

Eidgenössische Technische Hochschule Zürich

# lETHargy

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adapted from MIT's version of the KTH ACM Contest Template Library 2022-10-18

# Contest (1)

```
template.cpp
#include <bits/stdc++.h>
#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define all(x) x.begin(), x.end()
#define sz(x) (int)(x).size()
using pii = pair<int, int>
using vi = vector<int>
using 1l = long long;

hash.sh
tr -d '[:space:]' | md5sum

hash-cpp.sh
cpp -dD -P -fpreprocessed | tr -d '[:space:]' | md5sum
```

# 1.1 Notes

# 1.1.1 Vscode config

 $"editor.insertSpaces"\colon false,$ 

"window.titleBarStyle": "custom",

"window.customMenuBarAltFocus": false,

Also change the following shortcuts: CopyLineDown, CopyLineUp, cursorLineEnd, cursorLineStart.

# 1.1.2 Implementation Trick

Be cautious about the following:

• \_lg(0) might cause undefined behaviour, same for \_builtin\_ctz and \_builtin\_clz.

# $\underline{\mathrm{Misc}}$ (2)

hilbert-mos.cpp

**Description:** Hilbert curve sorting order for Mo's algorithm. Sorts queries  $(L_i, R_i)$  where  $0 \le L_i \le R_i < n$  into order  $\pi$ , such that  $\sum_i \left| L_{\pi_{i+1}} - L_{\pi_i} \right| + \left| R_{\pi_{i+1}} - R_{\pi_i} \right| = \mathcal{O}(n\sqrt{q})$ 

Usage: hilbertOrder(n, qs) returns  $\pi$  Time:  $\mathcal{O}(N \log N)$ .

# Data structure (3)

fenwick.cpp

**Description:** Fenwick tree with built in binary search. Can be used as a indexed set.

Usage: ??

Time:  $\mathcal{O}(\log N)$ .

35 lines

```
class Fenwick {
 private:
   vector<11> val;
  public:
   Fenwick(int n) : val(n+1, 0) {}
   // Adds v to index i
   void add(int i, ll v) {
      for (++i; i < val.size(); i += i & -i) {
        val[i] += v;
    // Calculates prefix sum up to index i
   11 get(int i) {
     11 \text{ res} = 0;
      for (++i; i > 0; i -= i & -i) {
       res += val[i];
      return res;
   11 get(int a, int b) { return get(b) - get(a-1); }
    // Assuming prefix sums are non-decreasing, finds last
      \hookrightarrow i s.t. get(i) <= v
   int search(ll v) {
      int res = 0;
      for (int h = 1 << 30; h; h >>= 1) {
       if ((res | h) < val.size() && val[res | h] <= v) {
         res |= h;
          v -= val[res];
      return res - 1;
}; // hash-cpp-all = 0d390772acaff4360d0f4d76da45148e
```

```
segtree.cpp
```

**Description:** Segment tree supporting range addition and range sum, minimum queries

Usage: ??

```
Time: \mathcal{O}(\log N).
```

```
58 lines
// Segment tree for range addition, range sum and range
class SegTree {
 private:
    vector<11> sum, minv, tag;
    int h = 1;
    // Returns length of interval corresponding to position
    11 len(int i) { return h >> (31 - builtin clz(i)); }
    void apply(int i, ll v) {
      sum[i] += v * len(i);
      minv[i] += v;
      if (i < h) tag[i] += v;</pre>
    void push(int i) {
      if (tag[i] == 0) return;
      apply(2*i, tag[i]);
      apply(2*i+1, tag[i]);
      tag[i] = 0;
    11 recGetSum(int a, int b, int i, int ia, int ib) {
      if (ib <= a || b <= ia) return 0;
      if (a <= ia && ib <= b) return sum[i];</pre>
      push(i);
      int im = (ia + ib) >> 1;
      return recGetSum(a, b, 2*i, ia, im) + recGetSum(a, b,
         \hookrightarrow 2*i+1, im, ib);
    11 recGetMin(int a, int b, int i, int ia, int ib) {
      if (ib <= a || b <= ia) return 4 * (11)1e18;
      if (a <= ia && ib <= b) return minv[i];</pre>
      push(i);
      int im = (ia + ib) >> 1;
      return min(recGetMin(a, b, 2*i, ia, im), recGetMin(a,
         \hookrightarrow b, 2*i+1, im, ib));
    void recapply (int a, int b, ll v, int i, int ia, int ib
      if (ib <= a || b <= ia) return;
      if (a <= ia && ib <= b) apply(i, v);</pre>
      else (
        push(i);
        int im = (ia + ib) >> 1;
        recApply(a, b, v, 2*i, ia, im);
        recApply(a, b, v, 2*i+1, im, ib);
        sum[i] = sum[2*i] + sum[2*i+1];
        minv[i] = min(minv[2*i], minv[2*i+1]);
 public:
    SegTree(int n) {
```

11 rangeSum(int a, int b) { return recGetSum(a, b+1, 1,

while  $(h < n) h \neq 2;$ 

sum.resize(2\*h, 0);

minv.resize(2\*h, 0);

tag.resize(h, 0);

 $\hookrightarrow$  0, h); }

 $\hookrightarrow$  0, h); }

```
void rangeAdd(int a, int b, ll v) { recApply(a, b+1, v,
       \hookrightarrow 1, 0, h); }
}; // hash-cpp-all = e3e31721068f2f6661b4302da9d50cb9
rmq.cpp
Description: range minimum query data structure with low memory
and fast queries
Usage: ??
Time: \mathcal{O}(N) preprocessing, \mathcal{O}(1) query.
                                                          63 lines
int firstBit(ull x) { return __builtin_ctzll(x); }
int lastBit(ull x) { return 63 - __builtin_clzll(x); }
// O(n) preprocessing, O(1) RMQ data structure.
template<class T>
class RMO {
  private:
    const int H = 6; // Block size is 2^H
    const int B = 1 \ll H;
    vector<T> vec; // Original values
    vector<ull> mins; // Min bits
    vector<int> tbl; // sparse table
    int n, m;
    // Get index with minimum value in range [a, a + len)
       \hookrightarrow for 0 <= 1en <= B
    int getShort(int a, int len) const {
      return a + lastBit(mins[a] & (-1ull >> (64 - len)));
    int minInd(int ia, int ib) const {
      return vec[ia] < vec[ib] ? ia : ib;</pre>
  public:
    RMQ(const vector<T>& vec_) : vec(vec_), mins(vec_.size
       \hookrightarrow ()) {
      n = vec.size();
      m = (n + B-1) >> H;
      // Build sparse table
      int h = lastBit(m) + 1;
      tbl.resize(h*m);
      for (int j = 0; j < m; ++j) tbl[j] = j << H;
      for (int i = 0; i < n; ++i) tbl[i >> H] = minInd(tbl[
         \hookrightarrowi >> H], i);
      for (int j = 1; j < h; ++j) {
        for (int i = j*m; i < (j+1)*m; ++i) {
          int i2 = min(i + (1 << (j-1)), (j+1)*m - 1);
           tbl[i] = minInd(tbl[i-m], tbl[i2-m]);
      // Build min bits
      ull cur = 0:
      for (int i = n-1; i >= 0; --i) {
        for (cur <<= 1; cur > 0; cur ^= cur & -cur) {
          if (vec[i + firstBit(cur)] < vec[i]) break;</pre>
        cur |= 1;
        mins[i] = cur;
    int argmin(int a, int b) const {
      ++b; // to make the range inclusive
      int len = min(b-a, B);
      int ind1 = minInd(getShort(a, len), getShort(b-len,
         \hookrightarrowlen));
```

ll rangeMin(int a, int b) { return recGetMin(a, b+1, 1,

```
int ax = (a >> H) + 1;
      int bx = (b >> H);
      if (ax >= bx) return ind1;
      else {
        int h = lastBit(bx-ax);
        int ind2 = minInd(tbl[h*m + ax], tbl[h*m + bx - (1)]
           \hookrightarrow<< h)1):
        return minInd(ind1, ind2);
    int get(int a, int b) const { return vec[argmin(a, b)];
}; // hash-cpp-all = 3dd48eb5fa928d12b0e5b263ce842625
sparse-table.cpp
Description: Sparse Table.
Time: \mathcal{O}(N \log N) for construction, \mathcal{O}(1) per query.
                                                          19 lines
template < class T, class F = function < T (const T&, const T&)
class SparseTable {
 int n;
 vector<vector<T>> st;
 const F func;
public:
  SparseTable(const vector<T> &a, const F &f): n(sz(a)),
     \hookrightarrow func(f) {
    assert(n > 0);
    st.assign(\underline{lg(n)} + 1, vector<T>(n));
    st[0] = a;
    rep(i, 1, _lg(n)) rep(j, 0, n - (1 << i)) st[i][j] =
        \rightarrow func(st[i - 1][j], st[i - 1][j + (1 << (i - 1))]);
  T ask(int 1, int r) {
    assert(0 <= 1 && 1 <= r && r < n);
    int k = __lg(r - 1 + 1);
    return func(st[k][1], st[k][r - (1 << k) + 1]);
}; // hash-cpp-all = b743d83364ed3febf454197dd9d6aa63
lichao.cpp
Description: Li Chao tree. Given x-coordinates, supports adding lines
and computing minimum Y-coordinate at a given input x-coordinate
Usage: ??
Time: \mathcal{O}(\log N).
struct Line {
 ll a, b;
 11 eval(11 x) const { return a*x + b; }
};
class LiChao {
  private:
    const static 11 INF = 4e18;
    vector<Line> tree; // Tree of lines
    vector<11> xs; // x-coordinate of point i
    int k = 1; // Log-depth of the tree
    int mapInd(int j) const {
      int z = __builtin_ctz(j);
      return ((1 << (k-z)) | (j>>z)) >> 1;
    bool comp(const Line& a, int i, int j) const {
      return a.eval(xs[j]) < tree[i].eval(xs[j]);</pre>
```

public:

```
LiChao(const vector<ll>& points) {
      while(points.size() >> k) ++k;
      tree.resize(1 << k, {0, INF});
      xs.resize(1 << k, points.back());</pre>
      for (int i = 0; i < points.size(); ++i) xs[mapInd(i</pre>
         \hookrightarrow+1)] = points[i];
    void addLine(Line line) {
      for (int i = 1; i < tree.size();) {</pre>
        if (comp(line, i, i)) swap(line, tree[i]);
        if (line.a > tree[i].a) i = 2*i;
        else i = 2 * i + 1;
    11 minVal(int j) const {
      j = mapInd(j+1);
      11 res = INF;
      for (int i = j; i > 0; i /= 2) res = min(res, tree[i
         \hookrightarrow ].eval(xs[i]));
      return res;
}; // hash-cpp-all = 51ad9045bff4d74f5c7b851530e02304
skew-heap.cpp
Description: Skew heap: a priority queue with fast merging
Usage: ??
Time: all operations \mathcal{O}(\log N).
                                                         38 lines
// Skew Heap
class SkewHeap {
  private:
    struct Node {
      11 \text{ val, inc} = 0;
      int ch[2] = \{-1, -1\};
      Node(ll v) : val(v) {}
    vector<Node> nodes;
  public:
    int makeNode(ll v) {
      nodes.emplace_back(v);
      return (int)nodes.size() - 1;
    // Increment all values in heap p by v
    void add(int i, ll v) {
      if (i == -1) return;
      nodes[i].val += v;
      nodes[i].inc += v;
    // Merge heaps a and b
    int merge(int a, int b) {
      if (a == -1 \mid | b == -1) return a + b + 1;
      if (nodes[a].val > nodes[b].val) swap(a, b);
      if (nodes[a].inc) {
        add(nodes[a].ch[0], nodes[a].inc);
        add(nodes[a].ch[1], nodes[a].inc);
        nodes[a].inc = 0;
      swap(nodes[a].ch[0], nodes[a].ch[1]);
      nodes[a].ch[0] = merge(nodes[a].ch[0], b);
      return a:
    pair<int, 11> top(int i) const { return {i, nodes[i].
       →val};
    void pop(int& p) { p = merge(nodes[p].ch[0], nodes[p].
```

 $\hookrightarrow$ ch[1]); }

# fast-prique persistent-segtree 2d-segtree

```
}; // hash-cpp-all = c72cc101090bd3027c2442ee11cee862
```

fast-prique.cpp

**Description:** Struct for priority queue operations on index set [0, n-1]. push(i, v) overwrites value at position i if one already exists. decKey is faster, but does nothing if the new key is smaller than the old one. top and pop can segfault if called on an empty priority queue.

Time:  $\mathcal{O}(\log N)$ . struct Prique { const 11 INF = 4 \* (11)1e18;vector<pair<11, int>> data; const int n; Prique(int siz) : n(siz), data(2\*siz, {INF, -1}) { data  $\hookrightarrow$  [0] = {-INF, -1}; } bool empty() const { return data[1].second >= INF; } pair<11, int> top() const { return data[1]; } void push(int i, ll v) {  $data[i+n] = \{v, (v >= INF ? -1 : i)\};$ for (i += n; i > 1; i >>= 1) data[i>>1] = min(data[i],  $\hookrightarrow$ data[i^1]); void decKey(int i, ll v) { for (int j = i+n; data[j].first > v; j >>= 1) data[j] =  $\hookrightarrow$  {v, i}; pair<ll, int> pop() { auto res = data[1]; push (res.second, INF); return res; }; // hash-cpp-all = 08f397034ba143af3dc3c98b96f9a634

#### persistent-segtree.cpp

Description: Persistent Segment Tree. Point apply and thus no lazy propogation.

Usage: Always define a global apply function to tell segment tree how you apply modification. Combine is set as plus so if you just let T be numerical type then you have range sum in the info and as range query result. To have something different, say rangeMin, define a struct with constructer and + operation.

**Time:**  $\mathcal{O}(\log N)$  per operation.

```
61 lines
template<class Info> class PersistSegtree {
// hash-cpp-1
  struct node { Info info; int ls, rs; };
  int n:
  vector<node> t;
public:
  // node 0 is left as virtual empty node.
  PersistSegtree(int n, int q): n(n), t(1) {
   assert(n > 0);
   t.reserve(q * (__lg(n) + 2) + 1);
  // pointApply returns the id of new root.
  template<class... T>
  int pointApply(int rt, int pos, const T&... val) {
    auto dfs = [&](auto &dfs, int &i, int l, int r) {
      t.push_back(t[i]);
      i = sz(t) - 1;
      ::apply(t[i].info, val...);
```

```
if (1 == r) return;
   int mid = (1 + r) >> 1;
   if (pos <= mid) dfs(dfs, t[i].ls, l, mid);</pre>
   else dfs(dfs, t[i].rs, mid + 1, r);
 dfs(dfs, rt, 0, n-1);
 return rt;
Info rangeAsk(int rt, int gl, int gr) {
 Info res{};
 auto dfs = [&](auto &dfs, int i, int l, int r) {
   if (i == 0 || qr < 1 || r < ql) return;
    if (ql <= 1 && r <= qr) {
      res = res + t[i].info;
      return;
   int mid = (1 + r) >> 1;
   dfs(dfs, t[i].ls, l, mid);
   dfs(dfs, t[i].rs, mid + 1, r);
 dfs(dfs, rt, 0, n-1);
 return res;
} // hash-cpp-1 = 920335506780ce4054d72e2496d81e6c
// lower_bound on prefix sums of difference between two

→ versions.

int lower_bound(int rt_l, int rt_r, Info val) { // hash-
  \hookrightarrow cpp-2
 Info sum{};
 auto dfs = [\&] (auto &dfs, int x, int y, int l, int r) {
   if (1 == r) return sum + t[v].info - t[x].info >= val
       \hookrightarrow ? 1 : 1 + 1;
    int mid = (1 + r) >> 1;
   Info s = t[t[y].ls].info - t[t[x].ls].info;
    if (sum + s >= val) return dfs(dfs, t[x].ls, t[y].ls,
       \hookrightarrow 1, mid);
    else {
      sum = sum + s;
      return dfs(dfs, t[x].rx, t[y].rs, mid + 1, r);
 };
 return dfs(dfs, rt_1, rt_r, 0, n - 1);
} // hash-cpp-2 = 8a719a17e052e3651546ac8d8a122c9c
```

## 2d-segtree.cpp

**Description:** 2D segment tree. Point apply and thus no lazy propoga-

Usage: Always define a global apply function to tell segment tree how you apply modification. Combine is set as plus so if you just let T be numerical type then you have range sum in the info and as range query result. To have something different, say rangeMin, define a struct with constructer and + operation.

78 lines

**Time:**  $\mathcal{O}(\log^2 N)$  per operation.

template<class T> struct SegTree2D { struct iNode { T info; int ls, rs; }; struct oNode { int id; int ls, rs; }; int oL, oR, iL, iR; // change to array to accelerate, since allocating takes  $\hookrightarrow$ time. (saves  $\sim$  200ms when allocating 1e7) vector<iNode> inner; vector<oNode> outer;

```
// node 0 is left as virtual empty node.
SegTree2D(int oL, int oR, int iL, int iR, int g): oL(oL),
   \hookrightarrow oR(oR), iL(iL), iR(iR), inner(1), outer(1) {
  inner.reserve(q * (\underline{\ \ } \lg(oR - oL + 1) + 2) * (\underline{\ \ } \lg(iR -
     \hookrightarrowiL + 1) + 2) + 1);
  outer.reserve(q * (\underline{\ } \lg(oR - oL + 1) + 2) + 1);
int newInner() { inner.push back({}}); return sz(inner) -
int newOuter() { outer.push_back({}); return sz(outer) -
   \hookrightarrow1; }
void pull(int i) {
  auto &[info, ls, rs] = inner[i];
  info = inner[ls].info + inner[rs].info;
void apply(int i, const T &val) {
  ::apply(inner[i].info, val);
// return new root id.
int pointApply(int rt, int op, int ip, const T &val) {
  auto idfs = [&](auto dfs, int &i, int l, int r) {
    if (!i) i = newInner();
    if (1 == r) {
      apply(i, val);
      return:
    int mid = (1 + r) >> 1;
    if (ip <= mid) dfs(dfs, inner[i].ls, l, mid);</pre>
    else dfs(dfs, inner[i].rs, mid + 1, r);
    pull(i);
  auto odfs = [&](auto dfs, int &i, int l, int r) {
    if (!i) i = newOuter();
    idfs(idfs, outer[i].id, iL, iR);
    if (1 == r) return;
    int mid = (1 + r) >> 1;
    if (op <= mid) dfs(dfs, outer[i].ls, l, mid);</pre>
    else dfs(dfs, outer[i].rs, mid + 1, r);
    return;
  odfs (odfs, rt, oL, oR);
  return rt;
T rangeAsk(int rt, int qol, int qor, int qil, int qir) {
  T res():
  auto idfs = [&](auto dfs, int i, int l, int r) {
    if (!i || gir < 1 || r < gil) return;</pre>
    if (gil <= 1 && r <= gir) {
      res = res + inner[i].info;
      return:
    int mid = (1 + r) >> 1;
    dfs(dfs, inner[i].ls, l, mid);
    dfs(dfs, inner[i].rs, mid + 1, r);
  auto odfs = [&](auto dfs, int i, int l, int r) {
    if (!i || qor < 1 || r < qol) return;</pre>
    if (qol <= 1 && r <= qor) {
      idfs(idfs, outer[i].id, iL, iR);
      return;
    int mid = (1 + r) >> 1;
    dfs(dfs, outer[i].ls, l, mid);
    dfs(dfs, outer[i].rs, mid + 1, r);
```

```
};
odfs(odfs, rt, oL, oR);
return res;
};
// hash-cpp-all = 8cacae47df103b5a46ee857150a26646

pq-tree.cpp
// TODO

treap.cpp
// TODO

matrix-seg.cpp
1 lines
```

1 lines

# Graph algorithms (4)

dinic.cpp

**Description:** Dinic algorithm for flow graph G = (V, E).

// TODO: segment tree for historic information

 $\begin{array}{ll} \textbf{Usage:} & \textbf{Always run } MaxFlow(src,sink) \text{ for some } src \text{ and } sink \\ \textbf{first.} & \textbf{Then you can run } getMinCut \text{ to obtain a Minimum Cut} \\ \textbf{(vertices in the same part as } src \text{ are returned).} \end{array}$ 

Time:  $\mathcal{O}\left(|V|^2|E|\right)$  for arbitrary networks.  $\mathcal{O}\left(|E|\sqrt{|V|}\right)$  for bipartite/unit network.  $\mathcal{O}\left(min|V|^(2/3),|E|^(1/2)|E|\right)$  for networks with only unit capacities.

```
template<class Cap = int, Cap Cap_MAX = numeric_limits<Cap</pre>
   \hookrightarrow>::\max()>
struct Dinic {
  int n; // hash-cpp-1
  struct E { int to; Cap a; }; // Endpoint & Admissible
    \hookrightarrow flow.
  vector<E> es;
  vector<vi> q;
  vi dis: // Put it here to get the minimum cut easily.
  Dinic(int n): n(n), q(n) {}
  void addEdge(int u, int v, Cap c, bool dir = 1) {
   g[u].push_back(sz(es)); es.push_back({v, c});
   g[v].push_back(sz(es)); es.push_back({u, dir ? 0 : c});
  Cap MaxFlow(int src, int sink) {
    auto revbfs = [&]() {
      dis.assign(n, -1);
      dis[sink] = 0;
      vi que{sink};
      rep(ind, 0, sz(que) - 1) {
        int now = que[ind];
        for (auto i: g[now]) {
          int v = es[i].to;
          if (es[i ^1].a > 0 && dis[v] == -1) {
            dis[v] = dis[now] + 1;
            que.push_back(v);
            if (v == src) return 1;
      return 0;
```

```
auto dfs = [&] (auto &dfs, int now, Cap flow) {
      if (now == sink) return flow;
      Cap res = 0;
      for (int &ind = cur[now]; ind < sz(g[now]); ind++) {</pre>
       int i = g[now][ind];
        auto [v, c] = es[i];
        if (c > 0 \&\& dis[v] == dis[now] - 1) {
          Cap x = dfs(dfs, v, min(flow - res, c));
          es[i].a -= x;
          es[i ^1].a += x;
        if (res == flow) break;
      return res:
   };
   Cap ans = 0;
   while (revbfs()) {
      cur.assign(n, 0);
      ans += dfs(dfs, src, Cap_MAX);
   return ans;
  } // hash-cpp-1 = 0099c35a07ab0465ecf3ddb9b105db6f
  // Returns a min-cut containing the src.
  vi getMinCut() { // hash-cpp-2
   vi res;
   rep(i, 0, n-1) if (dis[i] == -1) res.push_back(i);
  } // hash-cpp-2 = f8bc377d2af3ac0d3b75bbacb2e4f7e9
  // Gives flow on edge assuming it is directed/undirected.
    \hookrightarrow Undirected flow is signed.
  Cap getDirFlow(int i) { return es[i * 2 + 1].a; }
 Cap getUndirFlow(int i) { return (es[i * 2 + 1].a - es[i
     \hookrightarrow* 21.a) / 2; }
};
```

# costflow-successive-shortest-path.cpp

```
pair<Cost, Cap> mincostflow(int src, int sink, Cap
     \hookrightarrow mx_flow = Cap_MAX) {
    // Run Bellman-Ford first if necessary.
    h.assign(n, Cost_MAX);
    h[src] = 0;
    rep(rd, 1, n) rep(now, 0, n - 1) for (auto i: g[now]) {
      auto [v, c, w] = es[i];
      if (c > 0) h[v] = min(h[v], h[now] + w);
    // Bellman-Ford stops here.
    Cost cost = 0;
    Cap flow = 0;
    while (mx_flow) {
     priority_queue<pair<Cost, int>> pq;
      vector<Cost> dis(n, Cost_MAX);
      dis[src] = 0; pq.emplace(0, src);
      vi pre(n, -1), mark(n, 0);
      while (sz(pq)) {
        auto [d, now] = pq.top(); pq.pop();
        // Using mark[] is safer than compare -d and dis[
           \hookrightarrow now] when the Cost = double.
        if (mark[now]) continue;
        mark[now] = 1;
        for (auto i: g[now]) {
          auto [v, c, w] = es[i];
          Cost off = dis[now] + w + h[now] - h[v];
          if (c > 0 && dis[v] > off) {
            dis[v] = off;
            pq.emplace(-dis[v], v);
            pre[v] = i;
      if (pre[sink] == -1) break;
      rep(i, 0, n - 1) if (dis[i] != Cost_MAX) h[i] += dis[
         \hookrightarrowil;
      Cap aug = mx_flow;
      for (int i = pre[sink]; \sim i; i = pre[es[i ^ 1].to])
         \hookrightarrowaug = min(aug, es[i].a);
      for (int i = pre[sink]; \sim i; i = pre[es[i ^ 1].to]) es
         \hookrightarrow [i].a -= aug, es[i ^ 1].a += aug;
      mx flow -= aug;
      flow += aug;
      cost += aug * h[sink];
    return {cost, flow};
}: // hash-cpp-all = c69ec434ecc34aldb966fd1b901850d2
```

#### link-cut.cpp

1 lines

// TODO

binary-lifting.cpp

**Description:** Compute the sparse table for binary lifting of a tree T. **Time:**  $\mathcal{O}(|V| \log |V|)$  for precalculation and  $\mathcal{O}(\log |V|)$  for each lca query

```
assert (n > 0);
    anc.assign(n, vi(\underline{lg(n)} + 1));
    auto dfs = [&] (auto dfs, int now, int fa) -> void {
      assert (dep[now] == -1); // make sure it is indeed a
      dep[now] = fa == -1 ? 0 : dep[fa] + 1;
      anc[now][0] = fa;
      rep(i, 1, __lg(n)) {
        anc[now][i] = anc[now][i - 1] == -1 ? -1 : anc[anc[
           \hookrightarrownow][i - 1]][i - 1];
      for (auto v: g[now]) if (v != fa) dfs(dfs, v, now);
    dfs(dfs, rt, -1);
  int swim(int x, int h) {
    for (int i = 0; h \&\& x != -1; h >>= 1, i++) {
     if (h \& 1) x = anc[x][i];
   return x;
  int lca(int x, int y) {
   if (dep[x] < dep[y]) swap(x, y);
    x = swim(x, dep[x] - dep[y]);
   if (x == y) return x;
   for (int i = __lg(n); i >= 0; --i) {
      if (anc[x][i] != anc[y][i]) {
        x = anc[x][i];
        y = anc[y][i];
   return anc[x][0];
}; // hash-cpp-all = 1c314be79fc6dee496617d2ec4f13616
```

#### cut-and-bridge.cpp

**Description:** Given an undirected graph G = (V, E), compute all cut vertices and bridges.

```
Time: \mathcal{O}(|V| + |E|).
                                                       31 lines
auto CutAndBridge(int n, const vector<pii> es) {
  vvi q(n);
  rep(i, 0, sz(es) - 1) {
   auto [x, y] = es[i];
   g[x].push_back(i);
   g[y].push_back(i);
  vi cut, bridge, dfn(n, -1), low(n), mark(sz(es));
  int cnt = 0;
  auto dfs = [&] (auto &dfs, int now, int fa) -> void {
   dfn[now] = low[now] = cnt++;
   int sons = 0, isCut = 0;
   for (auto ind: q[now]) if (mark[ind] == 0) {
     mark[ind] = 1;
      auto [x, y] = es[ind];
      int v = now ^x ^y;
      if (dfn[v] == -1) {
       sons++;
        dfs(dfs, v, now);
        low[now] = min(low[now], low[v]);
        if (low[v] == dfn[v]) bridge.push_back(ind);
        if (low[v] >= dfn[now] && fa != -1) isCut = 1;
      } else low[now] = min(low[now], dfn[v]);
   if (fa == -1 && sons > 1) isCut = 1;
   if (isCut) cut.push_back(now);
```

```
rep(i, 0, n - 1) if (dfn[i] == -1) dfs(dfs, i, -1);
 return make_tuple(cut, bridge);
} // hash-cpp-all = c7b8c42c12ad0e48babb6cbda98c1c45
```

vertex-bcc.cpp

Description: Compute the Vertex-BiConnected Components of a graph G = (V, E) (not necessarily connected). Multiple edges and self loops are allowed. id[i] records the index of bcc the edge i is in. top[u]records the second highest vertex (which is unique) in the bcc which vertex u is in.

```
Time: O(|V| + |E|).
struct VertexBCC {
 int n, bcc;
 vi id, top, fa;
 vector<pii> bf; // edges of the block-forest.
  VertexBCC(int n, const vector<pii> &es): n(n), bcc(0), id
    \hookrightarrow (sz(es)), top(n), fa(n, -1) {
   vvi q(n);
   rep(ind, 0, sz(es) - 1) {
     auto [x, y] = es[ind];
     g[x].push_back(ind);
     g[y].push_back(ind);
   int cnt = 0;
   vi dfn(n, -1), low(n), mark(sz(es)), vsta, esta;
   auto dfs = [&](auto dfs, int now) -> void {
     low[now] = dfn[now] = cnt++;
     vsta.push back(now);
      for (auto ind: g[now]) if (mark[ind] == 0) {
       mark[ind] = 1;
       esta.push_back(ind);
       auto [x, y] = es[ind];
        int v = now ^x y;
        if (dfn[v] == -1) {
         dfs(dfs, v);
          fa[v] = now:
          low[now] = min(low[now], low[v]);
          if (low[v] >= dfn[now]) {
           bf.emplace_back(n + bcc, now);
            while (1) {
              int z = vsta.back();
              vsta.pop_back();
              top[z] = v;
              bf.emplace_back(n + bcc, z);
              if (z == v) break;
            while (1) {
              int z = esta.back();
              esta.pop_back();
              id[z] = bcc;
              if (z == ind) break;
           bcc++;
        } else low[now] = min(low[now], dfn[v]);
   };
   rep(i, 0, n - 1) if (dfn[i] == -1) {
     dfs(dfs, i);
     top[i] = i;
 bool SameBcc(int x, int y) {
   if (x == fa[top[y]] \mid | y == fa[top[x]]) return 1;
```

```
else return top[x] == top[y];
 vector<pii> getBlockForest() { return bf; }
}; // hash-cpp-all = 909e9d5a16dbb2ec4031065b0eaabecd
```

edge-bcc.cpp

Description: Compute the Edge-BiConnected Components of a connected graph. Multiple edges and self loops are allowed. Return the size of BCCs and the index of the component each vertex belongs to. Time:  $\mathcal{O}(|E|)$ .

```
auto EdgeBCC(int n, const vector<pii> &es, int st = 0) {
 vi dfn(n, -1), low(n), id(n), mark(sz(es), 0), sta;
 int cnt = 0, bcc = 0;
 vvi q(n);
 rep(ind, 0, sz(es) - 1) {
   auto [x, y] = es[ind];
   g[x].push_back(ind);
   g[y].push_back(ind);
  auto dfs = [&](auto dfs, int now) -> void {
   low[now] = dfn[now] = cnt++;
   sta.push_back(now);
   for (auto ind: g[now]) if (mark[ind] == 0) {
     mark[ind] = 1;
     auto [x, y] = es[ind];
     int v = now ^x y;
     if (dfn[v] == -1) {
       dfs(dfs, v);
       low[now] = min(low[now], low[v]);
      } else low[now] = min(low[now], dfn[v]);
    if (low[now] == dfn[now]) {
      while (sta.back() != now) {
       id[sta.back()] = bcc;
       sta.pop_back();
     id[now] = bcc;
      sta.pop_back();
     bcc++;
  };
 dfs(dfs, st);
 return make_tuple(bcc, id);
} // hash-cpp-all = ea66ad6c614370a1b88363aa23f553cd
```

dsu.cpp

**Description:** Disjoint set union. merge merges components which xand y are in respectively and returns 1 if x and y are in different com-

**Time:** amortized  $\mathcal{O}(\alpha(M,N))$  where M is the number of operations. Almost constant in competitive programming.

```
struct DSU {
 vi fa, siz;
 DSU(int n): fa(n), siz(n, 1) { iota(all(fa), 0); }
 int getcomp(int x) { return fa[x] == x ? x : fa[x] =
    // return 1 if x and y are in different component and
 bool merge(int x, int y) {
   int fx = getcomp(x), fy = getcomp(y);
   if (fx == fy) return 0;
```

```
if (siz[fx] < siz[fy]) swap(fx, fy);
fa[fy] = fx;
siz[fx] += siz[fy];
return 1;
}
}; // hash-cpp-all = d79908e5926d7bd63f242158624be7d7</pre>
```

## undo-dsu.cpp

**Description:** Undoable Disjoint Union Set for set 0, ..., N-1. Use top = top() to get a save point; use undo(top) to go back to the save point.

**Usage:** Fill in struct T, function join as well as choosing proper type (Z) for glob and remember to initialize it. To undo, do in the following way: Dsu dsu(n); ... int top = dsu.top(); ... // do merging here. dsu.undo(top);

Time: Amortized  $\mathcal{O}(\log N)$ .

```
struct UndoDSU {
  using Z = int; // choose some proper type (Z) for global
     \hookrightarrow variable glob.
  struct T {
   int siz:
    // add things you want to maintain here.
   T(int ind = 0): siz(1) {
      // initialize what you add here.
  };
  Z glob;
  void join(T &a, const T& b) {
   a.siz += b.siz;
   // maintain the things you added to struct T.
   // also remember to maintain glob here.
  vi fa:
  vector<T> ts;
  vector<tuple<int, int, T, Z>> sta;
  UndoDSU(int n): fa(n), ts(n) {
   iota(all(fa), 0);
   iota(all(ts), 0);
    // remember initializing glob here.
  int getcomp(int x) {
   while (x != fa[x]) x = fa[x];
   return x;
  bool merge(int x, int y) {
   int fx = getcomp(x), fy = getcomp(y);
   if (fx == fy) return 0;
   if (ts[fx].siz < ts[fy].siz) swap(fx, fy);</pre>
    sta.emplace_back(fx, fy, ts[fx], glob);
   fa[fy] = fx;
    join(ts[fx], ts[fy]);
   return 1:
  int top() { return sz(sta); }
  void undo(int top) {
```

```
while (sz(sta) > top) {
    auto &[x, y, dat, g] = sta.back();
    fa[y] = y;
    ts[x] = dat;
    glob = g;
    sta.pop_back();
}
};
// hash-cpp-all = 4895f51f00e324e4caf81d76afe751f6
```

# centroid-decomposition.cpp

Description: Centroid Decomposition.

Time:  $\mathcal{O}(N \log N)$ .

```
struct CentroidDecomposition {
 // anc[i]: ancestors of vertex i in centroid tree,
     \hookrightarrow including itself.
  // dis[i]: distances from vertex i to ancestors of vertex
     \hookrightarrow i in centroid tree, not necessarily monotone.
  vector<vi> anc, cdis;
  CentroidDecomposition(vector<vi> &g): n(sz(g)), anc(n),
     \hookrightarrowcdis(n) {
    vi siz(n);
    vector<bool> vis(n);
    function<void(int, int)> solve = [&](int _, int tot) {
      int mn = inf, cent = -1;
      function<void(int, int)> getcent = [&](int now, int
         →fa) {
        siz[now] = 1;
        int mx = 0;
        for (auto v: q[now]) if (v != fa && vis[v] == 0) {
          getcent (v, now);
          siz[now] += siz[v];
          mx = max(mx, siz[v]);
        mx = max(mx, tot - siz[now]);
        if (mn > mx) mn = mx, cent = now;
      getcent(, -1); vis[cent] = 1;
      function<void(int, int, int)> dfs = [&](int now, int
         \hookrightarrowfa, int dep) {
        anc[now].pb(cent);
        cdis[now].pb(dep);
        for (auto v: q[now]) if (v != fa && vis[v] == 0)
           \hookrightarrowdfs(v, now, dep + 1);
      dfs(cent, -1, 0);
      // start your work here or inside the function dfs.
      for (auto v: q[cent]) if (vis[v] == 0) solve(v, siz[v
         \hookrightarrow] < siz[cent] ? siz[v] : tot - siz[cent]);
    solve(0, n);
}; // hash-cpp-all = 09f707d97935f6e7de36c112672c8214
```

#### heavy-light-decomposition.cpp

**Description:** Heavy Light Decomposition for a tree T (can be modified easily for forest).

Usage: g should be the adjacent list of the tree T. rt for specifying the root of the tree T (default 0). chainApply(u,v,func,val) and chainAsk(u,v,func) are used for apply / query on the simple path from u to v on tree T. func is the function you want to use to apply / query on a interval. (Say rangeApply / rangeAsk of Segment tree.) Time:  $\mathcal{O}(|T|)$  for building.  $\mathcal{O}(\log N)$  for lca.  $\mathcal{O}(\log |T| \cdot A)$  for chainApply / chainAsk, where A is the running time of func in chainApply / chainAsk.

```
struct HLD {
 int n:
 vi fa, hson, dfn, dep, top;
 HLD(vvi \&g, int rt = 0): n(sz(g)), fa(n, -1), hson(n, -1)
     \hookrightarrow, dfn(n), dep(n, 0), top(n) {
    vi siz(n);
    auto dfs = [&] (auto &dfs, int now) -> void {
      siz[now] = 1;
      int mx = 0;
      for (auto v: g[now]) if (v != fa[now]) {
        dep[v] = dep[now] + 1;
        fa[v] = now;
        dfs(dfs, v);
        siz[now] += siz[v];
        if (mx < siz[v]) {
          mx = siz[v];
          hson[now] = v;
    dfs(dfs, rt);
    int cnt = 0;
    auto getdfn = [&](auto &dfs, int now, int sp) {
      top[now] = sp;
      dfn[now] = cnt++;
      if (hson[now] == -1) return;
      dfs(dfs, hson[now], sp);
      for (auto v: g[now]) {
        if(v != hson[now] && v != fa[now]) dfs(dfs, v, v);
    };
    getdfn(getdfn, rt, rt);
  int lca(int u, int v) {
    while (top[u] != top[v]) {
      if (dep[top[u]] < dep[top[v]]) swap(u, v);</pre>
      u = fa[top[u]];
    if (dep[u] < dep[v]) return u;
    else return v;
  template<class... T>
  void chainApply(int u, int v, const function<void(int,</pre>
     \hookrightarrowint, T...) > &func, const T&... val) {
    int f1 = top[u], f2 = top[v];
    while (f1 != f2) {
      if (dep[f1] < dep[f2]) swap(f1, f2), swap(u, v);</pre>
      func(dfn[f1], dfn[u], val...);
      u = fa[f1]; f1 = top[u];
    if (dep[u] < dep[v]) swap(u, v);</pre>
    func(dfn[v], dfn[u], val...); // change here if you
      \hookrightarrow want the info on edges.
```

60 lines

## 2sat.cpp

**Description:** 2SAT solver, returns if a 2SAT system of V variables and C constraints is satisfiable. If yes, it also gives an assignment.

}; // hash-cpp-all = fed861362ed14d707ccea2d6010bee89

**Usage:** For example, if you want to add clause  $\neg x \lor y$ , just call addClause(x, 0, y, 1);

```
Time: O(|V| + |C|).
                                                       46 lines
struct TwoSat {
 int n;
  vector<vi> e:
  vi ans;
  TwoSat(int n): n(n), e(n * 2), ans(n) {}
  void addClause(int x, bool f, int y, bool g) {
   e[x * 2 + !f].push_back(y * 2 + g);
   e[y * 2 + !q].push_back(x * 2 + f);
  bool satisfiable() {
    vi id(n * 2, -1), dfn(n * 2, -1), low(n * 2, -1), sta;
   int cnt = 0, scc = 0;
    auto dfs = [&](auto &dfs, int now) -> void {
      dfn[now] = low[now] = cnt++;
      sta.push_back(now);
      for (auto v: e[now]) {
        if (dfn[v] == -1) {
          dfs(dfs, v);
          low[now] = min(low[now], low[v]);
        } else if (id[v] == -1) low[now] = min(low[now],
           \hookrightarrowdfn[v]);
      if (low[now] == dfn[now]) {
        while (sta.back() != now) {
          id[sta.back()] = scc;
          sta.pop_back();
        id[sta.back()] = scc;
        sta.pop_back();
        scc++;
    1:
    rep(i, 0, n * 2 - 1) if (dfn[i] == -1) dfs(dfs, i);
    rep(i, 0, n - 1) {
      if (id[i * 2] == id[i * 2 + 1]) return 0;
      ans[i] = id[i * 2] > id[i * 2 + 1];
    return 1;
```

```
vi getAss() { return ans; }
}; // hash-cpp-all = 48021fb8f8e959774f7a861f2f294deb
```

#### hopcroft.cpp

**Description:** Fast bipartite matching for bipartite graph. You can also get a vertex cover of a bipartite graph easily.

```
Time: \mathcal{O}\left(|E|\sqrt{|V|}\right)
```

```
58 lines
```

```
struct Hopcroft {
// hash-cpp-1
 int L, R;
  vi lm, rm; // record the matched vertex for each vertex
     \hookrightarrowon both sides.
  vi ldis, rdis; // put it here so you can get vertex cover
     \hookrightarrow easily.
  Hopcroft(int L, int R, const vector<pii> &es): L(L), R(R)
     \hookrightarrow, lm(L, -1), rm(R, -1) {
    vector<vi> q(L);
    for (auto [x, y]: es) g[x].push_back(y);
    while (1) {
      ldis.assign(L, -1);
      rdis.assign(R, -1);
      bool ok = 0:
      vi que;
      rep(i, 0, L - 1) if (lm[i] == -1) {
        que.push back(i);
        ldis[i] = 0;
      rep(ind, 0, sz(que) - 1) {
        int i = que[ind];
        for (auto j: g[i]) if (rdis[j] == -1) {
          rdis[j] = ldis[i] + 1;
          if (rm[j] != -1) {
            ldis[rm[j]] = rdis[j] + 1;
            que.push_back(rm[j]);
          } else ok = 1:
      if (ok == 0) break;
      vi vis(R); // changing to static does not speed up.
      auto find = [&] (auto &dfs, int i) -> int {
        for (auto j: g[i]) if (vis[j] == 0 && rdis[j] ==
           \hookrightarrowldis[i] + 1) {
          vis[j] = 1;
          if (rm[j] == -1 || dfs(dfs, rm[j])) {
            lm[i] = j;
            rm[j] = i;
            return 1:
        return 0;
      rep(i, 0, L - 1) if (lm[i] == -1) find(find, i);
  } // hash-cpp-1 = 1bdeb27ebf133b92ed0dac89528c768e
  // returns vertices matched to left part, -1 means not
     \hookrightarrow matched.
  vi getMatch() { return lm; }
  pair<vi, vi> vertex_cover() { // hash-cpp-2
```

```
vi lvc, rvc;
   rep(i, 0, L - 1) if (ldis[i] == -1) lvc.push_back(i);
   rep(j, 0, R - 1) if (rdis[j] != -1) rvc.push_back(j);
   return {lvc, rvc};
  } // hash-cpp-2 = 4cfcc7973485543721e0bf5f6f67e3ce
};
```

#### hungarian.cpp

**Description:** Given a complete bipartite graph  $G = (L \cup, R, E)$ , where |L| < |R|, Finds minimum weighted perfect matching of L. Returns the matching.

**Usage:** ws[i][j] is the weight of the edge from i-th vertex in L to i-th vertex in R. Not sure how to choose safe T since I can not give a bound

on values in lp and rp. Seems safe to always use {long long \}.

template<class T = 11, T INF = numeric limits<T>::max()>

```
Time: \mathcal{O}(|L|^2|R|).
```

int i = que[ind];

rep(j, 0, R - 1) {

if (j == lm[i]) continue;

} else rp[j] -= d;

```
vector<pii> Hungarian(const vector<vector<T>> &ws) {
 int L = sz(ws), R = sz(ws[0]);
 vector<T> lp(L), rp(R); // left & right potential
 vi lm(L, -1), rm(R, -1); // left & right match
 rep(i, 0, L - 1) lp[i] = *min_element(all(ws[i]));
 auto step = [&](int src) {
   vi que{src}, pre(R, - 1); // bfs que & back pointers
   vector<T> sa(R, INF); // slack array; min slack from
      \hookrightarrownode in que
   auto extend = [&](int j) {
     if (sa[j] == 0) {
       if (rm[j] == -1) {
         while(j != -1) { // Augment the path
           int i = pre[j];
           rm[j] = i;
           swap(lm[i], j);
         return 1:
       } else que.push_back(rm[j]);
     return 0;
    rep(ind, 0, L - 1) { // BFS to new nodes
```

```
if (sa[j] > off) {
   sa[i] = off;
   pre[j] = i;
   if (extend(j)) return;
if (ind == sz(que) - 1) { // Update potentials
 T d = INF;
 rep(j, 0, R - 1) if (sa[j]) d = min(d, sa[j]);
 bool found = 0;
 for (auto i: que) lp[i] += d;
 rep(j, 0, R - 1) {
   if (sa[j]) {
     sa[j] -= d;
```

if (!found) found |= extend(j);

T off = ws[i][j] - lp[i] - rp[j]; // Slack in edge

47 lines

```
    if (found) return;
}

};

rep(i, 0, L - 1) step(i);

vector<pii> res;
    rep(i, 0, L - 1) res.emplace_back(i, lm[i]);
    return res;
} // hash-cpp-all = 1247de71554b1d4764b16a36de08a191
```

## euler-tour-nonrec.cpp

**Description:** For an edge set E such that each vertex has an even degree, compute Euler tour for each connected component. Note that this is a non-recursive implementation, which avoids stack size issue on some OJ and also saves memory (roughly saves 2/3 of memory) due to that.

Time:  $\mathcal{O}(|V| + |E|)$ . 52 lines

```
struct EulerTour {
 int n;
 vector<vi> tours;
 vi ori;
  EulerTour(int n, const vector<pii> &es, int dir = 0): n(n
    \hookrightarrow), ori(sz(es)) {
   vector<vi> q(n);
   int m = 0:
   for (auto [x, y]: es) {
     g[x].push_back(m);
     if (!dir) q[y].push_back(m);
   vi path, cur(n);
   vector<pii> sta;
   auto solve = [&](int st) {
     sta.emplace_back(st, -1);
      while (sz(sta)) {
        auto [now, pre] = sta.back();
        int fin = 1;
        for (int &i = cur[now]; i < sz(g[now]); ) {</pre>
          auto ind = g[now][i++];
          if (ori[ind]) continue;
          auto [x, y] = es[ind];
          ori[ind] = x == now ? 1 : -1;
          int v = now ^ x ^ y;
          sta.emplace_back(v, ind);
          fin = 0;
          break:
          if (pre != -1) path.push_back(pre);
          sta.pop_back();
   };
   rep(i, 0, n - 1) {
      path.clear();
      solve(i);
      if (sz(path)) {
        reverse (all (path));
        tours.push_back(path);
```

```
}
vector<vi> getTours() { return tours; }
vi getOrient() { return ori; }
}; // hash-cpp-all = b7e06cbd0d08b9923de36919e27d67d8
```

# String algorithms (5)

#### kmp.cpp

**Description:** Compute fail table of pattern string  $s = s_0...s_{n-1}$  in linear time and get all matched positions in text string t in linear time. fail[i] denotes the length of the border of substring  $p_0...p_i$ .

**Usage:** KMP kmp(s) for string s or vector<int> s.

Time:  $\mathcal{O}\left(|p|\right)$  for precalculation and  $\mathcal{O}\left(|p|+|t|\right)$  for matching. <sub>26 lines</sub>

```
template<class T> struct KMP {
 const T s;
 int n;
 vi fail:
  KMP(const T \&s): s(s), n(sz(s)), fail(n) {
   int j = 0;
   rep(i, 1, n - 1) {
     while (j > 0 \&\& s[j] != s[i]) j = fail[j-1];
     if (s[j] == s[i]) j++;
     fail[i] = j;
  // gets all matched (starting) positions.
 vi match(const T &t) {
   int m = sz(t), j = 0;
   vi res(m);
   rep(i, 0, m - 1) {
     while (j > 0 \&\& (j == n || s[j] != t[i])) j = fail[j]

→ - 1];
     if (s[j] == t[i]) j++;
     if (j == n) res[i - n + 1] = 1;
   return res:
}; // hash-cpp-all = 35226020a90976c8bef2bc77416a917c
```

#### z-algo.cpp

**Description:** Given string  $s=s_0...s_{n-1}$ , compute array z where z[i] is the lcp of  $s_0...s_{n-1}$  and  $s_i...s_{n-1}$ . Use function cal(t) (where |t|=m) to calculate the lcp of  $s_0...s_{n-1}$  and  $t_i...t_{m-1}$  for each i.

Usage: zAlgo za(s) for string s or vector<int> s.

**Time:**  $\mathcal{O}\left(|s|\right)$  for precalculation and  $\mathcal{O}\left(|s|+|t|\right)$  for matching. <sub>33 lines</sub>

#### aho-corasick.cpp

**Description:** Aho Corasick Automaton of strings  $s_0, ..., s_{n-1}$ . **Usage:** AhoCorasick<'a', 26> ac; for strings consisting of lowercase letters. Call ac.build() after you insert all strings  $s.0, ..., s.\{n-1\}$ .

```
Time: \mathcal{O}\left(\sum_{i=0}^{n-1}|s_i|\right).
template<char st, int C> struct AhoCorasick {
  struct node {
    int nxt[C];
    int fail;
    int cnt;
    node() {
      memset (nxt, -1, sizeof nxt);
      fail = -1:
      cnt = 0;
  };
  vector<node> t:
  AhoCorasick(): t(1) {}
  int insert(const string &s) {
    int now = 0;
    for (auto ch: s) {
      int c = ch - st;
      if (t[now].nxt[c] == -1) {
        t.emplace_back();
        t[now].nxt[c] = sz(t) - 1;
      now = t[now].nxt[c];
    t[now].cnt++;
    return now;
  void build() {
    vi que{0};
    rep(ind, 0, sz(que) - 1) {
      int now = que[ind], fa = t[now].fail;
      rep(c, 0, C - 1) {
        int &v = t[now].nxt[c];
```

int u = fa == -1 ? 0 : t[fa].nxt[c];

if (v == -1) v = u;

# suffix-array suffix-array-lcp sam general-sam

```
else {
      t[v].fail = u;
      que.push_back(v);
    }
    if (fa != -1) t[now].cnt += t[fa].cnt;
}

}; // hash-cpp-all = 3dca34c2bb5ab364d7abcab29a8c27f4
```

#### suffix-array.cpp

**Description:** Suffix Array for non-cyclic string  $s = s_0...s_{n-1}$ . rank[i] records the rank of the i-th suffix  $s_i...s_{n-1}$ . sa[i] records the starting position of the i-th smallest suffix. h[i] (also called height array or lcp array) records the lcp of the sa[i]-th suffix and the sa[i+1]-th suffix in s.

Time:  $\mathcal{O}(|s| \log |s|)$ .

49 lines

```
struct SA {
 int n;
 vi str, sa, rank, h;
  template < class T > SA(const T &s): n(sz(s)), str(n + 1),
    \hookrightarrowsa(n + 1), rank(n + 1), h(n - 1) {
   auto vec = s;
   sort(all(vec)); vec.erase(unique(all(vec)), vec.end());
   rep(i, 0, n-1) str[i] = rank[i] = lower_bound(all(vec
      \hookrightarrow), s[i]) - vec.begin() + 1;
   iota(all(sa), 0);
   n++;
   for (int len = 0; len < n; len = len ? len * 2 : 1) {
     vi cnt(n + 1);
      for (auto v : rank) cnt[v + 1]++;
     rep(i, 1, n - 1) cnt[i] += cnt[i - 1];
     vi nsa(n), nrank(n);
      for (auto pos: sa) {
       pos -= len;
        if (pos < 0) pos += n;
       nsa[cnt[rank[pos]]++] = pos;
      swap(sa, nsa);
      int r = 0, oldp = -1;
      for (auto p: sa) {
       auto next = [&](int a, int b) { return a + b < n ?</pre>
          \hookrightarrowa + b : a + b - n; };
       if (~oldp) r += rank[p] != rank[oldp] || rank[next(
          nrank[p] = r;
       oldp = p;
      swap(rank, nrank);
   sa = vi(sa.begin() + 1, sa.end());
   rank.resize(--n);
   rep(i, 0, n - 1) rank[sa[i]] = i;
    // compute height array.
    int len = 0:
   rep(i, 0, n - 1) {
     if (len) len--;
     int rk = rank[i];
     if (rk == n - 1) continue;
     while (str[i + len] == str[sa[rk + 1] + len]) len++;
```

```
h[rk] = len;
}
}; // hash-cpp-all = dc03be590b13b29f57b3250dc4634be7
```

## suffix-array-lcp.cpp

**Description:** Suffix Array with sparse table answering lcp of suffices. **Time:**  $\mathcal{O}(|s| \log |s|)$  for construction.  $\mathcal{O}(1)$  per query.

```
"suffix-array.cpp"
                                                           22 lines
struct SA lcp: SA {
 vector<vi> st;
  template < class T > SA_lcp(const T &s): SA(s) {
    assert (n > 0);
    st.assign(\underline{\phantom{a}}lg(n) + 1, vi(n));
    st[0] = h;
    st[0].push\_back(0); // just to make <math>st[0] of size n.
    rep(i, 1, _lg(n)) rep(j, 0, n - (1 << i)) {
      st[i][j] = min(st[i-1][j], st[i-1][j+(1 << (i-1)[j])
         \hookrightarrow 1))1);
  // return lcp(suff_i, suff_j) for i != j.
  int lcp(int i, int j) {
   if (i == n || j == n) return 0;
    assert(i != j);
    int 1 = rank[i], r = rank[j];
    if (1 > r) swap(1, r);
    int k = __lg(r - 1);
   return min(st[k][l], st[k][r - (1 << k)]);</pre>
}; // hash-cpp-all = ff57ad558a18576768e4c3b01e315c93
```

#### sam.cpp

**Description:** Suffix Automaton of a given string s. (Using map to store sons makes it 2 3 times slower but it should be fine in most cases.) len is the length of the longest substring corresponding to the state. fa is the father in the prefix tree. Note that fa[i] < i doesn't hold. occ is 0/1, indicating if the state contains a prefix of the string s. One can do a dfs/bfs to compute for each substring, how many times it occurs in the whole string s. (See function calOccurrence for bfs implementation.) root is set as 0.

**Usage:** Use SAM sam(s) for string s or vector<int> s. **Time:**  $\mathcal{O}(|s|)$ .

```
74 lines
template<class T> struct SAM {
  struct node { // hash-cpp-1
    map<int, int> nxt;
    int fa, len;
    int occ, pos; // # of occurrence (as prefix) & endpos.
   node (int fa = -1, int len = 0): fa(fa), len(len) {
      occ = pos = 0;
  };
 T s;
 int n;
  vector<node> t;
 vi at; // at[i] = the state at which the i-th prefix of s
  SAM(const T \&s): s(s), n(sz(s)), at(n) {
   t.emplace back();
    int last = 0; // create root.
    auto ins = [&](int i, int c) {
     int now = last;
```

```
t.emplace_back(-1, t[now].len + 1);
    last = sz(t) - 1;
    t[last].occ = 1;
    t[last].pos = i;
    at[i] = \overline{last};
    while (now !=-1 \&\& t[now].nxt.count(c) == 0) {
      t[now].nxt[c] = last;
      now = t[now].fa;
    if (now == -1) t[last].fa = 0; // root is 0.
      int p = t[now].nxt[c];
      if (t[p].len == t[now].len + 1) t[last].fa = p;
      else {
        auto tmp = t[p];
        tmp.len = t[now].len + 1;
        tmp.occ = 0; // do not copy occ.
        t.push back(tmp);
        int np = sz(t) - 1;
        t[last].fa = t[p].fa = np;
        while (now != -1 && t[now].nxt.count(c) && t[now
           \hookrightarrow].nxt[c] == p) {
          t[now].nxt[c] = np;
          now = t[now].fa;
  };
  rep(i, 0, n - 1) ins(i, s[i]);
} // hash-cpp-1 = 1c12eb7fbeec418a5befc77214c19b9b
void calOccurrence() { // hash-cpp-2
  vi sum(n + 1), que(sz(t));
  for (auto &it: t) sum[it.len]++;
  rep(i, 1, n) sum[i] += sum[i - 1];
  rep(i, 0, sz(t) - 1) que[--sum[t[i].len]] = i;
  reverse(all(que));
  for (auto now: gue) if (now != 0) t[t[now].fal.occ += t
      \hookrightarrow [now].occ;
 } // hash-cpp-2 = 34e98c4d6ea1e86aa5d52a582becf8a8
vector<vi> ReversedPrefixTree() { // hash-cpp-3
  vector<vi> q(sz(t));
  rep(now, 1, sz(t) - 1) g[t[now].fa].push_back(now);
  rep(now, 0, sz(t) - 1)
    sort(all(g[now]), [&](int i, int j) {
      return s[t[i].pos - t[now].len] < s[t[j].pos - t[</pre>
          →nowl.lenl;
    });
} // hash-cpp-3 = aadc726973415dfaac1e483d8fac558b
```

## general-sam.cpp

**Description:** General Suffix Automaton of a given Trie T. (Using map to store sons makes it 2 3 times slower but it should be fine in most cases. If T is of size  $> 10^6$ , then you should think of using int[] instead of map.) len is the length of the longest substring corresponding to the state. fa is the father in the prefix tree. Note that fa[i] < i doesn't hold. occ should be set manually when building Trie T. root is 0.

Usage: Use GSAM sam(T) for Trie T, where T is of type vector < GSAM :: node >.

Time:  $\mathcal{O}(|T|)$ .

52 lines

```
struct GSAM {
 struct node {
   map<int, int> nxt;
   int fa, len;
   int occ;
   node() \{ fa = -1; len = occ = 0; \}
  vector<node> t;
 GSAM(const vector<node> &trie): t(trie) { // swap(t, trie
    \hookrightarrow) here if TL and ML is tight
   auto ins = [&](int now, int c) {
     int last = t[now].nxt[c];
     t[last].len = t[now].len + 1;
     now = t[now].fa;
      while (now !=-1 \&\& t[now].nxt.count(c) == 0) {
       t[now].nxt[c] = last;
       now = t[now].fa;
      if (now == -1) t[last].fa = 0;
        int p = t[now].nxt[c];
        if (t[p].len == t[now].len + 1) t[last].fa = p;
        else { // clone a node np from node p.
          t.emplace_back();
          int np = sz(t) - 1;
          for (auto [i, v]: t[p].nxt) if (t[v].len > 0) {
           t[np].nxt[i] = v; // use emplace here?
          t[np].fa = t[p].fa;
          t[np].len = t[now].len + 1;
          t[last].fa = t[p].fa = np;
          while (now != -1 && t[now].nxt.count(c) && t[now
             \hookrightarrow].nxt[c] == p) {
            t[now].nxt[c] = np;
            now = t[now].fa;
   };
   vi que{0};
   rep(ind, 0, sz(que) - 1) {
     int now = que[ind];
     vi cs;
      for (auto [c, v]: t[now].nxt) {
        cs.push_back(c);
        que.push_back(v);
      for (auto c: cs) ins(now, c);
}; // hash-cpp-all = add4c78221df38584b76536f66703db7
```

#### manacher.cpp

**Description:** Manacher Algorithm for finding all palindrome subtrings of  $s = s_0...s_{n-1}$ . s can actually be string or vector (say vector $\lt$ int $\gt$ ). For returned vector len, len[i\*2] = r means that  $s_{i-r+1}...s_{i+r-1}$  is the maximal palindrome centered at position i. For returned vector len, len[i\*2+1] = r means that  $s_{i-r+1}...s_{i+r}$  is the maximal palindrome centered between position i and i+1.

Time:  $\mathcal{O}(|s|)$ .

```
vi Manacher(const T &s) {
  int n = sz(s), j = 0;
```

#### palindrome-tree.cpp

**Description:** Given string  $s = s_0...s_{n-1}$ , build the palindrom tree (automaton) for s. Each state of the automaton corresponds to a palindrome substring of s. Note that t[i].fa < i holds.

```
struct PalindromeTree {
 struct node {
   map<int, int> nxt;
   int fail, len;
   int cnt;
   node(int fail, int len): fail(fail), len(len) {
     cnt = 0;
  };
 vector<node> t;
  template<class T>
  PalindromeTree(const T &s) {
   int n = sz(s);
   t.emplace back(-1, -1); // Odd root -> state 0.
   t.emplace_back(0, 0); // Even root -> state 1.
   int now = 0:
   auto ins = [&](int pos) {
     auto get = [&](int i) {
       while (pos == t[i].len \mid \mid s[pos - 1 - t[i].len] !=
           \hookrightarrows[pos]) i = t[i].fail;
       return i;
     };
     int c = s[pos];
     now = get(now);
     if (t[now].nxt.count(c) == 0) {
       int q = now == 0 ? 1 : t[get(t[now].fail)].nxt[c];
       t.emplace_back(q, t[now].len + 2);
       t[now].nxt[c] = sz(t) - 1;
     now = t[now].nxt[c];
     t[now].cnt++;
   };
   rep(i, 0, n - 1) ins(i);
}; // hash-cpp-all = ca74a23e6dec05d3f4328aa98fd3d4d3
```

# hash-struct.cpp

12 lines

**Description:** Hash struct. 1000000007 and 1000050131 are good moduli.

```
template<int m1, int m2>
struct Hash {
  int x, y;
  Hash(l1 a, l1 b): x(a % m1), y(b % m2) {
    if (x < 0) x += m1;
    if (y < 0) y += m2;
  }
}</pre>
```

```
Hash(11 a = 0): Hash(a, a) \{ \}
       using H = Hash;
       static int norm(int x, int mod) { return x \ge mod ? x - mod ? x = mod ? x =
                  \hookrightarrow mod : x < 0 ? x + mod : x; }
        friend H operator +(H a, H b) \{ a.x = norm(a.x + b.x, m1) \}
                  \hookrightarrow; a.y = norm(a.y + b.y, m2); return a; }
        friend H operator - (H a, H b) { a.x = norm(a.x - b.x, m1)
                   \hookrightarrow; a.y = norm(a.y - b.y, m2); return a; }
        friend H operator *(H a, H b) { return H{111 * a.x * b.x,
                  \hookrightarrow 111 * a.y * b.y}; }
        friend bool operator == (H a, H b) { return tie(a.x, a.y)
                  \Rightarrow== tie(b.x, b.y); }
        friend bool operator !=(H a, H b) { return tie(a.x, a.y)
                  \hookrightarrow != tie(b.x, b.y); }
       friend bool operator <(H a, H b) { return tie(a.x, a.y) <</pre>
                 \hookrightarrow tie(b.x, b.y); }
}; // hash-cpp-all = ff126b1c842614ecc3db2080807d765e
```

#### de-bruijin.cpp

// TODO

#### lyndon.cpp

1 lines

1 lines

// TODO

# <u>Math</u> (6)

#### simplex.cpp

**Description:** Solves a general linear maximization problem: maximize  $c^T x$  subject to  $Ax \leq b$ ,  $x \geq 0$ . Returns  $\{res, x\}$ : res = 0 if the program is infeasible; res = 1 if there exists an optimal solution; res = 2 if the program is unbounded. x is valid only when res = 1.

**Time:**  $\mathcal{O}(NM * \#pivots)$ , where N is the number of constraints and M is the number of variables.

```
template<class T>
pair<int, vector<T>> Simplex(const vector<vector<T>> &A,
  const T eps = 1e-8;
  int n = sz(A);
 int m = sz(A[0]);
  vector < vector < T >> a(n + 1, vector < T > (m + 1));
  rep(i, 0, n-1) rep(j, 0, m-1) a[i+1][j+1] = A[i][
    \hookrightarrowj];
  rep(i, 0, n - 1) a[i + 1][0] = b[i];
 rep(j, 0, m-1) a[0][j+1] = c[j];
 vi left (n + 1), up (m + 1);
 iota(all(left), m);
 iota(all(up), 0);
  auto pivot = [&](int x, int y) {
   swap(left[x], up[y]);
   T k = a[x][y];
   a[x][y] = 1;
   vi pos:
   rep(j, 0, m) {
     a[x][j] /= k;
     if (fabs(a[x][j]) > eps) pos.push_back(j);
   rep(i, 0, n) {
```

if (fabs(a[i][y]) < eps || i == x) continue;</pre>

```
k = a[i][y];
      a[i][y] = 0;
      for (int j : pos) a[i][j] = k * a[x][j];
  };
  while (1) {
   int x = -1;
    rep(i, 1, n) if (a[i][0] < -eps && (x == -1 || a[i][0]
       \hookrightarrow < a[x][0])) {
    if (x == -1) break;
    int v = -1:
    rep(j, 1, m) if (a[x][j] < -eps && (y == -1 || a[x][j])
      \hookrightarrow < a[x][y])) {
      y = j;
    if (y == -1) return {0, vector<T>{}}; // infeasible
   pivot(x, y);
  while (1) {
    int y = -1;
    rep(j, 1, m) if (a[0][j] > eps && (y == -1 || a[0][j] >
      \hookrightarrow a[0][y])) {
      y = j;
    if (v == -1) break;
    int x = -1:
    rep(i, 1, n) if (a[i][y] > eps && (x == -1 || a[i][0] /
       \hookrightarrow a[i][y] < a[x][0] / a[x][y])) {
    if (x == -1) return {2, vector<T>{}}; // unbounded
    pivot(x, y);
  vector<T> ans(m);
  rep(i, 1, n) {
   if (1 <= left[i] && left[i] <= m) {</pre>
      ans[left[i] - 1] = a[i][0];
  return {1, ans};
} // hash-cpp-all = 65bffa3f1640fddb4ff878040d5c721c
```

## berlekamp-massey.cpp

// TODO

# fft.cpp

Description: Fast Fourier Transform. Time:  $\mathcal{O}(N \log N)$ 

```
// use T = double or long double.
template<class T> struct FFT {
  using cp = complex<T>;
  static constexpr T pi = acos(T{-1});
  vi r;
  int n2;
  void dft(vector<cp> &a, int is_inv) { // is_inv == 1 ->
     \hookrightarrow idft.
```

1 lines

73 lines

```
rep(i, 1, n2 - 1) if (r[i] > i) swap(a[i], a[r[i]]);
 for(int step = 1; step < n2; step <<= 1) {</pre>
   vector<cp> w(step);
   rep(j, 0, step-1) { // this has higher precision,
       \hookrightarrow compared to using the power of zeta.
     T theta = pi * j / step;
     if (is inv) theta = -theta;
     w[j] = cp\{cos(theta), sin(theta)\};
    for (int i = 0; i < n2; i += step << 1) {
     rep(j, 0, step - 1) {
        cp tmp = w[j] * a[i + j + step];
        a[i + j + step] = a[i + j] - tmp;
        a[i + j] += tmp;
 if (is inv) {
    for (auto &x: a) x \neq n2;
void pre(int n) { // set n2, r;
 int len = 0;
 for (n2 = 1; n2 < n; n2 <<= 1) len++;
 r.resize(n2);
 rep(i, 1, n2 - 1) r[i] = (r[i >> 1] >> 1) | ((i & 1) <<
     \hookrightarrow (len - 1));
template < class Z > vector < Z > conv(const vector < Z > & A,
   int n = sz(A) + sz(B) - 1;
 vector<cp> a(n2, 0), b(n2, 0);
 rep(i, 0, sz(A) - 1) a[i] = A[i];
 rep(i, 0, sz(B) - 1) b[i] = B[i];
 dft(a, 0); dft(b, 0);
 rep(i, 0, n2 - 1) a[i] *= b[i];
 dft(a, 1);
 vector < 7 > res(n):
 T eps = T{0.5} * (static_cast < Z > (1e-9) == 0);
 rep(i, 0, n-1) res[i] = a[i].real() + eps;
 return res:
vi conv(const vi &A, const vi &B, int mod) {
 int M = sqrt(mod) + 0.5;
 int n = sz(A) + sz(B) - 1;
 pre(n):
 vector<cp> a(n2, 0), b(n2, 0), c(n2, 0), d(n2, 0);
 rep(i, 0, sz(A) - 1) a[i] = A[i] / M, b[i] = A[i] % M;
 rep(i, 0, sz(B) - 1) c[i] = B[i] / M, d[i] = B[i] % M;
 dft(a, 0); dft(b, 0); dft(c, 0); dft(d, 0);
 vi res(n);
 auto work = [&] (vector<cp> &a, vector<cp> &b, int w,
    \hookrightarrowint mod) {
   vector<cp> tmp(n2);
   rep(i, 0, n2 - 1) tmp[i] = a[i] * b[i];
   dft(tmp, 1);
   rep(i, 0, n - 1) res[i] = (res[i] + (ll) (tmp[i].real)
       \hookrightarrow () + 0.5) % mod * w) % mod;
 work(a, c, 111 * M * M % mod, mod);
 work(b, d, 1, mod);
 work(a, d, M, mod);
 work(b, c, M, mod);
 return res;
```

```
}; // hash-cpp-all = 9e4b0b0ed2a6597eef170ecd23137484
ntt.cpp
Description: Number Theoretic Transform.
               class T should have static function getMod()
Usage:
to provide the mod. We usually just use modnum as the
template parameter.
To keep the code short we just set the primitive root as
3. However, it might be wrong when mod \neq 998244353. Here
is some commonly used mod and the corresponding primitive
root.
a \to mod \pmod{(\max \log(n))}
3 \rightarrow 104857601 (22), 167772161 (25), 469762049 (26),
998244353 (23), 1004535809 (21);
10 -> 786433 (18);
31 -> 2013265921 (27).
Time: \mathcal{O}(N \log N).
                                                         50 lines
template<class T> struct FFT {
 const T q: // primitive root.
 vi r;
 int n2;
 FFT(T _g = 3): g(_g) {}
  void dft(vector<T> &a, int is_inv) { // is_inv == 1 ->
    \hookrightarrow idft.
    rep(i, 1, n2 - 1) if (r[i] > i) swap(a[i], a[r[i]]);
    for(int step = 1; step < n2; step <<= 1) {</pre>
      vector<T> w(step);
      T zeta = g.pow((T::getMod() - 1) / (step << 1));
      if (is inv) zeta = 1 / zeta;
      w[0] = 1;
      rep(i, 1, step - 1) w[i] = w[i - 1] * zeta;
      for (int i = 0; i < n2; i += step << 1) {
        rep(j, 0, step - 1) {
          T tmp = w[j] * a[i + j + step];
          a[i + j + step] = a[i + j] - tmp;
          a[i + j] += tmp;
    if (is_inv == 1) {
     T inv = T\{1\} / n2;
      rep(i, 0, n2 - 1) a[i] *= inv;
  void pre(int n) { // set n2, r; also used in polynomial
    \hookrightarrowinverse.
    int len = 0:
    for (n2 = 1; n2 < n; n2 <<= 1) len++;
    r.resize(n2);
    rep(i, 1, n2 - 1) r[i] = (r[i >> 1] >> 1) | ((i & 1) <<
      \hookrightarrow (len - 1));
  vector<T> conv(vector<T> a, vector<T> b) {
   int n = sz(a) + sz(b) - 1;
    pre(n);
    a.resize(n2, 0);
   b.resize(n2, 0);
```

dft(a, 0); dft(b, 0);

rep(i, 0, n2 - 1) a[i] \*= b[i];

```
dft(a, 1);
   a.resize(n);
   return a;
}
}; // hash-cpp-all = c79d81db99fdb79f856409c48821f21c
```

## polynomial.cpp

**Description:** Basic polynomial struct. Usually we use modnum as template parameter.

```
template<class T> struct poly: vector<T> {
// hash-cpp-1
  using vector<T>::vector;
  poly(const vector<T> &vec): vector<T>(vec) {}
  friend poly& operator *=(poly &a, const poly &b) {
   FFT<T> fft;
    a = fft.conv(a, b);
    return a;
  friend poly operator *(const poly &a, const poly &b) {
    \hookrightarrowauto c = a; return c *= b; }
  poly inv(int n = 0) const {
    const poly &f = *this;
    assert(sz(f) > 0);
    if (n == 0) n = sz(*this);
    poly res{1 / f[0]};
    FFT<T> fft;
    for (int m = 2; m < n * 2; m <<= 1) {
      poly a(f.begin(), f.begin() + m);
      a.resize(m \star 2, 0);
      res.resize(m \star 2, 0);
      fft.pre(m * 2);
      fft.dft(a, 0); fft.dft(res, 0);
      rep(i, 0, m * 2 - 1) res[i] = (2 - a[i] * res[i]) *
         \hookrightarrowres[i];
      fft.dft(res, 1);
      res.resize(m);
    res.resize(n);
    return res:
  } // hash-cpp-1 = 9cecbacfe9d0d397fd8701b6594f8045
  // the following is seldom used.
  friend poly& operator += (poly &a, const poly &b) { //
     \hookrightarrow hash-cpp-2
    if (sz(a) < sz(b)) a.resize(sz(b), 0);
    rep(i, 0, sz(b) - 1) a[i] += b[i];
  friend poly operator +(const poly &a, const poly &b) {
    \hookrightarrowauto c = a; return c += b; }
  friend poly& operator -= (poly &a, const poly &b) {
   if (sz(a) < sz(b)) a.resize(sz(b), 0);
    rep(i, 0, sz(b) - 1) a[i] -= b[i];
  friend poly operator - (const poly &a, const poly &b) {
     \hookrightarrowauto c = a; return c -= b; }
// hash-cpp-2 = a4c680e717c3d8a211115bef9fb73e1e
};
```

```
linear-recurrence-kth-term.cpp
```

**Description:** Let Q(x) be the characteristic polynomial of our recurrence, and  $F(x) = \sum_{i=0}^{\infty} a_i x^i$  be the generating formal power series of our sequence. Then it can be seen that all nonzero terms of F(x)Q(x) are of at most (n-1)-st power. This means that F(x) = P(x)/Q(x) for some polynomial P(x). Moreover, we know what P(x) is: it is basically the first n terms of F(x)Q(x), that is, can be found in one multiplication of  $a_0 + \ldots + a_{n-1}x^{n-1}$  and Q(x), and then trimming to the proper degree.

```
degree. Usage: Suppose a_{-}i = \sum_{-}\{j = 1\}^{\hat{}}\{d\}a_{-}\{i - j\} * c_{-}j, then just let A = a_{-}0, \ldots, a_{-}\{d - 1 \text{ and } C = c_{-}1, \ldots, c_{-}d. "polynomial.cpp" 24 lines template<class T> T fps_coeff(poly<T> P, poly<T> Q, ll k) { while (k \ge sz(Q)) { auto nQ(Q); rep(i, 0, sz(nQ) - 1) if (i & 1) nQ[i] = 0 - nQ[i]; auto PQ = P * nQ; auto Q2 = Q * nQ; poly<T> R, S;
```

```
→ (PQ[i]);
rep(i, 0, sz(Q2) - 1) if (i % 2 == 0) S.push_back(Q2[i
→]);

swap(P, R);
swap(Q, S);
k >>= 1;
}
return (P * Q.inv())[k];
}

template<class T> T linear_rec_kth(const poly<T> &A, const
→poly<T> &C, 11 k) {
```

poly<T> Q{1}; // Q is characteristic polynomial.

} // hash-cpp-all = 320c2d19b585cfcec2a2bd545b5b8d99

rep(i, 0, sz(PO) - 1) if ((k + i) % 2 == 0) R.push back

fast-subset-transform.cpp

return fps\_coeff(P, Q, k);

auto P = A \* O;

P.resize(sz(Q) - 1);

**Description:** Fast Subtset Transform. Also known as fast zeta transform.

**Usage:** length of a should be a power of 2. **Time:**  $\mathcal{O}(N \log N)$ , where N is the length of a.

for (auto x: C) Q.push\_back(0 - x);

13 lines

```
fwht.cpp
```

**Description:** Fast Walsh-Hadamard Transform  $fwt(a)=(\sum_i (-1)^{pc(i\&0)}a_i,...,\sum_i (-1)^{pc(i\&n-1)}a_i).$  One can use it to do xor-convolution.

Usage: length of a should be a power of 2.

**Time:**  $\mathcal{O}(N \log N)$ , where N is the length of a.

```
template<class T> void fwt(vector<T> &a, int is_inv) {
  int N = sz(a);
  for (int s = 1; s < N; s <<= 1)
    for (int i = 0; i < N; i += s << 1)
      rep(j, 0, s - 1) {
      T x = a[i + j], y = a[i + j + s];
      a[i + j] = x + y;
      a[i + j + s] = x - y;
    }

if (is_inv) {
    for (auto &x: a) x = x / N;
}
// hash-cpp-all = 39548d4e5eba54c67b841c6f77a928ed</pre>
```

#### fwht-eval.cpp

**Description:** Let b = fwt(a). One can calculate  $b_{id}$  for some index id in O(N) time.

Usage: length of a should be a power of 2.

**Time:**  $\mathcal{O}(N)$ , where N is the length of a.

```
template<class T> T fwt_eval(const vector<T> &a, int id) {
  int N = sz(a);
  T res = 0;
  rep(i, 0, N - 1) {
    if (_builtin_popcount(i & id) & 1) res -= a[i];
    else res += a[i];
  }
  return res;
} // hash-cpp-all = 70afad3ebf9c5d79cb34009e63ceab27
```

#### matroid.cpp

// TODO

#### matrix.cpp

**Description:** Matrix struct. Used for Gaussian elimination or inverse of matrix.

Usage: To solve  $Ax = b^{\uparrow} \top$ , call SolveLinear(A, b).

Besides, you need function is Zero for your template T.

Time:  $\mathcal{O}(nm \min\{n, m\})$  for Gaussian, inverse and SolveLinear 98 lines

```
template<class T> struct Matrix {
    using Mat = Matrix;
    using Vec = vector<T>;

    vector<Vec> a;

Matrix(int n, int m) {
      assert(n > 0 && m > 0);
      a.assign(n, Vec(m));
}

Matrix(const vector<Vec> &a): a(a) {
      assert(sz(a) > 0 && sz(a[0]) > 0);
}

Vec& operator [](int i) const { return (Vec&) a[i]; }

Mat operator + (const Mat &b) const {
    int n = sz(a], m = sz(a[0]);
}
```

```
Mat c(n, m);
  rep(i, 0, n-1) rep(j, 0, m-1) c[i][j] = a[i][j] + b
     \hookrightarrow [i][j];
  return c;
Mat operator - (const Mat &b) const {
 int n = sz(a), m = sz(a[0]);
 Mat c(n, m);
  rep(i, 0, n-1) rep(j, 0, m-1) c[i][j] = a[i][j] - b
Mat operator *(const Mat &b) const {
 int n = sz(a), m = sz(a[0]), 1 = sz(b[0]);
  assert (m == sz(b.a));
 Mat c(n, 1);
  rep(i, 0, n-1) rep(k, 0, m-1) rep(j, 0, 1-1) c[i
     \hookrightarrow][j] += a[i][k] * b[k][j];
 return c;
Mat tran() const {
 int n = sz(a), m = sz(a[0]);
 Mat res(m, n);
 rep(i, 0, n-1) rep(j, 0, m-1) res[j][i] = a[i][j];
 return res;
// Do elimination for the first C columns, return the
int Gaussian(int C) {
 int n = sz(a), m = sz(a[0]), rk = 0;
 assert(C <= m);
  rep(c, 0, C - 1) {
   int id = rk;
    while (id < n && ::isZero(a[id][c])) id++;</pre>
    if (id == n) continue;
    if (id != rk) swap(a[id], a[rk]);
   T \text{ tmp} = a[rk][c];
    for (auto &x: a[rk]) x /= tmp;
    rep(i, 0, n - 1) if (i != rk) {
     T fac = a[i][c];
     rep(j, 0, m - 1) a[i][j] -= fac * a[rk][j];
    rk++;
 return rk;
Mat inverse() const {
 int n = sz(a), m = sz(a[0]);
 assert (n == m);
 auto b = *this;
 rep(i, 0, n - 1) b[i].resize(n * 2, 0), b[i][n + i] =
    \hookrightarrow1;
  assert(b.Gaussian(n) == n);
  for (auto &row: b.a) row.erase(row.begin(), row.begin()
  return b:
friend pair < bool, Vec > SolveLinear (Mat A, const Vec &b) {
  \#define revrep(i, a, n) for (auto i = n; i >= (a); --i)
```

```
int n = sz(A.a), m = sz(A[0]);
   assert(sz(b) == n);
   rep(i, 0, n - 1) A[i].push_back(b[i]);
   int rk = A.Gaussian(m);
   rep(i, rk, n - 1) if (!::isZero(A[i].back())) return
       \hookrightarrow {0, Vec{}};
   Vec res(m):
   revrep(i, 0, rk - 1) {
     T x = A[i][m];
      int last = -1;
      revrep(j, 0, m - 1) if (!::isZero(A[i][j])) {
       x \rightarrow A[i][j] * res[j];
       last = j;
      if (last != -1) res[last] = x;
   return {1, res};
}: // hash-cpp-all = c32ead126cef68d15e8988daa6882258
```

#### linear-base.cpp

Usage: keep w[] zero to use unweighted Linear Base.

```
Description: Maximum weighted of Linear Base of vector space \mathbb{Z}_2^{LG}
Time: \mathcal{O}\left(LG \cdot \frac{LG}{w}\right) for insertion; \mathcal{O}\left(LG^2 \cdot \frac{LG}{w}\right) for union.
// T is the type of vectors and Z is the type of weights.
// w[i] is the non-negative weight of a[i].
template<int LG, class T = bitset<LG>, class Z = int>
   ⇒struct LB {
// hash-cpp-1
  \#define revrep(i, a, n) for (auto i = n; i >= (a); --i)
  vector<T> a:
  vector<Z> w;
  T& operator [](int i) const { return (T&)a[i]; }
  LB(): a(LG), w(LG) {}
  // insert x. return 1 if the base is expanded.
  int insert(T x, Z val = 0) {
    revrep(i, 0, LG - 1) if (x[i]) {
      if (a[i] == 0) {
        a[i] = x;
        w[i] = val;
        return 1;
       } else if (val > w[i]) {
        swap(a[i], x);
        swap(w[i], val);
      x ^= a[i];
    return 0:
  } // hash-cpp-1 = a387f093648b516f28c7328018f56f16
  // min value we can get if we add vectors from linear
     \hookrightarrow base (with weight at least \text{$val$}) to \text{$x$}.
  T ask_min(T x, Z val = 0) { // hash-cpp-2}
    revrep(i, 0, LG - 1) {
      if (x[i] \&\& w[i] >= val) x ^= a[i]; // change x[i] to
          \hookrightarrow x[i] == 0 to ask maximum value we can get.
    return x;
  \frac{1}{2} // hash-cpp-2 = 97b49d40578d7eb5b1beb46eb3348463
  // take the union of two bases.
  friend LB operator + (LB a, const LB &b) { // hash-cpp-3
    rep(i, 0, LG - 1) if (b[i] != 0) a.insert(b[i]);
    return a;
```

```
} // hash-cpp-3 = 2cf1ecc88b178b24de182560d92f42d1
  // return the k-th smallest value spanned by vectors with

→ wieght at least $val$. k starts from 0.

  // Time: O(LG \cdot \frac{LG}{w}).
 T kth (unsigned long long k, Z val = 0) { // hash-cpp-4
   int N = 0;
    rep(i, 0, LG - 1) N += (a[i] != 0 \&\& w[i] >= val);
    if (k \ge (1ull << N)) return -1; // return -1 if k is
       \hookrightarrowtoo large.
    T res = 0:
    revrep(i, 0, LG - 1) if (a[i] != 0 \&\& w[i] >= val) {
      auto d = k \gg N \& 1;
      if (res[i] != d) res ^= a[i];
    return res:
 } // hash-cpp-4 = 0d7e2a5d390ca813f8cfef6ac98d30d4
};
```

# linear-base-intersect.cpp

**Description:** Intersection of two unweighted linear bases.

Usage: T should be of length at least  $2 \cdot LG$ . Time:  $\mathcal{O}\left(LG^2 \cdot \frac{LG}{2\pi}\right)$ .

```
template<int LG, class T = bitset<LG * 2>> LB<LG, T>
   ⇒intersect(LB<LG, T> a, const LB<LG, T> &b) {
 LB<LG, T> res;
 rep(i, 0, LG - 1) if (a[i] != 0) a[i][LG + i] = 1;
 T msk(string(LG, '1'));
 rep(i, 0, LG - 1) {
   T x = a.ask_min(b[i]);
   if ((x \& msk) != 0) a.insert(x);
   else {
     T v = 0;
     rep(j, 0, LG - 1) if (x[LG + j]) y ^= a[j];
     res.insert(y & msk);
 return res:
} // hash-cpp-all = ac77102be62217631c2b04f78b033fe2
```

# Z3-vector.cpp

**Description:** vector in  $\mathbb{Z}_3$ . Time:  $\mathcal{O}(L/w)$ .

38 lines

```
template<int L> struct v3 {
    bitset<L> a[3]:
    v3() { a[0].set(); }
    void set(int pos, int x) { rep(i, 0, 2) a[i][pos] = (i)
       \hookrightarrow == x);
    int operator [](int i) const {
        if (a[0][i]) return 0;
        else if (a[1][i]) return 1;
        else return 2;
    v3 operator + (const v3 &rhs) const {
        v3 res;
        res.a[0] = (a[0] & rhs.a[0]) | (a[1] & rhs.a[2]) |
            \hookrightarrow (a[2] & rhs.a[1]);
        res.a[1] = (a[0] \& rhs.a[1]) | (a[1] \& rhs.a[0]) |
            \hookrightarrow (a[2] & rhs.a[2]);
        res.a[2] = (\sim res.a[0] \& \sim res.a[1]);
        return res;
    v3 operator -(const v3 &rhs) const {
```

#### integrate.cpp

**Description:** Let f(x) be a continuous function over [a,b] having a fourth derivative,  $f^{(4)}(x)$ , over this interval. If M is the maximum value of  $|f^{(4)}(x)|$  over [a,b], then the upper bound for the error is  $O(\frac{M(b-a)^5}{N^4})$ . **Time:**  $O(N \cdot T)$ , where T is the time for evaluating f once.

#### integrate-adaptive.cpp

**Description:** It is somehow necessary to set the minimum depth of recursion. We use *dep* here. Change it smaller if Time Limit is tightines

#### recursive-ternary-search.cpp

**Description:** for convex function  $f: \mathbb{R}^d \to \mathbb{R}$ , we can approximately find the global minimum using ternary search on each coordinate recursively.

**Usage:** d is the dimension; mn, mx record the minimum and maximum possible value of each coordinate (the region you do ternary search); f is the convex function.

**Time:**  $\mathcal{O}\left(\log(1/\epsilon)^d \cdot T\right)$ , where T is the time for evaluating the function f.

```
// use T = double or long double.
template < class T > T rec ters (int d, const vector < T > & mn,
  vector<T> xs(d);
 auto dfs = [&] (auto dfs, int dep) {
   if (dep == d) return f(xs);
   T l = mn[dep], r = mx[dep];
   rep(_, 1, 60) {
    T m1 = (1 * 2 + r) / 3;
     T m2 = (1 + r * 2) / 3;
     xs[dep] = m1; T res1 = dfs(dfs, dep + 1);
     xs[dep] = m2; T res2 = dfs(dfs, dep + 1);
     if (res1 < res2) r = m2;
     else 1 = m1;
   xs[dep] = (1 + r) / 2;
   return dfs(dfs, dep + 1);
 return dfs(dfs, 0);
} // hash-cpp-all = 7463b827f8431abbabeed2f0528722ef
```

# Number theory (7)

# modnum.cpp

// hash-cpp-2

**Description:** Modular integer with  $mod \leq 2^{30}-1$ . Note that there are several advantages to use this code: 1. You do not need to keep writing % mod; 2. It is good to use this struct when doing Gaussian Elimination / Fast Walsh-Hadamard Transform; 3. Sometimes the input number is greater than mod and this code handles it. Do not write things like Mint1 / 3.pow(10) since 1 / 3 simply equals 0. Do not write things like Minta \* b where a and b are int since you might first have integer overflow.

```
Usage: mod should be a global variable (either const int or int) and should satisfy mod \leq 2^{3}-1. for exmaple you can use like this:
```

```
const int mod = 998244353;
using Mint = Z<mod>;
                                                         32 lines
template<const int &mod> struct Z {
// hash-cpp-1
 int x;
 Z(11 \ a = 0): x(a \% \ mod) \{ if (x < 0) x += mod; \}
  explicit operator int() const { return x; }
  Z\& operator +=(Z b) \{ x += b.x; if (x >= mod) x -= mod; \}
    →return *this; }
  Z\& operator -= (Z b) \{ x -= b.x; if (x < 0) x += mod; \}
    Z& operator \star = (Z b) \{ x = 111 \star x \star b.x \% mod; return \star \}
    →this; }
  friend Z operator +(Z a, Z b) { return a += b; }
  friend Z operator -(Z a, Z b) { return a -= b; }
  friend Z operator *(Z a, Z b) { return a *= b; }
// hash-cpp-1 = e5f2469d533a39d2945e75688e0b7e94
  // the followings are needed for ntt and polynomial
     \hookrightarrowoperations.
```

```
Z pow(11 k) const {
    Z res = 1, a = *this;
    for (; k; k >>= 1, a = a * a) if (k & 1) res = res * a;
    return res;
}
Z& operator /=(Z b) {
    assert(b.x != 0);
    return *this *= b.pow(mod - 2);
}
friend Z operator /(Z a, Z b) { return a /= b; }

static int getMod() { return mod; } // ntt need this.
// hash-cpp-2 = 25825dd33306e07c0d0faf87a0e74882

friend string to_string(Z a) { return to_string(a.x); }
    \[
    \times // just for debug.
};
```

#### factorization.cpp

**Description:** Fast Factorization. The mul function supports  $0 \le a, b < c < 7.268 \times 10^{18}$  and is a little bit faster than \_int128.

**Time:**  $\mathcal{O}\left(n^{1/4}\right)$  for pollard-rho and same for the whole factorization.

```
namespace Factorization {
 inline 11 mul(11 a, 11 b, 11 c) { // hash-cpp-1
    11 s = a * b - c * 11((long double)a / c * b + 0.5);
    return s < 0 ? s + c : s;
 11 mPow(11 a, 11 k, 11 mod) {
    for (; k; k >>= 1, a = mul(a, a, mod)) if (k \& 1) res =
       → mul(res, a, mod);
    return res:
 bool miller(ll n) {
    auto test = [&](ll n, int a) {
      if (n == a) return true;
      if (n % 2 == 0) return false;
      11 d = (n - 1) \gg \underline{\quad} builtin\_ctzll(n - 1);
      ll r = mPow(a, d, n);
      while (d < n - 1 \&\& r != 1 \&\& r != n - 1) d <<= 1, r
         \hookrightarrow= mul(r, r, n);
      return r == n - 1 || d & 1;
    };
    if (n == 2) return 1;
    for (auto p: vi\{2, 3, 5, 7, 11, 13\}) if (test(n, p) ==
       \hookrightarrow0) return 0;
    return 1:
 } // hash-cpp-1 = fdf01d99eff9d68a0b5ba775f3086359
// hash-cpp-2
 mt19937_64 rng(chrono::steady_clock::now().
     →time_since_epoch().count());
 ll myrand(ll a, ll b) { return uniform_int_distribution
     \hookrightarrow11>(a, b)(rng); }
 11 pollard(11 n) { // return some nontrivial factor of n.
    auto f = [\&](11 x) \{ return (( int128)x * x + 1) % n;
       \hookrightarrow };
```

11 x = 0, y = 0, t = 30, prd = 2;

while  $(t++ % 40 | | gcd(prd, n) == 1) {$ 

```
// speedup: don't take __gcd in each iteration.
      if (x == y) x = myrand(2, n - 1), y = f(x);
      11 \text{ tmp} = \text{mul}(\text{prd}, \text{ abs}(x - y), n);
      if (tmp) prd = tmp;
      x = f(x), y = f(f(y));
    return gcd(prd, n);
  vector<ll> work(ll n) {
    vector<11> res;
    function < void(11) > solve = [&](11 x) {
      if (x == 1) return;
      if (miller(x)) res.push_back(x);
      else {
        11 d = pollard(x);
        solve(d);
        solve(x / d);
    };
    solve(n);
    return res;
  } // hash-cpp-2 = e51a9b9919035e8e774f8e4cff6b8a8a
is-prime.cpp
                                                           1 lines
// TODO
cont-frac.cpp
// TODO
adleman-manders-miller.cpp
                                                           1 lines
// TODO
discrete-log.cpp
                                                           1 lines
// TODO
sieve.cpp
Description: Sieve for prime numbers / multiplicative functions in lin-
ear time.
Time: \mathcal{O}(N).
                                                          33 lines
struct LinearSieve {
  vi ps, minp;
  vi d, facnum, phi, mu;
  LinearSieve(int n): minp(n + 1), d(n + 1), facnum(n + 1),
     \hookrightarrow phi(n + 1), mu(n + 1) {
    facnum[1] = phi[1] = mu[1] = 1;
    rep(i, 2, n) {
      if (minp[i] == 0) {
        ps.push_back(i);
        minp[i] = i;
        d[i] = 1;
        facnum[i] = 2;
        phi[i] = i - 1;
        mu[i] = -1;
      for (auto p: ps) {
        11 v = 111 * i * p;
        if (v > n) break;
        minp[v] = p;
        if (i % p == 0) {
```

```
d[v] = d[i] + 1;
          facnum[v] = facnum[i] / (d[i] + 1) * (d[v] + 1);
         phi[v] = phi[i] * p;
         mu[v] = 0;
         break;
       d[v] = 1;
       facnum[v] = facnum[i] * 2;
       phi[v] = phi[i] * (p - 1);
       mu[v] = -mu[i];
}; // hash-cpp-all = 496b1c3a9df8a550e6022a4573bb36dd
```