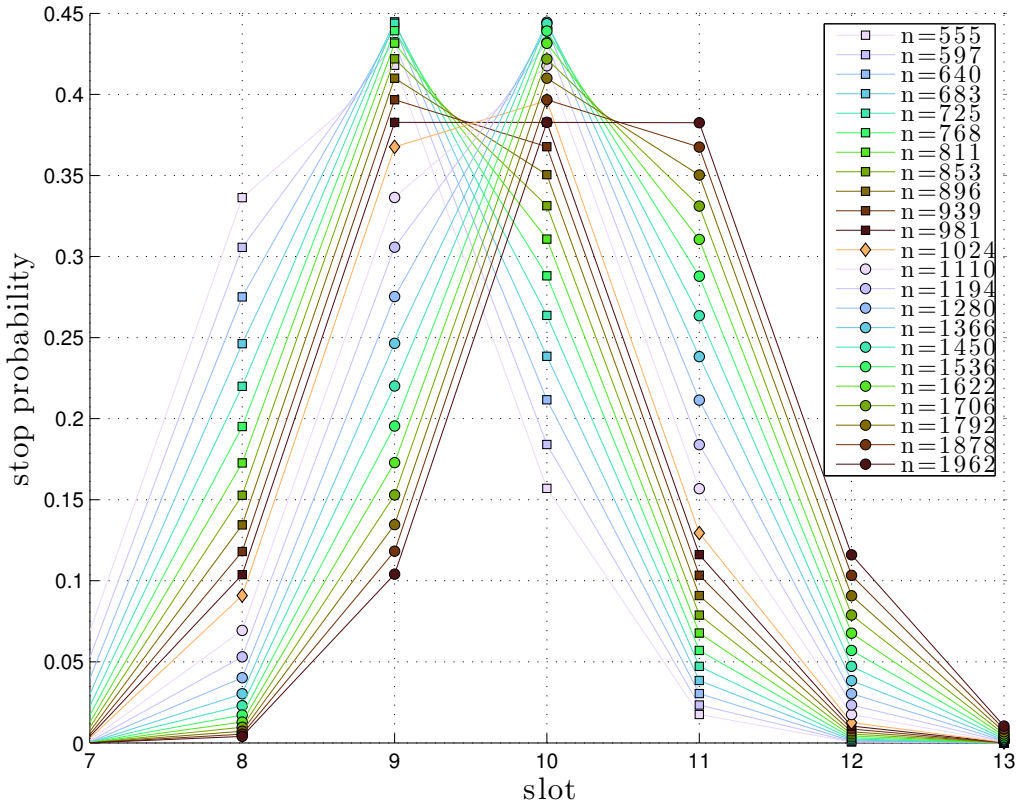


Figure 4.6: *Basic Greenberg*: General batch sizes distribution.

while resolving nodes would offer some advantages when the problem is not only the pure batch size estimation, but the batch resolution.

b) Basic Greenberg does not allow to achieve arbitrary precision in the estimate. In fact, we have that:

i. the estimate is, by construction, a power of 2. Only a small subset of batch sizes can be mapped without error.

ii. the estimate distribution is not sharp enough around the correct value of the batch size but it spans over a few slots: this is shown in Figure 4.5. We notice that using Greenberg algorithm it is difficult to discriminate between n and $\frac{n}{2}$.

c) Base b Greenberg could be used to get a tighter estimate. Anyway, to improve the accuracy, very small b has to be choosen which results in much worse running times (see Table 3.2): even if theoretically it remains $O(log_b n)$ which is sublinear in n, in practice estimate time can even overwhelm n when b is small.

4.3 EGA BSE

The *Enhached Greenberg Algorithm (EGA)* is a first improvement over the Basic Greenberg algorithm.

Let n be a batch size and p be a given transmission probability. As expressed by (4.12) (4.13) (4.14), varying n while p is fixed results in very different probabilities for *idle*, *successful* and *collided* slots.

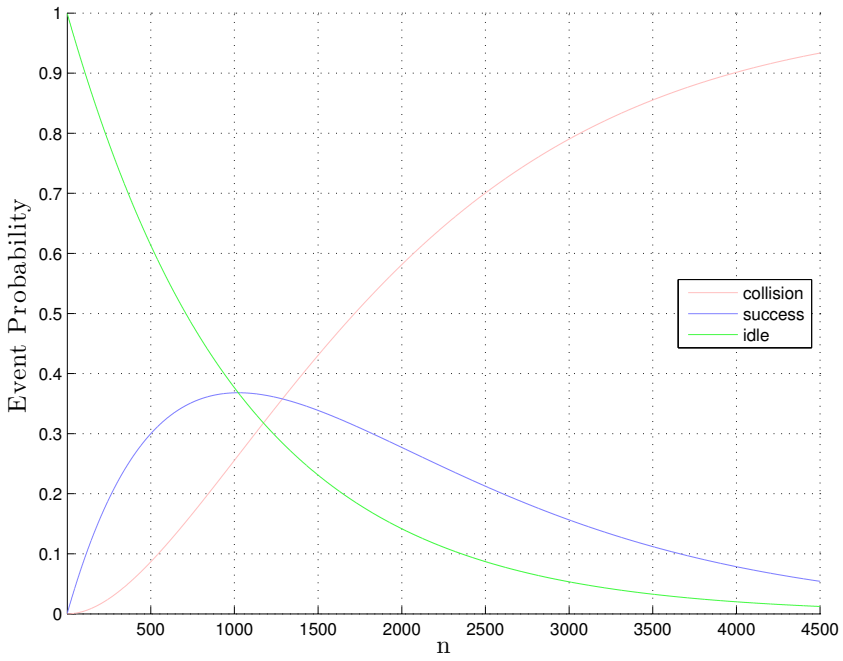


Figure 4.7: Probability of *idle*, *successful* or *collided* slots varying n while $p = 1/1024$. $q_0(p,n) \approx q_1(p,n)$ for $n = 1023$

Examining Figure 4.7, it is quite immediate to see that:

- $q_0(p,n) \approx q_1(p,n)$ when $p \approx \dfrac{1}{n}$ and, obviously,
- $q_0(p,n) \gg q_1(p,n)$ and $q_0(p,n) \gg q_{2+}(p,n)$ when $p \ll \dfrac{1}{n}$,
- $q_{2+}(p,n) \gg q_0(p,n)$ and $q_{2+}(p,n) \gg q_0(p,n)$ when $p \gg \dfrac{1}{n}$.

The collision probability $q_{2+}(p,n)$ is strictly monotonic increasing, while the empty slot probability $q_0(p,n)$ is strictly decreasing. Repeating a large number T of transmission with probability p, we can simply use the number of collisions or idle slots to uniquely determined the batch size. On the other hand, successful probability $q_1(p,n)$ is non-monotonic and hence

can not be used to uniquely determine the batch size.

The proposed technique to refine the estimate is the following.
Running *Basic Greenberg Algorithm* provides a raw estimate and the associated transmission probability p. Then we perform T more transmissions and use a *Maximum Likelihood Estimation (MLE)* to refine the estimate.

Let T be the number of slots we will use to refine the estimate and let N_0, N_1, N_{2+} be random variables which represent the total number of idle, successful and collided slots, respectively, observed after the T additional slots.
N_0, N_1, N_{2+} are binomial distributions with parameters $B(n, q_0(p,n))$, $B(n, q_1(p,n))$ and $B(n, q_{2+}(p,n))$, respectively. Of course $N_0 + N_1 + N_{2+} = T$ and

$$\begin{aligned} E[N_0] &= Tq_0(p,n), \\ E[N_1] &= Tq_1(p,n), \\ E[N_{2+}] &= Tq_{2+}(p,n). \end{aligned}$$

Let $f_T(i,s,c,p',n')$ denote the probability to get i idle, s success and c collision slots:

$$\begin{aligned} f_T(i,s,c,p',n') &= \Pr(N_0 = i, N_1 = s, N_{2+} = c | n = n', p = p') \\ &= \Pr(N_0 = i | n = n', p = p') \Pr(N_1 = s, N_{2+} = c | n = n', p = p', i), \end{aligned} \tag{4.18}$$

when we transmit with probability p' and the batch size is n'.
Let $\mathcal{N}$ be the set of the target batch sizes. Then, we can define

$$MLE(i,s,c,l) \leftarrow \underset{n' \in \mathcal{N}}{\arg\max} \Big(f_T\big(i,s,c,p(l),n'\big) \cdot f_G\big(n',l\big) \Big). \tag{4.19}$$

$MLE(i,s,c,l)$ is a $T \times T \times L$ matrix where L is the index of the "farthest" slot that can be visited during the *Basic Greenberg Algorithm*. L depends on the maximum cardinality n_{max} considered for the problem. Consequently L must be chosen so not to worse the performance of our estimator (if n_{max} is smaller than the real batch size n) and not to waste too much memory. Hence, our estimate is simply the result of a look-up in the MLE table,

$$\hat{n} \longleftarrow MLE(i,s,c,l). \tag{4.20}$$

Is it worth mentioning that:

- For performance tuning, the MLE table can be precomputed before running the estimate

algorithm. The computational cost to fill the whole MLE table depends on T and L and, if trivial exhaustive search on $\mathcal{N}$ is performed, also on $|\mathcal{N}|$. A fast way to compute the MLE table is described in Section 4.4.

- The size of the MLE table does not depend on $|\mathcal{N}|$ but only on T and L.
- The estimate deeply depends on the way the elements in $\mathcal{N}$ are chosen.

The high level pseudo-code of the algorithm is the following:

Algorithm 6 EGA $(\mathcal{B}, T, \mathcal{N})$

```
precompute or load the MLE table characterized by T, L, N
l ← 0
repeat
    l ← l + 1
    p ← 1/2^l
    choose to transmit with probability p
until no collision occurs
(i, s, c) ← events resulting from T transmissions with probability p
n̂ ← MLE(i, s, c, l)
```

Expected running time in slots follows from Greenberg expected running time and the further used slots. Therefore expected slot cost is is $\approx \log_2 n + T$.
We note also that only $MLE(i, s, c, l)$ is involved by the algorithm and not the whole MLE table. Hence, is it possible to choose wheter precomputing and storing the whole table or computing only $MLE(i, s, c, l)$ on-demand.
Once both the batch size n and the MLE table are fixed, the probability to estimate $\hat{n}$ can be expressed as:

$$\Pr(\hat{n} = n'|n) = \sum_l f_G(n, l) \sum_i \sum_s \hat{X}_{MLE(i,s,c,l)}(n') \cdot f_T\big(i, s, c, p(l), n\big), \tag{4.21}$$

where $\hat{X}_{MLE(i,s,c,l)}(n') = 1$ iff $MLE(i, s, c, l) = n'$, $\hat{X}_{MLE(i,s,c,l)}(n') = 0$ otherwise.
Hence the expected value of the estimate given n can be trivially computed as follows:

$$E[\hat{n}|n] = \sum_{n' \in \mathcal{N}} n' \cdot \Pr(\hat{n} = n'|n). \tag{4.22}$$

4.3.1 Performance

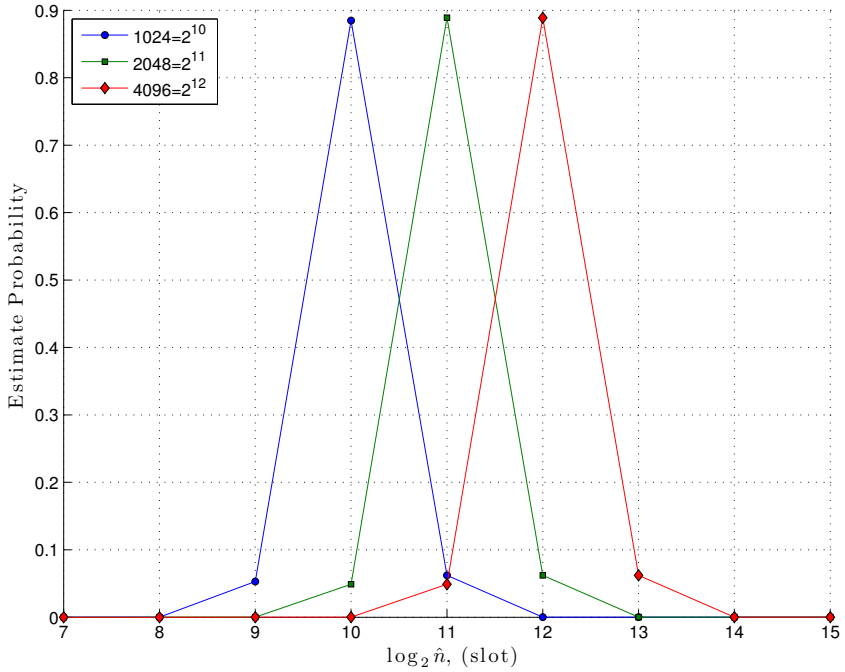


Figure 4.8: *EGA*: Large 2^k sizes distribution.

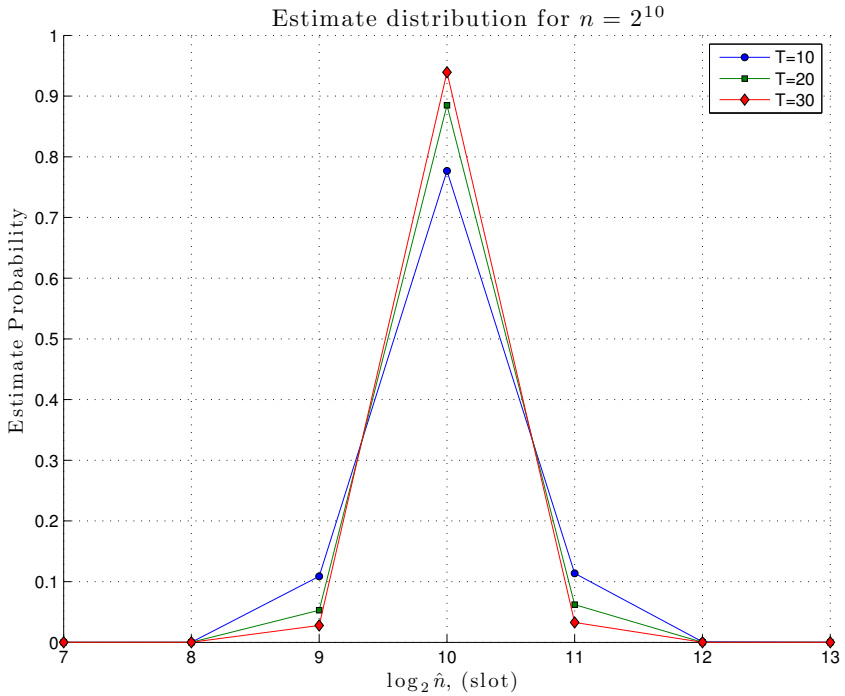


Figure 4.9: *EGA*: Estimate distribution when varying T. Plot refers to $n = 1024$.

Figures in this section were obtained by considering $\mathcal{N} = \{\text{powers of } 2\}$. Therefore, the target estimates are the same as for *basic Greenberg algorithm.*
Figure 4.8 shows that the proposed method peaks sharply near the right estimate. Comparing to the basic Greenberg method (see Figure 4.5) you can notice that there is only one extremely sharp peak located in a single slot, instead of a bell-like shape distributed among consecutive slots. Furthermore, you can also notice how the probability to overestimate the batch size is a bit higher than the probability to underestimate it.
Figure 4.9 shows how the peak behaves when varying T for a fixed batch size: increasing T the peak becomes more and more sharp.

4.4 GEGA BSE

The *Generalized Enhanced Greenberg Algorithm (GEGA)* is a less restrictive form of the EGA algorithm in the sense that we do not limit *a priori* the set of target estimates. This can be simply achieved by letting $\mathcal{N}$ be the set of all the positive integer numbers.

$$\mathcal{N} = \{1, 2, 3, ...\}.$$

In this case the algorithm can be interpreted as follows:

1. Find the transmission probability p for which the multiplicity of the activated set is 1 with very high probability,

2. Use T consecutive slots (a "window") to refine the estimate. The idea is similar to a window based estimate except that a node is allowed to transmit in each slot.

Recalling (4.19), given i, s, c and l, the estimate can be found by solving:

$$\frac{d\Big(f_T\big(i, s, c, p(l), n\big) \cdot f_G\big(n, l\big)\Big)}{dn} = 0, \tag{4.23}$$

which is not trivial to solve and involves numerical problems since $f_G(n, l)$ can present very small product terms. Then we solved the problem by using bisection method on (4.19) and taking the largest integer n for which

$$f_T\big(i, s, c, p(l), n+1\big) \cdot f_G\big(n+1, l\big) - f_T\big(i, s, c, p(l), n\big) \cdot f_G\big(n, l\big) \geq 0. \tag{4.24}$$

As an example, the results of this computation when $T = 10$ and $l = 10$ are presented in Table 4.1.

c \ s	0	1	2	3	4	5	6	7	8	9	10
0	352	413	477	545	616	689	765	843	922	1003	1086
1	489	559	632	709	787	868	951	1036	1122	1210	-
2	650	730	812	896	983	1072	1162	1254	1347	-	-
3	838	927	1017	1111	1206	1302	1401	1501	-	-	-
4	1055	1153	1254	1356	1461	1567	1675	-	-	-	-
5	1307	1416	1527	1641	1757	1875	-	-	-	-	-
6	1603	1726	1851	1979	2110	-	-	-	-	-	-
7	1962	2103	2248	2396	-	-	-	-	-	-	-
8	2416	2585	2761	-	-	-	-	-	-	-	-
9	3044	3267	-	-	-	-	-	-	-	-	-
10	4111	-	-	-	-	-	-	-	-	-	-

Table 4.1: *GEGA*: Possible estimates when $T = 10$ and $l = 10$.

Table 4.1 reveals that the estimate is always finite. This good feature follows from the bounded moments of the Greenberg algorithm. In general (see Table 3.3), simple window based MLE estimators can not achieve finite estimate when all the events are collisions.

The EGA MLE Table defined in (4.19) can be obtained as sub-case from GEGA MLE Table. In fact, once the GEGA MLE Table has been computed, for each tuple (i,s,c,l) providing estimate n', the corresponding EGA estimate can be obtained by considering the closest two EGA target estimates surrounding n' and by taking the best between the two by using (4.19) as usual.

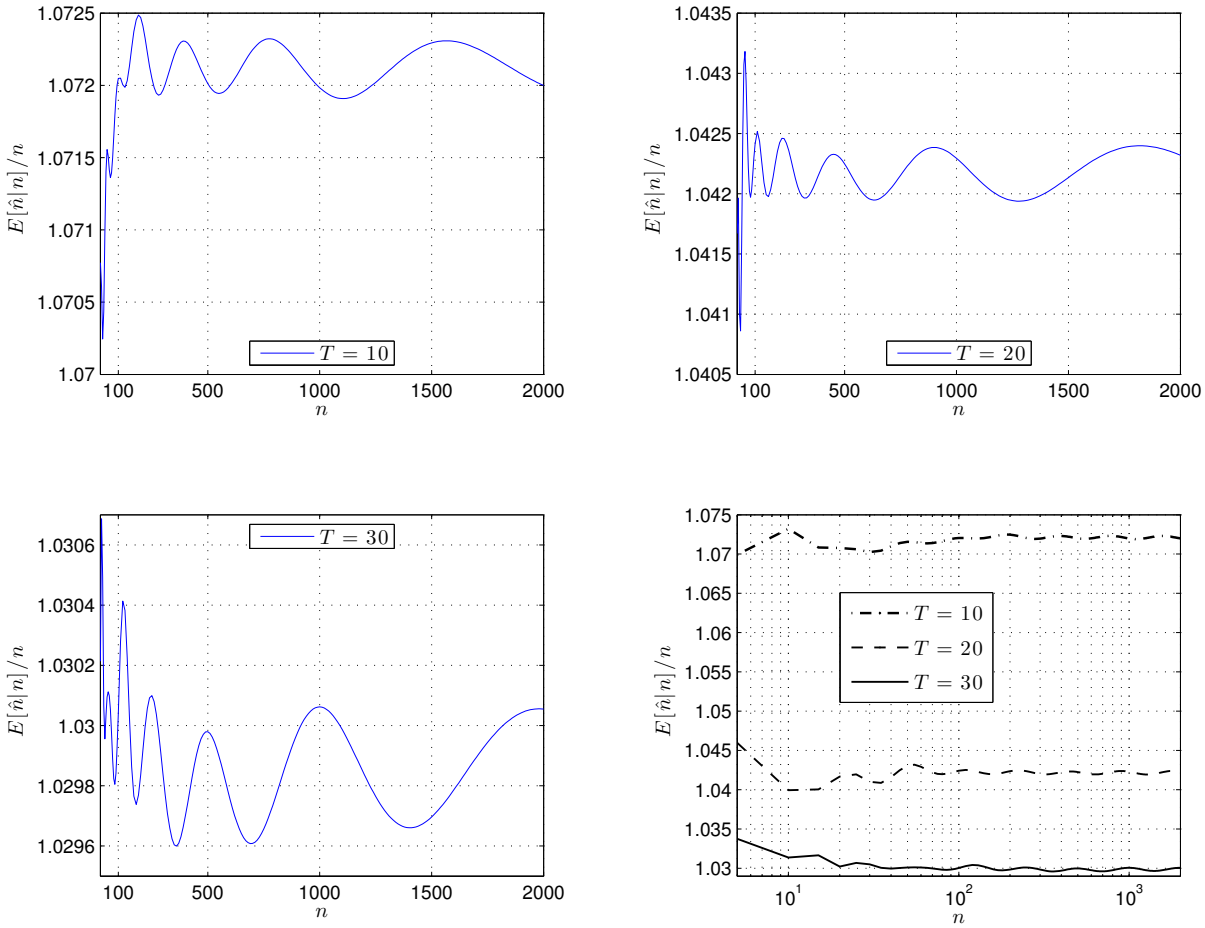


Figure 4.10: *GEGA*: Estimate Bias. The first tree plots show the estimate expected value over the real batch size for different T. Right below corner plot provides a summary in a log x-scale. Data reported were computed using formula (4.22).

4.4.1 Performance

The estimate provided by GEGA is biased. Figure 4.10 shows that the algorithm tends to overestimate the real batch size. Anyway, the bias can be easily corrected since it is always really near to the average value: the oscillations have very small amplitude. Increasing T reduces the bias as well as the oscillations amplitude. Very small batch sizes (less than 20) show higher oscillations compared to larger ones: this fact is negligible in practice since the oscillations are too small to affect the estimate and are hidden by the rounding operation. Table 4.2 reports the average bias $\phi(T)$ as a function of T. The results where computed by averaging

the bias estimate for batch sizes from 2 to 2000.

T	$\phi(T)$
10	0.0723
20	0.0422
30	0.0299

Table 4.2: *GEGA*: Average bias.

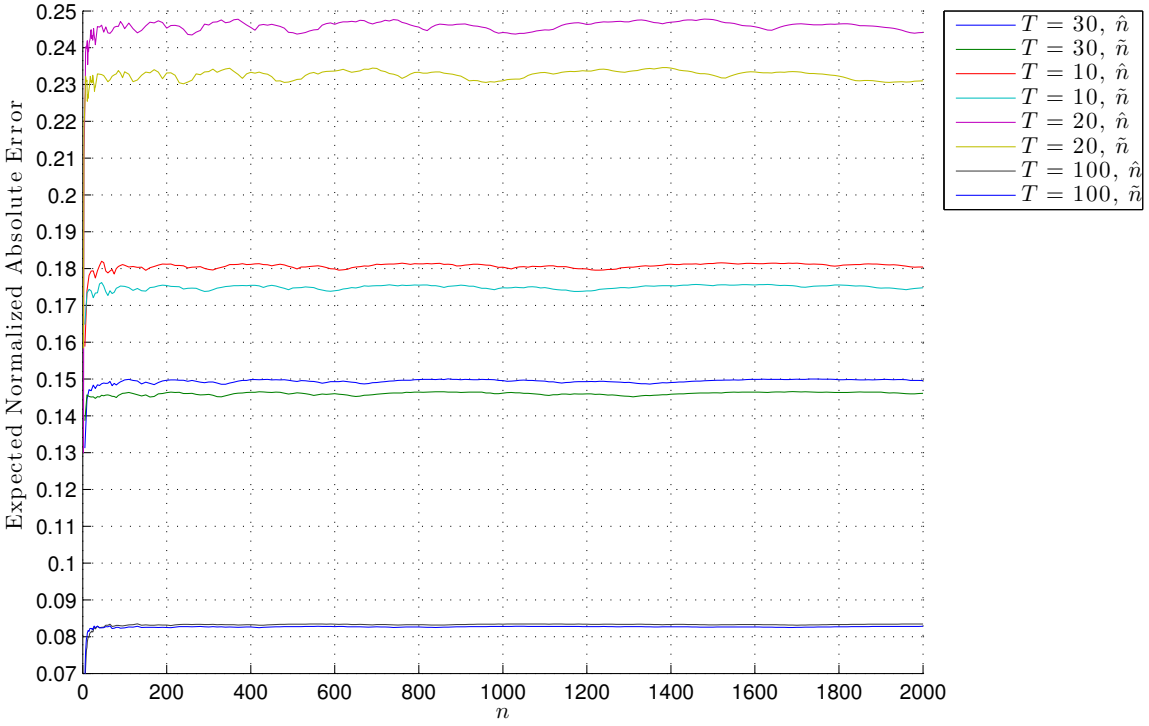


Figure 4.11: *GEGA*: biased/unbiased absolute normalized estimate error. The plot presents $e(\hat{n})$ and $e(\tilde{n})$ for different T.

Estimate bias can be corrected by considering $\tilde{n} = \dfrac{\hat{n}}{\phi(T)+1}$. We will see that in this way we get higher quality estimate.

To measure the performance of the proposed estimation method, we define the normalized absolute estimate error as

$$e(\hat{n}) = \frac{|\hat{n} - n|}{n}. \tag{4.25}$$

In the same manner we can define $e(\tilde{n})$. The parameter T is implicitly present in (4.25). Figure

4.11 shows that the bias-corrected estimate performs better than the biased estimate for any batch size and T. Furthermore, you can notice that the error is independent of the batch size.

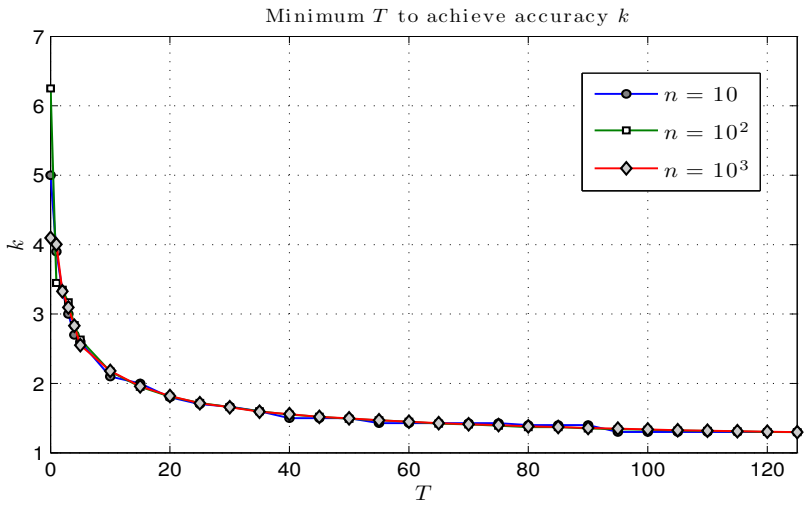


Figure 4.12: *GEGA*: Minimum T to achieve accuracy k

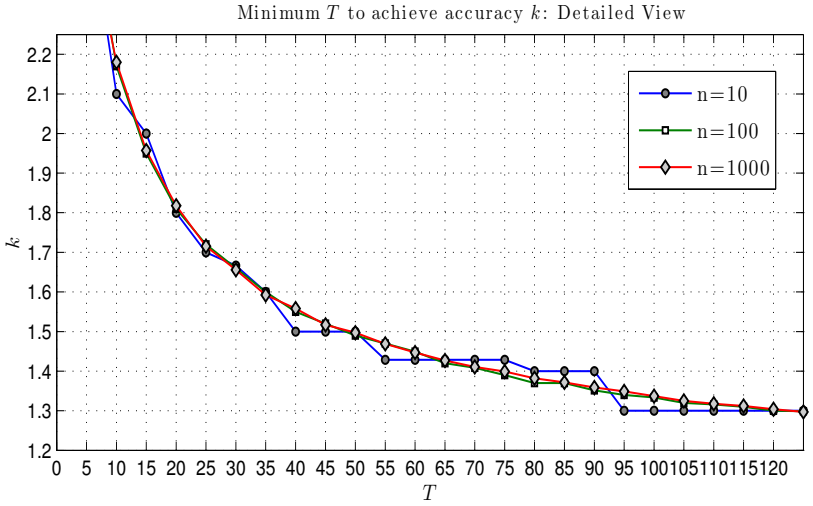


Figure 4.13: *GEGA*: Minimum T to achieve accuracy k, detailed view

Results presented in Figures 4.12 and 4.13 show that the accuracy in the estimate grows as T increases and it is independent of the batch size when the batch size is not too small (ex.: $n = 10$). In fact, for very small batch sizes, the accuracy shows a descending step trend since the probability distribution is discrete and the adopted algorithm performs rounding operations.

4.5 Comparison

Here we will provide the reader a comparison between the estimates obtained using *Cidon* and *GEGA* algorithms in terms of accuracy and time. In fact, in real life scenarios, we will be interested in getting a "tight enough" estimate of the real batch size in a reasonable time. We must always remember that there is a trade-off between the time taken by an explicit estimate phase and the following resolution: the goal is to minimize the total cost.
For the comparison we considered the dynamically adapting versions of the described algorithms:

- in *Cidon* this means that p_ϵ is not *a priori* fixed but, as the resolution goes on, gets updated in accordance to the resolved interval.
- in *GEGA* this simply means that T grows as the elapsed time.

In Figure 4.14 we try to answer the question: "elapsed time t, what is the accuracy of the estimate?". We considered the average running time for both the algorithms to provide a

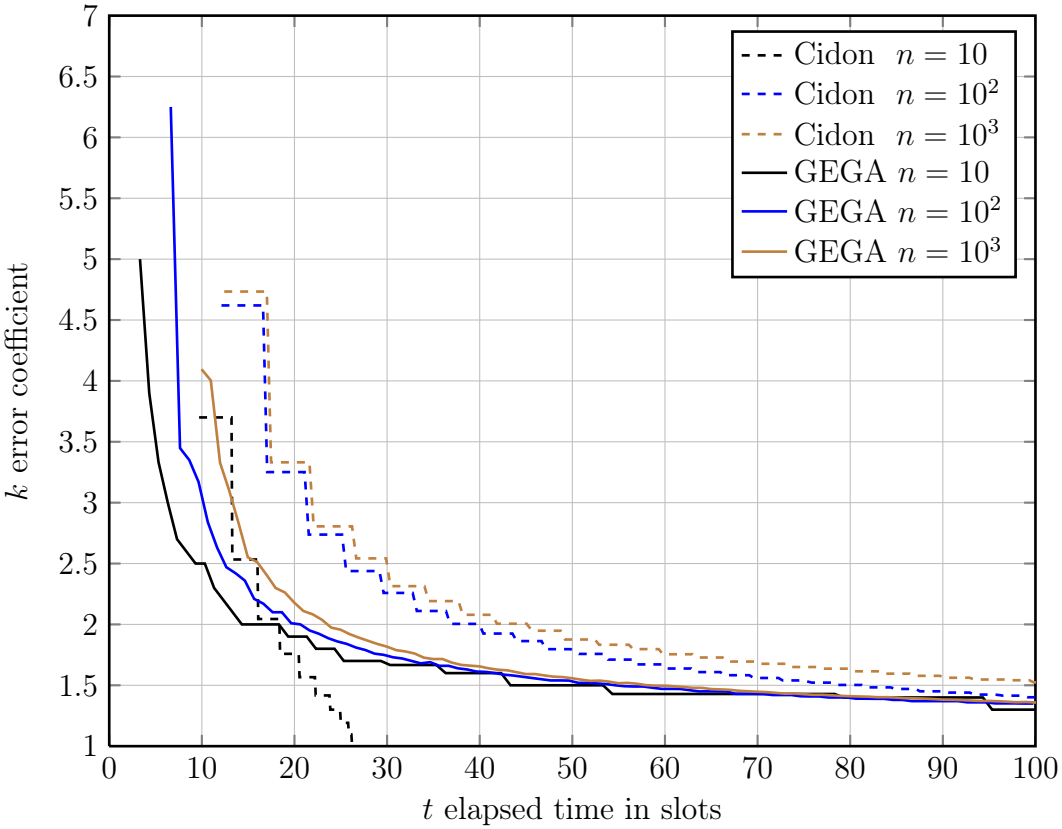


Figure 4.14: *GEGA vs Cidon*: Accuracy as function of elapsed time

meaningful and as fair as possible comparison: in *GEGA* $t \approx \log(n) + T$, in *Cidon* $t \approx np_\epsilon C$.

Figure 4.14 shows the minimum k for which θ_k in (4.8) is greater than or equal to 0.99. You can notice that the *GEGA* algorithm, since it provides always a finite estimate, gives a bound on the accuracy after just $\log(n)$ slots: as soon as phase 1 ends. Therefore, it results faster than *Cidon* in providing the first bound for any batch size. Considering $n = 10$ in the figure as an index of the behavior for very small batch sizes, you can see that initially *GEGA* estimate is more accurate than *Cidon* but later, for k smaller than two, *Cidon* takes less time. For larger sizes ($n = 100$ and $n = 1000$) *GEGA* achieves accuracy $k = 2$ in about the half of the time taken by *Cidon* , and $k = 1.5$ saving about $40 - 45\%$ of time.

In general, we can say that *GEGA* performs great for middle-large batch sizes when we require $\hat{n}$ to be, with high probability, in the interval $\frac{n}{k} \leq \hat{n} \leq kn$ with $1.5 \leq k \leq 2$.

Conclusions

The final goal of this work was to investigate *The Batch Size Estimate Problem* in order to develop, if possible, innovative techniques to solve the problem. Hence, we studied and analyzed both the *Batch Size Estimate* and some of the *Batch Resolution Algorithms* to understand in what measure the estimate can influence, and hopefully speedup, the resolution process. There are two possible ways to approach the matter: on one hand you can perform an explicit estimate phase, followed by a resolution one, on the other hand, you can choose to combine the two together. This second option is usually the case in window based scenarios where the result of the resolution is used to preform an estimate and where estimate and resolution processes coincide. Window based approach is really efficient when we have a good knowledge of the batch multiplicity but has the disadvantage that performs really poor when the batch size is different from the expected one. In fact, communication costs are reduced due to deferred feedback but, at the same time, a wrong estimate of the multiplicity results in really poor resolution performance. The big open problem of this approach is how to determine the size of the initial window. A large window results in good estimate and resolution of a possibly large amount of nodes but suffers poor performance if the batch to solve is much smaller than the windows size (empty slots) or much larger (collisions). Using *GEGA* is a great choice in this scenario to determine the batch size in terms of operational range.
The main advantages of *GEGA* are:

- Speed. In fact, for a wide range of minimum required accuracies and batch sizes, it is faster than any other estimate algorithm known to us.
- Ability to adapt to any batch size. In fact, the time required by the algorithm depends mostly on the user required accuracy and is quite independent of the real batch size (logarithmic addictive term).
- Estimate accuracy can be considered independent of the batch size.

However, there are also some limitation to the proposed estimate scheme:

- It is not able to achieve arbitrary precision in the estimate;

- When looking for a really tight (almost exact) estimate, other estimate techniques are faster.

Therefore we suggest to adopt it to:

- provide an initial estimate of the batch size which can be further refined using other techniques or that can be used to speedup dynamically estimating resolution processes.
- provide an estimate for algorithms that are "robust" in terms of resolution efficiency: their performance degrades smoothly when estimated batch size differs, but not too much, from the real one.

Understanding in details how to balance the estimate overhead with the improved resolution efficiency depends on the applied BRA and goes beyond the goals of this work. Just to provide you an idea on how things work, referring, as an example, to the *m Groups Split* in the CSMA scenario described in Section 2.1.4 we have that, following the notations used throughout this work, if estimate is within the interval $\frac{n}{k} \leq \hat{n} \leq kn$, maximum performance loss is below 3% when $k = 1.5$ and 7% when $k = 2$. Therefore, considering the estimate overhead and choosing accuracy in function of the batch size, using GEGA results convenient when the batch size is larger than 30-70.

Appendix A

A.1 Probability

This section reviews basic probability theory notions used in this work.

Chebyshev's inequality

Let X be a *random variable* with expected value μ and finite variance σ^2. Then for any real number $k > 0$,

$$\Pr(|X - \mu| \geq k) \leq \frac{\sigma^2}{k^2} \tag{A.1}$$

Binomial Distribution

The binomial distribution is the discrete probability distribution of the number of successes in a sequence of n independent yes/no experiments, each of which yields success with probability p. In general, let a random variable $K = B(n, p)$ be a binomial distribution with parameters n and p. The probability of getting exactly k successes in n trials is given by the probability mass function:

$$f(k; n, p) = \Pr(K = k) = \binom{n}{k} p^k (1-p)^{n-k} \tag{A.2}$$

for $k = 0, 1, \ldots, n$, where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.
The cumulative distribution function can be expressed as:

$$F(k; n, p) = \Pr(K \leq k) = \sum_{i=0}^{\lfloor k \rfloor} \binom{n}{i} p^i (1-p)^{n-i}. \tag{A.3}$$

The first and second moments are respectively given by:

$$\mathrm{E}[K] = np \tag{A.4}$$

and

$$Var[K] = np(1-p). \tag{A.5}$$

Normal Approximation

If n is large enough, then the skew of the distribution is not too great. In this case, if a suitable continuity correction[1] is used, then an excellent approximation to $B(n,p)$ is given by the normal distribution $\mathcal{N}(np, np(1-p))$
The approximation generally improves as n increases and is better when p is not near to 0 or 1. Various rules of thumb may be used to decide whether n is large enough, and p is far enough from the extremes of zero or unity: One rule is that both np and $n(1-p)$ must be greater than 5. However, the specific number varies from source to source, and depends on how good an approximation one wants; some sources give 10. Another easy rule is that $np(1-p) \geq 10$.

Poisson Approximation

The binomial distribution converges towards the Poisson distribution as the number of trials goes to infinity while the product np remains fixed. Therefore the Poisson distribution with parameter $\lambda = np$ can be used as an approximation to $B(n,p)$ of the binomial distribution if n is sufficiently large and p is sufficiently small. According to two rules of thumb, this approximation is good if $n \geq 20$ and $p \leq 0.05$, or if $n \geq 100$ and $np \leq 10$.

Numerical Computation

When n is large and p small, the numerical computation of the probability density function and cumulative distribution function show to be problematic if trivially computed by definition ($n!$ is huge).
The following trick can be used for the computation when $0 < k < n$:

$$f(k;n,p) = \Pr(K=k) = e^{\log(\frac{n!}{k!(n-k)!}p^k(1-p)^{n-k})} = e^{\phi}, \tag{A.6}$$

where

$$\phi = \sum_{i=k+1}^{n} \log i - \sum_{i=1}^{n-k} \log i + k\log p + (n-k)\log(1-p). \tag{A.7}$$

[1] when approximating a discrete random variable K using a continuos one X, then $\Pr(\mathrm{K} \leq k) = \Pr(\mathrm{K} < k+1) \approx \Pr(\mathrm{X} \leq k+1/2)$

Poisson Distribution

In probability theory and statistics, the Poisson distribution is a discrete probability distribution that expresses the probability of a number of events occurring in a fixed period of time if these events occur with a known average rate and independently of the time since the last event.
If the expected number of occurrences in this interval is λ, then the probability that there are exactly k occurrences (k being a non-negative integer, $k = 0, 1, 2, \ldots$) is equal to

$$f(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!} \tag{A.8}$$

Normal Distribution

Normal distribution is denoted as $\mathcal{N}(\mu, \sigma^2)$ where, as usual, μ identifies the mean and σ^2 the variance. The probability density function is defined as follows:

$$f(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{A.9}$$

A.2 Binary Trees Performance Summary

The following tables report the expected cost L_n in slots to solve a batch of given size n using BT, MBT, and CMBT. Performance reports in Chapter 2 plot these data.

Table A.1: *Basic Binary Tree*: Performance report

$L_2^{BT} = 5.0000$	$L_3^{BT} = 7.6667$	$L_4^{BT} = 10.5238$	$L_5^{BT} = 13.4190$	$L_6^{BT} = 16.3131$
$L_7^{BT} = 19.2009$	$L_8^{BT} = 22.0854$	$L_9^{BT} = 24.9690$	$L_{10}^{BT} = 27.8532$	$L_{11}^{BT} = 30.7382$
$L_{12}^{BT} = 33.6238$	$L_{13}^{BT} = 36.5096$	$L_{14}^{BT} = 39.3955$	$L_{15}^{BT} = 42.2812$	$L_{16}^{BT} = 45.1668$
$L_{17}^{BT} = 48.0522$	$L_{18}^{BT} = 50.9375$	$L_{19}^{BT} = 53.8227$	$L_{20}^{BT} = 56.7078$	$L_{21}^{BT} = 59.5930$
$L_{22}^{BT} = 62.4783$	$L_{23}^{BT} = 65.3636$	$L_{24}^{BT} = 68.2489$	$L_{25}^{BT} = 71.1344$	$L_{26}^{BT} = 74.0198$

Table A.2: *Modified Binary Tree*: Performance report with $p = 0.5$

$L_2^{MBT} = 4.5000$	$L_3^{MBT} = 7.0000$	$L_4^{MBT} = 9.6429$	$L_5^{MBT} = 12.3143$	$L_6^{MBT} = 14.9848$
$L_7^{MBT} = 17.6507$	$L_8^{MBT} = 20.3140$	$L_9^{MBT} = 22.9768$	$L_{10}^{MBT} = 25.6399$	$L_{11}^{MBT} = 28.3036$
$L_{12}^{MBT} = 30.9678$	$L_{13}^{MBT} = 33.6322$	$L_{14}^{MBT} = 36.2966$	$L_{15}^{MBT} = 38.9609$	$L_{16}^{MBT} = 41.6251$
$L_{17}^{MBT} = 44.2892$	$L_{18}^{MBT} = 46.9531$	$L_{19}^{MBT} = 49.6170$	$L_{20}^{MBT} = 52.2809$	$L_{21}^{MBT} = 54.9448$
$L_{22}^{MBT} = 57.6087$	$L_{23}^{MBT} = 60.2727$	$L_{24}^{MBT} = 62.9367$	$L_{25}^{MBT} = 65.6008$	$L_{26}^{MBT} = 68.2649$

Table A.3: *Modified Binary Tree*: Performance report with $p = 0.4175$

$L_2^{MBT} = 4.4143$	$L_3^{MBT} = 6.8847$	$L_4^{MBT} = 9.4826$	$L_5^{MBT} = 12.1078$	$L_6^{MBT} = 14.7352$
$L_7^{MBT} = 17.3605$	$L_8^{MBT} = 19.9841$	$L_9^{MBT} = 22.6069$	$L_{10}^{MBT} = 25.2293$	$L_{11}^{MBT} = 27.8519$
$L_{12}^{MBT} = 30.4745$	$L_{13}^{MBT} = 33.0972$	$L_{14}^{MBT} = 35.7201$	$L_{15}^{MBT} = 38.3430$	$L_{16}^{MBT} = 40.9659$
$L_{17}^{MBT} = 43.5889$	$L_{18}^{MBT} = 46.2118$	$L_{19}^{MBT} = 48.8347$	$L_{20}^{MBT} = 51.4577$	$L_{21}^{MBT} = 54.0806$
$L_{22}^{MBT} = 56.7034$	$L_{23}^{MBT} = 59.3263$	$L_{24}^{MBT} = 61.9492$	$L_{25}^{MBT} = 64.5721$	$L_{26}^{MBT} = 67.1949$

Table A.4: *Clipped Modified Binary Tree*: performance report. Experimental results obtained by running CMBT algorithm on 10 000 random generated instances.

$L_2^{CMBT} = 4.3423$	$L_3^{CMBT} = 6.5558$	$L_4^{CMBT} = 9.3585$	$L_5^{CMBT} = 12.1492$	$L_6^{CMBT} = 15.0521$
$L_7^{CMBT} = 17.9785$	$L_8^{CMBT} = 21.0389$	$L_9^{CMBT} = 24.1297$	$L_{10}^{CMBT} = 27.2045$	$L_{11}^{CMBT} = 30.4554$
$L_{12}^{CMBT} = 33.6398$	$L_{13}^{CMBT} = 36.9191$	$L_{14}^{CMBT} = 40.2243$	$L_{15}^{CMBT} = 43.5920$	$L_{16}^{CMBT} = 47.0131$
$L_{17}^{CMBT} = 50.4536$	$L_{18}^{CMBT} = 53.8787$	$L_{19}^{CMBT} = 57.2679$	$L_{20}^{CMBT} = 60.8329$	$L_{21}^{CMBT} = 64.3700$
$L_{22}^{CMBT} = 67.8123$	$L_{23}^{CMBT} = 71.4436$	$L_{24}^{CMBT} = 75.0234$	$L_{25}^{CMBT} = 78.7210$	$L_{26}^{CMBT} = 82.2599$

A.3 Greenberg bounded m-moments

In general, for *base b greenberg* algorithm the first and second moments are bounded by:

$$\phi(b) = \frac{1}{\log b} \int_0^\infty e^{-x}(1+x) \prod_{k=1}^{\infty} (1 - e^{-b^k x}(1 + b^k x)) x^{-2}\, dx \tag{A.10}$$

$$\Phi(b) = \frac{1}{\log b} \int_0^\infty e^{-x}(1+x) \prod_{k=1}^{\infty} (1 - e^{-b^k x}(1 + b^k x)) x^{-3}\, dx \tag{A.11}$$

Even if the integral is improper, the integrand function goes fast to 0 as k grows. We found that considering interval (0,40) for numerical integration provides good results.

A.4 CBT Estimate Experimental Distribution

Following table A.5 shows the behavior of CBT Algorithm (section 3.1) for estimation. The simulation was implemented in matlab. The reported distribution of $\hat{n}$ fixed n is the result of averaging 100 000 runs of the CBT algorithm applied on uniformly random generated nodes *id* batches.

Table A.5: Experimentally computed CBT Estimate Distributon. Table 1/3

n	$\hat{n}$:	2	4	8	16	32	64	128	256	512	1024	2048
2		0.499	0.253	0.125	0.061	0.031	0.015	0.007	0.004	0.002	9e-04	4e-04
4			0.189	0.303	0.225	0.133	0.072	0.038	0.020	0.010	0.005	0.002
8			0.055	0.212	0.261	0.201	0.126	0.071	0.037	0.019	0.009	0.005
16			8e-04	0.070	0.209	0.252	0.197	0.125	0.070	0.038	0.019	0.010
32				0.003	0.075	0.213	0.249	0.195	0.123	0.069	0.037	0.018
64					0.004	0.077	0.208	0.250	0.193	0.124	0.069	0.037
128						0.005	0.081	0.208	0.247	0.191	0.123	0.069
256						2e-05	0.006	0.079	0.209	0.246	0.193	0.123
512								0.005	0.081	0.207	0.245	0.193
1024									0.005	0.080	0.208	0.245
2048									2e-05	0.005	0.080	0.209
4096										1e-05	0.006	0.082
8192											1e-05	0.006
16384												2e-05
32768												

Table A.5: Experimentally computed CBT Estimate Distributon. Table 2/3

n	$\hat{n}$:	4096	8192	16384	32768	2^{16}	2^{17}	2^{18}	2^{19}	2^{20}	2^{21}	2^{22}
2		2e-04	1e-04	1e-04				1e-05				
4		0.001	5e-04	3e-04	1e-04	6e-05	4e-05		2e-05	1e-05		
8		0.002	0.001	6e-04	3e-04	9e-05	8e-05	4e-05	1e-05			
16		0.005	0.003	0.001	6e-04	3e-04	2e-04	6e-05	2e-05	2e-05		
32		0.009	0.005	0.003	0.001	6e-04	3e-04	1e-04	1e-04	6e-05	1e-05	1e-05
64		0.019	0.009	0.005	0.002	0.001	7e-04	3e-04	7e-05	4e-05		2e-05
128		0.037	0.019	0.010	0.005	0.003	0.001	5e-04	4e-04	2e-04	5e-05	4e-05
256		0.068	0.038	0.019	0.009	0.005	0.002	0.001	6e-04	3e-04	6e-05	8e-05
512		0.123	0.071	0.037	0.019	0.010	0.005	0.002	0.001	6e-04	3e-04	2e-04
1024		0.193	0.122	0.070	0.037	0.019	0.010	0.005	0.002	0.001	6e-04	2e-04
2048		0.246	0.194	0.123	0.068	0.037	0.019	0.010	0.005	0.002	0.001	6e-04
4096		0.210	0.246	0.193	0.121	0.068	0.037	0.019	0.009	0.004	0.003	0.001
8192		0.080	0.208	0.247	0.192	0.123	0.070	0.037	0.019	0.009	0.005	0.002
16384		0.006	0.080	0.208	0.247	0.192	0.123	0.069	0.037	0.019	0.010	0.005
32768			0.006	0.079	0.209	0.248	0.194	0.122	0.069	0.036	0.019	0.010

Table A.5: Experimentally computed CBT Estimate Distributon. Table 3/3

n	$\hat{n}$:	2^{23}	2^{24}	2^{25}	2^{26}	2^{27}	2^{28}	2^{29}	2^{30}	2^{31}	2^{32}
2											
4											
8											
16											
32		1e-05									
64		2e-05				1e-05					
128		4e-05		1e-05							
256		4e-05	1e-05		2e-05			1e-05			
512		7e-05	2e-05	1e-05	3e-05	1e-05					
1024		2e-04	1e-04	4e-05		2e-05	1e-05				
2048		3e-04	1e-04	3e-05	3e-05	3e-05	3e-05				
4096		6e-04	3e-04	9e-05	7e-05	1e-05					
8192		0.001	8e-04	3e-04	2e-04	8e-05	7e-05	2e-05	3e-05	1e-05	
16384		0.002	0.001	6e-04	3e-04	1e-04	1e-04	4e-05			
32768		0.005	0.002	0.001	6e-04	3e-04	2e-04	1e-04	1e-05	2e-05	1e-05

A.5 Greenberg Estimate Distribution

In following table A.6 we report how the end up probability (equation 4.16) is distributed among slots given a batch of size n. Column "n" lists the considered batch sizes. $\hat{n}$ is the resulting estimation (without corrections) when ending up in the underneath slot.
For sake of simplicity considered values are all powers of 2.
Datas presented were post-processed to become more accessible:

- values above 10^{-3} are reported in format (*'%1.3f'*);
- values below 10^{-12} are not presented since are tight close to 0.
- other values are presented in exponential notation and rounded to the first meaningful digit (*'%1.e'*)

n	$\hat{n}$ / slot:	2 / 1	4 / 2	8 / 3	16 / 4	32 / 5	64 / 6	128 / 7	256 / 8	512 / 9	1024 / 10	2048 / 11	4096 / 12	8192 / 13	16384 / 14	32768 / 15	65536 / 16
1		1.000															
2		0.750	0.234	0.015	2e-04	1e-06	9e-10										
4		0.312	0.508	0.166	0.014	3e-04	2e-06	2e-09									
8		0.035	0.354	0.450	0.147	0.013	3e-04	2e-06	4e-09	1e-12							
16		3e-04	0.063	0.363	0.422	0.138	0.013	3e-04	2e-06	4e-09	2e-12						
32		8e-09	0.001	0.078	0.366	0.409	0.134	0.013	3e-04	2e-06	4e-09	2e-12					
64			2e-07	0.002	0.084	0.367	0.402	0.131	0.013	3e-04	2e-06	4e-09	2e-12				
128				7e-07	0.002	0.088	0.367	0.399	0.130	0.013	3e-04	2e-06	5e-09	2e-12			
256					1e-06	0.003	0.090	0.368	0.397	0.130	0.013	3e-04	2e-06	5e-09	2e-12		
512						2e-06	0.003	0.090	0.368	0.397	0.130	0.012	3e-04	2e-06	5e-09	2e-12	
1024							2e-06	0.003	0.091	0.368	0.396	0.129	0.012	3e-04	2e-06	5e-09	2e-12

n	$\hat{n}$ / slot:	128 / 7	256 / 8	512 / 9	1024 / 10	2048 / 11	4096 / 12	8192 / 13	16384 / 14	32768 / 15	65536 / 16	2^{17} / 17	2^{18} / 18	2^{19} / 19	2^{20} / 20	2^{21} / 21	2^{22} / 22
2048		2e-06	0.003	0.091	0.368	0.396	0.129	0.012	3e-04	2e-06	5e-09	2e-12					
4096			2e-06	0.003	0.091	0.368	0.396	0.129	0.012	3e-04	2e-06	5e-09	2e-12				
8192				2e-06	0.003	0.091	0.368	0.396	0.129	0.012	3e-04	2e-06	5e-09	2e-12			
16384					2e-06	0.003	0.091	0.368	0.396	0.129	0.012	3e-04	2e-06	5e-09	2e-12		
32768						2e-06	0.003	0.091	0.368	0.396	0.129	0.012	3e-04	2e-06	5e-09	2e-12	
65536							2e-06	0.003	0.091	0.368	0.396	0.129	0.012	3e-04	2e-06	5e-09	2e-12

Table A.6: *Basic Greenberg*: Analytically computed estimate distribution

A.6 GEGA

c \ s	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	232	265	300	336	373	411	451	491	531	573	615	658	701	744	788	832	877	922	967	1012	1058
1	304	340	378	417	457	498	539	582	625	668	712	756	801	846	892	938	984	1030	1077	1124	-
2	383	423	464	505	548	591	635	679	724	769	815	861	908	955	1002	1049	1097	1145	1193	-	-
3	471	513	556	600	645	690	736	782	829	876	924	972	1020	1069	1118	1167	1217	1266	-	-	-
4	565	610	656	702	749	796	844	892	941	990	1040	1090	1140	1190	1241	1292	1343	-	-	-	-
5	667	714	762	811	860	909	959	1009	1060	1111	1163	1215	1267	1319	1372	1425	-	-	-	-	-
6	776	826	876	927	978	1029	1082	1134	1187	1240	1294	1348	1402	1457	1511	-	-	-	-	-	-
7	893	945	997	1050	1104	1158	1212	1267	1322	1378	1434	1490	1547	1604	-	-	-	-	-	-	-
8	1018	1073	1128	1183	1239	1296	1353	1410	1468	1526	1584	1643	1703	-	-	-	-	-	-	-	-
9	1153	1210	1268	1326	1385	1444	1504	1564	1624	1686	1747	1809	-	-	-	-	-	-	-	-	-
10	1298	1358	1419	1480	1542	1605	1667	1731	1795	1859	1924	-	-	-	-	-	-	-	-	-	-
11	1455	1519	1583	1648	1714	1780	1846	1914	1981	2050	-	-	-	-	-	-	-	-	-	-	-
12	1628	1695	1763	1832	1902	1972	2044	2115	2188	-	-	-	-	-	-	-	-	-	-	-	-
13	1817	1890	1963	2037	2112	2187	2264	2342	-	-	-	-	-	-	-	-	-	-	-	-	-
14	2030	2108	2187	2267	2348	2431	2515	-	-	-	-	-	-	-	-	-	-	-	-	-	-
15	2271	2356	2443	2532	2621	2713	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
16	2551	2647	2744	2844	2945	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
17	2888	2998	3111	3227	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
18	3314	3448	3587	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
19	3907	4085	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	4918	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table A.7: *GEGA*: Possible estimates when $T = 20$ and $l = 10$

Table A.8: *GEGA*: Possible estimates when $T = 30$ and $l = 10$

c \ s	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
0	179	202	226	250	276	302	328	355	383	411	439	468	497	526	555	585	615	645	675	705	736	767	797	828	859	890	922	953	984	1016	1047
1	228	253	278	305	331	359	387	415	444	473	502	531	561	591	621	652	683	713	744	775	807	838	869	901	933	964	996	1028	1060	1092	-
2	281	307	335	362	391	419	448	478	507	537	567	598	628	659	690	722	753	784	816	848	880	912	944	976	1009	1041	1074	1106	1139	-	-
3	338	366	395	424	453	483	513	543	574	604	636	667	698	730	762	794	826	858	891	923	956	988	1021	1054	1087	1120	1154	1187	-	-	-
4	399	428	458	488	518	549	580	611	643	675	706	739	771	803	836	869	902	935	968	1001	1034	1068	1102	1135	1169	1203	1237	-	-	-	-
5	463	493	524	555	587	618	650	682	715	747	780	813	846	880	913	947	980	1014	1048	1082	1116	1150	1185	1219	1254	1288	-	-	-	-	-
6	530	561	593	626	658	691	724	757	790	823	857	891	925	959	993	1027	1062	1097	1131	1166	1201	1236	1271	1306	1342	-	-	-	-	-	-
7	600	633	666	699	732	766	800	834	868	902	937	971	1006	1041	1076	1112	1147	1182	1218	1254	1289	1325	1361	1397	-	-	-	-	-	-	-
8	674	708	742	776	810	845	879	914	949	985	1020	1056	1091	1127	1163	1199	1235	1272	1308	1345	1381	1418	1455	-	-	-	-	-	-	-	-
9	751	786	821	856	891	927	962	998	1034	1070	1107	1143	1180	1217	1253	1290	1328	1365	1402	1440	1478	1515	-	-	-	-	-	-	-	-	-
10	831	867	903	939	976	1012	1049	1086	1123	1160	1197	1235	1272	1310	1348	1386	1424	1462	1501	1540	1578	-	-	-	-	-	-	-	-	-	-
11	916	952	989	1026	1064	1101	1139	1177	1215	1253	1292	1330	1369	1408	1447	1486	1525	1565	1604	1644	-	-	-	-	-	-	-	-	-	-	-
12	1004	1041	1080	1118	1156	1195	1234	1273	1312	1351	1391	1431	1470	1510	1551	1591	1632	1672	1713	-	-	-	-	-	-	-	-	-	-	-	-
13	1096	1135	1174	1214	1253	1293	1333	1373	1414	1454	1495	1536	1577	1618	1660	1702	1743	1786	-	-	-	-	-	-	-	-	-	-	-	-	-
14	1193	1233	1273	1314	1355	1396	1437	1479	1521	1563	1605	1647	1690	1733	1775	1819	1862	-	-	-	-	-	-	-	-	-	-	-	-	-	-
15	1294	1336	1378	1420	1462	1505	1548	1591	1634	1677	1721	1765	1809	1853	1898	1943	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
16	1402	1445	1488	1532	1576	1620	1664	1709	1754	1799	1844	1890	1936	1982	2028	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
17	1516	1560	1606	1651	1697	1742	1789	1835	1882	1929	1976	2024	2071	2120	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
18	1637	1683	1730	1778	1825	1873	1921	1970	2019	2068	2117	2167	2217	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
19	1766	1815	1864	1913	1963	2013	2064	2115	2166	2218	2270	2322	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	1905	1956	2008	2060	2112	2165	2218	2272	2326	2381	2436	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
21	2055	2110	2164	2219	2275	2331	2387	2444	2502	2560	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
22	2220	2277	2335	2394	2453	2513	2573	2634	2696	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
23	2401	2462	2525	2588	2651	2716	2781	2847	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
24	2603	2670	2738	2806	2876	2946	3018	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
25	2834	2907	2982	3058	3135	3214	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
26	3103	3185	3269	3355	3443	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
27	3427	3522	3620	3721	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
28	3838	3955	4076	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
29	4411	4570	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
30	5395	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

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