Chapter = 03
Measures of Central Tentency.

* ~~#~~ A measure of central tentency hlps to get a single representative value for a set of unequal values. This single is the point of location around which the individual values of the set cluster. Hence the avgs are also → measures of location.

* The imp M. of C. tentencies —
1) Arithemetic Mean
2) Median
3) Mode
4) Geometric Mean
5) Harmonic Mean.

I Arithemetic Mean (AM):- $(\bar{x})$

* Definition for a raw data :- (discreate)

$$\boxed{\bar{X} = \frac{x_1+x_2+x_3+\dots+x_n}{n}} \quad \boxed{\bar{x} = \frac{\Sigma x}{n}} ✓$$

where, $x_1, x_2, x_3 \dots x_n \rightarrow n$ observations.

* Definition for a freq data :-

$$\boxed{\bar{X} = \frac{x_1 f_1 + x_2 f_2 + x_3 f_2 \dots x_n f_n}{f_1+f_2+f_3 \dots f_n}}$$

$$\boxed{\bar{X} = \frac{\Sigma xf}{\Sigma f}} ✓$$

1) Calculate the AM of 12, 18, 14, 15, 16

A) $\bar{X} = \frac{x_1+x_2+\dots+x_n}{n} = \frac{12+18+14+15+16}{5}$

$= \frac{75}{5} = \underline{\underline{15}}$

2) cal. Avg income

Daily earnings	No. of workers	fx
5	3	15
6	8	48
7	12	84
8	10	80
9	7	63
	40	290

A) $AM = \frac{\Sigma xf}{\Sigma f} = \frac{290}{40} = \underline{\underline{7.25}}$

3) cal. AM

cls	f	x (ul+ll/2)	fx
0-4	1	2	2
4-8	4	6	24
8-12	3	10	30
12-16	2	14	28
	10		84

A) $\bar{x} = \frac{\Sigma xf}{\Sigma f} = \frac{84}{10} = \underline{\underline{8.4}}$

4) Cal. AM

cls	f	x	fx
0-7	8	3.5	28
8-15	15	11.5	172.5 ~~172.5~~
16-23	34	19.5	663
24-31	20	27.5	550
32-39	18	35.5	639
40-47	5	43.5	217.5
	100		~~[illegible]~~ 2270

$$\frac{\Sigma fx}{f} = \frac{\cancel{16733}}{\cancel{100}} = \cancel{16.733}$$

$$= \frac{2270}{100} = 22.7$$

→ Short cut method for calculating AM :-

1) $\boxed{\bar{x} = A + \frac{\Sigma d}{n}}$ $A \rightarrow$ Assumed mean ([illegible] center no)

$d = x - A$. (deviation).

(freq [illegible] data-[illegible] we can use this)

2) $\bar{x} = \boxed{A + \frac{\Sigma fd}{\Sigma f} \cdot c}$ c = width of cls interval

A = Assumed mean.

$d = \frac{x-A}{c}$ $\Sigma f = N$.

$\left(\frac{10}{0} = \text{infinity}\right)$

Q

1) cal. AM using short cut method.

cls	f	Mid value (x)	$d = \frac{x-A}{c} = \frac{x-25}{10}$	fd
0–10	3	5	−2	−6
10–20	12	15	−1	−12
20–30	20	25	0	0
30–40	10	35	1	10
40–50	5	45	2	10
	50			2

$AM = A + \frac{\Sigma fd}{\Sigma f} \cdot c$ $= 25 + \frac{2}{\cancel{50}_5} \cdot \cancel{10} = 25 + 0.4$

$A = 25$ $= 25.4$

$c = 10$

2) cal. AM.

cls	f	x	$d = \frac{x-19.5}{8}$	fd
0–7	8	3.5	−2	−16
8–15	15	11.5	−1	−15
16–23	34	19.5	0	0
24–31	20	27.5	1	20
32–39	18	35.5	2	36
40–47	5	43.5	3	15
	100			40

$\bar{x} = A + \frac{\Sigma fd}{\Sigma f} \cdot c$ $A = 19.5$

$\Sigma f = 100$

$c = 7 + 1 = 8$

$= 19.5 + \frac{40}{100} \cdot 8$

$= 19.5 + 3.2 = 22.7$

3) cal. AM – 305, 320, 332, 350

$\bar{x} = A + \frac{\Sigma d}{n}$ $A = 320$ $d = x - A$.

$n = 4$ $305 - 320 = -15$

$= 320 + \frac{27}{4}$ $\Sigma d = 27$ $320 - 320 = 0$

$= 320 + 6.75$ $332 - 320 = 12$

$= 326.75$ $350 - 320 = 30$

$\Sigma d = 27$

x	$d = x - 320$
305	−15
320	0
332	12
350	30
	27

→ properties :-

1) The AM is preserved under a linear transformation of scale

(i.e) if 'x_i' is changed to 'y_i' by a rule

$y_i = a + bx_i$

then $\boxed{\bar{y} = a + b\bar{x}}$ is also linear.

2) The mean of sum of variables is = the sum of means of the variable.

3) Algebraic sum of the deviations of every observations from the AM is 0.
(i.e) $\boxed{\sum(x-\bar{x}) = 0}$.

4) If n_1 observation hve an AM '$\bar{x}_1$' & n_2 observation hve an AM '$\bar{x}_2$', then the combined grp of n_1+n_2 observation is given by,

(avg) $\rightarrow$ $\boxed{\bar{x} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2}}$

Q

1) Avg mrk of 40 students of cls A be 38. The Avg mrk of 60 students of cls B is 42. wht is Avg mrk?

A) $\bar{x} = \dfrac{n_1x_1 + n_2x_2}{n_1+n_2}$

$= \dfrac{40 \times 38 + 60 \times 42}{40+60} = \dfrac{1520 + 2520}{100}$

$= \dfrac{4040}{100} = 40.4$

2) 3 samples of size 45, 40, 65 hving means 2, 2.5, 2 respectivly. find mean

A) $\bar{x} = \dfrac{n_1x_1 + n_2x_2 + n_3x_3}{n_1+n_2+n_3}$

$= \dfrac{45\times 2 + 40 \times 2.5 + 65 \times 2}{45+40+65}$

$= \dfrac{90 + 100 + 130}{150}$

$= \dfrac{320}{150} = 2.13$

(imp) 5) The Algebraic sum □'s of observation from AM is always min.
(i.e) $\sum_{i=1}^{n}(x_i - \bar{x})^2$ is always min

Proof -
let $x_1, x_2 \dots x_n$ be the set 'n' observation
Choose $x_0 = \bar{x}$

$\therefore f(x_0) = \sum_{i=1}^{n}(x_i - x_0)^2$

we hve to show that $f(x_0)$ ~~that~~ is min at $x_0 = \bar{x}$.

$\frac{d}{dx}(x_i-x_0)^2 = 2(x_i-x_0) \times -1$

$f'(x_0) = 0$ implies

$x^n = nx^{n-1}$

$= 2\sum_{i=1}^{n}(x_i - x_0) \times -1 = 0$

$\Rightarrow \sum_{i=1}^{n}(x_i - x_0) = 0$

$\Rightarrow \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_0 = 0$

$\frac{x_1+x_2 \dots x_n}{n} = \bar{x}$

$\Rightarrow \left(\frac{x_1+x_2 \dots x_n}{n}\right) - \left(\frac{x_0+x_0 \dots x_n}{n}\right) = 0$

$\Rightarrow n\bar{x} - nx_0 = 0$

$\Rightarrow n\bar{x} = nx_0$

$\bar{x} = x_0$

$f'(x_0) = 0$ at $x_0 = \bar{x}$

$f'(x_0) = -2\sum(x_i - x_0) = -2\left[x_1 x_0 + x_2 x_0 \cdots x_n x_0\right]$

$f''(x_0) = -2\left[\underbrace{(-1)+(-1)+\cdots(-1)}_{(n\ \text{times})}\right]$ $\sum_{i=1}^{n} -1 = -1n$

$= -2 \times n \times (-1)$ $\sum_{i=1}^{8} -1 = -1\times 8$

$f''(x_0) = 2n$

$2n > 0 \qquad f''(x_0) > 0$

Hence by the method of calculus $f(x_0)$ is min when $x_0 = \bar{x}$.

→ weighted AM :-

If $x_1, x_2 \ldots x_n$ are the values of items & $w_1, w_2 \ldots w_n$ are their respective weights. Then,

$$WAM = \frac{w_1 x_1 + w_2 x_2 \cdots w_n x_n}{w_1 + w_2 + \cdots w_n}$$

$$\boxed{WAM = \frac{\sum wx}{\sum w}}$$

1) A student obtains 60 mrk in statistic 48 mrk in economics, 72 in commerce 45 in taxation, 55 in law. cal. Simple AM & WAM ?.

A) ~~Simple AM,~~

~~AM = 60+72~~

The (w) of mrk are 2, 1, 3, 4, 2.

A) Simple AM, $AM = \frac{\sum fx}{\sum f}$

$$AM = \frac{60+72+48+45+55}{5} = \frac{280}{5}$$

$= 56$

$$WAM = \frac{\sum wx}{\sum w}$$

mrk	weight	wx
60	2	120
48	1	48
72	3	216
45	4	180
55	2	110
	12	674

$$WAM = \frac{674}{12} = 56.16$$

II Geometric Mean :- (GM)

1) Definition for raw data :- (No freq)

$$GM = \sqrt[n]{x_1 \times x_2 \times \cdots x_n}$$

$x_1, x_2, x_3 =$ are n observation.

$$\boxed{GM = \left[x_1 \times x_2 \times \cdots x_n\right]^{1/n}}$$

(or)

$$\boxed{GM = \text{Antilog}\left(\frac{\sum \log x}{n}\right)}$$

$\sqrt{x} = x^{1/2}$

$\sqrt[3]{x} = x^{1/3}$

$\frac{1}{\sqrt{x}} = x^{-1/2}$

$\frac{1}{\sqrt[3]{x}} = x^{-1/3}$

$\sqrt[n]{x} = x^{1/n}$

$\frac{1}{\sqrt[n]{x}} = x^{-1/n}$

2) Definition for freq distribution :-

$$GM = \sqrt[N]{x_1^{f_1} \times x_2^{f_2} \times \cdots x_n^{f_n}}$$

N = total freq.
↳ $N = \sum f$.

(or)

$$\boxed{GM = \text{Antilog}\left(\frac{\sum f \log x}{N}\right)}$$

$\text{Antilog}(x) = 10^x$

1) Cal. GM of 2, 4, 8 (shift + $\sqrt{}$ + no..)

A) $GM = \sqrt[3]{2 \times 4 \times 8}$
$= \sqrt[3]{64} = 4$

2) Cal. GM of 4, 6, 9, 11, 15

A) $GM = \text{Antilog}\left(\frac{\Sigma \log x}{n}\right)$ (log + no) ($\log_e \to \ln$)

x	$\log x$
4	0.6020
6	0.7781
9	0.9542
11	1.0413
15	1.1760
	4.5516

$GM = \text{Antilog}\left(\frac{4.5516}{5}\right)$
$= \text{Antilog}(0.91032)$ ($10^\wedge$ + no)
$= \underline{\underline{8.1342}}$

3) Cal. GM.

C/S	f	(mid.p) x	log x	f log x
1-3	8	2	0.3010	2.408
4-6	16	5	0.6989	11.1824
7-9	15	8	0.9030	13.545
10-12	3	11	1.0413	3.1239
	42			30.2593

A) $GM = \text{Antilog}\left(\frac{\Sigma f \log x}{N}\right) = \frac{30.2593}{42}$
$= \text{Antilog}(0.7204)$
$= \underline{\underline{5.252}}$

III Harmonic Mean :- (HM)

(HM) are of a set of observation is defined as the reciprocal of the AM of the reciprocal's of observations.

$\frac{1}{\frac{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}}{n}} = \frac{n}{\left(\frac{1}{x_1} + \frac{1}{x_2} + \dots \frac{1}{x_n}\right)}$ $\left(\frac{1}{7/8} = \frac{1 \times 8}{7}\right)$

$\boxed{HM = \frac{n}{\Sigma \frac{1}{x_i}}}$ → No freq.

* for a freq distribution :-

$\boxed{HM = \frac{N}{\Sigma\left(\frac{f_i}{x_i}\right)}}$ $N = \Sigma f$.

1) Cal. HM → 2, 3, 4, 5, 7

A) $HM = \frac{n}{\Sigma \frac{1}{x_i}} = \frac{5}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{7}}$
$= \frac{5}{0.5 + 0.33 + 0.25 + 0.2 + 0.14}$
$= \frac{5}{1.42} = \underline{\underline{3.52}}$

(or)

$\frac{5}{1.4261} = \underline{\underline{3.506}}$

2) Cal. HM →

GM

is a mean / Avg, which indicates the C. tendency by using the product of their values

wages	f	x	f_i/x_i
100–110	10	105	0.09522
110–120	25	115	0.21733
120–130	36	125	0.288
130–140	68	135	0.5037
140–150	32	145	0.2206
150–160	21	155	0.1354
160–170	8	165	0.0484
	200		1.5086

$$HM = \frac{N}{\Sigma f_i/x_i} = \frac{200}{1.5086} = 132.57$$

2) Cal. HM → 5, 11, 12, 16, 7, 9, 15, 13, 10, 8

A) $HM = \frac{n}{\Sigma 1/x_i}$

$= \frac{10}{1.0591}$

$= 9.44.$

x	1/x
5	0.2
11	0.090
12	0.083
16	0.0625
7	0.1428
9	0.1111
15	0.666
13	0.0769
10	0.1
8	0.125
	1.0591

3) cal GM & HM.

c/s	f
0–10	8
10–20	12
20–30	20
30–40	6
40–50	4

A)

c/s	f	x	log x	f log x	f/x
0–10	8	5	0.6990	5.5920	1.6000
10–20	12	15	1.1761	14.1132	0.8000
20–30	20	25	1.3979	27.9580	0.8000
30–40	6	35	1.5441	9.2646	0.1714
40–50	4	45	1.6532	6.6128	0.0889
	50			63.5406	3.4603

$$GM = \text{Antilog}\left(\frac{f \log x}{N}\right)$$

$$= \text{Antilog}\left(\frac{63.5406}{50}\right)$$

$$= \text{Antilog}(1.270812)$$

$$GM = 18.65$$

$$HM = \frac{N}{\Sigma\left(\frac{f_i}{x_i}\right)}$$

$$= \frac{50}{3.4603}$$

$$HM = 14.45$$

∴ GM is 18.65 &

HM is 14.45

IV) Median : (M)

It is the middle most observation when the observations are arranged in ascending / descending order.

1) Definition for raw data :-

If n is odd

$$\boxed{M = \left(\frac{n+1}{2}\right)^{th} \text{ term}}$$

If n is even

$$\boxed{M = \frac{\left(\frac{n}{2}\right)^{th} \text{ term} + \left(\frac{n}{2}+1\right)^{th} \text{ term}}{2}}$$

2) Definition for freq data :-

$$M = l + \frac{\left(\frac{N}{2} - m\right)}{f} \times c$$

CF.

$N = \Sigma f$

c = cls interval / width

l = lower limit of median cls.

f = freq of M.cls.

m = cumulative freq upto M.cls.

where the cls interval which contains $(N/2)^{th}$ observation → M.cls.

* Merits :-

a) Not unduly affected by extreme items.
b) can be cal. for open end data.
c) can be determined graphically
d) used to deal with qualitative data.

* Demerits :-

a) Not based on magnitude of all items.
b) Difficult to calculated when there are large no. of items which are to be arranged in order of magnitude.
c) Does not having sampling stability.

→ Merits of HM :-

*) It is rigidly defined.
*) Based on all the items.
*) Affected less by extreme items than AM i.e GM.
* can be algebraically manipulated.

→ Demerits of HM :-

* Neither simple to understand nor easy to cal.
* Has less sampling stability than AM.
* cannot be found graphically.
* Not defined for qualities.

* Merits of AM :-

a) It is rigidly defined.
b) Simple to understand i.e not difficult to cal.
c) Many forms of formula are available.
d) Very useful in day to day activities.

* **Demerits of AM :-**

a) Theoretically, it cannot be calculated for open end data.

b) cannot be found graphically.

c) Not defined to deal with qualities.

→ **Merits of GM :-**

* Base on all the items.
* Not as unduly affected by extreme items as A.M.
* rigidly defined.
* useful in avg ratios & % s.

→ **Demerits of GM :-**

* Neither simple to understand nor easy to cal.
* Cannot be cal. for open end data.
* cannot be found graphically.
* cannot be defined for qualities.

→ **(Pro) of GM :-**

- If G - GM ∴ G^n = product of n obsr..
- 2 set of obsrv.. same GM, if they equal no. of observ.. & equal products.
- In a set of obsrv.. if any 1 of obsrv.. is 0, then GM = 0.
- If the obser.. all ratios, then GM is the GM of numerators ÷ by GM of denominators.

$$\boxed{G = \text{Antilog}\left(\frac{n_1 \log G_1 + n_2 \log G_2}{n_1 + n_2}\right)}$$

Proof

we hve $G_1^{n_1}$ = product of n_1 obsrv

$G_1^{n_2}$ = " of n_2 obsrv.

$G_1^{n_1} \cdot G_2^{n_2}$ = " of n_1+n_2 obsrv.

$= G^{n_1+n_2}$

$$\therefore G = \left(G_1^{n_1} \cdot G_2^{n_2}\right)^{\frac{1}{n_1+n_2}}$$

$$\log G = \frac{1}{n_1+n_2} \log\left(G_1^{n_1} \cdot G_2^{n_2}\right)$$

$$= \frac{1}{n_1+n_2}\left(n_1 \log G_1 + n_2 \log G_2\right)$$

$$\boxed{G = \text{Antilog}\left(\frac{n_1 \log G_1 + n_2 \log G_2}{n_1 + n_2}\right)}$$

Cont--

* **Median cls :-**

The cls in which $\frac{N}{2}$ th observation falls.

1) cal. median 160, 180, 175, 179, 164, 178, 171, 164, 176.

A) Asnding ordr → 160, 164, 164, 171, 175, 176, 178, 179, 180.

$n = 9$ (odd)

$M = \left(\frac{n+1}{2}\right)^{th}$ term

$M = \frac{9+1}{2} = \frac{10}{2} = 5^{th}$ term → 175

$M = 175$

2) Cal. M → 75, 71, 73, 70, 74, 80, 85, 81, 86, 79.

A) 70, 71, 73, 74, 75, 79, 80, 81, 85, 86

$n = 10$ (even)

$$M = \frac{\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}}{2}$$

$$M = \frac{5^{th} \text{term} + 6^{th} \text{term}}{2}$$

$$M = \frac{75 + 79}{2} = \frac{154}{2} = 77$$

3) Cal. (M)

cls	f	cF
0-5	5	5
5-10	10	15
10-15	15	30
15-20	12	42
20-25	8	50
	50	

15 → m

30 → Median cls.

(freq should assending ordr)

$$M = l + \frac{\left(\frac{N}{2} - m\right)}{f} \times c$$

$N = 50$
$C = 5$
$N/2 = \frac{50}{2} = 25^{th}$
$f = 15$

M. cls → 10-15
$l = 10$
$m = 15$

$$M = 10 + \frac{(25 - 15)}{15} \times 5$$

$$= 10 + \frac{10}{15} \times 5$$

$$= 10 + 0.666 \times 5$$

$$M = 10 + 3.333 = 13.33$$

4) Cal. (M).

cls	f	cF
160	1	1
164	2	3
170	10	13
173	22	35
178	19	54
180	14	68
182	2	70
	70	

$N = 70$ (even)

$$M = \frac{\left(\frac{N}{2}\right)^{th} \text{term} + \left(\frac{N}{2}+1\right)^{th} \text{term}}{2}$$

$$= \frac{\left(\frac{70}{2}\right)^{th} \text{term} + \left(\frac{70}{2}+1\right)^{th} \text{term}}{2}$$

$$= \frac{35^{th} \text{term} + 36^{th}}{2}$$

(look at cF)

$$M = \frac{173 + 178}{2} = \frac{351}{2} = 175.5$$

5) cal (M)

cls	f	actual cls	cf
0-6	8	-0.5 - 6.5	8
7-13	17	6.5 - 13.5	25
14-20	28	13.5 - 20.5	53
21-27	15	20.5 - 27.5	68
28-34	9	27.5 - 34.5	77
35-41	3	34.5 - 41.5	80
	80		

$\frac{N}{2} = 40$

$l = 13.5$
$m = 25$
$f = 28$
$C = 7$

A) $M = l + \frac{\left(\frac{N}{2} - m\right)}{f} \times c$

$= 13.5 + \frac{(40-25)}{28} \times 7$

$= 13.5 + \frac{15}{28} \times 7$

$= 13.5 + 0.535 \times 7$

$M = 13.5 + 3.745 = \underline{\underline{17.25}}$

Mode :- $\boxed{Z}$ or $\boxed{M_0}$

* It is the value of the variable which occur max no. of times in a set of observations.

* For a freq distribution Mode is defined as the value of the variable having the max freq.

* For a continuous freq distribution it can be calculated using a formula

$$\boxed{Z = l + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times c}$$

Modal cls = cls having the max freq

l = lower limit of modal cls

c = cls interval

Δ_1 = Difference btw the freq of M.cls & that of the pre modal cls.

Δ_2 = Difference b/w freq of M.cls & that of post modal cls

Q
1) cal Mode —

cls	f	Actual cls.
0—9	5	-0.5 - 9.5
10—19	10	9.5 - 19.5
20—29	17	19.5 - 29.5
30—39	33	29.5 - 39.5
40—49	22	39.5 - 49.5
50—59	13	49.5 - 59.5

(33 = high freq)

A) $Z = l + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times c$

$c = 10$
$l = 29.5$
$\Delta_1 = 33 - 17 = \underline{16}$
$\Delta_2 = 33 - 22 = \underline{11}$

$= 29.5 + \frac{16}{16+11} \times 10$

$= 29.5 + 5.92$

$Z = \underline{35.42}$

2) Cal Mode-

Size =	3	4	5	6	7	8	9
No of pairs =	10	25	32	38	61	47	34

61 → high freq

$\therefore$ 7

A) Mode $= z = 7$

3) Cal. Mean, Median, Mode-

cls	f	x	fx	cf
4-8	3	6	18	3
8-12	7	10	70	10
12-16	16	14	224	26
16-20	8	18	144	34
20-24	2	22	44	36
	36		500	

A) Mean $\bar{x} = \frac{\Sigma fx}{f} = \frac{500}{36} = 13.89$

Median

$\frac{N}{2} = 18$

$l = 12$, $m = 10$, $f = 16$, $c = 4$

$M = l + \frac{\left(\frac{N}{2} - m\right)}{f} \times c$

$= 12 + \frac{(18 - 10)}{16} \times 4$

$= 12 + \frac{8}{16} \times 4$

$= 12 + 0.5 \times 4$

$M = 12 + 2 = 14$

Mode

$\Delta_1 = 16 - 7 = 9$

$\Delta_2 = 16 - 8 = 8$

$l = 12$

$c = 4$

$z = l + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times c$

$= 12 + \frac{9}{9+8} \times 4$

$= 12 + 0.529 \times 4$

$= 12 + 2.117 = 14.11$

☆ ⇒ Empirical Relation :-

$$\boxed{\text{Mean} - \underset{(z)}{\text{mode}} = 3(\text{Mean} - \text{Median})}$$

Q

1) In a moderatly assymmetrical distribution Mean is 24.6 & Median = 25.1, Mode = ?

A) Mean $- z = 3(\text{Mean} - \text{Median})$

$24.6 - z = 3(24.6 - 25.1)$

$24.6 - z = 3(-0.5)$

$24.6 - z = -1.5$

$z = 24.6 + 1.5 = 26.1$

2) For a moderatly assymmetrical disn Mean & mode is 15.6 & 16 Median = ?

A) Mean $- z = 3(\text{Mean} - \text{Median})$

$15.6 - 16 = 3(15.6 - M)$

$-0.4 = 46.8 - 3M$

$3M = 46.8 + 0.4 = 47.2$

$M = \frac{47.2}{3} = 15.73$

3) Median $= 41.6$, Mode $= 48.4$, Mean$(\bar{x})$ = ?

A) Mean $- Z = 3(\text{Mean} - M)$

Mean $- 48.4 = 3(\text{Mean} - 41.6)$

Mean $- 48.4 = 3\,\text{Mean} - 124.8$

$3\,\text{Mean} - \text{Mean} = 124.8 - 48.4$

$2\,\text{Mean} = 76.4$

Mean $= 38.2$

→ Relationships among avgs :-

1) whn all the obsr. in a series are identical, the mean, median, mode, GM & HM are equal.

eg :- x : 10, 10, 10, 10

2) whn the distribution is symmetrical

Mean = Median = Mode.

3) whn the data are asymmetrical median always lies b/w mean & mode.

4) If there are only 2 obsr. then,

$\boxed{GM^2 = AM \times HM}$

Proof

let a & b be the 2 obsr.

then, $AM = \frac{a+b}{2}$

$GM = \sqrt{ab}$

$HM = \frac{2ab}{a+b}$

$HM = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}}$

$= \frac{2}{\frac{1}{a} + \frac{1}{b}}$

$\frac{2}{\frac{b+a}{ab}} = \frac{2}{\frac{a+b}{ab}}$

$HM = \frac{2ab}{a+b}$

$AM \times HM = \frac{(a+b)}{2} \times \frac{2ab}{(a+b)}$

$AM \times HM = ab$

$ab = GM^2$

5) If all obsr. are not equal, GM lies b/w AM & HM.

$\boxed{HM < GM < AM}$

Proof

let a & b be 2 +ve obsr.

$(\sqrt{a} - \sqrt{b})^2 > 0$

$(a-b)^2$

$(\sqrt{a})^2 - 2 \times \sqrt{a} \times \sqrt{b} + (\sqrt{b})^2 > 0$

$a - 2\sqrt{ab} + b > 0$

$a + b > 2\sqrt{ab}$

$\frac{a+b}{2} > \sqrt{ab}$

$AM > GM$ or $\boxed{GM < AM}$ — ①

Consider,

$\left(\frac{1}{\sqrt{a}} - \frac{1}{\sqrt{b}}\right)^2 > 0$

$(a-b)^2$

$$= \left(\frac{1}{\sqrt{a}}\right)^2 - \cancel{*}2 \times \frac{1}{\sqrt{a}} \times \frac{1}{\sqrt{b}} + \left(\frac{1}{\sqrt{b}}\right)^2 > 0$$

$$= \frac{1}{a} - \frac{2}{\sqrt{ab}} + \frac{1}{b} > 0$$

$$= \frac{1}{a} + \frac{1}{b} > \frac{2}{\sqrt{ab}}$$

$$= \frac{a+b}{ab} > \frac{2}{\sqrt{ab}}$$

$$= \frac{a+b}{2ab} > \frac{1}{\sqrt{ab}}$$

$\sqrt{ab} = GM$

reverse the order i.e '>' become '<'

$$= \frac{2ab}{a+b} < \sqrt{ab}$$ $\boxed{HM < GM}$ — (2)

∴ from (1) & (2)

$HM < GM < AM.$

6) For a moderatly assymmetrical dis.. $\boxed{\text{Mean} - \text{Mode} = 3(\text{Mean} - \text{Median})}$

H.W.

1) cal. Mean, Median & mode.

Values	cf	cls	f	x	fx
< 10	4	0–10	4	5	20
< 20	18	10–20	14 (18-4)	15	210
< 30	38	20–30	20 (38-18)	25	500
< 40	89	30–40	51	35	1785
< 50	121	40–50	32	45	1440
< 60	138	50–60	17	55	935
< 70	144	60–70	6	65	390
< 80	148	70–80	4	75	300
			148		5580

A) Assume the lower limit of 1st cls = 0.

Mean $\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{5580}{148} = 37.70$

Median $M = l + \frac{\left(\frac{N}{2} - m\right)}{f} \times c$ $\frac{N}{2} = 74$

$= 30 + \frac{(74-38)}{51} \times 10$

$= 30 + 7.05 = 37.05$

Mode $Z = l + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times c$ $\Delta_1 = 51-20 = 31$ $\Delta_2 = 51-32 = 19$

$= 30 + \frac{31}{31+19} \times 10$

$= 30 + 6.2 = 36.2$

7) cal. GM & HM.

value	f
0–10	8
10–20	12
20–30	20
30–40	6
40–50	4

GM = 18.65

HM = 14.45.

(Already written)

⇒ Partition values :-

Total area under a freq curve is = Total freq.

we can divide the dis. / area under a curve into no. of equal parts.

They are → partition values / Quartiles.

The imp (P) values – quartiles, Deciles & percentiles.

1) Quartiles :- (Q)

Q are (Q) values which divide the distribution into 4 equal parts at 3 points namely Q_1, Q_2, Q_3

Q_1 → 1st quartil / lower Q

Q_2 → 2nd Q / middle Q / Median

Q_3 → 3rd Q / upper Q.

(4) $$\boxed{Q_1 = l_1 + \frac{\left(\frac{N}{4} - m\right)}{f} \times c}$$

Q_1 c/s → c/s in $\frac{N}{4}$th item falls.

(10) $$\boxed{Q_2 = l_2 + \frac{\left(\frac{N}{2} - m\right)}{f} \times c}$$

→ Median

(100) $$\boxed{Q_3 = l_3 + \frac{\left(\frac{3N}{4} - m\right)}{f} \times c}$$

Q_3 c/s → c/s in which $\frac{3N}{4}$th item falls.

2) Deciles :- (D)

Here the curve is divided into 10 equal parts at 9 points namely $D_1, D_2 \ldots D_9$.

$$\boxed{D_i = l_i + \frac{\left(\frac{iN}{10} - m\right)}{f} \times c}$$

$i = 1, 2, \ldots 9.$

3) Percentiles :- (P)

The curve is divided into 100 equal parts at 99 points namely $P_1, P_2 \ldots P_{99}$.

$$\boxed{P_i = l_i \frac{\left(\frac{iN}{100} - m\right)}{f} \times c}$$

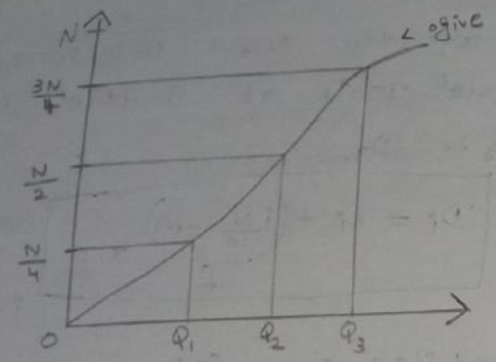
$i = 1, 2, \ldots 99$

→ Graphical determination of Q :-

Q can be determined graphically by drawing the ogives of given freq distribution. So draw < ogive of given data.

→ Weighted Geometric Mean :-

$$\boxed{W\,GM = \text{Antilog}\left(\frac{\Sigma w \log x}{\Sigma w}\right)}$$



Q

1) cal Q —

C|S	f	CF
30-35	10	10
35-40	16	26
40-45	18	44
45-50	27	71
50-55	18	89
55-60	8	97
60-65	3	100
	100	

Q1 Q2 Q3

$\frac{N}{4} = \frac{100}{4} = 25^{th}$

A) $Q_1 = l_1 + \frac{\left(\frac{N}{4} - m\right)}{f} \times c$

$= 35 + \frac{(25 - 10)}{16} \times 5$

$= 35 + \frac{15}{16} \times 5$

$= 35 + 4.6875 = 39.68$

$Q_2 = l_2 + \frac{\left(\frac{N}{2} - m\right)}{f} \times c$ $\quad \frac{N}{2} = 50$

$= 45 + \frac{(50 - 44)}{27} \times 5$

$= 45 + \frac{6}{27} \times 5$

$= 45 + 1.111 = 46.11$

$Q_3 = l_3 + \frac{\left(\frac{3N}{4} - m\right)}{f} \times c$ $\quad \frac{3N}{4} = \frac{300}{4} = 75$

$= 50 + \frac{(75 - 71)}{18} \times 5$

$= 50 + \frac{4}{18} \times 5$

$= 50 + 1.111 = 51.11$

2) find Q_1, Q_3, d_2, d_4, P_{16}, P_{65}

282, 754, 125, 765, 875, 645, 985, 235, 175, 895, 112, 155, 905.

A) 112, 125, 155, 175, 235, 282, 645, 754, ~~895, 985~~ 765, 875, 895, 905, 985

13 → odd

$Q_1 = \left(\frac{n+1}{4}\right)^{th}$ term $= \frac{13+1}{4} = \frac{14}{4} = 3.5^{th}$ term

$\left(\frac{155+175}{2}\right)$

$Q_1 = 165$

$Q_3 = \left(\frac{3(n+1)}{4}\right)^{th}$ term

$= \frac{3(13+1)}{4} = \cancel{\frac{40}{4}} = \cancel{10}$

$= 3 \times 3.5 = 10.5^{th}$

$\Rightarrow \frac{875+895}{2} = 885$

$(n+1) \to$ add $= 13$

$D_2 = \frac{2(n+1)}{10}$

$= 2 \times 1.4 = 2.8^{th}$ term

$(p.q)^{th}$ term $= p^{th} + \frac{q}{10}\left((p+1)^{th} - p^{th}\right)$

$(2.8)^{th} = 2^{nd} + \frac{8}{10}\left(3^{rd} - 2^{nd}\right)$

$= 125 + \frac{8}{10}(155 - 125)$

$= 125 + \frac{8}{10}(30)$

$= \frac{1250}{10} + \frac{240}{10} = \frac{1250+240}{10} = \frac{1490}{10}$

$= 149$

$D_9 = \frac{9(14)}{10} = 9 \times 1.4 = 12.6^{th}$.

$(12.6)^{th} = 12^{th} + \frac{6}{10}\left(13^{th} - 12^{th}\right)$

$= 905 + \frac{6}{10}(985 - 905)$

$= 905 + 48$

$= 953$

$P_{16} = 16 \times \frac{14}{100} = 16 \times 0.14 = 2.24^{th}$

$(2.24)^{th} = 2^{nd} + \frac{24}{100}\left(3^{rd} - 2^{nd}\right)$

$= 125 + \frac{24}{100}(155 - 125)$

$= 125 + 7.2 = 132.2$

calculate 9.10

$P_{65} = 65 \times \frac{14}{100} = 65 \times 0.14 = 9.10^{th}$

$(9.10)^{th} = 9^{th} + \frac{1}{100}\left(10^{th} - 9^{th}\right)$

$= 765 + \frac{1}{100}(875 - 765)$

$= 765 + 11 = 776$

3) Find $Q_1, Q_3, D_4, P_{30}, P_{99}$

Mark	F	CF	
25	3	3	
35	29	32	
40	32	64	
50	41	105	Q_1
52	49	154	
53	54	208	
67	38	246	Q_3
75	29	275	
80	27	302	
	302		

4) Cal. AM, Median, Mode

value	f	C.I.s	F	x	fx	CF
>50	250	50-100	10	75	750	10
>100	240	100-150	30	125	3750	40
>150	210	150-200	40	175	7000	80
>200	170	200-250	70	225	15750	150
>250	100	250-300	60	275	16500	210
>300	40	300-350	15	325	4875	225
>350	25	350-400	10	375	3750	235
>400	15	400-450	10	425	4250	245
>450	5	450-500	5	475	2375	250
>500	0		250		59000	

A) Mean

$\bar{x} = \frac{\Sigma f x}{f} = \frac{59000}{250} = 236$

Median

$M = l + \frac{\left(\frac{N}{2} - m\right)}{f} \times c$ $\quad \frac{N}{2} = 125$

$= 200 + \frac{(125 - 80)}{70} \times 50$

$= 200 + 0.642 \times 50$

$= 200 + 32.14 = 232.14$

Mode

$Z = l + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times c$

$= 200 + \frac{30}{30+10} \times 50$

$= 200 + 37.5 = 237.5$

3.A) $Q_1 = \frac{n+1}{4} = \frac{303}{4} = 75.75^{th} \Rightarrow 50$ $\quad$ ~~$\frac{N}{4} = \frac{302}{4} = 75.5^{th}$~~

(a) odd

$Q_3 = \frac{3(n+1)}{4} = \frac{3(303)}{4} = 3 \times 75.75$

$= 227.25^{th}$

$\Rightarrow 67$

$D_4 = \frac{4(n+1)}{10} = 4(30.3) = 121.2^{th}$

$\Rightarrow 52$

$P_{20} = 20\left(\frac{n+1}{100}\right) = 20\left(\frac{303}{100}\right) = 60.6^{th}$

$\Rightarrow 40$

$P_{99} = 99\left(\frac{n+1}{100}\right) = 99\left(\frac{303}{100}\right) = 299.97^{th}$

$\Rightarrow 80$

Haste 15/12/2021

Note

when actual values are known

* position of $Q_1 = \left(\frac{n+1}{4}\right)^{th}$
* position of $Q_3 = \frac{3(n+1)}{4}^{th}$
* " of $D_p = \frac{i(n+1)}{10}^{th}$
* " of $P_p = \frac{i(n+1)}{100}^{th}$.

Di/ value of $(p \cdot q)^{th}$ term $= p^{th}$ term $+ \frac{q}{10}\left((p+1)^{th}\right.$ term $-$ p^{th} term$\left.\right)$.

Pi/ * $(p \cdot q)^{th}$ term $= p^{th}$ term $+ \frac{q}{100}\left((p+1)^{th}\right.$ term $-$ p^{th} term$\left.\right)$.

3.A) $Q_1 = \frac{n+1}{4} = \frac{303}{4} = 75.75^{th}$

→ Weighted Harmonic Mean :-

$$\boxed{WHM = \frac{N}{\Sigma\left(\frac{w}{x}\right)}} \qquad N = \Sigma w$$