NUMBER SYSTEMS & CONVERSION

LEARNING OBJECTIVES

In this chapter you will learn about:

- What are Number Systems?
- Types of Number Systems
- What are the uses/significance of each number system?

WHAT ARE NUMBER SYSTEMS?

- A numeral system (or system of numeration) is a writing system for expressing numbers; it is a mathematical notation for consistently representing numbers in a set using digit or other symbols.

- Number systems help in representing the numbers in a small symbol set.

- Number systems are systems in mathematics that are used to express numbers in various forms and are understood by computers.

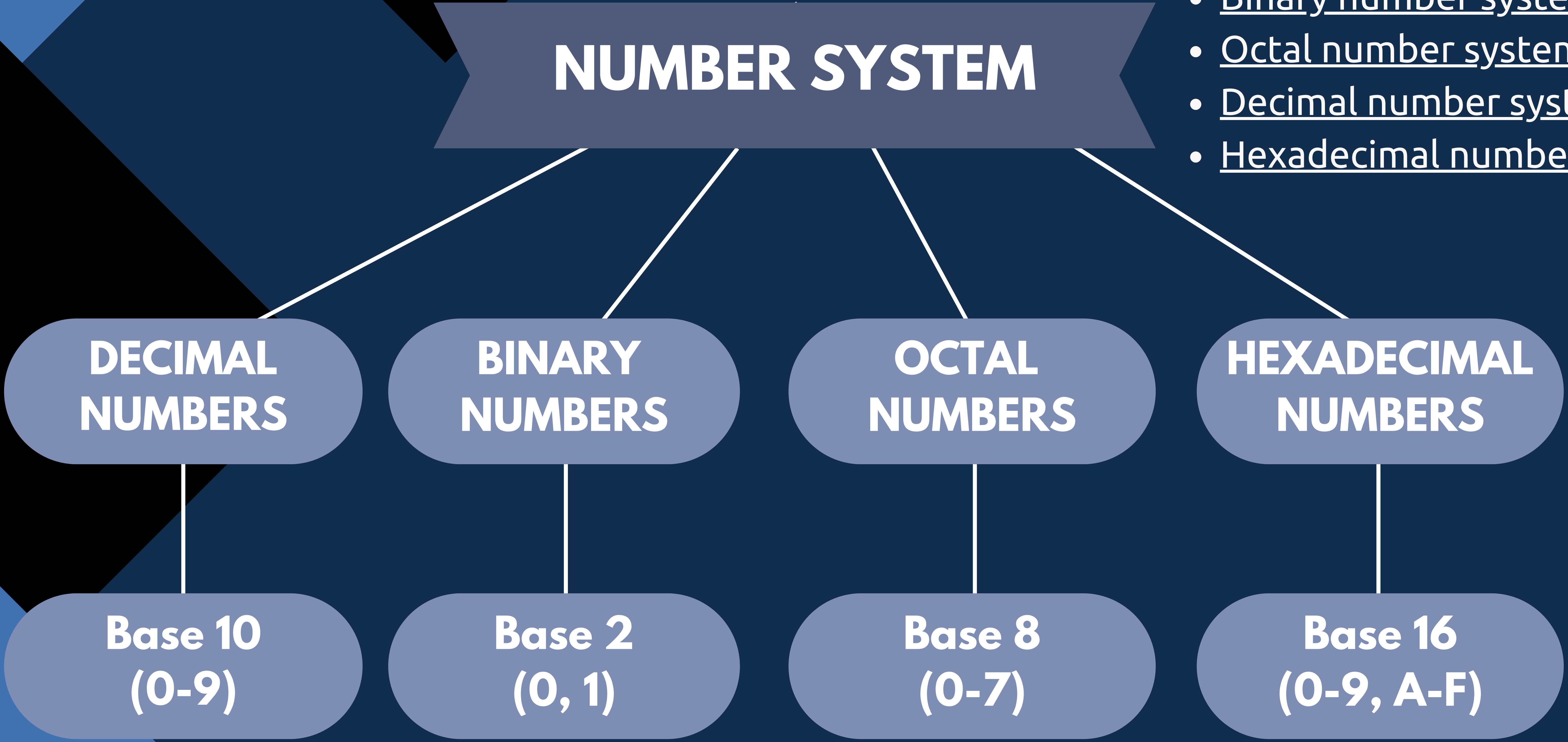
- Number Systems in Computer Science refer to the numeric base systems used for performing computations, storing and representing data.

WHAT IS THE IMPORTANCE OF THE NUMBER SYSTEM?

- It helps us keep count of things around us.
- Enables unique/accurate representation of several types of numbers.
- Used for computation in the banking sector.
- Helps us in encrypting data, avoiding hacking and misuse of data.
- Allows easy conversion of numbers for technical purposes.
- It should be noted that every fiber of data gets stored in the computer as a number.

TYPES OF NUMBER SYSTEMS

- Binary number system (Base - 2)
- Octal number system (Base - 8)
- Decimal number system (Base - 10)
- Hexadecimal number system (Base - 16)



NUMBER SYSTEM

DECIMAL NUMBERS

Base 10
(0-9)

BINARY NUMBERS

Base 2
(0, 1)

OCTAL NUMBERS

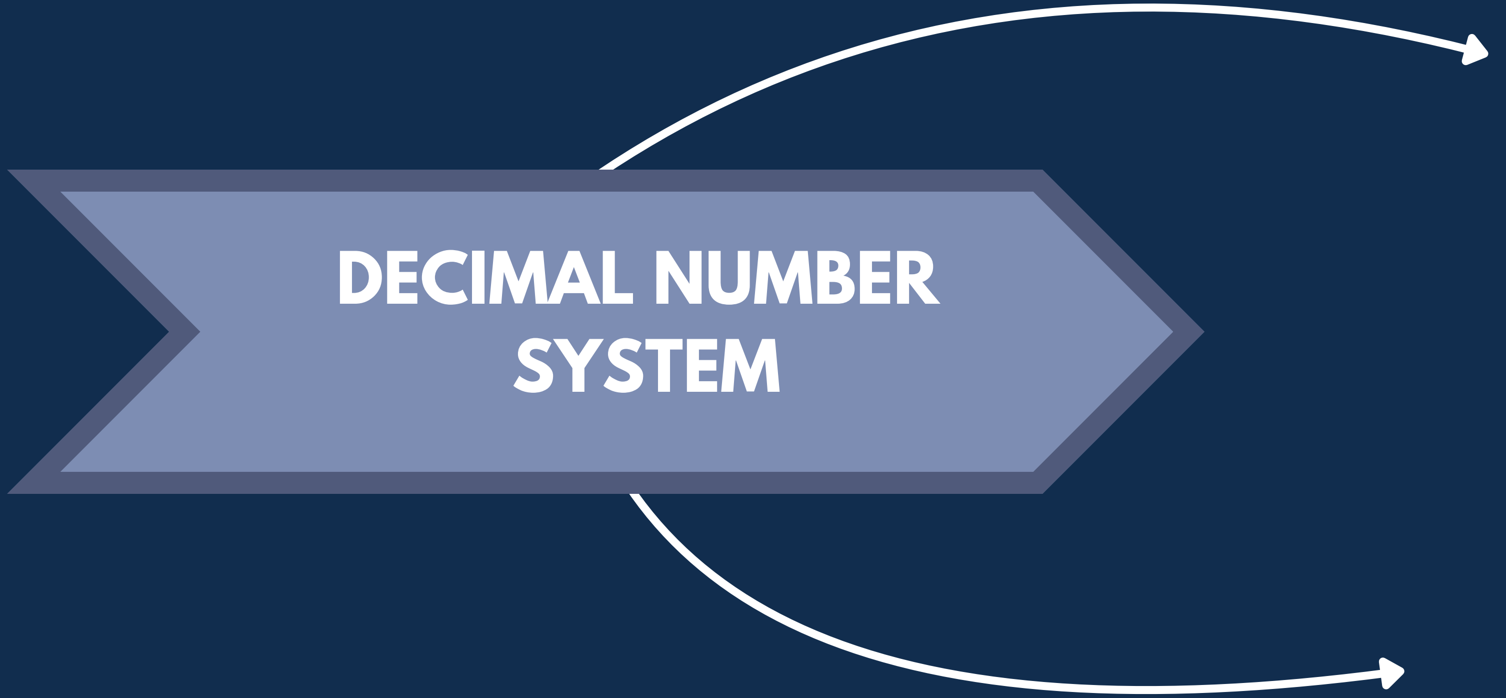
Base 8
(0-7)

HEXADECIMAL NUMBERS

Base 16
(0-9, A-F)

DECIMAL NUMBERS

With base value 10 is termed as Decimal number system. The decimal number system uses ten digits: 0,1,2,3,4,5,6,7,8 and 9.



DECIMAL NUMBER SYSTEM

Base 10

Upto 0 - 9 only

Examples:

243_{10}

62_{10}

8157_{10}

BINARY NUMBER SYSTEM

With base value 2 is termed as Binary number system. It uses 2 digits 0 and 1 for the creation of numbers. Digits 0 and 1 are called bits and 8 bits together make a byte. Binary number system is very useful in electronic devices and computer systems because it can be easily performed using just two states ON and OFF i.e. 0 and 1.

BINARY NUMBER SYSTEM

Base 2

Only 0 & 1

Examples:

01101_2

11101101_2

1011001_2

OCTAL NUMBER SYSTEM

The octal number system uses eight digits: 0,1,2,3,4,5,6 and 7 with the base of 8. The advantage of this system is that it has lesser digits when compared to several other systems, hence, there would be fewer computational errors.

OCTAL NUMBER SYSTEM

Base 8

Upto 0-7 only

Examples:

143_8

26_8

75_8

HEXADECIMAL NUMBER SYSTEM

The hexadecimal number system with the base number as 16. Digits from 0-9 are taken like the digits in the decimal number system but the digits from 10-15 are represented as A-F, like 10 is represented as A, 11 as B, 12 as C, 13 as D, 14 as E, and 15 as F. Hexadecimal Numbers are useful for handling memory address locations.

HEXADECIMAL NUMBER SYSTEM

Base 16

Upto 0 - 15 only

Examples:

$A1F_{16}$

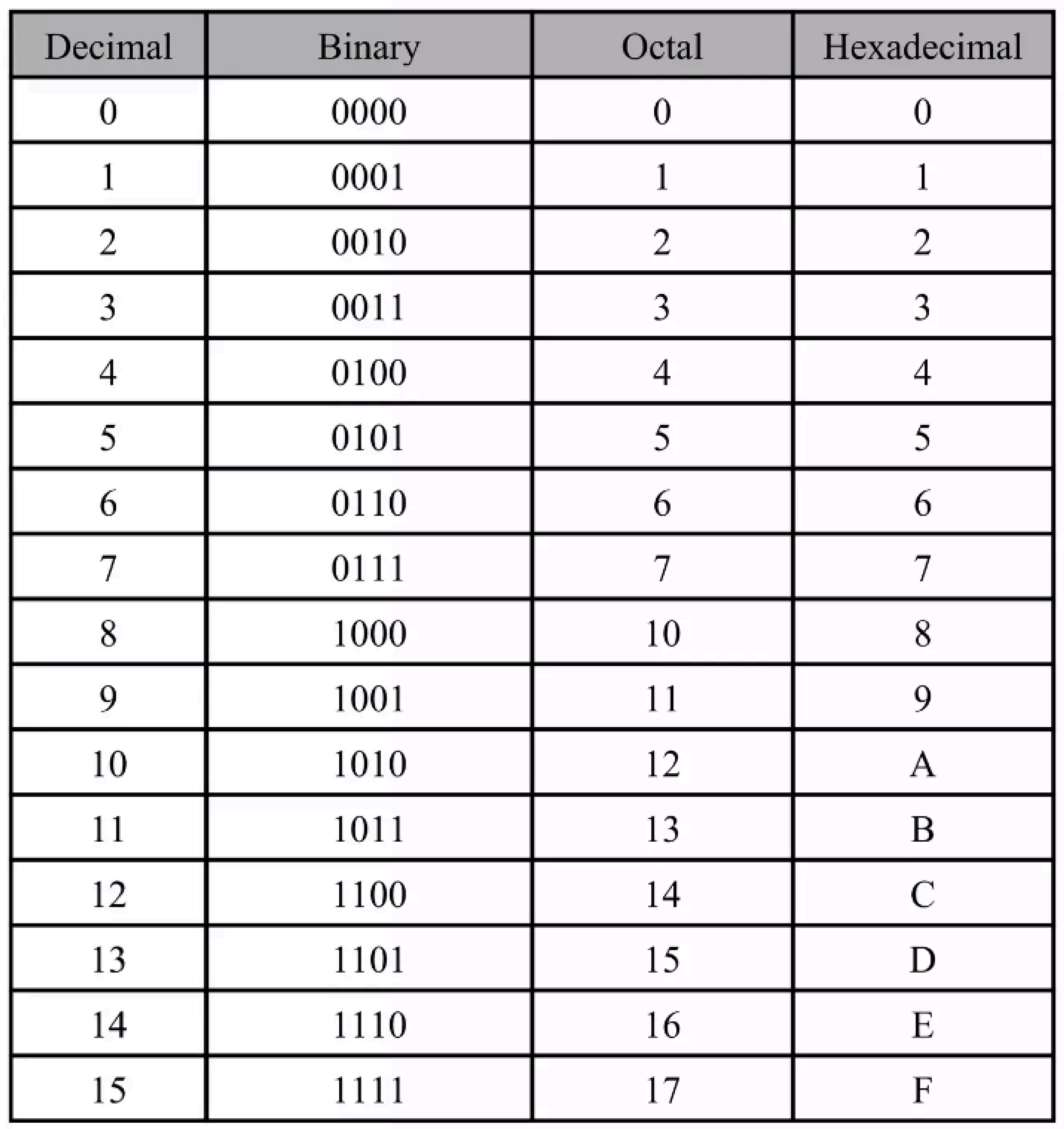
$49E_{16}$

$5D_{16}$

CONVERSION OF NUMBER SYSTEMS

A number can be converted from one number system to another number system using number system formulas.The types of conversions:

1. Decimal to Other Number Systems
2. Binary to Other Number Systems
3. Octal to Other Number Systems
4. Hexadecimal to Other Number Systems

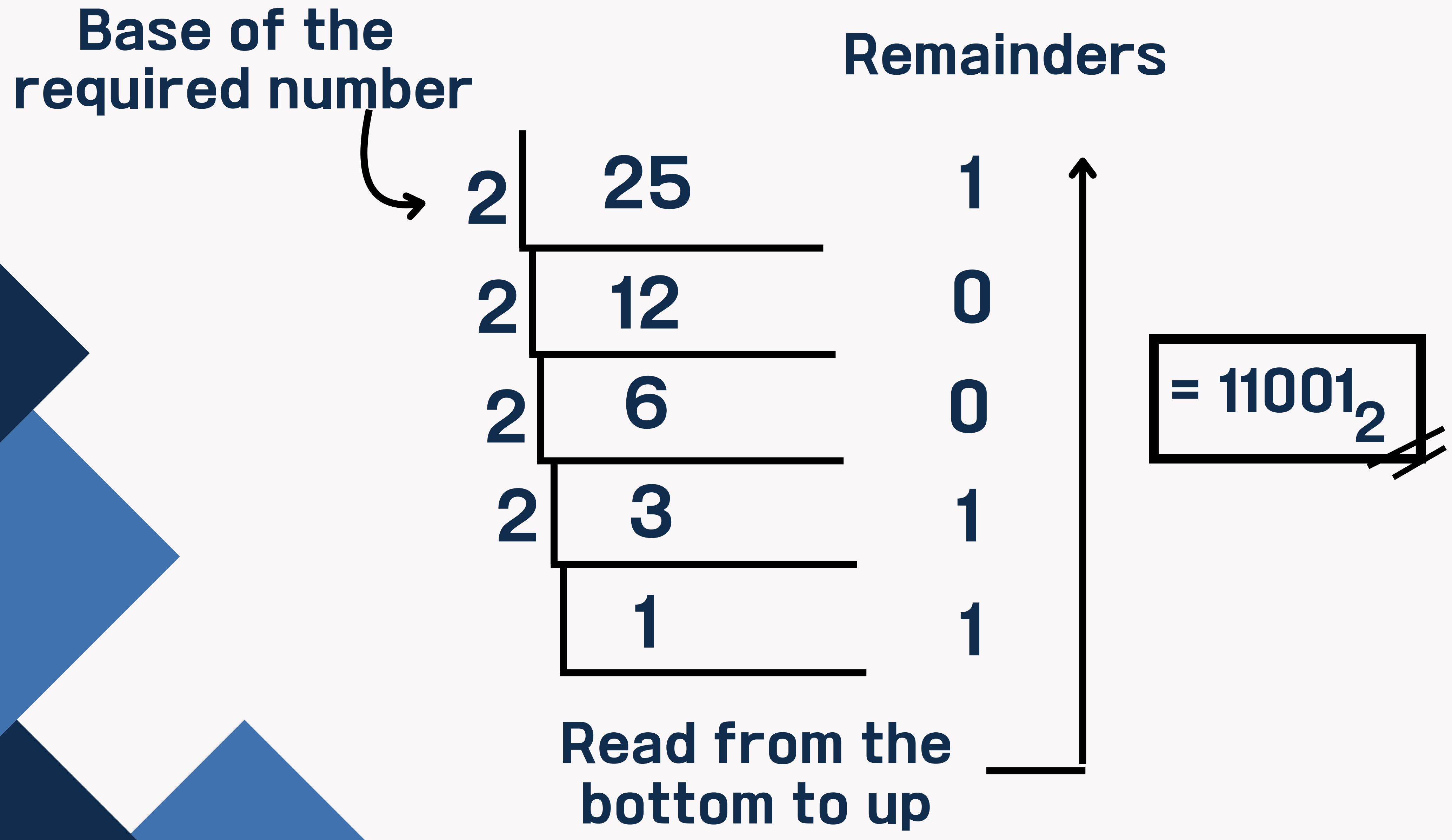


Decimal	Binary	Octal	Hexadecimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

DECIMAL TO BINARY

- Step 1: Divide the Decimal Number with the base of the number system to be converted to. Since the conversion is to binary, then the divisor will be 2.
- Step 2: The remainder obtained from the division will become the least significant digit of the new number.
- Step 3: The quotient obtained from the division will become the next dividend and will be divided by base 2.
- Step 4: The remainder obtained will become the second least significant digit, it will be added in the left of the previously obtained digit.

Example: $(25)_{10} \longrightarrow (?)_2$



Base of the required number

Remainders

2	25	1
2	12	0
2	6	0
2	3	1
	1	1

$= 11001_2$

Read from the bottom to up

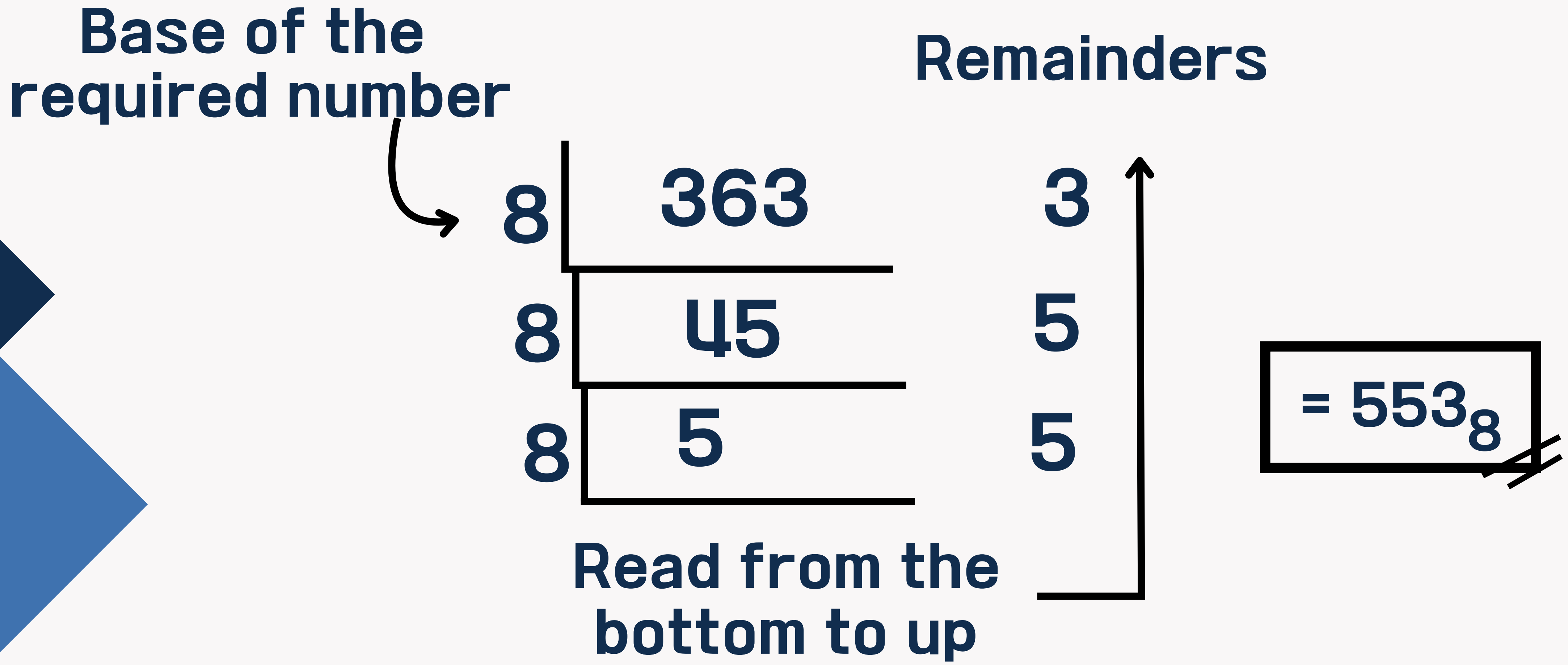
The steps 3 and 4 are repeated until the quotient obtained becomes 0, and the remainders obtained after each iteration are added to the left of the existing digits.

After all the iterations are over, the last obtained remainder will be termed as the Most Significant digit.

DECIMAL TO OCTAL

- Step 1: Divide the Decimal Number with the base of the number system to be converted to. Here the conversion is to octal, hence the divisor will be 8.
- Step 2: The remainder obtained from the division will become the least significant digit of the new number.
- Step 3: The quotient obtained from the division will become the next dividend and will be divided by base 8.
- Step 4: The remainder obtained will become the second least significant digit, it will be added in the left of the previously obtained digit.

Example: $(363)_{10} \longrightarrow (?)_8$



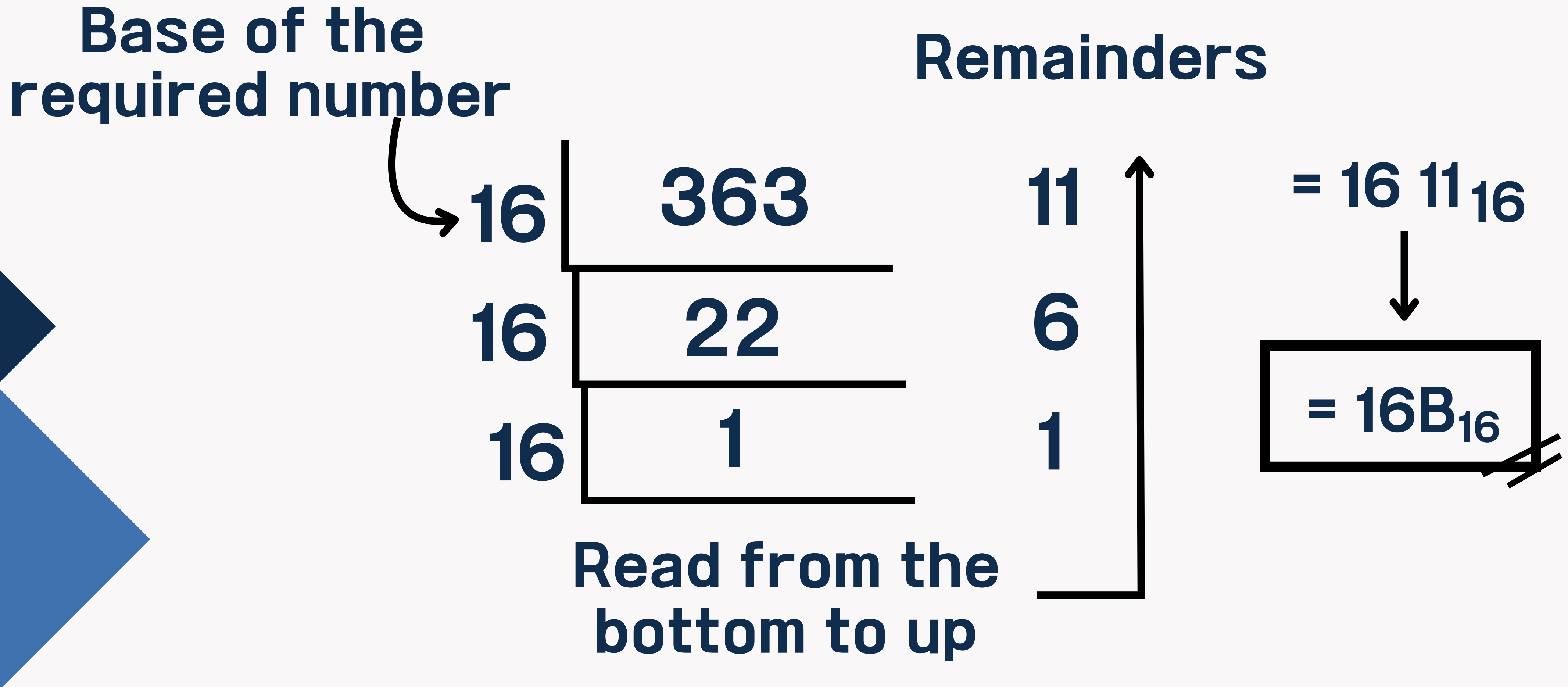
The steps 3 and 4 are repeated until the quotient obtained becomes 0, and the remainders obtained after each iteration are added to the left of the existing digits.

After all the iterations are over, the last obtained remainder will be termed as the Most Significant digit.

DECIMAL TO HEXADECIMAL

- Step 1: Divide the Decimal Number with the base of the number system to be converted to. Here the conversion is to octal, hence the divisor will be 16.
- Step 2: The remainder obtained from the division will become the least significant digit of the new number.
- Step 3: The quotient obtained from the division will become the next dividend and will be divided by base 16.
- Step 4: The remainder obtained will become the second least significant digit, it will be added in the left of the previously obtained digit.

Example: $(363)_{10} \longrightarrow (?)_{16}$



Base of the required number

Remainders

16 | 363 — 11

16 | 22 — 6

16 | 1 — 1

Read from the bottom to up

$= 16\ 11_{16}$

$\boxed{= 16B_{16}}$

The steps 3 and 4 are repeated until the quotient obtained becomes 0, and the remainders obtained after each iteration are added to the left of the existing digits.

After all the iterations are over, the last obtained remainder will be termed as the Most Significant digit.

BINARY TO DECIMAL

- Step 1: Multiply each digit of the Binary number with the place value of that digit, starting from right to left from least significant bit to most significant bit.
- Step 2: Add the result of this multiplication and the decimal number will be formed.

Example: $(1000100)_2 \longrightarrow (?)_{10}$

$$= 2^6 \times 1 + 2^5 \times 0 + 2^4 \times 0 + 2^3 \times 0 + 2^2 \times 1 + 2^1 \times 0 + 2^0 \times 0$$

$$= 64 + 0 + 0 + 0 + 4 + 0 + 0$$

$$\boxed{= \ 68_{10}}$$

2^6	2^5	2^4	2^3	2^2	2^1	2^0
64	32	16	8	4	2	1

BINARY TO OCTAL

- Step 1: Divide the binary number into groups of THREE digits starting from right to left, from least significant bit to most significant bit.
- Step 2: Convert these groups into equivalent octal digits.

Example: $(110101011)_2 \longrightarrow (?)_8$

$$\begin{array}{ccc} \underbrace{110} & \underbrace{101} & \underbrace{011} \\ \downarrow & \downarrow & \downarrow \\ 6 & 5 & 3 \end{array}$$

$$\boxed{= 653_8}$$

BINARY TO HEXADECIMAL

- Step 1: Divide the binary number into groups of FOUR digits starting from right to left, from least significant bit to most significant bit.
- Step 2: Convert these groups into equivalent hex digits.

Example: $(0111110111)_2 \longrightarrow (?)_{16}$

$$\underbrace{0001}_{\downarrow \atop 1}\ \underbrace{1111}_{\downarrow \atop 15}\ \underbrace{0111}_{\downarrow \atop 7}$$

$$\boxed{= 1F7_{16}}$$

OCTAL TO DECIMAL

- Step 1: Multiply each digit of the Binary number with the place value of that digit, starting from right to left from least significant bit to most significant bit.
- Step 2: Add the result of this multiplication and the decimal number will be formed.

8^3	8^2	8^1	8^0
512	64	8	1

Example: $(641)_8 \longrightarrow (?)_{10}$

$= 8^2 \times 6 + 8^1 \times 4 + 8^0 \times 1$

$= 384 + 32 + 1$

$\boxed{= 417_{10}}$

OCTAL TO BINARY

- Step 1: Write each digit of the octal number separately.
- Step 2: Convert each digit into an equivalent group of three binary digits.
- Step 3: Combine these groups to form the whole binary number.

Example: $(641)_8 \longrightarrow (?)_2$

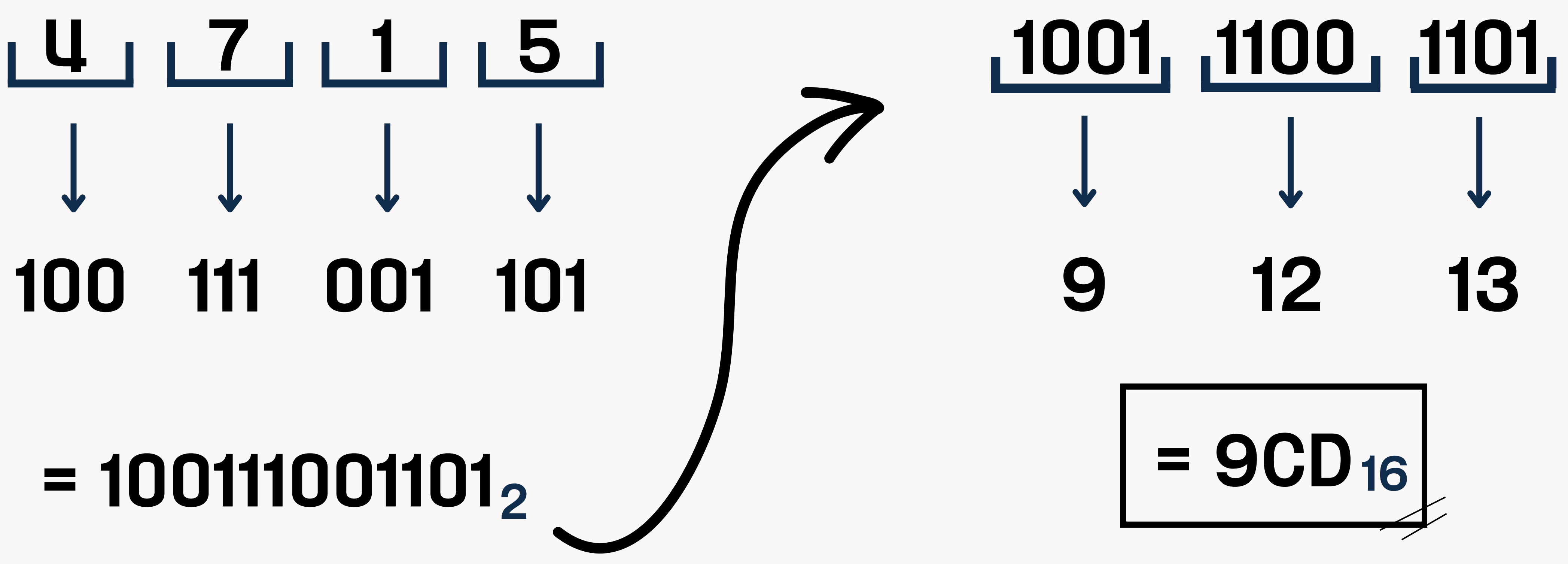
$$\begin{array}{ccc} \underline{6} & \underline{4} & \underline{1} \\ \downarrow & \downarrow & \downarrow \\ 110 & 100 & 001 \end{array}$$

$$\boxed{= 110100001_2}$$

OCTAL TO HEXADECIMAL

- Step 1: We need to convert the Octal number to Binary first. For that, follow the steps given in the above conversion.
- Step 2: Now to convert the binary number to Hex number, divide the binary digits into groups of four digits starting from right to left, from LSB to MSB.
- Step 3: Add zeros prior to MSB to make it a proper group of four digits(if required)
- Step 4: Now convert these groups into their relevant decimal values.
- Step 5: For values from 10-15, convert it into hex symbols, from A-F.

Example: $(4715)_8 \longrightarrow (?)_{16}$



Octal to Binary :

4 7 1 5

100 111 001 101

$= 100111001101_2$

Octal to Hexadecimal:

1001 1100 1101

9 12 13

$\boxed{= 9CD_{16}}$

HEXADECIMAL TO DECIMAL

- Step 1: Write the decimal values of the symbols used in the hex number, from A-F
- Step 2: Multiply each digit of the hex number with its place value. starting from right to left, LSB to MSB.
- Step 3: Add the result of multiplications and the final sum will be the decimal number.

Example: $(AF1)_{16} \longrightarrow (?)_{10}$

$$= 16^2 \times 10 + 16^1 \times 15 + 16^0 \times 1$$

$$= 2560 + 240 + 1$$

$$\boxed{= 2801_{10}}$$

16^3	16^2	16^1	16^0
4096	256	16	1

HEXADECIMAL TO BINARY

- Step 1: Convert the Hex symbols into its equivalent decimal values.
- Step 2: Write each digit of the Hexadecimal number separately.
- Step 3: Convert each digit into an equivalent group of four binary digits.
- Step 4: Combine these groups to form the whole binary number.

Example: $(AF1)_{16} \longrightarrow (?)_2$

A F 1

↓ ↓ ↓

10 15 1 → Equivalent Decimal Value

↓ ↓ ↓

1010 1111 0001 → Equivalent Binary Bits

$\boxed{= 101011110001_2}$

HEXADECIMAL TO OCTAL

- Step 1: We need to convert the Hexadecimal number to Binary first. For that, follow the steps given in the above conversion.
- Step 2: Now to convert the binary number to Octal number, divide the binary digits into groups of three digits starting from right to left, from LSB to MSB.
- Step 3: Add zeros prior to MSB to make it a proper group of three digits(if required)
- Step 4: Now convert these groups into their relevant decimal values.

Example: $(AF1)_{16}$ $(?)_8$

Hexadecimal to Binary :

$$\begin{array}{ccc} \underbrace{A} & \underbrace{F} & \underbrace{1} \\ \downarrow & \downarrow & \downarrow \\ 1010 & 1111 & 0001 \end{array}$$

$$= 101011110001_2$$

Binary to Octal:

$$\begin{array}{cccc} \underbrace{101} & \underbrace{011} & \underbrace{110} & \underbrace{001} \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 5 & 3 & 6 & 1 \end{array}$$

$$\boxed{= 5361_8}$$

WHAT ARE THE USES/SIGNIFICANCE OF EACH NUMBER SYSTEM?

- A number system is of great significance in our daily lives and even more than that in the subject of Mathematics. This system represents the specific identity that a number possesses on the number line and provides it with a significant notation. It also represents the arithmetic structure of the number. It helps us in calculating the numbers using which we can get a specific number.

- The number system is used in the computer system for better communication and representation. Computers can only understand numbers; therefore, it converts every letter and word into numbers for better understanding and processing.

THE USES/SIGNIFICANCE OF DECIMAL IN NUMBER SYSTEM

- The system is used extensively in everyday life to carry out routine tasks such as buying groceries, trading stocks, tracking football scores or scrolling through cable channels.

THE USES/SIGNIFICANCE OF BINARY IN NUMBER SYSTEM

- The binary number system is the base of all computing systems and operations. It enables devices to store, access and manipulate all types of information directed to and from the CPU or memory.

THE USES/SIGNIFICANCE OF OCTAL IN NUMBER SYSTEM

- The Octal Number system is widely used in computer application sectors and digital numbering systems. The computing systems use 16-bit, 32-bit or 64-bit word which is further divided into 8-bits words. The octal number is also used in the aviation sector in the form of a code.

THE USES/SIGNIFICANCE OF HEXADECIMAL IN NUMBER SYSTEM

- Hex numbers are compact and use less memory, so more numbers can be stored in computer systems. Their small size also makes input-output handling easier compared to other numbering formats. Because it's easy to convert hexadecimal to binary and vice versa, the system is widely used in computer programming.

ANALYSIS

Number systems are ways of representing and working with numbers. They help represent numbers in a small symbol set, provide different methods for expressing quantities and performing calculations, using various symbols and rules. In Computer Science it refers to the numeric base systems used for performing computations, storing and representing data. Each number system has its own base or foundational number, which determines how the numbers are structured.

As for the types of number systems, we'll start off with the most common one which is the Decimal Number System with the base of 10, uses the digits 0-9. The system is the one we use in our everyday lives such as buying groceries, tracking scores, measuring distances, and etc.

ANALYSIS

Second, we have the Binary Number System with the base of 2, it only contains two digits/symbols 0 and 1. This system is essential in computing and digital technology because it's how computers process and store data. Each digit in a binary number represents a power of 2, making it perfect for coding and other electronic data.

Third, the Octal Number System with the base of 8. it uses digits from 0-7. It isn't used that frequently as of today, but it was useful in the early computing for simplifying binary numbers. Since each octal digit corresponds to three binary digits, it helps making binary data easier to read.

ANALYSIS

Lasty, the Hexadecimal Number System with the base of 16, uses 0-9 digits, A-F symbols which makes it 16 symbols in total. This system is used in computing to make binary numbers more readable. Each hex digit represents four binary digits, Hex numbers are compact and use less memory, so more numbers can be stored in computer systems. Also, because it's easy to convert hexadecimal to binary and vice versa, the system is widely used in computer programming, allowing programmers to handle large numbers.

To conclude, each number system has its own importance because they provide in different ways to understand numbers, which is also essential in our daily lives and technology.

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