Eidgenössische Technische Hochschule Zürich

lETHargy

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adapted from MIT's version of the KTH ACM Contest Template Library

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Contest (1)

header.cpp

13 lines

```cpp
#include "bits/stdc++.h"
#define rep(i, a, n) for (auto i = a; i <= (n); ++i)
#define revrep(i, a, n) for (auto i = n; i >= (a); --i)
#define all(a) a.begin(), a.end()
#define sz(a) (int)(a).size()
using namespace std;
using ll = long long;
using pii = pair<int, int>;
using vi = vector<int>;

int main() {
  ios::sync_with_stdio(0); cin.tie(0);
}
```

debug-header.cpp

Description: debug header.

25 lines

```cpp
template<class A, class B> string to_string(const pair<A, B
   ↪> &p);
string to_string(const string s) { return '"' + s + '"'; }
string to_string(const char *s) { return to_string((string)
   ↪ s); }
string to_string(char c) { return "'" + string(1, c) + "'";
   ↪ }
string to_string(bool x) { return x ? "true" : "false"; }
template<class A> string to_string(const A &v) {
  bool first = 1;
  string res = "{";
  for (const auto &x: v) {
    if (!first) res += ", ";
    first = 0;
    res += to_string(x);
  }
  res += "}";
  return res;
}
template<class A, class B> string to_string(const pair<A, B
   ↪> &p) { return "(" + to_string(p.first) + ", " +
   ↪to_string(p.second) + ")"; }

void debug_out() { cerr << endl; }
template<class H, class... T> void debug_out(const H& h,
   ↪const T&... t) {
  cerr << " " << to_string(h);
  debug_out(t...);
}
#define debug(...) cerr << "[" << #__VA_ARGS__ << "]:",
   ↪debug_out(__VA_ARGS__)
// hash-cpp-all = 330e39ee93f62857c9eb153e5c322e29
```

1.1 MD5 checker

hash-cpp.sh

3 lines

```sh
# Hashes a cpp file, ignoring whitespace and comments.
# Usage: $ sh ./hash-cpp.sh < code.cpp
cpp -dD -P -fpreprocessed | tr -d '[:space:]' | md5sum
```

1.2 Input Layout

input-source.sh

2 lines

```sh
gsettings set org.gnome.desktop.input-sources sources "[('
   ↪xkb', 'us'), ('xkb', 'fi')]" # set input sources
gsettings set org.gnome.desktop.input-sources per-window
   ↪true # input sources are switched only in given window
```

1.3 Vscode config

vscode-settings.json

5 lines

```json
{
  "editor.insertSpaces": false,
  "window.titleBarStyle": "custom",
  "window.customMenuBarAltFocus": false,
}
```

Also change the following shortcuts: CopyLineDown, CopyLineUp, cursorLineEnd, cursorLineStart.

Misc (2)

random.cpp

6 lines

```cpp
mt19937 rng(chrono::steady_clock::now().time_since_epoch().
   ↪count());
template<class T>
T rand(T a, T b) { return uniform_int_distribution<T>(a, b)
   ↪(rng); }
template<class T>
T rand() { return uniform_int_distribution<T>()(rng); }
// shuffle(perm.begin(), perm.end(), rng);
```

fast-io.cpp

Description: Fast Read for int / long long.

20 lines

```cpp
namespace fastIO  {
  const int BUF_SIZE = 1 << 15;
  char buf[BUF_SIZE], *s = buf, *t = buf;
  inline char fetch() {
    if (s == t) {
      t = (s = buf) + fread(buf, 1, BUF_SIZE, stdin);
      if (s == t) return EOF;
    }
    return *s++;
  }

  template<class T> inline void read(T &x) {
    bool sgn = 1;
    T a = 0;
    char c = fetch();
    while (!isdigit(c)) sgn ^= (c == '-'), c = fetch();
    while (isdigit(c)) a = a * 10 + (c - '0'), c = fetch();
    x = sgn ? a : -a;
  }
} // hash-cpp-all = adf9f183d70e940e1930eb2081a1b271
```

hilbert-mos.cpp

Description: Hilbert curve sorting order for Mo's algorithm. Sorts queries (L_i, R_i) where $0 \le L_i \le R_i < n$ into order π, such that $\sum_i \left|L_{\pi_{i+1}} - L_{\pi_i}\right| + \left|R_{\pi_{i+1}} - R_{\pi_i}\right| = \mathcal{O}(n\sqrt{q})$

Usage: `hilbertOrder(n, qs)` returns π

Time: $\mathcal{O}(N \log N)$.

21 lines

```cpp
ll hilbertOrd(int y, int x, int h) {
  if (h == -1) return 0;
  int s = (1 << h), r = (1 << h) - 1;
  int y0 = y >> h, x0 = x >> h;
  int y1 = y & r, x1 = x & r;
  int ny = (y0 ? y1 : (x0 ? r - x1 : x1)); // x1 : r - x1))
   ↪;
  int nx = (y0 ? x1 : (x0 ? r - y1 : y1)); // y1 : r - y1))
   ↪; // r - y1 : y1));
  return s*s * (2*x0 + (x0 ^ y0)) + hilbertOrd(ny, nx, h-1)
   ↪;
}
vector<int> hilbertOrder(int n, const vector<pair<int, int
   ↪>>& qs) {
  int h = 0, q = qs.size();
  while((1 << h) < n) ++h;

  vector<pair<ll, int>> tmp(q);
  for (int i = 0; i < q; ++i) tmp[i] = {hilbertOrd(qs[i].
   ↪first, qs[i].second, h - 1), i};
  sort(tmp.begin(), tmp.end());

  vector<int> res(q);
  for (int qi = 0; qi < q; ++qi) res[qi] = tmp[qi].second;
  return res;
} // hash-cpp-all = 6467dd464ea41a6009895a50f6f12523
```

Data structure (3)

fenwick.cpp

Description: Fenwick tree with built in binary search. Can be used as a indexed set.

Usage: ??

Time: $\mathcal{O}(\log N)$.

35 lines

```cpp
class Fenwick {
  private:
    vector<ll> val;
  public:
    Fenwick(int n) : val(n+1, 0) {}

    // Adds v to index i
    void add(int i, ll v) {
      for (++i; i < val.size(); i += i & -i) {
        val[i] += v;
      }
    }

    // Calculates prefix sum up to index i
    ll get(int i) {
      ll res = 0;
      for (++i; i > 0; i -= i & -i) {
        res += val[i];
      }
      return res;
    }
    ll get(int a, int b) { return get(b) - get(a-1); }

    // Assuming prefix sums are non-decreasing, finds last
     ↪i s.t. get(i) <= v
    int search(ll v) {
      int res = 0;
      for (int h = 1<<30; h; h >>= 1) {
        if ((res | h) < val.size() && val[res | h] <= v) {
          res |= h;
          v -= val[res];
        }
      }
      return res - 1;
    }
}; // hash-cpp-all = 0d390772acaff4360d0f4d76da45148e
```

segtree.cpp

Description: Segment tree supporting range addition and range sum, minimum queries
Usage: ??
Time: $\mathcal{O}(\log N)$.

58 lines

```
// Segment tree for range addition, range sum and range
  ↪minimum.
class SegTree {
  private:
    vector<ll> sum, minv, tag;
    int h = 1;

    // Returns length of interval corresponding to position
      ↪ i
    ll len(int i) { return h >> (31 - __builtin_clz(i)); }

    void apply(int i, ll v) {
      sum[i] += v * len(i);
      minv[i] += v;
      if (i < h) tag[i] += v;
    }
    void push(int i) {
      if (tag[i] == 0) return;
      apply(2*i, tag[i]);
      apply(2*i+1, tag[i]);
      tag[i] = 0;
    }

    ll recGetSum(int a, int b, int i, int ia, int ib) {
      if (ib <= a || b <= ia) return 0;
      if (a <= ia && ib <= b) return sum[i];
      push(i);
      int im = (ia + ib) >> 1;
      return recGetSum(a, b, 2*i, ia, im) + recGetSum(a, b,
        ↪ 2*i+1, im, ib);
    }
    ll recGetMin(int a, int b, int i, int ia, int ib) {
      if (ib <= a || b <= ia) return 4 * (ll)1e18;
      if (a <= ia && ib <= b) return minv[i];
      push(i);
      int im = (ia + ib) >> 1;
      return min(recGetMin(a, b, 2*i, ia, im), recGetMin(a,
        ↪ b, 2*i+1, im, ib));
    }
    void recApply(int a, int b, ll v, int i, int ia, int ib
      ↪) {
      if (ib <= a || b <= ia) return;
      if (a <= ia && ib <= b) apply(i, v);
      else {
        push(i);
        int im = (ia + ib) >> 1;
        recApply(a, b, v, 2*i, ia, im);
        recApply(a, b, v, 2*i+1, im, ib);
        sum[i] = sum[2*i] + sum[2*i+1];
        minv[i] = min(minv[2*i], minv[2*i+1]);
      }
    }
  public:
    SegTree(int n) {
      while(h < n) h *= 2;
      sum.resize(2*h, 0);
      minv.resize(2*h, 0);
      tag.resize(h, 0);
    }
    ll rangeSum(int a, int b) { return recGetSum(a, b+1, 1,
      ↪ 0, h); }
    ll rangeMin(int a, int b) { return recGetMin(a, b+1, 1,
      ↪ 0, h); }
    void rangeAdd(int a, int b, ll v) { recApply(a, b+1, v,
      ↪ 1, 0, h); }
}; // hash-cpp-all = e3e31721068f2f6661b4302da9d50cb9
```

rmq.cpp

Description: range minimum query data structure with low memory and fast queries
Usage: ??
Time: $\mathcal{O}(N)$ preprocessing, $\mathcal{O}(1)$ query.

63 lines

```
int firstBit(ull x) { return __builtin_ctzll(x); }
int lastBit(ull x) { return 63 - __builtin_clzll(x); }

// O(n) preprocessing, O(1) RMQ data structure.
template<class T>
class RMQ {
  private:
    const int H = 6; // Block size is 2^H
    const int B = 1 << H;
    vector<T> vec; // Original values
    vector<ull> mins; // Min bits
    vector<int> tbl; // sparse table
    int n, m;

    // Get index with minimum value in range [a, a + len)
      ↪for 0 <= len <= B
    int getShort(int a, int len) const {
      return a + lastBit(mins[a] & (-1ull >> (64 - len)));
    }
    int minInd(int ia, int ib) const {
      return vec[ia] < vec[ib] ? ia : ib;
    }
  public:
    RMQ(const vector<T>& vec_) : vec(vec_), mins(vec_.size
      ↪()) {
      n = vec.size();
      m = (n + B-1) >> H;

      // Build sparse table
      int h = lastBit(m) + 1;
      tbl.resize(h*m);
      for (int j = 0; j < m; ++j) tbl[j] = j << H;
      for (int i = 0; i < n; ++i) tbl[i >> H] = minInd(tbl[
        ↪i >> H], i);
      for (int j = 1; j < h; ++j) {
        for (int i = j*m; i < (j+1)*m; ++i) {
          int i2 = min(i + (1 << (j-1)), (j+1)*m - 1);
          tbl[i] = minInd(tbl[i-m], tbl[i2-m]);
        }
      }
      // Build min bits
      ull cur = 0;
      for (int i = n-1; i >= 0; --i) {
        for (cur <<= 1; cur > 0; cur ^= cur & -cur) {
          if (vec[i + firstBit(cur)] < vec[i]) break;
        }
        cur |= 1;
        mins[i] = cur;
      }
    }
    int argmin(int a, int b) const {
      ++b; // to make the range inclusive
      int len = min(b-a, B);
      int ind1 = minInd(getShort(a, len), getShort(b-len,
        ↪len));
      int ax = (a >> H) + 1;
      int bx = (b >> H);
      if (ax >= bx) return ind1;
      else {
        int h = lastBit(bx-ax);
        int ind2 = minInd(tbl[h*m + ax], tbl[h*m + bx - (1
          ↪<< h)]);
        return minInd(ind1, ind2);
      }
    }
    int get(int a, int b) const { return vec[argmin(a, b)];
      ↪ }
}; // hash-cpp-all = 3dd48eb5fa928d12b0e5b263ce842625
```

cartesian-tree.cpp

Description: Cartesian Tree of array *as* (of distinct values) of length N. Node with smaller depth has smaller value. Set $gr = 1$ to have top with the greatest value. Returns the root of Cartesian Tree, left sons of nodes and right sons of nodes. (-1 means no left son / right son.)
Time: $\mathcal{O}(N)$ for construction.

14 lines

```
template<class T>
auto CartesianTree(const vector<T> &as, int gr = 0) {
  int n = sz(as);
  vi ls(n, -1), rs(n, -1), sta;
  rep(i, 0, n - 1) {
    while (sz(sta) && ((as[i] < as[sta.back()]) ^ gr)) {
      ls[i] = sta.back();
      sta.pop_back();
    }
    if (sz(sta)) rs[sta.back()] = i;
    sta.push_back(i);
  }
  return make_tuple(sta[0], ls, rs);
} // hash-cpp-all = 45ac593851f901756dd697a39dbbc90f
```

sparse-table.cpp

Description: Sparse Table of an array of length N.
Time: $\mathcal{O}(N \log N)$ for construction, $\mathcal{O}(1)$ per query.

19 lines

```
template<class T, class F = function<T(const T&, const T&)
  ↪>>
class SparseTable {
  int n;
  vector<vector<T>> st;
  const F func;
public:
  SparseTable(const vector<T> &init, const F &f): n(sz(init
    ↪)), func(f) {
    assert(n > 0);
    st.assign(__lg(n) + 1, vector<T>(n));
    st[0] = init;
    rep(i, 1, __lg(n)) rep(x, 0, n - (1 << i)) st[i][x] =
      ↪func(st[i - 1][x], st[i - 1][x + (1 << (i - 1))]);
  }

  T ask(int l, int r) {
    assert(0 <= l && l <= r && r < n);
    int k = __lg(r - l + 1);
    return func(st[k][l], st[k][r - (1 << k) + 1]);
  }
}; // hash-cpp-all = ba1bdd7413e0da2668e14467f92cf02d
```

lichao.cpp

Description: Li Chao tree. Given x-coordinates, supports adding lines and computing minimum Y-coordinate at a given input x-coordinate
Usage: ??
Time: $\mathcal{O}(\log N)$.

39 lines

```
struct Line {
  ll a, b;
  ll eval(ll x) const { return a*x + b; }
};
class LiChao {
  private:
    const static ll INF = 4e18;
    vector<Line> tree; // Tree of lines
    vector<ll> xs; // x-coordinate of point i
    int k = 1; // Log-depth of the tree

    int mapInd(int j) const {
      int z = __builtin_ctz(j);
      return ((1<<(k-z)) | (j>>z)) >> 1;
    }
    bool comp(const Line& a, int i, int j) const {
      return a.eval(xs[j]) < tree[i].eval(xs[j]);
    }
  public:
    LiChao(const vector<ll>& points) {
      while(points.size() >> k) ++k;
      tree.resize(1 << k, {0, INF});
      xs.resize(1 << k, points.back());
      for (int i = 0; i < points.size(); ++i) xs[mapInd(i
        ↪+1)] = points[i];
    }
    void addLine(Line line) {
      for (int i = 1; i < tree.size();) {
        if (comp(line, i, i)) swap(line, tree[i]);
        if (line.a > tree[i].a) i = 2*i;
        else i = 2*i+1;
      }
    }
    ll minVal(int j) const {
      j = mapInd(j+1);
      ll res = INF;
      for (int i = j; i > 0; i /= 2) res = min(res, tree[i
        ↪].eval(xs[j]));
      return res;
    }
}; // hash-cpp-all = 51ad9045bff4d74f5c7b851530e02304
```

skew-heap.cpp

Description: Skew heap: a priority queue with fast merging
Usage: ??
Time: all operations $\mathcal{O}(\log N)$.

38 lines

```
// Skew Heap
class SkewHeap {
  private:
    struct Node {
      ll val, inc = 0;
      int ch[2] = {-1, -1};
      Node(ll v) : val(v) {}
    };
    vector<Node> nodes;
  public:
    int makeNode(ll v) {
      nodes.emplace_back(v);
      return (int)nodes.size() - 1;
    }

    // Increment all values in heap p by v
    void add(int i, ll v) {
      if (i == -1) return;
      nodes[i].val += v;
      nodes[i].inc += v;
    }

    // Merge heaps a and b
    int merge(int a, int b) {
      if (a == -1 || b == -1) return a + b + 1;
      if (nodes[a].val > nodes[b].val) swap(a, b);
      if (nodes[a].inc) {
        add(nodes[a].ch[0], nodes[a].inc);
        add(nodes[a].ch[1], nodes[a].inc);
        nodes[a].inc = 0;
      }
      swap(nodes[a].ch[0], nodes[a].ch[1]);
      nodes[a].ch[0] = merge(nodes[a].ch[0], b);
      return a;
    }
    pair<int, ll> top(int i) const { return {i, nodes[i].
      ↪val}; }
    void pop(int& p) { p = merge(nodes[p].ch[0], nodes[p].
      ↪ch[1]); }
}; // hash-cpp-all = c72cc101090bd3027c2442ee11cee862
```

fast-prique.cpp

Description: Struct for priority queue operations on index set $[0, n-1]$.
Usage: `push(i, v) overwrites value at position i if one already exists. decKey is faster, but does nothing if the new key is smaller than the old one. top and pop can segfault if called on an empty priority queue.`
Time: $\mathcal{O}(\log N)$.

22 lines

```
struct Prique {
  const ll INF = 4 * (ll)1e18;
  vector<pair<ll, int>> data;
  const int n;

  Prique(int siz) : n(siz), data(2*siz, {INF, -1}) { data
    ↪[0] = {-INF, -1}; }
  bool empty() const { return data[1].second >= INF; }
  pair<ll, int> top() const { return data[1]; }

  void push(int i, ll v) {
    data[i+n] = {v, (v >= INF ? -1 : i)};
    for (i += n; i > 1; i >>= 1) data[i>>1] = min(data[i],
      ↪data[i^1]);
  }
  void decKey(int i, ll v) {
    for (int j = i+n; data[j].first > v; j >>= 1) data[j] =
      ↪ {v, i};
  }
  pair<ll, int> pop() {
    auto res = data[1];
    push(res.second, INF);
    return res;
  }
}; // hash-cpp-all = 08f397034ba143af3dc3c98b96f9a634
```

persistent-segtree.cpp

Description: Persistent Segment Tree of range $[0, N-1]$. Point apply and thus no lazy propogation. Always define a global *apply* function to tell segment tree how you apply modification. Combine is set as + operation. If you use your own struct, then please define constructor and + operation. In constructor, q is the number of *pointApply* you will use.
Usage: `Point Add and Range Sum.`

```
void apply(int &a, int b) { a += b; } // global
...
PersistSegtree<int> pseg(10, 1); // len = 10 and 1 update.
int rt = 0; // empty node.
int new_rt = pseg.pointApply(rt, 9, 1); // add 1 to last
position (position 9).
int sum = pseg.rangeAsk(new_rt, 7, 9); // ask the sum
between position 7 and 9, wrt version new_rt.
```

Time: $\mathcal{O}(\log N)$ per operation.

63 lines

```
template<class Info>
struct PersistSegtree {
  struct node { Info info; int ls, rs; }; // hash-cpp-1
  int n;
  vector<node> t;
  // node 0 is left as virtual empty node.
  PersistSegtree(int n, int q): n(n), t(1) {
    assert(n > 0);
    t.reserve(q * (__lg(n) + 2) + 1);
  }

  // pointApply returns the id of new root.
  template<class... T>
  int pointApply(int rt, int pos, const T&... val) {
    auto dfs = [&](auto &dfs, int &i, int l, int r) {
      t.push_back(t[i]);
      i = sz(t) - 1;

      if (l == r) {
        ::apply(t[i].info, val...);
        return;
      }
      int mid = (l + r) >> 1;
      if (pos <= mid) dfs(dfs, t[i].ls, l, mid);
      else dfs(dfs, t[i].rs, mid + 1, r);
      t[i].info = t[t[i].ls].info + t[t[i].rs].info;
    };
    dfs(dfs, rt, 0, n - 1);
    return rt;
  }

  Info rangeAsk(int rt, int ql, int qr) {
    Info res{};
    auto dfs = [&](auto &dfs, int i, int l, int r) {
      if (i == 0 || qr < l || r < ql) return;
      if (ql <= l && r <= qr) {
        res = res + t[i].info;
        return;
      }
      int mid = (l + r) >> 1;
      dfs(dfs, t[i].ls, l, mid);
      dfs(dfs, t[i].rs, mid + 1, r);
    };
    dfs(dfs, rt, 0, n - 1);
    return res;
  } // hash-cpp-1 = 9569f9abfb3ee296b5ea10a5f70b8ddb

  // lower_bound on prefix sums of difference between two
    ↪versions.
  int lower_bound(int rt_l, int rt_r, Info val) { // hash-
    ↪cpp-2
    Info sum{};
    auto dfs = [&](auto &dfs, int x ,int y, int l, int r) {
      if (l == r) return sum + t[y].info - t[x].info >= val
        ↪ ? l : l + 1;
      int mid = (l + r) >> 1;
      Info s = t[t[y].ls].info - t[t[x].ls].info;
      if (sum + s >= val) return dfs(dfs, t[x].ls, t[y].ls,
        ↪ l, mid);
      else {
        sum = sum + s;
        return dfs(dfs, t[x].rx, t[y].rs, mid + 1, r);
      }
    };
    return dfs(dfs, rt_l, rt_r, 0, n - 1);
  } // hash-cpp-2 = 8a719a17e052e3651546ac8d8a122c9c
};
```

segtree-2d.cpp

Description: 2D Segment Tree of range $[oL, oR] \times [iL, iR]$. Point apply and thus no lazy propogation. Always define a global *apply* function to tell segment tree how you apply modification. Combine is set as $+$ operation. If you use your own struct, then please define constructor and $+$ operation. In constructor, q is the number of *pointApply* you will use. oL, oR, Note that range parameters can be negative.
Usage: `Point Add and Range (Rectangle) Sum.`
```
void apply(int &a, int b) { a += b; } // global
...
SegTree2D<int> pseg(-5, 5, -5, 5, 1); // [-5, 5] * [-5, 5]
and 1 update.
int rt = 0; // empty node.
rt = pseg.pointApply(rt, 2, -1, 1); // add 1 to position
(2, -1).
int sum = pseg.rangeAsk(rt, 3, 4, -2, -1); // ask the sum
in rectangle [3, 4] * [-2, -1].
```
Time: $\mathcal{O}(\log(oR - oL + 1) \times \log(iR - iL + 1))$ per operation. 75 lines

```
template<class Info>
struct SegTree2D {
  struct iNode { Info info; int ls, rs; };
  struct oNode { int id; int ls, rs; };

  int oL, oR, iL, iR;
  // change to array to accelerate, since allocating takes
    ↪time. (saves ~ 200ms when allocating 1e7)
  vector<iNode> it;
  vector<oNode> ot;

  // node 0 is left as virtual empty node.
  SegTree2D(int oL, int oR, int iL, int iR, int q): oL(oL),
    ↪ oR(oR), iL(iL), iR(iR), it(1), ot(1) {
    it.reserve(q * (__lg(oR - oL + 1) + 2) * (__lg(iR - iL
      ↪+ 1) + 2) + 1);
    ot.reserve(q * (__lg(oR - oL + 1) + 2) + 1);
  }

  // return new root id.
  template<class... T>
  int pointApply(int rt, int op, int ip, const T&... val) {
    auto idfs = [&](auto &dfs, int &i, int l, int r) {
      if (!i) {
        it.push_back({});
        i = sz(it) - 1;
      }
      if (l == r) {
        ::apply(it[i].info, val...);
        return;
      }
      int mid = (l + r) >> 1;
      auto &[info, ls, rs] = it[i];
      if (ip <= mid) dfs(dfs, ls, l, mid);
      else dfs(dfs, rs, mid + 1, r);
      info = it[ls].info + it[rs].info;
    };
    auto odfs = [&](auto &dfs, int &i, int l, int r) {
      if (!i) {
        ot.push_back({});
        i = sz(ot) - 1;
      }
      idfs(idfs, ot[i].id, iL, iR);
      if (l == r) return;
      int mid = (l + r) >> 1;
      if (op <= mid) dfs(dfs, ot[i].ls, l, mid);
      else dfs(dfs, ot[i].rs, mid + 1, r);
    };
    odfs(odfs, rt, oL, oR);
    return rt;
  }

  Info rangeAsk(int rt, int qol, int qor, int qil, int qir)
    ↪ {
    Info res{};
    auto idfs = [&](auto &dfs, int i, int l, int r) {
      if (!i || qir < l || r < qil) return;
      if (qil <= l && r <= qir) {
        res = res + it[i].info;
        return;
      }
      int mid = (l + r) >> 1;
      dfs(dfs, it[i].ls, l, mid);
      dfs(dfs, it[i].rs, mid + 1, r);
    };
    auto odfs = [&](auto &dfs, int i, int l, int r) {
      if (!i || qor < l || r < qol) return;
      if (qol <= l && r <= qor) {
        idfs(idfs, ot[i].id, iL, iR);
        return;
      }
      int mid = (l + r) >> 1;
      dfs(dfs, ot[i].ls, l, mid);
      dfs(dfs, ot[i].rs, mid + 1, r);
    };
    odfs(odfs, rt, oL, oR);
    return res;
  }
}; // hash-cpp-all = abc3c0ce75b1b8cfcc9b974e0b8cfdfa
```

treap.cpp

Description: A Treap with lazy tag support. Default behaviour supports join, split, reverse and sum.
Time: All updates are $\mathcal{O}(logN)$ 60 lines

```
mt19937 rng(chrono::steady_clock::now().time_since_epoch().
  ↪count());
int rand() { return uniform_int_distribution<int>()(rng); }

struct Treap {
  private:
    const int pri;
    Treap *le = 0, *ri = 0;
    ll val, sum;
    int siz = 1, flip = 0;

    void update() {
      siz = 1 + getSiz(le) + getSiz(ri);
      sum = val + getSum(le) + getSum(ri);
    }
    void push() {
      if (flip) {
        swap(le, ri);
        reverse(le);
        reverse(ri);
        flip = 0;
      }
    }
  public:
    Treap(ll v) : val(v), sum(v), pri(rand()) {}
    ~Treap() { delete le; delete ri; }

    static int getSiz(Treap* x) { return x ? x->siz : 0; }
    static ll getSum(Treap* x) { return x ? x->sum : 0; }
    static void reverse(Treap* x) { if (x) x->flip ^= 1; }

    static Treap* join(Treap* a, Treap* b) {
      if (!a || !b) return a ? a : b;
      Treap* res = (a->pri < b->pri ? a : b);

      res->push();
      if (res == a) a->ri = join(a->ri, b);
      else b->le = join(a, b->le);
      res->update();
      return res;
    }

    // Split the treap into a left and right part, the left
      ↪ of size "le_siz"
    static pair<Treap*, Treap*> split(Treap* x, int le_siz)
      ↪ {
      if (!le_siz || !x) return {0, x};
      x->push();

      Treap *oth;
      int rem = le_siz - getSiz(x->le) - 1;
      if (rem < 0) {
        tie(oth, x->le) = split(x->le, le_siz);
        x->update();
        return {oth, x};
      } else {
        tie(x->ri, oth) = split(x->ri, rem);
        x->update();
        return {x, oth};
      }
    }

}; // hash-cpp-all = 4f72bba8689af456118ff9f9c60d6cf6
```

3.1 PBDS

pbds-hash-map.cpp

5 lines

```
#include<ext/pb_ds/assoc_container.hpp>
#include<ext/pb_ds/hash_policy.hpp>
using namespace __gnu_pbds;
template<class A, class B>
using HashMap = gp_hash_table<A, B>;
```

pbds-leftist-tree.cpp

5 lines

```
#include<ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;
template<class T>
using Heap = __gnu_pbds::priority_queue<T, greater<T>,
  ↪binomial_heap_tag>; // smallest value at the top.
// Use $a.join(b)$ to merge heap $b$ to heap $a$. After
  ↪merging, $b$ will be empty.
```

pbds-ordered-set.cpp

7 lines

```
#include<ext/pb_ds/assoc_container.hpp>
#include<ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template<class T>
using Oset = tree<T, null_type, less<T>, rb_tree_tag,
   ↪tree_order_statistics_node_update>; // if null_type
   ↪does not work, then use null_mapped_type instead.
// order_of_key(x) returns the number of elements which are
   ↪ smaller than x. (quite like lower_bound.)
// find_by_order(x) returns the x-th smallest element.
```

Graph algorithms (4)

4.1 Flows

dinic.cpp

Description: Dinic algorithm for flow graph $G = (V, E)$. You can get a minimum $src - sink$ cut easily. To get such minimum cut, first run $MaxFlow(src, sink)$. Then you can run $getMinCut()$ to obtain a Minimum Cut (vertices in the same part as src are returned).
Time: $\mathcal{O}\left(|V|^2|E|\right)$ for arbitrary networks. $\mathcal{O}\left(|E|\sqrt{|V|}\right)$ for bipartite/unit network. $\mathcal{O}\left(min|V|^{2/3}, |E|^{1/2}|E|\right)$ for networks with only unit capacities.

72 lines

```
template<class Cap = int, Cap Cap_MAX = numeric_limits<Cap
   ↪>::max()>
struct Dinic {
  int n; // hash-cpp-1
  struct E { int to; Cap a; }; // Endpoint & Admissible
     ↪flow.
  vector<E> es;
  vector<vi> g;
  vi dis; // Put it here to get the minimum cut easily.

  Dinic(int n): n(n), g(n) {}

  void addEdge(int u, int v, Cap c, bool dir = 1) {
    g[u].push_back(sz(es)); es.push_back({v, c});
    g[v].push_back(sz(es)); es.push_back({u, dir ? 0 : c});
  }

  Cap MaxFlow(int src, int sink) {
    auto revbfs = [&]() {
      dis.assign(n, -1);
      dis[sink] = 0;
      vi que{sink};

      rep(ind, 0, sz(que) - 1) {
        int now = que[ind];
        for (auto i: g[now]) {
          int v = es[i].to;
          if (es[i ^ 1].a > 0 && dis[v] == -1) {
            dis[v] = dis[now] + 1;
            que.push_back(v);
            if (v == src) return 1;
          }
        }
      }
      return 0;
    };

    vi cur;
    auto dfs = [&](auto &dfs, int now, Cap flow) {
      if (now == sink) return flow;
      Cap res = 0;
      for (int &ind = cur[now]; ind < sz(g[now]); ind++) {
        int i = g[now][ind];
        auto [v, c] = es[i];
        if (c > 0 && dis[v] == dis[now] - 1) {
          Cap x = dfs(dfs, v, min(flow - res, c));
          res += x;
          es[i].a -= x;
          es[i ^ 1].a += x;
        }
        if (res == flow) break;
      }
      return res;
    };

    Cap ans = 0;
    while (revbfs()) {
      cur.assign(n, 0);
      ans += dfs(dfs, src, Cap_MAX);
    }
    return ans;
  } // hash-cpp-1 = 0099c35a07ab0465ecf3ddb9b105db6f

  // Returns a min-cut containing the src.
  vi getMinCut() { // hash-cpp-2
    vi res;
    rep(i, 0, n - 1) if (dis[i] == -1) res.push_back(i);
    return res;
  } // hash-cpp-2 = f8bc377d2af3ac0d3b75bbacb2e4f7e9

  // Gives flow on edge assuming it is directed/undirected.
     ↪ Undirected flow is signed.
  Cap getDirFlow(int i) { return es[i * 2 + 1].a; }
  Cap getUndirFlow(int i) { return (es[i * 2 + 1].a - es[i
     ↪* 2].a) / 2; }
};
```

costflow-successive-shortest-path.cpp

Description: Successive Shortest Path for flow graph $G = (V, E)$. Run $mincostflow(src, sink)$ for some src and $sink$ to get the minimum cost and the maximum flow. For negative costs, Bellman-Ford is necessary.
Time: $\mathcal{O}\left(|F||E|\log|E|\right)$ for non-negative costs, where $|F|$ is the size of maximum flow. $\mathcal{O}\left(|V||E| + |F||E|\log|E|\right)$ for arbitrary costs.

61 lines

```
template<class Cap, class Cost, Cap Cap_MAX =
   ↪numeric_limits<Cap>::max(), Cost Cost_MAX =
   ↪numeric_limits<Cost>::max() / 4>
struct SuccessiveShortestPath {
  int n;
  struct E { int to; Cap a; Cost w; };
  vector<E> es;
  vector<vi> g;
  vector<Cost> h;

  SuccessiveShortestPath(int n): n(n), g(n), h(n) {}

  void addEdge(int u, int v, Cap c, Cost w) {
    g[u].push_back(sz(es)); es.push_back({v, c, w});
    g[v].push_back(sz(es)); es.push_back({u, 0, -w});
  }

  pair<Cost, Cap> mincostflow(int src, int sink, Cap
     ↪mx_flow = Cap_MAX) {
    // Run Bellman-Ford first if necessary.
    h.assign(n, Cost_MAX);
    h[src] = 0;
    rep(rd, 1, n) rep(now, 0, n - 1) for (auto i: g[now]) {
      auto [v, c, w] = es[i];
      if (c > 0) h[v] = min(h[v], h[now] + w);
    }
    // Bellman-Ford stops here.

    Cost cost = 0;
    Cap flow = 0;
    while (mx_flow) {
      priority_queue<pair<Cost, int>> pq;
      vector<Cost> dis(n, Cost_MAX);
      dis[src] = 0; pq.emplace(0, src);

      vi pre(n, -1), mark(n, 0);
      while (sz(pq)) {
        auto [d, now] = pq.top(); pq.pop();
        // Using mark[] is safer than compare -d and dis[
           ↪now] when the Cost = double.
        if (mark[now]) continue;
        mark[now] = 1;
        for (auto i: g[now]) {
          auto [v, c, w] = es[i];
          Cost off = dis[now] + w + h[now] - h[v];
          if (c > 0 && dis[v] > off) {
            dis[v] = off;
            pq.emplace(-dis[v], v);
            pre[v] = i;
          }
        }
      }
      if (pre[sink] == -1) break;

      rep(i, 0, n - 1) if (dis[i] != Cost_MAX) h[i] += dis[
         ↪i];
      Cap aug = mx_flow;
      for (int i = pre[sink]; ~i; i = pre[es[i ^ 1].to])
         ↪aug = min(aug, es[i].a);
      for (int i = pre[sink]; ~i; i = pre[es[i ^ 1].to]) es
         ↪[i].a -= aug, es[i ^ 1].a += aug;
      mx_flow -= aug;
      flow += aug;
      cost += aug * h[sink];
    }
    return {cost, flow};
  }
}; // hash-cpp-all = 2f6de2add5c8caaf0940e67ca83c82aa
```

4.2 Matchings

kuhn-matching.cpp

Description: Kuhn Matching algorithm for **bipartite** graph $G = (L \cup R, E)$. Edges E should be described as pairs such that pair (x, y) means that there is an edge between the x-th vertex in L and the y-th vertex in R. Returns a vector lm, where $lm[i]$ denotes the vertex in R matched to the i-th vertex in R.
Time: $\mathcal{O}\left((|L| + |R|)|E|\right)$.

22 lines

```
vi Kuhn(int n, int m, const vector<pii> &es) {
  vector<vi> g(n);
  for (auto [x, y]: es) g[x].push_back(y);
  vi rm(m, -1);
  rep(i, 0, n - 1) {
    vi vis(m);
    auto dfs = [&](auto &dfs, int x) -> int {
      for (auto y: g[x]) if (vis[y] == 0) {
        vis[y] = 1;
        if (rm[y] == -1 || dfs(dfs, rm[y])) {
          rm[y] = x;
          return 1;
        }
      }
      return 0;
    };
    dfs(dfs, i);
  }
  vi lm(n, -1);
  rep(i, 0, m - 1) if (rm[i] != -1) lm[rm[i]] = i;
  return lm;
} // hash-cpp-all = 799e88c72327efb98bd13f428b7ee8db
```

hopcroft.cpp

Description: Fast bipartite matching for **bipartite** graph $G = (L \cup R, E)$. Edges E should be described as pairs such that pair (x, y) means that there is an edge between the x-th vertex in L and the y-th vertex in R. You can also get a vertex cover of a bipartite graph easily.
Time: $\mathcal{O}\left(|E|\sqrt{|L|+|R|}\right)$. 56 lines

```
struct Hopcroft {
  int L, R; // hash-cpp-1
  vi lm, rm; // record the matched vertex for each vertex
    ↪on both sides.
  vi ldis, rdis; // put it here so you can get vertex cover
    ↪ easily.

  Hopcroft(int L, int R, const vector<pii> &es): L(L), R(R)
    ↪, lm(L, -1), rm(R, -1) {
    vector<vi> g(L);
    for (auto [x, y]: es) g[x].push_back(y);

    while (1) {
      ldis.assign(L, -1);
      rdis.assign(R, -1);
      bool ok = 0;
      vi que;
      rep(i, 0, L - 1) if (lm[i] == -1) {
        que.push_back(i);
        ldis[i] = 0;
      }
      rep(ind, 0, sz(que) - 1) {
        int i = que[ind];
        for (auto j: g[i]) if (rdis[j] == -1) {
          rdis[j] = ldis[i] + 1;
          if (rm[j] != -1) {
            ldis[rm[j]] = rdis[j] + 1;
            que.push_back(rm[j]);
          } else ok = 1;
        }
      }

      if (ok == 0) break;
      vi vis(R); // changing to static does not speed up.

      auto find = [&](auto &dfs, int i) -> int {
        for (auto j: g[i]) if (vis[j] == 0 && rdis[j] ==
          ↪ldis[i] + 1) {
          vis[j] = 1;
          if (rm[j] == -1 || dfs(dfs, rm[j])) {
            lm[i] = j;
            rm[j] = i;
            return 1;
          }
        }
        return 0;
      };
      rep(i, 0, L - 1) if (lm[i] == -1) find(find, i);
    }
  } // hash-cpp-1 = 1bdeb27ebf133b92ed0dac89528c768e

  vi getMatch() { return lm; } // returns lm.

  pair<vi, vi> vertex_cover() { // hash-cpp-2
    vi lvc, rvc;
    rep(i, 0, L - 1) if (ldis[i] == -1) lvc.push_back(i);
    rep(j, 0, R - 1) if (rdis[j] != -1) rvc.push_back(j);
    return {lvc, rvc};
  } // hash-cpp-2 = 4cfcc7973485543721e0bf5f6f67e3ce
};
```

blossom.cpp

Description: Maximum matching of a **general** graph $G = (V, E)$. Edges E should be described as pairs such that pair (u, v) means that there is an edge between vertex u and vertex v.
Time: $\mathcal{O}(|V||E|)$. 81 lines

```
vi Blossom(int n, const vector<pii> &es) {
  vector<vi> g(n);
  for (auto [x, y]: es) {
    g[x].push_back(y);
    g[y].push_back(x);
  }
  vi match(n, -1);

  auto aug = [&](int st) {
    vi fa(n), clr(n, -1), pre(n, -1), tag(n);
    iota(all(fa), 0);
    int tot = 0;
    vi que{st};
    clr[st] = 0;

    function<int(int)> getfa = [&](int x) {
      return fa[x] == x ? x : fa[x] = getfa(fa[x]);
    };

    auto lca = [&](int x, int y) {
      tot++;
      x = getfa(x);
      y = getfa(y);
      while (1) {
        if (x != -1) {
          if (tag[x] == tot) return x;
          tag[x] = tot;
          if (match[x] != -1) x = getfa(pre[match[x]]);
          else x = -1;
        }
        swap(x, y);
      }
    };
    auto shrink = [&](int x, int y, int f) {
      while (getfa(x) != f) {
        pre[x] = y;
        y = match[x];
        if (clr[y] == 1) {
          clr[y] = 0;
          que.push_back(y);
        }
        if (getfa(x) == x) fa[x] = f;
        if (getfa(y) == y) fa[y] = f;
        x = pre[y];
      }
    };

    rep(ind, 0, sz(que) - 1) {
      int now = que[ind];
      for (auto v: g[now]) {
        if (getfa(now) == getfa(v) || clr[v] == 1) continue
          ↪;
        if (clr[v] == -1) {
          clr[v] = 1;
          pre[v] = now;
          if (match[v] == -1) {
            while (now != -1) {
              int last = match[now];
              match[now] = v;
              match[v] = now;
              if (last != -1) {
                v = last;
                now = pre[v];
              } else break;
            }
            return;
          }
          clr[match[v]] = 0;
          que.push_back(match[v]);
        } else if (clr[v] == 0) {
          assert(getfa(now) != getfa(v));
          int l = lca(now, v);
          shrink(now, v, l);
          shrink(v, now, l);
        }
      }
    }
  };

  rep(i, 0, n - 1) if (match[i] == -1) aug(i);
  return match;
} // hash-cpp-all = cf7d426031408a38af90f44df608495e
```

hungarian.cpp

Description: Given a complete bipartite graph $G = (L \cup R, E)$, where $|L| \leq |R|$, Finds minimum weighted perfect matching of L. Returns the matching (a vector of pair<int, int>). $ws[i][j]$ is the weight of the edge from i-th vertex in L to j-th vertex in R. Not sure how to choose safe T since I can not give a bound on values in lp and rp. Seems safe to always use **long long**.
Time: $\mathcal{O}\left(|L|^2|R|\right)$. 60 lines

```
template<class T = ll, T INF = numeric_limits<T>::max()>
vector<pii> Hungarian(const vector<vector<T>> &ws) {
  int L = sz(ws), R = L == 0 ? 0 : sz(ws[0]);
  vector<T> lp(L), rp(R); // left & right potential
  vi lm(L, -1), rm(R, -1); // left & right match

  rep(i, 0, L - 1) lp[i] = *min_element(all(ws[i]));

  auto step = [&](int src) {
    vi que{src}, pre(R, - 1); // bfs que & back pointers
    vector<T> sa(R, INF); // slack array; min slack from
      ↪node in que

    auto extend = [&](int j) {
      if (sa[j] == 0) {
        if (rm[j] == -1) {
          while(j != -1) { // Augment the path
            int i = pre[j];
            rm[j] = i;
            swap(lm[i], j);
          }
          return 1;
        } else que.push_back(rm[j]);
      }
      return 0;
    };

    rep(ind, 0, L - 1) { // BFS to new nodes
      int i = que[ind];
      rep(j, 0, R - 1) {
        if (j == lm[i]) continue;
        T off = ws[i][j] - lp[i] - rp[j]; // Slack in edge
        if (sa[j] > off) {
          sa[j] = off;
          pre[j] = i;
          if (extend(j)) return;
        }
      }
      if (ind == sz(que) - 1) { // Update potentials
        T d = INF;
        rep(j, 0, R - 1) if (sa[j]) d = min(d, sa[j]);

        bool found = 0;
        for (auto i: que) lp[i] += d;
        rep(j, 0, R - 1) {
          if (sa[j]) {
            sa[j] -= d;
            if (!found) found |= extend(j);
          } else rp[j] -= d;
        }
        if (found) return;
      }
    }
  };

  rep(i, 0, L - 1) step(i);

  vector<pii> res;
  rep(i, 0, L - 1) res.emplace_back(i, lm[i]);
  return res;
} // hash-cpp-all = ec3fae2f44c4d2e8916ad89e33028e9a
```

4.3 Trees

binary-lifting.cpp

Description: Compute the sparse table for binary lifting of a rooted tree T. The root is set as 0 by default. g should be the adjacent list of the tree T.
Time: $\mathcal{O}(|V| \log |V|)$ for precalculation and $\mathcal{O}(\log |V|)$ for each lca query.

38 lines

```
struct BinaryLifting {
  int n;
  vi dep;
  vector<vi> anc;

  BinaryLifting(const vector<vi> &g, int rt = 0): n(sz(g)),
    ↪ dep(n, -1) {
    assert(n > 0);
    anc.assign(n, vi(__lg(n) + 1));
    auto dfs = [&](auto &dfs, int now, int fa) -> void {
      assert(dep[now] == -1); // make sure it is indeed a
        ↪tree.
      dep[now] = fa == -1 ? 0 : dep[fa] + 1;
      anc[now][0] = fa;
      rep(i, 1, __lg(n)) {
        anc[now][i] = anc[now][i - 1] == -1 ? -1 : anc[anc[
          ↪now][i - 1]][i - 1];
      }
      for (auto v: g[now]) if (v != fa) dfs(dfs, v, now);
    };
    dfs(dfs, rt, -1);
  }
  int swim(int x, int h) {
    for (int i = 0; h && x != -1; h >>= 1, i++) {
      if (h & 1) x = anc[x][i];
    }
    return x;
  }
  int lca(int x, int y) {
    if (dep[x] < dep[y]) swap(x, y);
    x = swim(x, dep[x] - dep[y]);
    if (x == y) return x;
    for (int i = __lg(n); i >= 0; --i) {
      if (anc[x][i] != anc[y][i]) {
        x = anc[x][i];
        y = anc[y][i];
      }
    }
    return anc[x][0];
  }
}; // hash-cpp-all = 49762913e2109a46ea1b423cd892c42b
```

heavy-light-decomposition.cpp

Description: Heavy Light Decomposition for a rooted tree T. The root is set as 0 by default. It can be modified easily for forest. g should be the adjacent list of the tree T. $chainApply(u, v, func, val)$ and $chainAsk(u, v, func)$ are used for apply / query on the simple path from u to v on tree T. $func$ is the function you want to use to apply / query on a interval. (Say rangeApply / rangeAsk of Segment tree.)
Time: $\mathcal{O}(|T|)$ for building. $\mathcal{O}(\log |T|)$ for lca. $\mathcal{O}(\log |T| \cdot A)$ for chainApply / chainAsk, where A is the running time of $func$ in chainApply / chainAsk.

69 lines

```
struct HLD {
  int n; // hash-cpp-1
  vi fa, hson, dfn, dep, top;
  HLD(vector<vi> &g, int rt = 0): n(sz(g)), fa(n, -1), hson
    ↪(n, -1), dfn(n), dep(n, 0), top(n) {
    vi siz(n);
    auto dfs = [&](auto &dfs, int now) -> void {
      siz[now] = 1;
      int mx = 0;
      for (auto v: g[now]) if (v != fa[now]) {
        dep[v] = dep[now] + 1;
        fa[v] = now;
        dfs(dfs, v);
        siz[now] += siz[v];
        if (mx < siz[v]) {
          mx = siz[v];
          hson[now] = v;
        }
      }
    };
    dfs(dfs, rt);

    int cnt = 0;
    auto getdfn = [&](auto &dfs, int now, int sp) {
      top[now] = sp;
      dfn[now] = cnt++;
      if (hson[now] == -1) return;
      dfs(dfs, hson[now], sp);
      for (auto v: g[now]) {
        if(v != hson[now] && v != fa[now]) dfs(dfs, v, v);
      }
    };
    getdfn(getdfn, rt, rt);
  } // hash-cpp-1 = 5c26a5d588a4413484d81b2ce1dd4fae

  int lca(int u, int v) { // hash-cpp-2
    while (top[u] != top[v]) {
      if (dep[top[u]] < dep[top[v]]) swap(u, v);
      u = fa[top[u]];
    }
    if (dep[u] < dep[v]) return u;
    else return v;
  } // hash-cpp-2 = c5c13283ffc68dacc37d3312019a26f8

  template<class... T> // hash-cpp-3
  void chainApply(int u, int v, const function<void(int,
    ↪int, T...)> &func, const T&... val) {
    int f1 = top[u], f2 = top[v];
    while (f1 != f2) {
      if (dep[f1] < dep[f2]) swap(f1, f2), swap(u, v);
      func(dfn[f1], dfn[u], val...);
      u = fa[f1]; f1 = top[u];
    }
    if (dep[u] < dep[v]) swap(u, v);
    func(dfn[v], dfn[u], val...); // change here if you
      ↪want the info on edges.
  } // hash-cpp-3 = e995d6fbf54395b102f90775b9a66a89

  template<class T> // hash-cpp-4
  T chainAsk(int u, int v, const function<T(int, int)> &
    ↪func) {
    int f1 = top[u], f2 = top[v];
    T ans{};
    while (f1 != f2) {
      if (dep[f1] < dep[f2]) swap(f1, f2), swap(u, v);
      ans = ans + func(dfn[f1], dfn[u]);
      u = fa[f1]; f1 = top[u];
    }
    if (dep[u] < dep[v]) swap(u, v);
    ans = ans + func(dfn[v], dfn[u]); // change here if you
      ↪ want the info on edges.
    return ans;
  } // hash-cpp-4 = 65ec12b740accde49b1ac20b95ea1de8
};
```

centroid-decomposition.cpp

Description: Centroid Decomposition of tree T. Here, $anc[i]$ is the list of ancestors of vertex i and the distances to the corresponding ancestor in centroid tree, including itself. Note that the distances are not monotone. Note that the top centroid is in the front of the vector.
Time: $\mathcal{O}(|T| \log |T|)$.

37 lines

```
struct CentroidDecomposition {
  int n;
  vector<vector<pii>> ancs;

  CentroidDecomposition(vector<vi> &g): n(sz(g)), ancs(n) {
    vi siz(n);
    vector<bool> vis(n);
    auto solve = [&](auto &solve, int st, int tot) -> void
      ↪{
      int mn = 0x3f3f3f3f, cent = -1;
      auto getcent = [&](auto &dfs, int now, int fa) ->
        ↪void {
        siz[now] = 1;
        int mx = 0;
        for (auto v: g[now]) if (v != fa && vis[v] == 0) {
          dfs(dfs, v, now);
          siz[now] += siz[v];
          mx = max(mx, siz[v]);
    }
    mx = max(mx, tot - siz[now]);
    if (mn > mx) mn = mx, cent = now;
   };
   getcent(getcent, st, -1);
   vis[cent] = 1;

   auto dfs = [&](auto &dfs, int now, int fa, int dep)
     ↪-> void {
    ancs[now].emplace_back(cent, dep);
    for (auto v: g[now]) if (v != fa && vis[v] == 0) {
     dfs(dfs, v, now, dep + 1);
    }
   };
   dfs(dfs, cent, -1, 0);
   // start your work here or inside the function dfs.

   for (auto v: g[cent]) if (vis[v] == 0) solve(solve, v
     ↪, siz[v] < siz[cent] ? siz[v] : tot - siz[cent])
     ↪;
  };
  solve(solve, 0, n);
 }
}; // hash-cpp-all = 8db9846c598845aeaba8d192e971b266
```

4.4 Connectivity

dsu.cpp

Description: Disjoint set union. *merge*(x, y) merges components which x and y are in respectively and returns 1 if x and y are in different components.
Time: amortized $\mathcal{O}(\alpha(M, N))$ where M is the number of operations. Almost constant in competitive programming.

18 lines

```
struct DSU {
 vi fa, siz;

 DSU(int n): fa(n), siz(n, 1) { iota(all(fa), 0); }

 int getcomp(int x) {
  return fa[x] == x ? x : fa[x] = getcomp(fa[x]);
 }

 bool merge(int x, int y) {
  int fx = getcomp(x), fy = getcomp(y);
  if (fx == fy) return 0;
  if (siz[fx] < siz[fy]) swap(fx, fy);
  fa[fy] = fx;
  siz[fx] += siz[fy];
  return 1;
 }
}; // hash-cpp-all = d79908e5926d7bd63f242158624be7d7
```

undo-dsu.cpp

Description: Undoable Disjoint Union Set for set $0, \ldots, N-1$. Fill in struct T, function *join* as well as choosing proper type Z for *glob* and remember to initialize it. Use $top = top()$ to get a save point; use $undo(top)$ to go back to the save point.
Usage: `UndoDSU dsu(n);`

```
...
int top = dsu.top(); // get a save point.
...  // do merging and other calculating here.
dsu.undo(top); // get back to the save point.
```

Time: Amortized $\mathcal{O}(\log N)$.

55 lines

```
struct UndoDSU {
 using Z = int; // choose some proper type (Z) for global
   ↪variable glob.
 struct T {
  int siz;
  // add things you want to maintain here.
  T(int ind = 0): siz(1) {
   // initialize what you add here.
  }
 };

 Z glob;
private:
 void join(T &a, const T& b) {
  a.siz += b.siz;
  // maintain the things you added to struct T.
  // also remember to maintain glob here.
 }

 vi fa;
 vector<T> ts;
 vector<tuple<int, int, T, Z>> sta;
public:
 UndoDSU(int n): fa(n), ts(n) {
  iota(all(fa), 0);
  iota(all(ts), 0);
  // remember initializing glob here.
 }

 int getcomp(int x) {
  while (x != fa[x]) x = fa[x];
  return x;
 }

 bool merge(int x, int y) {
  int fx = getcomp(x), fy = getcomp(y);
  if (fx == fy) return 0;
  if (ts[fx].siz < ts[fy].siz) swap(fx, fy);
  sta.emplace_back(fx, fy, ts[fx], glob);
  fa[fy] = fx;
  join(ts[fx], ts[fy]);
  return 1;
 }

 int top() { return sz(sta); }

 void undo(int top) {
  while (sz(sta) > top) {
   auto &[x, y, dat, g] = sta.back();
   fa[y] = y;
   ts[x] = dat;
   glob = g;
   sta.pop_back();
  }
 }
}; // hash-cpp-all = 20804d360ba467cdf1cd0b6125550c0f
```

cut-and-bridge.cpp

Description: Given an undirected graph $G = (V, E)$, compute all cut vertices and bridges. Cut vertices and bridges are returned in vectors containing indices.
Time: $\mathcal{O}(|V| + |E|)$.

31 lines

```
auto CutAndBridge(int n, const vector<pii> es) {
 vector<vi> g(n);
 rep(i, 0, sz(es) - 1) {
  auto [x, y] = es[i];
  g[x].push_back(i);
  g[y].push_back(i);
 }
 vi cut, bridge, dfn(n, -1), low(n), mark(sz(es));
 int cnt = 0;
 auto dfs = [&](auto &dfs, int now, int fa) -> void {
  dfn[now] = low[now] = cnt++;
  int sons = 0, isCut = 0;
  for (auto ind: g[now]) if (mark[ind] == 0) {
   mark[ind] = 1;
   auto [x, y] = es[ind];
   int v = now ^ x ^ y;
   if (dfn[v] == -1) {
    sons++;
    dfs(dfs, v, now);
    low[now] = min(low[now], low[v]);
    if (low[v] == dfn[v]) bridge.push_back(ind);
    if (low[v] >= dfn[now] && fa != -1) isCut = 1;
   } else low[now] = min(low[now], dfn[v]);
  }
  if (fa == -1 && sons > 1) isCut = 1;
  if (isCut) cut.push_back(now);
 };
 rep(i, 0, n - 1) if (dfn[i] == -1) dfs(dfs, i, -1);
 return make_tuple(cut, bridge);
} // hash-cpp-all = aa7a6da032272a8fb61b0196d4e44f91
```

vertex-bcc.cpp

Description: Compute the Vertex-BiConnected Components of a graph $G = (V, E)$ (not necessarily connected). Multiple edges and self loops are allowed. $id[i]$ records the index of bcc the i-th edge is in. $top[u]$ records the second highest vertex (which is unique) in the bcc which vertex u is in. (Just for checking if two vertices are in the same bcc.) This code also builds the block forest: bf records the edges in the block forest, where the i-th bcc corresponds to the $(n+i)$-th node. Call $getBlockForest()$ to get the adjacency list.
Time: $\mathcal{O}(|V| + |E|)$.

67 lines

```
struct VertexBCC {
 int n, bcc; // hash-cpp-1
 vi id, top, fa;
 vector<pii> bf; // edges of the block-forest.

 VertexBCC(int n, const vector<pii> &es): n(n), bcc(0), id
   ↪(sz(es)), top(n), fa(n, -1) {
  vector<vi> g(n);
  rep(ind, 0, sz(es) - 1) {
   auto [x, y] = es[ind];
   g[x].push_back(ind);
   g[y].push_back(ind);
  }

  int cnt = 0;
  vi dfn(n, -1), low(n), mark(sz(es)), vsta, esta;
  auto dfs = [&](auto dfs, int now) -> void {
   low[now] = dfn[now] = cnt++;
   vsta.push_back(now);
   for (auto ind: g[now]) if (mark[ind] == 0) {
    mark[ind] = 1;
    esta.push_back(ind);
    auto [x, y] = es[ind];
    int v = now ^ x ^ y;
    if (dfn[v] == -1) {
     dfs(dfs, v);
     fa[v] = now;
     low[now] = min(low[now], low[v]);
     if (low[v] >= dfn[now]) {
      bf.emplace_back(n + bcc, now);
      while (1) {
```

```cpp
       int z = vsta.back();
       vsta.pop_back();
       top[z] = v;
       bf.emplace_back(n + bcc, z);
       if (z == v) break;
      }
      while (1) {
       int z = esta.back();
       esta.pop_back();
       id[z] = bcc;
       if (z == ind) break;
      }
      bcc++;
     }
    } else low[now] = min(low[now], dfn[v]);
   }
  };
  rep(i, 0, n - 1) if (dfn[i] == -1) {
   dfs(dfs, i);
   top[i] = i;
  }
 } // hash-cpp-1 = 9fe76996c01d80b1cd996fc7fd090be7

 bool SameBcc(int x, int y) { // hash-cpp-2
  if (x == fa[top[y]] || y == fa[top[x]]) return 1;
  else return top[x] == top[y];
 } // hash-cpp-2 = 3cb78bd6aa7d389b1f6bb850cb631bb2

 vector<vi> getBlockForest() { // hash-cpp-3
  vector<vi> g(n + bcc);
  for (auto [x, y]: bf) {
   g[x].push_back(y);
   g[y].push_back(x);
  }
  return g;
 } // hash-cpp-3 = f03a2877586b955efaf91f9c09c7939d
};
```

edge-bcc.cpp

Description: Compute the Edge-BiConnected Components of a **connected** graph. Multiple edges and self loops are allowed. Return the size of BCCs and the index of the component each vertex belongs to.
Time: $\mathcal{O}(|E|)$.

35 lines

```cpp
auto EdgeBCC(int n, const vector<pii> &es, int st = 0) {
 vi dfn(n, -1), low(n), id(n), mark(sz(es), 0), sta;
 int cnt = 0, bcc = 0;
 vector<vi> g(n);
 rep(ind, 0, sz(es) - 1) {
  auto [x, y] = es[ind];
  g[x].push_back(ind);
  g[y].push_back(ind);
 }

 auto dfs = [&](auto dfs, int now) -> void {
  low[now] = dfn[now] = cnt++;
  sta.push_back(now);
  for (auto ind: g[now]) if (mark[ind] == 0) {
   mark[ind] = 1;
   auto [x, y] = es[ind];
   int v = now ^ x ^ y;
   if (dfn[v] == -1) {
    dfs(dfs, v);
    low[now] = min(low[now], low[v]);
   } else low[now] = min(low[now], dfn[v]);
  }
  if (low[now] == dfn[now]) {
   while (sta.back() != now) {
    id[sta.back()] = bcc;
    sta.pop_back();
   }
   id[now] = bcc;
   sta.pop_back();
   bcc++;
  }
 };
 dfs(dfs, st);
 return make_tuple(bcc, id);
} // hash-cpp-all = 2bdde4f70ef209337f1ec3d5666d1c64
```

tarjan.cpp

Description: Tarjan algorithm for directed graph $G = (V, E)$.

27 lines

```cpp
auto tarjan(const vector<vi> &g) {
 int n = sz(g);
 vi id(n, -1), dfn(n, -1), low(n, -1), sta;
 int cnt = 0, scc = 0;

 auto dfs = [&](auto &dfs, int now) -> void {
  dfn[now] = low[now] = cnt++;
  sta.push_back(now);
  for (auto v: g[now]) {
   if (dfn[v] == -1) {
    dfs(dfs, v);
    low[now] = min(low[now], low[v]);
   } else if (id[v] == -1) low[now] = min(low[now], dfn[
     ↪v]);
  }
  if (low[now] == dfn[now]) {
   while (1) {
    int z = sta.back();
    sta.pop_back();
    id[z] = scc;
    if (z == now) break;
   }
   scc++;
  }
 };
 rep(i, 0, n - 1) if (dfn[i] == -1) dfs(dfs, i);
 return make_tuple(scc, id);
} // hash-cpp-all = e9681d2c3fd78713716890417a465211
```

2sat.cpp

Description: 2SAT solver, returns if a 2SAT system of V variables and C constraints is satisfiable. If yes, it also gives an assignment. Call $addClause$ to add clauses. For example, if you want to add clause $\neg x \vee y$, just call $addClause(x, 0, y, 1)$.
Time: $\mathcal{O}(|V| + |C|)$.

46 lines

```cpp
struct TwoSat {
 int n;
 vector<vi> e;
 vi ans;

 TwoSat(int n): n(n), e(n * 2), ans(n) {}

 void addClause(int x, bool f, int y, bool g) {
  e[x * 2 + !f].push_back(y * 2 + g);
  e[y * 2 + !g].push_back(x * 2 + f);
 }

 bool satisfiable() {
  vi id(n * 2, -1), dfn(n * 2, -1), low(n * 2, -1), sta;
  int cnt = 0, scc = 0;

  auto dfs = [&](auto &dfs, int now) -> void {
   dfn[now] = low[now] = cnt++;
   sta.push_back(now);
   for (auto v: e[now]) {
    if (dfn[v] == -1) {
     dfs(dfs, v);
     low[now] = min(low[now], low[v]);
    } else if (id[v] == -1) low[now] = min(low[now],
      ↪dfn[v]);
   }
   if (low[now] == dfn[now]) {
    while (sta.back() != now) {
     id[sta.back()] = scc;
     sta.pop_back();
    }
    id[sta.back()] = scc;
    sta.pop_back();
    scc++;
   }
  };

  rep(i, 0, n * 2 - 1) if (dfn[i] == -1) dfs(dfs, i);
  rep(i, 0, n - 1) {
   if (id[i * 2] == id[i * 2 + 1]) return 0;
   ans[i] = id[i * 2] > id[i * 2 + 1];
  }
  return 1;
 }

 vi getAss() { return ans; }
}; // hash-cpp-all = 48021fb8f8e959774f7a861f2f294deb
```

link-cut.cpp

Description: Link-Cut tree
Time: TODO

175 lines

```cpp
struct Node { // hash-cpp-1
 Node* p = nullptr; // Parent (or path parent)
 Node* ch[2] = {nullptr, nullptr}; // Children
 bool flip = 0; // Should this node's subtree be flipped
 // Put other wanted data here

 void push() {
  if (flip) {
   swap(ch[0], ch[1]);
   if (ch[0]) ch[0]->flip ^= 1;
   if (ch[1]) ch[1]->flip ^= 1;
   flip = 0;
  }
 }

 void update() {
  // Do nothing by default
  // Note that while it is guaranteed that this node is
    ↪pushed,
  // its children haven't necessarily been pushed.
 }
}; // hash-cpp-1 = 8aa01493388a3707a742f56f04b4a804

// Link-Cut tree
// https://ocw.mit.edu/courses/electrical-engineering-and-
  ↪computer-science/6-854j-advanced-algorithms-fall-2008/
  ↪lecture-notes/lec6.pdf
// https://courses.csail.mit.edu/6.851/spring14/scribe/L19.
  ↪pdf
template<class T>
```

```cpp
class LinkCutTree {
  private:
    // Set a's child to be b, in direction d
    static void setChild(T* a, T* b, bool d) { // hash-cpp
        ↪-2
      a->ch[d] = b;
      if (b) b->p = a;
    }
    static bool isRoot(const T* a) {
      return a->p == nullptr || (a->p->ch[0] != a && a->p->
          ↪ch[1] != a);
    }
    // is right child?
    static bool irc(const T* a) {
      return a->p->ch[1] == a;
    }

    // Zigs or zags na up one step
    // na and its parent should be pushed beforehand
    static void rotate(T* a) {
      bool dir = irc(a);
      Node* b = a->p;
      Node* c = a->ch[dir ^ 1];

      if (isRoot(b)) a->p = b->p;
      else setChild(b->p, a, irc(b));
      setChild(b, c, dir);
      setChild(a, b, dir ^ 1);

      b->update();
      a->update();
    }

    // Splays on na. Path from na to it's root should be
        ↪pushed.
    static void splay(T* a) {
      while(! isRoot(a)) {
        if (! isRoot(a->p)) {
          if (irc(a) == irc(a->p)) rotate(a);
          else rotate(a->p);
        }
        rotate(a);
      }
    } // hash-cpp-2 = f4088c7eacddc2028cc46c0c1b13bcc8

    // Pushes all nodes on path from na to root of its
        ↪represented tree
    static void pushPath(T* a) { // hash-cpp-3
      if (a->p) pushPath(a->p);
      a->push();
    }
    // link-cut access operation
    // DOESN'T splay on na in the end!
    static T* access(T* a) {
      T* b = a;
      T* c = nullptr;
      pushPath(b);
      while(b) {
        splay(b);
        b->ch[1] = c;
        b->update();
        c = b;
        b = b->p;
      }
      splay(a);
      return a;
    }
    // Reroots at a, and makes the path from a to b active
    static T* expose(T* a, T* b) {
      access(a);
      a->flip = 1;
      access(b);
      return b;
    } // hash-cpp-3 = a01328162fb0a37773e5c1bc71a32358

    // Adds an edge between nodes a and b
    // returns false if a and b are in the same component
    static bool link(T* a, T* b) { // hash-cpp-4
      expose(a, b);
      if (a->p) return false;
      else {
        setChild(b, a, 1);
        b->update();
        return true;
      }
    }

    // Cuts the edge between nodes a and b
    // Returns false if no such edge exists
    static bool cut(T* a, T* b) {
      expose(a, b);
      if (b->ch[0] != a || a->ch[1]) return false;
      else {
        a->p = nullptr;
        b->ch[0] = nullptr;
        b->update();
        return true;
      }
    } // hash-cpp-4 = 7ba45468c79c7e359db83e04aac3a395

    vector<T> nodes;
  public:
    template<class... Args> // hash-cpp-5
    int emplace(Args&&... args) {
      nodes.emplace_back(forward<Args>(args)...);
      return (int)nodes.size() - 1;
    }
    T& expose(int ai, int bi) {
      return *expose(&nodes[ai], &nodes[bi]);
    }
    bool link(int ai, int bi) {
      if (ai == bi) return false;
      else return link(&nodes[ai], &nodes[bi]);
    }
    bool cut(int ai, int bi) {
      if (ai == bi) return false;
      else return cut(&nodes[ai], &nodes[bi]);
    } // hash-cpp-5 = af420521f57bed8f60a07710d3ec58dc
};

// Example usage
// Solves the tree dynamic connectivity problem
int main() {
  int n, q;
  cin >> n >> q;

  LinkCutTree<Node> lctree;
  for (int i = 0; i < n; ++i) lctree.emplace();

  for (int ti = 0; ti < q; ++ti) {
    char op;
    int a, b;
    cin >> op >> a >> b;
    --a; --b;
    if (op == 'l') {
      bool fail = ! lctree.link(a, b);
      if (fail) cout << "already connected\n";
      else cout << "link added\n";
    } else if (op == 'c') {
      bool fail = ! lctree.cut(a, b);
      if (fail) cout << "no such link\n";
      else cout << "link cut\n";
    } else {
      bool conn = ! lctree.link(a, b);
      if (conn) {
        cout << "connected\n";
      } else {
        lctree.cut(a, b);
        cout << "not connected\n";
      }
    }
  }
}
```

4.5 Paths

euler-tour-nonrec.cpp

Description: For an edge set E such that each vertex has an even degree, compute Euler tour for each connected component. *dir* indicates edges are directed or not (undirected by default). For undirected graph, $ori[i]$ records the orientation of the i-th edge $es[i] = (x, y)$, where $ori[i] = 1$ means $x \to y$ and $ori[i] = -1$ means $y \to x$. Note that this is a non-recursive implementation, which avoids stack size issue on some OJ and also saves memory (roughly saves 2/3 of memory) due to that.
Time: $\mathcal{O}(|V| + |E|)$.

52 lines

```cpp
struct EulerTour {
  int n;
  vector<vi> tours;
  vi ori;

  EulerTour(int n, const vector<pii> &es, int dir = 0): n(n
      ↪), ori(sz(es)) {
    vector<vi> g(n);
    int m = sz(es);
    rep(i, 0, m - 1) {
      auto [x, y] = es[i];
      g[x].push_back(i);
      if (dir == 0) g[y].push_back(i);
    }

    vi path, cur(n);
    vector<pii> sta;
    auto solve = [&](int st) {
      sta.emplace_back(st, -1);
      while (sz(sta)) {
        auto [now, pre] = sta.back();
        int fin = 1;
        for (int &i = cur[now]; i < sz(g[now]); ) {
          auto ind = g[now][i++];
          if (ori[ind]) continue;
          auto [x, y] = es[ind];
          ori[ind] = x == now ? 1 : -1;
          int v = now ^ x ^ y;
          sta.emplace_back(v, ind);
          fin = 0;
          break;
        }
        if (fin) {
          if (pre != -1) path.push_back(pre);
          sta.pop_back();
        }
      }
    };
    rep(i, 0, n - 1) {
      path.clear();
      solve(i);
      if (sz(path)) {
        reverse(all(path));
        tours.push_back(path);
      }
    }
  }

  vector<vi> getTours() { return tours; }

  vi getOrient() { return ori; }
}; // hash-cpp-all = e5f7e9e86d4e1d9d5aa0be753a0cb6e9
```

4.6 Others

max-clique.cpp

Description: Finding a Maximum Clique of a graph $G = (V, E)$. Should be fine with $|V| \leq 60$. (The algorithm actually enumberates all maximal clique, without double counting.)

26 lines

```
template<int L>
vi BronKerbosch(int n, const vector<pii> &es) {
  using bs = bitset<L>;
  vector<bs> nbrs(n);
  for (auto [x, y]: es) {
    nbrs[x].set(y);
    nbrs[y].set(x);
  }
  bs best;
  auto dfs = [&](auto &dfs, const bs &R, const bs &P, const
    ↪ bs &X) {
    if (P.none() && X.none()) {
      if (R.count() > best.count()) best = R;
      return;
    }
    bs tmp = P & ~nbrs[(P | X)._Find_first()];
    for (int v = tmp._Find_first(); v != L; v = tmp.
      ↪_Find_next(v)) {
      bs nR = R;
      nR.set(v);
      dfs(dfs, nR, P & nbrs[v], X & nbrs[v]);
    }
  };
  dfs(dfs, bs{}, bs{string(n, '1')}, bs{});
  vi res;
  rep(i, 0, n - 1) if (best[i]) res.push_back(i);
  return res;
} // hash-cpp-all = 32b465646370106ceb75c09e49f5f4e7
```

String algorithms (5)

5.1 String Matching

kmp.cpp

Description: Compute fail table of pattern string $s = s_0 \dots s_{n-1}$ in linear time and get all matched positions in text string t in linear time. $fail[i]$ denotes the length of the border of substring $s_0 \dots s_i$. In $match(t)$, $res[i] = 1$ means that $t_i \dots t_{i+n-1}$ matched to s.
Usage: `KMP kmp(s); // s can be string or vector.`
Time: $\mathcal{O}(|s|)$ for precalculation and $\mathcal{O}(|t|)$ for matching.

25 lines

```
template<class T> struct KMP {
  const T s; // hash-cpp-1
  int n;
  vi fail;

  KMP(const T &s): s(s), n(sz(s)), fail(n) {
    int j = 0;
    rep(i, 1, n - 1) {
      while (j > 0 && s[j] != s[i]) j = fail[j - 1];
      if (s[j] == s[i]) j++;
      fail[i] = j;
    }
  } // hash-cpp-1 = abad2ebf1bb7e6689c795bf074babcc6

  vi match(const T &t) { // hash-cpp-2
    int m = sz(t), j = 0;
    vi res(m);
    rep(i, 0, m - 1) {
      while (j > 0 && (j == n || s[j] != t[i])) j = fail[j
        ↪- 1];
      if (s[j] == t[i]) j++;
      if (j == n) res[i - n + 1] = 1;
    }
    return res;
  } // hash-cpp-2 = f586c1dee3650d26ab1db15140981c8b
};
```

z-algo.cpp

Description: Given string $s = s_0 \dots s_{n-1}$, compute array z where $z[i]$ is the lcp of $s_0 \dots s_{n-1}$ and $s_i \dots s_{n-1}$. Use function $cal(t)$ (where $|t| = m$) to calculate the lcp of of $s_0 \dots s_{n-1}$ and $t_i \dots t_{m-1}$ for each i.
Usage: `zAlgo za(s); // s can be string or vector.`
Time: $\mathcal{O}(|s|)$ for precalculation and $\mathcal{O}(|t|)$ for matching.

34 lines

```
template<class T>
struct zAlgo {
  const T s; // hash-cpp-1
  int n;
  vi z;

  zAlgo(const T &s): s(s), n(sz(s)), z(n) {
    z[0] = n;
    int l = 0, r = 0;
    rep(i, 1, n - 1) {
      z[i] = max(0, min(z[i - l], r - i));
      while (i + z[i] < n && s[z[i]] == s[i + z[i]]) z[i
        ↪]++;
      if (i + z[i] > r) {
        l = i;
        r = i + z[i];
      }
    }
  } // hash-cpp-1 = 0a5f9be882b336b6aa27f9ee79d633ec

  vi cal(const T &t) { // hash-cpp-2
    int m = sz(t);
    vi res(m);
    int l = 0, r = 0;
    rep(i, 0, m - 1) {
      res[i] = max(0, min(i - l < n ? z[i - l] : 0, r - i))
        ↪;
      while (i + res[i] < m && s[res[i]] == t[i + res[i]])
        ↪res[i]++;
      if (i + res[i] > r) {
        l = i;
        r = i + res[i];
      }
    }
    return res;
  } // hash-cpp-2 = 0a29c792be96f8c1ccdb699df9cfc984
};
```

aho-corasick.cpp

Description: Aho Corasick Automaton of strings $s_0, \dots, s_{n-1}$. Call $build()$ after you insert all strings $s_0, \dots, s_{n-1}$.
Usage: `AhoCorasick<'a', 26> ac; // for strings consisting of lowercase letters.`
```
ac.insert("abc"); // insert string "abc".
ac.insert("acc"); // insert string "acc".
ac.build();
```
Time: $\mathcal{O}\left(\sum_{i=0}^{n-1} |s_i|\right)$.

48 lines

```
template<char st, int C>
struct AhoCorasick {
  struct node {
    int nxt[C];
    int fail;
    int cnt;
    node() {
      memset(nxt, -1, sizeof nxt);
      fail = -1;
      cnt = 0;
    }
  };

  vector<node> t;

  AhoCorasick(): t(1) {}

  int insert(const string &s) {
    int now = 0;
    for (auto ch: s) {
      int c = ch - st;
      if (t[now].nxt[c] == -1) {
        t.emplace_back();
        t[now].nxt[c] = sz(t) - 1;
      }
      now = t[now].nxt[c];
    }
    t[now].cnt++;
    return now;
  }

  void build() {
    vi que{0};
    rep(ind, 0, sz(que) - 1) {
      int now = que[ind], fa = t[now].fail;
      rep(c, 0, C - 1) {
        int &v = t[now].nxt[c];
        int u = fa == -1 ? 0 : t[fa].nxt[c];
        if (v == -1) v = u;
        else {
          t[v].fail = u;
          que.push_back(v);
        }
      }
      if (fa != -1) t[now].cnt += t[fa].cnt;
    }
  }
}; // hash-cpp-all = 3dca34c2bb5ab364d7abcab29a8c27f4
```

5.2 Suffices & Substrings

suffix-array.cpp

Description: Suffix Array for non-cyclic string $s = s_0 ... s_{n-1}$. $rank[i]$ records the rank of the i-th suffix $s_i ... s_{n-1}$. $sa[i]$ records the starting position of the i-th smallest suffix. $h[i]$ (also called height array or lcp array) records the lcp of the $sa[i]$-th suffix and the $sa[i+1]$-th suffix in s.
Usage: `SA suf(s); // s can be string or vector.`
Time: $\mathcal{O}(|s| \log |s|)$.

49 lines

```
struct SA {
  int n;
  vi str, sa, rank, h;

  template<class T> SA(const T &s): n(sz(s)), str(n + 1),
    ↪sa(n + 1), rank(n + 1), h(n - 1) {
    auto vec = s;
    sort(all(vec)); vec.erase(unique(all(vec)), vec.end());
    rep(i, 0, n - 1) str[i] = rank[i] = lower_bound(all(vec
      ↪), s[i]) - vec.begin() + 1;
    iota(all(sa), 0);
    n++;

    for (int len = 0; len < n; len = len ? len * 2 : 1) {
      vi cnt(n + 1);
      for (auto v : rank) cnt[v + 1]++;
      rep(i, 1, n - 1) cnt[i] += cnt[i - 1];

      vi nsa(n), nrank(n);

      for (auto pos: sa) {
        pos -= len;
        if (pos < 0) pos += n;
        nsa[cnt[rank[pos]]++] = pos;
      }
      swap(sa, nsa);

      int r = 0, oldp = -1;
      for (auto p: sa) {
        auto next = [&](int a, int b) { return a + b < n ?
          ↪a + b : a + b - n; };
        if (~oldp) r += rank[p] != rank[oldp] || rank[next(
          ↪p, len)] != rank[next(oldp, len)];
        nrank[p] = r;
        oldp = p;
      }
      swap(rank, nrank);
    }
    sa = vi(sa.begin() + 1, sa.end());
    rank.resize(--n);
    rep(i, 0, n - 1) rank[sa[i]] = i;

    // compute height array.
    int len = 0;
    rep(i, 0, n - 1) {
      if (len) len--;
      int rk = rank[i];
      if (rk == n - 1) continue;
      while (str[i + len] == str[sa[rk + 1] + len]) len++;
      h[rk] = len;
    }
  }
}; // hash-cpp-all = dc03be590b13b29f57b3250dc4634be7
```

suffix-array-lcp.cpp

Description: Suffix Array with sparse table answering lcp of suffices.
Usage: `SA suf(s); // s can be string or vector.`
Time: $\mathcal{O}(|s| \log |s|)$ for construction. $\mathcal{O}(1)$ per query.

"suffix-array.cpp" 22 lines

```
struct SA_lcp: SA {
  vector<vi> st;

  template<class T> SA_lcp(const T &s): SA(s) {
    assert(n > 0);
    st.assign(__lg(n) + 1, vi(n));
    st[0] = h;
    st[0].push_back(0); // just to make st[0] of size n.
    rep(i, 1, __lg(n)) rep(j, 0, n - (1 << i)) {
      st[i][j] = min(st[i - 1][j], st[i - 1][j + (1 << (i -
        ↪ 1))]);
    }
  }
  // return lcp(suff_i, suff_j) for i != j.
  int lcp(int i, int j) {
    if (i == n || j == n) return 0;
    assert(i != j);
    int l = rank[i], r = rank[j];
    if (l > r) swap(l, r);
    int k = __lg(r - l);
    return min(st[k][l], st[k][r - (1 << k)]);
  }
}; // hash-cpp-all = ff57ad558a18576768e4c3b01e315c93
```

sam.cpp

Description: Suffix Automaton of a given string s. (Using map to store sons makes it 2∼3 times slower but it should be fine in most cases.) len is the length of the longest substring corresponding to the state. fa is the father in the prefix tree. Note that fa[i] < i doesn't hold. occ is 0/1, indicating if the state contains a prefix of the string s. One can do a dfs/bfs to compute for each substring, how many times it occurs in the whole string s. (See function *calOccurrence* for bfs implementation.) root is set as 0.
Usage: `SAM sam(s); // s can be string or vector<int>.`
Time: $\mathcal{O}(|s|)$.

75 lines

```
template<class T>
struct SAM {
  struct node { // hash-cpp-1
    map<int, int> nxt; // change this if it is slow.
    int fa, len;
    int occ, pos; // # of occurrence (as prefix) & endpos.
    node(int fa = -1, int len = 0): fa(fa), len(len) {
      occ = pos = 0;
    }
  };

  T s;
  int n;
  vector<node> t;
  vi at; // at[i] = the state at which the i-th prefix of s
    ↪ is.

  SAM(const T &s): s(s), n(sz(s)), at(n) {
    t.emplace_back();
    int last = 0; // create root.

    auto ins = [&](int i, int c) {
      int now = last;
      t.emplace_back(-1, t[now].len + 1);
      last = sz(t) - 1;
      t[last].occ = 1;
      t[last].pos = i;
      at[i] = last;

      while (now != -1 && t[now].nxt.count(c) == 0) {
        t[now].nxt[c] = last;
        now = t[now].fa;
      }
      if (now == -1) t[last].fa = 0; // root is 0.
      else {
        int p = t[now].nxt[c];
        if (t[p].len == t[now].len + 1) t[last].fa = p;
        else {
          auto tmp = t[p];
          tmp.len = t[now].len + 1;
          tmp.occ = 0; // do not copy occ.
          t.push_back(tmp);
          int np = sz(t) - 1;

          t[last].fa = t[p].fa = np;
          while (now != -1 && t[now].nxt.count(c) && t[now
            ↪].nxt[c] == p) {
            t[now].nxt[c] = np;
            now = t[now].fa;
          }
        }
      }
    };

    rep(i, 0, n - 1) ins(i, s[i]);
  } // hash-cpp-1 = 1c12eb7fbeec418a5befc77214c19b9b

  void calOccurrence() { // hash-cpp-2
    vi sum(n + 1), que(sz(t));
    for (auto &it: t) sum[it.len]++;
    rep(i, 1, n) sum[i] += sum[i - 1];
    rep(i, 0, sz(t) - 1) que[--sum[t[i].len]] = i;
    reverse(all(que));
    for (auto now: que) if (now != 0) t[t[now].fa].occ += t
      ↪[now].occ;
  } // hash-cpp-2 = 34e98c4d6ea1e86aa5d52a582becf8a8

  vector<vi> ReversedPrefixTree() { // hash-cpp-3
    vector<vi> g(sz(t));
    rep(now, 1, sz(t) - 1) g[t[now].fa].push_back(now);
    rep(now, 0, sz(t) - 1) {
      sort(all(g[now]), [&](int i, int j) {
        return s[t[i].pos - t[now].len] < s[t[j].pos - t[
          ↪now].len];
      });
    }
    return g;
  } // hash-cpp-3 = aadc726973415dfaac1e483d8fac558b
};
```

general-sam.cpp

Description: General Suffix Automaton of a given Trie T. (Using map to store sons makes it 2∼3 times slower but it should be fine in most cases. If T is of size $> 10^6$, then you should think of using int[] instead of map.) len is the length of the longest substring corresponding to the state. fa is the father in the reversed prefix tree. Note that fa[i] < i doesn't hold. occ should be set manually when building Trie T. root is 0.
Usage: `GSAM sam(T); // T should be vector<GSAM::node>.`
Time: $\mathcal{O}(|T|)$.

52 lines

```
struct GSAM {
  struct node {
    map<int, int> nxt; // change this if TL or ML is tight.
    int fa, len; // keep fa = -1 and len = 0 initially.
    int occ; // should be assigned when building the trie.
    node() { fa = -1; len = occ = 0; }
  };
  vector<node> t;
  GSAM(const vector<node> &trie): t(trie) { // swap(t, trie
    ↪) here if TL or ML is tight
    auto ins = [&](int now, int c) {
      int last = t[now].nxt[c];
      t[last].len = t[now].len + 1;
      now = t[now].fa;
      while (now != -1 && t[now].nxt.count(c) == 0) {
        t[now].nxt[c] = last;
        now = t[now].fa;
      }
      if (now == -1) t[last].fa = 0;
      else {
        int p = t[now].nxt[c];
        if (t[p].len == t[now].len + 1) t[last].fa = p;
        else { // clone a node np from node p.
          t.emplace_back();
          int np = sz(t) - 1;
          for (auto [i, v]: t[p].nxt) if (t[v].len > 0) {
            t[np].nxt[i] = v;
          }
          t[np].fa = t[p].fa;
          t[np].len = t[now].len + 1;

          t[last].fa = t[p].fa = np;
          while (now != -1 && t[now].nxt.count(c) && t[now
            ↪].nxt[c] == p) {
            t[now].nxt[c] = np;
            now = t[now].fa;
          }
        }
      }
    };

    vi que{0};
    rep(ind, 0, sz(que) - 1) {
      int now = que[ind];
      vi cs;
      for (auto [c, v]: t[now].nxt) {
        cs.push_back(c);
        que.push_back(v);
      }
      for (auto c: cs) ins(now, c);
    }
  }
}; // hash-cpp-all = add4c78221df38584b76536f66703db7
```

lyndon-factorization.cpp

Description: Lyndon factorization of string s. Return a vector of pairs (l, r), representing substring $s_l \dots s_r$.
Time: $\mathcal{O}(|s|)$.

17 lines

```
vector<pii> duval(string const& s) {
  int n = sz(s), i = 0;
  vector<pii> res;
  while (i < n) {
    int j = i + 1, k = i;
    while (j < n && s[k] <= s[j]) {
      if (s[k] < s[j]) k = i;
      else k++;
      j++;
    }
    while (i <= k) {
      res.emplace_back(i, i + j - k - 1);
      i += j - k;
    }
  }
  return res;
} // hash-cpp-all = 6fff07a96ae3b4e5c66e847abfeb48c6
```

5.3 Palindromes

manacher.cpp

Description: Manacher Algorithm for finding all palindrome subtrings of $s = s_0 \dots s_{n-1}$. s can actually be string or vector (say vector<int>). For returned vector len, $len[i*2] = r$ means that $s_{i-r+1} \dots s_{i+r-1}$ is the maximal palindrome centered at position i. $len[i*2+1] = r$ means that $s_{i-r+1} \dots s_{i+r}$ is the maximal palindrome centered between position i and $i+1$.
Usage: `vi rs = Manacher(s); // s can be string or vector.`
Time: $\mathcal{O}(|s|)$.

12 lines

```
template<class T>
vi Manacher(const T &s) {
  int n = sz(s), j = 0;
  vi len(n * 2 - 1, 1);
  rep(i, 1, n * 2 - 2) {
    int p = i / 2, q = i - p, r = (j + 1) / 2 + len[j] - 1;
    len[i] = r < q ? 0 : min(r - q + 1, len[j * 2 - i]);
    while (p > len[i] - 1 && q + len[i] < n && s[p - len[i
      ↪]] == s[q + len[i]]) len[i]++;
    if (q + len[i] - 1 > r) j = i;
  }
  return len;
} // hash-cpp-all = 4c6da773ee61b4d53dd654a4d0d04a4c
```

palindrome-tree.cpp

Description: Given string $s = s_0 \dots s_{n-1}$, build the palindrom tree (automaton) for s. Each state of the automaton corresponds to a palindrome substring of s. $t[i].fail$ is the state which is a border of state i. Note that $t[i].fail < i$ holds.
Usage: `Palindrome pt(s); // s can be string or vector.`
Time: $\mathcal{O}(|s|)$.

36 lines

```
struct PalindromeTree {
  struct node {
    map<int, int> nxt;
    int fail, len;
    int cnt;
    node(int fail, int len): fail(fail), len(len) {
      cnt = 0;
    }
  };
  vector<node> t;

  template<class T>
  PalindromeTree(const T &s) {
    int n = sz(s);
    t.emplace_back(-1, -1); // Odd root -> state 0.
    t.emplace_back(0, 0); // Even root -> state 1.

    int now = 0;
    auto ins = [&](int pos) {
      auto get = [&](int i) {
        while (pos == t[i].len || s[pos - 1 - t[i].len] !=
          ↪s[pos]) i = t[i].fail;
        return i;
      };
      int c = s[pos];
      now = get(now);
      if (t[now].nxt.count(c) == 0) {
        int q = now == 0 ? 1 : t[get(t[now].fail)].nxt[c];
        t.emplace_back(q, t[now].len + 2);
        t[now].nxt[c] = sz(t) - 1;
      }
      now = t[now].nxt[c];
      t[now].cnt++;
    };
    rep(i, 0, n - 1) ins(i);
  }
}; // hash-cpp-all = ca74a23e6dec05d3f4328aa98fd3d4d3
```

5.4 Hashes

hash-struct.cpp

Description: Hash struct. 1000000007 and 1000050131 are good moduli.

19 lines

```
template<int m1, int m2>
struct Hash {
  int x, y;
  Hash(ll a, ll b): x(a % m1), y(b % m2) {
    if (x < 0) x += m1;
    if (y < 0) y += m2;
  }
  Hash(ll a = 0): Hash(a, a) {}

  using H = Hash;
  static int norm(int x, int mod) { return x >= mod ? x -
    ↪mod : x < 0 ? x + mod : x; }
  friend H operator +(H a, H b) { a.x = norm(a.x + b.x, m1)
    ↪; a.y = norm(a.y + b.y, m2); return a; }
  friend H operator -(H a, H b) { a.x = norm(a.x - b.x, m1)
    ↪; a.y = norm(a.y - b.y, m2); return a; }
  friend H operator *(H a, H b) { return H{1ll * a.x * b.x,
    ↪ 1ll * a.y * b.y}; }

  friend bool operator ==(H a, H b) { return tie(a.x, a.y)
    ↪== tie(b.x, b.y); }
  friend bool operator !=(H a, H b) { return tie(a.x, a.y)
    ↪!= tie(b.x, b.y); }
  friend bool operator <(H a, H b) { return tie(a.x, a.y) <
    ↪ tie(b.x, b.y); }
}; // hash-cpp-all = ff126b1c842614ecc3db2080807d765e
```

string-hash.cpp

Description: Hash of a string.
Usage: `StringHash<unsigned long long> ha(s); // s can be string or vector<int>.`
Time: $\mathcal{O}(|s|)$.

15 lines

```
template<class hashv>
struct StringHash {
  const hashv base = 131; // change this if you hash a
    ↪vector<int>.
  int n;
  vector<hashv> hs, pw;

  template<class T>
  StringHash(const T &s): n(sz(s)), hs(n + 1), pw(n + 1) {
    pw[0] = 1;
    rep(i, 1, n) pw[i] = pw[i - 1] * base;
    rep(i, 0, n - 1) hs[i + 1] = hs[i] * base + s[i];
  }

  hashv get(int l, int r) { return hs[r + 1] - hs[l] * pw[r
    ↪ + 1 - l]; }
}; // hash-cpp-all = 6575c218c608958f097a71917dab22a9
```

Numerical (6)

6.1 Transforms & Polynomials

fft.cpp

Description: Fast Fourier Transform. T can be **double** or **long double**.
Usage: `FFT<double> fft;`
```
auto cs = fft.conv(vector<double>{1, 2, 3},
vector<double>{3, 4, 5});
vector<int> ds = fft.conv(vector<int>{1, 2, 3},
vector<int>{3, 4, 5}, 1000000007); // convolution of
integers wrt arbitrary mod ≤ 2^{31} − 1.
```
Time: $\mathcal{O}(N \log N)$.

73 lines

```
template<class T>
struct FFT {
  using cp = complex<T>; // hash-cpp-1
  static constexpr T pi = acos(T{-1});
  vi r;
  int n2;

  void dft(vector<cp> &a, int is_inv) { // is_inv == 1 ->
    ↪idft.
    rep(i, 1, n2 - 1) if (r[i] > i) swap(a[i], a[r[i]]);
    for(int step = 1; step < n2; step <<= 1) {
      vector<cp> w(step);
      rep(j, 0, step-1) { // this has higher precision,
        ↪compared to using the power of zeta.
        T theta = pi * j / step;
        if (is_inv) theta = -theta;
        w[j] = cp{cos(theta), sin(theta)};
      }
      for (int i = 0; i < n2; i += step << 1) {
        rep(j, 0, step - 1) {
          cp tmp = w[j] * a[i + j + step];
          a[i + j + step] = a[i + j] - tmp;
          a[i + j] += tmp;
        }
      }
    }
    if (is_inv) {
      for (auto &x: a) x /= n2;
    }
  }
  void pre(int n) { // set n2, r;
    int len = 0;
    for (n2 = 1; n2 < n; n2 <<= 1) len++;
    r.resize(n2);
    rep(i, 1, n2 - 1) r[i] = (r[i >> 1] >> 1) | ((i & 1) <<
      ↪ (len - 1));
  }
  template<class Z> vector<Z> conv(const vector<Z> &A,
    ↪const vector<Z> &B) {
    int n = sz(A) + sz(B) - 1;
    pre(n);
    vector<cp> a(n2, 0), b(n2, 0);
    rep(i, 0, sz(A) - 1) a[i] = A[i];
    rep(i, 0, sz(B) - 1) b[i] = B[i];

    dft(a, 0); dft(b, 0);
    rep(i, 0, n2 - 1) a[i] *= b[i];
    dft(a, 1);
    vector<Z> res(n);
    T eps = T{0.5} * (static_cast<Z>(1e-9) == 0);
    rep(i, 0, n - 1) res[i] = a[i].real() + eps;
    return res;
  } // hash-cpp-1 = 374598e5285b8ad9d80e16d7cdd26f15
  vi conv(const vi &A, const vi &B, int mod) { // hash-cpp
    ↪-2
    int M = sqrt(mod) + 0.5;
    int n = sz(A) + sz(B) - 1;
    pre(n);
    vector<cp> a(n2, 0), b(n2, 0), c(n2, 0), d(n2, 0);
    rep(i, 0, sz(A) - 1) a[i] = A[i] / M, b[i] = A[i] % M;
    rep(i, 0, sz(B) - 1) c[i] = B[i] / M, d[i] = B[i] % M;

    dft(a, 0); dft(b, 0); dft(c, 0); dft(d, 0);
    vi res(n);

    auto work = [&](vector<cp> &a, vector<cp> &b, int w,
      ↪int mod) {
      vector<cp> tmp(n2);
      rep(i, 0, n2 - 1) tmp[i] = a[i] * b[i];
      dft(tmp, 1);
      rep(i, 0, n - 1) res[i] = (res[i] + (ll)(tmp[i].real
        ↪() + 0.5) % mod * w) % mod;
    };
    work(a, c, 1ll * M * M % mod, mod);
    work(b, d, 1, mod);
    work(a, d, M, mod);
    work(b, c, M, mod);
    return res;
  } // hash-cpp-2 = 3f22907659cf5834b6b2c75333b3c777
};
```

ntt.cpp

Description: Number Theoretic Transform. class T should have static function $getMod()$ to provide the mod. We usually just use $modnum$ as the template parameter. To keep the code short we just set the primitive root as 3. However, it might be wrong when $mod \neq 998244353$. Here are some commonly used $mods$ and the corresponding primitive root.
$g \to mod$ (max $\log(n)$):
$3 \to$ 104857601 (22), 167772161 (25), 469762049 (26), 998244353 (23), 1004535809 (21);
$10 \to$ 786433 (18);
$31 \to$ 2013265921 (27).
Usage: `const int mod = 998244353;`
```
using Mint = Z<mod>; // Z is modnum struct.
...
FFT<Mint> ntt(3); // use 3 as primitive root.
vector<Mint> as = ntt.conv(vector<Mint>{1, 2, 3},
vector<Mint>{2, 3, 4});
```
Time: $\mathcal{O}(N \log N)$.

51 lines

```
template<class T>
struct FFT {
  const T g; // primitive root.
  vi r;
  int n2;

  FFT(T _g = 3): g(_g) {}

  void dft(vector<T> &a, int is_inv) { // is_inv == 1 ->
    ↪idft.
    rep(i, 1, n2 - 1) if (r[i] > i) swap(a[i], a[r[i]]);
    for(int step = 1; step < n2; step <<= 1) {
      vector<T> w(step);
      T zeta = g.pow((T::getMod() - 1) / (step << 1));
      if (is_inv) zeta = 1 / zeta;

      w[0] = 1;
      rep(i, 1, step - 1) w[i] = w[i - 1] * zeta;
      for (int i = 0; i < n2; i += step << 1) {
        rep(j, 0, step - 1) {
          T tmp = w[j] * a[i + j + step];
          a[i + j + step] = a[i + j] - tmp;
          a[i + j] += tmp;
        }
      }
    }

    if (is_inv == 1) {
      T inv = T{1} / n2;
      rep(i, 0, n2 - 1) a[i] *= inv;
    }
  }

  void pre(int n) { // set n2, r; also used in polynomial
    ↪inverse.
    int len = 0;
    for (n2 = 1; n2 < n; n2 <<= 1) len++;
    r.resize(n2);
    rep(i, 1, n2 - 1) r[i] = (r[i >> 1] >> 1) | ((i & 1) <<
      ↪ (len - 1));
  }

  vector<T> conv(vector<T> a, vector<T> b) {
    int n = sz(a) + sz(b) - 1;
    pre(n);
    a.resize(n2, 0);
    b.resize(n2, 0);
    dft(a, 0); dft(b, 0);
    rep(i, 0, n2 - 1) a[i] *= b[i];
    dft(a, 1);
    a.resize(n);
    return a;
  }
}; // hash-cpp-all = c79d81db99fdb79f856409c48821f21c
```

polynomial.cpp

Description: Basic polynomial struct. Usually we use $modnum$ as template parameter. $inv(k)$ gives the inverse of the polynomial $mod\ x^k$ (by default k is the highest power plus one).

48 lines

```
template<class T>
struct poly: vector<T> {
  using vector<T>::vector; // hash-cpp-1
  poly(const vector<T> &vec): vector<T>(vec) {}

  friend poly& operator *=(poly &a, const poly &b) {
    FFT<T> fft;
    a = fft.conv(a, b);
    return a;
  }
  friend poly operator *(const poly &a, const poly &b) {
    ↪auto c = a; return c *= b; }

  poly inv(int n = 0) const {
    const poly &f = *this;
    assert(sz(f) > 0);
    if (n == 0) n = sz(*this);
    poly res{1 / f[0]};
    FFT<T> fft;
    for (int m = 2; m < n * 2; m <<= 1) {
      poly a(f.begin(), f.begin() + m);
      a.resize(m * 2, 0);
      res.resize(m * 2, 0);
      fft.pre(m * 2);
      fft.dft(a, 0); fft.dft(res, 0);
      rep(i, 0, m * 2 - 1) res[i] = (2 - a[i] * res[i]) *
        ↪res[i];
      fft.dft(res, 1);
      res.resize(m);
    }
    res.resize(n);
    return res;
  } // hash-cpp-1 = 9cecbacfe9d0d397fd8701b6594f8045

  // the following is seldom used.
  friend poly& operator +=(poly &a, const poly &b) { //
     ↪hash-cpp-2
    if (sz(a) < sz(b)) a.resize(sz(b), 0);
    rep(i, 0, sz(b) - 1) a[i] += b[i];
    return a;
  }
  friend poly operator +(const poly &a, const poly &b) {
     ↪auto c = a; return c += b; }

  friend poly& operator -=(poly &a, const poly &b) {
    if (sz(a) < sz(b)) a.resize(sz(b), 0);
    rep(i, 0, sz(b) - 1) a[i] -= b[i];
    return a;
  }
  friend poly operator -(const poly &a, const poly &b) {
     ↪auto c = a; return c -= b; }
// hash-cpp-2 = a4c680e717c3d8a211115bef9fb73e1e
};
```

linear-recurrence-kth-term.cpp

Description: Suppose $a_i = \sum_{j=1}^{d} c_j * a_{i-j}$, then just let $A = \{a_0, ..., a_{d-1}\}$ and $C = \{c_1, ..., c_d\}$.
Here is how it works. Let $Q(x)$ be the characteristic polynomial of our recurrence, and $F(x) = \sum_{i=0}^{\infty} a_i x^i$ be the generating formal power series of our sequence. Then it can be seen that all nonzero terms of $F(x)Q(x)$ are of at most $(n-1)$-st power. This means that $F(x) = P(x)/Q(x)$ for some polynomial $P(x)$. Moreover, we know what $P(x)$ is: it is basically the first n terms of $F(x)Q(x)$, that is, can be found in one multiplication of $a_0 + \ldots + a_{n-1}x^{n-1}$ and $Q(x)$, and then trimming to the proper degree.
Time: $\mathcal{O}\left(d \log^2 d\right)$.

"polynomial.cpp" 26 lines

```
template<class T>
T fps_coeff(poly<T> P, poly<T> Q, ll k) {
  while (k >= sz(Q)) {
    auto nQ(Q);
    rep(i, 0, sz(nQ) - 1) if (i & 1) nQ[i] = 0 - nQ[i];
    auto PQ = P * nQ;
    auto Q2 = Q * nQ;
    poly<T> R, S;
    rep(i, 0, sz(PQ) - 1) if ((k + i) % 2 == 0) R.push_back
      ↪(PQ[i]);
    rep(i, 0, sz(Q2) - 1) if (i % 2 == 0) S.push_back(Q2[i
      ↪]);

    swap(P, R);
    swap(Q, S);
    k >>= 1;
  }
  return (P * Q.inv())[k];
}

template<class T>
T linear_rec_kth(const poly<T> &A, const poly<T> &C, ll k)
   ↪{
  poly<T> Q{1}; // Q is characteristic polynomial.
  for (auto x: C) Q.push_back(0 - x);
  auto P = A * Q;
  P.resize(sz(Q) - 1);
  return fps_coeff(P, Q, k);
} // hash-cpp-all = 320c2d19b585cfcec2a2bd545b5b8d99
```

berlekamp-massey.cpp

Description: Berlekamp Massey algorithm. 64 lines

```
template<int mod>
class BerlekampMassey {
private:
  static ll mPow(ll a, ll k) {
    ll res = 1;
    for (; k; k >>= 1, a = a * a % mod) if (k & 1) res =
       ↪res * a % mod;
    return res;
  }
  static void chadd(int &x, int y) { x += y; if (x >= mod)
     ↪x -= mod; }
  static void chsub(int &x, int y) { x -= y; if (x < 0) x
     ↪+= mod; }
  static void polyMulMod(vi &a, const vi &b, const vi &c) {
    int n = sz(c) - 1;
    revrep(i, 0, n * 2 - 2) {
      int v = 0;
      rep(x, max(0, i + 1 - n), min(n - 1, i)) chadd(v, 1ll
         ↪ * a[x] * b[i - x] % mod);
      a[i] = v;
    }
    revrep(i, n, n * 2 - 2) revrep(j, 0, n) chsub(a[i - j],
       ↪ 1ll * c[j] * a[i] % mod);
  }
  vi nxt, rec, old, seq;
  int t;
  // Given a sequence seq[0], ..., seq[n - 1] \in [0, P),
     ↪finds the minimum t and associated rec[0], ..., rec[
     ↪t] \in [0, P) s.t.
  // 1. rec[0] = 1 (mod P)
  // 2. \sum_{j = 0}^{t} rec[j] seq[i - j] = 0 (mod P) for
     ↪every i \in [t, n)
  // Time complexity: O(nt)
public:
  BerlekampMassey(const vi &s) : nxt(sz(s) + 1, 0), rec(sz(
     ↪s) + 1, 0), old(sz(s) + 1, 0), seq(s), t(0) {
    int old_t = 0, old_i = -1, old_d = 1;
    rec[0] = 1, old[0] = 1;
    rep(i, 0, sz(seq) - 1) {
      int d = s[i];
      rep(j, 1, t) chadd(d, 1ll * rec[j] * seq[i - j] % mod
         ↪);
      if (d == 0) continue;

      int mult = 1ll * d * mPow(old_d, mod - 2) % mod;
      rep(j, 0, t) nxt[j] = rec[j];
      rep(j, 0, old_t) chsub(nxt[j + i - old_i], 1ll * old[
         ↪j] * mult % mod);

      if (t * 2 <= i) {
        old_i = i, old_d = d, old_t = t;
        t = i + 1 - t;
        swap(old, rec);
      }
      swap(rec, nxt);
    }
    rec.resize(t + 1, 0);
  }
  // Returns seq[k], assuming \sum_{j = 0}^{t} rec[j] seq[i
     ↪ - j] = 0 (mod P) holds for i >= n
  // Time complexity: O(t^2 log k)
  int kthTerm(ll k) {
    if (t == 1) return 1ll * seq[0] * mPow((mod - rec[1]) %
       ↪ mod, k) % mod;

    vi cur(t * 2 + 2, 0), mult(t * 2 + 2, 0);
    cur[0] = 1, mult[1] = 1;
    for (; k > 0; k >>= 1) {
      if (k & 1) polyMulMod(cur, mult, rec);
      polyMulMod(mult, mult, rec);
    }
    int res = 0;
    rep(i, 0, t) chadd(res, 1ll * cur[i] * seq[i] % mod);
    return res;
  }
  vi getRec() { return rec; }
}; // hash-cpp-all = 4102b0a04d0946c47c0dd19c6739b09c
```

fast-subset-transform.cpp

Description: Fast Subtset Transform, which is also known as fast zeta transform. Length of a should be a power of 2.
Time: $\mathcal{O}(N \log N)$, where N is the length of a. 10 lines

```
template<class T>
void fst(vector<T> &a, int is_inv) {
  int n = sz(a);
  for (int s = 1; s < n; s <<= 1) {
    rep(i, 0, n - 1) if (i & s) {
      if (is_inv == 0) a[i] += a[i ^ s];
      else a[i] -= a[i ^ s];
    }
  }
} // hash-cpp-all = 06f39b727394293d6d6f6bbf5ac467db
```

subset-convolution.cpp

Description: Subset Convolution of array a and b. Resulting array c satisfies $c_z = \sum_{x,y:\, x|y=z, x\&y=0} a_x \cdot b_y$. Length of a and b should be same and be a power of 2.
Time: $\mathcal{O}\left(N \log^2 N\right)$, where N is the length of a.

"fast-subset-transform.cpp" 22 lines

```
template<class T>
vector<T> SubsetConv(const vector<T> &as, const vector<T> &
   ↪bs) {
  int n = sz(as);
  assert(n > 0 && sz(bs) == n);
  int k = __lg(n);
  vector<vector<T>> ps(k + 1, vector<T>(n)), qs(ps), rs(ps)
     ↪;
  rep(x, 0, n - 1) {
    ps[__builtin_popcount(x)][x] = as[x];
    qs[__builtin_popcount(x)][x] = bs[x];
  }
  for (auto &vec: ps) fst(vec, 0);
  for (auto &vec: qs) fst(vec, 0);
  rep(i, 0, k) rep(j, 0, k - i) {
    rep(x, 0, n - 1) rs[i + j][x] += ps[i][x] * qs[j][x];
  }
  for (auto &vec: rs) fst(vec, 1);
  vector<T> cs(n);
  rep(x, 0, n - 1) {
    cs[x] = rs[__builtin_popcount(x)][x];
  }
  return cs;
} // hash-cpp-all = 79c3cbd63fd24f3ecd9f93c66746f2ac
```

fwht.cpp

Description: Fast Walsh-Hadamard Transform of array a: $fwht(a) = (\sum_i (-1)^{pc(i\&0)} a_i, ..., \sum_i (-1)^{pc(i\&n-1)} a_i)$. One can use it to do xor-convolution. Length of a should be a power of 2.
Time: $\mathcal{O}(N \log N)$, where N is the length of a. 15 lines

```
template<class T>
void fwht(vector<T> &a, int is_inv) {
  int n = sz(a);
  for (int s = 1; s < n; s <<= 1)
    for (int i = 0; i < n; i += s << 1)
      rep(j, 0, s - 1) {
        T x = a[i + j], y = a[i + j + s];
        a[i + j] = x + y;
        a[i + j + s] = x - y;
      }

  if (is_inv) {
    for(auto &x: a) x = x / n;
  }
} // hash-cpp-all = 69be2c88185ff1254f92dea3f228137e
```

fwht-eval.cpp

Description: Let $b = fwt(a)$. One can calculate b_{id} for some index id in $O(N)$ time. Length of a should be a power of 2.
Time: $\mathcal{O}(N)$, where N is the length of a. 10 lines

```
template<class T>
T fwt_eval(const vector<T> &a, int id) {
  int n = sz(a);
  T res = 0;
  rep(i, 0, n - 1) {
    if (__builtin_popcount(i & id) & 1) res -= a[i];
    else res += a[i];
  }
  return res;
} // hash-cpp-all = 3803dcab58e34af9decd2a3be78a5724
```

6.2 Linear Systems

matrix.cpp

Description: Matrix struct. $Gaussian(C)$ eliminates the first C columns and returns the rank of matrix induced by first C columns. $inverse()$ gives the inverse of the matrix. $SolveLinear(A, b)$ solves linear system $Ax = b$ for matrix A and vector b. Besides, you need function $isZero$ for your template T.
Usage: For SolveLinear():

```
bool isZero(double x) { return abs(x) <= 1e-9; } // global
Matrix<double> A(3, 4);
vector<double> b(3);
... // set values for A and b.
vector<double> xs = SolveLinear(A, b);
```

Time: $\mathcal{O}(nm\min\{n, m\})$ for Gaussian, inverse and SolveLinear 98 lines

```
template<class T>
struct Matrix {
  using Mat = Matrix; // hash-cpp-1
  using Vec = vector<T>;

  vector<Vec> a;

  Matrix(int n, int m) {
    assert(n > 0 && m > 0);
    a.assign(n, Vec(m));
  }
  Matrix(const vector<Vec> &a): a(a) {
    assert(sz(a) > 0 && sz(a[0]) > 0);
  }

  Vec& operator [](int i) const { return (Vec&) a[i]; }
// hash-cpp-1 = 273826412c0415697d0c90ccf0130f7c

  Mat operator +(const Mat &b) const {
    int n = sz(a), m = sz(a[0]);
    Mat c(n, m);
    rep(i, 0, n - 1) rep(j, 0, m - 1) c[i][j] = a[i][j] + b
      ↪[i][j];
    return c;
  }

  Mat operator -(const Mat &b) const {
    int n = sz(a), m = sz(a[0]);
    Mat c(n, m);
    rep(i, 0, n - 1) rep(j, 0, m - 1) c[i][j] = a[i][j] - b
      ↪[i][j];
    return c;
  }

  Mat operator *(const Mat &b) const {
    int n = sz(a), m = sz(a[0]), l = sz(b[0]);
    assert(m == sz(b.a));
    Mat c(n, l);
    rep(i, 0, n - 1) rep(k, 0, m - 1) rep(j, 0, l - 1) c[i
      ↪][j] += a[i][k] * b[k][j];
    return c;
  }

  Mat tran() const {
    int n = sz(a), m = sz(a[0]);
    Mat res(m, n);
    rep(i, 0, n - 1) rep(j, 0, m - 1) res[j][i] = a[i][j];
    return res;
  }

  // Eliminate the first C columns, return the rank of
    ↪matrix induced by first C columns.
  int Gaussian(int C) { // hash-cpp-2
    int n = sz(a), m = sz(a[0]), rk = 0;
    assert(C <= m);
    rep(c, 0, C - 1) {
      int id = rk;
      while (id < n && ::isZero(a[id][c])) id++;
      if (id == n) continue;
      if (id != rk) swap(a[id], a[rk]);

      T tmp = a[rk][c];
      for (auto &x: a[rk]) x /= tmp;
      rep(i, 0, n - 1) if (i != rk) {
        T fac = a[i][c];
        rep(j, 0, m - 1) a[i][j] -= fac * a[rk][j];
      }
      rk++;
    }
    return rk;
  } // hash-cpp-2 = 1d0d00b2e87f9e2d7abb939d59db1202

  Mat inverse() const { // hash-cpp-3
    int n = sz(a), m = sz(a[0]);
    assert(n == m);
    auto b = *this;

    rep(i, 0, n - 1) b[i].resize(n * 2, 0), b[i][n + i] =
      ↪1;
    assert(b.Gaussian(n) == n);
    for (auto &row: b.a) row.erase(row.begin(), row.begin()
      ↪ + n);
    return b;
  } // hash-cpp-3 = 7f21877d9ac6d76d755d6b79b03be029

  friend pair<bool, Vec> SolveLinear(Mat A, const Vec &b) {
    ↪ // hash-cpp-4
    int n = sz(A.a), m = sz(A[0]);
    assert(sz(b) == n);
    rep(i, 0, n - 1) A[i].push_back(b[i]);
    int rk = A.Gaussian(m);
    rep(i, rk, n - 1) if (::isZero(A[i].back()) == 0)
      ↪return {0, Vec{}};
    Vec res(m);
    revrep(i, 0, rk - 1) {
      T x = A[i][m];
      int last = -1;
      revrep(j, 0, m - 1) if (::isZero(A[i][j]) == 0) {
        x -= A[i][j] * res[j];
        last = j;
      }
      if (last != -1) res[last] = x;
    }
    return {1, res};
  } // hash-cpp-4 = ca7ea2663b271d600d1d50cb6367eb72
};
```

linear-base.cpp

Description: Maximum weighted of Linear Base of vector space $\mathbb{Z}_2^d$. T is the type of vectors and Z is the type of weights. $w[i]$ is the non-negative weight of a[i]. Keep $w[]$ zero to use unweighted Linear Base.
Time: $\mathcal{O}\left(d \cdot \frac{d}{w}\right)$ for $insert$; $\mathcal{O}\left(d^2 \cdot \frac{d}{w}\right)$ for union; $\mathcal{O}\left(d \cdot \frac{d}{w}\right)$ for $kth()$. 52 lines

```
template<int d, class T = bitset<d>, class Z = int>
struct LB {
  vector<T> a; // hash-cpp-1
  vector<Z> w;

  T& operator [](int i) const { return (T&)a[i]; }
  LB(): a(d), w(d) {}

  // insert x. return 1 if the base is expanded.
  int insert(T x, Z val = 0) {
    revrep(i, 0, d - 1) if (x[i]) {
      if (a[i] == 0) {
        a[i] = x;
        w[i] = val;
        return 1;
      } else if (val > w[i]) {
        swap(a[i], x);
        swap(w[i], val);
      }
      x ^= a[i];
    }
    return 0;
  } // hash-cpp-1 = 18f5fb93fd62247833ec8b725ab4e689

  // View vecotrs as binary numbers. Then calculate the
    ↪minimum number we can get if we add vectors from
    ↪linear base (with weight at least $val$) to $x$.
  T ask_min(T x, Z val = 0) { // hash-cpp-2
    revrep(i, 0, d - 1) {
      if (x[i] && w[i] >= val) x ^= a[i]; // change x[i] to
        ↪ x[i] == 0 to ask maximum value we can get.
    }
    return x;
  } // hash-cpp-2 = 2abeaf37e03b3f853b1ccea025ec88ef

  // Compute the union of two bases.
  friend LB operator +(LB a, const LB &b) { // hash-cpp-3
    rep(i, 0, d - 1) if (b[i] != 0) a.insert(b[i]);
    return a;
  } // hash-cpp-3 = 9e0a459d8f20e3374e28ffb59a38c89e

  // Returns the k-th smallest number spanned by vectors of
    ↪ weight at least $val$. k starts from 0.
  T kth(unsigned long long k, Z val = 0) { // hash-cpp-4
    int N = 0;
    rep(i, 0, d - 1) N += (a[i] != 0 && w[i] >= val);
    if (k >= (1ull << N)) return -1; // return -1 if k is
      ↪too large.
    T res = 0;
    revrep(i, 0, d - 1) if (a[i] != 0 && w[i] >= val) {
      --N;
      auto bit = k >> N & 1;
      if (res[i] != bit) res ^= a[i];
    }
    return res;
  } // hash-cpp-4 = 3d8a0ecfd6a4e4f5ad30dafc3e1b6379
};
```

linear-base-intersect.cpp

Description: Intersection of two unweighted linear bases. T should be of length at least $2d$.
Time: $\mathcal{O}\left(d^2 \cdot \frac{d}{w}\right)$.

"linear-base.cpp" 16 lines

```
template<int d, class T = bitset<d * 2>>
LB<d, T> intersect(LB<d, T> a, const LB<d, T> &b) {
  LB<d, T> res;
  rep(i, 0, d - 1) if (a[i] != 0) a[i][d + i] = 1;
  T msk(string(d, '1'));
  rep(i, 0, d - 1) {
    T x = a.ask_min(b[i]);
    if ((x & msk) != 0) a.insert(x);
    else {
      T y = 0;
      rep(j, 0, d - 1) if (x[d + j]) y ^= a[j];
      res.insert(y & msk);
    }
  }
  return res;
} // hash-cpp-all = ef800af439fc0dc8b3438fa8b7a8af86
```

Z3-vector.cpp

Description: vector in $\mathbb{Z}_3$.
Time: $\mathcal{O}(d/w)$ for +, -, * and /.

45 lines

```
template<int d>
struct v3 {
  bitset<d> a[3]; // hash-cpp-1

  v3() { a[0].set(); }

  void set(int pos, int x) {
    rep(i, 0, 2) a[i][pos] = (i == x);
  }
  int operator [](int i) const {
    if (a[0][i]) return 0;
    else if (a[1][i]) return 1;
    else return 2;
  }
  v3 operator +(const v3 &rhs) const {
    v3 res;
    res.a[0] = (a[0] & rhs.a[0]) | (a[1] & rhs.a[2]) | (a
      ↪[2] & rhs.a[1]);
    res.a[1] = (a[0] & rhs.a[1]) | (a[1] & rhs.a[0]) | (a
      ↪[2] & rhs.a[2]);
    res.a[2] = (~res.a[0] & ~res.a[1]);
    return res;
  }
  v3 operator -(const v3 &rhs) const {
    v3 tmp = rhs;
    swap(tmp.a[1], tmp.a[2]);
    return *this + tmp;
  }
  v3 operator *(int rhs) const {
    if (rhs % 3 == 0) return v3{};
    else {
      auto res = *this;
      if (rhs % 3 == 2) swap(res.a[1], res.a[2]);
      return res;
    }
  }
  v3 operator /(int rhs) const {
    assert(rhs % 3 != 0);
    return *this * rhs;
  } // hash-cpp-1 = 0d5a33ef7c028d641716f6f8a1ebf1b5

  friend string to_string(const v3 &a) {
    string s;
    rep(i, 0, d - 1) s.push_back('0' + a[i]);
    return s;
  }
};
```

simplex.cpp

Description: Solves a general linear maximization problem: maximize $c^\top x$ subject to $Ax \le b$, $x \ge 0$. Returns $\{res, x\}$: $res = 0$ if the program is infeasible; $res = 1$ if there exists an optimal solution; $res = 2$ if the program is unbounded. x is valid only when $res = 1$. T can be **double** or **long double**.
Time: $\mathcal{O}(NM * \#pivots)$, where N is the number of constraints and M is the number of variables.

72 lines

```
template<class T>
pair<int, vector<T>> Simplex(const vector<vector<T>> &A,
   ↪const vector<T> &b, const vector<T> &c) {
  const T eps = 1e-8;

  assert(sz(A) > 0 && sz(A[0]) > 0);
  int n = sz(A);
  int m = sz(A[0]);
  vector<vector<T>> a(n + 1, vector<T>(m + 1));
  rep(i, 0, n - 1) rep(j, 0, m - 1) a[i + 1][j + 1] = A[i][
    ↪j];
  rep(i, 0, n - 1) a[i + 1][0] = b[i];
  rep(j, 0, m - 1) a[0][j + 1] = c[j];

  vi left(n + 1), up(m + 1);
  iota(all(left), m);
  iota(all(up), 0);

  auto pivot = [&](int x, int y) {
    swap(left[x], up[y]);
    T k = a[x][y];
    a[x][y] = 1;
    vi pos;
    rep(j, 0, m) {
      a[x][j] /= k;
      if (fabs(a[x][j]) > eps) pos.push_back(j);
    }
    rep(i, 0, n) {
      if (fabs(a[i][y]) < eps || i == x) continue;

      k = a[i][y];
      a[i][y] = 0;
      for (int j : pos) a[i][j] -= k * a[x][j];
    }
  };

  while (1) {
    int x = -1;
    rep(i, 1, n) if (a[i][0] < -eps && (x == -1 || a[i][0]
      ↪< a[x][0])) {
      x = i;
    }
    if (x == -1) break;

    int y = -1;
    rep(j, 1, m) if (a[x][j] < -eps && (y == -1 || a[x][j]
      ↪< a[x][y])) {
      y = j;
    }
    if (y == -1) return {0, vector<T>{}}; // infeasible
    pivot(x, y);
  }

  while (1) {
    int y = -1;
    rep(j, 1, m) if (a[0][j] > eps && (y == -1 || a[0][j] >
      ↪ a[0][y])) {
      y = j;
    }
    if (y == -1) break;

    int x = -1;
    rep(i, 1, n) if (a[i][y] > eps && (x == -1 || a[i][0] /
      ↪ a[i][y] < a[x][0] / a[x][y])) {
      x = i;
    }
    if (x == -1) return {2, vector<T>{}}; // unbounded
    pivot(x, y);
  }

  vector<T> ans(m);
  rep(i, 1, n) {
    if (1 <= left[i] && left[i] <= m) {
      ans[left[i] - 1] = a[i][0];
    }
  }
  return {1, ans};
} // hash-cpp-all = 1b84e92f161dc13c0d93359656b5b636
```

matroid-intersection.cpp

Description: Given a ground set E and two matroid $M_1 = (E, I_1)$ and $M_2 = (E, I_2)$, compute a largest independent set in their intersection $M = (E, I_1 \cap I_2)$, i.e. an element in $I_1 \cap I_2$ of largest size. Denote by as the ground set. $rebuild(A)$ rebuilds the data structure using elements in A. Then $check1(x)$ returns if $A \cup \{x\} \in I_1$ and $check2$ returns if $A \cup \{x\} \in I_2$ using the data structure just built before.
Time: $\mathcal{O}\left(r^2|E|\right)$, where $r = min(r(E, I_1), r(E, I_2))$.

56 lines

```
template<class T>
vector<T> MatroidIntersection(const vector<T> &as, function
   ↪<void(const vector<T>&)> rebuild, function<bool(const
   ↪T&)> check1, function<bool(const T&)> check2) {
  int n = sz(as);
  vi used(n);
  vector<vi> g;
  vector<T> A;

  auto augment = [&]() {
    int tot = n, s = tot++, t = tot++;
    g.assign(tot, {});
    A.clear();
    rep(i, 0, n - 1) if (used[i]) A.push_back(as[i]);
    rebuild(A);

    rep(y, 0, n - 1) if (used[y] == 0) {
      int cnt = 0;
      if (check1(as[y])) g[s].push_back(y), cnt++;
      if (check2(as[y])) g[y].push_back(t), cnt++;
      if (cnt == 2) { // if we have s -> y -> t, then we
        ↪could just augment via this path!
        used[y] = 1;
        return 1;
      }
    }
    rep(x, 0, n - 1) if (used[x]) {
      A.clear();
      rep(i, 0, n - 1) if (used[i] && i != x) A.push_back(
        ↪as[i]);
      rebuild(A);
      rep(y, 0, n - 1) if (used[y] == 0) {
        if (check1(as[y])) g[x].push_back(y);
        if (check2(as[y])) g[y].push_back(x);
      }
    }
    vi dis(tot, -1), pre(tot);
    vi que{s};
    dis[s] = 0;
    rep(ind, 0, sz(que) - 1) {
      int now = que[ind];
      for (auto v: g[now]) if (dis[v] == -1) {
        dis[v] = dis[now] + 1;
        que.push_back(v);
        pre[v] = now;
      }
    }
    if (dis[t] == -1) return 0;
    int now = pre[t];
    while (now != s) {
      used[now] ^= 1;
      now = pre[now];
    }
    return 1;
  };
  while (augment());
  vector<T> res;
  rep(i, 0, n - 1) if (used[i]) res.push_back(as[i]);
  return res;
}; // hash-cpp-all = 4f3a9cec511193b95a691457a8892ef8
```

6.3 Functions

integrate.cpp

Description: Let $f(x)$ be a continuous function over $[a,b]$ and have a fourth derivative, $f^{(4)}(x)$, over this interval. If M is the maximum value of $|f^{(4)}(x)|$ over $[a,b]$, then the upper bound for the error is $O\left(\frac{M(b-a)^5}{N^4}\right)$.

Time: $\mathcal{O}(N \cdot T)$, where T is the time for evaluating f once. 9 lines

```
template<class T = double>
T SimpsonsRule(const function<T(T)> &f, T a, T b, int N =
  ↪1000) {
  T res = 0;
  T h = (b - a) / (N * 2);
  res += f(a);
  res += f(b);
  rep(i, 1, N * 2 - 1) res += f(a + h * i) * (i & 1 ? 4 :
    ↪2);
  return res * h / 3;
} // hash-cpp-all = defd8926ebf2de40cd1a9e5dc26385c3
```

integrate-adaptive.cpp

Description: Adaptive Simpson's Rule. It is somehow necessary to set the minimum depth of recursion. We use *dep* here. Change it smaller if Time Limit is tight. 14 lines

```
template<class T = double>
T AdaptiveIntegrate(const function<T(T)> &f, T a, T b, T
  ↪eps = 1e-8, int dep = 5) {
  auto simpson = [&](T a, T b) {
    T c = (a + b) / 2;
    return (f(a) + f(c) * 4 + f(b)) * (b - a) / 6;
  };
  auto rec = [&](auto &dfs, T a, T b, T eps, T S, int dep)
    ↪-> T {
    T c = (a + b) / 2;
    T S1 = simpson(a, c), S2 = simpson(c, b), sum = S1 + S2
      ↪;
    if ((abs(sum - S) <= 15 * eps || b - a < 1e-10) && dep
      ↪<= 0) return sum + (sum - S) / 15;
    return dfs(dfs, a, c, eps / 2, S1, dep - 1) + dfs(dfs,
      ↪c, b, eps / 2, S2, dep - 1);
  };
  return rec(rec, a, b, eps, simpson(a, b), dep);
} // hash-cpp-all = c36fe3593b4c741c0e951ea53c574edd
```

recursive-ternary-search.cpp

Description: For convex function $f : \mathbb{R}^d \to \mathbb{R}$, we can approximately find the global minimum using ternary search on each coordinate recursively. d is the dimension; mn, mx record the minimum and maximum possible value of each coordinate (the region you do ternary search); f is the convex function. T can be **double** or **long double**.

Time: $\mathcal{O}\left(\log(1/\epsilon)^d \cdot C\right)$, where C is the time for evaluating the function f. 19 lines

```
template<class T> T RecTS(int d, const vector<T> &mn, const
  ↪ vector<T> &mx, function<T(const vector<T>&)> f) {
  vector<T> xs(d);
  auto dfs = [&](auto &dfs, int dep) {
    if (dep == d) return f(xs);
    T l = mn[dep], r = mx[dep];
    rep(_, 1, 60) { // change here if time is tight.
      T m1 = (l * 2 + r) / 3;
      T m2 = (l + r * 2) / 3;

      xs[dep] = m1; T res1 = dfs(dfs, dep + 1);
      xs[dep] = m2; T res2 = dfs(dfs, dep + 1);
      if (res1 < res2) r = m2;
      else l = m1;
    }
    xs[dep] = (l + r) / 2;
    return dfs(dfs, dep + 1);
  };
  return dfs(dfs, 0);
} // hash-cpp-all = cf72be7d40cc4f7693a87647aae4e6b4
```

Number Theory (7)

7.1 Modular Arithmetic

modnum.cpp

Description: Modular integer with $mod \le 2^{30} - 1$. Note that there are several advantages to use this code: 1. You do not need to keep writing $\%\ mod$; 2. It is good to use this struct when doing Gaussian Elimination / Fast Walsh-Hadamard Transform; 3. Sometimes the input number is greater than mod and this code handles it. Do not write things like $Mint\{1/3\}.pow(10)$ since $1/3$ simply equals 0. Do not write things like $Mint\{a * b\}$ where a and b are int since you might first have integer overflow.

Usage: Define the followings globally:
```
const int mod = 998244353;
using Mint = Z<mod>;
```
34 lines

```
template<const int &mod>
struct Z {
// hash-cpp-1
  int x;
  Z(ll a = 0): x(a % mod) { if (x < 0) x += mod; }
  explicit operator int() const { return x; }

  Z& operator +=(Z b) { x += b.x; if (x >= mod) x -= mod;
    ↪return *this; }
  Z& operator -=(Z b) { x -= b.x; if (x < 0) x += mod;
    ↪return *this; }
  Z& operator *=(Z b) { x = 1ll * x * b.x % mod; return *
    ↪this; }
  friend Z operator +(Z a, Z b) { return a += b; }
  friend Z operator -(Z a, Z b) { return a -= b; }
  friend Z operator *(Z a, Z b) { return a *= b; }
// hash-cpp-1 = e5f2469d533a39d2945e75688e0b7e94

  // the followings are for ntt and polynomials.
  Z pow(ll k) const { // hash-cpp-2
    Z res = 1, a = *this;
    for (; k; k >>= 1, a = a * a) if (k & 1) res = res * a;
    return res;
  }
  Z& operator /=(Z b) {
    assert(b.x != 0);
    return *this *= b.pow(mod - 2);
  }
  friend Z operator /(Z a, Z b) { return a /= b; }
  friend bool operator ==(Z a, Z b) { return a.x == b.x; }
  friend bool operator <(Z a, Z b) { return a.x < b.x; }

  static int getMod() { return mod; } // ntt need this.
// hash-cpp-2 = a71e6c1e407e60880f7d22fd35f9fcab

  friend string to_string(Z a) { return to_string(a.x); }
};
```

mod-sqrt.cpp

Description: Tonelli-Shanks algorithm for modular square roots. Formally, it solves $x^2 \equiv a(mod\ p)$ for prime p and return arbitrary solution if there exists. Usually we use *modnum* as template parameter.

Time: $\mathcal{O}(\log^2 p)$ worst case, often $\mathcal{O}(\log p)$. 16 lines

```
template<class Mint>
pair<bool, Mint> ModSqrt(Mint a) {
  int p = Mint::getMod();
  if (p == 2) return {true, a};
  if (a.pow((p - 1) / 2) == p - 1) return {false, 0};
  if (p % 4 == 3) return {true, a.pow((p + 1) / 4)};
  Mint b = 1;
  while (b.pow((p - 1) / 2) == 1) b += 1;
  int d = (p - 1) / 2, k = 0;
  while (d % 2 == 0) {
    d /= 2;
    k /= 2;
    if (a.pow(d) * b.pow(k) + 1 == 0) k += (p - 1) / 2;
  }
  return {true, a.pow((d + 1) / 2) * b.pow(k / 2)};
} // hash-cpp-all = 8f244ecec7738f76317b55ab798ca9c4
```

mod-log.cpp

Description: BSGS for discrete log. Formally, it solves $a^x \equiv b(\mod p)$ given integer a, b and a prime number p. Returns an solution x if there exists.
Time: $\mathcal{O}\left(\sqrt{p}\log p\right)$. 23 lines

```
template<class Mint>
pair<bool, int> ModLog(Mint a, Mint b) {
  int p = Mint::getMod();
  int sq = sqrt(p) + 0.5;
  while (1ll * sq * sq < p) sq++;
  Mint c = 1;
  vector<pair<Mint, int>> vec;
  rep(i, 1, sq) {
    c *= a;
    vec.emplace_back(b * c, -i);
  }
  sort(all(vec));

  Mint d = 1;
  rep(i, 1, sq) {
    d *= c;
    auto it = lower_bound(all(vec), make_pair(d, -p));
    if (it != vec.end() && it->first == d) {
      return {true, i * sq + it->second};
    }
  }
  return {false, 0};
} // hash-cpp-all = 2de150a4e247c2ec0a46d282e60f4d8e
```

get-primitive-root.cpp

Description: get the smallest primitive root of given integer n, assuming n has primitive roots.
Time: Roughly $\mathcal{O}\left(n^{1/4}\log^2 n\right)$ for $n \le 10^9$. Practically really fast. 32 lines

```
ll getPrimitiveRoot(ll n) {
  auto getps = [](ll x) {
    vector<ll> ps;
    for (ll i = 2; i * i <= x; i++) {
      if (x % i == 0) {
        ps.push_back(i);
        while (x % i == 0) x /= i;
      }
    }
    if (x > 1) ps.push_back(x);
    return ps;
  };
  auto ps = getps(n);
  ll phi = n;
  for (auto p: ps) phi = phi / p * (p - 1);
  auto qs = getps(phi);

  auto check = [&](ll x) {
    if (gcd(x, n) != 1) return 0;
    for (auto p: qs) {
      ll k = phi / p, a = x, res = 1;
      for (; k; k >>= 1, a = (__int128) a * a % n) {
        if (k & 1) res = (__int128) res * a % n;
      }
      if (res == 1) return 0;
    }
    return 1;
  };
  ll a = 1;
  while (check(a) == 0) a++;
  return a;
} // hash-cpp-all = 37f89d5b08432ac9455274dafc50ec12
```

primitive-root-condition.cpp

Description: Check if n has a primitive root. Only 2, 4, p^k and $2p^k$ have primitive roots (where p is some odd prime).
Time: $\mathcal{O}(\log n)$. 14 lines

```
bool hasPrimitiveRoot(ll n) {
  assert(n > 1);
  if (n % 4 == 0) return n == 4;
  if (n % 2 == 0) n /= 2;
  vector<ll> ps;
  for (ll i = 2; i * i <= n; i++) {
    if (n % i == 0) {
      ps.push_back(i);
      while (n % i == 0) n /= i;
    }
  }
  if (n > 1) ps.push_back(n);
  return sz(ps) < 2;
} // hash-cpp-all = 964f5ed68f358c4ecd7622ce0de7944c
```

7.2 Primality

factorization.cpp

Description: Primality test and Fast Factorization. The *mul* function supports $0 \le a, b < c < 7.268 \times 10^{18}$ and is a little bit faster than __int128.
Time: $\mathcal{O}\left(x^{1/4}\right)$ for pollard-rho and same for factorizing x. 64 lines

```
namespace Factorization {
  inline ll mul(ll a, ll b, ll c) { // hash-cpp-1
    ll s = a * b - c * ll((long double) a / c * b + 0.5);
    return s < 0 ? s + c : s;
  }

  ll mPow(ll a, ll k, ll mod) {
    ll res = 1;
    for (; k; k >>= 1, a = mul(a, a, mod)) if (k & 1) res =
      ↪ mul(res, a, mod);
    return res;
  }

  bool miller(ll n) {
    auto test = [&](ll n, int a) {
      if (n == a) return true;
      if (n % 2 == 0) return false;

      ll d = (n - 1) >> __builtin_ctzll(n - 1);
      ll r = mPow(a, d, n);
      while (d < n - 1 && r != 1 && r != n - 1) {
        d <<= 1;
        r = mul(r, r, n);
      }
      return r == n - 1 || d & 1;
    };

    if (n == 2) return 1;
    for (auto p: vi{2, 3, 5, 7, 11, 13}) if (test(n, p) ==
      ↪0) return 0;
    return 1;
  } // hash-cpp-1 = bb239644542d955fdb24ad66508e26d6

  mt19937_64 rng(chrono::steady_clock::now().
    ↪time_since_epoch().count()); // hash-cpp-2
  ll myrand(ll a, ll b) { return uniform_int_distribution<
    ↪ll>(a, b)(rng); }

  ll pollard(ll n) { // return some nontrivial factor of n.
    auto f = [&](ll x) { return ((__int128) x * x + 1) % n;
      ↪ };

    ll x = 0, y = 0, t = 30, prd = 2;
    while (t++ % 40 || gcd(prd, n) == 1) {
      // speedup: don't take __gcd in each iteration.
      if (x == y) x = myrand(2, n - 1), y = f(x);
      ll tmp = mul(prd, abs(x - y), n);
      if (tmp) prd = tmp;
      x = f(x), y = f(f(y));
    }
    return gcd(prd, n);
  }

  vector<ll> factorize(ll n) {
    vector<ll> res;

    auto dfs = [&](auto &dfs, ll x) {
      if (x == 1) return;
      if (miller(x)) res.push_back(x);
      else {
        ll d = pollard(x);
        dfs(dfs, d);
        dfs(dfs, x / d);
      }
    };
    dfs(dfs, n);
    return res;
  } // hash-cpp-2 = 11aa8a52e6d3fb6ce4aa98100d100a3c
}
```

sieve.cpp

Description: Sieve for prime numbers / multiplicative functions in $\{1, 2, \ldots, N\}$ in linear time.
Time: $\mathcal{O}(N)$. 33 lines

```
struct LinearSieve {
  vi ps, minp;
  vi d, facnum, phi, mu;
  LinearSieve(int n): minp(n + 1), d(n + 1), facnum(n + 1),
    ↪ phi(n + 1), mu(n + 1) {
    facnum[1] = phi[1] = mu[1] = 1;
    rep(i, 2, n) {
      if (minp[i] == 0) {
        ps.push_back(i);
        minp[i] = i;
        d[i] = 1;
        facnum[i] = 2;
        phi[i] = i - 1;
        mu[i] = -1;
      }
      for (auto p: ps) {
        ll v = 1ll * i * p;
        if (v > n) break;
        minp[v] = p;
        if (i % p == 0) {
          d[v] = d[i] + 1;
          facnum[v] = facnum[i] / (d[i] + 1) * (d[v] + 1);
          phi[v] = phi[i] * p;
          mu[v] = 0;
          break;
        }
        d[v] = 1;
        facnum[v] = facnum[i] * 2;
        phi[v] = phi[i] * (p - 1);
        mu[v] = -mu[i];
      }
    }
  }
}; // hash-cpp-all = 496b1c3a9df8a550e6022a4573bb36dd
```

7.3 Divisibility

euclidean.cpp

Description: Compute $\sum_{i=1}^{n}\lfloor\frac{ai+b}{c}\rfloor$ for integer numbers a, b, c, n.
Time: $\mathcal{O}(\log c)$.

15 lines

```
template<class T>
T Euclidean(ll a, ll b, ll c, ll n) {
  T res = 0;
  if (a >= c || b >= c) {
    res += T{a / c} * n * (n + 1) / 2;
    res += T{b / c} * (n + 1);
    a %= c;
    b %= c;
  }
  if (a != 0) {
    ll m = ((__int128)a * n + b) / c;
    res += T{m} * n - Euclidean<T>(c, c - b - 1, a, m - 1);
  }
  return res;
} // hash-cpp-all = 05c2bd1a556cb8149508fe555ca3d3f5
```

exgcd.cpp

Description: Solve the integer equation $ax+by = \gcd(a, b)$ for $a, b \geq 0$ and returns x and y such that $|x| \leq b$ and $|y| \leq a$. **Note that** retured value x and y are not guaranteed to be positive!
Time: $\mathcal{O}(\log\max\{a, b\})$.

6 lines

```
template<class T>
pair<T, T> exgcd(T a, T b)  {
  if (b == 0) return {1, 0};
  auto [x, y] = exgcd(b, a % b);
  return {y, x - a / b * y};
} // hash-cpp-all = f1ae06792ef3524ec6f5aff196c54a51
```

chinese.cpp

Description: Chinese Remainder Theorem for solveing equations $x \equiv a_i(mod\ m_i)$ for $i = 0, 1, \ldots, n-1$ such that all m_i-s are pairwise-coprime. Returns a such $x = a + k\cdot(\prod_{i=0}^{n-1} m_i))$, $k \in \mathbb{Z}$ are solutions. **Note that** you need to choose type T to fit $(\prod_i m_i)\cdot(\max_i m_i)$.
Time: $\mathcal{O}\left(n\log(\prod_{i=0}^{n-1} m_i)\right)$.

11 lines

```
template<class T>
T CRT(const vector<T> &as, const vector<T> &ms) {
  T M = 1, res = 0;
  for (auto x: ms) M *= x;
  rep(i, 0, sz(as) - 1) {
    T m = ms[i], Mi = M / m;
    auto [x, y] = exgcd(Mi, m);
    res = (res + as[i] % m * Mi * x) % M;
  }
  return (res + M) % M;
} // hash-cpp-all = 617e5d398d307d9d9399aff7908ae7ed
```

chinese-common.py

30 lines

```
# Author: Yuhao Yao
# Date: 22-10-24
def exgcd(a, b):
  if b == 0:
    return 1, 0
  x, y = exgcd(b, a % b)
  return y, x - a // b * y

# Returned A is the minimum non-negative integer satisfying
  ↪ given two equations.
def merge(a1, m1, a2, m2):
  if m1 == -1 or m2 == -1:
    return -1, -1
  y1, y2 = exgcd(m1, m2)
  g = m1 * y1 + m2 * y2
  if (a2 - a1) % g != 0:
    return -1, -1
  y1 = y1 * ((a2 - a1) // g) % (m2 // g)
  if y1 < 0:
    y1 += m2 // g
  M = m1 // g * m2
  A = m1 * y1 + a1
  return A, M

# Given a list of pairs (a_i, m_i) representing equations x
  ↪ = a_i (mod m_i)
# Return a, m such that a + m * k are solutions. -1, -1
  ↪means that there is no solution.
def general_chinese(ps):
  a, m = 0, 1
  for a2, m2 in ps:
    a, m = merge(a, m, a2, m2)
  return a, m
```

7.4 Others

continued-fraction.cpp

Description: Suppose x_0 satisfies that for $\frac{a}{b} < x_0$, $check(a, b) = 0$ and $\frac{a}{b} \geq x_0$, $check(a, b) = 1$. Then returned pair (p, q) satisfies $check(p, q) = 1$ and $\frac{p}{q} < x_0 + \epsilon$. Function *stop* is used to measure the error: $stop(a_mid, b_mid) = 1$ means we are satisfied with the error.
Time: might be $\mathcal{O}\left(\log^2\left(\frac{1}{\epsilon}\right)\cdot A\right)$, where A is the running time of *check*. Not sure.

24 lines

```
template<class T>
pair<T, T> ContinuedFrac(function<bool(T, T)> check,
  ↪function<bool(T, T)> stop) {
  vector<T> as{0, 1}, bs{1, 0};
  while (stop(as[0] + as[1], bs[0] + bs[1]) == 0) {
    T amid = as[0] + as[1];
    T bmid = bs[0] + bs[1];
    int d = check(amid, bmid);
    T step = 1;
    while (check(as[d] + as[d ^ 1] * step, bs[d] + bs[d ^
      ↪1] * step) == d) {
      as[d] += as[d ^ 1] * step;
      bs[d] += bs[d ^ 1] * step;
      step <<= 1;
      if (stop(as[0] + as[1], bs[0] + bs[1])) break;
    }
    while (step) {
      if (check(as[d] + as[d ^ 1] * step, bs[d] + bs[d ^ 1]
        ↪ * step) == d) {
        as[d] += as[d ^ 1] * step;
        bs[d] += bs[d ^ 1] * step;
      }
      step >>= 1;
    }
  }
  return {as[1], bs[1]};
} // hash-cpp-all = 1f41ae705907cfe85d26336d983d116b
```

Combinatorics (8)

8.1 Formulas

8.1.1 Möbius Inversion

$$g = f \star 1 \Leftrightarrow f = \mu \star g$$

Example:

$$\sum_{d|n}\phi(d) = n \Leftrightarrow \phi(n) = \sum_{d|n}\mu(d)\cdot\frac{n}{d}$$

8.1.2 Binomial Inversion

For $f_0, \ldots, f_n$ and $g_0, \ldots, g_n$:

$$f_i = \sum_{j=0}^{i}\binom{i}{j}g_j, \forall i \Leftrightarrow g_i = \sum_{j=0}^{i}(-1)^{i-j}\binom{i}{j}f_j, \forall i$$

$$f_i = \sum_{j=i}^{n}\binom{j}{i}g_j, \forall i \Leftrightarrow g_i = \sum_{j=i}^{n}(-1)^{j-i}\binom{j}{i}f_j, \forall i$$

8.1.3 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|}\sum_{g\in G}|X^g|,$$

where X^g are the elements fixed by g $(g.x = x)$. If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n}\sum_{k=0}^{n-1}f(\gcd(n, k)) = \frac{1}{n}\sum_{k|n}f(k)\phi(n/k).$$

8.2 Binomials

lucas.cpp

Description: Lucas's theorem: Let n, m be non-negative integers and p be a prime. Write $n = n_kp^k + \ldots + n_1p + n_0$ and $m = m_kp^k + \ldots + m_1p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k}\binom{n_i}{m_i} \pmod p$. It is used when p is not large but n, m are large. Usually we use *modnum* as template parameter.
Time: $\mathcal{O}(p)$ for preprosessing and $\mathcal{O}(\log_p n)$ for one query.

24 lines

```
template<class Mint>
struct Lucas {
  int p;
  vector<Mint> fac, ifac;
  Lucas(int p = Mint::getMod()): p(p), fac(p), ifac(p) {
    fac[0] = 1;
    rep(i, 1, p - 1) fac[i] = fac[i - 1] * i;
    ifac[p - 1] = 1 / fac[p - 1];
    revrep(i, 1, p - 1) ifac[i - 1] = ifac[i] * i;
  }

  template<class T = ll>
  Mint binom(T n, T m) {
    Mint res = 1;
    while (n || m) {
      T a = n % p, b = m % p;
      if (a < b) return 0;
      res *= fac[a] * ifac[b] * ifac[a - b];
      n /= p;
      m /= p;
    }
    return res;
  }
}; // hash-cpp-all = 3a1f01feffc32fab9df199768b786d4a
```

8.3 Numbers

8.3.1 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ \ c(0,0) = 1$$
$$\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\dots(x+n-1)$$

$c(8,k) =$
$8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) =$
$0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots$

8.3.2 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$
$$S(n,1) = S(n,n) = 1$$
$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

8.3.3 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

Geometry (9)

9.1 Geometric Primitives

point.cpp

Description: Class to handle points in 2D-plane. Avoid using T = *int*. 89 lines

```
template<class T>
struct Point {
  using P = Point; // hash-cpp-1
  using type = T;
  static constexpr T eps = 1e-9;
  static constexpr bool isInt = is_integral_v<T>;
  static int sgn(T x) { return (x > eps) - (x < -eps); }
  static int cmp(T x, T y) { return sgn(x - y); }

  T x, y;

  P operator +(P b) const { return P{x + b.x, y + b.y}; }
  P operator -(P b) const { return P{x - b.x, y - b.y}; }
  P operator *(T b) const { return P{x * b, y * b}; }
  P operator /(T b) const { return P{x / b, y / b}; }
  bool operator ==(P b) const { return cmp(x, b.x) == 0 &&
     ↪cmp(y, b.y) == 0; }
  bool operator <(P b) const { return cmp(x, b.x) == 0 ?
     ↪cmp(y, b.y) < 0: x < b.x; }

  T len2() const { return x * x + y * y; }
  T len() const { return sqrt(x * x + y * y); }
  P unit() const {
    if (isInt) return *this; // for long long
    else return len() <= eps ? P{} : *this / len(); // for
       ↪double / long double;
  }

  // dot and cross may lead to big relative error for
     ↪imprecise point when the result is relatively
     ↪smaller than the input magnitude.
  T dot(P b) const { return x * b.x + y * b.y; }
  T cross(P b) const { return x * b.y - y * b.x; }
// hash-cpp-1 = c8725433844eaae342bfd4d1db96a796

  int is_upper() const { return y > eps || (sgn(y) == 0 &&
     ↪x < -eps); } // hash-cpp-2

  // return -1 if a has smaller pollar; return 1 if a has a
     ↪ larger pollar; return 0 o.w.
  // Taking unit makes it slower but it performs as atan2.
  static int argcmp(P a, P b) {
    if (a.is_upper() != b.is_upper()) return a.is_upper() -
       ↪ b.is_upper();
    return sgn(b.cross(a));
  } // hash-cpp-2 = 56a1fd9b2a954e344fa3c7d65e5bfbbb

  P rot90() const { return P{-y, x}; }
  P rot270() const { return P{y, -x}; }

  // Possible precision error:
  // Absolute error is multiplied by the magnitude while
     ↪the resulting coordinates can have 0 as magnitude!
  P rotate(T theta) const { // hash-cpp-3
    P a{cos(theta), sin(theta)};
    return P{x * a.x - y * a.y, x * a.y + y * a.x};
  } // hash-cpp-3 = b7f233d9e27d21a3d96f77e9e270c695

  // Returns the signed distance to line $ab$. $a$, $b$
     ↪should be distinct.
  T dis_to_line(P a, P b) const { // hash-cpp-4
    assert((a - b).len2() > P::eps);
    if (isInt) return (*this - a).cross(b - a);
    else return (*this - a).cross(b - a) / (b - a).len();
  } // hash-cpp-4 = c0d0a82a07ba3cb98ce2fedd4231ff0e

  // Check if it is on line $ab$. $a$, $b$ should be
     ↪distinct.
  bool on_line(P a, P b) const { // hash-cpp-5
    return sgn(dis_to_line(a, b)) == 0;
  } // hash-cpp-5 = 36f390c4825f60ab790c53f9cfedb0f5

  // Returns the signed projected length onto line $ab$.
     ↪Return 0 if $a = b$.
  T project_len(P a, P b) const { // hash-cpp-6
    if (isInt) return (*this - a).dot(b - a);
    else if (a == b) return 0;
    else return (*this - a).dot(b - a) / (b - a).len();
  } // hash-cpp-6 = 1d7efd1f064a813aefd3df1162dda169

  // Calculate the projection to line $ab$. Return $a$ when
     ↪ $a = b$.
  // Only for double / long double.
  P project_to_line(P a, P b) const { // hash-cpp-7
    return a + (b - a).unit() * project_len(a, b);
  } // hash-cpp-7 = 5c70010192791fd0425a2059e613bbd8

  // Returns the distance to line segment $ab$. Safe when
     ↪$a = b$.
  // Only for double / long double.
  T dis_to_seg(P a, P b) const { // hash-cpp-8
    if (project_len(a, b) <= eps) return (*this - a).len();
    if (project_len(b, a) <= eps) return (*this - b).len();
    return fabs(dis_to_line(a, b));
  } // hash-cpp-8 = 447bbe88b5f46abfc682b046da4d57d4

  // Check if it is on segment ab. Safe when a == b.
  bool on_seg(P a, P b) const { // hash-cpp-9
    return dis_to_seg(a, b) <= eps;
  } // hash-cpp-9 = 18db720f414d96f96e122d04fc97b7b5

  friend string to_string(P p) { return "(" + to_string(p.x
     ↪) + ", " + to_string(p.y) + ")"; }
};
```

check-seg-seg-intersection.cpp

Description: check if Segment ab intersects Segment cd. Safe when segments degenerate. Returns 0 if they do not intersect; Returns 1 if they intersect properly; Returns 2 if they intersect o.w. (i.e. intersection is some endpoint). Can be used for long long, double and long double. **Needed function(s): dis_to_line, project_len, dis_to_seg, on_seg.** 13 lines

```
template<class P>
int checkcapSegSeg(P a, P b, P c, P d) {
  auto s1 = a == b ? 0 : P::sgn(c.dis_to_line(a, b));
  auto s2 = a == b ? 0 : P::sgn(d.dis_to_line(a, b));
  auto s3 = c == d ? 0 : P::sgn(a.dis_to_line(c, d));
  auto s4 = c == d ? 0 : P::sgn(b.dis_to_line(c, d));
  if (s1 * s2 < 0 && s3 * s4 < 0) return 1;
  if (c.on_seg(a, b)) return 2;
  if (d.on_seg(a, b)) return 2;
  if (a.on_seg(c, d)) return 2;
  if (b.on_seg(c, d)) return 2;
  return 0;
} // hash-cpp-all = 9523e422d21315b08ec123bfccbde5d6
```

check-ray-seg-intersection.cpp

Description: Check if **Ray** ab intersects Segment cd. ab should **not** degenerate but cd can degenerate. Returns 0 if they do not intersect; Returns 1 if they intersect properly; Returns 2 if they intersect o.w. (i.e. intersection is some endpoint). Can be used for long long, double and long double. Please make sure $a \neq b$. **Needed function(s): dis_to_line, on_line, project_len, dis_to_seg, on_seg.** 13 lines

```
template<class P>
int checkcapRaySeg(P a, P b, P c, P d) {
  assert(!(a == b));
  auto r1 = P::sgn(c.dis_to_line(a, b));
  auto r2 = P::sgn(d.dis_to_line(a, b));
  auto r3 = c == d ? 0 : P::sgn(a.dis_to_line(c, d));
  auto r4 = c == d ? 0 : P::sgn(b.dis_to_line(c, d) - a.
    ↪dis_to_line(c, d));
  if (r1 * r2 < 0 && r3 * r4 < 0) return 1;
  if (c.on_line(a, b) && c.project_len(a, b) >= -P::eps)
    ↪return 2;
  if (d.on_line(a, b) && d.project_len(a, b) >= -P::eps)
    ↪return 2;
  if (a.on_seg(c, d)) return 2;
  return 0;
} // hash-cpp-all = ad25a59ae79558b4f01674ef19bd3369
```

line-line-intersection.cpp

Description: Returns 1 and the intersection point if Line ab and Line cd do not degenerate and they are not parellel. Returns 0 (and an arbitrary point) otherwise. **Only** works for **double** or **long double**. 8 lines

```
template<class P>
pair<bool, P> capLineLine(P a, P b, P c, P d) {
  auto s0 = (a - c).cross(d - c);
  auto s1 = (a - b).cross(d - c);
  if (P::sgn(s1) == 0) return {false, P{}};
  return {true, a + (b - a) * s0 / s1};
}
// hash-cpp-all = 4747ab9ed16ac3e774577c05b7e32622
```

line-line-intersection-dis.cpp

Description: Compute the distance from Point a to the intersection point of Line ab and Line cd. 4 lines

```
template<class P>
auto discapLineLine(P a, P b, P c, P d) {
  return (c - a).cross(d - a) / (b - a).cross(d - c) * (b -
    ↪ a).len();
} // hash-cpp-all = 2cc9d1893b91e14c31e94e657ff39cde
```

closest-pair.cpp

Description: Given n points $p_0, \ldots, p_{n-1}$ on the plane, find the closest pair in euclidean distance. Returns the minimum squared distance.
Time: $\mathcal{O}\left(n \log^2 n\right)$. 25 lines

```
template<class P, class T = typename P::type>
T ClosestPair(vector<P> as) {
  sort(all(as), [](P a, P b) { return a.x < b.x; });
  assert(sz(as) > 1);
  T ans = (as[0] - as[1]).len2();
  auto dfs = [&](auto &dfs, int l, int r) -> void {
    if (l == r) return;
    int mid = (l + r) >> 1;
    dfs(dfs, l, mid);
    dfs(dfs, mid + 1, r);
    vector<P> bs;
    rep(i, l, r) {
      T dx = (as[i] - as[mid]).x;
      if (dx * dx <= ans) bs.push_back(as[i]);
    }
    sort(all(bs), [](P a, P b) { return a.y < b.y; });
    rep(i, 0, sz(bs) - 1) {
      rep(j, i + 1, min(sz(bs) - 1, i + 6)) {
        chmin(ans, (bs[i] - bs[j]).len2());
      }
    }
  };
  dfs(dfs, 0, sz(as) - 1);
  return ans;
} // hash-cpp-all = 04f9377c4561f0f354ccf41acefe3b1b
```

9.2 Polygons

poly-area.cpp

Description: Calculate the signed area of a simple Polygon $poly$. Positive area means counter-clockwise order.
Time: $\mathcal{O}\left(|poly|\right)$. 7 lines

```
template<class T>
T PolyArea(const vector<Point<T>> &poly) {
  if (poly.empty()) return 0;
  T sum = 0;
  rep(i, 0, sz(poly) - 1) sum += (poly[i] - poly[0]).cross(
    ↪poly[(i + 1) % sz(poly)] - poly[0]);
  return sum / 2;
} // hash-cpp-all = 0bd26dcb3506504f4871a9ef776dcbc5
```

poly-center.cpp

Description: Calculate the signed geometry center of a simple Polygon $poly$.
Time: $\mathcal{O}\left(|poly|\right)$. 12 lines

```
template<class P>
P PolyCenter(const vector<P> &poly) {
  auto S = PolyArea(poly);
  if (P::sgn(S) == 0) return P{}; // think twice here.
  P cen{};
  rep(i, 0, sz(poly) - 1) {
    P p = poly[i] - poly[0];
    P q = poly[(i + 1) % sz(poly)] - poly[0];
    cen = cen + (p + q) * (p.cross(q) / (S * 6));
  }
  return cen + poly[0];
} // hash-cpp-all = 5286b8054e36810580b712b418679ec5
```

poly-union-area.cpp

Description: Calculate the area of union of Simple c.c.w Polygons $polys$. Points of each polygon should be distinct. **Needed function(s): dis_to_line, project_len.**
Time: $\mathcal{O}\left(n^2 \log n\right)$, where n is the total number of points in all Polygons. 45 lines

```
template<class P, class T = typename P::type>
T PolyUnionArea(const vector<vector<P>> &polys) {
  T ans = 0;
  rep(ind, 0, sz(polys) - 1) {
    auto &poly = polys[ind];
    rep(i, 0, sz(poly) - 1) {
      P a = poly[i];
      P b = poly[(i + 1) % sz(poly)];

      vector<pair<T, int>> vec{{0, 1}, {1, -1}};
      rep(ind2, 0, sz(polys) - 1) {
        if (ind2 == ind) continue;
        auto &poly2 = polys[ind2];
        rep(j, 0, sz(poly2) - 1) {
          P c = poly2[j];
          P d = poly2[(j + 1) % sz(poly2)];
          int sgn1 = P::sgn(c.dis_to_line(a, b));
          int sgn2 = P::sgn(d.dis_to_line(a, b));
          if (sgn1 == 0 && sgn2 == 0) {
            if (P::sgn((d - c).cross(b - a)) < 0 || i < j)
              ↪{
              auto l = c.project_len(a, b) / (b - a).len();
              auto r = d.project_len(a, b) / (b - a).len();
              if (l > r) swap(l, r);
              vec.emplace_back(l, -1);
              vec.emplace_back(r, 1);
            }
          } else if ((sgn1 < 0) ^ (sgn2 < 0)) {
            vec.emplace_back((c - a).cross(d - a) / (b - a)
              ↪.cross(d - c), sgn1 < 0 ? -1 : 1);
          }
        }
      }
      sort(all(vec));
      int cnt = 0;
      T last = 0;
      for (auto [d, c]: vec) {
        chmax(d, T{0});
        chmin(d, T{1});
        if (cnt > 0) ans += a.cross(b) / 2.0 * (d - last);
        cnt += c;
        last = d;
      }
    }
  }
  return ans;
} // hash-cpp-all = 9acf5fa3fcef2b1ec4b1dcda6c9a77bb
```

check-in-poly.cpp

Description: check if point a is inside / on / outside the given simple (not necessarily convex) Polygon $poly$. Return 0 if outside; 1 if inside; 2 if on the border. $poly$ can be either clockwise or counter-clockwise and should not be self-intersecting. Consecutive collinear points in $poly$ should be fine **but all points should be distinct**. For c.c.w Polygon, $cnt = 2$ indicates strictly inside; for c.w Polygon, $cnt = -2$ indicates strictly inside. **Needed function(s): dis_to_line, project_len, dis_to_seg, on_seg.**
Time: $\mathcal{O}\left(|poly|\right)$. 16 lines

```
template<class P>
int checkinPoly(P a, const vector<P> &poly) {
  int cnt = 0;
  rep(i, 0, sz(poly) - 1) {
    P p = poly[i];
    P q = poly[(i + 1) % sz(poly)];
    if (a.on_seg(p, q)) return 2;

    int sgn1 = P::cmp(a.y, p.y);
    int sgn2 = P::cmp(a.y, q.y);
    if ((sgn2 - sgn1) * P::sgn(a.dis_to_line(p, q)) > 0) {
      cnt -= sgn2 - sgn1;
    }
  }
  return cnt == 0 ? 0 : 1;
} // hash-cpp-all = f908859140b8b07fe94c9c5472e66166
```

check-seg-in-poly.cpp

Description: check if Segment ab is inside the given simple (not necessarily convex) Polygon $poly$, (i.e. no part of the segment is outside the polygon). Return 0 if the segment has part outside the polygon, otherwise 1. $poly$ should be c.c.w and non-self-intersecting. Consecutive collinear points in $poly$ should be fine. If $a = b$, then we need the function **checkinPoly**. **Needed function(s): dis_to_line**.
Time: $\mathcal{O}(|poly| \log |poly|)$.

"check-in-poly.cpp" 30 lines

```
template<class P>
bool checkSeginPoly(P a, P b, const vector<P> &poly) {
  using T = typename P::type;
  if (a == b) return checkinPoly(a, poly) != 0;
  vector<pair<T, int>> res;
  int cnt = -1;
  rep(i, 0, sz(poly) - 1) {
    P p = poly[i];
    P q = poly[(i + 1) % sz(poly)];
    int sgn1 = P::sgn(p.dis_to_line(a, b));
    int sgn2 = P::sgn(q.dis_to_line(a, b));

    if ((sgn2 - sgn1) * P::sgn(a.dis_to_line(p, q)) > 0) {
      int c = sgn2 - sgn1;
      cnt -= c;
      if ((sgn2 - sgn1) * P::sgn(b.dis_to_line(p, q)) < 0)
        ↪{
        if (sgn1 * sgn2 == -1) return 0; // properly
          ↪intersect!
        if (sgn1 == 0) res.emplace_back((p - a).len2(), c);
        if (sgn2 == 0) res.emplace_back((q - a).len2(), c);
      }
    }
  }
  if (cnt == -1) return 0;
  sort(all(res));
  for (auto [_, c]: res) {
    cnt += c;
    if (cnt == -1) return 0;
  }
  return 1;
} // hash-cpp-all = ad41970934bd38d677b5dd4fe23be0dc
```

cut-poly.cpp

Description: Compute the intersection of a non-self-intersecting Polygon $poly$ and a Half Plane ab (i.e. the LHS of ab). The returned Polygon can be self intersecting (or say it can be a collection of separate pieces), so it can only be used for area relating problem. However, the shape is fine if Polygon $poly$ is convex. Only works for double or long double. **Needed function(s): dis_to_line**.
Time: $\mathcal{O}(|poly|)$.

18 lines

```
template<class P>
vector<P> cutPoly(const vector<P> &poly, P a, P b) {
  vector<P> res;
  rep(i, 0, sz(poly) - 1) {
    P p = poly[i];
    P q = poly[(i + 1) % sz(poly)];
    int sgn1 = P::sgn(p.dis_to_line(a, b));
    int sgn2 = P::sgn(q.dis_to_line(a, b));

    if (sgn1 <= 0) res.push_back(p);
    if (sgn1 * sgn2 == -1) {
      auto s0 = (p - a).cross(b - a);
      auto s1 = (p - q).cross(b - a);
      res.push_back(p + (q - p) * s0 / s1);
    }
  }
  return res;
} // hash-cpp-all = 03b8a44dc4e5c993ddd17d3a73708a67
```

poly-line-intersection.cpp

Description: Compute the intersection (Segments) of a non-self-intersecting Polygon $poly$ and a Line ab. Line ab should be non-degenerate. Returned Segments are not sorted in direction ab. Only works for double or long double. **Needed function(s): dis_to_line**.
Time: $\mathcal{O}(|poly| \log |poly|)$.

28 lines

```
template<class P>
vector<pair<P, P>> capPolyLine(const vector<P> &poly, P a,
  ↪P b) {
  using T = typename P::type;
  vector<tuple<T, P, int>> vec;
  rep(i, 0, sz(poly) - 1) {
    P p = poly[i];
    P q = poly[(i + 1) % sz(poly)];
    int sgn1 = P::sgn(p.dis_to_line(a, b));
    int sgn2 = P::sgn(q.dis_to_line(a, b));

    if (sgn1 != sgn2) {
      auto s0 = (p - a).cross(b - a);
      auto s1 = (q - a).cross(b - a);
      T d = (p - b).cross(q - b) / (b - a).cross(q - p) * (
        ↪b - a).len();
      vec.emplace_back(d, (q * s0 - p * s1) / (s0 - s1),
        ↪sgn2 - sgn1);
    }
  }
  sort(all(vec));
  vector<pair<P, P>> res;
  P last{};
  int cnt = -1;
  for (auto [_, p, c]: vec) {
    if (cnt < 0) last = p;
    cnt += c;
    if (cnt < 0) res.emplace_back(last, p);
  }
  return res;
} // hash-cpp-all = 6a4d21af54e97fc649f040dfce8a7a19
```

graham.cpp

Description: Given a set of **distinct** points, compute the Convex Hull of them. By setting $nonStrict = 1$, we also have the points on the border of the Convex Hull. When using double / long double the exact shape of returned Convex Hull might not be trustful (especially for imprecise points), so you should only use it for calculating the area / perimeter?
Time: $\mathcal{O}(|poly| \log |poly|)$.

23 lines

```
template<class P>
vector<P> Graham(vector<P> as, int nonStrict = 0) {
  int n = sz(as);
  if (n <= 1) return as;
  swap(as[0], *min_element(all(as)));
  P o = as[0];
  sort(as.begin() + 1, as.end(), [&](P a, P b) {
    auto res = P::sgn((b - o).cross(a - o));
    return res < 0 || (res == 0 && P::cmp((a - o).len2(), (
      ↪b - o).len2()) < 0);
  });
  vector<P> res{as[0], as[1]};
  rep(i, 2, n - 1) {
    while (sz(res) > 1 && P::sgn((as[i] - res.back()).cross
      ↪(res.back() - res.end()[-2])) >= nonStrict) res.
      ↪pop_back();
    res.push_back(as[i]);
  }
  if (nonStrict && P::sgn((as[1] - o).cross(as[n - 1] - o))
    ↪ != 0) {
    for (int i = n - 2; i >= 1; --i) {
      if (P::sgn((as[i] - o).cross(as[n - 1] - o)) != 0)
        ↪break;
      res.push_back(as[i]);
    }
  }
  return res;
} // hash-cpp-all = 843dacfcca1f42a9177388fa88e6499e
```

minkovski-sum.cpp

Description: Compute the Minkovski Sum of two **Convex Hulls** P and Q. The result is also a **Convex Hull**. Convex Hulls P and Q should **not** have duplicate (same) points while consecutive collinear points are allowed. The returned Convex Hull **may** have collinear points (on the borders), but **no** duplicate points. **Needed function(s): argcmp**.
Time: $\mathcal{O}(|P| + |Q|)$.

21 lines

```
template<class P>
vector<P> MinkovskiSum(vector<P> as, vector<P> bs) {
  auto pre = [](vector<P> &as) {
    auto it = min_element(all(as), [&](P a, P b) {
      return P::cmp(a.y, b.y) != 0 ? a.y > b.y : P::cmp(a.x
        ↪, b.x) < 0;
    });
    rotate(as.begin(), it, as.end());
    int n = sz(as);
    vector<P> res(n);
    rep(i, 0, n - 1) res[i] = as[(i + 1) % n] - as[i];
    return res;
  };
  vector<P> us = pre(as), vs = pre(bs), res(sz(as) + sz(bs)
    ↪);
  merge(all(us), all(vs), res.begin(), [](P a, P b) {
    ↪return P::argcmp(a, b) < 0; });
  P last = as[0] + bs[0];
  for (auto &p: res) {
    p = p + last; // accumulates error here when dealing
      ↪with imprecise points.
    last = p;
  }
  return res;
} // hash-cpp-all = 3fced7a37c051817d22eb9bd75d47a79
```

check-point-in-hull.cpp

Description: Given a c.c.w convex hull $p_0 ... p_{n-1}$, check if Point q is in the hull. $p_0, \cdots, p_{n-1}$ should be distinct points. (It should be fine that 3 of them are collinear.) Returns 0 if Point q is outside the hull; 1 if it is inside the hull; 2 if it is on the border of the hull. **Needed function(s): dis_to_line.**
Time: $\mathcal{O}(\log n)$.

23 lines

```
template<class P>
int PointInHull(const vector<P> &poly, P q) {
  int n = sz(poly);
  if (q.dis_to_line(poly[0], poly[1]) > P::eps) return 0;
  if (q.dis_to_line(poly[0], poly[n - 1]) < -P::eps) return
    ↪ 0;
  int l = 1, r = n;
  while (l < r) {
    int mid = (l + r) >> 1;
    if (q.dis_to_line(poly[0], poly[mid]) > P::eps) r = mid
      ↪;
    else l = mid + 1;
  }
  int id = r - 1;
  if (id == n - 1) {
    return (poly[n - 1] - poly[0]).len2() >= (q - poly[0]).
      ↪len2() ? 2 : 0;
  } else if (id == 1 && q.dis_to_line(poly[0], poly[1]) >=
    ↪-P::eps) {
    return (poly[1] - poly[0]).len2() >= (q - poly[0]).len2
      ↪() ? 2 : 0;
  } else {
    int s = P::sgn(q.dis_to_line(poly[id], poly[id + 1]));
    if (s > 0) return 0;
    else if (s == 0) return 2;
    else return 1;
  }
} // hash-cpp-all = 8da55b0e0aab9e46a263519cc261a3fa
```

check-hull-line-intersection.cpp

Description: Given a c.c.w convex hull $p_0 ... p_{n-1}$ and a vector of lines ls, for each line check if it intersects the hull. $p_0, \cdots, p_{n-1}$ should be distinct points. (It should be fine that 3 of them are collinear.) Returns 0 if the line does not intersect the hull; 1 if it intersects the hull properly; 2 if it passes through exactly a point or an edge of the hull. **Needed function(s): argcmp, dis_to_line.**
Time: $\mathcal{O}(\log n)$.

23 lines

```
template<class P>
vi capHullLine(vector<P> hull, const vector<pair<P, P>> &ls
  ↪) {
  auto it = min_element(all(hull), [&](P a, P b) {
    return P::cmp(a.y, b.y) != 0 ? a.y > b.y : P::cmp(a.x,
      ↪b.x) < 0;
  });
  rotate(hull.begin(), it, hull.end());
  int n = sz(hull);
  vector<P> vs(n);
  rep(i, 0, n - 1) vs[i] = hull[(i + 1) % n] - hull[i];

  vi res;
  for (auto [p, q]: ls) {
    auto dir = q - p;
    auto cmp = [](P a, P b) { return P::argcmp(a, b) < 0;
      ↪};
    int l = (lower_bound(all(vs), dir, cmp) - vs.begin()) %
      ↪ n;
    int r = (lower_bound(all(vs), P{} - dir, cmp) - vs.
      ↪begin()) % n;
    int s1 = P::sgn(hull[l].dis_to_line(p, q));
    int s2 = P::sgn(hull[r].dis_to_line(p, q));
    if (s1 == 0 || s2 == 0) res.push_back(2);
    else res.push_back(s1 != s2);
  }
  return res;
} // hash-cpp-all = 5205f4b317cd130dd518950349534ee4
```

convex-hull-tangent.cpp

Description: Compute the tangent lines of a Point q to c.c.w convex hull $p_0 ... p_{n-1}$. $p_0, \cdots, p_{n-1}$ should be distinct points. (It should be fine that 3 of them are collinear.) q should be strictly outside the convex hull. Returns a pair (l, r) such that edges $p_l p_{l+1}, \cdots, p_{r-1} p_r$ can be strictly seen from Point q. **Needed function(s): dis_to_line.**
Time: $\mathcal{O}(\log n)$.

23 lines

```
template<class P>
pii ConvexHullTangent(const vector<P> &poly, P q) {
  int n = sz(poly);
  auto solve = [&](function<bool(int i, int j)> onright) {
    bool up = onright(0, 1);
    int l = 1, r = n;
    while (l < r) {
      int mid = (l + r) >> 1;
      if (onright(0, mid)) {
        if (up) l = mid + 1;
        else r = mid;
      } else {
        if (onright(mid, (mid + 1) % n)) r = mid;
        else l = mid + 1;
      }
    }
    return l % n;
  };

  int l = solve([&](int i, int j) { return q.dis_to_line(
    ↪poly[i], poly[j]) > P::eps; });
  int r = solve([&](int i, int j) { return q.dis_to_line(
    ↪poly[i], poly[j]) < -P::eps; });
  return {l, r};
} // hash-cpp-all = 79b33ddd95aaa4699a9dfda3a9b59e8b
```

9.3 Circles

circle-circle-intersection.cpp

Description: Compute the intersection points of two circles. For two tangent circles, the tangent point is returned twice in the vector. **Needed function(s): rotate.**

12 lines

```
template<class T, class P = Point<T>>
vector<P> capCircleCircle(P o1, T r1, P o2, T r2) {
  if (P::cmp((o1 - o2).len(), r1 + r2) > 0) return {};
  if (P::cmp(max(r1, r2), min(r1, r2) + (o1 - o2).len()) >
    ↪0) return {};

  T val = (r1 * r1 + (o2 - o1).len2() - r2 * r2) / (2 * r1
    ↪* (o2 - o1).len());
  T theta = acos(max(min(val, T{1}), T{-1}));
  P u = (o2 - o1).unit() * r1;
  P p1 = o1 + u.rotate(-theta);
  P p2 = o1 + u.rotate(theta);
  return {p1, p2};
} // hash-cpp-all = df7fa762c02db9361a369d91cfe91268
```

circle-tangentline.cpp

Description: Compute the tangent points from Point a to Circle (o, r). Return empty vector if a is not outside the given Circle. Only works for double or long double. **Needed function(s): rotate.**

9 lines

```
template<class T, class P = Point<T>>
vector<P> PointCircleTangentPoints(P a, P o, T r) {
  P u = o - a;
  if (P::cmp(u.len2(), r * r) <= 0) return {};
  T d = sqrt(max(u.len2() - r * r, T{0}));
  T theta = asin(min(r / u.len(), T{1}));
  P v = u.unit();
  return {a + v.rotate(-theta) * d, a + v.rotate(theta) * d
    ↪};
} // hash-cpp-all = 5b7554d4d087f53c38dc3fe4cc554ecb
```

circle-circle-outer-tangentline.cpp

Description: Compute the outer two tangent lines of two circles. **Needed function(s): rotate.**

13 lines

```
template<class T, class P = Point<T>>
vector<pair<P, P>> CircleCirlceOuterTagentLine(P o1, T r1,
  ↪P o2, T r2) {
  if (P::cmp(r2, (o1 - o2).len() + r1) >= 0) return {};
  if (P::cmp(r1, (o1 - o2).len() + r2) >= 0) return {};

  T val = (r1 - r2) / (o1 - o2).len();
  T theta = acos(max(min(val, T{1}), T{-1}));
  vector<pair<P, P>> res;
  P v = (o2 - o1).unit();
  res.emplace_back(o1 + v.rotate(theta) * r1, o2 + v.rotate
    ↪(theta) * r2);
  res.emplace_back(o1 + v.rotate(-theta) * r1, o2 + v.
    ↪rotate(-theta) * r2);
  return res;
} // hash-cpp-all = 5039509f992cf65c604f6cd5fe9382b8
```

circumcircle.cpp

Description: Circumcircle of at most three points.

13 lines

```
template<class P, class T = typename P::type>
pair<P, T> circumcircle(const vector<P> &as) {
  assert(sz(as) > 0);
  if (sz(as) == 1) return {as[0], 0};
  else if (sz(as) == 2) return {(as[0] + as[1]) / 2, (as[1]
    ↪ - as[0]).len() / 2};
  else {
    P u = as[1] - as[0], v = as[2] - as[0];
    T r = u.len() * v.len() * (u - v).len() / abs(u.cross(v
      ↪) * 2);
    T B = u.len2(), C = v.len2();
    P w = P{v.y * B - u.y * C, u.x * C - v.x * B} / (u.
      ↪cross(v) * 2);
    return {as[0] + w, r};
  }
} // hash-cpp-all = c10c19ebbeb4a28bd5fe3c1cb3fb2b9e
```

enclosing-circle.cpp

Description: MinimumEnclosingCircle of points as.

"circumcircle.cpp" 17 lines

```
template<class P, class T = typename P::type>
pair<P, T> Welzl(vector<P> as) {
  mt19937_64 rng(chrono::steady_clock::now().
    ↪time_since_epoch().count());
  shuffle(all(as), rng);
  auto dfs = [&](auto &dfs, int n, vector<P> R) -> pair<P,
    ↪T> {
    auto [o, r] = sz(R) > 0 ? circumcircle(R) : pair<P, T>{
      ↪as[0], 0};
    rep(i, 0, n - 1) {
      if (P::cmp((as[i] - o).len(), r) > 0) {
        auto nR = R;
        nR.push_back(as[i]);
        tie(o, r) = dfs(dfs, i, nR);
      }
    }
    return {o, r};
  };
  return dfs(dfs, sz(as), vector<P>{});
} // hash-cpp-all = 50b32b4cce9ae8db25f84855d9b182ab
```

circles-hull-area.cpp

Description: Compute the area of Convex Hull of Union of Circles.
Usage: input *os* and *rs* should have same positive sizes.
Time: $\mathcal{O}(n^3)$, where n is the number of cycles.

"circle-circle-outer-tangentline.cpp", "graham.cpp" 61 lines

```
template<class T, class P = Point<T>>
T CirclesHullArea(const vector<P> &os, const vector<T> &rs)
  ↪ {
  vector<pair<P, T>> cs;
  revrep(i, 0, sz(os) - 1) {
    auto o1 = os[i];
    auto r1 = rs[i];
    int ok = 1;
    for (auto [o2, r2]: cs) if (o1 == o2 && r1 == r2) ok =
      ↪0;
    if (ok) cs.emplace_back(o1, r1);
  }
  vector<P> ps;
  rep(i, 0, sz(cs) - 1) {
    auto [o1, r1] = cs[i];
    rep(j, i + 1, sz(cs) - 1) {
      auto [o2, r2] = cs[j];
      auto tmp = CircleCirlceOuterTagentLine(o1, r1, o2, r2
        ↪);
      for (auto [a, b]: tmp) {
        ps.push_back(a);
        ps.push_back(b);
      }
    }
  }
  vector<P> nps;
  for (auto p: ps) {
    int ok = 1;
    for (auto [o, r]: cs) if (P::cmp((p - o).len(), r) < 0)
      ↪ ok = 0;
    if (ok) nps.push_back(p);
  }
  swap(ps, nps);
  static const T pi = acos(-1.0);
  if (ps.empty()) {
    auto r = *max_element(all(rs));
    return pi * r * r;
  } else {
    auto poly = Graham(ps);
    int n = sz(poly);
    vi ids(n);
    rep(i, 0, n - 1) {
      auto p = poly[i];
      rep(ind, 0, sz(cs) - 1) {
        auto [o, r] = cs[ind];
        if (P::cmp((p - o).len(), r) == 0) ids[i] = ind;
      }
    }
    T ans = 0;
    rep(i, 0, n - 1) {
      if (ids[i] == ids[(i + 1) % n]) {
        int ind = ids[i];
        auto [o, r] = cs[ind];
        auto a = poly[i] - o;
        auto b = poly[(i + 1) % n] - o;
        auto theta = atan2(b.y, b.x) - atan2(a.y, a.x);
        if (P::sgn(theta) < 0) theta += pi * 2;
        ans += theta * r * r / 2;
        ans += (poly[i] - poly[0]).cross(poly[(i + 1) % n]
          ↪- poly[0]) / 2;
        ans -= a.cross(b) / 2;
      } else ans += (poly[i] - poly[0]).cross(poly[(i + 1)
        ↪% n] - poly[0]) / 2;
    }
    return ans;
  }
} // hash-cpp-all = 95ca2393f754f5045caf7779d18f7635
```

circle-seg-intersection.cpp

Description: Compute the intersection points of a Circle and a Segment *ab*. *ab* can be degenerate. Only works for double or long double.
Needed function(s): dis_to_line, project_len, project_to_line, dis_to_seg, on_seg.

16 lines

```
template<class T, class P = Point<T>>
vector<P> capCircleSeg(P o, T r, P a, P b) {
  if (a == b) {
    vector<P> res;
    if ((a - o).len2() == r * r) res.push_back(a);
    return res;
  }
  T d = o.dis_to_line(a, b);
  if (abs(d) > r + P::eps) return {};
  P p = o.project_to_line(a, b), v = (b - a).unit();
  T len = sqrt(max(T{0}, r * r - d * d));
  vector<P> res;
  if ((p + v * len).on_seg(a, b)) res.push_back(p + v * len
    ↪);
  if ((p - v * len).on_seg(a, b)) res.push_back(p - v * len
    ↪);
  return res;
} // hash-cpp-all = 4a9a527c0b4872f0c200660571931d45
```

circle-poly-intersection.cpp

Description: Compute the intersection area of a Circle and a Polygon. Only works for double or long double.

24 lines

```
template<class T, class P = Point<T>>
T capCirclePoly(P o, T r, const vector<P> &poly) {
  auto tri = [&](P p, P q) {
    #define arg(p, q) atan2(p.cross(q), p.dot(q))

    T r2 = r * r;
    P d = q - p;

    if (p == q) return T{};
    T a = d.dot(p) / d.len2(), b = (p.len2() - r2) / d.len2
      ↪();
    T det = a * a - b;
    if (P::sgn(det) <= 0) return arg(p, q) * r2 / 2;

    T s = max(T{0}, -a - sqrt(det)), t = min(T{1}, -a +
      ↪sqrt(det));
    if (t < 0 || 1 < s) return arg(p, q) * r2 / 2;
    P u = p + d * s, v = p + d * t;
    return (p == u ? 0 : arg(p, u) * r2 / 2) + u.cross(v) /
      ↪ 2 + (v == q ? 0 : arg(v, q) * r2 / 2);

    #undef arg
  };
  T sum = 0;
  rep(i, 0, sz(poly) - 1) sum += tri(poly[i] - o, poly[(i +
    ↪ 1) % sz(poly)] - o);
  return sum;
} // hash-cpp-all = 8190150c002f60d579d29eeae2957086
```

9.4 HalfPlanes

halfplane-intersection.cpp

Description: Compute the intersection of Half Planes, which is a Convex hull. A Half Plane is represented by the left hind side of a directed line *ab* (i.e. counter-clockwise). Please make sure the intersection of Half Planes in *ls* is bounded. Also make sure that there is no HalfPlane with direction $dir() = (0, 0)$. If the intersection is empty, then the returned vector has a size at most 2. Otherwise a Convex hull is returned, which has no consecutive collinear points. Only works for **double** and **long double**. **Needed function(s): argcmp, dis_to_line.**

41 lines

```
template<class P> // hash-cpp-1
struct HalfPlane {
  P a, b; // make sure a != b.
  P dir() const { return b - a; }
  bool include(P p) const { return p.dis_to_line(a, b) < -P
    ↪::eps; }
  bool operator <(const HalfPlane &rhs) const {
    return P::argcmp(dir(), rhs.dir()) < 0;
  }
  pair<bool, P> capLL(const HalfPlane &rhs) const {
    auto s0 = (a - rhs.a).cross(rhs.dir());
    auto s1 = (a - b).cross(rhs.dir());
    if (P::sgn(s1) == 0) return {false, P{}};
    return {true, a + (b - a) * s0 / s1};
  }
}; // hash-cpp-1 = 8d7086fa1a8d7c32608ba9f76e7eed51

template<class P, class HP = HalfPlane<P>> // hash-cpp-2
vector<P> HPI(vector<HalfPlane<P>> hps) {
  // please make sure hps is closed.
  auto Samedir = [](HP &r, HP &s) { return (r < s || s < r)
    ↪ == 0; };
  sort(all(hps), [&](HP &r, HP &s) { return Samedir(r, s) ?
    ↪ s.include(r.a) : r < s; });
  // assuming hps is closed then the intersect function
    ↪should be fine.
  auto check = [](HP &w, HP &r, HP &s) {
    auto [res, p] = r.capLL(s);
    if (res == 0) return false; // if r and s are parallel
      ↪then it implies the intersection is empty.
    return w.include(p);
  };

  deque<HP> q;
  rep(i, 0, sz(hps) - 1) {
    if (i && Samedir(hps[i], hps[i - 1])) continue;
    while (sz(q) > 1 && !check(hps[i], q.end()[-2], q.end()
      ↪[-1])) q.pop_back();
    while (sz(q) > 1 && !check(hps[i], q[0], q[1])) q.
      ↪pop_front();
    q.push_back(hps[i]);
  }
  while (sz(q) > 2 && !check(q[0], q.end()[-2], q.end()
    ↪[-1])) q.pop_back();
  while (sz(q) > 2 && !check(q.back(), q[0], q[1])) q.
    ↪pop_front();
  vector<P> res;
  rep(i, 0, sz(q) - 1) res.push_back(q[i].capLL(q[(i + 1) %
    ↪ sz(q)]).second);
  return res;
} // hash-cpp-2 = cbb20be28f672362c72f1636eaa61a79
```