Problem Set 1

Econ 211C

Question 1 .... *25 points*

[Hamilton, Exercises 3.1 & 3.8] Consider the following $MA(2)$ process:

$$Y_t = (1 + 2.4L + 0.8L^2)\varepsilon_t,$$

where $\varepsilon_t \sim WN(0,1)$.

(a) (5 points) Is the process weakly stationary? If so, calculate its autocovariances.

(b) (10 points) Show that the process is not invertible and find an invertible representation for the process.

(c) (10 points) Calculate the autocovariances of the invertible representation. How do these relate to the autocovariances in part (a)?

Question 2 .... *33 points*

Consider two $MA(1)$ processes that differ only in the sign of their MA coefficient:

$$Y_{1,t} = 0.61 + \varepsilon_t + 0.95\varepsilon_{t-1}$$
$$Y_{2,t} = 0.61 + \varepsilon_t - 0.95\varepsilon_{t-1}$$

where $\varepsilon_t \sim WN(0, \sigma = 0.5)$. Simulate 1000 instances of 23 observations ($t = 1, 2, \ldots, 23$) of Y_1 and Y_2. For each simulation of $\{\varepsilon\}_{t=0}^{23}$, make sure to use *the same* values to compute Y_1 and Y_2 (i.e. do not simulate different ε sequences for the two MA processes). Plot the groups of time series in two different panels of a single figure: plot all time paths of Y_1 in the upper panel and all time paths of Y_2 in the lower panel. Set the transparency of each line to 0.2. You will find the `rgb` function to be useful in order to pass a color (and transparency value) to the `plot` function. *Bonus (5 points):* Write your code (including plotting) with no loops.

Question 3 .... *42 points*

Consider the $ARMA(2,5)$ model,

$$Y_t = 1.3Y_{t-1} - 0.4Y_{t-2} + \varepsilon_t + 0.7\varepsilon_{t-1} + 0.1\varepsilon_{t-3} - 0.5\varepsilon_{t-4} - 0.2\varepsilon_{t-5},$$

where $\varepsilon_t \sim WN(0,1)$.

(a) (7 points) Is this $ARMA$ process for Y_t weakly stationary?

(b) (7 points) Is this $ARMA$ process for Y_t invertible?

(c) (7 points) Calculate the first 5 autocovariances for Y_t. Derive a recursive equation that can be used to compute all subsequent autocovariances.

(d) (7 points) Calculate and plot the first 50 autocorrelations for Y_t.

(e) (7 points) Use `R` or Python to simulate $n = 1000$ values of Y_t. Do this without using any specialty time series functions (for example, do not use the `arima` function in `R`). What are the sample mean and variance of your simulation? What are your estimates of the first five autocovariances? How do these values compare with their theoretical counterparts computed in part (c)?

(f) (7 points) Repeat part (e) for $n = \{10000, 100000, 1000000\}$. How do your estimates of mean, variance and first five autocovariances compare with each other and with the true values that you have already computed?