MODULE :- 2.

Rate of changes and second derivative

2)

Rate of changes

* The eq of a line is $y = mx + b$ Eq- () $f(x) = mx + b$ is a linear (), the slope 'm' of a straight line represents the rate of change of y with respect x. (rate of change = m)

Q) Suppose that 'y' changes proportionally with x & rate of change is 3. If $y = 2$ when $x = 0$, find the eq relating y to x.

A)
$y = mx + b$
$= 3x + b$ $m = 3$.

3)

$x = 0,\ y = 2$

$\therefore 2 = 3x + b$
$2 = 3 \times 0 + b$ $\rightarrow 2 = 0 + b$, $\underline{2 = b}$
$\underline{b = 2}$

$\underline{y = 3x + 2}$

Q) A balloon is grown up has a volume $v = 3t^3 + 8t^2 + 16t$ cubic cm after 't' min. What is rate of change of volume at $t = 0.5$

$\frac{dx}{dy}$ same as $\frac{dv}{dt}$

$\frac{dv}{dt}\Big|_{t=0.5} = ?$

$t^3 = 3t^2 \times 3 = 9t^2$
$t^2 = 2t \times 8 = 16t$

$\frac{dv}{dt}(3t^3 + 8t^2 + 16t)$

$= \underline{9t^2 + 16t + 16}$

$\frac{dv}{dt}\Big|_{t=0.5} = 9 \times (0.5)^2 + 16 \times 0.5 + 16$
$= 2.25 + 8 + 16 = \underline{26.25}$ cubic m / min

Linear / proportional change :-

The variable y changes proportionally with x, when y is related to x by a linear (), $y = mx + b$ where $\frac{\Delta y}{\Delta x} = m$. The no. 'm' is the rate of change of 'y' with respect to x.

Rate of change :-

If 2 quantities x & y are related by $y = f(x)$ then the derivative $f'(x)$ represent the rate of change of y with respect to x at the point x_0.

It is measured in unit of y / unit of x

Q 1) A circle with radius 'r' millimeters has area $= \pi r^2$ square millimeters. find the rate of ~~change~~ of area with respect to radius at $r_0 = 5$

1) $A = \pi r^2$.

$$\left.\frac{dA}{dr}\right|_{r_0=5}$$

$$\frac{dA}{dr}(\pi r^2)$$

$$= \pi \cdot 2r$$

$$\left.\frac{dA}{dr}\right|_{r=5} = \pi \times 2 \times 5$$

$$= 10\pi \text{ millimeters.}$$

2) Suppose that the price of ~~pork~~ 'p' depends on the supply 's' by the formula. $p = 160 - 3s + (0.01)s^2$ find the rate of change 'p' with respect s, when $s = 50$.

A) $$\left.\frac{dp}{ds}\right|_{s=50}$$

$$\frac{dp}{ds}\left(160 - 3s + (0.01)s^2\right)$$

$$= 0 - 3 + (0.01)\,2s$$

$$= -3 + 0.01 \times 2s$$

$$\left.\frac{dp}{ds}\right|_{s=50} = \cancel{0 - 3 + 0.01 \times 2 \times 50}$$

$$= -3 + 0.01 \times 2 \times 50$$

$$= -3 + 0.01 \times 100$$

$$= -3 + 1 = -2$$

(-2) price is using when $s = 50$.

3) A reservoir contains $10^8 - 10^4 t - 80t^2 - 10t^3 + 5t^5$ litres of water at a time 't'. where t is the time in hrs. from when the gates are closed. How many litres (hr living the reservoir after 1 hr. t=1. (with respect to — $\frac{dv}{dt}$ here ...)

$$f(t) = \left(10^8 - 10^4 t - 80t^2 - 10t^3 + 5t^5\right)\frac{d}{dt} \qquad \frac{d}{dt}(10^8) = 0 \quad t^5 = 5t^4$$

$$f'(t) = 0 - 10^4 - 80 \times 2t - 10 \times 3t^2 + 5 \times 5t^4$$

$$= -10000 - 160t - 30t^2 + 25t^4$$

$t = 1$

$$= -10000 - 160 \times 1 - 30 \times 1^2 + 25 \times 1^4$$

$$= -10000 - 160 - 30 + 25 = 10165$$

$$\cancel{10160}$$

Suppose that $x = f(t) = \frac{1}{4}t^2 - t + 2$ denote the position of a bus at time t -

a) find the velocity as a f) of time?
b) find acceleration? (velocity - as $\frac{d}{dt}$ from Acce.)

2) a) $\boxed{\text{velocity} = \frac{dx}{dt}} = \frac{d}{dt}\left(\frac{1}{4}t^2 - t + 2\right)$

$= \frac{1}{4} 2t - 1 \cdot = \frac{\cancel{2}t}{\cancel{4}_2} - 1$

$= \frac{t}{2} - 1$

$V = \frac{t}{2} - 1$

b) $\boxed{\text{acceleration} = \frac{dv}{dt}} = \frac{d}{dt}\cdot\left(\frac{t}{2} - 1\right)$

$= \left(\frac{1}{2}t - 1\right)\frac{d}{dt}$

acceleration $= \frac{1}{2}$ $= \frac{1}{2}$.

3) ⇒ Second Derivatives :-

To compute 2nd deri- $f'(x)$.

Steps
* compute the 1st deri $f'(x)$
* cal. the deri- of $f'(x)$, the resul is $f''(x)$
* 2nd deri- of $y = f(x)$ is return as $\frac{d^2y}{dx^2}$

2 Acceleration :- 2nd deri- of position with respect to time.

1) cal. 2nd deri- of ⓐ $f(x) = x^4 + 2x^3 - 8x$.

ⓑ $f(x) = \frac{x+1}{\sqrt{x}}$

ⓒ $f(x) = 3x^2 - 2x + 1$ ⓓ $f(x) = 8x^2 + 2x + 10$

a) $f'(x) = x^4 + 2x^3 - 8x$ $x^3 = 3x^2 \times 2 = 6x^2$

$= 4x^3 + 6x^2 - 8$

$f''(x) = 4x^3 + 6x^2 - 8$. $x^3 = 3x^2 \times 4 = 12x^2$

$= 12x^2 + 12x$

$= 12\left(x^2 + x\right)$

b) $f'(x) = \frac{x+1}{\sqrt{x}}$ (Q. Rule)

$= \frac{\sqrt{x} \times \frac{d}{dx}(x+1) - (x+1)\frac{d}{dx}(\sqrt{x})}{(\sqrt{x})^2}$ $x = \sqrt{x}\cdot\sqrt{x}$

$= \frac{\sqrt{x} \times 1 - (x+1)\frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$ $\sqrt{x}\cdot\frac{\sqrt{x}\cdot\sqrt{x}}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}$

$= \frac{\sqrt{x} - \frac{x}{2\sqrt{x}} - \frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$ $\cancel{= \frac{\sqrt{x} - \frac{x+1}{2\sqrt{x}}}{(\sqrt{x})^2}}$

$\cancel{= \frac{\sqrt{x} - \frac{\sqrt{x}\cdot\sqrt{x}}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}}$ $= \frac{\frac{2\sqrt{x}\cdot\sqrt{x} - x - 1}{2\sqrt{x}}}{x}$

2)

$$\cancel{= \sqrt{x} - \frac{\sqrt{x}}{2} + \frac{1}{2\sqrt{x}}}$$

$$\cancel{f''(x) = \frac{x-1}{2x\sqrt{x}}}$$

$$f'(x) = \frac{\sqrt{x} \times 1 - (x+1)\frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$$

$$= \frac{\frac{\sqrt{x}\cdot 2\sqrt{x}}{1 \cdot 2\sqrt{x}} - \frac{x+1}{2\sqrt{x}}}{(\sqrt{x})^2} = \frac{2x-(x+1)}{2\sqrt{x}\cdot x} = \frac{2x-x-1}{2x\sqrt{x}}$$

$$= \frac{x-1}{2x\sqrt{x}} = \frac{x-1}{2x \cdot x^{1/2}} = \frac{x-1}{2x^{3/2}}$$

$$\frac{2x-x-1}{2x\sqrt{x}} = \frac{x-1}{2x\sqrt{x}} = \frac{x-1}{2x \cdot x^{1/2}}$$

$$f'(x) = \frac{x-1}{2x^{3/2}}$$

$$f''\left(\frac{x-1}{2x^{3/2}}\right)$$

$$\frac{d}{dx}\frac{1}{2}\left(\frac{x-1}{x^{3/2}}\right) \quad \text{(Q. Rule)}$$

3)

$$= \frac{1}{2}\frac{\left[x^{3/2}\frac{d}{dx}(x-1) - \left[(x-1)\frac{d}{dx}(x^{3/2})\right]\right]}{(x^{3/2})^2}$$

$$= \frac{1}{2}\frac{\left(x^{3/2}\cdot 1 - (x-1)\frac{3}{2}x^{1/2}\right)}{(x^{3/2})^2}$$

$$\frac{3}{2}x^{1/2} \qquad x^{3/2} =$$

$$= \frac{1}{2}\frac{\left[x^{3/2} - \left(\frac{3}{2}x^{3/2} - \frac{3}{2}x^{1/2}\right)\right]}{x^3}$$

$$= \frac{1}{2}\frac{\left[x^{3/2} - \frac{3}{2}x^{3/2} + \frac{3}{2}x^{1/2}\right]}{x^3}$$

$$f'(x) = 3x^2 - 2x + 1$$

$$\frac{d}{dx}(3x^2 - 2x + 1)$$

$$= 6x - 2$$

$$f''(x) = 6x - 2$$

$$\frac{d}{dx}(6x-2) = 6$$

$$f'(r) = 8r^2 + 2r + 10$$

$$= \frac{d}{dr}(8r^2 + 2r + 10)$$

$$= 16r + 2$$

$$f''(r) = 16r + 2$$

$$\frac{d}{dr}(16r+2) = 16$$

A race car travels in $\frac{1}{4}$ miles in 6 sec. Its distance from the start in feet after t sec being $f(t) = \frac{44t^2}{3} + 132t$.

find velocity & acc.. ~~at t=6.~~ to cross the finish line.

How fast was it going halfway down the track?

a) $v = f'(t) = \frac{44t^2}{3} + 132t$.

$\frac{d}{dx} = \frac{44}{3} 2t + 132$.

$= \frac{88t}{3} + 132$

$a = f''(t) = \frac{88t}{3} + 132 = \frac{88}{3}$

Sull $t = 6$, $\Rightarrow \frac{88 \times 6}{3} + 132$

$v = 308$ feet / sec.

$a \approx 29.3$ feet / sec^2

$88/3 = 29$

b) Total distance covered is

$f(t) = \frac{44t^2}{3} + 132t$.

$f(6) = \frac{44 \times 6^2}{3} + 132t = 1320$ feet.

h/f the distance is $\frac{1320}{2} = 660$ feet.

To find t corresponding to (dis) 660

$f(t) = 660$.

$\frac{44t^2}{3} + \frac{132t}{1} = 660$

$\frac{44t^2 + 396t}{3} = 660$

$44t^2 + 396t = 1980$. $\div 44$.

$t^2 + 9t = 45$ (or)

$t^2 + 9t - 45 = 0$

by quad. eq,

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $c = -45$

$= \frac{-9 \pm \sqrt{9^2 + 4 \times 1 \times 45}}{2}$

$t = \frac{-9 \pm \sqrt{81 + 180}}{2}$

$t = \frac{-9 \pm \sqrt{261}}{2}$

$t = \frac{-9 + \sqrt{261}}{2}$ $t = \frac{-9 - \sqrt{261}}{2}$

$= \frac{-9 + 16.15}{2}$ $= \frac{-9 - 16.15}{2}$

$= \frac{7.15}{2} = 3.58$ $= \frac{-25.15}{2}$

$t = 3.58$ $t = -12.5$

time always +ve, $t = 3.58$

$v = f'(t) = \frac{88t}{3} + 132$

$v \approx \frac{88 \times 3.58}{3} + 132$

≈ 237 feet / sec

Chain Rule :-

power of a f rule :-

To differenciate n^{th} power $[g(x)]^n$ of a f $g(x)$, where n is

2) the integer, take out the exponent as a factor.
reduce the exponent by 1 & x the derivative of $g(x)$.

$$\boxed{(g^n)'(x) = n\,(g(x))^{n-1} \cdot g'(x)}$$

$x^4 = nx$ $= ng(x)$

(or) $\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$.

1) $\frac{d}{dx}(g(x))^3$ where $g(x) = x^4 + 2x^2$

4) $\frac{d}{dx}(x^4+2x^2)^3 = 3(x^4+2x^2)^2 \cdot \frac{d}{dx}(x^4+2x^2)$

$x^3 = 3x^2$

$= 3(x^4+2x^2)^2 \cdot 4x^3 + 4x$

$= 3((x^4)^2 + 2x^4 \cdot 2x^2 + (2x^2)^2)(4x^3+4x)$

$= 3(x^8 + 4x^6 + 4x^4)(4x^3+4x)$

$= (3x^8 + 12x^6 + 12x^4)(4x^3 + 4x)$

$= 12x^{11} + 48x^9 + 48x^7 + 12x^9 + 48x^7 + 48x^5$

$= 12x^{11} + 60x^9 + 96x^7 + 48x^5$

3)

2) $\frac{d}{ds}(s^4+2s^3+3)^8$

$= 8(s^4+2s^3+3)^7 \frac{d}{dx}(s^4+2s^3+3)$

$= 8(s^4+2s^3+3)^7 \cdot (4s^3+6s^2)$ (The is 7 nominal step)

3) $y = (x^2+1)^{27}(x^4+3x+1)^8$

find rate of y with respect to x.

$y = (x^2+1)^{27}(x^4+3x+1)^8$ (by P. rule)

$= (x^2+1)^{27}\frac{d}{dx}(x^4+3x+1)^8 + (x^4+3x+1)^8 \frac{d}{dx}(x^2+1)^{27}$

~~$= (x^2+1)^{27} \cdot 8(x^4+3x+1)^7 + (x^4+3x+1)^8\ 27(x^2+1)^{26}$~~

chain R

$= (x^2+1)^{27} \cdot 8(x^4+3x+1)^7 \frac{d}{dx}(x^4+3x+1) + (x^4+3x+1)^8 \cdot 27(x^2+1)^{26}\frac{d}{dx}(x^2+1)$

$= (x^2+1)^{27} \cdot 8(x^4+3x+1)^7(4x^3+3) + (x^4+3x+1)^8\ 27(x^2+1)^{26}(2x)$

Composition of ( ) :-

If f & g are ( )s, defined for all real no. we defined their composition to be the ( ) which assigns to x, the no. $f(g(x))$.
The composition is denoted by $f \circ g$ ($\circ \to$ composite)

thus, $\boxed{(f \circ g)(x) = f(g(x))}$.

Q 1) If $f(x)=\sqrt{x}$ $g(x)=x^3-5$ find $f\circ g$ & $g\circ f$.

1) $f\circ g = ?$

$(f\circ g)(x) = f(g(x))$

$= f(x^3-5)$

$f(x)=\sqrt{x}$

$\Rightarrow f(x^3-5) = \sqrt{x^3-5}$

$g\circ f = ?$

$(g\circ f)(x) = g(f(x))$

$g(x)=x^3-5$ $= g(\sqrt{x})$

$g(\sqrt{x}) = (\sqrt{x})^3 - 5$

$x \to x^3$

So $x^3-5 \to (\sqrt{x})^3$

3)

2) $f(x)=x^3+2$ $g(x)=(x^2+1)^2$ $(f\circ g)($

$(f\circ g)(x) = f(g(x))$

$= f((x^2+1)^2)$

$f(x)=x^3+2$

$f(x^2+1)^2 = ((x^2+1)^2)^3+2 = (x^2+1)^6+2$

Chain rule :- (based on composition)

To differentiate a composition $f(g(x))$ differentiate g at x, differentiate f at $g(x)$. & multiply the result.

$$\boxed{(f\circ g)'(x) = f'(g(x))\cdot g'(x)}$$

1) Verify the chain rule for $f(u)=u^2$ $g(x)=x^3+1$

we need $\to$ (LHS = RHS)

A) $(f\circ g)'(x) = f'(g(x))\cdot g'(x)$

$g(x)=x^3+1$

$g'(x)=3x^2$

LHS

$(f\circ g)'(x) = ?$

$(f\circ g)(x) = f(g(x))$

$= f(x^3+1)$

$f(x^3+1) = (x^3+1)^2 \to f(u)=u^2$

$\therefore (f\circ g)'(x) = 2(x^3+1)\cdot \frac{d}{dx}(x^3+1)$

$= 2(x^3+1)\cdot 3x^2$

$= (2x^3+2)\cdot 3x^2$

$= 6x^5+6x^2$

$f'(g(x)) = 6x^5+6x^2$

$g'(x)=3x^2$

$f'(g(x))\cdot g'(x)$.

Hence Chain Rule Verified

2) let $f(x) = \frac{1}{(3x^2-2x+1)^{100}}$ find $f'(x)$

$\frac{1}{x^n} = x^{-n}$

$= (3x^2-2x+1)^{-100}$

$(n-1)$

$(-100-1)$

-101

$f'(x) = -100\,(3x^2-2x+1)^{-101} \cdot \frac{d}{dx}(3x^2-2x+1)$

$= -100\,(3x^2-2x+1)^{-101} \cdot (6x-2)$

3) differenciate $\sqrt{x^3-5}$

$\sqrt{x} = x^{1/2}$

$(x^3-5)^{1/2}$

$= \frac{1}{2}(x^3-5)^{1/2-1} \cdot \frac{d}{dx}(x^3-5)$

$= \frac{1}{2}(x^3-5)^{-1/2} \cdot (3x^2)$ or $\frac{1}{2}\,3x^2\,(x^3-5)^{-1/2}$

4) Use chain rule to differentiate

$f(x) = \left((x^2+1)^{20}+1\right)^4$.

$= 4\left((x^2+1)^{20}+1\right)^3 \cdot \frac{d}{dx}\left((x^2+1)^{20}+1\right)$

$= 4\left((x^2+1)^{20}+1\right)^3 \cdot \left(20\,(x^2+1)^{19} \cdot \frac{d}{dx}(x^2+1)\right)$

$= 4\left((x^2+1)^{20}+1\right)^3 \cdot \left(20\,(x^2+1)^{19}\,2x\right)$

$= 4\left((x^2+1)^{20}+1\right)^3 \cdot \left(40x\,(x^2+1)^{19}\right)$

$= 160\,x\left((x^2+1)^{19}+1\right)^3\,(x^2+1)^{19}$

→ Fractional powers & implicit differentiation

The power rule still ~~...~~ holds when exponent is a fraction.

$\sqrt[5]{x} = x^{1/5}$

$\sqrt{x} = x^{1/2}$

1) $\frac{d}{dx} \to f(x) = 3\sqrt[5]{x}$.

$= 3x^{1/5}$

$= \frac{1}{5}\,3x^{\frac{1}{5}-1}$

$\frac{1}{5}-\frac{1}{1} = \frac{1-5}{5} = -\frac{4}{5}$

$= \frac{1}{5}\,3x^{-\frac{4}{5}}$

$= \frac{3}{5}x^{-4/5}$

→ Fractional power rule :-

$\frac{d}{dx}(x^r) = r\,x^{r-1}$ ($r \to$ R.no.)

$\left(\frac{p}{q}\right) \to \frac{2}{5}, \frac{7}{5}$ rational no.

2) $f(x) = 3x^2 + \frac{(x^2+x^{1/3})}{\sqrt{x}}$.

3) $f'(x) = \frac{d}{dx}(3x^2) + \frac{d}{dx}\left(\frac{x^2+x^{1/3}}{\sqrt{x}}\right)$ → Q.Rule.

$= 6x + \frac{\sqrt{x}\,\frac{d}{dx}(x^2+x^{1/3}) - (x^2+x^{1/3})\frac{d}{dx}(\sqrt{x})}{(\sqrt{x})^2}$

$x^{\frac{1}{3}-1} = x^{\frac{1-3}{3}} = x^{-2/3}$

$= 6x + \frac{\sqrt{x}\left(2x + \frac{1}{3}x^{-2/3}\right) - \left[(x^2+x^{1/3})\frac{1}{2\sqrt{x}}\right]}{x}$

$= \frac{6x + 2x\sqrt{x} + \frac{1}{3}x^{-2/3} - \left[(x^2+x^{1/3})\frac{1}{2\sqrt{x}}\right]}{x}$.

$= \frac{6x + 2x\sqrt{x} + \frac{1}{3}x^{-2/3} - \left[\frac{x^2+x^{1/3}}{2\sqrt{x}}\right]}{x}$ //

→ Rational Power a () Rule :-

$$\frac{d}{dx}\left(f(x)\right)^r = r\left[f(x)\right]^{r-1} \cdot f'(x)$$

Q/1) $g(x) = \left(9x^3+10\right)^{5/3}$

A) $= \frac{5}{3}\left(9x^3+10\right)^{\frac{5}{3}-1} \cdot \frac{d}{dx}\left(9x^3+10\right)$

$= \frac{5}{3}\left(9x^3+10\right)^{2/3} \cdot \left(27x^2\right)$

$= \frac{5}{3}\left(9x^3+10\right)$

2) $\frac{d}{dx} \rightarrow \left(\frac{x^{1/2}+x^{3/2}}{x^{3/2}+1}\right)$ (Q. Rule)

$= \frac{x^{3/2}+1 \cdot \frac{d}{dx}\left(x^{1/2}+x^{3/2}\right) - \left(x^{1/2}+x^{3/2}\right) \cdot \frac{d}{dx}\left(x^{3/2}+1\right)}{\left(x^{3/2}+1\right)^2}$

3) $= \frac{\left(x^{3/2}+1\right)\cdot\left(\frac{1}{2}x^{\frac{1}{2}-1}+\frac{3}{2}x^{\frac{3}{2}-1}\right) - \left(x^{1/2}+x^{3/2}\right)\cdot\left(\frac{3}{2}x^{\frac{1}{2}}\right)}{\left(x^{3/2}+1\right)^2}$

$\frac{1}{2}-1 = \frac{1-2}{2}$

$\frac{3}{2}-1 = \frac{3-2}{2} = \frac{1}{2}$

$\cancel{\frac{3}{x+1}}$

$= \cancel{\frac{\left(x^{3/2}+1\right)\cdot\left(\frac{1}{2}x^{-1/2}+\frac{3}{2}x^{1/2}\right) - \left(x^{1/2}+x^{3/2}\right)\cdot\frac{3}{2}}{\left(x^{3/2}+1\right)}}$

$(a^n \cdot a^m = a^{n+m})$

$= \cancel{\left[\frac{1}{2}x^{-1/2}+\frac{3}{2}x^{1/2}\right] - \left[\frac{3}{2}x^{2/2}+\frac{3}{2}x^{4/2}\right]}$

$= \frac{\left(x^{3/2}+1\right)\cdot\left(\frac{1}{2}x^{-1/2}+\frac{3}{2}x^{1/2}\right) - \left(x^{1/2}+x^{3/2}\right)\cdot\frac{3}{2}x^{1/2}}{\left(x^{3/2}+1\right)^2}$

$= \frac{\left(\frac{1}{2}x^{-1/2}\cdot x^{\frac{3}{2}}\right)+\left(\frac{3}{2}x^{\frac{1}{2}}\cdot x^{\frac{3}{2}}\right)+\frac{1}{2}x^{-1/2}+\frac{3}{2}x^{1/2} - \left(x^{1/2}+x^{3/2}\right)\frac{3x^{1/2}}{2}}{\left(x^{3/2}+1\right)^2}$

$= \frac{\frac{1}{2}x^{-3/2}+\frac{3}{2}x^{3/2}+\frac{1}{2}x^{-1/2}+\frac{3}{2}x^{1/2} - \left[\frac{3}{2}x^{1/2}+\frac{3}{2}x^{1/2}\right]}{\left(x^{3/2}\right)^2+2x^{3/2}\times 1+1^2}$

$= \frac{\frac{1}{2}x^{\frac{2}{2}}+\left(\frac{1}{2}x^{-1/2}\right)+\frac{3}{2}\cancel{x^{4/2}}+\frac{3}{2}x^{1/2}-\frac{3}{2}x^{2/2}-\frac{3}{2}\cancel{x^{4/2}}}{\left(x^{3/2}\right)^2+2x^{3/2}+1}$

$= \frac{\frac{1}{2}x^{-1/2}+\frac{3}{2}x^{1/2}-\frac{3}{2}x^{2/2}+\frac{1}{2}x^{2/2}}{\left(x^{3/2}\right)^2+2x^{3/2}+1}$

$= \frac{\frac{1}{2}x^{-1/2}+\frac{3}{2}x^{1/2}-\frac{2x}{2}}{x^{3/2\times 2}+2x^{3/2}+1}$

$= \frac{\frac{x^{-1/2}+3x^{1/2}-2x}{2}}{x^3+2x^{3/2}+1}$

$= \frac{x^{-1/2}+3x^{1/2}-2x}{2\left(x^3+2x^{3/2}+1\right)}$

2) 1) If $y = f(x)$ and $x^2 + y^2 = 1$
express $\frac{dy}{dx}$ in terms of x & y?

A) $y = f(x)$ (give $\frac{d}{dx}$ on both sides)

$x^2 + y^2 = 1$

$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(1)$.

$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(1)$.

$\frac{d}{dx}(y^2)$ (y not related to x) $\Rightarrow 2y \cdot \frac{dy}{dx}$

$2x + 2y\frac{dy}{dx} = 0$ (-2)

$2y \cdot \frac{dy}{dx} = -2x$.

$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

3)

2) Find the eq of tangent line to the curve $2x^6 + y^4 = 9xy$ at the point $(1,2)$

A) $2x^6 + y^4 = 9xy$ at $(1,2)$

Tangent line eq,

$y_0 = 2$

$x_0 = 1$

$y - y_0 = m(x - x_0)$

m = slope.

$2x^6 + y^4 = 9xy$.

$\frac{d}{dx}(2x^6) + \frac{d}{dx}(y^4) = \frac{d}{dx}(9xy)$

$12x^5 + 4y^3 \cdot \frac{dy}{dx} = 9 \cdot (x \cdot y)$ Product

$\frac{d}{dx}(y) = 1 \cdot \frac{dy}{dx}$

$= 9\left(x \cdot \frac{dy}{dx} + y \cdot \frac{d}{dx}(x)\right)$

$= 9\left(x\frac{dy}{dx} + y\right)$

$12x^5 + 4y^3 \cdot \frac{dy}{dx} = 9x\frac{dy}{dx} + 9y$

$\Rightarrow 4y^3\frac{dy}{dx} - 9x\frac{dy}{dx} = 9y - 12x^5$

$\frac{dy}{dx}(4y^3 - 9x) = 9y - 12x^5$

$(1,2)$ $y = 2$ $x = 1$

$\frac{dy}{dx} = \frac{9y - 12x^5}{4y^3 - 9x}$

$= \frac{9\times2 - 12\times1}{4\times2^3 - 9\times1}$

$= \frac{18-12}{32-9} = \frac{6}{23}$

$\therefore m = \frac{6}{23}$

$\therefore y - y_0 = m(x - x_0)$

$y - 2 = \frac{6}{23}(x - 1)$

$$y - 2 = \frac{6}{23}x - \frac{6}{23}$$

$$y = 2 + \frac{6}{23}x - \frac{6}{23}$$

$$2 - \frac{6}{23} = \frac{46-6}{23} = \frac{40}{23}$$

$$y = \frac{6}{23}x + \frac{40}{23}$$

3) Suppose that x & y are () of t and that $x^4 + xy + y^4 = 1$. Relate $\frac{dx}{dt}$ and dy/dt?

A) $x^4 + xy + y^4 = 1$.

$$\frac{d}{dt}(x^4) + \frac{d}{dt}(xy) + \frac{d}{dt}(y^4) = \frac{d}{dt}(1).$$

$$4x^3 \frac{dx}{dt} + x \cdot \frac{dy}{dt} + y \cdot \frac{dx}{dt} + 4y^3 \frac{dy}{dt}$$

$$x \cdot \frac{dy}{dt} + 4y^3 \frac{dy}{dt} = -4x^3 \frac{dx}{dt} - y \frac{dx}{dt}$$

$$\frac{dy}{dt}(x + 4y^3) = \frac{dx}{dt} - (4x^3 - y)$$

$$\frac{dy}{dt} = \frac{-4x^3 - y}{x + 4y^3} \frac{dx}{dt}$$

Implicit Differenciation :-

To cal $\frac{dy}{dt}$ if x & y are related by an eq -

1) Differenciate both side of eq with respect to x, thinking of y as a () of x & using chain rule.
2) Solve the resulting eq for $\frac{dy}{dx}$.

Related Rates :-

To relate the rates $\frac{dx}{dt}$ & $\frac{dy}{dt}$ if x & y satisfy a given eq.

1) Differenciate both side of eq with respect to 't' thinking of x & y as () of 't'.
2) Solve the result in eq for $\frac{dy}{dt}$ in terms of $\frac{dx}{dt}$.

Parametric curves :- () always in 't'.

As t varies 2 eqs $x = f(t)$, $y = g(t)$ describe a curve in plane → p. curve

The slope of its tangent line is given by,

$$\boxed{\frac{dy}{dx} = \frac{dy/dt}{dx/dt}} \quad \text{if } \frac{dx}{dt} \neq 0$$

$m = \frac{dy}{dx}$

7) find the eq of the line tangent the P. curve given by the eqs

$x = (1+t^3)^4 + t^2$ at $t = 1$.

$y = t^5 + t^2 + 2$

T.C

$\boxed{y - y_0 = m(x - x_0)}$ — (a)

A) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ — (1)

$\frac{dy}{dt} = ?$

$\frac{d}{dt}(t^5 + t^2 + 2)$

$= 5t^4 + 2t$.

$\frac{dx}{dt} = ?$

$\frac{d}{dt}(1+t^3)^4 + t^2$

$= 4(1+t^3)^3 \cdot \frac{d}{dt}(1+t^3) + 2t$.

$= 4(1+t^3)^3 \, 3t^2 + 2t$.

3)

Sub in (1)

$\frac{dy}{dx} = \frac{5t^4 + 2t}{4(1+t^3)^3 3t^2 + 2t}$ $t = 1$.

$= \frac{5\times1 + 2\times1}{4(1+1)^3 3\times1 + 2\times1} = \frac{5+2}{96+2} = \frac{7}{98}$

$m = \frac{1}{14}$

y_0 ?

$y_0 = t^5 + t^2 + 2$

$= 1 + 1 + 2 = 4$

$t = 1$ x_0 ?

$x_0 = (1+t^3)^4 + t^2$

$= (1+1)^4 + 1^2$

$= 17$

Sub (a)

$y - y_0 = m(x - x_0)$

$y - 4 = \frac{1}{14}(x - 17)$

$y - 4 = \frac{1}{14}x - \frac{17}{14}$

$\cancel{\frac{1}{14}x = \frac{-17}{14}(y-4)}$

$y = \frac{1}{14}x - \frac{17}{14} + \frac{4\times14}{1\times14}$

$y = \frac{1x - 17 + 56}{14}$

$y = \frac{1}{14}x + \frac{39}{14}$

) Suppose that x & y are (). of time & that (x,y) moves on the circle $x^2+y^2=1$, if x is rising at 1 cm per sec. wht is the rate of change of y when $x = \frac{1}{\sqrt{2}}$ $y = \frac{1}{\sqrt{2}}$?

eq of circle

$x^2 + y^2 = r^2$

A) (based on time 't' → d/dt)

$x^2+y^2=1$

$\frac{d}{dt}(x^2)+\frac{d}{dt}(y^2)=\frac{d}{dt}(1)$

$2x\cdot\frac{dx}{dt}+2y\cdot\frac{dy}{dt}=0$

$2y\cdot\frac{dy}{dt}=-2x\cdot\frac{dx}{dt}$

$\therefore \frac{dy}{dt}=\frac{-2x}{2y}\cdot\frac{dx}{dt}$

$\frac{dy}{dt}=\frac{-x}{y}\cdot\frac{dx}{dt}$

at $x=\frac{1}{\sqrt{2}}$ $y=\frac{1}{\sqrt{2}}$

$\frac{dy}{dt}=\frac{-1/\sqrt{2}}{1/\sqrt{2}}\cdot\frac{dx}{dt}$.

$=-1\cdot\frac{dx}{dt}=-\frac{dx}{dt}$. → x rising 1 cm $=\underline{-1\ cm}$

(P.T)

⇒ **Antiderivatives :-**

An Antiderivative of 'f' is a () whose derivative is f

(to find anti find its ∫.)

1) find a () whose derivative is $2x+3$

A) $\int 2x+3\cdot dx$

$=\cancel{2}\cdot\frac{x^2}{\cancel{2}}+3x \Rightarrow \underline{x^2+3x}$

2) find the general anti derivative for the (). $f(x)=x^4+6$.

A) ⇒ $\underline{\frac{x^5}{5}+6x+C}$

⇒ Anti $\frac{d}{dx}$ & Indefinite integrals :-

* An A.D for f is a () 'F' such that $\boxed{F'(x)=f(x)}$. we write $F(x)=\int f(x)\,dx$.

* The (), $\int f(x)\cdot dx$ → The Indefinite of f.

f → The integrant.

* If 'F' is an anti deri~ $f(x)$, the general A.D has the form,

$F(x)+C$ (C → constnt)

→ **A.D rules :-**

1) Sum rule → $\int[f(x)+g(x)]\cdot dx=\int f(x)\,dx+\int g(x)\cdot dx$.

2) Constant M. rule → $\quad c \to$ Constant

$$\int a\, f(x)\, dx = a \int f(x)\, dx$$

3) Power rule → $\quad (n \neq -1)$

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + c$$

4) Polynomial →

$$\int (C_n x^n + \dots + C_1 x + C_0)\, dx = \frac{C_n}{n+1} x^{n+1} + \dots + \frac{C_1}{2} x^2 + C_0 x + c.$$

Q

1) find $\int \left[\frac{1}{x^2} + 3x + 2 - \frac{8}{\sqrt{x}} \right] dx.$

$$\boxed{\frac{1}{x^k} = x^{-k}}$$

$\frac{1}{x^2} = x^{-2}$

$\frac{8}{\sqrt{x}} = 8 \cdot \frac{1}{\sqrt{x}} = 8 \cdot x^{-1/2}$

A) $\int x^{-2} + 3\frac{x^2}{2} + 2x - 8 \cdot x^{-1/2}$

$= \frac{x^{-1}}{-1} + 3\frac{x^2}{2} + 2x - 8\frac{x^{-1/2}}{-1/2}$

$= \frac{-1}{x} + \frac{3x^2}{2} + 2x - \frac{8x^{1/2}}{1/2}$

$= \frac{-1}{x} + \frac{3x^2}{2} + 2x - 16\sqrt{x} + c$

$= -\frac{1}{x} + \frac{1}{1 \times 2} x^2$

$-\frac{1}{x} + \frac{x^2}{2}$

2) find $\int \frac{dx}{(3x+1)^5}$

$(3x+1)^{-5} \cdot dx.$

$\frac{(3x+1)^{-5+1}}{-5+1}$

$= \frac{\frac{(3x+1)^{-4}}{-4}}{\frac{d}{dx}(3x+1)}$ chain rule $= \frac{\frac{(3x+1)^{-4}}{-4}}{3} = \frac{(3x+1)^{-4}}{-4 \times 3}$

$= \frac{(3x+1)^{-4}}{-12} \Rightarrow \frac{1}{-12(3x+1)^4}$

$\frac{\frac{a}{b}}{c} = \frac{a}{b \times c}$

3) Prove the power rule $\int x^n\, dx = \frac{x^{n+1}}{n+1} + c.$

A) $F(x) = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$

$F'(x) = \frac{d}{dx}\left(\frac{x^{n+1}}{n+1} + c\right)$

$= \frac{1}{n+1}\left[(n+1)\, x^{n+1-1}\right] = x^n$

$F'(x) = x^n$

$\int x^n\, dx = F(x)$

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + c$$

4) $\int x^{3/2} - \sqrt{x} \xrightarrow{} x^{1/2} \; dx.$

$\frac{3}{2} + \frac{1}{1} = \frac{3+2}{2} = 5/2$

A) $\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} - \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}$

$\frac{x^{5/2}}{5/2} - \frac{x^{3/2}}{3/2}$

$= 2\frac{x^{5/2}}{5} - 2\frac{x^{3/2}}{3} + C$

5) $\int \frac{t^3 - t + 2}{t^6} \, dx.$

$\frac{a^m}{a^n} = a^{m-}$

A) $\int \frac{t^3}{t^6} - \frac{t}{t^6} + \frac{2}{t^6} \, dx.$

$= \int t^{3-6} - t^{1-6} + 2t^{-6} \, dx.$

$= \int t^{-3} - t^{-5} + 2t^{-6} \, dx.$

$= \frac{t^{-3+1}}{-3+1} - \frac{t^{-5+1}}{-5+1} + \frac{2t^{-6+1}}{-6+1}$

$= \frac{t^{-2}}{-2} - \frac{t^{-4}}{-4} + \frac{2t^{-5}}{-5}$

$= -\frac{1}{2t^2} + \frac{1}{4t^4} - \frac{2}{5t^5} + C$