$$P(x = 2 \text{ or } 3) = P(x=2) + P(x=3)$$

$$= \frac{e^{-3}\, 3^2}{2!} + \frac{e^{-3}\, 3^3}{3!}$$

$$= \frac{e^{-3}\, 9}{2} + \frac{e^{-3}\, 27}{6}$$

$$= 0.04979 \times 4.5 + 0.04979 \times 4.5$$

$$= \underline{\underline{0.448}}$$

$\Rightarrow$ **Mode of P.D :-**

Mode is the value of the random variable having max (prob)

If x is the mode, then $f(x)$ will be max. This gives 2 conditions.

1) $f(x) \geq f(x+1)$ $\quad f(x)$ always ↑.
2) $f(x) \geq f(x-1)$

Pf/

1) $f(x) \geq f(x+1)$

$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$\frac{\cancel{e^{-\lambda}}\lambda^x}{x!} \geq \frac{\cancel{e^{-\lambda}}\lambda^{x+1}}{(x+1)!}$ $\qquad \lambda^{x+1} = \lambda^x \lambda^1$

$\frac{\lambda^x}{x!} \geq \frac{\lambda^x \lambda}{(x+1)!}$ $\qquad x!$ & $(x+1)!$ big. $\therefore (x+1)\,x!$

$\frac{\lambda^x}{x!} \geq \frac{\lambda^x \lambda}{(x+1)x!}$

$1 \geq \frac{\lambda}{(x+1)}$ or

$(x+1) \geq \lambda$ or

$\lambda \leq (x+1)$ $\quad x \geq \lambda - 1$ — ①

2) $f(x) \geq f(x-1)$

$\frac{\cancel{e^{-\lambda}}\lambda^x}{x!} \geq \frac{\cancel{e^{-\lambda}}\lambda^{x-1}}{(x-1)!}$ $\qquad \lambda^{x-1} = \lambda^x \cdot \lambda^{-1} = \frac{\lambda^x}{\lambda}$

$\frac{\lambda^x}{x!} \geq \frac{\lambda^x}{\lambda(x-1)}$ $\qquad x!$ & $x-1$ big $\therefore x(x-1)$

~~$1 \geq \frac{\lambda}{(x-1)}$~~

~~$(x-1) \geq \lambda$~~

~~$\lambda \leq (x-1)$~~

$\frac{1}{x(x-1)!} \geq \frac{1}{\lambda(x-1)!}$

$\frac{1}{x} \geq \frac{1}{\lambda}$

$\lambda \geq x.$

$x \leq \lambda$ — ②

$x \to$ mode
$\lambda \to$ integr

from ① & ②,

$\lambda - 1 \leq x \leq \lambda$

~~$x = r$~~

~~$r \leq \lambda \leq r+1$~~

* If x is a integr. $\Rightarrow$ 2 mode
* if x is a float $\Rightarrow$ 1 mode. eg $\to$ 4.5 mode is 4

* If λ is an integr 'r' we gt 2 modes r & $r-1$.
* If λ is not an integr, the intgr part of λ will be mode.

$\Rightarrow$ Addive property :-

If x follows P.D λ_1 & y follows P.D λ_2,

If $x \to P(\lambda_1)$; $y \to P(\lambda_2)$ & if x & y are independent then $x+y \to P(\lambda_1+\lambda_2)$

$\boxed{M_x^{(t)} = e^{\lambda(e^t-1)} \text{ (moment)}}$

$x \to P(\lambda_1)$

$M_x^{(t)} = e^{\lambda_1(e^t-1)}$

$y \to P(\lambda_2)$

$M_y^{(t)} = e^{\lambda_2(e^t-1)}$

$x+y \to P(\lambda_1+\lambda_2)$

$M_{x+y}(t) = e^{(\lambda_1+\lambda_2)(e^t-1)} \longrightarrow$ To prove

Since x & y are independent,

$$\boxed{M_{x+y}(t) = M_x(t) \cdot M_y(t)}$$

$$= e^{\lambda_1(e^t-1)} \cdot e^{\lambda_2(e^t-1)}$$

$(a^m \cdot a^n = a^{m+n})$

$$= e^{\lambda_1(e^t-1)+\lambda_2(e^t-1)}$$

(common) $= e^{(e^t-1)(\lambda_1+\lambda_2)}$

$$\therefore \boxed{M_{x+y}(t) = e^{(\lambda_1+\lambda_2)(e^t-1)}}$$

$\Rightarrow$ Moment generative ( ) :-

$$M_x(t) = e^{\lambda(e^t-1)}$$

pf

$M_x(t) = E(e^{tx})$ $\qquad E(x) = \Sigma x \cdot f(x)$

$$= \sum_{x=0}^{\infty} e^{tx} \cdot f(x)$$

$$= \sum_{x=0}^{\infty} e^{tx} \cdot \frac{e^{-\lambda}\lambda^x}{x!}$$

$e^{tx}\lambda^x = (\lambda e^t)^x$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{e^{tx}\lambda^x}{x!} \qquad \boxed{\sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = e^{\lambda}}$$

$$= e^{-\lambda} \cdot e^{\lambda e^t}$$

$$= e^{-\lambda+\lambda e^t} \Rightarrow e^{\lambda e^t-\lambda}$$

$$\Rightarrow e^{\lambda(e^t-1)}$$

$\Rightarrow$ Recurrence relation for central moments :-

when $X \to P(\lambda)$

$$\boxed{\mu_{r+1} = \lambda\left[r\mu_{r-1} + \frac{d\mu_r}{d\lambda}\right]}$$

pf

$\mu_r = E(x-E(x))^r$ $\qquad E(x) = \Sigma x \cdot f x.$

$$= E(x-\lambda)^r$$

$$= \sum_{x=0}^{\infty} (x-\lambda)^r f(x).$$

$$\mu_r = \sum_{x=0}^{\infty} (x-\lambda)^r \cdot \frac{e^{-\lambda}\lambda^x}{x!}$$

$\frac{d\mu_r}{d\lambda} = ?$ (at a time $\frac{d}{d\lambda}$ ...)

$$\frac{d}{d\lambda}\left[\sum_{x=0}^{\infty} (x-\lambda)^r \frac{e^{-\lambda}\lambda^x}{x!}\right]$$

$$= \sum_{x=0}^{\infty} \frac{1}{x!}\left[\frac{d}{d\lambda} \underset{①}{(x-\lambda)^r}\ \underset{②}{e^{-\lambda}} \cdot \underset{③}{\lambda^x}\right]$$

$$= \sum_{x=0}^{\infty} \frac{1}{x!}\left[r(x-\lambda)^{r-1} \cdot -1 \cdot e^{-\lambda} \cdot \lambda^x + (x-\lambda)^r \cdot e^{-\lambda} \cdot -1 + \lambda^x + (x-\lambda)^r e^{-\lambda}\, x\lambda^{x-1}\right]$$

$$= -r\sum_{x=0}^{\infty} (x-\lambda)^{r-1} \frac{e^{-\lambda}\lambda^x}{x!} + \sum_{x=0}^{\infty} (x-\lambda)^r \frac{e^{-\lambda} \cdot -1 \cdot \lambda^x}{x!} + \sum_{x=0}^{\infty} x(x-\lambda)^r \frac{e^{-\lambda}\lambda^{x-1}}{x!}$$

common $\quad (\lambda^{x-1} = \lambda^x \cdot \lambda)$

$$= -r\,\mu_{r-1} + \sum_{x=0}^{\infty} (x-\lambda)^r \frac{e^{-\lambda}\lambda^x}{x!}\left[-1 + \frac{x}{\lambda}\right]$$

$$= -r\,\mu_{r-1} + \sum_{x=0}^{\infty} (x-\lambda)^r \frac{e^{-\lambda}\lambda^x}{x!}\left[\frac{x-\lambda}{\lambda}\right].$$

$$= -r\mu_{r-1} + \frac{1}{\lambda} \underbrace{\sum_{x=0}^{\infty} (x-\lambda)^{r+1} \frac{e^{-\lambda}\lambda^x}{x!}}_{\mu_{r+1}}$$

$$\frac{d\mu_r}{d\lambda} = -r\mu_{r-1} + \frac{1}{\lambda}\mu_{r+1}$$

$$\frac{1}{\lambda}\mu_{r+1} = \frac{d\mu_r}{d\lambda} + r\mu_{r-1}$$

$$\mu_{r+1} = \lambda\left[r\mu_{r-1} + \frac{d\mu_r}{d\lambda}\right]$$

Q 1) Fit a P.D to following data. →

No. of accident (x)	No. of men (f)	fx
0	95	0
1	75	75
2	44	88
3	18	54
4	2	8
5	1	5
	235	230

A) $\lambda \to$ mean of P.D.

$$\lambda = \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{230}{235} = 0.978$$

$$\lambda = 0.978 \approx 0.98$$

$$\boxed{f(x) = N \cdot \frac{e^{-\lambda}\lambda^x}{x!}}$$

$$= 235 \cdot \frac{e^{-0.98}\cdot(0.98)^x}{x!}$$

$f(x=0) = 235 \cdot \frac{e^{-0.98}\cdot(0.98)^0}{0!}$ $0! \to 1$.

$= 88.1$

$f(x=1) = 235 \cdot \frac{e^{-0.98}\cdot(0.98)^1}{1!}$

$= 86.37$

$f(x=2) = 235 \cdot \frac{e^{-0.98}\cdot(0.98)^2}{2!}$

$= 225.694 \times 2.664$ fact (5) $5! \to 5 + \text{shift} + x^{-1}$

$= 42.00$

$f(x=3) = 13.81$

$f(x=4) = 3.38$

$f(x=5) = 0.66$

2) The record of birth over the last 100 yrs maintained by the municiple council of a town showed that 200 children were born blind during that period. On the assumption that no. of children borned blind in a yr follows P.D. Estimate no. of yrs in which there were -

1) no blind births (4) more than 2 b. birth.
2) 1 " "
3) 2 " "

$(e^{-2} = 0.13534)$

A) (avg o/p (prob) × N.)

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

avg $= \lambda =$ mean $= \frac{200}{100} = 2$

1) $f(x=0) = \frac{e^{-2}\, 2^0}{0} = e^{-2} = 0.13534$

$\therefore$ no. of yrs $= 100 \times 0.13534$
$= 13.534 = 14$

2) $f(x=1) = \frac{e^{-2}\, 2^1}{1!} = 0.27$
yrs $= 100 \times 0.27 = 27.06 = 27$

3) $f(x=2) = \frac{e^{-2}\, 2^2}{2!} = 0.27$
yr $= 100 \times 0.27 = 27.06$
$= 27$

4) $f(x>2) = 1 - \left[f(x=0) + f(x=1) + f(x=2)\right]$
$= \cancel{1 - [14 + 27 + 27]}$
(no yrs)
$= 1 - [0.13534 + 0.27.06 + 0.27.06]$
$= 32$ yrs.

3) If x has PD with paramtr $\lambda > 0$
& $\mu_k' = E(x^k)$ then S.T
$$\mu_{k+1}' = \lambda\left[\mu_k' + \frac{d\mu_k'}{d\lambda}\right].$$

A) $\mu_k' = E(x^k)$
$= \sum x^k \cdot f(x)$
$\mu_k' = \sum_{x=0}^{\infty} x^k \cdot \frac{e^{-\lambda}\lambda^x}{x!}$

$$\frac{d\mu_k'}{d\lambda} = \frac{d}{d\lambda}\left[\sum_{x=0}^{\infty} x^k \cdot \frac{e^{-\lambda}\lambda^x}{x!}\right]$$

$\frac{d}{dx}(x-1)$
$1-0=1$

$= \sum_{x=0}^{\infty} \frac{1}{x!}\left[x^k \cdot e^{-\lambda}\lambda^x\right]$
$= \sum_{x=0}^{\infty} \frac{x^k}{x!}\left[e^{-\lambda}\cdot\lambda^x\right]$ (u.v rule).
$= \sum_{x=0}^{\infty} \frac{x^k}{x!}\left[e^{-\lambda}\cdot x\lambda^{x-1} + \left(-e^{-\lambda}\cdot\lambda^x\right)\right]$
$e^{-\lambda}\cdot x\lambda^{x-1} + (e^{-\lambda}\lambda^x)$
$= \sum_{x=0}^{\infty} \frac{x^k}{x!}\left[e^{-\lambda}\lambda^x\left[\frac{x}{\lambda}\right]-1\right]$
$\cancel{= \sum_{x=0}^{\infty} \frac{x^k}{x!}\left[e^{-\lambda}\frac{x}{\lambda}\left[\frac{x-\lambda}{\lambda}\right]\right]}$
$= \sum_{x=0}^{\infty} x^k \cdot \frac{e^{-\lambda}\lambda^x}{x!} \frac{(x-\lambda)}{\lambda}$
$= \sum \frac{x^k}{x!} e^{-\lambda}\lambda^x \cdot \left[-1 + \frac{x}{\lambda}\right]$
$= \sum \frac{x^k}{x!} e^{-\lambda}\lambda^x \cdot -1 + \sum \frac{x^k}{x!} e^{-\lambda}\lambda^x \cdot \frac{x}{\lambda}$
$\cancel{= \frac{1}{\lambda}\sum x^k}$
$= \sum \frac{x^k}{x!} e^{-\lambda}\lambda^x \frac{x}{\lambda} - \sum \frac{x^k}{x!} e^{-\lambda}\lambda^x - 1$

$$\frac{d\mu_k'}{d\lambda} = \frac{1}{\lambda}\underbrace{\sum x^{k+1}\frac{e^{-\lambda}\lambda^x}{x!}} - \underbrace{\sum x^k \frac{e^{-\lambda}\lambda^x}{x!}}_{\mu_k'}$$

$$\frac{d\mu_k'}{d\lambda} = \frac{1}{\lambda}\mu_{k+1}' - \mu_k'$$

$$\frac{1}{\lambda}\mu_{k+1} = \mu_k' \cdot \frac{d\mu_k'}{d\lambda}$$

$$\mu_{k+1} = r\left[\mu_k' + \frac{d\mu_k}{d\lambda}\right]$$

Q) If X & Y are indepent poisson variable & T conditional distri- of X gvn $x+y$ is binomial?.

A) $\cancel{P(X/X+Y) = f(x)}$?

To prove $\rightarrow P(X/X+Y) = {}^nC_x\, P^x q^{n-x}$?.

$$P(X|Y) = \frac{P(X,Y)}{P(Y)}$$

$$P(X=x/X+Y=n) = \frac{P(X=x, Y=n-x)}{P(X+Y=n)}$$

$$= \frac{P(X=x) + P(Y=n-x)}{P(X+Y=n)} \quad \text{— ①}$$

(a)

$X \rightarrow P(\lambda_1) \rightarrow P(x) = \dfrac{e^{-\lambda_1}\lambda_1^x}{x!}$

$Y \rightarrow P(\lambda_2)$

$y = n-x$ $\quad \rightarrow P(y) = \dfrac{e^{-\lambda_2}\lambda_2^{n-x}}{(n-x)!}$

$X+Y \rightarrow P(\lambda_1+\lambda_2)$

$\rightarrow P(X+Y) = \dfrac{e^{-(\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^n}{n!}$

— (a) $(x+y=n)$

① becomes,

$$= \frac{\dfrac{e^{-\lambda_1}\lambda_1^x}{x!} \cdot \dfrac{e^{-\lambda_2}\lambda_2^{n-x}}{(n-x)!}}{\dfrac{e^{-(\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^n}{n!}}$$

$$\cancel{= \frac{e^{-\lambda_1}\lambda_1^x \cdot e^{-\lambda_2}\lambda_2^{n-x}}{x!\,(n-x)!} \cdot \frac{n!}{e^{-(\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^n}}$$

$$\cancel{= \frac{n!}{x!(n-x)!}\,\lambda_1^x \lambda_2^{n-x}}$$

$$= \frac{e^{-\lambda_1}\cdot e^{-\lambda_2}\cdot \lambda_1^x \lambda_2^{n-x}}{x!\,(n-x)!} \cdot \frac{n!}{e^{-(\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^n}$$

$$= \frac{\cancel{e^{-(\lambda_1+\lambda_2)}}\,\lambda_1^x\,\lambda_2^{n+x}\; n!}{x!\,(n-x)!\;\cancel{e^{-(\lambda_1+\lambda_2)}}\,(\lambda_1+\lambda_2)^n}$$

$$= \frac{n!}{x!(n-x)!}\;\frac{\lambda_1^x\lambda_2^{n-x}}{(\lambda_1+\lambda_2)^n} \qquad \text{let } n = x+n-x.$$

$$= \frac{n!}{x!(n-x)!}\;\frac{\lambda_1^x\lambda_2^{n-x}}{(\lambda_1+\lambda_2)^{x+n-x}}.$$

$$= \frac{n!}{x!(n-x)!}\;\frac{\lambda_1^x\,\lambda_2^{n-x}}{(\lambda_1+\lambda_2)^x\,(\lambda_1+\lambda_2)^{n-x}}$$

$$= {}^nC_x \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^x \left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{n-x}.$$

p^x & q^{n-x}

$$= {}^nC_x\; p^x\; q^{n-x}.$$

where, $p = \dfrac{\lambda_1}{\lambda+\lambda_2}$

$q = \dfrac{\lambda_2}{\lambda_1+\lambda_2}$.

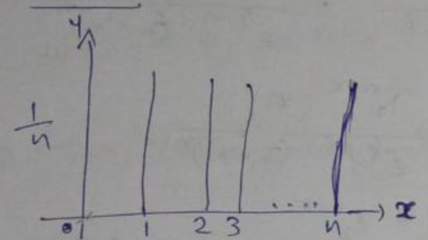
⇒ **Uniform Distribution :-**

This is the simplest type of discrete (prob) distri..., a discrete r.v 'x' is said to hve a uniform distri... if the (prob) density ( ) is gvn by,

$$\boxed{f(x) = \frac{1}{n}} \qquad x = 1, 2, \ldots, n.$$

$$= 0 \quad \text{elsewhere.}$$

graph



→ **Mean :-**

$$E(x) = \sum_{x=0}^{n} x \, f(x)$$

$$= \sum_{x=0}^{n} x \cdot \frac{1}{n}$$

$$= \frac{1}{n}\left[1+2+3+\cdots+n\right]$$

$$= \frac{1}{\cancel{n}}\left[\frac{\cancel{n}(n+1)}{2}\right] = \frac{n+1}{2}$$

$$\boxed{E(x) = \frac{n+1}{2}}$$

→ **Variance :-**

$$V(x) = E(x^2) - (E(x))^2.$$

$$E(x^2) = \sum_{x=0}^{n} x^2 \cdot f(x)$$

$$= \sum_{x=0}^{n} x^2 \cdot \frac{1}{n}$$

$$= \frac{1}{n}\left[1^2 + 2^2 + \cdots + n^2\right]$$

$$= \frac{1}{\cancel{n}}\left[\frac{\cancel{n}(n+1)(2n+1)}{6}\right]$$

$$E(x^2) = \frac{(n+1)(2n+1)}{6}$$

(common)

$$V(x) = \frac{(n+1)(2n+1)}{\underset{2\times3}{6}} - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{n+1}{2}\left[\frac{2n+1}{3} \times \frac{n+1}{2}\right]$$

$$= \frac{n+1}{2}\left[\frac{4n+2-3n+3}{6}\right]$$

$$= \frac{n+1}{2}\left[\frac{n-1}{6}\right]$$

$+ \cdot - = -$

$$\boxed{V(x) = \frac{n^2-1}{12}}$$

→ **Moment generating ( ) :-**

$$M_x^{(t)} = E(e^{tx})$$

$$= \sum_{x=0}^{n} e^{tx} \cdot f x = \sum e^{tx} \cdot \frac{1}{n}$$

$$= \sum_{x=0}^{n} \frac{1}{n}\left[e^{t} + e^{2t} + e^{3t} \cdots + e^{nt}\right]$$

$= \sum_{x=0}^{n} \frac{1}{n} e^t [\underbrace{1 + e^t + e^{2t} + \dots e^{nt}}_{\text{GP}}]$ —①

$S_n = \frac{a(r^n - 1)}{r-1}$ $\quad a = 1$ $\quad$ GP $= \frac{a(r^n-1)}{r-1}$

$r = e^t \rightarrow \frac{e^{2t}}{e^t} = e^t$)

$= \frac{1 (e^t)^n - 1}{e^t - 1}$

e^t

→①

$= \frac{1}{n} e^t \left[\frac{e^{tn} - 1}{e^t - 1}\right]$

$$\boxed{M_x(t) = \frac{e^t (e^{tn} - 1)}{n(e^t - 1)}}$$

⇒ Geometric Distribution :-

A r.v 'x' is defined to have (Geo) (pascal) distribution if pdf of x is given by,

$\boxed{f(x) = q^x \cdot p}$ $\quad x = 0, 1, 2 \dots$

$q + p = 1$

0, elsewhere.

(x — 0,1,2... [illegible] $q^x \cdot p$ [illegible] 0)

(Geo)

→ Mean :-

$E(x) = \sum x \cdot f(x)$

$= \sum_{x=0}^{\infty} x \cdot q^x \cdot p$ —ⓐ

$= p \sum_{x=0}^{\infty} x \cdot q^x$

$= p [0 + q + 2q^2 + 3q^3 + \dots]$

$= p \cdot q [1 + 2q + 3q^2 + \dots]$

$= p \cdot q \cdot \frac{1}{(1-q)^2}$

$1 - q = p$ $\quad = p \cdot q \cdot \frac{1}{p^2}$

$$\boxed{E(x) = \frac{q}{p}}$$

$1 + 2x + 3x^2 \dots = \frac{1}{(1-x)^2}$

$1 + 3x + 6x^2 \dots = \frac{1}{(1-x)^3}$

$1 + 4x + \dots = \frac{1}{(1-x)^4}$

→ Varience :-

$V(x) = E(x^2) - (E(x))^2$

$E(x^2) = \sum x^2 \cdot f(x) = \sum x^2 \cdot q^x \cdot p$

$[x^2 = x(x-1) + x]$

$= p \sum_{x=0}^{\infty} x^2 \cdot q^x$

$= p \sum_{x=0}^{\infty} [x(x-1) + x] \cdot q^x$

$= p \sum_{x=0}^{\infty} x(x-1) q^x + \sum_{x=0}^{\infty} x q^x$

$= p \sum_{x=0}^{\infty} x(x-1) q^x + \sum_{x=0}^{\infty} x q^x p$

$= p [2 \times 1 q^2 + 3 \cdot 2 q^3 + 4 \cdot 3 q^4 + \dots] + \frac{q}{p}$ —ⓐ

$= p [2q^2 + 6q^3 + 12q^4 + \dots] + \frac{q}{p}$

$= 2q^2 p [1 + 3q + 6q^2 + \dots] + \frac{q}{p}$

$= 2q^2 p \cdot \frac{1}{(1-q)^3} + \frac{q}{p}$

$= 2q^2 \cancel{p} \cdot \frac{1}{p^{\cancel{3}2}} + \frac{q}{p}$

$E(x^2) = \frac{2q^2}{p^2} + \frac{q}{p}$

$$V(x) = E(x^2) - (E(x))^2$$

$$= \frac{2q^2}{p^2} + \frac{qp}{p \times p} - \frac{q^2}{p^2}$$

$$= \frac{2q^2 + qp - q^2}{p^2} = \frac{q^2 + qp}{p^2}$$

$$= \frac{q(q+p)}{p^2}$$

$$\boxed{V(x) = \frac{q}{p^2}}$$

→ Moment generating ( ) :-

$$M_x(t) = E(e^{tx})$$

$$= \sum e^{tx} \cdot f(x) = \sum e^{tx} \cdot q^x \cdot p.$$

$$= p \sum e^{tx} \cdot q^x$$

$$= p \sum_{x=0}^{\infty} (qe^t)^x$$

$$= p \left[1 + (qe^t)^1 + (qe^t)^2 + \dots \right].$$

$$1 + x + x^2 + \dots = \frac{1}{(1-x)}$$

$$= p \frac{1}{(1-qe^t)}$$

$$\boxed{M_x(t) = \frac{p}{1-qe^t}}$$

⇒ Lack of Memory Property :-

If 'x' has Geometric density with paramtr 'p', then,

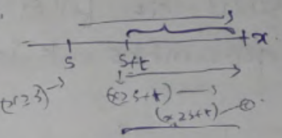
$$\boxed{P[x \geq s+t / x \geq s] = P[x \geq t)}$$

for $s, t = 0, 1, 2 \dots$

Pf/

$$P[\underset{(x)}{x \geq s+t} / \underset{(y)}{x \geq s}] = \frac{P(x \geq s+t, x \geq s)}{P(x \geq s)}$$

$$P(x/y) = \frac{P(x,y)}{P(y)}.$$



$$= \frac{P(x \geq s+t)}{P(x \geq s)}$$

$$= \frac{\sum_{x=s+t}^{\infty} q^x \cdot p}{\sum_{x=s}^{\infty} q^x \cdot p.}$$

$$= \frac{\cancel{p} \sum_{x=s+t}^{\infty} q^x}{\cancel{p} \sum_{x=s}^{\infty} q^x} = \frac{\sum_{x=s+t}^{\infty} q^x}{\sum_{x=s}^{\infty} q^x}.$$

$$= \frac{q^{s+t} + q^{s+t+1} + q^{s+t+2} + \dots}{q^s + q^{s+1} + q^{s+2} + \dots}$$

$$= \frac{q^{s+t} \left[\cancel{1 + q + q^2 + \dots} \right]}{q^s \left[\cancel{1 + q + q^2 + \dots} \right]}$$

$$= \frac{q^{s+t}}{q^s} = \frac{\cancel{q^s} \cdot q^t}{\cancel{q^s}} = q^t = P(X \geq t)$$

⇒ **Negative binomial distribution :-**

Let 'x' be the discrete R.V, if the assuming the values 0, 1, 2 ...., if the pmf of x is given by,

~~$p(x) = {}^{x+k-1}C_{k-1}$~~

$$\boxed{p(x) = {}^{x+k-1}C_{k-1}\, p^k q^x} \quad x = 0, 1, 2 \ldots \quad p + q = 1$$

then x is said to follow a -ve b.distri... with paramters k & p.

Note,

* -ve b.distri... can also be written as

$$\boxed{p(x) = -{}^{k}C_x\, p^k (-q)^x}, \quad x = 0, 1, 2 \ldots$$

* In the defi... of -ve b.distri...., if we take $\underline{k=1}$, $p(x)$ becomes

$$p(x) = {}^{x+1-1}C_{1-1}\, p^1 q^x.$$

$$= {}^{x}C_0\, p\, q^x$$

$$p(x) = p\, q^x \qquad \boxed{p(x) = q^x p} \quad x = 0, 1, 2 \ldots$$

This is the (prob) () of Geometric distri...

→ **Properties :-**

1) Mean = $\boxed{E(x) = \frac{kq}{p}}$

$$E(x^2) = \frac{k(k+1)q^2}{p^2} + \frac{kq}{p}$$

2) Variance = $\boxed{V(x) = \frac{kq}{p^2}}$

3) -ve b.distri... tends to poission distri... under certain condi...

4) Geometric distri... is a particular case of -ve b.distri....

5) -ve b.distri... → binomial waiting time distri....

Discrete

① Binomial ② poission ③ Geometric ④ Bernoulli

⑤ uniform ⑥ -ve binomial

⇒ **Continuous (prob) distri... :-**

I) **Normal distri.... :-**

A R.V 'x' is defined to be normally distributed, if its PDF is given by,

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-(x-\mu)^2}{2\sigma^2}}} \quad -\alpha < x < \alpha.$$

μ = mean

σ = S.D

μ & σ satisfy $-\infty < \mu < \infty$ & $\sigma > 0$

when x follows N. distri... we write its symbollically as,

$X \rightarrow N(\mu, \sigma)$ or

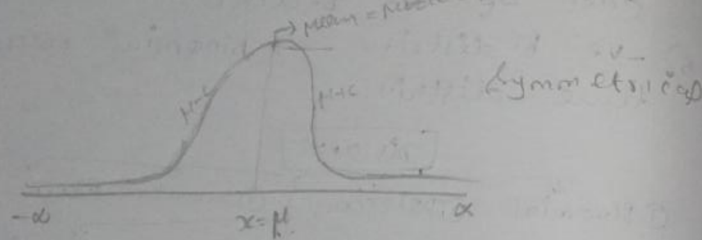
$x \rightarrow N(\mu, \sigma^2)$

→ Properties :-

* Graph of N. distri... is gvn by,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

is a bell-shaped smooth symmetrical curve → normal curve as shown below.



* Some of imp pro) are. →

* Normal curve symmetrical about $x=\mu$, (i.e) $f(\mu+c) = f(\mu-c)$ for any c.
* Mean, Mode & Median are identical.
* Normal curve f(x) has a max at $x=\mu$ & max value of the ordinates is $\frac{1}{\sigma\sqrt{2\pi}}$
* Normal curve extends from $-\infty$ to $+\infty$.
* curve touches the x axis only at $\pm\infty$. (ie) X-axis is an asymptote to the curve.
* for a N. distri... $\boxed{\beta_1 = 0, \beta_2 = 3}$

evn → 2r
odd → 2r+1

* $\boxed{QD = \frac{2}{3} \cdot SD, \quad MD = \frac{4}{5} \cdot SD}$
* All odd order central moments are vanish, (i.e) $\boxed{\mu_{2r+1} = 0}$, $r = 0, 1, 2 \ldots$
* The evn order central moments are gvn by, $\mu_{2r} = 1 : 3 : 5 : \ldots (2r-1)\sigma^{2r}$.
* The points of inflection of curve are $x = \mu \pm \sigma$
* The lower & upper quartiles are equidistance from median.

2) ~~Rectangular distribution~~ :-