26/09/16

DISCRETE TIME FOURIER TRANSFORM

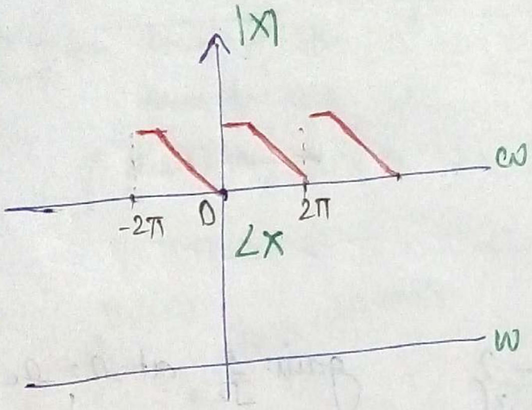
A: $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$

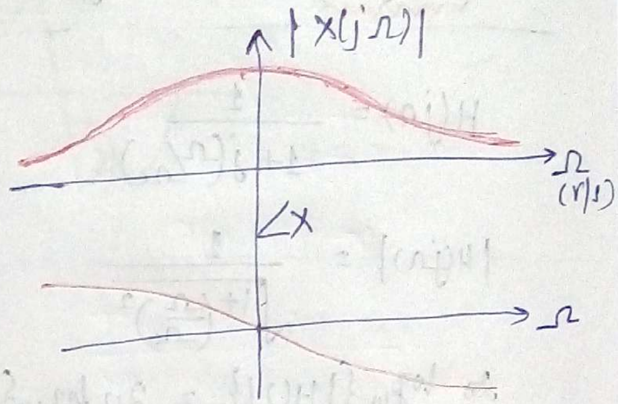
S: $x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$

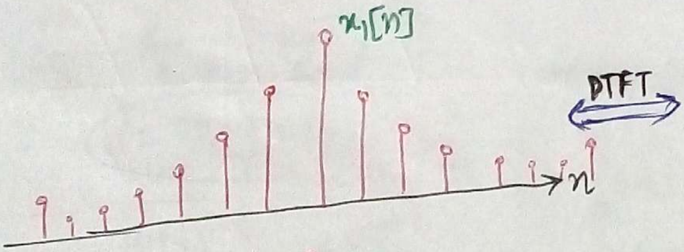
$X(e^{j(\omega+2\pi)}) = X(e^{j\omega})$

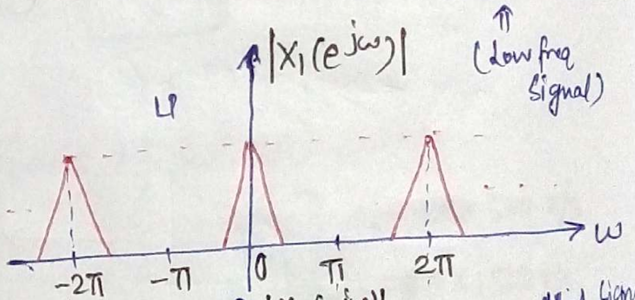
$F \to (0, 1, \ldots 10^3 \ldots 10^6 \ldots)$ Hz

$\Omega \to 2\pi(\quad)$ r/s.

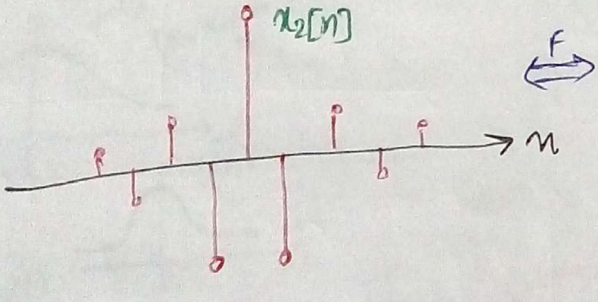


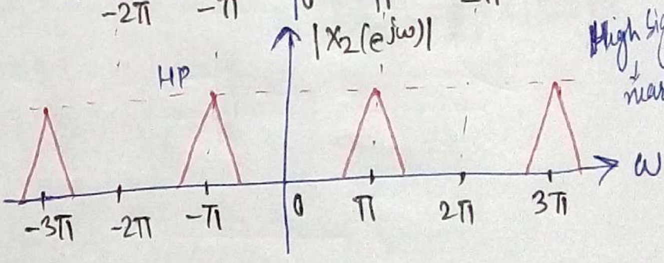






Concentrated near 0
⇑
(Low freq signal)





High signal near π

$x_1[n] = \{1, 1, 1, \ldots\}$ or $x_1[n] = \{1, a, a^2, \ldots\}$ $1 > a > 0$

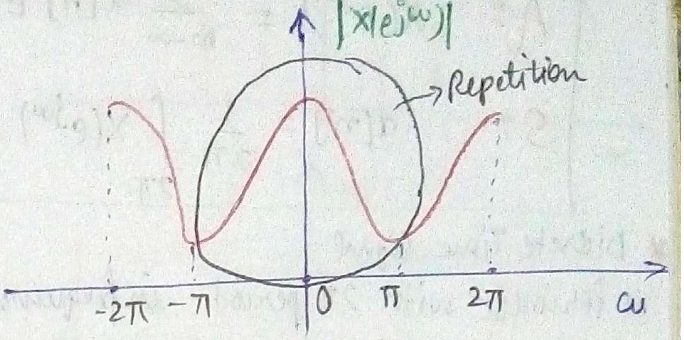
$x_2[n] = \{1, -1, 1, -1, \ldots\}$ or $x_2[n] = \{1, a, a^2, \ldots\}$ $-1 < a < 0$

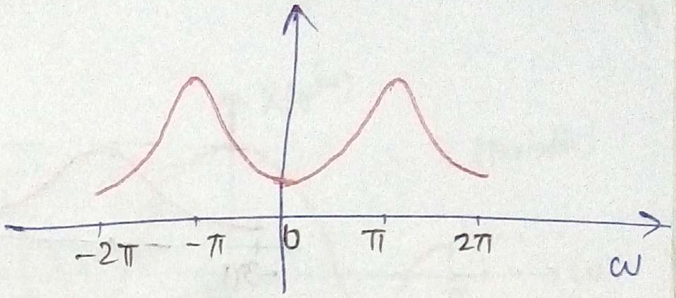
$X_1(e^{j\omega}) = ?$

$X_2(e^{j\omega}) = ?$

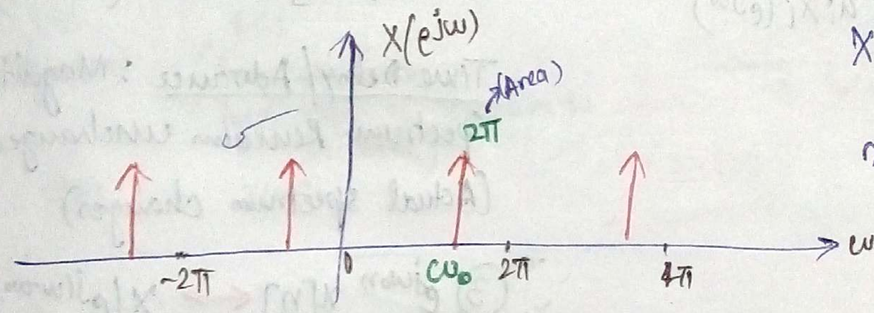
$x[n] = a^n u[n]$ $\quad |a| < 1$

$$X(e^{j\omega}) = \frac{1}{1 - ae^{-j\omega}}$$





* In some transforms, energy is lost
⇒ They are not reproducable back to original form.



$$X(e^{j\omega}) = 2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi l)$$

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega})\, e^{j\omega n}\, dn.$$

$$= \frac{1}{\cancel{2\pi}} \cancel{2\pi} \int_{2\pi} \delta(\omega - \omega_0 - 2\pi l)\, e^{j\omega n}\, d\omega$$

$$= e^{j\omega_0 n} \cdot \cancel{e^{j2\pi r n}} \rightarrow 1$$

$$= e^{j\omega_0 n}$$

$$x[n] = \sum_{k=\langle N\rangle} C_k\, e^{jk\left(\frac{2\pi}{N}\right)n}$$

$2\pi \rightarrow$ divided into 'N' points.

Let $N = 8$ $\quad \omega_0 = \frac{2\pi}{8} \times 0$

$\omega_1 = \frac{2\pi}{8} \times 1$

$\omega_8 = \frac{2\pi}{8} \times 8$

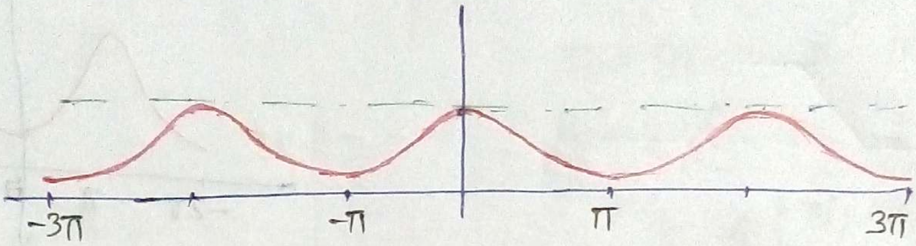
$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} 2\pi C_k\, \delta\left(\omega - 2\pi \frac{k}{N}\right)$$

28/09/16

A: $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$

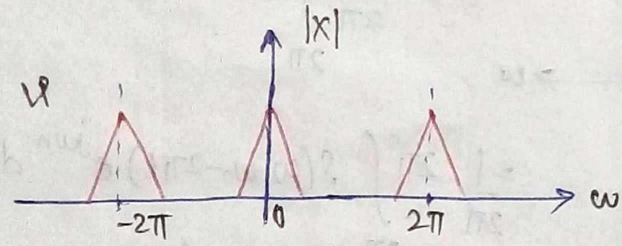
S: $x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega})\, e^{jn\omega}\, d\omega$

* Discrete Time Signal is Periodic with 2π period in frequency domain : $X(e^{j(\omega + 2m\pi)}) = X(e^{j\omega})$



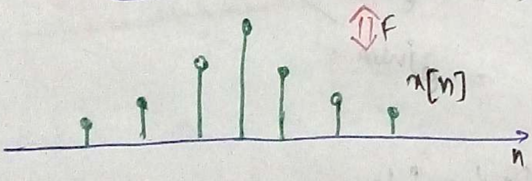
1. $x_i[n] \longleftrightarrow X_i(e^{j\omega})$

$$\sum_{i=1}^{P} a_i\, x_i[n] \longleftrightarrow \sum_{i=1}^{P} a_i X_i(e^{j\omega})$$



(Narrow Band Low Pass)

$\Updownarrow F$

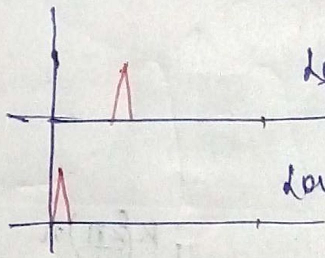


2. $x[n] \longleftrightarrow X(e^{j\omega})$

$x[n-n_0] \longleftrightarrow e^{-jn_0\omega}(X(e^{j\omega}))$

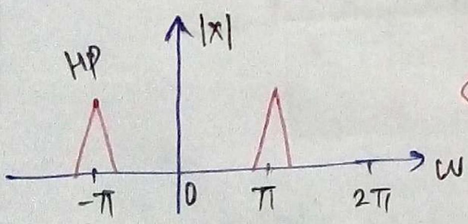
Time Delay/Advance : Magnitude Spectrum Remain unchanged (Actual spectrum changes)

3. $e^{j\omega_0 n}\, x[n] \longleftrightarrow X(e^{j(\omega-\omega_0)})$

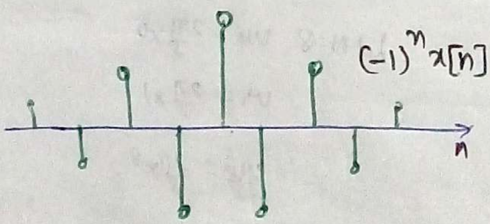


~~Low~~ Pass Narrow Band

Low Pass + Narrow Band



$\Leftrightarrow$



$(-1)^n x[n] \equiv e^{j\omega_0 n}\, x[n]$ (with $\omega_0 = \pi$)

$\Updownarrow$

$X(e^{j(\omega-\pi)})$

4. $x[n] \longleftrightarrow X(e^{j\omega})$

$x[-n] \longleftrightarrow X(e^{-j\omega})$

* Expansion in Time domain $\Leftrightarrow$ Compression in freq domain.

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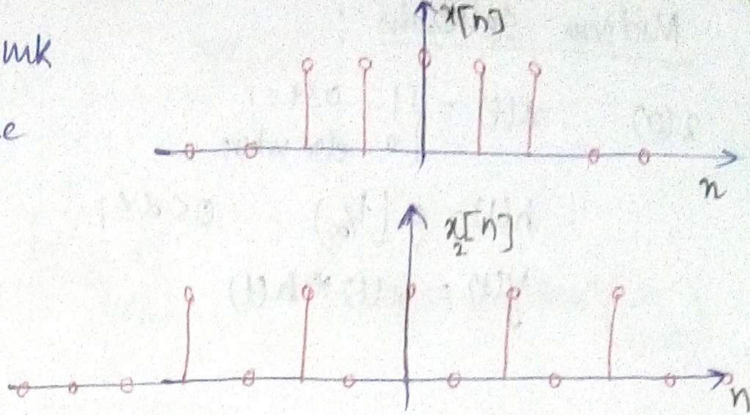
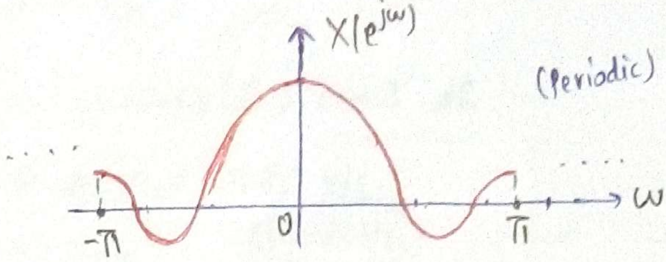
(5) * $x_k[n] = \begin{cases} x[n/k], & \text{if } n = mk \\ 0, & \text{otherwise} \end{cases}$



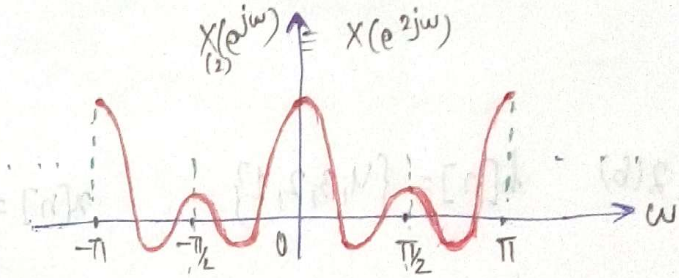
$$X_{(k)}(e^{j\omega}) = \sum_{m=-\infty}^{\infty} x_{(k)}[\underbrace{mk}_{n}]\, e^{-j\omega \underbrace{mk}_{n}}$$

$$= \sum_{m=-\infty}^{\infty} x[m]\, e^{-j(k\omega)m} = X(e^{jk\omega})$$

$$x_k[mk] = x\left[\frac{mk}{k}\right] = x[m]$$



- All signal processing is done in Time domain only.



LHS	RHS
* (Real Operation)	(For understanding implication of operation to be done)
* $x[n]$	$X(e^{j\omega})$

(6) $x[n] \longleftrightarrow X(e^{j\omega})$

$$n\,x[n] \longleftrightarrow j\frac{d}{d\omega} X(e^{j\omega})$$

(7) Parseval's Rel^n

ESD → (Energy Spectral Density)

$$\boxed{\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} \overbrace{|X(e^{j\omega})|^2}^{ESD}\, d\omega}$$

(8)

$$\boxed{\begin{aligned} y[n] &= h[n] * x[n] \\ Y(e^{j\omega}) &= H(e^{j\omega})\, X(e^{j\omega}) \end{aligned}}$$

$$y[n] = x_1[n]\, x_2[n]$$

$$Y(e^{j\omega}) = \frac{1}{2\pi}\int_{2\pi} X_1(e^{j\theta})\, X_2(e^{j(\omega-\theta)})\, d\theta$$

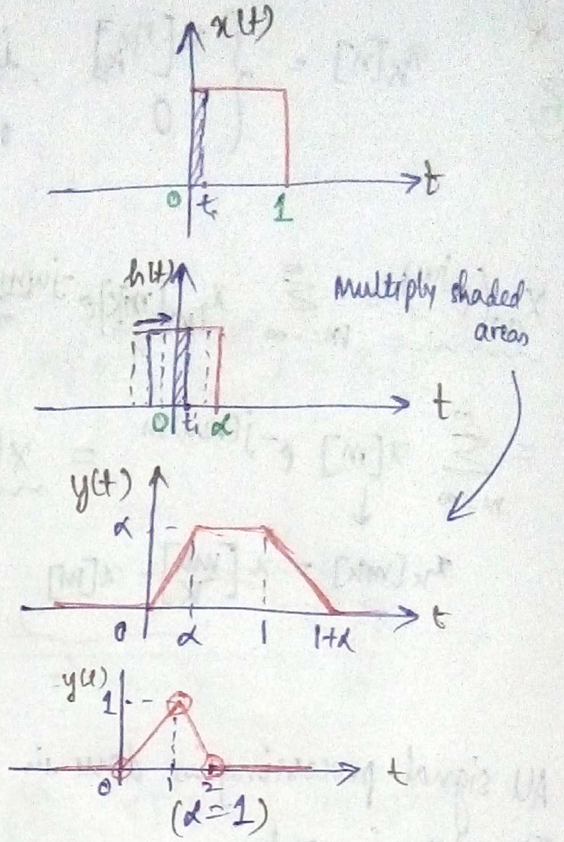
* Read Group Delay in Opp^{m} (+ Pictures)

Midsem Solutions :

2(a) $x(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0 & \text{else where} \end{cases}$

$h(t) = x(t/\alpha) \qquad 0 < \alpha \le 1$

$y(t) = x(t) * h(t)$

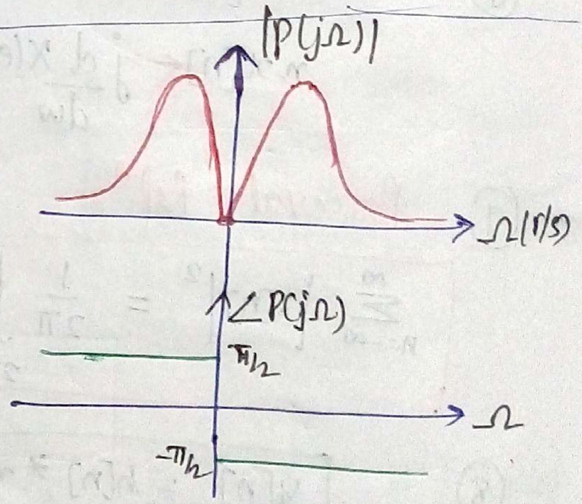


2(b) $h[n] = \{4, 3, 2, 1\}$ $\qquad x[n] = \{-3, 7, 4\}$

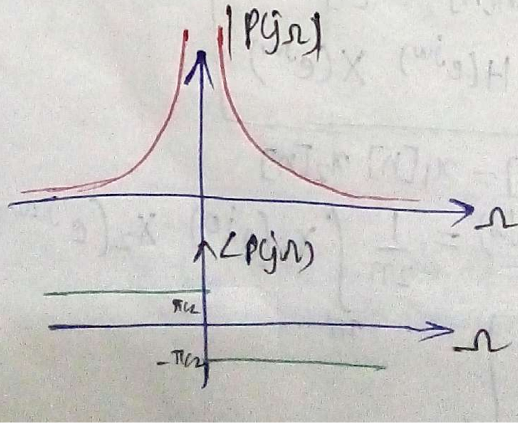
x \ h	−3	7	4
4	−12	28	16
3	−9	21	12
2	−6	14	8
1	−3	7	4

$y[n] = \{-12, 19, 31, 23, 15, 4\}$

3(a) $P(j\Omega) = \int_{-\infty}^{\infty} p(t)\, e^{-j\Omega t}\, dt = \dfrac{-2j\Omega}{\Omega^2 + a^2}$



3(b) $P(j\Omega) = -\dfrac{2j}{\Omega}$



3(c)

$$X(j\Omega) \longrightarrow \boxed{H(j\Omega)} \longrightarrow Y(j\Omega)$$

$$H(j\Omega) = \frac{1}{j\frac{\Omega}{\underbrace{2\pi 80}_{\Omega_c}} + 1}$$

$$\angle H(j\Omega) = -\tan^{-1}\left(\frac{\Omega}{2\pi 80}\right)$$

$$|H(j\Omega)| = \frac{1}{\sqrt{1+\left(\frac{\Omega}{\Omega_c}\right)^2}}$$

$$20\log|H(j\Omega)| = -20\log(\sqrt{2}) = -3 \text{ dB}.$$

$$\Rightarrow \Omega = \Omega_c = 2\pi 80 \text{ r/s}.$$

$F_c = 80\text{Hz}$

$F_i^{(1)} = 20\text{Hz}$ $\quad\quad$ $F_i^{(2)} = 200\text{Hz}$.

$x(t) = 5\cos(2\pi F_i t)$

$$|H|_{20\text{Hz}} = \frac{1}{\sqrt{1+\left(\frac{1}{4}\right)^2}} = \frac{4}{\sqrt{17}}$$

$$\angle H = -\tan^{-1}(1/4)$$

O/p Mag. = (I/p Mag) x (Gain at $F_i = \underset{20\text{Hz}}{\uparrow}$)

$$y(t) = 5\times\frac{4}{\sqrt{17}}\cos\left(2\pi\times 20t - 0.245\right)$$

4(a). 1. Shift
2. Scale/Multiply.

4(c). (i) No
(ii) Mathematically, $\frac{dy}{dt}$ is not causal (we can do forward diff^n or backward diff^n)
But here, Yes.
(iii) Yes
(iv) Yes

29/09/1

Laplace Transform

For continuous time case

(Generalized version of Fourier Transform)

→ Analysis can be done for unstable systems as well.

(in Resonance)

$$e^{st} \longrightarrow H(s)\, e^{st}$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\, e^{s(t-\tau)}\, d\tau$$

$$= e^{st} \int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau$$

$$= e^{st} H(s)$$

Analysis eqn

$$H(s) = \int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau$$

$$s = \sigma + j\Omega$$

$$H(s) = \int_{-\infty}^{\infty} h(\tau) e^{-\sigma\tau}\, e^{-j\Omega\tau}\, d\tau$$

$$\mathcal{L}\big(x(t)\big) \longleftrightarrow F\big(x(t)\, e^{-\sigma t}\big)$$

Laplace Transform — Fourier Transform

$$x(t) = e^{-at} u(t)$$

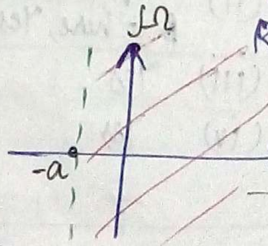
$$X(s) = ?$$

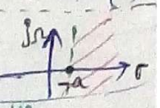
$$H(s) = \int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau$$

$$X(j\Omega) = \int_{-\infty}^{\infty} \left(e^{-at} u(t)\, e^{-j\Omega t}\right) dt = \int_{0}^{\infty} e^{-(a+j\Omega)t}\, dt = \frac{1}{a+j\Omega}, \quad a>0.$$

$$X(s) = X(\sigma + j\Omega) = \frac{1}{(a+\sigma)+j\Omega}, \quad a+\sigma > 0, \quad \sigma > -a$$

Region of Convergence → ROC





$$\boxed{e^{-at} u(t) \longleftrightarrow \frac{1}{s+a}, \quad Re\{s\} > -a}$$

$$x(t) = -e^{-at}\, u(-t)$$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= -\int_{-\infty}^{0} e^{-at} e^{-st}\, dt = \frac{e^{-(a+s)t}}{a+s}\Bigg|_{-\infty}^{0} = \frac{1 - e^{+(a+s)(+\infty)}}{a+s} = \frac{1}{s+a}$$

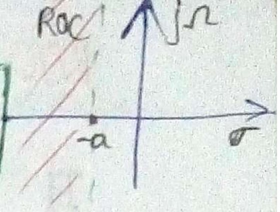
must be -ve

$Re\{s\} < -a$

ROC

Laplace Transform – Mandatory to specify ROC.

$$\boxed{-e^{-at}u(-t) \longleftrightarrow \frac{1}{s+a} \; ; \quad Re\{s\} < -a}$$

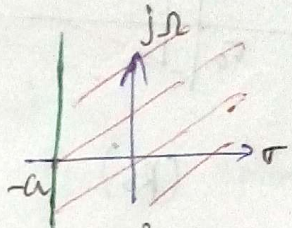


- L.T. can be computed only in ROC
  - If Imaginary axis is within ROC, then only F.T. exist
  - So, if $a<0$, F.T. does not exist in previous case when $x(t) = e^{-at}u(t)$.
  - ~~F.T. is done on imaginary plane~~.
  - ★ F.T. is laplace transform evaluated on $j\Omega$ axis.

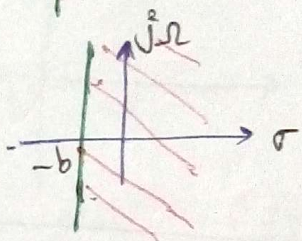
$$x(t) = k_1 e^{-at} u(t) + k_2 e^{-bt} u(t)$$

$$= x_1(t) + x_2(t)$$

$$X_1(s) = \frac{k_1}{s+a}, \quad ROC: Re\{s\} > -a$$



$$X_2(s) = \frac{k_2}{s+b}, \quad ROC: Re\{s\} > -b$$



$$X(s) = \frac{k_1(s+b) + k_2(s+a)}{(s+a)(s+b)}$$

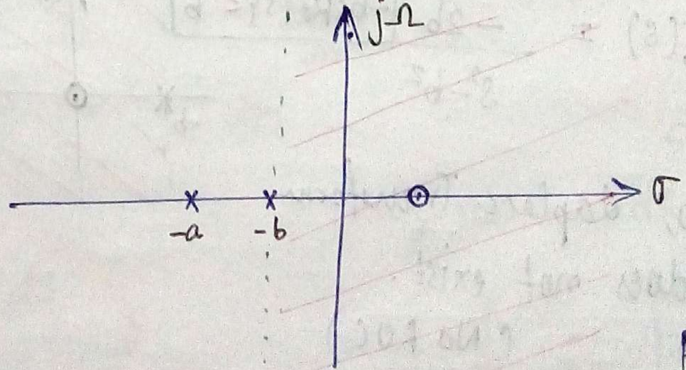
$$X(s) = \frac{(k_1+k_2)\left[s + \frac{k_1 b + k_2 a}{k_1+k_2}\right]}{(s+a)(s+b)}, \quad ROC\{X(s)\} > -b$$

$$= \frac{N(s)}{D(s)}$$

$D(s)$: $s=-a$, $s=-b$ → Poles ×

$N(s)$: $s = -\left(\frac{k_1 b + k_2 a}{k_1+k_2}\right)$ ← Zero o

Let $k_1=3$ $k_2=-2$
$a=2$ $b=1$



Pull zero plot.

★ Pole location is excluded from ROC.

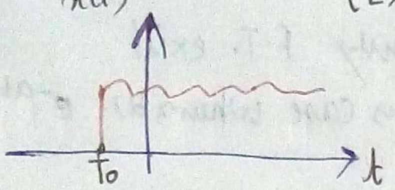
★ $x(t)$ → Finite duration, absolutely integratable
⇓
ROC = Entire Region

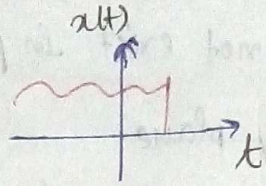
Equal Roots: $s=a$, $s=a$ ✱ (Double Cross)
2^{nd} order ◎

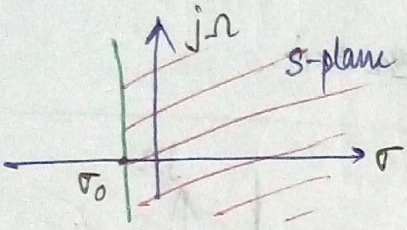
03/10/16

ROC in Laplace Transform

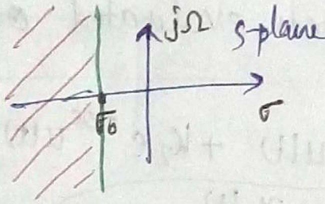
If $x(t)$ is RS (Right sided) (LS), if $Re\{s\} = \sigma_0$ is in ROC then $\forall s$, for which $Re\{s\} > \sigma_0$ will be in ROC. ($Re\{s\} < \sigma_0$)



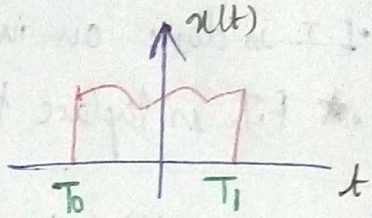


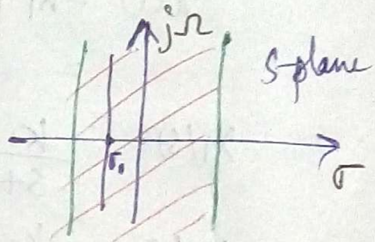


(RS)



(LS)

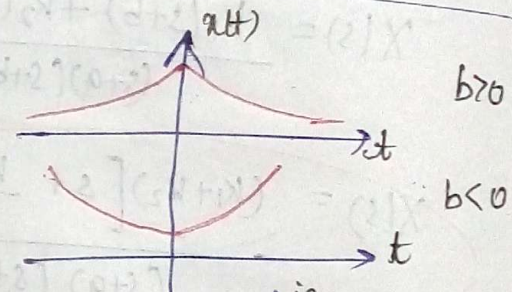




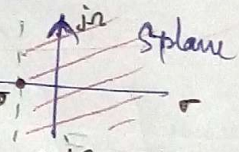
A strip contains σ_0 ⟵ (2 sided)

$x(t) = e^{-b|t|}$

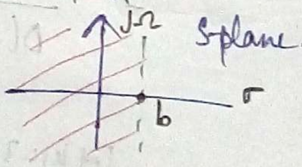
$= e^{-bt}u(t) + e^{+bt}u(-t)$



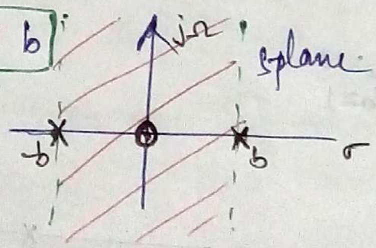
$e^{-bt}u(t) \overset{L}{\longleftrightarrow} \frac{1}{s+b}$ $\quad Re\{s\} > -b$



$e^{bt}u(-t) \overset{L}{\longleftrightarrow} \frac{-1}{s-b}$ $\quad Re\{s\} < b$



$X(s) = \frac{-2b}{s^2-b^2}$ $\boxed{-b < Re\{s\} < b}$



If $b \le 0$, Laplace Transform does not exist. (No ROC)

Here, ROC is bounded by poles.

Laplace Transform exists only for $b > 0$.

* If $x(t)$ is exponential ⇒ $X(s)$ is rational.

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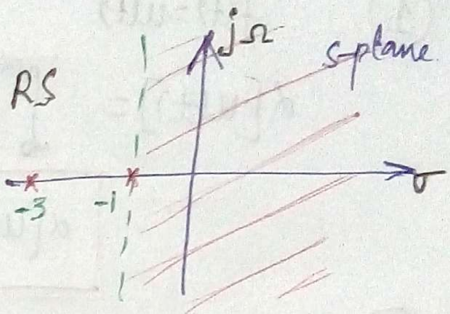
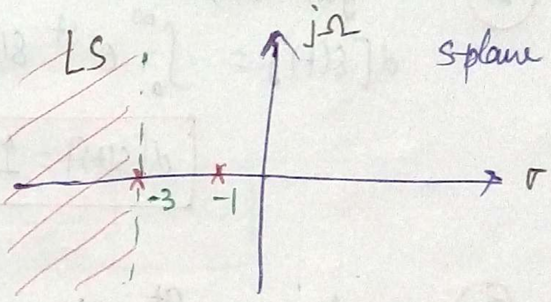
* Inside ROC, there should not be any poles.

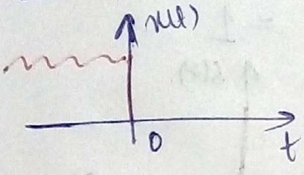
* If $X(s)$ rational &
→ If $x(t)$ RS
↳ ROC will be right of right most pole
→ If $x(t)$ LS ⟶ " " " left " left "

$$X(s) = \frac{1}{(s+1)(s+3)}$$



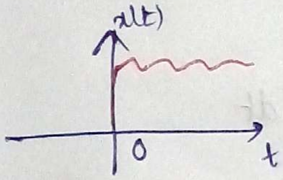


Anti-causal



for $t>0$, $x(t)=0$ ⇒ LS
$t>0$, $x(t)$ = finite

Causal



for $t<0$, $x(t)=0$ ⇒ RS
$t\geq 0$, $x(t)$ = finite

* $\mathcal{F}\{x(t)\} = \mathcal{L}\{x(t)\}_{\sigma=0}$

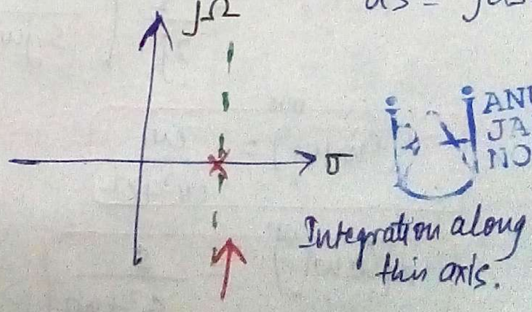
* $X(\sigma+j\Omega) = \mathcal{F}\{x(t)e^{-\sigma t}\} = \int_{t=-\infty}^{\infty}\{x(t)e^{-\sigma t}\}e^{-j\Omega t}\,dt$

⟶ $x(t)e^{-\sigma t} = \mathcal{F}^{-1}\{X(\sigma+j\Omega)\} = \frac{1}{2\pi}\int_{\Omega=-\infty}^{\infty} X(\sigma+j\Omega)e^{j\Omega t}\,d\Omega$

⇒ $x(t) = \frac{1}{2\pi}\int_{\Omega=-\infty}^{\infty} X(\sigma+j\Omega)e^{(\sigma+j\Omega)t}\,d\Omega$

$s = \sigma + j\Omega$
$ds = j\,d\Omega$

⇒ $x(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}\,ds$



05/10/16

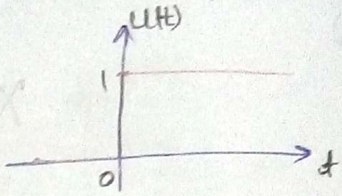
Circuit Analysis in s-domain

One sided L.T. $\qquad s = \sigma + j\omega$

$$\boxed{\mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t)\,dt = F(s)}$$

① $f(t) = u(t)$

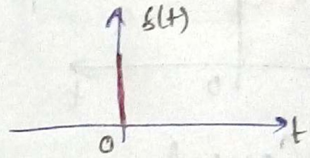
$$\mathcal{L}[u(t)] = \int_0^\infty e^{-st} u(t)\,dt = \left[\frac{e^{-st}}{-s}\right]_0^\infty$$



$$\boxed{\mathcal{L}[u(t)] = 1/s}$$

② $f(t) = \delta(t)$

$$\mathcal{L}[\delta(t)] = \int_0^\infty e^{-st}\,\delta(t)\,dt = \int_{0^-}^{0^+} e^{-st}\delta(t)\,dt = 1$$



$$\boxed{\mathcal{L}[\delta(t)] = 1}$$

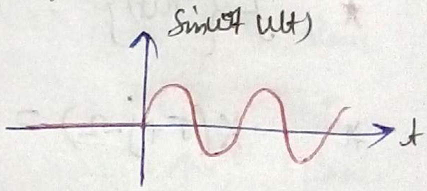
③ $f(t) = e^{at}\,u(t)$

$$\mathcal{L}[e^{at}u(t)] = \int_0^\infty e^{-st} e^{at} u(t)\,dt = \int_0^\infty e^{-(s-a)t}\,dt$$

$$\boxed{\mathcal{L}[e^{at}u(t)] = \frac{1}{s-a}}$$

④ $\mathcal{L}[\sin\omega t]$ & $\mathcal{L}[\cos\omega t]$

$$\hookrightarrow \int_0^\infty e^{-st} \sin\omega t\, u(t)\,dt$$



$$\sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$\mathcal{L}[\sin\omega t] = \frac{1}{2j}\int_0^\infty \left[e^{-(s-j\omega)t} - e^{-(s+j\omega)t}\right] u(t)\,dt$$

$$= \frac{1}{2j}\left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right] = \frac{s+j\omega - s + j\omega}{2j\,(s^2+\omega^2)}$$

$$\boxed{\mathcal{L}[\sin\omega t \; u(t)] = \frac{\omega}{\omega^2+s^2}}$$

$$\boxed{\mathcal{L}[\cos\omega t \; u(t)] = \frac{s}{s^2+\omega^2}}$$

$\mathcal{L}\left[\frac{df}{dt}\right] = ?$ $\qquad\qquad \mathcal{L}[f(t)] = F(s)$

5) $F(s) = \int_0^\infty e^{-st} f(t)\,dt$

$$\frac{dF}{ds} = \int_0^\infty \frac{d}{ds}(e^{-st})\, f(t)\,dt$$

$$= \int_0^\infty (-t)\, e^{-st} f(t)\,dt$$

$$\frac{dF}{ds} = -\int_0^\infty e^{-st}(t.f(t))\,dt$$

$$-\frac{dF}{ds} = \mathcal{L}[t f(t)]$$

$$\boxed{\mathcal{L}[t f(t)] = -\frac{dF}{ds}} \quad \& \quad \boxed{\mathcal{L}[f(t)] = F(s)}$$

6) $\mathcal{L}\left[\frac{df}{dt}\right] = \int_0^\infty e^{-st} \frac{df}{dt} u(t)\,dt$

$$= \int_0^\infty e^{-st}\frac{df}{dt}\,dt \quad = \left[e^{-st} f\right]_0^\infty - \int_0^\infty (-s)e^{-st} f(t)\,dt$$

$$= -f(0) + s\int_0^\infty e^{-st} f(t)\,dt$$

$$\boxed{\mathcal{L}\left[\frac{df}{dt}\right] = s F(s) - f(0)}$$

7) $\mathcal{L}\left[\frac{d^2f}{dt^2}\right] = \mathcal{L}\left[\frac{d}{dt}\left(\frac{df}{dt}\right)\right] = s\left[sF(s) - f_0\right] - \left(\frac{df}{dt}\right)_0$

$$= \boxed{s^2 F(s) - s f(0) - \frac{df}{dt}(0)}$$

$\rightarrow$ $\mathcal{L}\left[\int_0^t f(\tau)\,d\tau\right] = ?$

8 Let $\int_0^t f(\tau)\,d\tau = g(t)$ $\quad \therefore \frac{dg}{dt} = f(t)$.

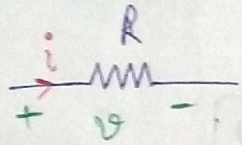
- $$\int_0^\infty \underset{\text{II}}{e^{-st}}\,\underset{\text{I}}{g(t)}\,dt = \left[g(t)\left(\frac{e^{-st}}{-s}\right)\right]_0^\infty - \int_0^\infty \frac{dg}{dt}\left(\frac{e^{-st}}{-s}\right)dt$$

$$= \frac{g(0)}{s} + \frac{1}{s}\int_0^\infty f(t)\,e^{-st}\,dt$$

$$\boxed{\mathcal{L}\left[\int_0^t f(\tau)\,d\tau\right] = \frac{\cancel{g(0)}^{\to 0}}{s} + \frac{F(s)}{s}}$$

$$\mathcal{L}^{-1}\left(\frac{\omega}{s^2+\omega^2}\right) = \sin\omega t\; u(t)$$

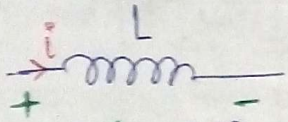
---

* 

$i = \frac{v}{R}$ (time domain) {Capital letters ⇒ Transformed}

$I(s) = \frac{V(s)}{R}$ ('s' domain)

or $\boxed{V(s) = R\,I(s)}$

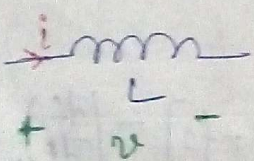
* 

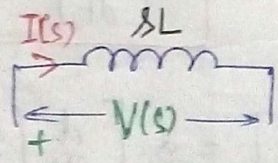
$v = L\frac{di}{dt}$ $\qquad \mathcal{L}[v] = V(s)$

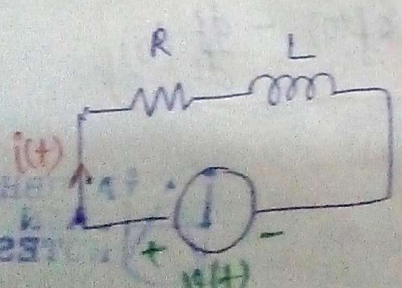
$V(s) = L[sI(s) - i(0)]$

if $i(0) = 0$

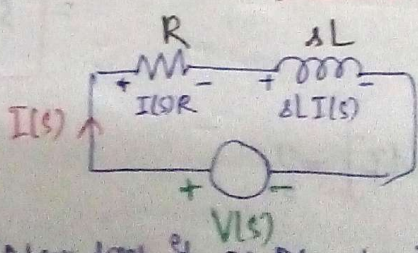
$\boxed{V(s) = Ls\,I(s)}$





- 

S Domain



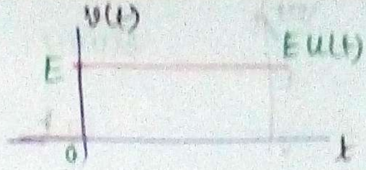
(Now treat it as DC ckt.)

$V(s) = RI(s) + sL\,I(s)$

$V(s) = (R+sL)\,I(s)$

$I(s) = \frac{V(s)}{R+sL}$

$$I(s) = \frac{E/s}{R+sL}$$



$v(t) = E\,u(t)$

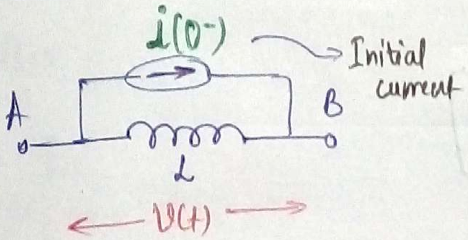
$\therefore \; V(s) = \frac{E}{s}$

$$I(s) = \frac{E/L}{s(s+R/L)}$$

$$= \frac{E}{L}\left[\frac{L/R}{s} - \frac{L/R}{s+R/L}\right]$$

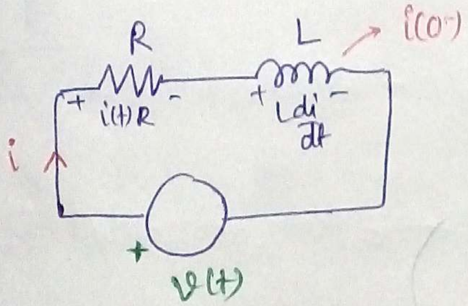
$$\boxed{I(s) = \frac{E}{R}\left[\frac{1}{s} - \frac{1}{s+R/L}\right]}$$

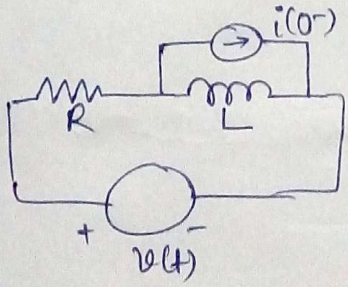
$$\boxed{i(t) = \frac{E}{R}\left[1-e^{-R/L\,t}\right]u(t)}$$

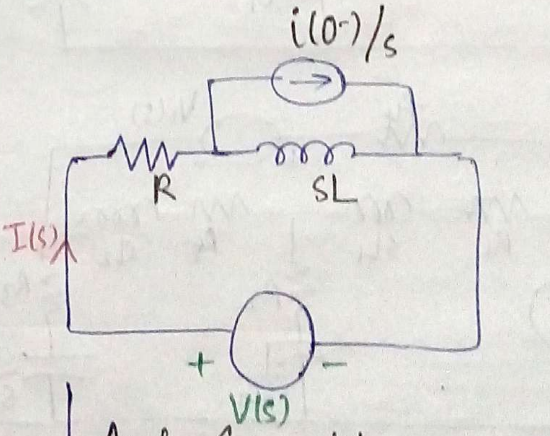


Initially charge inductor

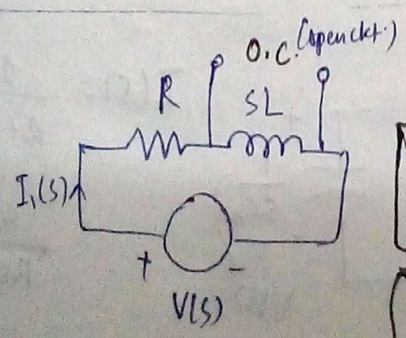
$\equiv$ (Initially uncharged) || (Current Source)



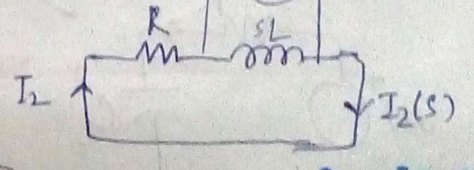




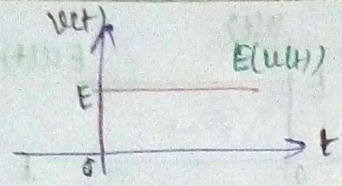
↓ Apply Superposition.



$$\boxed{I_1(s) = \frac{V(s)}{R+sL}}$$



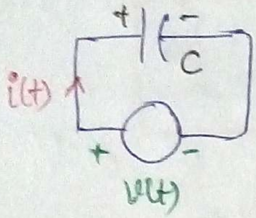
$$\boxed{I_2(s) = \frac{i(0^-)}{\cancel{s}}\,\frac{\cancel{s}L}{R+sL}}$$



- $\mathcal{L}^{-1}\left\{\frac{V(s)}{R+sL}\right\} = \mathcal{L}^{-1}\left\{\frac{E/s}{R+sL}\right\} = \frac{E}{R}\left(1-e^{-R/L\,t}\right)$
- $\mathcal{L}^{-1}\left\{\frac{i(0^-)\,L}{R+sL}\right\} = i(0^-)\,\mathcal{L}^{-1}\left\{\frac{1}{s+R/L}\right\} = i(0^-)\,e^{-R/L\,t}$

$$\boxed{i(t) = \frac{E}{R}\left(1-e^{-R/L\,t}\right) + i(0^-)\,e^{-R/L\,t}}$$

*



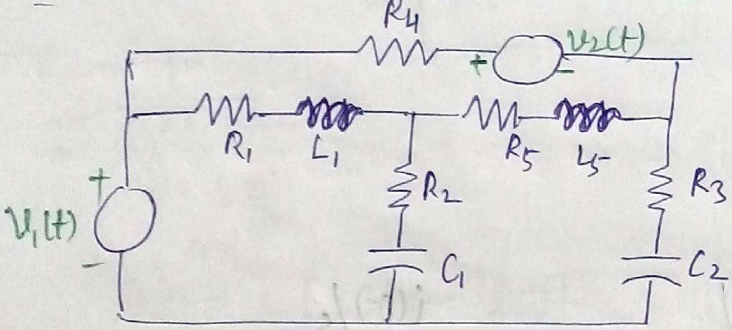
$$C\frac{dv}{dt} = i$$

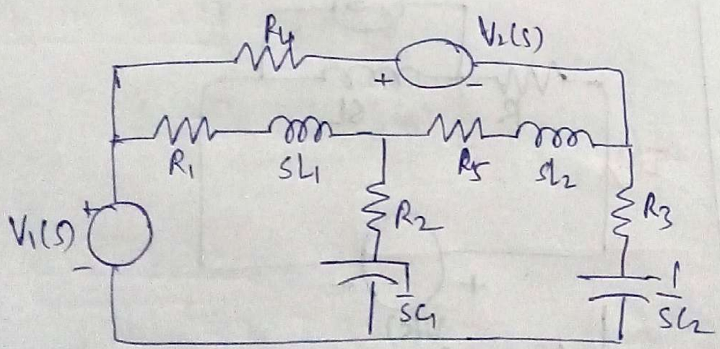
$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i\,dt = v(0^-) + \frac{1}{C}\int_{0}^{t} i\,dt$$

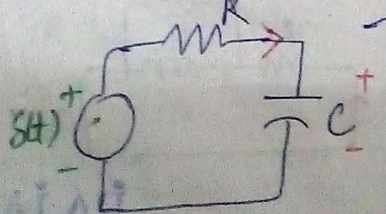
$$V(s) = \frac{v(0^-)}{s} + \frac{I(s)}{sC}$$

if $v(0)=0$

$$\boxed{V(s) = \frac{1}{sC}\,I(s)}$$

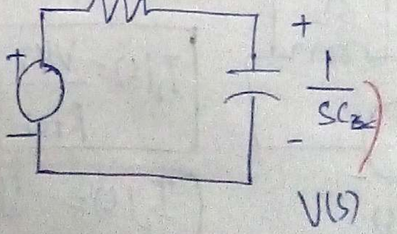






$V_i(s)$

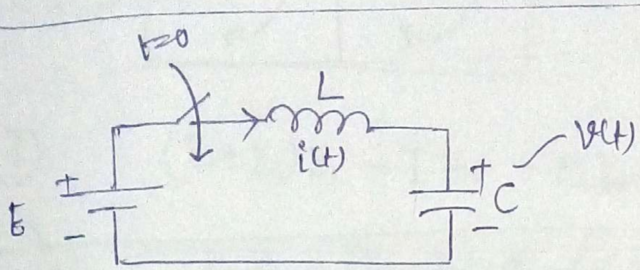
$= \delta(s) = 1$



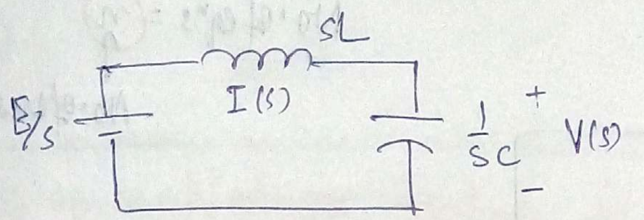
$$I(s) = \frac{1 = V_i(s)}{R + \frac{1}{sC}} = \frac{sC}{RsC+1}$$

$$V(s) = I(s)\,\frac{1}{sC} = \frac{1}{1+RCs} = \frac{1}{RC\left(s+\frac{1}{RC}\right)}$$

$$v(t) = \frac{1}{RC}\,e^{-t/RC}$$



$$\omega^2 = \frac{1}{LC}$$



$$\frac{E}{s} = I(s)\left\{sL + \frac{1}{sC}\right\}$$

$$I(s) = \frac{E/s}{sL+\frac{1}{sC}} = \frac{E}{s^2L+\frac{1}{C}}$$

$$= \frac{E/L}{(s-j\omega)(s+j\omega)}$$

$$I(s) = \frac{E}{L}\left\{\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right\}\left(\frac{1}{2j\omega}\right)$$

$$i(t) = \frac{E}{2Lj\omega}\left(e^{j\omega t} - e^{-j\omega t}\right)(u(t))$$

$$\boxed{i(t) = \frac{E}{L\omega}\sin\omega t\; u(t)}$$

$$V(s) = \frac{1}{sC}\,I(s)$$

$$= \frac{E}{LC\,s(s-j\omega)(s+j\omega)}$$

$$= \frac{E}{LC\,s\,(s^2+\omega^2)}$$

$$= \frac{E}{\cancel{LC}}\left(\frac{1}{s} - \frac{s}{s^2+\omega^2}\right)\frac{1}{\cancel{\omega^2}}$$

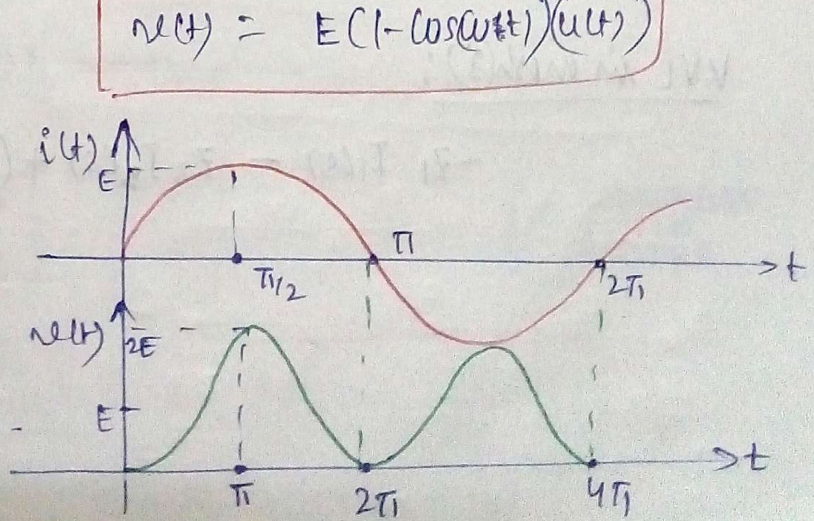
$$v(t) = E\left(u(t) - \cos\omega t\, u(t)\right)$$

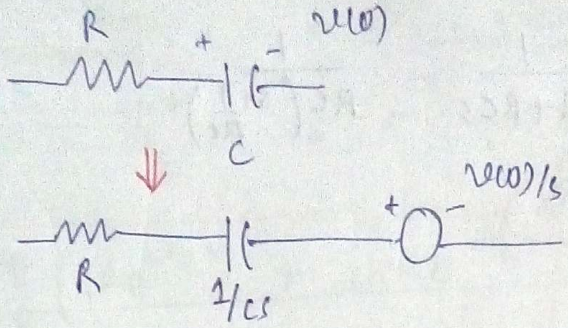
$$v(t) = E(1-\cos\omega t)\,u(t)$$

$$v(t) = E^{u(t)} - L\frac{di}{dt}$$

$$= E^{u(t)} - L\left(\frac{E}{\cancel{L\omega}}\right)\cancel{\omega}\cos\omega t\; u(t)$$

$$\boxed{v(t) = E(1-\cos(\omega t))(u(t))}$$

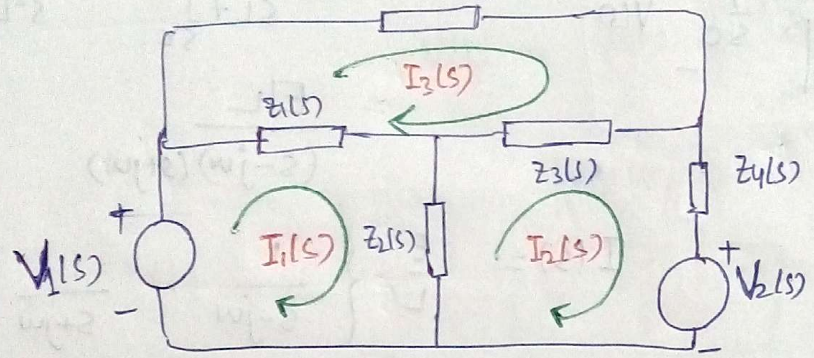




$$Z(s) = R + sL + \frac{1}{sC}$$

Mesh Analysis

No. of eqⁿs = n ← No. of Mesh.



KVL in mesh ①:

$$V_1(s) - (I_1(s) - I_3(s))Z_1(s) - (I_1(s) - I_2(s))Z_2(s) = 0$$

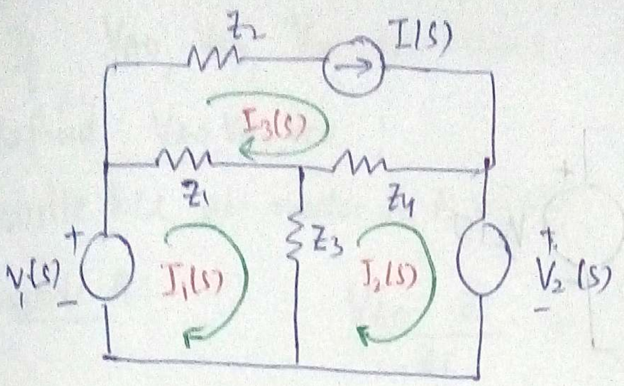
$$(Z_1(s) + Z_2(s))I_1(s) - Z_2(s)I_2(s) - Z_1(s)I_3(s) = V_1(s)$$

KVL in mesh ②:

$$-Z_2 I_1(s) + (Z_2 + Z_3 + Z_4)I_2(s) - Z_3 I_3(s) = -V_2(s)$$

KVL in mesh ③:

$$-Z_1 I_1(s) - Z_3 I_2(s) + (Z_1 + Z_3 + Z_5)I_3(s) = 0.$$



$I_3 = I$

① : $(Z_1+Z_3)I_1 - IZ_1 - Z_3 I_2 = V_1(s)$

② : $-Z_3 I_1 + (Z_3+Z_4) I_2 - Z_4 I = -V_2(s)$

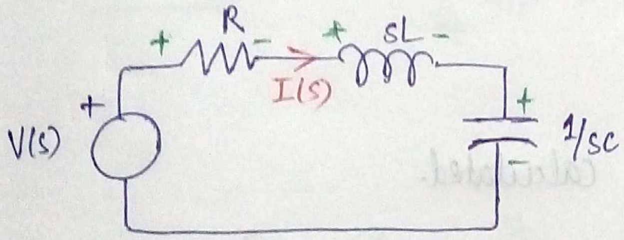
③ : $I_3 = I$.

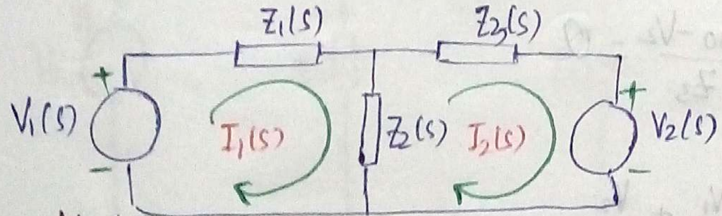
Network Analysis in s-domain

06/10/16

$Z(s) = R + sL + \frac{1}{sC}$

$V(s) = Z(s)\,I(s)$





Mesh: in s domain

Mesh 1: $(Z_1+Z_2)I_1 - Z_2 I_2 = V_1$

Mesh 2: $-Z_2 I_1 + (Z_3+Z_2) I_2 = -V_2$

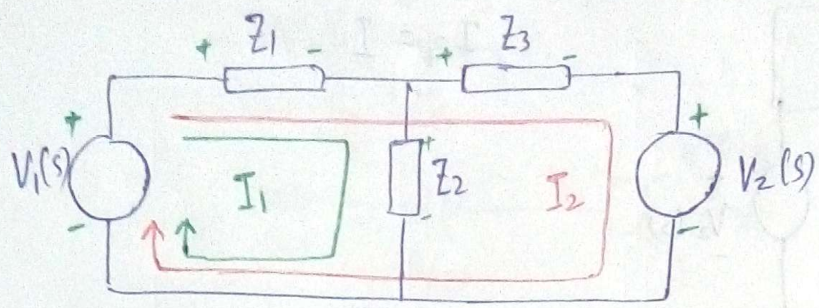
$$\begin{bmatrix} Z_1+Z_2 & -Z_2 \\ -Z_2 & Z_2+Z_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

ANUBHAV JAIN NOTES

(Cramer's Rule)

$$I_1(s) = \frac{\begin{vmatrix} V_1 & -Z_2 \\ -V_2 & Z_2+Z_3 \end{vmatrix}}{|Z|}$$

$$I_2(s) = \frac{\begin{vmatrix} Z_1+Z_2 & V_1 \\ -Z_2 & -V_2 \end{vmatrix}}{|Z|}$$

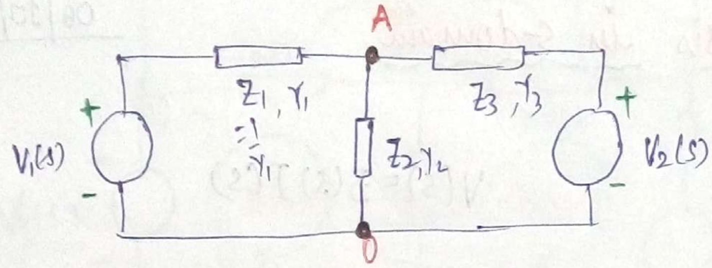


1: $(Z_1 + Z_2)\, I_1 + Z_1 I_2 = +V_1$

2: $+Z_1 I_1 + (Z_1 + Z_3) I_2 = V_1 - V_2$

Nodal Method (s-domain)

Node: Junction where more than 2 elements are joined.



★ No. of eqⁿs = $n - 1$
Total no. of nodes.

If V_{AO} is known,
all the currents can be calculated.

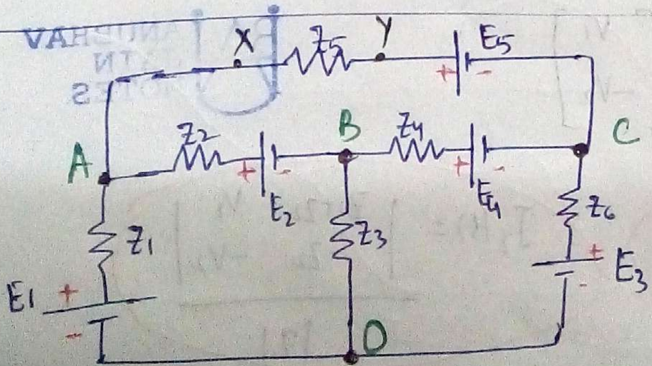
At node (A) write KCL

$$\frac{V_{AO} - V_1}{Z_1} + \frac{V_{AO}}{Z_2} + \frac{V_{AO} - V_2}{Z_3} = 0$$

$$\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right) V_{AO} = \frac{V_1}{Z_1} + \frac{V_2}{Z_3}$$

$$\boxed{(Y_1 + Y_2 + Y_3)\, V_{AO} = V_1 Y_1 + V_2 Y_3}$$

$Z^{-1} \equiv Y$
$(\text{Impedence})^{-1} = \text{Admittance}$



$$\frac{\vec{V}_{XY}}{Z_5} = \frac{V_{XO} - V_{YO}}{Z_5} = \frac{V_{AO} - (V_{CO} + E_5)}{Z_5}$$

$$\frac{\vec{V}_{XY}}{Z_5} = \frac{V_{AO} - V_{CO} - E_5}{Z_5}$$

If V_{AO}, V_{BO}, V_{CO} are known, all currents can be calculated.

To find V_{AO}, V_{BO}, V_{CO}:

write KCL at nodes A, B and C.

at node A:

$$\frac{V_{AO}-E_1}{Z_1}+\frac{\overset{V_{AO}-(V_{BO}+E_2)}{V_{AO}-E_2-V_{BO}}}{Z_2}+\frac{V_{AO}-E_5-V_{CO}}{Z_5}=0$$

$$\left(\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_5}\right)V_{AO}-\frac{1}{Z_2}V_{BO}-\frac{1}{Z_5}V_{CO}=\frac{E_1}{Z_1}+\frac{E_2}{Z_2}+\frac{E_5}{Z_5}$$

Coefficients
→ Sum of all admittance connected at node A.

negative of sum of all admittance between nodes A&B

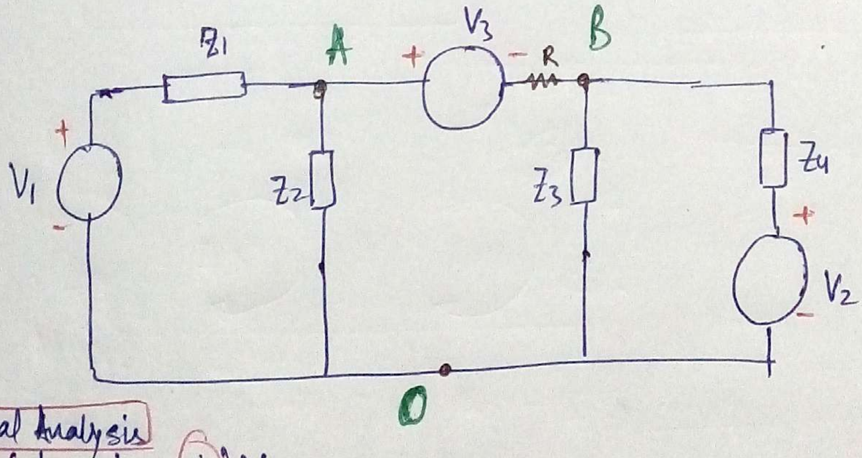
== A&C

at node B:

$$-\frac{1}{Z_2}V_{AO}+\left(\frac{1}{Z_2}+\frac{1}{Z_3}+\frac{1}{Z_4}\right)V_{BO}-\frac{1}{Z_4}V_{CO}=-\frac{E_2}{Z_2}+\frac{E_4}{Z_4}$$

at node C:

$$-\frac{1}{Z_5}V_{AO}-\frac{1}{Z_4}V_{BO}+\left(\frac{1}{Z_5}+\frac{1}{Z_4}+\frac{1}{Z_6}\right)V_{CO}=\frac{-E_5}{Z_5}-\frac{E_4}{Z_4}+\frac{E_3}{Z_6}$$



Nodal Analysis

A: $\left(\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{0}\right)V_{AO}$ ×

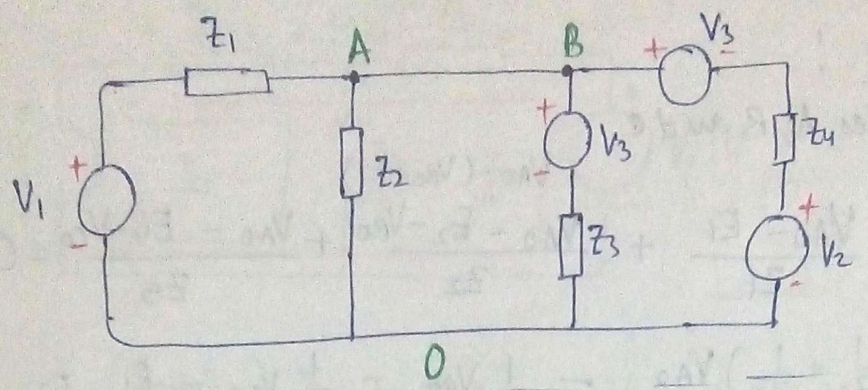
Method 1 ⇒ Assume impedence between A & B → R. } Solve completely & put R=0.

Method 2 → Choose 'B' as reference. (Change Ref.)

Method 3 → (See next page)

Draw equivalent ckt diagram (assuming ideal voltage sources).

→ Replace voltage source by short ckt & transfer it in subsequent branches.



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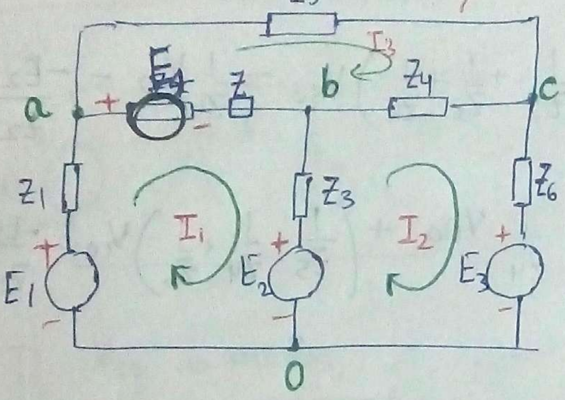
* Steady State → DC : $s=0$

Sinusoidal : $s=j\omega$

Here, we will get $\frac{1}{0}$ coefficients for V_{ao}, V_{bo}

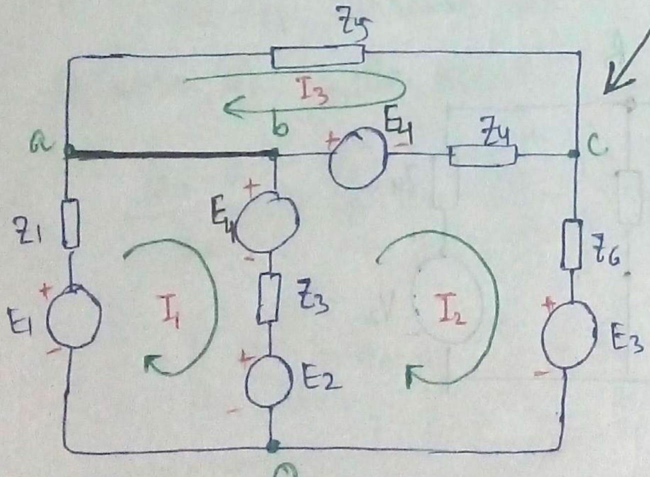


Method 1: Assume some Z in ab branch.

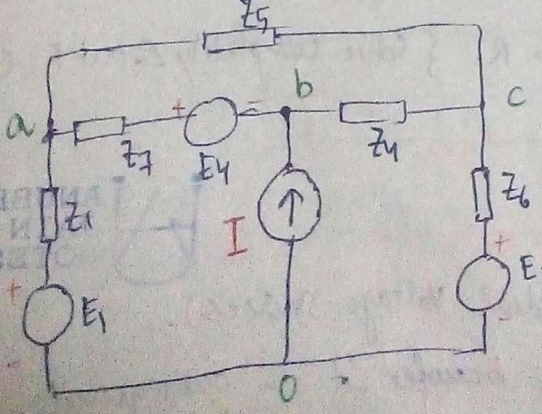
After solving all equations, put $Z=0$.

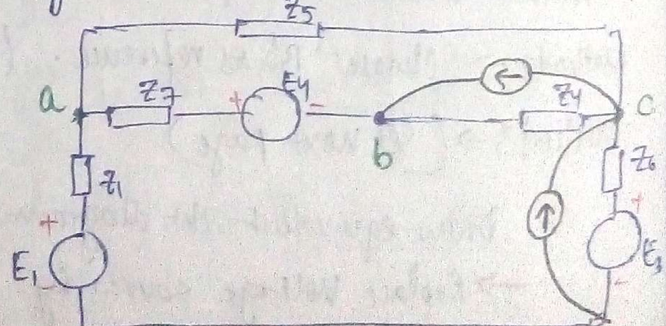
Method 2: Short Voltage source E_4 & put in other connected branches.

Here, all loop (KVL) equations are unchanged.



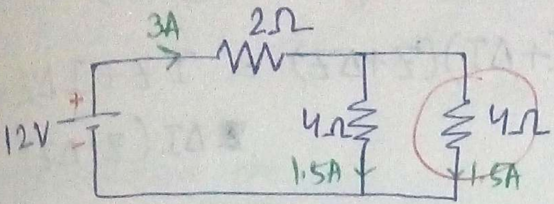
Problem: In Mesh Analysis, we don't know voltage across current source.



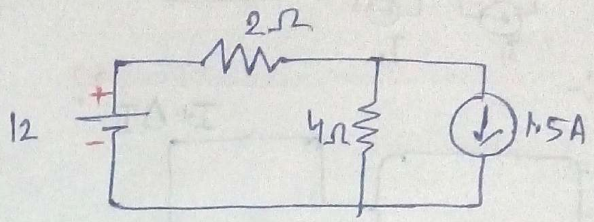


Open Circuit Current Source
and transfer it to subsequent branches (parallel connection).

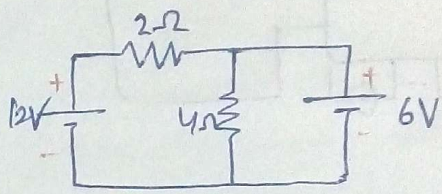
SUBSTITUTION TH.



⇒



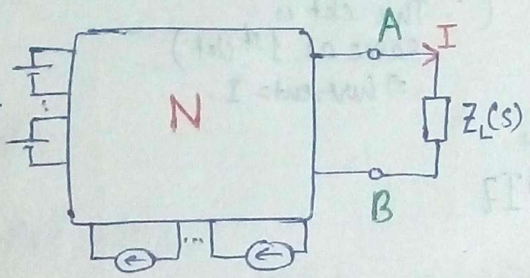
⇓



Identical ckts
(As long as current distribution is concerned)

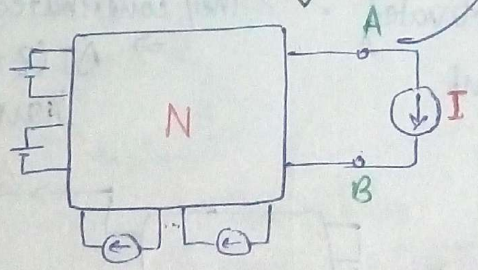
* Proof of Thevenin's Th.

Many Voltage & Current sources + Impedence elements



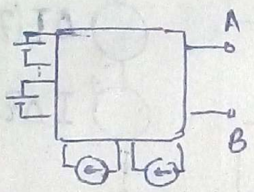
$V_{AB} = I Z_L$

⇓



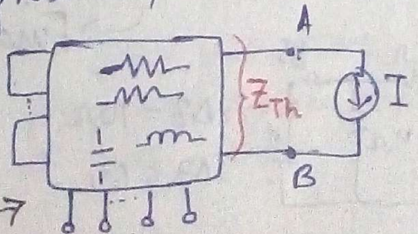
→ Find V_{AB} by Superposition Th^m.

1) All sources of the network are active & I is absent.
Calculate v_{AB}?



$v_{AB} = v_{OC} = v_{Th}$
(open ckt.)

2) All sources of the network deactivated only
I is present, calculate v_{AB}?



$v_{AB} = -I Z_{Th}$

Only Impedence present

ANUBHAV JAIN NOTES

* Total Voltage : $V_{AB} = V_{Th} - I Z_{Th} = I Z_L$ $\boxed{I = \dfrac{V_{Th}}{Z_{Th} + Z_L}}$

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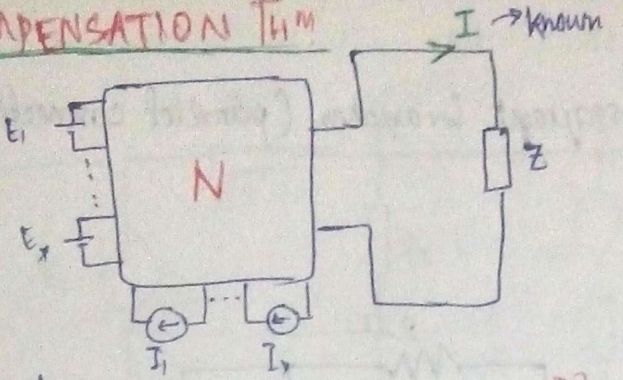
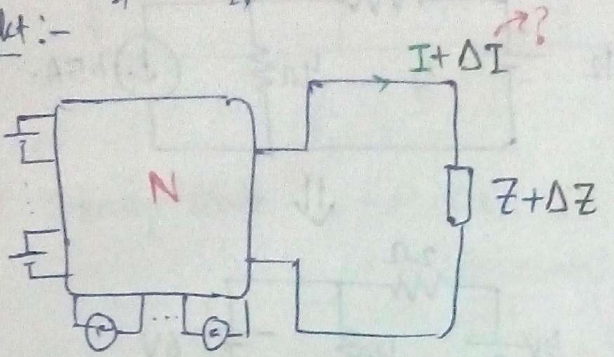
COMPENSATION Thm

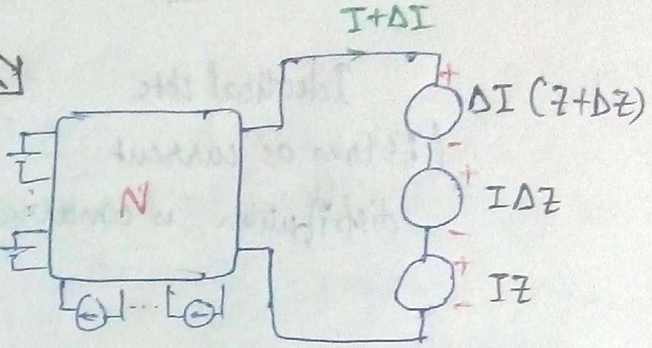


{ All in 's' domain }

$$(I+\Delta I)(Z+\Delta Z) = IZ + I\Delta Z + \Delta I(Z+\Delta Z)$$

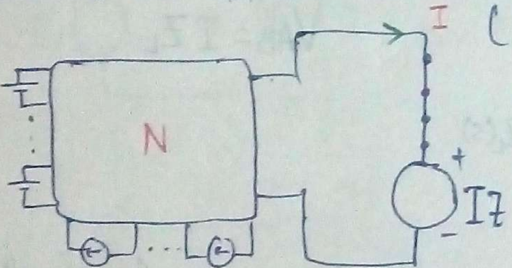
New ckt :-





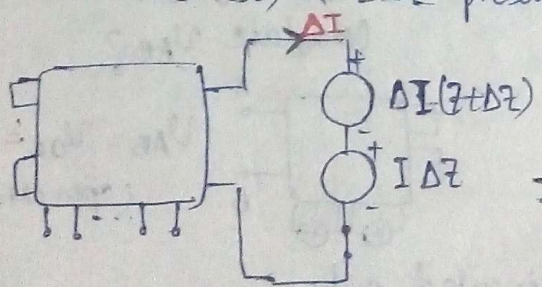
Apply Superposition Thm.

(1) Consider all network sources and IZ.

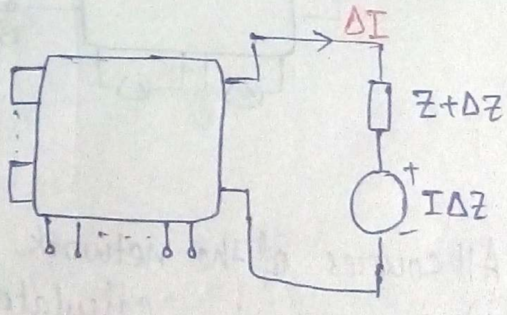


(∵ This ckt is same as 1st ckt) ⇒ current = I.

(2) All network sources & IZ : deactivated, ΔI(Z+ΔZ) & IΔZ present → They contributed I, Total I+ΔI ⇒ ΔI(Z+ΔZ) & IΔZ must have produced ΔI.

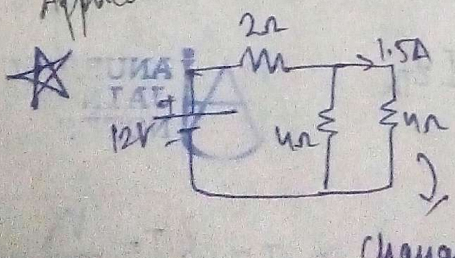


⇒

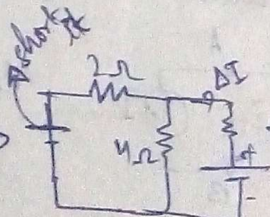


↳ Find ΔI ✓

Application :-



→



$Z+\Delta Z = 10\Omega$

$I\Delta Z = 6V$

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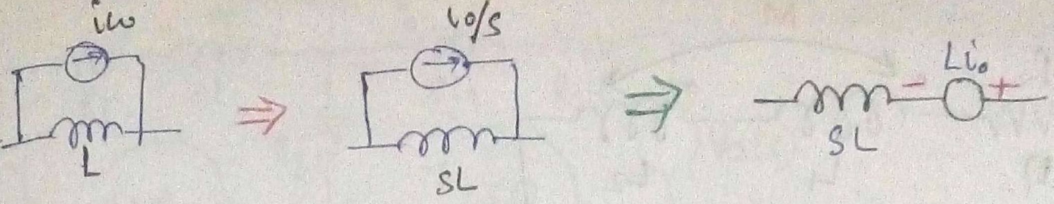
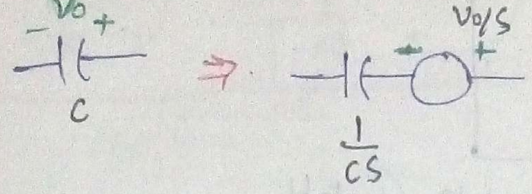
Topics already covered:





1. Transformed ckt.
2. Mesh and Nodal Analysis
3. Transferring voltage or current
* sources in unusual situations.
4. Substitution Th. – its application.
5. Compensation Th.

* Key word: Laplace Transform and inverse L.T. for standard f^ns.

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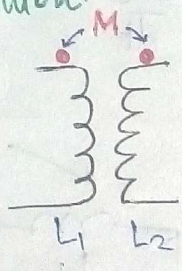
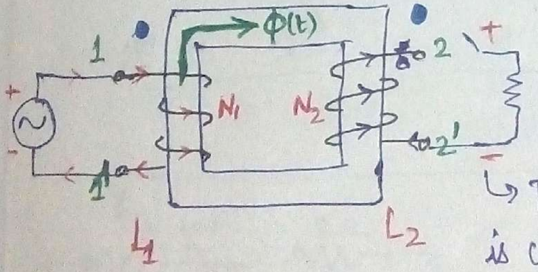
* Coupled ckt.

Mutually coupled coils

* Tellegen's Th.

Dot Convention

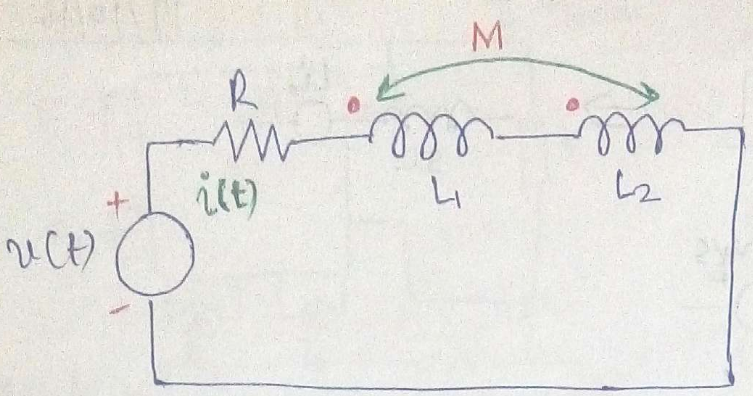




↳ This polarity is consistent with Lenz's Law.
(Since flux produced opposes the cause of induction ⇒ $\phi(t)$)

Dots → Both sides will have same polarity at a given time. (Both + or both –)

⇒ Dots represent like polarity.



$$v(t) = Ri + \left(L_1 \frac{di}{dt} \pm M\frac{di}{dt}\right) + \left(L_2 \frac{di}{dt} \pm M \frac{di}{dt}\right)$$

due to L_2 due to L_1

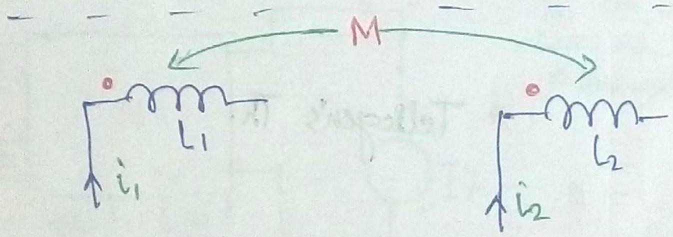
('+' here)

$$V(s) = RI(s) + sL_1 I(s) + MsI(s) + sL_2 I(s) + sMI(s)$$

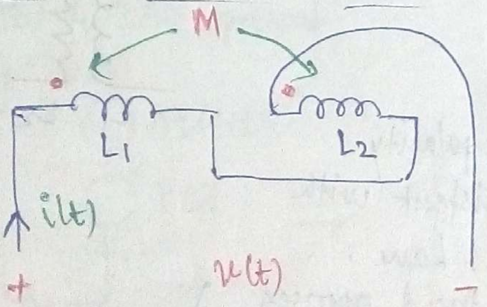
When current is entering L_2, $\bullet_2 = +$

$\Rightarrow \bullet_1 = +$

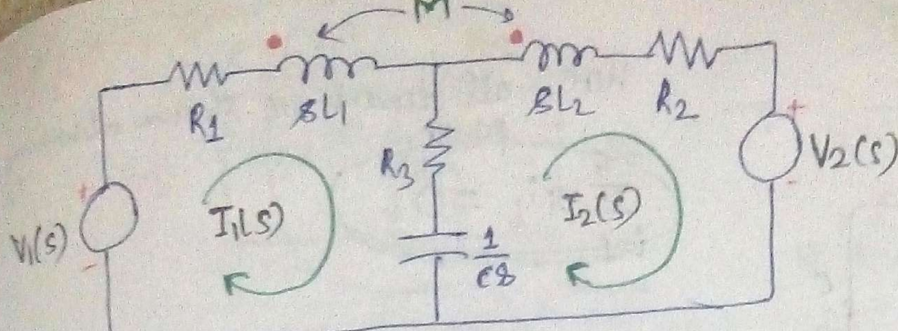
$\Rightarrow \pm M\frac{di}{dt} = +M\frac{di}{dt}$



$L_1 \frac{di_1}{dt} + M\frac{di_2}{dt}$ $\qquad$ $L_2 \frac{di_2}{dt} + M\frac{di_1}{dt}$



$$v(t) = \left(L_1 \frac{di}{dt} - M\frac{di}{dt}\right) + \left(L_2 \frac{di}{dt} - M\frac{di}{dt}\right)$$



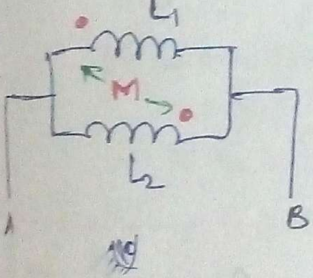
Mesh1:-

$$\left(R_1 + sL_1 + R_3 + \frac{1}{Cs}\right) I_1(s) = V_1(s) - sMI_2(s) - \left(R_3+\frac{1}{Cs}\right) I_2(s)$$

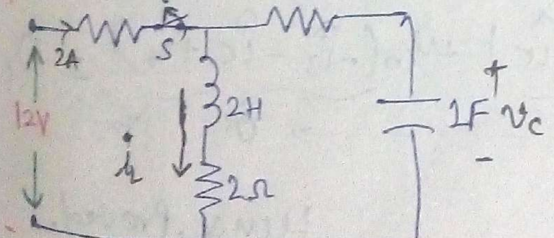
Think in this way: Due to M, there will be additional voltage drop.
Assume it as a voltage source, write it with $V_1(s)$

Mesh2:-

$$-\left(R_3+\frac{1}{Cs}\right) I_1(s) + \left(R_2+sL_2+\frac{1}{Cs}+R_3\right)I_2(s) = -V_2(s) - sMI_1(s)$$



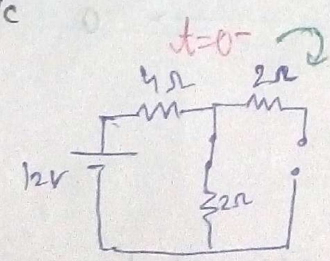
Leq ?

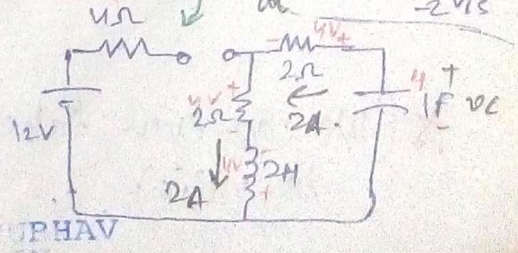


at t=0, S is opened.

$i_L(0+) = 2A$ Can't change
$v_c(0+) = 4V$
$\frac{di_L}{dt}(0+) = -2$ A/s
$\frac{dv_c}{dt}(0+) = -2$ A/F or -2 V/s

$i_L(0^-) = 2A$
$v_c(0^-) = 4V$





$* \; C\frac{dv_c}{dt}\Big|_{t=0^+} = -2A$

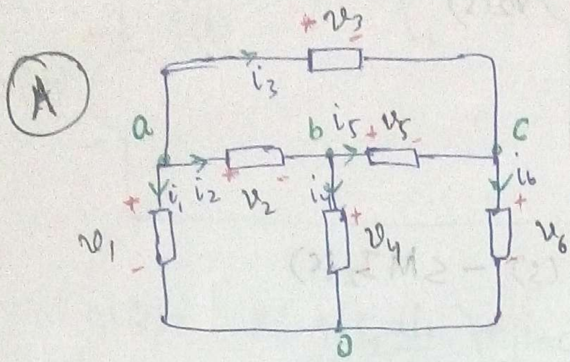
$* \; L\frac{di_L}{dt}\Big|_{t=0^+} = -4V$

TELLEGEN'S Th^m.

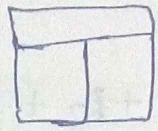
Here, all elements are shown absorbing power.

$$\sum_{i=1}^{6} v_i i_i = 0$$

(A)

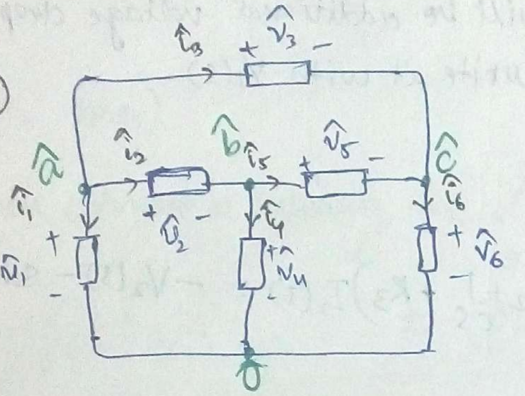


→ skeleton



Ckt. Topology Same.

(B)



$$\sum_{i=1}^{6} \hat{v}_i \hat{i}_i = 0$$

* $v_1 \hat{i}_1 + v_2 \hat{i}_2 + \ldots + v_6 \hat{i}_6 = \boxed{\sum_{i=1}^{6} v_i \hat{i}_i = 0}$

* $\hat{v}_1 i_1 + \hat{v}_2 i_2 + \ldots + \hat{v}_6 i_6 = \boxed{\sum_{i=1}^{6} \hat{v}_i i_i = 0}$

Proof of Tellegen's Th^m.

- $v_{a0} \hat{i}_1 + \underset{\downarrow\,(v_{a0} - v_{b0})}{v_{ab}} \hat{i}_2 + (v_{a0} - v_{c0}) \hat{i}_3 + v_{b0} \hat{i}_4 + (v_{b0} - v_{c0}) \hat{i}_5 + v_{c0} \hat{i}_6$

$$= v_{a0}\underbrace{(\hat{i}_1 + \hat{i}_2 + \hat{i}_3)}_{0} + v_{b0}\underbrace{(-\hat{i}_2 + \hat{i}_4 + \hat{i}_5)}_{0} + v_{c0}\underbrace{(-\hat{i}_3 - \hat{i}_5 + \hat{i}_6)}_{0} = 0.$$

KCL in 2^{nd} ckt.

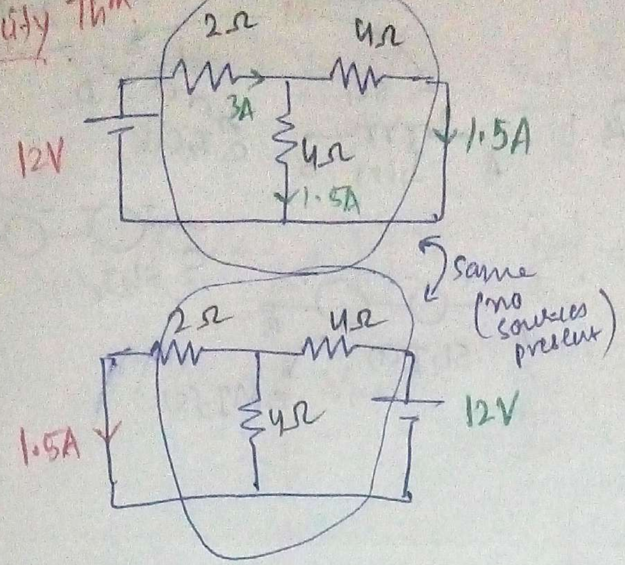
Hence, proved.

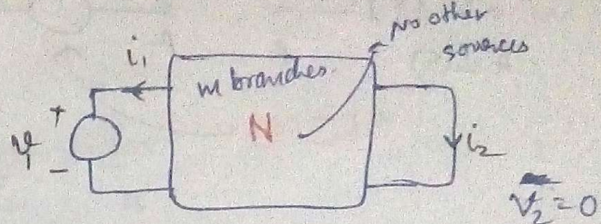
(Similar for other part).

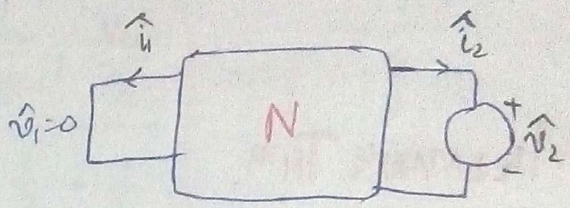
* (We can even take v_i at t_1 and $\hat{i}_i$ at t_2, still th^m will be valid).

Reciprocity Thm

Theory of Reciprocity.







$$v_1 \hat{i}_1 + (0)\hat{i}_2 + \sum_{k=3}^{m} v_k \hat{i}_k = 0$$

$$(0) i_1 + \hat{v}_2 i_2 + \sum_{k=3}^{m} \hat{v}_k i_k = 0$$

or

$$v_1 \hat{i}_1 + \cancel{\sum_{k=3}^{m} R_k i_k \hat{i}_k} = \hat{v}_2 i_2 + \cancel{\sum_{k=3}^{m} R_k \hat{i}_k i_k} = 0.$$

$$\Rightarrow \boxed{v_1 \hat{i}_1 = \hat{v}_2 i_2}$$

$$\boxed{\frac{i_2}{v_1} = \frac{\hat{i}_1}{\hat{v}_2}}$$

As in our case,

$v_1 = \hat{v}_2 = 12V$

$\Rightarrow i_2 = \hat{i}_1 = 1.5A.$

Q Find $\mathcal{L}^{-1}\left[\frac{1}{s(s^2+2s+3)}\right]$

$$\frac{1}{s((s+1)^2+2)} = \frac{\cancel{1}}{s\cancel{(s+1+\sqrt{2})(s+1-\sqrt{2})}} \quad ?$$

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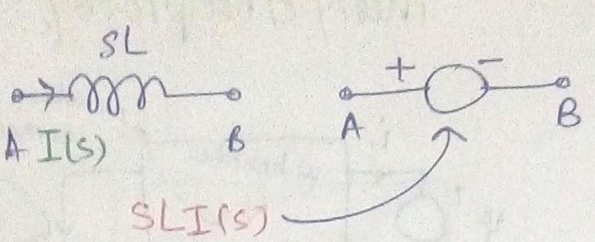
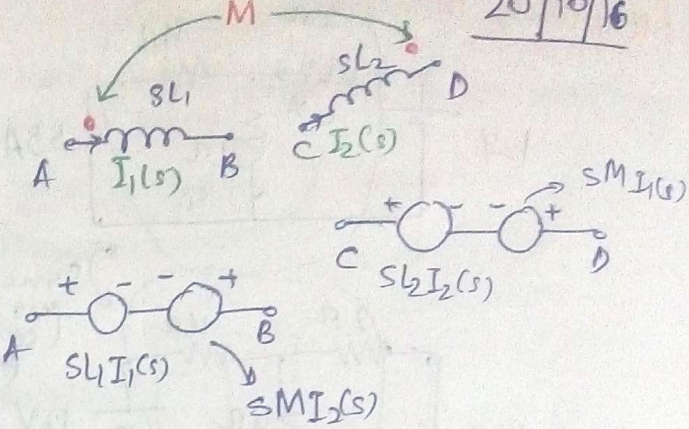
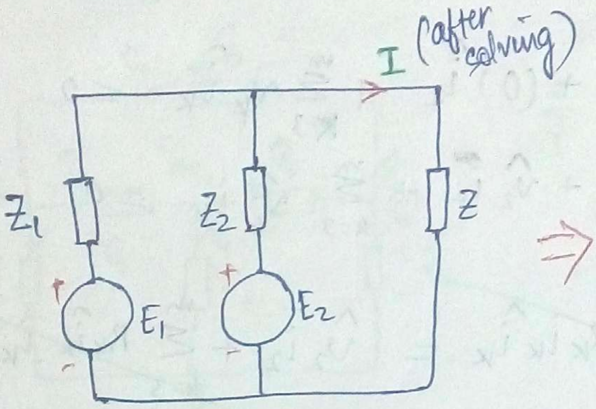
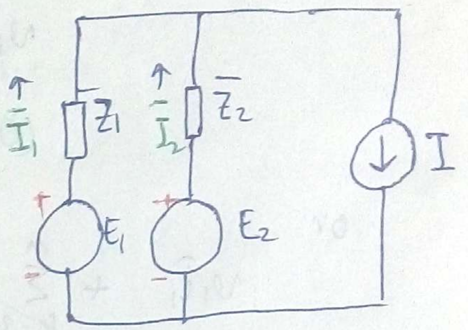




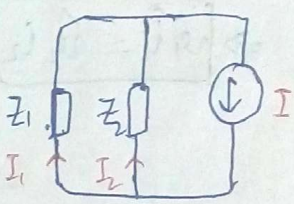
MILLMAN'S THM.



$\Rightarrow$



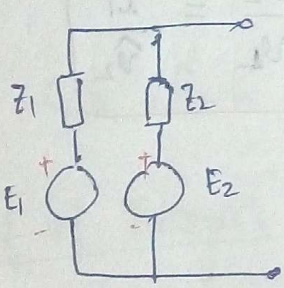
Sup.



due to I

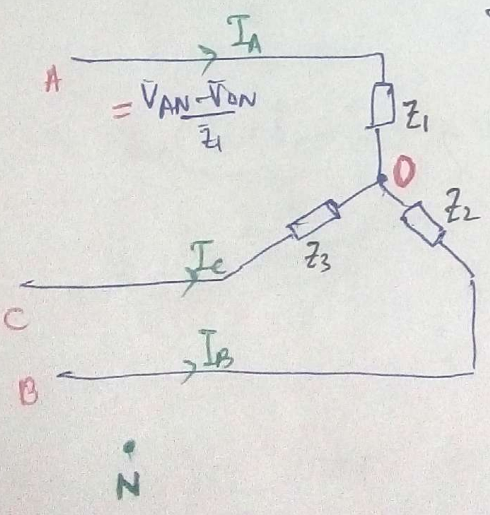
$I_1 = I \dfrac{Z_2}{Z_1+Z_2}$

$I_2 = I \dfrac{Z_1}{Z_1+Z_2}$.



$$\frac{E_1-E_2}{Z_1+Z_2} + \frac{IZ}{Z_1+Z_2}$$

Steady state solⁿ $\cancel{s} = j\omega$.



V_{AB}, V_{BC}, V_{CA}	V_{AN}, $\bar{V}_{BN}$, $\bar{V}_{CN}$

$\bar{I}_A = \dfrac{\bar{V}_{AO}}{\bar{Z}_1}$	$\bar{I}_B = \dfrac{\bar{V}_{BO}}{\bar{Z}_2}$	$\bar{I}_C = \dfrac{\bar{V}_{CO}}{\bar{Z}_3}$

Apply KCL at O :-

$\bar{I}_A + \bar{I}_B + \bar{I}_C = 0$

$$\frac{\bar{V}_{AO}}{\bar{Z}_1} + \frac{\bar{V}_{BO}}{\bar{Z}_2} + \frac{\bar{V}_{CO}}{\bar{Z}_3} = 0$$

($\bar{V} \rightarrow$ rms value of v)

$$\frac{\bar{V}_{AN}-\bar{V}_{ON}}{\bar{Z}_1}+\frac{\bar{V}_{BN}-\bar{V}_{ON}}{\bar{Z}_2}+\frac{\bar{V}_{CN}-\bar{V}_{ON}}{\bar{Z}_3}=0$$

$$\bar{V}_{ON}\left(\frac{1}{\bar{Z}_1}+\frac{1}{\bar{Z}_2}+\frac{1}{\bar{Z}_3}\right)=\frac{\bar{V}_{AN}}{\bar{Z}_1}+\frac{\bar{V}_{BN}}{\bar{Z}_2}+\frac{\bar{V}_{CN}}{\bar{Z}_3}$$

$$\Rightarrow \boxed{\bar{V}_{ON}=\frac{\frac{\bar{V}_{AN}}{\bar{Z}_1}+\frac{\bar{V}_{BN}}{\bar{Z}_2}+\frac{\bar{V}_{CN}}{\bar{Z}_3}}{\frac{1}{\bar{Z}_1}+\frac{1}{\bar{Z}_2}+\frac{1}{\bar{Z}_3}}}$$

* if $\bar{Z}_1=\bar{Z}_2=\bar{Z}_3$

$\bar{V}_{ON}=0$ (as $(\bar{V}_{AN}+\bar{V}_{BN}+\bar{V}_{CN})=0$)

---

* L.T.

$$\mathcal{L}^{-1}\left[\frac{1}{s(s^2+2s+3)}\right]$$

* Method 1

factorise

$$s=\frac{-2\pm\sqrt{4-12}}{2}=-1\pm j\sqrt{2}$$

a, b ↳ complex.

$$\frac{1}{s(s^2+2s+3)}=\frac{1}{s(s-a)(s-b)}=\frac{A}{s}+\frac{B}{s-a}+\frac{C}{s-b}$$

* Method 2

$$F(s)=\frac{1}{s(s^2+2s+3)}=\frac{A}{s}+G(s)$$

☆ Put $s=0$ $\rightarrow A=\frac{1}{3}$ $\Rightarrow F(s)=\frac{1}{3s}+G(s)=\frac{1}{s(s^2+2s+3)}$

$$G(s)=\frac{1}{s}\left\{\frac{1}{s^2+2s+3}-\frac{1}{3}\right\}\Rightarrow G(s)=\frac{-(s^2+2s)}{3s(s^2+2s+3)}=\frac{-s-2}{3(s^2+2s+3)}$$

$$\Rightarrow F(s)=\frac{1}{3}\frac{1}{s}-\frac{s+2}{3(s^2+2s+3)}$$

$$=\frac{1}{3}\frac{1}{s}-\frac{1}{3}\frac{(s+2)}{(s+1)^2+2}$$

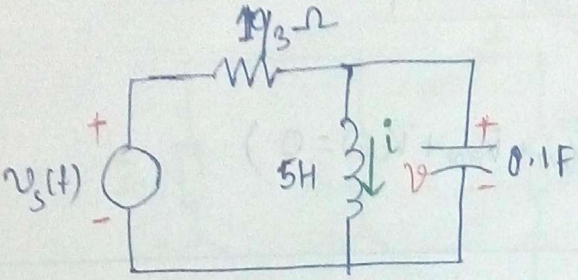
$$=\frac{1}{3}\frac{1}{s}-\frac{1}{3}\left[\frac{s+1}{(s+1)^2+(\sqrt{2})^2}+\frac{1}{(s+1)^2+(\sqrt{2})^2}\right]$$

* $\mathcal{L}\cos\omega t = \frac{s}{s^2+\omega^2}$
* $\mathcal{L}\, e^{-at}\cos\omega t = \frac{s+a}{(s+a)^2+\omega^2}$
* $\mathcal{L}\, e^{-at}\sin\omega t = \frac{\omega}{(s+a)^2+\omega^2}$

$\mathcal{L}[f(t)] = F(s)$

$\mathcal{L}[e^{-at}f(t)] = F(s+a)$

$$\Rightarrow \boxed{\mathcal{L}^{-1}[F(s)] = \frac{1}{3}u(t) - \frac{1}{3}e^{-t}\cos\sqrt{2}t - \frac{1}{3\sqrt{2}}e^{-t}\sin\sqrt{2}t}$$

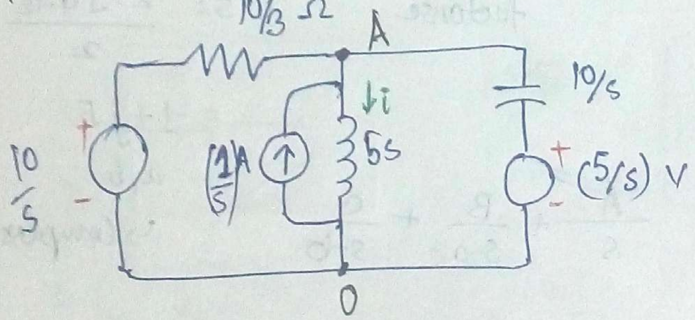


$i(0) = -1A$

$v(0^-) = +5V$

Calculate $v(t)$ for $t \geq 0$ when $v_s(t) = 10\,u(t)$

For $t \geq 0$



KCL at A :-

$$V_{A0}\left(\frac{3}{10} + \frac{1}{5s} + \frac{s}{10}\right) = \frac{10/s}{10/3} + \frac{1}{s} + \frac{5/s}{10/s}$$

$$= \frac{4}{s} + \frac{1}{2}$$

$$V_{A0}\left(\frac{3s+2+s^2}{10s}\right) = \frac{8+s}{2s}$$

$$V_{A0} = \frac{5(s+8)}{(s^2+3s+2)} = \frac{5(s+8)}{(s+1)(s+2)}$$

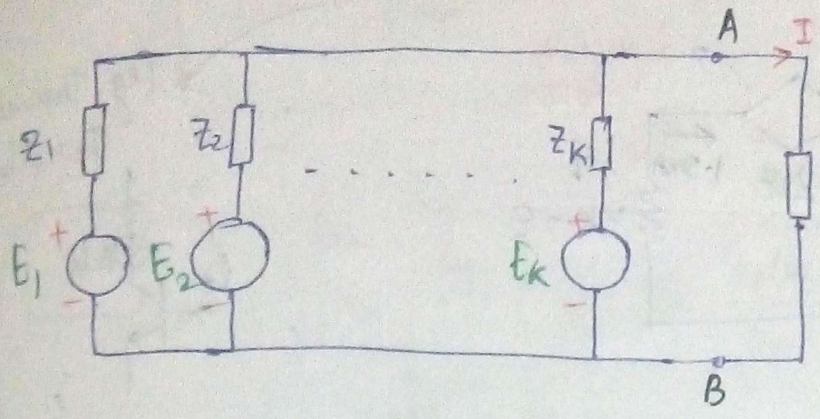
$$= \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = \frac{5\times 7}{1} = 35$$

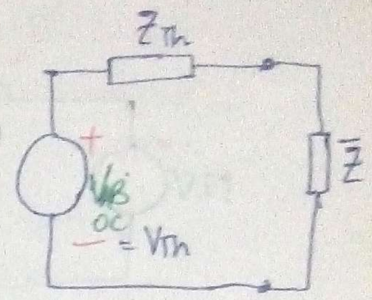
$$B = \frac{5\times 6}{-1} = -30.$$

MILLMAN'S THM:-

24/10/16



Thev
$\Longrightarrow$
equiv.
at AB



KCL at A   with Z removed:-

$V_{Th} = V_{AB\,(O.C)}$

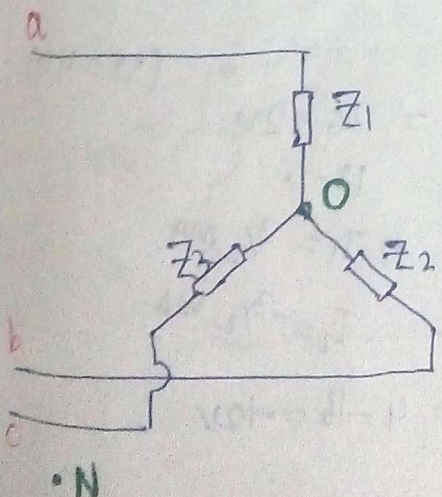
$$\frac{V_{Th}-E_1}{Z_1} + \frac{V_{Th}-E_2}{Z_2} + \dots + \frac{V_{Th}-E_K}{Z_K} = 0$$

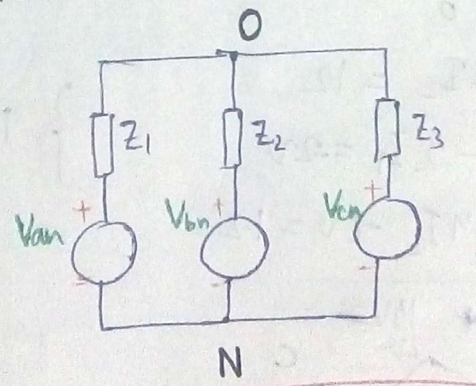
$$Y_1(V_{Th}-E_1) + Y_2(V_{Th}-E_2) + \dots + Y_K(V_{Th}-E_K) = 0$$

$$\boxed{V_{Th} = \frac{E_1Y_1 + E_2Y_2 + \dots + E_KY_K}{Y_1+Y_2+Y_3+\dots+Y_K}}$$

$$\boxed{Z_{Th} = \frac{1}{Y_1+Y_2+\dots+Y_K}}$$

$$I = \frac{V_{Th}}{Z_{Th}+Z}$$

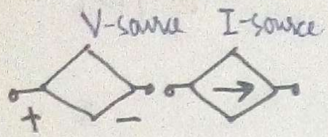




$$V_{ON} = \frac{V_{an}Y_1 + V_{bn}Y_2 + V_{cn}Y_3}{Y_1+Y_2+Y_3}$$

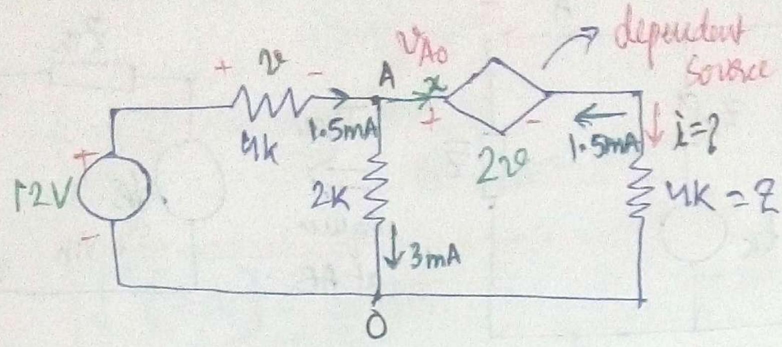
$$I_1 = \frac{V_{an}-V_{on}}{Z_1}$$

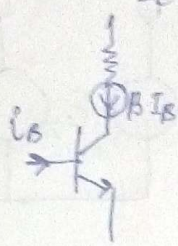
Circuit with dependent sources.



(eg. Transistor)

$I_c = \beta I_B$





$$\frac{v_{AO} - 12}{4} + \frac{v_{AO}}{2} + x = 0$$

All currents in mA

$$x = \frac{v_{AO} - 2v}{4}$$

$* \; -12 + v + v_{AO} = 0$

$v = 12 - v_{AO}$

$* \; v_{AO} - 2v$

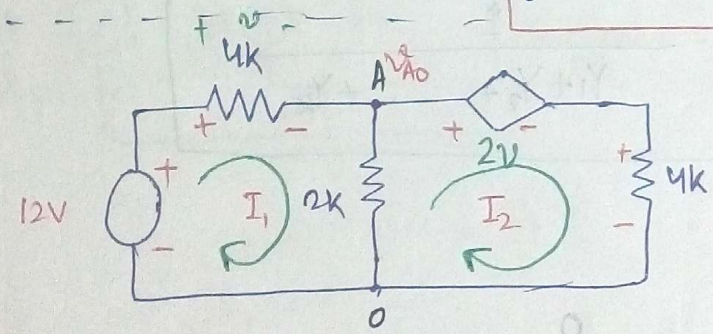
$= v_{AO} - 24 + 2v_{AO}$

$$\frac{v_{AO}}{4} - 3 + \frac{v_{AO}}{2} + \frac{3v_{AO}}{4} - 6 = 0$$

$$\frac{3v_{AO}}{2} = 9$$

$$\boxed{v_{AO} = 6V}$$

$$\Rightarrow \boxed{v = 12 - 6 = 6V}$$



$6I_1 - 2I_2 = 12$

$6I_2 - 2I_1 = -2v$

$4I_1 + 4I_2 - 2v = 12$

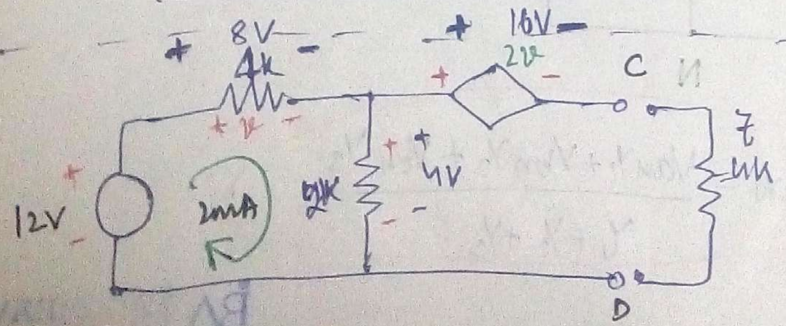
$(v = 4I_1)$

$16I_1 = 36 - 2v$

$8I_1 = 18 - v$

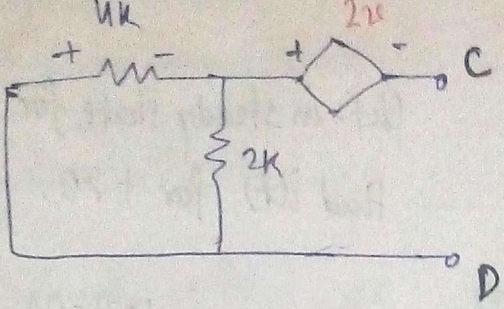
$\Rightarrow I_1 = 3/2 \text{ mA}$

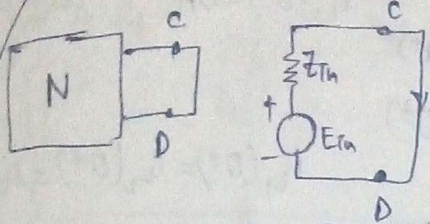
$I_2 = -3/2 \text{ mA}$



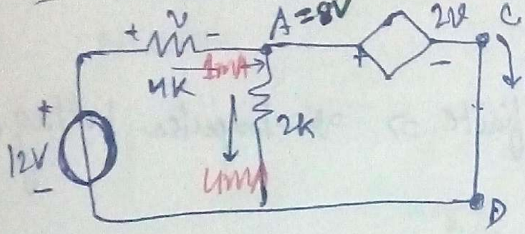
$V_{CD} = 4 - 16 = -12V$
(O.C.)

While calculating Thevenin's impedance, we deactivate the sources.





$I_{sc} = \frac{E_{Th}}{Z_{Th}} \Rightarrow \frac{V_{CD}(OC)}{I_{sc}} = Z_{Th}$.



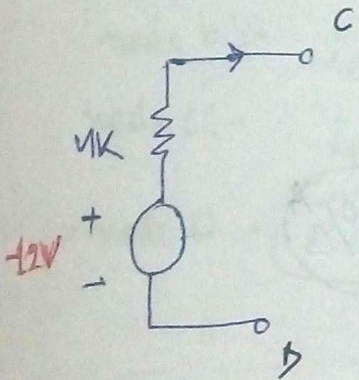
$i_{sc} = -3mA$

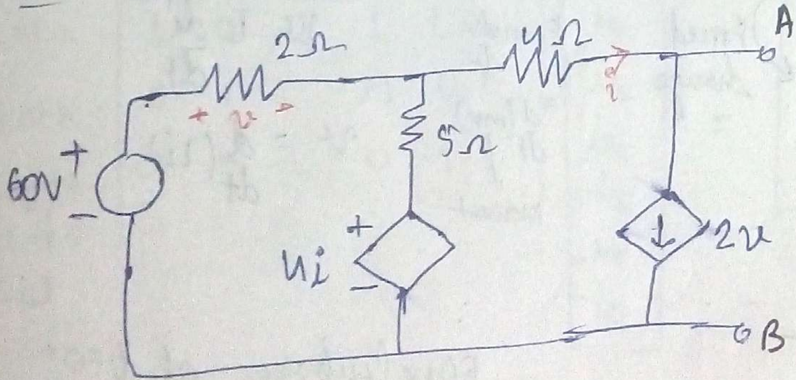
$\Rightarrow i_{sc} = -3mA$.

$V_{AD} = 2v$

Outer loop: $12 - v - 2v = 0 \Rightarrow v = 4V$

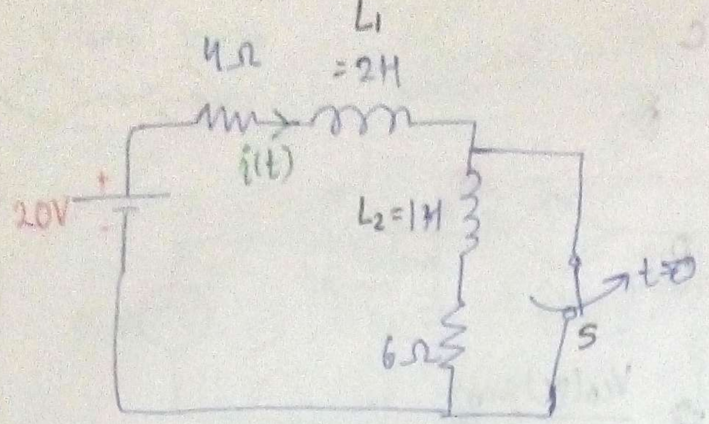
$\Rightarrow Z_{Th} = \frac{-12}{-3} = 4K$





Get Thev. equiv. between A&B.

26/10/16



Ckt. in steady state for $t<0$

Find $i(t)$ for $t>0$.

$i_{L_2}(0^-) = 0A$ $\quad i_{L_1}(0^-) = 5A$

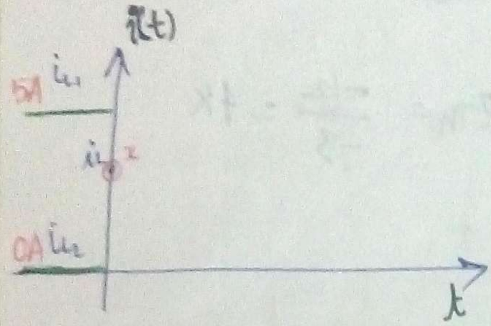
$i_L(0^-) = i_L(0^+)$

At $t(0^+)$

$i_{L_1}(0^+) = i_{L_2}(0^+) = i_L(0^+)$

Let $i(0^+) = x$

$$i = \frac{1}{L}\int_{0^-}^{0^+} v\,dt = \text{finite} \Rightarrow v = \text{Impulse Voltage}$$



for L_1:-

$$5 - x = \frac{1}{L_1}\,\delta(t)$$

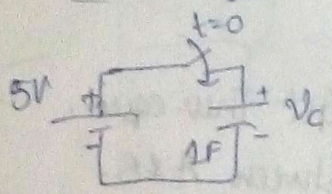
for L_2:-

$$x - 0 = \frac{1}{L_2}\,\delta(t)$$

$$(5-x)L_1 = xL_2$$

$$(5-x)2 = x \times 1$$

$$\text{or } 3x = 10 \Rightarrow x = 10/3\ A$$



$v_C(0^-) = 0V$

$v_C(0^+) = 5V$ Forced change

$$v_C = \frac{1}{C}\int_{0^-}^{0^+} \underbrace{i}_{\text{Impulse}}\,dt = 5V$$

$F = m\dfrac{dv}{dt} = \dfrac{d(mv)}{dt}$ → Moment

$v = L\dfrac{di}{dt}$

$v = \dfrac{d}{dt}(Li)$

$L = \dfrac{N\phi}{i}$

$Li = \phi$

Flux linkage at $t=0^-$

$2\times5 + 1\times0 = 10$

Flux linkage at $t=0^+$

$2\times x + 1\times x = 3x$

$10 = 3x$

$\Rightarrow x = \dfrac{10}{3}\ A$

Flux linkage through ckt cannot change instantly $\Rightarrow$ More fundamental