01: Probability Distribution

→ Binomial Distri... = A random variable 'x' is defined to have a binomial Distri...., if the (prob). density ( ) (pdf) is givn by,

$$\boxed{f(x) = {}^nC_x\, p^x q^{n-x}}$$

$x = 0, 1, 2, \ldots, n$ $\qquad p + q = 1 \qquad 0 \le p \le 1$

here n and p → parametrs of BD, whn x follows BD we can write

$X \rightarrow B(n, p)$ or

$x \sim B(n, p)$

coin tossing / throw a die

n = total trial
x = no. of success
p = (prob) of success
q = (prob) of failure

The symbol $b(x; n, p)$ also represent the (prob) for x, success in n trials with (prob) of success p eg → 1 coin tossed 10 times, 5 get H. $\boxed{{}^nC_x\, p^x q^{n-x}}$

$n = 10 \quad x = 5$

$p = 1/2 \quad q = 1/2$

$p + q = 1$

$1/2 + 1/2 = 1$

$$(q+p)^n = q^n + {}^nC_1 q^{n-1} p + {}^nC_2 q^{n-2} p^2 + \ldots + p^n$$

$$= \sum \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$(q+p)^{n-1} = \sum \frac{(n-1)!\, p^{x-1} q^{n-x}}{(x-1)!(n-x)!}$$

$$(q+p)^{n-2} = \sum \frac{(n-2)!\, p^{x-2} q^{n-x}}{(x-2)!(n-x)!}$$

$= \sum_{x=0}^{n} {}^nC_x\, p^x q^{n-x}$

$= \sum_{x=0}^{n} \frac{n!}{x!(n-x)!} p^x q^{n-x}$ $\qquad {}^nC_x = \frac{n!}{x!(n-x)!}$

$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \ldots$

Binomial expansion

→ Moments of BD :-

1) Mean, – $\boxed{E(x) = np}$ (discrete)

Pf

$$E(x) = \sum x \cdot f(x)$$
$$= \sum_{x=0}^{n} x \cdot {}^nC_x\, p^x q^{n-x}$$
$$= \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$\boxed{{}^nC_x = \frac{n!}{x!(n-x)!}}$

$$= \sum \frac{x\, n(n-1)!}{x(x-1)!(n-x)!} p^x q^{n-x}$$

$n! = n(n-1)!$

$x! = x(x-1)!$

$$= \sum \frac{n(n-1)!}{(x-1)!(n-x)!} p^1 \cdot p^{x-1} q^{n-x}$$

p^x can be written as →

$p^x = p^1 \cdot p^{x-1}$
$\Rightarrow p^{1+x-1} = p^x$

(np out)

(we want to get np so np outside)

$$= np \sum \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} q^{n-x}$$

we know $\sum \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} p^{n-x}$
$= (q+p)^{n-1}$

$$E(x) = np \cdot (q+p)^{n-1}$$

$p+q = 1$

So, $E(x) = np$

2) variance :- $\boxed{V(x) = npq}$

Pf

$$V(x) = E(x^2) - (E(x))^2.$$

$E(x) = np.$

$E(x^2)$?

$$E(x^2) = E[x(x-1) + x].$$
$$= E[x(x-1)] + E(x).$$

x^2 can be written as →
$x^2 = x(x-1) + x$
$\therefore = x^2 - x + x = x^2$

$E(x) = \sum x \cdot f(x)$

$E(x(x-1)) = \sum x(x-1) f(x)$

$E(x)$ can be used to find $E(x(x-1))$

$E(x(x-1)) = \sum_{x=0}^{n} x(x-1) \cdot f(x)$

$$= \sum x(x-1) \cdot f(x) + np$$
$$= \sum x(x-1) \cdot {}^nC_x\, p^x q^{n-x} + np$$
$$= \sum x(x-1) \frac{n!}{x!(n-x)!} p^x q^{n-x} + np$$

${}^nC_x = \frac{n!}{x!(n-x)!}$

$$= \sum \frac{\cancel{x(x-1)}\, n!}{\cancel{x(x-1)}(x-2)!(n-x)!} p^x q^{n-x} + np$$

$p^x = p \cdot p^{x-1}$

$x! = x(x-1)(x-2)!$

$n! = n(n-1)(n-2)!$

$p^2 = p^2 \cdot p^{x-2}$

$$= \sum \frac{n(n-1)(n-2)!}{(x-2)!(n-x)!} p^2 \cdot p^{x-2} q^{n-x} + np$$
$$= n(n-1)p^2 \sum \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} q^{n-x} + np$$
$$= n(n-1)\, p^2 \cdot (q+p)^{n-2} + np$$

$p + q = 1$

$q = 1-p.$

$$E(x^2) = n(n-1)\, p^2 + np$$

$$V(x) = E(x^2) - (E(x))^2$$
$$= n(n-1)p^2 + np - (np)^2$$
$$= (n^2 - n)p^2 + np - n^2 p^2$$
$$= \cancel{n^2 p^2} - np^2 + np - \cancel{n^2 p^2}$$
$$= -np^2 + np = np(-p+1)$$
$$= np(1-p)$$

$p+q=1$
$1-p=q$

$$V(x) = npq$$

$\boxed{\begin{array}{l} p+q=1 \\ q=1-p \end{array}}$

Standard Deviation, $SD = \sqrt{V(x)}$

$SD = \sqrt{npq}$

* mean, $E(x) > V(x)$ in binomial distri...

(V(x) ... E(x) ...)

1) cal μ_3 and μ_4 (binomial distri...)

mean, $E(x) = \mu_1' = np$

$E(x^2) = \mu_2' = n(n-1)p^2 + np$

$E(x^3) = \mu_3' = n(n-1)(n-2)p^3 + 3n(n-1)p^2 + np$

μ_1' = mean in binomial distri...

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

$= npq(q-p)$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

$= 3n^2p^2q^2 + npq(1-6pq)$

$\mu_2 = V(x) = npq$

⇒ Beta and Gamma Coefficients :-

$\boxed{\gamma_1 = \sqrt{\beta_1}}$, $\boxed{\beta_1 = \frac{\mu_3^2}{\mu_2^3}}$

$= \frac{(npq(q-p))^2}{(npq)^3}$

$\beta_1 = \frac{n^2p^2q^2(q-p)^2}{n^3p^3q^3} = \frac{(q-p)^2}{npq}$

$\therefore \gamma_1 = \sqrt{\frac{(q-p)^2}{npq}}$

$\boxed{\gamma_1 = \frac{q-p}{\sqrt{npq}}}$

Skewness

∴ A binomial distri... is +vly skewed, symmetric & -vly skewed according as,

$\gamma_1 > 0$ → +vly. s	⇒	$q > p$
$\gamma_1 < 0$ → -vly. s	⇒	$q < p$
$\gamma_1 = 0$ → symm. s	⇒	$q = p$

Kurtosis

$\boxed{\beta_2 = \frac{\mu_4}{\mu_2^2}}$ $= \frac{3n^2p^2q^2 + npq(1-6pq)}{n^2p^2q^2}$

$= \frac{3\cancel{n^2}\cancel{p^2}\cancel{q^2}}{\cancel{n^2}\cancel{p^2}\cancel{q^2}} + \frac{\cancel{npq}(1-6pq)}{n^2p^2q^2}$

$= 3 + \frac{(1-6pq)}{npq}$

$\beta_2 > 3$ → lepto kurtic	→ $pq > \frac{1}{6}$
$\beta_2 < 3$ → platy kurtic	→ $pq < \frac{1}{6}$
$\beta_2 = 3$ → meso kurtic	→ $pq = \frac{1}{6}$

→ Moment generating ( ) :- (mgf).

$M_x(t)$ →

$M_x^{(t)} = E(e^{tx})$

$= \sum e^{tx} \cdot f(x)$.

$= \sum e^{tx} \cdot {}^nC_x p^x q^{n-x}$.

$e^{tx} \cdot p^x = (pe^t)^x$

$= \sum {}^nC_x (pe^t)^x q^{n-x}$.

$$\boxed{M_x^{(t)} = (q + pe^t)^n}$$

$E(x) = \sum x \cdot f(x)$.

$f(x) = {}^nC_x p^x q^{n-x}$

$E(x^2) = \sum x^2 \cdot f(x)$

$V(x) = E(x^2) - (E(x))^2$

$\sum {}^nC_x p^x q^{n-x} = (p+q)^n$.

${}^nC_x p^x (e^t)^x q^{n-x}$

$(q + p(e^t))^n = (q+pe^t)^n$

To get mean, μ_1' or $E(x)$.

$\frac{d}{dt}\left[M_x^{(t)}\right]_{t=0}$

$\mu_1' = E(x) = \left[\frac{d}{dt} M_x^{(t)}\right]$.

$= \frac{d}{dt}(q + pe^t)^n$

$= n(q+pe^t)^{n-1} \cdot \frac{d}{dt}(q+pe^t)$

$= n(q+pe^t)^{n-1} \cdot (0 + pe^t)$

$= n(q+pe^t)^{n-1} \cdot pe^t$.

at $t=0$,

$= n(q+pe^0)^{n-1} \cdot pe^0$. $\quad e^0 = 1$

$= n(q+p)^{n-1} \cdot p$ $\quad \because p+q=1$.

$$\boxed{\mu_1' = E(x) = n \cdot p}$$

$V(x) = E(x^2) - (E(x))^2$.

$E(x)^2 = \mu_2' = \left[\frac{d^2}{dt^2} M_x^{(t)}\right]_{t=0}$.

$= \frac{d}{dt}\left[n(q+pe^t)^{n-1} \cdot pe^t\right]$ (p. rule)

$\frac{pe^t}{}$

$\frac{d}{dt}(pe^t)$

$p \cdot \frac{d}{dt} e^t = pe^t$

$= n(q+pe^t)^{n-1} \cdot pe^t + pe^t \cdot n(n-1)(q+pe^t)^{n-2}(0+pe^t)$

at $t=0$,

$= n(q+pe^0)^{n-1} \cdot pe^0 + pe^0 \cdot n(n-1)(q+pe^0)^{n-2}(pe^0)$

$(q+p=1)$

$= n(q+p)^{n-1} \cdot p + p \cdot n(n-1)(q+p)^{n-2} \cdot p$

~~$np + p$~~

$= n \cdot 1^{n-1} \cdot p + p \cdot n(n-1) 1^{n-2} \cdot p$

$E(x^2) = np + p^2 n(n-1)$

$V(x) = E(x^2) - (E(x))^2$

$= np + p^2 n(n-1) - (np)^2$

$= np + \cancel{p^2n^2} - p^2n - \cancel{n^2p^2}$

$= np - np^2$

$= np(1-p)$.

$V(x) = npq$

→ Additive property of binomial distribution

(binomial distribution with n and p)

If x is $B(n_1, p)$ & y is $B(n_2, p)$ & they are independent then their sum $x+y$ also follows binomial distribution $B(n_1+n_2, p)$

(Mgf) ~~pdf~~

$X \to B(n_1, p) \implies M_x^{(t)} = (q+pe^t)^{n_1}$

$Y \to B(n_2, p) \implies M_y^{(t)} = (q+pe^t)^{n_2}$

$X+Y \to B(n_1+n_2, p) \implies M_{x+y}^{(t)} = (q+pe^t)^{n_1+n_2}$ to prove

To Show $M_{x+y}^{(t)} = (q+pe^t)^{n_1+n_2}$.

$M_{x+y} = M_x^{(t)} + M_y^{(t)}$ (x & y are indpnt)

$= (q+pe^t)^{n_1} + (q+pe^t)^{n_2}$ $\quad a^n \cdot a^m = a^{m+n}$.

$M_{x+y} = (q+pe^t)^{n_1+n_2}$.

$X+Y \longrightarrow B(n_1+n_2, p)$

⇒ Recurrence relation for Binomial (prob) :-

$$\boxed{b(x+1;n,p) = \frac{n-x}{x+1} \cdot \frac{p}{q}\, b(x;n,p).}$$

binomial distri..

Pf/ we know, ① $b(x,n,p) = {}^nC_x\, p^x q^{n-x}$.

② $b(x+1,n,p) = {}^nC_{x+1}\, p^{x+1} q^{n-(x+1)}$

$n-x-1$

$$\frac{②}{①}\quad \frac{b(x+1,n,p)}{b(x,n,p)} = \frac{{}^nC_{x+1}\, p^{x+1}\, q^{n-x-1}}{{}^nC_x\, p^x\, q^{n-x}}$$

$$= \frac{\dfrac{\cancel{n!}}{(x+1)!\,n-(x+1)!}\ \cancel{p^x}\cdot p\cdot \cancel{q^{n-x}}\cdot q^{-1}}{\dfrac{\cancel{n!}}{(x)!\,(n-x)!}\ \cancel{p^x}\ \cancel{q^{n-x}}}$$

${}^nC_x = \dfrac{n!}{(x)!\,(n-x)!}$

$p^{x+1} = p^x \cdot p^1$

$q^{n-x-1} = q^{n-x}\cdot q^{-1}$

$x!\,(n-x)! = x!\,(n-x)(n-x-1)!$

$(n-x)! = (n-x)(n-x-1)!$

$$= \frac{1}{(x+1)!\,(n-x-1)!}\ p\, q^{-1}\ \frac{x!\,(n-x)!}{1}$$

$$= \frac{1 \cdot p\, q^{-1}\, x!\,(n-x)\cancel{(n-x-1)!}}{(x+1)x!\ \cancel{(n-x-1)!}}$$

$q^{-1} = \frac{1}{q}$

$$= \frac{p\, q^{-1}\,(n-x)}{(x+1)} = \frac{p\,(n-x)}{q\,(x+1)}$$ (split)

$$= \frac{p}{q}\ \frac{n-x}{x+1}$$

$$\frac{b(x+1,n,p)}{b(x,n,p)} = \frac{p}{q}\ \frac{n-x}{x+1}$$

$$b(x+1,n,p) = \left(\frac{n-x}{x+1}\right)\frac{p}{q}\ b(x;n,p)$$

⇒ Recurrence relation for central moment :-

when $x \longrightarrow B(n,p)$

$$\boxed{\mu_{r+1} = pq\left[nr\mu_{r-1} + \frac{d\mu_r}{dp}\right]}$$

binomial discrete (Σ)

Pf/ $\mu_r = E(x-E(x))^r$ $\qquad \mu = np$.

$= E(x-\mu)^r = E(x-np)^r$ $\qquad E(x) = \Sigma x f(x)$

$= \Sigma (x-np)^r f(x)$.

$= \Sigma (x-np)^r\ {}^nC_x\, p^r q^{n-x}$.

$$\frac{d\mu_r}{dp} = \frac{d}{dp}\left[\Sigma (x-np)^r\ {}^nC_x\, p^r q^{n-x}\right] \qquad q = (1-p)$$

$$= \frac{d}{dp}\left[\Sigma\, {}^nC_x\, p^x (1-p)^{n-x} (x-np)^r\right]$$

$$= \sum_{x=x}^{n}\left[{}^nC_x\, p^x q^{n-x}\, r(x-np)^{r-1}\cdot -n + {}^nC_x\, q^{n-x}(x-np)^r\, x\, p^{x-1} + {}^nC_x\, p^x (x-np)^r (n-x)(1-p)^{n-x-1}\, x-1\right]$$

$$= \sum_{x=0}^{n}\left[(-nr)\ {}^nC_x\, p^x q^{n-x} (x-np)^{r-1} + x\, {}^nC_x\, p^{x-1} q^{n-x} (x-np)^r + (-1)\ {}^nC_x\, p^x q^{n-x-1} (n-x)(x-np)^r\right]$$

$$= \sum_{x=0}^{n} (-nr) \cdot {}^nC_x p^x q^{n-x} (x-np)^{r-1} + \sum_{x=0}^{n} x \, {}^nC_x p^{x-1} q^{n-x} (x-np)^r + \sum_{x=0}^{n} (-1) \, {}^nC_x p^x q^{n-x-1} (n-x) (x-np)^r$$

(gives)

common out

$$= -nr \sum_{x=0}^{n} {}^nC_x p^x q^{n-x} (x-np)^{r-1} + \sum {}^nC_x p^x q^{n-x} (x-np)^r \left[\frac{x}{p} + \frac{-1}{q}(n-x) \right]$$

common dist

$$= -nr \underbrace{\sum_{x=0}^{n} {}^nC_x p^x q^{n-x} (x-np)^{r-1}}_{\mu_{r-1}} + \sum {}^nC_x p^x q^{n-x} (x-np)^r \left[\frac{x}{p} + \frac{-n+x}{q} \right]$$

$$\boxed{\mu_{r-1} = \sum_{x=0}^{n} {}^nC_x p^x q^{n-x} (x-np)^{r-1}}$$

$$= -nr \, \mu_{r-1} + \sum {}^nC_x p^x q^{n-x} (x-np)^r \left[\frac{qx + p(-n+x)}{pq} \right]$$

(μ_r not bcz we want to get μ_{r+1})

$$= -nr \, \mu_{r-1} + \sum {}^nC_x p^x q^{n-x} (x-np)^r \left[\frac{qx - np + px}{pq} \right]$$

($q = 1-p$)

$$= -nr \, \mu_{r-1} + \sum {}^nC_x p^x q^{n-x} (x-np)^r \left[\frac{(1-p)x - np + px}{pq} \right]$$

$$= \ldots \ldots \left[\frac{x - xp - np + px}{pq} \right]$$

$$= -nr \, \mu_{r-1} + \sum {}^nC_x p^x q^{n-x} (x-np)^r \left[\frac{x-np}{pq} \right]$$

$$= -nr \, \mu_{r-1} + \sum {}^nC_x p^x q^{n-x} (x-np)^{r+1} \frac{1}{pq}$$

$$\frac{d\mu_r}{dp} = -nr \, \mu_{r-1} + \mu_{r+1} \frac{1}{pq}$$

$$\frac{1}{pq} \mu_{r+1} = \frac{d\mu_r}{dp} + nr \, \mu_{r-1}$$

$$\mu_{r+1} = pq \left[\frac{d\mu_r}{dp} + nr \, \mu_{r-1} \right]$$

→ **Fitting of Binomial distribution :-**

(Q → fitting B.d = expected freq)

By fitting B.d means to determine expected / theoretical binomial freq against the gvn observed freq is givn by,

$$\boxed{\begin{aligned} f(x) &= N \cdot b(x; n, p) \\ &= N \cdot {}^nC_x p^x q^{n-x} \end{aligned}}$$

$x = 0, 1, 2 \ldots n$

N = Total freq $p + q = 1$.

observed freq   expected freq

→ **Bernoulli distri..:-**

$p(x=0) = q$

$p(x=1) = p$

If $x=0$ → failure → q

If $x=1$ → success → p

mean → $\boxed{E(x) = p}$ → Bernoli distri.. $E(x) = \sum_{x=0,1} x \cdot f(x) = \sum_{x=0} 0 \cdot q + 1 \times p = p$

$E(x^2) = p$

$V(x) = E(x^2) - (E(x))^2$

$= p - p^2 = p(1-p)$

$\boxed{V(x) = pq}$

let 'X' be discreate R.variable taking only 2 values 0 & 1, if the pmf of X is gvn by,

$$\boxed{P(X=x) = p(x) = p^x q^{1-x}}, \quad x = 0, 1; \; 0 < p < 1; \; p + q = 1$$

Then x is said to follow Bernoli distri.., it is also called point binomial distri.. $(B(1,p))$

$B \to$ Binomial D of 1, p.

* Mean :- $E(x) = \sum_{x=0,1} x\,p(x)$

$= 0\cdot q + 1\times p$

$\boxed{E(x) = p}$

$\boxed{\begin{array}{l} \text{if } x=0 \\ p(x=0)=q \\ \text{if } x=1 \\ p(x=1)=p. \end{array}}$

* Varience :-

$E(x^2) = \sum x^2 p(x) = 0^2\cdot q + 1^2\cdot p$

$= 0 + p$

$\underline{\underline{E(x^2) = p}}$

$V(x) = E(x^2) - (E(x))^2$

$= p - p^2 = p(1-p)$

$\boxed{V(x) = pq}$

→ Moment generating function :- (Bernoli D)

$M_x(t) = E(e^{tx})$

$= \sum_{x=0,1} e^{tx}\cdot p(x)$

$= e^0\,p(x=0) + e^t\cdot p(x=1)$

$= 1\cdot q + e^t\cdot p$

$\boxed{M_x(t) = (q + pe^t)}$

Q/ 1) Mean & varience of B.D are 2.5 & 1.875 respectively, obtain the binomial probability distri...

A) mean, $E(x) = 2.5$

$V(x) = 1.875$

$\left.\begin{array}{l} E(x) = np = 2.5 \\ V(x) = npq = 1.875 \end{array}\right\}$ B.D

B.D = ? $= nC_x\, p^x q^{n-x}$?.

$np = 2.5$

$npq = 1.875$

$2.5\,q = 1.875$

$q = \frac{1.875}{2.5} = \underline{\underline{0.75}}$

$p + q = 1$

$p = 1 - q$

$p = 1 - q = 1 - 0.75 = \underline{\underline{0.25}}$

$np = 2.5$

$n \times 0.25 = 2.5$

$n = \frac{2.5}{0.25} = \underline{\underline{10}}$

(x - q- [illegible])

∴ BD is

$f(x) = nC_x\, p^x q^{n-x}$.

$= 10 \times C_x (0.25)^x\cdot(0.75)^{10-x}, \quad x = 0,1,\dots 10$

$\downarrow$

$n=10$.

2) If mean & varience of B.D are 4 & 2. find (prob).. of,

a) Exactly 2 success

b) <2 success

c) > 6 success.

d) atleast 2 success

A) $E(x) = np = 4$

$V(x) = npq = 2$

$npq = 4 \cdot q = 2$

$q = 2/4 = \underline{0.5}$

$p = 1 - q = 1 - 0.5 = \underline{0.5}$

$np = 4$

$n \times 0.5 = 4$

$n = 4/0.5 = \underline{8}$

$f(x) = n C_x \, p^x q^{n-x}$.

$= 8 C_x (0.5)^x (0.5)^{8-x}$. $\quad x = 0, 1, \dots 18.$

a) x = no. of success

$x = 2$

$p(x=2)\; 8 C_2 (0.5)^2 (0.5)^{8-2}$

$\Rightarrow 8 C_2 \cdot 0.25 \cdot 0.015625$

$28 \cdot 3.90625 = \underline{0.1092}$

b) $p(x<2) \Rightarrow$ $p(x=0) + p(x=1)$

$= 8 C_0 (0.5)^0 (0.5)^8 +$

$8 C_1 (0.5)^1 (0.5)^7$

$= 1 \cdot 3.90625 +$

0.03125

$3.90 \times 10^{-3} + 0.03125$

$0.00390 + 0.03125$

$= \underline{0.03515}$

eg 3.04×10^{-2} $= 0.0304$

c) >6. $\quad x = 7, 8$

$p(x > 6) = p(x=7) + p(x=8)$

$= 8 C_7 (0.5)^7 \cdot (0.5)^1 +$

$8 C_8 (0.5)^8 (0.5)^0$

~~8× 3.90625 + 1×~~

$= 0.03125 + 0.00390$

$= \underline{0.03515}$

d) $x = 0, 1, 2, 3 \dots 8$. $\quad$ atleast 2 → $2 \geq$.

$p(x \geq 2) =$

for easy purpose $(x \geq 2) \to 1 - p(x<2)$

$\boxed{p(x \geq 2) = 1 - p(x<2)}$

$= 1 - [p(x=0) + p(x=1)]$

$= 1 - [0.03515] = \underline{0.96485}$

3) Given the mgf of the binomial variable $M_x(t) = \left(\frac{1}{3}\right)^5 (2 + e^t)^5$, obtain mean & variance.

A) Mean, $E(x) = np$

$V(x) = npq$

$M_x(t) = (q + pe^t)^n$

$M_{x}(t) = \left(\frac{1}{3}\right)^5 (2+e^t)^5$ [illegible] $(q+pe^t)^n$ [illegible] formula [illegible]

$M_x(t) \Rightarrow \left[\frac{1}{3}(2+e^t)\right]^5 \Rightarrow \left(\frac{2}{3} + \frac{e^t}{3}\right)^5 \Rightarrow \left(\frac{2}{3} + \frac{1}{3}e^t\right)^5 = (q+pe^t)^n$

$\therefore$ we get $q = \frac{2}{3}$, $p = \frac{1}{3}$, $n = 5$

mean, $E(x) = np$

$= 5 \times \frac{1}{3} = 5/3$

$V(x) = npq = \frac{5}{3} \times \frac{2}{3} = \frac{10}{9}$

4) Comment on the statement the mean of binomial distribution is 3 & $V(x)$ is 4 ?

A) $E(x) = 3$ $\qquad$ $V(x) = 4$

$np = 3$ $\qquad$ $npq = 4$

$\therefore 3q = 4$

$q = \frac{4}{3}$

$\therefore p = 1 - q$

$= 1 - \frac{4}{3}$

$= \frac{3-4}{3} = \frac{-1}{3}$ $\qquad$ (prob) not be -ve

$\therefore$ It is not possible, $\qquad$ $0 < p < 1$, $0 < q < 1$

mean always be > variance

$\boxed{E(x) > V(x)}$

here only bcz $E(x) < V(x)$

$3 < 4$

5) In 256 sets of 12 tosses of a coin, In how many cases I may expect 8 heads & 4T.

A) $\frac{\text{possibility of}}{\text{[illegible]}}$ $\qquad$ cases = possi. × total case

$N = 256$ (Total freq)

$n = 12$ $\quad$ $(8H + 4T = 12)$ $\quad$ [illegible] expected freq

let success = Head

no. of success, $x = 8$.

$p = 1/2$ $\quad$ [illegible]

$q = 1/2$

$P(x) = nC_x \, p^x q^{n-x}$

$= 12C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^4$

$= 0.1208$

$\therefore$ no. of cases $= 0.1208 \times 256$

$= 30.9 \rightarrow 31$

$30.9 \rightarrow >5 \Rightarrow 31$
$30.2 \rightarrow <5 \Rightarrow 30$

6) In litters of 4 mice the no. of litters which contained 0, 1, 2, 3, 4 females where noted, the figures are below.

(a) No. of female mice :	0	1	2	3	4	Total
(f) No. of litters :	8	32	34	24	5	103

If chance of obtaining female in a single trial assumed constant. Estimate this constant of unknown (prob). find also Expected freq.?
(Always do in this method)

A)

x	f	fx
0	8	0
1	32	32
2	34	68
3	24	72
4	5	20
	103	192

$$\text{mean} = \frac{\Sigma xf}{\Sigma f} = \frac{192}{103} = 1.864$$

$f(x) = n\,C_x\, p^x q^{n-x}$
$x = 0, 1, \dots n$

binomial dist. $\rightarrow$ $E(x) = 1.864 = np$

$x = 0, 1, 2, 3, 4 \rightarrow n$ $\quad$ $n = 4$

$np = 1.864$

$4p = 1.864$

$p = \frac{1.864}{4} = 0.466$

$q = 1 - p = 0.534$

Ex. freq,

$f(x) = n\,C_x\, p^x q^{n-x}$
$= 4\,C_x (0.466)^x (0.534)^{4-x}$

$\therefore$ exp. freq

$$\boxed{\begin{aligned} f(x) &= N \cdot b(x; n, p) \\ &= N \cdot nC_x\, p^x q^{n-x} \end{aligned}}$$

$\therefore f(x=0) = 103 \cdot 4C_0 (0.466)^0 (0.534)^4$
$= 8.375 \Rightarrow 8.3 \rightarrow 5 < 8.3 \Rightarrow 8$

$f(x=1) = 103 \cdot 4C_1 (0.466)^1 (0.534)^3$
$= 29.23 \rightarrow 29$

$f(x=2) = 103 \cdot 4C_2 (0.466)^2 (0.534)^2$
~~0.2851856~~
~~0.061923336~~
$= 38.26 \Rightarrow 38$

$f(x=3) = 103 \cdot 4C_3 (0.466)^3 (0.534)^1$
$= 22.2 \Rightarrow 22$

$f(x=4) = 103 \cdot 4C_4 (0.466)^4 (0.534)^0$
$= 4.85 \rightarrow 5$

~~obtaining female in a single trail~~
~~$\rightarrow f(x=1) = 29$~~

2) A manfacture knows that 5% of his product is ~~deffective~~ , he guarantees that not more than 10 out of 100 items in a box will be deffective what is the prob) that a box will fail to meet the guarenteed quality?

A) (q [illegible] main [illegible] → success)
(q [illegible] % [illegible] p.)

$\cdot\ p = 5\% = \frac{5}{100} = \underline{0.05}$

$q = 1 - p = 1 - 0.05 = \underline{0.95}$.

10 out of $100 \Rightarrow$ max $10 = x$.

$x = 0, 1, 2, \dots, 10$.

success = defective.

$\therefore n = 100$.

10 → [illegible] deffective [illegible], so $x > 10$.

$P(x > 10) = ?$ ($>10 \Rightarrow px = 11, x = 12 \dots$ not possible.)

$= 1 - p(x \le 10)$

$= 1 - [p(x=0) + p(x=1) + \dots\ p(x=10)]$

$= 1 - \left[\sum_{x=0}^{10} n c_x (0.05)^x (0.95)^{100-x}\right]$

10 [illegible] stop here.

3) It has been observed from past exp that the prob) of an electrical compound being defective is 5%

a) If sample of 25 items is taken what is the prob) of

i) Exactly 1 being defective

2) > 1 being defective.

b) If 100 sample of size are taken in how many samples can be expect to hve no defective?

$x = 0, 1, \dots, 25$

A)

a) $p = \frac{5}{100} = 0.05$

$q = 1 - p = 1 - 0.05 = 0.95$

i) $p(x=1) = n c_x (0.05)^1 \cdot (0.95)^{25-1}$

$= 25 c_1 (0.05)^1 (0.95)^{24}$

$= \underline{1.25} \times (0.95)^{24}$

2) $p(x > 1) = 1 - p(x \le 1)$

$= 1 - [p(x=0) + p(x=1)]$

$= 1 - [25 c_0 (0.05)^0 (0.95)^{25} + 1.25 \times (0.95)^{24}]$

$= 1 - [(0.95)^{25} + 1.25 (0.95)^{24}]$

$= 1 - [(0.95)^{24} (0.95 + 1.25)]$

$= 1 - [(0.95)^{24}\ 2.20]$

b) $N = 100$

Expected no $= N \times p(x=0)$.

no defective → $x = 0$.

$= 100 \cdot p(x=0)$

$= 100 \cdot 25 c_0 (0.05)^0 (0.95)^{25}$

$= \underline{100 \cdot (0.95)^{25}}$

⇒ Poisson Distribution :-

binomial $X \to B(n, p)$ parameter

(P.D)

$X \to p(\lambda)$ parameter

A discrete R.V x is defined to hve poisson distri... if the (prob) density () of X is gvn by,

$$\boxed{f(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}} \qquad x = 0, 1, 2, \ldots \qquad \lambda > 0$$

where $\lambda \to$ paramtr of P.D.

In this case, we can write

$X \xrightarrow[\text{(follows)}]{} P(\lambda)$

eg → The no. of total traffic accidnts per week in a gvn ~~stage~~ state,

the no. of telephone calls per hr coming into the switch board,

The no. of defects per unit of some material.

⇒ Moments of P.D :-

1) Mean :-

$$\boxed{E(x) = \lambda}$$

$$e^{\lambda} = \sum \frac{\lambda^{x-1}}{(x-1)!}$$

Same

Pf

$$E(x) = \sum x \, f(x).$$

$$= \sum x \cdot \frac{e^{-\lambda} \lambda^{x}}{x!} \qquad \lambda^{x} = \lambda \, \lambda^{x-1}.$$

$$= e^{-\lambda} \lambda \sum_{x=0}^{\infty} \frac{x \, \lambda^{x-1}}{x!} \qquad x! = x(x-1)$$

$$= e^{-\lambda} \lambda \sum_{x=0}^{\infty} \frac{\cancel{x} \, \lambda^{x-1}}{\cancel{x}(x-1)!}$$

$$= e^{-\lambda} \lambda \underbrace{\sum_{x=0}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}}_{e^{\lambda}}$$

$$= e^{-\lambda} \lambda \cdot e^{\lambda}$$

$$= \lambda \cdot e^{-\lambda+\lambda} = \lambda \cdot e^{0} = \lambda$$

2) Varience :-

$$\boxed{V(x) = \lambda}$$

$$\boxed{\begin{aligned} S.D &= \sqrt{V(x)} \\ S.D &= \sqrt{\lambda} \end{aligned}}$$

$$V(x) = E(x^2) - (E(x))^2$$

$$E(x^2) = \sum x^2 \cdot f(x) \qquad x^2 = x(x-1) + x$$

$$= \sum_{x=0}^{\infty} \left[x(x-1) + x \right] \cdot f(x)$$

$$= \sum_{x=0}^{\infty} x(x-1) \cdot f(x) + \underbrace{\sum_{x=0}^{\infty} x \cdot f(x)}$$

$E(x) = \lambda$.

$$= \sum_{x=0}^{\infty} x(x-1) \cdot \frac{e^{-\lambda} \lambda^{x}}{x!} + E(x).$$

$$= \sum_{x=0}^{\infty} x(x-1) \cdot \frac{e^{-\lambda} \lambda^{x}}{x!} + \lambda$$

$$= e^{-\lambda} \lambda^{2} \sum_{x=0}^{\infty} \frac{x(x-1) \, \lambda^{x-2}}{x!} + \lambda.$$

$$= e^{-\lambda} \lambda^{2} \sum_{x=0}^{\infty} \frac{\cancel{x(x-1)} \, \lambda^{x-2}}{\cancel{x(x-1)}(x-2)!} + \lambda.$$

$$= e^{-\lambda} \lambda^{2} \underbrace{\sum_{x=0}^{\infty} \frac{\lambda^{x-2}}{(x-2)!}} + \lambda$$

$$= e^{-\lambda} \lambda^{2} \cdot e^{\lambda} + \lambda.$$

$$= \lambda^{2} \cdot e^{-\lambda+\lambda} + \lambda = \lambda^{2} e^{0} + \lambda.$$

$$E(x^2) = \lambda^{2} + \lambda$$

$$V(x) = E(x^2) - (E(x))^2$$

$= \lambda^2 + \lambda - \lambda^2$

$V(x) = \lambda$

(Study $\mu_1', \mu_2', \mu_3', \mu_4'$)

3) **Calculation of μ_3 & μ_4 :-**

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$. $\mu_1 \to$ central

here $\mu_1' = E(x) = \lambda$

$\mu_2' = E(x^2) = \lambda^2 + \lambda$

$\mu_3' = E(x^3) = ?$

$E(x^3) = \sum x^3 \cdot f(x)$

$x^3 = x(x-1)(x-2) + 3x^2 - 2x$

$= \sum_{x=0}^{\infty} \left[x(x-1)(x-2) + 3x^2 - 2x\right] f(x)$

$= \sum_{x=0}^{\infty} x(x-1)(x-2) f(x) + \sum_{x=0}^{\infty} 3x^2 f(x) - \sum_{x=0}^{\infty} 2x f(x)$

$= \sum_{x=0}^{\infty} x(x-1)(x-2) \frac{e^{-\lambda}\lambda^x}{x!} + 3\sum_{x=0}^{\infty} x^2 \frac{e^{-\lambda}\lambda^x}{x!} - 2\sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!}$

$= e^{-\lambda}\lambda^3 \sum_{x=0}^{\infty} \frac{\cancel{x(x-1)(x-2)}\lambda^{x-3}}{\cancel{x(x-1)(x-2)}(x-3)!} + 3[\lambda^2+\lambda] - 2\lambda$

$= e^{-\lambda}\lambda^3 \sum_{x=0}^{\infty} \frac{\lambda^{x-3}}{(x-3)!} + 3\lambda^2 + 3\lambda - 2\lambda$

$= e^{-\lambda}\lambda^3 \cdot e^{\lambda} + 3(\lambda^2+\lambda) - 2\lambda$ $\boxed{e^{-\lambda+\lambda} = e^0 = 1}$

$= \lambda^3 e^{-\lambda+\lambda} + 3\lambda^2 + 3\lambda - 2\lambda$

$= \lambda^3 + 3\lambda^2 + \lambda$ //

$\mu_3' = \lambda^3 + 3\lambda^2 + \lambda$

$\therefore \mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

$= \lambda^3 + 3\lambda^2 + \lambda - 3(\lambda^2+\lambda)\lambda + 2\lambda^3$

$= \cancel{\lambda^3} + \cancel{3\lambda^2} + \lambda - \cancel{3\lambda^3} - \cancel{3\lambda^2} + \cancel{2\lambda^3}$

$\mu_3 = \lambda$

Similarly we can find μ_4.

$\therefore \mu_4 = 3\lambda^2 + \lambda$. (proof $\to$ last pg)

$\to$ **Poisson distri... as a limiting form of B.D :-**

Poisson distri... is obtained as an approximation to the B.D under the condi...

a) n is very large $(n \to \infty)$

b) p is very small $(p \to 0)$

c) $np = \lambda$, a finite quantity

Pf/

$f(x) = {}^nC_x p^x q^{n-x}$ ${}^nC_x = \frac{n!}{x!(n-x)!}$

$q^{n-x} = (1-p)^{n-x} = \frac{(1-p)^n}{(1-p)^x}$

$= \frac{n!}{x!(n-x)!} p^x q^{n-x}$

$= \frac{n(n-1)(n-2)\dots(n-x-1)\cancel{(n-x)!}}{x!\cancel{(n-x)!}} p^x (1-p)^{n-x}$

$= \frac{n(n-1)(n-2)\dots(n-x-1)(1-p)^n}{x!(1-p)^x}$

$$= \frac{n^x \left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x-1}{n}\right) p^x (1-p)^n}{x!\,(1-p)^x}$$

$$f(x) = \frac{(np)^x \left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x-1}{n}\right)\right](1-p)^n}{x!\,(1-p)^x} \quad (1)$$

apply 3 condi~

$$\lim_{n\to\infty} \left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x-1}{n}\right) = 1$$

$np = \lambda \quad \therefore \quad p = \frac{\lambda}{n}. \qquad (np)^x = \lambda^x$

$$\lim_{n\to\infty} (1-p)^x = \lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^x = 1$$

$$\lim_{n\to\infty} (1-p)^n = \boxed{\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^n = e^{-\lambda}} \text{ (def)}$$

(1) becomes

$$f(x) = \frac{\lambda^x \times 1 \times e^{-\lambda}}{x!}$$

$$\boxed{f(x) = \frac{e^{-\lambda}\cdot\lambda^x}{x!}} \longrightarrow \text{poisson distri~}$$

⇒ Fitting of poisson distri.... :- (P.D)

(multiply N with f(x))

By fitting (P.D), we mean to calculate the expected poisson freq against the gvn observed freq.

If f(x) denotes the poisson freq given by,

$f(x) = N \cdot p(X = x)$

$$\boxed{f(x) = N\cdot\frac{e^{-\lambda}\cdot\lambda^x}{x!}} \qquad x = 0, 1, \ldots, \quad \lambda > 0$$

1) A random variable X follows a P.D with mean = 1. cal prob that

a) $X = 0$ (b) $X = 1$ (c) $X \geq 2$

A)

$E(x) = \lambda = 1 \qquad p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

a) $p(X=0) = \frac{e^{-1}\cdot 1^0}{0!} = e^{-1} = 0.3678 \qquad \boxed{0! = 1}$

b) $p(X=1) = \frac{e^{-1}\cdot 1^1}{1} = e^{-1} = 0.3678$

$e^{-1} = 0.367$

c) $p(X \geq 2) = p(X=2) + p(X=3) \ldots$ (difficult)

$\therefore = 1 - [p(X=0) + p(X=1)]$

$= 1 - [0.3678 + 0.3678]$

$= 0.2644$

1) If x & y are Poisson variates such that $P(x=1) = P(x=2)$ &
$P(y=2) = P(y=3)$
find variance of $x-2y$?

A) $V(x-2y) =$ always + in $V(x)$
$V(x) + V(2y)$

$V(ax \pm b) = a^2 V(x)$

$V(2y) = 2^2 V(y)$
$= 4 V(y)$

$= V(x) + 4V(y)$ —— (1)

$V(x), V(y)$? $\quad x \to P(\lambda_1)$
$\quad y \to P(\lambda_2)$

$$P(x) = \frac{e^{-\lambda_1} \cdot \lambda_1^x}{x!}$$

$$P(y) = \frac{e^{-\lambda_2} \cdot \lambda_2^y}{y!}$$

→ $P(x=1) = P(x=2)$ (given)

$$\frac{\cancel{e^{-\lambda_1}} \cdot \cancel{\lambda_1}}{1!} = \frac{\cancel{e^{-\lambda_1}} \cdot \cancel{\lambda_1} \lambda_1}{2!}$$

$2! \to 1 \times 2 = 2$

$$1 = \frac{\lambda_1}{2}$$

$\lambda_1 = 2$

→ $P(y=2) = P(y=3)$ (given)

$$\frac{\cancel{e^{-\lambda_2}} \cancel{\lambda_2^2}}{2!} = \frac{\cancel{e^{-\lambda_2}} \cancel{\lambda_2^2} \lambda_2}{3!}$$

$$= \frac{1}{2} = \frac{\lambda_2}{6}$$

$6 = \lambda_2 2$
$\lambda_2 = 6/2 = 3$

$\lambda_2 = \frac{6}{2} = 3$

$V(x) = 2$
$V(y) = 3$

$V(x-2y) = V(x) + 4V(y)$ —— (1)
$= 2 + 4 \times 3$
$= 2 + 12 \quad = 14$

2) A P.D has double mode at $x=2$ & $x=3$. what prob that x will have 1 | the other value of the 2 values ?

A) Since P.D has double mode at $x=2$ & $x=3$ clearly $\lambda = 3$
compliment 2 mode 2,3, take highest no. as $\lambda \to \lambda = 3$

$$P(x = 2 \text{ or } 3) = P(x=2) + P(x=3)$$
$$= \frac{e^{-3} 3^2}{2!} + \frac{e^{-3} 3^3}{3!}$$
$$= \frac{e^{-3} 9}{2} + \frac{e^{-3} 27}{6}$$
$= 0.04979 \times 4.5 +$
0.04979×4.5
$= 0.448$