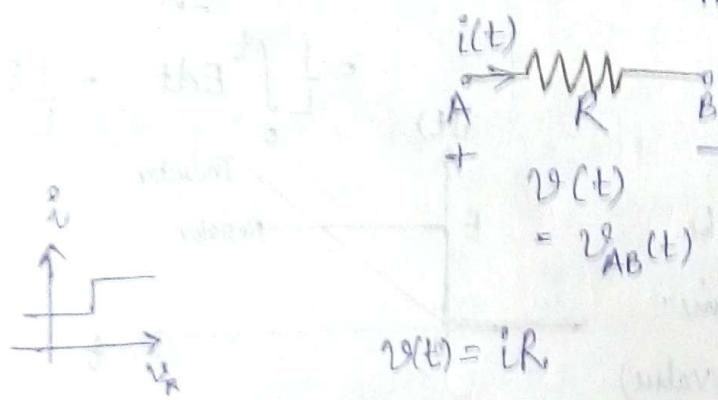


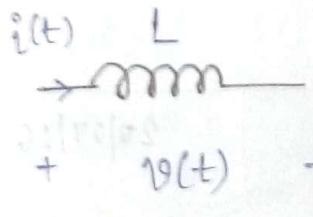
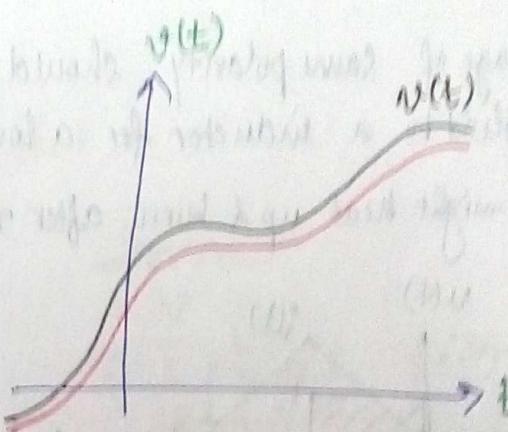
Resistive element:

$$I = \frac{V}{R}$$

The value of current in a purely resistive network depends on the current value of voltage (and not on voltage applied previously).



$$V(t) = iR$$



$$V(t) = L \frac{di}{dt}$$

$$di = \frac{1}{L} V(t) dt$$

$$i(t) = \frac{1}{L} \int_{-\infty}^t V(t) dt$$

Current through inductor cannot jump instantaneously.

$$i(t^-) = i(t^+)$$

$$i(t) = \int_{-\infty}^{0^-} V(t) dt + \frac{1}{L} \int_{0^-}^{0^+} V(t) dt + \frac{1}{L} \int_{0^+}^t V(t) dt$$

$$i(t) = \frac{1}{L} \int_{-\infty}^0 V(t) dt + 0 + \frac{1}{L} \int_0^t V(t) dt$$

$$i(t) = i(0^-) + \frac{1}{L} \int_{0^+}^t V(t) dt$$

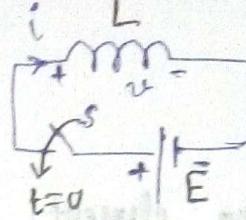
Current in an inductor at time t

$V(t)$ is a normal function
(not an impulse function)
any well behaved function

at $t=0^+$

$$i(0^+) = i(0^-) + \frac{1}{L} \int_{0^+}^{t=0^+} v(t) dt$$

$\Rightarrow i(0^+) = i(0^-)$ Property of Inductor



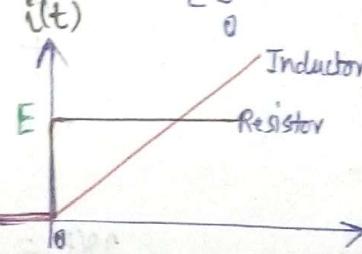
* Inductor is a device with memory.

- It remembers what you did earlier to it.

$i(0^-) = 0$

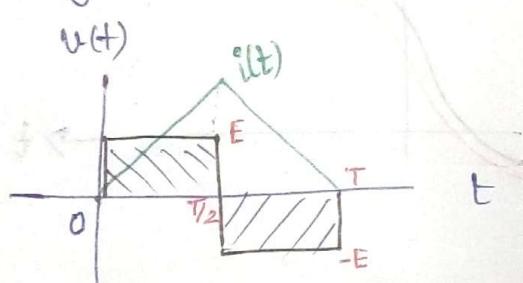
$$\Rightarrow i(t) = \frac{1}{L} \int_{-\infty}^t v(t) dt$$

$$= \frac{1}{L} \int_0^t Edt = \frac{1}{L} Et$$



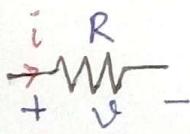
* Voltage of same polarity should not be applied to a inductor for a long time.

(It might heat up & burn after rated value)

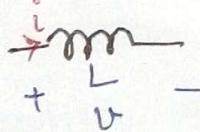


$i(t)$ goes back to zero if volt-sec is same
(Area is same)

20/07/16



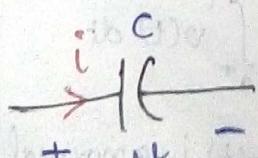
$$i = \frac{v}{R}$$



$$i(t) = \frac{1}{L} \int_{-\infty}^t v dt = \frac{1}{L} \int_{-\infty}^0 v dt + \frac{1}{L} \int_0^{0^+} v dt + \frac{1}{L} \int_{0^+}^t v dt$$

$i(t) = i(0^-) + \frac{1}{L} \int_{0^+}^t v dt$

$i(0^+) = i(0^-)$



$$v = q/c$$

$$i = C \frac{dq}{dt}$$

$$v(t) = \frac{1}{C} \int_{-\infty}^t i dt$$

$$V(t) = \frac{1}{C} \int_{-\infty}^{0^-} i dt + \frac{1}{C} \int_0^{0^+} i dt + L \int_{0^+}^t i dt$$

i = Excitation = Input

v = Output = Voltage

for normal well behaved function,

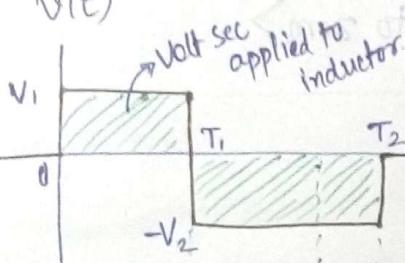
$$\int_{0^-}^{0^+} i dt = 0.$$

$$\Rightarrow v(t) = \frac{1}{C} \int_{-\infty}^{0^-} i dt + \frac{1}{C} \int_{0^+}^t i dt$$

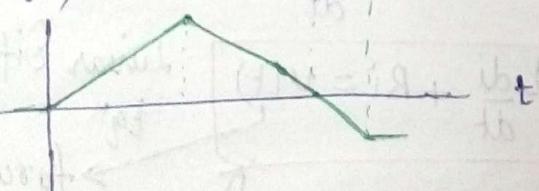
$$v(0^+) = \frac{1}{C} \int_{-\infty}^{0^-} i dt + \frac{1}{C} \int_{0^+}^{0^+} i dt = \frac{1}{C} \int_{-\infty}^{0^-} i dt$$

$$\Rightarrow v(0^+) = v(0^-)$$

v(t)



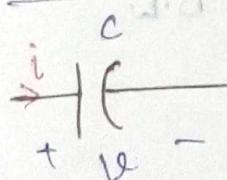
i(t)



i \rightarrow mm

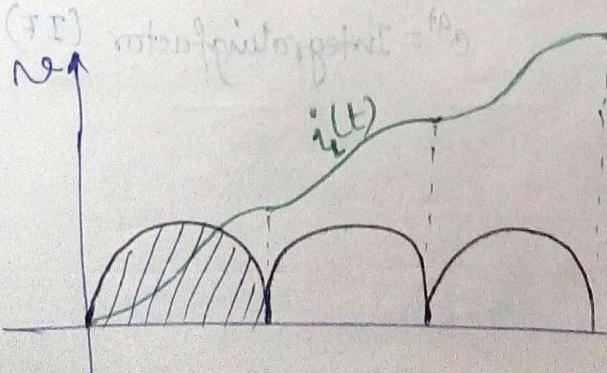
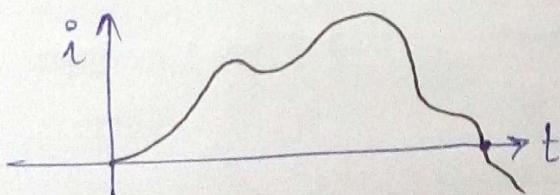
$$i = \frac{1}{L} \int_v^t v dt \quad \{ i(0^-) = 0. \}$$

$$v = L \frac{di}{dt} \quad \frac{di}{dt} = \frac{v}{L}$$

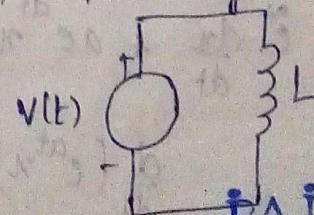


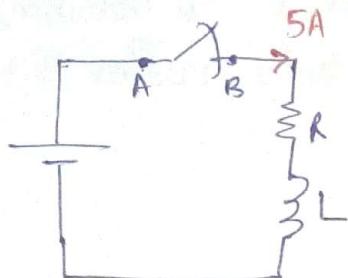
$$i = C \frac{dv}{dt}$$

$$\frac{dv}{dt} = \frac{i}{C}$$



If positive voltage is applied to inductor, current in a inductor keeps on increasing.



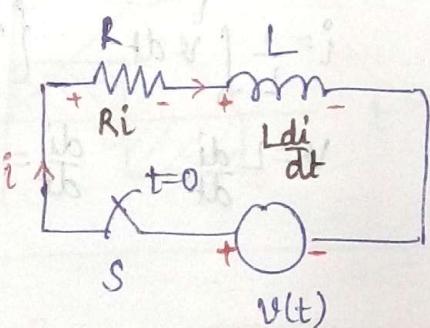


at $t=0$, circuit is opened.

$\Rightarrow \frac{di}{dt}$ is very large.

as voltage develops between A,B which may result in flash (due to air breakdown).

< Because current in an inductor cannot instantly turn out to zero >



$$i(0) = 0$$

$$V(t) - R_i i - \frac{L di}{dt} = 0$$

$$\boxed{\frac{L di}{dt} + R_i i = V(t)}$$

Linear Differential Eqn

forcing f^n.

1 Energy storing element \Rightarrow 1st order D.E.

Inductor
Capacitor

1st order Linear D.E.

$$\frac{dx}{dt} + a x = f(t)$$

$$e^{at} \frac{dx}{dt} + a e^{at} x = e^{at} f(t)$$

$$\frac{d}{dt} (e^{at} x) = e^{at} f(t)$$

e^{at} = Integrating factor. (IF)

$$\int d(e^{at}x) = \int e^{at} f(t) dt$$

$$e^{at}x = \int e^{at} f(t) dt + C \quad \text{const.}$$

$$x(t) = e^{-at} \underbrace{\int e^{at} f(t) dt}_{PI} + ce^{-at}$$

PI (Particular Integral) $\downarrow x_n(t)$
 Response due to Natural
 forcing f. response

To get $x_n(t)$, $f(t) = 0$

$$\frac{dx}{dt} + ax = 0 \quad \text{or} \quad D \equiv \frac{d}{dt}$$

$$(D+a)x = 0$$

$$\text{Ch. eqn. } M+a=0$$

$$M=-a$$

$$x_n(t) = Ce^{Mt} = Ce^{-at}$$

$$\Rightarrow \frac{d}{dt}(Ce^{-at}) + a(Ce^{-at}) = 0.$$

PI or Response due to forcing f.

$$x_f(t) = e^{-at} \int e^{at} f(t) dt$$

$$= e^{-at} \left[f(t) \frac{e^{at}}{a} - \int \frac{df}{dt} \frac{e^{at}}{a} dt \right]$$

$$= \frac{1}{a} f(t) - \frac{e^{-at}}{a} \int \frac{df}{dt} \frac{e^{at}}{a} dt$$

$$= \frac{1}{a} f(t) - \frac{e^{-at}}{a} \left[\frac{df}{dt} \frac{e^{at}}{a} - \int \frac{d^2f}{dt^2} \frac{e^{at}}{a} dt \right]$$

$$= \frac{1}{a} f(t) - \frac{1}{a^2} \frac{df}{dt} + \frac{e^{-at}}{a^2} \int \frac{d^2f}{dt^2} e^{at} dt$$

$x_f(t)$

(Response due to forcing function) will be linear combination of forcing function and its higher order derivatives

⇒

$$x_f(t) = K_0 f(t) + K_1 \frac{df}{dt} + K_2 \frac{d^2f}{dt^2} + \dots \infty$$

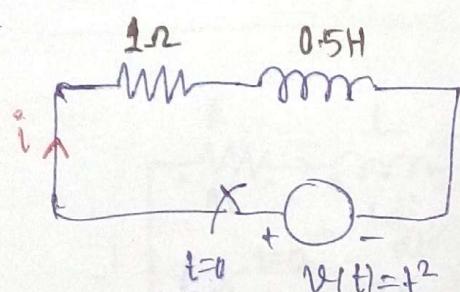
$$K_0 = \frac{1}{a}, K_1 = -\frac{1}{a^2}, \dots (K_n = (-1)^n a^{-(n+1)})$$

* $\frac{dx}{dt} + 3x = f(t)$

$$x(t) = ce^{-3t} + K_0 f + K_1 \dot{f} + K_2 \ddot{f} + \dots \infty$$

$$(f = \frac{df}{dt})$$

$$(\ddot{f} = \frac{d^2f}{dt^2})$$

* 

$$i(0^-) = 0 \Rightarrow i(0^+) = 0$$

$$1i + 0.5 \frac{di}{dt} = t^2$$

$$\frac{di}{dt} + 2i = 2t^2$$

Ch. eqⁿ. $m+2=0 \Rightarrow m=-2$

$$i(t) \equiv x(t) = ce^{-2t} + \underbrace{K_0 t^2}_{(1)} + \underbrace{K_1 t}_{(2)} + K_2$$

$$i(0) = 0 = ce^{-0} + 0 + K_2 \quad \text{by}$$

$$\Rightarrow c = -k_2 - \textcircled{1}$$

$$\frac{d}{dt}(ce^{-2t} + K_0 t^2 + K_1 t + K_2) + 2(ce^{-2t} + K_0 t^2 + K_1 t + K_2) = 2t^2$$

$$2K_0 t + K_1 + 2K_0 t^2 + 2K_1 t + 2K_2 = 2t^2$$

$$2(K_0 - 1)t^2 + 2(K_0 + K_1)t + (K_1 + 2K_2) = 0$$

$$\boxed{K_0 = 1}$$

$$\boxed{K_1 = -K_0 = -1}$$

$$2K_2 = K_1 \Rightarrow \boxed{K_2 = -\frac{1}{2} = c}$$

$$* \frac{di}{dt} + 2i = 5 \sin ct$$

$$\Rightarrow i = ce^{-2t} + A \sin ct + B \cos ct$$

2nd order diff. eqn

$$\frac{d^2x}{dt^2} + a \frac{dx}{dt} + bx = f(t)$$

$$(D^2 + aD + b)x = f(t)$$

$$(D+m_1)(D+m_2)x = f(t)$$

$y(t)$

$$y(t) = C_1 e^{-m_1 t} + K_0 f(t) + K_1 \dot{f}(t) + K_2 \ddot{f}(t) + \dots$$

$$(D+m_2)x = y(t)$$

$$\begin{aligned} u(t) &= C_2 e^{-m_2 t} + K_0' y(t) + K_1' \dot{y}(t) + \dots \\ &= C_2 e^{-m_2 t} + K_0' (C_1 e^{-m_1 t} + K_0 f(t) + K_1 \dot{f}(t) + \dots) \end{aligned}$$

$$\Rightarrow \frac{d^2x}{dt^2} + a \frac{dx}{dt} + bx = f(t)$$

$$(D^2 + aD + b)x = f(t)$$

$$(m+m_1)(m+m_2) = 0$$

$$x(t) = C_1 e^{-m_1 t} + C_2 e^{-m_2 t} + K_0 f(t) + K_1 \dot{f}(t) + K_2 \ddot{f}(t) + \dots$$

21/07/16

$$* \frac{dx}{dt} + ax = f(t)$$

$$x(t) = ce^{-at} + [K_0 f(t) + K_1 \dot{f}(t) + K_2 \ddot{f}(t) + \dots]$$

$$* \frac{d^2x}{dt^2} + a \frac{dx}{dt} + bx = f(t)$$

$$(D+m_1)(D+m_2)x = f(t)$$

Ch. eqn: $m^2 + am + b = 0$
Assume $m_1 \neq m_2$

$$(D+M_2)y * (D+M_2)x = y(t)$$

$$\Rightarrow n(t) = C_2 e^{-M_2 t} + \bar{K}_0 y + \bar{K}_1 \dot{y} + \dots \infty$$

$$* (D+m_1) y(t) = f(t)$$

$$\Rightarrow y(t) = C_1 e^{-m_1 t} + \bar{K}_0 f + \bar{K}_1 \dot{f} + \dots \infty$$

$$n(t) = \boxed{C_1 e^{-m_1 t} + C_2 e^{-M_2 t} + K_0 f + K_1 \dot{f} + \dots \infty}$$

$$\frac{dx}{dt} + ax = f(t) = A e^{-at}$$

- a = Characteristic Root

→ Expected Soln:-

$$n(t) = C_1 e^{-at} + K_0 f + K_1 \dot{f} + \dots \infty$$

Actual Soln:-

$$e^{at} \frac{dx}{dt} + e^{at} ax = A$$

Remember how we derived it

$$\frac{d}{dt}(x e^{at}) = A dt$$

$$x e^{at} = A t + C$$

$$n(t) = A t e^{-at} + C e^{-at}$$

$$\frac{d^2x}{dt^2} + a \frac{dx}{dt} + bx = f(t)$$

Suppose $m_1 = m_2 = m$.

$$* (D+m) n = y(t)$$

$$n(t) = C_2 e^{-mt} + \bar{K}_0 y + \bar{K}_1 \dot{y} + \dots \infty$$

$$* (D+m) y(t) = f(t)$$

$$y(t) = C_1 e^{-mt} + \bar{K}_0 f + \bar{K}_1 \dot{f} + \dots \infty$$

Similar type of eqn

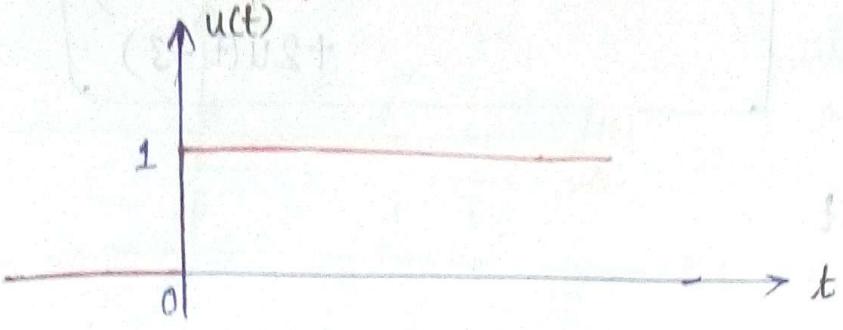
$$\boxed{(D+m)n = y(t) = C_1 e^{-mt} + \bar{K}_0 f + \bar{K}_1 \dot{f} + \dots \infty \quad (-m = \text{Chr. Root})}$$

$$n(t) = (A t e^{-mt}) + \dots$$

$$\boxed{n(t) = \underbrace{(A t + B)}_{\text{Natural Response}} e^{-mt} + \bar{K}_0 f + \bar{K}_1 \dot{f} + \dots \infty}$$

Natural Response Response due to forcing f .

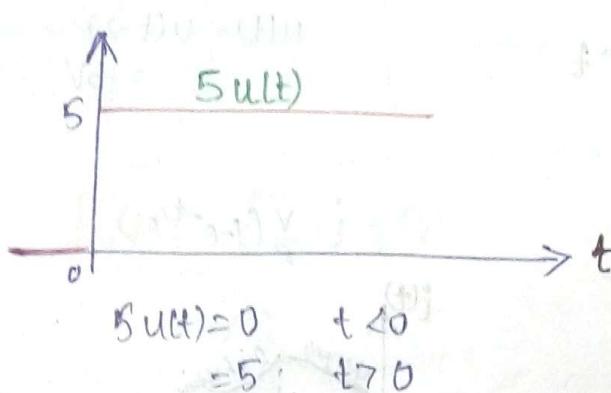
ABOUT f^n 'S (SINGULARITY $f^n \dots$)



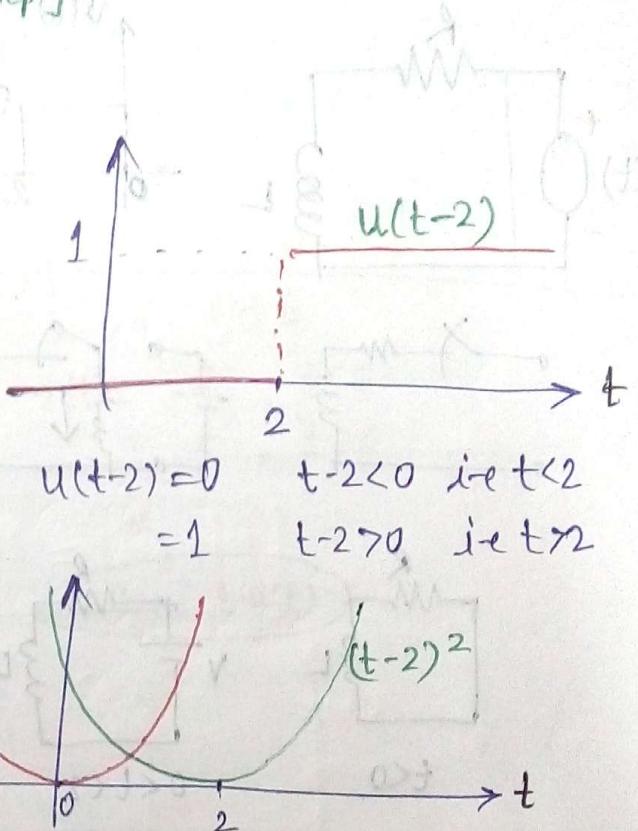
$$u(t) = 0 \quad t < 0 \\ = 1 \quad t \geq 0$$

Unit Step f^n

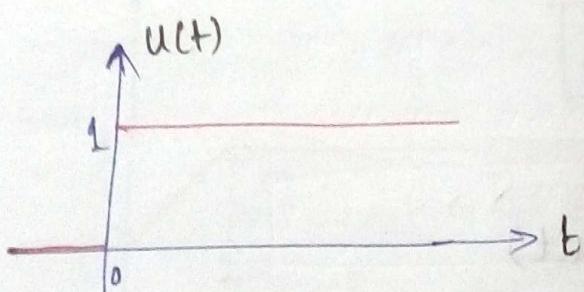
$u(t)$



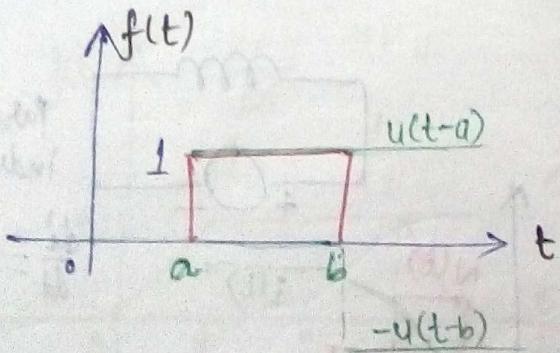
$$u(t-T) = 0 \quad t < T \\ = 1 \quad t \geq T$$



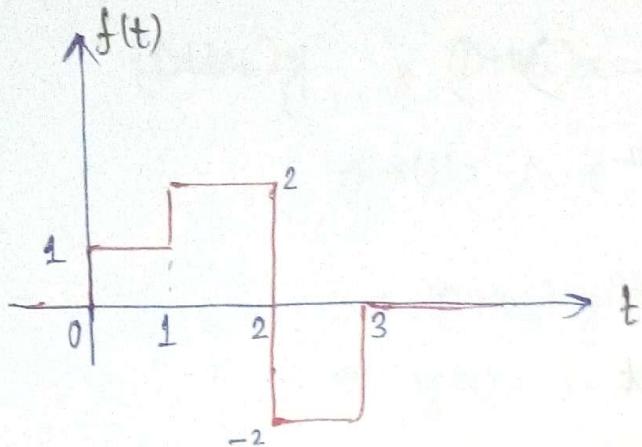
$$u(t-2) = 0 \quad t-2 < 0 \text{ i.e. } t < 2 \\ = 1 \quad t-2 \geq 0 \text{ i.e. } t \geq 2$$



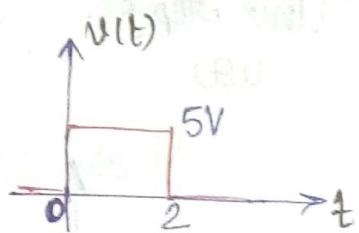
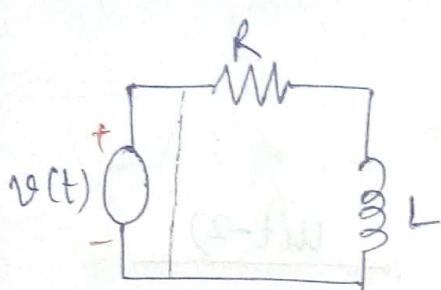
$$f(t) = 0 \quad t < a \\ = 1 \quad a < t < b \\ = 0 \quad t > b$$



$$\Rightarrow f(t) = u(t-a) - u(t-b)$$

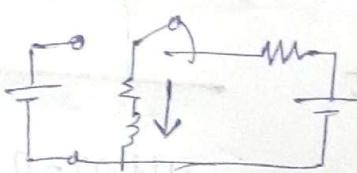
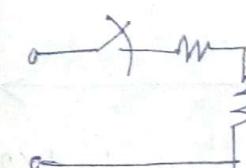


$$f(t) = u(t) + u(t-1) - 4u(t-2) + 2u(t-3)$$

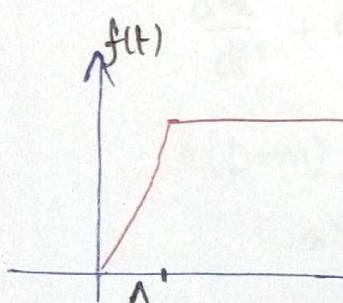
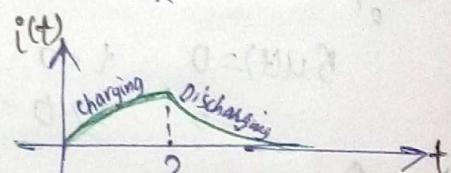
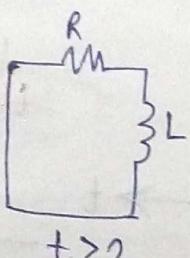
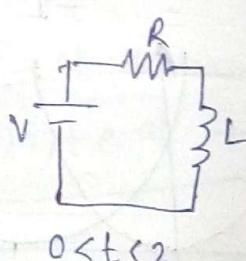
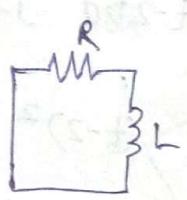


$$v(t) = 5u(t) - 5u(t-2)$$

$$u(t) - u(t-2) = 5V$$

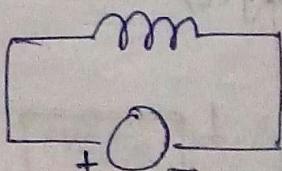
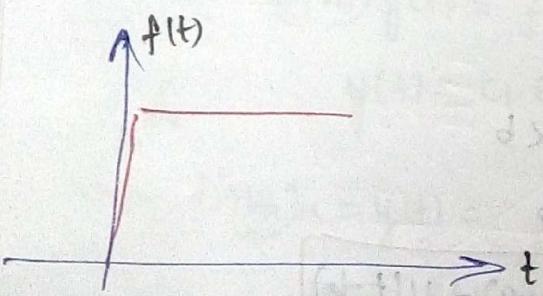


$$i = \frac{V}{R} (1 - e^{-t/\tau})$$

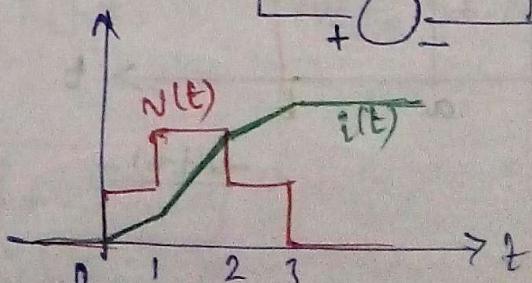


$$f(t) \rightarrow u(t)$$

as $\Delta \rightarrow 0$



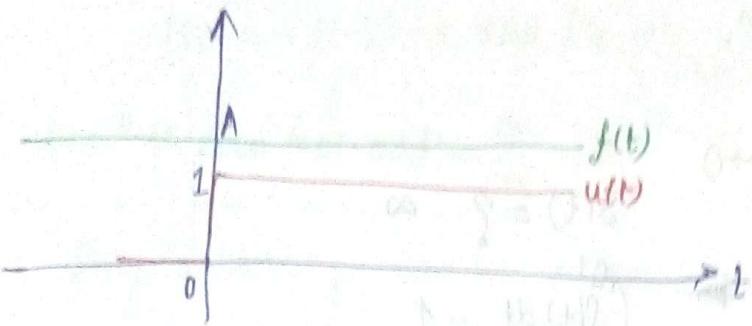
$$\frac{di}{dt} = \frac{v}{L}$$



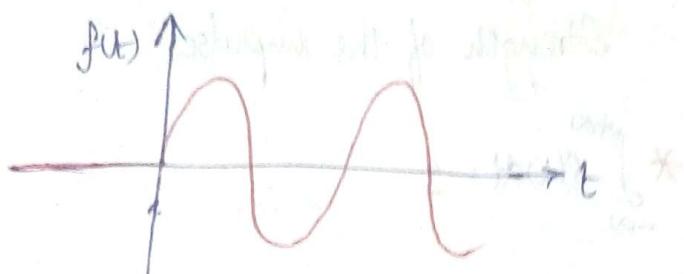
$$f(t) = A \rightarrow \text{const}$$

25/07/16

$u(t)$ must not be confused with a constant function.



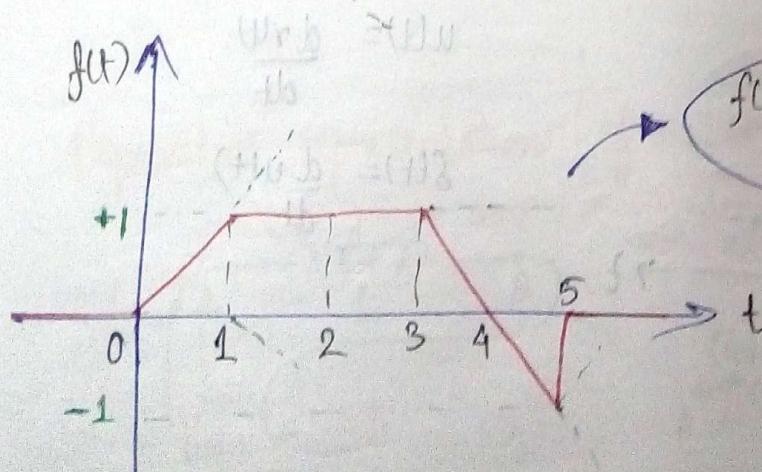
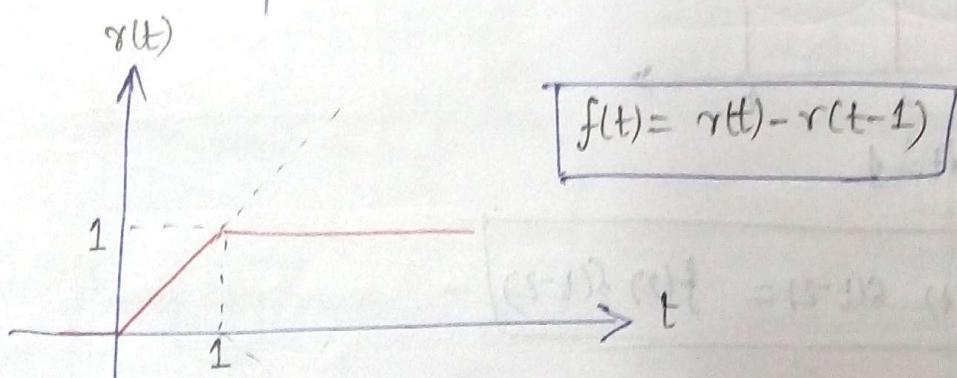
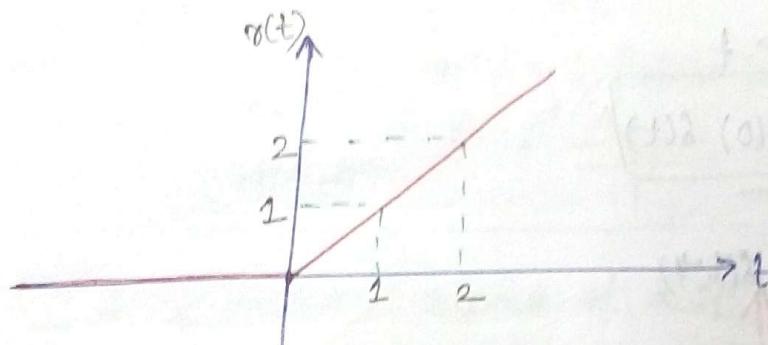
$$f(t) = \sin \omega t + u(t)$$



RAMP f. : $r(t)$

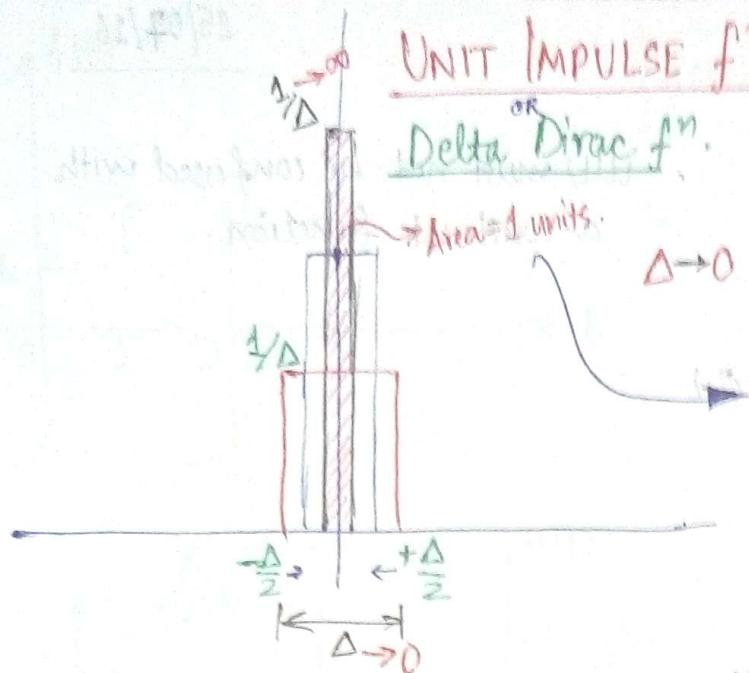
$$r(t) = \begin{cases} 0, & t < 0 \\ t, & t \geq 0 \end{cases}$$

or
 $r(t) = t \cdot u(t)$



UNIT IMPULSE f^n .

or
Delta, Dirac f^n .

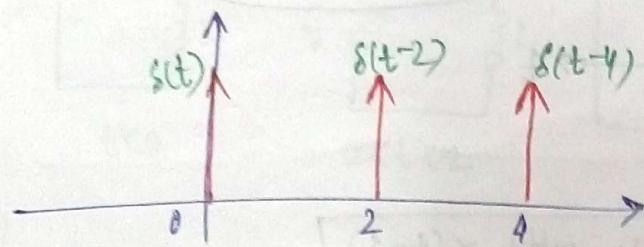
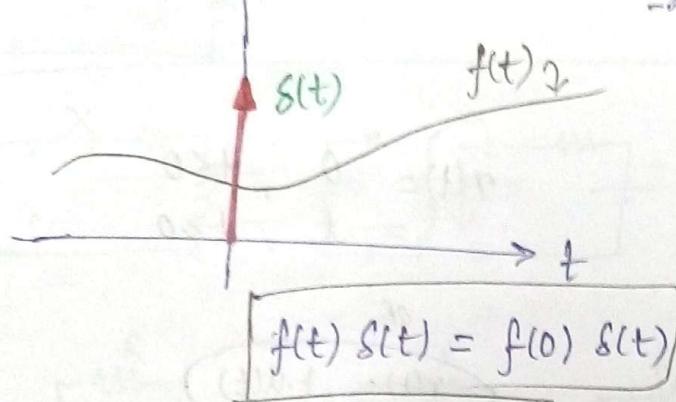


$$\delta(t) = ? \infty$$

$$\int_{0^-}^{0^+} \delta(t) dt = 1$$

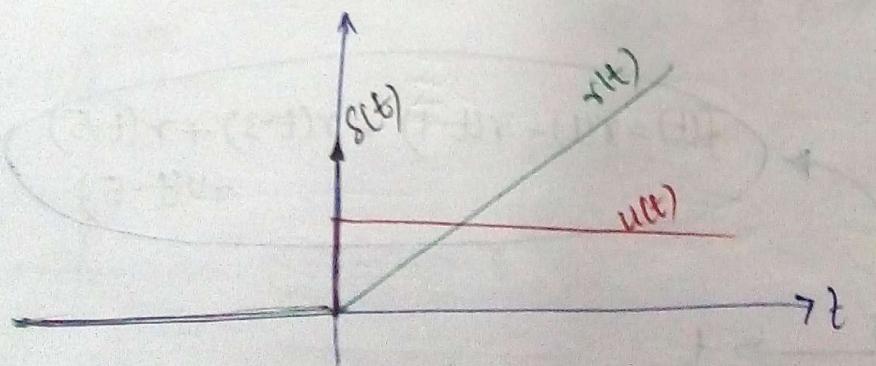
Strength of the impulse

$$*\int_{-\infty}^{+\infty} \delta(t) dt = 1$$



$$*\int_{-\infty}^{+\infty} f(t-2) dt = 1.$$

$$f(t) \delta(t-2) = f(2) \delta(t-2)$$



$$u(t) = \frac{d r u(t)}{dt}$$

$$\delta(t) = \frac{d u(t)}{dt}$$

$f(t) = f(-t) \rightarrow$ Even fn
 $f(t) = -f(-t) \rightarrow$ Odd fn
 Neither even nor odd:-

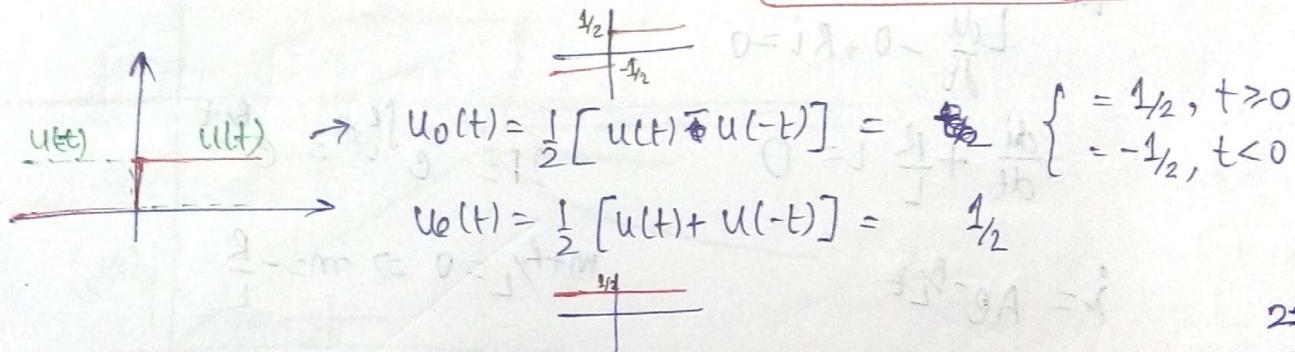
fold along y-axis: Superposition.

★ Any arbitrary can be broken into sum of an even & odd function.
 let $f(t)$ be a general fn \rightarrow neither even nor odd.

$$f(t) = f_o(t) + f_e(t)$$

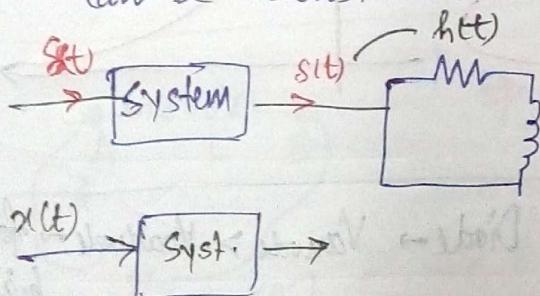
$$f(-t) = f_o(t) + f_e(-t) = -f_o(t) + f_e(t)$$

$$\Rightarrow f_e(t) = \frac{f(t) + f(-t)}{2} \quad \& \quad f_o(t) = \frac{f(t) - f(-t)}{2}$$

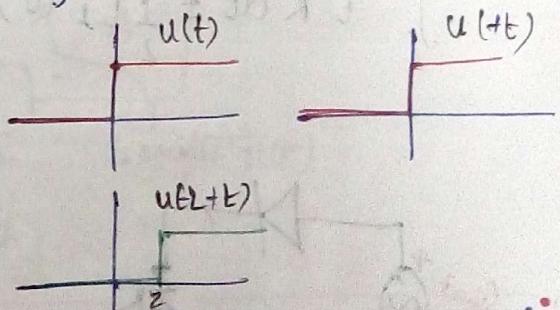


27/7/16

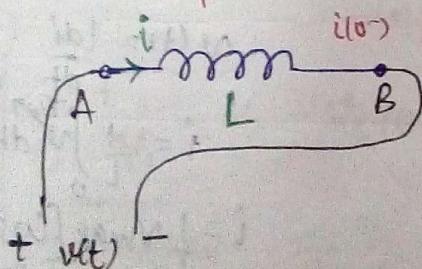
Note : If you know response of a system, then ~~for~~ any arbitrary function can be reconstructed.



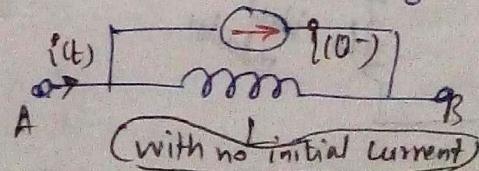
$$f(t) s(t) = f(0) s(t).$$

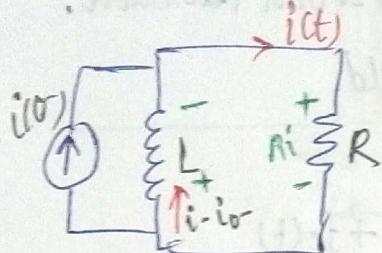
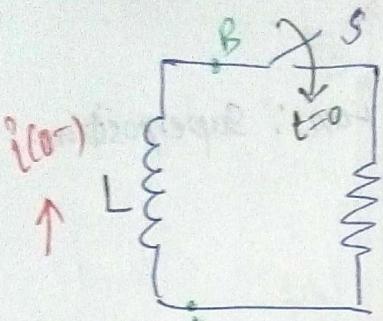


Equivalent representation of an inductor having initial current (i_{0-})



$$i(t) = i(0-) + \frac{1}{L} \int_0^t v(t') dt$$





$$L \frac{di - i(0-)}{dt} + RI = 0$$

$$L \frac{di}{dt} - i(0-) + RI = 0$$

$$\frac{di}{dt} + \frac{R}{L} i = 0$$

$$i = Ae^{-\frac{R}{L}t}$$

$$i(0) = ? = i(0-)$$

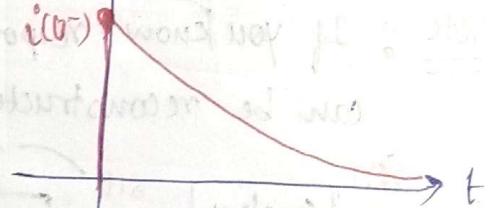
$$i - i(0-) = 0$$

$$\Rightarrow i(t) = e^{-\frac{R}{L}t} i(0-).$$

$$IF = C \int e^{\frac{R}{L}t} dt = e^{\frac{R}{L}t}$$

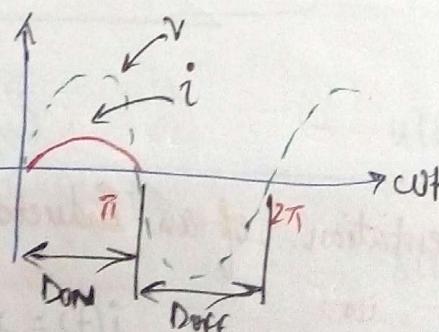
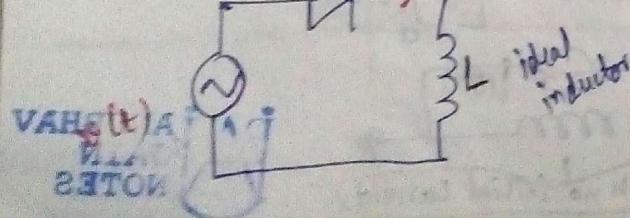
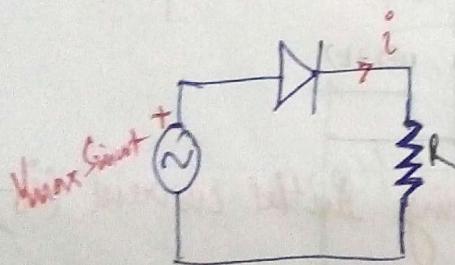
$$m + \frac{R}{L} = 0 \Rightarrow m = -\frac{R}{L}$$

$$i(t)$$



$$\int_0^\infty i^2 R dt = \frac{1}{2} L i(0)^2$$

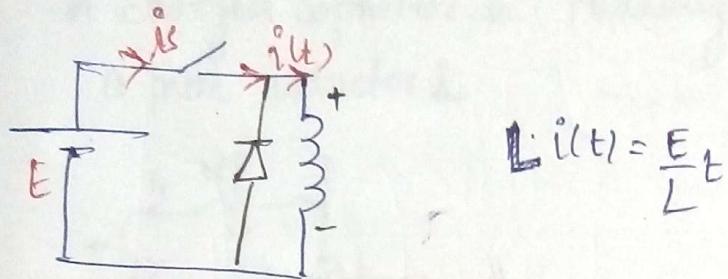
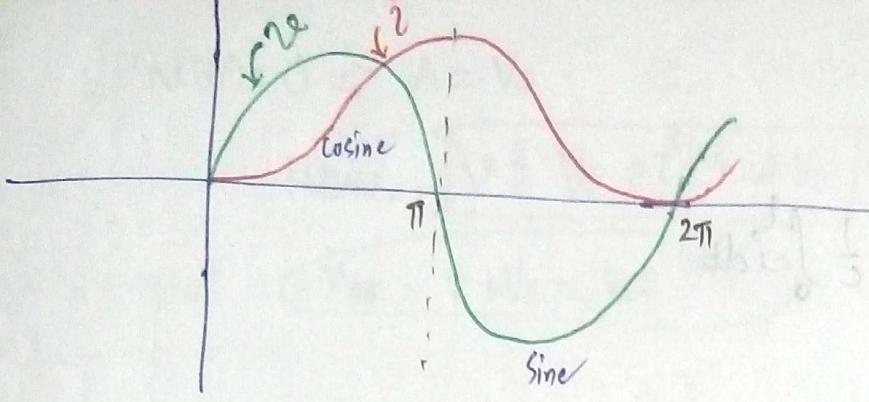
Diode \rightarrow Anode $>$ Cathode \rightarrow forward biased.



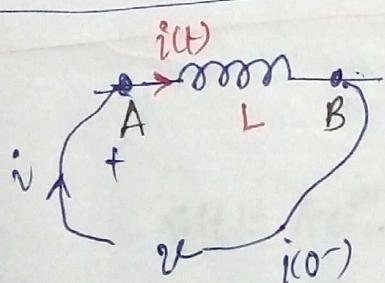
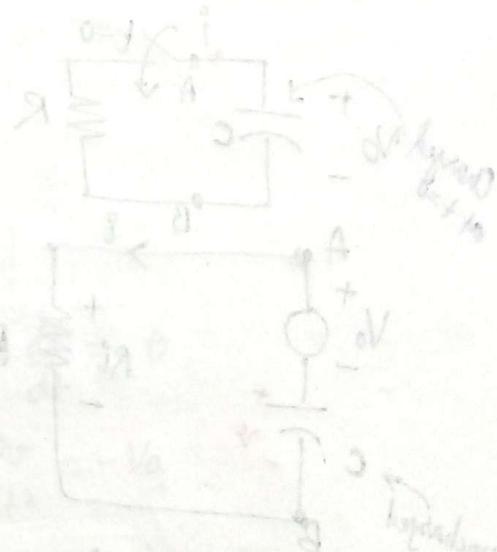
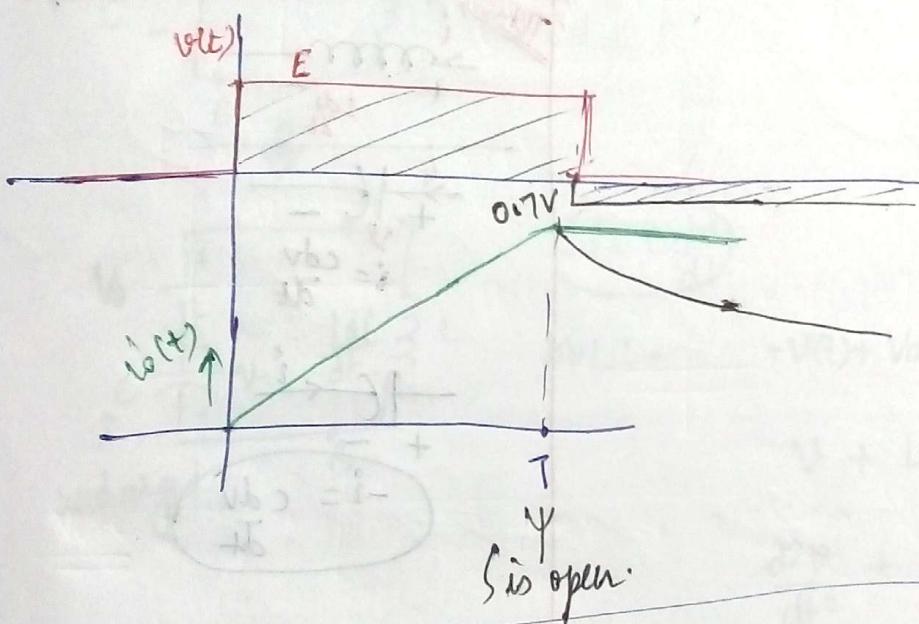
$$\Delta \theta = \frac{1}{2} \frac{dI}{dt}$$

$$i = \frac{1}{L} \int_{0}^{2\pi} v_{max} \cos \omega t dt$$

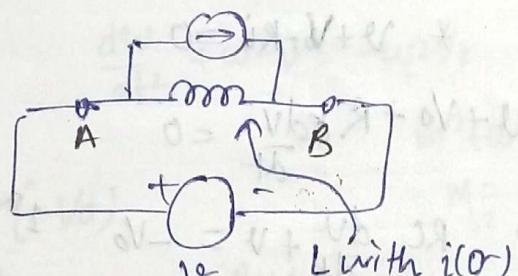
$$i = \frac{1}{L} v_{max} \left[\cos \omega t \right]_0^{2\pi} = \frac{v_{max}}{\omega} (1 - \cos \omega t)$$



$$L \cdot i(t) = \frac{E}{L} t$$

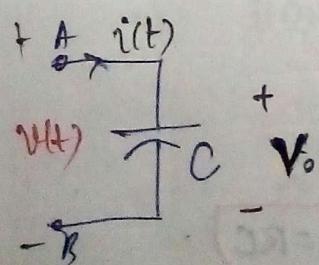


⇒



$$i(t) = i(0) + \frac{1}{L} \int_0^t V dt$$

How to represent a capacitor having some initial voltage in form of equivalent ckt.



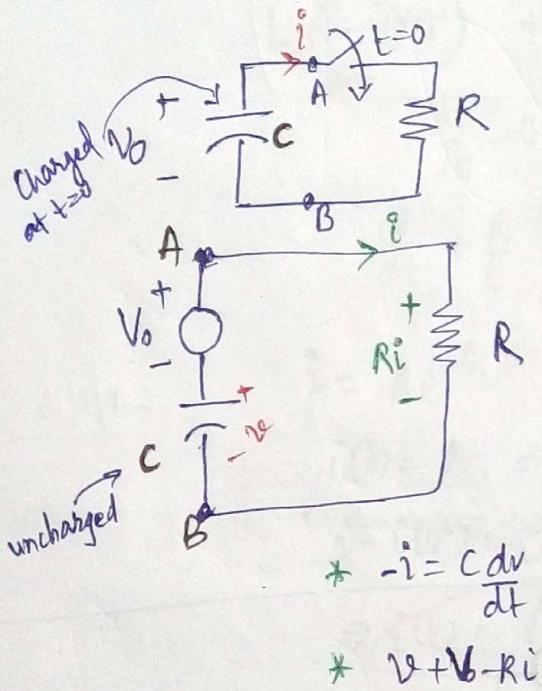
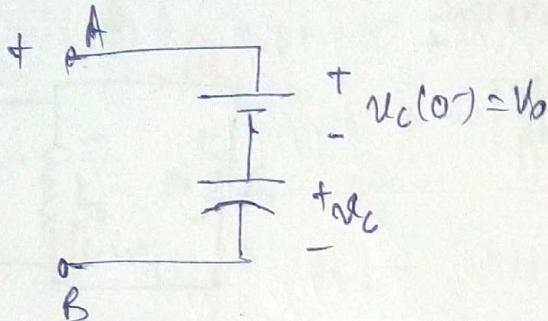
$$V_C(0-) = V_0$$

$$i = C \frac{dV}{dt}$$

$$V(t) = \frac{1}{C} \int_{-\infty}^t i dt$$

$$= V_c(0^-) + \frac{1}{C} \int_0^t i dt.$$

Eg: Ckt



$$\Rightarrow V + V_0 + R C \frac{dv}{dt} = 0$$

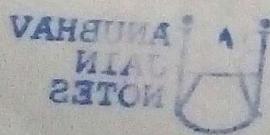
$$\Rightarrow RC \frac{dv}{dt} + v = -V_0 \quad \text{1st order Diff. eqn.}$$

$$\frac{dv}{dt} + \frac{1}{RC} v = -\frac{V_0}{RC} \quad \text{(1)}$$

$$v = Ae^{-t/RC} - K_0 \frac{V_0}{RC}$$

$$Ae^{-t/RC} \cdot \left(-\frac{1}{RC}\right) + \frac{1}{RC} \left(Ae^{-t/RC}\right) = -\frac{V_0}{RC}$$

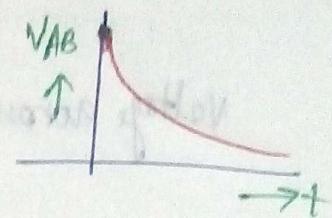
$$-\frac{K_0 V_0}{(RC)^2} = \frac{V_0}{RC} \Rightarrow K_0 = RC$$



$$V(0) = 0 \Rightarrow A = V_0$$

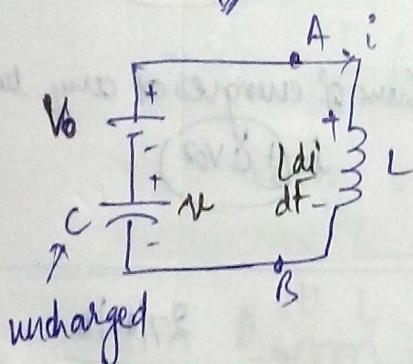
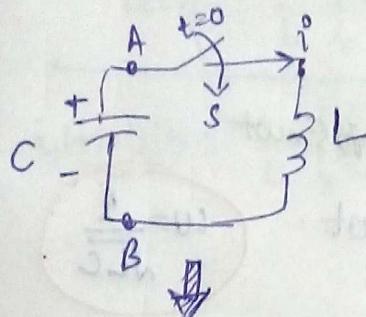
Hence, $V = V_0 e^{-t/RC} - V_0$

$$V_{AB} = V + V_0 = V_0 e^{-t/RC}$$



A charged capacitor is suddenly connected (at $t=0$) across a pure inductor L .

L is uncharged initially.



$$-i = C \frac{dV}{dt}$$

$$\text{KVL: } +V(A) + V_0 - L \frac{di}{dt} = 0$$

$$V + L \frac{d^2V}{dt^2} = -V_0$$

$$\frac{d^2V}{dt^2} + \frac{1}{LC} V = -\frac{V_0}{LC}$$

$$\frac{d^2V}{dt^2} + \omega^2 V = -\omega^2 V_0$$

(Let $\frac{1}{LC} = \omega^2$)

$$\text{CE: } m^2 + \omega^2 = 0$$

$$m_{1,2} = \pm j\omega \quad j = \sqrt{-1}$$

$$V(t) = A e^{j\omega t} + B e^{-j\omega t} + (-V_0)$$

$$i = -C \frac{dV}{dt} = -A c j \omega e^{j\omega t} + B c j \omega e^{-j\omega t} + 0$$

$$V(0) = 0$$

$$\therefore A + B = +V_0$$

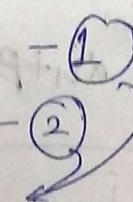
$$i(0) = 0$$

$$-A + B = 0$$

$$\therefore A = B = +\frac{V_0}{2}$$

RHS \rightarrow const.

$$\text{Do } \omega^2 V = -\omega^2 V_0 \Rightarrow V = -V_0$$



$$\Rightarrow V(t) = +\frac{V_0}{2} e^{j\omega t} + \frac{V_0}{2} e^{-j\omega t} - V_0$$

$$V(t) = \frac{V_0}{2} (2 \cos \omega t) - V_0$$

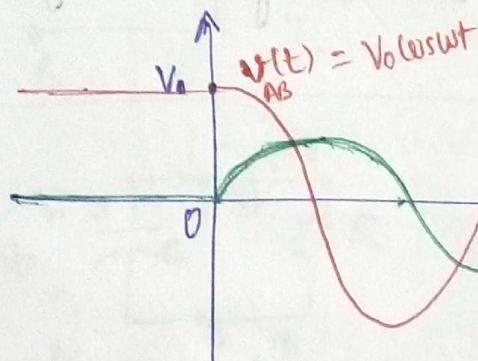
$$\Rightarrow V(t) = V_0 \cos \omega t - V_0$$

Voltage across initially charged capacitor \rightarrow

$$V(t) + V_0 = V_{AB}(t) = V_0 \cos \omega t$$

$$i(t) = j C \omega V_0 \left(-e^{j\omega t} + e^{-j\omega t} \right)$$

$$i(t) = C \omega V_0 \sin \omega t$$



$$i(t) = C \omega V_0 \sin \omega t \rightarrow \text{cut.}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$* \frac{1}{2} CV_0^2 = \frac{1}{2} L V_0^2 \omega^2 C^2$$

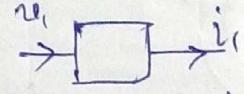
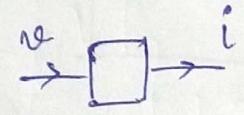
Sum of energies at any time
is $\frac{1}{2} CV_0^2$

$$\begin{array}{c} i \\ \rightarrow \end{array} \begin{array}{c} R \\ | \\ \text{---} \\ | \\ \text{---} \\ + \quad - \end{array} \quad i = \frac{v}{R}$$

$$i_1 = \frac{v_1}{R}$$

$$i_2 = \frac{v_2}{R}$$

$$i_1 + i_2 = \frac{v_1 + v_2}{R}$$

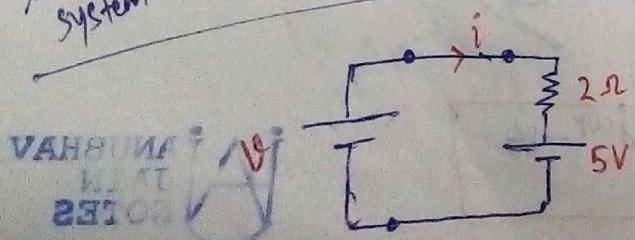


$$v_1 + v_2 \rightarrow \frac{v_1 + v_2}{R}$$

Principle of Superposition.

$$\alpha v_1 + \beta v_2 = \alpha i_1 + \beta i_2$$

Linear system.



$$V = V_1 = 10V$$

$$i = \frac{V-5}{2}$$

$$i(t) = i(0^-) + \frac{1}{L} \int_0^t v dt$$

$i(0^-) = 0$

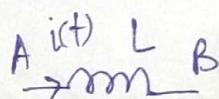
$$\dot{i}_1(t) = \frac{1}{L} \int_0^t v_1 dt$$

$$\dot{i}_2(t) = \frac{1}{L} \int_0^t v_2 dt$$

$$\dot{i}_1(t) + \dot{i}_2(t) = \frac{1}{L} \int_0^t (v_1 + v_2) dt$$

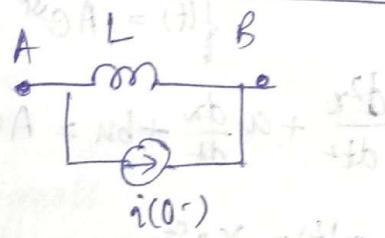
$$v(t) = \frac{1}{C} \int_{-\infty}^t i dt$$

$$v(t) = v(0) + \frac{1}{C} \int_0^t i dt$$

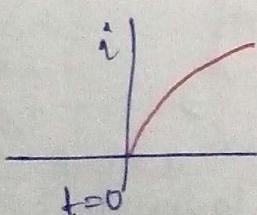
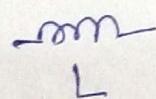
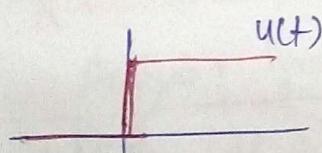


$$i(0^-)$$

$$-i(t)$$



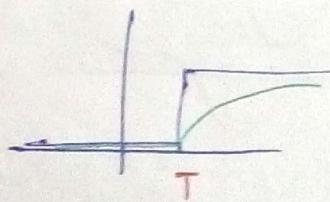
Linear & Time invariant system.

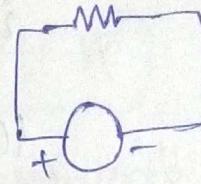
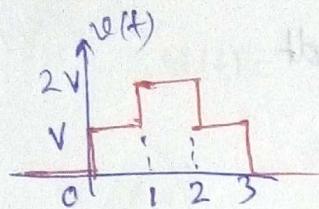


$$x(t) \rightarrow y(t)$$

I/P O/P.

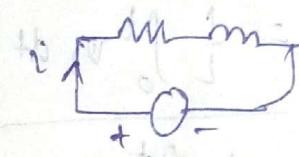
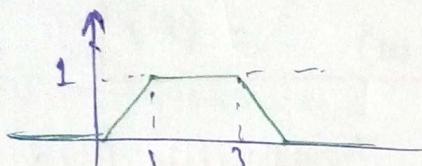
$\Rightarrow x(t-T) \rightarrow y(t-T)$





$$v(t) = u(t) + u(t-1) + \dots$$

v_1



$$v(t) = r(t) - r(t-1) \dots$$

Causal.

$$(D^2 + aD + b)x = f(t)$$

for a class of f 's

$$f(t) = A e^{st}$$

$$\frac{d^2x}{dt^2} + a \frac{dx}{dt} + bx = A e^{st}$$

(P.I.)

$$\text{let } x(t) = X e^{st}$$

$$X e^{st} s^2 + X e^{st} sa + X e^{st} b = A e^{st}$$