Time Series And Index Numbers.

chapter = 1

Analysis of time series.

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* Semi Avg Method :-

Q 1) Draw a trend line by semi-avg method for following data -

year	production
2002	55
2003	62
2004	65
2005	58
2006	65
2007	72
2008	75
2009	68.

n = 8

∴ 8/2 = 4

4 parts ÷

$55 + 62 + 65 + 58$

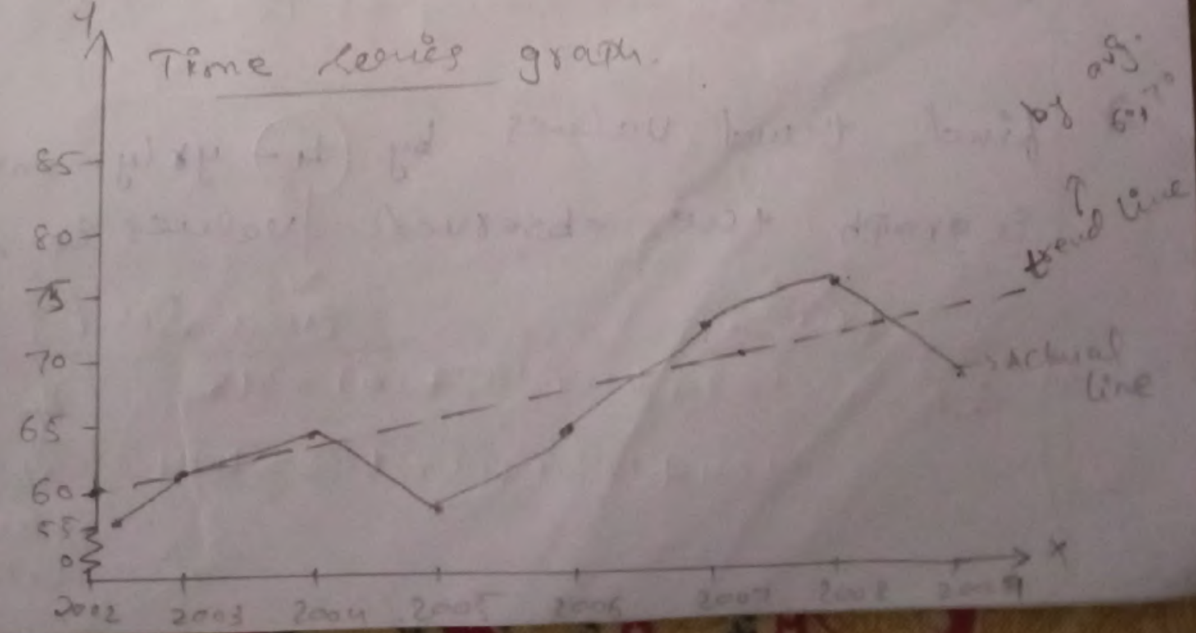
$= \frac{240}{4} = 60$

$60 \to 2004$

$65 + 72 + 75 + 68$

$= \frac{280}{4} = 70$

$70 \to 2008$



Method of Moving Avg :-

1) Compute trend values by finding 3-yearly moving avg —

yr.	population	3-yearly moving total	Trend values ÷ 3
2000	412	—	—
2001	438	1296	432 (1296 ÷ 3)
2002	446	1338	446
2003	454	1370	456.67
2004	470	1407	469
2005	483	1443	481
2006	490	—	—

- when we taking 412 → no no. in up, so blank
- 3-yr moving → means we need 3 no. nearby
- when we taking 438 → have up & down no. and the 3 no. → 412 + 438 + 446 = 1296
- as it is 3 yr moving → Trend value must be ÷ 3

2) find trend values by 4-yrly moving avg. & graph the observed values & trend values.

Even Series

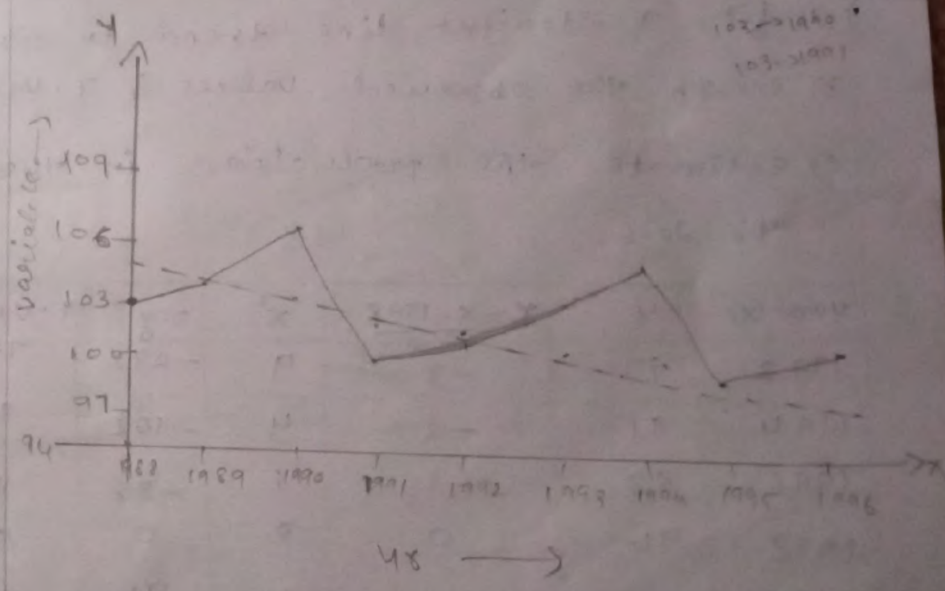
trend

÷ 8 ⇒ 103 + 104 + 107 + 101 = 415

104 + 107 + 101 + 102 = 414

4 + 4 = 8

yr.	value	4-yr ly moving total	2 item m. total	Trend
1988	103	—	—	—
1989	104	415 (centre)	—	—
1990	107	414	829	103.6
1991	101	414	828	103.5
1992	102	412	826	103.3
1993	104	410	822	102.8
1994	105	408	818	102.3
1995	99	—	—	—
1996	100	—	—	—



* Method of least squares :-

1) Linear (Straight line)
2) Quadratic (parabolic)
3) Exponential

1) Straight line trend by method of least squares —

$y = a + bx$. & give Σ

$na + b\Sigma x = \Sigma y$ $\times x$.

$\Rightarrow a\Sigma x + b\Sigma x^2 = \Sigma xy$

y = observed values.

$\boxed{\Sigma x = 0}$

then values of a & b →

$\boxed{a = \frac{\Sigma y}{n}}$ $\boxed{b = \frac{\Sigma xy}{\Sigma x^2}}$

Q/

1) a) fit a straight line trend to data.

b) Graph the observed values & T. values.

c) Estimate the production in the yr 2006.

$x = \frac{\Sigma x}{n} = 1998$

year (X)	y	$x = \frac{X-1998}{2}$	x^2	xy	T. values.
1992	77	−3	9	−231	78.87
1994	81	−2	4	−162	82.48
1996	88	−1	1	−88	86.09
1998	94	0	0	0	89.70
2000	94	1	1	94	93.31
2002	96	2	4	192	96.92
2004	98	3	9	296	100.53
	628	0	28	101	

$\Sigma x = 0$

a) $a = \frac{\Sigma y}{n} = \frac{628}{7} = 89.7$

Let $y = a + bx$
$\Sigma x = 0$
$n = 7$ (odd)

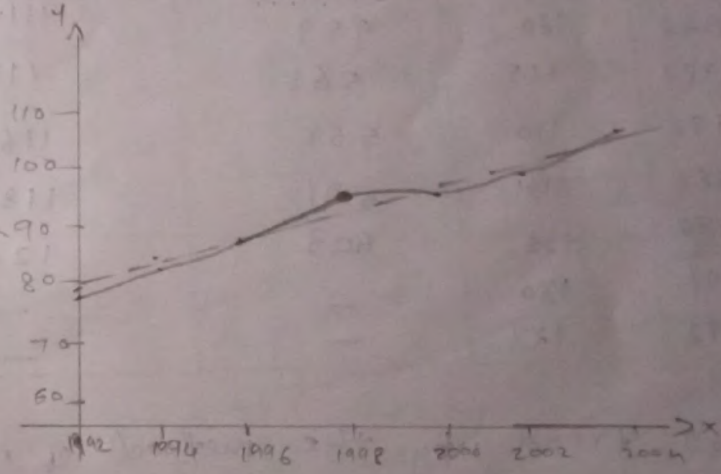
$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{101}{28} = 3.61$

T. eq is, $y = 89.7 + 3.61x$ — ①

Sub x values in eq ① from table so we get T-values

eg = $x = -2$, $y = 89.7 + 3.61 \times (-2) = 82.48$

b)



c) value in x corresponding to the yr 2006 is 4.

$y = 89.7 + 3.61x$

$x = 4$ $y = 89.7 + 3.61 \times 4 = 104.14$ thousand units

1) cal 5 yrly moving avg for the data —

yr (x)	values	5-yrly m. totals	Trend values ÷5
1981	110	—	—
1982	104	—	—
1983	98	526	105.2
1984	105	536	107.2
1985	109	547	109.4
1986	120	559	111.8
1987	115	568	113.6
1988	110	568	116.2
1989	114	591	118.2
1990	121	603	120.6
1991	130	—	—
1992	127	—	—

2) cal trend by the method of moving avg from data —

yr	sales	4 yr m. total	centered 4 yrly m. total	centered 4 yrly m. avg.
1985	108	—	—	—
1986	112	450	—	—
1987	110	482	932 ÷8 →	116.5
1988	120	400	882	121.5
1989	140	480	880	121.25
1990	120	495	975	121.87
1991	100	—	—	—
1992	135	—	—	—

3) For the follo.. time series fit a linear trend by the method of L. Square. Estimate the sales for the yr 1996.

yr (X)	y (sales)	x (x-1987)	x^2	xy
1980	103	-7	49	-721
1982	106	-5	25	-530
1984	95	-3	9	-285
1986	93	-1	1	-93
1988	98	1	1	98
1990	93	3	9	279
1992	90	5	25	450
1994	86	7	49	602
	764	0	168	-200

x = X-1987

$$a = \frac{\Sigma y}{n} = \frac{764}{8} = 95.5$$

$$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{-200}{168} = -1.19$$

T. eq is —

$$y = a + bx$$

$$y = 95.5 + (-1.19)x$$

Estimated sales for 1996 is obtained by putting x = 11,

$$y = 95.5 - 1.19 \times 11 = 82.41$$

$y = 82.41 \times 1000 = 82410$ units

(in sales is in 000 units)

2) find straight line trend eq. & tabulate against each yr.

yr (X)	value (y)	$x = X - 2004$	xy	x^2	Trend
2000	380	−4	−1520	16	398
2001	400	−3	−1200	9	468.5
2002	650	−2	−1300	4	539
2003	720	−1	−720	1	609.5
2004	690	0	0	0	680
2005	620	1	620	1	750.5
2006	670	2	1340	4	821
2007	950	3	2850	9	891.5
2008	1040	4	4160	16	962
	6120		4230	60	

$$a = \frac{\Sigma y}{n} = \frac{6120}{9} = 680$$

$$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{4230}{60} = 70.5$$

T. eq. is, $y = a + bx$.

$y = 680 + 70.5x$

→ Fitting of Parabola :-

$$y = a + bx + cx^2$$

$$\Sigma y = na + b\Sigma x + c\Sigma x^2$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$

$$\Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

1) Fit quadratic trend for the following series. Estimate the population for the yr 2001.

yr	population (y)	x	x^2	x^3	x^4	xy	x^2y
1951	36	−2	4	−8	16	−72	144
1961	44	−1	1	−1	1	−44	44
1971	55	0	0	0	0	0	0
1981	68	1	1	1	1	68	68
1991	84.	2	4	8	16	168	336
2001	287	0	10	0	34	120	592

$$x_1 = \frac{1951 - 1971}{10} = \frac{-20}{10} = -2$$

$$x = \frac{X - \bar{X}}{\ldots}$$

$$x_2 = \frac{1961 - 1971}{10} = \frac{-10}{10} = -1$$

$\bar{X} = 1971$

$$= \frac{X - 1971}{10}$$

$$x_3 = \frac{1971 - 1971}{10} = \frac{0}{10} = 0$$

$$x_4 = \frac{10}{10} = 1$$

$$x_5 = \frac{20}{10} = 2$$

$287 = 5a + \cancel{b} + 10c$ — ① $\times 2$

$120 = \cancel{a} + 10b + \cancel{c}$ — ②

$592 = 10a + \cancel{b} + 34c$ — ③

from ②

$120 = 10b$

$b = \frac{120}{10} = 12$

$\cancel{574 = 10a + 20c}$

$\cancel{592 = 10a + 34c}$

-18

$592 = 10a + 34c$

$574 = 10a + 20c$

$18 = 14c$

$c = \frac{18}{14} = \frac{9}{7}$

$= 1.29$

$5a + 10 \times 1.29 = 287$

$5a + 12.9 = 287$

$5a = 287 - 12.9$

$5a = 274.1$

$a = 54.82$

$y = a + bx + cx^2$

$= 54.82 + 12x + 1.29x^2$

value of x ~~population~~ for 2001 is $x = 3$ (i.e) $x = 3$

∴ estimate the population for 2001 is

$y = 54.82 + 12 \times 3 + 1.29 \times 3^2$

$y = 54.82 + 36 + \cancel{\ldots}$

$= 54.82 + 36 + 11.61$

$y = 102.43$

→ Exponential trend :-

$\boxed{y = ab^x}$

$\log y = \log a + x \log b$

$\log y = Y$
$\log a = A$
$\log b = B$

$Y = A + Bx$

$\Sigma Y = nA + B\Sigma x$

$\Sigma xY = A\Sigma x + B\Sigma x^2$

a = antilog A
b = antilog B

7) fit an exponential trend to the following time series.

no. of values
so x = 1, 2, 3

yr	value (y)	x	x^2	$Y = \log y$	xY
1990	2	1	1	0.301	0.301
1991	3	2	4	0.477	0.954
1992	4	3	9	0.602	1.806
1993	6	4	16	0.778	3.112
1994	9	5	25	0.954	4.77
1995	13	6	36	1.113	6.678
	37	21	91	4.225	17.621

$4.225 = 6A + 21B$ $\times 7$

$17.621 = 21A + 91B$ $\times 2$

$\cancel{= 88.725}$

$$42A + 147B = 29.575$$
$$42A \pm 182B = 35.242$$

$$35B = 5.667$$
$$B = 0.1619$$

$$21A + 91 \times 0.1619 = 17.621$$
$$21A + 14.73 = 17.621$$
$$21A = 17.621 - 14.73$$
$$21A = 2.891$$
$$A = 0.13$$

a = antilog A

$= A\ (0.13)$

$a = 1.37$

$b = 1.451$

$$y = ab^{x}$$
$$= 0.13 \cdot 1.37^{x}$$

1) Compute simple Index no.

Commodity :	Rice	wheat	ghee	sugar
price in 1985 :	20	30	20	10
price in 1990 :	40	45	50	30

A)

Commodity	price in 1985 (P_0)	price in 1990 (P_1)	price relative (P_1/P_0)
Rice	20	40	40/20 = 2.
wheat	30	45	1.5
ghee	20	50	2.5
sugar	10	30	3
	80	165	9

$$\boxed{\text{Simple Aggregative I.No} = \frac{\Sigma P_1}{\Sigma P_0} \times 100}$$

$$= \frac{165}{80} \times 100 = 206.25$$

$$\boxed{\text{Simple Avg of relatives I.No} = \frac{\Sigma \frac{P_1}{P_0}}{n} \times 100}$$

$n = 4.$

$$= \frac{9}{4} \times 100$$
$$= 225$$

Equations :-

1) Simple I.No → (2)

* Simple Aggregative I.No $= \frac{\Sigma P_1}{\Sigma P_0} \times 100$.

* Simple Avg I.No $= \frac{\Sigma \frac{P_1}{P_0}}{n} \times 100$

2) weighted I.No →

① weighted Aggregative I.No ⇒ (6)

1) Laspeyre's Method =

$$L \cdot I \cdot No = \frac{\Sigma q_0 P_1}{\Sigma q_0 P_0} \times 100$$

2) Paasche's Method =

$$P \cdot I \cdot N = \frac{\Sigma q_1 P_1}{\Sigma q_1 P_0} \times 100$$

3) Fisher's ideal Method =

$$F.I \cdot I \cdot No = \sqrt{L \times P}$$

L = Laspeyre's (M)

P = Paasche's (M)

$$\boxed{I.No = \sqrt{\frac{\Sigma q_0 P_1}{\Sigma q_0 P_0} \times \frac{\Sigma q_1 P_1}{\Sigma q_1 P_0}} \times 100}$$

4) Marshal - Edgeworth Method =

$$M\text{-}E \cdot I \cdot No = \frac{\Sigma (q_0+q_1) P_1}{\Sigma (q_0+q_1) P_0} \times 100$$

(or)

$$I \cdot No = \frac{\Sigma q_0 P_1 + \Sigma q_1 P_1}{\Sigma q_0 P_0 + \Sigma q_1 P_0} \times 100.$$

5) Dorbish & Bowley's Method =

$$I \cdot No = \frac{L+P}{2} \times 100$$

(i.e) $$\boxed{I \cdot No = \frac{\left[\frac{\Sigma q_0 P_1}{\Sigma q_0 P_0} + \frac{\Sigma q_1 P_1}{\Sigma q_1 P_0}\right]}{2} \times 100.}$$

6) Kelley's Method =

$$K \cdot I \cdot No = \frac{\Sigma P_1 q}{\Sigma P_0 q} \times 100.$$

$$q = \frac{q_0 + q_1}{2}$$

q_1 → current yr quantity
q_0 → Base yr quantity
P_1 → current yr price
P_0 → Base yr price

② weighted Avg I.No =

* using AM,

$$P_{01} = \frac{\Sigma wP}{\Sigma w}$$

$$Q_{01} = \frac{\Sigma wq}{\Sigma w}$$

* using GM,

$$P_{01} = \text{Antilog}\left(\frac{\Sigma w \log P}{\Sigma w}\right)$$

$$P = \frac{P_1}{P_0} \times 100$$

2) obtain trend of the form $y = ab^x$

$x \to$	1	2	3	4	5
$y \to$	5.9	11.8	24.2	47	95

A)

$y = ab^x$.

$\log y = \log a + x \log b$

$Y = A + Bx$.

$\log y = Y$
$\log a = A$
$\log b = B$

∴ normal eq is,

$\Sigma Y = nA + B \Sigma x$ —— ①

$\Sigma xY = A \Sigma x + B \Sigma x^2$ —— ②

x	y	$Y = \log y$	x^2	xY
1	5.9	0.7709	1	0.7709
2	11.8	1.0719	4	2.1438
3	24.2	1.38586	9	4.1568
4	47	1.6721	16	6.6884
5	95	1.9777	25	9.8885
15		6.8782	55	23.6484

subs in eq ① & ②

we get,

$6.8782 = 5A + 15B$ $\times 3$

$23.6484 = 15A + 55B$

~~23.6484 = 15A + 55B = 23.6484~~
~~20.6346 = 15A + 45B = 20.6346~~
~~3.0138 = 10B = 3.0138~~
~~B = 0.30138~~

$15A + 55B = 23.6484$

$15A + 45B = 20.6346$

$10B = 3.0138$

$B = 0.30138$

$5A + 15 \times 0.30138 = 6.8782$

$5A + 4.5207 = 6.8782$

$5A = 2.3575$

$A = 0.4715$

$a = \text{antilog } A$

$= \text{antilog}(0.4715)$

$a = 2.961$

$b = \text{antilog}(0.30138)$

$b = 2.002$

∴ $y = ab^x$

$y = (2.961)(2.002)^x$

The trend values can be obtained by putting $x = 1,2,3,4$ & 5 in eq

3) cal. Laspeyre's, (P) & fisher's I. no.

Commodities	Base yr		Current yr.	
	price (P_0)	q (q_0)	price (P_1)	q (q_1)
A	10	12	12	15
B	7	15	5	20
C	5	24	9	20
D	16	5	14	5

A)

q_0P_0	q_0P_1	q_1P_0	q_1P_1
120	144	150	180
105	75	140	100
120	216	100	180
80	70	80	70
425	505	470	530

L. I. No, $P_{01} = \frac{\Sigma q_0 P_1}{\Sigma q_0 P_0} \times 100$

$= \frac{505}{425} \times 100 = 118.82$

P. I. No, $P_{01} = \frac{\Sigma q_1 P_1}{\Sigma q_1 P_0} \times 100$

$= \frac{530}{470} \times 100 = 112.76$

$\sqrt{L \times P}$ F. I. No, $P_{01} = \sqrt{\frac{\Sigma q_0 P_1}{\Sigma q_0 P_0} \times \frac{\Sigma q_1 P_1}{\Sigma q_1 P_0}}$

$= \sqrt{\frac{505}{425} \times \frac{530}{470}} \times 100$

$= 115.8$

4) cal. Marshall - Edge worth I. No which is = F. I. no.

Commodities	1990		1994	
	P_0	q_0	P_1	q_1
A	2	74	3	82
B	5	125	4	140
C	7	40	6	33

(to find F. I. No & M. E. I. No)

A)

q_0P_0	q_0P_1	q_1P_0	q_1P_1
148	222	164	246
625	500	700	560
280	240	231	198
1053	962	1095	1004

$F.I.No\ P_{01} = \sqrt{\frac{\Sigma q_0 p_1}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_1 p_0}} \times 100$

$= \sqrt{\frac{962}{1053} \times \frac{1004}{1095}} \times 100$

$= \underline{\underline{91.524}}$

$M.E.I.No;\ P_{01} = \frac{\Sigma q_0 p_1 + \Sigma q_1 p_1}{\Sigma q_0 p_0 + \Sigma q_1 p_0} \times 100$

$= \frac{962 + 1004}{1053 + 1095} \times 100$

$= \underline{\underline{91.527}}$

5) cal I.No of prices for 1993 on the base of 1990 —

c	weight(w)	price 1990 (P_0)	c.y price 1993 (P_1)
A	40	16	20
B	25	40	60
C	5	2	3
D	20	5	7
E	10	2	4
	100		

$P = \frac{P_1}{P_0} \times 100$	WP	log P	w log P
125	5000	2.0969	83.87
150	3750	2.1761	54.40
150	750	2.1761	10.88
140	2800	2.1461	42.92
200	2000	2.3010	23.01
	14300		215.0910

using AM, $P_{01} = \frac{\Sigma WP}{\Sigma W}$

$= \frac{14300}{100} \cancel{\times 100} = \underline{\underline{143}}$

using GM, $P_{01} = \text{Antilog}\left[\frac{\Sigma w \log P}{\Sigma w}\right]$

$= \text{Antilog}\left(\frac{215.0910}{100}\right)$

$= \underline{\underline{141.55}}$

6) Construct cost of living index for 1980 taking 1979 as b.yr from follo. data using 'Aggregate expenditure' Method.

Articles	q 1979 (q_0)	p 1979 (p_0)	p 1980 (p_1)	q_0p_1	q_0p_0
A	6	5.75	6	36	34
B	1	5	8	8	5
C	6	6	9	54	36
D	4	8	10	40	32
E	2	2	1	3	4
f	1	20	15	15	20
				156.60	131.50

$$\text{Cost of living index} = \frac{\sum q_0p_1}{\sum q_0p_0} \times 100$$

$$= \frac{156.60}{131.50} \times 100$$

$$= 119.09$$

7) Cal cost of L. index. no. —

Item	B.yr.p (p_0)	C.yr.p (p_1)	weight (w)	$P = \frac{p_1}{p_0} \times 100$	wP
food	39	47	4	120.51	482.04
fuel	8	12	1	150	150
cloth	14	18	3	128.57	385.71
House	12	15	2	125	250
Miscellanous	25	30	1	120	120
			11		1387.75

$$\text{Cost of L.I.no} = \frac{\sum wP}{\sum w} = \frac{1387.75}{11}$$

$$= 126.16$$

⇒ Cost of Living Index No :-

① Aggregate Expenditure Method / weighted Aggregatives Method =

[A, B, C, D ...] ↓ use this method

$$\text{Cost of L.I.No} = \frac{\sum q_0P_1}{\sum q_0P_0} \times 100$$

② Family Budget method / weighted Avg of relatives Method =

$$\text{Cost of L.I.No} = \boxed{\frac{\sum wP}{\sum w}}$$

[fd, cloth, house] ↓ use this method.

If 'w' not given

$\boxed{W = q_0P_0}$ $\boxed{P = \frac{P_1}{P_0} \times 100}$

Hw

Construct I.No for 1995 taking 1990 as base

C	P.1990 (P_0)	P.1995 (P_1)	$P = P_1/P_0 \times 100$	log P
A	50	70	140	2.1461
B	40	60	150	2.1761
C	80	90	112.50	2.0512
D	110	120	109.09	2.0378
E	20	20	100	2.
	300	260	611.59	10.4112

$= \frac{1235}{} \times 100 = \underline{123.5}$

$$M.E.I.No = \frac{\Sigma q_0 P_1 + \Sigma q_1 P_1}{\Sigma q_0 P_0 + \Sigma q_1 P_0} \times 100$$

$$= \frac{310 + 365}{225 + 330} \times 100 = \underline{121.6}$$

4) Construct cost of living I.No for 1987 on the basis of 1986. using Aggregate expenditure (M) & family budget (M) —

Articles	q consumed in 1986	unit	P.1986	P.1987
wheat	20ktl	perqtl	150	165
Gram	10ktl	"	80	100
Rice	10ktl	"	120	150
Bajra	1.5ktl	"	60	90
Ashar	1.5ktl	"	100	140
oil	10 kg	kg	10	12
Gur	40 kg	kg	2	3

A) Aggregate E. Method →

Articles	q 1986 (q_0)	P.1986 (P_0)	P.1987 (P_1)	q_0P_0	q_0P_1
W	2	150	165	300	330
Gr	1	80	100	80	100
R	1	120	150	120	150
B	1.5	60	90	90	135
Ar	1.5	100	140	150	210
oil	10 kg	10	12	100	120
Gur	40 kg	2	3	80	120
				920	1165

$$\text{cost of L.I.No} = \frac{\Sigma q_0 P_1}{\Sigma q_0 P_0} \times 100$$

$$= \frac{1165}{920} \times 100$$

$$= \underline{126.63}$$

by Family budget (M) →

$P = \frac{P_1}{P_0} \times 100$	$W = q_0 P_0$	WP
110	300	33000
125	80	10000
125	120	15000
150	90	13500
140	150	21000
120	100	12000
150	80	12000
	920	116500

$$\text{cost of L.I.No} = \frac{\Sigma WP}{\Sigma W} = \frac{116500}{920} = \underline{126.63}$$

A.I

5) S.T F.I. index satisfies both the time reversal & factor reversal test —

C	Base yr. p_0	q_0	Current yr p_1	q_1	q_0p_0	q_0p_1	q_1p_0	q_1p_1
A	6	50	10	56	300	500	336	560
B	2	100	2	120	200	200	240	240
C	4	60	6	60	240	360	240	360
D	10	30	12	24	300	360	240	288
E	8	40	12	36	320	480	288	432
					1360	1900	1344	1880

By F.I. no.

$P_{01} \times P_{10} = 1$ To prove

$$P_{01} = \sqrt{\frac{\Sigma q_0p_1}{\Sigma q_0p_0} \times \frac{\Sigma q_1p_1}{\Sigma q_1p_0}}$$

$$= \sqrt{\frac{1900}{1360} \times \frac{1880}{1344}}$$

$$P_{10} = \sqrt{\frac{\Sigma q_1p_0}{\Sigma q_1p_1} \times \frac{\Sigma q_0p_0}{\Sigma q_0p_1}} = \sqrt{\frac{1344}{1880} \times \frac{1360}{1900}}$$

Time reversal →

$$P_{01} \times P_{10} = \sqrt{\frac{\cancel{1900}}{\cancel{1360}} \times \frac{\cancel{1880}}{\cancel{1344}}} \times \sqrt{\frac{\cancel{1344}}{\cancel{1880}} \times \frac{\cancel{1360}}{\cancel{1900}}}$$

$$= \sqrt{1} = 1$$

Factor reversal

$$P_{01} \times Q_{01} = \sqrt{\frac{\Sigma q_0p_1}{\Sigma q_0p_0} \times \frac{\Sigma q_1p_1}{\Sigma q_1p_0}} \times \sqrt{\frac{\Sigma q_1p_0}{\Sigma q_0p_0} \times \frac{\Sigma q_1p_1}{\Sigma q_0p_1}}$$

$$= \sqrt{\frac{\cancel{1900}}{1360} \times \frac{1880}{\cancel{1344}}} \times \sqrt{\frac{\cancel{1344}}{1360} \times \frac{1880}{\cancel{1900}}}$$

$$= \sqrt{\frac{1880 \times 1880}{1360 \times 1360}} = \frac{1880}{1360} = \frac{\Sigma q_1p_1}{\Sigma q_0p_0}$$

(or pick any 1)

$$= \frac{3534400}{1849600} = \frac{\sqrt{3534400}}{\sqrt{1849600}} = \frac{1880}{1360}$$

6) Construct fixed base & chain base index no. relating to production of (e).

yr	production	F B(Q_{01})	LR	CB (Q_{01})
1981	25 FB	100	100	100
1982	27	108	= 108	108
1983	30	120	111.11	120
1984	24	96	80	96
1985	28	112	116.65	112
1986	29	116	103.57	116
1987	31	124	106.90	124
1988	35	140	112.90	140

FB → Fixed base
LR → Linked relatives
CB → Chain base

CB

LR → FB

FB

5) S.T F.I. index satisfies both the time reversal & factor reversal test —

C	Base yr p_0	q_0	Current yr p_1	q_1	q_0p_0	q_0p_1	q_1p_0	q_1p_1
A	6	50	10	56	300	500	336	560
B	2	100	2	120	200	200	240	240
C	4	60	6	60	240	360	240	360
D	10	30	12	24	300	360	240	288
E	8	40	12	36	320	480	288	432
					1360	1900	1344	1880

By F.I. no.

$P_{01} \times P_{10} = 1$ → To prove

$$P_{01} = \sqrt{\frac{\Sigma q_0 p_1}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_1 p_0}}$$

$$= \sqrt{\frac{1900}{1360} \times \frac{1880}{1344}}$$

$$P_{10} = \sqrt{\frac{\Sigma q_1 p_0}{\Sigma q_1 p_1} \times \frac{\Sigma q_0 p_0}{\Sigma q_0 p_1}} = \sqrt{\frac{1344}{1880} \times \frac{1360}{1900}}$$

Time reversal →

$$P_{01} \times P_{10} = \sqrt{\frac{1900}{1360} \times \frac{1880}{1344}} \times \sqrt{\frac{1344}{1880} \times \frac{1360}{1900}}$$

$$= \sqrt{1} = 1$$

Factor reversal

$$P_{01} \times Q_{01} = \sqrt{\frac{\Sigma q_0 p_1}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_1 p_0}} \times \sqrt{\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}}$$

$$= \sqrt{\frac{1900}{1360} \times \frac{1880}{1344}} \times \sqrt{\frac{1344}{1360} \times \frac{1880}{1900}}$$

$$= \sqrt{\frac{1880 \times 1880}{1360 \times 1360}} = \frac{1880}{1360} = \frac{\Sigma p_1 q_1}{\Sigma q_0 p_0}$$

6) Construct fixed base & chain base index no. relating to production of ...

Yr	Production	FB (Q_{01})	LR	CB (Q_{01})
1981	25	100	100	100
1982	27	108	108	108
1983	30	120	111.11	120
1984	24	96	80	96
1985	28	112	116.65	112
1986	29	116	103.57	116
1987	31	124	106.90	124
1988	35	140	112.90	140

FB → Fixed base
LR → linked relatives
CB → Chain base

CB

1981 → $\frac{100 \times 100}{100} = 100$

1983 → $\frac{111.11 \times 108}{100} = 120$

1984 → $\frac{80 \times 120}{100} = 96$

1987 → $\frac{106.90 \times 116}{100} = 124$

LR → FB

100 → $\frac{100}{100} \times 100$

120 → $\frac{120}{108} \times 100 = 111.11$

124 → $\frac{124}{116} \times 100 = 106.90$

140 → $\frac{140}{124} \times 100 = 112.90$

FB

$\frac{25}{25} \times 100 = 100$

27 → $\frac{27}{25} \times 100 = 108$

30 → $\frac{30}{25} \times 100$

35 → $\frac{35}{25} \times 100 = 140$

⇒ Test for an Ideal No =

1) Time Reversal Test :-

$P_{01} = \frac{\Sigma q_0 P_1}{\Sigma q_0 P_0}$ $\qquad P_{10} = \frac{\Sigma q_1 P_0}{\Sigma q_1 P_1}$

(interchanging 1 by 0 & 0 by 1)

$P_{01} \times P_{10} = \frac{\Sigma q_0 P_1}{\Sigma q_0 P_0} \times \frac{\Sigma q_1 P_0}{\Sigma q_1 P_1} = 1$

(i.e) $\boxed{P_{01} \times P_{10} = 1}$ → To prove.

2) Factor Reversal Test :-

$\boxed{P_{01} \times Q_{01} = \frac{\Sigma q_1 P_1}{\Sigma q_0 P_0}}$ → To prove

(i.e) $P_{01} = \sqrt{\frac{\Sigma q_0 P_1}{\Sigma q_0 P_0} \times \frac{\Sigma q_1 P_0}{\Sigma q_1 P_1}}$ (interchanging P by q & q by P)

$Q_{01} = \sqrt{\frac{\Sigma P_0 q_1}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_0}{\Sigma P_1 q_1}}$

$P_{01} \times Q_{01} = \frac{\Sigma q_1 P_1}{\Sigma q_0 P_0}$.

⇒ Chain base & Fixed Base I.no :-

Chain Base I.(CBI) = $\frac{\text{Current yr LR} \times \text{preceding yr}}{100}$.

Current yr F.B.I = $\frac{\text{Current CBI} \times \text{Preceding year}}{100}$

LR → Link Relative

⇒ Base shifting :-

Q) shift the base of following indices from 1971 to 1974 —

yr	1971 Index No	1974 Index No.
1971	100	80
1972	105	84
1973	110	88
1974	125	100
1975	120	96
1976	150	120.

I.no
1971 → 100
1974 → 125
i.e. $\frac{100}{125}$

$124 \times \frac{100}{125}$

$= 100$

$150 \times \frac{100}{125}$

$= 120$

⇒ Splicing :-

① formula for forward splicing = $\frac{\text{dum}}{100} \times \text{I.N}$

required Index = $\frac{\text{I.N of yr of intertwining}}{100} \times$ Index No

② formula for backward splicing = $\frac{100}{\text{dum}}$ I.N

R.I = $\frac{100}{\text{In of yr of intrwining}} \times$ Index No.

Q) Splice the following 2 I.no series. continuing series A forward & series B backwards.

Yr :	1968	1969	1970	1971	1972	1973
S→A :	100	120	150			
S→B :			100	110	120	150

A)

Yr	S(A)	S(B)	S(A) with forward conti.	S(B) with backward conti.
1968	100	?	100	$\frac{100}{150}\times 100 = 66.67$
1969	120	?	120	$\frac{100}{150}\times 120 = 80$
1970	150	100	150	100
1971	?	110	$\frac{150}{100}\times 100 = 150$	110
1972	?	120	$\frac{150}{100}\times 120 = 180$	120
1973	?	150	$\frac{150}{100}\times 150 = 225$	150

$\frac{100}{150}\times 225$

$= \underline{150}$

31/3/22