An Equilibrium Asset Pricing Model with Labor Market Search

Lars-Alexander Kuehn*
Carnegie Mellon University

Nicolas Petrosky-Nadeau†
Carnegie Mellon University

Lu Zhang‡
The Ohio State University
and NBER

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Abstract

Frictions in the labor market are important for understanding the equity premium in the financial market. We embed the Diamond-Mortensen-Pissarides search framework into a dynamic stochastic general equilibrium model with recursive preferences. The model produces realistically high equity premium and stock market volatility, as well as low interest rate and interest rate volatility. The equity premium is also strongly countercyclical, and forecastable with labor market tightness, a pattern we confirm in the data. Intriguingly, three key ingredients in the model — small profits, large job flows, and fixed matching costs — combine to give rise endogenously to rare economic disasters à la Rietz (1988) and Barro (2006).

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*Tepper School of Business, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh PA 15213. Tel: (412) 268-8501; e-mail: kuehn@cmu.edu.

†Tepper School of Business, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh PA 15213. Tel: (412) 268-4198; e-mail: npn@cmu.edu.

‡Fisher College of Business, The Ohio State University, 760A Fisher Hall, 2100 Neil Avenue, Columbus OH 43210; and NBER. Tel: (614) 292-8644; e-mail: zhanglu@fisher.osu.edu.

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1 Introduction

Modern asset pricing research has been successful in specifying preferences and cash flow dynamics to explain the equity premium, its volatility, and its cyclical variation in endowment economies. Accounting for the equity premium in production economies with endogenous cash flows has proven more difficult (e.g., Rouwenhorst (1995); Jermann (1998); Kaltenbrunner and Lochstoer (2010)). However, prior studies treat cash flows mostly as dividends. In the data labor income accounts for about two thirds, while dividends only account for a small fraction of aggregate disposable income. As such, an equilibrium macroeconomic model of asset prices should take the labor market seriously.

We study aggregate asset prices by embedding search frictions in the labor market (e.g., Diamond (1982); Mortensen (1982); Pissarides (1985, 2000)) into a dynamic stochastic general equilibrium economy with recursive preferences. A representative household pools incomes from its employed and unemployed workers, and decides on optimal consumption and asset allocation. The unemployed workers search for job vacancies posted by a representative firm. The rate at which a job vacancy is filled is affected by the congestion in the labor market. The degree of congestion is measured by labor market tightness, defined as the ratio of the number of job vacancies over the number of unemployed workers. Deviating from Walrasian equilibrium, search frictions create rents to be divided between the firm and the employed workers through the wage rate, which is in turn determined by the outcome of a generalized Nash bargaining process.

We find that labor market frictions are important for understanding the equity premium. Quantitatively, the search economy reproduces an equity premium of 5.70% and an average stock market volatility of 10.83% per annum. Both moments are adjusted for financial leverage, and are close to the moments in the data, 5.07% and 12.94%, respectively. The equity premium is also strongly countercyclical in the model. The vacancy-unemployment ratio forecasts stock market excess returns with a significantly negative slope, a pattern we confirm in the data. In the model, the average interest rate is 2.90%, which is somewhat high relative to 0.59% in the data. The interest rate volatility is 1.34%, which is close to 1.87% in the data. Finally, the model is also broadly consistent with business cycle moments as well as labor market moments.

Intriguingly, the search economy shows rare but deep disasters. In the stationary distribution from the model's simulations, the unemployment rate is positively skewed with a long right tail. The mean unemployment rate is 8.51%, and its skewness is 7.83. The 2.5 percentile is 5.87%, which is not far from the median of 7.30%, but the 97.5 percentile is far away, 19.25%. As such, output and consumption are both negatively skewed with a long left tail, giving rise endogenously to rare economic disasters per Rietz (1988) and Barro (2006). Applying Barro and Ursúa's (2008) peak-to-trough measurement on the simulated data, we find that the consumption and GDP disasters in the model have the same average magnitude, about 20%, as in the data. The consumption disaster probability is 3.08% in the model, which is close to 3.63% in the data. The GDP disaster probability is 4.66%, which is somewhat high relative to 3.69% in the data. However, both disaster probabilities in the data are within one cross-simulation standard deviation from the disaster probabilities in the model.

We show via comparative statics that three key ingredients of the model (small profits, large job flows, and fixed matching costs), when combined, are capable of producing disasters and a realistic equity premium. First, we calibrate the value of unemployment activities to be relatively high, implying small profits (output minus wages). Also, a high value of unemployment makes wages less elastic to labor productivity, giving rise to operating leverage. In bad times, output falls, but wages do not fall as much, causing profits to drop disproportionately more than output. As such, by dampening the procyclical covariation of wages, the small profits magnify the procyclical covariation (risk) of dividends, causing the equity premium to rise. Further, the impact of the inelastic wages is especially stronger in worse economic conditions, when the profits are even smaller (because of lower productivity). This time-varying operating leverage mechanism turbocharges the risk and risk premium, making the equity premium and the stock market volatility strongly countercyclical.

Second, job flows are large in the model. The U.S. labor market is characterized by large job flows in and out of employment. In particular, whereas the rate of capital depreciation is around 1% per month (e.g., Cooper and Haltiwanger (2006)), the worker separation rate is 5% in the data (e.g., Davis, Faberman, Haltiwanger, and Rucker (2010)). As such, contrary to swings in investment that have little impact on the disproportionately large capital stock, cyclical variations in job flows cause large fluctuations in aggregate employment. In particular, the large job flows out of employment put a tremendous strain on the labor market to reallocate unemployed workers back to work. Any frictions that disrupt this process in the labor market are likely to have a major impact on the macroeconomy. Consequently, economies with labor market frictions can be substantially riskier than baseline production economies without labor market frictions.

Third, we incorporate fixed matching costs per Mortensen and Nagypál (2007) and Pissarides (2009). The net effect is to make the unit cost of vacancy posting asymmetric (countercyclical) in our economy, whereas the unit cost is constant in the standard search model. This asymmetry impacts the economy via two channels. First, because wages contain a fraction of total vacancy costs, the asymmetry makes wages even more inelastic, increasing the operating leverage. Second, the asymmetry strengthens the amplification mechanism in the model. Intuitively, when a large negative shock hits the economy, the small profits become even smaller. To make a bad situation worse, the unit cost of vacancy goes up. As a result, the firm suppresses the incentives of hiring, stifling job creation flows. In the mean time, jobs continue to be destroyed at a high rate. Consequently, aggregate employment falls off a cliff, giving rise to economic disasters.

Our work provides two new insights to the macro finance literature. First, labor market frictions are important, if not critical, for equilibrium asset prices. In the baseline production economies, often with capital as the only productive input, the amount of endogenous risk is too small, giving rise to a negligible and time-invariant equity premium.[1] We show that labor market frictions seem to overcome many of these difficulties in production economies. Second, labor market frictions are capable of *endogenizing* rare economic disasters in production economies.[2] To our knowledge, most, if not all, the studies in the existing disasters risk literature specify disasters exogenously either on aggregate total factor productivity or on both aggregate productivity and capital stock. However, while there exists some evidence on consumption and GDP disasters, direct evidence on productivity disasters seems scarce. In our model, productivity follows a standard autoregressive process with homoscedastic shocks. As such, our endogenous disaster mechanism helps reconcile the existing exogenous disaster models with the lack of evidence on productivity disasters.

Our work is related to Danthine and Donaldson (2002) and Favilukis and Lin (2011), who also discuss the importance of wage contracting for asset prices. Instead of specifying wages exogenously, we differ by using the standard search framework with period-by-period Nash bargaining

[1] Rouwenhorst (1995) shows that the standard real business cycle model fails to explain the equity premium because of consumption smoothing. With internal habit preferences, Jermann (1998) and Boldrin, Christiano, and Fisher (2001) use capital adjustment costs and cross-sector immobility, respectively, to restrict consumption smoothing to explain the equity premium. However, both models struggle with excessively high interest rate volatilities. Using recursive preferences to curb interest rate volatility, Tallarini (2000) and Kaltenbrunner and Lochstoer (2010) show that baseline production economies with and without capital adjustment costs still fail to match the equity premium (see also Campanale, Castro, and Clementi (2010)).

[2] Rietz (1988) argues that rare disasters help explain the equity premium puzzle. Barro (2006), Barro and Ursúa (2008), and Barro and Jin (2011) examine long-term international data that include many disasters (see also Reinhart and Rogoff (2009)). Gourio (2010) embeds disasters exogenously into a production economy, and argues that disaster risks help explain the equity premium. However, Gourio defines dividends as levered output and treats the claim to the levered output as equity. The return on capital, which is the stock return in production economies, still has a low risk premium and a small volatility. Our search model goes one step further by producing a high and time-varying equity premium via endogenous disaster risks, while defining dividends as the net residual payout to shareholders.

to break the link between wages and marginal product of labor endogenously. Bazdresch, Belo, and Lin (2009) show that labor adjustment costs help explain the cross section of expected stock returns. We differ by examining equilibrium asset prices in a search economy.

Our work also adds to the labor search literature.[3] Methodologically, we solve the search model using a globally nonlinear projection algorithm with parameterized expectations, while imposing a nonnegativity constraint on vacancies. This solution method is new to the search literature. We find that the constraint does bind occasionally in the model's simulations, especially in parametrizations that imply small profits. More important, the globally nonlinear algorithm allows us to characterize disaster dynamics in the search model. In contrast, these dynamics have been missed so far in the search literature, in which models have traditionally been solved with local linearization methods.

The rest of the paper is organized as follows. Section 2 constructs the model. Section 3 calibrates and solves the model. Section 4 discusses quantitative results. Finally, Section 5 concludes.

2 The Model

We embed the standard Diamond-Mortensen-Pissarides (DMP hereafter) search model into a dynamic stochastic general equilibrium economy with recursive preferences.

2.1 Search and Matching

The model is populated by a representative household and a representative firm that uses labor as the single productive input. Following Merz (1995), we use the representative family construct, which implies perfect consumption insurance. In particular, the household has a continuum (of mass one) of members who are, at any point in time, either employed or unemployed. The fractions of employed and unemployed workers are representative of the population at large. The household pools the income of all the members together before choosing per capita consumption and asset holdings.

The representative firm posts a number of job vacancies, V_t, to attract unemployed workers, U_t. Vacancies are filled via a constant returns to scale matching function, $G(U_t, V_t)$, specified as:

$$G(U_t, V_t) = \frac{U_t V_t}{(U_t^{\iota} + V_t^{\iota})^{1/\iota}}, \tag{1}$$

[3] Merz (1995) and Andolfatto (1996) embed search frictions into the real business cycle framework. Shimer (2005) argues that the unemployment volatility in the search model is too low relative to that in the data. Hagedorn and Manovskii (2008) use small profits, and Mortensen and Nagypál (2007) and Pissarides (2009) use fixed matching costs to address the Shimer puzzle. We instead study equilibrium asset prices.

in which $\iota > 0$ is a constant parameter. This matching function, originated from Den Haan, Ramey, and Watson (2000), has the desirable property that matching probabilities fall between zero and one.

Specifically, define $\theta_t \equiv V_t/U_t$ as the vacancy-unemployment (V/U) ratio. The probability for an unemployed worker to find a job per unit of time (the job finding rate), $f(\theta_t)$, is:

$$f(\theta_t) \equiv \frac{G(U_t, V_t)}{U_t} = \frac{1}{\left(1 + \theta_t^{-\iota}\right)^{1/\iota}}, \tag{2}$$

and the probability for a vacancy to be filled per unit of time (the vacancy filling rate), $q(\theta_t)$, is:

$$q(\theta_t) \equiv \frac{G(U_t, V_t)}{V_t} = \frac{1}{(1 + \theta_t^{\iota})^{1/\iota}}. \tag{3}$$

It follows that $f(\theta_t) = \theta_t q(\theta_t)$ and $\partial q(\theta_t)/\partial \theta_t < 0$, meaning that an increase in the scarcity of unemployed workers relative to vacancies makes it harder for a firm to fill a vacancy. As such, θ_t is labor market tightness from the firm's perspective, and $1/q(\theta_t)$ is the average duration of vacancies.

The representative firm incurs costs in posting vacancies. Following Mortensen and Nagypál (2007) and Pissarides (2009), we assume that the unit cost per vacancy, denoted κ_t, contains two components, the proportional cost, κ_0, and the fixed cost, κ_1. Specifically,

$$\kappa_t \equiv \kappa_0 + \kappa_1 q(\theta_t), \tag{4}$$

in which $\kappa_0, \kappa_1 > 0$. The proportional cost is standard in the labor search literature. The fixed cost aims to capture matching costs, such as training, interviewing, negotiation, and administrative setup costs of adding a worker to the payroll. These costs are paid after a hired worker arrives but before the wage bargain takes place. The cost of hiring a worker arising from the proportional cost, $\kappa_0/q(\theta_t)$, increases with the mean duration of vacancies, $1/q(\theta_t)$. In contrast, the hiring cost arising from the fixed cost, κ_1, is "fixed" as it is independent of the duration of vacancies.

Once matched, jobs are destroyed at a constant rate of s per period. Employment, N_t, evolves as:

$$N_{t+1} = (1 - s)N_t + q(\theta_t)V_t. \tag{5}$$

The functional form of new hires, $q(\theta_t)V_t$, is reminiscent of the capital installation function (e.g., Jermann (1998, equation (2.2))). The key difference is that the labor installation costs (to be distinguished from the unit cost of vacancy) are endogenously procyclical. In particular, the labor market is tighter for the firm in good times (θ_t is higher), meaning that the vacancy filling rate, $q(\theta_t)$, is lower. As such, it is more costly for the firm to fill a given vacancy. This procyclicality propagates negative shocks in the model, making slumps longer and more painful. In contrast, this endogenous propagation mechanism is absent in the standard installation function for capital adjustment.

Finally, because the size of the population is normalized to be unity, $U_t = 1 - N_t$. As such, N_t and U_t can also be interpreted as the rates of employment and unemployment, respectively.

2.2 The Representative Firm

The firm takes aggregate labor productivity, X_t, as given. The law of motion of $x_t \equiv \log(X_t)$ is:

$$x_{t+1} = \rho x_t + \sigma \epsilon_{t+1}, \tag{6}$$

in which $\rho \in (0, 1)$ is the persistence, $\sigma > 0$ is the conditional volatility, and ϵ_{t+1} is an independently and identically distributed (i.i.d.) standard normal shock. The firm uses labor to produce output, Y_t, with a constant returns to scale production technology,

$$Y_t = X_t N_t. \tag{7}$$

The dividends to the firm's shareholders are given by:

$$D_t = X_t N_t - W_t N_t - [\kappa_0 + \kappa_1 q(\theta_t)] V_t, \tag{8}$$

in which W_t is the wage rate (to be determined later in Section 2.4). Let $M_{t+\triangle t}$ be the representative household's stochastic discount factor from period t to $t+\triangle t$. Taking the matching probability, $q(\theta_t)$, and the wage rate, W_t, as given, the firm posts an optimal number of job vacancies to maximize the cum-dividend market value of equity, denoted S_t:

$$S_t \equiv \max_{\{V_{t+\triangle t}, N_{t+\triangle t+1}\}_{\triangle t=0}^{\infty}} E_t \left[\sum_{\triangle t=0}^{\infty} M_{t+\triangle t} \left[X_{t+\triangle t} N_{t+\triangle t} - W_{t+\triangle t} N_{t+\triangle t} - [\kappa_0 + \kappa_1 q(\theta_t)] V_{t+\triangle t} \right] \right], \tag{9}$$

subject to the employment accumulation equation (5) and a nonnegativity constraint on vacancies:

$$V_t \geq 0. \tag{10}$$

Because $q(\theta_t) > 0$, this constraint is equivalent to $q(\theta_t) V_t \geq 0$. As such, the only source of job destruction in the model is the exogenous separation of employed workers from the firm.[4]

[4]The vacancy nonnegativity constraint has been ignored so far in the labor search literature, in which models are typically solved via local linearization methods. However, using global projection methods, we find that the

Let μ_t denote the Lagrange multiplier on the employment accumulation equation (5), and λ_t the multiplier on the nonnegativity constraint $q(\theta_t)V_t \geq 0$. The first-order conditions with respect to V_t and N_{t+1} in maximizing the market value of equity are given by, respectively:

$$\mu_t = \frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t, \tag{11}$$

$$\mu_t = E_t \left[M_{t+1} \left[X_{t+1} - W_{t+1} + (1-s)\mu_{t+1} \right] \right]. \tag{12}$$

Combining the two first-order conditions yields the intertemporal job creation condition:

$$\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t = E_t \left[M_{t+1} \left[X_{t+1} - W_{t+1} + (1-s) \left[\frac{\kappa_0}{q(\theta_{t+1})} + \kappa_1 - \lambda_{t+1} \right] \right] \right]. \tag{13}$$

The optimal vacancy policy also satisfies the Kuhn-Tucker conditions:

$$q(\theta_t)V_t \geq 0, \quad \lambda_t \geq 0, \quad \text{and} \quad \lambda_t q(\theta_t) V_t = 0. \tag{14}$$

Intuitively, equation (11) says that the marginal cost of hiring a worker with the nonnegativity constraint accounted for, $\kappa_0/q(\theta_t) + \kappa_1 - \lambda_t$, equals the marginal value of employment, μ_t. When the firm posts vacancies, $V_t > 0$ and $\lambda_t = 0$. Equation (11) says that the marginal cost of hiring, which is the unit cost of vacancy divided by the probability of a successful match, $q(\theta_t)$, is equal to the marginal value of employment. When the nonnegativity constraint is binding, $V_t = 0$, $\lambda_t > 0$, $\theta_t = V_t/U_t = 0$, and $q(\theta_t) = (1+\theta_t^\iota)^{-1/\iota} = 1$. As such, equation (11) reduces to $\mu_t = \kappa_0 + \kappa_1 - \lambda_t$.

The intertemporal job creation condition (13) is intuitive. The marginal cost of hiring at period t equals the marginal value of employment, μ_t, which in turn equals the marginal benefit of hiring at period $t+1$, discounted to t with the stochastic discount factor, M_{t+1}. The marginal benefit at period $t+1$ includes the marginal product of labor, X_{t+1}, net of the wage rate, W_{t+1}, plus the marginal value of employment, μ_{t+1}, which in turn equals the marginal cost of hiring at $t+1$, net of separation.

Recalling S_t is the cum-dividend equity value, we define the stock return as $R_{t+1} \equiv S_{t+1}/(S_t - D_t)$. The constant returns to scale assumption implies that (see Appendix A for derivations):

$$R_{t+1} = \frac{X_{t+1} - W_{t+1} + (1-s)\left[\kappa_0/q(\theta_{t+1}) + \kappa_1 - \lambda_{t+1}\right]}{\kappa_0/q(\theta_t) + \kappa_1 - \lambda_t}. \tag{15}$$

Intuitively, the stock return is the quantitative tradeoff between the marginal benefit of hiring at period $t+1$ and the marginal cost of hiring at period t, as in Cochrane (1991).

nonnegativity constraint is occasionally binding in the search model, especially when the profits are small. As such, we impose the nonnegativity constraint on vacancies to solve the model more accurately.

2.3 The Representative Household

The household maximizes utility, denoted J_t, over consumption using recursive preferences (e.g., Kreps and Porteus (1978); Epstein and Zin (1989)) by trading risky shares issued by the representative firm and a risk-free bond. Let C_t denote consumption. The recursive utility function is given by:

$$J_t = \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left(E_t\left[J_{t+1}^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}, \tag{16}$$

in which β is time discount factor, ψ is the elasticity of intertemporal substitution, and γ is relative risk aversion. This utility function separates ψ from γ, allowing the model to produce a high equity premium and a low interest rate volatility simultaneously.

The household's first-order condition gives rise to the fundamental equation of asset pricing:

$$1 = E_t[M_{t+1}R_{t+1}]. \tag{17}$$

In particular, the stochastic discount factor, M_{t+1}, is given by:

$$M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{J_{t+1}}{E_t[J_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}. \tag{18}$$

Finally, the risk-free rate is given by $R_{t+1}^f = 1/E_t[M_{t+1}]$.

2.4 Wage Determination

The wage rate is determined endogenously by applying the sharing rule per the outcome of a generalized Nash bargaining process between the employed workers and the firm. Let $\eta \in (0,1)$ be the workers' relative bargaining weight and b the workers' value of unemployment activities. The equilibrium wage rate is given by (see Appendix B for derivations):

$$W_t = \eta\left(X_t + [\kappa_0 + \kappa_1 q(\theta_t)]\,\theta_t\right) + (1-\eta)b. \tag{19}$$

The wage rate is increasing in labor productivity, X_t, and in the savings of the total vacancy costs per unemployed worker, $\kappa_t\theta_t = \kappa_t V_t/U_t$. Intuitively, the more productive the workers are, and the more costly for the firm to fill a vacancy, the higher the wage rate is for employed workers.

Also, the value of unemployment activities, b, and the workers' bargaining weight, η, affect the elasticity of wage with respect to productivity. The lower η is, and the higher b is, the more the wage is tied with the constant unemployment value, inducing a lower wage elasticity to productivity. Finally, because θ_t is procyclical and $q(\theta_t)$ is countercyclical, the presence of the $\kappa_1 q(\theta_t)$ term in the wage equation due to fixed matching costs makes the wage rate more inelastic to productivity.

2.5 Competitive Equilibrium

In equilibrium, the financial markets clear. The risk-free asset is in zero net supply, and the household holds all the shares of the representative firm. As such, the equilibrium return on wealth equals the stock return, and the household's financial wealth equals the cum-dividend equity value of the firm. The goods market clearing condition is then given by:

$$C_t + [\kappa_0 + \kappa_1 q(\theta_t)]\, V_t = X_t N_t. \tag{20}$$

The competitive equilibrium in the search economy consists of vacancy posting, $V_t^\star \geq 0$; multiplier, $\lambda_t^\star \geq 0$; consumption, $C_t^\star$; and indirect utility, $J_t^\star$; such that (i) $V_t^\star$ and $\lambda_t^\star$ satisfy the intertemporal job creation condition (13) and the Kuhn-Tucker conditions (14), while taking the stochastic discount factor in equation (18) and the wage equation (19) as given; (ii) $C_t^\star$ and $J_t^\star$ satisfy the intertemporal consumption-portfolio choice condition (17), in which the stock return is given by equation (15); and (iii) the goods market clears as in equation (20).

3 Calibration, Computation, and the Model's Solution

We calibrate the model in Section 3.1, discuss our global solution algorithm in Section 3.2, and describe the basic properties of the model's solution in Section 3.3.

3.1 Calibration

We calibrate the model in monthly frequency. Table 1 lists the parameter values in the benchmark calibration. For the five tastes and technology parameters, our general calibration strategy is to use values that are (largely) standard in the literature. In particular, following Bansal and Yaron (2004), we set the risk aversion, γ, to be 10, and the elasticity of intertemporal substitution, ψ, to be 1.5. Following Gertler and Trigari (2009), we set the time discount factor, β, to be $0.99^{1/3}$, the persistence of the (log) aggregate productivity, ρ, to be $0.95^{1/3}$, and the conditional volatility of the aggregate productivity, σ, to be 0.0077. In particular, the σ value is chosen so that the volatilities of consumption growth and output growth in the model are largely in line with those in the data.

Table 1 : Parameter Values in the Benchmark Monthly Calibration

Notation	Parameter	Value
β	Time discount factor	$0.99^{1/3}$
γ	Relative risk aversion	10
ψ	The elasticity of intertemporal substitution	1.5
ρ	Aggregate productivity persistence	0.983
σ	Conditional volatility of productivity shocks	0.0077
η	Workers' bargaining weight	0.052
b	The value of unemployment activities	0.85
s	Job separation rate	0.05
ι	Elasticity of the matching function	1.25
κ_0	The proportional cost of vacancy posting	0.6
κ_1	The fixed cost of vacancy posting	0.4

For the labor market parameters, our general calibration strategy is to use existing evidence and quantitative studies as much as we can to restrict their values. For the parameters whose values are important in driving our quantitative results, we conduct extensive comparative statics to evaluate their impact and to understand the underlying mechanism. It is worthwhile pointing out that our calibration strategy differs from the standard practice in the search literature that relies only on steady state relations. In our highly nonlinear asset pricing model, steady state restrictions hold very poorly in the model's simulations. This nonlinearity means that matching a given moment precisely in simulations is virtually impossible. As such, our strategy is to report a wide range of simulated moments to compare with moments in the data.

Our calibration of the workers' bargaining weight, η, and the value of unemployment activities, b, is in the same spirit as in Hagedorn and Manovskii (2008). We set η to be 0.052, which is the same value in Hagedorn and Manovskii. The calibration of b is more controversial in the search literature. Shimer (2005) pins down $b = 0.4$ by assuming that the only benefit for an unemployed worker is the government unemployment insurance. In contrast, Hagedorn and Manovskii argue that in a perfect competitive labor market, b should equal the value of employment. In particular, the value of unemployment activities measures not only unemployment insurance, but also the total value of home production, self-employment, disutility of work, and leisure. In the model, the average marginal product of labor is unity, to which b should be close. We set b to be 0.85, which is the same value in Rudanko (2011). Although high, this b value is not as extreme as 0.955 in Hagedorn and Manovskii.

More generally, we view the low-η-high-b calibration only as a parsimonious modeling device to obtain small profits and inelastic wages, both of which turn out to be critical for asset prices and disaster dynamics. This parsimony is valuable, both conceptually as a first stab in embedding the DMP structure into an equilibrium asset pricing framework, as well as pragmatically as a first step toward solving the search model nonlinearly (see Section 3.2 for our solution algorithm). We have nothing new to say about the Shimer puzzle. Rather, our insight is that a search model with small profits and inelastic wages has intriguing implications for asset prices and disaster dynamics. Other specifications of the search model with small profits and inelastic wages are likely to have similar implications. However, to what extent this statement is true, quantitatively, is left for future research.

We calibrate the job separation rate, s, to be 5%. This value, which is also used in Andolfatto (1996), is estimated in Davis, Faberman, Haltiwanger, and Rucker (2010, Table 5.4), and is within the range of estimates from Davis, Faberman, and Haltiwanger (2006). This estimate is higher than 3.7% from the publicly available Job Openings and Labor Turnover Survey (JOLTS). As pointed out by Davis, Faberman, Haltiwanger, and Rucker, the JOLTS sample overweights relatively stable establishments with low rates of hires and separations and underweights establishments with rapid growth or contraction (see also Hall (2010)). For the elasticity parameter in the matching function, ι, we set it to be 1.25, which is close to the value in Den Haan, Ramey, and Watson (2000). We also conduct a comparative static experiment by varying its value to 0.9.

To pin down the two vacancy cost parameters, κ_0 and κ_1, we first experiment so that the unit cost of vacancy posting is on average around 0.8 in the model's simulations. This level of the average unit cost is necessary for the model to reproduce a realistic unemployment rate. The average unemployment rate for the United States over the 1920–2009 period is about 7%. However, flows in and out of nonparticipation in the labor force as well as discouraged workers not accounted for in the pool of individuals seeking employment suggest that the unemployment rate should be somewhat higher. In the benchmark calibration, the unemployment rate is 8.51% in simulations. The evidence on the relative weights of the proportional cost and the fix cost out of the total unit cost of vacancy seems scarce. To pin down κ_0 and κ_1 separately, we set the weight of the fixed cost to be 25%, meaning $\kappa_0 = 0.6$ and $\kappa_1 = 0.4$. We also conduct a comparative static experiment in which the weight of the fixed cost is zero, meaning the unit cost of vacancy is constant, around 0.8.

Is the magnitude of the vacancy (hiring) costs in the model empirically plausible? The model implies that the marginal cost of vacancy in terms of labor productivity (output per worker) is

0.815, which is the average of $\kappa_0 + \kappa_1 q(\theta_t)$ in simulations (the average labor productivity is unity). The marginal cost of hiring is 1.588, which is the average of $\kappa_0/q(\theta_t) + \kappa_1$. Merz and Yashiv (2007) estimate the marginal cost of hiring to be 1.48 times the average output per worker with a standard error of 0.57. As such, the value of 1.588 is well within the plausible range of empirical estimates. For the total costs of vacancy, $\kappa_t V_t$, the average in the model's simulations is about 0.67% of annual wages. This magnitude does not appear implausibly high. In particular, the estimated labor adjustment costs in Bloom (2009) imply "limited hiring and firing costs of about 1.8% of annual wages" and "a high fixed cost of around 2.1% of annual revenue (p. 663)."

3.2 Computational Methods

Although analytically transparent, solving the model numerically is quite challenging for several reasons. First, the search economy is not Pareto optimal because the competitive equilibrium does not correspond to the social planner's solution. Intuitively, the firm in the decentralized economy does not take into account the congestion effect of posting a new vacancy on the labor market when maximizing the equity value, whereas the social planner does when maximizing social welfare. As such, we must solve for the competitive equilibrium from the optimality conditions directly. Unlike value function iteration, algorithms that approximate the solution to the optimality conditions do not have nice convergence properties. Second, because of the occasionally binding constraint on vacancy, standard perturbation methods cannot be used. As such, we solve for the competitive equilibrium using a globally nonlinear projection algorithm, while applying the Christiano and Fisher (2000) idea of parameterized expectations to handle the vacancy constraint.

Third, because of the model's nonlinearity and our focus on nonlinearity-sensitive asset pricing and disaster moments, we must solve the model on a large, fine grid to ensure accuracy. We must also apply homotopy to visit the parameter space in which the model exhibits strong nonlinearity. Because many economically interesting parameterizations imply strong nonlinearity, we can only update the parameter values very slowly to ensure the convergence of the projection algorithm.

The state space of the model consists of employment and productivity, (N_t, x_t). The goal is to solve for the optimal vacancy function: $V_t^\star = V(N_t, x_t)$, the multiplier function: $\lambda_t^\star = \lambda(N_t, x_t)$, and an indirect utility function: $J_t^\star = J(N_t, x_t)$ from two functional equations:

$$J(N_t, x_t) = \left[(1-\beta)C(N_t, x_t)^{1-\frac{1}{\psi}} + \beta \left(E_t\left[J(N_{t+1}, x_{t+1})^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}} \tag{21}$$

$$\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda(N_t, x_t) = E_t\left[M_{t+1}\left[X_{t+1} - W_{t+1} + (1-s)\left[\frac{\kappa_0}{q(\theta_{t+1})} + \kappa_1 - \lambda(N_{t+1}, x_{t+1})\right]\right]\right]. \tag{22}$$

$V(N_t, x_t)$ and $\lambda(N_t, x_t)$ must also satisfy the Kuhn-Tucker conditions (14).

The standard projection method would approximate $V(N_t, x_t)$ and $\lambda(N_t, x_t)$ to solve equations (21) and (22), while obeying the Kuhn-Tucker conditions. With the occasionally binding constraint, the vacancy and multiplier functions are not smooth, making the standard projection method tricky and cumbersome to apply. As such, we adapt the Christiano and Fisher (2000) parameterized expectations method by approximating the right-hand side of equation (22):

$$\mathcal{E}_t \equiv \mathcal{E}(N_t, x_t) = E_t\left[M_{t+1}\left[X_{t+1} - W_{t+1} + (1-s)\left[\frac{\kappa_0}{q(\theta_{t+1})} + \kappa_1 - \lambda(N_{t+1}, x_{t+1})\right]\right]\right]. \tag{23}$$

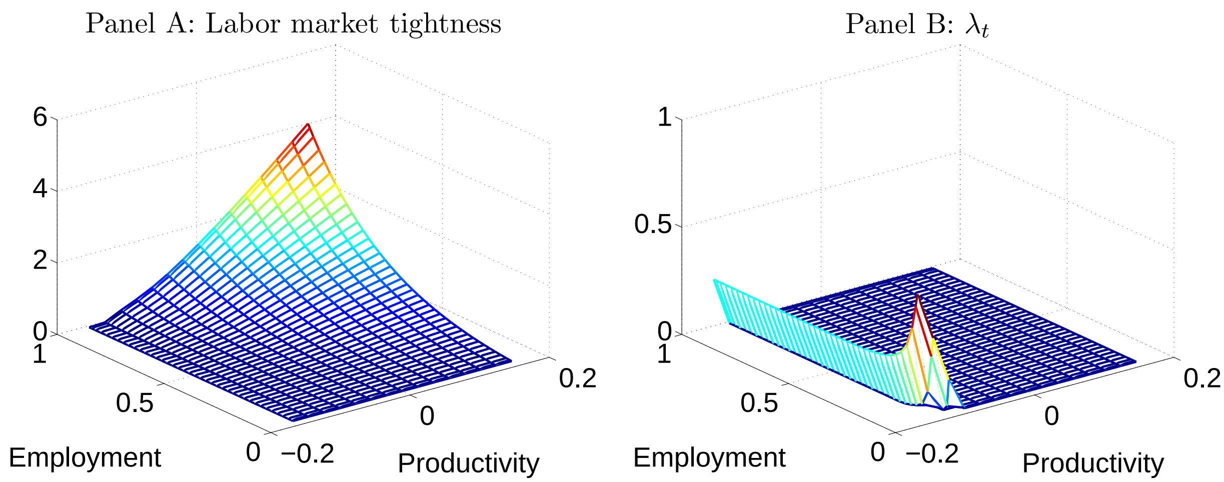
We then exploit a convenient mapping from the conditional expectation function to policy and multiplier functions, thereby eliminating the need to parameterize the multiplier function separately. Specifically, after obtaining the parameterized $\mathcal{E}_t$, we first calculate $\tilde{q}(\theta_t) = \kappa_0/\left(\mathcal{E}_t - \kappa_1\right)$. If $\tilde{q}(\theta_t) < 1$, the nonnegativity constraint is not binding, we set $\lambda_t = 0$ and $q(\theta_t) = \tilde{q}(\theta_t)$. We then solve $\theta_t = q^{-1}(\tilde{q}(\theta_t))$, in which $q^{-1}(\cdot)$ is the inverse function of $q(\cdot)$ defined in equation (3), and $V_t = \theta_t(1 - N_t)$. If $\tilde{q}(\theta_t) \geq 1$, the nonnegativity constraint is binding, we set $V_t = 0$, $\theta_t = 0$, $q(\theta_t) = 1$, and $\lambda_t = \kappa_0 + \kappa_1 - \mathcal{E}_t$. This approach is computationally convenient because it enforces the Kuhn-Tucker conditions automatically, without the need of parameterizing the multiplier function. Appendix C contains additional details of our computational methods.

3.3 Basic Properties of the Model's Solution

Panel A of Figure 1 plots the vacancy-unemployment ratio (labor market tightness) against employment and labor productivity. We see that labor market tightness is increasing in both states. The labor market is tighter when there are fewer unemployed workers searching for jobs (employment is high), and when the demand for workers is high (productivity is high). More generally, increases in productivity lead the firm to post more vacancies to create more jobs.

From Panel B, the multiplier of the occasionally binding constraint on vacancy, λ_t, is countercyclical. λ_t equals zero for most values of productivity, but turns positive as productivity approaches

Figure 1 : Labor Market Tightness and the Multiplier of the Occasionally Binding Constraint on Vacancy (λ_t)



its lowest level. The multiplier is also convex in employment. λ_t is flat across most values of employment, but rises with an increasing speed as it approaches the lowest level. Intuitively, when the constraint is binding, $V_t = 0, \theta_t = 0$, $q(\theta_t) = 1$, and $\lambda_t = \kappa_0 + \kappa_1 - \mathcal{E}_t$, in which $\mathcal{E}_t$ is the expectation in equation (23). As the economy approaches the low-employment-low-productivity states, $\mathcal{E}_t$ drops rapidly. This nonlinearity is due to the stochastic discount factor, M_{t+1}. As consumption approaches zero, marginal utility blows up, causing $\mathcal{E}_t$ to fall and λ_t to rise precipitously. As noted, the vacancy nonnegativity constraint has been so far ignored in the labor search literature. Armed with our global (albeit computationally intensive) solution algorithm, we show that this vacancy constraint should be properly accounted for. As shown in Panel B, the constraint is most binding in extreme bad times, to which asset pricing and disaster moments are most sensitive.

4 Quantitative Results

We present basic business cycle and asset pricing moments in Section 4.1 and labor market moments in Section 4.2. In Section 4.3, we examine the linkage between the labor market and the financial market by using labor market tightness to forecast stock market excess returns. In Section 4.4, we quantify the model's endogenous disaster risks that are important for asset prices. We study the model's implications for long run risks and uncertainty shocks in Section 4.5, and cyclical dividend dynamics in Section 4.6. Finally, Section 4.7 reports several comparative static experiments.

4.1 Basic Business Cycle and Financial Moments

Panel A of Table 2 reports the standard deviation and autocorrelations of log consumption growth and log output growth, as well as unconditional financial moments in the data. Consumption is annual real personal consumption expenditures, and output is annual real gross domestic product from 1929 to 2010 from the National Income and Product Accounts (NIPA) at Bureau of Economic Analysis. The annual consumption growth in the data has a volatility of 3.04%, and a first-order autocorrelation of 0.38. The autocorrelation drops to 0.08 at the two-year horizon, and turns negative, -0.21, at the three-year horizon. The annual output growth has a volatility of 4.93% and a high first-order autocorrelation of 0.54. The autocorrelation drops to 0.18 at the two-year horizon, and turns negative afterward: -0.18 at the three-year horizon and -0.23 at the five-year horizon.

We obtain monthly series of the value-weighted market returns including all NYSE, Amex, and Nasdaq stocks, one-month Treasury bill rates, and inflation rates (the rates of change in Consumer Price Index) from Center for Research in Security Prices (CRSP). The sample is from January 1926 to December 2010 (1,020 months). The mean of real interest rates (one-month Treasury bill rates minus inflation rates) is 0.59% per annum, and the annualized volatility is 1.87%.

The equity premium (the average of the value-weighted market returns in excess of one-month Treasury bill rates) in the 1926–2010 sample is 7.45% per annum. Because we do not model financial leverage, we adjust the equity premium in the data for leverage before matching with the equity premium from the model. Frank and Goyal (2008) report that the aggregate market leverage ratio of U.S. corporations is stable around 0.32. As such, we calculate the leverage-adjusted equity premium as $(1-0.32)\times 7.45\% = 5.07\%$ per annum. The annualized volatility of the market returns in excess of inflation rates is 18.95%. Adjusting for leverage (taking the leverage-weighted average of real market returns and real interest rates) yields an annualized volatility of 12.94%.

Panel B of Table 2 reports the model moments. From the initial condition of zero for log productivity, x_t, and 0.90 for employment, N_t, we first simulate the economy for 6,000 monthly periods to reach its stationary distribution. We then repeatedly simulate 1,000 artificial samples, each with 1,020 months. On each artificial sample, we calculate the annualized monthly averages of the equity premium and the real interest rate, as well as the annualized monthly volatilities of the market returns and the real interest rate. We also take the first 984 monthly observations of consumption and output, and time-aggregate them into 82 annual observations. (We add up 12 monthly obser-

Table 2 : Basic Business Cycle and Financial Moments

In Panel A, consumption is annual real personal consumption expenditures (series PCECCA), and output is annual real gross domestic product (series GDPCA) from 1929 to 2010 (82 annual observations) from NIPA (Table 1.1.6) at Bureau of Economic Analysis. σ^C is the volatility of log consumption growth, and σ^Y is the volatility of log output growth. Both volatilities are in percent. $\rho^C(\tau)$ and $\rho^Y(\tau)$, for $\tau = 1, 2, 3$, and 5, are the τ-th order autocorrelations of log consumption growth and log output growth, respectively. We obtain monthly series from January 1926 to December 2010 (1,020 monthly observations) for the value-weighted market index returns including dividends, one-month Treasury bill rates, and the rates of change in Consumer Price Index (inflation rates) from CRSP. $E[R - R^f]$ is the average (in annualized percent) of the value-weighted market returns in excess of the one-month Treasury bill rates, adjusted for the long-term market leverage rate of 0.32 reported by Frank and Goyal (2008). (The leverage-adjusted average $E[R - R^f]$ is the unadjusted average times 0.68.) $E[R^f]$ and σ^{R^f} are the mean and volatility, both of which are in annualized percent, of real interest rates, defined as the one-month Treasury bill rates in excess of the inflation rates. σ^R is the volatility (in annualized percent) of the leverage-weighted average of the value-weighted market returns in excess of the inflation rates and the real interest rates. In Panel B, we simulate 1,000 artificial samples, each of which has 1,020 monthly observations, from the model in Section 2. In each artificial sample, we calculate the mean market excess return, $E[R - R^f]$, the volatility of the market return, σ^R, as well as the mean, $E[R^f]$, and volatility, σ^{R^f}, of the real interest rate. All these moments are in annualized percent. We time-aggregate the first 984 monthly observations of consumption and output into 82 annual observations in each sample, and calculate the annual volatilities and autocorrelations of log consumption growth and log output growth. We report the mean and the 5 and 95 percentiles across the 1,000 simulations. The p-values are the percentages with which a given model moment is larger than its data moment.

	Panel A: Data	Panel B: Model			
		Mean	5%	95%	p-value
σ^C	3.036	3.633	1.922	8.121	0.458
$\rho^C(1)$	0.383	0.183	−0.035	0.463	0.100
$\rho^C(2)$	0.081	−0.141	−0.346	0.094	0.059
$\rho^C(3)$	−0.206	−0.126	−0.347	0.097	0.730
$\rho^C(5)$	0.062	−0.073	−0.279	0.141	0.142
σ^Y	4.933	4.133	2.472	8.393	0.172
$\rho^Y(1)$	0.543	0.185	−0.026	0.447	0.025
$\rho^Y(2)$	0.178	−0.132	−0.329	0.082	0.020
$\rho^Y(3)$	−0.179	−0.122	−0.332	0.093	0.681
$\rho^Y(5)$	−0.227	−0.075	−0.276	0.141	0.891
$E[R - R^f]$	5.066	5.695	4.869	6.517	0.894
$E[R^f]$	0.588	2.898	2.495	3.176	1.000
σ^R	12.942	10.830	9.935	11.824	0.001
σ^{R^f}	1.872	1.342	0.800	2.264	0.106

vations within a given year, and treat the sum as the year's annual observation.) For each data moment, we report the average as well as the 5 and 95 percentiles across the 1,000 simulations. The p-values are the frequencies with which a given model moment is larger than its data counterpart.

The model predicts a consumption growth volatility of 3.63% per annum, which is somewhat higher than 3.04% in the data. However, this data moment lies within the 90% confidence interval of the model's bootstrapped distribution with a bootstrapped p-value of 0.46. The model also implies a positive first-order autocorrelation of 0.18 for consumption growth, which is lower than 0.38 in the data. At longer horizons, consumption growth in the model are all negatively autocorrelated. All the autocorrelations in the data are within 90% confidence interval of the model. The output growth volatility implied by the model is 4.13% per annum, which is somewhat lower than 4.93% in the data. The model implies a positive first-order autocorrelation of 0.19 for the output growth, which is lower than 0.54 in the data. The model also implies a negative second-order autocorrelation of -0.13, but this autocorrelation is 0.18 in the data. Both correlations in the data are outside the 90% confidence interval of the model's bootstrapped distribution. However, at longer horizons, the autocorrelations are negative in the model, consistent with the data.

The model seems to perform well in matching financial moments. The equity premium is 5.70% per annum, which is somewhat higher than the leverage-adjusted equity premium of 5.07% in the data. This data moment lies within the 90% confidence interval of the model's bootstrapped distribution. The volatility of the stock market return in the model is 10.83% per annum, which is close to the leverage-adjusted market volatility of 12.94% in the data. The volatility of the interest rate in the model is 1.34%, close to 1.87% in the data. However, the model implies an average interest rate of 2.90% per annum, which is somewhat high relative to 0.59% in the data. Overall, the model's fit of the financial moments, especially the stock market volatility, seems notable. As shown in Tallarini (2000) and Kaltenbrunner and Lochstoer (2010), baseline production economies with recursive preferences and capital adjustment costs struggle to reproduce a high stock market volatility.

4.2 Labor Market Moments

To evaluate the model's fit for labor market moments, we first document these moments per Hagedorn and Manovskii (2008, Table 3) using an updated sample. We obtain seasonally adjusted monthly unemployment (thousands of persons 16 years of age and older) from the Bureau of Labor Statistics (BLS), and seasonally adjusted help wanted advertising index from the Conference Board.

The sample is from January 1951 to June 2006.[5] We take quarterly averages of the monthly series to obtain 222 quarterly observations. The average labor productivity is seasonally adjusted real average output per person in the nonfarm business sector from BLS.

Hagedorn and Manovskii (2008) report all variables in log deviations from the Hodrick-Prescott (1997, HP) trend with a smoothing parameter of 1,600. In contrast, we detrend all variables as the HP-filtered cyclical component of proportional deviations from the mean (with the same smoothing parameter).[6] We do not take logs because vacancies can be zero in the model's simulations when the nonnegativity constraint on vacancy is binding. In the data, the two detrending methods yield quantitatively similar results, which are in turn close to Hagedorn and Manovskii's. In particular, from Panel A of Table 3, the standard deviation of the V/U ratio is 0.26. The V/U ratio is procyclical with a positive correlation of 0.30 with labor productivity. Finally, vacancy and unemployment have a negative correlation of -0.91, indicating a downward-sloping Beveridge curve.

To evaluate the model's fit with the labor market moments, we simulate 1,000 artificial samples, each with 666 months. We take the quarterly averages of the monthly unemployment, U, vacancy, V, and labor productivity, X, to obtain 222 quarterly observations for each series. We then apply the exactly same procedures as in Panel A of Table 3 on the artificial data, and report the cross-simulation averages (as well as standard deviations) for the model moments.

Panel B reports the model's quantitative performance. The standard deviations of U and V in the model are 0.15 and 0.12, respectively, which are close to those in the data. The model implies a standard deviation of 0.17 for the V/U ratio, which is lower than 0.26 in the data. The model also generates a Beveridge curve with a negative U-V correlation of -0.57, but its magnitude is lower than -0.91 in the data. The correlation between the V/U ratio and labor productivity is 0.99, which is higher than 0.30 in the data.

4.3 The Linkage between the Labor Market and the Financial Market

A large literature in financial economics shows that the equity risk premium is time-varying (countercyclical) in the data (e.g., Lettau and Ludvigson (2001)). In the labor market, as vacancies are procyclical and unemployment is countercyclical, the V/U ratio is strongly procyclical (e.g., Shimer

[5]The sample ends in June 2006 because the Conference Board switched from help wanted advertising index to help wanted online index around that time. The two indexes are not directly comparable. As such, we follow the standard practice in the labor search literature in using the longer time series before the switch.

[6]Specifically, for any variable Z, the HP-filtered cyclical component of proportional deviations from the mean is calculated as $(Z-\bar{Z})/\bar{Z}-\text{HP}[(Z-\bar{Z})/\bar{Z}]$, in which $\bar{Z}$ is the mean of Z, and $\text{HP}[(Z-\bar{Z})/\bar{Z}]$ is the HP trend of $(Z-\bar{Z})/\bar{Z}$.

Table 3 : Labor Market Moments

In Panel A, seasonally adjusted monthly unemployment (U, thousands of persons 16 years of age and older) is from the Bureau of Labor Statistics. The seasonally adjusted help wanted advertising index, V, is from the Conference Board. The series are monthly from January 1951 to June 2006 (666 months). Both U and V are converted to 222 quarterly averages of monthly series. The average labor productivity, X, is seasonally adjusted real average output per person in the nonfarm business sector from the Bureau of Labor Statistics. All variables are in HP-filtered proportional deviations from the mean with a smoothing parameter of 1,600. In Panel B, we simulate 1,000 artificial samples, each of which has 666 monthly observations. We take the quarterly averages of monthly U, V, and X to convert to 222 quarterly observations. We implement the exactly same empirical procedures as in Panel A on these quarterly series, and report the cross-simulation averages and standard deviations (in parentheses) for all the model moments.

	U	V	V/U	X	
		Panel A: Data			
Standard deviation	0.119	0.134	0.255	0.012	
Quarterly autocorrelation	0.902	0.922	0.889	0.761	
Correlation matrix		−0.913	−0.801	−0.224	U
			0.865	0.388	V
				0.299	V/U
		Panel B: Model			
Standard deviation	0.151	0.116	0.173	0.016	
	(0.082)	(0.019)	(0.033)	(0.002)	
Quarterly autocorrelation	0.843	0.685	0.788	0.772	
	(0.058)	(0.055)	(0.038)	(0.040)	
Correlation matrix		−0.568	−0.647	−0.637	U
		(0.123)	(0.147)	(0.151)	
			0.949	0.974	V
			(0.036)	(0.016)	
				0.992	V/U
				(0.015)	

(2005)). As such, the V/U ratio should forecast stock market excess returns with a negative slope at business cycle frequencies. Panel A of Table 4 documents such predictability in the data.

Specifically, we perform monthly long-horizon regressions of log excess returns on the CRSP value-weighted market returns, $\sum_{h=1}^{H} R_{t+3+h} - R^f_{t+3+h}$, in which $H = 1, 3, 6, 12, 24$, and 36 is the forecast horizon in months. When $H > 1$, we use overlapping monthly observations of H-period holding returns. The regressors are *two*-month lagged values of the V/U ratio. The BLS takes less than one week to release monthly employment and unemployment data, and the Conference Board takes about one month to release monthly help wanted advertising index data.[7] We impose the two-month lag between the V/U ratio and market excess returns to guard against look-ahead bias in

[7] We verify this practice through a private correspondence with the Conference Board staff.

Table 4 : Long-Horizon Regressions of Market Excess Returns on Labor Market Tightness

Panel A reports long-horizon regressions of log excess returns on the value-weighted market index from CRSP, $\sum_{h=1}^{H} R_{t+3+h} - R_{t+3+h}^{f}$, in which H is the forecast horizon in months. The regressors are two-month lagged values of the V/U ratio. We report the ordinary least squares estimate of the slopes (Slope), the Newey-West corrected t-statistics (t_{NW}), and the adjusted R^2s in percent. The seasonally adjusted monthly unemployment (U, thousands of persons 16 years of age and older) is from the Bureau of Labor Statistics, and the seasonally adjusted help wanted advertising index (V) is from the Conference Board. The sample is from January 1951 to June 2006 (666 monthly observations). We multiply the V/U series by 50 so that its average is close to that in the model. In Panel B, we simulate 1,000 artificial samples, each of which has 666 monthly observations. On each artificial sample, we implement the exactly same empirical procedures as in Panel A, and report the cross-simulation averages and standard deviations (in parentheses) for all the model moments.

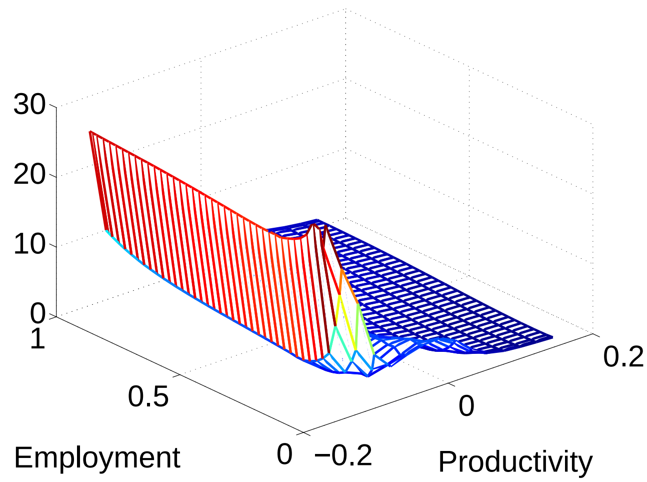
	Forecast horizon (H) in months					
	1	3	6	12	24	36
	Panel A: Data					
Slope	−1.425	−4.203	−7.298	−10.312	−9.015	−10.156
t_{NW}	−2.575	−2.552	−2.264	−1.704	−0.970	−0.861
Adjusted R^2	0.950	2.598	3.782	3.672	1.533	1.405
	Panel B: Model					
Slope	−0.504	−1.483	−2.876	−5.405	−9.615	−13.073
	(0.295)	(0.849)	(1.609)	(2.946)	(4.966)	(6.405)
t_{NW}	−2.060	−2.161	−2.285	−2.559	−3.219	−3.796
	(0.836)	(0.884)	(0.953)	(1.119)	(1.486)	(1.784)
Adjusted R^2	0.609	1.784	3.444	6.440	11.478	15.677
	(0.449)	(1.273)	(2.390)	(4.367)	(7.481)	(9.766)

predictive regressions. To make the regression slopes comparable to those in the model, we also scale up the V/U series in the data by a factor of 50 to make its average close to that in the model. This scaling is necessary because the vacancy and unemployment series in the data have different units.

Panel A of Table 4 shows that the V/U ratio forecasts stock market excess returns at business cycle frequencies. At the one-month horizon, the slope is -1.43, which is more than 2.5 standard errors from zero. The standard error is adjusted for heteroscedasticity and autocorrelations of 12 lags per Newey and West (1987). The slopes are significant at the three-month and six-month horizons but turn insignificant afterward. The adjusted R^2s peak at 3.78% at the six-month horizon, and decline to 3.67% at the one-year horizon and further to 1.41% at the three-year horizon.

To see how the model can explain this predictability, we plot in Figure 2 the equity premium in annual percent in the state space. We observe that the risk premium is strongly countercyclical. The equity premium is low in good times when both employment and productivity are high, but

Figure 2 : The Equity Premium in Annual Percent, $E_t[R_{t+1} - R^f_{t+1}]$



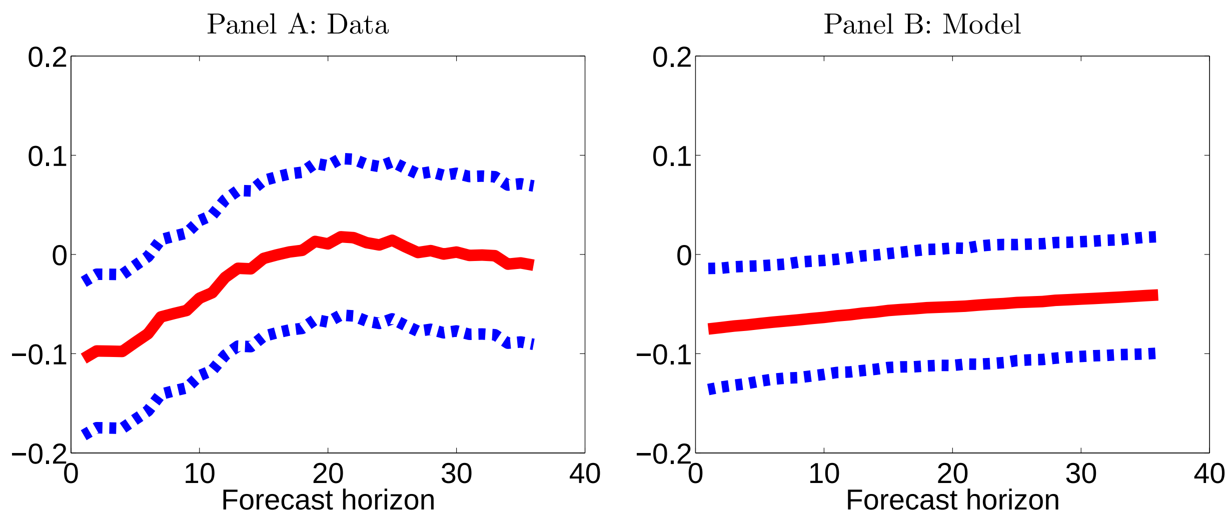
high in bad times when both employment and productivity are low. From Panel A of Figure 1, labor market tightness is strongly procyclical. The V/U ratio is low in bad times, but high in good times. The joint cyclical properties of the equity premium and the V/U ratio imply that the ratio should forecast stock market excess returns with a negative slope in the model.

Panel B of Table 4 reports the model's quantitative fit for the predictive regressions. Consistent with the data, the model predicts that the V/U ratio forecasts market excess returns with a negative slope. Specifically, at the one-month horizon, the predictive slope is -0.50 ($t = -2.06$). At the six-month horizon, the slope is -2.88 ($t = -2.29$). However, the model exaggerates the predictive power of the V/U ratio relative to that in the data. Both the t-statistic of the slope and the adjusted R^2 peak at the six-month horizon but decline afterward in the data. In contrast, both moments increase monotonically with the forecast horizon in the model, probably because it only has one shock.

Panel A of Figure 3 plots the cross-correlations and their two standard-error bounds between the V/U ratio, V_t/U_t, and future market excess returns, $R_{t+H} - R^f_{t+H}$, for $H = 1, 2, \ldots, 36$ months in the data. No overlapping observations are used. The panel shows that the correlations are significantly negative for forecast horizons up to six months, consistent with the predictive regressions in Table 4. Panel B reports the cross-correlations and their two cross-simulation standard-deviation bounds from the model's bootstrapped distribution. Consistent with the data, the model predicts significantly negative cross-correlations between V_t/U_t and future stock market excess returns at short horizons. However, although the cross-correlations are insignificant at long horizons, the correlations decay more slowly than those in the data, consistent with Panel B of Table 4.

Figure 3 : Cross-Correlations between the V/U Ratio and Future Market Excess Returns

We report the cross-correlations (in red) between labor market tightness, V_t/U_t, and future market excess returns, $R_{t+H} - R^f_{t+H}$, in which $H = 1, 2, \ldots, 36$ is the forecast horizon in months, as well as their two standard-error bounds (in blue broken lines). In Panel A, V_t is the seasonally adjusted help wanted advertising index from the Conference Board, and U_t is the seasonally adjusted monthly unemployment (thousands of persons 16 years of age and older) from the BLS. The sample is from January 1951 to June 2006. The market excess returns are the CRSP value-weighted market returns in excess of one-month Treasury bill rates. In Panel B, we simulate 1,000 artificial samples, each with 666 monthly observations. On each artificial sample, we calculate the cross-correlations between V_t/U_t and $R_{t+H} - R^f_{t+H}$, and plot the cross-simulation averaged correlations (in red) and their two cross-simulation standard-deviation bounds (in blue broken lines).

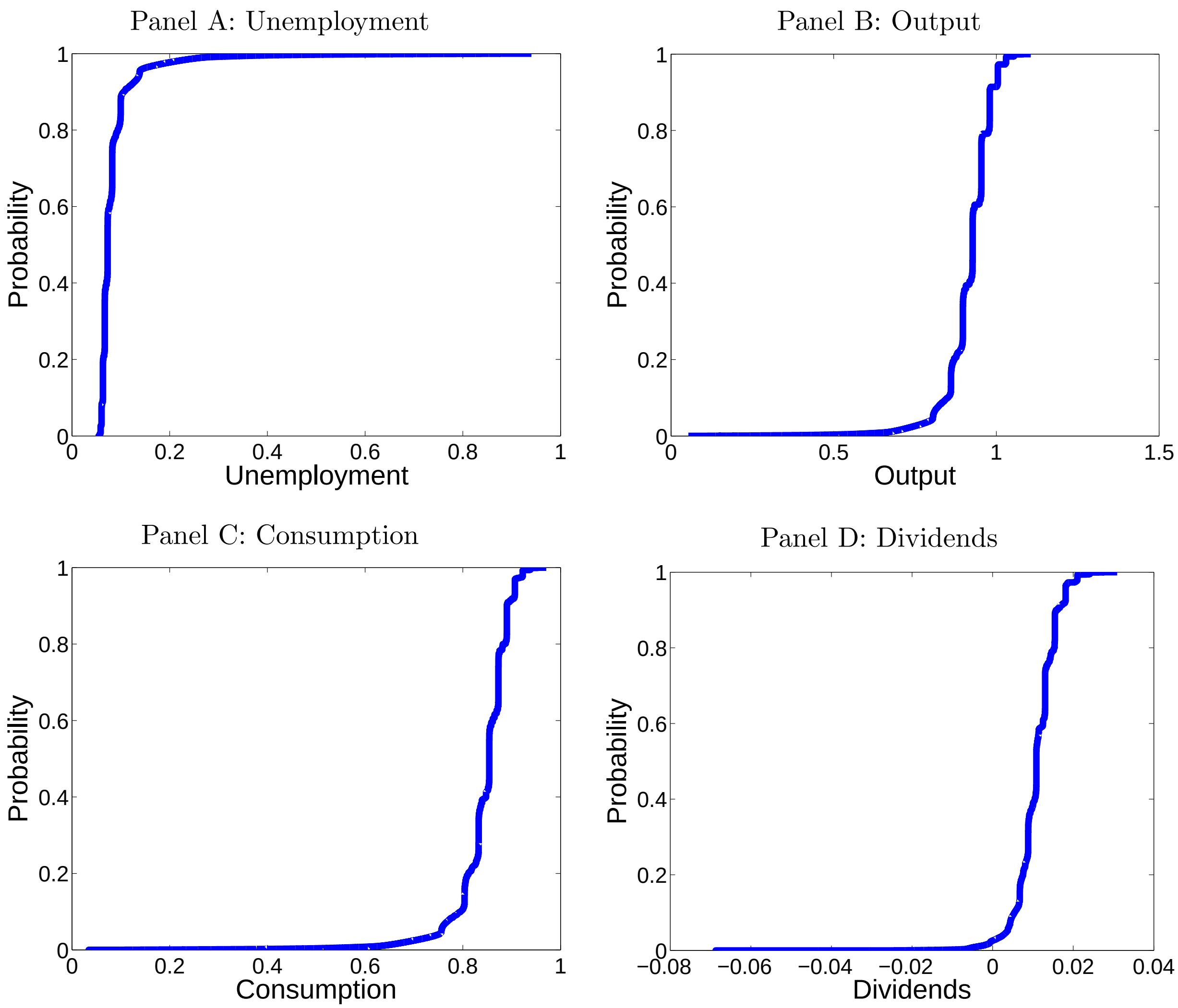


4.4 Endogenous Rare Disasters

The search economy gives rise endogenously to rare disaster risks à la Rietz (1988) and Barro (2006). We simulate 1,006,000 monthly periods from the model, discard the first 6,000 periods, and treat the remaining one million months as the model's stationary distribution. Figure 4 reports the empirical cumulative distribution functions for unemployment, output, consumption, and dividends. From Panel A, unemployment is positively skewed with a long right tail. As the population moments, the mean unemployment rate is 8.51%, the median is 7.30%, and the skewness is 7.83. The 2.5 percentile of unemployment is close to the median, 5.87%, whereas the 97.5 percentile is far away, 19.25%. As a mirror image, the employment rate is negatively skewed with a long left tail. As a result, output, consumption, and dividends all show rare but deep disasters (Panels B, C, and D, respectively). With small probabilities, the model economy falls off a cliff.

The disasters in macroeconomic quantities reflect in asset prices as rare upward spikes in the

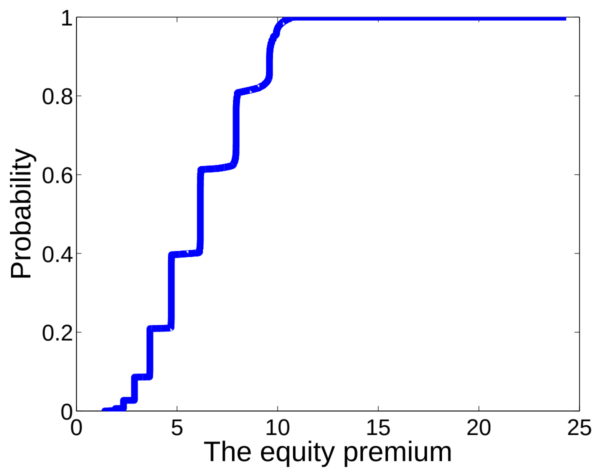
Figure 4 : Empirical Cumulative Distribution Functions of Unemployment, Output, Consumption, and Dividends Simulated from the Model's Stationary Distribution



equity premium, $E_t[R_{t+1} - R^f_{t+1}]$. From Figure 5, its stationary distribution is positively skewed with a long right tail. The conditional equity premium has a mean of 6.29% per annum and a median of 6.16%. The 2.5 and 97.5 percentiles are 2.34% and 10.04%, respectively. However, with small probabilities, the conditional equity premium can reach close to 25% in simulations.

Do the economic disasters arising endogenously from the model resemble those in the data? Barro and Ursúa (2008) apply a peak-to-trough method on international data from 1870 to 2006 to identify economic crises, defined as cumulative fractional declines in per capita consumption or GDP of at least 10%. Suppose there are two states, normalcy and disaster. The disaster probability measures the likelihood with which the economy shifts from normalcy to disaster in a given year. The number of disaster years is defined as the number of years in the interval between peak and trough

Figure 5 : Empirical Cumulative Distribution Function of the Equity Premium from the Model's Stationary Distribution



for each disaster event. The number of normalcy years is the total number of years in the sample minus the number of disaster years. The disaster probability is the ratio of the number of disasters over the number of normalcy years. Barro and Ursúa estimate the disaster probability to be 3.63%, the average size 22%, and the average duration 3.6 years for consumption disasters. For GDP disasters, the disaster probability is 3.69%, the average size 21%, and the average duration 3.5 years.

To quantify the magnitude of the disasters in the model, we first simulate the economy for 6,000 monthly periods to reach the stationary distribution. We then repeatedly simulate 1,000 artificial samples, each with 1,644 months (137 years). The sample size matches the average sample size in Barro and Ursúa (2008). On each artificial sample, we time-aggregate the monthly observations of consumption and output into annual observations. We apply Barro and Ursúa's measurement on each artificial sample, and report the cross-simulation averages and the 5 and 95 percentiles for the disaster probability, size, and duration for both consumption and GDP (output) disasters.

Table 5 reports the detailed results. For consumption disasters, Panel A shows that the disaster probability and the average disaster size are 3.08% and 20.21% in the model, which are close to 3.63% and 22% in the data, respectively. The average duration is 4.81 years, which is longer than 3.6 years in the data. The cross-simulation standard deviation of the average duration is 1.71 years, meaning that the data duration is within one standard deviation from the model's estimate.

From Panel B of Table 5, the average size of GDP disasters in the model, 19.12%, is close to that in the data, 21%. However, the disaster probability of 4.66% is somewhat higher than 3.69% in the data. The cross-simulation standard deviation of the GDP disaster probability is 2.01%, meaning

Table 5 : Moments of Macroeconomic Disasters

The data moments are from Barro and Ursúa (2008). The model moments are from 1,000 simulations, each with 1,644 monthly observations. We time-aggregate these monthly observations of consumption and output into 137 annual observations. On each artificial sample, we apply Barro and Ursúa's peak-to-trough method to identify economic crises, defined as cumulative fractional declines in per capita consumption or GDP of at least 10%. We report the mean and the 5 and 95 percentiles across the 1,000 simulations. The p-values are the percentages with which a given model moment is higher than its data moment. The disaster probabilities and average size are all in percent, and the average duration is in terms of years.

	Data	Model			
		Mean	5%	95%	p-value
Panel A: Consumption disasters					
Probability	3.63	3.084	0.752	6.422	0.304
Average size	22	20.206	11.486	39.369	0.254
Average duration	3.6	4.808	3.000	7.000	0.832
Panel B: GDP disasters					
Probability	3.69	4.661	1.575	8.290	0.297
Average size	21	19.121	12.336	34.051	0.279
Average duration	3.5	4.508	3.250	6.182	0.844

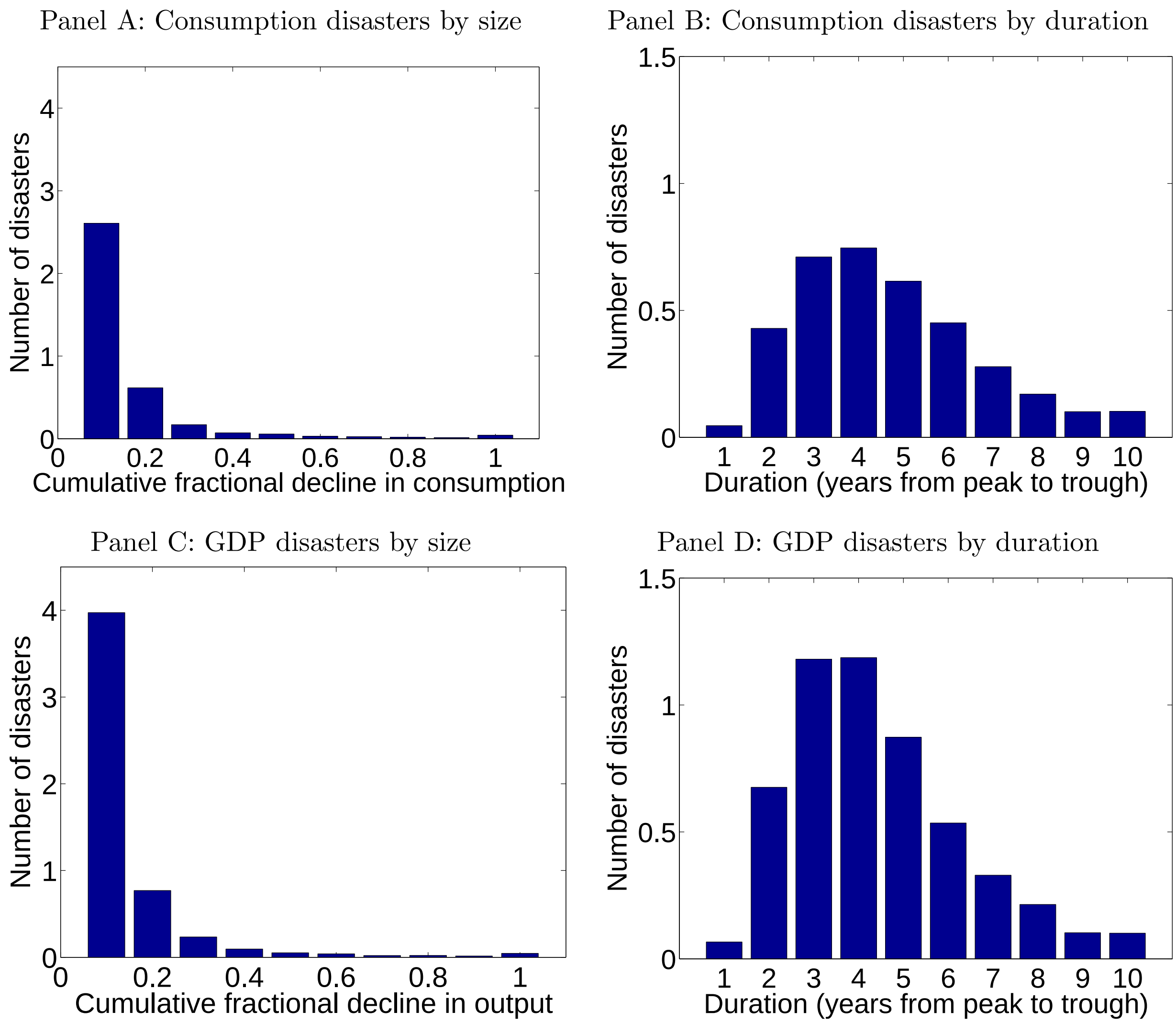
that the probability in the data is within one standard deviation from the model. In addition, the average duration of the GDP disasters in the model is 4.51 years, which is longer than 3.5 years in the data. The cross-simulation standard deviation of the duration is 0.88 years, meaning that the duration in the data is slightly more than one standard deviation from the model.

Figure 6 reports the frequency distributions of consumption and GDP disasters by size and duration averaged across 1,000 economies (each with 137 years) simulated from the model. We observe that the size and duration distributions for consumption and GDP disasters in the model display roughly similar patterns as those in the data (see Barro and Ursúa's (2008) Figures 1 and 2). In particular, the size distributions seem to follow a power-law density per Barro and Jin (2011).

4.5 Endogenous Long Run Risks and Endogenous Uncertainty Risks

We also explore the model's implications for long run risks per Bansal and Yaron (2004) and uncertainty shocks per Bloom (2009). Bansal and Yaron propose long-run consumption risks to explain

Figure 6 : Distributions of Consumption and GDP Disasters by Size and Duration



aggregate asset prices. Specifically, monthly consumption growth is assumed to follow:

$$z_{t+1} = .979 z_t + .044 \sigma_t e_{t+1}, \tag{24}$$

$$g_{t+1} = .0015 + z_t + \sigma_t \eta_{t+1}, \tag{25}$$

$$\sigma_{t+1}^2 = .0078^2 + .987(\sigma_t^2 - .0078^2) + .23 \times 10^{-5} w_{t+1}, \tag{26}$$

in which g_{t+1} is the consumption growth, z_t is the expected consumption growth, σ_t is the conditional volatility of g_{t+1}, and $e_{t+1}, u_{t+1}, \eta_{t+1}$, and w_{t+1}, are i.i.d. standard normal shocks, which are mutually uncorrelated. Bansal and Yaron argue that the stochastic slow-moving component, z_t, of the consumption growth is crucial for explaining the level of the equity premium, and that the mean-reverting stochastic volatility helps explain the time-variation in the risk premium.

Kaltenbrunner and Lochstoer (2010) argue that long-run risks can arise endogenously via consumption smoothing. Within a production economy with capital as the only productive input, Kaltenbrunner and Lochstoer (Table 6) show that the endogenous consumption growth follows:

$$z_{t+1} = .986 z_t + .093 \sigma e_{t+1}, \tag{27}$$

$$g_{t+1} = .0013 + z_t + \sigma \eta_{t+1}, \tag{28}$$

with transitory productivity shocks. With permanent productivity shocks, the z_t process follows:

$$z_{t+1} = .990 z_t + .247 \sigma e_{t+1}. \tag{29}$$

However, their models fail to reproduce time-varying volatilities in expected and realized consumption growth as well as stock market returns.

We investigate how consumption dynamics in our search economy compare with those in the Kaltenbrunner and Lochstoer (2010) economy and with those calibrated in Bansal and Yaron (2004). This economic question is important because different parameterizations of the consumption process specified in equations (24)–(26) can be largely consistent with observable moments of consumption growth such as volatility and autocorrelations (see Table 2). Yet, different parameterizations can give rise to vastly different economic mechanisms for the equity premium and its dynamics.

We simulate the search economy for 6,000 monthly periods to reach the stationary distribution, and then simulate one million monthly periods. We calculate the expected consumption growth and the conditional volatility of the realized consumption growth in the employment-productivity state space, and use the solutions to simulate these moments. Fitting the consumption growth process specified by Bansal and Yaron (2004) on the simulated data, we obtain:

$$z_{t+1} = .697 z_t + .598 \sigma_t e_{t+1}, \tag{30}$$

$$g_{t+1} = z_t + \sigma_t \eta_{t+1}, \tag{31}$$

$$\sigma^2_{t+1} = .0026^2 + .658(\sigma^2_t - .0026^2) + 1.91 \times 10^{-5} w_{t+1}. \tag{32}$$

In addition, the unconditional correlation between e_{t+1} and η_{t+1} is 0.340, that between e_{t+1} and w_{t+1} is zero, and the correlation between η_{t+1} and w_{t+1} is 0.123.

Although the consumption growth is not i.i.d. in our economy, the persistence in the expected consumption growth is only 0.697, which is substantially lower than those in Bansal and Yaron

(2004) and Kaltenbrunner and Lochstoer (2010). However, the expected consumption growth is more volatile in our economy. The conditional volatility of the expected consumption growth about 60% of the conditional volatility of the realized consumption growth. This percentage is higher than 9.3% and 24.7% in Kaltenbrunner and Lochstoer as well as 4.4% in Bansal and Yaron. For the stochastic variance, its persistence is 0.658 in our economy, which is lower than 0.987 in Bansal and Yaron. However, the volatility of our stochastic variance is more than eight times of theirs.

These results suggest that disaster risks (and uncertainty risks) play a more important role than long-run risks in the sense of high persistence of the expected consumption growth in the search economy. As the economy occasionally falls into disasters, shocks to both expected consumption growth and the conditional variance of consumption growth are large. Disasters also give rise to lower persistence for the expected consumption growth and the conditional variance.

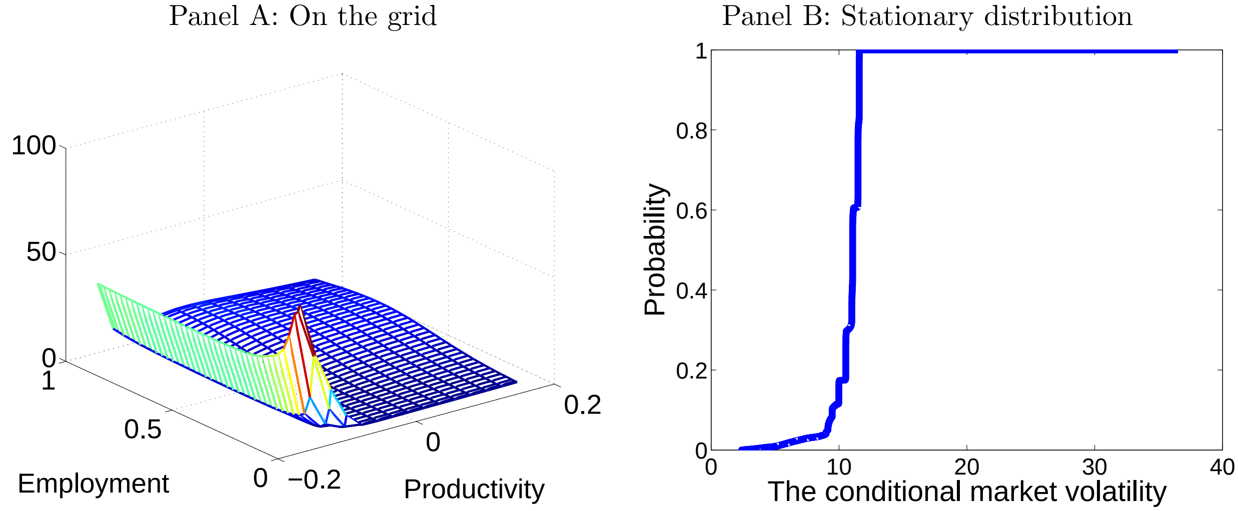
To further characterize the endogenous uncertainty risks in the search economy, we plot the conditional market volatility, σ_t^R, in the employment-productivity state space. From Panel A of Figure 7, the stock market volatility is strongly countercyclical. The volatility is low in good times and high in bad times. Panel B reports the empirical cumulative distribution of the stock market volatility in simulations. We simulate 1,006,000 monthly periods, discard the first 6,000 periods, and treat the remaining one million periods as from the stationary distribution. We observe that the conditional volatility hovers around its median about 11% per annum. However, with small probabilities, the market volatility can jump to more than 35%. The endogenous time-varying uncertainty is another dimension of the search economy that is distinctive from the baseline production economy in Kaltenbrunner and Lochstoer (2010). As noted, without the powerful propagation mechanism from the labor market, their economy cannot generate time-varying volatilities.

4.6 Dividend Dynamics

Rouwenhorst (1995) shows that dividends are countercyclical in baseline production economies. Intuitively, dividends equal profits minus investment, and profits equal output minus wages. With frictionless labor markets, wages equal the marginal product of labor, meaning profits are proportional to (and as procyclical as) output. Because investment is more procylical than output and profits due to consumption smoothing, dividends (profits minus investment) must be countercyclical.

However, this dividend countercyclicality is counterfactual. Dividends in the production economies correspond to net payout (dividends plus stock repurchases minus new equity issues)

Figure 7 : The Conditional Market Volatility in Annual Percent, σ_t^R, on the Employment-productivity Grid and Empirical Stationary Distribution in Simulations



in the data. Following Jermann and Quadrini (2010), we measure the net payout using aggregate data from the Flow of Funds Accounts of the Federal Reserve Board.[8] The sample is quarterly from the fourth quarter of 1951 to the fourth quarter of 2010. We obtain quarterly real GDP (NIPA Table 1.1.6) and quarterly implicit price deflator for GDP (NIPA Table 1.1.9) used to deflate net payout. We detrend real net payout and real GDP as HP-filtered proportional deviations from the mean with a smoothing parameter of 1,600. We do not take logs because the net payout can be negative in the data. Consistent with Jermann and Quadrini, we find that the cyclical components of real net payout and real GDP have a positive correlation of 0.55.

The search economy avoids the pitfall of counterfactual dividend dynamics in the baseline production economies. Intuitively, the wage rate from the generalized Nash bargaining process is no longer equal to the marginal product of labor. Because the value of unemployment activities, b, is positive (and large), wages are less procyclical than productivity. As such, profits are more procyclical than output. In effect, the inelastic wages work as operating leverage to magnify the procyclicality (and volatility) of profits. This amplified procyclicality of profits is sufficient to overcome the procyclicality of total vacancy costs to turn dividend dynamics procyclical.

Quantitatively, the model replicates the dividend procyclicality. We first simulate the economy for 6,000 monthly periods to reach the stationary distribution, and then repeatedly simulate 1,000

[8]Specifically, we calculate the net payout as net dividends of nonfarm, nonfinancial business (Table F.102, line 3) plus net dividends of farm business (Table F.7, line 24) minus net increase in corporate equities of nonfinancial business (Table F.101, line 35) minus proprietors' net investment of nonfinancial business (Table F.101, line 39).

artificial samples, each with 711 months (237 quarters). The sample size matches the quarterly series from the fourth quarter of 1951 to the fourth quarter of 2010 in Jermann and Quadrini (2010). On each artificial sample, we time-aggregate monthly observations of dividends and output into quarterly observations. After detrending the quarterly series as HP-filtered proportional deviations from the mean, we calculate the correlation between the cyclical components of dividends and output. We find that in the model this correlation is 0.561, which is close to 0.55 in the data.

We also compare the wage dynamics in the model to those in the data. Following Hagedorn and Manovskii (2008), we measure wages as labor share times labor productivity from BLS. The sample is quarterly from the first quarter of 1947 to the four quarter of 2010 (256 quarters). We take logs and HP-detrend the series with a smoothing parameter of 1,600. We find that the wage elasticity to labor productivity is 0.46, close to Hagedorn and Manovskii's estimate, meaning that a one percentage point increase in labor productivity delivers a 0.46 percentage increase in real wages. In addition, we measure (quarterly) output as real GDP (NIPA Table 1.1.6). We find that the correlation between the log wage growth and the log output growth is 0.48, and that the ratio of the wage growth volatility over the output growth volatility is 0.70.

To see the model's performance, we first simulate the economy for 6,000 monthly periods to reach the stationary distribution, and then repeatedly simulate 1,000 artificial samples, each with 768 months (256 quarters). On each artificial sample, we take quarterly averages of monthly wages and labor productivity to obtain quarterly series. Implementing the same empirical procedure used on the real data, we find that the wage elasticity to productivity is 0.577 in the model, which is not far from 0.46 in the data. However, the correlation between the log wage growth and the log output growth in the model is 0.896, which is higher than 0.48 in the data. At the same time, the relative volatility of the wage growth is 0.378, which is lower than 0.70 in the data.

4.7 Comparative Statics

To shed further light on the economic mechanisms underlying the key results in the model, Table 6 conducts four comparative statics: (i) the value of unemployment activities, b, changed from 0.85 in the benchmark calibration to 0.4; (ii) the job separation rate, s, from 0.05 to 0.035; (iii) the proportional cost of vacancy, κ_0, from 0.6 to 0.815, and simultaneously, the fixed cost of vacancy, κ_1, from 0.4 to zero (0.815 is the average κ_t in the simulations from the benchmark economy); and (iv) the elasticity of the matching function, ι, from 1.25 to 0.9. In each experiment, all the other parameters remain the same as in the benchmark calibration.

Small Profits versus Large Profits

In the first experiment, the value of unemployment activities, $b = 0.4$, is set to be the low value in Shimer (2005). Because unemployment is less valuable to workers, the unemployment rate drops to 5%, which is lower than the historical average in the U.S. data. A lower b also means that the wage rate is more sensitive to labor productivity shocks. The wage elasticity to productivity increases to 0.68 from 0.56 in the benchmark economy. As a result, profits and vacancies are less sensitive to shocks. Employment and output are also less sensitive, giving rise to a lower consumption growth volatility of 1.69% per annum and a lower output growth volatility of 2.05%.

The low-b economy shows essentially no disaster risks. In simulations, the unemployment rate hovers around 5%. Neither output nor consumption has a long left tail in its empirical distribution. Applying Barro and Ursúa's (2008) peak-to-trough measurement, we find that the disaster probabilities are no greater than 0.7% and the average disaster size no greater than 13.5%. The equity premium drops to only 0.12% per annum, and the V/U ratio shows no predictive power for market excess returns. The stock market volatility drops only to 3.87%.

Consistent with Shimer (2005), the standard deviation of the V/U ratio in the low-b model is only 2.7%, which is an order of magnitude smaller than that in the data. As such, a high value of unemployment activities "explains" the Shimer puzzle as well as the equity premium puzzle simultaneously. Intuitively, by dampening the procyclical covariation of wages with productivity, a high value of b magnifies the procyclical variation of profits and vacancies to increase the volatility of the V/U ratio. This operating leverage mechanism also impacts the financial moments as the high b amplifies the procyclical variation of dividends, raising the equity premium and the stock market volatility.

Large Job Flows versus Small Job Flows

In the second experiment, we reduce the separation rate, s, from 5% to 3.5% per month. Because employment is destructed at a lower rate, the disasters in the low-s economy are less extreme and less frequent than those in the benchmark economy. The consumption and GDP disaster probabilities are 1.11% and 1.95%, respectively, which are more than halved relative to those in the benchmark economy. The average magnitudes of the disasters are also smaller: 15.40% and 15.66%, respectively. Because of the dampened disaster risks, the equity risk premium is close to zero.

Table 6 : Comparative Statics

We report four experiments: (i) $b = .4$ is for the value of unemployment activities set to 0.4; (ii) $s = .035$ is for the job separation rate set to 0.035; (iii) $\kappa_t = .815$ is for the proportional cost of vacancy $\kappa_0 = .815$ and the fixed cost $\kappa_1 = 0$, in which .815 is the average κ_t in the benchmark calibration; and (iv) $\iota = .9$ is for the elasticity of the matching function set to .9. In each experiment, all the other parameters are identical to those in the benchmark calibration. See Table 2 for the description of Panel A. See Table 3 for the description of Panel B: σ^U, σ^V, and $\sigma^{V/U}$ are the standard deviations of unemployment, vacancy, and the vacancy-unemployment ratio, respectively. $\rho^{U,V}$ is the correlation between unemployment and vacancy. See the caption of Table 4 for the description of Panel C: (1) and (12) denote for forecast horizons of one and 12 months, respectively. See the caption of Table 5 for the description of Panel D: (C) and (Y) denote consumption and GDP disasters, respectively. $\rho^{D,Y}$ is the correlation between the cyclical components of quarterly dividends and output, and $e^{W,X}$ is the wage elasticity to productivity.

	Data	Benchmark	$b = .4$	$s = .035$	$\kappa_t = .815$	$\iota = .9$
	Panel A: Basic business cycle and financial moments					
σ^C	3.036	3.633	1.688	2.211	2.750	4.260
$\rho^C(1)$	0.383	0.183	0.133	0.146	0.146	0.217
$\rho^C(3)$	−0.206	−0.126	−0.102	−0.099	−0.112	−0.133
σ^Y	4.933	4.133	2.047	2.674	3.366	4.825
$\rho^Y(1)$	0.543	0.185	0.133	0.144	0.149	0.216
$\rho^Y(3)$	−0.179	−0.122	−0.100	−0.097	−0.109	−0.128
$E[R - R^f]$	5.066	5.695	0.116	0.002	2.338	6.068
$E[R^f]$	0.588	2.898	3.963	3.956	3.743	3.012
σ^R	12.942	10.830	3.869	11.468	12.293	10.787
σ^{R^f}	1.872	1.342	0.133	0.629	1.040	1.510
	Panel B: Labor market moments					
σ^U	0.119	0.151	0.002	0.092	0.113	0.147
σ^V	0.134	0.116	0.025	0.098	0.093	0.125
$\sigma^{V/U}$	0.255	0.173	0.027	0.135	0.138	0.185
$\rho^{U,V}$	−0.913	−0.568	−0.950	−0.653	−0.626	−0.573
	Panel C: Forecasting market excess returns with the V/U ratio					
Slope(1)	−1.425	−0.504	−0.072	−0.187	−0.613	−0.617
Slope(12)	−10.312	−5.405	−0.790	−2.072	−6.628	−6.577
$t_{NW}(1)$	−2.575	−2.060	−0.673	−0.646	−1.709	−2.337
$t_{NW}(12)$	−1.704	−2.559	−0.873	−0.832	−2.130	−2.914
	Panel D: Moments of macroeconomic disasters					
Probability(C)	3.63	3.084	0.414	1.110	1.939	4.248
Size(C)	22	20.206	11.654	15.396	18.204	21.450
Duration(C)	3.6	4.808	6.041	5.555	5.059	4.619
Probability(Y)	3.69	4.661	0.684	1.946	3.494	6.287
Size(Y)	21	19.121	13.226	15.656	17.461	20.589
Duration(Y)	3.5	4.508	5.748	5.174	4.673	4.374
	Panel E: Dividend dynamics					
$\rho^{D,Y}$	0.546	0.561	1.000	0.841	0.590	0.406
$e^{W,X}$	0.463	0.577	0.683	0.608	0.551	0.528

With versus Without Fixed Cost of Vacancy

In the benchmark calibration, the proportional cost of vacancy, κ_0, is 0.6, the fixed cost, κ_1, is 0.4, and the average of $\kappa_0+\kappa_1 q(\theta_t)$ is 0.815 in simulations. In the third experiment, we remove the fixed cost, while maintaining the average cost. That is, we set $\kappa_1 = 0$ but increase κ_0 from 0.6 to 0.815.

It is easier for the firm to fill a vacancy in bad times because more unemployed workers are available. That is, the vacancy filling rate, $q(\theta_t)$, is countercyclical. As such, removing fixed matching costs affects the results in two ways. First, with $\kappa_1 = 0$, equation (19) implies that wages are more elastic than wages in the benchmark economy with $\kappa_1 > 0$. This effect weakens the operating leverage mechanism. Second, the unit cost of vacancy is countercyclical in the benchmark economy. Because it is more difficult for the firm to hire in bad times to counteract large job destruction flows, disasters are more frequent and deeper. Without the countercyclical component of the unit cost, disasters are less frequent and less severe. Table 6 shows that without fixed matching costs, consumption and GDP disaster probabilities drop to 1.94% and 3.49%, respectively. The average disaster size is also lowered, but only slightly. The equity premium falls to 2.34% per annum, the output growth volatility falls to 3.37% per annum, and the consumption growth volatility to 2.75%.

High versus Low Elasticity of the Matching Function

The evidence on the elasticity parameter in the matching function seems scarce. In the benchmark calibration, we follow Den Haan, Ramey, and Watson (2000) to set the elasticity to 1.25. In the final experiment, we quantify the impact of this parameter by setting it to 0.9.

The last column of Table 6 shows that, sensibly, lowering the elasticity of the matching function strengthens the risk dynamics in the model. As the labor market becomes less efficient in matching vacancies with unemployed workers, the consumption growth volatility goes up to 4.26% per annum, and the output growth volatility to 4.83%. The consumption and GDP disaster probabilities also increase to 4.25% and 6.29%, respectively, although the average disaster size grows only slightly. Not surprisingly, the equity premium goes up to 6.07%.

5 Conclusion

We study aggregate asset pricing by embedding the standard Diamond-Mortensen-Pissarides search model of the labor market into a dynamic stochastic general equilibrium economy with recursive preferences. We find that labor market frictions are important for understanding the equity premium in the financial market. Quantitatively, the model reproduces a realistic equity premium, a high stock market volatility, and a low interest rate volatility. The equity premium is also strongly countercyclical, and is forecastable by labor market tightness, a prediction that we confirm in the data. Intriguingly, three key ingredients in the model (small profits, large job flows, and fixed matching costs) combine to create endogenously rare disaster risks as in Rietz (1988) and Barro (2006).

As a first stab in embedding labor market frictions into equilibrium asset pricing, we have tried to keep the model parsimonious. We do not interpret our work as saying that the baseline search model "explains" the equity premium puzzle. Nevertheless, the rich dynamics displayed even in this model, many of which are conducive to understanding the equity premium, yet are entirely absent from baseline production economies, suggest that labor market frictions might be essential for equilibrium asset pricing. Several directions are possible for future work. First, the high value of unemployment activities is not the only mechanism that can produce small profits to "explain" the unemployment volatility puzzle in the labor market. One can explore asset pricing implications of, for example, wage rigidity that seems important for labor market dynamics (e.g., Hall and Milgrom (2008)). Second, more generally, one can introduce endogenous labor supply and capital accumulation into our model to develop a unified equilibrium framework for both asset prices and business cycles. Third, one can introduce financial frictions such as defaultable debt into the model to study the interaction between search frictions and financial frictions in endogenizing economic disasters. Finally, with the equity premium in sight in production economies, one can introduce firm heterogeneity to develop an equilibrium framework for the cross-section of expected returns.

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A The Stock Return Equation

We prove equation (15) following the same logic in Liu, Whited, and Zhang (2009) in the context of the q-theory of investment. Rewrite the equity value maximization problem as:

$$S_t = \max_{V_{t+\triangle t}, N_{t+\triangle t+1}} E_t \left[\sum_{\triangle t=0}^{\infty} M_{t+\triangle t} \left[\begin{array}{c} X_{t+\triangle t} N_{t+\triangle t} - W_{t+\triangle t} N_{t+\triangle t} - [\kappa_0 + \kappa_1 q(\theta_t)] V_{t+\triangle t} \\ -\mu_{t+\triangle t} [N_{t+\triangle t+1} - (1-s) N_{t+\triangle t} \\ -V_{t+\triangle t} q(\theta_{t+\triangle t})] + \lambda_{t+\triangle t} q(\theta_{t+\triangle t}) V_{t+\triangle t} \end{array} \right] \right], \tag{A.1}$$

in which μ_t is the Lagrange multiplier on the employment accumulation equation, and λ_t is the Lagrange multiplier on the occasionally binding constraint on job creation. The first order conditions are equations (11) and (12) as well as the Kuhn-Tucker conditions (14).

Define dividends as $D_t \equiv X_t N_t - W_t N_t - [\kappa_0 + \kappa_1 q(\theta_t)] V_t$ and the ex-dividend equity value as $P_t \equiv S_t - D_t$. Expanding S_t yields:

$$\begin{aligned} P_t + X_t N_t - W_t N_t - [\kappa_0 + \kappa_1 q(\theta_t)] V_t = S_t = X_t N_t - W_t N_t - [\kappa_0 + \kappa_1 q(\theta_t)] V_t + \lambda_t q(\theta_t) V_t \\ -\mu_t [N_{t+1} - (1-s) N_t - V_t q(\theta_t)] + E_t M_{t+1} [X_{t+1} N_{t+1} - W_{t+1} N_{t+1} - [\kappa_0 + \kappa_1 q(\theta_t)] V_{t+1} \\ -\mu_{t+1} [N_{t+2} - (1-s) N_{t+1} - V_{t+1} q(\theta_{t+1})] + \lambda_{t+1} q(\theta_{t+1}) V_{t+1}] + \ldots \end{aligned} \tag{A.2}$$

Recursively substituting equations (11) and (12) yields:

$$P_t + X_t N_t - W_t N_t - [\kappa_0 + \kappa_1 q(\theta_t)] V_t = X_t N_t - W_t N_t + \mu_t (1-s) N_t. \tag{A.3}$$

Using equation (11) to simplify further:

$$P_t = [\kappa_0 + \kappa_1 q(\theta_t)] V_t + \mu_t (1-s) N_t = \mu_t [(1-s) N_t + q(\theta_t) V_t] + \lambda_t q(\theta_t) V_t = \mu_t N_{t+1}, \tag{A.4}$$

in which the last equality follows from equation (14).

To show equation (15), we expand the stock returns:

$$\begin{aligned} R_{t+1} &= \frac{S_{t+1}}{S_t - D_t} = \frac{\mu_{t+1} N_{t+2} + X_{t+1} N_{t+1} - W_{t+1} N_{t+1} - [\kappa_0 + \kappa_1 q(\theta_t)] V_{t+1}}{\mu_t N_{t+1}} \\ &= \frac{X_{t+1} - W_{t+1} - [\kappa_0 + \kappa_1 q(\theta_t)] V_{t+1}/N_{t+1} + \mu_{t+1} [(1-s) + q(\theta_{t+1}) V_{t+1}/N_{t+1}]}{\mu_t} \\ &= \frac{X_{t+1} - W_{t+1} + (1-s)\mu_{t+1}}{\mu_t} + \frac{\mu_{t+1} q(\theta_{t+1}) V_{t+1} - [\kappa_0 + \kappa_1 q(\theta_t)] V_{t+1}}{\mu_t N_{t+1}} \\ &= \frac{X_{t+1} - W_{t+1} + (1-s)\mu_{t+1}}{\mu_t}, \end{aligned} \tag{A.5}$$

in which the last equality follows because the Kuhn-Tucker conditions imply:

$$\mu_{t+1} q(\theta_{t+1}) V_{t+1} - [\kappa_0 + \kappa_1 q(\theta_t)] V_{t+1} = -\lambda_{t+1} q(\theta_{t+1}) V_{t+1} = 0. \quad \blacksquare \tag{A.6}$$

B Wage Determination under Nash Bargaining

Let $\eta \in (0,1)$ denote the relative bargaining weight of the worker, J_{Nt} the marginal value of an employed worker to the representative family, J_{Ut} the marginal value of an unemployed worker to the representative family, ϕ_t the marginal utility of the representative family, S_{Nt} the marginal value of an employed worker to the representative firm, and S_{Vt} the marginal value of an unfilled vacancy to the representative firm. A worker-firm match turns an unemployed worker into an employed worker for the representative household as well as an unfilled vacancy into a filled vacancy (an employed worker) for the firm. As such, we can define the total surplus from the Nash bargain as:

$$\Lambda_t \equiv \frac{J_{Nt} - J_{Ut}}{\phi_t} + S_{Nt} - S_{Vt}. \tag{B.1}$$

The wage equation (19) is determined via the Nash worker-firm bargain:

$$\max_{\{W_t\}} \quad \left(\frac{J_{Nt} - J_{Ut}}{\phi_t}\right)^{\eta} \left(S_{Nt} - S_{Vt}\right)^{1-\eta}, \tag{B.2}$$

The outcome of maximizing equation (B.2) is the surplus-sharing rule:

$$\frac{J_{Nt} - J_{Ut}}{\phi_t} = \eta \Lambda_t = \eta \left(\frac{J_{Nt} - J_{Ut}}{\phi_t} + S_{Nt} - S_{Vt}\right). \tag{B.3}$$

As such, the worker receives a fraction of η of the total surplus from the wage bargain. In what follows, we derive the wage equation (19) from the sharing rule in equation (B.3).

B.1 Workers

To derive J_{Nt} and J_{Ut}, we need to specify the details of the representative household's problem. Tradeable assets consist of risky shares and a risk-free bond. Let R^f_{t+1} denote the risk-free interest rate, which is known at the beginning of period t, Π_t the household's financial wealth, χ_t the fraction of the household's wealth invested in the risky shares, $R^{\Pi}_{t+1} \equiv \chi_t R_{t+1} + (1-\chi_t) R^f_{t+1}$ the return on wealth, and T_t the taxes raised by the government. The household's budget constraint is given by:

$$\frac{\Pi_{t+1}}{R^{\Pi}_{t+1}} = \Pi_t - C_t + W_t N_t + U_t b - T_t. \tag{B.4}$$

Note that the household's dividends income, D_t, is included in the current financial wealth, Π_t. Finally, the government balances its budget, meaning that $T_t = U_t b$.

Let ϕ_t denote the Lagrange multiplier for the household's budget constraint (B.4). The household's maximization problem is given by:

$$J_t = \left[(1-\beta) C_t^{1-\frac{1}{\psi}} + \beta \left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}} - \phi_t \left(\frac{\Pi_{t+1}}{R^{\Pi}_{t+1}} - \Pi_t + C_t - W_t N_t - U_t b + T_t\right), \tag{B.5}$$

The first-order condition for consumption yields:

$$\phi_t = (1-\beta)C_t^{-\frac{1}{\psi}} \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta \left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}-1}, \tag{B.6}$$

which gives the marginal utility of consumption.

Recalling $N_{t+1} = (1-s)N_t + f(\theta_t)U_t$ and $U_{t+1} = sN_t + (1-f(\theta_t))U_t$, we differentiate J_t in equation (B.5) with respect to N_t:

$$\begin{aligned} J_{Nt} &= \phi_t W_t + \frac{1}{1-\frac{1}{\psi}} \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta \left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}-1} \\ &\times \frac{1-\frac{1}{\psi}}{1-\gamma}\beta \left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}-1} E_t\left[(1-\gamma)J_{t+1}^{-\gamma}[(1-s)J_{Nt+1} + sJ_{Ut+1}]\right]. \end{aligned} \tag{B.7}$$

Dividing both sides by ϕ_t:

$$\frac{J_{Nt}}{\phi_t} = W_t + \frac{\beta}{(1-\beta)C_t^{-\frac{1}{\psi}}} \left[\frac{1}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma} E_t\left[J_{t+1}^{-\gamma}[(1-s)J_{Nt+1} + sJ_{Ut+1}]\right]. \tag{B.8}$$

Dividing and multiplying by ϕ_{t+1}:

$$\begin{aligned} \frac{J_{Nt}}{\phi_t} &= W_t + E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left[\frac{J_{t+1}}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma} \left[(1-s)\frac{J_{Nt+1}}{\phi_{t+1}} + s\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right] \\ &= W_t + E_t\left[M_{t+1}\left[(1-s)\frac{J_{Nt+1}}{\phi_{t+1}} + s\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right]. \end{aligned} \tag{B.9}$$

Similarly, differentiating J_t in equation (B.5) with respect to U_t yields:

$$\begin{aligned} J_{Ut} &= \phi_t b + \frac{1}{1-\frac{1}{\psi}} \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta \left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}-1} \\ &\times \frac{1-\frac{1}{\psi}}{1-\gamma}\beta \left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}-1} E_t\left[(1-\gamma)J_{t+1}^{-\gamma}[f(\theta_t)J_{Nt+1} + (1-f(\theta_t))J_{Ut+1}]\right]. \end{aligned} \tag{B.10}$$

Dividing both sides by ϕ_t:

$$\frac{J_{Ut}}{\phi_t} = b + \frac{\beta}{(1-\beta)C_t^{-\frac{1}{\psi}}} \left[\frac{1}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma} E_t\left[J_{t+1}^{-\gamma}[f(\theta_t)J_{Nt+1} + (1-f(\theta_t))J_{Ut+1}]\right]. \tag{B.11}$$

Dividing and multiplying by ϕ_{t+1}:

$$\begin{aligned}
\frac{J_{Ut}}{\phi_t} &= b + E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left[\frac{J_{t+1}}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma}\left[f(\theta_t)\frac{J_{Nt+1}}{\phi_{t+1}} + (1-f(\theta_t))\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right] \\
&= b + E_t\left[M_{t+1}\left[f(\theta_t)\frac{J_{Nt+1}}{\phi_{t+1}} + (1-f(\theta_t))\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right]. \qquad \text{(B.12)}
\end{aligned}$$

B.2 The Firm

We start by rewriting the infinite-horizon value-maximization problem of the firm recursively as:

$$S_t = X_t N_t - W_t N_t - [\kappa_0 + \kappa_1 q(\theta_t)] V_t + \lambda_t q(\theta_t) V_t + E_t[M_{t+1} S_{t+1}], \qquad \text{(B.13)}$$

subject to $N_{t+1} = (1-s)N_t + q(\theta_t)V_t$. The first-order condition with respect to V_t says:

$$S_{Vt} = -[\kappa_0 + \kappa_1 q(\theta_t)] + \lambda_t q(\theta_t) + E_t[M_{t+1} S_{Nt+1} q(\theta_t)] = 0. \qquad \text{(B.14)}$$

Equivalently,

$$\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t = E_t[M_{t+1} S_{Nt+1}]. \qquad \text{(B.15)}$$

In addition, differentiating S_t with respect to N_t yields:

$$S_{Nt} = X_t - W_t + (1-s)E_t[M_{t+1} S_{Nt+1}]. \qquad \text{(B.16)}$$

Combining the last two equations yields the intertemporal job creation condition in equation (13).

B.3 The Wage Equation

From equations (B.9), (B.12), and (B.16), the total surplus of the worker-firm relationship is:

$$\begin{aligned}
\Lambda_t &= W_t + E_t\left[M_{t+1}\left[(1-s)\frac{J_{Nt+1}}{\phi_{t+1}} + s\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right] - b \\
&\quad - E_t\left[M_{t+1}\left[f(\theta_t)\frac{J_{Nt+1}}{\phi_{t+1}} + (1-f(\theta_t))\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right] + X_t - W_t + (1-s)E_t[M_{t+1} S_{Nt+1}] \\
&= X_t - b + (1-s)E_t\left[M_{t+1}\left(\frac{J_{Nt+1} - J_{Ut+1}}{\phi_{t+1}} + S_{Nt+1}\right)\right] - f(\theta_t)E_t\left[M_{t+1}\frac{J_{Nt+1} - J_{Ut+1}}{\phi_{t+1}}\right] \\
&= X_t - b + (1-s)E_t[M_{t+1}\Lambda_{t+1}] - \eta f(\theta_t) E_t[M_{t+1}\Lambda_{t+1}], \qquad \text{(B.17)}
\end{aligned}$$

in which the last equality follows from the definition of Λ_t and the surplus sharing rule (B.3).

The surplus sharing rule implies $S_{Nt} = (1-\eta)\Lambda_t$, which, combined with equation (B.16), yields:

$$(1-\eta)\Lambda_t = X_t - W_t + (1-\eta)(1-s)E_t[M_{t+1}\Lambda_{t+1}]. \qquad \text{(B.18)}$$

Combining equations (B.17) and (B.18) yields:

$$\begin{aligned} X_t - W_t + (1-\eta)(1-s)E_t\left[M_{t+1}\Lambda_{t+1}\right] &= (1-\eta)(X_t - b) + (1-\eta)(1-s)E_t\left[M_{t+1}\Lambda_{t+1}\right] \\ &\quad -(1-\eta)\eta f(\theta_t)E_t\left[M_{t+1}\Lambda_{t+1}\right] \\ X_t - W_t &= (1-\eta)(X_t - b) - (1-\eta)\eta f(\theta_t)E_t\left[M_{t+1}\Lambda_{t+1}\right] \\ W_t &= \eta X_t + (1-\eta)b + (1-\eta)\eta f(\theta_t)E_t\left[M_{t+1}\Lambda_{t+1}\right]. \end{aligned}$$

Using equations (B.3) and (B.15) to simplify further:

$$W_t = \eta X_t + (1-\eta)b + \eta f(\theta_t)E_t\left[M_{t+1}S_{Nt+1}\right] \tag{B.19}$$

$$= \eta X_t + (1-\eta)b + \eta f(\theta_t)\left[\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t\right]. \tag{B.20}$$

Using the Kuhn-Tucker conditions, when $V_t > 0$, then $\lambda_t = 0$, and equation (B.20) reduces to the wage equation (19) because $f(\theta_t) = \theta_t q(\theta_t)$. On the other hand, when the nonnegativity constraint is binding, $\lambda_t > 0$, but $V_t = 0$ means $\theta_t = 0$ and $f(\theta_t) = 0$. Equation (B.20) reduces to $W_t = \eta X_t + (1-\eta)b$. Because $\theta_t = 0$, the wage equation (19) continues to hold.

C Details of the Globally Nonlinear Solution Algorithm

In the numerical algorithm, the two functional equations (21) and (23) should be expressed only in terms of two state variables N_t and x_t. As noted, we exploit a convenient mapping from the conditional expectation function, $\mathcal{E}_t$, to policy and multiplier functions to eliminate the need to parameterize the multiplier separately. After obtaining $\mathcal{E}_t$, we first calculate $\tilde{q}(\theta_t) = \kappa_0/\left(\mathcal{E}_t - \kappa_1\right).$ If $\tilde{q}(\theta_t) < 1$, the nonnegativity constraint is not binding, we set $\lambda_t = 0$ and $q(\theta_t) = \tilde{q}(\theta_t)$. We then solve $\theta_t = q^{-1}(\tilde{q}(\theta_t))$, in which $q^{-1}(\cdot)$ is the inverse function of $q(\cdot)$ defined in equation (3), and $V_t = \theta_t(1-N_t)$. If $\tilde{q}(\theta_t) \geq 1$, the nonnegativity constraint is binding, we set $V_t = 0$, $\theta_t = 0$, $q(\theta_t) = 1$, and $\lambda_t = \kappa_0 + \kappa_1 - \mathcal{E}_t$. We then perform the following set of substitutions:

$$U_t = 1 - N_t \tag{C.1}$$

$$N_{t+1} = (1-s)N_t + \frac{U_t V_t}{\left(U_t^{\iota} + V_t^{\iota}\right)^{1/\iota}} \tag{C.2}$$

$$x_{t+1} = \rho x_t + \sigma\epsilon_{t+1} \tag{C.3}$$

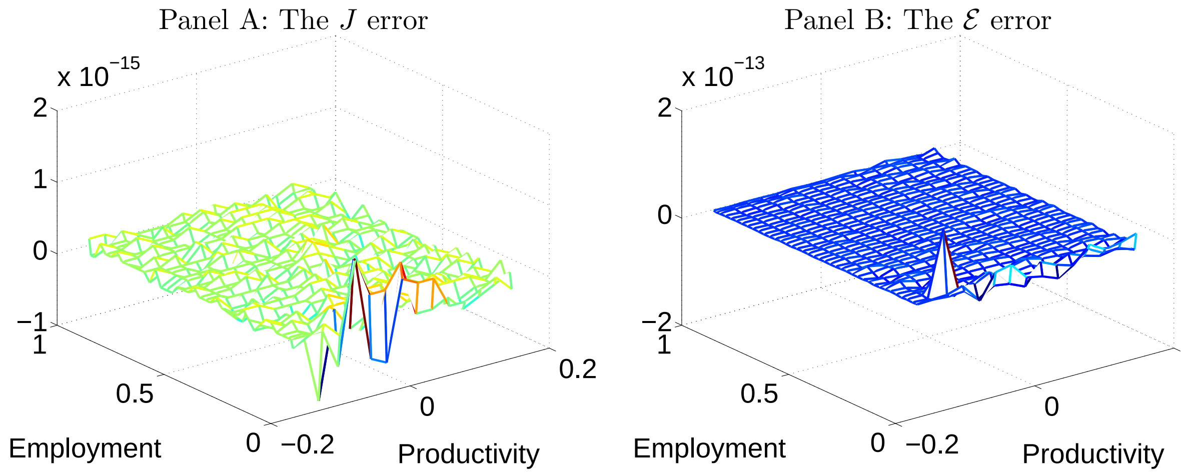
$$C(N_t, x_t) = \exp(x_t)N_t - \left[\kappa_0 + \kappa_1 q(\theta_t)\right]V(N_t, x_t) \tag{C.4}$$

$$M_{t+1} = \beta\left[\frac{C(N_{t+1}, x_{t+1})}{C(N_t, x_t)}\right]^{-\frac{1}{\psi}}\left[\frac{J(N_{t+1}, x_{t+1})}{E_t[J(N_{t+1}, x_{t+1})^{1-\gamma}]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma}, \tag{C.5}$$

and

$$W_t = \eta\left[\exp(x_t) + \left[\kappa_0 + \kappa_1 q(\theta_t)\right]\theta_t\right] + (1-\eta)b. \tag{C.6}$$

Figure C.1 : Errors in the J and $\mathcal{E}$ Functional Equations



We approximate the x_t process in equation (6) based on the discrete state space method of Rouwenhorst (1995) with 15 grid points.[9] This grid is large enough to cover the values of x_t within four unconditional standard deviations from its unconditional mean of zero. We set the minimum value of N_t to be 0.0365 and the maximum value to be 0.99. This range is large enough so that N_t never hits one of the boundaries in simulations. We use cubic splines with 40 basis functions on the N space to approximate $J(N_t, x_t)$ and $\mathcal{E}(N_t, x_t)$ on each grid point of x_t. We use extensively the approximation took kit in the CompEcon Toolbox in Matlab of Miranda and Fackler (2002). To obtain an initial guess for the projection algorithm, we use the social planner's solution via value function iteration. Solving the nonlinear model takes a lot of care, otherwise the projection algorithm would not converge. (Unlike the value function, iterating on the first-order conditions is not a contraction mapping.) The idea of homotopy continuation methods (e.g., Judd (1998, p. 179)) is used extensively to ensure convergence for a wide range of parameter values.[10]

Figure C.1 reports the error in the J functional equation (21), defined as $J(N_t, x_t)^{1-\frac{1}{\psi}} - (1 - \beta)C(N_t, x_t)^{1-\frac{1}{\psi}} - \beta\left(E_t\left[J(N_{t+1}, x_{t+1})^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}$, and the error in the $\mathcal{E}$ functional equation (23), defined as $\mathcal{E}(N_t, x_t) - E_t\left[M_{t+1}\left[X_{t+1} - W_{t+1} + (1-s)\left(\kappa_0/q(\theta_{t+1}) + \kappa_1 - \lambda(N_{t+1}, x_{t+1})\right)\right]\right]$. These errors, in the magnitude no higher than 10^{-13}, are extremely small. As such, our nonlinear algorithm does an accurate job in characterizing the competitive equilibrium in the search economy.

[9] Kopecky and Suen (2010) show that the Rouwenhorst (1995) method is more reliable and accurate than other methods in approximating highly persistent first-order autoregressive processes.

[10] In practice, when we solve the model with a new set of parameters, we set the lower bound of N_t to be 0.4 to alleviate the burden of nonlinearity on the solver. After obtaining the model's solution, we then apply homotopy to gradually reduce the lower bound to 0.0365 or whatever level that N_t never hits in simulations. Time-wise, with all the trial and error that comes with homotopy, solving the model with a parametrization that admits strong nonlinearity can take almost a week. Indeed, we have encountered some specifications and parametrizations of the model for which the projection solver has failed to converge at all.