01: INTRODUTION TO DIFFERENTIAL EQUATION.

⇒ ordinary & partial diff~ eq =

A D.eq involving a single independent variable & hence only ordinary deri~ → ordinary D.eq (ODE)

eg → ① $(y^2+x)\dfrac{d^2y}{dx^2} + 2y\left(\dfrac{dy}{dx}\right)^2 = 7.$ → dependent → x-നെ depent ചെയ്യുന്നതാണ് y.

② $y'' + (8x+3)y' + e^y \sin x = 0$

③ $y\left(\dfrac{dy}{dt}\right)^2 + 2t\dfrac{dy}{dt} - y = 0$

* A D.eq involving more than 1 independent variable, & hence partial deri~ → partial diff~ eq (PDE)

eg → ① $t^2 \dfrac{\partial^2 u}{\partial t^2} - x\left(\dfrac{\partial u}{\partial x}\right)^2 - \sin t \dfrac{\partial u}{\partial t} = 0$ u → dependent, t, x → independent

② $U_{xx} + U_{yy} + U_{zz} = 0$ → $\dfrac{\partial^2 u}{\partial x^2} = U_{xx}$ u → dependent, x, y, z → indi~

* Rmrk
1) Leibniz notation = $\dfrac{dy}{dx}, \dfrac{d^2y}{dx^2}, \dfrac{d^3y}{dx^3}$

2) prime notation = $y', y'', y''', y^{(4)}, y^{(5)}$ (4) → 4^{th} deri~

n^{th} deri~ of f $= f^{(n)}$

☆ order & degree of D.eq =

* The order of a D.eq is the order of highest deri~ occuring in the eq.
* The degree of a D.eq is the degree of the highest deri~ which occurs in it.

eg for order →

① $(y^2+x)\dfrac{d^2y}{dx^2} + 2y\left(\dfrac{dy}{dx}\right)^2 = 7$. ($\dfrac{d^2y}{dx^2}$ → 2nd deri. $\dfrac{dy}{dx}$ → 1st deri.)

here order = 2.

Degree = 3 → ($\dfrac{d^2y}{dx^2}$ ...times ...)

② $(y'')^3 + 5xy' - 5xy = 8$

O = 2

D = 3

③ $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$ O = 2 D = 1 (... power → 1)

④ $\dfrac{\partial^2 z}{\partial x^2} + \left(\dfrac{\partial^2 z}{\partial y^2}\right)^2 = 0$ O = 2 D = 2

\+ Rmrk

we can express n^{th} order ODE in 1 independent variable by the general form,

$$\boxed{F(x, y, y', y'', \dots, y^{(n)}) = 0} \; — ①$$

1st order ODE → $y' + x = 0$

2nd " → $y'' + x = 0$

F → real valued ( ) with n+2 variables.

($x, y, y', \dots y^n = n+2$)

* The D.eq $\dfrac{d^n y}{dx^n} = f(x, y, y', \dots, y^{(n-1)})$

f → real valued contin ( ) of n+1 variables. $x, y, y', \dots y^{(n-1)}$ is referred to as normal form of eq — ①

⇒ Linear & Non-linear D.eq =

$x' + 3 = 1$ → linear D.F.	$x + 3 = 1$ → linear
$y'' + 3y + x = 0$ → L.DF.	$x^2 + 3 = 1$? → non-linear
$(y'')^3 + 3y' = 0$ → non-L.	$x^3 + 3 = 1$

* The ODE of order n $F(x, y, y', \dots y^{(n)}) = 0$ is said to be linear if F is linear ( ) of variables $y, y', y'', \dots y^{(n)}$.

* Any linear ODE of degree n can be return as,

$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \dots + a_1(x)y' + a_0(x)y = \phi(x)$.

$a_n(x), a_{n-1}(x), \dots a_1(x), a_0(x)$ & $\phi(x)$ are ( )s of independent variable x.

$a_n(x)y^n$

$a_2(x)y''$

* 2 imp sple cases are linear 1st order & linear 2nd order ODE'S,

1st o ← $a_1(x)\dfrac{dy}{dx} + a_0(x)y = \phi(x)$ — ① &

2nd o ← $a_2(x)\dfrac{d^2y}{dx^2} + a_1(x)\dfrac{dy}{dx} + a_0(x)y = \phi(x)$ — ②

format

* eg of linear D.eq →

1) $\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + 6y = 0$. by — ②

2) $\dfrac{d^4y}{dt^4} + t^2\dfrac{d^3y}{dt^3} + 5t^3\dfrac{d^2y}{dt^2} + 6\sin t\, y = t e^t$

*eq for non-linear D.eq →

1) $\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + 6y^2 = 0$ (y - ... here y^2.)

2) $\dfrac{d^2y}{dx^2} + 5\left(\dfrac{dy}{dx}\right)^2 + 6y = 0$. [in general eq no whole square]

3) $\dfrac{d^2y}{dx^2} + 5y\dfrac{dy}{dx} + 6y = 0$

$\Rightarrow$ **Solution of a D.eq =**

Any ( ) ϕ defined on an interval 'I' & processing atleast n deri.. that are continu.. on I, which when substituted into an n^{th} order ODE reduces the eq to an identity, it is said to be a soln of the eq

Q
1) S.T the ( ) ϕ is defined by
$\phi(t) = 2\sin t + 3\cos t$ is a soln of the follo.. D.eq for all real t. by,
$y'' + y = 0$.

$x - 2 = 0$
$x = 0 + 2$
$x = 2$
(or)
$x^2 - 2 = 0$
$2 - 2 = 0$

A) gvn $\phi(t) = 2\sin t + 3\cos t$

$y = \phi(t)$ $\quad y = \phi(t) = 2\sin t + 3\cos t$ — ①

$y' = \phi'(t) = 2\cos t - 3\sin t$

$y'' = \phi''(t) = -2\sin t - 3\cos t$ — ②

② + ① $\cancel{② + ① = 0}$

$y'' + y = -2\cancel{\sin t} - 3\cancel{\cos t} + 2\cancel{\sin t} + 3\cancel{\cos t} = 0$

hence $\phi(t)$ is a soln of gvn D.Eq.

2) verify that the ( ) $y = xe^x$ is a soln of the D.eq by $y'' - 2y' + y = 0$

A) $y = x \cdot e^x$

(proof) $y' = x \cdot e^x + e^x = xe^x + e^x$

$y'' = x \cdot e^x + e^x + e^x$ or $xe^x + 2e^x$

$y'' - 2y' + y = xe^x + 2e^x - 2(xe^x + e^x) + xe^x$
$= xe^x + 2e^x - 2xe^x - 2e^x + xe^x$
$= x e^x - 2xe^x + xe^x$
$= 2xe^x - 2xe^x = 0$

hence y is the soln gvn D.eq

Rmrk

* A soln of a D.eq that is identically 0 on an interval 'I' is said to be a trivial soln

3) Determine the value of r for which the D.eq $y''' - 3y'' + 2y' = 0$ has the soln of the form $y = e^{rx}$?

A) Since y satisfies the gvn D.eq
$y = e^{rx}$
$y' = re^{rx}$
$y'' = r^2 e^{rx}$
$y''' = r^3 e^{rx}$

$y''' - 3y'' + 2y' = 0 \Rightarrow r^3 e^{rx} - 3(r^2 e^{rx}) + 2(re^{rx}) = 0$
$= r^3 e^{rx} - 3r^2 e^{rx} + 2re^{rx} = 0$
$= e^{rx}[r^3 - 3r^2 + 2r] = 0$
① ②

ഒന്നിലെങ്കിൽ ① 0 ആകണം / ② ആകണം then only we gt 0 as answer bt $e^x \neq 0$ (always), so

$\Rightarrow r^3 - 3r^2 + 2r = 0$

$\Rightarrow r[r^2 - 3r + 2] = 0$ $\quad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow r(r-2)(r-1) = 0$

$r = 0$ or $r - 2 = 0$ or $r - 1 = 0$

$r = 0 \quad r = 2 \quad r = 1$

$\therefore$ r values 0, 1, 2