(P) → probability

Chapter : 02

probability Distribution.

let (x, y) be a pair of discreate bivariate random variable assuming pairs values $(x_1, y_1), (x_2, y_2) \dots (x_n, y_n)$ from the real plane then $f(x_i, y_j)$ or P_{ij}

→ **Joint probability mass ( ) of (x, y)** satisfies the (pro).

(1) $f(x_i, y_j) \geq 0$ for all (x_i, y_j)

(2) $\sum_{x_i} \sum_{y_j} f(x_i, y_j) = 1$.

→ **Joint probability Density ( ) :-**

Let x & y be 2 contin Random variable if $P\{x \leq X \leq x+dx, y \leq Y \leq y+dy\} = f(x,y)\, dx\, dy$. Then $f(x,y)$ → J.P.D ( ). of (x,y) & it has the following (pro).

(1) $f(x,y) \geq 0$ for all (x,y).

(2) $\int_x \int_y f(x,y) \cdot dx\, dy = 1$

Note –

$P(a \leq X \leq b,\ c \leq y \leq d) = \int_a^b \int_c^d f(x,y) \cdot dx\, dy$

→ **Marginal probability ( ) :-**

If x & y are 2 (dis) R.V with joint pdf $f(x,y)$ ~~then~~ the marginal (P) density ( ) of x is given by,

$f_1(x) = \sum_y f(x,y)$ → x, y → (dis)

$f_1(x) = \int_y f(x,y)\, dy$ → x, y → contin

$f_2(y) = \sum_x f(x,y)$ → x, y → (dis)

$f_2(y) = \int_x f(x,y)\, dx$ → x, y → contin.

→ **Conditional (P) ( ) :-**

The condit^n (P) distribution of x given $Y = y$ is given by,

$$\boxed{f(x|y) = \frac{f(x,y)}{f_2(y)}}$$

(f of x given by y), $f_2(y) > 0$.

& the condit^n (P) of y given $X = x$,

$$\boxed{f(y|x) = \frac{f(x,y)}{f_1(x)}}$$

Note →

$P(a \leq x \leq b \mid Y = y) = \int_a^b \frac{f(x,y)}{f_2(y)} \cdot dx$

$P(c \leq y \leq d \mid X = x) = \int_c^d \frac{f(x,y)}{f_1(x)} \cdot dy$

⇒ Independence of 2 R.V :-

2 R.V x & y with joint pdf $f(x,y)$ & marginal pdf $f_1(x)$ & $f_2(y)$ respectively → statistically independent if & only if $\boxed{f(x,y) = f_1(x) \cdot f_2(y)}$

The variables which are not statistically independent → statistically dependent.

Note

* when condit- pdf are known the independence of R.V can be decided when ever $\boxed{\begin{array}{l} f(x|y) = f_1(x) \\ f(y|x) = f_2(y). \end{array}}$

→ Joint Distribution (C) :-

Let (x,y) be a pair of r.v's either (dis) / contin. Joint distri- (C) of (x,y) is denoted by

$F(x,y)$ –

$$F(x,y) = P(X \le x, Y \le y).$$

$$= \sum_{-\infty}^{x} \sum_{-\infty}^{y} f(x,y) \qquad X,Y \to (dis)$$

$$= \int_{-\infty}^{x} \int_{-\infty}^{y} f(x,y)\,dx\,dy \qquad X,Y \to \text{contin.}$$

(pro) →

1) $F(+\infty, +\infty) = 1$
2) $F(-\infty, -\infty) = 0$
3) $F(-\infty, y) = F(x, -\infty) = 0$
4) $0 \le F(x,y) \le 1$
5) $F(x, \infty) = F(x)$
   $F(\infty, y) = F(y)$
6) whenever $F(x,y)$ is differentiate,
   $f(x,y) = \dfrac{\partial^2 F(x,y)}{\partial x \partial y}$
7) when X & Y are indepnt → $F(x,y) = F(x)\,F(y)$
8) for real no. a,b,c,d →
   $P\{a < X \le b,\ c < Y \le d\} = F(b,d) + F(a,d) - F(a,c) - F(b,c)$.

Q/ 1) x & y are R.V with joint pdf $f(x,y) = \dfrac{x+2y}{18}$ where $(x,y) = (1,1), (1,2), (2,1)\ (2,2)$ $= 0$, else where

Are the variables Independent ?.

A) $f(x,y) = \dfrac{x+2y}{18}$

$f(x,y) = f_1(x) \cdot f_2(y)$

$f_1(x) = ?$ $\qquad f_2(y) = ?$.

$x = 1,2 \qquad y = 1,2.$

$f_1(x) = \sum_y f(x,y)$

$= \sum_{y=1,2} \dfrac{x+2y}{18}$

$\overset{(y=1)}{=} \dfrac{x+2}{18} + \overset{(y=2)}{\dfrac{x+4}{18}}$

$$\frac{x+2+x+4}{18} = \frac{2x+6}{18}$$ (dis)

$$f_2(y) = \sum_x f(x,y)$$
$$= \sum_{x=1,2} \frac{x+2y}{18}$$
$$= \frac{1+2y}{18} + \frac{2+2y}{18}$$
$$= \frac{1+2y+2+2y}{18} = \frac{3+4y}{18}$$

$$f(x,y) = f_1(x) \cdot f_2(y)$$
$$= \left(\frac{2x+6}{18}\right)\left(\frac{3+4y}{18}\right) \neq \frac{x+2y}{18} = f(x,y).$$

(when x · 4y = 8xy not in ans. so ≠)

(ie) $f_1(x) \cdot f_2(y) \neq f(x,y)$

∴ X & Y are not Independent

2) If X & Y are R.V with joint pdf $f(x,y) = \frac{4}{5}\frac{x+y}{x^3}$, $0 \le y \le 1$, $1 \le x < \infty$

S.T $P\left\{(0 < y < \frac{1}{2}) \middle| X=2\right\} = \frac{9}{20}$

$P(y/x) =$

A) $f(y/x) = \frac{f(x,y)}{f_1(x)}$ (contin)

$f_1(x) = ?$.

$$f_1(x) = \int_y f(x,y) \cdot dy.$$
$$= \int_0^1 \frac{4}{5}\left(\frac{x+y}{x^3}\right) \cdot dy$$ (No ∫ with dy so constant)
$$= \frac{4}{5x^3} \int_0^1 (x+y) \cdot dy.$$ (x is constant ← so dy give xy)
$$= \frac{4}{5x^3}\left[xy + \frac{y^2}{2}\right]_0^1$$
$$= \frac{4}{5x^3}\left[\left(x+\frac{1}{2}\right) - (0+0)\right]$$
$$= \frac{4}{5x^3}\left[\frac{x}{1} + \frac{1}{2}\right]$$
$$= \frac{4}{5x^3} \cdot \frac{2x+1}{2}$$
$$f_1(x) = \frac{4}{10x^3}(2x+1)$$

$$P\left\{(0 < y < \frac{1}{2}) \middle| X=2\right\} = \int_0^{1/2} \frac{f(x,y)}{f_1(x)} dy.$$
$$= \frac{\frac{4}{5}\left(\frac{x+y}{x^3}\right)}{\frac{4}{10x^3}(2x+1)}$$

$x=2$. $$= \frac{\frac{4}{5}\,\frac{2+y}{8}}{\frac{4}{10 \cdot 8} \cdot 5} \cdot dy$$

$$= \int_0^{1/2} \frac{\frac{1}{10}(2+y)}{\frac{1}{4}} \cdot dy.$$

$$= \frac{\frac{1}{10}}{1/4} \int_0^{1/2} (2+y) \cdot dy$$

$\frac{4}{10} = \frac{2}{5}$

$$= \frac{1}{\cancel{10}_5} \cdot \frac{\cancel{4}^2}{1} \left(2y + \frac{y^2}{2}\right)_0^{1/2}$$

$$= \frac{2}{5} \left(2y + \frac{y^2}{2}\right)_0^{1/2}$$

$$= \frac{2}{5} \left(2 \times \frac{1}{2} + \frac{(1/2)^2}{2} - 0\right)$$

$$= \frac{2}{5} \left(1 + \frac{1/4}{2}\right)$$

$$= \frac{2}{5} \left(\frac{1}{1} + \frac{1}{8}\right) = \frac{2}{5} \left(\frac{8+1}{8}\right)$$

$$= \frac{\cancel{2}}{5} \cdot \frac{9}{\cancel{8}_4}$$

$$= \frac{9}{20}$$

3) 2 R.V x & y have joint pdf

$$p(x,y) = \begin{cases} k(x^2+y^2); & 0<x<2, \ 1<y<4 \\ 0, & \text{otherwise} \end{cases}$$

find k?

A) $\int_x \int_y f(x,y)\, dx\, dy = 1.$ (for finding constant) [find k in use this eq (contin..)].

$$\int_0^2 \int_1^4 k(x^2+y^2)\, dx\, dy = 1.$$

1st $\int$ with x then y.

or $\int$ y then x

$$k\int_0^2 \int_1^4 (x^2+y^2)\, dy\, dx = 1. \quad \text{with respect to } y.$$

(y) $$k\int_0^2 \left(x^2 y + \frac{y^3}{3}\right)_1^4 dx = 1.$$

x^2 const, $y \to \int 1$ with y

$y = 4, 1$ $$k\int_0^2 \left(4x^2 + \frac{64}{3}\right) - \left(x^2 + \frac{1}{3}\right) \cdot dx = 1.$$

$$k\int_0^2 \left(4x^2 + \frac{64}{3} - x^2 - \frac{1}{3}\right) dx = 1$$

$\frac{64}{3} - \frac{1}{3} = \frac{63}{3} = 21$

$$k\int_0^2 (3x^2 + 21)\, dx = 1.$$

($\int$ with x)

(x) $$k\left[\cancel{3}\frac{x^3}{\cancel{3}} + 21x\right]_0^2 = 1$$

$$k\left[2^3 + 21 \times 2\right] - 0 = 1$$

$$k\left[8 + 42\right] = 1$$

$$k\left[50\right] = 1$$

$$k = \frac{1}{50}$$

4) If x & y have joint pdf

$f(x,y) = x+y \quad 0<x,\ y<1$

find $P\{0<x<\frac{1}{2},\ \frac{1}{2}<y<1\}$.

A) $f(x,y) = x+y \quad 0<x,\ y<1$

$\quad\quad\quad\quad\quad\quad 0<x<1 \quad 0<y<1.$

$P\left(0<x<\frac{1}{2},\ \frac{1}{2}<y<1\right\} =$

$\int_0^{1/2}\int_{1/2}^{1} f(x,y)\,dx\,dy.$ (here not finding constant so. no = 1.)

$= \int_0^{1/2}\int_{1/2}^{1} (x+y)\,dy\,dx.$

$= \int_0^{1/2}\left(xy+\frac{y^2}{2}\right)_{1/2}^{1} dx.$

$y = 1, \frac{1}{2}$

$= \int_0^{1/2}\left[\left(x+\frac{1}{2}\right) - \left(\frac{x}{2} + \frac{(1/2)^2}{2}\right)\right] dx.$

$= \int_0^{1/2}\left(x+\frac{1}{2} - \frac{x}{2} - \frac{\frac{1}{4}}{2}\right) dx.$

$1-\frac{1}{2} = \frac{1}{2}$

$x - \frac{x}{2}$

$\frac{x}{2}$

$= \int_0^{1/2}\left(x+\frac{1}{2} - \frac{x}{2} - \frac{1}{4}\cdot\frac{1}{2}\right)$

$\frac{1}{8} \to \frac{1}{2} - \frac{1}{8} = \frac{8-2}{16} = \frac{6}{16} = \frac{3}{8}$

$= \int_0^{1/2}\left(\frac{x}{2}+\frac{3}{8}\right) dx$

$x/2 \to \frac{1}{2}x.$

$\int_0^{1/2}\left(\frac{1}{2}x+\frac{3}{8}\right) dx.$

$\left(\frac{1}{2}\frac{x^2}{2} + \frac{3}{8}x\right)_0^{1/2} \cdot \to \left(\frac{1}{2}\frac{x^2}{2}+\frac{3}{8}x\right)$

$= \left[\frac{1}{4}(1/2)^2 + \left(\frac{3}{8}\right)\left(\frac{1}{2}\right)\right] - 0$

$= \frac{1}{4}\cdot\frac{1}{4} + \frac{3}{16}$

$= \frac{1}{16} + \frac{3}{16} = \frac{4}{16} = \frac{1}{4}$

5) The joint pdf of 2 R.V X & Y

$\to f(x,y) = 24x(1-y), \quad 0<x<y<1$

$\quad = 0$, otherwise

Find marginal pdf's of X & Y?

A) $f(x,y) = 24x(1-y) \quad 0<x<y<1$

$\int_y \to \int_x$

$f_1(x) = ?$

$f_1(x) = \int_y f(x,y)\,dy$

$= \int_x^1 24x(1-y)\,dy.$

$= 24x\int_x^1 (1-y)\,dy.$

$= 24x\left(y-\frac{y^2}{2}\right)_x^1.$

$\Rightarrow 24x\left[\left(1-\frac{1}{2}\right) - \left(x-\frac{x^2}{2}\right)\right]$

$= 24x\left[\underbrace{\frac{1}{2} - x + \frac{x^2}{2}}\right]$

$= \cancel{24}^{12} x \cdot \frac{1}{\cancel{2}} - 24x^2 + \frac{24x^3}{2}$

$= 12x - 24x^2 + 12x^3$

$f_2(y) = ?$

$f_2(y) = \int_x f(x,y)\, dx.$ $\quad 0 < x < y < 1$

$= \int_0^y 24x(1-y)\, dx.$

$= (1-y)24\int_0^y x \cdot dx.$

$= (1-y)24\left(\frac{x^2}{2}\right) = 24(1-y)\left(\frac{x^2}{2}\right)_0^y$

~~$= (1-y)\cdot 12x^2$~~

$\Rightarrow 24(1-y)\left[\frac{y^2}{2} - 0\right] = \cancel{24}^{12}(1-y)\cdot\frac{y^2}{\cancel{2}}$

~~$24(1-y)\left[\frac{y^2}{2}\right]$~~ $= 12(1-y)y^2$

6) Let joint pdf of (X,Y) be

$(x,y) = (1,1)\ (1,2)\ (2,1)\ (2,2)\ (3,1)\ (3,3)$

$f(x,y) = \frac{1}{10}\ \frac{1}{5}\ \frac{2}{5}\ \frac{3}{20}\ \frac{1}{10}\ \frac{1}{20}$

a) verify whether X & Y are independent

b) compute $P(X \le 2 \mid Y \ge 2)$.

A) $f(x,y) = f_1(x)\cdot f_2(y).$

values of $x = 1,2,3$

" $y = 1,2,3.$

y\x	1	2	3	$f_2(y)$.
1	1/10	2/5	1/10	6/10
2	1/5	3/20	0	7/20
3	0	0	1/20	1/20
$f_1(x)$	3/10	11/20	3/20	1

always.

$x \to f_2(y) \to \Sigma$

$y \to f_1(x) \to \Sigma$

$(x,y) \to (1,1) \to \frac{1}{10}$

(P).

$f_1(x)$

$\frac{1}{10} + \frac{1}{5}\frac{\times 2}{\times 2} = \frac{1+2}{10} = \frac{3}{10}$

$\frac{2\times 4}{5\times 4} + \frac{3}{20} = \frac{8+3}{20} = 11/20.$

$\frac{1\times 2}{10\times 2} + \frac{1}{20} = \frac{2+1}{20} = 3/20$

$f_2(y)$

$\frac{1}{10} + \frac{2}{5}\frac{\times 2}{\times 2} + \frac{1}{10} = \frac{1+4+1}{10} = 6/10$

a) $f(x,y) = f_1(x) \cdot f_2(y)$.

$f_1(1) \cdot f_2(1) = \frac{3}{10} \cdot \frac{6}{10} = \frac{18}{100} \neq \frac{1}{10} = f(1,1)$

$f_1(1) \cdot f_2(1) \neq f(1,1)$

$\therefore$ x & y are not independent.

b) $P(x \le 2 \mid y \ge 2)$

$f(x|y) = \sum \frac{f(x,y)}{f_2(y)}$

$x = 1,2$; $y = 2,3$

$P(x \le 2 \mid y \ge 2) = \sum \frac{P(x \le 2,\ y \ge 2)}{P(y \ge 2)}$ $\rightarrow (2,3)$

(1,2) (2,3)
(1,3) (2,2)

$\therefore = \frac{P(1,2) + P(1,3) + P(2,2) + P(2,3)}{P(y=2) + P(y=3)}$

(table-ൽ y=2 and sum)

In table find (1,2), (2,3), (1,3), (2,2)

$= \frac{\frac{1}{5} + 0 + \frac{3}{20} + 0}{\frac{7}{20} + \frac{1}{20}} = \frac{\frac{1}{5} + \frac{3}{20}}{\frac{7}{20} + \frac{1}{20}}$

$= \frac{\frac{1}{5} + \frac{3}{20}}{8/20} = \frac{7/20}{8/20} = 7/8$

7) For the follow... bivariate (p) distri...

$y \backslash x$	1	2	3	4	5	6	$f_2(y)$
0	0	0	1/32	2/32	2/32	3/32	8/32
1	1/16	1/16	1/8	1/8	1/8	1/8	10/16
2	1/32	1/32	1/64	1/64	0	2/64	8/64
$f_1(x)$	3/32	3/32	11/64	13/64	6/32	16/64	1

Determine,

a) $P(X \le 4,\ y = 2)$

b) $P(x = 1,\ y \le 1)$

c) $P(X \ge 3)$

(d) $P(y < 2)$

(e) $P(X + y = 6)$

(f) $P(x = 4)$

(g) $P(x + y = 5 \mid y = 2)$

(h) $P(x + y > 4)$

(i) $P(x - y = 2)$

A) a) $P(x \le 4,\ y = 2)$

$x = 1,2,3,4$; $y = 2$.

$= P(1,2) + P(2,2) + P(3,2) + P(4,2)$

$= \frac{1 \times 2}{32 \times 2} + \frac{1 \times 2}{32 \times 2} + \frac{1}{64} + \frac{1}{64}$

$= \frac{2 + 2 + 1 + 1}{64} = \frac{6}{64}$

b) $P(x=1, y \le 1)$ $(1,0)$ $(1,1)$

$x=1$ $y=0,1$

$= P(1,0) + P(1,1)$

$= 0 + \frac{1}{16} = \frac{1}{16}$

c) $P(x \ge 3)$ $(3,4,5,6.)$

$= P(x=3) + P(x=4) + P(x=5) + P(x=6)$

$= \frac{11}{64} + \frac{13}{64} + \frac{6 \times 2}{32 \times 2} + \frac{16}{64}$

$= \frac{11 + 13 + 12 + 16}{64} = \frac{52}{64}.$

d) $P(y<2)$ $(y=0,1)$

$= P(y=0) + P(y=1)$

$= \frac{8}{32} + \frac{10 \times 2}{16 \times 2} = \frac{8+20}{32} = \frac{28}{32}$

e) $P(x+y=6)$

0, 1, 2, 3, 4, 5, 6

(0,6) (5,1) (4,2)

0+6=6 5+1=6 4+2=6

$= P(6,0) + P(5,1) + P(4,2)$

$\cancel{= \frac{8 \times 2}{32 \times 2} + \frac{1 \times 8}{8 \times 8} + \frac{1}{64} = \frac{16+8+1}{64} =}$

$= \frac{3 \times 2}{32 \times 2} + \frac{1 \times 8}{8 \times 8} + \frac{1}{64} = \frac{6+8+1}{64} = \frac{15}{64}$

f) $P(x=y)$

$x = 1,2$
$y = 1,2$
$(1,1)$ $(2,2)$
$(x=y)$
$1=1$
$2=2$

$= P(1,1) + P(2,2)$

$= \frac{1 \times 2}{16 \times 2} + \frac{1}{32} = \frac{2+1}{32} = \frac{3}{32}$

g) $P(x+y=5 \mid y=2)$

$f(x \mid y) = \sum \frac{f(x,y)}{f_2(y)}$

$x+y=5$ $y=2$
$3+2=5$
$x=3$

$P(x+y=5 \mid y=2) = \frac{P(x+y=5, y=2)}{P(y=2)}$

$= \frac{P(x=3, y=2)}{P(y=2)}$

$= \frac{\frac{11}{64} + \frac{8}{64}}{8/64} = \frac{19/64}{8/64}$

$= \frac{19}{8}$

h) $P(x+y>4)$ $(>4 \to 5, 6, 7, 8.)$

$= P(x+y=5) + P(x+y=6) + P(x+y=7) + P(x+y=8)$

(5,0) (4,1) (3,2) (6,0) (5,1) (4,2) (6,1) (5,2) (6,2)

$= P(5,0) + P(4,1) + P(3,2) + P(6,0) + P(5,1) + P(4,2) +$
$P(6,1) + P(5,2) + P(6,2)$

$= \frac{2^{\times 2}}{32^{\times 2}} + \frac{1^{\times 8}}{8 \times 8} + \frac{1}{64} + \frac{3^{\times 2}}{32^{\times 2}} + \frac{1^{\times 8}}{8 \times 8} + \frac{1}{64} +$

$\frac{1^{\times 8}}{8 \times 8} + 0 + \frac{2}{64} = \frac{4+8+1+6+8+1+8+2}{64}$

$= \frac{38}{64}$

i) $P(x-y=2)$ $(2,0)$ $(3,1)$ $(4,2)$

$= P(2,0) + P(3,1) + P(4,2)$

$= 0 + \frac{1^{\times 8}}{8 \times 8} + \frac{1}{64} = 9/64$

8) If (x,y) has joint density ().

$$f(x,y) = \begin{cases} \frac{1}{8}(6-x-y), & 0<x<2,\ 2<y<4 \\ 0, & \text{otherwise} \end{cases}$$

a) Determine Marginal densities?

b) Test the independence of x & y?

c) compute $P(X<1, Y<3)$, $P(X+Y<3)$, $P(X<1 / Y<3)$

A) $f(x,y) = \frac{1}{8}(6-x-y)$ $\int_0^2 x$ $\int_2^4 y$

$f_1(x) = \int_y f(x,y)\, dy$

$= \int_2^4 \frac{1}{8}(6-x-y) \cdot dy$

$= \frac{1}{8} \int_2^4 (6-x-y) \cdot dy$

$= \frac{1}{8}\left[6y - xy - \frac{y^2}{2}\right]_2^4$

$= \frac{1}{8}\left[6\times4 - x\cdot4 - \frac{4^2}{2} - \left(6\times2 - x\cdot2 - \frac{2^2}{2}\right)\right]$

$= \frac{1}{8}\left[24 - 4x - 8 - (12 - 2x - 2)\right]$

$= \frac{1}{8}\left[16 - 4x - (10 - 2x)\right]$

$= \frac{1}{8}\left[16 - 4x - 10 + 2x\right]$ $-4+2=-2$

$= \frac{1}{8}\left[6 - 2x\right]$

(or)

$\Rightarrow \frac{\cancel{2}(3-x)}{\cancel{8}\,4}$ $\boxed{f_1(x) = \frac{3-x}{4}}$ $0<x<2$

$f_2(y) = \int_x f(x,y)\, dx$

$= \int_0^2 \frac{1}{8}(6-x-y)\, dx$

$= \frac{1}{8}\int_0^2 (6-x-y)\, dx$

$= \frac{1}{8}\left[6x - \frac{x^2}{2} - xy\right]_0^2$

$= \frac{1}{8}\left[6\times2 - \frac{2^2}{2} - 2\times y - 0\right]$

$= \frac{1}{8}\left[12 - 2 - 2y\right]$

$= \frac{1}{8}\left[10 - 2y\right].$

(or)

$\Rightarrow \frac{2(5-y)}{\cancel{8}\,4}$ $\boxed{f_2(y) = \frac{5-y}{4}.}$ $2<y<4$

b) $f(x,y) = f_1(x) \cdot f_2(y)$

$f_1(x) = \frac{3-x}{4}$ $f_2(y) = \frac{5-y}{4}.$

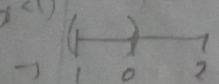
$f_1(x) \cdot f_2(y) = \left(\frac{3-x}{4}\right) \cdot \left(\frac{5-y}{4}\right)$ xy is not in $\frac{1}{8}(6-x-y)$

$\therefore$ x & y are not independent.

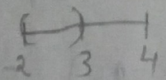
c) (i) $P(x<1,\ y<3)$

$\int_0^2$ $\int_2^4$

(x<1)



(y<3)



$\int_0^1$ $\int_2^3$

$P(x<1,\ y<3) = \int_0^1 \int_2^3 f(x,y)\,dx\,dy.$

$= \int_0^1 \int_2^3 \frac{1}{8}(6-x-y)\,dx\,dy.$

$= \int_0^1 \left[\int_2^3 \frac{1}{8}(6-x-y)\,dy\right] dx.$

$= \frac{1}{8}\int_0^1 \left[6y - xy - \frac{y^2}{2}\right]_2^3 dx.$

$= \frac{1}{8}\int_0^1 \left[6\times 3 - x\cdot 3 - \frac{3^2}{2} - \left(6\cdot 2 - x\cdot 2 - \frac{2^2}{2}\right)\right] dx$

$= \frac{1}{8}\int_0^1 \left[18 - 3x - \frac{9}{2} - \left(12 - 2x - 2\right)\right] dx.$

$= \frac{1}{8}\int_0^1 \left[18 - \frac{9}{2} - 3x - \left(10 - 2x\right)\right] dx.$

$= \frac{1}{8}\int_0^1 \left[\frac{27}{2} - 3x - 10 + 2x\right] dx.$

$= \frac{1}{8}\int_0^1 \left[\frac{27}{2} - 10 - 3x + 2x\right] dx.$

$= \frac{1}{8}\int_0^1 \left[\frac{7}{2} - x\right] dx.$

$= \frac{1}{8}\left[\frac{7}{2}x - \frac{x^2}{2}\right]_0^1$

$= \frac{1}{8}\left[\frac{7}{2}\times 1 - \frac{1}{2} - 0\right]$

$= \frac{1}{8}[3] = \frac{3}{8}$

$P(x<1 \mid y<3)$

$f(x|y) = \frac{f(x,y)}{f_2(y)}$

$= \frac{P(x<1, y<3)}{P(y<3)}$ — ①

$2<y<4$

$y<3$

$2<y<3$

$f_2(y<3) = ?$

$f_2(y) = \int_x f(x,y)\, dx$

$= \int_2^3 f(x,y)\, dx$

$= \int_2^3 \frac{1}{8}(6-x-y)\, dx$

$= \frac{1}{8}\int_2^3 (6-x-y)\, dx$

$= \frac{1}{8}\left[6x - \frac{x^2}{2} - xy\right]_2^3$

$= \frac{1}{8}\left[6\cdot 3 - \frac{3^2}{2} - 3y - \left(6\cdot 2 - \frac{2^2}{2} - 2y\right)\right]$

$= \frac{1}{8}\left[18 - \frac{9}{2} - 3y - (12 - 2 - 2y)\right]$

$= \frac{1}{8}\left[\frac{27}{2} - 3y - 10 + 2y\right]$

$f_2(y) = \frac{1}{8}\left[\frac{7}{2} - y\right]$

$P(x<1 \mid y<3) = \frac{P(x<1, y<3)}{P(y<3)}$

$\boxed{P(y<3) = \int_2^3 f_2(y)\, dy}$

$= \int_2^3 \frac{5-y}{4}\, dy$

$\frac{y}{4} \Rightarrow \frac{1}{4}\int y = \frac{1}{4}\frac{y^2}{2}$

$= \int_2^3 \left(\frac{5}{4} - \frac{y}{4}\right) dy$

$= \left[\frac{5}{4}y - \frac{1}{4}\cdot\frac{y^2}{2}\right]_2^3$

$= \left(\frac{5}{4}\times\frac{3}{1} - \frac{1}{4}\cdot\frac{9}{2}\right) - \left(\frac{5}{4}\cdot 2 - \frac{1}{4}\cdot 2\right)$

$= \frac{15}{4} - \frac{9}{8} - \left(\frac{10}{4} - \frac{2}{4}\right)$

$= \frac{30-9}{8} - \frac{8}{4} = \frac{21}{8} - \frac{2}{1}$

$= \frac{21-16}{8}$

$P(y<3) = \frac{5}{8}$

Sule ①

$= \frac{3/8}{5/8} = \frac{3}{5}$

9) Given $f(x,y)$ $\frac{c_1 x}{y^2}$, $0<x<y<1$

& $f_2(y) = c_2 y^4$, obtain c_1, c_2

& also get joint pdf ?

A) $f(x,y) = \frac{c_1 x}{y^2}$

$\int f(x|y)\,dx = 1 \Rightarrow \int_{-\infty}^{\infty} \frac{f(x,y)}{f_2(y)} \cdot dx = 1$

$\int \left(\frac{c_1 x}{y^2}\right) dx = 1$

$\Rightarrow c_1 \int \frac{x}{y^2}\, dx = 1$

$\Rightarrow c_1 \left[\frac{x^2}{2y^2}\right] = 1$

$\underline{\underline{c_1 = 2}}$

$\approx$

$\int_0^1 c_2 y^4\, dy = 1.$

$c_2 \left[\frac{y^5}{5}\right]_0^1 = 1$

$\underline{\underline{c_2 = 5}}$

Joint pdf, $f(x,y) = f(y) \cdot f(x|y)$

$= c_2 y^4 \cdot \frac{c_1 x}{y^2}$

$= \underline{\underline{10xy^2}}, \quad 0 < x < y < 1$

8) (A) (ii) $P(X+Y<3)$

$0<x<2 \quad \int_0^1$

$2<y<4 \quad \int_2^{3-x}$

$x+y<3$

$y<3-x$

$\begin{cases} 2<y<3-x \\ 0<x<1 \end{cases}$

$= P(0<x<1,\ 2<y<3-x)$

$= \int_0^1 \int_2^{3-x} f(x,y) \cdot dy\, dx.$

$= \int_0^1 \left[\int_2^{3-x} \frac{1}{8}(6-x-y)\, dy\right] dx.$

$= \frac{1}{8} \int_0^1 \left(6y - xy - \frac{y^2}{2}\right)_2^{3-x} dx.$

$= \frac{1}{8} \int_0^1 \left[6(3-x) - x\cdot(3-x) - \frac{(3-x)^2}{2} - \left(6\cdot 2 - x\cdot 2 - \frac{2^2}{2}\right)\right].$

$= \frac{1}{8} \int_0^1 \left[18 - 6x - 3x + x^2 - \frac{(9-6x-x^2)}{2} - (12 - 2x - 2)\right].$

$= \frac{1}{8} \int_0^1 \left[18 - 6x - 3x + x^2 - \frac{(9-6x-x^2)}{2} - (10-2x)\right].$

$= \frac{1}{8} \int_0^1 \left[18 - 9x + x^2 - \frac{(9-6x-x^2)}{2} - (10-2x)\right].$

$$= \frac{1}{8}\int_0^1 \left[18 - 9x + x^2 - \frac{9}{2} + 3x - \frac{x^2}{2} - 10x + 2x\right) dx.$$

$$= \frac{1}{8}\left[18x - 9\frac{x^2}{2} + \frac{x^3}{3} - \frac{9}{2}x + 3\frac{x^2}{2} - \frac{1}{2}\frac{x^3}{3} - 10x + \cancel{2}\frac{x^2}{\cancel{2}}\right]_0^1.$$

$$= \frac{1}{8}\left[\frac{18\times6}{1\times6} - \frac{9\times3}{2\times3} + \frac{1\times2}{3\times2} - \frac{9\times3}{2\times3} + \frac{3\times3}{2\times3} - \frac{1}{6} - \frac{10\times6}{1\times6} + \frac{1\times6}{1\times6} - 0\right].$$

$$= \frac{1}{8}\left[\frac{108 - 27 + 2 - 27 + 9 - 1 - 60 + 6}{6}\right]$$

$$= \frac{1}{8}\cdot\frac{10}{6} = \frac{10}{48} = \frac{5}{24}$$

q) If $f(x,y) = \frac{1}{72}(2x + 3y)$, $x = 0,1,2$; $y = 1,2,3$

is joint pdf of (x,y).

1) find distri— of $x+y$.

2) find the conditional distri— of x.

3) Examine whether x & y are indepnt.

A) let $Z = X+Y$

x:	0	1	2
y:	1	2	3

$0+1,\ 0+2,\ 0+3$

$1+1,\ 1+2,\ 1+3$

$2+1,\ 2+2,\ 2+3$

$x+y \Rightarrow 1,2,3,4,5$

$\frac{1}{72}(2x+3y)$

a) $Z = x+y = 1,2,3,4,5$

$f(z) = ?$

$f(1) = f(0,1)$ → $x=0$, $y=1$

$$= \frac{1}{72}(2\times0 + 3\cdot1)$$

$$= \frac{1}{72}\times3 = \frac{3}{72}$$

$$f(2) = f(0,2) + f(1,1)$$

$$= \frac{1}{72}(2\times0 + 3\times2) + \frac{1}{72}(2+3)$$

$$= \frac{1}{72}(6) + \frac{5}{72} = \frac{6}{72} + \frac{5}{72}$$

$$= \frac{11}{72}$$

$$f(3) = f(0,3) + f(1,2) + f(2,1)$$

$$= \frac{1}{72}(0+9) + \frac{1}{72}(2+6) + \frac{1}{72}(4+3)$$

$$= \frac{9}{72} + \frac{8}{72} + \frac{7}{72}$$

$$= \frac{24}{72}$$

$$f(4) = \cancel{f(0,4)} + f(2,2) + \cancel{f(3,1)} + f(1,3)$$

$$= \frac{1}{72}(2+9) + \cancel{\frac{1}{72}(0+12)} + \cancel{\frac{1}{72}(4+6)} + \frac{1}{72}(6+3)$$

$$= \frac{11}{72} + \frac{10}{72} = \frac{21}{72}$$

$f(5) = \frac{13}{72}$ $\quad f(5) = f(2,3) = \frac{1}{72}(4+9) = \frac{13}{72}$

Now table,

Z	0	1	2	3	4	5	total
f(z)		3/72	$\frac{11}{72}$	$\frac{24}{72}$	$\frac{21}{72}$	$\frac{13}{72}$	1

d) $P(x \mid x+y=3) = ?$

$x+y=3$
$y=3-x$

$$= \frac{P(x, y=3-x)}{P(x+y=3)} \longrightarrow (1)$$

$P(x+y=3) = \frac{24}{72}$

$P(x, y=3-x) = ?$

$$= \frac{1}{72}\left(2x + 3(3-x)\right)$$

$$= \frac{1}{72}\left(2x + 9 - 3x\right)$$

$$= \frac{1}{72}(9-x)$$

Sub (1)

$$= \frac{\frac{1}{\cancel{72}}(9-x)}{\frac{24}{\cancel{72}}} = \frac{9-x}{24}$$

c) $f(x,y) = f_1(x) \cdot f_2(y)$.

$f_1(x) = \sum_y f(x,y)$

$$= \sum_{y=1,2,3} \frac{1}{72}(2x+3y)$$

$$= \frac{1}{72}(2x+3) + \frac{1}{72}(2x+6) + \frac{1}{72}(2x+9)$$

$$= \frac{2x+3 + 2x+6 + 2x+9}{72}$$

$$= \frac{6x+18}{72} = \frac{\cancel{6}(x+3)}{\cancel{72}_{12}} = \frac{x+3}{12}$$

$f_2(y) = \sum_x f(x,y)$

$$= \sum_{x=0,1,2} \frac{1}{72}(2x+3y)$$

$$= \frac{1}{72}(0+3y) + \frac{1}{72}(2+3y) + \frac{1}{72}(4+3y)$$

$$= \frac{3y + 2+3y + 4+3y}{72} = \frac{6+9y}{72}$$

$$= \frac{\cancel{3}(2+3y)}{\cancel{72}_{24}} = \frac{2+3y}{24}$$

$f(x,y) \quad \left(\frac{x+3}{12}\right) \cdot \left(\frac{2+3y}{24}\right)$

$2x + 3xy + 6 + 9y$

not independent