

# Intercomparison of data-driven and learning-based interpolations of along-track Nadir and wide-swath SWOT altimetry observations

Maxime Beauchamp<sup>1</sup>, Ronan Fablet<sup>1</sup>, Clément Ubelmann<sup>2</sup>, Maxime Ballarotta<sup>3</sup>  
and Bertrand Chapron<sup>4</sup>,

1. IMT Atlantique Bretagne-Pays de la Loire, Brest, France
2. Ocean Next, Grenoble, France
3. Collecte Localisation Satellites (CLS), Ramonville St-Agne, France
4. IFREMER, Brest

[maxime.beauchamp@imt-atlantique.fr](mailto:maxime.beauchamp@imt-atlantique.fr)



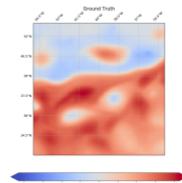
2020, October 13

- Ground truth dataset  $\mathbf{x}$  : high-resolution  $1/60^\circ$  NATL60 configuration of the NEMO (Nucleus for European Modeling of the Ocean) model
- A  $10^\circ \times 10^\circ$  GULFSTREAM region is used with downgraded resolution to  $1/20^\circ$

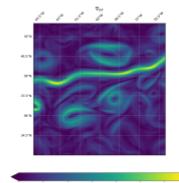


GULFSTREAM domain

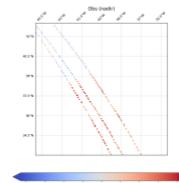
- OSSE : pseudo-altimetric nadir and SWOT observational datasets  $\mathbf{y} = \{\mathbf{y}_k\}$  at time  $t_k$  are generated by a realistic sub-sampling satellite constellations on subdomain  $\Omega = \{\Omega_k\}$  of the grid.



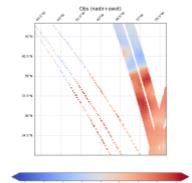
(a) Ground Truth (SSH)



(b) Ground Truth  
( $\nabla_{\text{SSH}}$ )



(c) Observations  
(nadir)

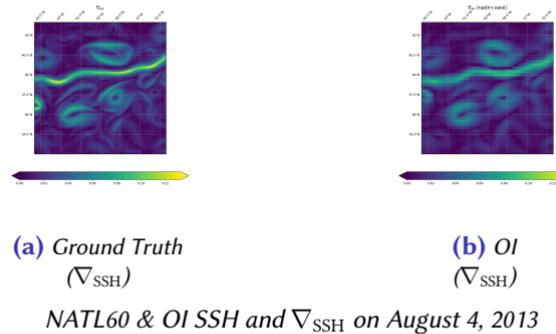


(d) Observations  
(nadir+swot)

Ground Truth (SSH &  $\nabla_{\text{SSH}}$ ) and pseudo-observations (nadir & nadir+swot) on August 4, 2013

## Methods

DUACS OI  $\bar{x}$  (Taburet et al.) as a baseline : significant smooting, solving spatial scales up to 150km :



All the interpolations methods used here will work on the anomaly field  $d\mathbf{x}$  :

$$\mathbf{x} = \bar{\mathbf{x}} + d\mathbf{x} + \epsilon$$

### Data-driven and learning-based approaches

- **VE-DINEOF** is a state-of-the-art interpolation approach (Ping et al., 2016) using an EOF-based iterative filling strategy. Typically the large-scale component provided by the OI is used (or 0 values if working on the anomaly) as a first guess to fill in the missing data over  $\Omega$ ;
- **The Analog Data Assimilation (AnDA)** (Lguensat et al., 2017) is a purely data-driven data assimilation method introducing a statistical operator  $\mathcal{A}$  as a substitute for the dynamical model  $\mathcal{M}$  in a classic state-space formulation;
- **Convolutional Neural Networks (CNN)** : specifically dedicated to spatio-temporal interpolation problems (Fablet et al., 2019), neural DINEOF extensions + an explicit link with variational data assimilation

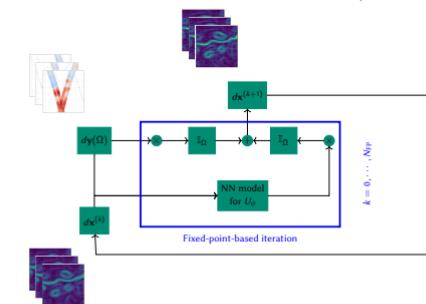
## End-to-end learning

- An end-to-end learning representation has recently been introduced in Fablet et al. (2019) to deal with image sequences involving potentially large missing data rates. An energy-based representation  $U_\psi = \|d\mathbf{x} - \psi(d\mathbf{x})\|_\Omega^2$  to minimize is introduced where the operator  $\psi = \psi_\theta$  denotes a NN-based representation (Convolutional autoencoders **ConvAE** or Gibbs energy related NN **GENN**) of the underlying processes.
- For a specific definition of the hidden state interpolator  $\widehat{d\mathbf{x}_k} = I_{U_\psi}(d\mathbf{y}_k(\Omega_k))$  based on the irregular space-time dataset  $\{d\mathbf{y}_k(\Omega_k)\}$ , the learning problem for optimizing parameters  $\theta$  of  $\psi$  is stated as the minimization of the reconstruction error :

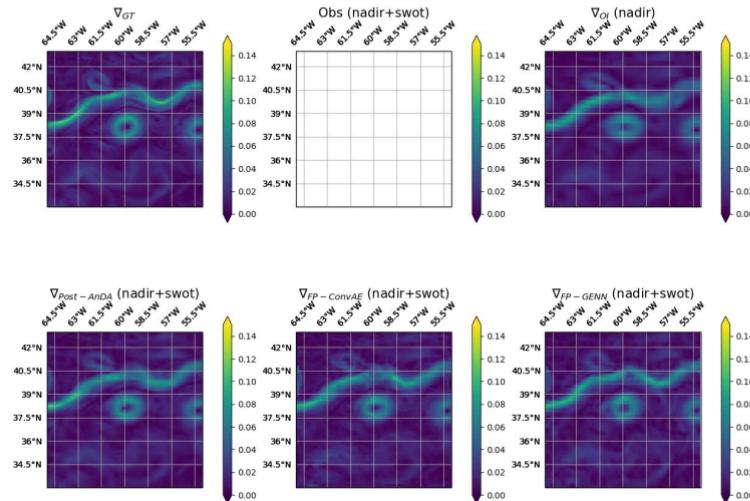
$$\widehat{\theta} = \arg \min_{\theta} \sum_k \|d\mathbf{y}_k(\Omega_k) - I_{U_\psi}(d\mathbf{y}_k(\Omega_k))\|_{\Omega_k}^2$$

An iterative fixed-point (FP) solver is used to optimize parameters  $\theta$  of the NN-model  $\psi$  w.r.t cost  $U_\psi$  :

$$\begin{cases} \mathbf{x}^{(i+1)} &= \psi(\mathbf{x}^{(i)}) \\ \mathbf{x}^{(i+1)}(\Omega) &= \mathbf{y}(\Omega) \\ \mathbf{x}^{(i+1)}(\bar{\Omega}) &= \mathbf{x}^{(i+1)}(\bar{\Omega}) \end{cases}$$



*Sketch of the iterative fixed-point algorithm*

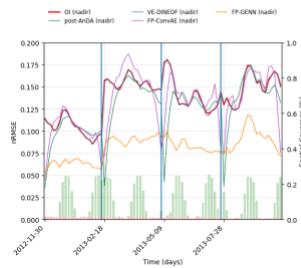


*Global SSH gradient field reconstruction obtained for a joint assimilation/learning of along-track nadir with wide-swath SWOT data*

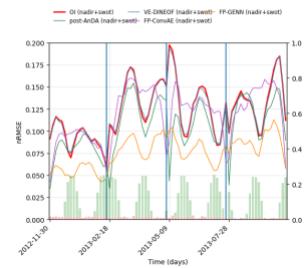
## Scores (I)

The scores are computed on four 20-day validation periods over the one-year NATL60 daily dataset :

**Up to 40% relative gain on the SSH daily root mean squared error with FP-GENN**  
**Up to 30% relative gain when using 2D SWOT vs 1D along-track nadir**



(a) nadir



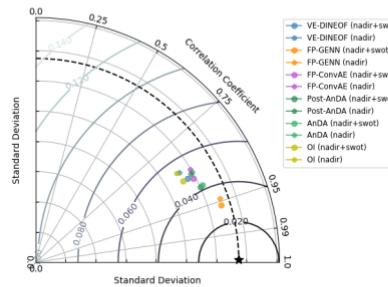
(b) nadir+swot

*Daily spatial nRMSE computed on the 80-days non-continuous validation period. The spatial coverage of 0-days accumulated along-track nadir and wide-swath SWOT data are given by the red and green-colored barplots*

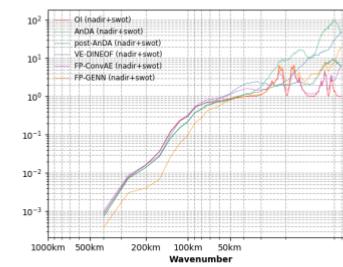
## Scores (II)

**Reconstruction (R)-score (over  $\Omega$ ) and Interpolation (I)-score (over  $\bar{\Omega}$ )**  
**FP-GENN always better on I-scores**

**Reconstruction of the spatial scales up to 50km which is an important improvement compared to the scales that OI is handling by now**



(a) Taylor diagram



(b) Signal-to-noise ratio

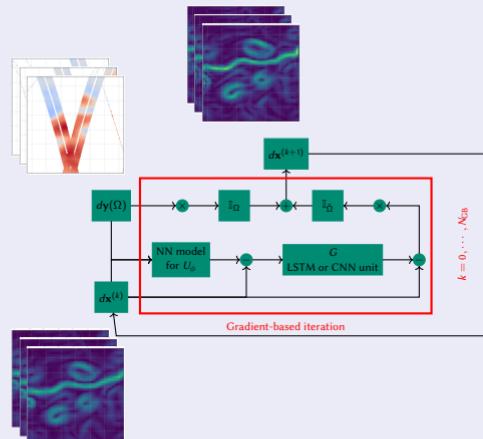
*Taylor diagram and signal-to-noise ratio computed on the 80-days non-continuous validation period for a joint assimilation/learning with wide-swath SWOT data*

## Perspectives

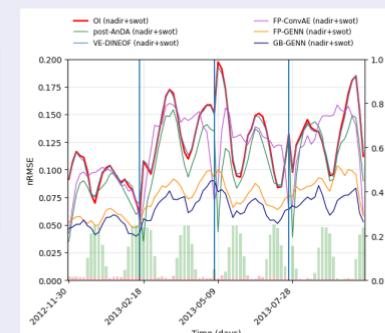
Replace the fixed-point solver by an iterative NN gradient-based descent to optimize parameters  $\theta$  of the NN-model (ConvAE or GENN)  $\psi$  w.r.t cost  $U_\psi$ :

$$J_{U_\psi} = J_\psi(\mathbf{x})(\mathbf{x} - \psi(\mathbf{x})) \quad (4.1)$$

where  $J_{U_\psi}$ , the gradient of  $U_\psi$ , is finally replaced by a ConvNet or LSTM unit  $G(\mathbf{x} - \psi(\mathbf{x}))$ , thus enabling to solve jointly for the parametrization of  $\psi$  and  $G$ :



(a) Sketch of the iterative gradient-based algorithm



(b) nRMSE

Daily spatial nRMSE computed on the 80-days non-continuous validation period with gradient-based solver

## References I

- R. Fablet, L. Drumetz, and F. Rousseau. End-to-end learning of optimal interpolators for geophysical dynamics. 2019.
- R. Lguensat, P. Huynh Viet, M. Sun, G. Chen, T. Fenglin, B. Chapron, and R. Fablet. Data-driven Interpolation of Sea Level Anomalies using Analog Data Assimilation. Oct. 2017. URL <https://hal.archives-ouvertes.fr/hal-01609851>.
- B. Ping, F. Su, and Y. Meng. An Improved DINEOF Algorithm for Filling Missing Values in Spatio-Temporal Sea Surface Temperature Data. *PLOS ONE*, 11(5) :e0155928, 2016. ISSN 1932-6203. doi : 10.1371/journal.pone.0155928. URL <http://journals.plos.org/plosone/article?id=10.1371/journal.pone.0155928>.
- G. Taburet, A. Sanchez-Roman, M. Ballarotta, M.-I. Pujol, J.-F. Legeais, F. Fournier, Y. Faugere, and G. Dibarbour. DUACS DT2018 : 25 years of reprocessed sea level altimetry products. 15(5) :1207–1224. ISSN 1812-0784. doi : <https://doi.org/10.5194/os-15-1207-2019>. URL <https://www.ocean-sci.net/15/1207/2019/>. Publisher : Copernicus GmbH.