Computer Vision

Scale spaces

Silvia Pintea

Scale spaces: Reading material

- **Article**:
  - Lindeberg, Tony. "Scale-space." (2009): 2495-2504.
  - https://www.diva-portal.org/smash/get/diva2:441147/FULLTEXT01.pdf
- UvA tutorial on scale-space:

https://staff.fnwi.uva.nl/r.vandenboomgaard/IPCV20172018/LectureNotes/IP/ScaleSpace/index.html

- Prof. Kristen Grauman slides: https://www.cs.utexas.edu/~grauman/courses/378/slides/lecture13_full.pdf

Find objects across images

- How would you describe an object in an image? e.g. which features can you use?

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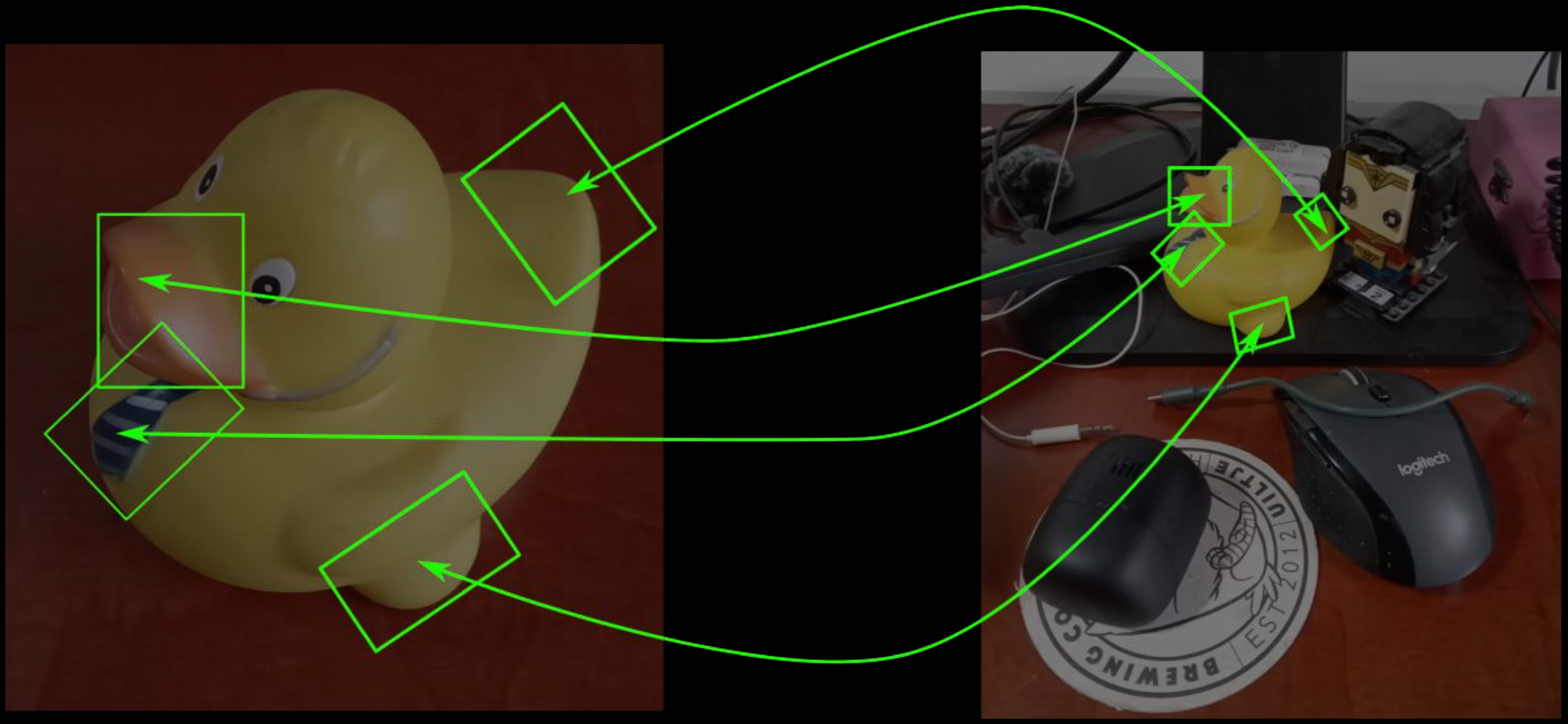

- Will this work? Why yes/no?

Scale

What do I mean by "scale"?

Where do we deal with multiple scales?

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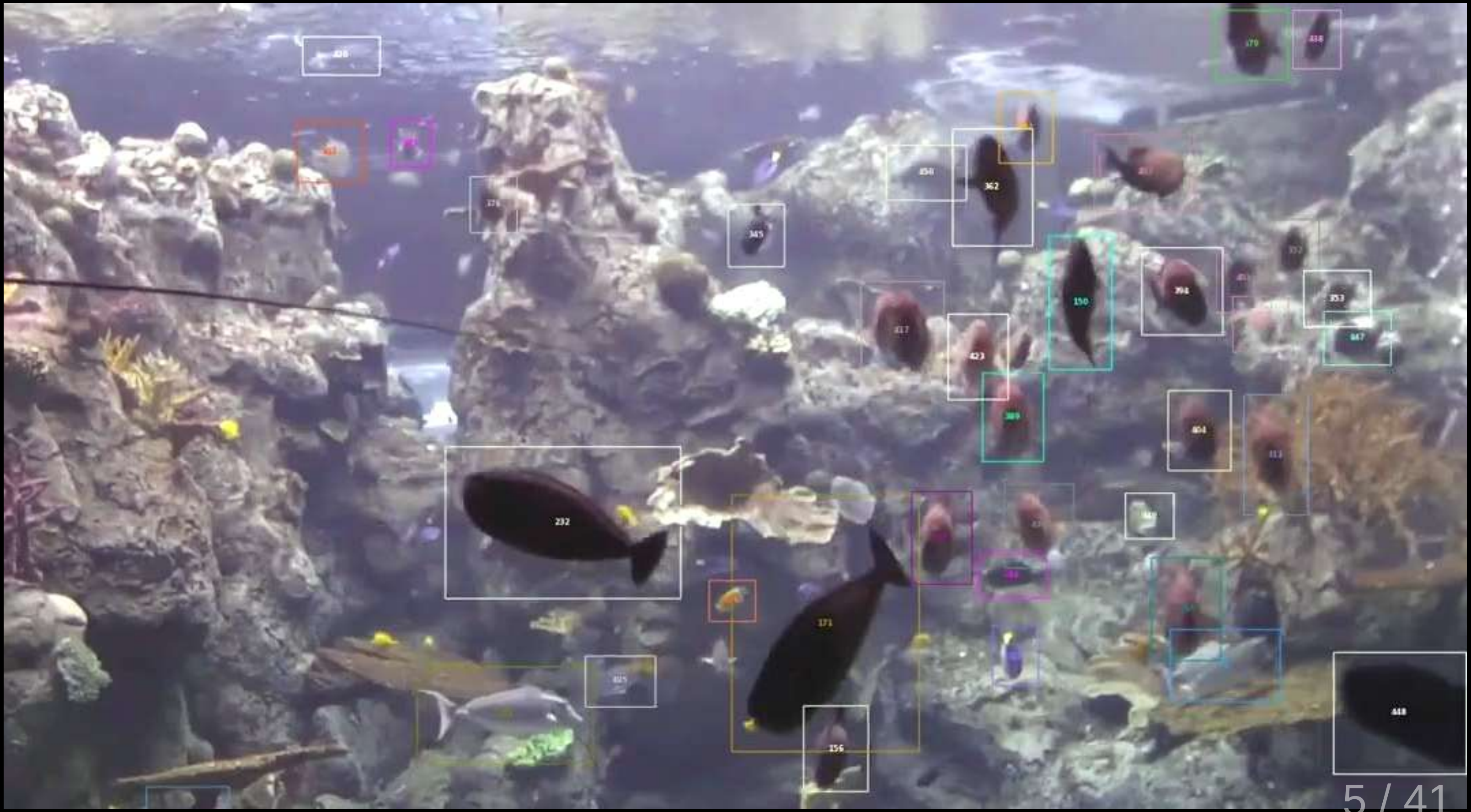
Where do we deal with multiple scales?



High / low scale

What information do we have at high scale and not at low scale?

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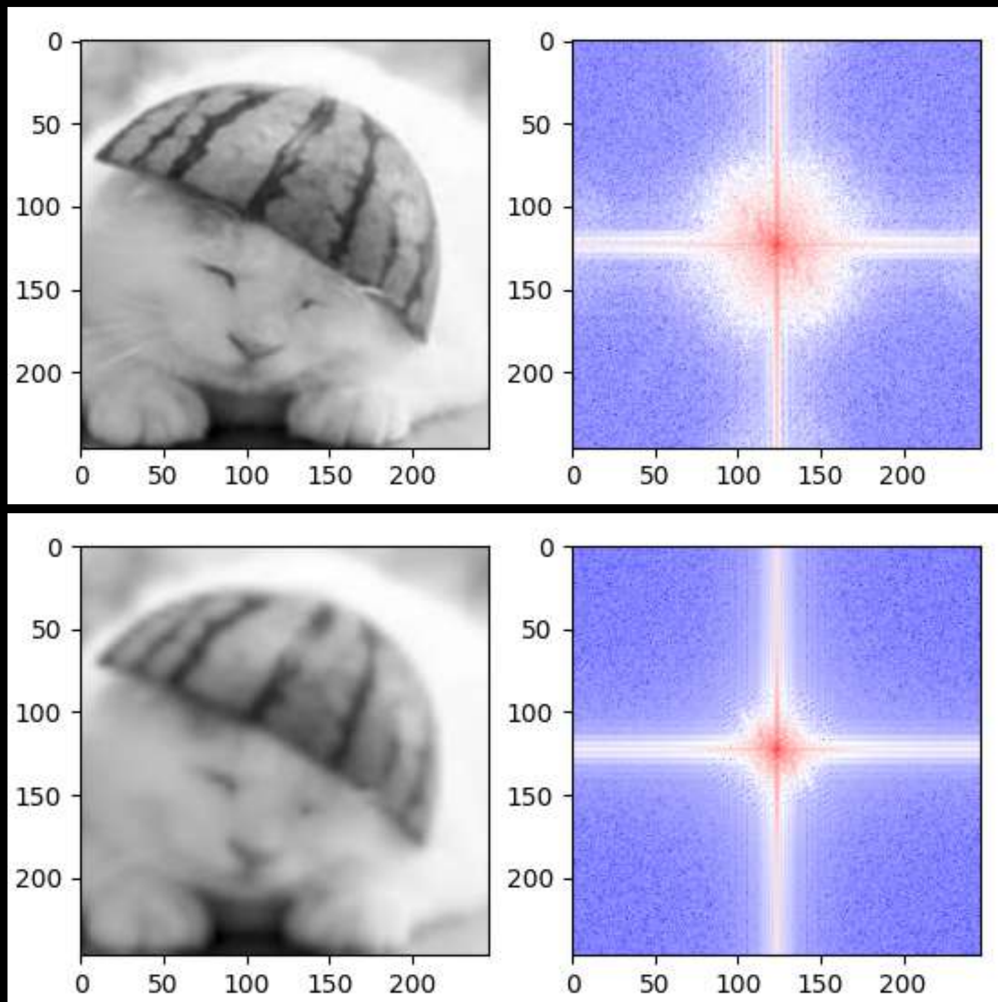

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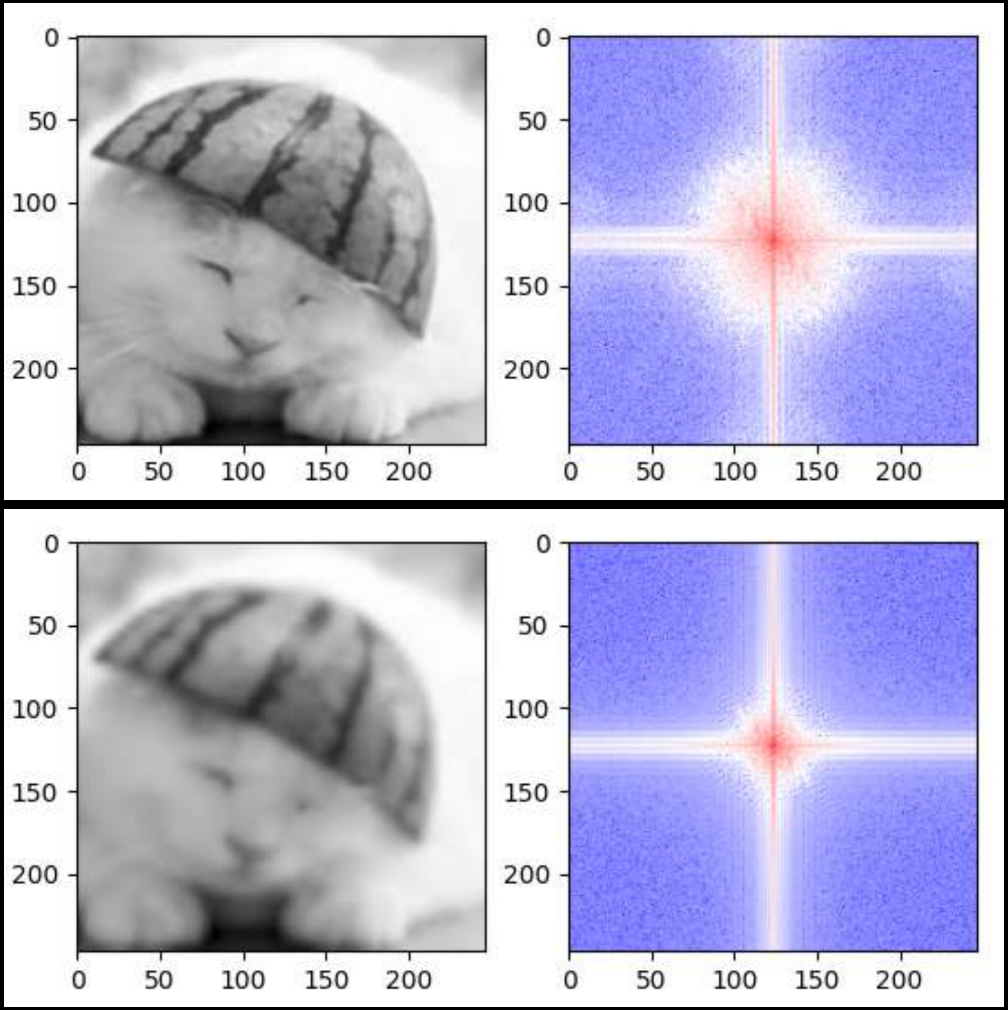
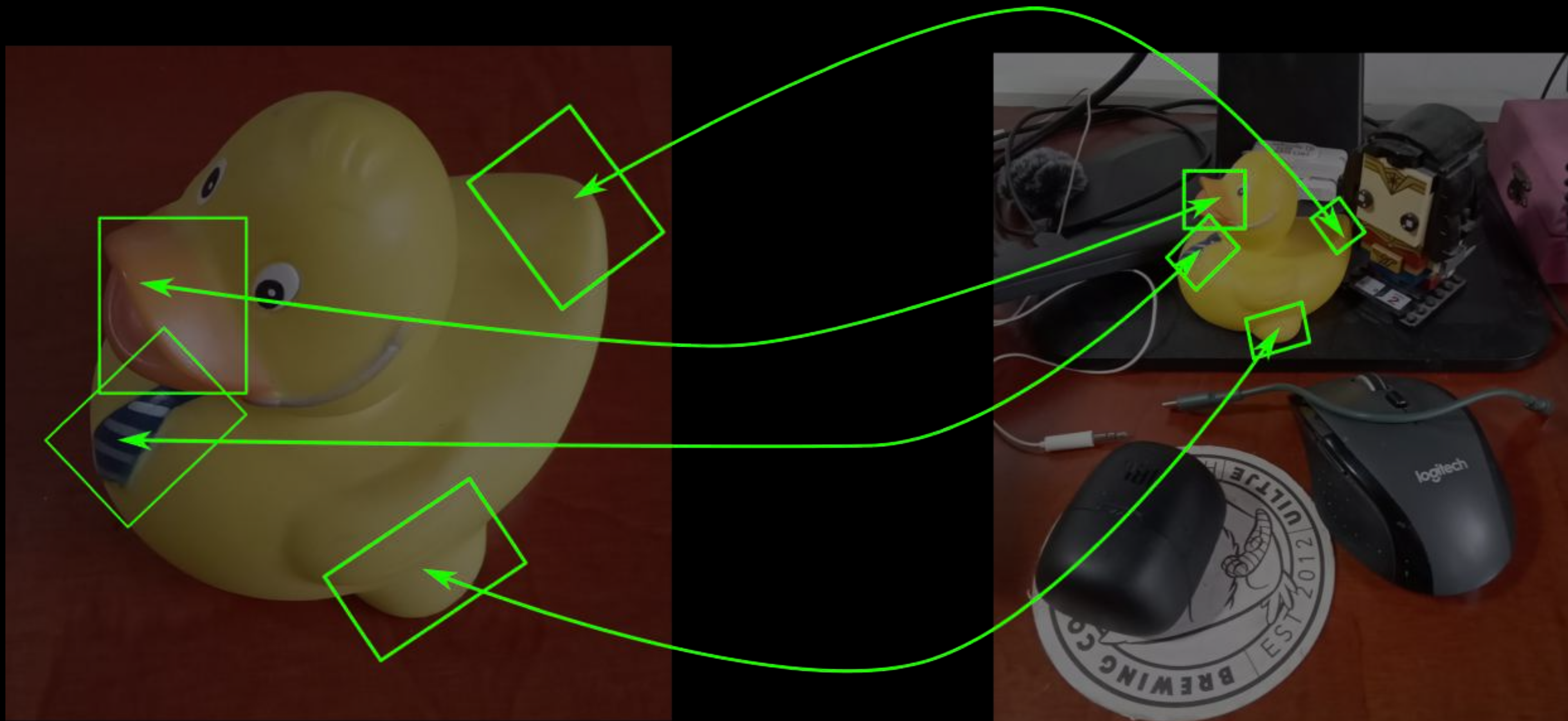


Scale

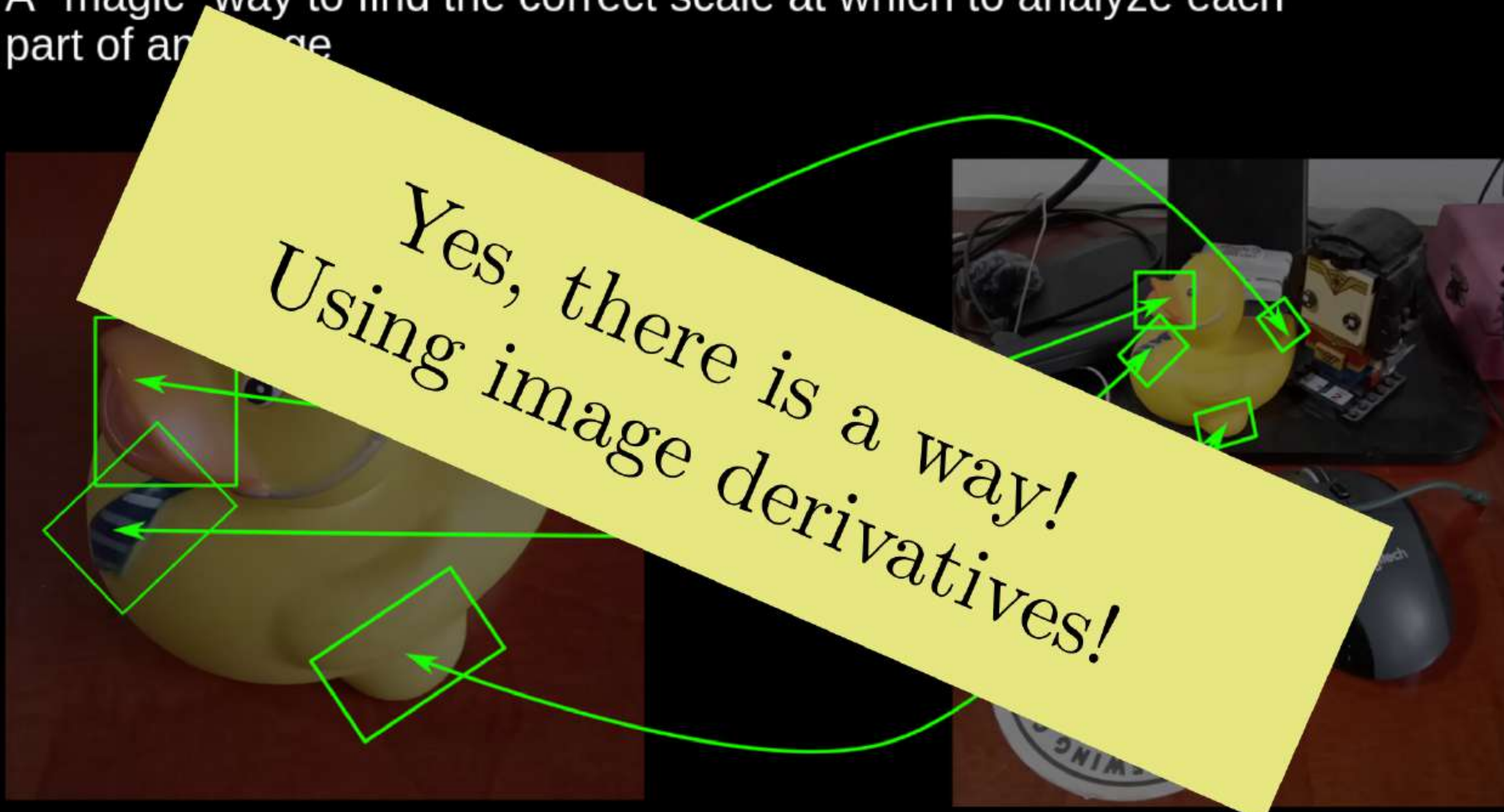
A "magic" way to find the correct scale at which to analyze each part of an image



Find blobs = keypoints / interest points.

Scale

A "magic" way to find the correct scale at which to analyze each part of an image.



Find blobs = keypoints / interest points.

Recap: Edge detection

- What causes an edge in an image?

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Edges are changes or discontinuities in brightness or color.

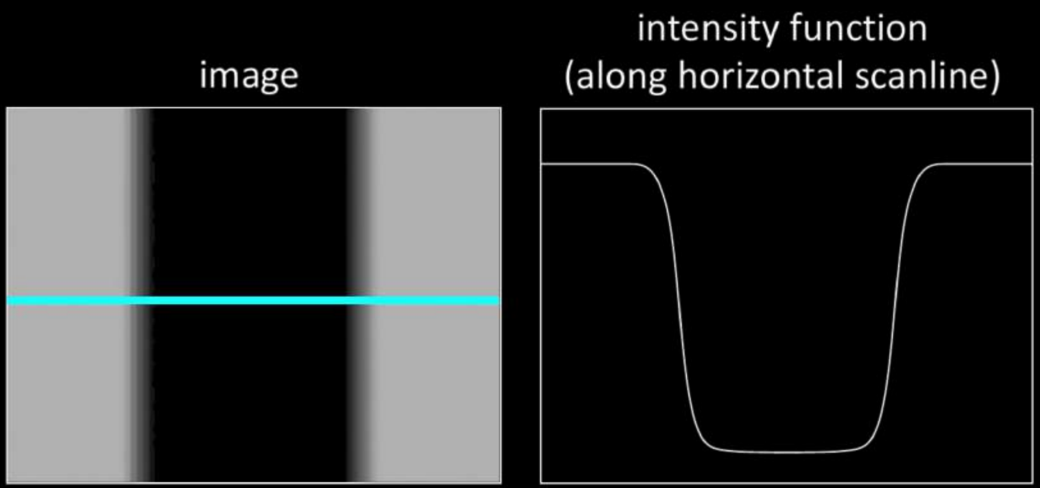
Recap: Edge detection

- What causes an edge in an image?

Edges are changes or discontinuities in brightness or color.

- What mathematical operator detects rapid **changes** in a signal?

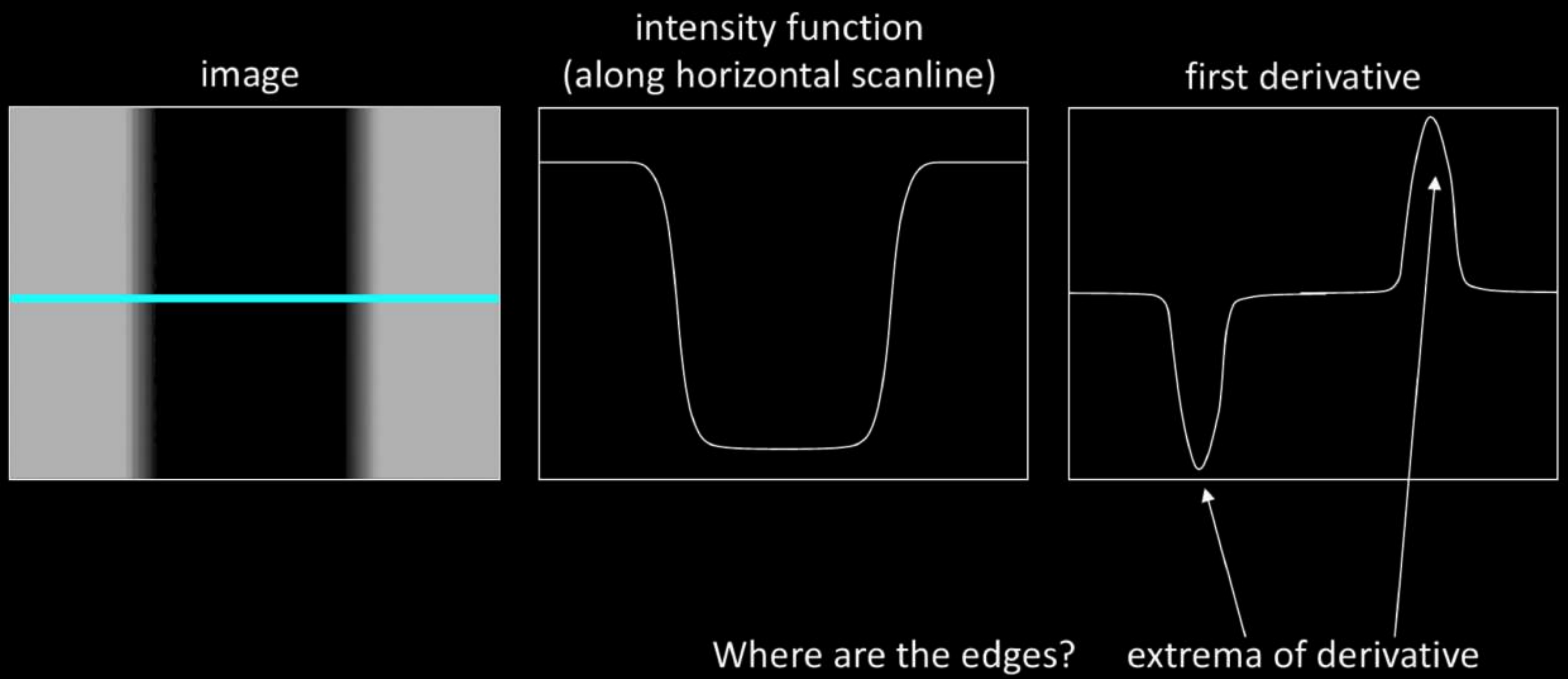
Recap: Edge detection



Where are the edges?

From Fei-Fei Li

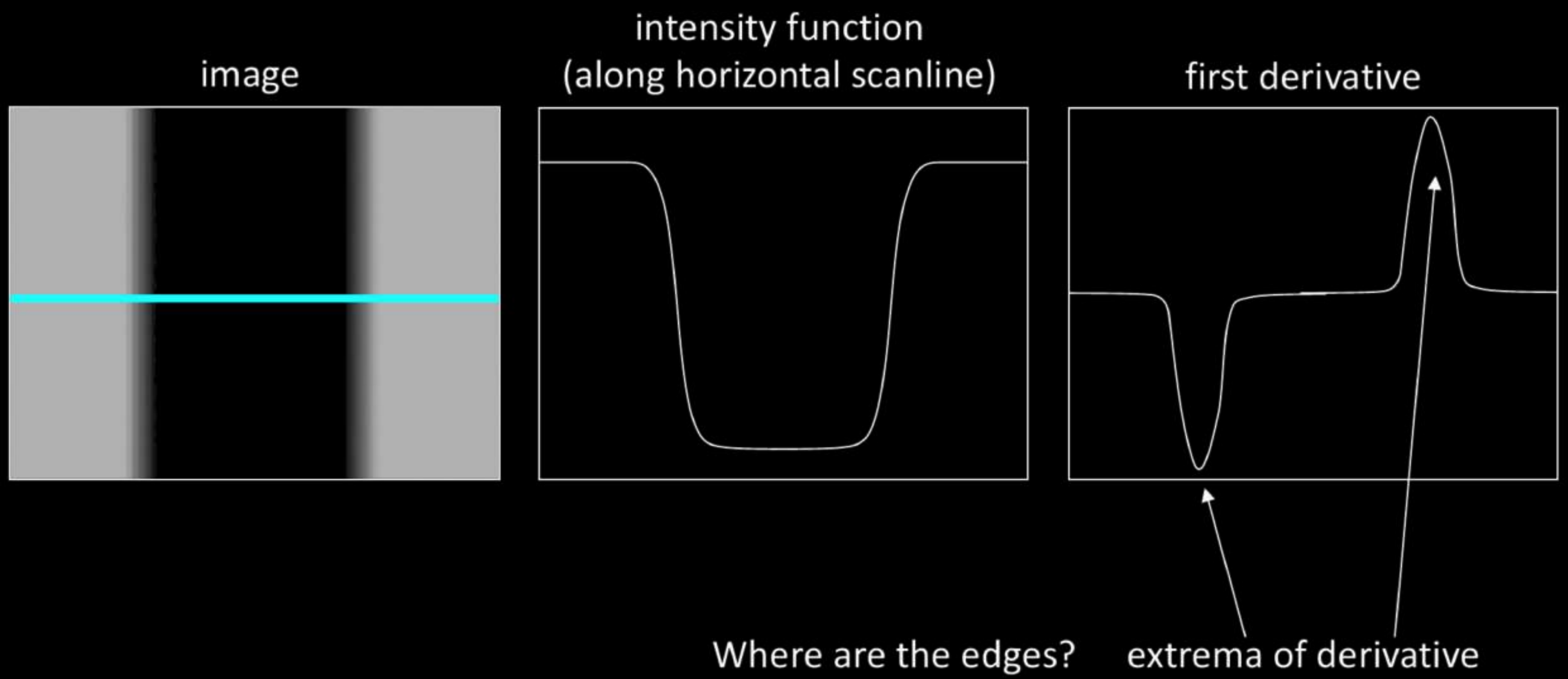
Recap: Edge detection



From Fei-Fei Li

- What is a good approximation of a derivative?

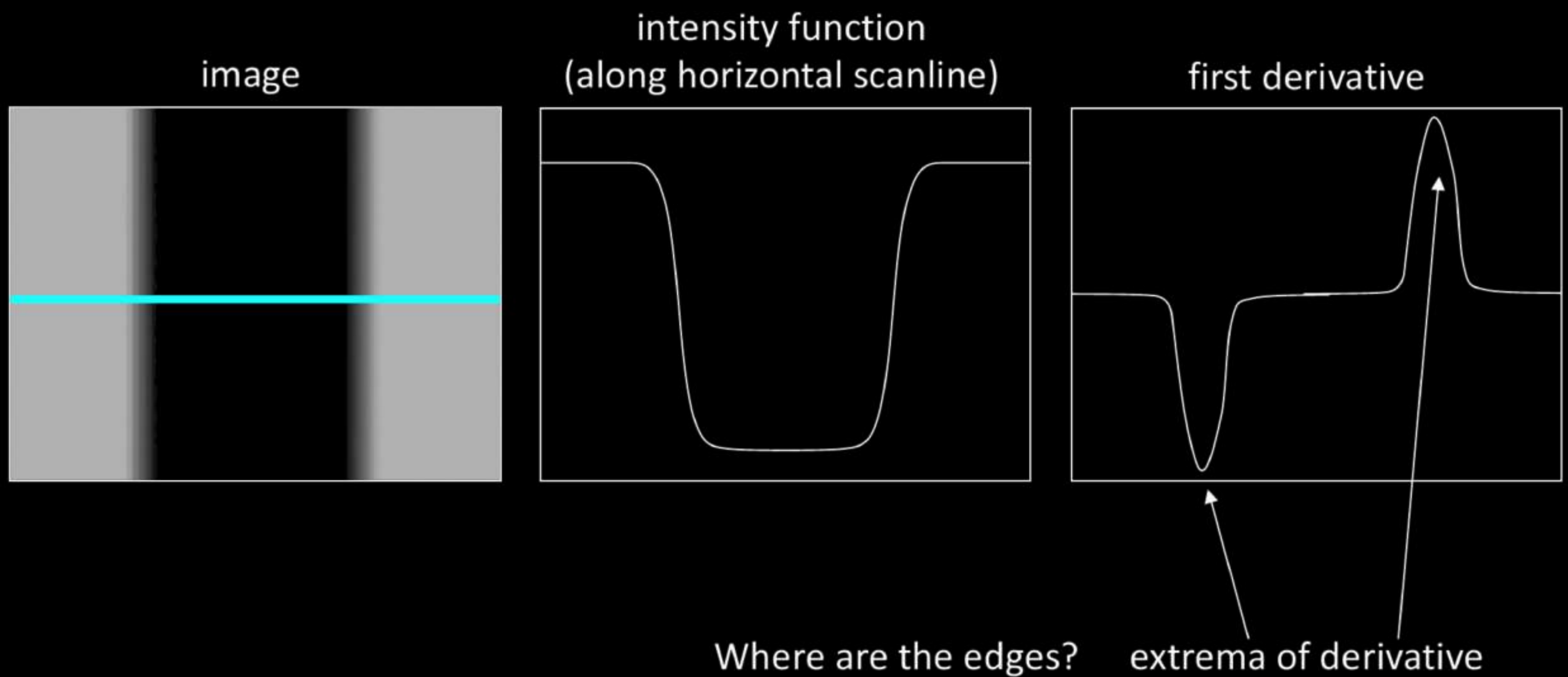
Recap: Edge detection



From Fei-Fei Li

- What is a good approximation of a derivative?
- Finite differences: $\frac{\partial f(x)}{\partial x} \approx \frac{f(x+\epsilon)-f(x)}{\epsilon}$

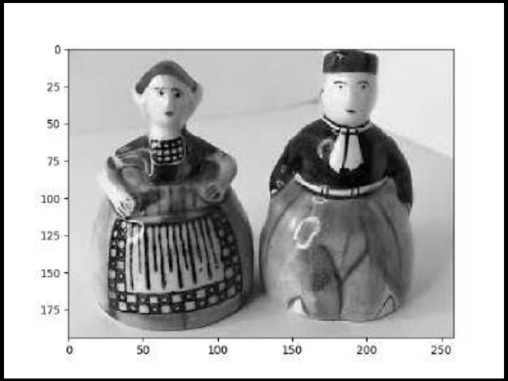
Recap: Edge detection

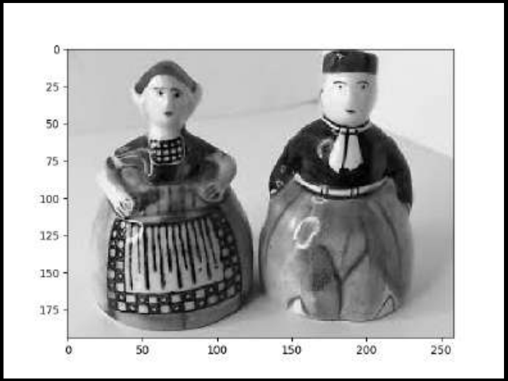


From Fei-Fei Li

- What is a good approximation of a derivative?
- Finite differences: $\frac{\partial f(x)}{\partial x} \approx \frac{f(x+\epsilon)-f(x)}{\epsilon}$
- How can I implement finite differences efficiently?

Recap: Edge detection





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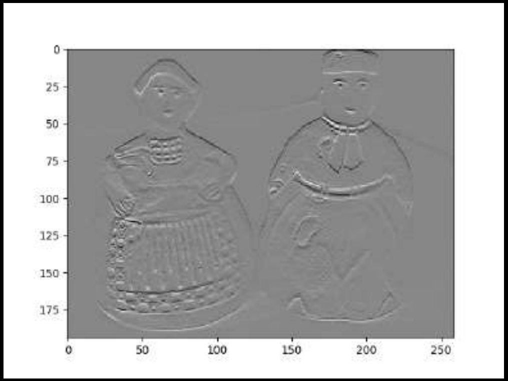
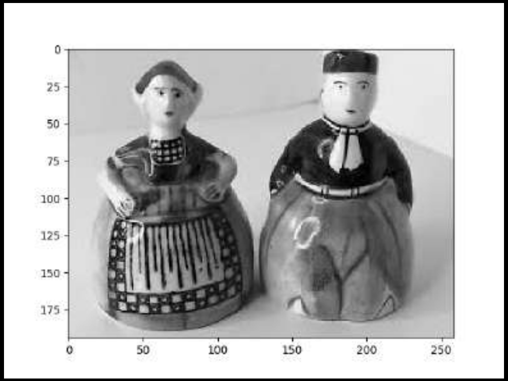


Recap: Edge detection



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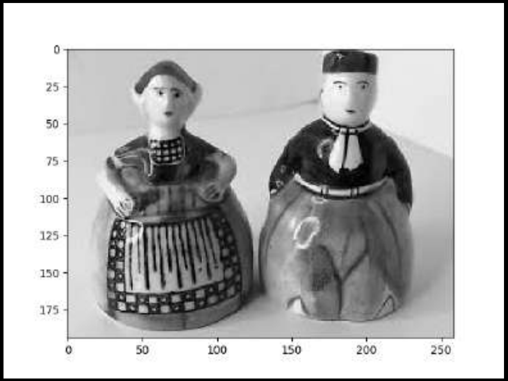
$$\begin{bmatrix} 1 & -1 \end{bmatrix}$$



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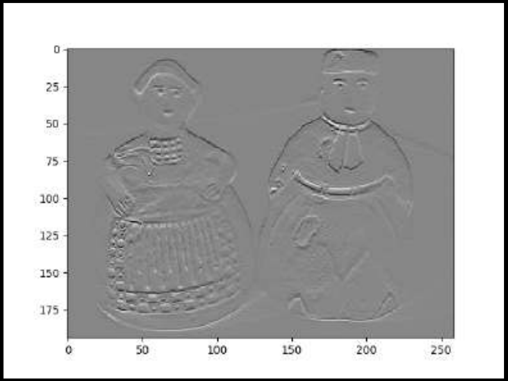
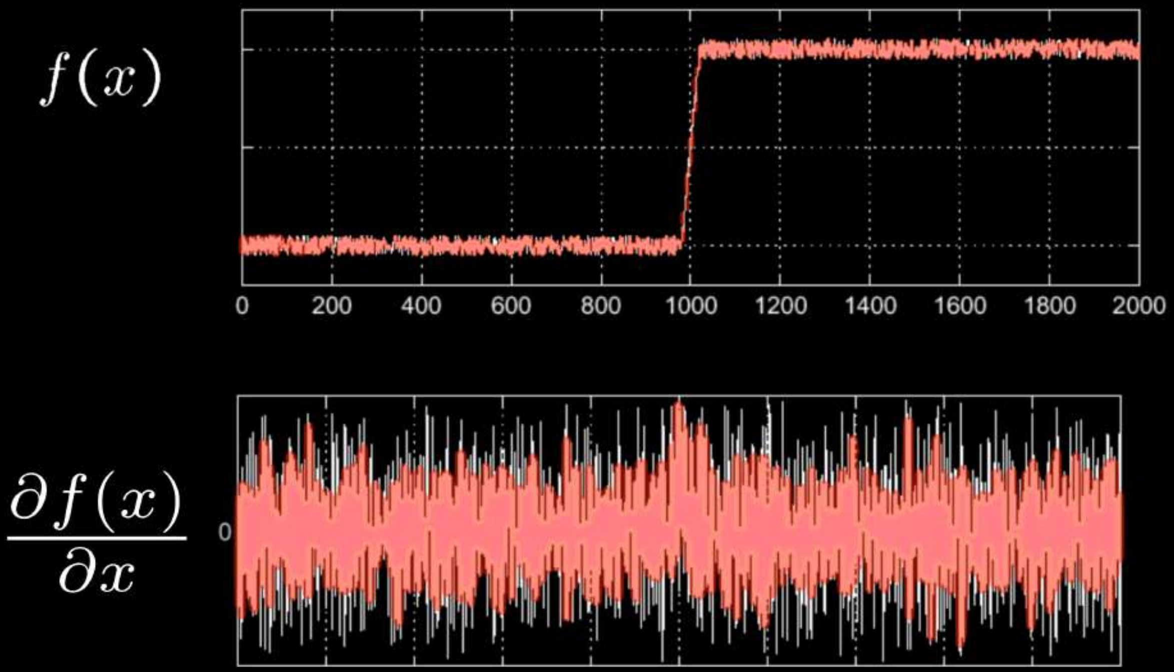


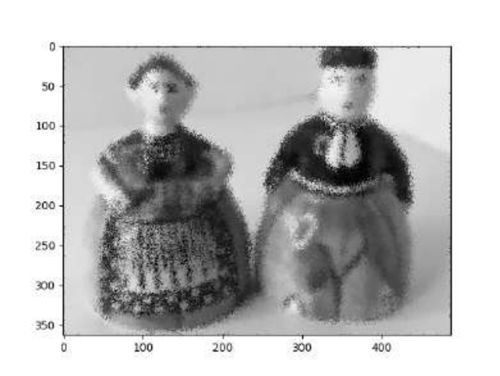
Recap: Noisy edge detection

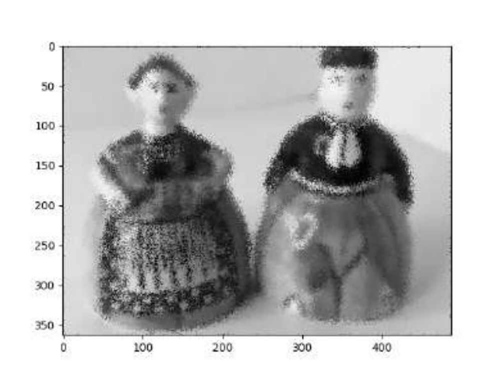


From Fei-Fei Li

- Why does this happen?

Recap: Noisy edge detection





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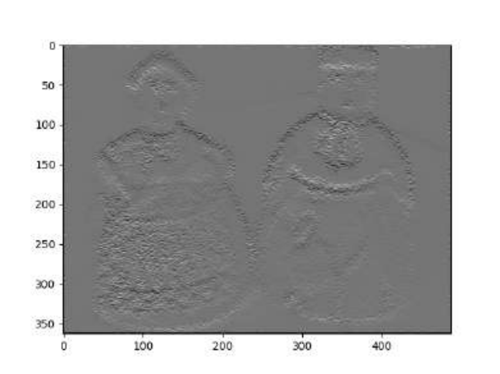
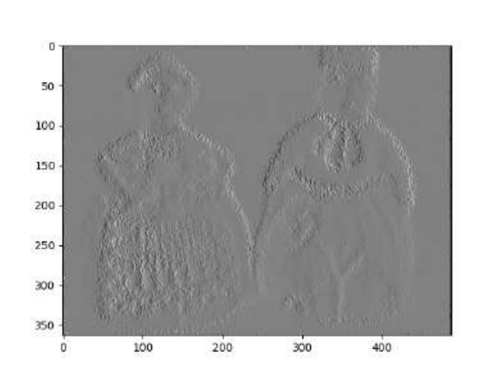




How can we fix this?

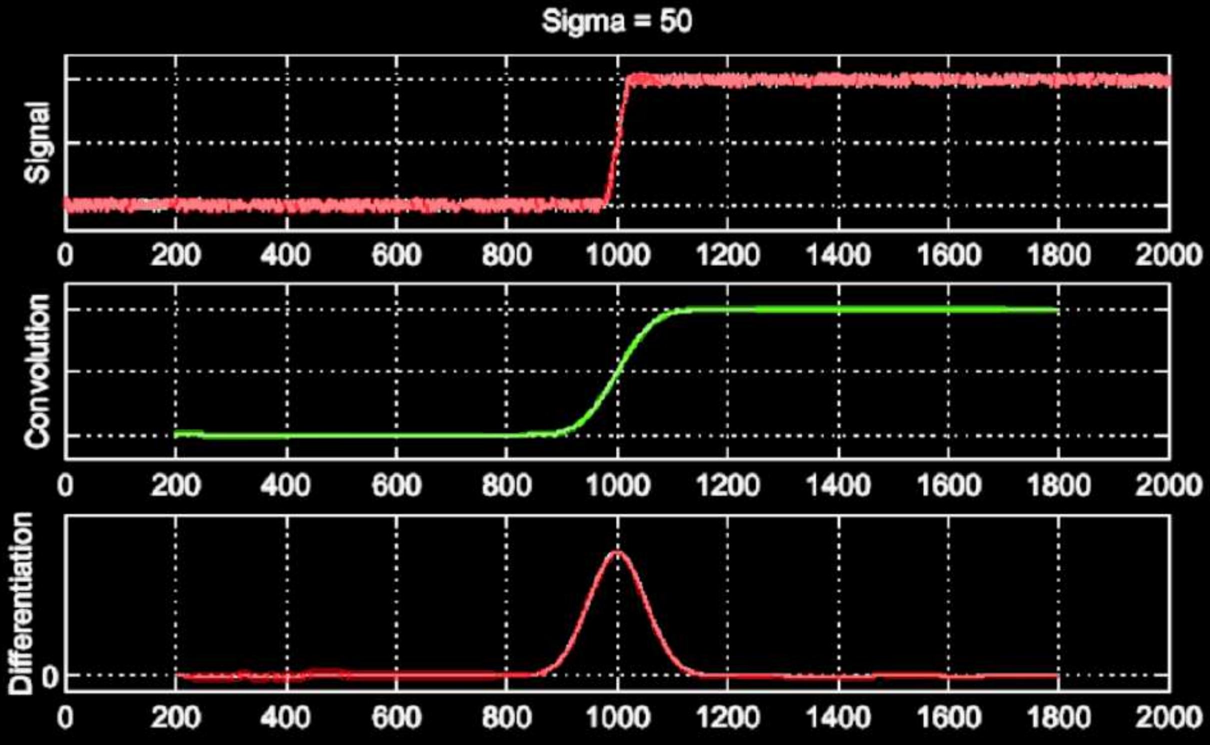
Recap: Noisy edge detection

First smooth then take the image derivative

f

$h \star f$

$\frac{\partial}{\partial x}(h \star f)$



From Fei-Fei Li

- Where $h(x)$ is a Gaussian smoothing kernel

Derivative of convolution theorem

$$\frac{\partial\left(h(x)\star f(x)\right)}{\partial x} = \frac{\partial h(x)}{\partial x} \star f(x)$$

- What does this say?

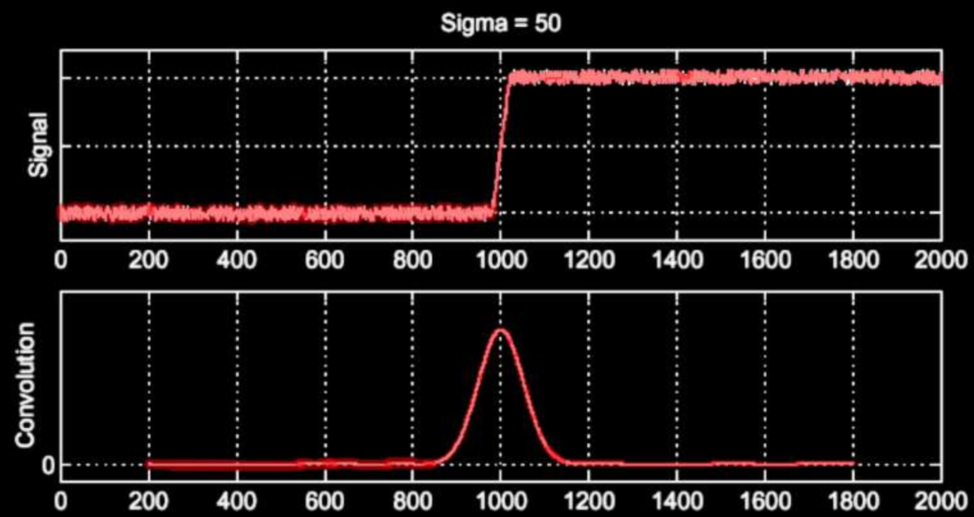
Derivative of convolution theorem

$$\frac{\partial\Big(h(x)\star f(x)\Big)}{\partial x} = \frac{\partial h(x)}{\partial x} \star f(x)$$

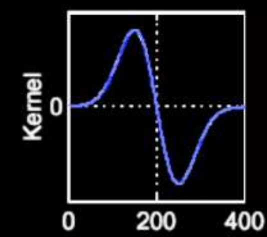
- What does this say?
  Derivative of convolution = convolution with derivative

f

$(\frac{\partial}{\partial x}h) \star f$



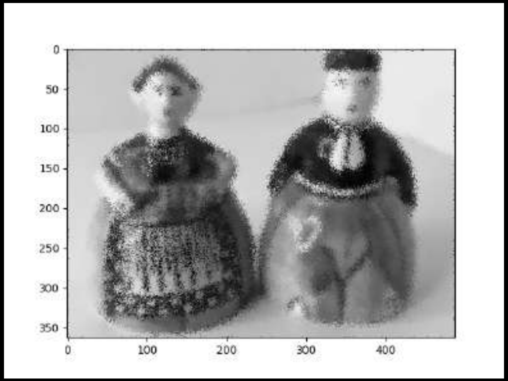
$\frac{\partial}{\partial x}h$

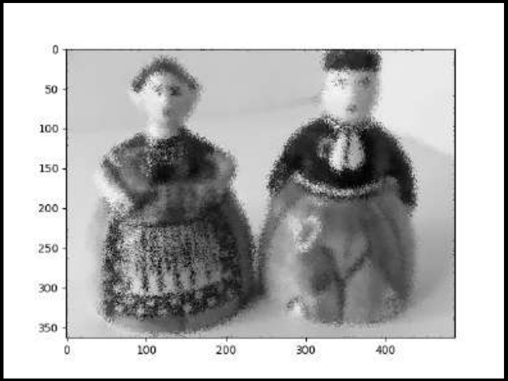


From Fei-Fei Li

- Why is this useful?

Noisy edge detection





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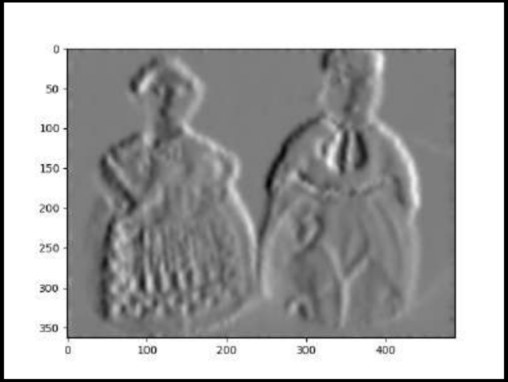
$$\frac{\partial h(x,y)}{\partial x}$$



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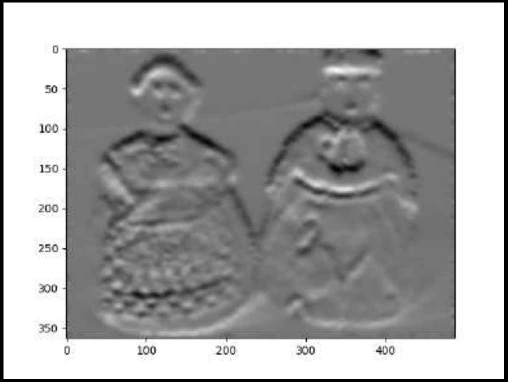
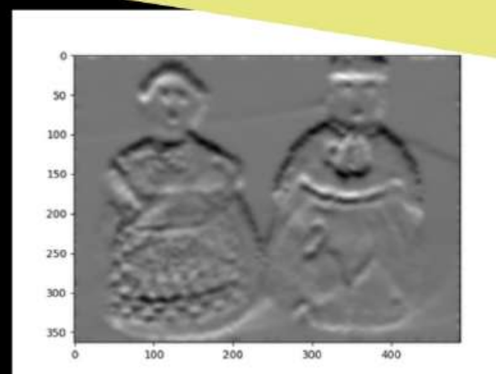


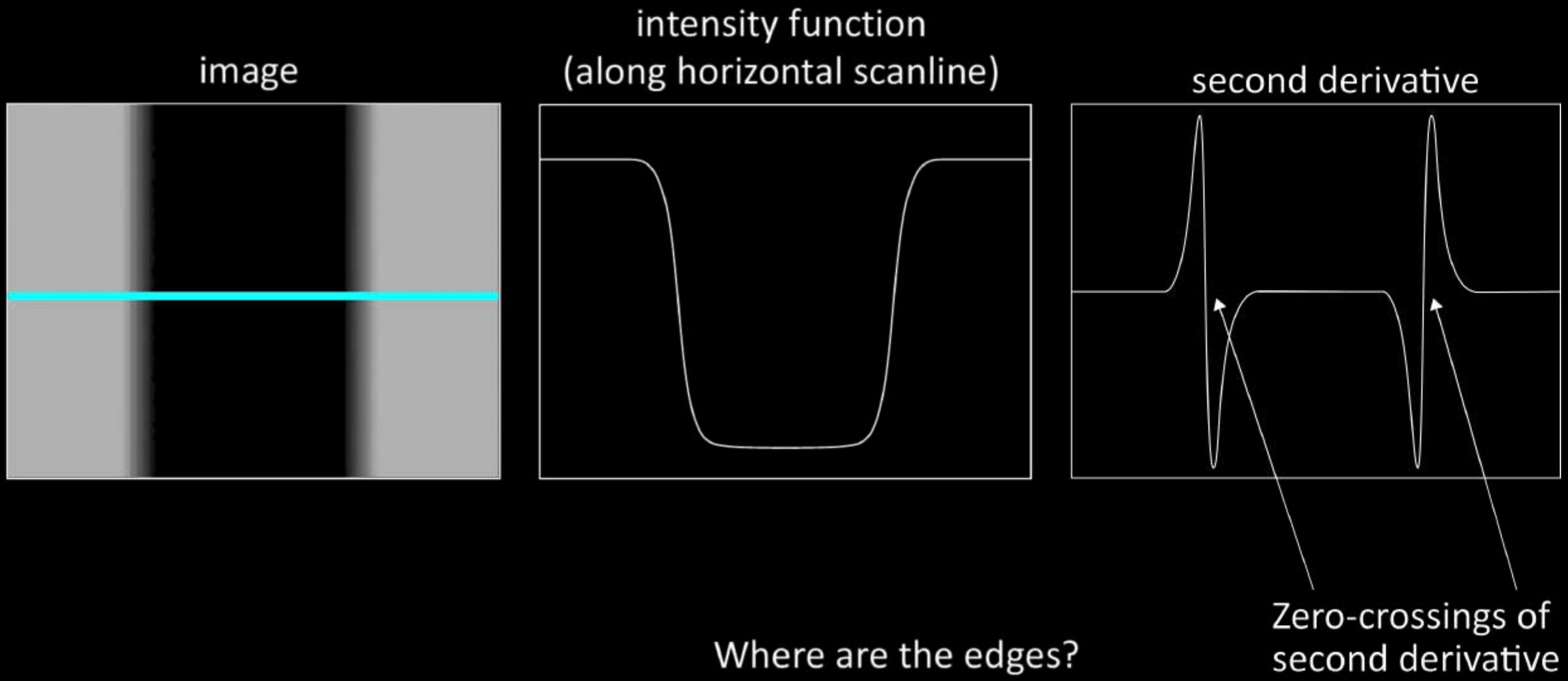
Noisy edge detection

Gradient: $\nabla h(x,y) = \left[\frac{\partial h(x,y)}{\partial x}, \frac{\partial h(x,y)}{\partial y}\right]$

$$\frac{\partial h(x,y)}{\partial x}$$

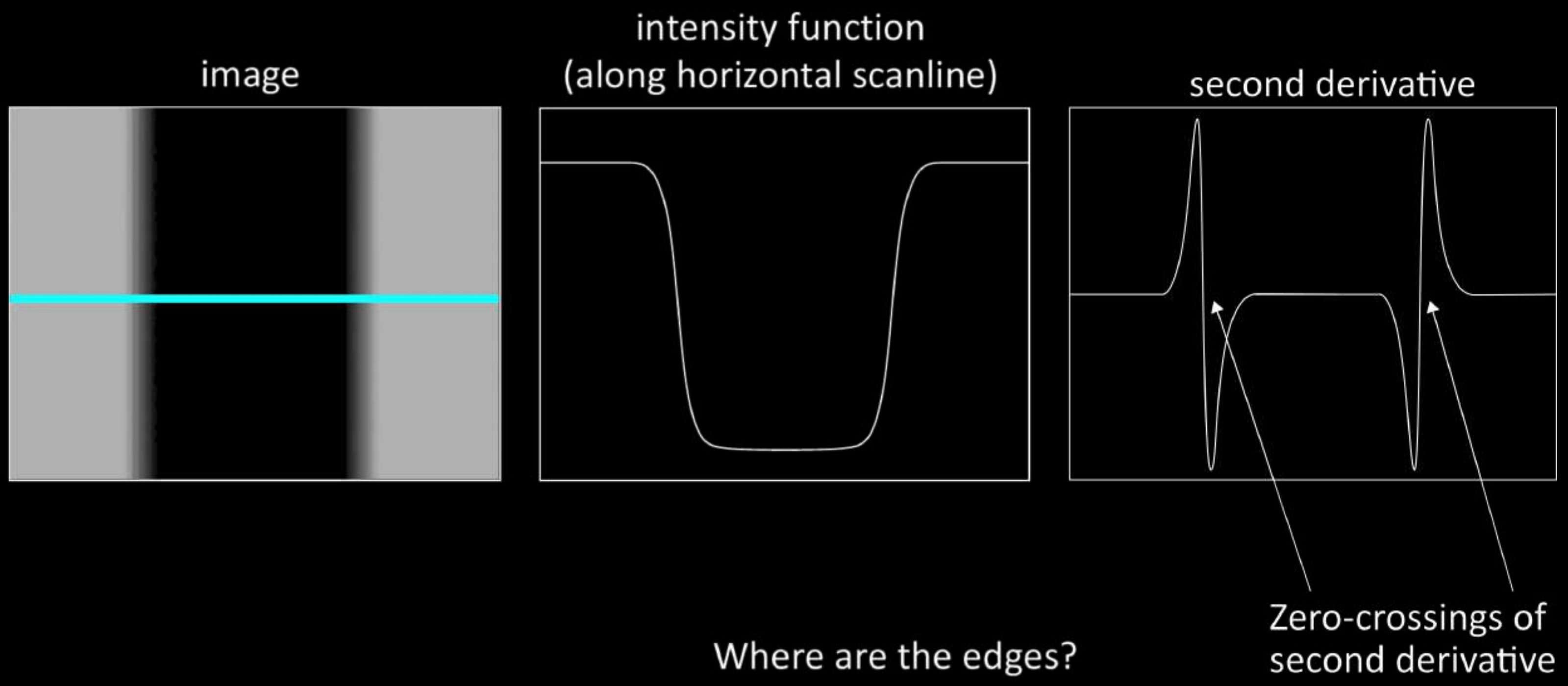


Edge detection



From Fei-Fei Li

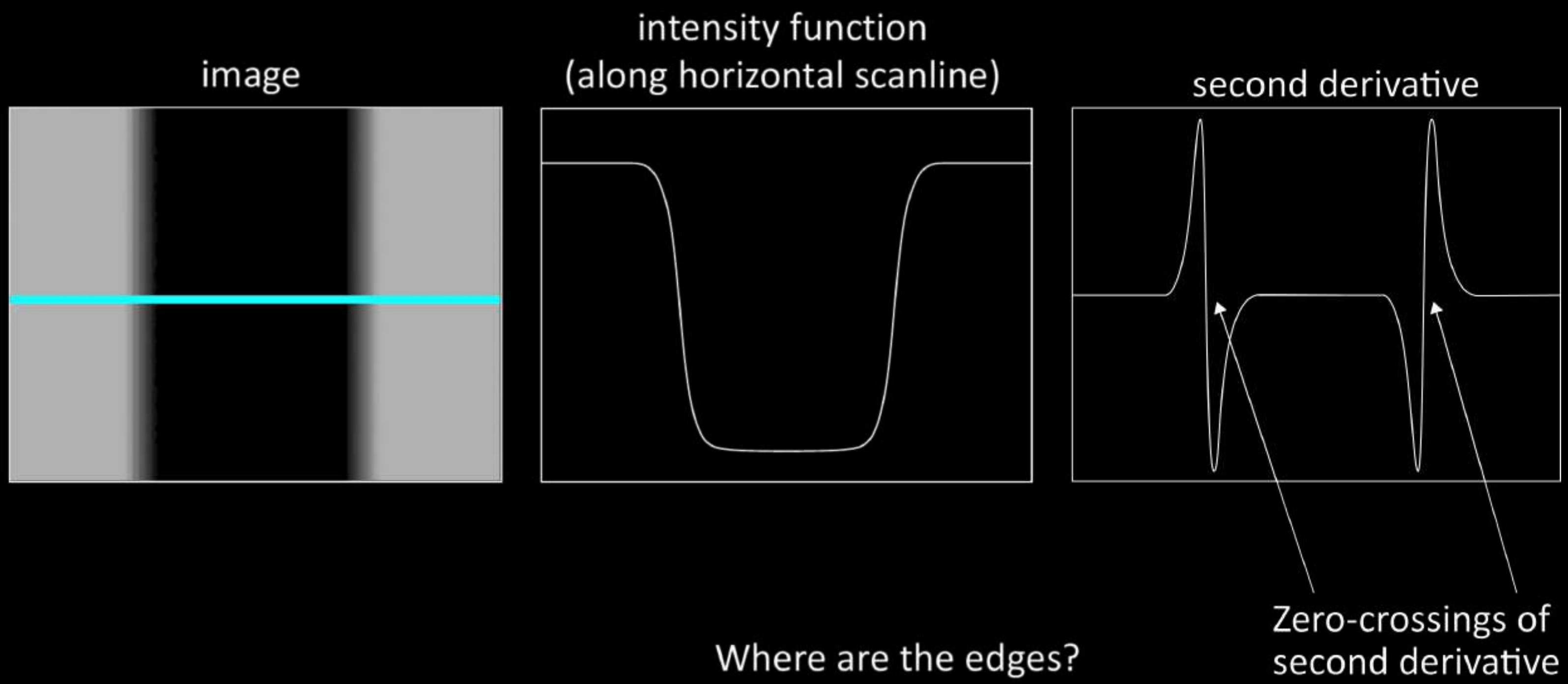
Edge detection



From Fei-Fei Li

- How can I take a second order image derivative?

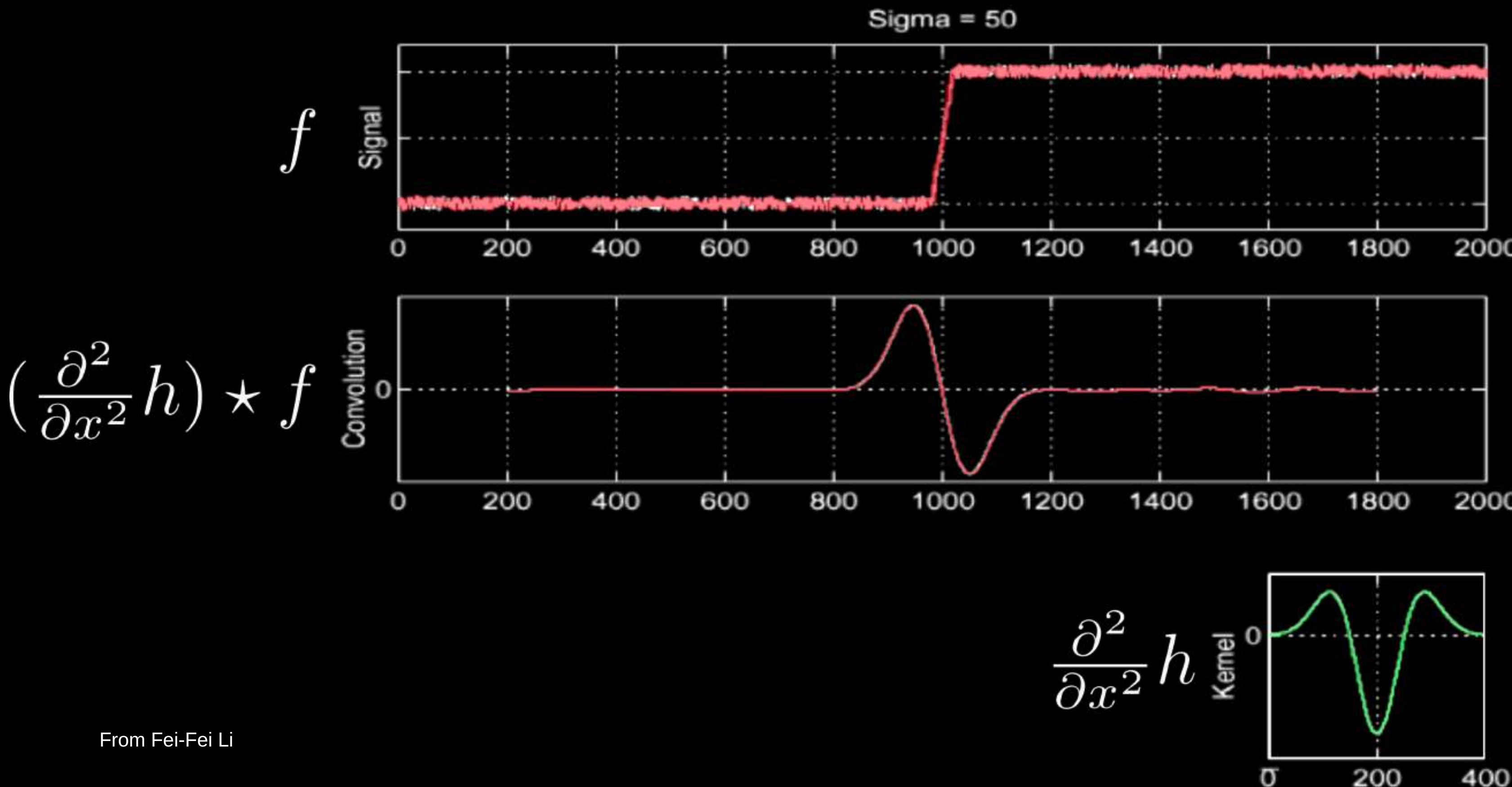
Edge detection



From Fei-Fei Li

- How can I take a second order image derivative?
- Convolve with $[1, -2, 1]$. Is there a problem with this?

Second order derivative

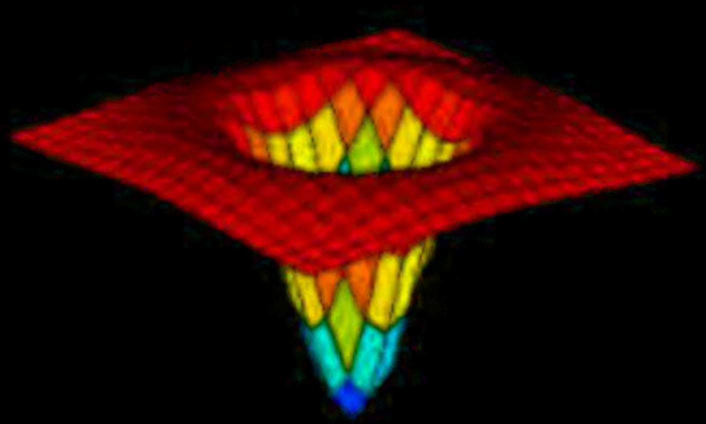


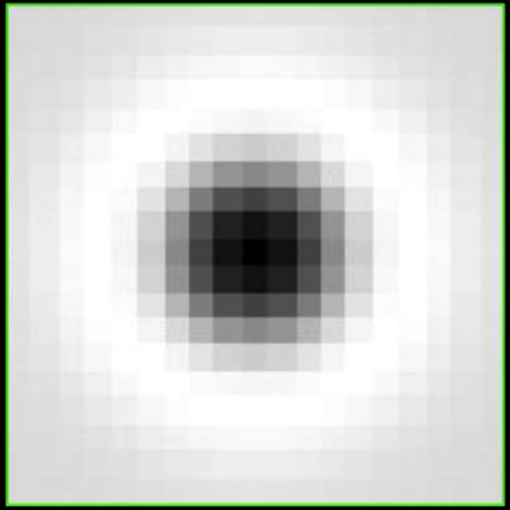
From Fei-Fei Li

- Convolve with the 2nd order Gaussian derivative.

Laplacian of Gaussian

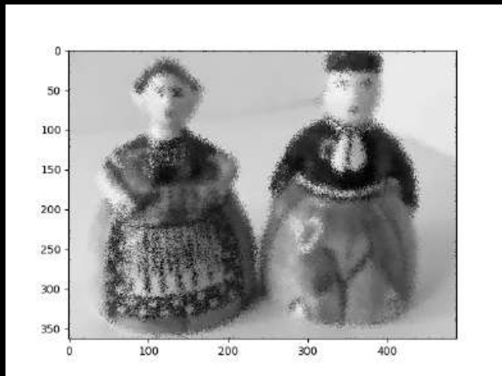
- $LoG(x, y) = \frac{\partial^2 h(x,y)}{\partial^2 x} + \frac{\partial^2 h(x,y)}{\partial^2 y}$





- What is controls the scale of the Laplacian of Gaussian?

Noisy edge detection



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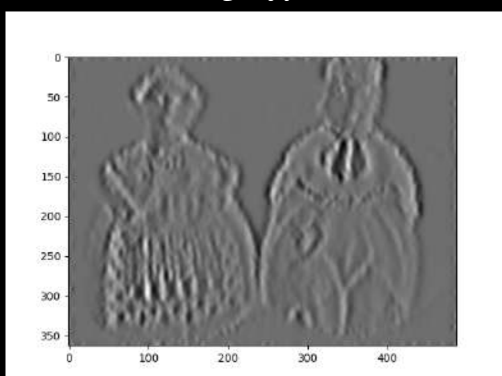
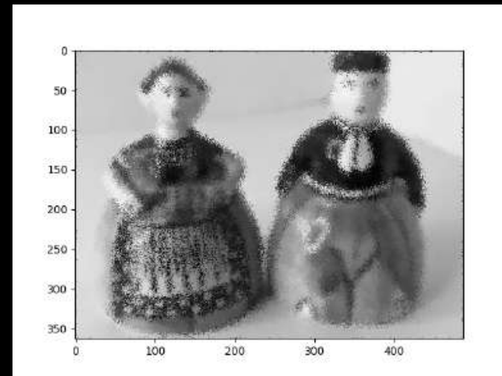
$$\frac{\partial^2 h(x, y)}{\partial^2 x}$$





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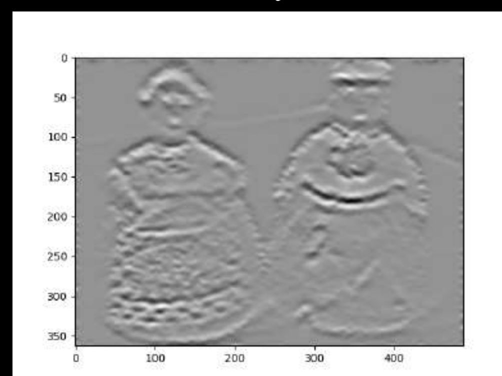
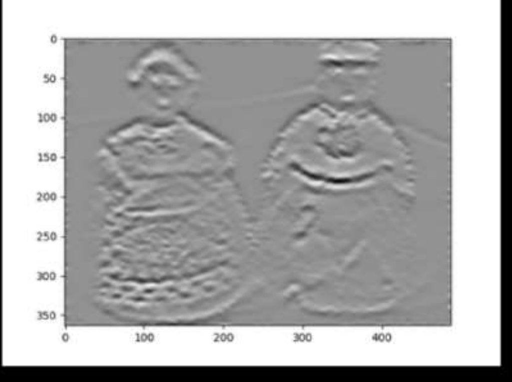
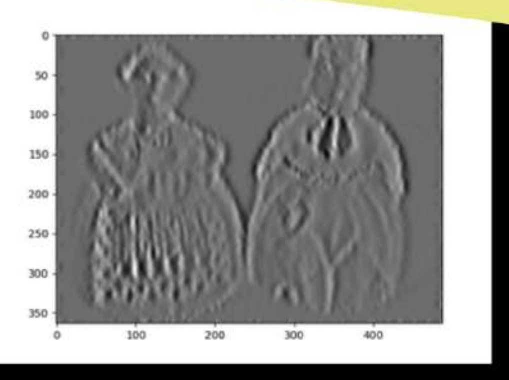


Noisy edge detection

$$\text{Laplacian: } \frac{\partial^2 h(x,y)}{\partial^2 x} + \frac{\partial^2 h(x,y)}{\partial^2 y}$$

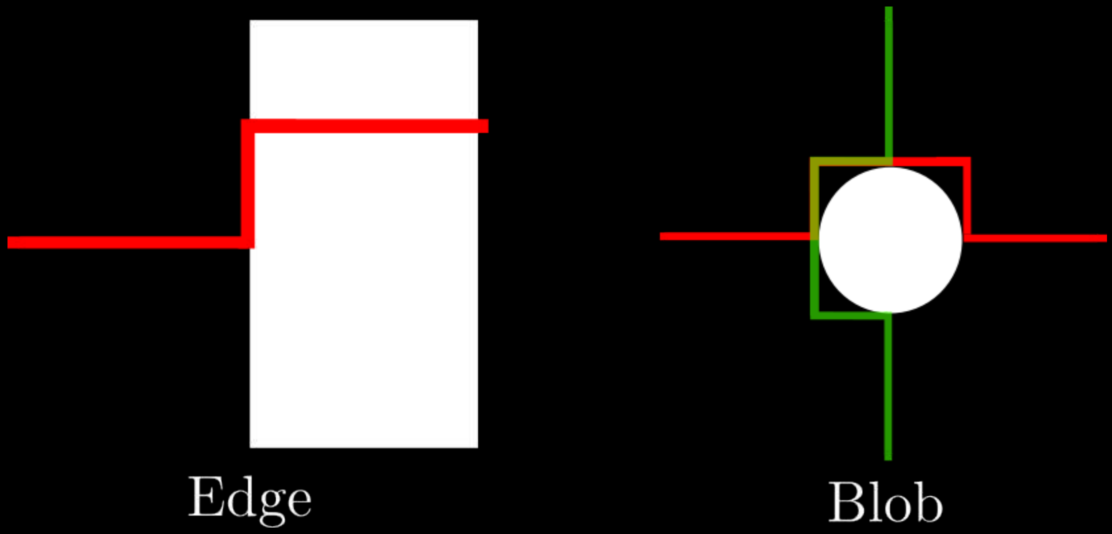
$\partial^2 x$





Blob detection

- An edge is a ripple in the image.

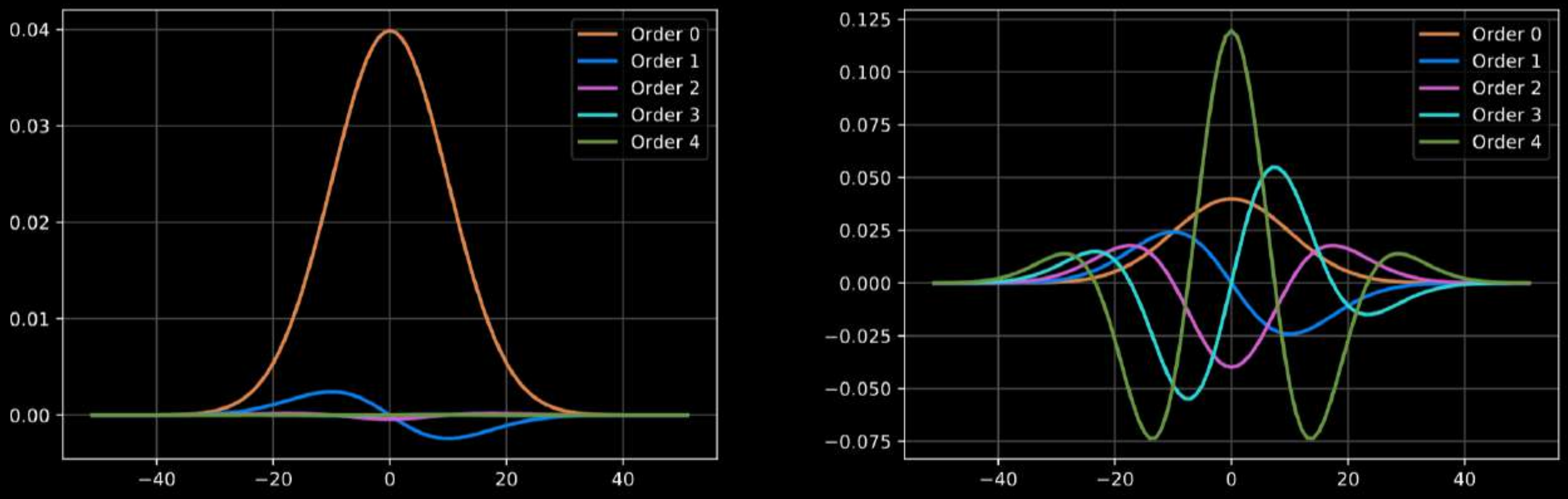


- We can find blobs at the intersection of ripples.
- So we can use the Laplacian of Gaussian to find blobs.

Laplacian normalization

The "scale" of the Laplacian is σ.

- **Problem:** the response of a derivative of Gaussian decreases as σ increases
- **Solution:** multiple the mth derivative by σ^m.



(a) Unnormalized (b) Normalized

- How should we normalize the Laplacian of Gaussian?

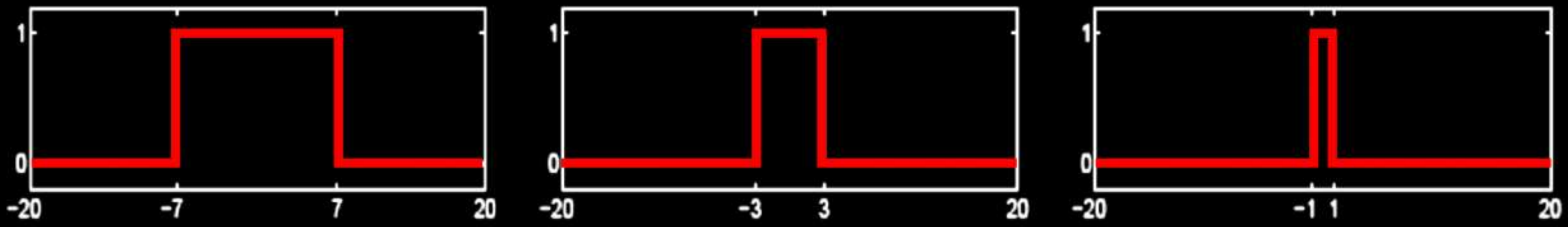
Scale selection

So the normalized Laplacian is: $\sigma^2 \left(\frac{\partial^2}{\partial x^2} h(x, y) + \frac{\partial^2}{\partial y^2} h(x, y) \right)$

Scale selection

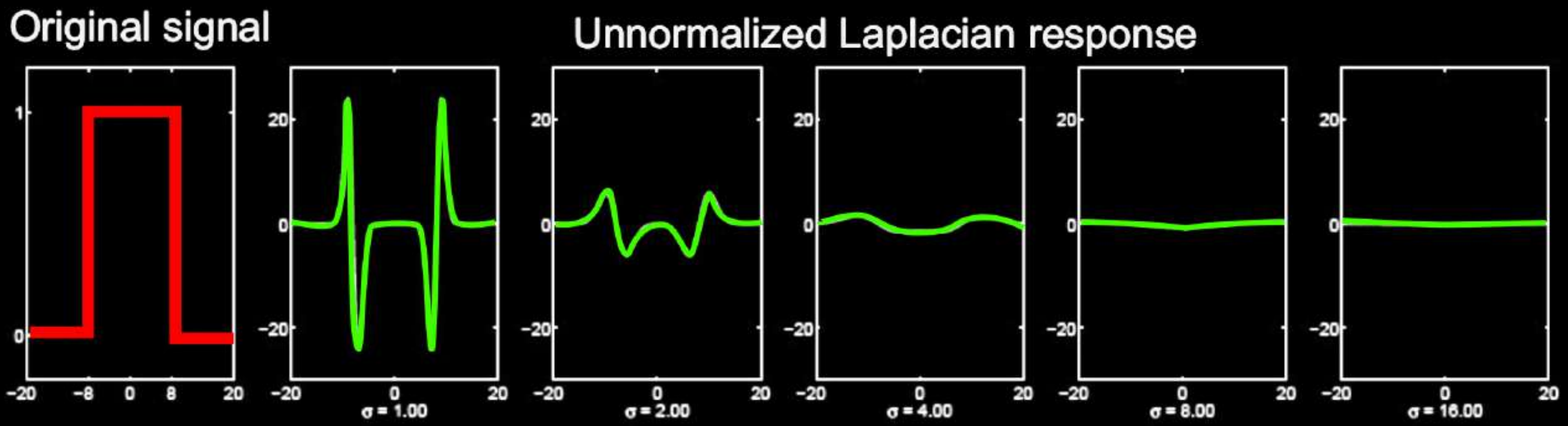
So the normalized Laplacian is: $\sigma^2 \left(\frac{\partial^2}{\partial x^2} h(x, y) + \frac{\partial^2}{\partial y^2} h(x, y) \right)$

- The magnitude of the Laplacian response will achieve **a maximum/minimum** at the center of the blob, provided the **scale of the Laplacian** is "matched" to the **scale of the blob**.

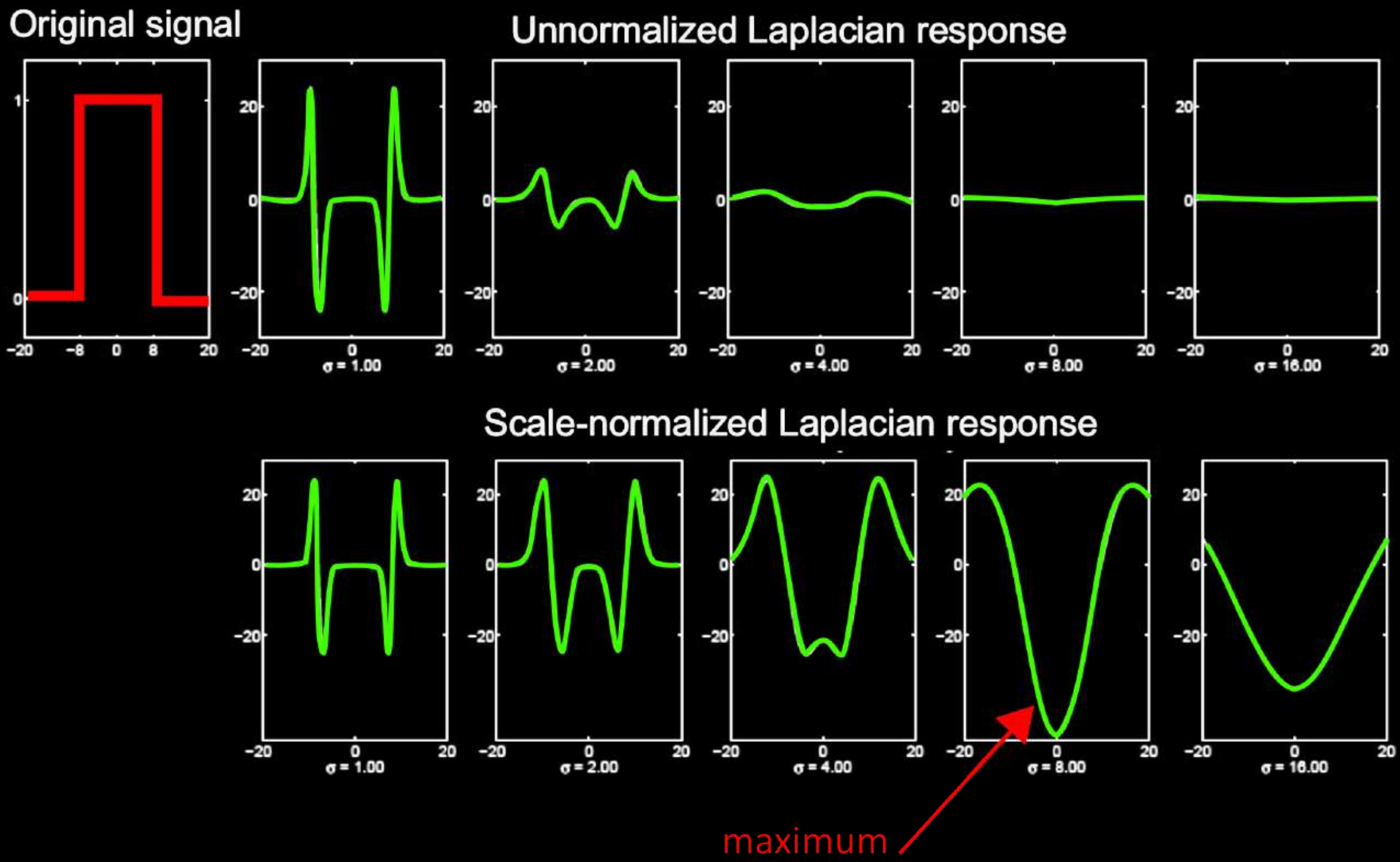


What does this say?

Normalized Laplacian of Gaussian



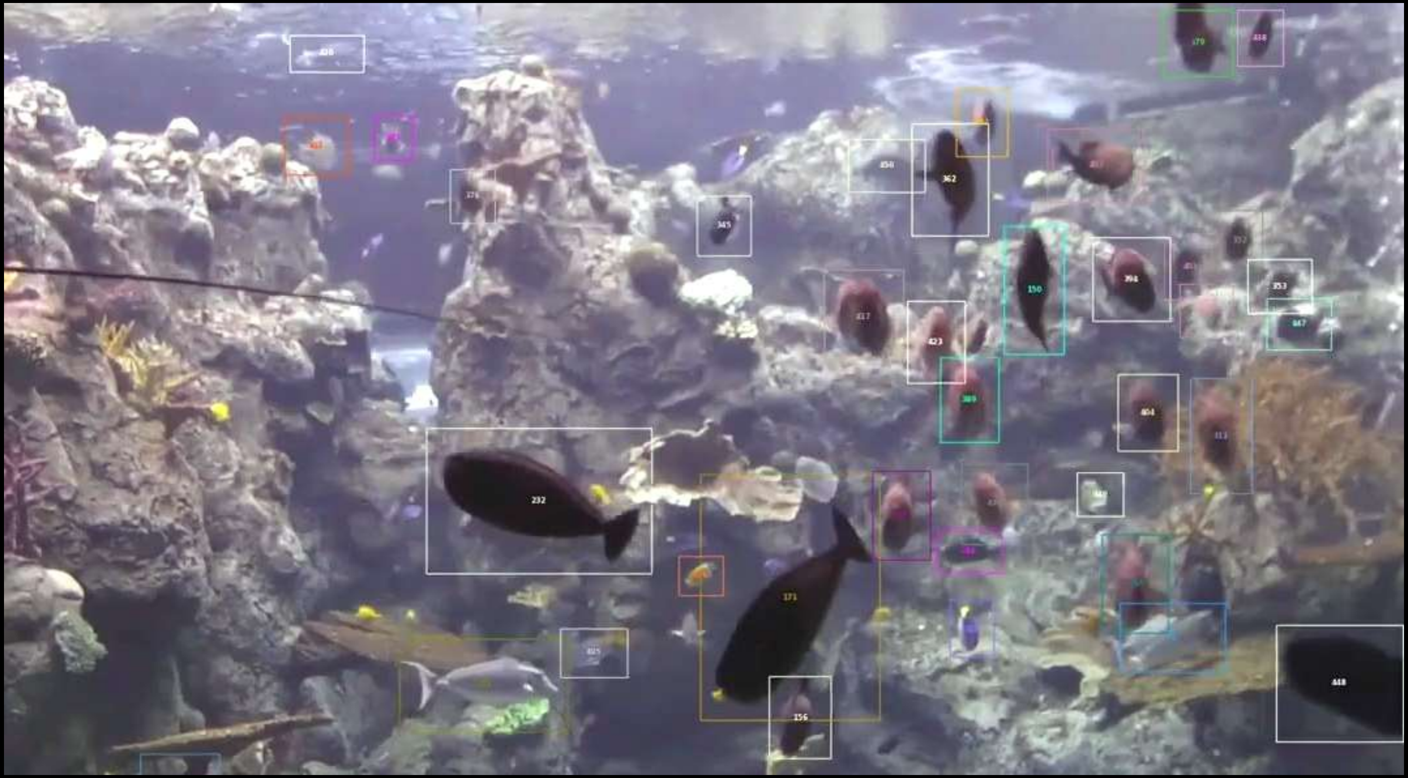
Normalized Laplacian of Gaussian



Questions?

Scale

A "magic" way to find the correct scale at which to analyze each part of an image



Scale selection

- Each circle is centered at a Laplacian extremum and has a radius proportional to the Laplacian scale, σ.

Which scale σ to use?

Scale selection

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Scale selection

- Each circle is centered a Laplacian extremum and has a radius proportional to the Laplacian scale, σ.

Which scale σ to use? Use a range of values.

Scale selection

- We can use a range of σ values in the Laplacian.
- Alternatively what can we do? (hint: "derivative of convolution" theorem).

Scale selection

- We can use a range of σ values in the Laplacian.
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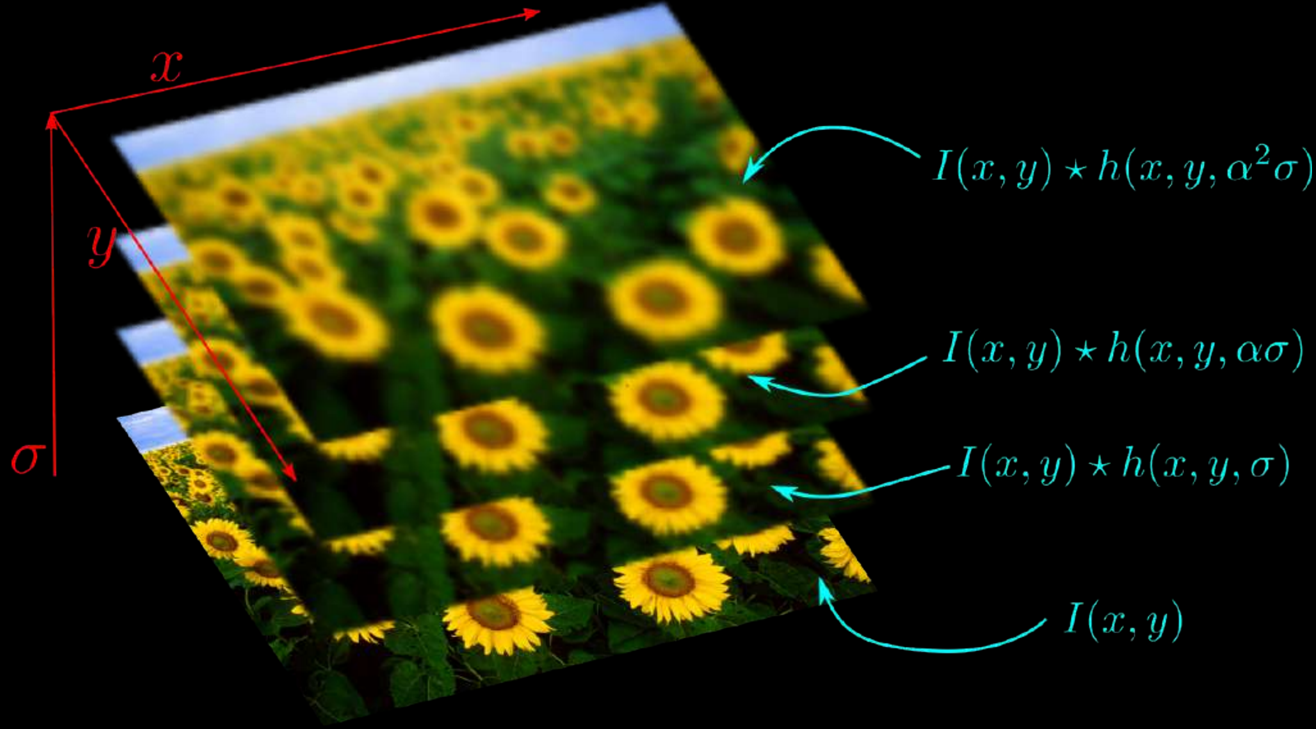
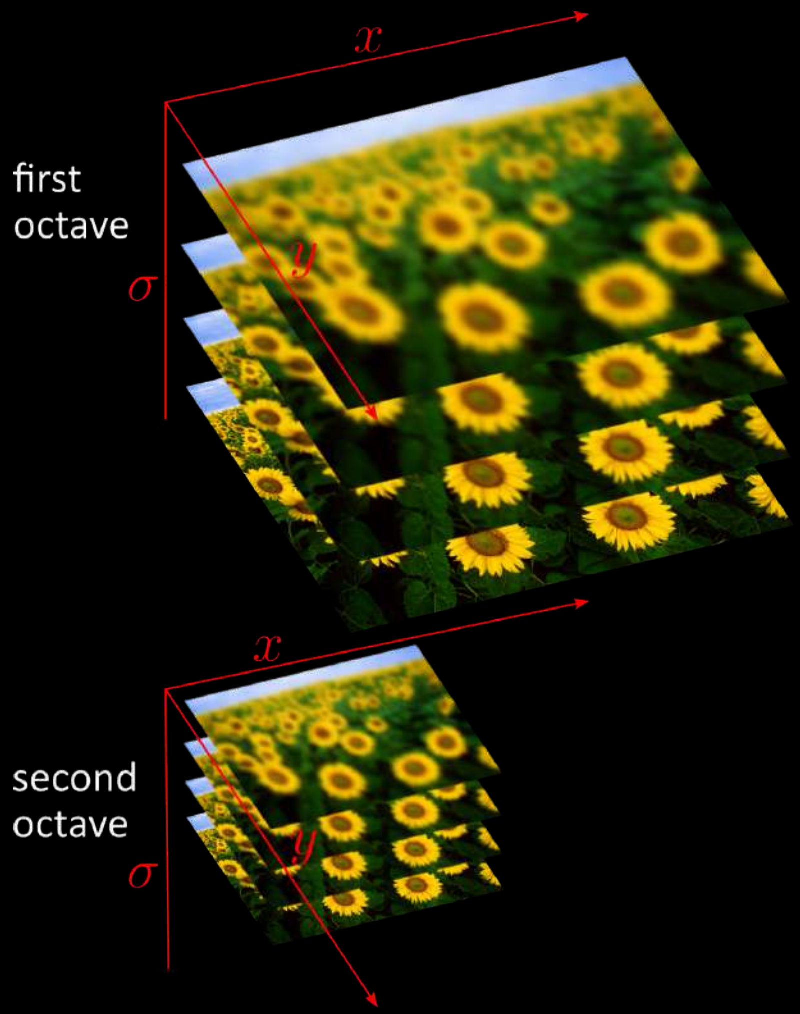


We can blur the image with increasing values of σ and then compute the Laplacian responses.

Scale spaces



Build a scale-space:

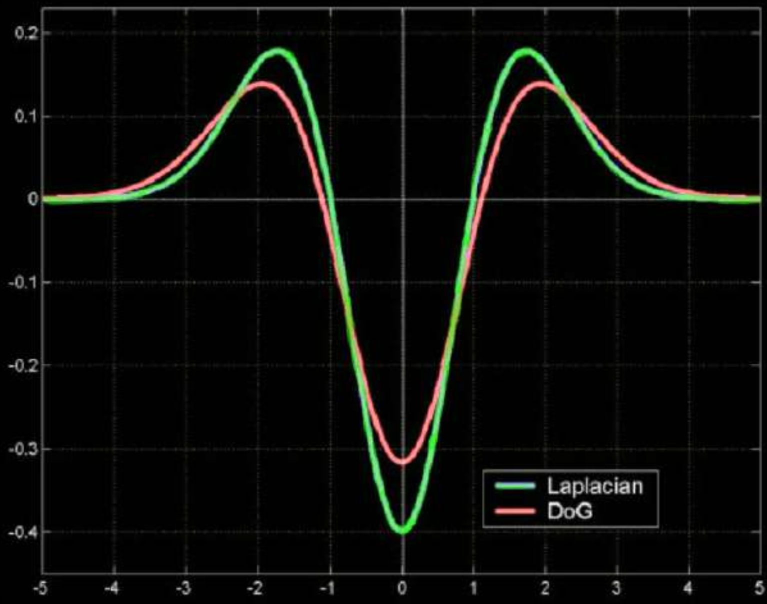
- 1) **Octave:** Take the original image and progressively blur it N times with increasing σ such that: $\sigma_k = \alpha^k \sigma_0$

- 2) Resize the image to its half and repeat the process.

Fast Laplacian responses

Compute Laplacian responses at every layer in the "scale space".

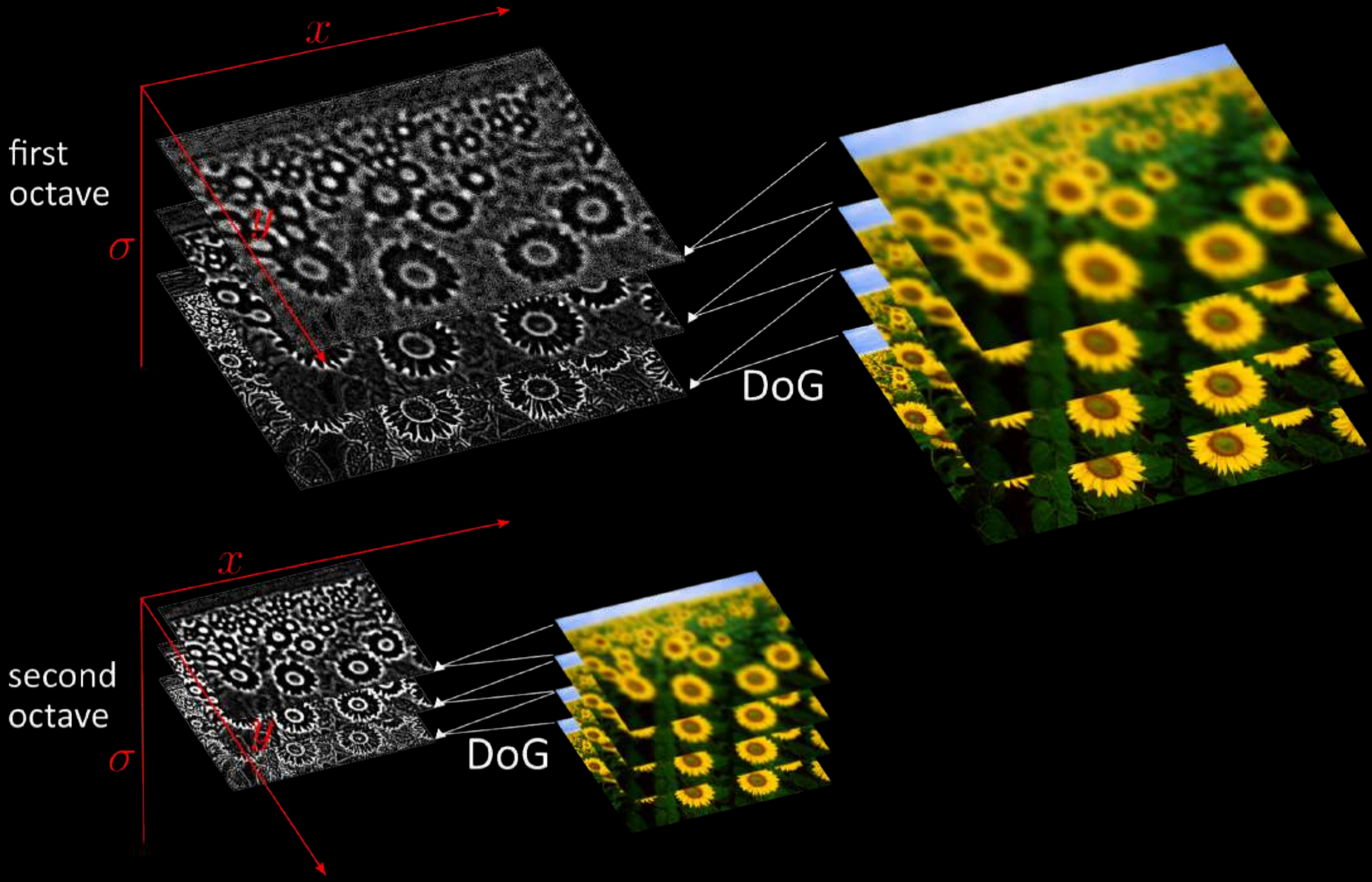
- Laplacian of Gaussian ≈ Difference of Gaussians



$$LoG = \sigma^2 \left(\frac{\partial^2}{\partial x^2} h(x, y) + \frac{\partial^2}{\partial y^2} h(x, y) \right)$$

$$DoG = h(x, y, k\sigma) - h(x, y, \sigma)$$

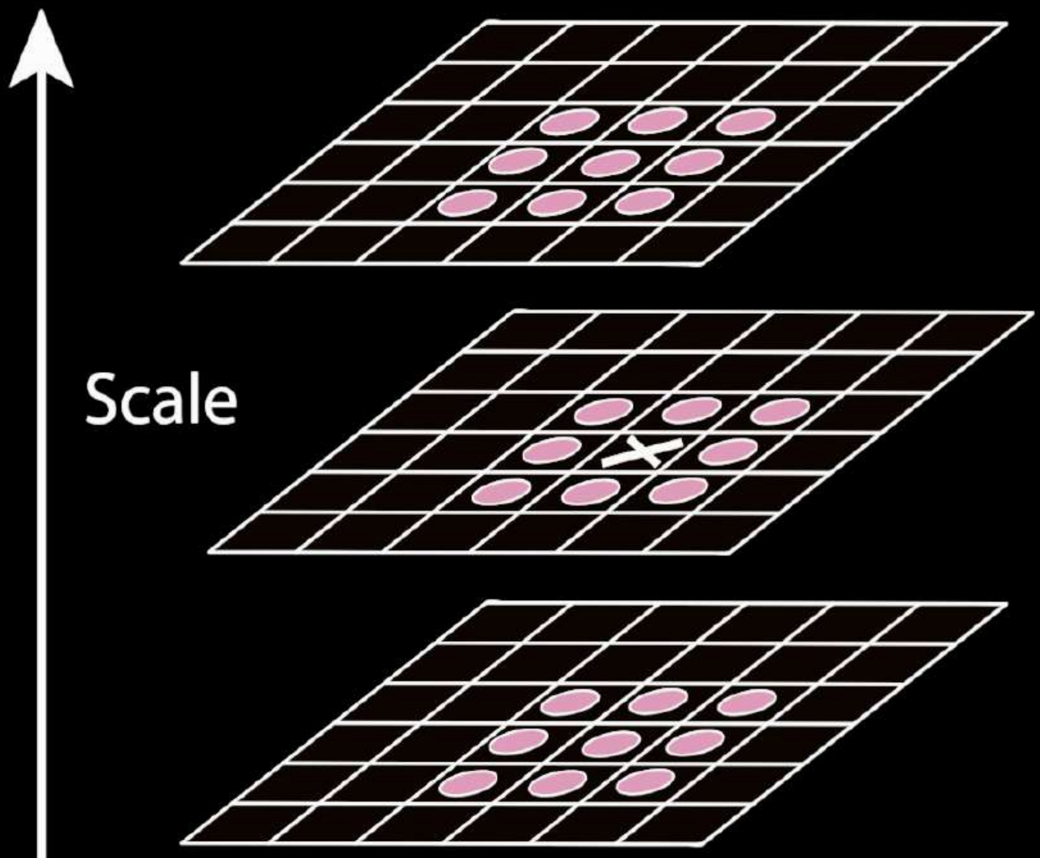
Fast Laplacian responses



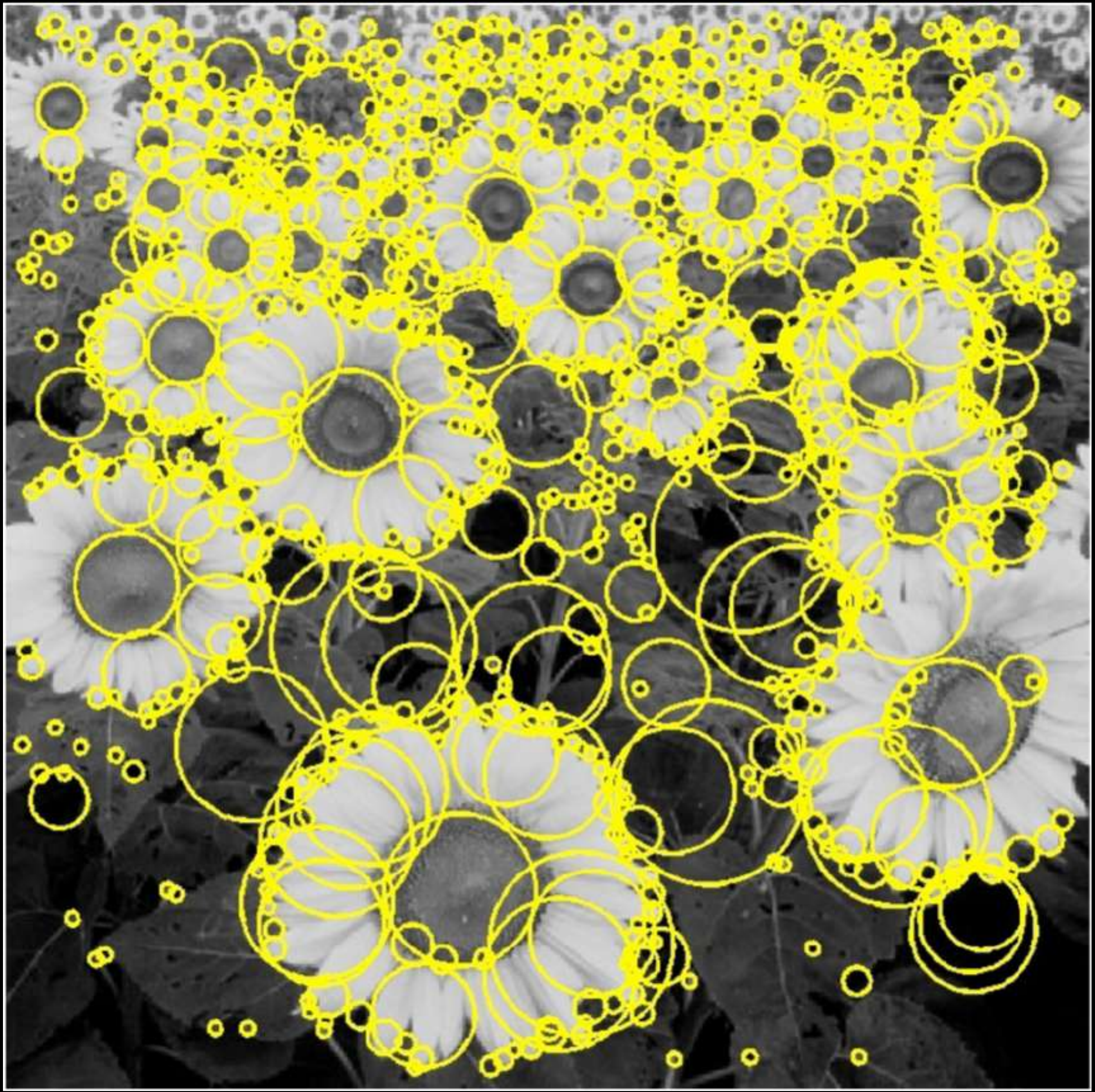
- How do we find the blobs in the image?

Multi-scale blob detection in images

- Find the local extrema (minima/maxima) by looking at all neighbors across scales.



Multi-scale blob detection in images



Questions?

Reading material next lecture:

- **Article**:
  - *Distinctive Image Features from Scale-Invariant Keypoints*, David G. Lowe, IJCV 2024.
  - https://www.cs.ubc.ca/~lowe/papers/ijcv04.pdf

- Section 7.1.2 Feature descriptors, in Szeliski book

- Video lecture Mubarak Shah: https://www.youtube.com/watch?v=NPcMS49V5hg