Linear Algebra (5th edition)

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Chapter 03: Elementary matrix operations and systems of linear equations

Assignments

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3.1 Elementary matrix operations and elementary matrices

- **Problems**

1. Label the following statements as true or false.

(a) An elementary matrix is always square.
(b) The only entries of an elementary matrix are zeros and ones.
(c) The $n \times n$ identity matrix is an elementary matrix.
(d) The product of two $n \times n$ elementary matrices is an elementary matrix.
(e) The inverse of an elementary matrix is an elementary matrix.
(f) The sum of two $n \times n$ elementary matrices is an elementary matrix.
(g) The transpose of an elementary matrix is an elementary matrix.
(h) If B is a matrix that can be obtained by performing an elementary row operation on a matrix A, then B can also be obtained by performing an elementary column operation on A.
(i) If B is a matrix that can be obtained by performing an elementary row operation on a matrix A, then A can be obtained by performing an elementary row operation on B.

3.1 Elementary matrix operations and elementary matrices

- **Problems**

2. Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 1 & -1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 3 \\ 1 & -2 & 1 \\ 1 & -3 & 1 \end{pmatrix}, \quad \text{and } C = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & -2 \\ 1 & -3 & 1 \end{pmatrix}.$$

Find an elementary operation that transforms A into B and an elementary operation that transforms B into C. By means of several additional operations, transform C into I_3.

3. Use the proof of Theorem 3.2 to obtain the inverse of each of the following elementary matrices.

(a) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ **(b)** $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ **(c)** $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$