$$\Pi = \begin{bmatrix} \Pi_1 = [P(X_1 = \omega_1)/\lambda] \\ \Pi_2 = [P(X_1 = \omega_2)/\lambda] \\ \Pi_3 = [P(X_1 = \omega_3)/\lambda] \\ \Pi_4 = [P(X_1 = \omega_4)/\lambda] \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \\ 0.4 \end{bmatrix} \text{ Donc } \widehat{\Pi} = \begin{bmatrix} \widehat{\Pi}_1 = \gamma_1(\omega_1) \\ \widehat{\Pi}_2 = \gamma_1(\omega_2) \\ \widehat{\Pi}_3 = \gamma_1(\omega_3) \\ \widehat{\Pi}_4 = \gamma_1(\omega_4) \end{bmatrix} = \begin{bmatrix} 0,050295 \\ 0,050295 \\ 0,050295 \\ 0,849115 \end{bmatrix}$$

$$\gamma_t(i) = \frac{\alpha_t(i)\beta_t(i)}{P(Y/\lambda)} = \frac{\alpha_t(i)\beta_t(i)}{\sum_{i=1}^{N} \alpha_t(i)\beta_t(i)}$$

$$\theta_t(i,j) = \frac{\alpha_t(i)a_{ij}b_{\omega_j}(Y_{t+1})\beta_{t+1}(j)}{P(Y = Y_{1\to T}/\lambda)}$$

$\alpha_1(\omega_1) = \alpha_1(\omega_2) = \alpha_1(\omega_3)$	$\alpha_1(\omega_4)$
0,02	0,28

$\alpha_1(1)\beta_1(1)$	$\alpha_1(2)\beta_1(2)$	$\alpha_1(3)\beta_1(3)$	$\alpha_1(4)\beta_1(4)$	
6,12464E − 08	6,12464E − 08	6,12464E − 08	1,03398E − 06	S = 1,21772E − 06

$\gamma_1(\omega_1) = 0,05029$	$\gamma_1(\omega_2) = 0,05029$	$\gamma_1(\omega_3) = 0,05029$	$\gamma_1(\omega_4) = 0,84913$

$$\theta_t(1,1) = \frac{\alpha_t(1)a_{11}b_{\omega_1}(Y_{t+1})\beta_{t+1}(1)}{\sum_{i=1}^{N} \alpha_t(i)\beta_t(i)}$$

$$\sum_{t=1}^{9} \theta_t(1,1) = \frac{a_{11}}{\sum_{i=1}^{N} \alpha_t(i)\beta_t(i)} \times \sum_{t=1}^{9} \alpha_t(1)b_{\omega_1}(Y_{t+1})\beta_{t+1}(1)$$

	$\alpha_t(1)$	$b_{\omega_1}(Y_{t+1}$	$\beta_{t+1}(1)$	Le produit
t=1	0,02	0,1	1,17093E − 05	2,34E − 08

$$\hat{a}_{11} = \frac{\sum_{t=1}^{T-1} \theta_t(1,1)}{\sum_{t=1}^{T-1} \gamma_t(1)}$$

$$\hat{a}_{11} = a_{11} \times \frac{2{,}51750E-06}{1{,}21772E-06} \times \frac{1}{1{,}94462} = 0.31934$$