Recurrence Relations

Sequence

Rs 10/- 1st km

Rs 5/- for each km next.

$C_1 = 10$

$C_n = 10 + 5(n-1) \quad n \geq 1.$

$C_2 = 15$

$C_3 = 20$

$\vdots$

Sequence → 10, 15, 20, 25, ...

Sequence is a special type of function.

domain = integers.

1. Start with integer 5
2. add 3 in each step.

$c_1 \quad c_2 \quad c_3 \ldots c_n$

$c_1 = 5 \qquad c_2 = 8 \qquad c_3 = 11 \qquad \Rightarrow c_n = 5 + 3(n-1)$

S – $\{S_n\}$
C_n – n^{th} term
n – index.

Increasing sequence $\equiv c_n < c_{n+1}$

if $S_n < S_{n+1}$ for a sequence $\{S_n\}$

Non Decreasing if $S_n \leq S_{n+1}$

Decreasing if $S_n > S_{n+1}$

Non-Increasing if $S_n \geq S_{n+1}$

Let $\{S_n\}$ be a sequence defined for $n = m, m+1, \ldots$

If the domain of sequence is infinite, then it is called infinite sequence.

$\{S_n\}_{n=k}^{\infty}$

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If domain $\{S_n\}_{n=1}^{i}$ is finite, it is finite sequence.

Let $n_1, n_2 \ldots$ are some increasing sequence (index) and these are values from $\{m, m+1, \ldots\}$
then $\{S_{n_k}\}$ is the subsequence of sequence $\{S_n\}$

* Sum $= \sum_{n=i}^{j} c_n$

Sequence : $c_1, c_2, \ldots c_n \ldots$

* Product $= \prod_{n=1}^{i} c_n$

Recurrence Relation :-

$c_n = 10 + 5(n-1)$

$c_n = 5 + c_{n-1}$

$c_n = c_{n-1} + c_{n-2}$.

Recurrence relation is an equation that generate the n_{th} value from certain of its predecessors (previous terms) initial conditions must be explicitly defined.

$\{S_n\} - c_1, c_2, c_3, \ldots, c_n$

$c_n = c_i, c_j \quad i, j < n.$

Fibonacci Sequence:

$f_n = f_{n-1} + f_{n-2} \quad , n \geq 3$

$f_1 = 1$

$f_2 = 1 \qquad 1, 1, 2, 3, 5, \ldots$

OR

$f_0 = 0, f_1 = 1$

f_n for $n \geq 2$

Let S_n be the number of subsets of set having n elements

$S_{n-1} \rightarrow (n-1)$ elements $\rightarrow$ subsets $= 2^{n-1}$

$\downarrow$

subsets $= 2^n$.

$S_n = 2 \cdot S_{n-1}$

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Let S_n be the number of n bit strings not containing the pattern 111

0, 1

00, 01, 11, 10

110

a) String begins with 0 ____ $\{S_{n-1}\}$ → Recurrence 1 → cannot say anything.

b) If string begins with 10 $\{S_{n-2}\}$

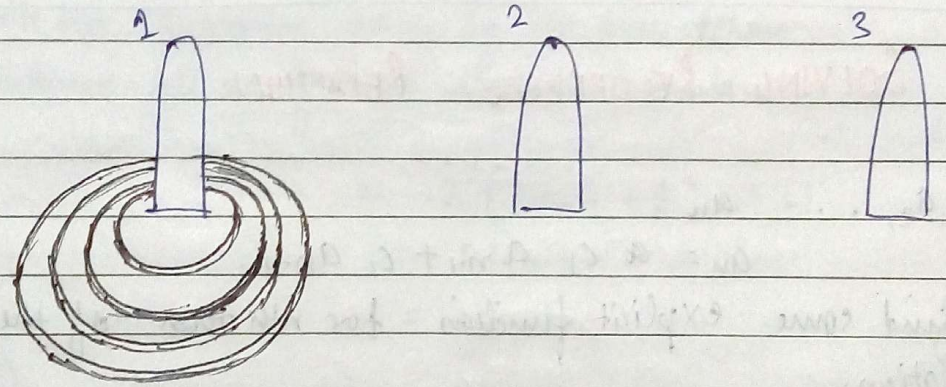
c) String begins with 110 $\{S_{n-3}\}$

Disjoint Sets

$$\boxed{S_n = S_{n-1} + S_{n-2} + S_{n-3}}$$

Initial conditions
$$\begin{cases} S_0 = 0 \\ S_1 = 2 \\ S_2 = 4 \\ S_3 = 7 \end{cases}$$

Tower of Hanoi



★ Problem: Transfer all disks from peg 1 to peg 2 1) One disc at a time
2) Whenever we assume a disk in a peg, then disc with small diameter only can be placed on the disc with larger diameter

3 pegs – n disc with different size, stacked in a peg

Sol^n. Sequences – no. of moves.

$C_1 = 1$
$C_2 = 3$

(largest) n^{th} → 1 to 2
all $(n-1)$ to 3.

$n-1$ from 1 to 3 & then from 3 to 2.

$$\boxed{C_n = 1 + 2 \cdot C_{n-1}.}$$

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How to prove that this gives optimal no. of moves

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BS: Let d_n represents the no. optimal moves for n no. of discs.

$c_n = 1$ $\quad$ $c_n = d_n$, by inspection

$d_n = 1$ $\quad$ at $n=1$.

IS:

for $(n-1)$ no. discs $\quad c_{n-1} = d_{n-1}$, c_{n-1} is optimal no. of moves

$\Rightarrow d_{n-1} \geq 2d_{n-2}$

if c_n does not give optimal soln

$\hookrightarrow c_n \geq d_n$

$d_n \geq 2d_{n-1} + 1 = 2(c_{n-1}+1)$

$d_n \geq c_n$

$c_n = d_n$.

Hence, Proved.

SOLVING RECURRENCE RELATION

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$a_1, a_2, \ldots a_n$

$$a_n = c_1 a_{n-1} + c_2 a_{n-2}.$$

To find some explicit function for nth term of the recurrence relation.

1. Start with a value 5, $a_1 = 5$.
2. add 3.

Recurrence Relⁿ: $a_n = a_{n-1} + 3$

$= a_{n-2} + 3 + 3 = a_{n-2} + 2(3)$

$= \vdots$

$= a_1 + 3(n-1)$

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$S_0 = 1$

$S_n = 2 S_{n-1}$

$= 2 \cdot 2 S_{n-2}$

$= 2^n S_{n-n} = 2^n S_0$

$\boxed{S_n = 2^n}$

Methods:-

1. Iteration
2. Linear homogeneous recurrence relation.

Tower of Hanoi:-

$C_n = 2(C_{n-1}) + 1$

$= 2(2C_{n-2} + 1) + 1 \quad = 2^2 C_{n-2} + 2 + 1$

* Write the nth term in term of predecessors
* Write the successive terms by the prev. terms
* Continue till we get an explicit formula.

$= 2^2(2C_{n-3} + 1) + 2 + 1$

$= 2^3 C_{n-3} + 2^2 + 2^1 + 1$

$= ,$

$= 2^{n-1} C_1 + (2^{n-2} + \dots + 2^2 + 2^1 + 2^0)$

$C_n = (2^{n-1} + 2^{n-2} + \dots + 2^0) = \boxed{2^n - 1}$

$C_1 = 1$

RR

$S_n = 2^n + 4 \cdot 3^n \qquad n \geq 0$

$S_0 = 2^0 + 4 \cdot 3^0 = 5$

$S_1 = 2^1 + 4 \cdot 3^1 = 14$

$S_{n-1} = 2^{n-1} + 4 \cdot 3^{n-1}$

$S_{n-2} = 2^{n-2} + 4 \cdot 3^{n-2}$

$S_n = a \cdot S_{n-1} + b \cdot S_{n-2} + c \dots \rightarrow$ (LHRR)

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$f_n = 1 \cdot f_{n-1} + 1 \cdot f_{n-2}$

$S_n = 2 S_{n-1}$

$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \ldots\ldots$

for prev example, Prove → $S_n = 5S_{n-1} - 6S_{n-2}$
(Put & satisfy).

Let a_n be a linear homogeneous recurrence relation of order k with constant coefficients.

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \ldots c_k a_{n-k}, \quad c_k \neq 0.$$

$c_1, c_2 \ldots c_k$ → Constant coefficients.

- $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ – 2nd order l.h.r.r.

Ex. 1. $a_n = 2n\, a_{n-1}$

Not a l.h.r.r

$a_n = a_{n-1} + 2n$

$a_n = c_1 a_{n-1} a_{n-2}$ → Not l.h.r.r but homogeneous

It is a non-linear recurrence relation.

Ex. 2 $a_n = 5a_{n-1} - 6a_{n-2}$

$a_0 = 7, \quad a_1 = 16.$

Let the sol^n be $V_n = t^n$

(We are trying to get the sol^n of a simplified recurrence relation & then to develop the final one.)

$V_n = 5V_{n-1} - 6V_{n-2}$

$t^n = 5t^{n-1} - 6t^{n-2}$ divide it by t^{n-2}

$t^2 = 5t - 6$

$t^2 - 5t + 6 = 0$

$(t-2)(t-3) = 0$

$t = 2, 3.$

$S_n = 2 \cdot S_{n-1}$
$S_n = 2^n$

If S & T are the solution then $bS + dT$ will be the sol^n also. $S_n = 2^n$, $T = 3^n$.

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$$U_n = bS_n + dT_n$$

$U_0 = bS_0 + dT_0 \Rightarrow 7 = b + d$

$U_1 = bS_1 + dT_1 \Rightarrow 16 = 2b + 3d$

$14 = 2b + 2d$

$d = 2 \quad b = 5$

$$\Rightarrow \boxed{U_n = 5 \cdot 2^n + 2 \cdot 3^n}$$

~~2nd order~~

Theorem: (a) Let $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ — (1) be a second order linear homogeneous recurrence relation with constant coefficients, If S and T are solutions of (1) then $U = bS + dT$ is also a sol^n.

(b) Let r is a root of $t^2 - c_1 t - c_2 = 0$ — (2) then sequence r^n, $n = 0, 1, \ldots$ is a sol^n of (1).

(c) If a is the sequence defined by eq^n (1), with $a_0 = c_0$, $a_1 = c_1$ and r_1 & r_2 are root of eq^n (2) with $r_1 \neq r_2$ then there exists constants b and d such that

$$a_n = b r_1^n + d r_2^n.$$

Proof:

$S = $ sol$^n \Rightarrow S_n = c_1 S_{n-1} + c_2 S_{n-2}$

$T = $ sol$^n \Rightarrow T_n = c_1 T_{n-1} + c_2 T_{n-2}$

$$U_n = bS_n + dT_n$$
$$= b(c_1 S_{n-1} + c_2 S_{n-2}) + d(c_1 T_{n-1} + c_2 T_{n-2})$$
$$= c_1(bS_{n-1} + dT_{n-1}) + c_2(bS_{n-2} + dT_{n-2})$$

$\boxed{U_n = c_1 U_{n-1} + c_2 U_{n-2}}$ Hence, U_n is also sol^n.

$\rightarrow$ r is a root of $t^2 - c_1 t - c_2 = 0 \rightarrow r^2 - c_1 r - c_2 = 0$

$$c_1 r^{n-1} + c_2 r^{n-2} = r^{n-2}(c_1 r + c_2) = r^{n-2} \cdot r^2 = r^n.$$

$\Rightarrow r^n$ is a sol^n of eq (1) $(n = 0, 1, \ldots)$

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1

Let $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ be a 2nd order linear.

$U_n = br_1^n + dr_2^n$ $\quad r_1, r_2$ are roots of ② , $r_1 \neq r_2$.

initial condn: $a_0 = C_0$, $a_1 = C_1$

$U_0 = br_1^0 + dr_2^0 = C_0$ $\quad , n=0, \; a_0 = C_0$

$= b + d$

$U_1 = br_1 + dr_2 = C_1$

Mult. U_0 by $r_1 - U_1 \rightarrow d(r_1 - r_2) = C_0 r - C_1$

$d = \dfrac{C_0 r - C_1}{r_1 - r_2}$ $\quad b = ✓$

Hence, Proved.

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Ex: Assume the growth of elephant in India. Let there are 1000 elephants at $n=0$, Growth rate is 10% i.e population at time n is 10% of that of at time $(n-1)$

$\longrightarrow d_n$ represents the population at time n.

$d_n - d_{n-1} = 0.1\, d_{n-1}$

$d_n = (1.1)\, d_{n-1}$

$= (1.1)^2 d_{n-2}$

$\vdots$

$d_n = (1.1)^n d_0$

$= (1.1)^n \times 1000$

* The growth rate from $(n-1)$ to n is double (twice) that of $(n-2)$ to $(n-1)$

$d_0 = 1000$

$d_1 = 1200$

Recurrence Relation $\boxed{d_n - d_{n-1} = 2(d_{n-1} - d_{n-2})}$

$d_n = 3d_{n-1} - 2d_{n-2}$

Characteristic Polynomial of this Recurrence Relation $\rightarrow t^2 - 3t + 2$

$t = 1, t = 2$

$S = 1, T = 2.$

$U_n = b \cdot 1^n + d \cdot 2^n$

$\boxed{U_n = 800 + 200 \cdot 2^n}$

$b + d = 1000$

$b + 2d = 1200$

$d = 200$

$b = 800$

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Ex.2 Solve the recurrence relation of fibonacci sequence.

$$f_n = f_{n-1} + f_{n-2}, \quad f_1 = 1, \; f_2 = 1$$

Ch. Polynomial → $t^2 - t - 1 = 0$

$$t = \frac{-(-1) \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$U = bS^n + dT^n$

$$U_n = b\left(\frac{1+\sqrt{5}}{2}\right)^n + d\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$1 = b\left(\frac{1+\sqrt{5}}{2}\right) + d\left(\frac{1-\sqrt{5}}{2}\right)$$

$$1 = b\left(\frac{1+\sqrt{5}}{2}\right)^2 + d\left(\frac{1-\sqrt{5}}{2}\right)^2$$

$$d\left(\frac{1-\sqrt{5}}{2}\right)\left(\frac{1+\sqrt{5}}{2} - \frac{1-\sqrt{5}}{2}\right) = \frac{1+\sqrt{5}}{2} - 1$$

$$d\left(\frac{1-\sqrt{5}}{2}\right) \cdot \sqrt{5} = \frac{\sqrt{5}-1}{2}$$

$$d = \frac{1}{\sqrt{5}} \frac{(\sqrt{5}-1)}{2} \times \frac{2}{-(\sqrt{5}-1)}$$

$$\boxed{b = \frac{1}{\sqrt{5}}} \quad \boxed{d = -\frac{1}{\sqrt{5}}}$$

$$\Rightarrow \boxed{U_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n}$$

Theorem: Let $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ be a 2nd order linear homogeneous recurrence relation with constant coefficients.

If both roots of $(t^2 - c_1 t - c_2 = 0)$ are equal to 'r' then there exists b & d such that

$\boxed{a_n = br^n + dnr^n}$ exist and gives the sol^n of above relation.

Proof:- $t^2 - c_1 t - c_2 = (t-r)^2 = t^2 - 2tr + r^2$ → r^n = Sol^n 1.

$c_1 = 2r$

$c_2 = -r^2$

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$$c_1[(n-1)r^{n-1}] + c_2[(n-2)r^{n-2}]$$
$$= 2r(n-1)r^{n-1} - r^2(n-2)r^{n-2}$$
$$= 2r^n(n-1) - r^n(n-2) = r^n(2n-2-n+2)$$
$$= \underbrace{nr^n}_{2^{nd}\text{ soln.}}$$

$S = r^n$
$T = nr^n$

Ex. Solve $d_n = 4(d_{n-1} - d_{n-2})$

$t^2 - 4t + 4 = 0$ $\qquad d_0 = 1$

$t = 2$ $\qquad d_1 = 1.$

$U = bS + dT$

$= b \cdot 2^n + d_n \cdot 2^n$

$1 = b + 0.d \Rightarrow b = 1$

$1 = 2b + 2d \Rightarrow d = -1/2$

Soln:- $\boxed{U = 2^n - n.2^{n-1}}$

Theorem: Let $c_1, c_2 \cdots c_k$ are real numbers. suppose that the characteristic equation $r^k - c_1 r^{k-1} - c_2 r^{k-2} - \cdots - c_k = 0$ has k roots $r_1, r_2, \cdots, r_k$.
Then a sequence $\{a_n\}$ is a solution of recurrence relation $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}$
iff $a_n = \underset{\alpha_1}{a_1} r_1^n + \underset{\alpha_2}{a_2} r_2^n + \cdots + \underset{\alpha_k}{a_k} r_k^n$ for $n = 0, 1, 2 \cdots$ where $\overset{\alpha_1}{a_1}, \overset{\alpha_2}{a_2} \cdots$ are con

Ex. Find the solution of the relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$

$$\left.\begin{aligned} a_0 &= 2 \\ a_1 &= 5 \\ a_2 &= 15 \end{aligned}\right\}$$

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Characteristic Polynomial:-

$$t^3 - 6t^2 + 11t - 6 = 0 \quad = (t-a_1)(t-a_2)(t-a_3)$$

$$t = 1, 2, 3$$

$$a_n = \overset{c_1}{a_1}\, 1^2 + \overset{c_2}{a_2}\, 2^n + \overset{c_3}{a_3}\, 3^n$$

Solve a_1, a_2, a_3 for $2, 5, 15$, $n = 0, 1, 2$. $\longrightarrow a_1 = 1,\ a_2 = -1,\ a_3 = 2.$

$$\boxed{a_n = 1 - 2^n + 2 \cdot 3^n}$$

Theorem:- Let $c_1, c_2, \ldots c_k$ be real numbers

$$r^k - c_1 r^{k-1} - c_2 r^{k-2} - \cdots - c_k = 0$$

has t distinct roots $r_1, r_2, \ldots, r_t$ with multiplicities $m_1, m_2, \ldots, m_t$

$$m_1 + m_2 + \cdots + m_k = k$$

$\{a_n\}$ is a sol^n

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}$$

iff

$$a_n = (a_{1,0} + a_{1,1} n + \cdots + a_{1,m_1-1} n^{m_1-1}) r_1^n +$$
$$(a_{2,0} + a_{2,1} n + \cdots + a_{2,m_2-1} n^{m_2-1}) r_2^n +$$
$$\vdots$$
$$(a_{t,0} + a_{t,1} n + \cdots + a_{t,m_t-1} n^{m_t-1}) r_t^n$$

for $n = 0, 1, 2 \ldots$ where $a_{i,j}$ are constants

$$1 \le i \le t, \quad 0 \le j \le m_i - 1$$

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Linear Non-Homogeneous Recurrence Relations

$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k} + F(n)$

$F_n = F_{n-1} + n$

n	F_{n-1}	a_n
1	1	2
2	2	4
3	4	7
4	7	11
5	11	16

Examples:-

$a_n = \boxed{a_{n-1} + a_{n-2}} + n!$

$a_n = \boxed{a_{n-3}} + n^2+5$

$a_n = \boxed{a_{n-1}} + 5^n$

Theorem: If $\{a_n^{(p)}\}$ is a particular solution of the nonhomogeneous linear recurrence relation with constant coefficient

$a_n = c_1 a_{n-1} + \dots + c_k a_{n-k} + F(n)$, then every solution is of the form $\{a_n^{(p)} + a_n^{(h)}\}$

where $\{a_n^{(h)}\}$ is a solution of the associated homogeneous recurrence relation $a_n = c_1 a_{n-1} + \dots + c_k a_{n-k}$.

$\rightarrow a_n^{(p)} = c_1 a_{n-1}^{(p)} + c_2 a_{n-2}^{(p)} + \dots + c_k a_{n-k}^{(p)} + F(n) \leftarrow$

Proof:- Since $a_n^{(p)}$ is the particular sol^n, so

Let b_n is the sol^n of non-homogeneous r.r.

$$\boxed{b_n = c_1 b_{n-1} + c_2 b_{n-2} + \dots + c_k b_{n-k} + F(n)}$$

$$\underline{b_n - a_n^{(p)}} = c_1 (\underline{b_{n-1} - a_{n-1}^{(p)}}) + c_2 (\underline{b_{n-2} - a_{n-2}^{(p)}}) + \dots + c_k (\underline{b_{n-k} - a_{n-k}^{(p)}})$$

$\hookrightarrow$ Sol^n of homogenous recurrence relation $= a_n^{(h)}$

$b_n - a_n^{(p)} = a_n^{(h)}$

$b_n = a_n^{(p)} + a_n^{(h)}$

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Ex. 1 Solve $a_n = \underbrace{3a_{n-1}}_{h.r.r} \underbrace{+2n}_{F(n)}$, $a_1 = 3$

$a_n^{(h)} = 3^n \cdot \alpha$ $\hookrightarrow a_n^{(h)}$ $\qquad a_n^{(p)} = cn + d$

$a_n = 3a_{n-1} + 2n$

$cn + d = 3(c(n-1) + d) + 2n$

$cn + d = 3cn - 3c + 3d + 2n$

$2cn + 2n + 2d - 3c = 0$

$2n(c+1) + (2d - 3c) = 0$

$\Rightarrow c + 1 = 0$ and $2d - 3c = 0$

$\boxed{c = -1}$ and $d = +\frac{3c}{2} = \boxed{\frac{-3}{2}}$

$a_n = \alpha \cdot 3^n - n - 3/2$

$n = 1$ $\quad a_1 = 3 = 3\alpha - 1 - 3/2$

$3\alpha = 4 + 3/2 = 11/2$

$\boxed{\alpha = 11/6}$

$\Rightarrow \boxed{a_n = \frac{11}{6} 3^n - n - 3/2}$ or $11 \cdot 3^{n-2} - n - 3/2$.

Ex. 2 Solve $a_n = \underbrace{5a_{n-1} - 6a_{n-2}}_{\alpha_1 3^n + \alpha_2 2^n} \underbrace{+7^n}_{+F(n)}$

$a_n^{(p)} = c \cdot 7^n$

$c \cdot 7^n = 5c\, 7^{n-1} - 6c\, 7^{n-2} + 7^n$

$49(c-1) = 35c - 6c = 29c$

$20c = 49 \Rightarrow \boxed{c = 49/20}$

$a_n = \alpha_1 3^n + \alpha_2 2^n + \frac{49}{20} \cdot 7^n$

Find α_1, α_2 for given a_0, a_1

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Theorem: Suppose that $\{a_n\}$ satisfies the linear non-homogeneous recurrence relation $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k} + F(n)$, c_i's are real numbers and

$F(n) = (b_t n^t + b_{t-1} n^{t-1} + \dots + b_1 n + b_0) s^n$ where b_i's and s are real numbers.

When s is not a root of characteristic equation of the associated linear homogeneous recurrence relation, there is a particular solution of the form $(p_t n^t + p_{t-1} n^{t-1} + \dots + p_1 n + p_0) s^n$.

When s is a root of this characteristic equation and its multiplicity is 'm' there is a particular sol^n of the form $n^m (p_t n^t + p_{t-1} n^{t-1} + \dots + p_1 n^1 + p_0) s^n$.

Ex. 3 Solve $a_n = 6a_{n-1} - 9a_{n-2} + F(n)$

$a_n - 6a_{n-1} + 9a_{n-2} = 0$

$r^2 - 6r + 9 = 0 \Rightarrow r = 3 \Rightarrow m = 2$ (Multiplicity)

$a_n = P_n s^n$

$F(n) = 3^n$
$F(n) = n \cdot 3^n$
$F(n) = n^2 \cdot 3^n$
$F(n) = (n^2+1) 3^n$
$F(n) = (n^2+n+1) s^n$

$F(n) = 3^n$

$p_2 n^2 + p_1 n + p_0$

* $p_0 n^2 \cdot 3^n$, if $F(n) = 3^n$
* $n^2 (p_1 n + p_0) \cdot 3^n$, if $F(n) = n \cdot 3^n$.
* $n^2 (p_2 n^2 + p_1 n + p_0) 3^n$, if $F(n) = n^2 \cdot 3^n$

* Solving Recurrence Relation using Substitution.

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Ex. Solve $a_n^2 - 2a_{n-1}^2 = 1$, $n \geq 1$, $a_0 = 1$

Substitute $b_n = a_n^2$ $\leftarrow$ $b_0 = 1$
$b_1 = 3$

$b_n - 2b_{n-1} = 1$
$\underbrace{b_n - 2b_{n-1} - 1 = 0}$

$b_n = d \cdot 2^n + c$ $\Rightarrow$ $d + c = 1$
$2d + c = 3$
$\Rightarrow \boxed{d = 2 \mid c = -1}$

$\Rightarrow b_n = 2^{n+1} - 1$
$a_n = \pm\sqrt{2^{n+1} - 1}$
Since $a_0 = +1$ $\Rightarrow$ $\boxed{a_n = \sqrt{2^{n+1} - 1}}$

Ex. Several person order laptops.
Their name together with their order codes are entered into a spreadsheet. The records are to be sorted in alphabetical order. An error occurs when the names are sorted but the order codes are unchanged. As a result, no one receives the correct laptop. Let D_n denote the no. of ways that n persons can all receive the wrong orders

(a) Show that the sequence $D_1, D_2, \ldots$ satisfy R.R.
$D_n = (n-1)(D_{n-1} + D_{n-2})$

(b) Show the R.R. of part (a) by substitution $C_n = D_n - nD_{n-1}$.

Name	Order Code		Name	Order Code
Hardik	S35		Aayush	S35
Aayush	R12	After Sorting Names	Avinash	R12
Avinash	R05		Bisesh	R05
Shikha	S01		Hardik	S01
Bisesh	M31		Shikha	M31.

ANUBHAV JAIN NOTES

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Initial conditions

$n=1$ $\quad D_1=0$

$n=2$ $\quad D_2=1$

$n=3$ $\quad D_3=2=(3-1)(1+0)$

$n=4$ $\quad D_4=9=(4-1)(2+1)$

$$D_n=(n-1)(D_{n-1}+D_{n-2})$$

a 1
b 2
c 3

a	b	c	
2	1	3	✗
2	3	1	✓
3	1	2	✓
3	2	1	✗

a 1
b 2
c 3
d 4

a	b	c	d	
2	3	4	1	✗
2	3	1	4	✓
2	1	3	4	✗
2	1	4	3	✓
2	4	1	3	✓
2	4	3	1	✗

3 cases

3 → 3 cases
4 → 3 cases

9 cases in total

Let there are n person who order
We consider the order the person 'p' places.

p, q $\quad$ p receives q's order , $p, q \in n$

& — q receives p's order → D_{n-2}

— q does not receive p's order. → D_{n-1}

$(n-2)$ orders in possession with $(n-2)$ persons as p & q both receives wrong order.

$(n-1)$ orders ——— $(n-1)$ ——— persons.

Among n persons, order of p

$$\boxed{D_n=(n-1)(D_{n-1}+D_{n-2})}$$

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