Tranching, CDS, and Asset Prices: How Financial Innovation Can Cause Bubbles and Crashes[†]

By Ana Fostel and John Geanakoplos*

We show how the timing of financial innovation might have contributed to the mortgage bubble and then to the crash of 2007–2009. We show why tranching and leverage first raised asset prices and why CDS lowered them afterward. This may seem puzzling, since it implies that creating a derivative tranche in the securitization whose payoffs are identical to the CDS will raise the underlying asset price, while the CDS outside the securitization lowers it. The resolution of the puzzle is that the CDS lowers the value of the underlying asset since it is equivalent to tranching cash. (JEL E32, E44, G01, G12, G13, G21).

In this paper, we propose the possibility that the mortgage boom and bust crisis of 2007–2009 might have been caused by financial innovation. We suggest that the astounding rise in subprime and Alt A leverage from 2000 to 2006, together with the remarkable growth in securitization and tranching throughout the 1990s and early 2000s, raised the prices of the underlying assets such as houses and mortgage bonds. We further raise the possibility that the introduction of Credit Default Swaps (CDS), in 2005 and 2006 brought those prices crashing down with just the tiniest spark.

Securitization and tranching did not happen over night. The securitization of mortgages by the government agencies Fannie Mae and Freddie Mac began in earnest in the 1970s, when the first pools of mortgages were assembled and shares were sold to investors. In 1986, Salomon and First Boston created the first tranches, buying Fannie and Freddie pools and cutting them into four pieces. This was no simple task because it involved not only special tax treatment by the government but also the creation of special legal entities and trusts which would collect the homeowner payments and then divide them up among the bondholders. By the middle 1990s, the greatest mortgage powerhouse was the investment bank Kidder Peabody, cutting hundreds of billions of dollars worth of mortgage pools into over 90 types of tranches called CMOs (collateralized mortgage obligations). These tranches bore esoteric names like floater, inverse floater, IO, PO, inverse IO, Pac, Tac, etc. The young traders, often in their mid-20s, who collectively engineered

this multi-trillion dollar operation were motivated by the profits they could make buying pools of mortgages and cutting them up into more valuable tranches. They would find out the needs of various buyers and tailor make the tranches to deliver money in just those states of nature that the buyers wanted them. In short, they exploited the heterogeneous needs of their buyers by creating heterogeneous pieces out of a homogeneous pie.

The impetus driving the tranching machine was not a demand for riskless assets; on the contrary, it was a demand for contingent assets. The Fannie and Freddie principal mortgage payments were guaranteed against homeowner default by Fannie and Freddie, enabling the tranches to be rated AAA. But that hardly meant they were riskless. Changes in interest rates or changes in prepayments by the underlying homeowners could radically alter the cash flows of the tranches. Gradually, Wall Street came to see that default risk was just one among many risks, and pools and tranching began to be undertaken without government guarantees, for example, for jumbo mortgages that were not eligible for purchase by Fannie and Freddie and for credit cards and other assets.

Spurred on by these private securitizations, Wall Street dreamt up the idea in the mid-1990s of pooling and tranching subprime mortgages, with no government guarantees at all. Through a cleverly constructed architecture of pooling, senior pieces were still able to get AAA bond ratings because they were protected by junior tranches that absorbed the losses in case of homeowner defaults. The subprime mortgage market grew from a few million dollars to a trillion dollars by 2006.

In the 1990s, credit default swaps were invented for corporate bonds and sovereign bonds. It was not until 2005, however, that credit default swaps were standardized for mortgages. CDS are a kind of insurance on an asset or bond. It is the promise to take back the underlying asset at par once there is a default, that is, to make up the losses of the underlying asset.

Our approach, like many papers in economics that take technological innovation as exogenous, is to take the financial innovations in the mortgage market between 1986 and 2010 as exogenous and investigate their consequences for asset pricing. Under this view, the tranching of subprime mortgages couldn't have begun earlier because it had to wait for the innovation of CMO tranching. In later work, we hope to explain why the innovations came when they did and why, for example, CDS seem to appear in various markets only after the risk of default is generally recognized to be significant.

The size of these financial innovations is certainly staggering, and leaves one wondering what their effects might have been. Consider first the history of subprime and Alt A leverage and housing prices from 2000 to 2008 shown in Figure 1, taken from Geanakoplos (2010b). Leverage went from about 7 in 2000 (14 percent average down payment for the top half of households) to about 35 in the second quarter of 2006 (2.7 percent down payment on average for the top half of households). Next, consider the growth of securitization and tranching presented in Figure 2, especially in the late 1990s and early 2000s. These amounted to trillions of dollars a year. Finally, consider the growth of the CDS market presented in Figure 3, especially after 2005. The available numbers are not specific to mortgages, since most of these were over the counter, but the fact that subprime CDS were not standardized until
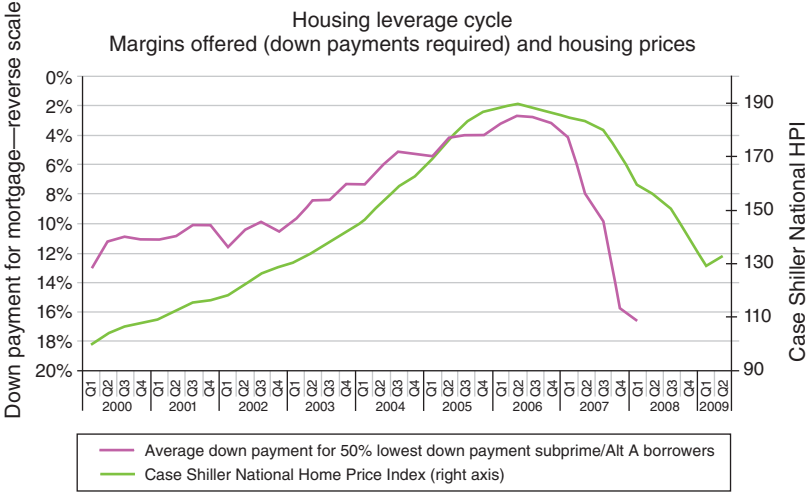
FIGURE 1

Notes: Observe that the down payment axis has been reversed because lower down payment requirements are correlated with higher home prices. For every Alt A or subprime first loan originated from 2000:Q1 to 2008:Q1, down payment percentage was calculated as appraised value (or sale price if available) minus total mortgage debt, divided by appraised value. For each quarter, the down payment percentages were ranked from highest to lowest, and the average of the bottom half of the list is shown in the diagram. This number is an indicator of down payment required. Clearly many homeowners put down more than they had to, and that is why the top half is dropped from the average. A 13 percent down payment in 2000:Q1 corresponds to leverage of about 7.7, and a 2.7 percent down payment in 2006:Q2 corresponds to leverage of about 37. Subprime/Alt A issuance stopped in 2008:Q1.
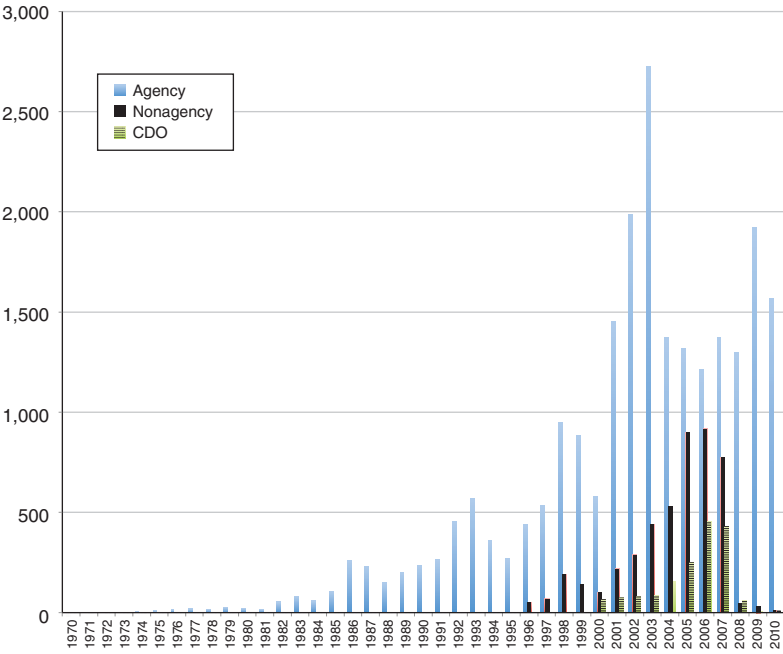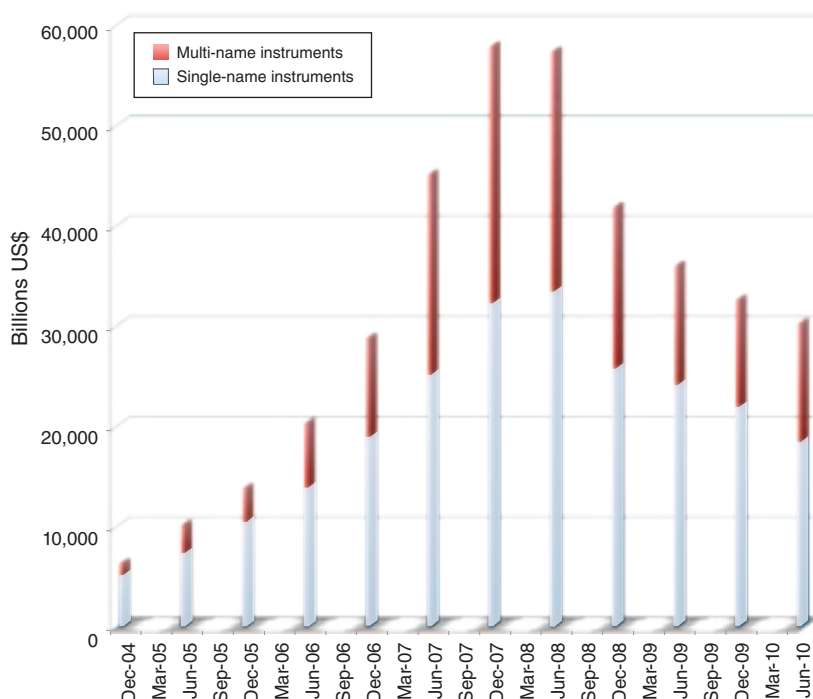

FIGURE 2. SECURITIZATION/TRANCHING

FIGURE 3. CDS MARKETS, OUTSTANDING NOTIONAL AMOUNT

Source: IBS OTC Derivatives Market Statistics

late 2005 suggests that the growth of mortgage CDS in 2006 is likely even sharper than Figure 3 suggests. What is clear is that the explosive growth of the CDS market came after the explosive growth of securitization.

Many people who are aware of these numbers have linked securitization and CDS to the crisis of 2007–2009. While we agree with much of their view, our analysis is based on entirely different considerations. And we wish to explain the boom as well as the bust.

Some problems that many critics have noted with tranching are: the standing opportunity for the original lender to sell his loans into a securitization destroys his incentive to choose good loans, and once a pool is tranched, it becomes very difficult for the bondholders to negotiate with each other (for example, to write down principal).

Many observers have pointed to the creation of CDS as the source of many problems, to mention a few: important financial institutions wrote trillions of dollars of CDS insurance—the economy could not run smoothly after they lost so much money on their bad bets; writers of CDS insurance did not even post enough collateral to cover their bets, forcing the government to bail out the beneficiaries; CDS were traded on OTC markets, with a lack of transparency that enabled price gouging; CDS give investors (at least those who wrote much more insurance than the underlying assets were worth) the incentive to manipulate markets, for example, to avoid paying off a big insurance amount by directly paying off the bonds. George

Soros went as far as calling CDS "instruments of destruction that should be outlawed" and claimed that " … some derivatives ought not to be allowed to be traded at all. I have in mind credit default swaps. The more I've heard about them, the more I've realized they're truly toxic …"[1]

All the difficulties with tranching and CDS pointed out by others in the last paragraphs rely on the pernicious effect of securitization and tranching on the basic cash flows. Our thesis is quite different. We argue that the very splitting of the same cash flows changes the underlying collateral price, generally making it higher if the splitting is done properly. Leverage and tranching therefore raise asset prices, and CDS lowers them.

The first main contribution of this paper is to show how tranching and leverage raise the price of the underlying collateral even if they have no effect on the total cash flows coming from the collateral. Though not generally recognized by economic theory, this is really common sense. Indeed the historically enthusiastic government support for the tranching of mortgages was doubtless due to an understanding that it raises the price of the underlying mortgage assets, and therefore reduces the borrowing costs to the homeowner.

Tranching makes the underlying collateral more valuable because it can be broken into pieces that are tailor made for different parts of the population, just as the traders in the 1990s realized. Splitting plain vanilla into strawberry for one group and chocolate for another should raise the value of the scarce ice cream. Leverage is an imperfect form of tranching, and therefore one would guess that leverage would not raise the asset price as much as tranching. But our analysis shows that things are a bit subtler.

We first build a static two-period, two-state model with heterogeneous agents in which we can investigate the circumstances under which these common sense conclusions hold true. We compute equilibrium prices without leverage, with leverage on assets, with tranching of assets, then with tranching and CDS. We find that once the tranching technology is invented and freely available, it will inevitably proceed in equilibrium all the way to Arrow securities, or at least all the way to commonly verifiable events. Leverage and asset tranching always raise asset prices above their CDS and no-leverage levels. Somewhat surprisingly, however, we find that this fine tranching does not always raise the asset price above the leverage price when all the general equilibrium effects are taken into account. We prove that if there is more heterogeneity among the pessimists than among the optimists, then tranching always yields a higher price than leverage.

Furthermore, we show that with tranching, the price of the underlying collateral can rise above what any agent thinks it is worth, while, with leverage, the price of the asset typically rises to what a more optimistic agent thinks it is worth. This tranched hyper price fits the definition of a bubble given in Harrison and Kreps (1978), where a bubble is defined as an asset price that is higher than any agent thinks the asset is worth. In Harrison and Kreps (1978), the explanation for bubbles turned on the ability of agents to resell the asset to others who would think it was worth more, whereas, in our two-period model, there is no resale of the asset.

---

[1] George Soros. 2009. "One Way to Stop Bear Raids Credit Default Swaps Need Much Stricter Regulation." *Wall Street Journal*, March 24, 2009.

Third, we show that the introduction of CDS dramatically lowers the asset price, even below the nonleverage level, provided that the median investor thinks that the asset is more than 50 percent likely to have a good payoff. This seems counterintuitive at first glance. Tranching creates derivatives of the underlying asset, and presumably one of those tranches could have exactly the same payouts as the CDS. Indeed that happens in our model. The tranche and the CDS are perfect substitutes in every agent's mind. Yet, when the CDS is created exclusively inside the securitization as a tranche of the asset, it raises the asset price. When the CDS is created outside the securitization, it lowers the asset price. How could this be?

We show that on second thought this is not surprising at all. When agents sell CDS and put up cash as collateral, they are effectively tranching cash! That raises the value of cash relative to the reference asset. When every asset (more precisely, when all future cash flows) can be perfectly tranched, we get the Arrow-Debreu equilibrium, and all asset bubbles disappear. The depressing effect of CDS on asset prices is most dramatic when the asset is not tranched, but is held outright or levered, because in that case the buyers of the asset will divert their wealth into writing CDS, which is a perfect substitute for holding the asset.

In Section I, we present our two-period model of collateral equilibrium. In Section II, we show how specifying the collateral technology in different ways allows the same model to encompass leverage, tranching, and CDS in one simple framework. In Section III, we explain why leverage and securitization raise asset prices, and why CDS lowers them. Finally, in Section IV, we describe a dynamic model in which a nonlevered initial situation is followed by the unexpected introduction of leverage, then securitization and tranching, and finally CDS. All the way through very small bad shocks are occurring. Nevertheless, prices rise dramatically during the initial three phases, then come crashing down with the introduction of CDS.

The timing of the financial innovation was crucial. Tranching and securitization came first, raising asset prices, then CDS followed much later, crushing their prices. Figures 1, 2, and 3 show empirical evidence suggesting that this was indeed the timing in securitization and CDS markets.[2] The timing of innovation was disastrous because it caused a crash, forcing many people into bankruptcy or underwater. Had the CDS come at the same time as the securitization, asset prices would never have gotten so high, and the crash would have been milder, as we show in Section IV.

Our financial innovation theory of booms and busts complements the Leverage Cycle theory proposed in Geanakoplos (2003, 2010a, and 2010b) and developed further in Fostel and Geanakoplos (2008, 2011, and forthcoming). The financial innovation theory is similar in many ways to the leverage cycle, but unlike that theory, it does not rely on any kind of shock, much less on a shock that also increases the volatility of future shocks. In the financial innovation story of the boom and bust told here, innovation could have generated the entire cycle on its own, with no external triggers.

In the leverage cycle story of the recent boom and bust told in Geanakoplos (2010a and 2010b), a prolonged period of low volatility led to high leverage and therefore high asset prices and the concentration of assets in the hands of the most

---

[2] Academic papers describing the financial crisis all agree on this fact, as in Brunnermeier (2009), Gorton (2009), Gorton and Metrick (forthcoming), Geanakoplos (2010b), and Stultz (2009).

optimistic investors. When bad news came in 2007, in the form of a spike in delinquencies of subprime homeowners, it not only directly reduced prices, but it also reduced leverage because it created more uncertainty about what would happen next, which had an indirect effect on asset prices. The combination of bad news, losses by the hyper-leveraged optimists, and plummeting leverage led to a huge fall in asset prices, much bigger than could be explained by the bad news alone.

Geanakoplos (2010b) attributed the rise in leverage to many factors besides low volatility. One of these was the almost explicit government guarantees to government agencies like Fannie Mae and Freddie Mac, and another was the implicit guarantees to the big banks that were too big to fail. Yet another was low interest rates and the resulting pursuit of yield. But most important, he said, was securitization. Yet he provided no model for the connection between securitization and leverage. Geanakoplos (2010a, 2010b), also suggested that the introduction of standardized credit default swaps in 2005 had a sharp negative impact on the prices of assets.

Here, we make rigorous the connection between leverage, tranching, and asset prices by extending the model in Geanakoplos (2003). In the language of Fostel and Geanakoplos (2008), tranching increases the collateral value of the underlying asset. Leverage is an imperfect form of tranching, and so raises the underlying asset value less than ideal tranching. CDS is a form of tranching cash, and so raises the relative value of cash, thus lowering the value of the reference asset.

Our paper is more generally related to a literature on leverage as in Araújo, Kubler, and Schommer (forthcoming); Acharya and Viswanathan (2011); Adrian and Shin (2010); Brunnermeier and Pedersen (2009); Cao (2010); Fostel and Geanakoplos (2008, 2011, and forthcoming); Geanakoplos (1997, 2003, and 2010b); Gromb and Vayanos (2002); and Simsek (2010). It is also related to work that studies the asset price implications of leverage as Hindy (1994); Hindy and Huang (1995); and Garleanu and Pedersen (2011).

Our paper is also part of a growing theoretical literature on CDS. Bolton and Oehmke (forthcoming) study the effect of CDS on the debtor-creditor relationship. The proposition that CDS tends to lower asset prices was demonstrated in Geanakoplos (2010a), and confirmed in exactly the same model by Che and Sethi (2010).

I. General Equilibrium Model with Collateral

The model is a two-period general equilibrium model, with time $t = 0, 1$. Uncertainty is represented by a tree $S = \{0, U, D\}$ with a root $s = 0$ at time 0 and two states of nature $s = U, D$ at time 1.

There are two assets in the economy which produce dividends of the consumption good at time 1. The riskless asset X produces $X_U = X_D = 1$ unit of the consumption good in each state, and the risky asset Y produces $Y_U = 1$ unit in state U and $0 < Y_D = R < 1$ unit of the consumption good in state D. Figure 4 shows asset payoffs.

Each investor in the continuum $h \in H = (0, 1)$ is risk-neutral and characterized by a linear utility for consumption of the single consumption good x at time 1,
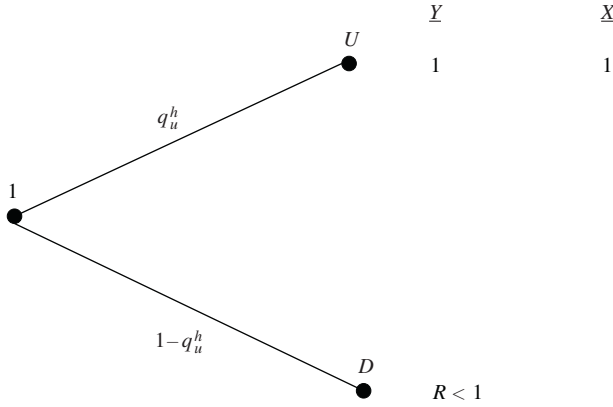
$$
\begin{array}{ccc}
 & \underline{Y} & \underline{X} \\
U & 1 & 1 \\
\end{array}
$$

FIGURE 4. ASSET PAYOFFS

and subjective probabilities, $(q_U^h, q_D^h = 1 - q_U^h)$. The von-Neumann-Morgenstern expected utility to agent h is

$$
(1) \qquad\qquad U^h(x_U, x_D) = q_U^h x_U + q_D^h x_D.
$$

We shall suppose that q_U^h is strictly monotonically increasing and continuous in h. Examples are $q_U^h = 1 - (1 - h)^2, q_D^h = (1 - h)^2$, and $q_U^h = h, q_D^h = 1 - h$.

Each investor $h \in (0, 1)$ has an endowment of one unit of each asset at time 0 and nothing else. Since only the output of Y depends on the state and $1 > R$, higher h denotes more optimism. Heterogeneity among the agents stems entirely from the dependence of q_U^h on h.

The reader may be aghast by the simplicity of the model, and in particular by heterogeneous priors, risk neutrality, and the lack of endowment of the consumption good in states 1 and 2. We hasten to assure such a reader that we are using the simplest model we can to illustrate our point. None of the results depend on risk neutrality or heterogeneous priors. By assuming common probabilities and strictly concave utilities, and adding large endowments in state D versus state U for agents with high h and low endowments in state D versus state U for agents with low h, we could reproduce the distribution of marginal utilities we get from differences in prior probabilities. We have chosen to replace the usual marginal analysis of consumers who have interior consumption with a continuum of agents and a marginal buyer. Our view is that the slightly unconventional modeling is a small price to pay for the simple tractability of the analysis.

A. *Arrow Debreu Equilibrium*

Arrow-Debreu equilibrium is easy to describe for our simple economy. It is given by present value consumption prices (p_U, p_D), which, without loss of generality, we can normalize to add up to 1, and by consumption $(x_U^h, x_D^h)_{h \in H}$ satisfying:

1. $\displaystyle\int_0^1 x_U^h \, dh = 1 + 1$

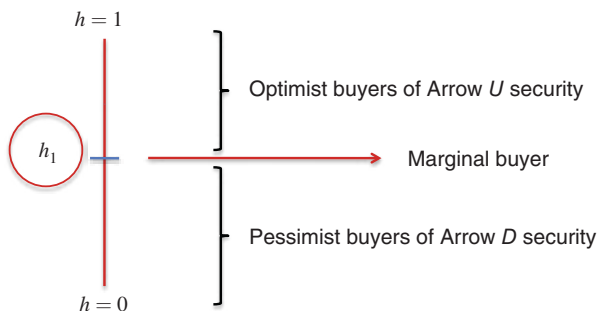2. $\displaystyle\int_0^1 x_D^h \, dh = 1 + R$

FIGURE 5. ARROW-DEBREU EQUILIBRIUM REGIME

3. $(x_U^h, x_D^h) \in B_W^h(p_U, p_D) = \{(x_U, x_D) \in R_+^2 : p_U x_U + p_D x_D$

$$\leq p_U(1 + 1) + p_D(1 + R)\}$$

4. $(x_U, x_D) \in B_W^h(p_U, p_D) \Rightarrow U^h(x) \leq U^h(x^h), \forall h.$

The interpretation of Arrow-Debreu equilibrium is that at time 0 agents trade contingent commodities forward. An agent with high h, for example, might sell a future claim for D consumption in exchange for U consumption. It is taken for granted that h will deliver the goods in D if that state occurs.

We can easily compute Arrow-Debreu equilibrium. Because of linear utilities, and the continuity of utility in h, and the connectedness of the set of agents $H = (0, 1)$, at state $s = 0$ there will be a *marginal buyer*, h_1, who will be indifferent between buying the Arrow U and the Arrow D security. All agents $h > h_1$ will sell everything and buy only the Arrow U security. Agents $h < h_1$ will sell everything and buy only the Arrow D security. This regime is shown in Figure 5.

At $s = 0$, aggregate revenue from sales of the Arrow U security is given by $p_U \times 2$. On the other hand, aggregate expenditure by the buyers $h \in [h_1, 1)$ is given by $(1 - h_1)(2p_U + (1 + R)(1 - p_U))$. Equating, we have

(2) $$(2p_U + (1 + R)(1 - p_U))(1 - h_1) = 2p_U.$$

The next equation states that the marginal buyer is indifferent between buying the Arrow U and the Arrow D security:

(3) $$q_U^{h_1}/p_U = (1 - q_U^{h_1})/(1 - p_U).$$

Hence, we have a system of two equations and two unknowns: the price of the Arrow U security, p_U; and the marginal buyer, h_1. For the probabilities $q_U^h = 1 - (1 - h)^2$ and $R = 0.2$, we get $h_1 = 0.33$ and $p_U = 0.55$. The implicit prices of X and Y are $p_X = p_U 1 + p_D 1 = 1$ and $p_Y = p_U 1 + p_D R = 0.64$.

B. *Financial Contracts and Collateral*

The heart of our analysis involves contracts and collateral. In Arrow-Debreu equilibrium, the question of why agents repay their loans is ignored. We suppose from now on that the only enforcement mechanism is collateral.

At time 0, agents can trade financial contracts. A financial contract (A, C) consists of both a promise, $A = (A_U, A_D)$, and an asset acting as collateral backing it, $C \in \{X, Y\}$. The lender has the right to seize as much of the collateral as will make him whole once the promise comes due, but no more; the contract therefore delivers $(\min(A_U, C_U), \min(A_D, C_D))$ in the two states. The significance of the collateral is that the borrower must own the collateral C at time 0 in order to make the promise A.

We shall suppose every contract is collateralized either by one unit of X or by one unit of Y. The set of promises j backed by one unit of X is denoted by J^X and the set of contracts backed by one unit of Y is denoted by J^Y. In the next section, we will analyze different economies obtained by varying the set $J = J^X \cup J^Y$.

We shall denote the sale of promise j by $\varphi_j > 0$ and the purchase of the same contract by $\varphi_j < 0$. The sale of a contract corresponds to borrowing the sale price, and the purchase of a promise is tantamount to lending the price in return for the promise. The sale of $\varphi_j > 0$ units of contract type $j \in J^X$ requires the ownership of φ_j units of X, whereas the purchase of the same number of contracts does not require any ownership of X.

C. *Budget Set*

Each contract $j \in J^C$ will trade for a price π^j. An investor can borrow π^j today by selling contract j in exchange for a promise of A^j tomorrow, provided he owns C.

We can always normalize one price in each state $s \in S = \{0, U, D\}$, so we take the price of X in state 0 and the price of consumption in each state U, D to be one. Thus, X is both riskless and the numeraire. Hence, it is in some ways analogous to a durable consumption good like gold, or to money, in our one commodity model. Given asset and contract prices at time 0, $(p, (\pi^j)_{j \in J})$, each agent $h \in H$ decides his asset holdings, x of X and y of Y and contract trades φ_j in state 0, in order to maximize utility (1) subject to the budget set defined by

$$
B^h(p, \pi) = \Big\{ (x, y, \varphi, x_U, x_D) \in R_+ \times R_+ \times R^{J^X} \times R^{J^Y} \times R_+ \times R_+ :
$$

$$
(x - 1) + p(y - 1) \leq \sum_{j \in J} \varphi_j \pi^j
$$

$$
\sum_{j \in J^X} \max(0, \varphi_j) \leq x, \sum_{j \in J^Y} \max(0, \varphi_j) \leq y
$$

$$
x_U = x + y - \sum_{j \in J^X} \varphi_j \min(A^j_U, 1) - \sum_{j \in J^Y} \varphi_j \min(A^j_U, 1)
$$

$$
x_D = x + yR - \sum_{j \in J^X} \varphi_j \min(A^j_D, 1) - \sum_{j \in J^Y} \varphi_j \min(A^j_D, R) \Big\}.
$$

At time 0, expenditures on the assets purchased (or sold) can be at most equal to the money borrowed selling contracts using the assets as collateral. The assets put up as collateral must indeed be owned. In the final states, consumption must equal dividends of the assets held minus debt repayment.

Notice that there is no sign constraint on φ_j; a positive (negative) φ_j indicates the agent is selling (buying) contracts or borrowing (lending) π^j. Notice also that we are assuming that short selling of assets is not possible, $x, y \geq 0$.

D. *Collateral Equilibrium*

We suppose that agents are uniformly distributed in $(0,1)$, that is they are described by Lebesgue measure. A *Collateral Equilibrium* in this economy is a price of asset Y, contract prices, asset purchases, contract trade and consumption decisions by all the agents $((p, \pi), (x^h, y^h, \varphi^h, x_U^h, x_D^h)_{h \in H}) \in (R_+ \times R_+^J) \times (R_+ \times R_+ \times R^{J^X} \times R^{J^Y} \times R_+ \times R_+)^H$, such that

$$\int_0^1 x^h \, dh \; = \; 1$$

$$\int_0^1 y^h \, dh \; = \; 1$$

$$\int_0^1 \varphi_j^h \, dh \; = \; 0 \;\; \forall j \in J$$

$$(x^h, y^h, \varphi^h, x_U^h, x_D^h) \in B^h(p, \pi), \;\; \forall h$$

$$(x, y, \varphi, x_U, x_D) \in B^h(p, \pi) \;\Rightarrow\; U^h(x) \; \leq \; U^h(x^h), \;\; \forall h.$$

Markets for the consumption good in all states clear. Assets and promises clear in equilibrium at time 0, and agents optimize their utility in their budget sets. As shown by Geanakoplos and Zame (1997), equilibrium in this model always exists under the assumptions we made so far.

E. *Tranching*

One of the most important financial innovations has been the "tranching" of assets or collateral. In tranched securitizations, the collateral dividend payments are divided among a number of bonds, which are sold off to separate buyers. So far in our analysis, we have assumed that each collateral can back just one promise, so tranching seems out of the picture. But, in fact, the collateral holder gets the residual payments after the promise is paid. So effectively, we have been tranching into two bonds all along. And with two states of nature, we shall show that there is no reason to have more pieces. So as long as there is no restriction on the nature of the promise, our collateral equilibrium includes tranching.

In practice, houses have been tranched into first and second mortgages, and sometimes third mortgages. These tranches have the property that they all move in the same direction—good news for the house value is good news for all the tranches. But when mortgages are tranched, the tranche values often move in opposite directions. The more a floater pays, the less an inverse floater pays and so on. Even when the tranches of subprime mortgages appear to have the form of debt for the higher tranches and equity for the lower tranches, the presence of various triggers, which move cash flows from one tranche to another, can make the payoffs go in opposite directions.

Thus, in what follows, we shall assume in our analysis that tranching has reached a degree of perfection that permits Arrow security tranches to be created.[3] The tranching of mortgages in the CMO revolution of the 1990s moved far along in that direction. And to the extent that the mortgage principal amount is nearly as high as the house price, as often occurred in the 2000s, the mortgage already includes the entire future value of the house. Thus, we shall not distinguish between tranching the asset or tranching a mortgage written on the asset. In short, we shall assume that the tranching is directly backed by the asset.

II. Leverage, Securitization, and CDS

In this section, we study the effect of leverage and derivatives on equilibrium by considering four different versions of the collateral economy introduced in the last section, each defined by a different set of feasible contracts J. We describe each variation and the system of equations that characterizes the equilibrium. In the next section, we compare equilibrium asset prices across all the economies.

A. *No-Leverage Economy*

We consider first the simplest possible scenario, where no promises can be made, $J = \emptyset$. Agents can only trade assets Y and X. They cannot borrow using the assets as collateral.

Let us describe the system of equations that characterizes the equilibrium. Because of the strict monotonicity and continuity of q_U^h in h, and the linear utilities and the connectedness of the set of agents $H = (0,1)$ at state $s = 0$, there will be a unique *marginal buyer*, h_1, who will be indifferent between buying or selling Y. In equilibrium, it turns out that all agents $h > h_1$ will buy all they can afford of Y, while selling all their endowment of X. Agents $h < h_1$ will sell all their endowment of Y. This regime is shown in Figure 6.

At $s = 0$, aggregate revenue from sales of the asset is given by $p \times 1$.[4] On the other hand, aggregate expenditure on the asset is given by $(1 - h_1)(1 + p)$, which

---

[3] Of course, in reality, Arrow securities cannot be created. But the reason has to do with the lack of verifiability and the cost of writing complex contingencies into a contract, both of which are ignored in our analysis.

[4] All asset endowments add to one, and, without loss of generality, are put up for sale even by those who buy them.
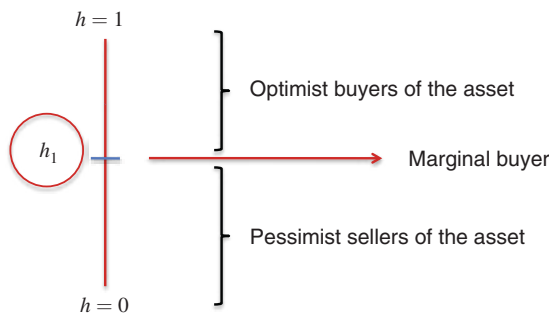
FIGURE 6. NON-LEVERAGE ECONOMY EQUILIBRIUM REGIME

is total income from the endowment of one unit of X, plus revenue from the sale of one unit of asset Y, by buyers $h \in [h_1, 1)$. Equating supply and demand, we have

$$(4) \qquad\qquad p = (1 - h_1)(1 + p).$$

The next equation states that the price at $s = 0$ is equal to the marginal buyer's valuation of the asset's future payoff:

$$(5) \qquad\qquad p = q_U^{h_1} 1 + (1 - q_U^{h_1})R.$$

Hence, we have a system of two equations and two unknowns: the price of the asset, p, and the marginal buyer, h_1. For the probabilities $q_U^h = 1 - (1 - h)^2$ and $R = 0.2$, we get $h_1 = 0.54$ and $p = 0.83$.

B. *Leverage Economy*

Agents now are allowed to borrow money to buy more of the risky asset Y. We let them issue noncontingent promises using the asset as collateral. In this case $J = J^Y$, and each $A_j = (j,j)$ for all $j \in J = J^Y$. The following result regarding leverage holds.

PROPOSITION 1: *Suppose that in equilibrium the max min contract j^* $= \min_{s=U,D}\{Y_s\} = R$ is available to be traded, that is $j^* \in J = J^Y$. Then j^* is the only contract traded, and the riskless interest rate is equal to zero, this is, $\pi^{j^*} = j^* = R$.*

PROOF:
See Geanakoplos (2003) and Fostel and Geanakoplos (forthcoming and 2011).
Leverage is endogenously determined in equilibrium. In particular, the proposition *derives* the conclusion that although all contracts will be priced in equilibrium, the only contract actively traded is the max min contract, which corresponds to the *Value at Risk equal zero rule*, *VaR* = 0, assumed by many other papers in the literature. Hence, there is no default in equilibrium.
Taking the proposition as given, let us describe the system of equations that characterizes the equilibrium. As before, there will be a *marginal buyer*, h_1, who will be indifferent between buying or selling Y. In equilibrium, all agents $h > h_1$ will buy
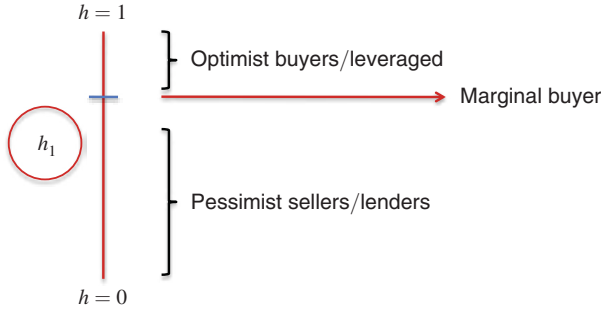
FIGURE 7. LEVERAGE ECONOMY EQUILIBRIUM REGIME

all they can afford of Y, i.e., they will sell all their endowment of the X and borrow to the max min using Y as collateral. Agents $h < h_1$ will sell all their endowment of Y and lend to the more optimistic investors. The regime is showed in Figure 7.

At $s = 0$, aggregate revenue from sales of the asset is given by $p \times 1$. On the other hand, aggregate expenditure on the asset is given by $(1 - h_1)(1 + p) + R$. The first term is total income (endowment of X plus revenues from asset sales) of buyers $h \in [h_1, 1)$. The second term is borrowing, which from proposition 1 is R (recall that the interest rate is zero). Equating, we have

$$(6) \qquad p = (1 - h_1)(1 + p) + R.$$

The next equation states that the price at $s = 0$ is equal to the marginal buyer's valuation of the asset's future payoff:

$$(7) \qquad p = q_U^{h_1} 1 + (1 - q_U^{h_1}) R.$$

We have a system of two equations and two unknowns: the price of the asset, p; and the marginal buyer, h_1. Notice how equation (6) differs from equation (4). Optimists now can borrow R. This will imply that in equilibrium a fewer number of optimists can afford to buy all the asset in the economy. Hence, the marginal buyer in the Leverage economy will be someone more optimistic than the marginal buyer in the No-Leverage economy. We will discuss this in detail in Section III. For the probabilities $q_U^h = 1 - (1 - h)^2$ and $R = 0.2$, we get $h_1 = 0.63$ and $p = 0.89$.

Finally, notice that buying the asset while leveraging to the max min is equivalent to buying the Arrow U security. Since the owner needs to pay back R in period 1, his net payoffs are $1 - R$ at U and 0 at D. Hence, optimistic investors who are desperate to transfer their wealth to the U state can very effectively do that by leveraging the asset to the max min. In the example with $q_U^h = 1 - (1 - h)^2$, the implicit price of the Arrow U security is given by $p_U = (p - R)/(1 - R) = 0.86$.

C. *Asset-Tranching Economy*

In this economy, we suppose that the risky asset Y can be *tranched* into arbitrary contingent promises, including the riskless promises from the last section and all
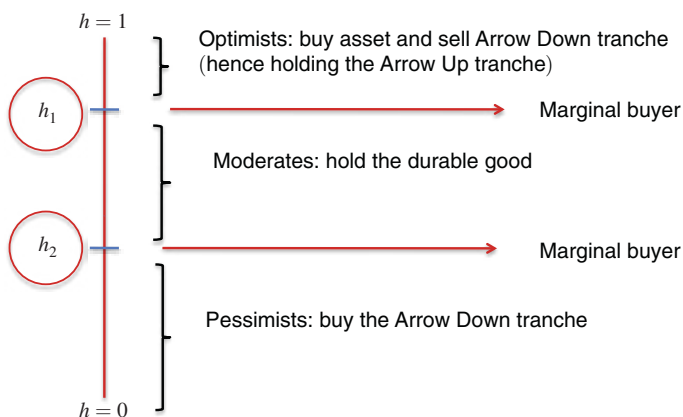
FIGURE 8. ASSET TRANCHING ECONOMY EQUILIBRIUM REGIME

Arrow promises. The holder of the asset can sell off any of the tranches he does not like and retain the rest. This is a step forward from the Leverage economy, in which investors holding a leveraged position on the asset could synthetically create the Arrow U security. Now they can also synthetically create the Arrow D security.

To simplify the analysis, we suppose, at first, that $J = J^Y$ consists of the single promise $A = (0, R)$, tantamount to a multiple of the Arrow D security. Notice that by buying the asset Y and selling off the tranche $(0, R)$, any agent can obtain the Arrow U security. Our parsimonious description of $J = J^Y$ therefore already includes the possibility of tranching Y into Arrow securities. We shall see shortly that once that is possible, there is no reason to consider further tranches.

Let us describe the system of equations that characterizes the tranching equilibrium with the single tranche $A = (0, R)$. In this case, it is easy to see that there will be two *marginal buyers* h_1 and h_2. In equilibrium, all agents $h > h_1$ will buy all of Y, and sell the down tranche $A = (0, R)$, hence effectively holding only the Arrow U security. Agents $h_2 < h < h_1$ will sell all their endowment of Y and purchase all of the durable consumption good X. Finally, agents $h < h_2$ will sell their assets Y and X and buy the down tranche from the most optimistic investors. The regime is showed in Figure 8.

The system of equations that characterizes equilibrium is the following: let π_D denote the price of the down tranche. Equation (8) states that money spent on the asset should equal the aggregate revenue from its sale. The top $1 - h_1$ agents are buying the asset and selling off the down tranche. They each have wealth $1 + p$ plus the revenue from the tranche sale π_D. Finally, there is one unit of total supply of the asset. Hence, we have

$$(8) \qquad\qquad (1 - h_1)(1 + p) + \pi_D = p.$$

Notice that the implicit price of the Arrow U security, which the top $1 - h_1$ agents are effectively buying, equals $p_U = p - \pi_D$, the price of the asset minus the price of the down tranche $A = (0, R)$.

Equation (9) states that total money spent on the down tranche should equal aggregate revenues from their sale. The bottom h_2 agents spend all their endowments to buy all the down tranches available in the economy (which is one since there is one asset), at the price of π_D:

$$(9) \qquad\qquad\qquad h_2(1 + p) = \pi_D.$$

Equation (10) states that h_1 is indifferent between buying the Arrow U security and holding the durable consumption good. So his expected marginal utility from buying the Arrow U security, the probability $q_U^{h_1}$ multiplied by the delivery of 1, divided by its price, $p - \pi_D$, equals the expected marginal utility of holding the durable consumption good divided by its price, 1:

$$(10) \qquad\qquad\qquad \frac{q_U^{h_1}}{p - \pi_D} = 1.$$

Finally, equation (11) states that h_2 is indifferent between holding the down tranche and the durable consumption good X. So his expected marginal utility from buying the down tranche, which is the probability $1 - q_U^{h_2}$ multiplied by the pay-off R, divided by its price, π_D, equals the expected marginal utility of holding the durable consumption good divided by its price, 1:

$$(11) \qquad\qquad\qquad \frac{(1 - q_U^{h_2})R}{\pi_D} = 1.$$

We have a system of four equations and four unknowns: the price of the asset, p; the price of the down tranche π_D; and the two marginal buyers, h_1 and h_2.

Finally, notice that despite the fact that both Arrow securities are present, markets are not complete. Arrow securities are *created through the asset*. Hence, agents cannot sell all the Arrow securities they desire and the Arrow-Debreu allocation cannot be implemented. Tranching the asset is not enough to complete markets.

For the probabilities $q_U^h = 1 - (1 - h)^2$ and $R = 0.2$, we get $h_1 = 0.58$, $h_2 = 0.08$, $p = 1$, and $\pi_D = 0.17$. The asset price is much higher even than it was with leverage. The simple reason is that leverage is an imperfect form of tranching. When the owner of the asset Y can create pieces even better suited to heterogenous buyers, it makes the asset still more attractive.

One important conclusion to be drawn from combining equations (10) and (11) is that the tranching asset price is

$$(12) \qquad\qquad\qquad p = q_U^{h_1}1 + q_D^{h_2}R.$$

Interestingly, the asset price can be higher than any agent in the economy thinks it is worth! Defining the implicit Arrow security prices $p_U = p - \pi_D = 0.83$ and $p_D = \pi_D/R = 0.83$, we see that $p = p_U + p_D R > 1$. We discuss how this could happen in Section III.

A moment's reflection should convince the reader that in our two-state economy, completely tranching Y is tantamount to allowing the asset to back a promise
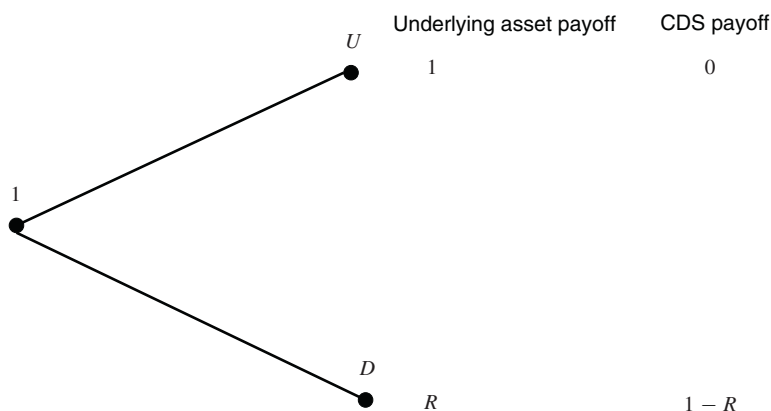
FIGURE 9. CDS PAYOFFS

of R in the down state. The asset holder on net then retains the Arrow U security. By buying y units of Y and selling off y units of the tranche $A = (0, R)$, and also buying z/R units of the down tranche (perhaps created by somebody else), any agent who has enough wealth can effectively purchase the arbitrary consumption $x_U = y, x_D = z$. If it were possible to create different tranches beyond the two Arrow securities, no agent would have anything to gain by doing so. In the end, his new tranches would not offer a potential buyer anything the buyer could not obtain for himself via the Arrow tranches, as we just saw. With unlimited and costless tranching, tranching into Arrow securities always drives out all alternative tranching schemes, a point made in Geanakoplos and Zame (2011). Since $J^Y = \{(0, R)\}$ already embodies Arrow tranching, there is no reason to consider any more complicated tranching schemes.

D. *CDS Economy*

A CDS on the asset Y is a contract that promises to pay 0 at $s = U$ when Y pays 1, and promises $1 - R$ at $s = D$ when Y pays only R. Figure 9 describes a comparison between the underlying asset payoffs and the CDS payoffs.

A CDS is thus an insurance policy for Y. A seller of a CDS must post collateral, typically in the form of money. In a two-period model, buyers of the insurance would insist on $1 - R$ of X as collateral. Thus, for every one unit of payment, one unit of X must be posted as collateral. We can therefore incorporate CDS into our economy by taking J^X to consist of one contract called c promising $(0, 1)$.

We shall maintain our hypothesis that the asset Y itself can be tranched, so we continue to suppose that J^Y consists of the single promise $(0, R)$ called the down tranche or D. Of course that is equivalent to supposing that $(1 - R)/R$ units of the asset Y can be put up as collateral for 1 CDS promising $1 - R$ in state D. In other words, the down tranche in the securitization of Y is identical to the CDS. Yet, we shall show that the two have very different effects on the price of Y.
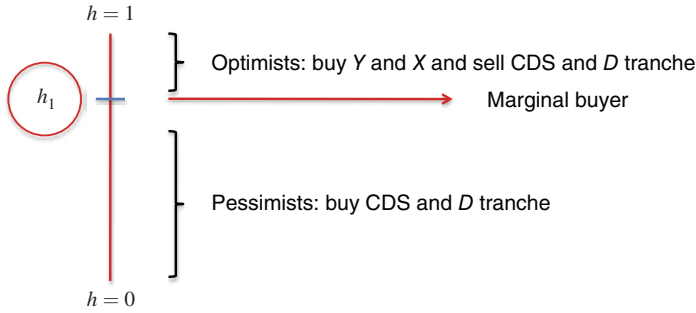
FIGURE 10. CDS ECONOMY EQUILIBRIUM REGIME

Equilibrium requires that buyers recognize that D and c, the CDS, are essentially proportional, and hence, in equilibrium, their prices must be in the same proportion. Equation (13) states that

$$(13) \qquad\qquad \pi_D = R\pi_c.$$

Once equation (13) holds, it must also be the case that buyers recognize that there are two equivalent ways of effectively buying the Arrow U security: *tranching the asset Y* and *tranching cash X*. Hence, we must have equation (14)

$$(14) \qquad\qquad \frac{1}{p - \pi_D} = \frac{1}{1 - \pi_c}.$$

Given these identities, it is evident that in equilibrium there will be a *marginal buyer* h_1, such that all agents $h > h_1$ will buy all of Y and X and sell the down tranche D backed by the Y and the CDS contract c collateralized with X. Agents $h < h_1$ will sell all their endowment of Y and X and buy D and c. The regime is shown in Figure 10.

Equation (15) states that the total money spent on Y and X has to equal the revenues from their sale. Agents $1 - h_1$ buy both. They use their endowments as before, but now they also receive income from the sales of D and c, using Y and X as collateral, respectively. The total wealth represented by the endowments of X and Y is $1 + p$ per person, and the total revenue from the sale of D and C is $\pi_D + \pi_c$. This must equal the purchase cost of all the X and Y in the economy, which is also $1 + p$. Hence,

$$(15) \qquad\qquad (1 - h_1)(1 + p) + \pi_D + \pi_c = 1 + p.$$

Equation (16) states that the marginal buyer should be indifferent between buying the Arrow U security (either way he can) and buying the down tranche or the CDS.

$$(16) \qquad\qquad \frac{q_U^{h_1}}{p - \pi_D} = \frac{(1 - q_U^{h_1})}{\pi_c}.$$
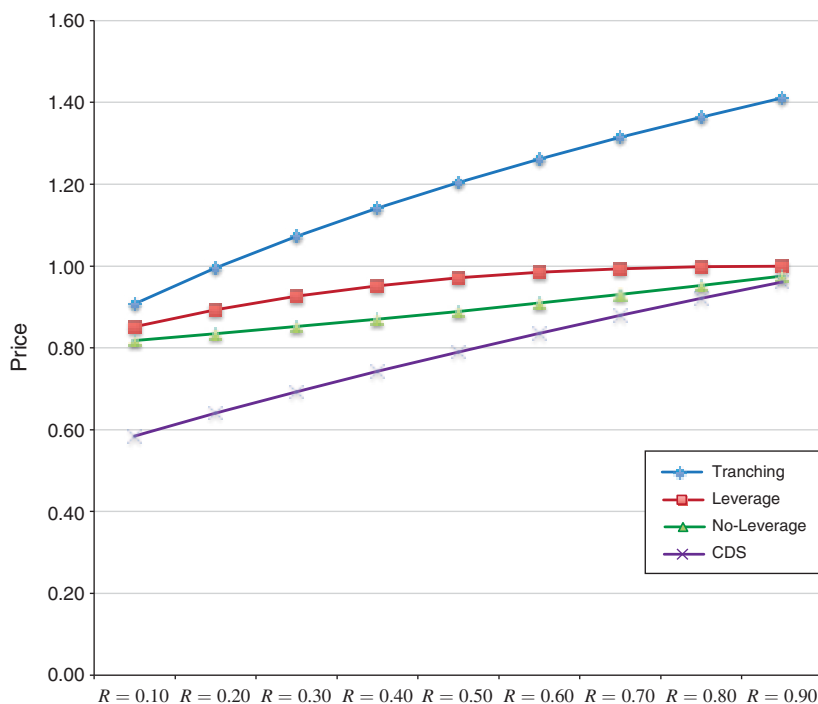
FIGURE 11. ASSET PRICES IN ALL ECONOMIES FOR DIFFERENT VALUES OF R

A succinct way of describing the difference between this economy and the previous one is that CDS allow for the tranching of cash in addition to the previous tranching of assets. As a consequence, with only two states of the world, CDS and tranching allow the economy to implement the Arrow-Debreu equilibrium. Hence, for the probabilities $q_U^h = (1 - h)^2$ and $R = 0.2$, we get the same equilibrium as the one described in the Arrow-Debreu Section IA. The price of the asset is given by $p = p_U + Rp_D = 0.64$.

Finally, a CDS can be "covered" or "naked" depending on whether the buyer of the CDS needs to hold the underlying asset. Our previous discussion corresponds to the case of "naked" CDS. When CDS must be "covered," agents willing to buy CDS need to hold the asset. But notice that holding the asset and buying a CDS is equivalent to holding the riskless bond, which was already available without CDS. "Covered" CDS have no effect on equilibrium. For the rest of the paper, we will focus only on the "naked" CDS case.

III. Financial Innovation and Asset Pricing

We solve for equilibrium with probabilities $q_U^h = 1 - (1 - h)^2$ in all the economies just described as R varies. Figure 11 displays the Y asset prices p for different values of R.[5]

---

[5] Complete results are presented in Appendix C, Table C1.

For all economies the asset price increases as R increases and disagreement disappears. This is not surprising, since the asset clearly makes more payments the higher R is; and so naturally its price should increase. We come back to how far it should increase shortly.

By far the most important implication of our numerical simulations is that leverage and tranching make the asset price higher than it would be without leverage, and still higher than it would be in the CDS or Arrow-Debreu economy. Leverage and derivatives thus have a profound effect on asset prices. And, therefore, so does financial innovation. We now investigate how general these results are.

PROPOSITION 2: *The asset price in the Leverage economy is higher than in the No-Leverage economy for all strictly monotonic and continuous q_U^h, and all $0 < R < 1$.*

PROOF:

From equation (4), we see that in the No-Leverage economy

$$\frac{1}{h_1^{NL}} = 1 + p^{NL},$$

while in the Leverage economy, from equation (6),

$$\frac{1}{h_1^{L}} + \frac{R}{h_1^{L}} = 1 + p^{L}.$$

Assuming $R > 0$, $p^{NL} \geq p^{L}$ only if $h_1^{L} > h_1^{NL}$. But from equations (5) and (7) (which say that the asset price is equal to the marginal buyer's valuation), these last two inequalities are not compatible.

As discussed before, the possibility of borrowing against the asset makes it possible for fewer investors to hold all the assets in the economy. Hence, the marginal buyer is someone more optimistic than in the No-Leverage economy, raising the price of the asset. This effect was first identified in Geanakoplos (1997, 2003). This connection between leverage and asset prices is precisely the Leverage Cycle theory discussed in Geanakoplos (2003, 2010a, 2010b) and Fostel and Geanakoplos (2008, 2011, and forthcoming). This theory can rationalize the housing market behavior during the crisis. Leverage on housing increased dramatically from 2000 to 2006, and housing prices increased dramatically during the same time. Leverage on housing collapsed in 2007, and the same happened to housing prices.

The boost in the price of Y from leverage is greatest for intermediate values of R, a region that can be characterized as "normal times" and when disagreement is not negligible. For too low values of R, agents can borrow very little against the asset, and hence the marginal buyer will be someone very close to the marginal buyer when borrowing is not possible. On the other extreme, when R is very high, though borrowing is very important, agents almost agree on the outcome of the asset, pushing even the no-leverage price up near to one. Next, we turn to tranching.

PROPOSITION 3: *The asset price in the Tranching economy is higher than in the No-Leverage economy for all strictly monotonic and continuous q_U^h, and for all $0 < R < 1$.*

PROOF:
From equation (8) in the Tranching economy, we see that

$$\frac{1}{h_1^T} + \frac{\pi_D^T}{h_1^T} = 1 + p^T.$$

From equation (4), we see that in the No-Leverage economy

$$\frac{1}{h_1^{NL}} = 1 + p^{NL}.$$

Then $p^{NL} \geq p^T$ only if $h_1^L > h_1^{NL}$. But from equations (5) and (12), these last two inequalities are not compatible.

In the numerical simulations, tranching raised the asset price even above the Leverage economy price. But this need not always be the case. Consider, for example, the beliefs given by

$$q_U^h = \max\{1 - (1 - h)^2, 1 - (1 - 0.60)^2\}.$$

The Leverage economy equilibrium calculated earlier is still the same, since the marginal buyer was $h_1^L = 0.63$. But in the Tranching economy, the marginal buyer is $h_1^T < 0.60$, and the price will therefore be $0.84(1) + 0.16(0.2) = 0.872 < 0.89$. In general, $h_1^T < h_1^L$. The tranching price becomes higher than the leverage price when $q_U^{h_1^T} + q_D^{h_2^T} >> 1$ because then the tranching price $p^T = q_U^{h_1^T} 1 + q_D^{h_2^T} R$ can be very large. In the example just given, the probabilities are cooked up so that $q_U^{h_1^T} = q_U^{h_2^T}$.

All this suggests that the tranching price is higher than the leverage price when there is more heterogeneity at the bottom, among the pessimists, than there is at the top, among the optimists. Indeed the following is true.

PROPOSITION 4: *If the probabilities q_U^h are concave in h, as well as strictly monotonic and continuous, then the asset price in the Tranching economy is higher than in the Leverage economy for all $0 < R < 1$.*

PROOF:
From equations (7) and (12), and the fact that $h_1^T > h_2^T$, we have that

$$p^L = q_U^{h_1^L} 1 + q_D^{h_1^L} R = R + q_U^{h_1^L}(1 - R)$$

$$p^T = q_U^{h_1^T} 1 + q_D^{h_2^T} R > R + q_U^{h_1^T}(1 - R).$$

Assume temporarily that $p^L \geq p^T$. Then we must have $q_U^{h_1^L} > q_U^{h_1^T}$, so $h_1^L > h_1^T$, and, hence, $(q_D^{h_1^T} - q_D^{h_1^L}) > 0$. Putting the above two equations together:

$$p^T - p^L = -\left(q_U^{h_1^L} - q_U^{h_1^T}\right)1 + \left[\left(q_D^{h_2^T} - q_D^{h_1^T}\right) + \left(q_D^{h_1^T} - q_D^{h_1^L}\right)\right]R.$$

To get our desired contradiction, it suffices to show that $(q_U^{h_1^L} - q_U^{h_1^T}) < (q_D^{h_2^T} - q_D^{h_1^T})R$. From the hypothesized concavity of q^h in h and from $h_1^L > h_1^T > h_2^T$, it suffices to show that $h_1^L - h_1^T < (h_1^T - h_2^T)R$. From equations (6) and (8), we know that

$$\left(1 - h_1^L\right)\left(1 + p^L\right) + R = p^L$$

$$\frac{1 + R}{1 + p^L} = h_1^L$$

$$\left(1 - h_1^T\right)(1 + p^T) + \pi_D = p^T$$

$$\frac{1 + \pi_D}{1 + p^T} = h_1^T.$$

If $p^L \geq p^T$, then

$$h_1^L - h_1^T \leq \frac{R - \pi_D}{1 + p^T} < \frac{R}{1 + p^T}.$$

On the other hand, from equations (10) and (11) and Walras Law, we know that in the tranching equilibrium, the agents between h_1^T and h_2^T must hold all the X, hence,

$$h_1^T - h_2^T = \frac{1}{1 + p^T}.$$

From the last two equations and the concavity of q_U^h,

$$q_U^{h_1^L} - q_U^{h_1^T} < R\left(q_U^{h_1^T} - q_U^{h_2^T}\right) = R\left(q_D^{h_2^T} - q_D^{h_1^T}\right).$$

From the equation above and our earlier calculations, we conclude $p^T - p^L > 0$.

The idea of the proof is that the tranching price tends to be lower than the leverage price because $q_U^{h_1^L} > q_U^{h_1^T}$, since $h_1^L > h_1^T$. But the tranching asset price rises because $q_D^{h_2^T} > q_D^{h_1^T}$, since $h_1^T > h_2^T$. By concavity, the gap between h_1^T and h_2^T has a bigger effect than the gap between h_1^L and h_1^T.

Our examples, where $q_U^h = 1 - (1 - h)^2$ or $q_U^h = h$, both satisfy the concavity hypothesis. A striking consequence of the power of securitization to raise the asset price can be seen as R increases in our example with $q_U^h = 1 - (1 - h)^2$. As the graph shows, the price of the asset with tranching goes even above one. This seems puzzling since the durable good X delivers at least as much as the asset in every state and its price in equilibrium equals one. With leverage, the price rose because the marginal buyer became a more optimistic agent, and the price came to reflect his beliefs instead of the more pessimistic marginal buyer that obtained without

leverage. However, with leverage, the asset price can never rise above one, since no agent values the asset at more than one. But with tranching, the asset price can rise above what any agent thinks it is worth. How can this be? The answer is that with tranching there are two marginal buyers instead of one. The marginal buyer with leverage was indifferent between the asset and the two cash flows into which leverage split it, and his beliefs determined the price of the asset. In the tranching equilibrium, the cash flows into which the asset is split are held by different people, and nobody who wants one flow would touch the other.

The tranching of the asset *Y* that was begun with leverage is perfected by tranching into Arrow securities. Leverage is a precursor or primitive form of tranching. With leverage, the asset can be used as collateral to issue noncontingent promises. In the tranching economy, the asset can be used as collateral to issue contingent promises raising even further its value as collateral. *Y* becomes so valuable because it can be broken into pieces that are tailor-made for different parts of the population. Splitting plain vanilla into strawberry for one group and chocolate for another raises the value of the scarce ice cream.

This result parallels the result in Harrison and Kreps (1978), who defined a bubble as a situation in which an asset is priced higher than any agent thinks its cash flows are worth. But we obtain the result in a static context without resale of the asset. They displayed bubbles in a dynamic context with heterogeneous agents; the most optimistic agent would buy the asset; but next period, instead of suffering bad cash flows, he could resell the asset to a different agent. In our context, tranching alone, without resale, creates collateral value and potentially bubbles. The gap in prices between leveraged (or securitized) assets and unleveraged assets is what Fostel and Geanakoplos (2008) called *Collateral Value*. When assets can be used as collateral to borrow and not just as investment, there are deviations from classical forms of Law of One Price. An asset with identical or inferior payoffs in the future can be priced higher if it has higher collateral capacities.

This result is in tune with developments in the financial market. For more than five years securitized mortgages traded at negative OAS.[6] Securitization and tranching dramatically increased from the 1990s to 2006, along with leverage, explaining, in our view, much of the rise in housing and related securities prices.

Finally, we turn our attention to the CDS economy. The numerical simulations show that CDS can dramatically lower the asset price, even below the no-leverage level. In the simulations, the lower the *R*, the bigger the price reduction from the introduction of the CDS. For very high *R*, the introduction of CDS has little effect on asset prices when compared to the no-leverage level. We have the following proposition.

---

[6]Option Adjusted Spread (OAS) is the fudge factor Wall Street firms add to their pricing model when all their risk factors are unable to explain market prices. Typically this number is positive, suggesting that the market is willing to pay less than the model values because of the fear of some unknown risk the modelers may have missed. But for mortgages around 2000, this OAS number turned negative. We are suggesting here the reason is that Fannie Mae and Freddie Mac were willing to pay more for the mortgages than their cash flows warranted because they could use them to create pools which would then be cut by Wall Street into more valuable tranches. An alternative explanation is that because of their implicit government guarantees, Fannie Mae and Freddie Mac could borrow at cheaper rates than anybody else and, hence, were willing to overpay for their assets.

PROPOSITION 5: *The asset price in the CDS economy is lower than in the Leverage economy and lower than in the Tranching economy, for all strictly monotonic and continuous q_U^h, and for all $0 < R < 1$.*

PROOF:

Recall from equations (6) and (8) that

$$\frac{1 + R}{1 + p^L} = h_1^L$$

$$\frac{1 + q_D^{h_1^T} R}{1 + p^T} = h_1^T.$$

Next, recall that the CDS equilibrium is the same as the Arrow-Debreu equilibrium. Hence, $p^{CDS} = p^{AD} = p_U 1 + p_D R$. From equation (3), $p_D = q_D^{h_1^{CDS}}$. From equation (2) and Walras Law,

$$\frac{q_D^{h_1^{CDS}}(1 + R)}{1 + p^{CDS}} = h_1^{CDS}.$$

Suppose that $p^{CDS} \geq p^T$. Then the marginal buyer $h_1^{CDS} > h_1^T$, hence, $q_D^{h_1^{CDS}} < q_D^{h_1^T}$. But this contradicts the last two equations above. Similarly, if $p^{CDS} \geq p^L$, then the marginal buyer $h_1^{CDS} > h_1^L$, contradicting the first and third equations above.

The stunning fact that the introduction of CDS dramatically lowers the asset price below the leverage price and below the tranching price seems counterintuitive at first glance. Tranching creates exactly the same derivative payouts as the CDS. They are perfect substitutes in every agent's mind. Yet, when the CDS is created exclusively inside the securitization as a tranche of the asset, it raises the asset price. When the CDS is created outside the securitization, it lowers the asset price.

On second thought this is not surprising at all. CDS are a way of tranching *X*. When agents sell CDS and put up cash as collateral, they are effectively tranching cash! That raises the value of cash relative to other assets, lowering the others' prices.

However, the CDS price need not be lower than the no-leverage price. Indeed, if agents think the bad state is very likely, then in the limit, by equation (4) and Walras Law, we would find that $h_1^{NL} \to 1/(1 + R)$, while by equation (2) and Walras Law, $h_1^{CDS} \to 1$. But if the median agent regards the good state as more likely than the bad state, then the CDS price must be lower than the no-leverage price, as we show in the next proposition.

PROPOSITION 6: *If the q_U^h are strictly monotonic and continuous in h and if $q_U^{1/2} \geq$ ½, then the asset price in the CDS economy is lower than in the No-Leverage economy for all $0 < R < 1$.*

PROOF:

Recall that in the No-Leverage economy, the top $1 - h_1^{NL}$ agents hold all the assets and the bottom h_1^{NL} agents hold all the durable good X, which is more valuable. Hence, $h_1^{NL} > ½$. In the CDS economy,

$$h_1^{CDS} = \frac{(1 + R)q_D^{h_1^{CDS}}}{2q_U^{h_1^{CDS}} + (1 + R)q_D^{h_1^{CDS}}}.$$

If $h_1^{CDS} \geq ½$, then, by hypothesis, $q_U^{h_1^{CDS}} \geq ½$. But that contradicts the above equation for $R < 1$. Hence, $h_1^{CDS} < ½ < h_1^{NL}$, and therefore $p^{NL} > p^{CDS}$.

Putting the last five propositions together, we get:

THEOREM 1: *If the q_U^h are strictly monotonic, concave, and continuous in h, and if $q_U^{1/2} \geq ½$, then the tranching asset price is greater than the leverage asset price, which is greater than the no-leverage asset price, which is greater than the CDS asset price, for all $0 < R < 1$.*

In Appendix A, we give the equations for two more economies, which allow us to see the depressing effect of CDS on the asset price even more clearly. First, we consider what happens if cash X can be tranched by CDS, but Y cannot be tranched or held as collateral for any kind of loan or derivative. Now the tranching boost to price goes exclusively toward X, and the price of Y relative to X plummets still further. In Figure 16, we see that the asset price is much lower.

Finally, we consider the situation which pertains today to many assets like sovereign bonds. The underlying bond-asset is not tranched, but people can leverage their purchases of it. On the other hand they can use cash as collateral to write CDS on the asset-bond. As we see in Appendix A, Figure 16, the Y asset price is below the complete markets, tranching-CDS price, but above the price where we have CDS backed by cash alone. The reason is that the asset Y can only be imperfectly tranched via leverage, whereas the cash is perfectly tranched.

IV. Dynamic Asset Prices: Bubbles and Crashes

As we said at the outset, securitization emerged over a period of 20 years, and then the CDS mortgage market suddenly exploded at the end of the securitization boom. To model the implications of that dynamic we need to examine a multi-period model. Needless to say, as long as nobody gets utility from holding collateral, leverage, tranching, and CDS are irrelevant without uncertainty. Hence, the dynamic model must incorporate risk at each stage. To keep the model tractable (and thus to avoid exponential growth in the number of states), we suppose that in every period with high probability a tiny bit of bad news is received, which leaves the world a little worse but much like it was; or with low probability all the uncertainty is resolved and the good outcome obtains for sure from there on out. The most likely and most interesting history occurs along the path of consecutive pieces of bad news. This single history contains *all* the nodes at which the asset price is nontrivial. As we
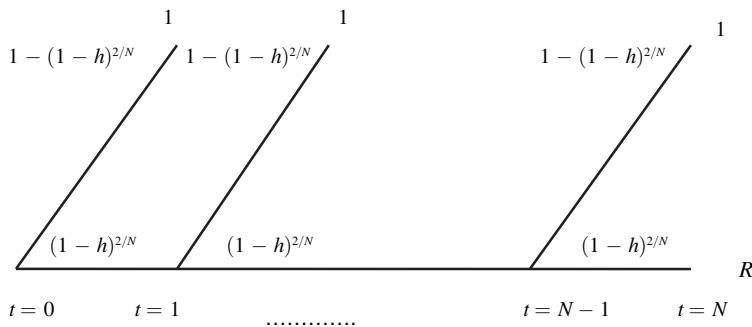
FIGURE 12. DYNAMIC ASSET PAYOFFS

shall see, this model is tractable, it assumes volatility increases with bad news (and decreases with good news), and it makes leverage pro-cyclical.[7]

The risky asset payoffs are described in Figure 12, as are the probabilities of the agents. At each point in time, there is either good news (with low probability $1 - (1 - h)^{2/N}$) or bad news (with high probability $(1 - h)^{2/N}$). After good news, uncertainty is completely resolved and the risky asset Y pays 1. However, after bad news, the economy proceeds to the next period. After N consecutive periods of bad news, output materializes at $R < 1$. Notice that each agent believes that the final output of Y will be R with probability $(1 - h)^2$ and 1 with probability $1 - (1 - h)^2$, exactly as we had in the two-period model of Section I. Each agent in the continuum $h \in (0, 1)$ begins at time 0 with one unit of Y and one unit of X, and has no further endowments. As before, X produces 1 unit of the consumption good in every terminal state. We suppose that agents care only about expected consumption in the terminal states.

Notice that as bad news comes, future output volatility increases. In other words, bad news is revealed very slowly and each piece of bad news comes attached with a spike in volatility. This type of technology, in which first moments and second moments of the asset distribution are negatively correlated, was introduced in Geanakoplos (2003) in a three-period tree, and studied extensively by Fostel and Geanakoplos (forthcoming). This kind of economy induces pro-cyclical leverage, meaning that each piece of bad news also decreases leverage. One potential drawback (or advantage) to the model is that it presumes that if at the beginning one agent is more optimistic than another, then he remains more optimistic throughout.[8]

The No-Leverage, Leverage, Tranching, and CDS economies are defined by their contract structure, as in the static model. In each node s of the tree, we are given a set of one period contracts $J^X(s)$ using one unit of X as collateral and another set of one period contracts $J^Y(s)$ using one unit of Y as collateral. For the No-Leverage economy, these sets are empty. For the Leverage economy, $J^X(s)$ is empty and

---

[7]Fostel and Geanakoplos (forthcoming) show that this stochastic structure is not something crazy or ad hoc. They show that investors, given the opportunity to choose between technologies that exhibit positive or negative correlations between first and second moments, would mostly choose the latter.

[8]This may be the reason why in our model agents would buy CDS contracts at the beginning if they were available, rather than waiting until default becomes more likely.
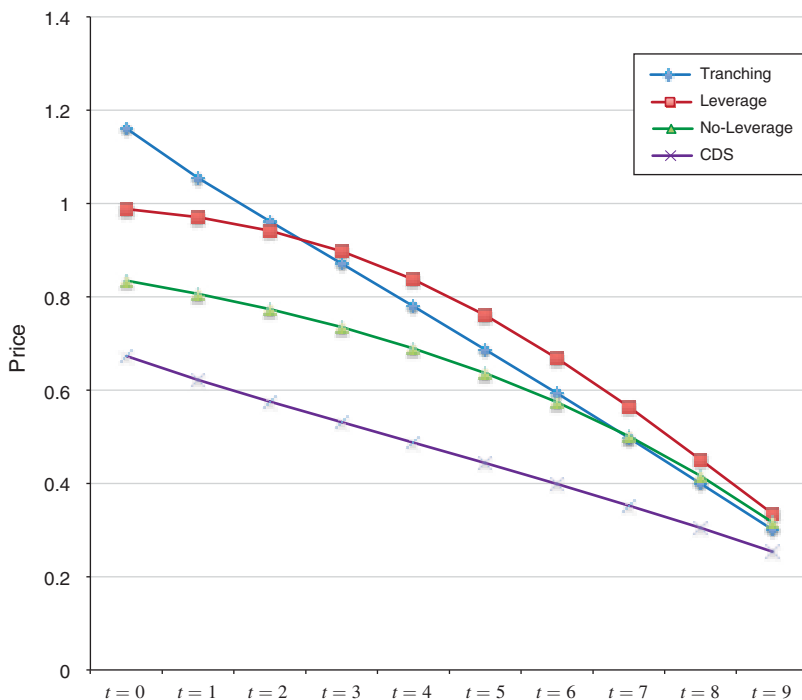
FIGURE 13. ASSET PRICE DYNAMICS FOR $N = 10$

$J^Y(s)$ consists of all promises (j,j) in the following two successor states. For the Tranching economy, $J^X(s)$ is empty and $J^Y(s)$ consists of all possible promises in the two successor states, which, recalling our previous discussion, reduces to trading the single promise $(0, p^Y_{sD})$. In the CDS economy, we would like to define the CDS as a contract promising $1 - R$ in the terminal state following all bad news. But then the question is: how much X collateral will the market require at each node s? The answer is just enough to cover the market value of the CDS at node sD, the successor node of s after bad news. With that insight we can reproduce the CDS equilibrium by assuming that $J^X(s)$ consists of the one period promise $(0,1)$ for all s, and that $J^Y(s)$ consists of all possible promises in the two successor states.

Given the contract structure, equilibrium is defined by analogy to what we did in Section I for the two-period model in which $N = 1$. However, for longer horizons, agent optimization is much more complicated because agents must be forward looking, anticipating what the price of Y will be in the successor states and taking into account that they might not want to buy the risky asset Y today, even if its expected payoffs exceed its price, because they might do better by waiting to buy it next period when its price might be cheaper. All the equations that characterize the equilibrium for each economy are presented in Appendix B.

Figure 13 shows the asset price evolution for the different economies for $R = 0.2$ and $N = 10$.[9] Note first that at time $t = 0$ the leverage price and the tranching price

---

[9] Complete results are presented in Appendix C, Tables C2 and C3.
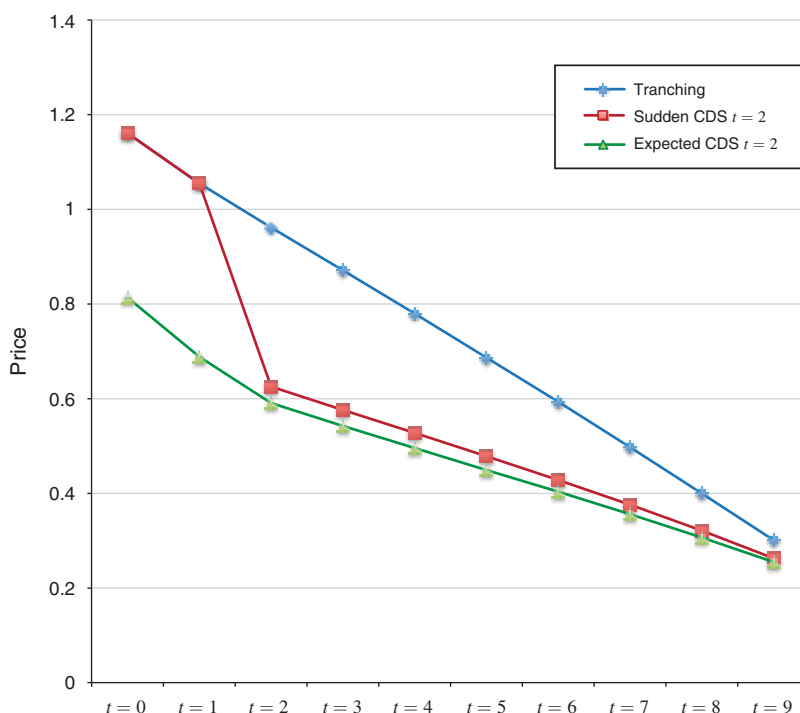
FIGURE 14. LATE INTRODUCTION OF CDS AT $t = 2$

are much higher than they were at time $t = 0$ in the two-period economy. Multiple periods increase the power of leverage and tranching to raise asset prices. The no-leverage price at time $t = 0$ is exactly the same in the 10-period economy as in the two-period economy.

As before, the tranching price starts out higher than all the others, followed by the leverage price, then the non-leverage price, then the very low CDS price. As time goes and bad news keeps occurring, the prices all fall (there is no point in presenting the price after good news, since that is always one). Since all the price lines end up at nearly the same price near 0.2, the fall is inversely related to the starting point. In the Leverage economy, prices start higher, and, hence, the price crash is much bigger than in the No-Leverage and CDS economies. Since the no-leverage marginal buyer never changes (he has no reason to sell after bad news), the price always reflects his opinion of expected output. Thus, in the Leverage economy the bigger price decline must be from feedback effects through deleveraging and through wealth loss among the optimists. It is interesting that at the very beginning the leverage price is more stable than the non-leverage price, but picks up steam fast as more bad news comes in. However, the really dramatic fall in price on just a tiny bit of bad news happens in the Tranching economy.

By contrast, the fall is much more modest in the CDS economy. Early securitization and tranching created a huge increase in asset prices. Had CDS been available from the very beginning this overvaluation would not have happened.

However, as discussed in the introduction, CDS were actually introduced later. Figure 14 shows the effect on asset prices if CDS were introduced to a Tranching
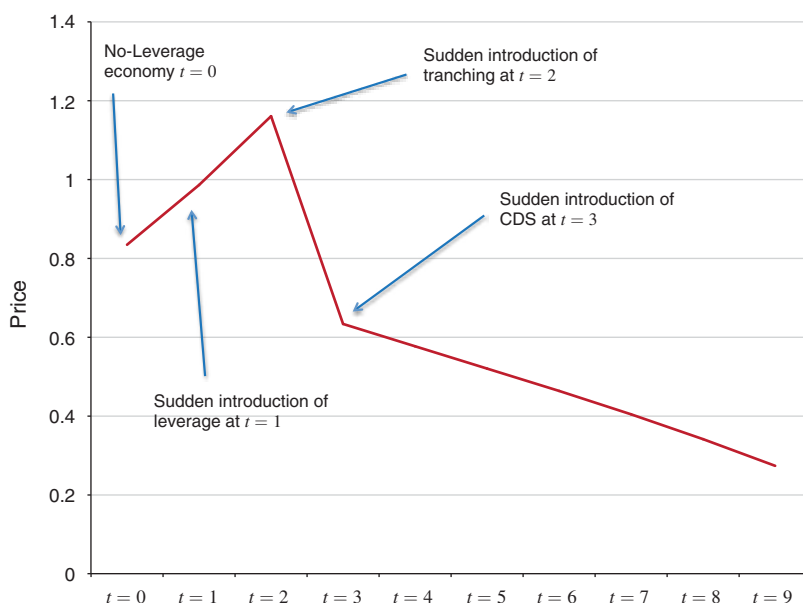
FIGURE 15. ANATOMY OF THE BUBBLE AND CRASH

economy in period $t = 2$. We distinguish two different cases: a sudden, unexpected introduction in which the economy is proceeding along as if CDS will never exist; and an expected introduction in which, from the very beginning, it is known that CDS will appear in period $t = 2$.

Without CDS, there would be a 17 percent drop in prices in the Tranching economy after two pieces of bad news, as we saw in Figure 13. This drop becomes much bigger if CDS appear in period $t = 2$, namely 28 percent, even if the appearance of CDS at time $t = 2$ is anticipated from the very beginning at time $t = 0$. The crash becomes a horrific 46 percent if the CDS appear in period $t = 2$ as a surprise.

In our view the key to the size of the crisis is the order of the financial innovation that materialized. Securitization, with all the tranching of CDOs and leverage created a bubble and the introduction of CDS burst it, pushing pricing faster and further down than they would have gone had there never been tranching or leverage or CDS. Had CDS been there from the beginning, asset prices would never have gotten so high.

The most dramatic drop in price occurs with the historical timing. In Figure 15, we see the evolution of the risky asset price assuming that the economy starts at $t = 0$ with no leverage. At $t = 1$, leverage is unexpectedly introduced and price goes up, even though some bad news arrives. In $t = 2$, the risky asset tranching technology is unexpectedly introduced and the price climbs even further, despite more bad news. Finally, at $t = 3$, CDS are unexpectedly introduced, and with it the price crashes. Just one piece of bad news, on top of the introduction of CDS, reduces the price by nearly 50 percent. In our dynamic model, we only keep track of price after bad news, yet the introduction of leverage and tranching is still strong enough to offset the bad news and raise prices. Obviously in reality, the leverage and tranching part of the cycle occurred mostly during pieces of "good news," which made the bubble even more violent.

It would be very interesting to endogenize the order of financial innovation. We conjecture that with a transactions cost, CDS would not be traded at the beginning because there is a very small probability of paying off. As the likelihood increases and more disagreement is created, more people will want to trade them. This common sense thought does not really obtain in our model, as far as we can see, because in the model agents who are going to be the most pessimistic at the end are already the most pessimistic at the beginning. They would buy their CDS from the start if there were no transactions costs. Indeed CDS volume would decline, not increase, over time. Clearly this merits more investigation.

APPENDIX A. TWO MORE CDS ECONOMIES

A. *CDS without Tranching*

For many assets, like sovereign bonds nowadays, direct tranching is not available, but CDS are written. As we said earlier, if the asset itself can be used as collateral by the writer of CDS insurance, then indeed the assets are indirectly tranched, and the analysis of the last section applies. There is good reason why the asset might be used as collateral for its own insurance (puzzling as that sounds), because the optimists most likely to own the bonds will be the same people writing the insurance. But in practice cash seems to be the collateral of choice, so, for completeness, we also analyze the case where only cash can be used as collateral for the CDS. We also do not permit leverage of the assets. In the next section, we do.

To formalize our assumption that there is no tranching and that CDS must be collateralized with cash, we take $J = J^X$ to consist of just the CDS c that promises $(0,1)$. In equilibrium, there will be two marginal buyers $h_1 > h_2$. Agents $h > h_1$ will hold all of the "cash" X and use it all as collateral to write CDS c on Y, hence, effectively holding only the Arrow U security. Agents $h_2 < h < h_1$ will sell all their endowment of X and purchase all of the asset Y. Finally, agents $h < h_2$ will sell their assets Y and X and buy the CDS from the most optimistic investors.

Equation (A1) says that the top group of agents indeed holds all of X, and writes the maximal number of CDS:

$$(A1) \qquad (1 - h_1)(1 + p) + \pi_c = 1.$$

Note that these agents are effectively buying the Arrow Up security at an implicit price of $p_U = 1 - \pi_c$. Equation (A2) says that the bottom group of agents holds all the CDS.

$$(A2) \qquad h_2(1 + p) = \pi_c.$$

Equation (A3) says that h_1 is indifferent between writing CDS backed by X (and thus synthetically holding the Arrow Up security) and holding the asset:

$$(A3) \qquad \frac{q_U^{h_1}}{1 - \pi_c} = \frac{q_U^{h_1} + (1 - q_U^{h_1})R}{p}.$$

Equation (A4) says that h_2 is indifferent between holding the asset and holding the CDS c:

$$\text{(A4)} \qquad \frac{q_U^{h_2} + (1 - q_U^{h_2})R}{p} = \frac{1 - q_U^{h_2}}{\pi_c}.$$

For the probabilities $q_U^h = (1 - h)^2$ and $R = 0.2$, we get $h_1 = 0.65, h_2 = 0.29$, $p = 0.55$, and $\pi_c = 0.36$. The implicit Arrow prices are given by $p_U = 0.63$ and $p_D = 0.45$.

B. *CDS with Leverage*

Finally, we add the possibility of leverage for Y, maintaining the hypothesis that Y cannot be tranched and that only X can be used as collateral for the CDS. To formalize this assumption, we take J^X to consist of just the CDS c that promises $(0, 1)$, and we take J^Y to be all promises of the form (j, j) for any $j > 0$. As we saw before, only the promise $(j^*, j^*) = (R, R)$ will be traded in equilibrium. We denote its price by π_{j^*}.

In equilibrium, there will, as usual, be two marginal buyers, $h_1 > h_2$. Agents $h > h_1$ will hold all of Y, purchased entirely via leverage, and all of the cash X, using it all as collateral to write CDS c on Y, hence, effectively holding only the Arrow U security. Agents $h_2 < h < h_1$ will sell all their endowment of X and Y and lend it to the optimists, protected by the collateral of Y. Finally, agents $h < h_2$ will sell their assets Y and X and buy the CDS from the most optimistic investors.

Equation (B1) says that the top group of agents indeed holds all of Y via leverage and also all of X, and writes the maximal number of CDS:

$$\text{(B1)} \qquad (1 - h_1)(1 + p) + \pi_c + \pi_{j^*} = 1 + p.$$

Note that these agents are effectively buying the Arrow Up security at an implicit price of $p_U = 1 - \pi_c$. But they are equally buying all the Arrow Up security via their leveraged purchase of Y. Hence, equation (B2) requires that

$$\text{(B2)} \qquad \frac{1 - R}{p - \pi_{j^*}} = \frac{1}{1 - \pi_c}.$$

Equation (B3) says that the bottom group of agents holds all the CDS

$$\text{(B3)} \qquad h_2(1 + p) = \pi_c.$$

Equation (B4) says that h_1 is indifferent between writing CDS backed by X (and thus synthetically holding the Arrow Up security) and lending

$$\text{(B4)} \qquad \frac{q_U^{h_1}}{1 - \pi_c} = \frac{R}{\pi_{j^*}}.$$
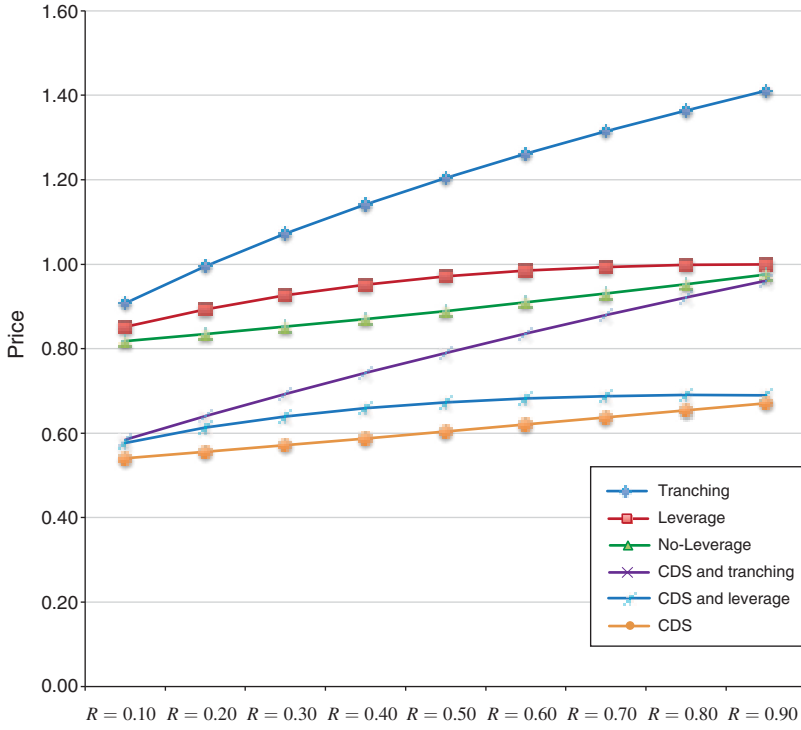
FIGURE 16. ASSET PRICES IN ALL ECONOMIES FOR DIFFERENT VALUES OF R

Equation (B5) says that h_2 is indifferent between lending and holding the CDS c

$$(B5) \qquad \frac{R}{\pi_{j^*}} = \frac{1 - q_U^{h_2}}{\pi_c} .$$

For the probabilities $q_U^h = (1 - h)^2$ and $R = 0.2$, we get $h_1 = 0.38, h_2 = 0.28,$ $p = 0.61$, $\pi_c = 0.45$, and $\pi_{j^*} = 0.17$. The implicit arrow prices are $p_U = 0.54$ and $p_D = 0.56$.

Figure 16 presents asset prices for different values of R, as we did in the main text, but for the six different cases. Notice how much lower the asset price is in these last two cases.

APPENDIX B

A. Equilibrium Equations for Different Economies in Section IV

CDS and Tranching Economy.—As noted before, this equilibrium corresponds to the Arrow-Debreu equilibrium. The equations are the traditional equations in which we solve for Arrow-prices for $N + 1$ final states.

No-Leverage Economy.—We use the fact that the marginal buyer rolls over his debt at every node to build up the system and then verify that the guess is correct. Notice that the probability of good news in period k is given by $1 - (1 - h)^{2/N}$.

$$p_1 = \left(1 - (1 - h)^{2/N}\right)^N + \left(1 - (1 - (1 - h)^{2/N})^N\right)R$$

$$p_1 = \frac{(1 - h_1) + R}{h_1}$$

$$\vdots$$

$$p_k = \left(1 - (1 - h)^{2/N}\right)^{N-k} + \left(1 - (1 - (1 - h)^{2/N})^{N-k}\right)R$$

$$\vdots$$

$$p_{N-1} = \left(1 - (1 - h)^{2/N}\right)^{N-(N-1)}$$

$$+ \left(1 - (1 - (1 - h)^{2/N})^{N-(N-1)}\right)R.$$

Leverage Economy.—Notice that since the final probability of disaster is constant (regardless of N), the probability of bad news in period k is given by $(1 - h_k)^{2/k}$. What is crucial is that the marginal buyers are decreasing in time. There is a perpetual wealth redistribution toward more pessimistic investors in each period after bad news since previous leveraged investors go bankrupt.

$$p_{N+1} = R$$

$$p_N = \left(1 - (1 - h_N)^{2/N}\right) + (1 - h_N)^{2/N}R$$

$$h_{N-1} = \frac{h_N(1 + p_N)}{1 + p_{N+1}}$$

$$p_{N-1} = \frac{\left(1 - (1 - h_{N-1})^{2/N}\right) + (1 - h_{N-1})^{2/N}\frac{\left(1-(1-h_{N-1})^{2/N}\right)}{\left(1-(1-h_N)^{2/N}\right)}p_N}{\left(1 - (1 - h_{N-1})^{2/N}\right) + (1 - h_{N-1})^{2/N}\frac{\left(1-(1-h_{N-1})^{2/N}\right)}{\left(1-(1-h_N)^{2/N}\right)}}$$

$$h_{N-2} = \frac{h_{N-1}(1 + p_{N-1})}{1 + p_N}$$

$$\vdots$$

$$p_1 = \frac{\left(1 - (1 - h_1)^{2/N}\right) + (1 - h_1)^{2/N}\frac{\left(1-(1-h_1)^{2/N}\right)}{\left(1-(1-h_2)^{2/N}\right)}p_2}{\left(1 - (1 - h_1)^{2/N}\right) + (1 - h_1)^{2/N}\frac{\left(1-(1-h_1)^{2/N}\right)}{\left(1-(1-h_2)^{2/N}\right)}}$$

$$h_0 = \frac{h_1(1 + p_1)}{1 + p_2} = 1.$$

Tranching Economy.—The probabilities are as before. The equilibrium in this economy is more challenging due to the multiplicity of marginal buyers in each period. It turns out that both marginal buyers are decreasing with time. Buyers of Arrow U security last only for one period. The second marginal buyer wealth evolution turns out to be more complicated. They buy gold in the future, but eventually, as time goes by, they buy Arrow U. In our example for $N = 10$, h_2^0 buys gold from $t = 1$ to $t = 8$, and buys Arrow U in the last trading period. Define $q_U^h = (1 - (1 - h)(2/N))$ and $q_D^h = 1 - q_U^h$.

At $t = 0$,

$$p_0 - p_1 p_{D1} = (1 - h_0^1)(1 + p_0)$$

$$h_0^2(1 + p_0) = p_1 p_{D1}$$

$$\frac{q_U^{h_0^1}}{p_0 - p_1 p_{D1}} = q_U^{h_0^1} + q_D^{h_0^1} \frac{q_U^{h_0^1}}{p_1 - p_2 p_{D2}}$$

$$\frac{q_D^{h_0^2}}{p_{D1}} C_0 = q_U^{h_0^2} + q_D^{h_0^2} C_0,$$

where p_{D1} is the price of the Arrow down (which pays in period 1 after bad news), and C_0 is the continuation value of h_0^2 given his future investment.

For $t = k$ for $k = 2, \ldots, N - 2$,

$$p_k - p_{k+1} p_{Dk+1} = \frac{h_{k-1}^1 - h_k^1}{h_{k-1}^1 - h_{k-1}^2}$$

$$\frac{h_k^2}{h_{k-1}^2} p_k = p_{k+1} p_{Dk+1}$$

$$\frac{q_U^{h_k^1}}{p_k - p_{k+1} p_{Dk+1}} = q_U^{h_k^1} + q_D^{h_k^1} \frac{q_U^{h_k^1}}{p_{k+1} - p_{k+2} p_{Dd+2}}$$

$$\frac{q_D^{h_k^2}}{p_{Dk+1}} C_k = q_U^{h_k^2} + q_D^{h_k^2} C_k.$$

At $t = N - 1$,

$$p_{N-1} - p_N p_{DN} = \frac{h_{N-2}^1 - h_{N-1}^1}{h_{N-2}^1 - h_{N-2}^2}$$

$$\frac{h_{N-1}^2}{h_{N-2}^2} p_{N-1} = p_N p_{DN}$$

$$\frac{q_U^{h_{N-1}^1}}{p_{N-1} - R p_{DN}} = 1$$

$$\frac{q_D^{h_{N-1}^2}}{p_{DN}} = 1.$$

APPENDIX C

Table 1, 2, and 3 show the complete equilibrium information from Section III and IV, respectively.

TABLE C1—ASSET PRICES IN THE STATIC MODEL

	No-Leverage	Leverage	Tranching	CDS
$R = 0.10$	0.82	0.85	0.91	0.58
$R = 0.20$	0.83	0.89	1.00	0.64
$R = 0.30$	0.85	0.93	1.07	0.69
$R = 0.40$	0.87	0.95	1.14	0.74
$R = 0.50$	0.89	0.97	1.20	0.79
$R = 0.60$	0.91	0.99	1.26	0.84
$R = 0.70$	0.93	0.99	1.31	0.88
$R = 0.80$	0.95	1.00	1.36	0.92
$R = 0.90$	0.98	1.00	1.41	0.96

TABLE C2—ASSET PRICES IN THE DYNAMIC MODEL

	No-Leverage	Leverage	Tranching	CDS
$t = 0$	0.83	0.99	1.16	0.67
$t = 1$	0.81	0.97	1.05	0.62
$t = 2$	0.77	0.94	0.96	0.58
$t = 3$	0.73	0.90	0.87	0.53
$t = 4$	0.69	0.84	0.78	0.49
$t = 5$	0.64	0.76	0.69	0.44
$t = 6$	0.57	0.67	0.59	0.40
$t = 7$	0.50	0.56	0.50	0.35
$t = 8$	0.42	0.45	0.40	0.30
$t = 9$	0.32	0.34	0.30	0.25

TABLE C3—FINANCIAL INNOVATION AND ASSET PRICES

	Sudden CDS $t = 2$	Expected CDS $t = 2$	Anatomy of bubble and crash
$t = 0$	1.16	0.81	0.83
$t = 1$	1.05	0.69	0.99
$t = 2$	0.63	0.59	1.16
$t = 3$	0.58	0.54	0.63
$t = 4$	0.53	0.50	0.58
$t = 5$	0.48	0.45	0.52
$t = 6$	0.43	0.40	0.46
$t = 7$	0.38	0.36	0.40
$t = 8$	0.32	0.31	0.34
$t = 9$	0.26	0.25	0.27

REFERENCES

Acharya, Viral V., and S. Viswanathan. 2011. "Leverage, Moral Hazard, and Liquidity." *Journal of Finance*, 66(1): 99–138.

Adrian, Tobias, and Hyun Song Shin. 2010. "Liquidity and Leverage." *Journal of Financial Intermediation*, 19(3): 418–37.

Araújo, Aloisio, Felix Kubler, and Susan Schommer. Forthcoming. "Regulating Collateral-Requirements When Markets are Incomplete." *Journal of Economic Theory*.

Bolton, Patrick, and Martin Oehmke. Forthcoming. "Credit Default Swaps and the Empty Creditor Problem." *Review of Financial Studies*.

Brunnermeier, Markus K. 2009. "Deciphering the Liquidity and Credit Crunch 2007–2008." *Journal of Economic Perspectives*, 23(1), 77–100.

Brunnermeier, Markus K., and Lasse Heje Pedersen. 2009. "Market Liquidity and Funding Liquidity." *Review of Financial Studies*, 22(6): 2201–38.

Cao, Dan. 2010. "Collateral Shortages, Asset Price and Investment Volatility with Heterogenous Beliefs." Unpublished.

Che, Yeon-Koo, and Rajiv Sethi. 2010. "Credit Derivatives and the Cost of Capital." Unpublished.

Fostel, Ana, and John Geanakoplos. Forthcoming. "Why Does Bad News Increase Volatility and Decrease Leverage." *Journal of Economic Theory*.

Fostel, Ana, and John Geanakoplos. 2008. "Leverage Cycles and the Anxious Economy." *American Economic Review*, 98(4): 1211–44.

Fostel, Ana, and John Geanakoplos. 2011. "Endogenous Leverage: VaR and Beyond." Yale University, Cowles Foundation for Research in Economics, Discussion Paper 1800.

Fostel, Ana, and John Geanakoplos. 2012. "Tranching, CDS, and Asset Prices: How Financial Innovation Can Cause Bubbles and Crashes; Dataset." *American Economic Journal: Macroeconomics*. http://dx.doi.org/10.1257/mac.4.1.190

Garleanu, Nicolae, and Lasse Heje Pedersen. 2011. "Margin-Based Asset Pricing and Deviations from the Law of One Price." *Review of Financial Studies*, 24(6): 1980–2022.

Geanakoplos, John. 1997. "Promises, Promises." In *The Economy as an Evolving Complex System II*, ed. W. Brian Arthur, Steven N. Durlauf, and David A. Lane, 28–320. Reading, MA: Addison-Wesley.

Geanakoplos, John. 2003. "Liquidity, Default, and Crashes: Endogenous Contracts in General Equilibrium." In *Advances in Economics and Econometrics: Theory and Applications, Eighth World Congress*, Vol. 2, ed. Mathias Dewatripont, Lars Peter Hansen, and Stephen J. Turnovsky, 170–205. Cambridge, UK: Cambridge University Press.

Geanakoplos, John. 2010a. "The Leverage Cycle." In *NBER Macroeconomics Annual 2009*, ed. Daron Acemoglu, Kenneth Rogoff, and Michael Woodford, 1–65. Chicago: University of Chicago Press.

Geanakoplos, John. 2010b. "Solving the Present Crisis and Managing the Leverage Cycle." *Federal Reserve Bank of New York Economic Policy Review*, 16(1): 101–31.

Geanakoplos, John, and William Zame. 1997. "Collateral Equilibrium." Unpublished.

Geanakoplos, John, and William Zame. 2011. "Collateralized Security Markets." Unpublished.

Gorton, Gary. 2009. "The Subprime Panic." *European Financial Management*, 15(1): 10–46.

Gorton, Gary, and Andrew Metrick. Forthcoming. "Securitized Banking and the Run on Repo." *Journal of Financial Economics*.

Gromb, Denis, and Dimitri Vayanos. 2002. "Equilibrium and Welfare in Markets with Financially Constrained Arbitrageurs." *Journal of Financial Economics*, 66(2–3): 361–407.

Harrison, J. Michael, and David M. Kreps. 1978. "Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations." *Quarterly Journal of Economics*, 92(2): 323–36.

Hindy, Ayman. 1994. "Viable Prices in Financial Markets with Solvency Constraints." *Journal of Mathematical Economics*, 24(2): 105–35.

Hindy, Ayman, and Ming Huang. 1995. "Asset Pricing with Linear Collateral Constraints." Unpublished.

Simsek, Alp. 2010. "When Optimists Need Credit: Asymmetric Filtering of Optimism and Implications for Asset Prices." www.econ.yale.edu/seminars/jr-fac/jf10/simsek-JMP.pdf.

Stultz, Rene. 2009. "Credit Default Swaps and the Credit Crisis." Charles A. Dice Center Working Paper 2009-16.