LECTURE 10
Euclidean space

A linear (vector) space R is called *Euclidean* if there is a rule enabling for any two vectors x and $y \in R$ to construct a real number called the *scalar product* of vectors and denoted by (x, y), and this rule satisfies the following conditions: (1) $(x, y) = (y, x)$ – commutative property;
(2) $(x, y + z) = (x, y) + (x, z)$ – distributive property;
(3) $(\alpha x, y) = \alpha (x, y)$ for every real number α;
(4) $(x, x) > 0$ if x is a non-zero vector.

The conditions (1)-(4) imply the following:

a) $(y + z, x) = (y, x) + (z, x)$; $(x, \alpha y) = \alpha (x, y)$; $(0, x) = 0$ for every vector x.

The scalar product of every vector $x \in R$ on itself is called the *scalar square* of the vector x.

The length (norm) of a vector x in a Euclidean space is the square root of its scalar square: $|x| = \sqrt{(x,x)}$.

If λ is a real number and x is an arbitrary vector of a Euclidean space then $|\lambda x| = |\lambda|\cdot|x|$.

A vector of which the length is equal to 1 is called *normalized*. If $x \in R$ is a non-zero vector then it isn't difficult to see that the vector $\frac{1}{|x|}\cdot x$ is normalized.

For any two vectors x and y the inequality $(x,y)^2 \le (x,x)(y,y)$ called *Cauchy-Bunyakovskii inequality* holds in a Euclidean space.

Prove this inequality: Let E be a Euclidean space. Take arbitrary $x, y \in E$ and $\tau \in R$. Since E is a linear space, the element $x - \tau y \in E$. By (4) we have:

$$0 \le (x - \tau y, x - \tau y) = (x,x) - 2(x,y)\tau + (y,y)\tau^2 \quad \text{for every } \tau$$

The obtained quadratic trinomial is nonnegative for every τ iff its discriminant is non-positive, i.e. $(x,y)^2 - (x,x)(y,y) \le 0$. □

Obviously, the equality $(x,y)^2 = (x,x)(y,y)$ holds iff the vectors x and y are linearly dependent.

The Cauchy-Bunyakovskii inequality implies that $-1 \le \frac{(x,y)}{|x||y|} \le 1$.

An angle determined by the equality $\cos\varphi = \frac{(x,y)}{|x||y|}$ and belonging the segment $[0; \pi]$ is called *the angle between vectors* x and y.

If x and y are non-zero vectors and $\varphi = \pi/2$ then $(x, y) = 0$. In this case we say that vectors x and y are *orthogonal* and denote this by $x \perp y$. The zero element of a Euclidean space is assumed to be orthogonal to every other element of the space.

For arbitrary vectors x and y of a Euclidean space the following important formulas hold:

1. $|x + y| \le |x| + |y|$ (*triangle inequality*).

Prove it: From the axioms of a Euclidean space and the Cauchy-Bunyakovskii inequality we have: $|x+y|^2 = (x+y, x+y) = (x,x) + 2(x,y) + (y,y) \le |x|^2 + 2|x||y| + |y|^2 = (|x| + |y|)^2$

Since the numbers $|x+y|$ and $|x|+|y|$ are nonnegative, we obtain the triangle inequality. □

2. Let φ be the angle between vectors x and y. Then

$|x-y|^2 = |x|^2 + |y|^2 - 2|x|\cdot|y|\cos\varphi$ (*theorem of cosines*).

If $x \perp y$ then we obtain the following equality $|x-y|^2 = |x|^2 + |y|^2$. Replacing y by $(-y)$ in the last equality we obtain $|x+y|^2 = |x|^2 + |y|^2$ (*Pythagor theorem*).

Example 1. Consider the following linear space: $\{(\xi_1, \xi_2, \ldots, \xi_n), (\eta_1, \eta_2, \ldots, \eta_n), (\zeta_1, \zeta_2, \ldots, \zeta_n), \ldots\}$ – the set of all n–tuples of real numbers so that the sum of every two elements is determined by $(\xi_1, \xi_2, \ldots, \xi_n) + (\eta_1, \eta_2, \ldots, \eta_n) = (\xi_1 + \eta_1, \xi_2 + \eta_2, \ldots, \xi_n + \eta_n)$, and the product of every element on a real number is determined by $\lambda(\xi_1, \xi_2, \ldots, \xi_n) = (\lambda\xi_1, \lambda\xi_2, \ldots, \lambda\xi_n)$. Can we define scalar product of two arbitrary vectors $x = (\xi_1, \xi_2, \ldots, \xi_n)$ and $y = (\eta_1, \eta_2, \ldots, \eta_n)$ by the equality $(x, y) = \xi_1\eta_1 + \xi_2\eta_2 + \ldots + \xi_n\eta_n$, in order to this space become Euclidean?

Solution: Check the conditions (1)-(4).

(1) Since $(y, x) = \eta_1\xi_1 + \eta_2\xi_2 + \ldots + \eta_n\xi_n$ then $(x, y) = (y, x)$.

(2) Let $z = (\zeta_1, \zeta_2, \ldots, \zeta_n)$. Then $y + z = (\eta_1 + \zeta_1, \eta_2 + \zeta_2, \ldots, \eta_n + \zeta_n)$ and $(x, y + z) = \xi_1\eta_1 + \xi_1\zeta_1 + \xi_2\eta_2 + \xi_2\zeta_2 + \ldots + \xi_n\eta_n + \xi_n\zeta_n = (\xi_1\eta_1 + \xi_2\eta_2 + \ldots + \xi_n\eta_n) + (\xi_1\zeta_1 + \xi_2\zeta_2 + \ldots + \xi_n\zeta_n) = (x, y) + (x, z)$.
(3) $(\lambda x, y) = \lambda\xi_1\eta_1 + \lambda\xi_2\eta_2 + \ldots + \lambda\xi_n\eta_n = \lambda(\xi_1\eta_1 + \xi_2\eta_2 + \ldots + \xi_n\eta_n) = \lambda(x, y)$.
(4) $(x, x) = \xi_1^2 + \xi_2^2 + \ldots + \xi_n^2 > 0$ if there is at least one from numbers $\xi_1, \xi_2, \ldots, \xi_n$ that differs from zero.
Thus, this space is Euclidean.
Observe that scalar product defined in the last example has an economic sense: If $x = (x_1, x_2, \ldots, x_n)$ is a vector of volumes of various goods, and $y = (y_1, y_2, \ldots, y_n)$ is a vector of their prices, the scalar product (x, y) expresses the total cost of these goods.

Orthogonal basis and procedure of orthogonalization

A basis $e_1, e_2, \ldots, e_n$ of a Euclidean space is called *orthogonal* if $(e_i, e_k) = 0$ for $i \neq k$.
Theorem. *There is an orthogonal basis in any Euclidean space.*
If an orthogonal basis consists of normalized vectors then this basis is called *orthonormal*.
For an orthonormal basis $e_1, e_2, \ldots, e_n$ the following equalities hold: $(e_i, e_k) = \begin{cases} 0 & if \quad i \neq k, \\ 1 & if \quad i = k. \end{cases}$

Examples of orthonormal bases: 1) In the space V_3 of geometric vectors any three unit pairwise orthogonal vectors i, j, k form an orthonormal basis.
2) In Euclidean space of n-tuples of real numbers (Example 1) an orthonormal basis is the system of n unit vectors e_i at which the i-th component equals 1 and the rest components equal 0:

$$e_1 = (1, 0, \ldots, 0),\ e_2 = (0, 1, \ldots, 0),\ \ldots,\ e_n = (0, 0, \ldots, 1).$$

We can construct an orthonormal basis in an n-dimensional Euclidean space on the base of an arbitrary basis by procedure of orthogonalization. Describe this procedure.
Let k linearly independent elements $x_1, x_2, \ldots, x_k$ of an n-dimensional Euclidean space be given. Construct pairwise orthogonal elements $e_1, e_2, \ldots, e_k$ representing linear combinations of elements $x_1, x_2, \ldots, x_k$ as follows:
Put $e_1 = x_1$, $e_2 = x_2 - a_{12}e_1$ where $a_{12} = (x_2, e_1)\cdot(e_1, e_1)^{-1}$.
Such a choice of the coefficient a_{12} provides an orthogonality of e_1 and e_2: $(e_1, e_2) = 0$.
Further, put $e_3 = x_3 - a_{13}e_1 - a_{23}e_2 = x_3 - \sum_{i=1}^{2} a_{i3}e_i$ where

$$a_{13} = (x_3, e_1)\cdot(e_1, e_1)^{-1}, \quad a_{23} = (x_3, e_2)\cdot(e_2, e_2)^{-1}.$$

Such a choice of the coefficients a_{13} and a_{23} provides an orthogonality e_3 to the elements e_1 and e_2. And so on. At the m-th step ($m \leq k$) we put $e_m = x_m - \sum_{i=1}^{m-1} a_{im}e_i$ where $a_{im} = (x_m, e_i)\cdot(e_i, e_i)^{-1}$.
Such a choice of coefficients a_{im} provides an orthogonality e_m to elements $e_1, \ldots, e_{m-1}$. In result of k steps the described procedure gives pairwise orthogonal elements $e_1, e_2, \ldots, e_k$. We can prove that these elements are linearly independent.
Applying the procedure of orthogonalization to an arbitrary basis $f_1, f_2, \ldots, f_n$ of an n-dimensional Euclidean space we obtain a basis of n pairwise orthogonal elements $e_1, e_2, \ldots, e_n$ (an orthogonal basis). In order to make it orthonormal it is necessary to multiply every element e_i on the number $\frac{1}{|e_i|}$. The obtained elements $g_1 = \frac{e_1}{|e_1|}, \ldots, g_n = \frac{e_n}{|e_n|}$ form an orthonormal basis of the Euclidean space.

The coordinate representation of scalar product

A useful instrument of studying properties of some collection of elements $\{f_1, f_2, \ldots, f_k\}$ in a Euclidean space is the Gram matrix. In a Euclidean space the Gram matrix of a system of elements $\{f_1, f_2, \ldots, f_k\}$ is called the matrix $\Gamma = \begin{pmatrix} (f_1, f_1) & (f_2, f_1) & \dots & (f_k, f_1) \\ (f_1, f_2) & (f_2, f_2) & \dots & (f_k, f_2) \\ \dots & \dots & \dots & \dots \\ (f_1, f_k) & (f_2, f_k) & \dots & (f_k, f_k) \end{pmatrix}$.

Let in E^n a basis $\{g_1, g_2, ..., g_n\}$ be given. The scalar product of elements $x = \sum_{i=1}^{n} \xi_i g_i$ and $y = \sum_{j=1}^{n} \eta_j g_j$ is presented as $(x, y) = \left(\sum_{i=1}^{n} \xi_i g_i, \sum_{j=1}^{n} \eta_j g_j \right) = \sum_{i=1}^{n} \sum_{j=1}^{n} \xi_i \eta_j (g_i, g_j) = \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \xi_i \eta_j$, where $\gamma_{ij} = (g_i, g_j)$, $i, j = 1, ..., n$ are components of the matrix Γ_g named the *basis matrix of Gram*. Then the coordinate representation of scalar product can be written as follows:

$$(x, y) = (x)_g^T \Gamma_g (y)_g = (\xi_1 \quad \xi_2 \quad \dots \quad \xi_n) \begin{pmatrix} (g_1, g_1) & (g_2, g_1) & \dots & (g_n, g_1) \\ (g_1, g_2) & (g_2, g_2) & \dots & (g_n, g_2) \\ \dots & \dots & \dots & \dots \\ (g_1, g_n) & (g_2, g_n) & \dots & (g_n, g_n) \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \dots \\ \eta_n \end{pmatrix}$$

Observe that in an orthonormal basis $\Gamma_e = E$, and the formula for scalar product takes the following form: $(x, y) = \sum_{i=1}^{n} \xi_i \eta_i$.

Theorem. For the basis matrix of Gram Γ_g in any basis $\det \Gamma_g > 0$.

Proof. At transition from a basis $\{g_1, g_2, ..., g_n\}$ to a basis $\{g_1', g_2', ..., g_n'\}$ with a transition matrix S the following holds: $\Gamma_{g'} = S^T \Gamma_g S$, $\det \Gamma_{g'} = \det \Gamma_g (\det S)^2$ where $\det S \neq 0$. This implies that the value $\operatorname{sgn}(\det \Gamma_g)$ is invariant, i.e. doesn't change at replacement of a basis. At last, taking in account that $\det \Gamma_e = 1$ we obtain that $\det \Gamma_g > 0$ in any basis. □

Any vector *x* of a Euclidean space given in an orthonormal basis e_1, e_2, ..., e_n is determined by the equality: $x = \xi_1 e_1 + \xi_2 e_2 + ... + \xi_n e_n$.

The coordinates ξ_i of an element *x* are expressed by the formula: $\xi_i = (x, e_i)$, $i = 1, ..., n$.

The scalar product (x, e_i) is called *projection of the element x on the element e_i*. Thus, the coordinates of an arbitrary element of a Euclidean space in an orthonormal basis are equal to projections of this element on corresponding basic elements.

The length of a vector *x* is found by the formula: $|x| = \sqrt{\xi_1^2 + \xi_2^2 + ... + \xi_n^2}$.

Two vectors $x = \xi_1 e_1 + \xi_2 e_2 + ... + \xi_n e_n$ and $y = \eta_1 e_1 + \eta_2 e_2 + ... + \eta_n e_n$ are linearly dependent (collinear, proportional) if and only if $\xi_1 / \eta_1 = \xi_2 / \eta_2 = ... = \xi_n / \eta_n$.

The condition of orthogonality of vectors *x* and *y* has the following form: $\xi_1 \eta_1 + \xi_2 \eta_2 + ... + \xi_n \eta_n = 0$.

The angle between two vectors *x* and *y* is found by the formula:

$$\cos\varphi = \frac{\xi_1 \eta_1 + \xi_2 \eta_2 + ... + \xi_n \eta_n}{\sqrt{\xi_1^2 + \xi_2^2 + ... + \xi_n^2} \cdot \sqrt{\eta_1^2 + \eta_2^2 + ... + \eta_n^2}}.$$

Example: Vectors e_1, e_2, e_3, e_4, e_5 form an orthonormal basis. Find the scalar product and the lengths of vectors $x = e_1 - 2e_2 + e_5$, $y = 3e_2 + e_3 - e_4 + 2e_5$.

Solution: $(x, y) = 1 \cdot 0 + (-2) \cdot 3 + 0 \cdot 1 + 0 \cdot (-1) + 1 \cdot 2 = -4$.

$|x| = \sqrt{1^2 + (-2)^2 + 0^2 + 0^2 + 1^2} = \sqrt{6}$, $|y| = \sqrt{0^2 + 3^2 + 1^2 + (-1)^2 + 2^2} = \sqrt{15}$.

Orthogonal complements and orthogonal projections in a Euclidean space

Let in a Euclidean space E some subspace E_1 be given. Consider the set $E_2 \subseteq E$ of elements x that are orthogonal to all elements of E_1. In a Euclidean space E the set of elements x such that $(x, y) = 0$ for every $y \in E_1 \subset E$ is called the *orthogonal complement* of the set E_1.

Theorem. The orthogonal complement of a k-dimensional subspace $E_1 \subset E^n$ is a subspace of dimension $n-k$.

Proof: Let in E^n with standard scalar product an orthonormal basis be given, and let E_2 be the orthogonal complement to E_1. Choose some basis in E_1: $\{g_1, g_2, ..., g_k\}$. Then by orthogonality of an arbitrary element $x \in E_2$ to every element of E_1 $(x, g_i) = 0, i = 1,..., n$ or in coordinate form:

$$\begin{cases} \varepsilon_{11}\xi_1 + \varepsilon_{12}\xi_2 + ... + \varepsilon_{1n}\xi_n = 0 \\ \varepsilon_{21}\xi_1 + \varepsilon_{22}\xi_2 + ... + \varepsilon_{2n}\xi_n = 0 \\ .... \\ \varepsilon_{k1}\xi_1 + \varepsilon_{k2}\xi_2 + ... + \varepsilon_{kn}\xi_n = 0 \end{cases} \text{ where } g_j = \begin{pmatrix} \varepsilon_{j1} \\ \varepsilon_{j2} \\ ... \\ \varepsilon_{jn} \end{pmatrix}, \ j = 1,..., k \text{ and } x = \begin{pmatrix} \xi_1 \\ \xi_2 \\ ... \\ \xi_n \end{pmatrix}$$

This is homogeneous system of linear equations defining the orthogonal complement E_2 has the rank k by linear independence of elements $\{g_1, g_2, ..., g_k\}$. Then it has $n-k$ linearly independent solutions forming a basis of the subspace E_2. □

Theorem. If E_2 is the orthogonal complement of a subspace $E_1 \subset E$ then E_1 is the orthogonal complement of E_2.

Proof: For every element $x \in E_2$ the following holds: $(y, x) = 0$ for all $y \in E_1$. But this means that for every element $y \in E_1$ the following holds: $(x, y) = 0$ for all $x \in E_2$, i.e. E_1 is the orthogonal complement of E_2. □

In a Euclidean space E an element y is called an *orthogonal projection* of an element x on a subspace E^* if 1) $y \in E^*$; 2) $(x - y, z) = 0$ for all $z \in E^*$.

Consider a method of constructing an orthogonal projection of some element $x \in E$ on a subspace $E^* \subset E$ if E^* is a k-dimensional subspace. In this case in E^* there is a basis $\{g_1, g_2, ..., g_k\}$ and every element $z \in E^*$ (and also y) can be represented as a linear combination of the basis vectors. Let $y = \sum_{i=1}^{k} \xi_i g_i$. Choose the numbers $\xi_i, \ i = 1,..., k$ so that $(x - y, z) = 0$ for all $z \in E^*$. For this it is necessary and sufficient that the element y would be orthogonal to each of the basis elements of the subspace E^*, i.e. $(x - y, g_j) = 0$ for every $j = 1,...., k$. Then the numbers $\xi_i, \ i = 1,..., k$ are found from the system of linear equations:

$$\left(x - \sum_{i=1}^{k} \xi_i g_i, g_j \right) = 0 \text{ for every } j = 1,...., k \text{ or } \sum_{i=1}^{k} (g_i, g_j)\xi_i = (x, g_j) \text{ for every } j = 1,...., k.$$

Since the basic matrix of this system (as Gram matrix of a collection of linearly independent elements $g_1, g_2, ..., g_k$) is regular, there is a unique solution of the system. □

If a basis $\{e_1, e_2, ..., e_k\}$ in a subspace E^* is orthonormal then the orthogonal projection of an element x on E^* is the element $y = \sum_{i=1}^{k} (x, e_i) e_i$.

Glossary

Euclidean space – евклидово пространство; **quadratic trinomial** – квадратный трехчлен
orthonormal – ортонормированный

Exercises for Seminar 10

10.1. Consider the following linear space: $\{x = (\xi_1, \ \xi_2, ..., \ \xi_n) \mid \xi_1, \ \xi_2, ..., \xi_n$ are real positive numbers$\}$ is the set of all systems consisting of n positive numbers. The sum of every two vectors $x = (\xi_1, \ \xi_2, ..., \xi_n)$ and $y = (\eta_1, \ \eta_2, ..., \eta_n)$ is determined by $x + y = (\xi_1\eta_1, \xi_2\eta_2, \ldots, \xi_n\eta_n)$, and the product of every element $x = (\xi_1, \ \xi_2, ..., \xi_n)$ on a real number λ is determined by $\lambda x = (\xi_1^{\lambda}, \ \xi_2^{\lambda}, ..., \xi_n^{\lambda})$. Can we define the scalar product of two vectors by the equality $(x, y) = \ln \xi_1 \ln \eta_1 + \ln \xi_2 \ln \eta_2 + ... + \ln \xi_n \ln \eta_n$ in order to this space become Euclidean?

10.2. Consider the following Euclidean space: $\{(\xi_1, \xi_2, \ldots, \xi_n), (\eta_1, \eta_2, \ldots, \eta_n), (\zeta_1, \zeta_2, \ldots, \zeta_n), \ldots\}$ – the set of all *n*–tuples of real numbers so that the sum of every two elements is determined by $(\xi_1, \xi_2, \ldots, \xi_n) + (\eta_1, \eta_2, \ldots, \eta_n) = (\xi_1 + \eta_1, \xi_2 + \eta_2, \ldots, \xi_n + \eta_n)$, the product of every element on a real number is determined by $\lambda(\xi_1, \xi_2, \ldots, \xi_n) = (\lambda\xi_1, \lambda\xi_2, \ldots, \lambda\xi_n)$ and the scalar product is determined by $(x, y) = \xi_1\eta_1 + \xi_2\eta_2 + \ldots + \xi_n\eta_n$. Let $n = 4$. Determine the angle between vectors $x = (4; 1; 2; 2)$ and $y = (1; 3; 3; -9)$.
10.3. Consider a Euclidean space given in 10.2. Determine the angle between vectors $x = (1; \sqrt{3}; \sqrt{5}; \ldots; \sqrt{2n-1})$ and $y = (1; 0; 0; \ldots; 0)$.
10.4. Consider a Euclidean space given in 10.2 for $n = 6$. Check the validity of the Pythagor theorem for orthogonal vectors $x = (1; 0; 2; 0; 2; 0)$ and $y = (0; 6; 0; 3; 0; 2)$.
10.5. Find the length of vector $x = 4e_1 - 2e_2 + 2e_3 - e_4$.
10.6. Normalize vector $x = e_1 + 2\sqrt{2}e_2 + 3\sqrt{3}e_3 + 8e_4 + 5\sqrt{5}e_5$.
10.7. What value λ is a basis formed by vectors $g_1 = \lambda e_1 + e_2 + e_3 + e_4$, $g_2 = e_1 + \lambda e_2 + e_3 + e_4$, $g_3 = e_1 + e_2 + \lambda e_3 + e_4$, $g_4 = e_1 + e_2 + e_3 + \lambda e_4$ orthogonal for? Normalize this basis.
10.8. A subspace E_1 of a Euclidean space E be given as a linear hull of vectors having in an orthonormal basis the following coordinates: 1) $(3, 1, 2)$; 2) $(1, -5, 1), (-1, 1, 1)$.
Find: a) the matrix of the system of equations defining the orthogonal complement of E_1;
b) a basis in the orthogonal complement of E_1.
10.9. Orthogonalize the following system of vectors of the arithmetic space with standard scalar product: $(1, 3, -2), (3, 7, -2)$.
10.10. A subspace E_1 of a Euclidean space E is the linear hull of the vector $a = (10, 5, 5)$, and let $x = (3, 0, 0)$ in an orthonormal basis. Find the orthogonal projections of x on both E_1 and its orthogonal complement respectively.

Exercise for Homework 10

10.11. In the linear space of polynomials of degree $\le n$ for two polynomials p and q is put the number $F(p,q) = \int_{-1}^{1} p(t)q(t)dt$. Prove that this number satisfies the properties of scalar product.
10.12. Normalize vector $x = e_1 \sin^3\alpha + e_2 \sin^2\alpha\cos\alpha + e_3 \sin\alpha\cos\alpha + e_4 \cos\alpha$.
10.13. Determine the angle between vectors $x = e_1\sqrt{7} + e_2\sqrt{5} + e_3\sqrt{3} + e_4$ and $y = e_1\sqrt{7} + e_2\sqrt{5}$.
10.14. Find a normalized vector that is orthogonal to vectors $x = 3e_1 - e_2 - e_3 - e_4$, $y = e_1 - 3e_2 + e_3 + e_4$, $z = e_1 + e_2 - 3e_3 + e_4$.
10.15. What value λ do the vectors $x = \lambda e_1 + \lambda e_2 - e_3 - \lambda e_4$ and $y = e_1 - e_2 + \lambda e_3 - e_4$ have the same lengths for?
10.16. What values α and β is a basis formed by vectors $e_1' = \frac{\alpha}{3}e_1 + \frac{1-\alpha}{3}e_2 + \beta e_3$, $e_2' = \frac{1-\alpha}{3}e_1 + \beta e_2 + \frac{\alpha}{3}e_3$, $e_3' = \beta e_1 + \frac{\alpha}{3}e_2 + \frac{1-\alpha}{3}e_3$ orthonormal for?
10.17. A subspace E_1 of a Euclidean space E be given as a linear hull of vectors having in an orthonormal basis the following coordinates: 1) $(3, 1, -2)$; 2) $(3, -15, 9, 1), (3, -6, -3, 2)$.
Find: a) the matrix of the system of equations defining the orthogonal complement of E_1;
b) a basis in the orthogonal complement of E_1.
10.18. Orthogonalize the following system of vectors of the arithmetic space with standard scalar product: $(2, 1, 0, -1), (3, 6, 2, 6)$.
10.19. A subspace E_1 of a Euclidean space E is the linear hull of the vectors $a_1 = (6, 1, 5)$ and $a_2 = (4, -1, 3)$, and let $x = (1, 3, -2)$ in an orthonormal basis. Find the orthogonal projections of x on both E_1 and its orthogonal complement respectively.