(3) Straight line

$r(t) = (a_1+b_1t)i + (a_2+b_2t) + (a_3+b_3t)$

(4) Circle

General form

$\frac{x^2}{a} + \frac{y^2}{b} = 1$

(5) Ellipse

$r(t) = a\cos t\, i + b\sin t\, j$

· → 1 dimension

L → 2 d

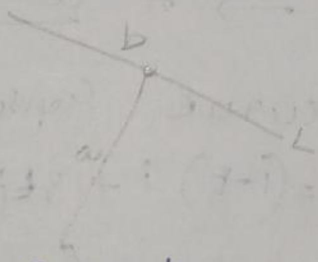
→ 3 d

01: Vector Valued functions

It is a ( ) 'r' defined by $r(t) = f(t)i + g(t)j + h(t)k$, where component ( ) f, g and h of 'r' are real valued ( ) of parametr 't' lying in a paramtr interval 'i'.

→ Curves defined by V.( ) :-

1) Straight line :- 'L' through a point 'A' with position vector,



V.eq →

$(a_1+b_1t)i + (a_2+b_2t)j + (a_3+b_3t)k.$

$a = a_1 i + a_2 j + a_3 k$

$b = b_1 i + b_2 j + b_3 k$

$$\boxed{r(t) = a + tb}$$

Circle

$x^2+y^2 = a^2$

$r(t) = [a\cos t + a\sin t]$

2) Ellipse ~~Circle~~ :-

The paramtric eq $x = a\cos t$, $y = b\sin t$, whose vector eq

$$\boxed{r(t) = a\cos t\, i + b\sin t\, j} \quad (E)$$

$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$ ⇒ ~~circle v.eq.~~ ⇒ ~~$x^2+y^2=r^2$~~

3) circular helix :-

v.eq ⟶ $\boxed{a \cos t\, i + b \sin t\, j + ct\, k}$

4) Elliptical helix :-

$\boxed{a \cos t\, i + b \sin t\, j + ct\, k.}$

5) parameteric eq of the twisted cubic curve :-

$x = t$, $y = t^2$, $z = t^3$

* A plane curve is a curve that lies in space. A curve that is not plane ⟶ A twisted curve

Q/ 1) Discribe the curve represented by the vector eq $r(t) = (1-t)\, i + 3tj + 2tk$.

↳ straight line.

A) $(a_1 + b_1 t) i + (a_2 + b_2 t) j + (a_3 + b_3 t) k$.

straight line

$(1-t)\, i + 3tj + 2tk$. ↑ in the form.

$a = a_1 i + a_2 j + a_3 k$. ; $a_1 = 1$ $a_2 = 0$ $a_3 = 0$.

$b = b_1 i + b_2 j + b_3 k$. $b_1 = -1$ $b_2 = 3$ $b_3 = 2$

a is passing through the point $a_1, a_2 .. a_3$ & || to the (v) to b, hence the curve represented by the v.eq

$r(t) = (1-t)\, i + 3tj + 2tk$ is a straight line in 3 space (x,y,z) passing through $(1,0,0)$ & || to the (v) $-i + 3j + 2k$

when $t = 1$, $x = 0$, $y = 3$, $z = 2$

hence line represented by the given eq passes through $(0,3,2)$ then the required curve is a straight line $(1,0,0)$ & $(0,3,2)$

2) Sketch the graph & radius (v) of $r(t) = \cos t\, i + \sin t\, j$ $0 \le t \le 2\pi$

A) general form, $\boxed{r(t) = xi + yj}$

$r(t) = xi + yj$

$r(t) = \cos t\, i + \sin t\, j$

∴ $x = \cos t$

$y = \sin t$

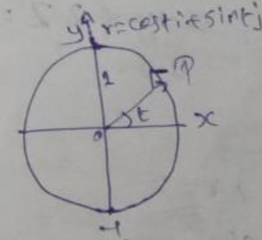
○ total angle $= 2\pi$

$\cos^2 x + \sin^2 x = 1$

$x^2 + y^2 = r^2$

$\cos^2 t + \sin^2 t = 1$

parametric eq corresponding to the given v.eq r $x = \cos t$ $y = \sin t$, ~~since~~ curve represented by the above v.eq is a ○ of radius 1 centered at origin in x,y plane. Both cos & sin () as t rises from $0 \to 2\pi$



$r = 1$

~~2) $r(t) = \cos t\, i + \sin t\, j + 2k$~~ ~~$0 \le t \le 2\pi$~~

the curve winds around the cylinder $x^2+y^2=a^2$,

$\boxed{\text{v.eq of cylinder, } a^2\sin^2 t + a^2\cos^2 t = a^2}$

$x^2+y^2=a^2$

$x = a\sin t \quad y = a\cos t$

$(a\sin t)^2 + (a\cos t)^2$

$\Rightarrow a^2\sin^2 t + a^2\cos^2 t = a^2$

Q) $r(t) = ((t^2+1)i + \cos t\, j - 3k)$

$\lim_{t\to 0}(\ldots)$

$= i\lim_{t\to 0}(t^2+1) + j\lim_{t\to 0}\cos t - 3k\lim_{t\to 0} 1$

$= i(0+1) + j\cos 0 + 3k \cdot 1$

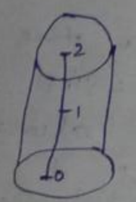
$= i + j - 3k$

3) $r(t) = \cos t\, i + \sin t\, j + 2k \qquad 0 \le t \le 2\pi$

A) parametric eq $\rightarrow$ $x = \cos t$, $y = \sin t$, $z = 2$

$x^2+y^2 = a^2 \Rightarrow (\cos t)^2 + (\sin t)^2 = 1$

$\cos^2 t + \sin^2 t = 1$



It is ○ of $r=1$ centered at origin in the plane $z=2$

4) Find a v.() that describes the curve intersection of cylinder $x^2+y^2=4$ & the plane $x+y+2z=4$.

A) $x^2+y^2=r^2$

$\therefore r^2 = 4, \; r = 2$

$P(x,y,z)$ be intersecting point.

$x = a\cos t = 2\cos t$

$y = a\sin t = 2\sin t$

$x+y+2z=4$.

$2\cos t + 2\sin t + 2z = 4$.

$2z = 4 - 2\cos t - 2\sin t \Rightarrow z = \frac{4}{2} - \frac{2\cos t}{2} - \frac{2\sin t}{2}$

$z = 2 - \cos t - \sin t$

v.eq $\boxed{r(t) = xi + yj + zk}$

$\Rightarrow 2\cos t\, i + 2\sin t\, j + (2 - \cos t - \sin t)k$.

→ limit of a vector valued ():-

Let $r(t) = f(t)i + g(t)j + h(t)k$ be a (v). valued () & L be a (v), r has a limit l, ~~as~~ $\lim_{t\to t_0} r(t) = L$,

For every $\varepsilon > 0$ (epsilon), there exist a corresponding no. $\delta > 0$ (delta), such that $\forall t$ $0 < |t - t_0| < \delta \Rightarrow |r(t) - L| < \varepsilon$.

If $L = L_1 i + L_2 j + L_3 k$, then $\boxed{\lim_{t\to t_0} r(t) = L}$

~~$\Rightarrow \lim_{t\to t_0} f(t)i = L_1 i$~~ $\Rightarrow \lim_{t\to t_0} f(t)i + \lim_{t\to t_0} g(t)j + \lim_{t\to t_0} h(t)k = L_1 i + L_2 j + L_3 k$.

* $r(t) = a\cos t\, i + a\sin t\, j + ct\, k$.

give limit → $\lim_{t\to\pi} r(t) = \lim_{t\to\pi} a\cos t\, i + \lim_{t\to\pi} a\sin t\, j + \lim_{t\to\pi} ct\, k$.

$= ai\lim_{t\to\pi}\cos t + aj\lim_{t\to\pi}\sin t + ck\lim_{t\to\pi} t$

$= ai\cos\pi + aj\sin\pi + ck\pi$

$= ai \cdot -1 + aj \times 0 + c\pi k$

$\boxed{\sin\pi = 0, \; \cos\pi = -1}$

$ax - 1 \times i = -ai$

$= -ai + 0j + c\pi k$.

$= -ai + ck\pi$

* $r(t) = t^2 i + e^t j - 2\cos\pi t\, k$.

$\lim_{t\to 0} r(t) = \lim_{t\to 0} t^2 i + \lim_{t\to 0} e^t j - \lim_{t\to 0} 2\cos\pi t\, k$.

$2\cos\pi \times 0\, k$

$2\cos 0\, k$

$2 \times 1\, k = 2k$

$2 \cdot \cos 0 \cdot k$

$2k$

$= 0i + j - 2k$

→ **Continuity of a (v) valued ( ) :-**

(v) valued ( ) $r(t)$ is continous at a point $t = a$ in its domain if

3 condi..

1) $r(a)$ is defined
2) $\lim_{t \to a} r(t)$ exist
3) $\lim_{t \to a} r(t) = r(a)$

→ **Component test for continuity at a point :-**

(v) ( ) ~~are~~ $r(t) = f(t)i + g(t)j + h(t)k$ is continous at $t = a$, if and only if f, g, h are continous at a.

eg→ $r(t) = \cos t\, i + e^t j + t^2 k$.

A) → The component ( ) $\cos t, e^t, t^2$ are continous at every real value of t, hence $r(t)$ also contin~ everywhere.

eg→ $r(t) = t^3 i + 5t j + \frac{1}{t} k$. $\frac{1}{t} = \frac{1}{0} = \infty$ so ∄

Not contin~ at $t = 0$, since the component ( ), $h(t) = \frac{1}{t}$ is not contin~ at $t = 0$.

1) Find the intrvals on which the (v) valued ( ) ~~are~~ 'r' defined below is contin~

$r(t) = e^{-t} i + \cos\sqrt{4-t}\, j + \frac{1}{t^2-1} k$.

A) $f(t) = e^{-t}$ $g(t) = \cos\sqrt{4-t}$ $h(t) = \frac{1}{t^2-1}$

$t = \pm 1$ not contin

$(-\infty,-1)\ (-1,1)\ (1,4)$

f(t), g(t), h(t) } must contins.

$f(t) \to$ contin

$g(t) \to$ contin till 4. $(-\infty, 4)$.

$h(t) \to$ contin only above, $(-1,1)$

Contin on the intrvals $(-\infty,-1), (-1,1), (1,4]$

→ **Derivative of a (v) valued ( ) :-**

$$\boxed{r'(t) = \lim_{\Delta t \to 0} \frac{r(t+\Delta t) - r(t)}{\Delta t}}$$

→ **Therom :-**

If $r(t)$ is a (v) valued ( ), then 'r' is differentiable at t, if and only if each of its component ( ) are differentiable at t. In which case the component ( ) of $r'(t)$ are the deriv of the corresponding component ( ) of $r(t)$.

Pf/ $r(t) = f(t)i + g(t)j + h(t)k$.

$$\frac{r(t+\Delta t) - r(t)}{\Delta t} = \frac{\cancel{f(t+\Delta t) - f(t) + g(t+\Delta t) - g(t) +} h(t+\Delta t) - h(t)}{\Delta t}$$

$$= \frac{f(t+\Delta t) - f(t)}{\Delta t} + \frac{g(t+\Delta t) - g(t)}{\Delta t} + \frac{h(t+\Delta t) - h(t)}{\Delta t}$$

$$\lim_{\Delta t\to 0} \frac{r(t+\Delta t)-r(t)}{\Delta t} = \lim_{\Delta t\to 0} \frac{f(t+\Delta t)-f(t)}{\Delta t} +$$

$$\lim_{\Delta t\to 0} \frac{g(t+\Delta t)-g(t)}{\Delta t} + \lim_{\Delta t\to 0} \frac{h(t+\Delta t)-h(t)}{\Delta t}.$$

$$\boxed{r'(t) = f'(t)i + g'(t)j + h'(t)k.}$$

Q

1) $r(t) = a\cos t\, i + a\sin t\, j + ct\, k.$

A) $r'(t) = -a\sin t\, i + a\cos t\, j + c\, k.$

2) $r(t) = t^2 i + e^t j - 2\cos\pi t\, k.$

A) $r'(t) = 2t\, i + e^t j + 2\cdot\sin\pi t\, k.$

$-2\cos(\pi t)k.$
$+2\sin(\pi t)\times\pi\times k.$

Rules :-

1) ~~const~~ constant (c) rule,

$\frac{d}{dt}(c) = 0$

2) scalar multiple rule.

$\frac{d}{dt}[c\cdot u(t)] = c\frac{d}{dt}(u(t)).$ &

$\frac{d}{dt}(f(t)\cdot u(t)) = \frac{df}{dt}\cdot u(t) + \frac{du}{dt}\cdot f(t).$
(p. rule)

3) Sum rule,

$\frac{d}{dt}[u(t)+v(t)] = \frac{d}{dt}(u(t)) + \frac{d}{dt}(v(t))$

4) Difference rule,

$\frac{d}{dt}[u(t)-v(t)] = \frac{d}{dt}(u(t)) - \frac{d}{dt}(v(t))$

5) Dot product.

$\frac{d}{dt}u(t)\cdot v(t) = \frac{du}{dt}\cdot v(t) + u(t)\cdot\frac{dv}{dt}.$

6) Cross product.

$\frac{d}{dt}(u(t)\times v(t)) = \frac{du}{dt}\times v(t) + \frac{dv}{dt}\times u(t).$

7) Chain rule

$\frac{dr}{dt} = \frac{dr}{ds}\cdot\frac{ds}{dt}.$

→ 2nd derivative :-

$r(t) = f(t)i + g(t)j + h(t)k.$

$r''(t) = f''(t)i + g''(t)j + h''(t)k.$

Q 1) $r(t) = ti + t^2 j + t^3 k.$

find $\frac{d}{dt}\left(r(t)\cdot[r'(t)\times r''(t)]\right).$
dot(p) cross(p).

A) $r'(t) = i + 2t\, j + 3t^2 k.$

$r''(t) = 2j + 6t\, k.$

cross (p) →

$$r'(t)\times r''(t) = \begin{vmatrix} t & t^2 & t^3 \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix}$$
→ 1st deri
→ 2nd deri

then find determinant $||$.

$t\begin{vmatrix} 6t & 3t^2 \\ 2 & 6t \end{vmatrix} - t^2\begin{vmatrix} 1 & 3t^2 \\ 0 & 6t \end{vmatrix} + t^3\begin{vmatrix} 1 & 2t \\ 0 & 2 \end{vmatrix}$

$t|12t^2-6t^2| - t^2|6t-0| + t^3|2-0|$

$t|12t^2-6t^2| - t^2|6t| + t^3|2|$.

$\cancel{12t^3-6t^3-6t^3+2t^3}$

$\cancel{6t^3}$

$t|6t^2| - t^2|6t| + t^3|2|$

$6t^3 - 6t^3 + 2t^3 = 2t^3$

dot (i)

$r(t)\cdot[r'(t)\times r''(t)] =$

$\frac{d}{dt}(2t^3) = 6t^2$

2) $r_1(t) = \cos t\, i + \sin t\, j + t\, k$

$r_2(t) = i + tk$

P.T $\frac{d}{dt}[r_1(t)\cdot r_2(t)] = \frac{dr_1}{dt}\cdot r_2(t) + r_1(t)\cdot\frac{dr_2}{dt}$

A)

$r_1(t)\cdot r_2(t) = (\cos t\, i + \sin t\, j + t k)\cdot(i + tk)$

$= \cos t\cdot i + \sin t\, j\cdot 0 + t\cdot t$.

$= \cos t + t^2$

$\therefore \frac{d}{dt}[r_1(t)\cdot r_2(t)] = \frac{d}{dt}[\cos t + t^2]$.

$= -\sin t + 2t$.

Now, $\frac{dr_1}{dt} = \frac{d}{dt}(\cos t\, i + \sin t\, j + t k)$

$= -\sin t\, i + \cos t\, j + k$.

Now, $\frac{dr_1}{dt}\cdot r_2(t) = (-\sin t\, i + \cos t\, j + k)\cdot(i+tk)$

$= -\sin t + t$ — (1)

$\approx \frac{dr_2}{dt} = \frac{d}{dt}(i + tk) = k$

$r_1 t\cdot\frac{dr_2}{dt} = (\cos t\, i + \sin t\, j + t k)\cdot k = t$

$\therefore \frac{dr_1}{dt}\cdot r_2(t) + \frac{dr_2}{dt}\cdot r_1(t) = -\sin t + 2t$. (2)

$\therefore$ from (1) & (2)

$\frac{d}{dt}[r_1(t)\cdot r_2(t)] = \frac{dr_1}{dt}\cdot r_2(t) + r_1(t)\frac{dr_2}{dt}$

3) $\frac{d}{dt}[r_1(t)\times r_2(t)] = \frac{dr_1}{dt}\times r_2(t) + r_1(t)\times\frac{dr_2}{dt}$.

4) find $\frac{dr}{dt}$ if $r(s) = \cos 2s\, i + \sin 2s\, j + e^{-3s}k$, $s = t^4$.

A) by chain rule,

$\frac{dr}{dt} = \frac{dr}{ds}\cdot\frac{ds}{dt}$

$\frac{dr}{ds} = -\sin 2s\cdot 2\cdot i + \cos 2s\cdot 2j + e^{-3s}\cdot -3k$

$$\frac{ds}{dt} = 4t^3$$

$$\frac{dr}{dt} = \left(-2\sin 2s\, i + 2\cos 2s\, j - 3e^{-3s} k\right) \cdot 4t^3$$

$s = t^4$

$$= -8\sin 2t^4 t^3 i + 8\cos 2t^4 t^3 j - 3e^{-3s} t^3 k$$

$$= -8\sin 2t^4 t^3 i + 8\cos 2t^4 t^3 j - 12 e^{-3t^4} t^3 k$$

$$= -8t^3 \sin 2t^4 i + 8t^3\cos 2t^4 j - 12t^3 e^{-3t^4} \cdot k.$$

→ Integrals of (v) ( ) :-

$\boxed{\text{Vector form} \to r(t) = f(t)\,i + g(t)\,j + h(t)\,k}$

Indefinite → no limit

Definite → limit will be there.

Indefinite

$$\int r(t) = f(t)\,i + g(t)\,j + h(t)\,k.$$

$$\int r(t)\,dt = \int f(t)\,dt\; i + \int g(t)\,dt\; j + \int h(t)\,dt\; k.$$

→ definite $\int$ of r over interval $[a, b]$

$$\int_a^b r(t)\,dt = \int_a^b f(t)\,dt\; i + \int_a^b g(t)\,dt\; j + \int_a^b h(t)\,dx\; k.$$

$$* \int r'(t)\,dt = r(t) + c \qquad \left(\int + \frac{d}{dt} = \text{Cancel}\right)$$

⇒ Rules of $\int$:-

1) Constant scalar multiple rule :-

$$\int_a^b k\, r(t)\,dx = k\int_a^b r(t)\,dx.$$

2) Sum & Difference rule :-

$$\int_a^b \left(r_1(t) \pm r_2(t)\right) dt = \int_a^b r_1(t)\,dt \pm \int_a^b r_2(t)\,dt$$

3) Constant (v) multiple rule :-

$$\int_a^b c \cdot r(t)\,dt = c\int_a^b r(t)\,dt.$$

4) Cross product :-

$$\int_a^b c \times r(t)\,dt = c \times \int_a^b r(t)\,dt.$$

Q 1) Evaluate the indefinite $\int \to \int \left(t^2 i - 4t j + \frac{1}{t} k\right) dt$

here - and + is there, so use sum & diff rule

A) $\int \left(t^2 i - 4t j + \frac{1}{t} k\right) dt.$

$$\int t^2 i\,dt - 4\int t\,j\,dt + \int \frac{1}{t} k\,dt.$$

$$\frac{t^3}{3} i - 4\frac{t^2}{2} j + \ln t\, k \to \frac{t^3}{3} + c_1 - 4\frac{t^2}{2} + c_2 + \ln t + c_3$$

$$\frac{t^3}{3} - 2t^2 j + \ln t\, k + c$$

$c = c_1 + c_2 + c_3$ (optional)

$$c = c_1 i + c_2 j + c_3 k.$$

2) Find antiderivative ($\int$) of $r'(t) = \cos t\, i + e^{-t} j + \sqrt{t}\, k$ satisfying initial condition $r(0) = i + 2j + 3k.$

A) $$\int r'(t)\,dx = \int \cos t\, i\,dt + \int e^{-t} j\,dt + \int \sqrt{t}\, k\,dt.$$

$$= (\sin t + c)\,i + (-e^{-t} + c_2) + \left(\frac{2}{3} t^{3/2}\right) k$$

$$r(t) = \left(\sin t\, i - e^{-t} + \frac{2}{3} t^{3/2}\right) + c$$

r(t)

$C = c_1 i + c_2 j + c_3 k.$

$r(t) = \left(\sin t\, i - e^{-t} j + \frac{2}{3} t^{3/2} k\right) + C$

$r(0) = i + 2j + 3k$: $= \left(\sin 0\, i - e^{0} j + \frac{2}{3}\, 0\, k\right) + C$

$\Rightarrow -j + C = i + 2j + 3k.$

$C = i + 3j + 3k$

$r(t) = \sin t\, i - e^{-t} j + \frac{2}{3} t^{3/2} k + i + 3j + 3k$

$\Rightarrow (1+\sin t)\, i + (-e^{-t}+3)\, j + \left(\frac{2}{3} t^{3/2} + 3\right) k.$

3) Evaluate definite $\int$

$$\int_{-\pi/4}^{\pi/4} \left(\sin t\, i + (1+\cos t)\, j + \sec^2 t\, k\right) dt.$$

A) $\int \sin t\, i\, dt + \int (1+\cos t)\, j\, dt + \int \sec^2 t\, k\, dt$:

$[\cos t]_{-\pi/4}^{\pi/4} i + [t + \sin t]_{-\pi/4}^{\pi/4} j + [\tan t]_{-\pi/4}^{\pi/4} k$

$\left[\cos\frac{\pi}{4} - \cos(\pi/4)\right] + \left[\frac{\pi}{4} + \sin\frac{\pi}{4} - \left(\frac{-\pi}{4} + \sin\left(\frac{-\pi}{4}\right)\right)\right] + \left[\tan\frac{\pi}{4} - \tan\left(\frac{-\pi}{4}\right)\right]$

$= \left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right] + \left[\frac{\pi}{4} + \frac{1}{\sqrt{2}} + \frac{\pi}{4} + \frac{1}{\sqrt{2}}\right] + [1 + 1]$

$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}}$

$= 0 i + \left[\cancel{2}\frac{\pi}{\cancel{4}_2} + \frac{2}{\sqrt{2}}\right] j + 2k.$

$\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}$

$= 0 i + \left[\frac{\pi}{2} + \frac{2}{\sqrt{2}}\right] j + 2K$

3) find $r(t) \to r''(t) = 12t\, i - 3t^{-1/2} j + 2k.$

$r'(1) = j, \quad r(1) = 2i - k.$

A) $r'(t) = \int r''(t)\, dt = \int (12t i - 3t^{-1/2} + 2k)\, dt.$

$= \left[\int 12t\, dt\right] i - \left[\int 3t^{-1/2} dt\right] j + \left[\int 2\, dt\right] k.$

$= \cancel{12}^{6} \cdot \frac{t^2}{\cancel{2}} i - 3 \cdot \frac{t^{1/2}}{1/2} j + 2t\, k + c$

$= 6t^2 i - 6t^{1/2} j + 2t k + c.$

To find c,

$r''(t) = j$ when $t = 1 \to r'(1) = j$

$r'(1) = j \Rightarrow 6i - 6j + 2k + c = j$

$\Rightarrow c = -6i + 7j - 2k.$

$r'(t) = 6t^2 i - 6t^{1/2} j + 2t k (-6i + 7j - 2k)$

$= (6t^2 - 6) i + (7 - 6t^{1/2}) j + (2t - 2) k$

$\int$-ing $r'(t)$ with respect to t,

$r(t) = \int r'(t)\, dt = \int \left[(6t^2 - 6) i + (7 - 6t^{1/2}) j + (2t - 2) k\right] dt.$

$= \left[\int (6t^2 - 6)\, dt\right] i + \left[\int (7 - 6t^{1/2})\, dt\right] j + \left[\int (2t - 2)\, dt\right] k.$

$= (2t^3 - 6t) i + (7t - 4t^{3/2}) j + (t^2 - 2t) k + D.$

where D is a constant for $\int$.

To find D we use, $r(1) = 2i - k$

when $t=1 \rightarrow r(1) = 2i-k$.

$r(1) = 2i-k \Rightarrow -4i+3j-k+D = 2i-k$

$\Rightarrow D = 6i-3j$.

$r(t) = (2t^3-6t)i + (7t-4t^{3/2})j +$

$(t^2-2t)k + 6i - 3j$

$= (2t^3-6t+6)i + (7t-4t^{3/2}-3)j +$

$(t^2-2t)k$.

⇒ ~~Geometric Interpretation of derivative~~ :-

Derivative $r'(t)$ of the vector of $r(t)$ interpreted as the tangent (v) defined by, r at the point t, $r'(t) \neq 0$,

$$\boxed{\begin{array}{l} \text{The unit tangent (v), } T(t) = \dfrac{r'(t)}{\|r'(t)\|} \\ \text{Tangent line, } r(t) = r(t_0) + t \cdot r'(t_0) \end{array}}$$

where (v) valued () $r(t)$ at t with position (v) $r(t_0)$

$t_0 \rightarrow$ point.

Q/

1) Find derivative of $r(t) = (t^2+1)i + e^{-t}j - \sin 2t\, k$.

b) find the point of tangency & unit tangent (v) at the point on the curve corresponding to $t=0$

1) a) $r(t) = (t^2+1)i + e^{-t}j - \sin 2t\, k$

$r'(t) = (2t)i - e^{-t}j - \cos(2t) \times 2k$

$\frac{d}{dx} e^t = e^t$

$\frac{d}{dx} e^{-t} = -e^{-t}$

b) ~~r(t)~~ $t=0$,

$\therefore r(0) = i + j + 0 = i + j \rightarrow (1,1,0)$

$r(t)$-ൽ

point of tangency $\rightarrow (1,1,0)$

ഓരോന്നിന്റെ coefficient മാത്രം.

unit tangent (v), $T(t) = \dfrac{r'(t)}{\|r'(t)\|} = \dfrac{(2t)i - e^{-t}j - \cos 2t \cdot 2k}{\sqrt{(-1)^2+(-2)^2}}$

$r'(t) = 2ti - e^{-t}j - \cos 2t\ 2k$.

$r'(0) = 0 - j - 2k = -j-2k$

$T(t) = \dfrac{r'(t)}{\|r'(t)\|} = \dfrac{-j-2k}{\sqrt{(-1)^2+(-2)^2}} = \dfrac{-j-2k}{\sqrt{1+4}}$

$= \dfrac{-j-2k}{\sqrt{5}} = \dfrac{-1}{\sqrt{5}}j - \dfrac{2}{\sqrt{5}}k$.

2) Find parametric eq for the line (i.e) tangent to the curve, $r(t) = \sin t\, i + (t^2-\cos t)j + e^t k$, $t=0$.

A) $r(t) \rightarrow$ position (v)

$r(t) = \sin t\, i + (t^2 - \cos t)j + e^t k$.

~~$r(t) = \sin t\, i + (2t-1)j + e^t k$.~~

~~$r(0) = 0 + (2t-1)j + e^t k$.~~

$r(0) = 0 + (0-1)j + 1 \times k$.

$r(0) = -j + k$

$-\cos t = \sin t$

$r'(t) = \cos t\, i + (2t + \sin t)j + e^t k$.

$r'(0) = 1i + (0+0)j + 1 \times k$.

$r'(0) = i + k$

Tangnt line eq,

$k + tk.$
$k(1+t)$

$r(t) = r(t_0) + t\, r'(t_0)$

$= (-j + k) + t \cdot (i + k) = -j + k + ti + tk.$

$\cancel{= (j+k) + ti + tk}$ $-j + ti + (1+t)k.$

$= ti - j + (1+t)k$

parametric form $\Rightarrow$ $xi + yi + zi$
$ti - j + (1+t)k.$

$x = t \quad y = -1 \quad z = 1+t.$

3) find parametric eq of tangnt to the ellipse $\frac{x^2}{4} + y^2 = 1$ at P $(\sqrt{2}, \frac{1}{\sqrt{2}})$

A) parametric eq of givn ellipse —

$x(t) = 2\cos t \quad y(t) = \sin t \quad z(t) = 0$

vector eq —

$r(t) = 2\cos t\, i + \sin t\, j$

$r'(t) = \frac{dr}{dt} = -2\sin t\, i + \cos t\, j$

At the point P, $x(t) = 2\cos t = \sqrt{2}$ &

$y(t) = \sin t = 1/\sqrt{2}$

$\therefore$ at P, $t = \frac{\pi}{4}$ & so tangnt (v) at P is

$r'\left(\frac{\pi}{4}\right) = \frac{dr}{dt}\Big|_{\pi/4} = -2\sin\left(\frac{\pi}{4}\right) i + \cos\left(\frac{\pi}{4}\right) j$

$r(\pi/4) = -\sqrt{2}\, i + \frac{1}{\sqrt{2}}\, j.$

& is $\parallel$ to (v) $r'(\pi/4) = -\sqrt{2}\, i + (1/\sqrt{2})\, j$

$r(t) = r(\pi/4) + t\, r'(\pi/4)$

$r(t) = \sqrt{2}\, i + \frac{1}{\sqrt{2}}\, j + t\left[-\sqrt{2}\, i + \frac{1}{\sqrt{2}}\, j\right]$

$r(t) = \sqrt{2}(1-t)\, i + \frac{1}{\sqrt{2}}(1+t)\, j$

$\therefore$ P.eq of tangnt line at P is

$x = \sqrt{2}(1-t) \quad y = \frac{1}{\sqrt{2}}(1+t) \quad z = 0$

4) If $r(t)$ is a differenciable (v) valued (f) in 2 space / 3 space & is of constnt for all 't'. They $r(t) \cdot r'(t) = 0$, (i.e) $r(t)$ & it's tangnt (v) $r'(t)$ are orthogonal (v) for all 't'

A) $\|r(t)\| =$ constnt. ($r(t)$ has constnt length)

$\therefore$ $r(t) \cdot r(t) = \|r(t)^2\| =$ constnt

differ.. on both side,

$\frac{d}{dt}(r(t) \cdot r(t)) = \frac{d}{dt}(\text{const}) \rightarrow 0.$

(product rule)

$\frac{dr}{dt} \cdot r(t) + r(t)\frac{dr}{dt} = 0$

$2\, r(t)\frac{dr}{dt} = 0.$ $\qquad \frac{dr}{dt} = r'(t)$

$2\, r(t) \cdot r'(t) = 0$

$\therefore$ $r(t) \cdot r'(t) = 0$

⇒ Length of a Space Curve :-

$r(t) = f(t)i + g(t)j + h(t)k$

[r(t) [illegible] Length [illegible] $\|r(t)\|$]

$$\boxed{L = \int_a^b \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2}\, dt}$$

$r'(t) = f'(t)i + g'(t)j + h'(t)k$

~~$L = \int_a^b \sqrt{\|r'(t)\|}$~~ $$\boxed{L = \int_a^b \|r'(t)\|\, dt}$$

Q/ 1) find the L of the catillary $r(t) = ti + \cosh t\, j$ from 0 to 1

A) $L = \int_a^b \|r'(t)\|\, dt$

$r(t) = ti + \cosh t\, j$ $\boxed{1 + \sinh^2 h = \cosh^2 t}$

$r'(t) = 1i + \sinh t\, j$

$\therefore L = \int_0^1 \sqrt{1^2 + \sinh^2 t} = \int_0^1 \sqrt{\cosh^2 t}$

$= \int_0^1 \cosh t \Rightarrow [\sinh t]_0^1$

$= \sinh 1 - \sinh 0 = \underline{\sinh 1}$

2) find arc length b/w [0,0,1] & [1,0,1] of [illegible] helix, $y = \sin 2\pi x$, $z = \cos 2\pi x$.

A) 1st find parametric eq. [illegible] z [illegible] x [illegible] para. eq [illegible] (i.e) $x = t$,

eq becoms → $y = \sin 2\pi t$, $z = \cos 2\pi t$, $x = t$.

[0,0,1] [1,0,1]

$r(t) = xi + yj + zk$

$r(t) = ti + \sin 2\pi t\, j + \cos 2\pi t\, k$.

$r'(t) = 1i + \cos 2\pi t \cdot 2\pi j - \sin 2\pi t \cdot 2\pi k$

$L = \int_a^b \|r'(t)\|\, dt$.

$= \int_0^1 \sqrt{1^2 + (\cos 2\pi t \cdot 2\pi)^2 + (2\pi \cdot \sin 2\pi t)^2}\, dt$.

$(\cos 2\pi t)^2 \cdot (2\pi)^2$

$= \int_0^1 \sqrt{1 + 4\pi^2 \cdot \cos^2 2\pi t + 4\pi^2 \cdot \sin^2 2\pi t}$

$= \int_0^1 \sqrt{1 + 4\pi^2 (\underbrace{\cos^2 2\pi t + \sin^2 2\pi t}_{1})}\, dt$. $\left[\cos^2 x + \sin^2 x = 1\right]$

$= \int_0^1 \sqrt{1 + \underbrace{4\pi^2}_{\text{constant}}}\, dt$ $\int dt = t$

$= \sqrt{1 + 4\pi^2} \int_0^1 dt \Rightarrow \sqrt{1 + 4\pi^2}\, [t]_0^1$

$\Rightarrow \sqrt{1 + 4\pi^2}\, (1 - 0) = \underline{\sqrt{1 + 4\pi^2}}$

3) find L of one 3rd of helix
$r(t) = \cos t\, i + \sin t\, j + t k$.

⇒ Arc length parameter :-

Arc length function :-

Suppose c is a smooth curve described by,

$r(t) = f(t) i + g(t) j + h(t) k$.

$\boxed{\text{arc length, } s(t) = \int_{t_0}^{t} \| r'(u) \| \, du}$

$s(t) = \int_{t_0}^{t} \sqrt{(f'(u))^2 + (g'(t))^2 + (h'(t))^2} \, dt$.

1) The arc length parametrication of $x^2+y^2 = a^2$ with counter clock wise orientation.

A) we know, param. eq of

$x = a\cos t$ $y = a\sin t$

$r(t) = a\cos t\, i + a \sin t\, j$

$r'(t) = -a\sin t\, i + a\cos t\, j$

$t_0 = 0$ arc length, $s(t) = \int_{t_0}^{t} \| r'(u) \| \, du$. Starting → 0 ending → we don't know

$t \to u$

$= \int_0^t \sqrt{(-a\sin t)^2 + (a\cos t)^2} \, du$.

$= \int_0^t \sqrt{a^2 \sin^2 u + a^2 \cos^2 u} \, du$.

$= \int_0^t \sqrt{a^2 [\sin^2 u + \cos^2 u]} \, du$.

$= \int_0^t \sqrt{a^2} \, du = \int_0^t a \, du$

$= a\int_0^t du = a[u]_0^t \Rightarrow a(t-0) = at$

$s = at \to t = s/a$

arc length parametriazation

$0 \le t \le 2\pi$

$r(t(s)) = a\cos(s/a) i + a\sin(s/a) j$

$s(t) = at$.

$\therefore s(0) = 0$

$s(2\pi) = 2\pi a$. interval ⇒ $[0, 2a\pi]$.

2) Reparametrise following parameter in terms of length →

a) $r(t) = \frac{t^2}{2} i + \frac{t^3}{3} k$. $0 \le t \le 2$

A) $r'(t) = \frac{1}{2} 2t\, i + \frac{1}{3} \cdot 3t^2 k$

$r'(t) = t i + t^2 k$

$t_0 = 0$.

$t \to u$.

$r'(u) = u i + u^2 k$

$t \to u$ $s = \int_0^t \| r'(t) \| \, du$.

$= \int_0^t \sqrt{u^2 + u^4} \, du$.

$= \int_0^t \sqrt{u^2(1+u^2)} \, du$

$= \int_0^t u\sqrt{1+u^2}\, du.$

Substitution (m)

put $1+u^2 = v.$

$\frac{d}{du}(1+u^2) = \frac{d}{du}(v)$

$2u\, du = dv$

$u\, du = \frac{dv}{2}$

$v = 1+u^2 = 1+0 = 1$

$v = 1+t^2$

Limit

$u = 0 \quad , \quad v = 1$

$u = t \quad , \quad v = 1+t^2$

$\therefore \int_1^{1+t^2} \sqrt{v}\, \frac{dv}{2} = \frac{1}{2}\int_1^{1+t^2} \sqrt{v}\, dv.$

$= \frac{1}{2}\int_0^{1+t^2} v^{1/2}\, dv = \frac{1}{2}\left[\frac{v^{3/2}}{3/2}\right]_1^{1+t^2}$

$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} \cdot \left[v^{3/2}\right]_1^{1+t^2}$

$= \frac{1}{3}\left[v^{3/2}\right]_1^{1+t^2} = \frac{1}{3}\left[(1+t^2)^{3/2} - 1^{3/2}\right]$

$S = \frac{(1+t^2)^{3/2}}{3} - \frac{1}{3}$

$S = \frac{(1+t^2)^{3/2} - 1}{3}$

$3S = (1+t^2)^{3/2} - 1.$

$3S + 1 = (1+t^2)^{3/2}$

(3/2 [illegible] 2/3 [illegible])

$(3S+1)^{2/3} = 1+t^2.$

$(3S+1)^{2/3} - 1 = t^2$

$\sqrt{(3S+1)^{2/3} - 1} = t.$ (or)

$\left((3S+1)^{2/3} - 1\right)^{1/2} = t$

$\gamma(t(s)) = \left(\left((3S+1)^{2/3} - 1\right)^{1/2}\right)^2 \ldots$

$= \frac{(3S+1)^{2/3} - 1}{2}\, i + \frac{\left[(3S+1)^{2/3} - 1\right]^{3/2}}{3} k.$

$\gamma(t) = \frac{t^2}{2} i + \frac{t^3}{3} k.$

$= \frac{\left(\left[(3S+1)^{2/3} - 1\right]^{1/2}\right)^2}{2}\, i + \frac{\left(\left((3S+1)^{2/3} - 1\right)^{1/2}\right)^3}{3}$

$= \frac{(3S+1)^{2/3} - 1}{2}\, i + \frac{\left[(3S+1)^{2/3} - 1\right]^{3/2}}{3}$

4) $\gamma(t) = (2\cos t)\, i + (2\sin t)\, j, \quad 0 \le t \le 2\pi$

5) Find arc length param of helix $r(t) = \cos t\, i + \sin t\, j + t\, k$, then has reference point $r(0) = [1, 0, 0]$ & same orientation has the gun helix.

A) $\gamma'(t) = -\sin t\, i + \cos t\, j + k.$

$= -\sin u\, i + \cos u\, j + [illegible]\, k$

$$s = \int_{t_0}^{t} \| r'(u) \| \, du.$$

$$= \int_0^t \sqrt{(-\sin u)^2 + (\cos u)^2 + 1}\, du.$$

$$= \int_0^t \sqrt{\sin^2 u + \cos^2 u + 1}\, du.$$

(To find arc length / find till $t = \ldots$)

$$= \int_0^t \sqrt{1+1} \Rightarrow \cancel{[u]_0^t}$$

$$S = \int_0^t \sqrt{2} \Rightarrow S = \sqrt{2}\, t.$$

$\rightarrow \sqrt{2}\int_0^t 1 \longrightarrow t = S/\sqrt{2}.$

$\therefore$

$$r(t(s)) = \cos(s/\sqrt{2})\, i + \sin(s/\sqrt{2})\, j + (s/\sqrt{2})\, k.$$

HW 3) L of $r(t) = \cos t\, i + \sin t\, j + t k.$

A) $r'(t) = -\sin t\, i + \cos t\, j + k.$

helix make 1 full turn as t runs from 0 to 2π,

$$L = \int_a^b \| r'(t) \| \, dt$$

$$= \int_0^{2\pi} \sqrt{(-\sin t)^2 + (\cos t)^2 + 1^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{\sin^2 t + \cos^2 t + 1}\, dt = \int_0^{2\pi} \sqrt{1+1}\, dt.$$

$$= \int_0^{2\pi} \sqrt{2}\, dt = \underline{\underline{2\pi\sqrt{2}}}$$

4) Arc L of $r(t) = (2\cos t)\, i + (2\sin t)\, j$ $\quad 0 \le t \le 2\pi.$

A) velocity $\boxed{v(t) = \dfrac{dr}{dt}} = -2\sin t\, i + 2\cos t\, j.$

$t_0 = 0,$

$$s = s(t) = \int_0^t \| v(t) \| \, dt.$$

$$= \int_0^t \sqrt{(-2\sin t)^2 + (2\cos t)^2}\, dt.$$

$$= \int_0^t 2\, dt = 2t.$$

$s = 2t.$

$t = s/2$

$\therefore$ s rises from 0 to 4π.

$u(s) = r(s/2) = 2\cos(s/2) + 2\sin(s/2)\, j$

$(0 \le s \le 4\pi)$

$\Rightarrow$ **Motion on a curve :-**

$\boxed{(\text{dir}) \Rightarrow \dfrac{v(t)}{\|v(t)\|}}$

(v) func $\rightarrow r(t) = f(t)\, i + g(t)\, j + h(t)\, k$

velocity $\rightarrow v(t) = r'(t) = f'(t)\, i + g'(t)\, j + h'(t)\, k.$

acc $\rightarrow a(t) = r''(t) = f''(t)\, i + g''(t)\, j + h''(t)\, k.$

Speed $\rightarrow v = \| r'(t) \| = \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2}$

$\boxed{\cos\theta = \dfrac{u \cdot v}{\|u\|\,\|v\|}}$

Q 1) position (v) of a particle in space at time 't' is $r(t) = (t+1)\, i + (t^2 - 1)\, j + 2t\, k$, acc (v)?

a) find particle velocity & acc (v)?

b) find particle speed & (dir) of motion at $t = 1$?

c) find angle b/w the velocity & acc (v) at $t=1$?

A) ~~$v(t) = r'(t) =$~~

$r(t) = (t+1)\,i + (t^2-1)\,j + 2t\,k$

a) $v(t) = r'(t) = 1\,i + 2t\,j + 2\,k$

$a(t) = r''(t) = 0\,i + 2\,j = 2j$

b) Speed $= \|r'(t)\| = \sqrt{1^2 + (2t)^2 + 2^2}$

$= \sqrt{1 + 4t^2 + 4} = \sqrt{5 + 4t^2}$

Speed at $t=1$

$v \Rightarrow \sqrt{5+4t^2} = \sqrt{5+4\times 1} = \sqrt{5+4} = \sqrt{9} = 3$

(dir) of motion →

$\Rightarrow \dfrac{v(t)}{\|v(t)\|}$ at $t=1$ $\quad v(t) = i + 2t\,j + 2k$, $v(1) = i + 2j + 2k$.

$\dfrac{v(1)}{\|v(1)\|} \Rightarrow \dfrac{i + 2j + 2k}{3} = \dfrac{i}{3} + \dfrac{2j}{3} + \dfrac{2}{3}k$.

c) $\boxed{\cos\theta = \dfrac{v(1)\; a(1)}{\|v(1)\|\,\|a(1)\|}} = \dfrac{(i+2j+2k)\,2j}{\sqrt{3^2}\cdot\sqrt{2^2}}$

Same component

$= \dfrac{(i+2j+2k)\,2j}{3\cdot 2} = \dfrac{4}{3\cdot 4}\ldots = \dfrac{2}{3}$

$\cos\theta = 2/3$, $\theta = \cos^{-1}(2/3)$

2) Suppose $r(t) = t^2 i + (t^3-2t)\,j + (t^2-5t)\,k$ is the position (v) of a moving particle at wht points does the particle pass through the (x y) plane? wht are its v & acc at this points?

A) $r(t) = t^2 i + (t^3-2t)\,j + (t^2-5t)\,k$.

param eq → $r(t) = x i + y j + z k$.

$x = t^2 \quad y = t^3 - 2t \quad z = t^2 - 5t$

* (x y) plane
$z = 0$

$t^2 - 5t = z$

$t^2 - 5t = 0$

$t(t-5) = 0$

$t = 0, \quad t - 5 = 0$

$t = 5$

at $t=0$, $x = y = z = 0$.

at $t = 5$, $x = t^2 = 25$

$y = t^3 - 2t = 125 - 10 = 115$

$z = 0$

∴ points → $(0,0,0)$ & $(25, 115, 0)$

* $v(t) = r'(t) = 2t\,i + (3t^2-2)\,j + (2t-5)\,k$

$a(t) = r''(t) = 2\,i + 6t\,j + 2\,k$

at $t=0$, $v(t)$? $a(t)$?

$v(t) = v(0) = 2\times 0\, i + (3\times 0 - 2)\,j + (2\times 0 - 5)\,k$.

$= -2j - 5k$

$a(t) = a(0) = 2i + 2k$

at $t = 5$,

$v(t) = v(5) = 10i + [illegible]j + 5k$.

$a(t) = a(5) = 2i + 30j + 2k$

3) A person hand glader is ~~spider~~ spirelling upward due to rapidly racing air on a path having position (r), $r(t) = 3\cos t\, i + 3\sin t\, j + t^2 k$, $0 \le t \le 4\pi$ then evaluate following,

a) v & accn (v)

b) gladeris speed at any time 't'.

c) The times, if any whn the gladeris accn is orthogonal to its velocity ($v(t) \cdot a(t) = 0$)

4) If a particle moves with a constant speed, S.T its velocity (v) 'v' is ⊥ to its accn (v) 'a' ? $[v \perp a\,?]$

A) let speed $= \|v(t)\| = c$ → constant

$v(t) \cdot v(t) = \|v(t)^2\| = c$

$v(t) \cdot v(t) = c$

$\frac{d}{dt}\underset{(\text{P. rule})}{(v(t) \cdot v(t))} = \frac{d}{dt}(c) = 0$

$\frac{dv}{dt} v(t) + v(t)\frac{dv}{dt} = 0$.

$2\frac{dv}{dt} v(t) = 0$

$\frac{dv}{dt} \cdot v(t) = 0$ $\qquad \frac{dv}{dt} = a(t)$

$a(t) \cdot v(t) = 0$

means $a(t) \perp v(t)$

⇒ centripetal Accelation :-

$r(t) = ?$

$r(t) = r_0 \cos\omega t\, i + r_0 \sin\omega t\, j$

$v(t) = r'(t) = -r_0 \sin\omega t \cdot \omega\, i + r_0 \cos\omega t \cdot \omega\, j$

~~$a(t) = r''(t) = -r_0 \cos\omega^2 t\, i + r_0 \sin\omega^2 t\, j$~~

~~$= -\omega^2 [r_0 \cos t\, i + r_0 \sin\omega t\, j]$~~

$a(t) = r''(t) = -r_0 \cos\omega t \cdot \omega^2 i + -r_0 \sin\omega t \cdot \omega^2 j$.

$= -\omega^2 [r_0 \cos\omega t\, i + -r_0 \sin\omega t\, j]$

$= -\omega^2 r(t)$

$v = \|v(t)\| = \sqrt{(-r_0 \sin\omega t \cdot \omega)^2 + (r_0 \cos\omega t\, \omega)^2}$

$= \sqrt{r_0^2 \sin^2\omega t \cdot \omega^2 + r_0^2 \cos^2\omega t \cdot \omega^2}$

$= \sqrt{r_0^2 \omega^2 (\sin^2\omega t + \cos^2\omega t)}$

$= \sqrt{r_0^2 \omega^2} = r_0 \omega$

$a = \|a(t)\| = \sqrt{(-r_0 \cdot \cos\omega t \cdot \omega^2)^2 + (-r_0 \sin\omega t \cdot \omega^2)^2}$

$= \sqrt{r_0^2 \cos^2\omega t\, \omega^4 + r_0^2 \sin^2\omega t \cdot \omega^4}$

$\sqrt{r_0^2 \omega^4 (\cos^2\omega t + \sin^2\omega t)}$

$\sqrt{r_0^2 \omega^4} = r_0 \omega^2$

$$\boxed{\begin{aligned} v(t) &= r_0 \omega \\ a(t) &= r_0 \omega^2 \end{aligned}}$$

3.A) hand glader :-

a) $r(t) = 3\cos t\, i + 3\sin t\, j + t^2 k$

$v(t) = r'(t) = -3\sin t\, i + 3\cos t\, j + 2t\, k$

$a(t) = r''(t) = -3\cos t\, i - 3\sin t\, j + 2\, k$

b) Speed $= \|v(t)\| = \sqrt{(-3\sin t)^2 + (3\cos t)^2 + (2t)^2}$

$= \sqrt{9\sin^2 t + 9\cos^2 t + 4t^2}$

$= \sqrt{9(\sin^2 t + \cos^2 t) + 4t^2}$

$= \sqrt{9 + 4t^2}$

c) we look for the values of t for which $v(t) \cdot a(t) = 0$

$v(t) \cdot a(t) = (-3\sin t\, i + 3\cos t\, j + 2t\, k) \cdot (-3\cos t\, i + -3\sin t\, j + 2k)$

$= 9\sin t\cos t - 9\cos t\sin t + 4t$

$= 4t$

Hence $v(t) \cdot a(t) = 0$ if and only if $t = 0$. Thus the only time the acc (v) is orthogonal to v is when $t=0$.

$4t$
$4 \times 0 = 0$

⇒ Motion of a projectile :-

position () for a projectile →

The trajectory of a (proj) fired from a height 'h' with an initial speed V_0 & an angle of elevation α is gvn by the position vector (),

$$r(t) = (V_0 \cos\alpha)t\, i + \left[h + (V_0 \sin\alpha)t - \frac{1}{2}gt^2\right] j$$

g is constnt of acceleration due to gravity.

Q.1) A shell fired from a cannon, has a muzzle speed of 320 ft/sec. The barrel makes an angle of 30° with the horizontal & for simplicity, the barrel opening is assumed to be at ground level.

a) find para. eq for shell's trajectory?

b) How high does the shell rise?

c) How far does the shell travel horizontally?

d) Speed of the shell at its point of impact with the ground?

A) a) $h = 0$ Speed $V_0 = 320$ ft/sec

$\alpha = 30°$ (angle of elevation)

acc due to gravity, $g = 32$ ft/sec²

by defn, $r(t) = (V_0 \cos\alpha)t\, i + \left[h + (V_0 \sin\alpha)t - \frac{1}{2}gt^2\right] j$

$= (320 \times \cos 30°)t\, i + \left[0 + (320 \times \sin 30°)t - \frac{1}{2} \times 32t^2\right] j$

$= \left(320 \times \frac{\sqrt{3}}{2}\right) t \hat{i} + \left[\left(320 \times \frac{1}{2}\right) t - 16t^2\right] \hat{j}$

$= 160\sqrt{3}\, t\, \hat{i} + (160t - 16t^2)\hat{j}$.

hence para.. eq :-

$x(t) = 160\sqrt{3}\,t$, $y(t) = 160t - 16t^2$, $t \geq 0$.

b) $x(t) = 160\sqrt{3}\,t$ $\quad y(t) = 160t - 16t^2 \quad t \geq 0$.

max height of shell is max value of y.

To find the value of t whn y is max,

$\frac{d}{dt}(160t - 16t^2) \longrightarrow \frac{dy}{dt} = 160 - 32t$ & $\frac{d^2y}{dt^2} = -32 \to < 0$

$160 - 32t \Rightarrow -32$

hence $\frac{dy}{dt} = 0$ whn $t = \frac{160}{32} = 5$ &

$\Rightarrow 160 - 32t = 0$

$160 = 32t$

$\frac{160}{32} = t$

$5 = t$

$\frac{d^2y}{dt^2} < 0$.

$\frac{d^2y}{dt^2} \to 0$ maximum
$\frac{dy}{dt} = 0$ minimum

$\therefore$ y is max whn $t = 5$ sec i.e max

value of y is $160 \times 5 - 16 \times 5^2 = 800 - 400$
$= 400$ ft

hence max h to which shell rise is 400 ft

c) $x(t) = 160\sqrt{3}\,t \quad y(t) = 160t - 16t^2 \quad t \geq 0$.

$y = 0 \Rightarrow 160t - 16t^2 = 0$

$\Rightarrow t(160 - 16t) = 0$

$t = 0$, ~~160-16t=0~~

$t = 10$. $\longrightarrow 10 = t$

$160 - 16t = 0$
$160 = 16t$
$\frac{160}{16} = t$

(as how tog)

Soln $t = 0$ corresponds to initial position of shell i.e the soln $t = 10$ corresponds to time of impact. Then horizontal distance travelled by the shell is the value of x whn $t = 10$. we want to find horiz - also x.

$\therefore$ horizontal distance $\to$

$x(10) = 160\sqrt{3} \times 10 = 1600\sqrt{3} \approx 2771.28$ ft

d) (v) eq to shell $\to$

$r(t) = 160\sqrt{3}\, t\, \hat{i} + (160t - 16t^2)\hat{j}$.

$v(t) = r'(t) = 160\sqrt{3}\, \hat{i} + (160 - 32t)\hat{j}$.

Speed?

impact with the ground occurs whn $t = 10$. i.e velocity (v) ,

$v(10) = 160\sqrt{3}\, \hat{i} + (160 - 320)\hat{j}$

$= 160\sqrt{3}\, \hat{i} - 160\hat{j}$. $\quad \therefore$ speed $= \|v(10)\| =$

$\sqrt{(160\sqrt{3})^2 + (160)^2} = 160\sqrt{3+1} = 160 \times 2 = 320$ ft/sec

$\Rightarrow$ Curvature of a plane curve :-

The rate at which 'T' turns per unit of length along the curve $\to$ curvature.

curvature $(\,) \longrightarrow \kappa$ (kappa)

def $\to$ Let C be a smooth curve defined by $r(s)$, where s is the arc length of the paramtr. Then the curvature of C at s is ,

$$\boxed{\kappa(s) = \left\|\frac{dT}{ds}\right\| = \|T'(s)\|}$$

$T \to$ tangent $(\vec{v})$.

$$\boxed{\kappa(t) = \frac{\|T'(t)\|}{\|r'(t)\|}}$$

Q/ 1) Find curvature of a ○ of radius a

A) $r(t) = a\cos t\, i + a\sin t\, j \qquad 0 \le t \le 2\pi$

$r'(t) = -a\sin t\, i + a\cos t\, j$

$\|r'(t)\| = \sqrt{(-a\sin t)^2 + (a\cos t)^2} = a.$

$\sqrt{a^2\sin^2 t + a^2\cos^2 t}$

$\sqrt{a^2(\sin^2 t + \cos^2 t)} = \sqrt{a^2} = a$

$\therefore \boxed{T(t) = \frac{r'(t)}{\|r'(t)\|}} = \frac{-a\sin t\, i + a\cos t\, j}{a}$

$= -\frac{a\sin t\, i}{a} + \frac{a\cos t\, j}{a}$

$= -\sin t\, i + \cos t\, j$

$T'(t) = -\cos t\, i - \sin t\, j$

$\|T'(t)\| = \sqrt{(-\cos t)^2 + (\sin t)^2}$

$= \sqrt{\cos^2 t + \sin^2 t} = \sqrt{1} = 1$

$\kappa(t) = \frac{T'(t)}{\|r'(t)\|} = \frac{1}{a}$

$\therefore$ curvature of a ○ of radius a is $1/a$.

2) Curvature of plane curve?

$r(t) = t\, i + (\ln\cos t)\, j \quad (-\frac{\pi}{2} < t < \pi/2)$

A) $r'(t) = i + \frac{-\sin t}{\cos t} j$

$= i - \tan t\, j$

$\|r'(t)\| = \sqrt{1 + (-\tan t)^2} = \sqrt{1 + \tan^2 t} = \sec t$

$T(t) = \frac{r'(t)}{\|r'(t)\|} = \frac{i - \tan t\, j}{\sec t} = \frac{1}{\sec t} i - \frac{\tan t}{\sec t} j$

$= \cos t\, i - \sin t\, j$

$\frac{d}{dx}(\ln\cos t)$ $= \frac{-\sin t}{\cos t}$.
$\frac{\tan t}{\sec t} = \sin t$

$T'(t) = -\sin t\, i - \cos t\, j$

$\therefore \kappa(t) = \frac{\|T'(t)\|}{\|r'(t)\|} = \frac{\sqrt{(-\sin t)^2 + (-\cos t)^2}}{\sec t}$

$= \frac{1}{\sec t} = \cos t$

$\Rightarrow$ Radius of curvature :-

radius $\Rightarrow \boxed{\rho = \frac{1}{\kappa}}$

* Center of curvature of curve at P is the center of the ○ of curvature at P.

$\Rightarrow$ Unit Normal. $(\vec{N})$:-

$$\boxed{N(t) = \frac{T'(t)}{\|T'(t)\|}}$$

Q/ 1) find the Unit normal (v) $N(t)$ for the helix, $x = a\cos t$, $y = a\sin t$, $z = ct$, $a>0$.

A) To find $T(t) = \dfrac{r'(t)}{\|r'(t)\|}$

$r(t) = a\cos t\, i + a\sin t\, j + ct\, k.$

$r'(t) = -a\sin t\, i + a\cos t\, j + c\, k.$

$\|r'(t)\| = \sqrt{(-a\sin t)^2 + (a\cos t)^2 + (c)^2}$

$= \sqrt{a^2\sin^2 t + a^2\cos^2 t + c^2}$

$= \sqrt{a^2(\sin^2 t + \cos^2 t) + c^2}$

$= \sqrt{a^2 + c^2}$ $\quad \cancel{a+c}$ ([illegible] a^2+c^2 [illegible])

$T(t) = \dfrac{-a\sin t\, i + a\cos t\, j + ck}{\sqrt{a^2+c^2}}$

$T(t) = -\dfrac{a\sin t\, i}{\sqrt{a^2+c^2}} + \dfrac{a\cos t\, j}{\sqrt{a^2+c^2}} + \dfrac{ck}{\sqrt{a^2+c^2}}$

$T'(t) = \dfrac{-a\cos t\, i - a\sin t\, j}{\sqrt{a^2+c^2}}$

$= \dfrac{-a\cos t\, i}{\sqrt{a^2+c^2}} - \dfrac{a\sin t\, j}{\sqrt{a^2+c^2}}$

$\|T'(t)\| = \sqrt{\left(\dfrac{-a\cos t}{\sqrt{a^2+c^2}}\right)^2 + \left(\dfrac{a\sin t}{\sqrt{a^2+c^2}}\right)^2}$

$= \sqrt{\dfrac{a^2\cos^2 t + a^2\sin^2 t}{a^2+c^2}}$

$= \sqrt{\dfrac{a^2(\cos^2 t + \sin^2 t)}{a^2+c^2}} = \dfrac{a}{\sqrt{a^2+c^2}}.$

$N(t) = \dfrac{T'(t)}{\|T'(t)\|}$

$= \dfrac{-\dfrac{a\cos t\, i}{\sqrt{a^2+c^2}} - \dfrac{a\sin t\, j}{\sqrt{a^2+c^2}}}{\dfrac{a}{\sqrt{a^2+c^2}}}$

$= \dfrac{-a\cos t\, i - a\sin t\, j}{\cancel{\sqrt{a^2+c^2}}} \cdot \dfrac{\cancel{\sqrt{a^2+c^2}}}{a}$

$= \cancel{a}(-\cos t\, i - \sin t\, j) \cdot \dfrac{1}{\cancel{a}}$

$= -\cos t\, i - \sin t\, j$

2) $r(t) = (e^t\cos t)i + (e^t\sin t)j + 2k.$

$T(t) = \dfrac{r'(t)}{\|r'(t)\|}$

~~$r'(t) = -e^t\sin t)i + e^t\cos t\, j$~~

pro. rule

$r'(t) = (-e^t\sin t + e^t\cos t)i + (e^t\cos t + e^t\sin t)j + 0k.$

$= (-e^t\sin t + e^t\cos t)i + (e^t\sin t + e^t\cos t)j$

$\|r'(t)\| = \sqrt{(-e^t\sin t + e^t\cos t)^2 + (e^t\sin t + e^t\cos t)^2}$

$(e^t)^2 = e^{2t}$

$= \sqrt{(e^t\sin t)^2 + (e^t\cos t)^2 + (e^t\sin t)^2 + (e^t\cos t)^2}$

$= \sqrt{e^{2t}\cdot\sin^2 t + e^{2t}\cdot\cos^2 t + e^{2t}\sin^2 t + e^{2t}\cos^2 t}$

$$= \sqrt{e^{2t}(\cos^2 t + \sin^2 t) + e^{2t}(\sin^2 t + \cos^2 t)}$$

$$e^{2t} = (e^t)^2 \qquad = \sqrt{e^{2t} + e^{2t}} = \sqrt{2e^{2t}} = \sqrt{2(e^t)^2} = e^t\sqrt{2}$$

$$T(t) = \frac{(-e^t \sin t + e^t \cos t)\,i + (e^t \sin t + e^t \cos t)\,j}{e^t\sqrt{2}}$$

$$= \frac{-\cancel{e^t} \sin t + \cancel{e^t} \cos t}{\cancel{e^t}\sqrt{2}} + \frac{\cancel{e^t} \sin t + \cancel{e^t} \cos t}{\cancel{e^t}\sqrt{2}}$$

$$= \frac{-\sin t + \cos t + \sin t + \cos t}{\sqrt{2}}$$

$$T(t) = \frac{-\sin t + \cos t}{\sqrt{2}} + \frac{\sin t + \cos t}{\sqrt{2}}$$

$$T'(t) = \frac{-\cos t - \sin t}{\sqrt{2}} + \frac{\cos t - \sin t}{\sqrt{2}}$$

$$\|T'(t)\| = \sqrt{\left(\frac{-\cos t + (-\sin t)}{\sqrt{2}}\right)^2 + \left(\frac{\cos t + (-\sin t)}{\sqrt{2}}\right)^2}$$

$$= \frac{(\cos^2 t + \sin^2 t) + (\cos^2 t + \sin^2 t)}{2}$$

$$= \frac{1+1}{2} = \frac{2}{2} = 1$$

$$N(t) = \frac{T'(t)}{\|T'(t)\|}$$

$$= \left(\frac{-\cos t - \sin t}{\sqrt{2}}\right) i + \left(\frac{\cos t - \sin t}{\sqrt{2}}\right) j$$

⇒ Binormal (v) in 3-space :-

If c is the ~~length~~ graph of a smooth (v) valued ( ) $\gamma(t)$ in 3-space, then we define the binormal (v) to c at t,

$$\boxed{B(t) = T(t) \times N(t)}$$

Q) position of a moving particle,

$\gamma(t) = 3\cos t\, i + 3\sin t\, j + 4t\, k$.

find (v) → T, N, B & curvature ?

A) $\gamma'(t) = -3\sin t\, i + 3\cos t\, j + 4k$

$$\|\gamma'(t)\| = \sqrt{(-3\sin t)^2 + (3\cos t)^2 + (4)^2}$$

$$= \sqrt{3^2 \sin^2 t + 3^2 \cos^2 t + 16}$$

$$= \sqrt{9(\sin^2 t + \cos^2 t) + 16} = \sqrt{9+16}$$

$$= \sqrt{25} = 5$$

$$T(t) = \frac{\gamma'(t)}{\|\gamma'(t)\|} = \frac{-3\sin t\, i + 3\cos t\, j + 4k}{5}$$

$$T'(t) = \frac{1}{5}\left[-3\cos t\, i - 3\sin t\, j\right]$$

$$= \frac{-3}{5}\left[\cos t\, i - \sin t\, j\right]$$

$$\|T'(t)\| = \frac{3}{5}\sqrt{(\cos t)^2 + (\sin t)^2} = \frac{3}{5}$$

$$\|T'(t)\| = \sqrt{\left(\frac{-3}{5}\right)^2 (\cos t)^2 + (\sin t)^2}$$

$$= \sqrt{\frac{9}{25}\left(\cos^2 t + \sin^2 t\right)} = \frac{3}{5}\sqrt{1}$$

$$= \frac{3}{5}$$

$$N(t) = \frac{T'(t)}{\|T'(t)\|} = \frac{-\frac{3}{5}[\cos t\, i + \sin t\, j]}{3/5}$$

$$= -(\cos t\, i + \sin t\, j) \quad (\text{or}) \; -\cos t\, i - \sin t\, j$$

crossproduct ~~answer~~ matrix form

$$\therefore B(t) = T(t) \times N(t)$$

$$= \left[-\frac{3}{5}\sin t\, i + \frac{3}{5}\cos t\, j + \frac{4}{5}k\right] \times$$

$$[-\cos t\, i - \sin t\, j]$$

find determinant

$$= \begin{vmatrix} i & j & k \\ -\frac{3}{5}\sin t & \frac{3}{5}\cos t & \frac{4}{5} \\ -\cos t & -\sin t & 0 \end{vmatrix}$$

$$= \left(0 + \frac{4}{5}\sin t\right) i - \left(0 + \frac{4}{5}\cos t\right) j +$$

$$\left(\frac{3}{5}\sin^2 t + \frac{3}{5}\cos^2 t\right) k.$$

$$= \frac{4}{5}\sin t\, i - \frac{4}{5}\cos t\, j + \frac{3}{5}k$$

<u>curvature</u>

$$\gamma(t) = k(t) = \frac{\|T'(t)\|}{\|r'(t)\|} = \frac{3/5}{5} = \frac{3}{25}$$

⇒ <u>formulas for a_T, a_N & curvature :-</u>

curvature

$$\boxed{a = a_T T + a_N N}\text{; where } a_T = v', \; a_N = kv^2.$$

a = accelm
T = Tangential (v)
N = normal (v)

$$v(t) = r'(t) = \|r'(t)\| T = vT \quad \text{— (1)}$$

$$a = v' = \frac{d}{dt}(v) = \frac{d}{dt}(vT) = v'T + vT' \quad \text{— (2)}$$

from (1) & (2)

$$v \cdot a = (vT) \cdot (v'T + kv^2 N)$$

$$= vv'\, T\cdot T + kv^3\, T \cdot N$$

But $T \cdot T = \|T\|^2 = 1$. Since T is a unit (v) & $T \cdot N = 0$. Since T & N are orthogonal.

$$\therefore v \cdot a = vv'$$

$$\boxed{a_T = v' = \frac{v \cdot a}{v} = \frac{r'(t) \cdot r''(t)}{\|r'(t)\|}}$$

$$a = v'T + vT' = v'T + v\, kvN$$

$$= v'T + kv^2 N \quad \text{— (3)}$$

from (1) & (3),

$$v \times a = (vT) \times (v'T + kv^2 N)$$

$$= vv'(T \times T) + kv^3 (T \times N)$$

Since $T \times T = 0$, & $T \times N = B$.

$$v \times a = kv^3 B \;\; \& \;\; \|v \times a\| = kv^3\|B\| = kv^3$$

$$(\because \|B\| = 1)$$

$$\boxed{a_N = kv^2 = \frac{kv^3}{v} = \frac{\|v \times a\|}{\|v\|} = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|}}$$

$$\text{curvature} \rightarrow k = \frac{a_N}{v^2} = \frac{a_N}{\|r'(t)\|^2}$$

$$\boxed{K = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3}}$$

2) A particle moves along a curve described by the (v) (f) $r(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$. find tangential scalar & normal scalar components of acc.... & of the particle at any time t, Also find of curvature of the path at any time t

A) $r(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$.

$r'(t) = \hat{i} + 2t\hat{j} + 3t^2\hat{k}$.

$r''(t) = 2\hat{j} + 6t\hat{k}$.

$\|r'(t)\| = \sqrt{1^2 + (2t)^2 + (3t^2)^2} = \sqrt{1 + 4t^2 + 9t^4}$

$r'(t) \cdot r''(t) = (\hat{i} + 2t\hat{j} + 3t^2\hat{k}) \cdot (2\hat{j} + 6t\hat{k})$ $\quad 3t^2 \cdot 6t = 18t^3$

$= 4t + 18t^3$

$$r'(t) \times r''(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix} \quad \begin{matrix} (12t^2 - 6t^2)\hat{i} - (6t - 0)\hat{j} + (2-0)\hat{k} \\ (12t^2 - 6t^2)\hat{i} - 6t\hat{j} + 2\hat{k} \\ 6t^2\hat{i} - 6t\hat{j} + 2\hat{k} \end{matrix}$$

$= 6t^2\hat{i} - 6t\hat{j} + 2\hat{k}$

∴ tangential scalar component a_T & normal scalar component a_N of acc... of particle at any time t,

$$\boxed{a_T = \frac{r'(t) \cdot r''(t)}{\|r'(t)\|}} = \frac{4t + 18t^3}{\sqrt{1 + 4t^2 + 9t^4}}$$

$$\boxed{a_N = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|}}$$

$$= \frac{\sqrt{(6t^2)^2 + (6t)^2 + (2)^2}}{\sqrt{1 + 4t^2 + 9t^4}} = \frac{\sqrt{36t^4 + 36t^2 + 4}}{\sqrt{1 + 4t^2 + 9t^4}}$$

$$= \sqrt{\frac{36t^4 + 36t^2 + 4}{1 + 4t^2 + 9t^4}}$$

$$= 2\sqrt{\frac{9t^4 + 9t^2 + 1}{9t^4 + 4t^2 + 1}}$$

Curvature

$$K = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3} = \frac{\sqrt{(6t^2)^2 + (6t)^2 + (2)^2}}{\left[\sqrt{1 + 4t^2 + 9t^4}\right]^3}$$

$$= 2 \cdot \frac{\sqrt{9t^4 + 9t^2 + 1}}{\left[9t^4 + 4t^2 + 1\right]^{3/2}}$$

3) A particle moves along a circular helix that is described by (v) (f),

$r(t) = a\cos t\,\hat{i} + a\sin t\,\hat{j} + bt\,\hat{k}$,

$a, b \geq 0$, $a^2 + b^2 \neq 0$

find tangential & normal scalar components of acce... & of particle at any time t, Also find curvature of the path at any time 't'.

A) $r'(t) = (-a\sin t)\hat{i} + (a\cos t)\hat{j} + b\hat{k}$

$r''(t) = (-a\cos t)\hat{i} + (-a\sin t)\hat{j}$.

$$\|r'(t)\| = \sqrt{(-a\sin t)^2 + (a\cos t)^2 + b^2}$$

$$= \sqrt{a^2+b^2}$$

$$r'(t) \cdot r''(t) = [(-a\sin t)i + (a\cos t)j + bk] \cdot [(-a\cos t)i + (-a\sin t)j]$$

$$= a^2 \sin t\cos t - a^2 \sin t \cos t = 0$$

$$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ -a\sin t & a\cos t & b \\ -a\cos t & -a\sin t & 0 \end{vmatrix}$$

$$= ab\sin t\, i - ab\cos t\, j + (a^2\sin^2 t + a^2\cos^2 t)k$$

$$= ab\sin t\, i - ab\cos t\, j + a^2 k$$

Tangential ⟶

$$a_T = \frac{r'(t)\cdot r''(t)}{\|r'(t)\|} = \frac{0}{\sqrt{a^2+b^2}} = 0.$$

$$a_N = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|} = \frac{\|ab\sin t\, i - ab\cos t\, j + a^2 k\|}{\sqrt{a^2+b^2}}$$

$$= \frac{\sqrt{(ab\sin t)^2 + (ab\cos t)^2 + a^4}}{\sqrt{a^2+b^2}} = \frac{a\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}}$$

$$= a$$

Curvature

$$K(t) = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3}$$

$$= \frac{\sqrt{(ab\sin t)^2 + (-ab\cos t)^2 + a^4}}{[\sqrt{a^2+b^2}]^3}$$

$$= \frac{a\sqrt{a^2+b^2}}{[\sqrt{a^2+b^2}]^3} = \frac{a}{[\sqrt{a^2+b^2}]^2} = \frac{a}{a^2+b^2}$$

4) Find eq for osculating, normal & rectifying planes of curve, $r(t) = \cos t\, i + \sin t\, j - k$, at $t = \frac{\pi}{4}$

A) $r'(t) = -\sin t\, i + \cos t\, j$.

$$\|r'(t)\| = \sqrt{(-\sin t)^2 + (\cos t)^2} = 1$$

$$T(t) = \frac{r'(t)}{\|r'(t)\|} = \frac{-\sin t\, i + \cos t\, j}{1}$$

$$= -\sin t\, i + \cos t\, j$$

$$T'(t) = -\cos t\, i - \sin t\, j$$

$$\|T'(t)\| = 1$$

$$N(t) = \frac{T'(t)}{\|T'(t)\|} = \frac{-\cos t\, i - \sin t\, j}{1}$$

$$= -\cos t\, i - \sin t\, j$$

$$B(t) = T(t) \times N(t)$$

$$= (-\sin t\, i + \cos t\, j) \times (-\cos t\, i - \sin t\, j)$$

$$= \begin{vmatrix} i & j & k \\ -\sin t & \cos t & 0 \\ -\cos t & -\sin t & 0 \end{vmatrix}$$

$$= (0-0)i - (0-0)j + (\sin^2 t + \cos^2 t)k$$

$$= k$$

hence $T\left(\frac{\pi}{4}\right) = -\sin(\pi/4)i + \cos(\pi/4)j.$

$$= \frac{1}{\sqrt{2}}(-i+j)$$

$$N(\pi/4) = -\cos(\pi/4) - \sin(\pi/4)j$$

$$= \left(-\frac{1}{\sqrt{2}}\right)(i+j) \quad \& \quad B(\pi/4) = k$$