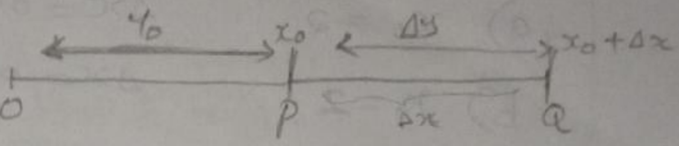
Derivaties And Limits

* **velocity :-**



$$\text{velocity} = \frac{\text{distance travelled}}{\text{time}}$$

$$= \frac{\Delta y}{\Delta x} = \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

A particle moving along a straight line from a fixed point ~~of~~ 'O' & that fun $y=f(x)$ describes the position of the moving particle at time 'x'.

Here 'x' denote the timing seconds &
'y' denote the (dis) travelled in 'x' sec.

Suppose the particle passes ~~from~~ point P & Q at the time x_0 & $x_0+\Delta x$ respectively.

If 'y_0' & $y_0+\Delta y$ are the respective (dis) of P & Q from fixed point 'O'. Then $y_0 = f(x_0)$
& $y_0+\Delta y = f(x_0+\Delta x)$.

The avg velocity of the particle during time interval 'Δx' is

$$\boxed{\text{Avg velocity} = \frac{\text{Dis travelled}}{\text{time}} = \frac{\Delta y}{\Delta x} = \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}}$$

1) Let $y = 3x^2+x$ represents the (dis) a bus has travelled in mtrs after x sec. find Δy & Avg v during the time interval Δx for the following situations -

a) $x_0 = 2$ $\Delta x = 0.5$

b) $x_0 = 2$ $\Delta x = 0.01$

c) $x_0 = 4$ $\Delta x = 0.5$

A) a) $y_0 = f(x_0)$ $y = f(x)$

$y = 3x^2+x$.

$f(x) = 3x^2+x$.

$x_0 = 2$. $f(2) = 3\times 2^2+2$

$= 3\times 4+2 = \underline{14}$ m.

$x_0+\Delta x = 2+0.5 = \underline{2.5}$ sec

$f(x_0+\Delta x) = f(2.5)$

$= 3\times(2.5)^2+2.5$

$= \underline{21.25}$ m/s

Avg v $= \dfrac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$

$= \dfrac{21.25 - 14}{0.5} = \underline{14.5}$ m/s

b) $f(x) = 3x^2+x$.

$x_0=2$. $f(2) = 14$ m.

$x_0+\Delta x = 2+0.01 = \underline{2.01}$ sec

$0.01 + 2 = 2.01$

$f(x_0+\Delta x) = f(2.01)$

$= 3\times(2.01)^2+2.01$

$= 3\times 4.0401+2.01 = 12.1203+2.01$

$= \underline{14.1303}$ m/s

Avg v $= \dfrac{14.1303-14}{0.01}$

$= \dfrac{0.1303}{0.01} = \underline{13.03}$ m/s.

c) $x_0 = 4$.

$f(4) = 3\times 4^2+4$

$= 3\times 16+4 = 48+4 = \underline{52}$ m

$x_0+\Delta x = 4+0.5 = \underline{4.5}$ sec

$f(x_0+\Delta x) = f(4.5)$

$= 3\times(4.5)^2+4.5$

$= 3\times 20.25+4.5$

$= 60.75+4.5 = \underline{65.25}$ m/s

Avg v $= \dfrac{65.25-52}{0.5}$

$= \dfrac{13.25}{0.5} = \underline{26.5}$ m/s

2) A bus travels $2x^2$ m in x sec find Δx, Δy & Avg velocity during the time interval Δx for the following situations -

a) $x_0 = 3$ $x = 4$

b) $x_0 = 3$ $x = 3.1$

c) $x_0 = 3$ $x = 3.01$

A) a) $x_0 = 3$ $\quad x = 4$

$\Delta x = 4 - 3 = 1$

$f(x) = 2x^2$

$f(x_0) = 2 \times 3^2 = 2 \times 9 = \underline{\underline{18}}$

$f(x_0 + \Delta x) = f(3+1)$
$= f(4) \rightarrow 2 \times 4^2 = 2 \times 16 = \underline{\underline{32}}$

$\Delta y = \dfrac{f(x_0+\Delta x) - f(x_0)}{\Delta x} = \dfrac{32 - 18}{1} = \underline{\underline{14\,m}}$

Avg velo $\dfrac{\Delta y}{\Delta x} = \dfrac{14}{1} = \underline{\underline{14\,m/s}}$

b) $x_0 = 3$ $\quad x = 3.1$

$\Delta x = 3.1 - 3 = \underline{\underline{0.1}}$

$f(x_0) = 2x^2$
$= 2 \times 3^2 = \underline{\underline{18}}$

$f(x_0 + \Delta x) = f(3 + 0.1) = \underline{\underline{f(3.1)}} = 2 \times 3.1^2$
$= 2 \times 9.61 = \underline{\underline{19.22\,m}}$

$\Delta y = \dfrac{f(x_0+\Delta x) - f(x_0)}{\Delta x} = \dfrac{\cancel{32}^{19.22} - 18}{0.1} = \dfrac{\cancel{14}\,1.22}{0.1} = \underline{\underline{\cancel{14}\;12.2\,m/s}}$

$\dfrac{\Delta y}{\Delta x} = \dfrac{\cancel{14}}{0.1}\ldots$ $\cancel{14\,9\,0}$

c) $x_0 = 3$ $\quad x = 3.01$

$\Delta x = 3.01 - 3 = \underline{\underline{0.01}}$

$f(x_0) = 2x^2 = 2 \times 3^2 = 18$

$f(x_0 + \Delta x) = (3 + 0.01) fx = fx(3.01)$
$= 2 \times 3.01^2 = 2 \times 9.0601$
$= 18.1202\,m$

$\Delta y = \dfrac{f(x_0+\Delta x) - f(x_0)}{\Delta x} = \dfrac{18.1202 - 18}{0.01}$
$= \dfrac{0.1202}{0.01} = \underline{\underline{12.02\,m/s}}$

→ **Instantaneous velocity :-**

To calculate I.V at x_0 when the position at time x is $y = f(x)$

1) For avg.v over the interval from x_0 to $x_0 + \Delta x$,

$$\frac{\Delta y}{\Delta x} = \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

2) Simplify your expression for $\frac{\Delta y}{\Delta x}$ as much as possible, cancelling Δx from numerator & denomenator, where ever you can.

3) Find the no 'V' that is approximated by $\frac{\Delta y}{\Delta x}$ for Δx small.

eg:- The ~~bus~~ has gone a $f(x) = 2x^2$ m at time x(sec). cal. its I.V at $x_0 = 3$.

A)- $f(x) = 2x^2$
$f(3) = 2 \times 3^2 = 18$

$f(3+\Delta x) = 2 \times (3+\Delta x)^2$
$= 2 \times (9 + 6\Delta x + (\Delta x)^2)$
$= 18 + 12\Delta x + 2(\Delta x)^2$

$(a+b)^2 = a^2 + 2ab + b^2$
$(3+\Delta x)^2 = 3^2 + 2 \times 3 \times \Delta x + \Delta x^2$
$= 9 + 6\Delta x + \Delta x^2$

$\dfrac{\Delta y}{\Delta x} = \dfrac{f(x_0+\Delta x) - f(x_0)}{\Delta x} = \dfrac{\cancel{18} + 12\Delta x + 2(\Delta x)^2 - \cancel{18}}{\Delta x}$

$= \dfrac{12\Delta x + 2(\Delta x)^2}{\Delta x}$ (Take Δx common)

$$\frac{\cancel{\Delta x}(12+2\Delta x)}{\cancel{\Delta x}} = 12+2\Delta x$$

$\Delta x \to 0 \Rightarrow 12 + 2\times 0 = 12.$

$v = 12\,m/s.$ (v = instantaneous No)

Thus I.v at $x_0 = 3$, ~~thus the required~~ is 12 m/s

2) The position of a bus at time x is $y = 3x^2 + 8x$, $x \geq 0$,

a) Find the I.v at an arbitrary time x_0

b) At wht time is I.v 11 m/s.

A) $f(x) = 3x^2 + 8x$.

a)
$$f(x_0) = (3x_0^2 + 8x_0)$$
$$f(x_0+\Delta x) = 3\times(x_0+\Delta x)^2 + 8(x_0+\Delta x)$$
$$= 3\times(x_0^2 + 2\Delta x x_0 + (\Delta x)^2) + 8x_0 + 8\Delta x.$$
$$= 3x_0^2 + 6x_0\Delta x + 3(\Delta x)^2 + 8x_0 + 8\Delta x.$$

$$\frac{\Delta y}{\Delta x} = \frac{\cancel{3x_0^2} + 6x_0\Delta x + 3(\Delta x)^2 + \cancel{8x_0} + 8\Delta x - (\cancel{3x_0^2} + \cancel{8x_0})}{\Delta x}$$

$$= \frac{6x_0\Delta x + 3(\Delta x)^2 + 8\Delta x}{\Delta x}$$

$$= \frac{\cancel{\Delta x}(6x_0 + 3\Delta x + 8)}{\cancel{\Delta x}} = 6x_0 + 3\Delta x + 8$$

$\Delta x \to 0.$

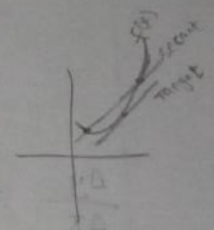
$$= 6x_0 + 3\times 0 + 8$$

$$\frac{\Delta y}{\Delta x} = 6x_0 + 8 \; m/s$$

b)
$$6x_0 + 8 = 11$$
$$6x_0 = 11 - 8$$
$$6x_0 = 3$$
$$x_0 = \frac{3}{6} = \frac{1}{2}\text{ sec.}$$

→ Slope (m) of Tangent Line :-

Given a (?) $y = f(x)$, this slope 'm' of the line tangnt to its graph at (x_0, y_0) is calculated as follows:



a) for the slope of secant line

$$\frac{\Delta y}{\Delta x} = \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

b) Simplify the expression for $\frac{\Delta y}{\Delta x}$, cancelling Δx if possible.

c) Find the no. 'm', that is approximated by $\frac{\Delta y}{\Delta x}$ for Δx small.

Q
1) Cal. the slope of (T) line to the graph of $f(x) = x^2 + 1$ at $x_0 = -1$

$x_0 \to$ a point

A)
$$\frac{\Delta y}{\Delta x} = \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}.$$

$$f(-1) = -1^2 + 1 = 1 + 1 = 2$$

$$f(x_0) = x_0^2 + 1$$
~~$f(x_0+\Delta x) = x_0^2 + (x_0+\Delta x)^2 + 1$~~
~~$= (x_0^2 + 2\Delta x x_0 + (\Delta x)^2) + 1$~~

$$\frac{\Delta y}{\Delta x} = \frac{f(1+\Delta x) - f(1)}{\Delta x}$$

$f(x_0+\Delta x) = (x_0+\Delta x)^2 + 1$

$= x_0^2 + 2x_0\Delta x + \Delta x^2 + 1$

$= -1^2 + 2\times(-1)\Delta x + -1^2 + 1$

$= 1 + -2\Delta x + 1 + 1$

$f(-1+\Delta x) = (-1+\Delta x)^2 + 1$

$= (-1)^2 + 2\times(-1)\times\Delta x + (\Delta x)^2 + 1$

$= 1 - 2\Delta x + (\Delta x)^2 + 1$

$= 2 - 2\Delta x + (\Delta x)^2$

$$\frac{\Delta y}{\Delta x} = \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

$$= \frac{\cancel{2} - 2\Delta x + (\Delta x)^2 - \cancel{2}}{\Delta x}$$

$$= \frac{-2\Delta x + (\Delta x)^2}{\Delta x}$$

$$= \frac{\cancel{\Delta x}(-2+\Delta x)}{\cancel{\Delta x}} = -2 + \Delta x$$

$\Delta x \to 0$

$\frac{\Delta y}{\Delta x} = -2$ Slope, $m = -2$

⇒ Derivatives :-

To calculate the derivative $f'(x_0)$ of a ( ) $y = f(x)$ at x_0

a) find $\frac{\Delta y}{\Delta x} = \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$

b) Simplify $\frac{\Delta y}{\Delta x}$, cancelling Δx if possible.

c) The derivative is the no. $f'(x_0)$ that $\frac{\Delta y}{\Delta x}$ approximates for Δx small.

Q

i) Suppose that 'm' is constant differenciate $f(x) = mx + 2$ at $x_0 = 10$.

A) $\frac{\Delta y}{\Delta x} = \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$

$f(x_0) = f(10) = m\times 10 + 2 = 10m + 2$

$$\frac{\Delta y}{\Delta x} = \frac{m(10+\Delta x) + 2 - (10m+2)}{\Delta x}$$

$$= \frac{\cancel{10m} + m\Delta x + \cancel{2} - (\cancel{10m} + \cancel{2})}{\Delta x}$$

$$= \frac{m\cancel{\Delta x}}{\cancel{\Delta x}} = m$$

→ Quadratic ( ) rule :-

✯ let $f(x) = ax^2 + bx + c$, where a, b, c are constant. & let x_0 be any real No..

then, $\boxed{f'(x_0) = 2ax_0 + b}$

Q

i) find the derivative of $f(x) = 3x^2 + 8x$. at (a) $x_0 = -2$ (b) $x_0 = \frac{1}{2}$.

A) Applying quadratic rule,

$f(x) = ax^2 + bx + c$

$f(x) = 3x^2 + 8x + 0$.

$a = 3$

$b = 8$

$c = 0$

(a) $f'(x_0) = 2ax_0 + b$

$= 2\times 3(-2) + 8$

$= -12 + 8 = -4$

$x_0 = \frac{1}{2}$

(b) $f'(x_0) = 2ax_0 + b$
$= 2 \times 3 \times \frac{1}{2} + 8$
$= 3 + 8 = 11$

☆ → Differentiating the simplest ( ) :-

* The derivative of quadratic ( ), $f(x) = ax^2+bx+c$ is the linear ( ) $f'(x) = 2ax + b$.
* The derivative of linear ( ), $f(x) = bx + c$ is the constant ( ) $f'(x) = b$.
* The derivative of constant ( ) $f(x) = c$ is the zero ( ), $f'(x) = 0$.

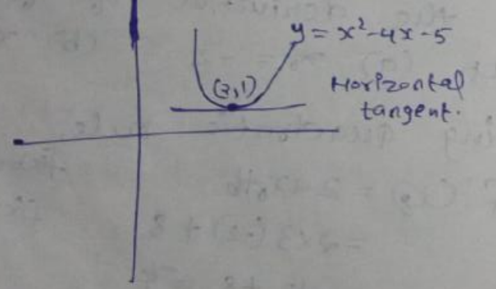
$$\boxed{\begin{array}{ll} f(x) = ax^2+bx+c, & f'(x) = 2ax+b \\ f(x) = bx+c, & f'(x) = b \\ f(x) = c, & f'(x) = 0. \end{array}}$$

Q

1) There is 1 point on the graph of parabola $y = f(x) = x^2 - 4x + 5$, where the slope is 0, so that tangent line is horizontal. Find the point using

a) Derivatives (b) Algebra.

A)



$f(x) = x^2 - 4x + 5$
$a = 1 \quad b = -4 \quad c = 5$
$f'(x) = 2ax_0 + b$
$= 2 \times 1\, x_0 + (-4)$
$= 2x - 4$

for zero slope,
$f'(x) = 0 \Rightarrow 2x - 4 = 0$
$2x = 4$
$x = 2$

when $x = 2$,
$y = f(x) = x^2 - 4x + 5$
$= 2^2 - 4 \times 2 + 5$
$= 4 - 8 + 5$
$= -4 + 5 = 1$

$x = 2$
$y = 1$

our point is $(2, 1)$
This point → vertex of parabola.

(or)
$f(x) = x^2 - 4x + 5$
$= x^2 - 4x + 4 + 1$
$= (x-2)^2 + 1$

$x^2 - 4x + 4$
$(x-2)^2 = x^2 + 2x2 + 2^2$
$= x^2 + 4x + 4$
$(x-2)^2 = 0$
$x = 2$

$(x-2)^2$ is zero for $x = 2$.
$y = 1$

2) A stunt women on a moving passenger train, her (lis) ( ) is $3t^2 + t$, on the adjacent track is a long moving freight train. The (lis) ( ) for the center of this train is $t^2 + 7t$. She must jumb to freight train, wht

time is best ?

A) Safe time to jump is whn the stunt women has the same velocity as train.

v = instantaneous velocity = derivative of $3t^2+t$

$3t^2+t = 2\times 3t+1 = 6t+1$

then by quadratic formula,

$v = 6t+1$

instan.. velocity of train is (derivative of t^2+7t) $2t+7$

$t^2+7t = 2t+7\times 1 = 2t+7$

Safest time whn velocity are equal,

$6t+1 = 2t+7$

$6t-2t = 7-1$

$4t = 6$

$t = \frac{6}{4}$

$t = \frac{3}{2}$

→ Limits :-

Limit of a () $f(x)$ at a point $x = x_0$ is the value which $f(x)$ approximate for x close to x_0

1) using numerical commutation guess the value of $\lim_{x\to 4}\left[\frac{1}{4x-2}\right]$

x	4.1	4.01	4.001	3.9	3.99	3.999
$\frac{1}{4x-2}$	0.0694	0.0712	0.0714	0.0735	0.0716	0.0714

$\left(\frac{1}{4\times 4.1-2} = \frac{1}{14.4} = 0.069\right)$

Limit is a no.. which when rounded to 3 decimal places is 0.071

2) Let $\phi(x) = |x|$

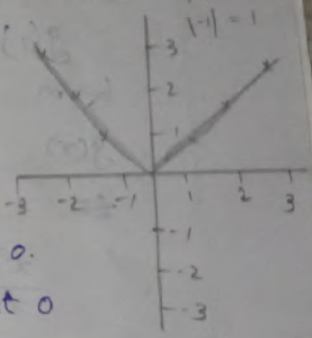
$|x| \to +ve$

whn $\phi(x)$ is defined for all real valued of $f(x)$.

A) $|x| = \begin{cases} -x & x<0 \\ x & x>0 \end{cases}$

$|1| = 1$

$|-1| = 1$



$\phi(x)$ approches arbitrarily close to 0 as x approches 0.

(or) $\phi(x)$ approches the Limit 0 as x approaches 0.

$|x| \longrightarrow 0$ as $x \longrightarrow 0$

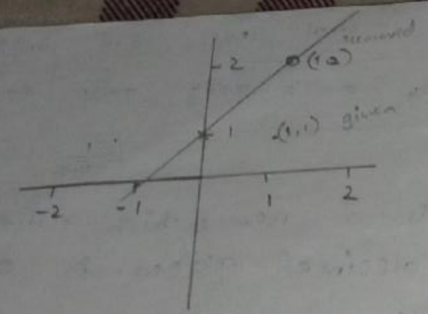
$\lim_{x\to 0} |x| = 0$

3) $g(x) = \begin{cases} \frac{x^2-1}{x-1} & x \neq 4 \\ 1 & x=1 \end{cases}$

$(a+b)(a-b) = a^2-b^2$

A) for, $x \neq 1$

$g(x) = \frac{x^2-1}{x-1} = \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}} = x+1$



The graph of g is the line $y = x+1$ with 1 point removed $(1,2)$ together with $(1,1)$.

$g(1)$ is defined to be 1.

From graph of g, it is clr that $g(x)$ approaches to 2 as x approaches 1.

$$\frac{x^2-1}{x-1} \to 2 \quad \text{as} \quad x \to 1$$

$$\lim_{x\to 1} \frac{x^2-1}{x-1} = 2$$

→ The Notion of Limit :-

(Remarks)

* The quantity $\lim_{x\to x_0} f(x)$ depends upon the value of $f(x)$ for x near x_0 but not for $x = x_0$ even if $f(x_0)$ is ~~not~~ defined. It can be change arbitarily without affecting the value of the limit.

* As x approaches to x_0 the value of $f(x)$ might not approach any fixed no. in this case we say that $f(x)$ has no limit as $x \to x_0$.

* In determining $\lim_{x\to x_0} f(x)$ we must consider values of $f(x)$ on both side of x_0.

⟹ Basic (pro) of limits :-

* Assume $\lim_{x\to x_0} f(x)$ & $\lim_{x\to x_0} g(x)$ exist.

1) Sum rule =

$$\lim_{x\to x_0} \left(f(x) + g(x)\right) = \lim_{x\to x_0} f(x) + \lim_{x\to x_0} g(x).$$

2) Product rule =

$$\lim_{x\to x_0} \left(f(x) \cdot g(x)\right) = \lim_{x\to x_0} f(x) \cdot \lim_{x\to x_0} g(x).$$

3) Reciprocal rule =

$$\lim_{x\to x_0} \frac{1}{f(x)} = \frac{1}{\lim_{x\to x_0} f(x)}.$$

4) Constant Sum rule =

$$\lim_{x\to x_0} c = c$$

5) Identity () rule =

$$\lim_{x\to x_0} x = x_0$$

6) Replacement rule =

If the () f & g hve the same values for all x near x_0, bt not necessarily including $x = x_0$ then $\lim_{x\to x_0} f(x) = \lim_{x\to x_0} g(x)$.

1) use basic (Pro) of limits (a) find

$\lim_{x\to3}(x^2+2x+5) =$

$= \lim_{x\to3}(x^2) + \lim_{x\to3}(2x) + \lim_{x\to3}(5)$. (Sum rule)

constant

$= \lim_{x\to3}(x\cdot x) + 2\lim_{x\to3} x + 5$ (constnt Sum rule)

$= \lim_{x\to3} x \lim_{x\to3} x + 2\times3 + 5$ (product rule & Identity (I) rule)

$= 3\cdot3 + 2\cdot3 + 5$ (Identity (I) rule).

$= 9 + 6 + 5 = 20$

2) Show that $\lim_{x\to3}\left(\frac{2x^2-7x+3}{x-3}\right) = 5$.

$(3-3=0) \to$ factorise

~~$2x^2-7x+3$~~

we cannot use quatient rule,
Since $\lim_{x\to3}(x-3) = 0$
So factorising $2x^2-7x+3$

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

$a=2$
$b=-7$
$c=3$

$x = \frac{-(-7)\pm\sqrt{49-4\times2\times3}}{2\times2}$

$= \frac{7\pm\sqrt{49-24}}{4}$

$= \frac{7\pm\sqrt{25}}{4}$

$= \frac{7\pm5}{4}$

$= \frac{7+5}{4} = \frac{12}{4} = 3$ $= \frac{7-5}{4} = \frac{2}{4} = \frac{1}{2}$

$x = 3$ $x = \frac{1}{2}$ as $2x-1$

$(x-3)(2x-1) =$ ~~factors,~~ $2x^2-7x+3$

$\lim_{x\to3}\frac{\cancel{(x-3)}(2x-1)}{\cancel{x-3}}$

$\lim_{x\to3}(2x-1) = 2\times3-1 = 6-1 = 5$

Hence proved

3) Find $\lim_{u\to2}\left(\frac{8u^2+2}{u-1}\right)$

$\lim_{u\to2}(8u^2+2)$

$= \lim_{u\to2}(8u^2) + \lim_{u\to2}(2)$. (Sum rule)

$= 8\lim_{u\to2} u^2 + 2$ (constnt sum rule)

$= 8\lim_{u\to2} u \cdot \lim_{u\to2} u + 2$ (product rule)

$= 8\times2\cdot2 + 2$

$= 8\times4+2 = 34$

$\lim_{u\to2}(8u^2-2) = 34$.

$\lim_{u\to2}(u-1) = 2-1 = 1$

$\Rightarrow \frac{\lim_{u\to2} 8u^2+2}{\lim_{u\to2} u-1} = \frac{34}{1} = 34$

Soumya

4) find $\lim_{x\to 1} \dfrac{x^3 - 3x^2 + 14x}{x^6 + x^3 + 2} = \dfrac{f(x)}{g(x)}$

$1^6 + 1^3 + 2 = 1+1+2 = 4$, $4 \neq 0$

$f(x) = x^3 - 3x^2 + 14x$

$\lim_{x\to 1} f(x) = x^3 - 3x^2 + 14x$

$= \lim_{x\to 1}(x^3) - \lim_{x\to 1}(3x^2) + \lim_{x\to 1}(14x)$

$= \lim_{x\to 1}(x\cdot x\cdot x) - 3\lim_{x\to 1}(x^2) + 14\lim_{x\to 1}(x)$

$= 1\cdot 1\cdot 1 - 3 \lim_{x\to 1} x \cdot \lim_{x\to 1} x + 14 \times 1$

$= 1 - 3 \times 1\cdot 1 + 14$

$= \cancel{27 - 3 + 14}$

$= 1 - 3 + 14$ $\cancel{= 12 \quad 1 - 11}$

$-2+14 = 12$

$= 11 + 1 = \underline{\underline{12}}$

$\lim_{x\to 1} g(x) = \lim_{x\to 1}(x^6 + x^3 + 2) = 4$

$\dfrac{\lim f(x)}{\lim g(x)} = \dfrac{12}{4} = \underline{\underline{3}}$

⇒ **Definition of Continuity :-**

A ( ) $f(x)$ is said to be continuous at $x = x_0$, if $\lim_{x\to x_0} f(x) = f(x_0)$.

If $f(x)$ is continous at x_0, 2 things are true –

1) $\lim_{x\to x_0} f(x)$ exist

2) limit can be calculated by setting $x = x_0$ in $f(x)$

⇒ **Continuity of rational ( ) :-**

If $f(x)$ is a polynomial / ratio of a polynomial & $f(x_0)$ is defined, then $\lim_{x\to x_0} f(x) = f(x_0)$.

1) Find $\lim_{x\to 2} \dfrac{x^2 + x - 6}{x^2 + 2x - 8}$.

$z = 0$ so factorise both.

$= \lim_{x\to 2} \dfrac{(x+3)\cancel{(x-2)}}{(x+4)\cancel{(x-2)}}$

$= \lim_{x\to 2} \dfrac{x+3}{x+4}$

$= \dfrac{2+3}{2+4} = \underline{\underline{\dfrac{5}{6}}}$

$x^2 + x - 6$; $a = 1$, $b = 1$, $c = -6$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -1 \pm \dfrac{\sqrt{1^2 - 4\times 1 \times 6}}{2\times 1} = -1 \pm \dfrac{\sqrt{1+24}}{2} = -1 \pm \dfrac{\sqrt{25}}{2} = -1 \pm \dfrac{5}{2}$

$= \dfrac{-1+5}{2} = \dfrac{4}{2} = 2$, $= \dfrac{-1-5}{2} = \dfrac{-6}{2} = -3$

$x = -3$, $x = 2$

$x^2 + 2x - 8$; $a = 1$, $b = 2$, $c = -8$

$x = -b \pm \dfrac{\sqrt{b^2 - 4ac}}{2a} = -2 \pm \dfrac{\sqrt{2^2 - 4\times 1 \times -8}}{2\times 1} = -2 \pm \dfrac{\sqrt{4+32}}{2} = -2 \pm \dfrac{\sqrt{36}}{2} = -2 \pm \dfrac{6}{2}$

$= \dfrac{-2+6}{2} = \dfrac{4}{2} = 2$, $= \dfrac{-2-6}{2} = \dfrac{-8}{2} = -4$

$x = 2$, $x = -4$

~~we cannot~~ At $x = 2$ denominator vanishes we cannot use the continuity of rational ( ), so factorise

2) $\lim_{\Delta x\to 0} \dfrac{(\Delta x)^2 + 2\Delta x}{(\Delta x)^2 + \Delta x}$.

$= \lim_{\Delta x\to 0} \dfrac{\cancel{\Delta x}(\Delta x + 2)}{\cancel{\Delta x}(\Delta x + 1)}$

$= \lim_{\Delta x \to 0} \frac{\Delta x + 2}{\Delta x + 1}$

$= \frac{0+2}{0+1} = \underline{\underline{2}}$

3) $\lim_{x \to 3} \frac{8x^2}{1+\sqrt{x}}$ (Denominator ≠ 0 so direct give)

$= \frac{8 \times 3^2}{1+\sqrt{3}} = \underline{\underline{\frac{72}{1+\sqrt{3}}}}$

4) Does $\lim_{x \to 0} \left(\frac{|x|}{x}\right)$ exist

$\cancel{x \to 0}$ • $\frac{|x|}{x}$ has value 1 for $x > 0$.

$x = 1 \quad \frac{|x|}{x} = \frac{1}{1} = 1$

for $x < 0$ $\quad \frac{|x|}{x} = -1 \quad x = -1$

$\frac{|x|}{x} = \frac{|-1|}{-1} = \frac{1}{-1} = -1$

when $x = 0$, ( ) not defined.
There is no number 'l' which is approximated by $\frac{|x|}{x}$ as $x \to 0$. Since $\frac{|x|}{x}$ is sometimes 1 & sometimes -1 according sign of x. So we conclude that $\lim_{x \to 0} \frac{|x|}{x}$ does not exist.

5) find

1) $\lim_{x \to \infty} \frac{1}{x} = \underline{\underline{0}}$

2) $\lim_{x \to \infty} \frac{2x+1}{3x+1}$

3) $\lim_{x \to \infty} \frac{5x^2 - 3x + 2}{x^2+1}$

As x get very large $\frac{1}{x}$ gets very small

1.A) $\lim_{x \to \infty} \frac{1}{x} = \underline{\underline{0}}$

2.A) $\lim_{x \to \infty} \frac{2x+1}{3x+1}$

$1 \to x$
$\cancel{x} \times \frac{1}{\cancel{x}} = 1.$
$\therefore 1 = \frac{1}{x}$

$\lim_{x \to \infty} \left(\frac{\cancel{x}(2 + 1/x)}{\cancel{x}(3 + 1/x)}\right)$

$\lim_{x \to \infty} \left(\frac{2 + 1/x}{3 + 1/x}\right)$

$\lim_{x \to \infty} \frac{1}{x} = 0$

$\lim_{x \to \infty} \left(\frac{2+0}{3+0}\right) = \underline{\underline{\frac{2}{3}}}$

3.A) $\lim_{x \to -\infty} \frac{5x^2 - 3x + 2}{x^2+1}$

$\lim_{x \to -\infty} \left(\frac{\cancel{x^2}\left(5 - \frac{3}{x} + \frac{2}{x^2}\right)}{\cancel{x^2}\left(1 + \frac{1}{x^2}\right)}\right)$

$\frac{3}{x} = 3 \cdot \frac{1}{x}$
$= 3 \times 0 = 0$

$\frac{1}{x} = 0$
$\therefore \frac{1}{x^2} = 0$

$\lim_{x \to -\infty} \left(\frac{5 - 0 + 0}{1 + 0}\right) = \underline{\underline{5}}$

4) $\lim_{x \to 2} \frac{-3x}{x^2 - 4x + 4}$

Denomenator = 0. factorise.

$\lim_{x \to 2} \frac{-3x}{(x-2)^2}$

$= -\infty$ //

5) $\lim_{x \to 0} \frac{3x+2}{x}$

$$\frac{3x+2}{x} = \frac{\cancel{x}\left(3+\frac{2}{x}\right)}{\cancel{x}}$$

$$= 3 + \frac{2}{x}$$

when x tends to 0, $\frac{2}{x}$ is either large & +ve. or large & -ve, according to sign of x. Hence $\lim_{x \to 0} \frac{3x+2}{x}$ does not have any value finite / Infinite

→ Formal definition of derivatives :-

let $f(x)$ be a () whose domains contains an open interval about x_0, we say that f is differentiable at x_0 when the following limit exist.

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}.$$

$f'(x_0)$ → Derivative of $f(x)$ at x_0.

Q
1) Suppose that $f(x) = x^2$ then $f'(3) = 6$ by quadratic () rule. with ~~A~~
$a=1$, $b=c=0$ & $x_0 = 3$. justify that ~~F~~ $f'(3) = 6$ directly from the formal definition of derivatives & rules of limit ?

A)
$$\frac{\Delta y}{\Delta x} = \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

$f(x) = x^2$
$x_0 = 3$

$$f(x_0+\Delta x) = f(3+\Delta x)$$

$f(x) = x^2$

$$\therefore = (3+\Delta x)^2 = 9 + 6\Delta x + \Delta x^2$$

$(a+b)^2$

$$f(x_0) = f(3)$$

$f(x) = x^2$ $\therefore f(3)^2 = 9$

$$\frac{\Delta y}{\Delta x} = \frac{\cancel{9} + 6\Delta x + \Delta x^2 - \cancel{9}}{\Delta x}$$

$$= \frac{6\Delta x + \Delta x^2}{\Delta x} = \frac{\cancel{\Delta x}(6+\Delta x)}{\cancel{\Delta x}}$$

$$= 6 + \Delta x$$

$$f'(3) = \lim_{x \to 0}\left(\frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}\right)$$

$$= \lim_{\Delta x \to 0}(6+\Delta x) = 6+0 = 6$$

2) use the formal definition of derivative & rules of limits to differenciate $f(x) = x^3$?

A)
$$f(x) = x^3$$

$$f(x_0+\Delta x) = f(x_0+\Delta x)^3$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(x_0+\Delta x)^3 = x_0^3 + 3x_0^2\Delta x + 3x_0\Delta x^2 + (\Delta x)^3$$

$$\frac{\Delta y}{\Delta x} = \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x} \qquad f(x_0) = x_0^3$$

$$= \frac{\cancel{x_0^3} + 3x_0^2\Delta x + 3x_0\Delta x^2 + \Delta x^3 - \cancel{x_0^3}}{\Delta x}$$

$$= \frac{3x_0^2\Delta x + 3x_0\Delta x^2 + \Delta x^3}{\Delta x}$$

$$= \frac{\cancel{\Delta x}\left(3x_0^2 + 3x_0\Delta x + \Delta x^2\right)}{\cancel{\Delta x}}$$

$$= 3x_0^2 + 3x_0\Delta x + \Delta x^2$$

$$\lim_{\Delta x\to 0}\left(3x_0^2 + 3x_0\Delta x + \Delta x^2\right)$$

$$\lim_{\Delta x\to 0}\left(3x_0^2 + 0 + 0\right) = 3x_0^2$$

3) $f(x) = \frac{1}{x}$ find $f'(x)$, for $x \neq 0$.

$$f'(x) = \lim_{\Delta x\to 0}\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(x+\Delta x) = \frac{1}{x+\Delta x}$$

$$f(x) = \frac{1}{x}$$

$$f'(x) = \lim_{\Delta x\to 0}\frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{\frac{\cancel{x} - (\cancel{x}+\Delta x)}{(x+\Delta x)x}}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\left(\frac{-\cancel{\Delta x}}{x(x+\Delta x)} \times \frac{1}{\cancel{\Delta x}}\right)$$

$$= \lim_{\Delta x\to 0}\left(\frac{-1}{x(x+\Delta x)}\right)$$

$$= \lim_{\Delta x\to 0}\frac{-1}{x(x+0)} = \frac{-1}{x^2}$$

Derivaties
$\frac{1}{x} = \frac{-1}{x^2}$
$\sqrt{x} = \frac{1}{2\sqrt{x}}$
$x^3 = 3x^2$
$x^n = nx^{n-1}$
$x^2 = 2x$
$\frac{1}{x^3} = \frac{-3}{x^4}$

4) Differenciate $\sqrt{x}$, $x > 0$

A) $f(x) = \sqrt{x}$.

$$\frac{\Delta y}{\Delta x} = \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

$$f(x_0+\Delta x) = \sqrt{x_0+\Delta x}$$

$$f(x_0) = \sqrt{x_0}$$

$$\frac{\Delta y}{\Delta x} = \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}.$$

multiply Num & Deno with $\sqrt{x+\Delta x} + \sqrt{x}$.

$$\frac{\Delta y}{\Delta x} = \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \times \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$(a-b)(a+b) = a^2 - b^2$$

$$= \frac{\left(\sqrt{x+\Delta x}\right)^2 - \left(\sqrt{x}\right)^2}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)} = \frac{\cancel{x} + \Delta x - \cancel{x}}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$

$$= \frac{\cancel{\Delta x}}{\cancel{\Delta x}\left(\sqrt{x+\Delta x} + \sqrt{x}\right)} = \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}}.$$

$$f'(x) = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}.$$

$$= \lim_{\Delta x\to 0}\left(\frac{1}{\sqrt{x+\Delta x} + \sqrt{x}}\right)$$

$$= \lim_{\Delta x \to 0} \left(\frac{1}{\sqrt{x+0} + \sqrt{x}} \right)$$

$$= \lim_{\Delta x \to 0} \left(\frac{1}{\sqrt{x} + \sqrt{x}} \right) = \frac{1}{2\sqrt{x}}$$

⇒ Theorem (Differentiability implies continuity) :-

If $f'(x_0)$ exist, then f is continous at x_0.

(i.e) $\lim_{x \to x_0} f(x) = f(x_0)$

proof

Note that

$\lim_{x \to x_0} f(x) = f(x_0) \Rightarrow \lim_{x \to x_0} f(x) - f(x_0) = 0$

$\Delta x = x - x_0$

$\Delta y = f(x_0 + \Delta x) - f(x_0)$

$$\lim_{\Delta x \to 0} \Delta y = f(x_0 + 0) - f(x_0)$$
$$= f(x_0) - f(x_0) = 0$$

$\frac{\Delta y}{\Delta x}$ Multiply N & Deno by Δx,

$$\lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta x} \times \Delta x \right) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \times \lim_{\Delta x \to 0} \Delta x$$

$\frac{\Delta y}{\Delta x} = f'(x)$

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = f'(x)$

$$= f'(x) \times 0 = 0$$

* Remark :-

Converse of the above theorem need not be true.

eg = $f(x) = |x|$ is continuous but it is not differenciable at $x_0 = 0$.

⇒ Leibniz Notation :-

If $y = f(x)$, the derivative $f'(x)$ may be return $\frac{dy}{dx}$, $\frac{df(x)}{dx}$, $\frac{d}{dx}(f(x))$, $\left(\frac{d}{dx}\right)(f(x))$

If we wish to denote the value $f'(x_0)$ of f' at a specified point x_0, we may write $\left.\frac{dy}{dx}\right|_{x_0}$ and $\left.\frac{df(x)}{dx}\right|_{x_0}$

Q

1) find the slope 'm' of the graph $y = \sqrt{x}$ at $x = 4$.

$\boxed{\text{Slope} = \frac{dy}{dx}}$

A) $y = \sqrt{x}$ derivative of $\sqrt{x} = \frac{1}{2\sqrt{x}}$.

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\left.\frac{dy}{dx}\right|_{x=4} = \frac{1}{2\sqrt{4}} = \frac{1}{2 \times 2} = \frac{1}{4}$$

2) find velocity of a bus whose distance (~~formula~~) is t^3.

$\frac{d}{dx} x^n = nx^{n-1}$

A) $f(t) = t^3$

$$\frac{df}{dt} = \frac{d}{dt}(t^3) = 3t^2$$

⇒ Differenciating polynomial :-

* Power rule :-

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$

(i.e) $\frac{d}{dx}(x^n) = n x^{n-1}$ $n = 1, 2, 3 \ldots$

1) Compute derivative

1) $f(x) = x^8$ $\quad = 8\frac{d}{dx}(x^8) = 8x^7$

2) $f(x) = x^{12}$ $\quad = 12x^{11}$

3) $f(x) = x^{99}$ $\quad = 99x^{98}$

⇒ **Constant Multiple Rule :-**

$$(kf)'(x) = kf'(x)$$

or

$$\frac{d}{dx}(kf(x)) = k\frac{d}{dx}(f(x))$$

$h(x) = k f(x)$

$h(x+\Delta x) = k f(x+\Delta x)$

$$h'(x) = \lim_{\Delta x \to 0} \frac{k f(x+\Delta x) - kf(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{k[f(x+\Delta x) - f(x)]}{\Delta x}$$

$$= k\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$h'(x) = k f'(x)$$

Q

1) Differenciate

a) $-3x^7$

b) $5\sqrt{x}$

c) $\frac{8}{x}$

d) $-6ax^2$

A)

$k f(x) = k f'x.$

a) $\frac{d}{dx}(-3x^7) = -3\frac{d}{dx}(x^7)$ (Const × rule)

$= -3 \cdot 7x^6$ (power rule)

$= -21x^6$

b) $\frac{d}{dx}(5\sqrt{x}) = 5\frac{d}{dx}\left(\frac{1}{2\sqrt{x}}\right)$

$= 5 \cdot \frac{1}{2\sqrt{x}}$

~~$= 5 \cdot \frac{1}{2} \cdot \frac{1}{x^2}$~~

~~$= 5 \cdot \frac{1}{2} - \frac{1}{x^2}$~~

~~$= 5 \cdot 2 - \frac{1}{x^2}$~~

c) $\frac{d}{dx}\left(\frac{8}{x}\right) = 8\frac{d}{dx}\left(\frac{1}{x}\right)$

$= 8 - \frac{1}{x^2}$ or $-8/x^2$

d) $\frac{d}{dx}(-6ax^2) = -6a\frac{d}{dx}(x^2)$

$= -6a \cdot 2x$

$= -12ax$

2) Let $f(x) = 3x^2 + 5x + 9$

$g(x) = 2x^2 + 5x$

use quadratic (+) rule to verify

$(f+g)'(x) = f'(x) + g'(x)$ Q (+) rule

A) $f(x) + g(x) = 5x^2 + 10x + 9$ $f(x) = ax^2 + bx + c$

$(f+g)'(x) = 5 \cdot 2x + 10 + 0$ $\therefore f'(x) = 2ax + b$

$= 10x + 10$

$f(x) = 3x^2 + 5x + 9$

$f'(x) = 6x + 5$

$g(x) = 2x^2 + 5x$

$g'(x) = 4x + 5$

$f'(x) + g'(x) = 6x + 5 + 4x + 5$

$= 10x + 10$

$10x + 10 = 10x + 10$

⟹ Sum rule :-

Let $f(x)$

$(f+g)'(x) = f'(x) + g'(x)$ or

$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x))$

Proof

let f & g are differentiable at x_0 then by definition,

$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$

$g'(x_0) = \lim_{\Delta x \to 0} \frac{g(x_0 + \Delta x) - g(x_0)}{\Delta x}$.

$(f+g)'(x_0) = f'(x_0) + g'(x_0)$

$(f+g)'(x_0) = \lim_{\Delta x \to 0} \frac{(f+g)(x_0 + \Delta x) - (f+g)(x_0)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) + g(x_0 + x_0) - [f(x_0) - g(x_0)]}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0) + g(x_0 + \Delta x) - g(x_0)}{\Delta x}$.

given that to all.

$= \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} + \lim_{\Delta x \to 0} \frac{g(x_0 + \Delta x) - g(x_0)}{\Delta x}$.

$(f+g)'(x_0) = f'(x_0) + g'(x_0)$

1) find the formula for the derivatives $8f(x) - 10g(x)$.

1) $\frac{d}{dx}(8f(x)) + \frac{d}{dx}(-10g(x))$. (Sum rule)

$= 8\frac{d}{dx}(f(x)) - 10\frac{d}{dx}(g(x))$

$= 8f'(x) - 10g'(x)$

2) find $\frac{d}{dx}(x^{95} + x^{23} + 2x^2 + 4x + 1)$

$\frac{d}{dx}(x^{95}) + \frac{d}{dx}(x^{23}) + \frac{d}{dx}(2x^2) + \frac{d}{dx}(4x) + \frac{d}{dx}(1)$.

$\Downarrow$ 0

$= 95x^{94} + 23x^{22} + 4x + 4$

3) $\frac{d}{dx}(4x^9 - 6x^5 + 3x)$

$= \frac{d}{dx}(4x^9) + \frac{d}{dx}(-6x^5) + \frac{d}{dx}(3x)$

$(4 \cdot 9x^8)$

$= 36x^8 - 30x^4 + 3$

4) $\frac{d}{dx}(x^3 + 5x^2 - 9x + 2)$

$= \frac{d}{dx}(x^3) + \frac{d}{dx}(5x^2) - \frac{d}{dx} 9x + \frac{d}{dx}(2)$

$= 3x^2 + 10x - 9$

5) $\frac{d}{dx}\left(10x^3 - \frac{8}{x} + 5\sqrt{x}\right)$

$$= \frac{d}{dx}(10x^3) - \frac{d}{dx}\left(\frac{8}{x}\right) + \frac{d}{dx}(5\sqrt{x})$$

$$= 30x^2 + \frac{8}{x^2} + \frac{5}{2\sqrt{x}}$$

$8 \cdot \left(\frac{1}{x}\right) = -\frac{1}{x^2}$

$\frac{d}{dx}(x^3) = 3x^2$

6) find slope of tangent line —

$y = x^4 - 2x^3 + 1$ at $x = 1$

$$\frac{dy}{dx} = \frac{d}{dx}(x^4 - 2x^3 + 1)$$

$$= 4x^3 - 6x^2 \rightarrow \text{slope.}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = 4 \times 1^3 - 6 \times 1^2$$

$$= 4 - 6 = -2$$

7) A train has its its position

$x = 3t^2 + 2 - \sqrt{t}$ at $t = 2$. find velocity?

$\int t^2 = 2t \times 3.$

$$\frac{d}{dt}(3t^2 + 0 - \sqrt{t}) = 6t - \frac{1}{2\sqrt{t}}.$$

$$\left.\frac{dx}{dt}\right|_{t=2} \quad 6 \times 2 - \frac{1}{2\times\sqrt{2}} = 12 - \frac{1}{2\sqrt{2}}$$

$\Rightarrow$ product rule :-

$$\frac{d}{dx}\left[\overset{I}{f(x)} \cdot \overset{II}{g(x)}\right] = f(x) \cdot \underset{(\text{deri})}{g'(x)} + g(x) \cdot \underset{(\text{deri})}{f'(x)}.$$

eg $= (s^3 + 3)(s^3 + 2s + 1)$

$$= \frac{d}{dx}\left[(s^3+3)(s^3+2s+1)\right]$$

$$= (s^3+3)\frac{d}{ds}(s^3+2s+1) + (s^3+2s+1)\frac{d}{ds}(s^3+3)$$

$$= (s^3+3)(3s^2+2) + (s^3+2s+1)(3s^2)$$

$$= (3s^5 + 2s^3 + 9s^2 + 6)(3s^5 + 6s^3 + 3s^2)$$

$$= 6s^5 + 8s^3 + 12s^2 + 6$$

(8) $\frac{d}{dx}\left((s^3+3)(s^2+2s+1)\right).$

$\cancel{\frac{d}{dx}(s^3) + \frac{d}{dx}3}$ $= s^5 + 2s^4 + s^3 + 3s^2 + 6s + 3$

$$= \frac{d}{dx}(s^5) + \frac{d}{dx}(2s^4) + \frac{d}{dx}(s^3) + \frac{d}{dx}(3s^2) + \frac{d}{dx}(6s) + \frac{d}{dx}(3)$$

$2 \cdot 4s^3$

$$= 5s^4 + 8s^3 + 3s^2 + 6s + 6$$

$\cancel{= 5s^4 + 14s + 3s^2 + 6} \Rightarrow \cancel{5s^4 + 3s^2 + 14s + 6}.$

9) $f(x) = x^2$ $\quad g(x) = x^3$.

find $\frac{d}{dx}(\overset{①}{x^2} \cdot \overset{②}{x^3})$ using product rule.

$$\frac{d}{dx}(x^2 \cdot x^3) = x^2 \cdot \frac{d}{dx}(x^3) + x^3 \cdot \frac{d}{dx}(x^2)$$

$$= x^2 \cdot 3x^2 + x^3 \cdot 2x$$

$$= 3x^4 + 2x^4 = 5x^4$$

10) using product rule

$$\frac{d}{dx}(x^m \cdot x^n) = x^m \cdot \frac{d}{dx}(x^n) + x^n \cdot \frac{d}{dx}(x^m)$$

$$= x^m\, n x^{n-1} + x^n \cdot m x^{m-1}$$

11) using product rule differenciate $(x^2+2x-1)\ (x^3-4x^2)$ check yr answer while multiplied out 1st.

A) $\frac{d}{dx}\left((x^2+2x-1)(x^3-4x^2)\right) = (x^2+2x-1)\frac{d}{dx}(x^3-4x^2) + (x^3-4x^2)\frac{d}{dx}(x^2+2x-1)$

(by product (P))

$$= (x^2+2x-1)\cdot(3x^2-8x) + (x^3-4x^2)(2x+2)$$

$$= (3x^4 - 8x^3 + 6x^3 - 16x^2 - 3x^2 + 8x) + (2x^4 + 2x^3 - 8x^3 - 8x^2)$$

$$= (3x^4 - 2x^3 - 19x^2 + 8x) + (2x^4 - 6x^3 - 8x^2)$$

$$= 5x^4 - 8x^3 - 27x^2 + 8x.$$

$$(x^2+2x-1)(x^3-4x^2) = (x^5 - 4x^4 + 2x^4 - 8x^3 - x^3 + 4x^2)$$

$$= (x^5 - 2x^4 - 9x^3 + 4x^2)$$

$$\frac{d}{dx}(x^5 - 2x^4 - 9x^3 - 4x^2)$$

$$= 5x^4 - 8x^3 - 27x^2 + 8x$$

12) using product rule differenciate $(s^3+3)\ (s^2+2s+1)$ check yr answer by multiplied 1st.

A) $\frac{d}{ds}\left[(s^3+3)(s^2+2s+1)\right] = (s^3+3)\cdot\frac{d}{dx}(s^2+2s+1) + (s^2+2s+1)\cdot\frac{d}{dx}(s^3+3)$

$$= (s^3+3)\cdot(2s+2) + (s^2+2s+1)\cdot 3s^2$$

$$= (2s^4 + 2s^3 + 6s + 6) + (3s^4 + 6s^3 + 3s^2)$$

$$\cancel{= 5s^4 + 2s^3 + 8s + 1}$$

$$= 5s^4 + 8s^3 + 3s^2 + 6s + 6$$

by multipliny we get same answer.

13) Differenciate $x^{\frac{3}{2}}$ by writing $x^{\frac{3}{2}} = x \cdot \sqrt{x}$ by product rule

$(\sqrt{x} = x^{1/2})$

A) $\frac{d}{dx}(x^{\frac{3}{2}}) = \frac{d}{dx}(x \cdot \sqrt{x})$

$$= x \cdot \frac{d}{dx}(\sqrt{x}) + \sqrt{x} \cdot \frac{d}{dx}(x).$$

$$= x \cdot \frac{1}{2\sqrt{x}} + \sqrt{x} \cdot 1$$

$x = \sqrt{x} \cdot \sqrt{x}.$

$$= \frac{x}{2\sqrt{x}} + \sqrt{x}.$$

$$= \frac{\sqrt{x}\cdot\cancel{\sqrt{x}}}{2\cancel{\sqrt{x}}} + \sqrt{x}.$$

$$= \frac{\sqrt{x}}{2} + \sqrt{x} = \frac{\sqrt{x} + 2\sqrt{x}}{2} = \frac{3\sqrt{x}}{2}$$

$$\Rightarrow \frac{3}{2}\sqrt{x}$$

$$\frac{d}{dx}(x^{\frac{3}{2}}) = \frac{3}{2} x^{\frac{3}{2}-1}$$

$$= \frac{3}{2} x^{\frac{1}{2}}$$

⇒ Quotient Rule :-

Let $f(x)$ & $g(x)$ are 2 (?)., $g(x) \neq 0$, then,

$$\boxed{\left(\frac{f}{g}\right)'(x) = \frac{g(x) \cdot f'(x) - f(x)\, g'(x)}{(g(x))^2}}$$

Q

1) find $\frac{d}{dx}\left(\frac{x^2}{x^3+5}\right)$. (numerator & Deno - efo x ... use this rule)

$$= \frac{(x^3+5)\frac{d}{dx}(x^2) - \left(x^2 \frac{d}{dx}(x^3+5)\right)}{(x^3+5)^2}$$

$$= \frac{(x^3+5)\,2x - (x^2 \cdot 3x^2)}{(x^3+5)^2}$$

$$= \frac{(2x^4+10x) - (x^2 \cdot 3x^2)}{(x^3+5)^2}$$

$$= \frac{2x^4+10x-3x^4}{(x^3+5)^2} = \frac{-x^4+10x}{(x^3+5)^2}$$

H.W

2) $\frac{d}{dx}\left(\frac{2x+1}{x^2-2}\right)$

3) $\frac{d}{dx}\left(\frac{\sqrt{x}}{1+3x^2}\right)$.

→ proof of Quotient rule :-

$$\left(\frac{f}{g}\right)' x = \frac{g(x) \cdot f'(x) - f(x)\, g'(x)}{(g(x))^2}$$

$\frac{d}{dx} \frac{f(x)}{g(x)}$

$$h'(x_0) = \lim_{\Delta x \to 0} \frac{h(x_0+\Delta x) - h(x_0)}{\Delta x} \qquad h(x) = \frac{f(x)}{g(x)}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{f(x_0+\Delta x)}{g(x_0+\Delta x)} - \frac{f(x_0)}{g(x_0)}}{\Delta x} \qquad h(x_0+\Delta x) = \frac{f(x_0+\Delta x)}{g(x_0+\Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{f(x_0+\Delta x)\, g(x_0) - f(x_0)\, g(x_0+\Delta x)}{g(x_0+\Delta x)\, g(x_0)}}{\Delta x}$$

adding & sub $f(x_0)\, g(x_0)$

$$= \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x)\, g(x_0) - f(x_0)\, g(x_0+\Delta x) + f(x_0)\, g(x_0) - f(x_0)\, g(x_0)}{g(x_0+\Delta x)\, g(x_0) \cdot \Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x)\, g(x_0) - f(x_0)\, g(x_0) - \left[f(x_0)\, g(x_0+\Delta x) - f(x_0)\, g(x_0)\right]}{g(x_0+\Delta x)\, g(x_0)\, \Delta x}$$

Take common $g(x_0)$ & $f(x_0)$

$$= \lim_{\Delta x \to 0} \frac{g(x_0)\left(f(x_0+\Delta x) - f(x_0)\right) - f(x_0)\left(g(x_0+\Delta x) - g(x_0)\right)}{g(x_0+\Delta x)\, g(x_0)\, \Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{g(x_0+\Delta x)\, g(x_0)} \left[\frac{f(x_0+\Delta x) - f(x_0)}{\Delta x} \times g(x_0)\right] - \left[\frac{g(x_0+\Delta x) - g(x_0)}{\Delta x} \times f(x_0)\right]$$

$$\cancel{= \lim_{\Delta x \to 0} \frac{1}{g(x_0+\Delta x)\, g(x_0)} \left[f'(x_0) \cdot g(x_0) - g'(x_0)\, f(x_0)\right]}$$

$$= \lim_{\Delta x \to 0} \frac{1}{g(x_0)\, g(x_0+\Delta x)} \left[\lim_{\Delta x \to 0} \left(\frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}\right) g(x_0) - \lim_{\Delta x \to 0} \left(\frac{g(x_0+\Delta x) - g(x_0)}{\Delta x}\right) \times f(x_0)\right]$$

$g(x_0)\, g(x_0)$

$$= \frac{1}{(g(x_0))^2} \left[f'(x_0) \cdot g(x_0) - g'(x_0)\, f(x_0)\right].$$

$$= \frac{f'(x_0) \cdot g(x_0) - g'(x_0)\, f(x_0)}{(g(x_0))^2}$$

2) $\frac{d}{dx}\left(\frac{2x+1}{x^2-2}\right) = \left[\frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{(g(x))^2}\right]$

$= \cancel{(x^2-2)\frac{d}{dx}(2x+1) - (2x+1)\frac{d}{dx}(x^2-2)}$

$\cancel{(x^2-2)\,2 - x^2}$

$= \frac{(x^2-2)\frac{d}{dx}(2x+1) - (2x+1)\frac{d}{dx}(x^2-2)}{(x^2-2)^2}$

$= \frac{(x^2-2)\,2 - (2x+1)\,2x}{(x^2-2)^2}$

$= 2x^2 - 4 - 4x^2 + 2x$

$= \frac{-2x^2+2x-4}{(x^2-2)^2} = \frac{-2(x^2+x-2)}{(x^2-2)^2}$

3) $\frac{d}{dx}\left(\frac{\sqrt{x}}{1+3x^2}\right)$

$= \frac{(1+3x^2)\frac{d}{dx}(\sqrt{x}) - (\sqrt{x})\frac{d}{dx}(1+3x^2)}{(1+3x^2)^2}$

$= \frac{(1+3x^2)\cdot\frac{1}{2\sqrt{x}} - \sqrt{x}\;6x}{(1+3x^2)^2}$

$= \cancel{\frac{\frac{1}{2\sqrt{x}} + 6x\sqrt{x}}{(1+3x^2)}}$ $= \frac{\frac{1+3x^2}{2\sqrt{x}} - \frac{6x\sqrt{x}\times 2\sqrt{x}}{1\times 2\sqrt{x}}}{(1+3x^2)^2}$

$= \frac{\frac{1+3x^2-6x\sqrt{x}}{2\sqrt{x}}}{(1+3x^2)^2}$

$\frac{1+3x^2-12x^2}{2\sqrt{x}\,(1+3x^2)^2} = \frac{9x^2-1}{2\sqrt{x}\,(1+3x^2)^2}$

⇒ Reciprocal Rule :-

let $g(x)$ be a (), $g(x) \neq 0$

then,

$$\boxed{\left(\frac{1}{g}\right)'(x) = \frac{-g'(x)}{(g(x))^2}}$$ or

$$\frac{d}{dx}\left(\frac{1}{u}\right) = -\frac{1}{u^2}\cdot\frac{du}{dx}.$$

1) Differenciate $\frac{1}{(x^3+3x^2)}$

$g(x) = x^3+3x^2$

$g'(x) = 3x^2+6x$

$\left(\frac{1}{g}\right)'(x) = \frac{-g'(x)}{(g(x))^2}$

$= \frac{-3x^2+6x}{(x^3+3x^2)^2}$

2) $\frac{1}{\sqrt{x}+2}$

$g(x) = \sqrt{x}+2$

$g'(x) = \frac{1}{2\sqrt{x}}$

$\left(\frac{1}{g}\right)'x = \frac{-g'(x)}{(g(x))^2}$

$= \frac{-\frac{1}{2\sqrt{x}}}{(\sqrt{x}+2)^2} = \frac{-1}{2\sqrt{x}\,(\sqrt{x}+1)^2}$

2) :

Remark :-
$$\boxed{x^{-k} = \frac{1}{x^k}}$$

=> **Integer power rule :-**

If n is any (+ve, -ve or 0) integer

$$\boxed{\frac{d}{dx}(x^n) = nx^{n-1}.}$$ (when $n<0$ must be $\neq 0$)

i) $\frac{d}{dx}\left(\frac{1}{x^6}\right) = \frac{d}{dx}(x^{-6}) = -6x^{-7}$

=> **Quotient rule from product rule & Reciprocal rule :-**

3) $\left(\frac{f}{g}\right)'(x) = \left(f \cdot \frac{1}{g}\right)'(x)$.

by p. Rule

$= f(x) \cdot \left(\frac{1}{g}\right)'(x) + \frac{1}{g(x)} f'(x)$

R. rule

$= f(x) \frac{-g'(x)}{g(x)^2} + \frac{f'(x)}{g(x)}$.

$= \frac{f'(x)}{g(x)} + \frac{f(x) \cdot (-g'(x))}{g(x)^2}$

$= \frac{f'(x) \cdot g(x)^2 + f(x)(-g'(x)) \cdot g(x)}{(g(x))^3}$ rule

Common

$= \frac{g(x)\left[f'(x) g(x) - f(x) g'(x)\right]}{(g(x))^{3}}$

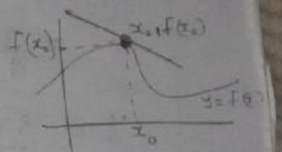
$= \frac{f'(x) \cdot g(x) - f(x) g'(x)}{(g(x))^2}$

> **Linear Approximation & Tangent lines :-**

eq of tangent line,

$y = f(x)$ at $(x_0, f(x_0))$ is

$$\boxed{y = f(x_0) + f'(x_0)(x - x_0)}$$



1) Find the eq of line tangent to the graph $y = \sqrt{x} + \frac{1}{2(x+1)}$ at $x = 1$

$x_0 = 1$

$f(x) = \sqrt{x} + \frac{1}{2(x+1)}$

$f(1) = \sqrt{1} + \frac{1}{2(1+1)} = \frac{1}{1} + \frac{1}{4} = \frac{5}{4}$

$f'(x_0) = ?$

$f(x) = \sqrt{x} + \frac{1}{2(x+1)}$

$f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{2(x+1)^2}$

$f(1) = \frac{1}{2\sqrt{1}} - \frac{1}{2(1+1)^2}$

$= \frac{1}{2} - \frac{1}{8} = \frac{8-2}{16} = \frac{6}{16} = \frac{3}{8}$

R. Rule

$\frac{1}{(x+1)} = \frac{1}{g(x)} = \frac{-g'(x)}{g(x)^2}$

$= \frac{-1}{(x+1)^2}$

$y = f(x_0) + f'(x_0)(x - x_0)$

$= \frac{5}{4} + \frac{3}{8}(x-1)$

$$= \frac{5}{4} + \frac{3}{8}x - \frac{3}{8}$$

$$\frac{5}{4} - \frac{3}{8} = \frac{40-12}{32}$$

$$= \frac{3}{8}x + \left(\frac{5}{4} - \frac{3}{8}\right)$$

$$= \frac{3}{8}x + \frac{7}{8}$$

$$y = \frac{3x+7}{8}$$

$$8y = 3x+7$$

2) $f(x) = \frac{(2x+1)}{3x+1}$ at $x = 1$

A) $f(x) = \frac{2x+1}{3x+1}$; $f(1) = \frac{2+1}{3+1} = \frac{3}{4}$

3) $f'(x) = \frac{2x+1}{3x+1}$ (by Q-Rule)

$$= \frac{(3x+1)\frac{d}{dx}(2x+1) - (2x+1)\frac{d}{dx}(3x+1)}{(3x+1)^2}$$

$$= \frac{(3x+1)\,2 - (2x+1)\,3}{(3x+1)^2}$$

$$f'(1) = \frac{(3+1)\,2 - (2+1)\,3}{(3+1)^2}$$

$$= \frac{8-9}{16} = -\frac{1}{16}$$

Rule $y = f(x_0) + f'(x_0)(x - x_0)$

$$= \frac{3}{4} - \frac{1}{16}(x-1)$$

$$= \frac{3}{4} - \frac{1}{16}x + \frac{1}{16}$$

$$= -\frac{1}{16}x + \left(\frac{3}{4} + \frac{1}{16}\right)$$

$$= -\frac{1}{16}x + \frac{13}{16}$$

$$y = \frac{-1x + 13}{16}$$

$$16y = 13 - x$$

$\frac{1}{x^3} = x^{-3} \to -3x^{-2}$; $-\frac{3}{x^4} \times 5 = \frac{-15}{x^4}$

$$\frac{d}{dx}\left(3x^4 + \frac{2}{x} - \frac{5}{x^3}\right)$$

$$= \frac{d}{dx}(3x^4) + \frac{d}{dx}\left(\frac{2}{x}\right) - \frac{d}{dx}\left(\frac{5}{x^3}\right)$$

$\frac{1}{x^3} = \frac{3}{x^4}$

$$= 12x^3 - \frac{2}{x^2} + \frac{15}{x^4}$$

$$\frac{d}{dx}\left(\frac{1}{(x^2+3)(x^2+4)}\right)$$

to the form $\Rightarrow \left(\frac{1}{g}\right)'x = \frac{-g'(x)}{(g(x))^2}$

$$= \frac{-(x^2+3)(x^2+4)}{((x^2+3)(x^2+4))^2}$$

$$= -\frac{(x^2+3)\frac{d}{dx}(x^2+4) + (x^2+4)\frac{d}{dx}(x^2+3)}{(x^2+3)^2(x^2+4)^2}$$

$$= \frac{-(x^2+3)\,2x + (x^2+4)\,2x}{(x^2+3)^2\,(x^2+4)^2}$$

2)

$$= \frac{-(2x^3+6x) + (2x^3+8x)}{(x^2+3)^2\,(x^2+4)^2}$$

$$= -\frac{4x^3+14x}{(x^2+3)^2(x^2+4)^2} = \frac{-2x(2x^2+7)}{(x^2+3)^2(x^2+4)^2}$$

5) where does the line tangent to $y=\sqrt{x}$ at $x=2$ cross the x-axis.

A) $f(x) = \sqrt{x}$ $\quad x_0 = 2$

$f(x_0) = f(2) = \sqrt{2}$

3)

$f'(x) = \frac{1}{2\sqrt{x}}$ $\quad f'(2) = \frac{1}{2\sqrt{2}}$

$y = f(x_0) + f'(x_0)(x-x_0)$

$= \sqrt{2} + \frac{1}{2\sqrt{2}}(x-2)$

$= \frac{2\sqrt{2}\sqrt{2} + 1(x-2)}{2\sqrt{2}} = \frac{4 + (x-2)}{2\sqrt{2}}$

$$y = \frac{2+x}{2\sqrt{2}}$$

x across means $y=0$.

$0 = \frac{2+x}{2\sqrt{2}}$

$0 \times 2\sqrt{2} = 2+x$

$0 = 2+x$

$x = -2$

Linear approximation :-

for x near x_0, $f(x_0) + f'(x_0)(x-x_0)$ is a gd approximation for $f(x)$.

$$\boxed{f(x_0+\Delta x) \approx f(x_0) + f'(x_0)\Delta x}$$

(or)

$$\Delta y \approx f'(x_0)\,\Delta x.$$

$\sqrt{x} = \frac{1}{2\sqrt{x}}$

The error becomes arbit~~rarely~~ rarily small compared with Δx as $\Delta x \to 0$.

Cal. the approximate value for following quantities using linear approximation ~~aton~~ around $x_0 = 9$ compare with the values on your calculator.

$\sqrt{9.02}$ $\qquad$ given $x_0 = 9$

$\sqrt{2}.\;\sqrt{10}$ $\qquad 10-9 = 1 \to \Delta x$.

$= \sqrt{8.82}$ $\qquad 8-9 = -1\ \Delta x$.

$\sqrt{8}$.

a) $\sqrt{9.02}$ $\sqrt{9+\Delta x}$ $9+0.02 = 9.02$

$f(x_0+\Delta x) \approx f(x_0) + f'(x_0)\Delta x$

$x_0 = 9$ $\Delta x = 0.02$

$f(x) = \sqrt{x}$

$f(x_0+\Delta x) = \sqrt{x_0+\Delta x}$

$= \sqrt{9+\Delta x}$

$= \sqrt{9+0.02}$

$= \sqrt{9.02}$

$x_0+\Delta x = 9.02$

$9+\Delta x = 9.02$

$\Delta x = 9 - 9.02 = -0.02$

$-0.02+9 \neq 9.02$

$\therefore 0.02+9 = 9.02$

$f(x_0) = \sqrt{9} = 3$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(x_0) = f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{2\cdot 3} = \frac{1}{6}$

$f(x_0+\Delta x) \approx f(x_0) + f'(x_0)\Delta x$

$\sqrt{9.02} \approx 3 + \frac{1}{6}\cdot 0.02$

$\approx \cancel{\frac{18+1}{6} \times 0.02}$

$\approx \cancel{\frac{19}{6} \times 0.02}$

~~3.0033~~

$\approx 3 + 3.333 = 3.003$

Cal. va

$\sqrt{9}$

$= 3.$

$\sqrt{10}$ $\Rightarrow x_0 = 9$ $\Delta x = 1$

$f(x) = \sqrt{x}$

$f(x_0+\Delta x) = \cancel{\sqrt{x_0+\Delta x}} = \sqrt{x_0+\Delta x}$

$\cancel{\sqrt{10+1}}$ $= \sqrt{9+1}$

$\cancel{\sqrt{11}}$ $= \sqrt{10}$

$f(x_0) = \sqrt{9} = 3$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(x_0) = f'(9)$

$= \frac{1}{2\sqrt{9}} = \frac{1}{2\times 3} = \frac{1}{6}$

$f(x_0+\Delta x) \approx f(x_0) + f'(x_0)\Delta x$

$\sqrt{10} \approx 3 + \frac{1}{6}\cdot 1$

3.1622 $\approx \frac{3}{1} + \frac{1}{6} = \frac{18+1}{6} = \frac{19}{6} = 3.16$

$\sqrt{8.82}$

$x_0 = 9$ ~~$\Delta x = 0.82$~~

$(x_0+\Delta x) = 8.82$

$9+\Delta x = 8.82$

$\Delta x = 8.82 - 9$

$\Delta x = -0.18$

$f(x) = \sqrt{x}$

$\cancel{f(x_0+\Delta x) = \sqrt{x_0+\Delta x}}$

$\cancel{= \sqrt{9+}}$

$f(x_0+\Delta x) = 9+(-0.18) = \sqrt{8.82}$

$-0.18+9$

$f(x_0) = \sqrt{9} = 3$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(x_0) = \frac{1}{2\sqrt{9}} = \frac{1}{6}$

$$f(x_0+\Delta x) \approx f(x_0) + f'(x_0)\,\Delta x.$$

$$\sqrt{8.82} \approx 3 + \frac{1}{6} \times -0.18$$

$$\approx 2.97$$

d) $\sqrt{8}$.

$x_0 = 9$ $\Delta x = -1$

$x_0 + \Delta x = 8$
$9 + \Delta x = 8$
$\Delta x = 8 - 9$

$f(x_0+\Delta x) = \sqrt{x_0+\Delta x}$ $\sqrt{9-1} = 8$

$f(x_0) = \sqrt{9} = 3$

$f'(x) = \frac{1}{2\sqrt{x}}$

Sub

$\sqrt{8} \approx 3 + \frac{1}{6} \times -1 = 3 - 0.166$

$= 2.833$

2) cal. an $\approx$ value for $\frac{2}{\sqrt{0.99} + (0.99)^2}$.
& compare with numerical value on yr calculator.

3) A) $\frac{2}{\sqrt{0.99} + (0.99)^2}$

$f(x) = \frac{2}{\sqrt{x} + x^2}$ $f(x_0) = \frac{2}{\sqrt{x_0} + x_0^2}$

$x_0 + \Delta x = 0.99$

$f(0.99) = \frac{2}{\sqrt{0.99} + (0.99)^2}$ $0.99 \to 1$

interval of $x_0 + \Delta x$

$f(1 - 0.01) = f(1 + (-0.01))$ $x_0 = 1$, $\Delta x = -0.01$

$(x_0 + \Delta x)$

$f(x_0) = f(1)$

$= \frac{2}{\sqrt{1} + 1^2} = \frac{2}{2}$

$f(x) = \frac{2}{\sqrt{x} + x^2}$ (Q. Rule)

$$f'(x) = \frac{(\sqrt{x}+x^2)\frac{d}{dx}(2) - \left[2 \cdot \frac{d}{dx}(\sqrt{x}+x^2)\right]}{(\sqrt{x}+x^2)^2}$$

$\frac{2}{2\sqrt{x}} = \frac{1}{\sqrt{x}}$

$$= \frac{(\sqrt{x}+x^2) \times 0 - \left[2 \times \left(\frac{1}{2\sqrt{x}} + 2x\right)\right]}{(\sqrt{x}+x^2)^2}$$

$$f'(x) = -\frac{\left[\frac{1}{\sqrt{x}} + 4x\right]}{(\sqrt{x}+x^2)^2}$$

$$= -\frac{(1+4x)}{(\sqrt{x}+x^2)^2\sqrt{x}}$$

$$f'(x_0) = f'(1) = \frac{-(1+4\times 1)}{(\sqrt{1}+1^2)^2 \times \sqrt{1}} = -\frac{5}{4}$$

Sub.

$$\frac{2}{\sqrt{0.99} + (0.99)^2} \approx 1 + \left(-\frac{5}{4}\right) \times (-0.01).$$

$$\approx 1 + \frac{5 \times 0.01}{4}$$

$$\approx 1.0125$$