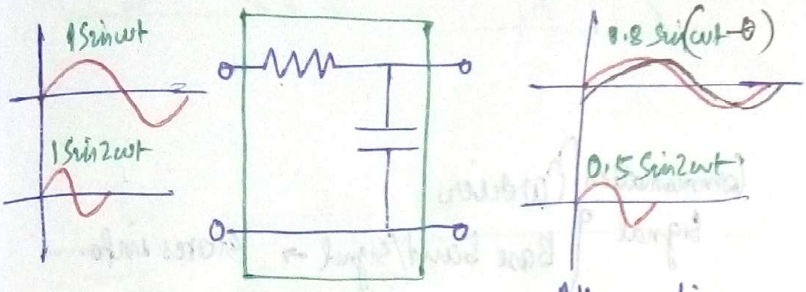
Signals & Systems

01/08/17

Signal – I/p
System – Black box.



Gain = $\frac{0.8}{1}$

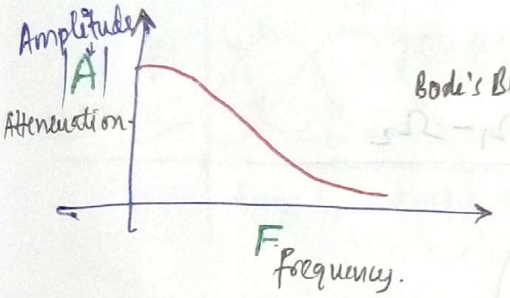
Gain = $\frac{0.5}{1}$

Attenuation.
⇒ Gain ↓es.

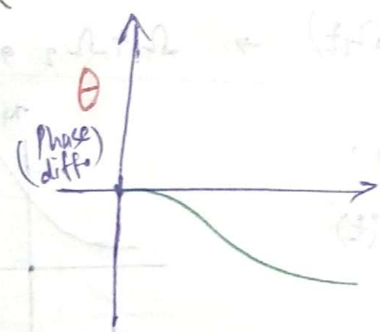
We always plot.

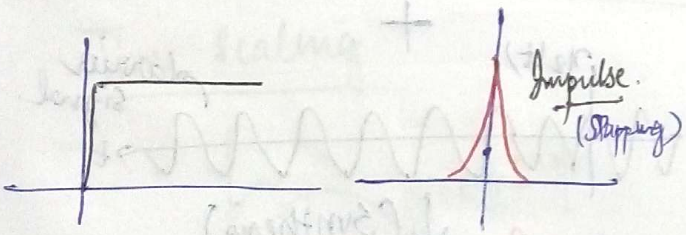
* Audio Systems :– Energy v/s Time

* We only give sinusoids. for observing freq. Response



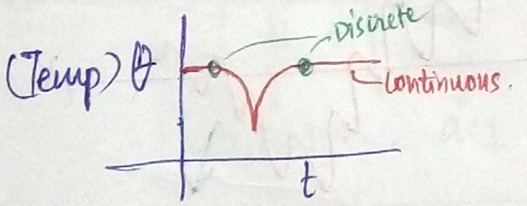
Bode's Block

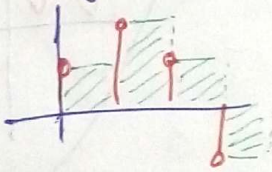




Install MATLAB ('13) older version

* Continuous and discrete-time signals.
↳ eg. Stock market





03/08/17

Mathematical Modeling of Signals.

Analysis + Synthesis.

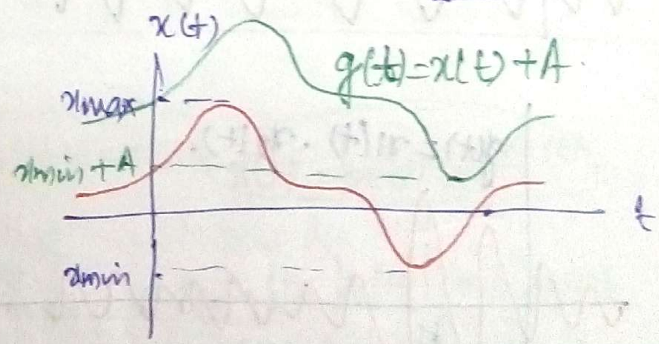
SIGNAL OPERATIONS

Addition of a constant offset.

$g(t) = x(t) + A$

* DC Shift
* Amplitude Shift

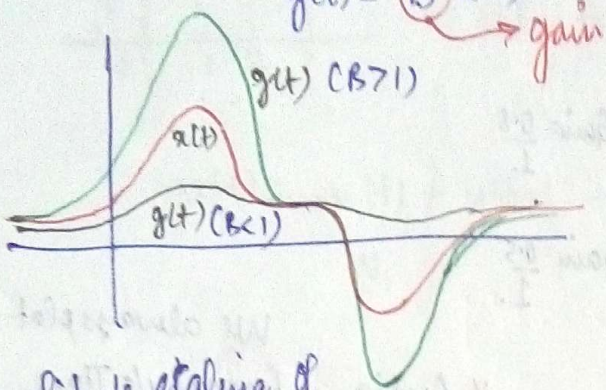
Amplitude (Shifting of signal)



Multiplying by a constant gain factor.

$g(t) = B\, x(t)$ → gain

$x_{max} - x_{min}$ = Dynamic Range = P-P value.



Communication Signal { Carrier; Base band/signal → stores info.

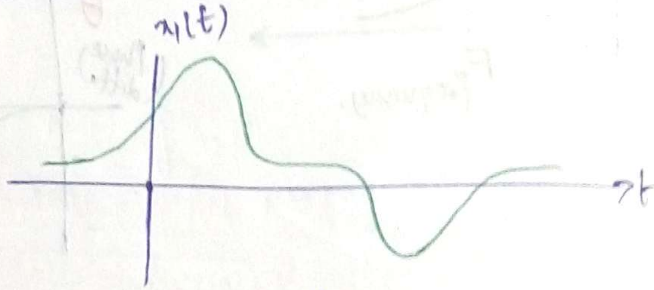
(Amplitude scaling of signal)

$g(t) = b(t) \cdot x(t)$

$\sin(\Omega_2 t)$ $\quad \sin(\Omega_1 t)$ → $\Omega_1 + \Omega_2$, $\Omega_1 - \Omega_2$

Adding two signals.

$g(t) = x_1(t) + x_2(t)$



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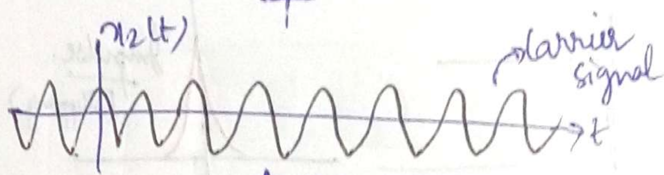
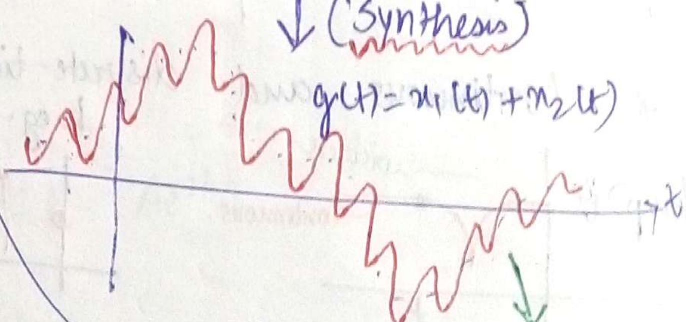
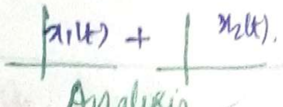


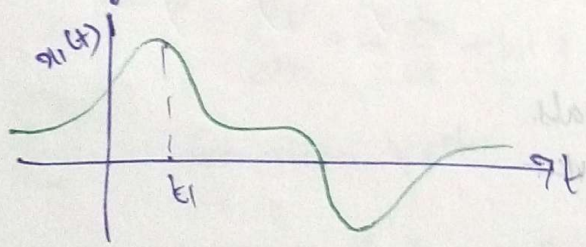
↓ (Synthesis)

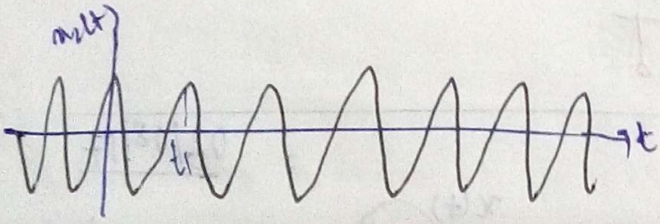


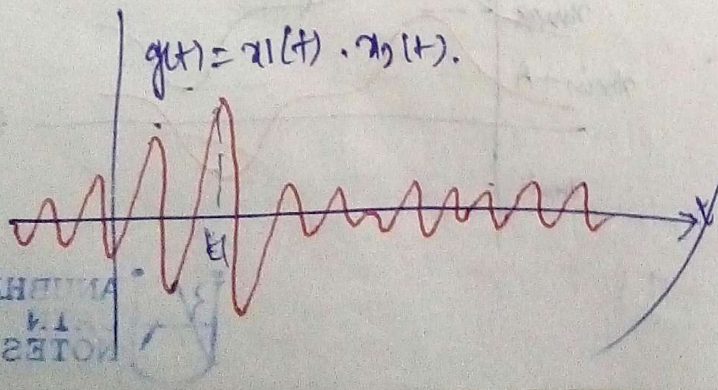


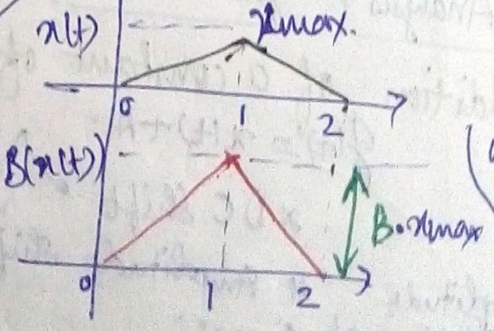
Multiplying two signals.

$g(t) = x_1(t)\, x_2(t)$

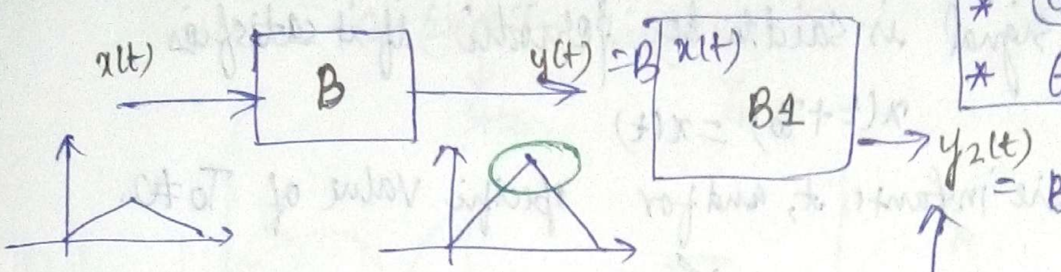






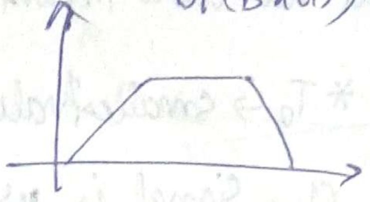


dynamic Range Res.

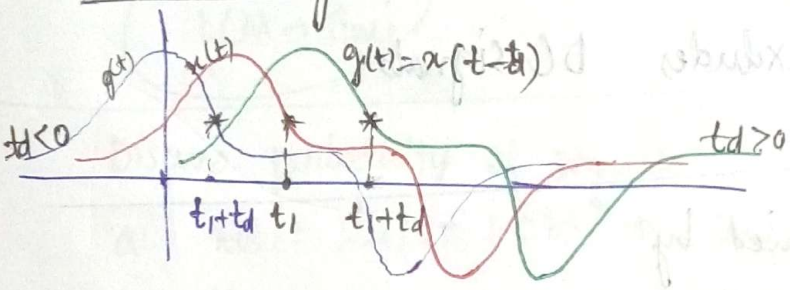


- (Gain < 1) ≡ Attenuation
- Gain ≡ B > 0

Do not allow values above a certain value



Time shifting



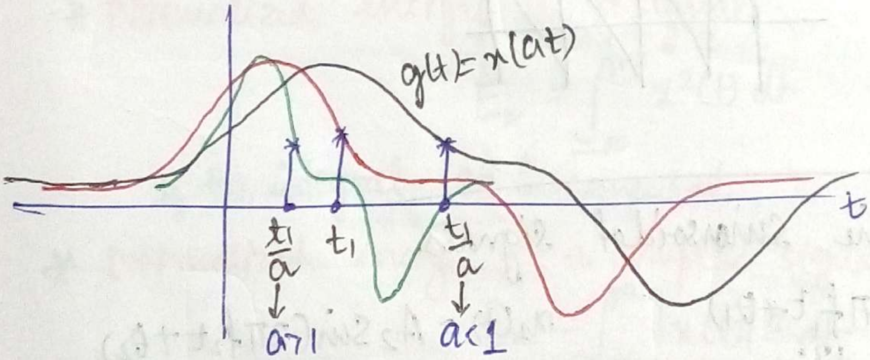
RADAR → warfare applications.

- Noise
- Interference.

For good Reception, Signal : Noise >> 1

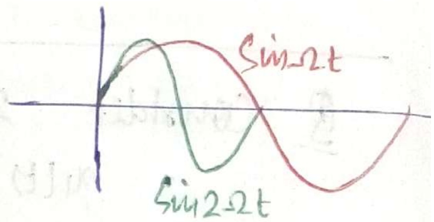
Signal : Interference >> 1

Time Scaling



Cont. time signal → Ω

Discrete → ω



Time Reversal (Reflection about vertical axis) → $g(t) = x(-t)$

Complex signal in Cartesian form.

$$x(t) = x_r(t) + j x_i(t)$$

$$x(t) = |x(t)| \, e^{j \angle x(t)}$$

- $|x(t)| = \left[x_r^2(t) + x_i^2(t) \right]^{1/2}$
- $\angle x(t) = \tan^{-1} \left[\frac{x_i(t)}{x_r(t)} \right]$
- $x_r(t) = |x(t)| \cos(\angle x(t))$
- $x_i(t) = |x(t)| \sin(\angle x(t))$

ANUBHAV JAIN NOTES

Periodic - A signal is said to be periodic if it satisfies

$$x(t+T_0) = x(t)$$

at all time instants t, and for specific value of $T_0 \neq 0$.

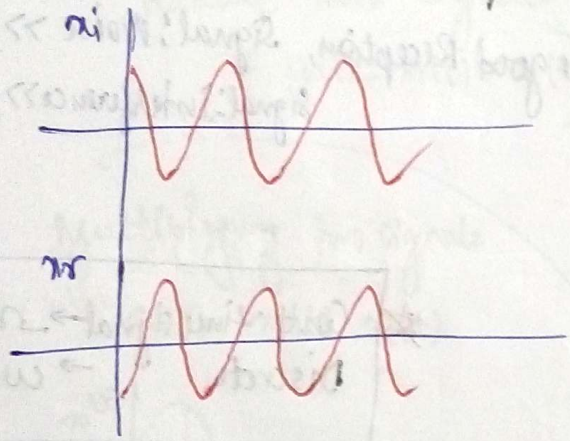
* $T_0 \rightarrow$ smallest value $(2\pi, 4\pi, 6\pi, \dots)$

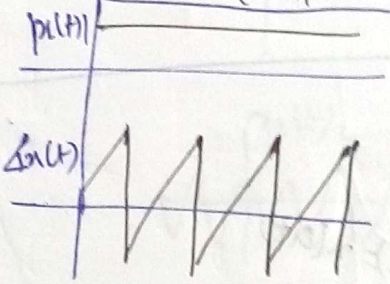
* If a signal is periodic T_0, then it is also periodic with periods of $2T_0, 3T_0, \dots KT_0, \dots$ where $\{K \in Z\}$

* Definition of periodicity excludes DC signals.

Q Consider a signal defined by

$$x(t) = x_r(t) + j x_i(t)$$
$$= A\cos(2\pi f_0 t + \theta) + j A \sin(2\pi f_0 t + \theta)$$





Q Consider 2 cont.-time sinusoidal signals.

$$x_1(t) = A_1 \sin(2\pi f_1 t + \theta_1) \qquad x_2(t) = A_2 \sin(2\pi f_2 t + \theta_2)$$

Determine the conditions under which the sum signal

$$x(t) = x_1(t) + x_2(t)$$

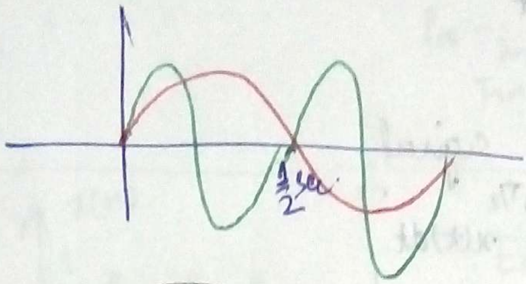
is also periodic. Also determine the fundamental period of the signal $x(t)$ as a function of the relevant parameters of $x_1(t)$ & $x_2(t)$.

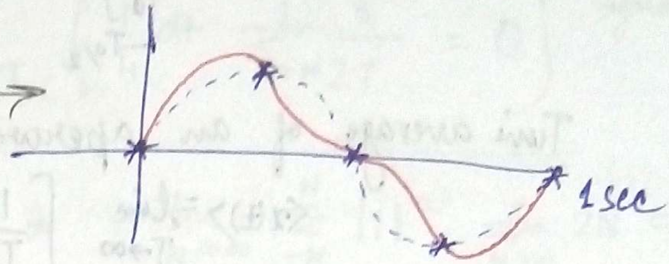
Ans:

$$x_1(t + m_1 T_1) = x_1(t) \qquad T_1 = 1/f_1$$
$$x_2(t + m_2 T_2) = x_2(t) \qquad T_2 = 1/f_2$$
$$x_1(t+T_0) + x_2(t+T_0) = x_1(t) + x_2(t)$$
$$T_0 = m_1 T_1 = m_2 T_2 \Rightarrow \frac{1}{f_0} = \frac{m_1}{f_1} = \frac{m_2}{f_2}$$

$x(t) = \sin(2\pi f_1 t) + \sin(2\pi f_2 t)$

$f_1 \rightarrow 1Hz$, $f_2 \rightarrow 2Hz$





HCF → freq

LCM → Time

04/08/16

Discuss periodicity of signals.

a. $x(t) = \sin(2\pi\, 1.5 t) + \sin(2\pi\, 2.5t)$

b. $y(t) = \sin(2\pi\, 1.5t) + \sin(2\pi\, 2.75t)$

$f_0 = 0.5$

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* Normalized energy of a signal

$$E_x = \int_{-\infty}^{\infty} x^2(t)\, dt$$

if the integral can be computed.

* Normalized energy of a complex signal

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

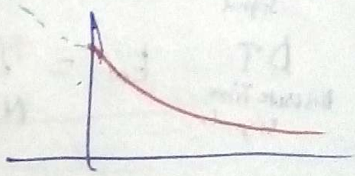
if the integral can be computed.

* Some sinusoid signals → energy cannot be calculated ($E_x \rightarrow \infty$)

can be realized but

Energy of a right-sided exponential signal

$$x(t) = Ae^{-\alpha t} u(t)$$



(Compute Normalized energy).

$$E_x = \int_0^{\infty} A^2 e^{-2\alpha t}\, dt = A^2 \left| \frac{e^{-2\alpha t}}{-2\alpha} \right|_0^{\infty} = \frac{A^2}{2\alpha}$$

ANUBHAV JAIN NOTES

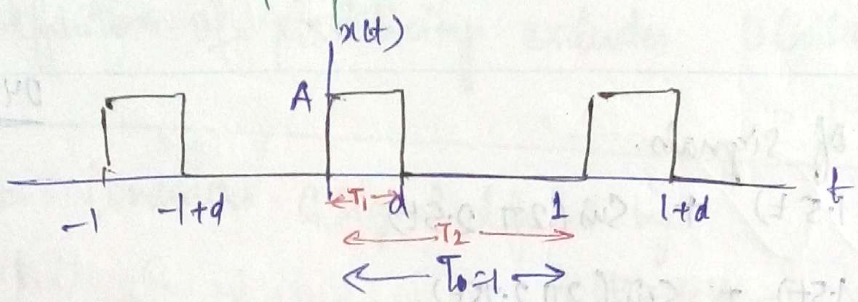
Time average of a signal periodic with period T_0.

$$\langle x(t) \rangle = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t)\,dt$$

Time average of an aperiodic signal

$$\langle x(t) \rangle = \lim_{T \to \infty} \left[\frac{1}{T} \int_{-T/2}^{T/2} x(t)\,dt \right]$$

Time av. of a pulse train



$x(t + k T_0) = x(t)$.

$$\langle x(t) \rangle = \int_0^1 x(t)\,dt = \int_0^d (A)\,dt + \int_d^1 0\,dt = Ad.$$

Duty Ratio :- $\frac{T_1 \to \text{On time}}{T_2 \to \text{Total time}} = d$

$d = 1 \to$ DC signal.

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finite energy ~~Normalized~~

Energy signals are those that have finite energy & zero power.

$E_x < \infty$ and $P_x = 0$

Power signals are those that have finite power and infinite energy.

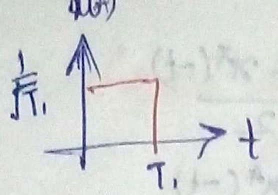
$E_x \to \infty$ and $P_x < \infty$.

CT (Continuous Time Signal) $E_\infty = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2\,dt$

DT (Discrete Time Signal.) $E_\infty = \lim_{N \to \infty} \sum_{n=-N}^{N} |x(n)|^2$

CT $P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\,dt$

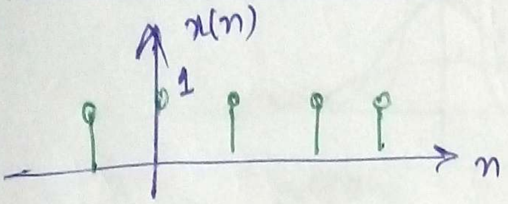
DT $P_\infty = \lim_{N \to \infty} \frac{1}{2N} \sum_{n=-N}^{N} |x(n)|^2$



$$E_\infty = \int_0^{T_1} \frac{1}{T_1}\, dt = 1$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_0^{T_1} \frac{1}{T_1}\, dt = \lim_{T\to\infty} \frac{1}{2T} = 0$$

Finite Energy Signal.



$$E_\infty = \infty \equiv \lim_{N\to\infty} \sum_{-N}^{N} |1|^2 = \lim_{N\to\infty} 2N = \infty$$

$$P_\infty = 1 \equiv \lim_{N\to\infty} \frac{1}{2N} \sum_{-N}^{N} |1|^2 = \lim_{N\to\infty} \frac{2N}{2N} = 1$$

RMS value of a multitone signal.

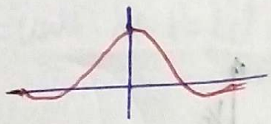
$$\langle x^2(t) \rangle = \frac{a_1^2}{2} + \frac{a_2^2}{2} \qquad x_1 = a_1 \cos(\,-\,) + a_2 \cos(\,-\,)$$

$$X_{RMS} = \sqrt{\frac{a_1^2}{2} + \frac{a_2^2}{2}}$$

$$P_x = \frac{a_1^2}{2} + \frac{a_2^2}{2}$$

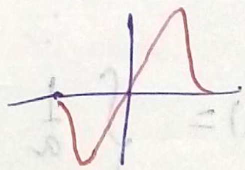
Even symmetry

$x(-t) = x(t)$



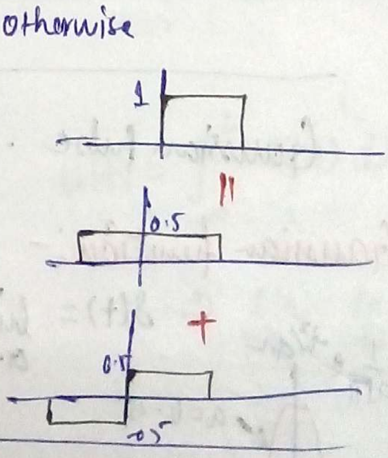
Odd symmetry

$x(-t) = -x(t)$



Representation of pulse

Pulse :- $x(t) = \Pi(t - 1/2) = \begin{cases} 1, & 0<t<1 \\ 0, & \text{otherwise} \end{cases}$



Conjugate Symmetry $x(t) = x^*(-t)$ $\equiv$ Even

Conjugate Antisymmetry $x(-t) = -x^*(t)$ $\equiv$ odd.

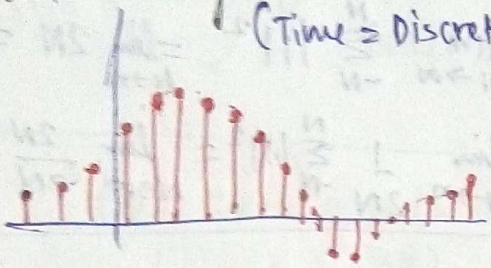
Conjugate symmetric component $\Rightarrow$ $x_E(t) = \dfrac{x(t) + x^*(-t)}{2}$

Antisymmetric ——— $\Rightarrow$ $x_o(t) = \dfrac{x(t) - x^*(-t)}{2}$

$$x(t) = x_E(t) + x_o(t)$$

Discrete Time signal:-

(Time = Discrete, value $\neq$ Discrete)



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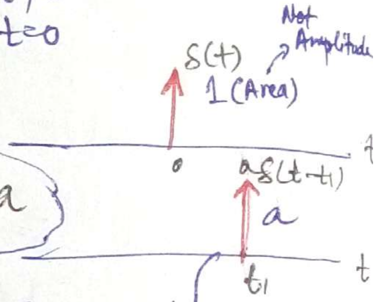
* Unit-impulse function $\rightarrow$ * $\delta(t) = \begin{cases} 0 & , \text{if } t \neq 0 \\ \underset{\rightarrow\infty}{\text{undefined}} & , \text{if } t = 0 \end{cases}$
* Unit-step function
* Unit-pulse function
* Unit-ramp function
* Unit-triangle function
* Sinusoidal signals.

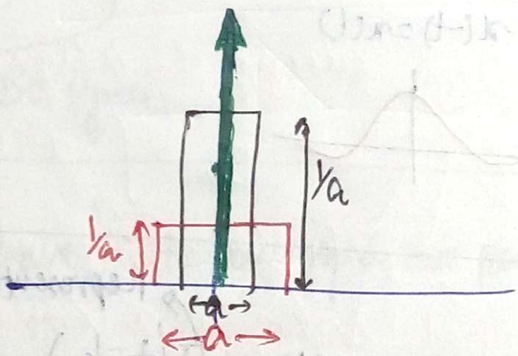
* $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

$$\int_{-\infty}^{\infty} a\,\delta(t-t_1)\,dt = a$$



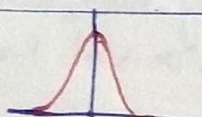
{Amplitude scaled and time shifted}

$$q(t) = \begin{cases} \frac{1}{a} & , |t| < \frac{a}{2} \\ 0 & , |t| > a/2 \end{cases}$$



$$\delta(t) = \lim_{a \to 0} [q(t)]$$

Gaussian Pulse.

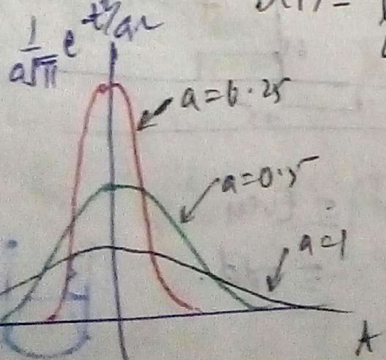


Gaussian function:-

$$\delta(t) = \lim_{a \to 0} \left[\frac{1}{a\sqrt{\pi}} e^{-t^2/a^2} \right]$$

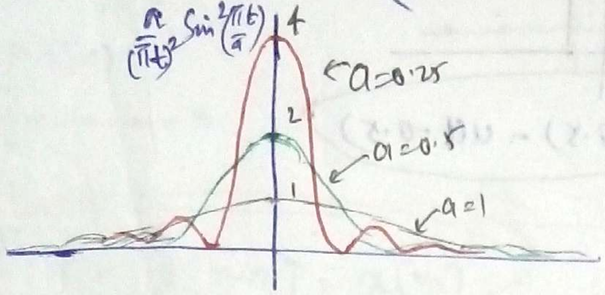
$2\sigma^2 = a^2$

$\mu = 0$



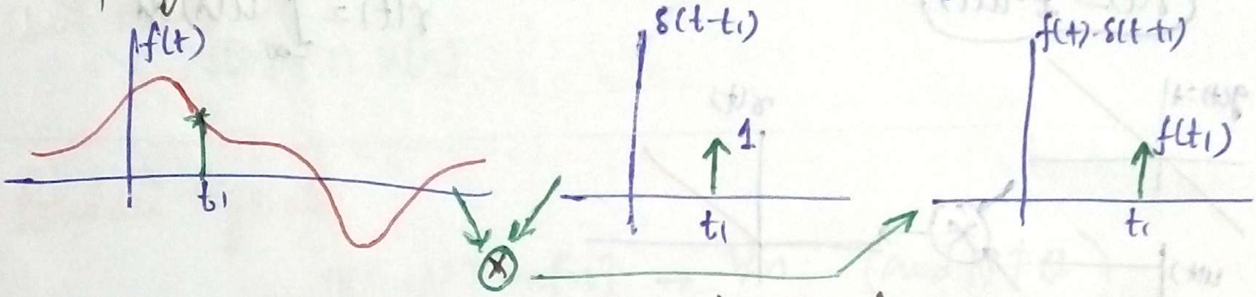
Square sinc pulse

$$\delta(t) = \lim_{a\to 0}\left[\frac{a}{(\pi t)^2}\sin^2\left(\frac{\pi t}{a}\right)\right] = \lim_{a\to 0}\frac{1}{a}\left[\frac{\sin^2(\pi t/a)}{(\pi t/a)^2}\right]$$



08/08/16

Sampling property of the unit-impulse function:- $f(t)\,\delta(t-t_1) = f(t_1)\,\delta(t-t_1)$



Shifting property of the unit-impulse function:-

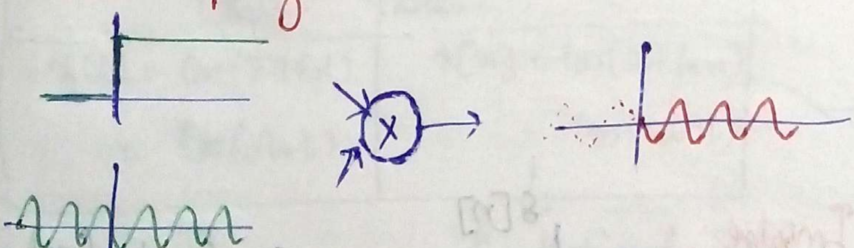
shift: To examine something very carefully to decide what is important or useful.

($f(t)$ = cont. at $t=t_1$)

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_1) = f(t_1)$$

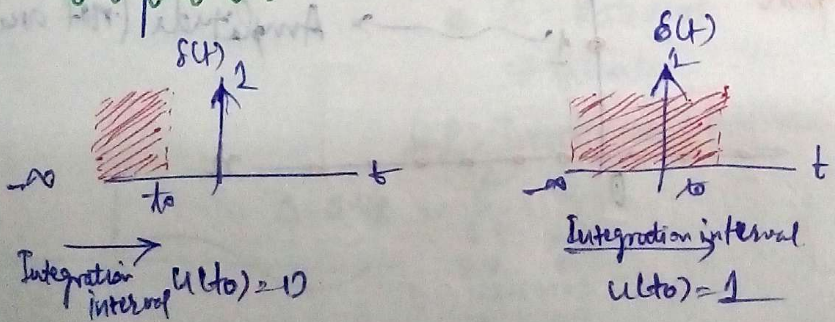
$$\int_{t_1-\Delta t}^{t_1+\Delta t} f(t)\,\delta(t-t_1) = f(t_1)$$

Unit Step Signal

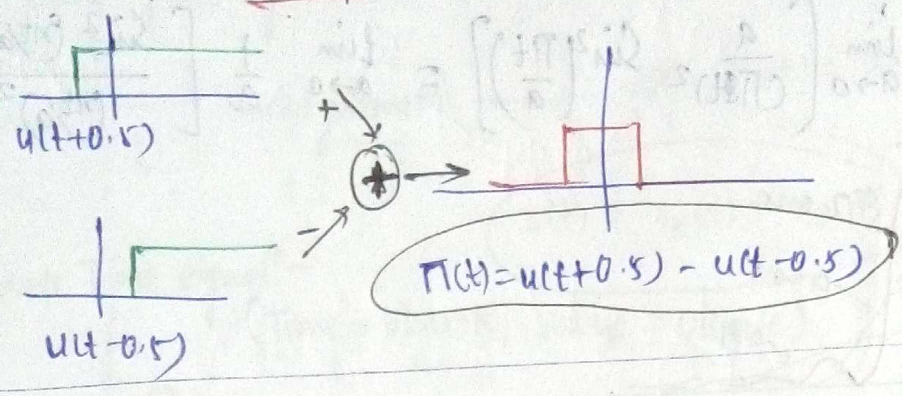


$$u(t) = \int_{-\infty}^{t} \delta(\lambda)\,d\lambda$$

$$\Rightarrow \delta(t) = \frac{du}{dt}$$



Pulse f^n or Box f^n :-

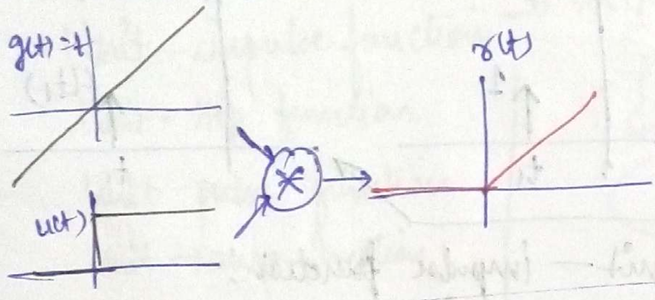


$$\Pi(t) = u(t+0.5) - u(t-0.5)$$

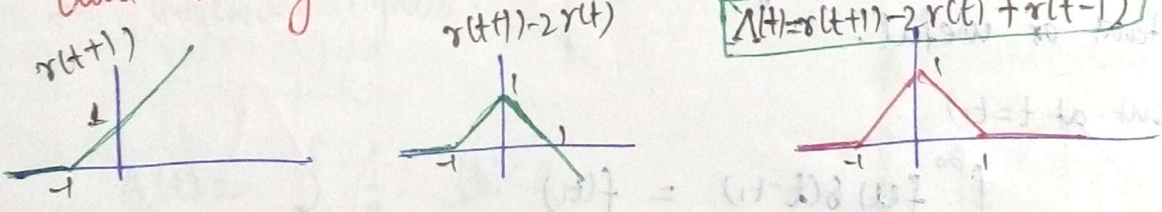
Unit Ramp f^n :-

$$r(t) = t \cdot u(t)$$

$$r(t) = \int_{-\infty}^{t} u(\lambda)\, d\lambda$$



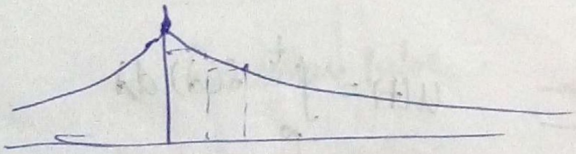
Unit Triangle f^n :-



$$\Lambda(t) = r(t+1) - 2r(t) + r(t-1)$$

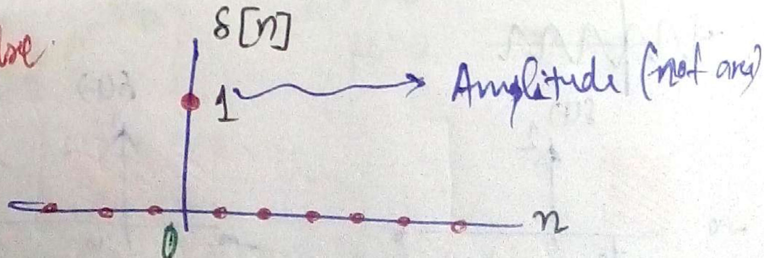
Impulse decomposition for continuous-time signals.

$$\tilde{x}(t) = \sum_{n=-\infty}^{\infty} x(n\Delta)\, \Pi\left(\frac{t-n\Delta}{\Delta}\right)$$



Discrete time case :- Unit Impulse

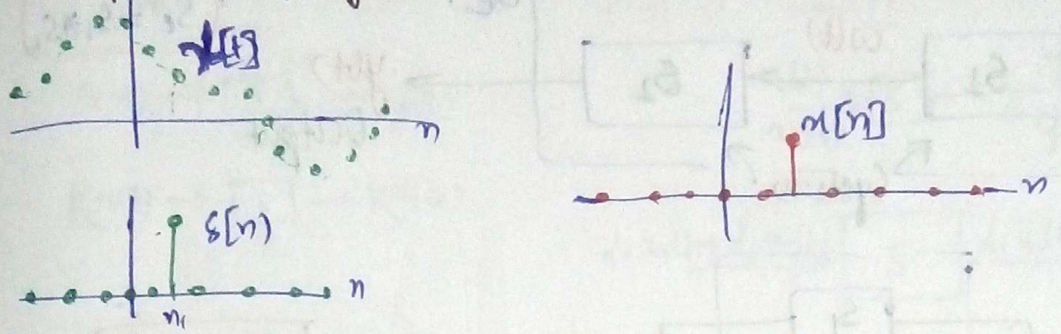
$$\delta[n] = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$$



$$a\,\delta[n-n_1] = \begin{cases} a, & n=n_1 \\ 0, & n \neq n_1 \end{cases}$$

a ↓ Scaling factor

Sampling property -



$$\sum_{-\infty}^{\infty} x[n]\, \delta[n-n_1] = x[n_1]$$

Cont – ( )

Discrete – [ ]

Unit Ramp f^n.

$r[n] = n\, u[n]$

Periodic Signals

$x[n+N] = x[n] \rightarrow \forall n.$ (and $N \neq 0$)

* If a signal is periodic with period N, then it is also periodic with periods 2N, 3N, ... kN, $k \in \mathbb{Z}$

Check Periodicity! -

a. $x[n] = \cos(0.2n) \longrightarrow f_0 = \frac{0.2}{2\pi} \Rightarrow N = \frac{k}{f_0} = 10\pi k \rightarrow \notin \mathbb{Z}$ $\Rightarrow$ Not periodic

b. $x[n] = \cos(0.2\pi n + \pi/5) \rightarrow f_0 = \frac{0.2\pi}{2\pi} = 10k \rightarrow$ Periodic with N = 10 samples

c. $x[n] = \cos(0.3\pi n - \pi/10) \rightarrow f_0 = 0.15$ $\quad N = \frac{k}{0.15} \Rightarrow$ Period with N = 20 samples.

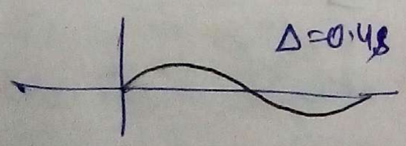
Cont.	Disc
$x(t) = \cos(2\pi f_0 t)$	$x[n] = \cos(2\pi f_0 n)$
$= \cos(\Omega_0 t)$	$= \cos(\omega_0 n)$

$x[n] = \cos\left(2\pi \left(\frac{k}{N}\right) n\right)$ → f_0

* f_0 = Rational

$\Rightarrow$ Periodic

* f_0 - Irrational $\Rightarrow$ Aperiodic



$\Delta = 0.48$

$n\Delta$	
0	n = 0
0.4	n = 1
0.8	n = 2
1.2	n = 3
1.6	n = 4
2.0	n = 5

ANUBHAV JAIN NOTES

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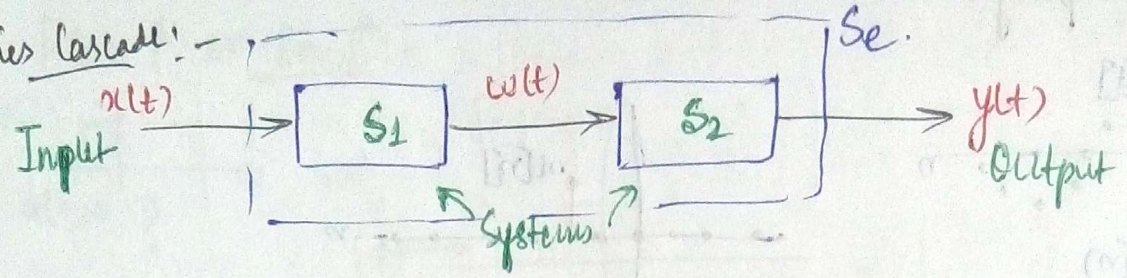
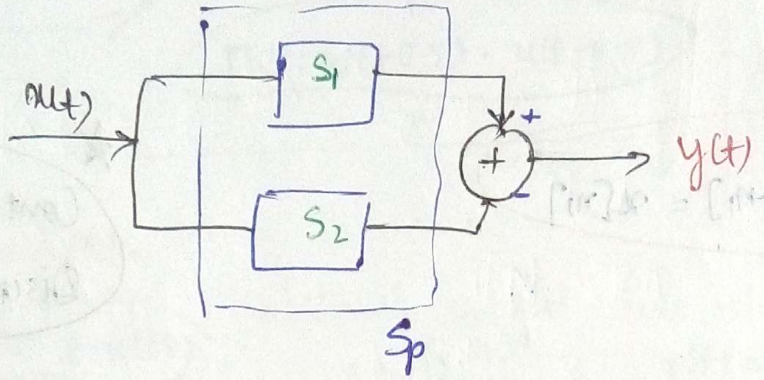
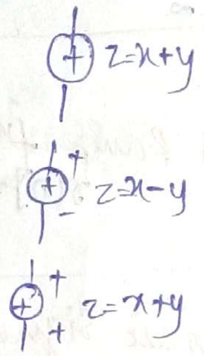
Series Cascade :-



$\boxed{S_e = S_1 * S_2}$



$\boxed{S_p = S_1 + S_2}$

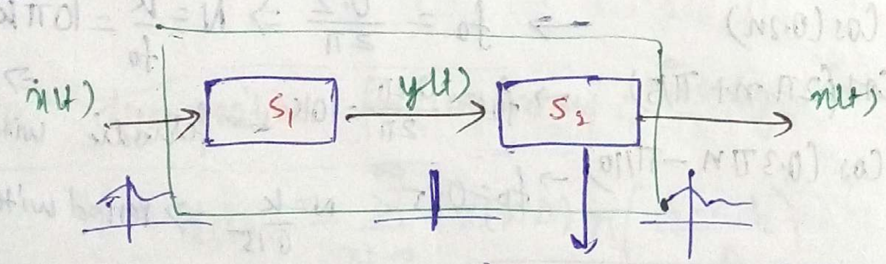


$y(t) = f[x(t)]$

Memory system → if output is dependent on the present value and anything which is not present (past or future). $y(t) = x(t) + x(t-1)$

Memory → Anything not instantaneous.

(Read Opp. →)



Inverse system (May or may not exist);

10/8/16

* Causality — depends on present & past.

* Anti-causal — Present & future

Causal System :-

$$y[n] = f\left[x[n], x[n-1], \ldots \ldots\right]$$

Linear System :- ① Homogenity , ② Additivity.

I/P O/P

$x(t) \rightarrow y_1(t)$

$Kx_1(t) \rightarrow Ky_1(t)$

1 H 2 A

Pg - 53-55 (examples) Opp. Pg-27 Fig 1.27

$x(t) \rightarrow \boxed{S} \rightarrow y(t)$

$Kx(t) \rightarrow \boxed{S} \rightarrow Ky(t)$

$x_1(t), x_2(t) \rightarrow \boxed{S} \rightarrow y_1(t), y_2(t)$

$\alpha_1 x_1(t) + \alpha_2 x_2(t) \rightarrow \boxed{S} \rightarrow \alpha_1 y_1(t) + \alpha_2 y_2(t)$

Time invariance System.

$I/p \rightarrow \boxed{S} \rightarrow O/p$

If I/p is delayed by $\frac{1 hr}{(t)}$, O/p is also delayed by $\frac{1 hr}{(t)}$.

$x(t) \rightarrow \boxed{S} \rightarrow y(t)$

$x(t-t_0) \rightarrow \boxed{S} \rightarrow y(t-t_0)$

$t_0 > 0$ - Delay
$t_0 < 0$ - Advance

LTI System

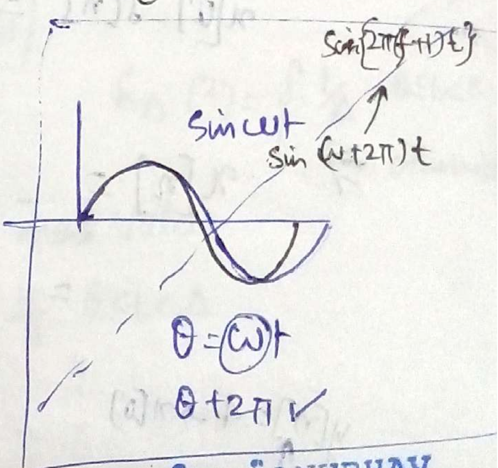
Linear Time Invariant System (If all above properties hold).

Discrete Time Complex Sinusoid.

$x[n] = e^{j\omega_0 n} = e^{j(\omega_0 + 2\pi)n}$

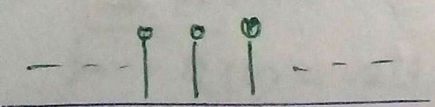
$\omega_0 \rightarrow$ Radians. [$n \rightarrow$ no unit]

(Cont. time $\rightarrow \omega_0$ = rad/s)

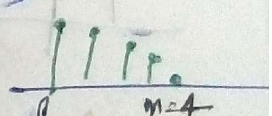


$x[n] = R\{e^{j\omega_0 n}\} = \cos(\omega_0 n)$

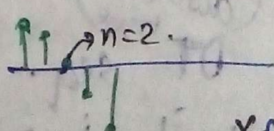
$\omega_0 = 0, 2\pi$ $x[n] = 1$



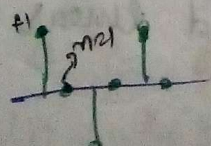
* $\omega_0 = \pi/8, \frac{15\pi}{8}$ $x[n] = \cos[\frac{\pi}{8} n]$



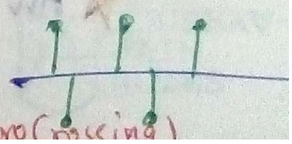
* $\omega_0 = \frac{\pi}{4}$



* $\omega_0 = \pi/2$



* $\omega_0 = \frac{8\pi}{8} = \pi$



(No zero crossing)

ANUBHAV JAIN NOTES

ω_0 ↑es from 0 to π, freq of signal observed → ↑es.
ω_0 ↓es from π to 2π, ———————— → ↓es.

⇒ f_{max} (observed) = f at ($\omega_0 = \pi$)

$$x[n] = e^{j\omega_0 n} = e^{j\omega_0 (n+N)} \rightarrow (N \text{ periodic})$$

$$\Rightarrow e^{j\omega_0 N} = 1$$

$$\omega_0 N = 2K\pi \qquad \{K \in Z\}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{K}{N} \qquad \{N \in Z\}.$$

→ if it is rational → f^n is periodic in n

(rad/s) $\Omega_0 = 2\pi F_0$ (Hz)

(rad) $\omega_0 = 2\pi f_0$ (No unit)

Multitone signal → multiple No of pure sinusoids.

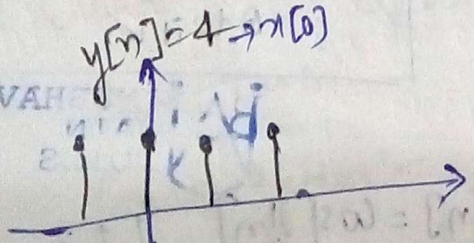
Convolution

$$x[-1]\,\delta[n+1] = \begin{cases} x[-1], & n=-1 \\ 0 & n \neq -1 \end{cases}$$

$$x[0]\,\delta[n] = \begin{cases} x[0], & n=0 \\ 0, & n \neq 0 \end{cases}$$

$$\Rightarrow x[n] = \dots + x[-1]\,\delta[n+1] + x[0]\,\delta[n] + x[1]\,\delta[n-1] + \dots$$

$$= \sum_{K=-\infty}^{+\infty} x[K]\,\delta[n-K]$$



★ An arbitrary (discrete time) DT seq. is a weighted linear combination of shifted unit impulses $\delta[n-k]$

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$\delta[n-k] \rightarrow \boxed{S} \rightarrow h_k[n]$

Linear System (TV)

(Output of LTI System)

H property: $x[k]\ \delta[n-k] \longrightarrow x[k]\ h_k[n]$

A property: $\sum_{k=-\infty}^{\infty} x[k]\ \delta[n-k] \rightarrow \sum_{k=-\infty}^{\infty} x[k]\ h_k[n]$

$x[n] \rightarrow y[n]$

for,

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h_k[n]$$

for TI system,

$h_k[n] = h_0[n-k]$

$= h[n-k]$

$x \rightarrow$ I/p sig.

System characteristic

h: Impulse Response of System

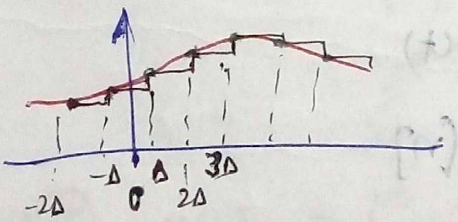
$$\Rightarrow\ y[n] = \sum_{-\infty}^{\infty} x[k].\, h[n-k]$$

(k=0) $y[n] = \ldots + x[-1]\, h[n+1] + x[0]\, h[n] + x[1]\, h[n-1] + \ldots$

$y[n] = x[n] * h[n] = h[n] * x[n]$ (commutative)

Staircase approx of $x(t)$ [Continuous signals]

$$\delta_\Delta(t) = \begin{cases} 1/\Delta, & 0<t<\Delta \\ 0, & \text{otherwise} \end{cases}$$



Area $\therefore\ \Delta\, \delta_\Delta(t)$ has value unity if $0<t<\Delta$

$$\hat{x}(t) = \sum_{k=-\infty}^{+\infty} x(k\Delta)\, \delta_\Delta(t-k\Delta)\, \Delta$$

$$\lim_{\Delta\to 0} \hat{x}(t) = x(t)$$

$$x(t) = \lim_{\Delta\to 0} \sum_{-\infty}^{\infty} x(k\Delta)\, \delta_\Delta(t-k\Delta)\, \Delta$$

$$x(t) = \int x(\tau)\, \delta(t-\tau)\, d\tau$$

Shifted (approx) Input $\qquad$ Impulse Response (O/P)

$$\delta_\Delta(t-k\Delta) \longrightarrow \hat{h}_{k\Delta}(t)$$

$$\underbrace{\sum_{-\infty}^{\infty} x(k\Delta)\,\delta_\Delta(t-\Delta k)\Delta}_{\hat{x}(t)} \quad \text{(weighted L.C.)} \longrightarrow \underbrace{\sum_{-\infty}^{\infty} x(k\Delta)\,\hat{h}_{k\Delta}(t)\,\Delta}_{\hat{y}(t)}$$

$$\hat{y}(t) = \sum_{-\infty}^{\infty} x(k\Delta)\,\hat{h}_{k\Delta}(t)\,\Delta$$

$$\therefore \text{O/p} = \lim_{\Delta\to 0} \sum_{-\infty}^{\infty} x(k\Delta)\,\hat{h}_k(t)\,\Delta$$

$$= \int_{-\infty}^{\infty} x(\tau)\,h_\tau(t)\,d\tau$$

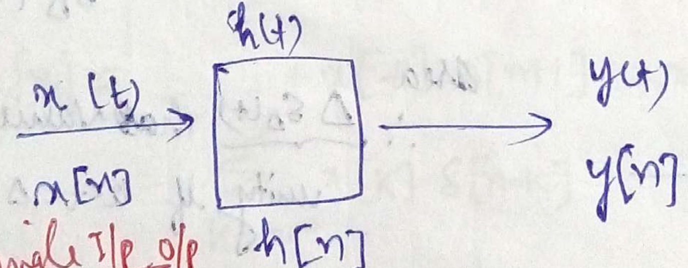
The system is time invariant.

$$h_\tau(t) = h_0(t-\tau) = h(t-\tau)$$

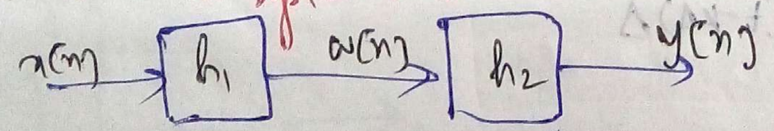
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] \equiv x[n] * h[n]$$

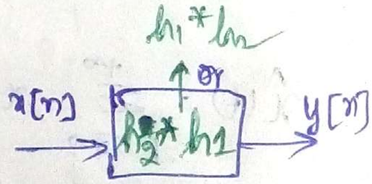
$\hookrightarrow$ Not multiplication

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$



Valid for Single I/P, O/P System only.





$\rightarrow$ convolution

$$y[n] = h_2[n] * \omega[n]$$

$$\omega[n] = h_1[n] * x[n]$$

$$y[n] = h_2[n] * h_1[n] * x[n]$$

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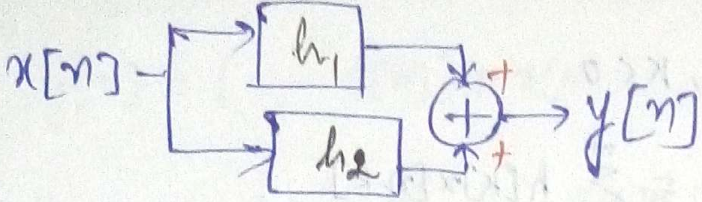


$$y[n] = [h_1[n] + h_2[n]] * x[n]$$

11/08/16

Invertibility

DT $\quad h[n] * h_I[n] = \delta[n]$ (Inverse)

CT $\quad h(t) * h_I(t) = \delta(t)$

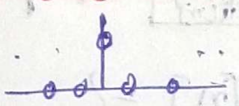
Causality of LTI System.

O/p → dependent on value of x only at that particular instant.

$$y[n] = (n+1)\begin{cases} x[n] \\ -x[n-1] \end{cases}$$

$$\delta[n] \longrightarrow h[n]$$

For causal LTI system,



$$\Rightarrow h[n] = 0, \quad n < 0$$

DT $\quad y[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$

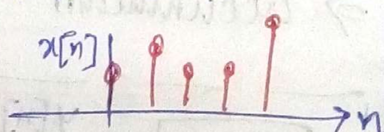
$$= \sum_{k=-\infty}^{n} x[k]\, h[n-k] \quad - F1$$

$$= \sum_{k=0}^{\infty} h(k)\, x[n-k] \quad - F2$$

CT $\quad y(t) = \int_{\tau=-\infty}^{t} x(\tau)\, h(t-\tau)\, d\tau = \int_{\tau=0}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$

Inverse Accumulator : $h[n] = u[n]$ — (1)

Inverse: $h_I[n] = \delta[n] - \delta[n-1]$ — (2)



(1) $y[n] = \sum_{k=0}^{\infty} x[n-k]$

Accu... $x[n] = \{1, 2, 1, 1, 3\}$ $\quad$ $y[n]\ \{1, 3, 4, 5, 8\}$

$n=0$

LTI Causality

$h[k]=0, \; k<0$

DT) $$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$

$$= \dots x[n]h[n-n] + \cancel{x[n+1]}\,h[-1] + \cancel{x[n+2]}\,h[-2] + \dots$$

$$= \dots \cancel{x} + h[-1]\cancel{x[n+1]} + h[0]x[n] + h[1]x[n-1] + \dots$$

(all h(-ve) terms will be cancelled $\because h(x)=0$ for $x<0$)

$$= \sum_{k=0}^{\infty} h[k]\,x[n-k]$$

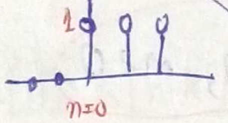
↑ (vertical arrow) represents $n=0$

$$x[n] = \{1, 2, 1, 1, 3\}$$

$\uparrow$ $n=0$ ← $\uparrow$ $n=0$

(always 'n' may not be time)

* $h[n] = u[n]$



$$x[n] \rightarrow \boxed{h[n] = u[n]} \rightarrow y[n]$$

$y[n] = ?$

$$y[n] = \sum_{k=0}^{\infty} h[k]\,x[n-k]$$

$$y[n] = \cancel{h[0]}^{1}\,x[n] + \cancel{h[1]}^{1}\,x[n-1] + \cancel{h[2]}^{1}\,x[n-2] + \dots$$

$$y[n] = x[n] + x[n-1] + x[n-3] + \dots$$

(acting like accumulator)

$$y[n] = \sum_{k=0}^{\infty} x[n-k] = \{1, 3, 4, 5, 8\}$$

$$\boxed{y[n] = y[n-1] + x[n]}$$

$\Rightarrow$ accumulator has a memory

$$x[n] \rightarrow \boxed{h[n]} \xrightarrow{y[n] = x_I[n]} \boxed{h_I[n]} \rightarrow y_I[n] = x[n].$$

$$h[n] * h_I[n] = \delta[n]$$

$$y[n] = (\quad \delta[n]) * x[n]$$

$$= \ldots + \cancel{\delta[-1]\, x[n+1]} + \delta[0]\, x[n] + \cancel{\delta[1]\, x[n-1]} + \ldots$$

$$= x[n]$$

$$x[n] \longrightarrow \boxed{h[n] = \delta[n]} \longrightarrow y[n] = x[n]$$

Output of inverse is original input.

Accu. $\rightarrow$ $\quad x[n] \longrightarrow y[n] = \sum_{k=0}^{\infty} x[n-k]$

Inverse sys. $\rightarrow$ $\quad y[n] = x_I[n] \rightarrow y_I[n] = x[n]$

$$y_I[n] = \sum_{k=0}^{\infty} h_I[k]\, x_I[n-k]$$

$$= \sum_{k=0}^{\infty} \{\delta[k] - \delta[k-1]\}\, x_I[n-k]$$

$$= \sum_{k=0}^{\infty} \{\delta[k]\, x_I[n-k] - \delta[k-1]\, x_I[n-k]\}$$

$$= x_I[n] - x_I[n-1]$$

$$x_I[n] = y[n] = \{1, 3, 4, 5, 8\}.$$

$$y_I[n] = x_I[n] - x_I[n-1] = \{1, 2, 1, 1, 3\} = x[n].$$

$$\boxed{h[n] * h_I[n] = \delta[n]}$$

$h[n] \rightarrow u[n]$ $\qquad$ $h_I[n] \rightarrow \delta[n] - \delta[n-1]$

$$u[n] * (\delta[n] - \delta[n-1]) = u[n] * \delta[n] - u[n] * \delta[n-1]$$

$$= u[n] - u[n-1]$$

$$= \delta[n].$$

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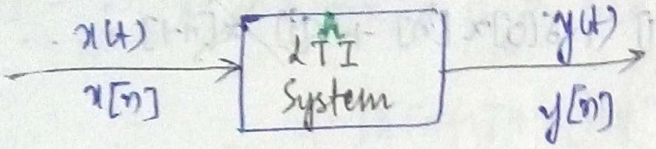
Stability of LTI System :-



BIBO
Bounded Input Bounded Output stability

$$|x[n]| < B$$

$$y[n] = \sum_{-\infty}^{\infty} h[k]\, x[n-k]$$

$$|y[n]| \leq \sum_{k=-\infty}^{\infty} |h[k]|\, |x[n-k]|$$

$$|y[n]| \leq B \left\{ \sum_{k=-\infty}^{\infty} |h[k]| \right\}$$

(Stability $\to |y[n]| =$ finite)

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

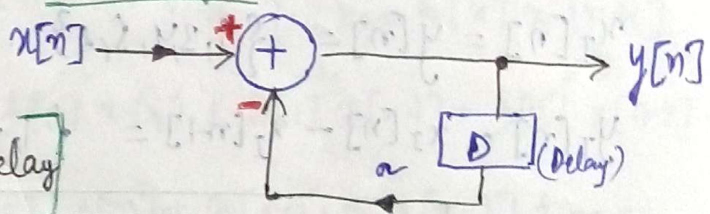
$$y[n] + a\,y[n-1] = b\,x[n]$$

$$y[n] = -a\,y[n-1] + b\,x[n]$$

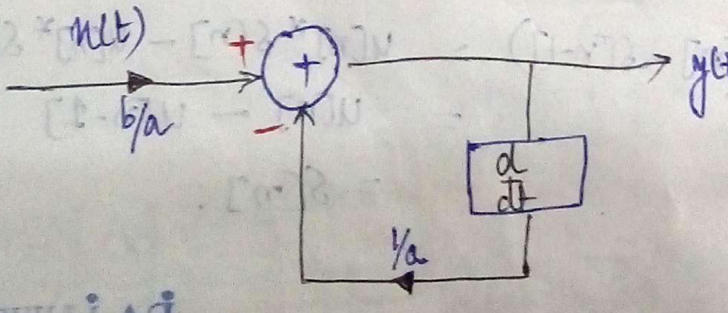
$\rightarrow x[n]$

$\xrightarrow{2} 2x[n]$ (Multiplier)

Block Diagram:-



* Discrete time Signal → Delay
* CT Signal → Differentiation operator $\dfrac{dy(t)}{dt} + a\,y(t) = b\,x(t)$



* In any ckt./algo., never blindly use differentiator.
Always enhances noise.
↓
High freq. component

Signal-to-Noise Ratio

$$x(t) = A\sin(2\pi f_0 t) + \frac{A}{10}\sin(2\pi 10^3 f_0 t)$$

$$x(t) = s(t) + w(t)$$

SNR $= 10^2$ (in dB)

$$\frac{dx}{dt} = 2\pi f_0 A\cos(2\pi f_0 t) + \frac{A}{10} 2\pi 10^3 f_0 \cos(2\pi 10^3 f_0 t)$$

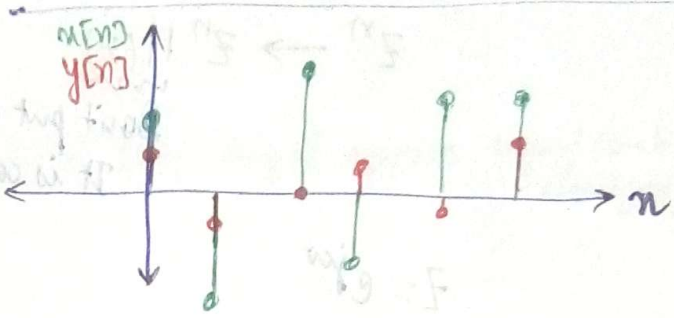
$$SNR = \frac{(2\pi f_0 A)^2}{(A\, 2\pi 10^2 f_0)^2} = 10^{-4}$$

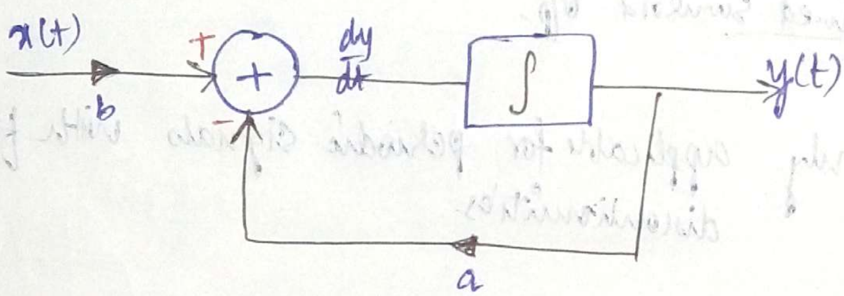
$$y[n] = \frac{1}{2}\left[x[n] + x[n-1]\right]$$

$*\ x[0-1] = 0$

$\Rightarrow y[0] = \frac{1}{2}x[0]$

$\Rightarrow y[1] = \frac{1}{2}\left[x[1] + x[0]\right]$



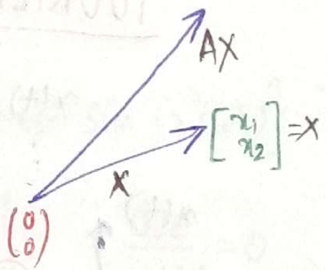


forward part

* Putting integrator in forward part is better than differentiator at feedback part

---

$$Ax = \lambda x$$



I/p x to system 'A', o/p is scaled version of I/p.

Orientation of eigen vectors do not change on multiplication with matrix.

Response of LTI Systems : IP - Complex Exponentials.

ANUBHAV JAIN NOTES

CT-LTI $\quad x(t) \longrightarrow y(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$

$e^{(-1+j2)t} = e^{-t} \cdot e^{j2t}$

Precedence
$\rightarrow$ shift (1), Scale (2)

$$e^{st} \longrightarrow y(t) = e^{st}\int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau$$

$$e^{st} \longrightarrow e^{st} H(s)$$

Any signal can not be eigen signal.

$e^{j\Omega t} \rightarrow e^{j\Omega t} H(j\Omega) \Big|_{s = j\Omega}$

DT-LTI

$$x[n] = z^n \longrightarrow y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

$$= \underbrace{z^n}_{I/p} \underbrace{\left[\sum_{k=-\infty}^{\infty} h[k]\, z^{-k}\right]}_{\text{scale} = H(z)}$$

$$z^n \longrightarrow z^n H(z)$$

Don't put in verticle bracket. It is const f^n.

$$z = e^{j\omega}$$

I/p → Sustained sinusoid O/p.

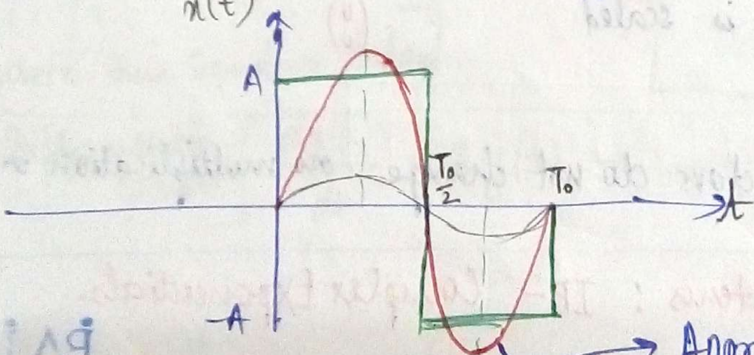
Fourier Series → Only applicable for periodic signals with finite discontinuities.

* Dirichle Conditions

FOURIER SERIES

$$x(t) = x(t + T_0) \quad \forall t$$

$$\Omega_0 = \frac{2\pi}{T_0}$$



Approximation of signal

$$x^{(1)}(t) = b_1^{(1)} \sin(\Omega_0 t)$$

Approx error :- $\varepsilon_1(t) = x(t) - x^{(1)}(t) = x(t) - b_1^{(1)} \sin(\Omega_0 t)$