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1  #Forward Algorithm
2  alpha[[2.16428571e-01 1.36400000e-02 1.42400000e-02]
3      [1.95769992e-02 4.62921929e-02 0.00000000e+00]
4      [4.24880513e-03 0.00000000e+00 7.85375567e-03]
5      [9.76003523e-04 0.00000000e+00 3.12326315e-03]
6      [3.29921234e-04 6.47449665e-04 0.00000000e+00]
7      [3.20008815e-04 1.60363730e-05 5.13796939e-06]
8      [1.37170014e-04 2.66283431e-06 3.30297118e-06]
9      [1.16064059e-05 2.54283690e-05 0.00000000e+00]
10     [2.40343339e-06 0.00000000e+00 4.51873505e-06]
11     [2.79097917e-06 4.61284874e-08 7.44634171e-08]]
12
13  #Backward Algorithm
14  beta[[1.15016146e-05 1.96386385e-05 1.08442431e-05]
15      [7.79366660e-05 2.99360418e-05 8.44869264e-05]
16      [2.28668211e-04 9.05609248e-05 2.47016139e-04]
17      [7.50866995e-04 1.54509881e-03 6.97578825e-04]
18      [3.65329877e-03 2.63537125e-03 3.59649833e-03]
19      [8.63440960e-03 6.54014081e-03 8.48657256e-03]
20      [2.00995806e-02 3.45288760e-02 1.89425333e-02]
21      [1.35055084e-01 5.28569859e-02 1.46092673e-01]
22      [4.25017143e-01 3.10388571e-01 4.18274286e-01]
23      [1.00000000e+00 1.00000000e+00 1.00000000e+00]]
24
25  #Viterbi Algorithm
26  viterbi ([0, 0, 2, 2, 1, 0, 0, 0, 2, 0], 1.5974935002286267e-07)
27
28  #Baum Welch
29
30  baum_welch(n_iteration=1)
31  {'A_hat': array([[0.59672972, 0.19376389, 0.20950638],
32      [0.61225714, 0.07742379, 0.31031907],
33      [0.53655268, 0.2211738 , 0.24227352]]),
34  'B_hat': array([[0.60591698, 0.24115553, 0.15292749],
35      [0.10326011, 0.89673989, 0.          ],
36      [0.05281582, 0.          , 0.94718418]]),
37  'Pi_hat': array([0.85496042, 0.09200223, 0.05303735])}
38
39  baum_welch(n_iteration=2)
40  {'A_hat': array([[0.87748492, 0.01295527, 0.1095598 ],
41      [0.91165126, 0.05360651, 0.03474223],
42      [0.88492554, 0.01770503, 0.09736943]]),
43  'B_hat': array([[0.41014493, 0.34136941, 0.24848565],
44      [0.78979552, 0.21020448, 0.          ],
45      [0.10654666, 0.          , 0.89345334]]),
46  'Pi_hat': array([0.53475317, 0.39416723, 0.0710796 ])}
47
48  baum_welch(n_iteration=5)
49  {'A_hat': array([[8.21783182e-01, 1.45954055e-02, 1.63621412e-01],
50      [9.98560495e-01, 1.43171283e-03, 7.79208236e-06],
51      [9.15074687e-01, 1.47527742e-02, 7.01725392e-02]]),
52  'B_hat': array([[0.35848839, 0.3826689 , 0.2588427 ],
53      [0.99881539, 0.00118461, 0.          ],
54      [0.29252822, 0.          , 0.70747178]]),
55  'Pi_hat': array([0.18747245, 0.66804596, 0.14448159])}
56
57  baum_welch(n_iteration=10)
58  {'A_hat': array([[9.99999987e-001, 6.71847722e-014, 1.31617802e-008],

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59     [1.00000000e+00, 2.19211172e-115, 7.69527435e-118],
60     [1.00000000e+00, 9.01932542e-074, 7.47681502e-070]]),
61     'B_hat': array([[3.42803960e-001, 3.28598020e-001, 3.28598020e-001],
62     [1.00000000e+00, 9.72262592e-116, 0.00000000e+00],
63     [1.00000000e+00, 0.00000000e+00, 9.81666974e-064]]),
64     'Pi_hat': array([0.12969602, 0.42578717, 0.44451681]))
65
66     baum_welch(n_iteration=50)
67     {'A_hat': array([[9.99999906e-01, 1.90801523e-13, 9.40088605e-08],
68     [1.00000000e+00, 0.00000000e+00, 0.00000000e+00],
69     [1.00000000e+00, 0.00000000e+00, 0.00000000e+00]]),
70     'B_hat': array([[0.34280391, 0.32859804, 0.32859804],
71     [1.          , 0.          , 0.          ],
72     [1.          , 0.          , 0.          ]]),
73     'Pi_hat': array([0.129696 , 0.42578718, 0.44451682]))
74
75     baum_welch(n_iteration=100)
76     {'A_hat': array([[9.99973281e-01, 3.41971526e-10, 2.67188056e-05],
77     [1.00000000e+00, 0.00000000e+00, 0.00000000e+00],
78     [1.00000000e+00, 0.00000000e+00, 0.00000000e+00]]),
79     'B_hat': array([[0.34278777, 0.32860611, 0.32860611],
80     [1.          , 0.          , 0.          ],
81     [1.          , 0.          , 0.          ]]),
82     'Pi_hat': array([0.12968827, 0.42579096, 0.44452077]))
83
84     baum_welch(n_iteration=500)
85     {'A_hat': array([[9.13783677e-01, 1.74376327e-06, 8.62145789e-02],
86     [1.00000000e+00, 0.00000000e+00, 0.00000000e+00],
87     [1.00000000e+00, 0.00000000e+00, 0.00000000e+00]]),
88     'B_hat': array([[0.28912734, 0.35543633, 0.35543633],
89     [1.          , 0.          , 0.          ],
90     [1.          , 0.          , 0.          ]]),
91     'Pi_hat': array([0.10302077, 0.43883776, 0.45814147]))
92
93     baum_welch(n_iteration=1000)
94     {'A_hat': array([[9.13783677e-01, 2.74981838e-06, 8.62135728e-02],
95     [1.00000000e+00, 0.00000000e+00, 0.00000000e+00],
96     [1.00000000e+00, 0.00000000e+00, 0.00000000e+00]]),
97     'B_hat': array([[0.28912734, 0.35543633, 0.35543633],
98     [1.          , 0.          , 0.          ],
99     [1.          , 0.          , 0.          ]]),
100     'Pi_hat': array([0.10302077, 0.43883776, 0.45814147]))
101
102     baum_welch(n_iteration=1500)
103     {'A_hat': array([[9.13783677e-01, 4.32349122e-06, 8.62119992e-02],
104     [1.00000000e+00, 0.00000000e+00, 0.00000000e+00],
105     [1.00000000e+00, 0.00000000e+00, 0.00000000e+00]]),
106     'B_hat': array([[0.28912734, 0.35543633, 0.35543633],
107     [1.          , 0.          , 0.          ],
108     [1.          , 0.          , 0.          ]]),
109     'Pi_hat': array([0.10302077, 0.43883776, 0.45814147]))
110
111     baum_welch(n_iteration=2000)
112     {'A_hat': array([[9.13783677e-01, 6.76965149e-06, 8.62095530e-02],
113     [1.00000000e+00, 0.00000000e+00, 0.00000000e+00],
114     [1.00000000e+00, 0.00000000e+00, 0.00000000e+00]]),
115     'B_hat': array([[0.28912734, 0.35543633, 0.35543633],
116     [1.          , 0.          , 0.          ],

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117         [1.          , 0.          , 0.          ])),
118     'Pi_hat': array([0.10302077, 0.43883776, 0.45814147]))
119
120     baum_welch(n_iteration=2500)
121     {'A_hat': array([[9.13783677e-01, 1.05390393e-05, 8.62057836e-02],
122                    [1.00000000e+00, 0.00000000e+00, 0.00000000e+00],
123                    [1.00000000e+00, 0.00000000e+00, 0.00000000e+00]]),
124     'B_hat': array([[0.28912734, 0.35543633, 0.35543633],
125                    [1.          , 0.          , 0.          ],
126                    [1.          , 0.          , 0.          ])),
127     'Pi_hat': array([0.10302077, 0.43883776, 0.45814147]))
128
129     baum_welch(n_iteration=5000)
130     {'A_hat': array([[9.13783677e-01, 3.74788927e-05, 8.61788438e-02],
131                    [1.00000000e+00, 0.00000000e+00, 0.00000000e+00],
132                    [1.00000000e+00, 0.00000000e+00, 0.00000000e+00]]),
133     'B_hat': array([[0.28912734, 0.35543633, 0.35543633],
134                    [1.          , 0.          , 0.          ],
135                    [1.          , 0.          , 0.          ])),
136     'Pi_hat': array([0.10302077, 0.43883776, 0.45814147]))
137
138
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