

✔ Congratulations! You passed!

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higher

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Retake the assignment in **7h 59m**

1. Which of the following is the expression for the Fourier Transform?

1 / 1 point

- ☐ $F(u) = \int_0^{\infty} f(x)e^{-i2\pi ux} dx$
- ☐ $F(u) = \int_{-\infty}^{\infty} f(x)e^{-\frac{i2\pi u}{x}} dx$
- ☐ $F(u) = \int_{-\infty}^{\infty} f(x+u)e^{-i2\pi ux} dx$
- ☒ $F(u) = \int_{-\infty}^{\infty} f(x)e^{-i2\pi ux} dx$

✔ **Correct**

See definition of the Fourier Transform.

2. Which of the following must be a property of functions for which Fourier's Theorem applies?

1 / 1 point

- ☐ Bounded
- ☒ Periodic
- ☐ Differentiable
- ☐ Continuous

✔ **Correct**

Fourier's Theorem applies to periodic functions which are reasonably continuous. This includes discontinuous functions with a finite number of discontinuities and unbounded functions (see Dirichlet Conditions for further reading).

3. What is the expression for the frequency of the following function?

2 / 2 points

$$f(x) = B \sin \left(\frac{x}{T} + \psi \right)$$

☐ $\frac{1}{T}$

☐ T

☐ $\frac{2\pi}{T}$

☒ $\frac{1}{2\pi T}$

✓ **Correct**

The expression for the frequency of a sinusoidal expression is given as the coefficient of the angle divided by 2π .

4. What is the expression for the phase shift of the following function with respect to $f(x) = \sin(x)$?

3 / 3 points

$$g(x) = A \cos(\pi u + \psi)$$

$$x = \pi u$$

☐ ψ

☐ $\frac{\psi}{2}$

☒ $\psi + \frac{\pi}{2}$

☐ $\frac{\psi + \pi}{2}$

✓ **Correct**

$g(x)$ can be rewritten as follows:

$$g(x) = A \cos(\pi u + \psi)$$

$$= A \sin \left(\pi u + \left(\psi + \frac{\pi}{2} \right) \right).$$

From this the phase shift can be read off as $\psi + \frac{\pi}{2}$.

5. Recall that the Fourier transformation of a standard Gaussian is $\sqrt{\pi}e^{-\pi^2 u^2}$. 2 / 2 points

What would be the Fourier transformation of the second derivative of the Gaussian?

- ☒ $-4\pi^{5/2}u^2 e^{-\pi^2 u^2}$
- ☐ $2\pi^{3/2}iu^2 e^{-\pi^2 u^2}$
- ☐ $2\pi^{5/2}(2\pi^2 u^2 - 1)e^{-\pi^2 u^2}$
- ☐ $-2\pi^{3/2}iue^{-\pi^2 u^2}$

✓ **Correct**

Recall that the Fourier transform of the n th derivative of a function f is given by

$(i2\pi u)^n F$, where F is the Fourier transform of f . If we apply this property we get the above result.

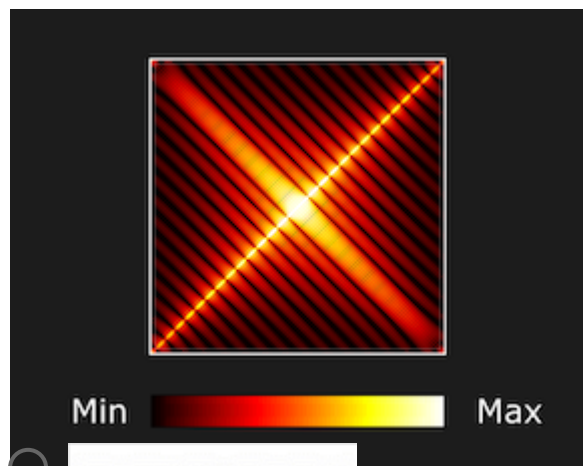
6. Which of the following is not a property of the Fourier transform? 1 / 1 point

- ☐ $\alpha f(x) + \beta g(x) \iff \alpha F(u) + \beta G(u)$
- ☐ $f(ax) \iff \frac{1}{|a|} F\left(\frac{u}{a}\right)$
- ☐ $\frac{d^n}{dx^n} f(x) \iff (i2\pi u)^n F(u)$
- ☒ $h(x) = f(x) \cdot g(x) \iff H(u) = F(u) \cdot G(u)$

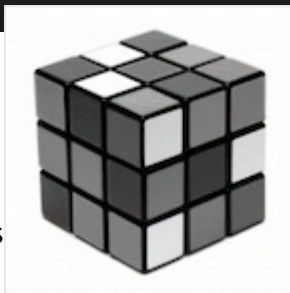
✓ **Correct**

The first answer choice represents the linearity property. The second answer choice is the scaling property. The third answer choice is the differentiation property. The last answer choice is not valid.

7. 2 / 2 points



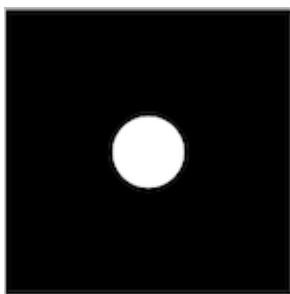
☐



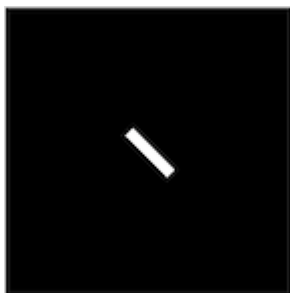
The
thes

rier transform of an image is shown above. Which of
ier transform of?

☐



☒



☐



✓ **Correct**

This object has four sets of strong edges (4 sides of the rectangle). That is, all the edges are aligned with two directions, where the directions are perpendicular to each other. This manifests as two lines of non-zero frequencies in the Fourier transform.

8. If the Fourier transformation of a function f is ever 0 then convolving a signal with f is irreversible.

1 / 1 point

☒ True

☐ False

✓ **Correct**

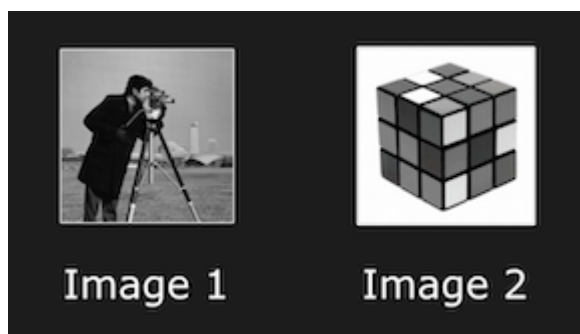
This is a consequence of the Convolution Theorem. Let $g(x)$ be some signal and suppose $F(v) = 0$ for some v . For any constant C ,

$$\begin{aligned} F(u) \cdot G(u) &= F(u) \cdot (G(u) + C\delta(v)) \\ \implies f(x) * g(x) &= f(x) * (g(x) + Ce^{2\pi iuv}) \end{aligned}$$

To better understand this, let us think of a constant signal in time domain $f(x)$ (Fourier Transform = $\delta(u)$), and some other signal $g(x)$ (Fourier Transform = $G(u)$). Once we are in the frequency domain and multiply $G(u)$ with the impulse response, we only get $G(0)$. If we apply inverse Fourier transform, we would not arrive at the original $f(x) * g(x)$, since we only have values at frequency 0.

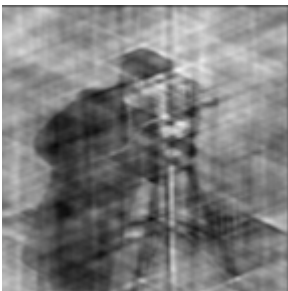
9.

1 / 1 point



If the magnitude of the frequencies in image 1 were combined with the phases of the frequencies in image 2, which of these options would it most closely

resemble?

**Correct**

The phases of the frequencies have more of an impact on an object's recognizability than the magnitudes. Changing the magnitudes will, however, affect the saturation of various parts of the image.

10. The maximum frequency in a continuous (optical) image is u_{\max} . To recover the continuous signal from its discrete samples, the sampling frequency f_s must satisfy:

2 / 2 points

☒ $f_s \geq 2u_{\max}$

☐ $f_s \geq u_{\max}$

☐ $f_s \leq \frac{u_{\max}}{2}$

☐ $f_s \geq u_{\max}^2$

☒ **Correct**

This result follows directly from the Nyquist Theorem which states that to be able to fully recover a continuous signal, it must be sampled at a frequency that is at least twice the maximum frequency of the signal.