A (very) short introduction to Group Representation Theory

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ABSTRACT. This is a short introduction to Group Theory and represents the second part of the course *Group Theory II* at the University of Sussex.

Contents

Introduction

This comprises the lecture notes for the second part of the course *Group Theory II*. It is based mainly on [**1**], in particular Chapters 1 and 2, as well as lecture notes of a course the author took a few years ago.

Part 1

Group Representations

CHAPTER 1

Group Theory Prerequisites

In this chapter, we are introducing the basic group theoretical notions required for this course. We are going to focus on the Symmetric Group in particular. This chapter is based on [**2**], in particular chapter I.1 and II.4. We will assume familiarity with the basic group theoretical definitions. As a reminder, S_n consists of all bijections from $\mathbb{Z}_n$ to itself, using composition as the group operation. Throughout the course, G is written multiplicatively with identity $\mathfrak{e}$.

DEFINITION 1. *Let* $a, b \in \mathbb{Z}_n$*,* $a \neq b$*. The* transposition $\tau = (a\ b)$ *is the permutation defined by* $\tau a = b$*,* $\tau b = a$ *and* $\tau x = x$ *for all* $x \neq a, b$*.*

PROPOSITION 1. *Every permutation* $\pi \in S_n$ *is the product of transpositions.*

The proof is not important for the course and left as an exercise.

EXERCISE 1. *Show that* $(1; 2), (2\ 3), \ldots, (n-1\ n)$ *form a basis and, indeed, generate* S_n*.*

[1] If π is a permutation, there are three different notations we can use. The *two line notation* is probably the most common and is as follows:

$$\pi = \begin{matrix} 1 & 2 & \ldots & n \\ \pi(1) & \pi(2) & \ldots & \pi(n) \end{matrix}$$

In order to get the *one line notation*, we drop the first line, as it is fixed. We can also display π using *cycle notation*.

[1]could include Braid groups? maybe as an alternative to π_0? Certainly easier to do questions for the exam with.

APPENDIX A

The First Appendix

APPENDIX B

The Second Appendix

Bibliography

[1] Bruce E. Sagan: *The Symmetric Group - Representations, Combinatorial Algorithms and Symmetric Functions*, **Springer Verlag**, New York, 2001.
[2] Pierre Antoine Grillet: *Abstract Algebra*, **Springer Science and Media**, New York, 2007.