Computer Vision

Gabor filters

Silvia Pintea

Gabor filters -- reading material

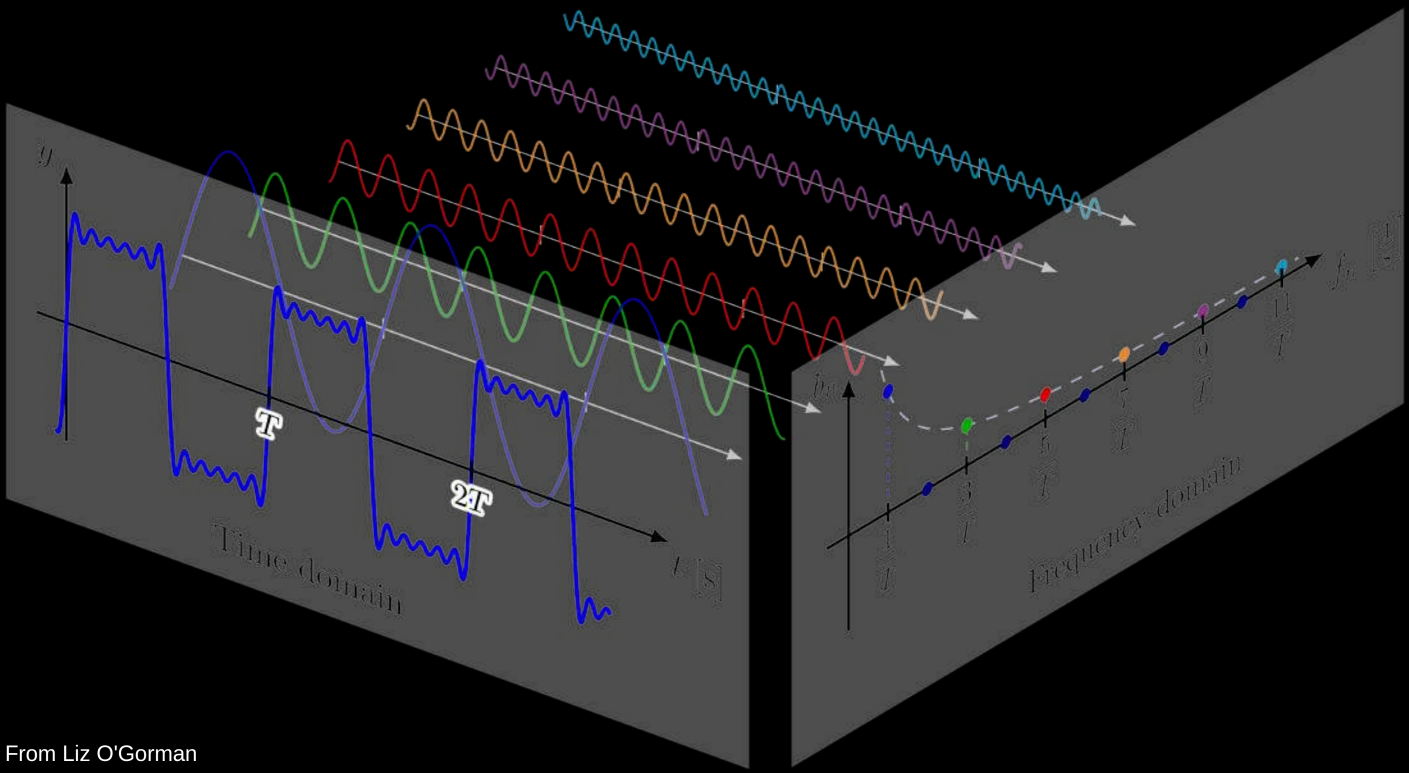
- **Article**:
  - Chengjun Liu and Harry Wechsler, "A Gabor Feature Classifier for Face Recognition", ICCV (2001)
  - https://web.archive.org/web/20060903125211id_/http://www.cs.njit.edu/liu/papers/iccv01.pdf
- Section 3.5.4 Wavelets from the Szeliski book
- Jupyter notebook: Test the effect of the different Gabor parameters both in image and Fourier domains.

Recap

What does Fourier transform do?

Recap

What does Fourier transform do?

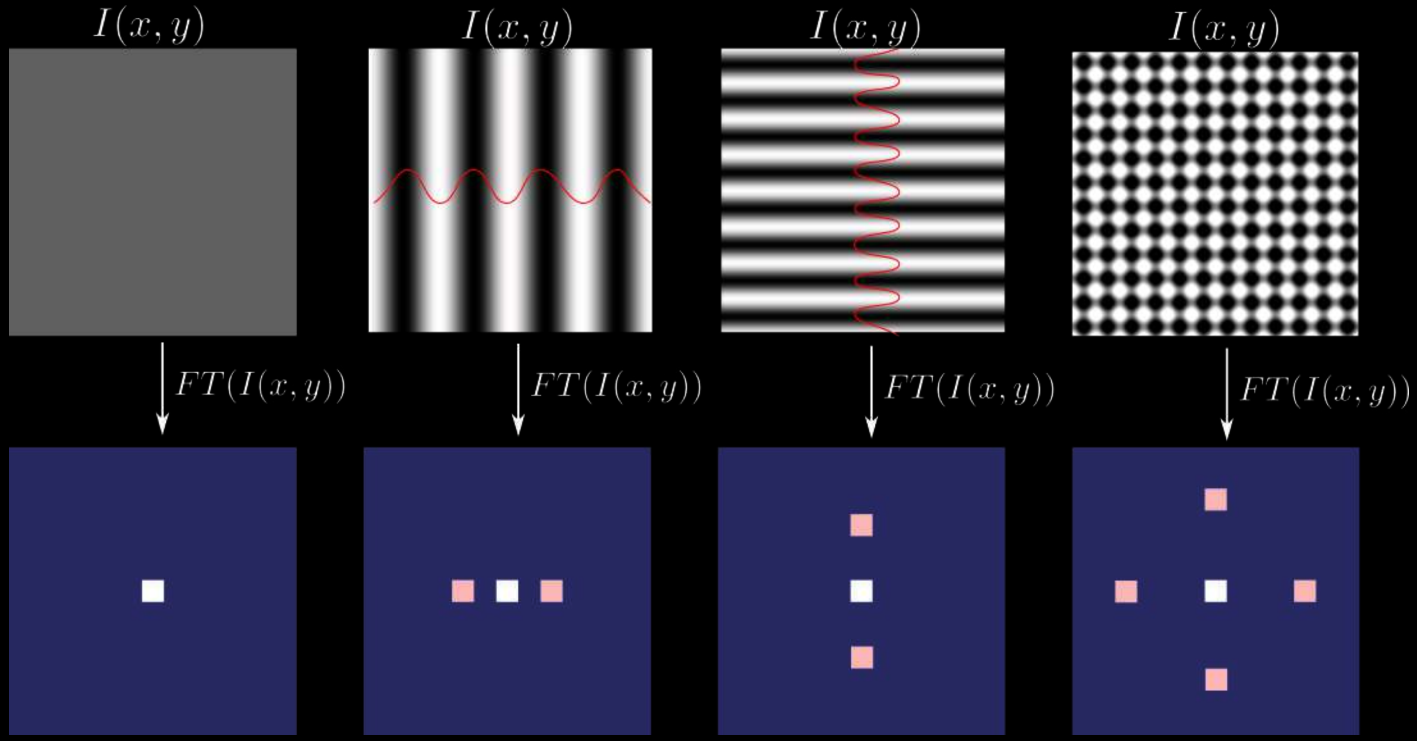


From Liz O'Gorman

Decomposes a signal (e.g. an image) into linear combination of frequency components.

Recap

Fourier on images: sines on x-direction and y-direction

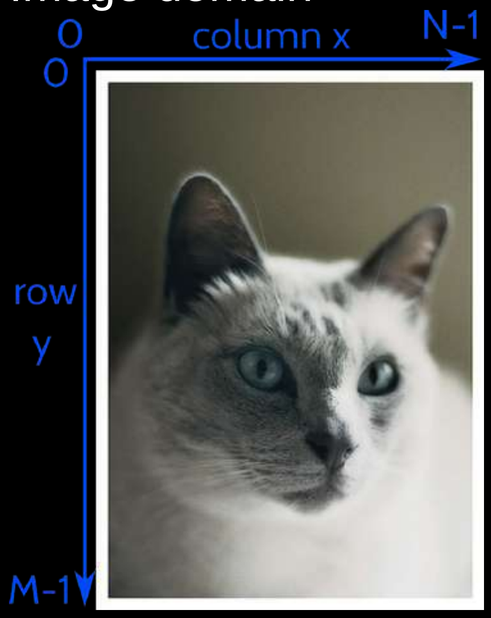


Recap

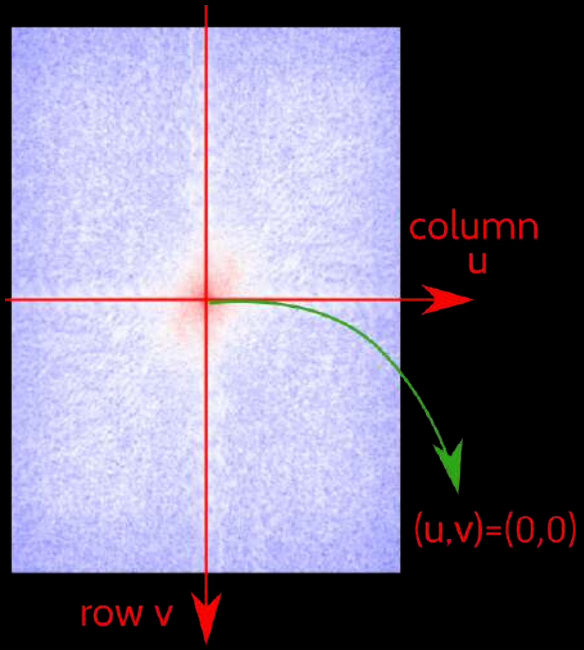
Fourier transform moves the image from the (x, y) coordinate domain to the (u, v) frequency domain.

- $FT(u, v) = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} I(x, y)\, e^{-i2\pi\left(\frac{ux}{N}+\frac{vy}{M}\right)}$

Image domain



Fourier domain



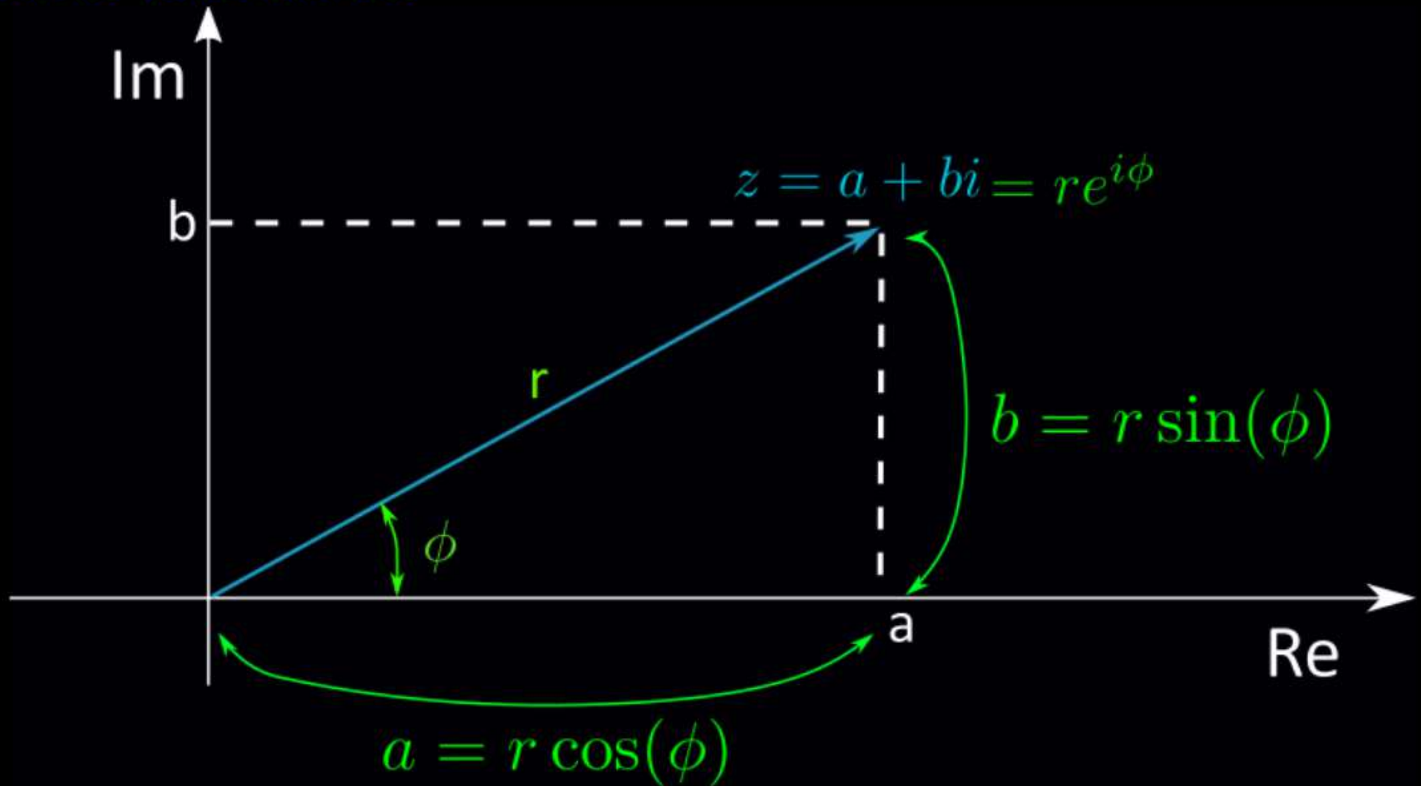
Recap

Fourier transform decomposes an image into linear combination of frequency components.

- $FT(u,v) = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} I(x,y)\, e^{-i2\pi\left(\frac{ux}{N}+\frac{vy}{M}\right)}$

- This $FT(u,v)$ is a complex number.
  It has a <u>real</u> and <u>imaginary</u> part.

Euler's identities



- $e^{i\phi} = \cos(\phi) + i\sin(\phi)$
- $e^{-i\phi} = \cos(\phi) - i\sin(\phi)$
- Euler's number (or Napier's constant): $e \approx 2.71828$

Recap

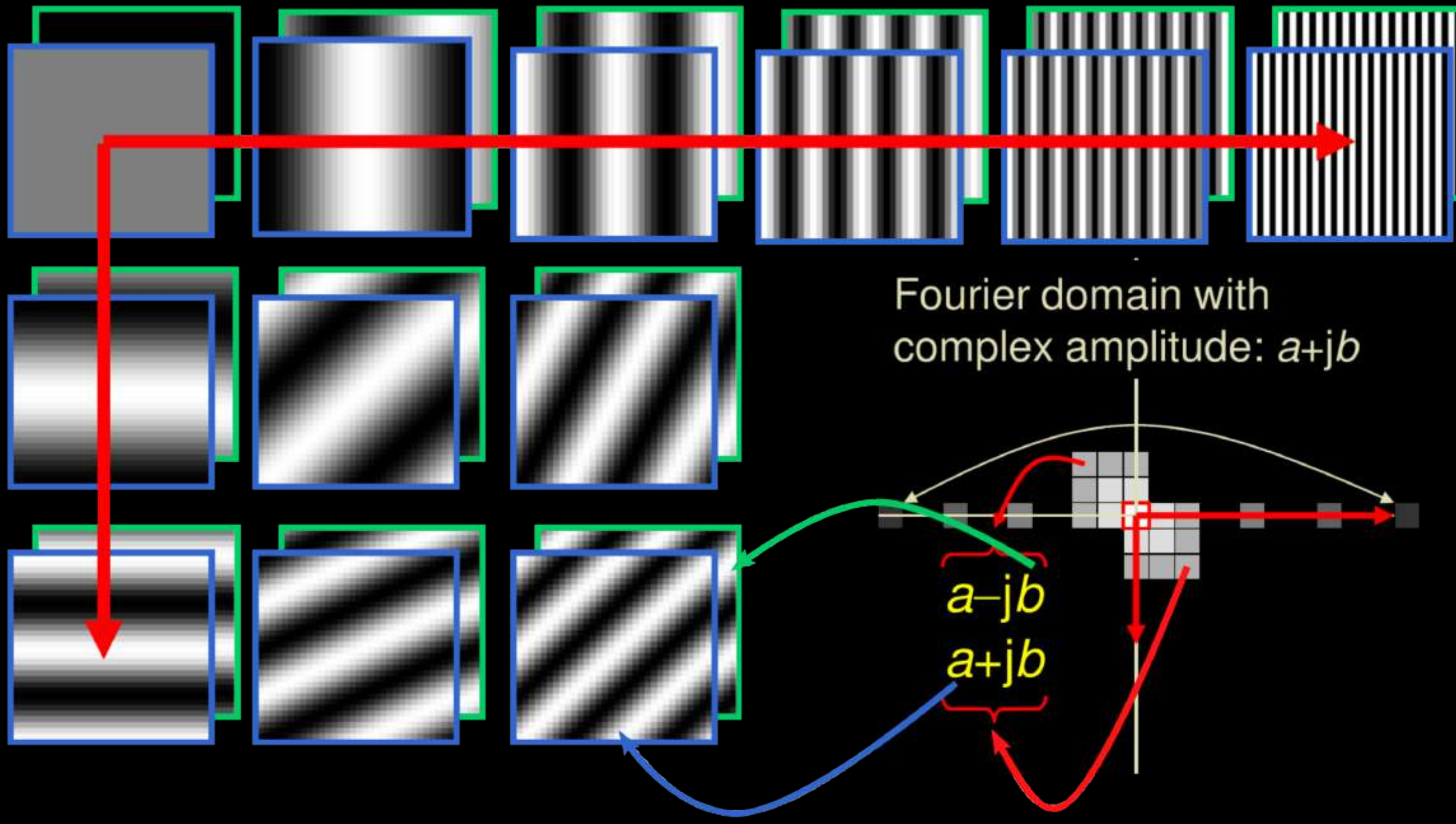
Fourier transform decomposes an image into linear combination of frequency components.

- $FT(u,v) = \sum_{x,y} I(x,y)\; e^{-i2\pi\left(\frac{ux}{N}+\frac{vy}{M}\right)}$
- This $FT(u,v)$ is a complex number.
  It has a <u>real</u> and <u>imaginary</u> part.

$$FT(u,v) = \sum_{x,y} I(x,y) \left(\cos\left(2\pi\left(\frac{ux}{N}+\frac{vy}{M}\right)\right) - i\sin\left(2\pi\left(\frac{ux}{N}+\frac{vy}{M}\right)\right)\right)$$

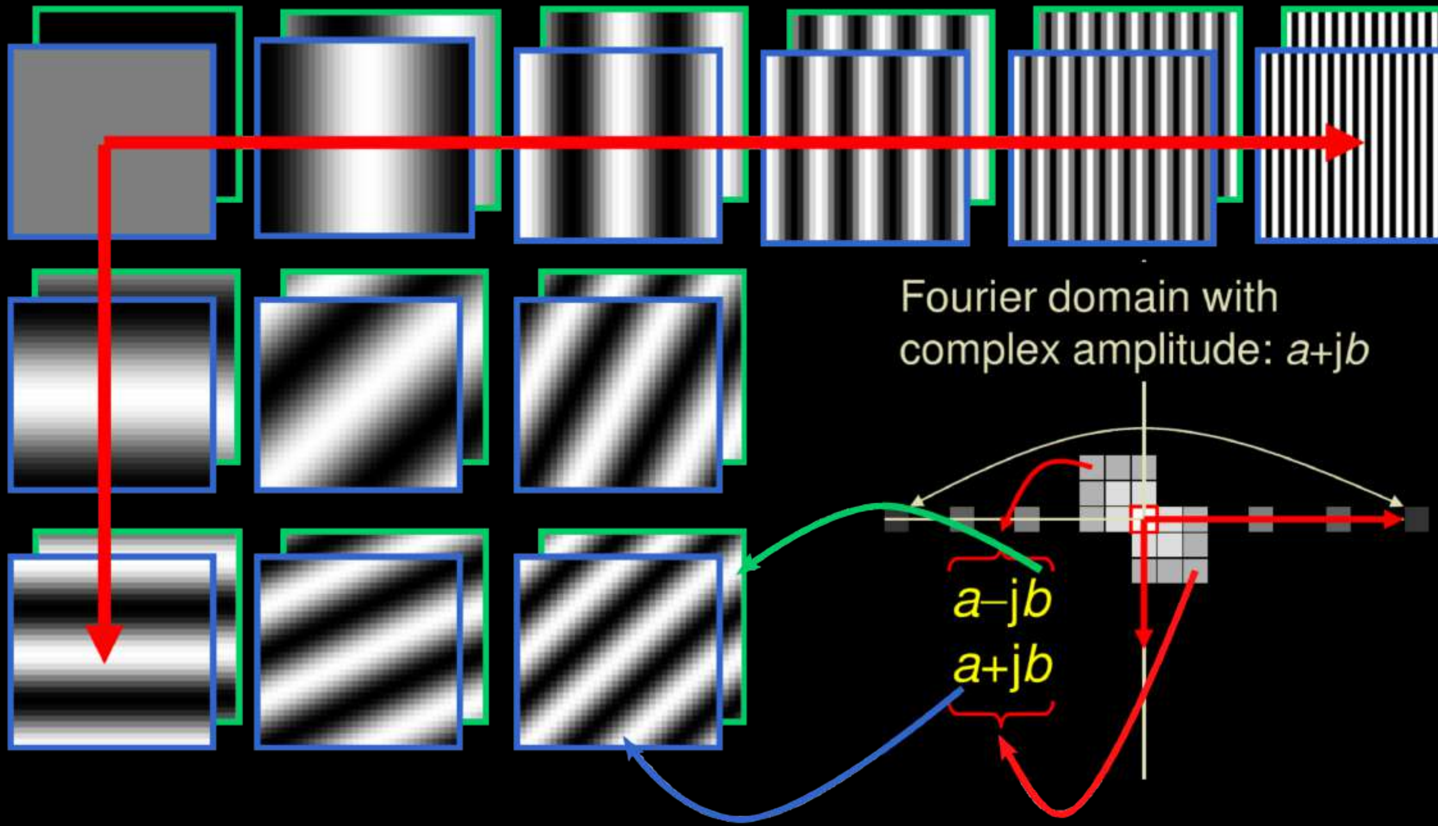
Recap

A weighted sum of cos (even) and sin (odd) images.



Recap

A weighted sum of cos (even) and sin (odd) images.



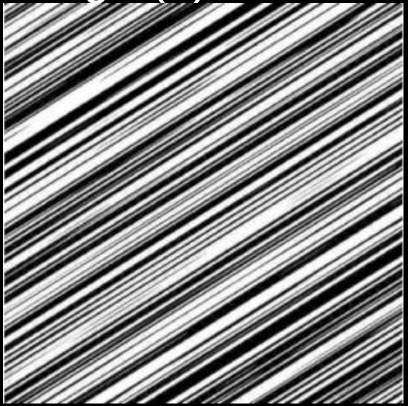
$$\cos\left(\frac{\pi}{2} - \phi\right) = \sin(\phi)$$

Recap

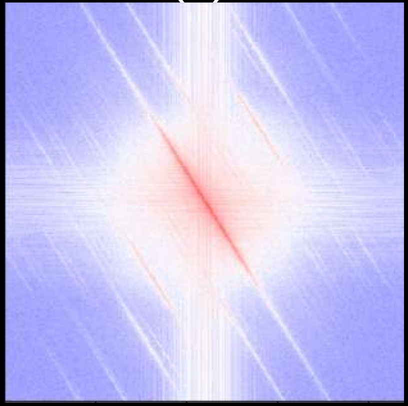
Image (1)

Image (2)

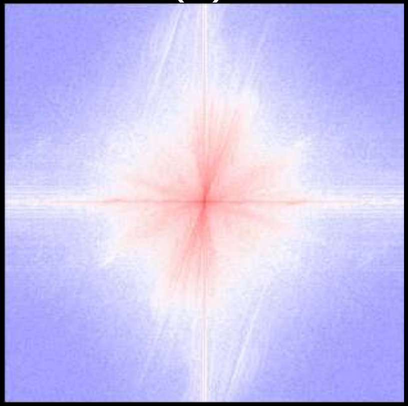
Image (3)



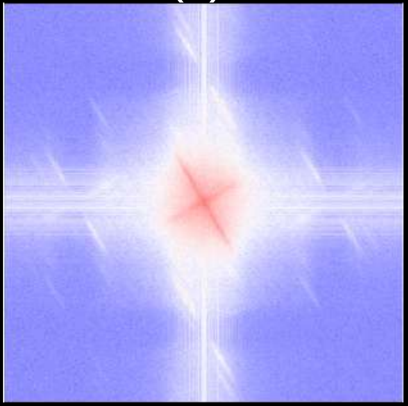
Fourier (a)



Fourier (b)



Fourier (c)

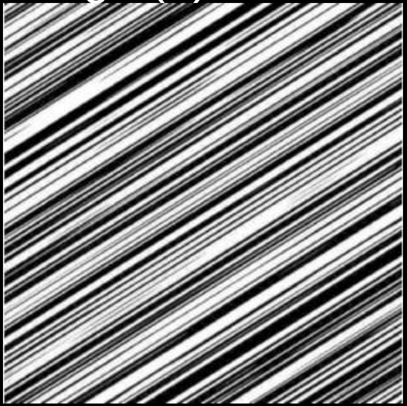


Recap

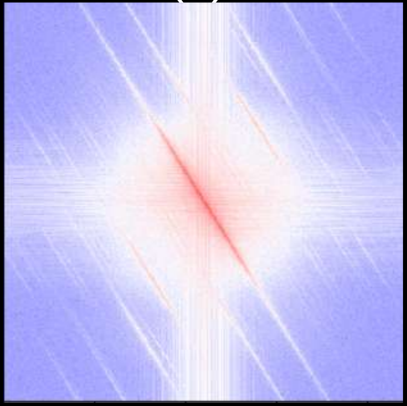
Image (1)

Image (2)

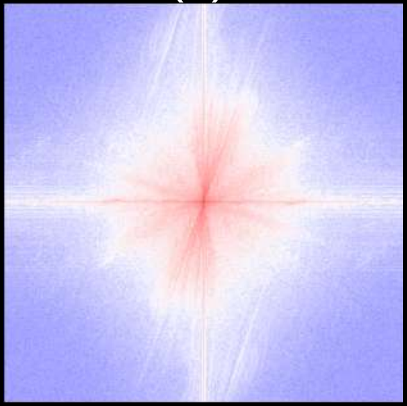
Image (3)



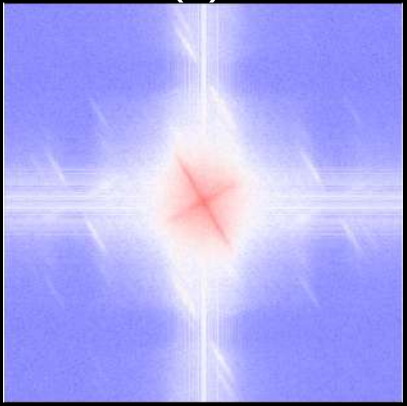
Fourier (a)



Fourier (b)

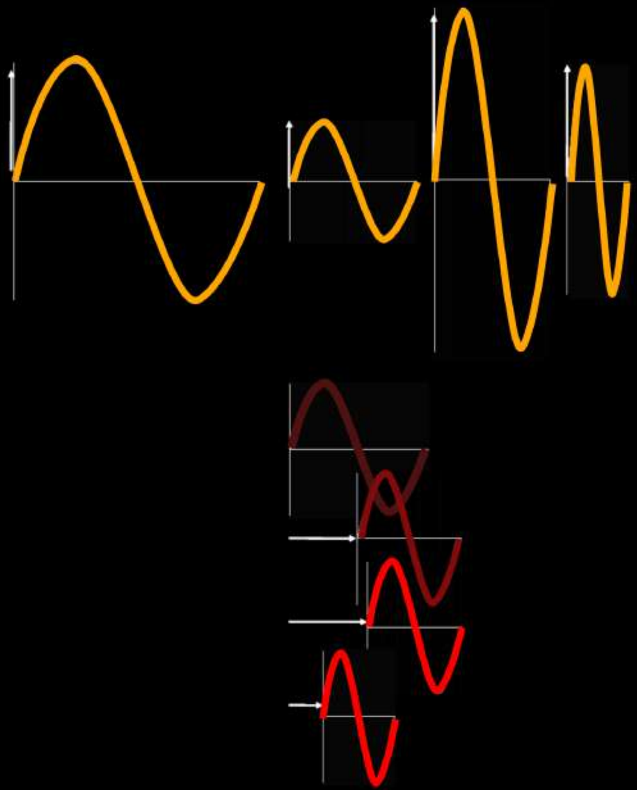


Fourier (c)



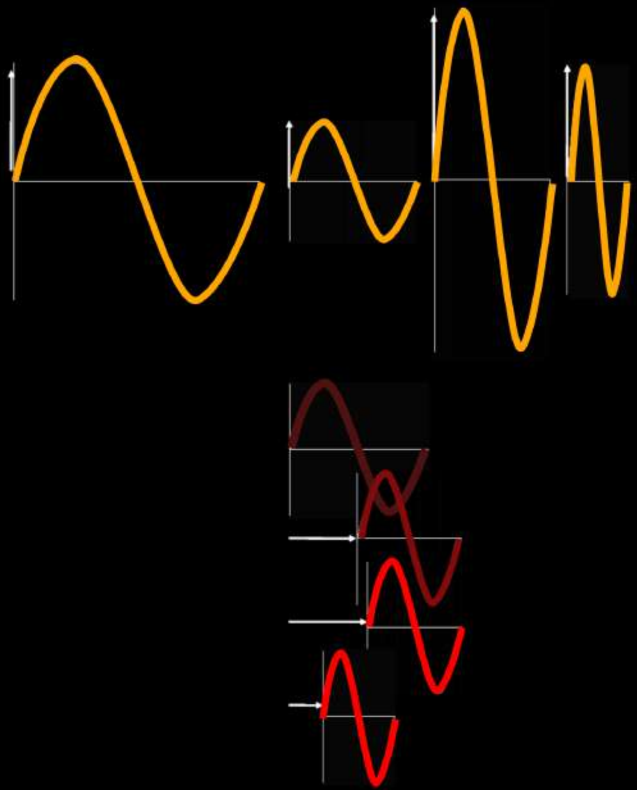
Answer: Image (1) - Fourier (c); Image (2) - Fourier (b); Image (3) - Fourier (a)

Magnitude and phase



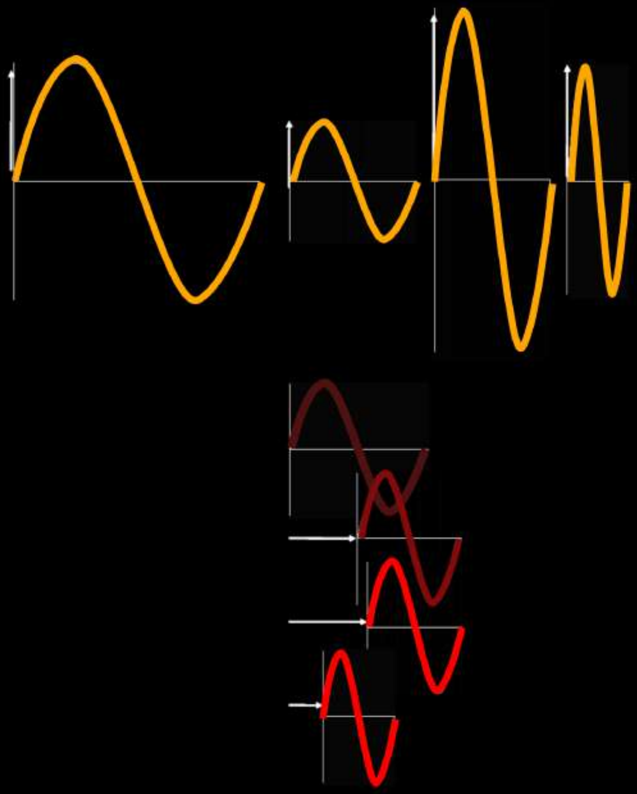
- What does the magnitude of Fourier transform encode?

Magnitude and phase



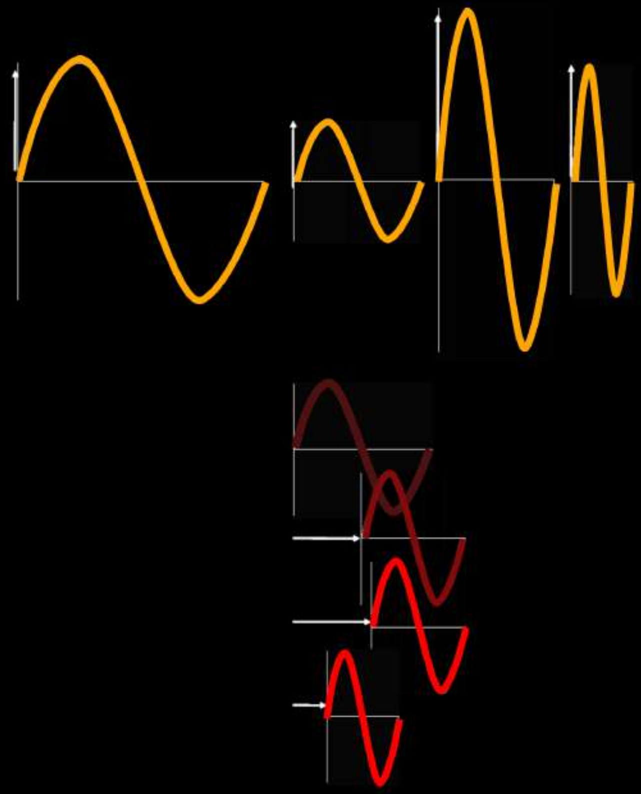
- What does the magnitude of Fourier transform encode?
  - How much each sinusoid <u>contributes</u> to form the whole image $I(x, y)$.

Magnitude and phase



- What does the magnitude of Fourier transform encode?
  - How much each sinusoid <u>contributes</u> to form the whole image $I(x, y)$.
- What does the phase of Fourier transform encode?

Magnitude and phase



- What does the magnitude of Fourier transform encode?
  - How much each sinusoid contributes to form the whole image $I(x, y)$.
- What does the phase of Fourier transform encode?
  - How each sinusoid lines up relative to one another to form the whole image $I(x, y)$

Magnitude and phase

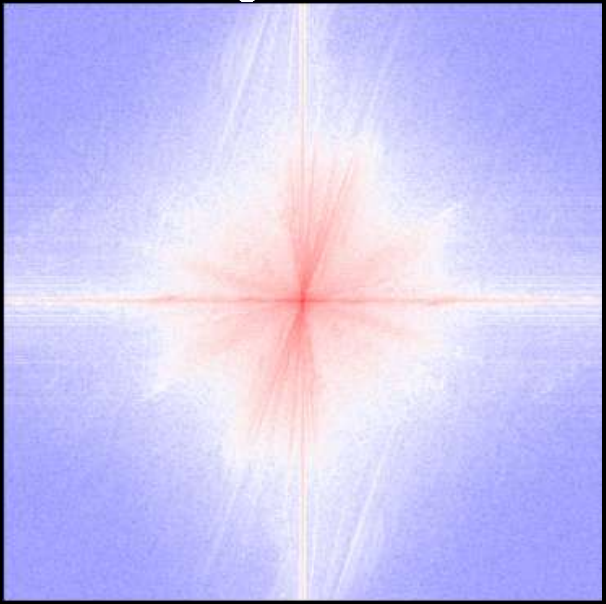
- How do we compute the magnitude (amplitude) of the Fourier transform?

Magnitude and phase

- How do we compute the magnitude (amplitude) of the Fourier transform?
- The absolute of Fourier transform: $\left| FT\left(I(x,y)\right) \right|$
  - For a complex number $z = a + bi$ this is $|z| = \sqrt{a^2 + b^2}$

Image

Fourier magnitude



Magnitude and phase

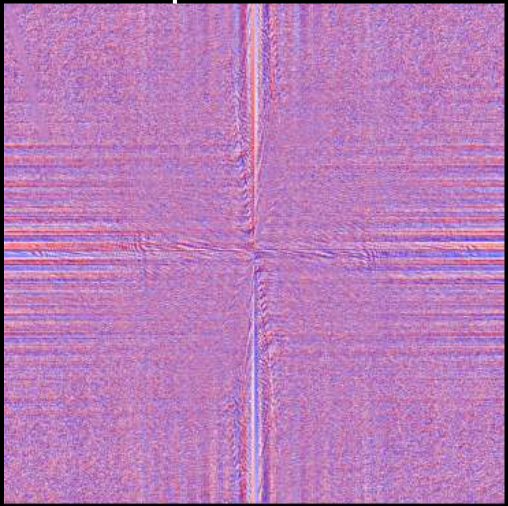
- What do we compute the phase of the Fourier transform?

Magnitude and phase

- What do we compute the phase of the Fourier transform?
- The angle of Fourier transform: $\phi = tan^{-1} \frac{Im\left(FT\left(I(x,y)\right)\right)}{Re\left(FT\left(I(x,y)\right)\right)}$
  - For a complex number $z = a+bi$ this is $\phi = tan^{-1} \frac{b}{a}$

Image

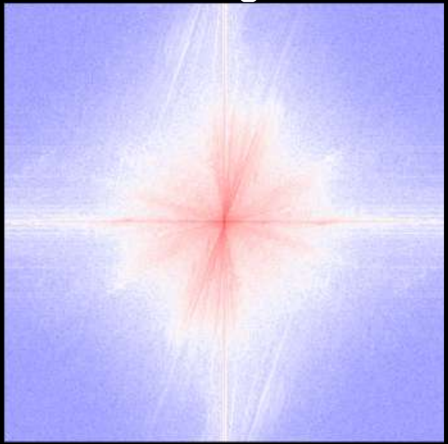
Fourier phase



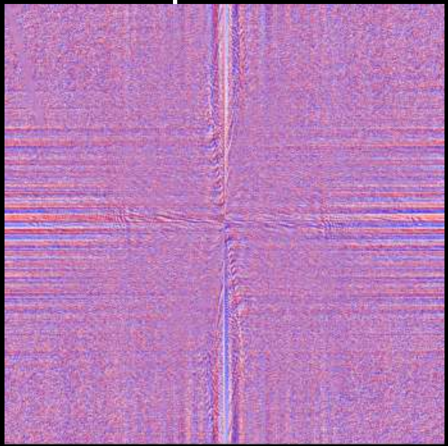
Spatial information

Image

Fourier magnitude



Fourier phase

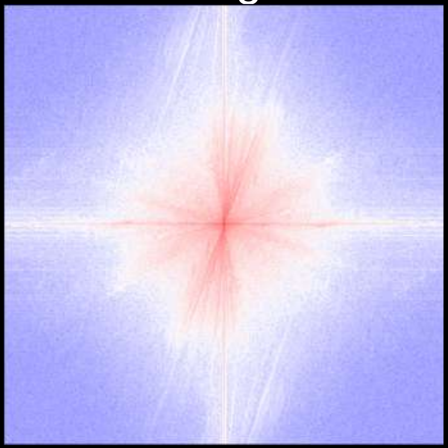


- Looking at the Fourier transform can we say something about the frequencies present in the top-right corner?

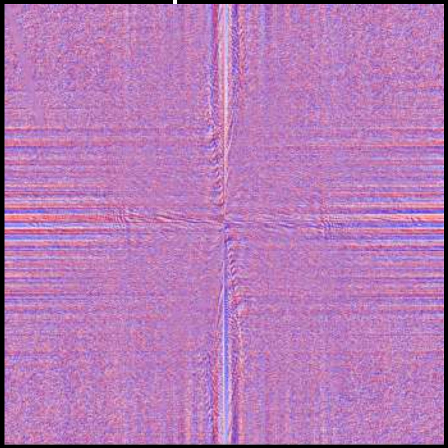
Spatial information

Image

Fourier magnitude

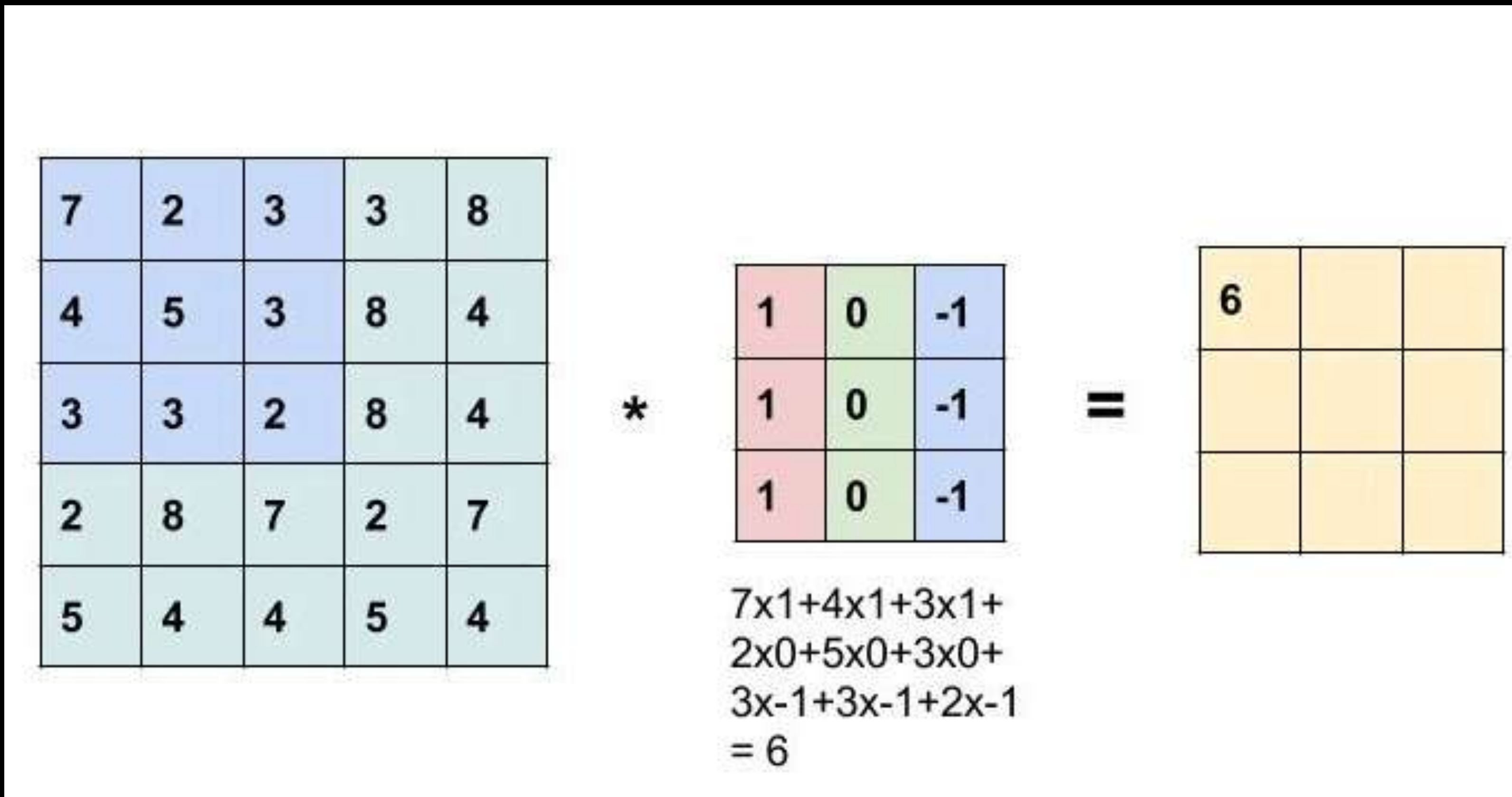


Fourier phase



- Looking at the Fourier transform can we say something about the frequencies present in the top-right corner?
- Fourier transform is a <u>global</u> transformation (in the formula there is a sum over all pixels)
- Do you know a local transformation?

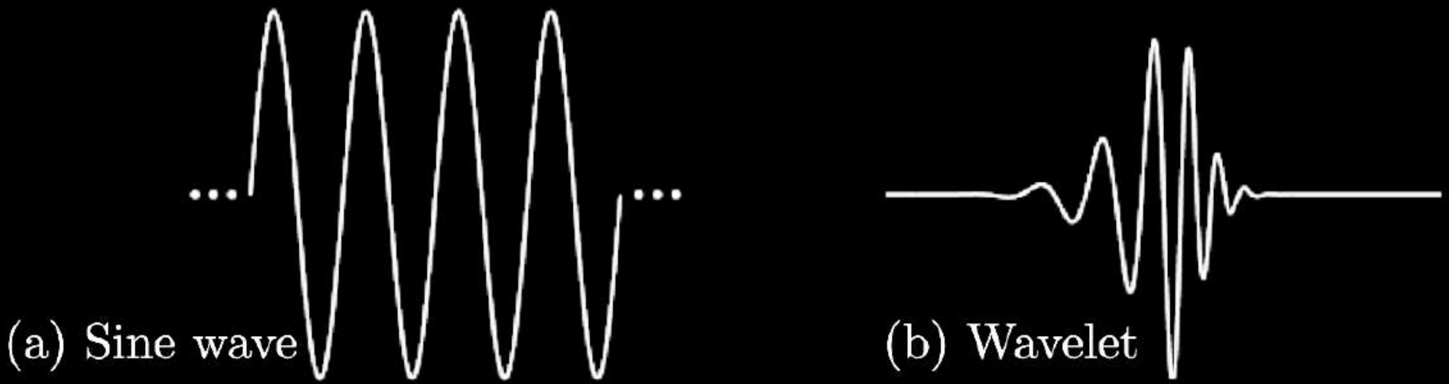
Convolution



- Convolution is local: the result is per pixel, not across all pixels.

Wavelets

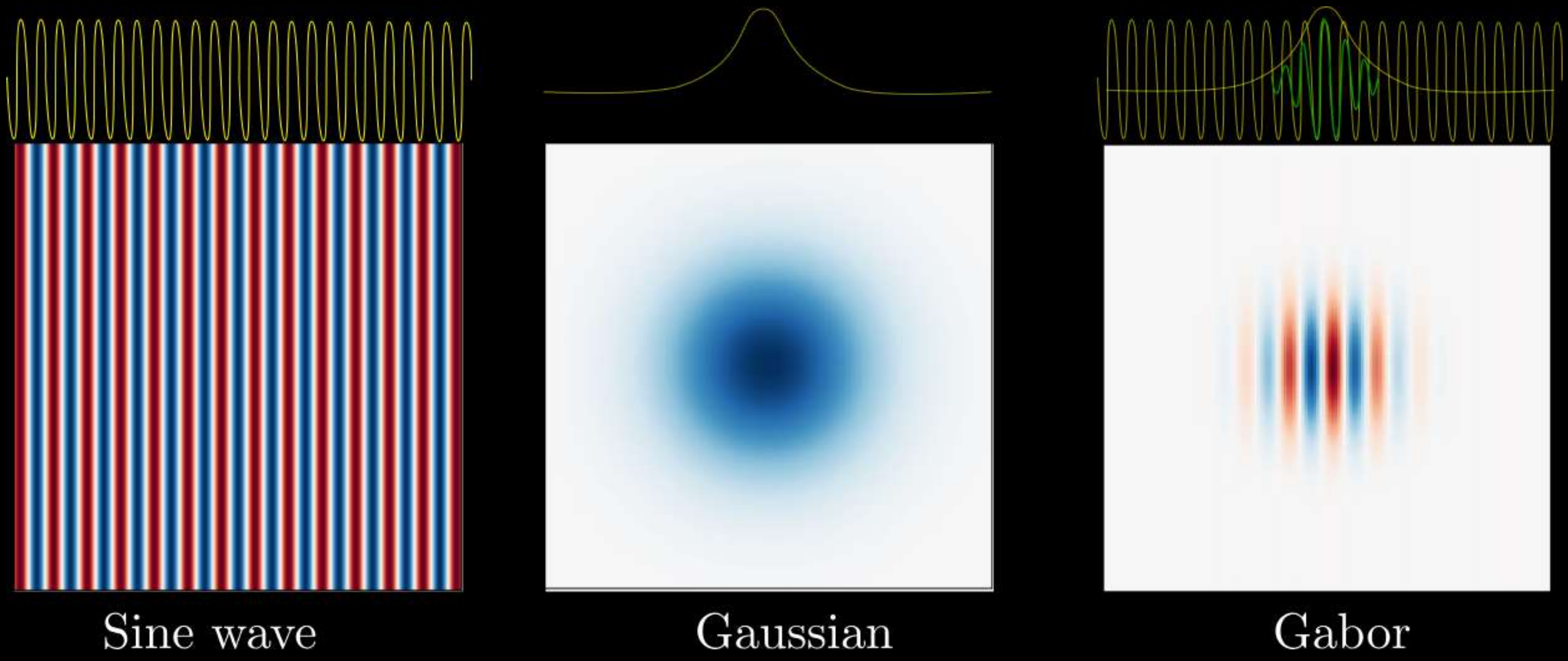
- Wavelet = "short wave"



- Gabor filters are wavelets

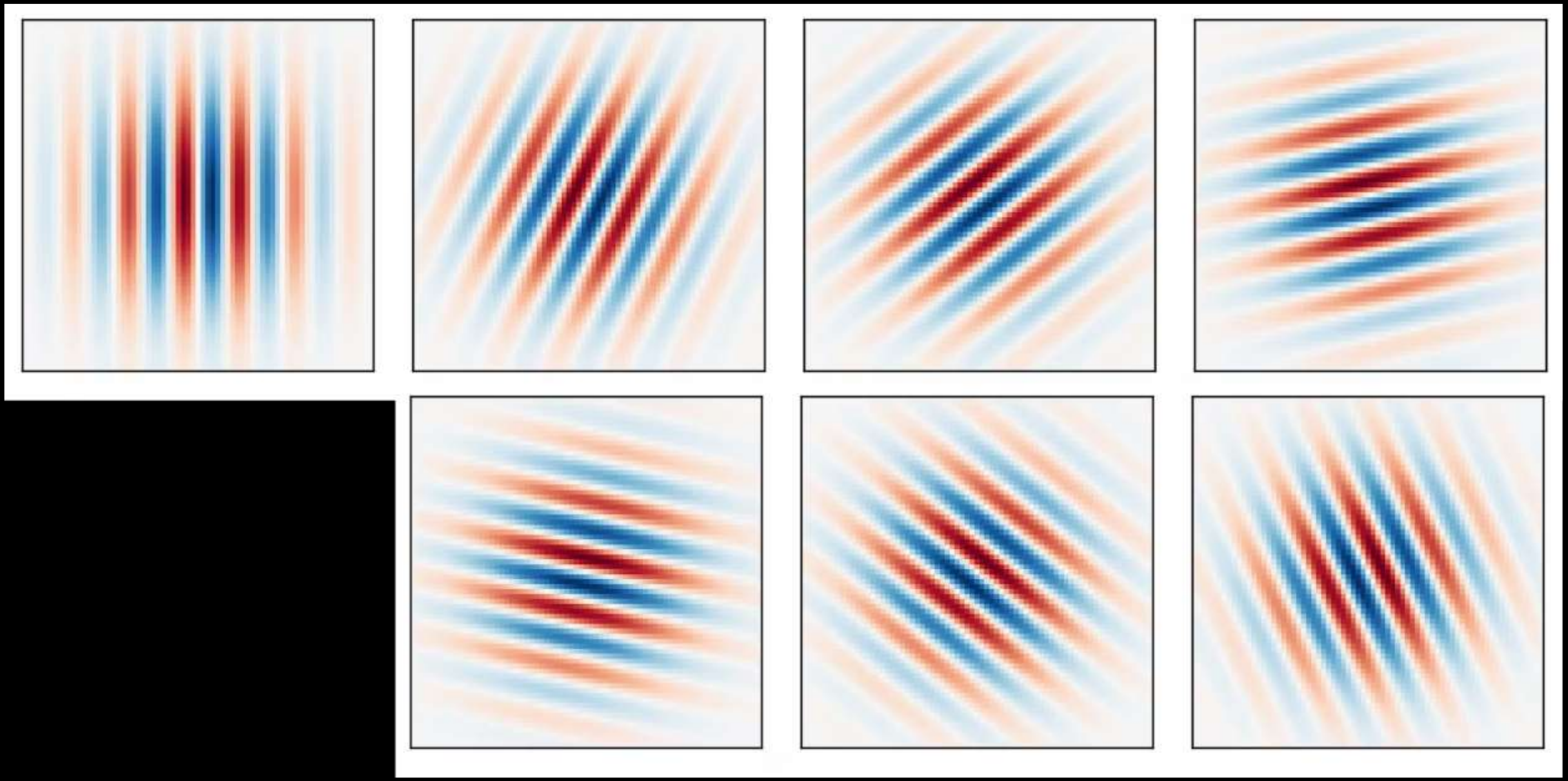
Gabor filters

- Gabor filters focus on **localized frequencies**



Gabor filters

- And also **localized orientations**



Gabor filters

- Formally a Gabor filter at location (x, y) is
  - $G_{\lambda,\theta,\phi,\sigma,\gamma}(x, y) = e^{\left(-\frac{\bar{x}^2+\gamma^2\bar{y}^2}{2\sigma^2}\right)} e^{i\left(\frac{2\pi+\bar{x}}{\lambda}+\phi\right)}$
  - with $\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

- What does the green part look like?

Gabor filters

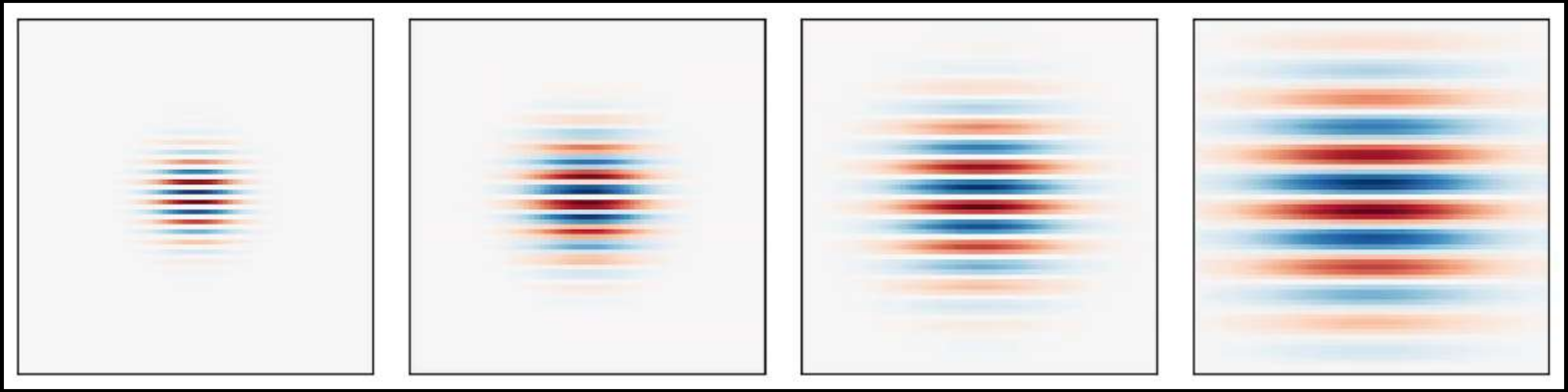
- Formally a Gabor filter at location (x, y) is
  - $G_{\lambda,\theta,\phi,\sigma,\gamma}(x, y) = e^{\left(-\frac{\bar{x}^2+\gamma^2\bar{y}^2}{2\sigma^2}\right)} e^{i\left(\frac{2\pi+\bar{x}}{\lambda}+\phi\right)}$
  - with $\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

Gabor filters

- Formally a Gabor filter at location (x, y) is
  - $G_{\lambda,\theta,\phi,\sigma,\gamma}(x, y) = e^{\left(-\frac{\bar{x}^2+\gamma^2\bar{y}^2}{2\sigma^2}\right)} e^{i\left(\frac{2\pi+\bar{x}}{\lambda}+\phi\right)}$
  - with $\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$
- σ -

Gabor filters

- Formally a Gabor filter at location (x, y) is
  - $G_{\lambda,\theta,\phi,\sigma,\gamma}(x, y) = e^{\left(-\frac{\bar{x}^2+\gamma^2\bar{y}^2}{2\sigma^2}\right)} e^{i\left(\frac{2\pi+\bar{x}}{\lambda}+\phi\right)}$
  - with $\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$
- σ - the scale given by the Gaussian envelope



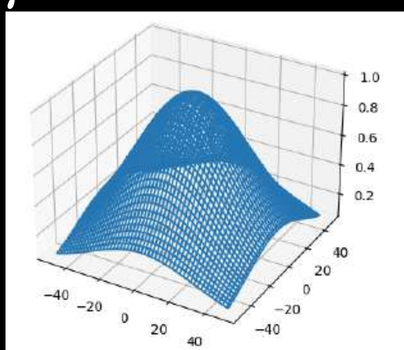
Gabor filters

- Formally a Gabor filter at location (x, y) is
  - $G_{\lambda,\theta,\phi,\sigma,\gamma}(x, y) = e^{\left(-\frac{\bar{x}^2+\gamma^2\bar{y}^2}{2\sigma^2}\right)} e^{i\left(\frac{2\pi+\bar{x}}{\lambda}+\phi\right)}$
  - with $\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$
- γ -

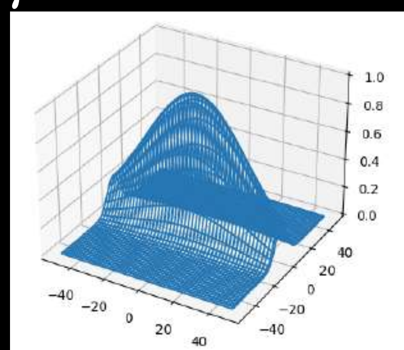
Gabor filters

- Formally a Gabor filter at location (x, y) is
  - $G_{\lambda,\theta,\phi,\sigma,\gamma}(x, y) = e^{\left(-\frac{\bar{x}^2+\gamma^2\bar{y}^2}{2\sigma^2}\right)} e^{i\left(\frac{2\pi+\bar{x}}{\lambda}+\phi\right)}$
  - with $\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$
- γ - the ratio of the Gaussian envelope

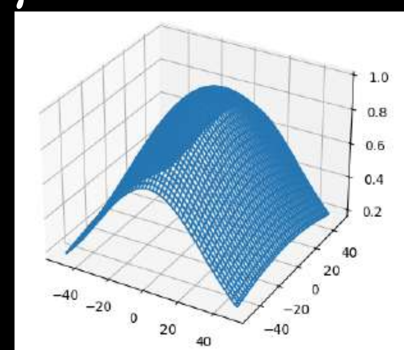
$\gamma = 1$



$\gamma = 5$



$\gamma = 0.5$



Gabor filters

- Formally a Gabor filter at location (x, y) is
  - $G_{\lambda,\theta,\phi,\sigma,\gamma}(x, y) = e^{\left(-\frac{\bar{x}^2+\gamma^2\bar{y}^2}{2\sigma^2}\right)} e^{i\left(\frac{2\pi+\bar{x}}{\lambda}+\phi\right)}$
  - with $\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

- What does the violet part look like?

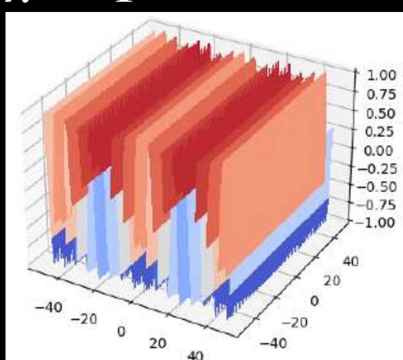
Gabor filters

- Formally a Gabor filter at location (x, y) is
  - $G_{\lambda,\theta,\phi,\sigma,\gamma}(x, y) = e^{\left(-\frac{\bar{x}^2+\gamma^2\bar{y}^2}{2\sigma^2}\right)} e^{i\left(\frac{2\pi+\bar{x}}{\lambda}+\phi\right)}$
  - with $\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$
- λ -

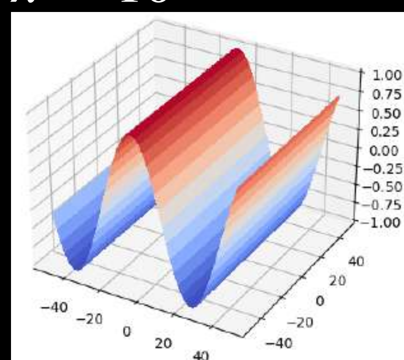
Gabor filters

- Formally a Gabor filter at location (x, y) is
  - $G_{\lambda,\theta,\phi,\sigma,\gamma}(x, y) = e^{\left(-\frac{\bar{x}^2+\gamma^2\bar{y}^2}{2\sigma^2}\right)} e^{i\left(\frac{2\pi+\bar{x}}{\lambda}+\phi\right)}$
  - with $\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$
- λ - the wavelength of the sine wave

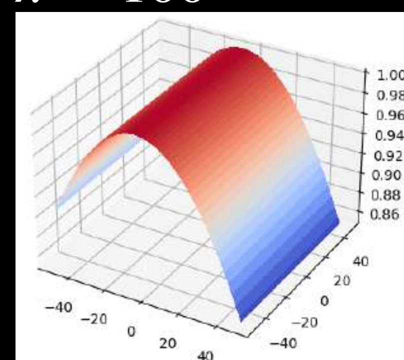
$\lambda = 1$



$\lambda = 10$



$\lambda = 100$



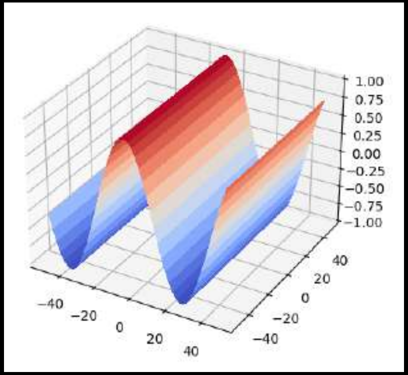
Gabor filters

- Formally a Gabor filter at location (x, y) is
  - $G_{\lambda,\theta,\phi,\sigma,\gamma}(x, y) = e^{\left(-\frac{\bar{x}^2+\gamma^2\bar{y}^2}{2\sigma^2}\right)} e^{i\left(\frac{2\pi+\bar{x}}{\lambda}+\phi\right)}$
  - with $\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$
- ϕ -

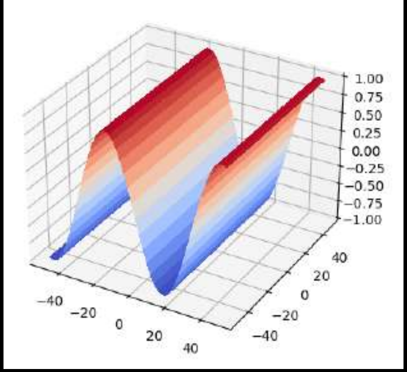
Gabor filters

- Formally a Gabor filter at location (x, y) is
  - $G_{\lambda,\theta,\phi,\sigma,\gamma}(x, y) = e^{\left(-\frac{\bar{x}^2+\gamma^2\bar{y}^2}{2\sigma^2}\right)} e^{i\left(\frac{2\pi+\bar{x}}{\lambda}+\phi\right)}$
  - with $\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$
- ϕ - the phase offset

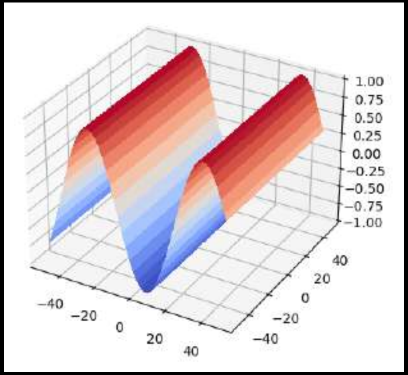
$\phi = 0.$



$\phi = 1.$



$\phi = 2.$



Gabor filters

- Formally a Gabor filter at location (x, y) is
  - $G_{\lambda,\theta,\phi,\sigma,\gamma}(x, y) = e^{\left(-\frac{\bar{x}^2+\gamma^2\bar{y}^2}{2\sigma^2}\right)} e^{i\left(\frac{2\pi+\bar{x}}{\lambda}+\phi\right)}$
  - with $\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

- What does the yellow part look like?

Gabor filters

- Formally a Gabor filter at location (x, y) is
    - $G_{\lambda,\theta,\phi,\sigma,\gamma}(x, y) = e^{\left(-\frac{\bar{x}^2+\gamma^2\bar{y}^2}{2\sigma^2}\right)} e^{i\left(\frac{2\pi+\bar{x}}{\lambda}+\phi\right)}$
    - with $\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$
- θ -

Gabor filters

- Formally a Gabor filter at location (x, y) is
  - $G_{\lambda,\theta,\phi,\sigma,\gamma}(x, y) = e^{\left(-\frac{\bar{x}^2+\gamma^2\bar{y}^2}{2\sigma^2}\right)} e^{i\left(\frac{2\pi+\bar{x}}{\lambda}+\phi\right)}$
  - with $\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$
- θ - the orientation of the Gabor filter

Gabor filters

In which data domain are the Gabor filters: e.g. $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}$, etc.?

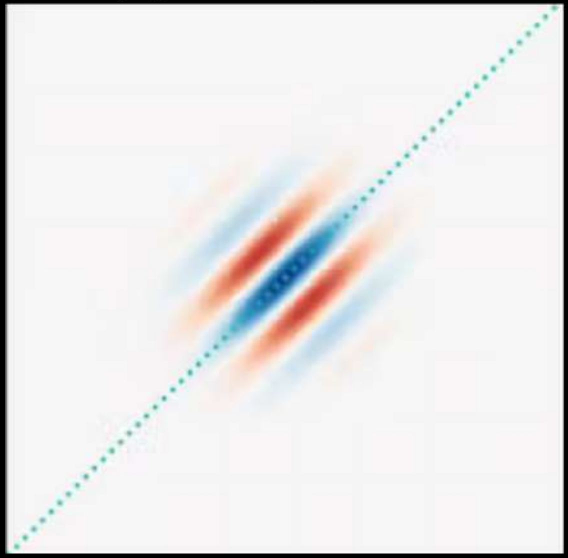
Gabor filters

In which data domain are the Gabor filters: e.g. $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}$, etc.?

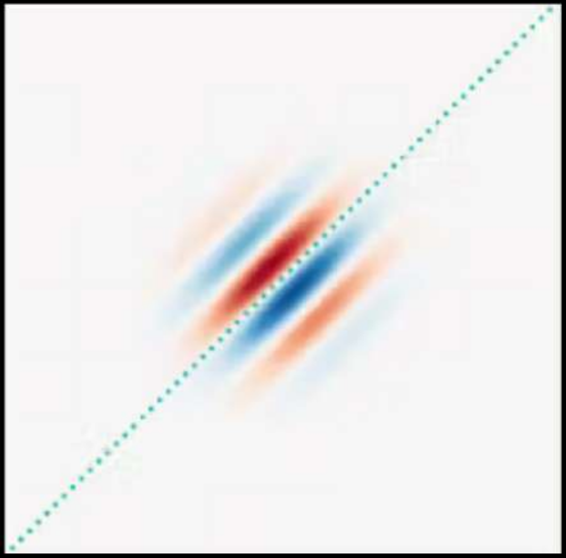
The Gabor filter is a complex filter:

- (a) the real part is **even-symmetric** and responds to lines;
- (b) the imaginary part is **odd-symmetric** and responds to edges.

(a) Real part



(a) Imaginary part

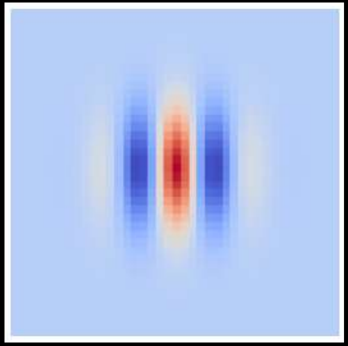


Questions?

Fourier of Gabor filters

- What do we expect if we take the Fourier transform of the real part of a Gabor filter?

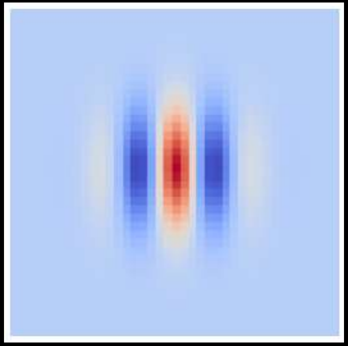
$\theta = 0°$



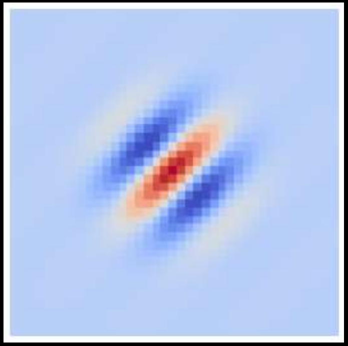
Fourier of Gabor filters

- What do we expect if we take the Fourier transform of the real part of a Gabor filter?

$\theta = 0^\circ$



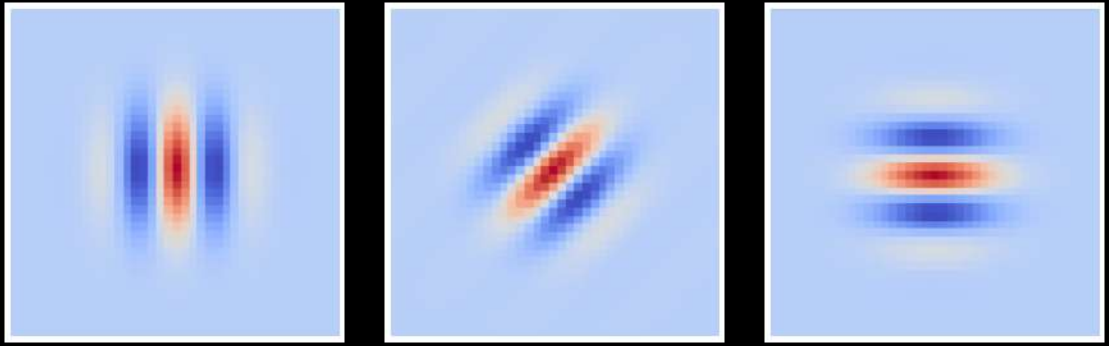
$\theta = 45^\circ$



Fourier of Gabor filters

- What do we expect if we take the Fourier transform of the real part of a Gabor filter?

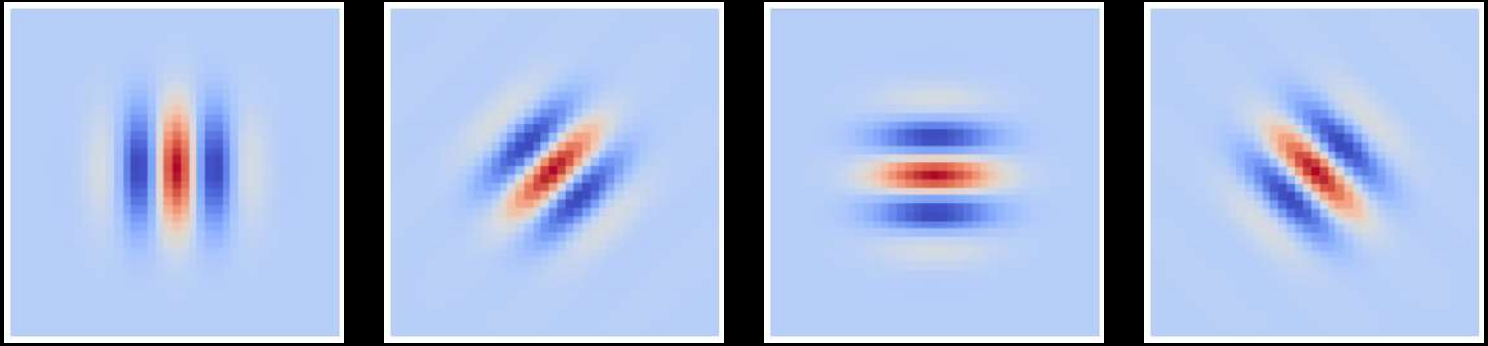
$\theta = 0°$ $\theta = 45°$ $\theta = 90°$



Fourier of Gabor filters

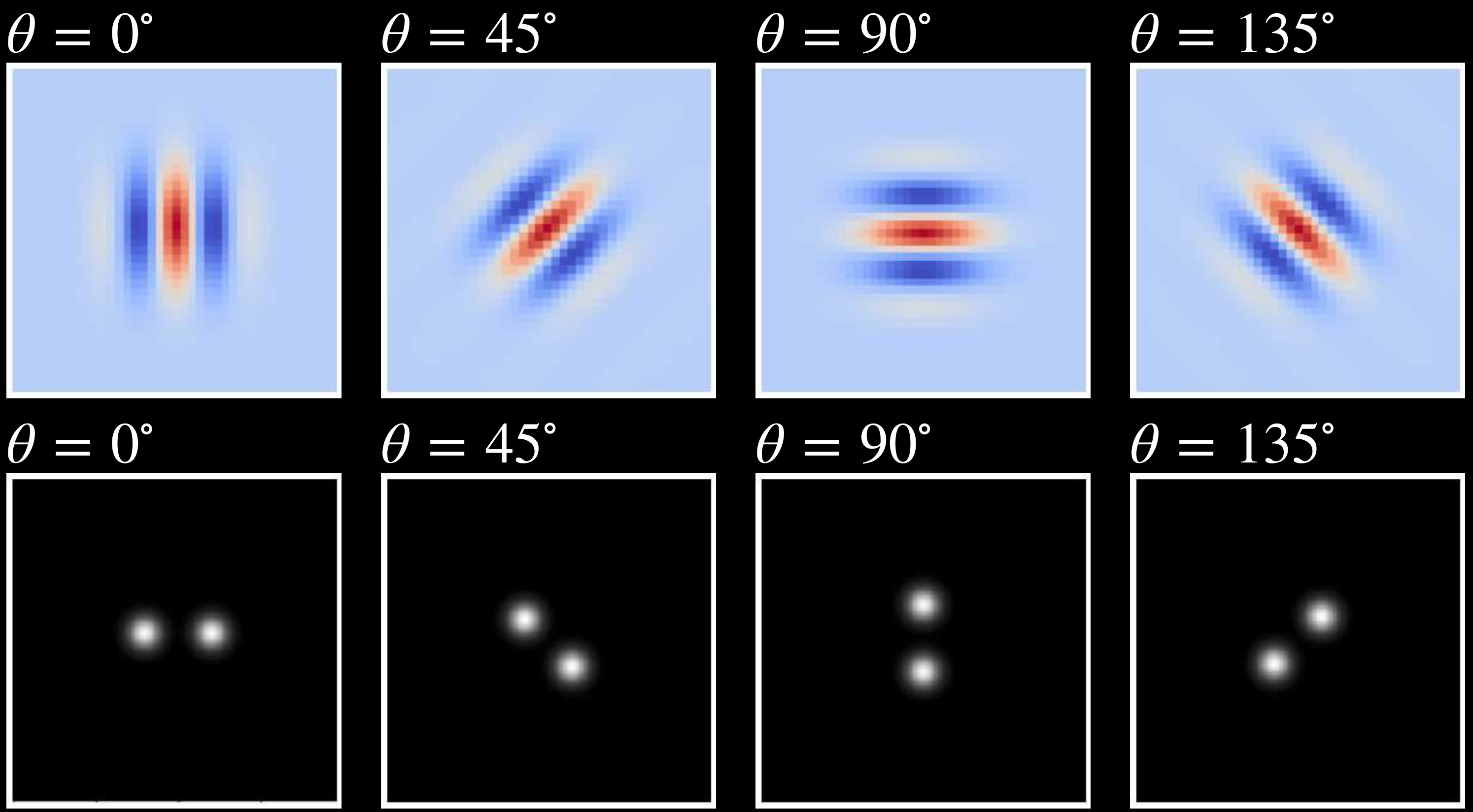
- What do we expect if we take the Fourier transform of the real part of a Gabor filter?

$\theta = 0^\circ$ $\theta = 45^\circ$ $\theta = 90^\circ$ $\theta = 135^\circ$



Fourier of Gabor filters

- What do we expect if we take the Fourier transform of the real part of a Gabor filter?

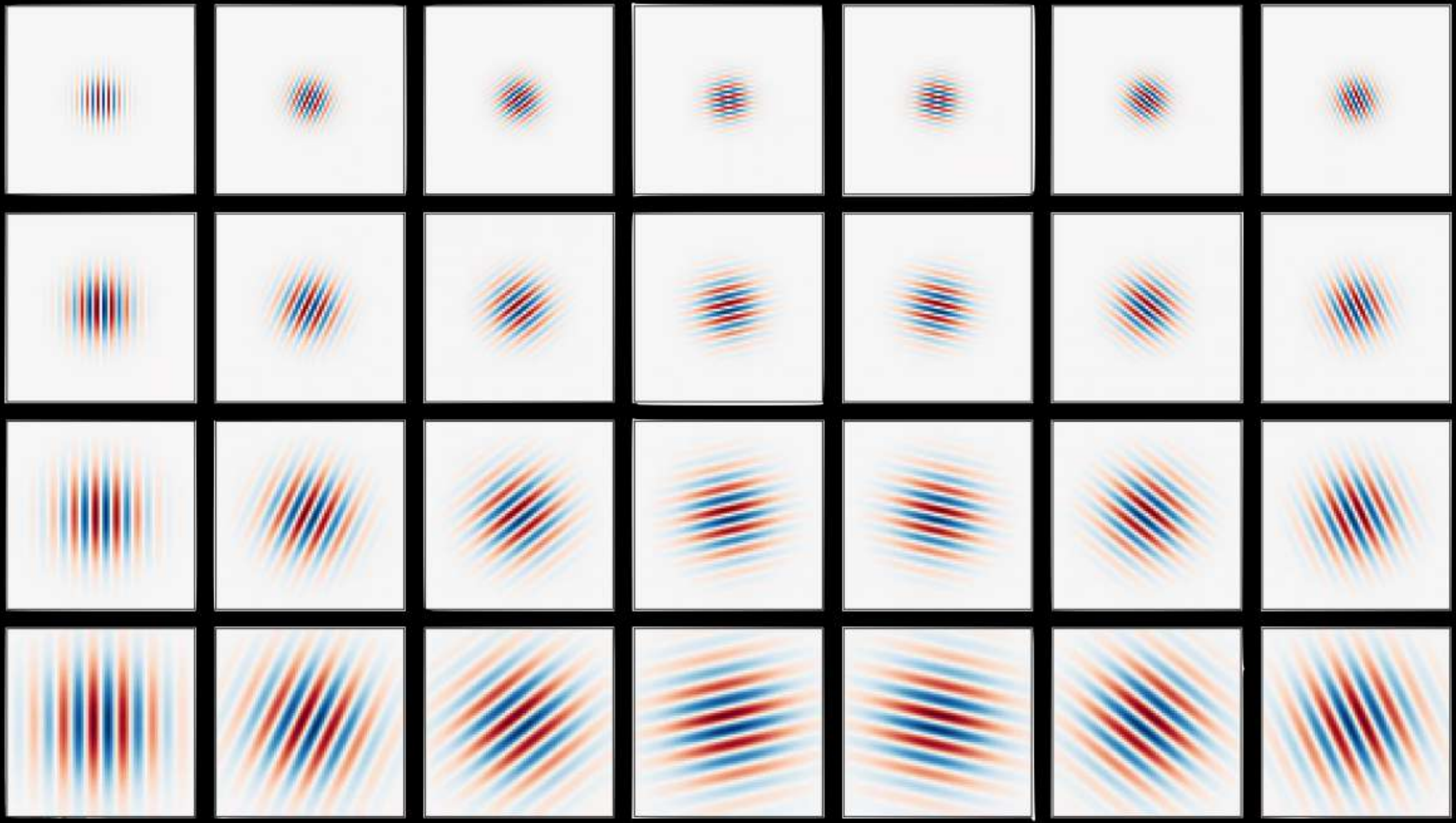


Multi-scale & orientation Gabors: Filter banks

- Gabor filter responses can be used as a *"feature"* to describe an image locally
- Which scale and which orientation should we pick?

Multi-scale & orientation Gabors: Filter banks

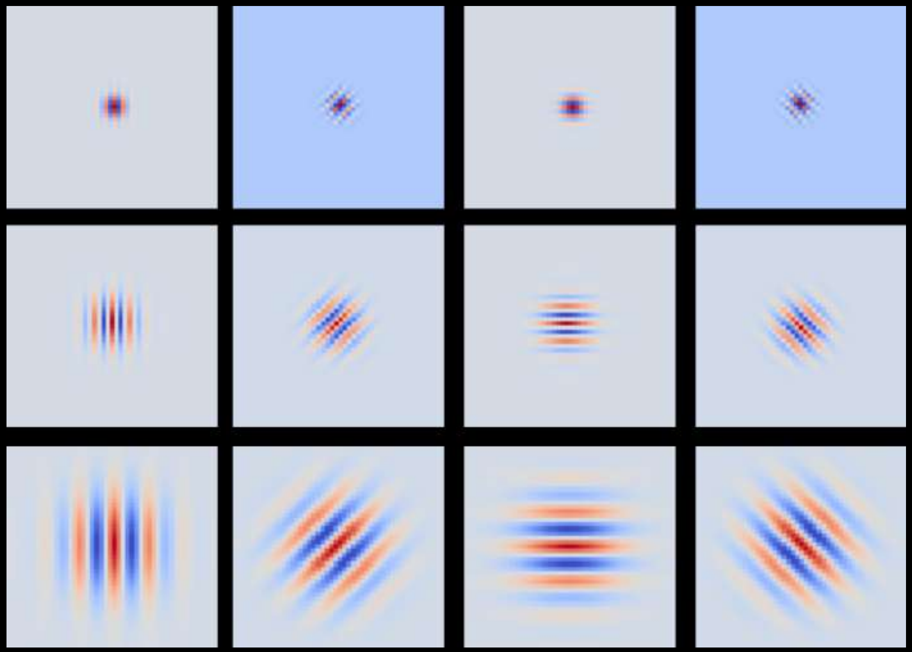
- Gabor filter responses can be used as a *"feature"* to describe an image locally
- Which scale and which orientation should we pick?



Fourier of Gabor filters

What does the sum of Fourier of this Gabor filter bank look like?

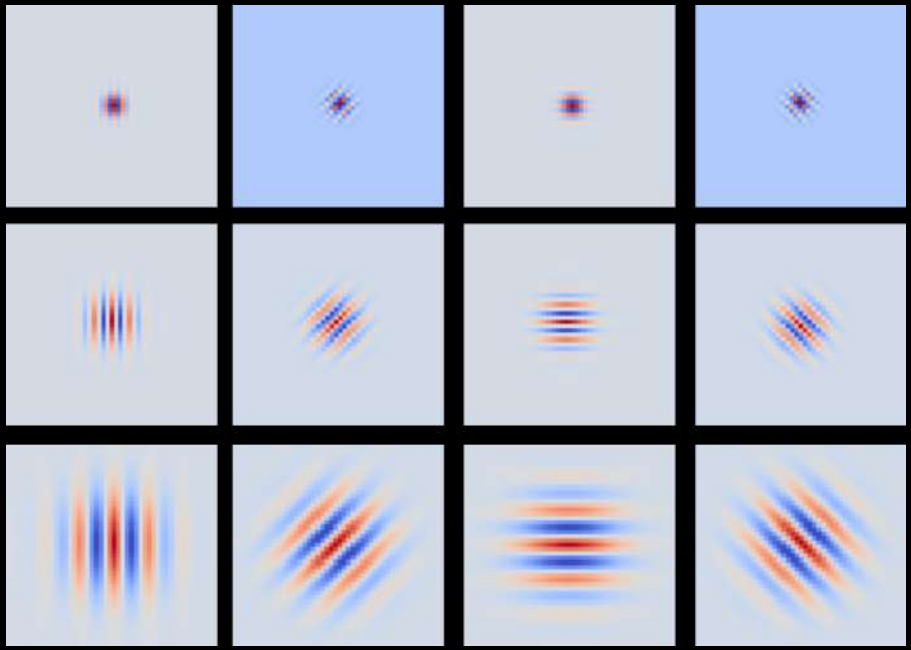
- $\sum_{\sigma,\theta} FT\left(G_{\lambda,\theta,\phi,\sigma,\gamma}(x,y)\right)$

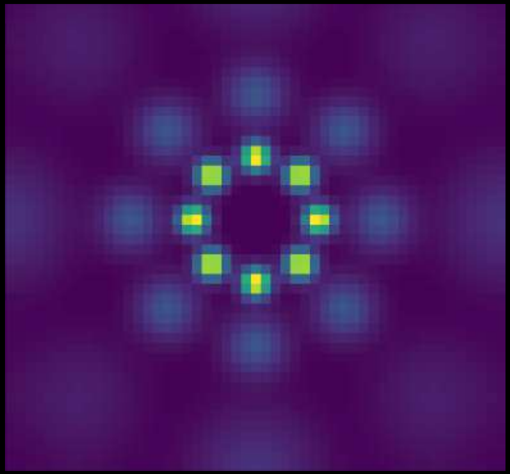


Fourier of Gabor filters

What does the sum of Fourier of this Gabor filter bank look like?

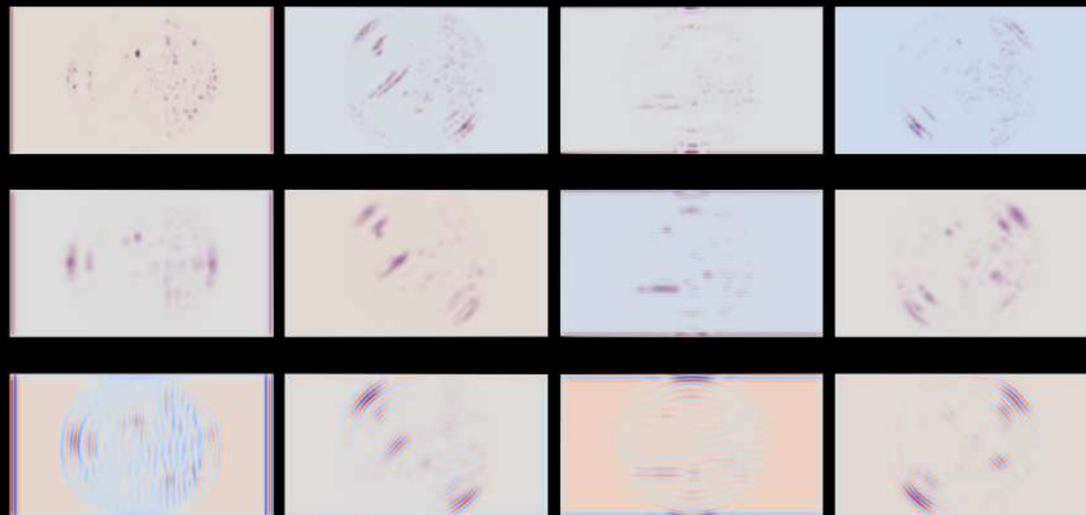
- $\sum_{\sigma,\theta} FT\left(G_{\lambda,\theta,\phi,\sigma,\gamma}(x, y)\right)$





Gabor features

- Gabor filter responses can be used as a *"feature"* to describe an image locally



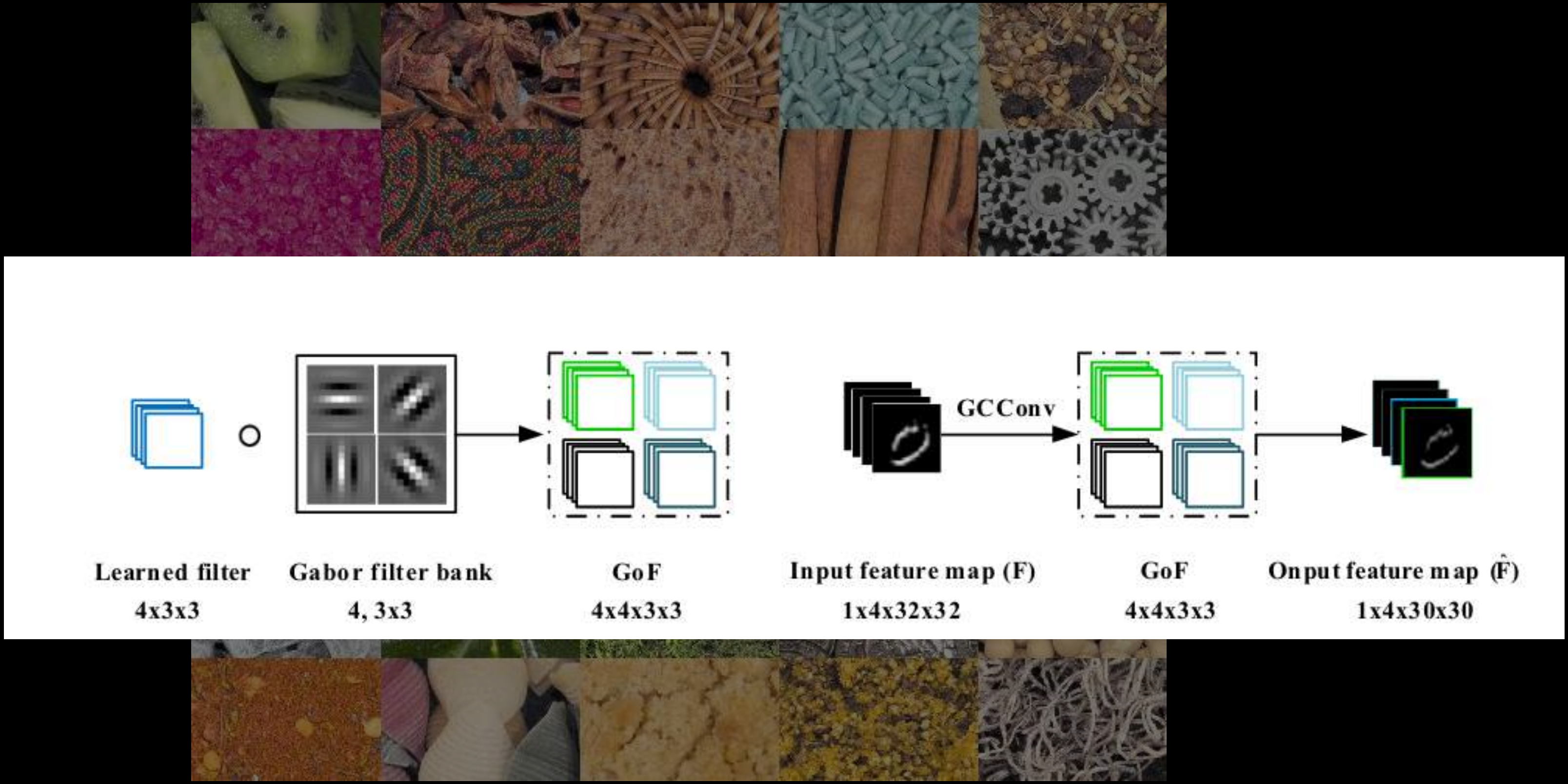
Gabor features

Used to recognize different types of textures:

ALOT dataset: https://aloi.science.uva.nl/public_alot/

Gabor features

Used to recognize different types of textures:



ALOT dataset: https://aloi.science.uva.nl/public_alot/

Luan, Shangzhen, et al. "Gabor convolutional networks." IEEE Transactions on Image Processing(2018).

Reading material for next lecture

- **Article**:
    - Simoncelli, Eero P., and Bruno A. Olshausen. "Natural image statistics and neural representation." Annual review of neuroscience 24.1 (2001): 1193-1216.

- Information Theory Book (https://www.inference.org.uk/itprnn/book.pdf): Section 4.1 (pp. 67-73)