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SISTEMAS DE CONTROL – INGENIERÍA MATEMÁTICA

Inverted Pendulum Stabilization using PID Control `[Proyecto Final]`

Firstly, we install the `control` package in Python, which provides tools for control systems analysis and design.

In [177…
```
!pip install control
```

```
Requirement already satisfied: control in /usr/local/lib/python3.10/dist-pac
kages (0.10.0)
Requirement already satisfied: numpy>=1.23 in /usr/local/lib/python3.10/dist
-packages (from control) (1.25.2)
Requirement already satisfied: scipy>=1.8 in /usr/local/lib/python3.10/dist-
packages (from control) (1.11.4)
Requirement already satisfied: matplotlib>=3.6 in /usr/local/lib/python3.10/
dist-packages (from control) (3.8.4)
Requirement already satisfied: contourpy>=1.0.1 in /usr/local/lib/python3.1
0/dist-packages (from matplotlib>=3.6->control) (1.2.1)
Requirement already satisfied: cycler>=0.10 in /usr/local/lib/python3.10/dis
t-packages (from matplotlib>=3.6->control) (0.12.1)
Requirement already satisfied: fonttools>=4.22.0 in /usr/local/lib/python3.1
0/dist-packages (from matplotlib>=3.6->control) (4.51.0)
Requirement already satisfied: kiwisolver>=1.3.1 in /usr/local/lib/python3.1
0/dist-packages (from matplotlib>=3.6->control) (1.4.5)
Requirement already satisfied: packaging>=20.0 in /usr/local/lib/python3.10/
dist-packages (from matplotlib>=3.6->control) (24.0)
Requirement already satisfied: pillow>=8 in /usr/local/lib/python3.10/dist-p
ackages (from matplotlib>=3.6->control) (9.4.0)
Requirement already satisfied: pyparsing>=2.3.1 in /usr/local/lib/python3.1
0/dist-packages (from matplotlib>=3.6->control) (3.1.2)
Requirement already satisfied: python-dateutil>=2.7 in /usr/local/lib/python
3.10/dist-packages (from matplotlib>=3.6->control) (2.8.2)
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.10/dist-pa
ckages (from python-dateutil>=2.7->matplotlib>=3.6->control) (1.16.0)
```

The following libraries will also be needed for the execution of the program:

In [178…
```
import numpy as np
import control as ctrl
```

1. MODELING

To model an inverted pendul system, we must define its transfer function given by: $$ G(s) = \frac{1}{s^2 - \frac{g}{l}} $$

In [179…
```
g = 9.81 # gravity (m/s^2)
l = 1.0 # length of the pendulum (m)
m = 1.0 # mass of the pendulum (kg)
# Transfer function G(s) = 1 / (s^2 - g/l)
num = [1]
den = [1, 0, -g/l]
G = ctrl.TransferFunction(num, den)
```

The transfer function is the result of linearalising and applying Laplace to the equations of motion of the inverted pendulum.

2. STABILITY ANALISIS

The roots of the denominator of the transfer function G determine the stability of the system. With Pyhon function `poles()` we obtain these roots.

In [180…
```
poles = ctrl.poles(G)
print("Poles of the system:", poles)
```

```
Poles of the system: [-3.13209195+0.j  3.13209195+0.j]
```

For our system, we have obtained the poles printed above, we identify that one of them is positive.

According to the `principle of Routh-Hurwitz`, in order for a linear system to be stable, all of its poles must have negative real parts, that is they must all lie within the left-half of the s-plane.

Therefore, the system is unstable.To stabilise it, we will use a PID controller.

3. PID CONTROLLER DESIGN

We are going to test different parameters for the gains `(Kp,Ki,Kd)` of a PID controller to stabilize the inverted pendulum.

In [181…
```
Kp = [1.0, 10.0, 100.0, 100.0] # Proportional gain
Ki = [1.0, 1.0, 1.0, 1.0]      # Integral gain
Kd = [2.0, 1.0, 1.0, 20.0]     # Derivative gain
```

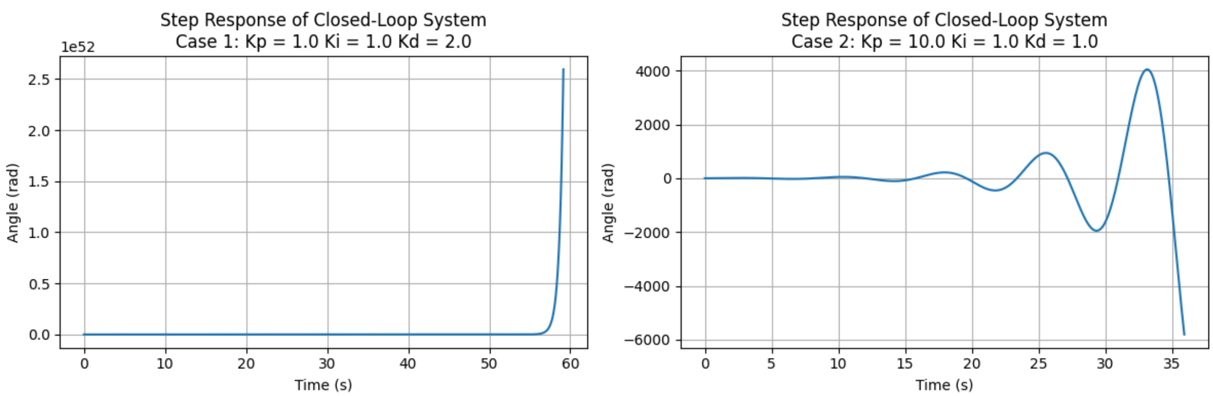
4. SIMULATION

We observe the response of the system to different initial conditions and disturbances.

First results

In [182…
```
import matplotlib.pyplot as plt
# Create a figure with subplots
fig, axs = plt.subplots(1,2, figsize=(12, 4))

# Iterate over each subplot
for i in range(2):
  # PID controller
  controller = ctrl.TransferFunction([Kd[i], Kp[i], Ki[i]], [1, 0])
  # Closed-loop system
  closed_loop = ctrl.feedback(G * controller)
  # Time response
  t, y = ctrl.step_response(closed_loop)
  # Plotting
  axs[i].plot(t, y)
  axs[i].set_xlabel('Time (s)')
  axs[i].set_ylabel('Angle (rad)')
  axs[i].set_title(f'Step Response of Closed-Loop System\nCase {i+1}: Kp = {
  axs[i].grid(True)
plt.tight_layout()
plt.show()
```



These responses are not stable. We modify the response by increasing the proportional and derivative gain.

In [183…
```
import matplotlib.pyplot as plt
# Specify the size of the plot
plt.figure(figsize=(12, 4))
x_limit = [40,1]
# Iterate over each subplot
for i in range(2,4):
  # PID controller
  controller = ctrl.TransferFunction([Kd[i], Kp[i], Ki[i]], [1, 0])
  # Closed-loop system
  closed_loop = ctrl.feedback(G * controller)
  # Time response
  t, y = ctrl.step_response(closed_loop)
  # Plotting
  # Specify the size of the plot
  plt.figure(figsize=(12, 4))
  plt.plot(t, y)
```

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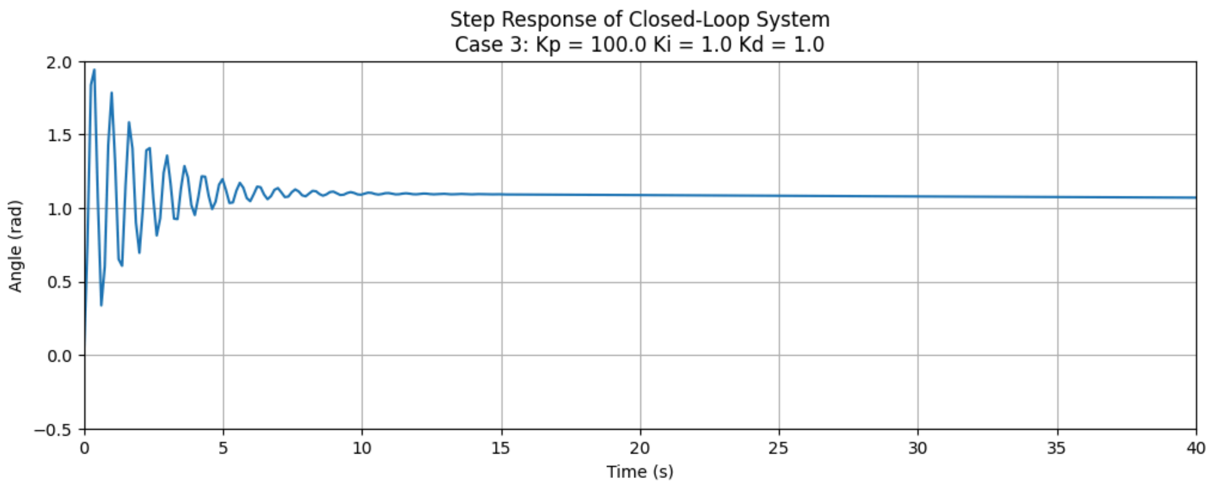
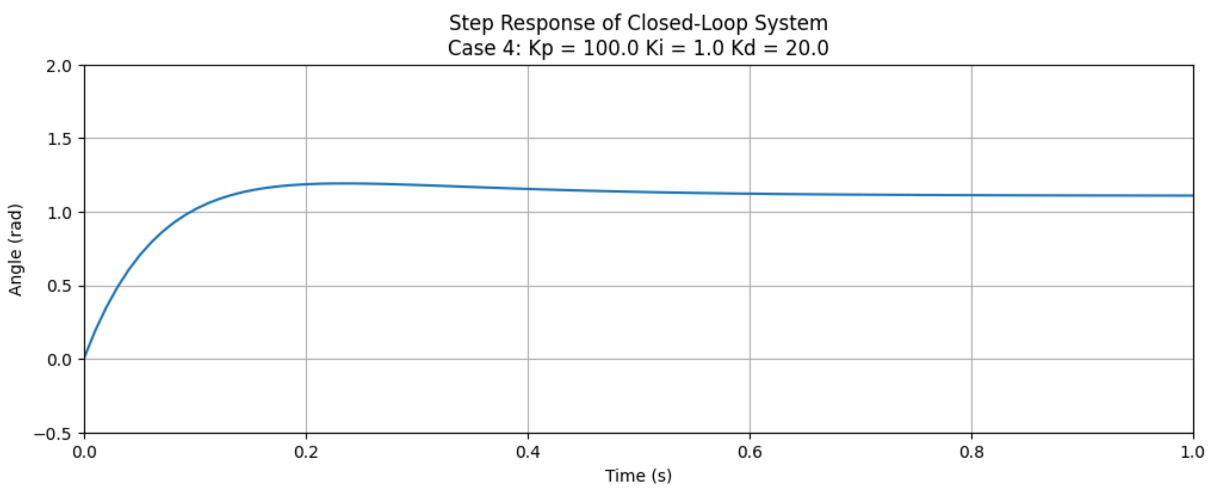
```
    # Labels
    plt.xlabel('Time (s)')
    plt.ylabel('Angle (rad)')
    # Tittle and grid
    plt.title(f'Step Response of Closed-Loop System\nCase {i+1}: Kp = {Kp[i]}
    plt.grid(True)
    # Specify the limits of the x-axis and y-axis
    plt.xlim(0, x_limit[i-2])
    plt.ylim(-0.5, 2)
    plt.show()
```

<Figure size 1200x400 with 0 Axes>





In case 3, we only increased the K_p variable and got a stable response, specifically it stabilises in aproximately 15 seconds. By also increasing the K_d variable we have gotten a stable response in less than 1 second for case 4, which is a remarkable improvement.

5. PERFORMANCE ANALYSIS

Unitary Step

Lastly, we will introduce a unitary step entry to all the previous PID systems.

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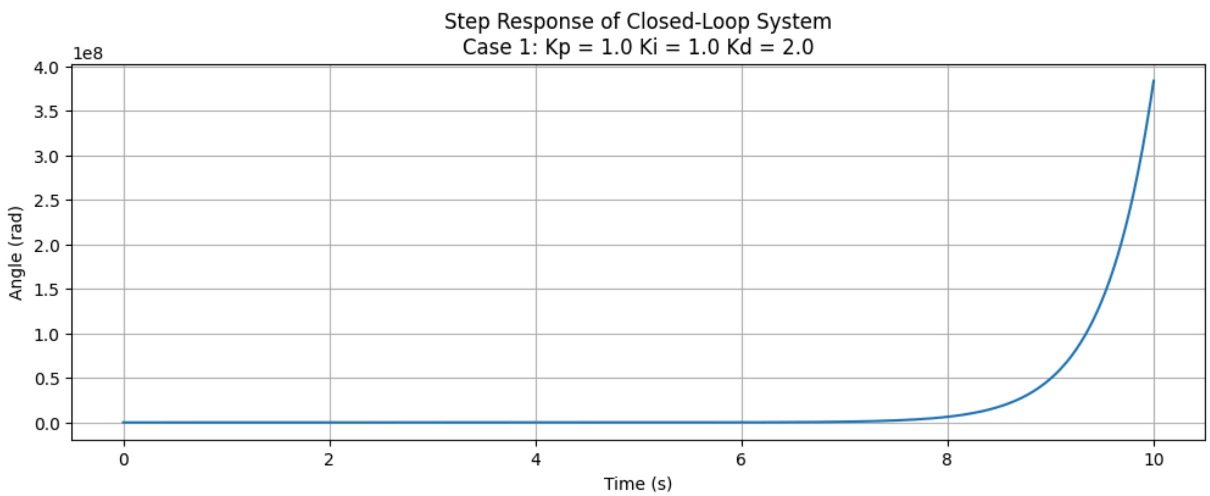
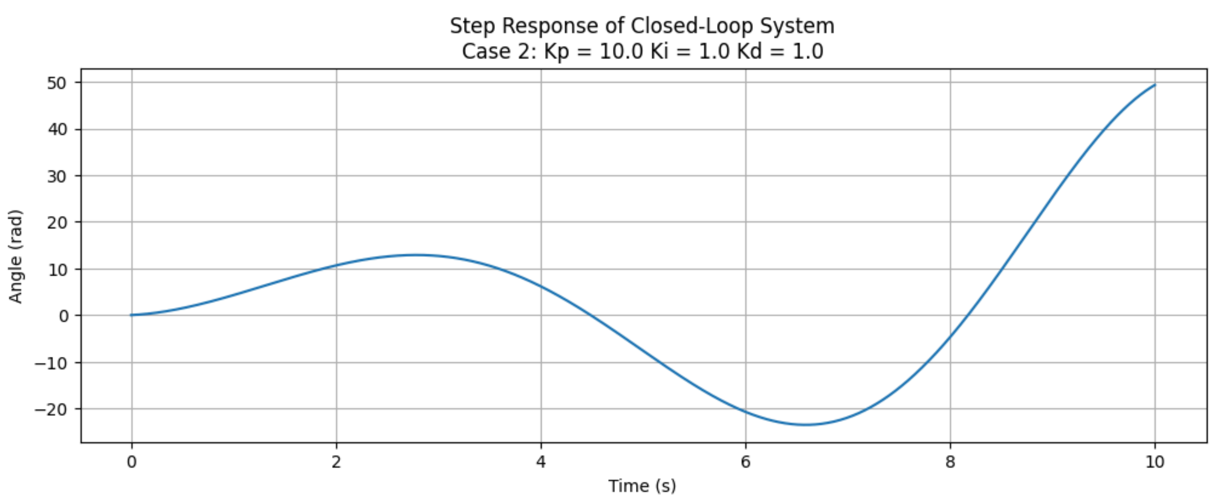
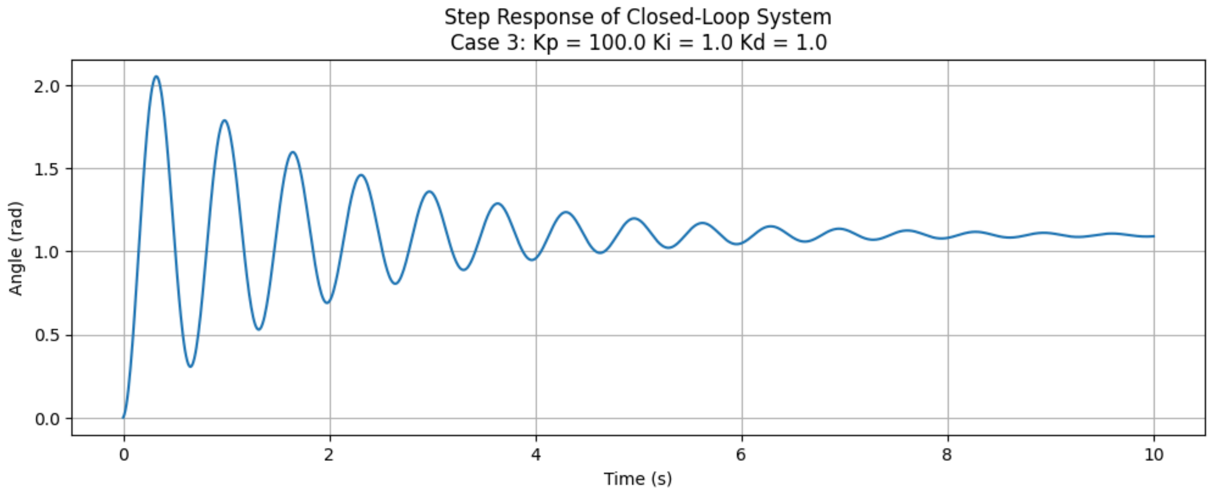
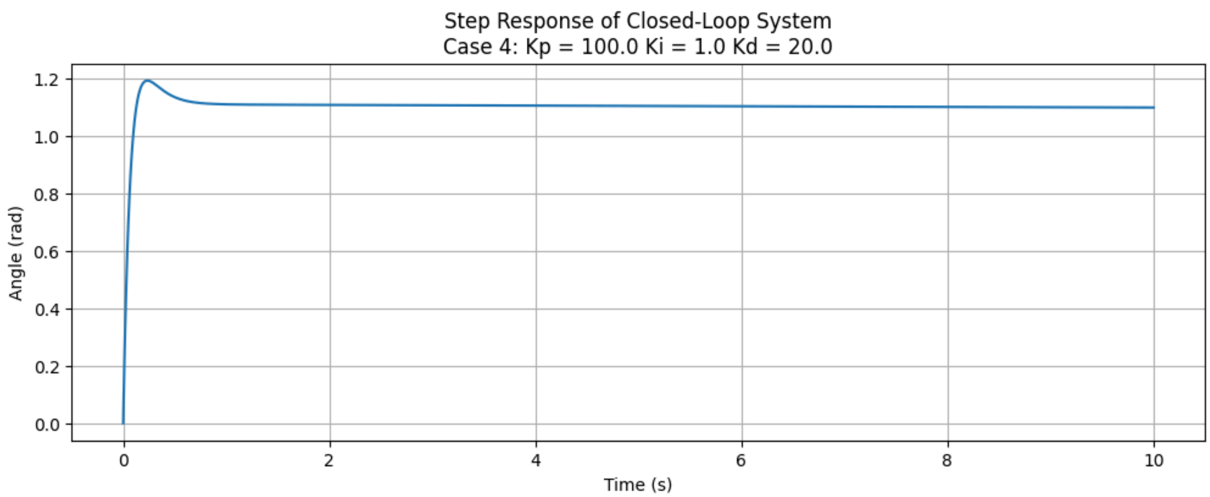
In [184…

```python
for i in range(4):
  controller = ctrl.TransferFunction([Kd[i], Kp[i], Ki[i]], [1, 0])
  # closed-loop system
  closed_loop = ctrl.feedback(G * controller)

  # Apply unitary step
  t = np.linspace(0, 10, 1000)
  t, y = ctrl.step_response(closed_loop, T=t)

  # Plotting
  # Specify the size of the plot
  plt.figure(figsize=(12, 4))
  plt.plot(t, y)
  # Labels
  plt.xlabel('Time (s)')
  plt.ylabel('Angle (rad)')
  # Tittle and grid
  plt.title(f'Step Response of Closed-Loop System\nCase {i+1}: Kp = {Kp[i]}
  plt.grid(True)
```









For cases 1 and 2, both unstable, we identify a difference in the behaviour of the step response when using a unitary step entry. There is a considerable time reduction, for example, in case 1, the first result had an exponential behaviour rapidly increasing values between 50 and 60 seconds. Meanwhile after using a one step entry, the step response increases values between 8 and 10 seconds.

For cases 2 and 3, both stable, there is no disturbance in the response after using a unitary step entry, the graphs remain the same as before.

6. VISUALIZATION

We are going to visualise the best PID obtained (case 4 was the one that had a stable behaviour and became stactionary in lesser time). And we will use once again the unitary step entry to test it.

In [196…

```
controller = ctrl.TransferFunction([Kd[3], Kp[3], Ki[3]], [1, 0])
  # closed-loop system
  closed_loop = ctrl.feedback(G * controller)

  # Apply unitary step
```

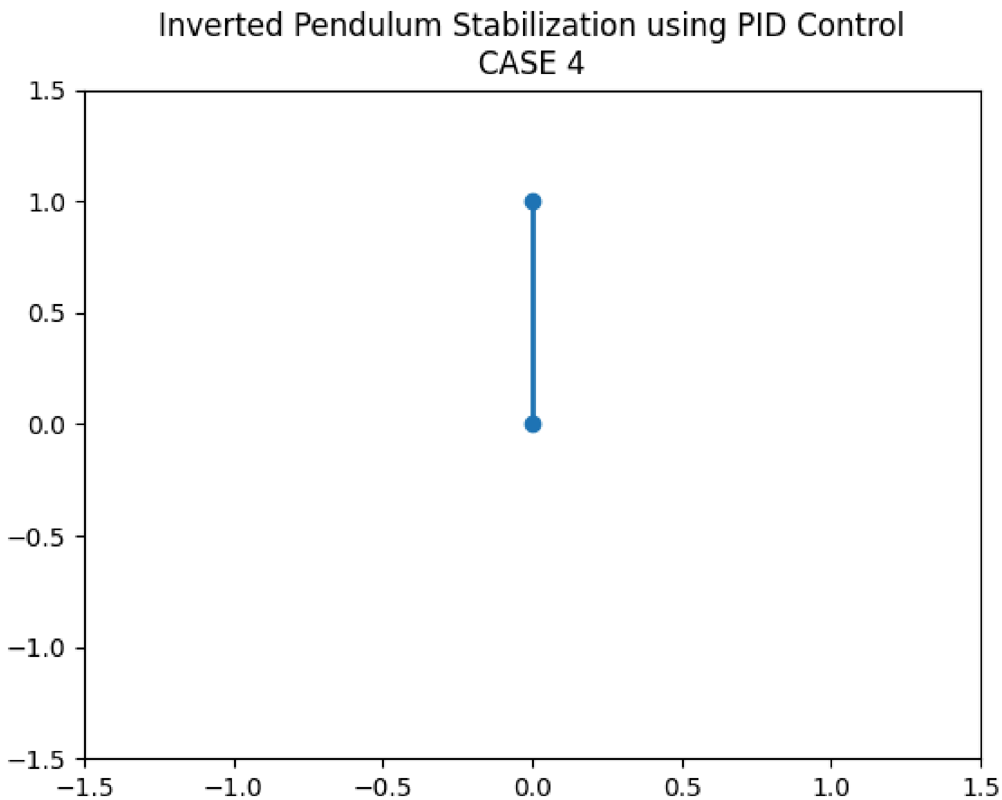
```
t = np.linspace(0, 10, 1000)
t, y = ctrl.step_response(closed_loop, T=t)
```

We proceed with the animation:

In [199…

```
# Libraries for ploting animations
from IPython.display import HTML
import matplotlib.pyplot as plt
import matplotlib.animation as animation

# Function to animate the pendule
def animate(i):
    line.set_data([0, np.sin(y[i])], [0, np.cos(y[i])])
    return line,

# Configuration of the figure and axes
fig, ax = plt.subplots()
ax.set_xlim(-1.5, 1.5)
ax.set_ylim(-1.5, 1.5)

# Tittle
ax.set_title(f'Inverted Pendulum Stabilization using PID Control\nCASE 4')
line, = ax.plot([], [], 'o-', lw=2)

# Reduce the number of frames
frames = len(y) // 2

# Create the animation
ani = animation.FuncAnimation(fig, animate, frames=frames, interval=20, blit

# Show the animation
HTML(ani.to_jshtml())
```

Out[199…



Once Loop Reflect

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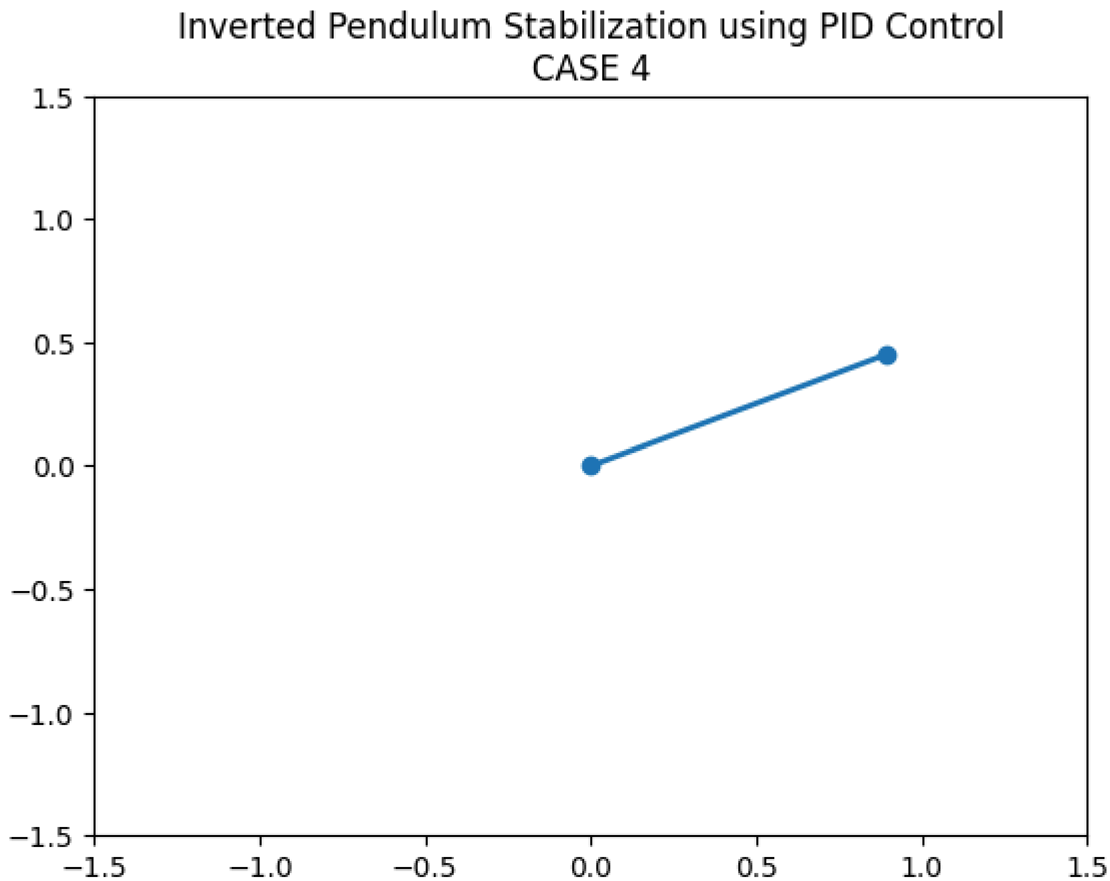


We can see it takes very little time and movements before it becomes stactionary.

For case 3, the other stable PID system:

In [200…

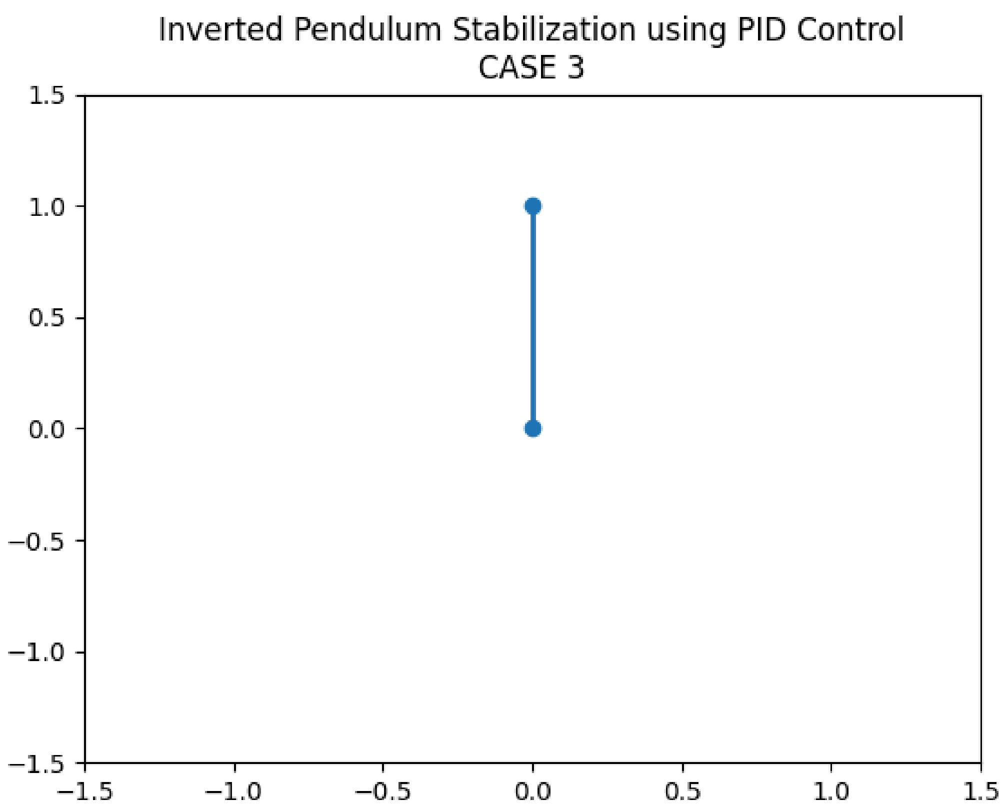
```
controller = ctrl.TransferFunction([Kd[2], Kp[2], Ki[2]], [1, 0])
# closed-loop system
closed_loop = ctrl.feedback(G * controller)

# Apply unitary step
t = np.linspace(0, 10, 1000)
t, y = ctrl.step_response(closed_loop, T=t)
```

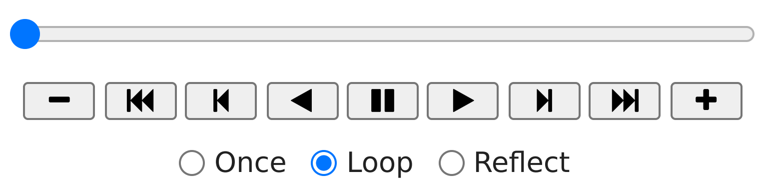
In [201…

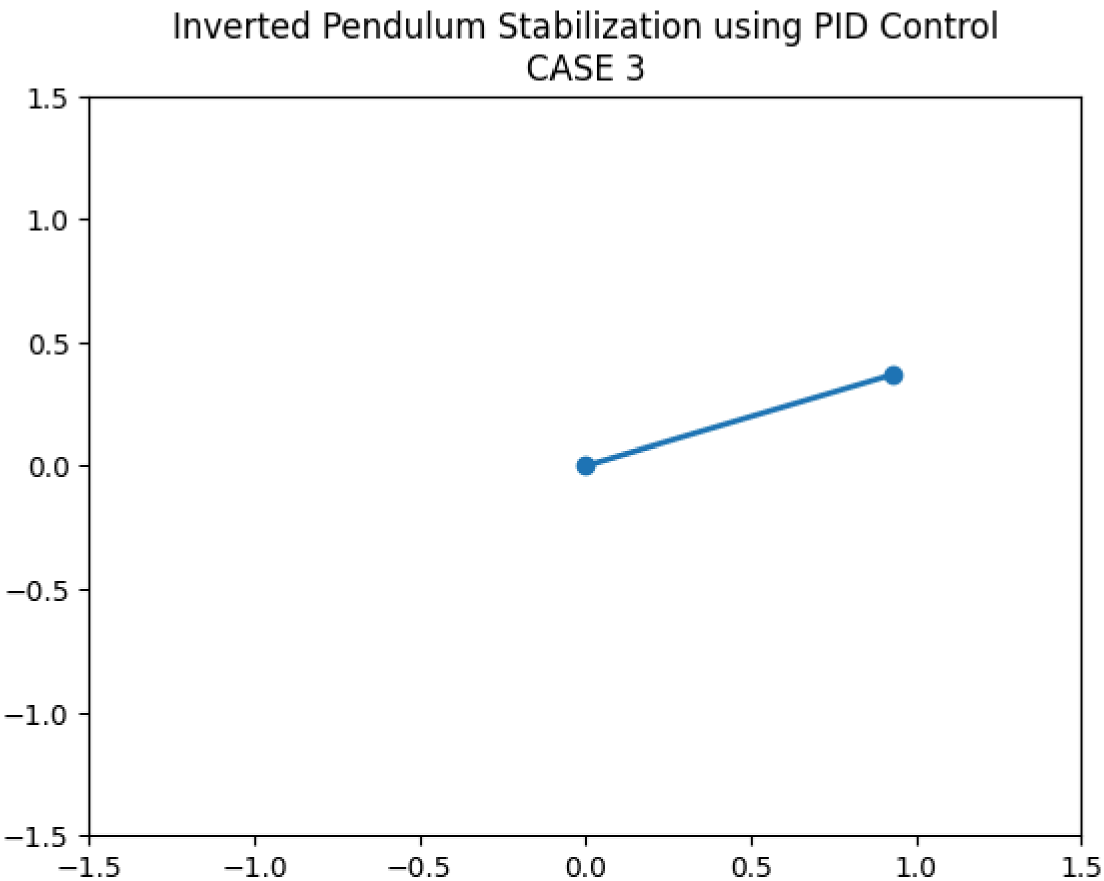
```
# Libraries for ploting animations
from IPython.display import HTML
import matplotlib.pyplot as plt
import matplotlib.animation as animation

# Function to animate the pendule
def animate(i):
    line.set_data([0, np.sin(y[i])], [0, np.cos(y[i])])
    return line,

# Configuration of the figure and axes
fig, ax = plt.subplots()
ax.set_xlim(-1.5, 1.5)
im(-1.5, 1.5)
```

```python
# Tittle
ax.set_title(f'Inverted Pendulum Stabilization using PID Control\nCASE 3')

line, = ax.plot([], [], 'o-', lw=2)

# Reduce the number of frames
frames = len(y) // 2

# Create the animation
ani = animation.FuncAnimation(fig, animate, frames=frames, interval=30, blit

# Show the animation
HTML(ani.to_jshtml())
```

Out[201…







There is a singnificant difference in the behaviour compared to case 4. Now the pendule has a more oscilatory behaviour and takes more time and oscilations to become stactionary.

For case 2, unstable PID system, we expect the pendule to never become stactionary and keep oscilating around the center.

In [203…

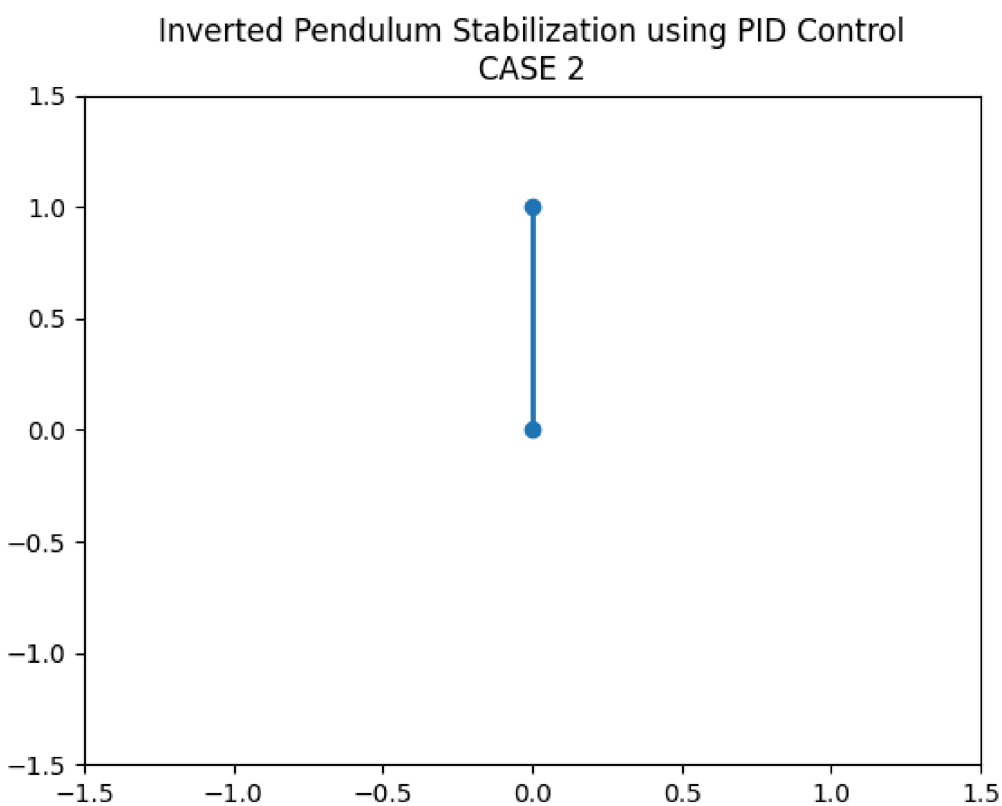
```
controller = ctrl.TransferFunction([Kd[1], Kp[1], Ki[1]], [1, 0])
# closed-loop system
closed_loop = ctrl.feedback(G * controller)

# Apply unitary step
t = np.linspace(0, 10, 1000)
t, y = ctrl.step_response(closed_loop, T=t)
```

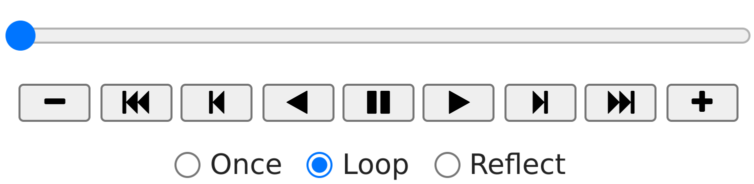
In [204…

```
# Libraries for ploting animations
from IPython.display import HTML
import matplotlib.pyplot as plt
import matplotlib.animation as animation

# Function to animate the pendule
def animate(i):
    line.set_data([0, np.sin(y[i])], [0, np.cos(y[i])])
    return line,

# Configuration of the figure and axes
```

```python
fig, ax = plt.subplots()
ax.set_xlim(-1.5, 1.5)
ax.set_ylim(-1.5, 1.5)

# Tittle
ax.set_title(f'Inverted Pendulum Stabilization using PID Control\nCASE 2')

line, = ax.plot([], [], 'o-', lw=2)

# Reduce the number of frames
frames = len(y) // 2

# Create the animation
ani = animation.FuncAnimation(fig, animate, frames=frames, interval=20, blit

# Show the animation
HTML(ani.to_jshtml())
```

Out[204…





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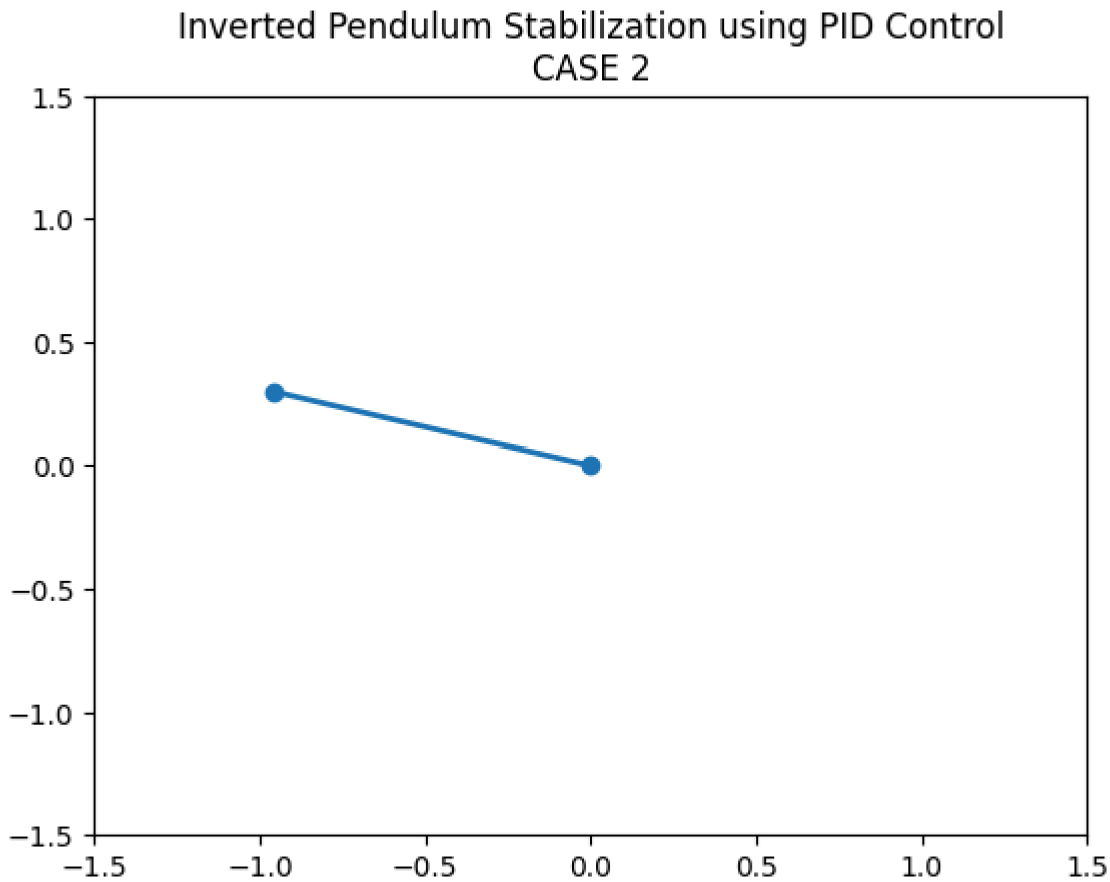

As expected, the pendule won't stabilise.

7. CONCLUSIONS

In conclusion, we have obtained 2 unstable systems (cases 1 and 2) that fluctuate to infinite and 2 stable systems (cases 3 and 4) that stabilise at approximately 1rad.

The last case, showed the best performance, needing less than 2 seconds to stabilise. This means the inverted pendule system would be stationary in very little time.

When we introduced a unitary step function to each case, we noticed that for the unstable cases, the fluctuations would appear quicker (and would still be unstable). However, the last 2 cases tend to the same stability as previously showcased.

BIBLIOGRAPHY:

- MIT handout on pole-zero plots:

  Author: Massachusetts Institute of Technology

Title: Pole-Zero Analysis

URL: https://web.mit.edu/2.14/www/Handouts/PoleZero.pdf

- University of Michigan Control Tutorials for MATLAB and Simulink, specifically the Inverted Pendulum example:

  Author: University of Michigan

  Title: Inverted Pendulum - Control Tutorials for MATLAB and Simulink

  URL: https://ctms.engin.umich.edu/CTMS/index.php?example=InvertedPendulum&section=ControlPID

- PID real life example:

  Author: @mci_mechatronik (Instagram)

  URL: https://www.instagram.com/reel/C5qVb4ciN_E/?igsh=a29ydWJ6a3BwbXlp

- LibreTexts page on PID tuning via classical methods:

  Author: LibreTexts

  Title: PID Tuning via Classical Methods

  Source: Chemical Process Dynamics and Controls (Woolf)

  URL:
  https://eng.libretexts.org/Bookshelves/Industrial_and_Systems_Engineering/Cher
  Integral-Derivative_(PID)_Control/9.03%3A_PID_Tuning_via_Classical_Methods