Les calculs pour t=1

$$\delta_1(\omega_1) = \Pi_{\omega_1}\, b_{\omega_1}(Y_1) = 0.2 \times 0.1 = 0{,}02; \quad \text{donc } \psi_1(\omega_1) = 0$$
$$\delta_1(\omega_2) = \Pi_{\omega_2}\, b_{\omega_2}(Y_1) = 0.2 \times 0.1 = 0{,}02; \quad \text{donc } \psi_1(\omega_2) = 0$$
$$\delta_1(\omega_3) = \Pi_{\omega_3}\, b_{\omega_3}(Y_1) = 0.2 \times 0.3 = 0{,}02; \quad \text{donc } \psi_1(\omega_3) = 0$$
$$\delta_1(\omega_4) = \Pi_{\omega_4}\, b_{\omega_4}(Y_1) = 0.4 \times 0.1 = 0{,}28; \quad \text{donc } \psi_1(\omega_4) = 0$$

pour t=2

$$\delta_2(j) = \max_{1 \le i \le 4}[a_{ij}\, \delta_1(i)]b_{\omega_j}(Y_2)$$

$$\delta_2(\omega_1) = \max \begin{bmatrix} [a_{11}\, \delta_1(\omega_1)] \\ [a_{21}\, \delta_1(\omega_2)] \\ [a_{31}\, \delta_1(\omega_3)] \\ [a_{41}\, \delta_1(\omega_4)] \end{bmatrix} b_{\omega_1}(Y_1 = v_4) = \max \begin{bmatrix} [0.3 \times 0.02] \\ [0.2 \times 0.02] \\ [0.2 \times 0.02] \\ [\mathbf{0.2 \times 0.28}] \end{bmatrix} \times 0.1 = 0{,}0056\,; \text{ donc } \psi_2(\omega_1) = \omega_4$$

$$\delta_2(\omega_2) = \max \begin{bmatrix} [a_{12}\, \delta_1(\omega_1)] \\ [a_{22}\, \delta_1(\omega_2)] \\ [a_{32}\, \delta_1(\omega_3)] \\ [a_{42}\, \delta_1(\omega_4)] \end{bmatrix} b_{\omega_2}(v_4) = \max \begin{bmatrix} [0.2 \times 0.02] \\ [0.3 \times 0.02] \\ [0.2 \times 0.02] \\ [\mathbf{0.2 \times 0.28}] \end{bmatrix} \times 0.1 = 0{,}0056\,; \text{ donc } \psi_2(\omega_2) = \omega_4$$

$$\delta_2(\omega_3) = \max \begin{bmatrix} [a_{13}\, \delta_1(\omega_1)] \\ [a_{23}\, \delta_1(\omega_2)] \\ [a_{33}\, \delta_1(\omega_3)] \\ [a_{43}\, \delta_1(\omega_4)] \end{bmatrix} b_{\omega_3}(v_4) = \max \begin{bmatrix} [0.2 \times 0.02] \\ [0.2 \times 0.02] \\ [0.3 \times 0.02] \\ [\mathbf{0.2 \times 0.28}] \end{bmatrix} \times 0.1 = 0{,}0056\,; \text{ donc } \psi_2(\omega_3) = \omega_4$$

$$\delta_2(\omega_4) = \max \begin{bmatrix} [a_{14}\, \delta_1(\omega_1)] \\ [a_{24}\, \delta_1(\omega_2)] \\ [a_{34}\, \delta_1(\omega_3)] \\ [a_{44}\, \delta_1(\omega_4)] \end{bmatrix} b_{\omega_4}(v_4) = \max \begin{bmatrix} [0.3 \times 0.02] \\ [0.3 \times 0.02] \\ [0.3 \times 0.02] \\ [\mathbf{0.4 \times 0.28}] \end{bmatrix} \times 0.7 = 0{,}0784; \text{ donc } \psi_2(\omega_4) = \omega_4$$

$$\delta_{10}(\omega_1) = \max \begin{bmatrix} [a_{11}\, \delta_9(\omega_1)] \\ [a_{21}\, \delta_9(\omega_2)] \\ [a_{31}\, \delta_9(\omega_3)] \\ [a_{41}\, \delta_9(\omega_4)] \end{bmatrix} b_{\omega_1}(Y_{10}) = \max \begin{bmatrix} [0.3 \times 8{,}34E-10] \\ [0.2 \times 8{,}34E-10] \\ [0.2 \times 8{,}34E-10] \\ [0.2 \times 5{,}84E-09] \end{bmatrix} \times 0.3 = 3{,}50E-10\,; \ \psi_{10}(\omega_1) = \omega_4$$

$$\delta_{10}(\omega_2) = \max \begin{bmatrix} [a_{12}\, \delta_9(\omega_1)] \\ [a_{22}\, \delta_9(\omega_2)] \\ [a_{32}\, \delta_9(\omega_3)] \\ [a_{42}\, \delta_9(\omega_4)] \end{bmatrix} b_{\omega_2}(Y_{10}) = \max \begin{bmatrix} [0.2 \times 8{,}34E-10] \\ [0.3 \times 8{,}34E-10] \\ [0.2 \times 8{,}34E-10] \\ [0.2 \times 5{,}84E-09] \end{bmatrix} \times 0.3 = 3{,}50E-10\,; \ \psi_{10}(\omega_2) = \omega_4$$

$$\delta_{10}(\omega_3) = \max \begin{bmatrix} [a_{13}\, \delta_9(\omega_1)] \\ [a_{23}\, \delta_9(\omega_2)] \\ [a_{33}\, \delta_9(\omega_3)] \\ [a_{43}\, \delta_9(\omega_4)] \end{bmatrix} b_{\omega_3}(Y_{10}) = \max \begin{bmatrix} [0.2 \times 8{,}34E-10] \\ [0.2 \times 8{,}34E-10] \\ [0.3 \times 8{,}34E-10] \\ [0.2 \times 5{,}84E-09] \end{bmatrix} \times 0.3 = 3{,}50E-10\,; \ \psi_{10}(\omega_3) = \omega_4$$

$$\delta_{10}(\omega_4) = \max \begin{bmatrix} [a_{14}\, \delta_9(\omega_1)] \\ [a_{24}\, \delta_9(\omega_2)] \\ [a_{34}\, \delta_9(\omega_3)] \\ [a_{44}\, \delta_9(\omega_4)] \end{bmatrix} b_{\omega_4}(Y_{10}) = \max \begin{bmatrix} [0.3 \times 8{,}34E-10] \\ [0.3 \times 8{,}34E-10] \\ [0.3 \times 8{,}34E-10] \\ [0.4 \times 5{,}84E-09] \end{bmatrix} \times 0.1 = 2{,}034E-10\,; \ \psi_{10}(\omega_4) = \omega_4$$

Backtracking des chemins :

$$\mathbf{P}^* = \max_{1 \le i \le 4} \delta_T(i) \text{ et } X_T^* = \operatorname{argmax}_{1 \le i \le 4} \delta_T(i)$$

Donc $X_{10}^* = \max \begin{bmatrix} \delta_{10}(\omega_1) \\ \delta_{10}(\omega_2) \\ \delta_{10}(\omega_3) \\ \delta_{10}(\omega_4) \end{bmatrix} = \omega_4$; On applique $X_t^* = \psi_{t+1}(X_{t+1}^*)$, pour $t = T-1, T-2, \ldots, 1$

$X_9^* = \psi_{10}(X_{10}^*) = \psi_{10}(\omega_4) = \omega_4$

Les états les plus probables sont :

$\boldsymbol{\omega_4}$, $\boldsymbol{\omega_4}$, $\boldsymbol{\omega_1}$, $\boldsymbol{\omega_1}$, $\boldsymbol{\omega_1}$, $\boldsymbol{\omega_1}$, $\boldsymbol{\omega_1}$, $\boldsymbol{\omega_1}$, $\boldsymbol{\omega_4}$, $\boldsymbol{\omega_4}$

$\boldsymbol{\omega_4}$, $\boldsymbol{\omega_4}$, $\boldsymbol{\omega_2}$, $\boldsymbol{\omega_2}$, $\boldsymbol{\omega_2}$, $\boldsymbol{\omega_2}$, $\boldsymbol{\omega_2}$, $\boldsymbol{\omega_2}$, $\boldsymbol{\omega_4}$, $\boldsymbol{\omega_4}$

$\boldsymbol{\omega_4}$, $\boldsymbol{\omega_4}$, $\boldsymbol{\omega_3}$, $\boldsymbol{\omega_3}$, $\boldsymbol{\omega_3}$, $\boldsymbol{\omega_3}$, $\boldsymbol{\omega_3}$, $\boldsymbol{\omega_3}$, $\boldsymbol{\omega_4}$, $\boldsymbol{\omega_4}$