A Compound-Multifractal Model for High-Frequency Asset Returns

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- Almost all asset returns are better characterized by a fat-tailed distribution.
- This was first observed in Mandelbrot (1963).
- After decades of research, there is still no consensus regarding which family of fat-tailed distributions best characterizes asset returns.

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- We do this in the context of high-frequency, trade-by-trade data.

En route to our final model we highlight two insights:

1. Returns distributions are effectively categorized into two groups: those occurring during prescheduled news announcement periods, and those that do not.
2. Outside of news announcement periods, returns measured in trade-time (not clock-time) follow a Gaussian distribution.

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- This model is based on the Multifractal Model of Asset Returns developed by Mandelbrot, Calvet and Fisher (1997) as well as subsequent work by Calvet and Fisher.

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- Our model can be characterized as a mixture of Gaussian distributions.

We show that our resulting mixture of Gaussians provides a very good fit to the data.

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- It is exemplary of a large class of assets (including equities).
  - This is attributed to its liquidity and the relationship of price formation between the futures and equities exchanges (Laughlin, Aguirre and Grundfest (2013)).

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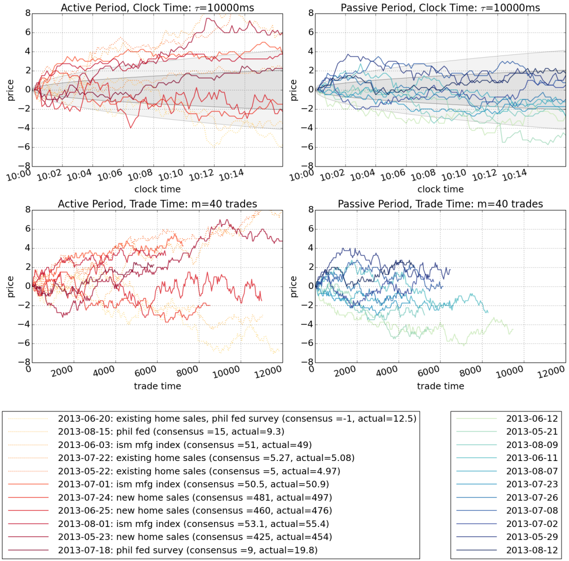
- The resulting sorted subsamples are roughly equal in size: 191,127 active trades and 174,041 passive trades.

Data



Active Period, Clock Time: τ=10000ms
Passive Period, Clock Time: τ=10000ms
Active Period, Trade Time: m=40 trades
Passive Period, Trade Time: m=40 trades
price
clock time
trade time
10:00
10:02
10:04
10:06
10:08
10:10
10:12
10:14
0
2000
4000
6000
8000
10000
12000
8
6
4
2
0
−2
−4
−6
−8
2013-06-20: existing home sales, phil fed survey (consensus =-1, actual=12.5)
2013-08-15: phil fed (consensus =15, actual=9.3)
2013-06-03: ism mfg index (consensus =51, actual=49)
2013-07-22: existing home sales (consensus =5.27, actual=5.08)
2013-05-22: existing home sales (consensus =5, actual=4.97)
2013-07-01: ism mfg index (consensus =50.5, actual=50.9)
2013-07-24: new home sales (consensus =481, actual=497)
2013-06-25: new home sales (consensus =460, actual=476)
2013-08-01: ism mfg index (consensus =53.1, actual=55.4)
2013-05-23: new home sales (consensus =425, actual=454)
2013-07-18: phil fed survey (consensus =9, actual=19.8)
2013-06-12
2013-05-21
2013-08-09
2013-06-11
2013-08-07
2013-07-23
2013-07-26
2013-07-08
2013-07-02
2013-05-29
2013-08-12

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$$r_m(n) = p(n) - p(n - m),$$

  where n denotes the n-th trade and m is the number of trades in a unit of time.

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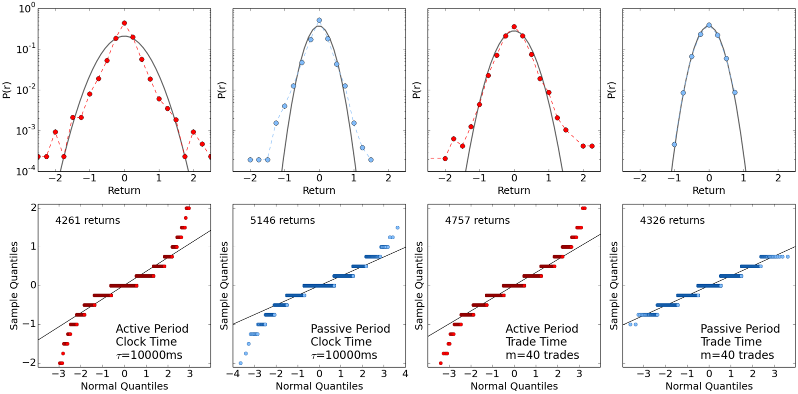
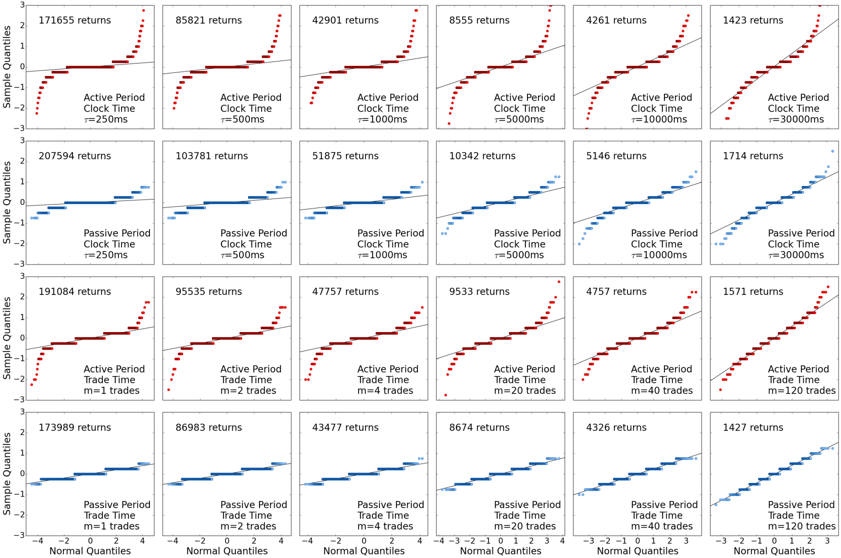
- There are 43 passive 1000-second intervals with 191,127 observations.
  - This is an average of 4.44 trades per second.
- So we roughly equate a trade-time interval of $m = 1$ trade to a clock-time interval of $\tau = 0.25$ seconds.

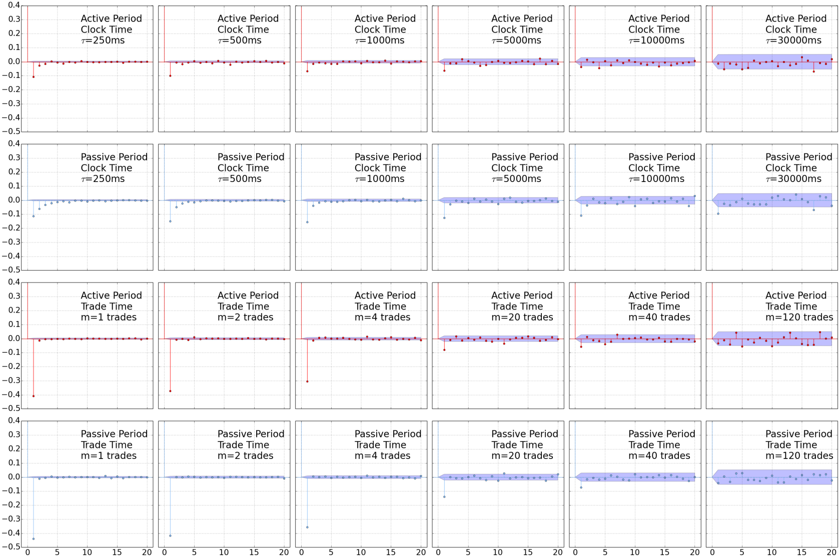
Empirical Densities, $\tau = 10,000$ ms and $m = 40$



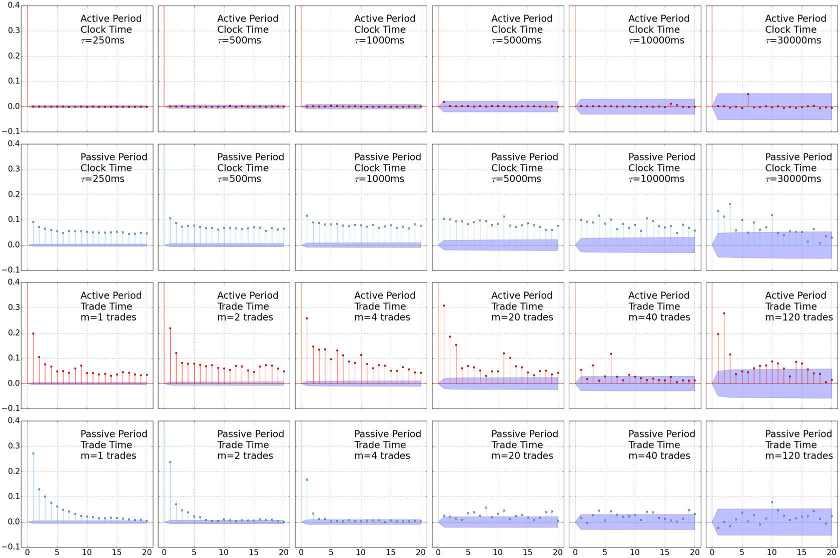
Q-Q Plots of Clock-Time and Trade-Time Returns



ACFs of Clock-Time and Trade-Time Returns



ACFs of Clock-Time and Trade-Time Squared Returns



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- For clock-time duration τ, denote the number of m-period executed trades as $N_m(\tau)$ with probability $P(N_m(\tau) = k)$.

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The probability density function of $r_\tau(t)$ is,

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- This is a finite Normal mixture model.
- The mixture weights vary according to the probability distribution of $N_m(\tau)$.

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- The resulting distribution of $r_\tau(t)$ will have fatter tails than a Gaussian.
- The tail fatness will be intimately related to the distribution of $N_m(\tau)$.

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where λ is the arrival intensity parameter.

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- Poisson trade arrivals are associated with Exponential inter-trade arrival times.

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- $\bar{k}$ latent state variables, $M_{k,i}$, that obey two-state Markov-switching processes.
- Persistence parameters, γ_k, for each latent variable $M_{k,i}$, $k = 1, 2, \ldots, \bar{k}$.

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- As time evolves the latent states, $M_{k,i}$, switch values with varying degrees of persistence, γ_k.
- This causes the actual distribution of intra-day trade durations to be a mixture of Exponentials.
- The latent states can be interpreted as shocks that have varying impacts over diverse timescales.

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- We can simulate from the distribution of the counting process.
- We can pair simulations from the counting density with Gaussian random variables to obtain simulations for clock-time returns $r_\tau(t)$ associated with the MSMD model.

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- We estimate the model using trade-time unit $m = 4$.

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We use the passive-market E-mini data to estimate the Exponential and MSMD duration models.

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Estimates

λ	$\gamma_{\bar{k}}$	b	m_0	ν	λ	μ	σ
15.71	0.1906	3.039	1.623	1.189	0.8408	-0.0006324	0.1393

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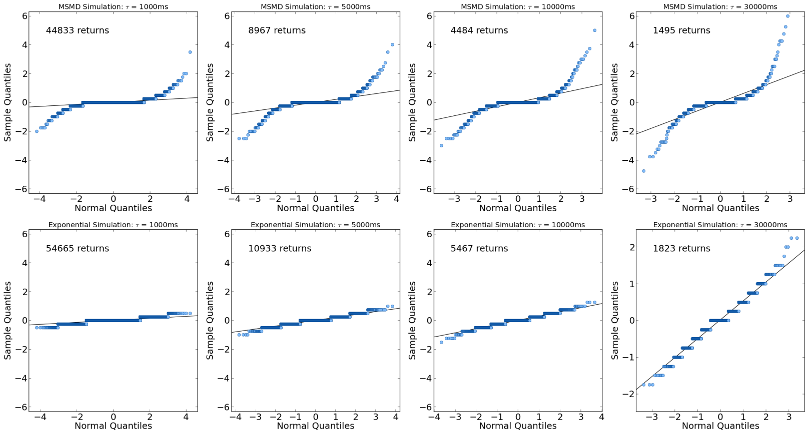
- The resulting number of clock-time returns, under both models roughly corresponds to the number of observations in our passive-period data for corresponding values of τ.

Q-Q Plots of Simulated Clock-Time Returns



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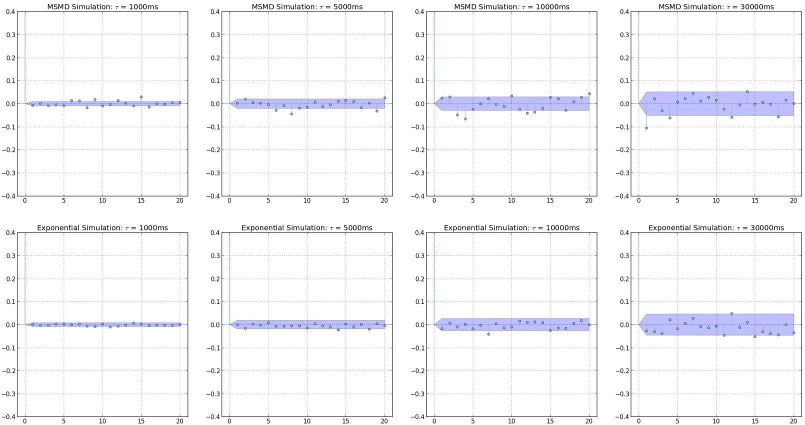
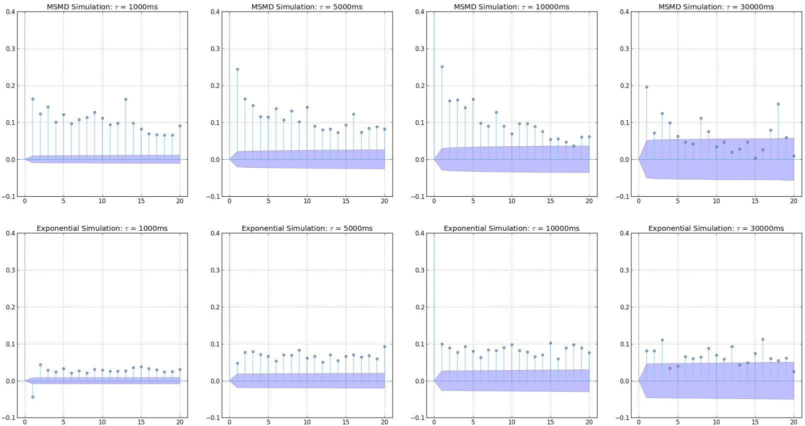


ACFs of Simulated Clock-Time Squared Returns



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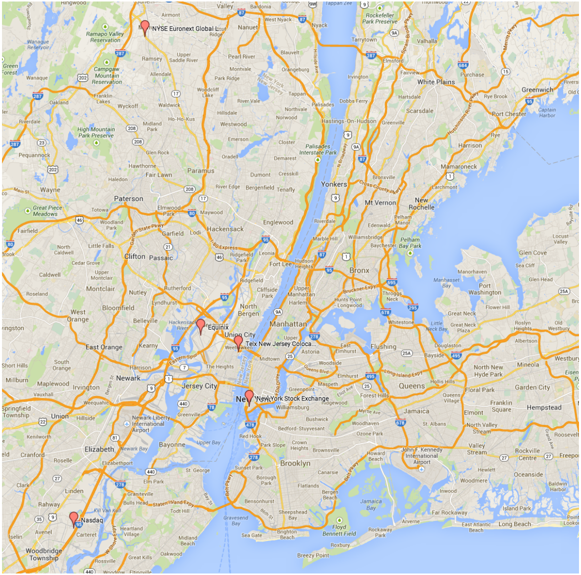
Exchanges in Metro NY



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$$0.7 \times 10 \text{ ms} + 0.3 \times 0.66 \text{ ms} \approx 7 \text{ ms}.$$

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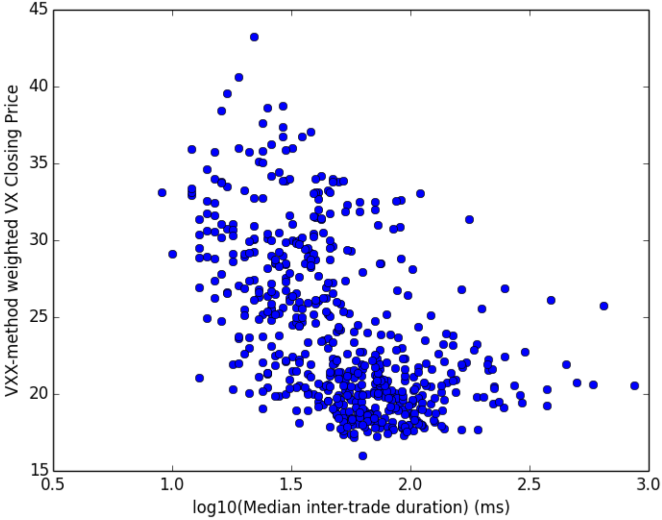

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- We are working on generating model implied volatilities that are associated with inter-trade durations.

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- Further, co-locating all NY exchanges would reduce the total market latency to roughly 1 microsecond, or a VIX upper bound of 6500.
- Market fragmentation forces an implicit threshold which bounds market volatility.

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- The clock-time returns distribution has fat tails that are commensurate with the data and also exhibits volatility clustering.
- Finally, our work with duration data suggests that market fragmentation is desirable from the perspective of placing an implicit bound on market volatility.