

Chapter 1

Exercise Solutions

E1.1

$$n_i = BT^{3/2} \exp\left(-\frac{E_g}{2kT}\right)$$

GaAs :

$$n_i = (2.1 \times 10^{14})(300)^{3/2} \exp\left(\frac{-1.4}{2(86 \times 10^{-6})(300)}\right)$$

$$\underline{n_i = 1.8 \times 10^6 \text{ cm}^{-3}}$$

Ge :

$$n_i = (1.66 \times 10^{15})(300)^{3/2} \exp\left(\frac{-0.66}{2(86 \times 10^{-6})(300)}\right)$$

$$\underline{n_i = 2.40 \times 10^{13} \text{ cm}^{-3}}$$

E1.2

Si :

$$n_i = (5.23 \times 10^{15})(400)^{3/2} \exp\left(\frac{-1.1}{2(86 \times 10^{-6})(400)}\right)$$

$$\underline{n_i = 4.76 \times 10^{12} \text{ cm}^{-3}}$$

GaAs :

$$n_i = (2.1 \times 10^{14})(400)^{3/2} \exp\left(\frac{-1.4}{2(86 \times 10^{-6})(400)}\right)$$

$$\underline{n_i = 2.44 \times 10^3 \text{ cm}^{-3}}$$

Ge :

$$n_i = (1.66 \times 10^{15})(400)^{3/2} \exp\left(\frac{-0.66}{2(86 \times 10^{-6})(400)}\right)$$

$$\underline{n_i = 9.06 \times 10^{14} \text{ cm}^{-3}}$$

E1.3

a. majority carrier: $\underline{p_0 = 10^{17} \text{ cm}^{-3}}$
minority carrier:

$$n_0 = \frac{n_i^2}{N_a} = \frac{(1.5 \times 10^{10})^2}{10^{17}} = 2.25 \times 10^3 \text{ cm}^{-3}$$

b. $n_0 = N_d - N_a = 5 \times 10^{15}$

majority carrier: $\underline{n_0 = 5 \times 10^{15} \text{ cm}^{-3}}$

minority carrier:

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{15}} = 4.5 \times 10^4 \text{ cm}^{-3}$$

E1.4

(a) $n_o = 5 \times 10^{16} \text{ cm}^{-3}, p_o \ll n_o$
 $\sigma \equiv e\mu_n n_o = (1.6 \times 10^{-19})(1350)(5 \times 10^{16}) \Rightarrow$
 $\underline{\sigma = 10.8 (\Omega \cdot \text{cm})^{-1}}$

(b) $p_o = 5 \times 10^{16} \text{ cm}^{-3}, n_o \ll p_o$
 $\sigma \equiv e\mu_p p_o = (1.6 \times 10^{-19})(480)(5 \times 10^{16}) \Rightarrow$
 $\underline{\sigma = 3.84 (\Omega \cdot \text{cm})^{-1}}$

E1.5

a. $n_0 = N_d = 8 \times 10^{15} \text{ cm}^{-3}$
 $p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{8 \times 10^{15}} \Rightarrow$
 $\underline{p_0 = 2.81 \times 10^4 \text{ cm}^{-3}}$

b. $n = n_0 + \delta n = 8 \times 10^{15} + 0.1 \times 10^{15} \Rightarrow$
 $\underline{n = 8.1 \times 10^{15} \text{ cm}^{-3}}$
 $p = p_0 + \delta p \Rightarrow$
 $\underline{p \approx 10^{14} \text{ cm}^{-3}}$

E1.6

$$J = \sigma E = (10)(15) \Rightarrow$$

$$\underline{J = 150 \text{ A/cm}^2}$$

E1.7

a. $V_{bi} = V_T \ln\left(\frac{N_a N_d}{n_i^2}\right)$
 $V_{bi} = (0.026) \ln\left[\frac{(10^{15})(10^{17})}{(1.5 \times 10^{10})^2}\right]$
 $\underline{V_{bi} = 0.697 \text{ V}}$

b. $V_{bi} = (0.026) \ln\left[\frac{(10^{17})(10^{17})}{(1.5 \times 10^{10})^2}\right]$
 $\underline{V_{bi} = 0.817 \text{ V}}$

E1.8

$$V_{bi} = V_T \ln \left[\frac{N_a N_d}{n_i^2} \right] = (0.026) \ln \left[\frac{(10^6)(10^{17})}{(1.8 \times 10^{10})^2} \right]$$

$$\underline{V_{bi} = 1.23 \text{ V}}$$

E1.9

$$C_J = C_{J0} \left(1 + \frac{V_R}{V_{bi}} \right)^{-1/2}$$

$$V_{bi} = V_T \ln \left[\frac{N_a N_d}{n_i^2} \right]$$

$$= (0.026) \ln \left[\frac{(10^{17})(10^{16})}{(1.8 \times 10^{10})^2} \right] = 0.757 \text{ V}$$

$$0.8 = C_{J0} \left(1 + \frac{5}{0.757} \right)^{-1/2} = C_{J0}(7.61)^{-1/2}$$

$$= C_{J0}(0.362)$$

$$\underline{C_{J0} = 2.21 \text{ pF}}$$

E1.10

a. $I = I_S \left[\exp \left(\frac{V_D}{V_T} \right) - 1 \right]$

For $V_D = 0.5$: $I \approx 10^{-14} \exp \left(\frac{0.5}{0.026} \right)$

For $V_D = 0.6$: $I = 10^{-14} \exp \left(\frac{0.6}{0.026} \right)$

For $V_D = 0.7$: $I = 10^{-14} \exp \left(\frac{0.7}{0.026} \right)$

So we have

| $\frac{V_D}{V_T}$ | I |
|-------------------|--------------------|
| 0.5 | $2.25 \mu\text{A}$ |
| 0.6 | 0.105 mA |
| 0.7 | 4.93 mA |

b. 10^{-14} A both cases

E1.11

$$I = I_S \left[\exp \left(\frac{V_D}{V_T} \right) - 1 \right]$$

$$10^{-3} = (10^{-13}) \left[\exp \left(\frac{V_D}{0.026} \right) - 1 \right]$$

$$(0.026) \ln(10^{10}) \approx V_D$$

$$\underline{V_D = 0.599 \text{ V}}$$

E1.12

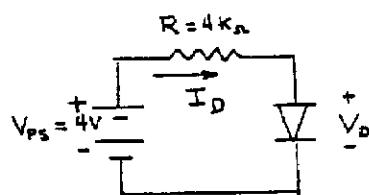
$$\Delta T = 100^\circ \text{C}$$

$$\Delta V_D \approx 2 \times 100 = 200 \text{ mV}$$

$$\Rightarrow V_D = 0.650 - 0.2$$

$$\Rightarrow \underline{V_D = 0.450 \text{ V}}$$

E1.13



$$I_S = 10^{-12} \text{ A}$$

$$V_{PS} = I_D R + V_D \text{ and } I_D \approx I_S \exp \left(\frac{V_D}{V_T} \right)$$

So

$$4 = I_D(4) + V_D \Rightarrow I_D = (4 - V_D)/4 \text{ mA}$$

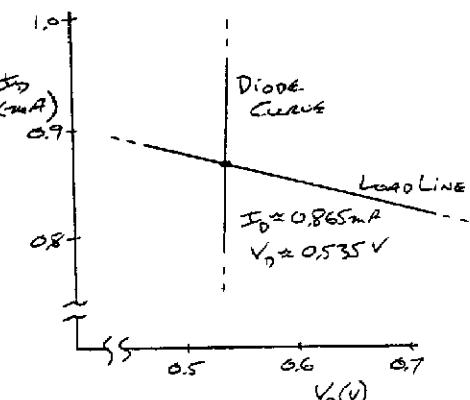
and

$$I_D \approx I_S \exp \left(\frac{V_D}{V_T} \right) \Rightarrow I_D = 10^{-9} \exp \left(\frac{V_D}{0.026} \right) \text{ mA}$$

By trial and error:

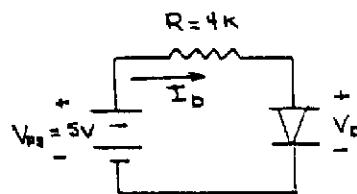
$$I_D = 0.864 \text{ mA} \text{ and } V_D = 0.535 \text{ V}$$

E1.14



E1.15

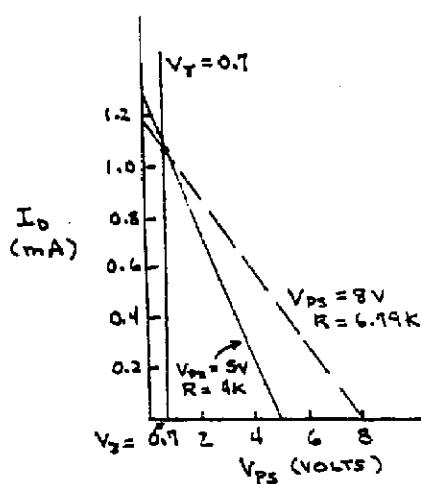
a.



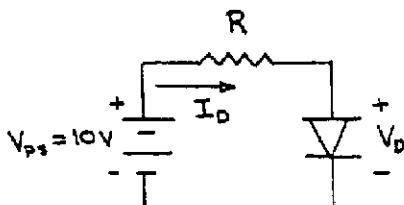
$$I_D = \frac{V_{PS} - V_\gamma}{R} = \frac{5 - 0.7}{4} \Rightarrow I_D = 1.08 \text{ mA}$$

b. $I_D = \frac{V_{PS} - V_\gamma}{R} \Rightarrow R = \frac{V_{PS} - V_\gamma}{I_D}$
 $R = \frac{8 - 0.7}{1.075} \Rightarrow R = 6.79 \text{ k}\Omega$

c.



E1.16

Power dissipation in diode = $I_D V_D$

$$1.05 \text{ mW} = I_D (0.7) \Rightarrow I_D = 1.5 \text{ mA}$$

$$R = \frac{V_{PS} - V_\gamma}{I_D} = \frac{10 - 0.7}{1.5} \Rightarrow R = 6.3 \text{ k}\Omega$$

E1.17

$$g_d = \frac{I_D}{V_T} = \frac{0.8}{0.026} = 30.8 \text{ mS}$$

E1.18

$$r_d = \frac{V_T}{I_D} \Rightarrow 50 = \frac{0.026}{I_D} \Rightarrow I_D = \frac{0.026}{50}$$

$$I_D = 0.52 \text{ mA}$$

E1.19

$$I_D \cong I_S \exp\left(\frac{V_D}{V_T}\right)$$

$$V_D = V_T \ln\left(\frac{I_D}{I_S}\right)$$

$$\text{pn junction: } V_D = (0.026) \ln\left(\frac{10^{-3}}{10^{-12}}\right)$$

$$V_D = 0.539 \text{ V}$$

$$\text{Schottky Diode: } V_D = (0.026) \ln\left(\frac{10^{-3}}{10^{-8}}\right)$$

$$V_D = 0.299 \text{ V}$$

E1.20

For the pn junction diode

$$V_D = V_T \ln\left(\frac{I_D}{I_S}\right) = (0.026) \ln\left(\frac{1.2 \times 10^{-3}}{4 \times 10^{-15}}\right)$$

$$V_D = 0.6871 \text{ V}$$

Schottky diode voltage will be smaller

$$\Rightarrow V_D = 0.5871 - 0.265 = 0.4221 \text{ V}$$

$$I_D = I_S \exp\left(\frac{V_D}{V_T}\right)$$

$$I_S = \frac{1.2 \times 10^{-3}}{\exp\left(\frac{0.4221}{0.026}\right)} \Rightarrow I_S = 1.07 \times 10^{-10} \text{ A}$$

E1.21

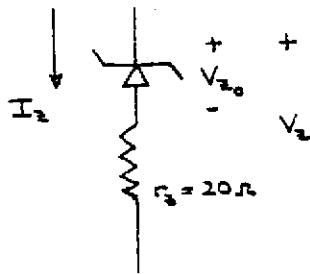
$$\text{Power} = I \cdot V_z$$

$$10 = I(5.6) \Rightarrow I = 1.79 \text{ mA}$$

$$I = \frac{10 - 5.6}{R} = 1.79$$

$$R = \frac{10 - 5.6}{1.79} \Rightarrow R = 2.46 \text{ k}\Omega$$

E1.22



$$V_z = V_{zo} + I_z r_z$$

$$\text{So } V_{zo} = V_z - I_z r_z$$

$$V_{zo} = 5.20 - (10^{-3})(20) = 5.20 - 0.02 = 5.18$$

Then

$$V_z = 5.18 + (10 \times 10^{-3})(20) \Rightarrow V_z = 5.38 \text{ V}$$

Chapter 1

Problem Solutions

1.1

(a) $n_i = BT^{3/2} e^{-E_g/2kT}$

(i) Silicon, T=275K

$$n_i = (5.23 \times 10^{15})(275)^{3/2} e^{-1.1/2(86 \times 10^{-6})(275)}$$

$$n_i = 1.90 \times 10^9 \text{ cm}^{-3}$$

(ii) T=325K

$$n_i = (5.23 \times 10^{15})(325)^{3/2} e^{-1.1/2(86 \times 10^{-6})(325)}$$

$$n_i = 8.71 \times 10^{10} \text{ cm}^{-3}$$

(b) GaAs

(i) T=275K

$$n_i = (2.1 \times 10^{14})(275)^{3/2} e^{-1.4/2(86 \times 10^{-6})(275)}$$

$$n_i = 1.34 \times 10^5 \text{ cm}^{-3}$$

(ii) T=325K

$$n_i = (2.10 \times 10^{14})(325)^{3/2} e^{-1.4/2(86 \times 10^{-6})(325)}$$

$$n_i = 1.63 \times 10^7 \text{ cm}^{-3}$$

1.2

a. $n_i = BT^{3/2} \exp\left(\frac{-E_g}{2kT}\right)$

$$10^{12} = 5.23 \times 10^{15} T^{3/2} \exp\left(\frac{-1.1}{2(86 \times 10^{-6})(T)}\right)$$

$$1.91 \times 10^{-4} = T^{3/2} \exp\left(-\frac{6.40 \times 10^3}{T}\right)$$

By trial and error, $T \approx 367^\circ \text{ K}$

b. $n_i = 10^9 \text{ cm}^{-3}$

$$10^9 = 5.23 \times 10^{15} T^{3/2} \exp\left(\frac{-1.1}{2(86 \times 10^{-6})(T)}\right)$$

$$1.91 \times 10^{-7} = T^{3/2} \exp\left(-\frac{6.40 \times 10^3}{T}\right)$$

By trial and error, $T \approx 268^\circ \text{ K}$

1.3

a. $N_d = 5 \times 10^{15} \text{ cm}^{-3} \Rightarrow n - \text{type}$

$$n_0 = N_d = 5 \times 10^{15} \text{ cm}^{-3}$$

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{15}} \Rightarrow p_0 = 4.5 \times 10^4 \text{ cm}^{-3}$$

b. $N_d = 5 \times 10^{15} \text{ cm}^{-3} \Rightarrow n - \text{type}$

$$n_0 = N_d = 5 \times 10^{15} \text{ cm}^{-3}$$

$$n_i = (2.10 \times 10^{14})(300)^{3/2} \exp\left(\frac{-1.4}{2(86 \times 10^{-6})(300)}\right)$$

$$= (2.10 \times 10^{14})(300)^{3/2} (1.65 \times 10^{-12})$$

$$= 1.80 \times 10^6 \text{ cm}^{-3}$$

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.8 \times 10^6)^2}{5 \times 10^{15}} \Rightarrow p_0 = 6.48 \times 10^{-4} \text{ cm}^{-3}$$

1.4

a. $N_d = 10^{16} \text{ cm}^{-3} \Rightarrow p - \text{type}$

$$p_0 = N_d = 10^{16} \text{ cm}^{-3}$$

$$n_0 = \frac{n_i^2}{p_0} = \frac{(1.5 \times 10^{10})^2}{10^{16}} \Rightarrow n_0 = 2.25 \times 10^4 \text{ cm}^{-3}$$

b. Germanium

$$N_d = 10^{16} \text{ cm}^{-3} \Rightarrow p - \text{type}$$

$$p_0 = N_d = 10^{16} \text{ cm}^{-3}$$

$$n_i = (1.66 \times 10^{15})(300)^{3/2} \exp\left(\frac{-0.66}{2(86 \times 10^{-6})(300)}\right)$$

$$= (1.66 \times 10^{15})(300)^{3/2} (2.79 \times 10^{-6})$$

$$= 2.4 \times 10^{13} \text{ cm}^{-3}$$

$$n_0 = \frac{n_i^2}{p_0} = \frac{(2.4 \times 10^{13})^2}{10^{16}} \Rightarrow n_0 = 5.76 \times 10^{10} \text{ cm}^{-3}$$

1.5

(a) $n_0 = 5 \times 10^{15} \text{ cm}^{-3}$

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{15}} \Rightarrow p_0 = 4.5 \times 10^4 \text{ cm}^{-3}$$

(b) $n_0 \gg p_0 \Rightarrow n - \text{type}$

(c) $n_0 \equiv N_d = 5 \times 10^{15} \text{ cm}^{-3}$

1.6

a. Add Donors

$$N_d = 7 \times 10^{15} \text{ cm}^{-3}$$

b. Want $p_0 = 10^6 \text{ cm}^{-3} = n_i^2/N_d$

So $n_i^2 = (10^6)(7 \times 10^{15}) = 7 \times 10^{21}$

$$= B^2 T^3 \exp\left(\frac{-E_g}{kT}\right)$$

$$7 \times 10^{21} = (5.23 \times 10^{15})^2 T^3 \exp\left(\frac{-1.1}{(86 \times 10^{-6})(T)}\right)$$

By trial and error, $T \approx 324^\circ \text{ K}$

1.7

$$I = J \cdot A = \sigma E A$$

$$I = (2.2)(15)(10^{-4}) \Rightarrow I = 3.3 \text{ mA}$$

$$\text{a. } V_{bi} = (0.026) \ln \left[\frac{(10^{15})(10^{15})}{(1.8 \times 10^6)^2} \right] \Rightarrow V_{bi} = 1.05 \text{ V}$$

$$\text{b. } V_{bi} = (0.026) \ln \left[\frac{(10^{15})(10^{18})}{(1.8 \times 10^6)^2} \right] \Rightarrow V_{bi} = 1.23 \text{ V}$$

1.8

$$J = \sigma E \Rightarrow \sigma = \frac{J}{E} = \frac{85}{12}$$

$$\sigma = 7.08 \text{ (ohm - cm)}^{-1}$$

$$\text{c. } V_{bi} = (0.026) \ln \left[\frac{(10^{18})(10^{18})}{(1.8 \times 10^6)^2} \right] \Rightarrow V_{bi} = 1.41 \text{ V}$$

1.9

(a) For n-type,

$$\sigma \equiv e\mu_n N_d = (1.6 \times 10^{-19})(8500)N_d$$

$$\text{For } 10^{15} \leq N_d \leq 10^{19} \text{ cm}^{-3} \Rightarrow$$

$$1.36 \leq \sigma \leq 1.36 \times 10^4 \text{ } (\Omega \text{-cm})^{-1}$$

$$\text{(b) } J = \sigma E = \sigma(0.1) \Rightarrow$$

$$0.136 \leq J \leq 1.36 \times 10^3 \text{ A/cm}^2$$

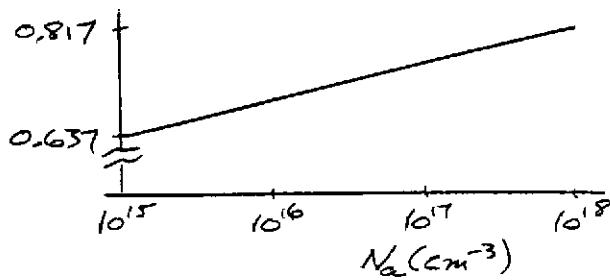
1.10

$$\text{a. } N_a = 10^{17} \text{ cm}^{-3} \Rightarrow p_0 = 10^{17} \text{ cm}^{-3}$$

$$n_0 = \frac{n_i^2}{p_0} = \frac{(1.8 \times 10^6)^2}{10^{17}} \Rightarrow n_0 = 3.24 \times 10^{-5} \text{ cm}^{-3}$$

$$\text{b. } n = n_0 + \delta n = 3.24 \times 10^{-5} + 10^{15} \Rightarrow n = 10^{15} \text{ cm}^{-3}$$

$$p = p_0 + \delta p = 10^{17} + 10^{15} \Rightarrow p = 1.01 \times 10^{17} \text{ cm}^{-3}$$



1.11

$$V_{bi} = V_T \ln \left(\frac{N_a N_d}{n_i^2} \right)$$

$$\text{a. } V_{bi} = (0.026) \ln \left[\frac{(10^{15})(10^{15})}{(1.5 \times 10^{10})^2} \right] \Rightarrow V_{bi} = 0.578 \text{ V}$$

$$\text{b. } V_{bi} = (0.026) \ln \left[\frac{(10^{15})(10^{18})}{(1.5 \times 10^{10})^2} \right] \Rightarrow V_{bi} = 0.757 \text{ V}$$

$$\text{c. } V_{bi} = (0.026) \ln \left[\frac{(10^{18})(10^{18})}{(1.5 \times 10^{10})^2} \right] \Rightarrow V_{bi} = 0.937 \text{ V}$$

| T | kT | $(kT)^{3/2}$ |
|-----|---------|--------------|
| 200 | 0.01733 | 2828.4 |
| 250 | 0.02167 | 3952.8 |
| 300 | 0.026 | 5196.2 |
| 350 | 0.03033 | 6547.9 |
| 400 | 0.03467 | 8000.0 |
| 450 | 0.0390 | 9545.9 |
| 500 | 0.04333 | 11180.3 |

1.12

$$V_{bi} = V_T \ln \left(\frac{N_a N_d}{n_i^2} \right)$$

$$n_i = (2.1 \times 10^{14}) \left(T^{3/2} \right) \exp \left(\frac{-1.4}{2(86 \times 10^{-6})(T)} \right)$$

$$V_{bi} = V_T \ln \left(\frac{N_a N_d}{n_i^2} \right)$$

| T | n_i | V_{bi} |
|-----|-----------------------|----------|
| 200 | 1.256 | 1.405 |
| 250 | 6.02×10^3 | 1.389 |
| 300 | 1.80×10^6 | 1.370 |
| 350 | 1.09×10^8 | 1.349 |
| 400 | 2.44×10^9 | 1.327 |
| 450 | 2.80×10^{10} | 1.302 |
| 500 | 2.00×10^{11} | 1.277 |

$$C_j(\text{avg}) = \frac{0.00743 + 0.0118}{2} = 0.00962 \text{ pF}$$

$$v_c(t) = v_c(\text{final}) + (v_c(\text{initial}) - v_c(\text{final}))e^{-t/\tau}$$

where

$$\tau = RC = RC_j(\text{avg}) = (47 \times 10^3)(0.00962 \times 10^{-12})$$

or

$$\tau = 4.52 \times 10^{-10} \text{ s}$$

Then

$$v_c(t) = 1.5 = 0 + (5 - 0)e^{-t_1/\tau}$$

$$\frac{5}{1.5} = e^{+t_1/\tau} \Rightarrow t_1 = \tau \ln\left(\frac{5}{1.5}\right)$$

$$t_1 = 5.44 \times 10^{-10} \text{ s}$$

(b) For $V_R = 0 \text{ V}$,

$$C_j = C_{j0} = 0.02 \text{ pF}$$

For $V_R = 3.5 \text{ V}$,

$$C_j = (0.02) \left(1 + \frac{3.5}{0.8}\right)^{-1/2} = 0.00863 \text{ pF}$$

$$C_j(\text{avg}) = \frac{0.02 + 0.00863}{2} = 0.0143 \text{ pF}$$

$$\tau = RC_j(\text{avg}) = 6.72 \times 10^{-10} \text{ s}$$

$$v_c(t) = v_c(\text{final}) + (v_c(\text{initial}) - v_c(\text{final}))e^{-t/\tau}$$

$$3.5 = 5 + (0 - 5)e^{-t_2/\tau} = 5(1 - e^{-t_2/\tau})$$

$$\text{so that } t_2 = 8.09 \times 10^{-10} \text{ s}$$

1.15

$$C_j = C_{j0} \left(1 + \frac{V_R}{V_{bi}}\right)^{-1/2}$$

$$V_{bi} = (0.026) \ln \left[\frac{(2 \times 10^{18})(2 \times 10^{15})}{(1.5 \times 10^{10})^2} \right] = 0.673 \text{ V}$$

a. $V_R = 1 \text{ V}$

$$C_j = (1) \left(1 + \frac{1}{0.673}\right)^{-1/2} \Rightarrow C_j = 0.634 \text{ pF}$$

b. $V_R = 5 \text{ V}$

$$C_j = (1) \left(1 + \frac{5}{0.673}\right)^{-1/2} \Rightarrow C_j = 0.344 \text{ pF}$$

1.16

$$(a) C_j = C_{j0} \left(1 + \frac{V_R}{V_{bi}}\right)^{-1/2}$$

For $V_R = 5 \text{ V}$,

$$C_j = (0.02) \left(1 + \frac{5}{0.8}\right)^{-1/2} = 0.00743 \text{ pF}$$

For $V_R = 1.5 \text{ V}$,

$$C_j = (0.02) \left(1 + \frac{1.5}{0.8}\right)^{-1/2} = 0.0118 \text{ pF}$$

1.17

$$V_{bi} = (0.026) \ln \left[\frac{(10^{18})(10^{15})}{(1.5 \times 10^{10})^2} \right] = 0.757 \text{ V}$$

a. $V_R = 1 \text{ V}$

$$C_j = (0.25) \left(1 + \frac{1}{0.757}\right)^{-1/2} = 0.164 \text{ pF}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(2.2 \times 10^{-3})(0.164 \times 10^{-12})}}$$

$$f_0 = 8.38 \text{ MHz}$$

b. $V_R = 10 \text{ V}$

$$C_J = (0.25) \left(1 + \frac{10}{0.757} \right)^{-1/2} = 0.0663 \text{ pF}$$

$$f_0 = \frac{1}{2\pi\sqrt{(2.2 \times 10^{-3})(0.0663 \times 10^{-12})}}$$

$$\underline{f_0 = 13.2 \text{ MHz}}$$

1.18

a. $I = I_S \left[\exp \left(\frac{V_D}{V_T} \right) - 1 \right]$

$$-0.90 = \exp \left(\frac{V_D}{V_T} \right) - 1$$

$$\exp \left(\frac{V_D}{V_T} \right) = 1 - 0.90 = 0.10$$

$$V_D = V_T \ln(0.10) \Rightarrow \underline{V_D = -0.0599 \text{ V}}$$

b.

$$\left| \frac{I_F}{I_R} \right| = \frac{I_S}{I_S} \cdot \frac{\left[\exp \left(\frac{V_F}{V_T} \right) - 1 \right]}{\left[\exp \left(\frac{V_R}{V_T} \right) - 1 \right]} = \left| \frac{\exp \left(\frac{0.2}{0.026} \right) - 1}{\exp \left(\frac{-0.2}{0.026} \right) - 1} \right|$$

$$= \left| \frac{2190}{-1} \right|$$

$$\underline{\frac{I_F}{I_R} = 2190}$$

1.19

a.

$$I \approx (10^{-11}) \exp \left(\frac{0.5}{0.026} \right) \Rightarrow \underline{I = 2.25 \text{ mA}}$$

$$I = (10^{-11}) \exp \left(\frac{0.6}{0.026} \right) \Rightarrow \underline{I = 0.105 \text{ A}}$$

$$I = (10^{-11}) \exp \left(\frac{0.7}{0.026} \right) \Rightarrow \underline{I = 4.93 \text{ A}}$$

b.

$$I \approx (10^{-13}) \exp \left(\frac{0.5}{0.026} \right) \Rightarrow \underline{I = 22.5 \mu\text{A}}$$

$$I = (10^{-13}) \exp \left(\frac{0.6}{0.026} \right) \Rightarrow \underline{I = 1.05 \text{ mA}}$$

$$I = (10^{-13}) \exp \left(\frac{0.7}{0.026} \right) \Rightarrow \underline{I = 49.3 \text{ mA}}$$

1.20

(a) $I = I_S \left(e^{\frac{V_D}{V_T}} - 1 \right)$

$$150 \times 10^{-6} = 10^{-11} \left(e^{\frac{V_D}{V_T}} - 1 \right) \Rightarrow 10^{-11} e^{\frac{V_D}{V_T}}$$

Then

$$V_D = V_T \ln \left(\frac{150 \times 10^{-6}}{10^{-11}} \right) = (0.026) \ln \left(\frac{150 \times 10^{-6}}{10^{-11}} \right)$$

Or

$$\underline{V_D = 0.430 \text{ V}}$$

(b)

$$V_D = V_T \ln \left(\frac{150 \times 10^{-6}}{10^{-13}} \right)$$

Or

$$\underline{V_D = 0.549 \text{ V}}$$

1.21

a. $I_D \approx I_S \exp \left(\frac{V_D}{nV_T} \right)$

$$10^{-3} = I_S \exp \left(\frac{0.7}{2(0.026)} \right) \Rightarrow \underline{I_S = 1.42 \times 10^{-9} \text{ A}}$$

b. $I_D = (1.42 \times 10^{-9}) \exp \left(\frac{0.8}{2(0.026)} \right)$

$$\underline{I_D = 6.82 \text{ mA}}$$

c. $10^{-3} = I_S \exp \left(\frac{0.7}{0.026} \right)$

$$\underline{I_S = 2.03 \times 10^{-15} \text{ A}}$$

$$I_D = (2.03 \times 10^{-15}) \exp \left(\frac{0.8}{0.026} \right)$$

$$\underline{I_D = 46.8 \text{ mA}}$$

1.22

I_S doubles for every 5°C increase in temperature.

$$I_S = 10^{-12} \text{ A} \quad \text{at } T = 300\text{K}$$

$$\text{For } I_S = 0.5 \times 10^{-12} \text{ A} \Rightarrow \underline{T = 295\text{K}}$$

$$\text{For } I_S = 50 \times 10^{-12} \text{ A}, \quad (2)^n = 50 \Rightarrow n = 5.64$$

Where n equals number of 5°C increases.

Then

$$\Delta T = (5.64)(5) = 28.2 \text{ K}$$

So

$$\underline{295 \leq T \leq 328.2 \text{ K}}$$

1.23

$$\frac{I_S(T)}{I_S(-55)} = 2^{\Delta T/5}, \quad \Delta T = 155^\circ C$$

$$\frac{I_S(100)}{I_S(-55)} = 2^{155/5} = 2.147 \times 10^3$$

$$V_T @ 100^\circ C \Rightarrow 373^\circ K \Rightarrow V_T = 0.03220$$

$$V_T @ -55^\circ C \Rightarrow 216^\circ K \Rightarrow V_T = 0.01865$$

$$\frac{I_D(100)}{I_D(-55)} = (2.147 \times 10^3) \times \frac{\exp\left(\frac{0.6}{0.0322}\right)}{\exp\left(\frac{0.6}{0.01865}\right)}$$

$$= \frac{(2.147 \times 10^3)(1.237 \times 10^8)}{(9.374 \times 10^{13})}$$

$$\frac{I_D(100)}{I_D(-55)} = 2.83 \times 10^3$$

1.24

$$a. \frac{I_{D2}}{I_{D1}} = 10 = \exp\left(\frac{V_{D2} - V_{D1}}{V_T}\right)$$

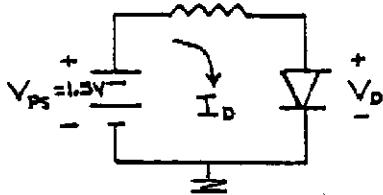
$$\Delta V_D = V_T \ln(10) \Rightarrow \Delta V_D = 59.9 \text{ mV} \approx 60 \text{ mV}$$

$$b. \Delta V_D = V_T \ln(100) \Rightarrow \Delta V_D = 119.7 \text{ mV} \approx 120 \text{ mV}$$

1.25

a.

$$R = 10 M\Omega$$



$$1.5 = I_D (10 \times 10^6) + V_D \text{ and}$$

$$I_D = I_S \left[\exp\left(\frac{V_D}{V_T}\right) - 1 \right]$$

$$1.5 = (10 \times 10^6) (30 \times 10^{-9}) \left[\exp\left(\frac{V_D}{V_T}\right) - 1 \right] + V_D$$

$$= 0.3 \left[\exp\left(\frac{V_D}{V_T}\right) - 1 \right] + V_D$$

By trial and error, $V_D = 0.046 \text{ V}$

$$\text{Then } I_D = \frac{1.5 - 0.046}{10} \Rightarrow I_D = 0.145 \mu\text{A}$$

b. Reverse-Bias

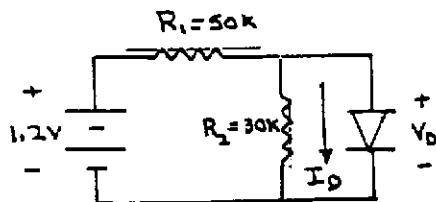
$$I = I_S = 30 \text{ nA}$$

$$V_R = (30 \times 10^{-9})(10 \times 10^6) = 0.30 \text{ V}$$

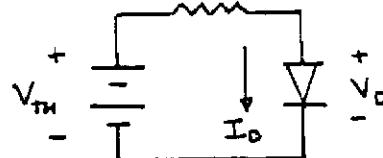
$$V_D = -1.5 + 0.3 \Rightarrow V_D = -1.2 \text{ V}$$

1.26

$$I_S = 5 \times 10^{-13} \text{ A}$$



$$R_{TH} = R_1 || R_2 = 18.75 \text{ k}\Omega$$



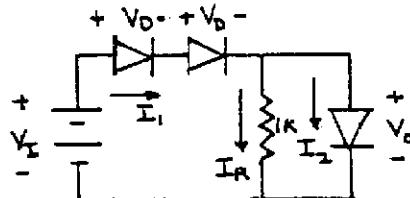
$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (1.2) = \left(\frac{30}{80} \right) (1.2) = 0.45 \text{ V}$$

$$0.45 = I_D R_{TH} + V_D, \quad V_D = V_T \ln\left(\frac{I_D}{I_S}\right)$$

By trial and error:

$$I_D = 2.6 \mu\text{A}, \quad V_D = 0.402 \text{ V}$$

1.27



$$I_S = 2 \times 10^{-13} \text{ A}$$

$$V_G = 0.60 \text{ V}$$

$$I_2 = I_S \exp\left(\frac{V_G}{V_T}\right) = (2 \times 10^{-13}) \exp\left(\frac{0.60}{0.026}\right)$$

$$= 2.105 \text{ mA}$$

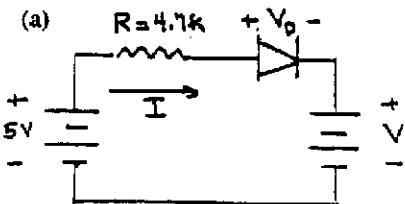
$$I_R = \frac{0.6}{1 \text{ K}} = 0.60 \text{ mA}$$

$$I_1 = I_2 + I_R = 2.705 \text{ mA}$$

$$V_D = V_T \ln\left(\frac{I_1}{I_S}\right) = (0.026) \ln\left(\frac{2.705 \times 10^{-3}}{2 \times 10^{-13}}\right) = 0.6065$$

$$V_I = 2V_D + V_G \Rightarrow V_I = 1.81 \text{ V}$$

1.28



(a) $R = 4.1k\Omega$

$I = \frac{5}{4.1k} = 1.2 \text{ mA}$

$V_o = 5 - 1.2 = 3.8 \text{ V}$

$I_s = 5 \times 10^{-12} \text{ A}$

$I = 0.50 \text{ mA}$

$V_D = (0.026) \ln \left(\frac{0.5 \times 10^{-3}}{5 \times 10^{-12}} \right)$

$\underline{V_D = 0.479 \text{ V}}$

$5 = IR + V_D + V$

$= (0.5 \times 10^{-3})(4.1 \times 10^3) + 0.479 + V$

$\underline{V = 2.17 \text{ V}}$

(b) $P = I_D V_D = (0.5)(0.479)$

or

$\underline{P = 0.24 \text{ mW}}$

(b) Diode is conducting

$5 = V_r + I_D(10) - 5$

or

$I_D = \frac{10 - 0.6}{10} \Rightarrow I_D = 0.94 \text{ mA}$

$V_o = I_D R - 5 = (0.94)(10) - 5 \Rightarrow \underline{V_o = 4.4 \text{ V}}$

(c) Diode is reverse biased

$I_D = 0 \quad V_D = -10 \text{ V}$

1.31

Minimum diode current for $V_{ps}(\min)$

$I_D(\min) = 2 \text{ mA}, \quad V_D = 0.7 \text{ V}$

$I_2 = \frac{0.7}{R_2}, \quad I_1 = \frac{5 - 0.7}{R_1} = \frac{4.3}{R_1}$

We have

$I_1 = I_2 + I_D$

so

(1) $\frac{4.3}{R_1} = \frac{0.7}{R_2} + 2$

Maximum diode current for $V_{ps}(\max)$

$P = I_D V_D - 10 = I_D(0.7) \Rightarrow I_D = 14.3 \text{ mA}$

$I_1 = I_2 + I_D$

or

(2) $\frac{9.3}{R_1} = \frac{0.7}{R_2} + 14.3$

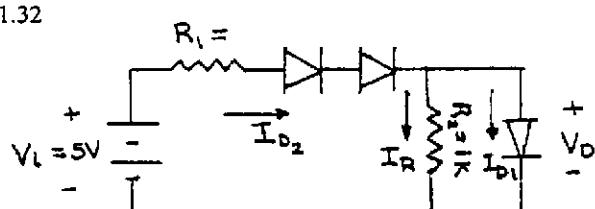
Using Eq. (1),

$\frac{9.3}{R_1} = \frac{4.3}{R_1} - 2 + 14.3 \Rightarrow \underline{R_1 = 0.41 \text{ k}\Omega}$

Then

$\underline{R_2 = 82.5 \Omega}$

1.32



$ID_1 = \frac{1}{2}ID_2 \Rightarrow ID_1 = I_R = \frac{0.65}{1} = 0.65 \text{ mA}$

$ID_2 = 1.3 \text{ mA} = \frac{5 - 3(0.65)}{R_1}$

$\underline{R_1 = 2.35 \text{ k}\Omega, \quad ID_1 = 0.65 \text{ mA, \quad ID_2 = 1.30 \text{ mA}}$

1.29

(a) Assume diode is conducting.

Then, $V_D = V_r = 0.7 \text{ V}$

So that $I_{R2} = \frac{0.7}{30} = 23.3 \mu\text{A}$

$I_{R1} = \frac{1.2 - 0.7}{10} = 50 \mu\text{A}$

Then $I_D = I_{R1} - I_{R2} = 50 - 23.3$

Or

$\underline{I_D = 26.7 \mu\text{A}}$

(b) Let $R_1 = 50 \text{ k}\Omega$. Diode is cutoff.

$V_D = \frac{30}{30+50} \cdot (1.2) = 0.45 \text{ V}$

Since $V_D < V_r$, $I_D = 0$

1.30

(a) Diode is conducting

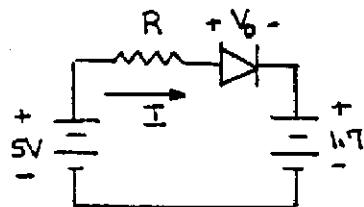
$S = I_D(10) + V_r - 5$

or

$I_D = \frac{10 - 0.6}{10} \Rightarrow I_D = 0.94 \text{ mA}$

$V_o = V_r - 5 = 0.6 - 5 \Rightarrow \underline{V_o = -4.4 \text{ V}}$

1.33



$$V_T = 0.65 \text{ V}$$

$$\text{Power} = I \cdot V_T = 0.2 \text{ mW} = I(0.65)$$

$$\Rightarrow I = 0.308 \text{ mA}$$

$$I = \frac{5 - 0.65 - 1.7}{R} = 0.308$$

$$\Rightarrow R = \frac{2.65}{0.308} \Rightarrow R = 8.60 \text{ k}\Omega$$

1.34

For forward bias

$$I_D = \frac{1.5 - 0.7}{10 \text{ M}\Omega} = 0.08 \mu\text{A}$$

For reverse bias

$$I_D = 0, V_D = -1.5 \text{ V}$$

1.35

$$\text{a. } r_d = \frac{V_T}{I_{DQ}} = \frac{(0.026)}{1} = 0.026 \text{ k}\Omega = 26\Omega$$

$$i_d = 0.05 I_{DQ} = 50 \mu\text{A peak-to-peak}$$

$$v_d = i_d r_d = (26)(50) \mu\text{V}$$

$$\Rightarrow v_d = 1.30 \text{ mV peak-to-peak}$$

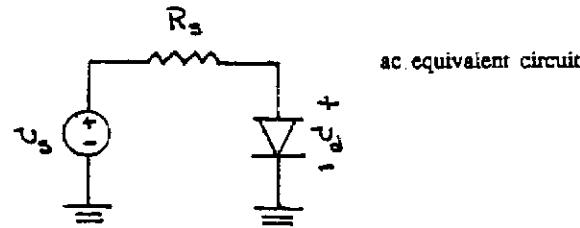
$$\text{b. For } I_{DQ} = 0.1 \text{ mA} \Rightarrow r_d = \frac{(0.026)}{0.1} = 260\Omega$$

$$i_d = 0.05 I_{DQ} = 5 \mu\text{A peak-to-peak}$$

$$v_d = i_d r_d = (260)(5) \mu\text{V}$$

$$\Rightarrow v_d = 1.30 \text{ mV peak-to-peak}$$

1.36



$$\text{a. diode resistance } r_d = V_T/I$$

$$v_d = \left(\frac{r_d}{r_d + R_s} \right) v_s = \left(\frac{V_T/I}{V_T/I + R_s} \right) v_s$$

$$v_d = \left(\frac{V_T}{V_T + IR_s} \right) v_s = v_0$$

$$\text{b. } R_s = 260\Omega$$

$$I = 1 \text{ mA}, \frac{v_0}{v_s} = \left(\frac{V_T}{V_T + IR_s} \right) = \frac{0.026}{0.026 + (1)(0.26)}$$

$$\Rightarrow \frac{v_0}{v_s} = 0.0909$$

$$I = 0.1 \text{ mA}, \frac{v_0}{v_s} = \frac{0.026}{0.026 + (0.1)(0.26)}$$

$$\Rightarrow \frac{v_0}{v_s} = 0.50$$

$$I = 0.01 \text{ mA}, \frac{v_0}{v_s} = \frac{0.026}{0.026 + (0.01)(0.26)}$$

$$\Rightarrow \frac{v_0}{v_s} = 0.909$$

1.37

$$I \approx I_S \exp \left(\frac{V_a}{V_T} \right), V_a = V_T \ln \left(\frac{I}{I_S} \right)$$

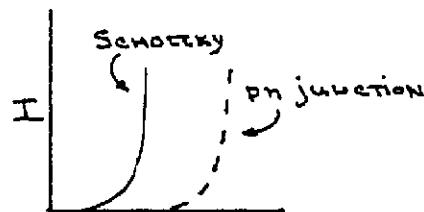
$$\text{pn junction, } V_a = (0.026) \ln \left(\frac{100 \times 10^{-6}}{10^{-14}} \right)$$

$$V_a = 0.599 \text{ V}$$

$$\text{Schottky diode, } V_a = (0.026) \ln \left(\frac{100 \times 10^{-6}}{10^{-9}} \right)$$

$$V_a = 0.299 \text{ V}$$

1.38



$$\text{Schottky: } I \approx I_S \exp \left(\frac{V_a}{V_T} \right)$$

$$V_a = V_T \ln \left(\frac{I}{I_S} \right) = (0.026) \ln \left(\frac{0.5 \times 10^{-3}}{5 \times 10^{-17}} \right) \\ = 0.1796 \text{ V}$$

Then

$$V_a \text{ of pn junction} = 0.1796 + 0.30 \\ = 0.4796$$

$$I_S = \frac{I}{\exp \left(\frac{V_a}{V_T} \right)} = \frac{0.5 \times 10^{-3}}{\exp \left(\frac{0.4796}{0.026} \right)}$$

$$I_S = 4.87 \times 10^{-12} \text{ A}$$

1.39

pn junction $I_D = 0.5 \text{ mA}$

$$I_D \approx I_s e^{V_D V_T} \Rightarrow V_D = V_T \ln\left(\frac{I_D}{I_s}\right)$$

Then

$$V_D = (0.026) \ln\left(\frac{0.5 \times 10^{-3}}{10^{-12}}\right) = 0.521 \text{ V}$$

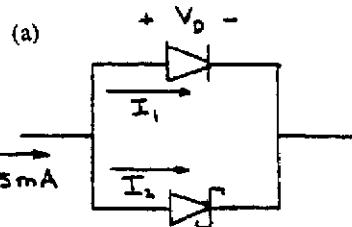
Schottky diode

$$V_D = (0.026) \ln\left(\frac{0.5 \times 10^{-3}}{10^{-8}}\right) = 0.281 \text{ V}$$

Then

$$R = \frac{V_D(\text{pn}) - V_D(S)}{0.5} = \frac{0.521 - 0.281}{0.5} \Rightarrow R = 480 \Omega$$

1.40



$$I_1 + I_2 = 0.5 \times 10^{-3}$$

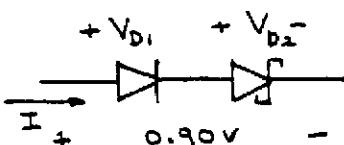
$$5 \times 10^{-8} \exp\left(\frac{V_D}{V_T}\right) + 10^{-12} \exp\left(\frac{V_D}{V_T}\right) = 0.5 \times 10^{-3}$$

$$5.001 \times 10^{-8} \exp\left(\frac{V_D}{V_T}\right) = 0.5 \times 10^{-3}$$

$$V_D = (0.026) \ln\left(\frac{0.5 \times 10^{-3}}{5.001 \times 10^{-8}}\right) \Rightarrow V_D = 0.2395$$

Schottky diode, $I_2 = 0.49999 \text{ mA}$ pn junction, $I_1 = 0.00001 \text{ mA}$

(b)



$$I = 10^{-12} \exp\left(\frac{V_{D1}}{V_T}\right) = 5 \times 10^{-8} \exp\left(\frac{V_{D2}}{V_T}\right)$$

$$V_{D1} + V_{D2} = 0.9$$

$$10^{-12} \exp\left(\frac{V_{D1}}{V_T}\right) = 5 \times 10^{-8} \exp\left(\frac{0.9 - V_{D1}}{V_T}\right)$$

$$= 5 \times 10^{-8} \exp\left(\frac{0.9}{V_T}\right) \cdot \exp\left(-\frac{V_{D1}}{V_T}\right)$$

$$\exp\left(\frac{2V_{D1}}{V_T}\right) = \left(\frac{5 \times 10^{-8}}{10^{-12}}\right) \exp\left(\frac{0.9}{0.026}\right)$$

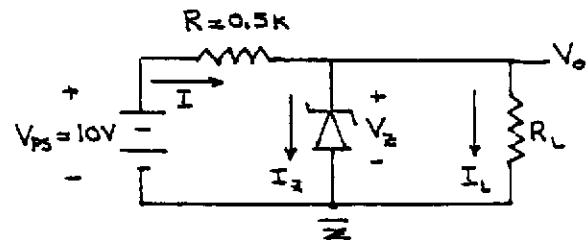
$$2V_{D1} = V_T \ln\left(\frac{5 \times 10^{-8}}{10^{-12}}\right) + 0.9 = 1.1813$$

$$V_{D1} = 0.591 \text{ pn junction}$$

$$V_{D2} = 0.309 \text{ Schottky diode}$$

$$I = 10^{-12} \exp\left(\frac{0.5907}{0.026}\right) \Rightarrow I = 7.36 \text{ mA}$$

1.41



$$V_Z = V_{Z0} = 5.6 \text{ V at } I_Z = 0.1 \text{ mA}$$

$$r_Z = 10 \Omega$$

$$I_Z r_Z = (0.1)(10) = 1 \text{ mV}$$

$$V_{Z0} = 5.599$$

a. $R_L \rightarrow \infty \Rightarrow$

$$I_Z = \frac{10 - 5.599}{R + r_Z} = \frac{4.401}{0.50 + 0.01} = 8.63 \text{ mA}$$

$$V_Z = V_{Z0} + I_Z r_Z = 5.599 + (0.00863)(10)$$

$$V_Z = V_0 = 5.685 \text{ V}$$

$$\text{b. } V_{PS} = 11 \text{ V} \Rightarrow I_Z = \frac{11 - 5.599}{0.51} = 10.59 \text{ mA}$$

$$V_Z = V_0 = 5.599 + (0.01059)(10) = 5.705 \text{ V}$$

$$V_{PS} = 9 \text{ V} \Rightarrow I_Z = \frac{9 - 5.599}{0.51} = 6.669 \text{ mA}$$

$$V_Z = V_0 = 5.599 + (0.006669)(10) = 5.666 \text{ V}$$

$$\Delta V_0 = 5.705 - 5.666 \Rightarrow \Delta V_0 = 0.039 \text{ V}$$

e. $I = I_Z + I_L$

$$I_L = \frac{V_0}{R_L}, \quad I = \frac{V_{PS} - V_0}{R}, \quad I_Z = \frac{V_0 - V_{Z0}}{r_Z}$$

$$\frac{10 - V_0}{0.50} = \frac{V_0 - 5.599}{0.010} + \frac{V_0}{2}$$

$$\frac{10}{0.50} + \frac{5.599}{0.010} = V_0 \left[\frac{1}{0.50} + \frac{1}{0.010} + \frac{1}{2} \right]$$

$$20.0 + 559.9 = V_0(102.5)$$

$$\underline{V_0 = 5.658 \text{ V}}$$

1.42

a. $I_Z = \frac{9 - 6.8}{0.2} \Rightarrow \underline{I_Z = 11 \text{ mA}}$

$$P_Z = (11)(6.8) \Rightarrow \underline{P_Z = 74.8 \text{ mW}}$$

b. $I_Z = \frac{12 - 6.8}{0.2} \Rightarrow \underline{I_Z = 26 \text{ mA}}$

$$\% = \frac{26 - 11}{11} \times 100 \Rightarrow \underline{136\%}$$

$$P_Z = (26)(6.8) = 176.8 \text{ mW}$$

$$\% = \frac{176.8 - 74.8}{74.8} \times 100 = \underline{136\%}$$

1.43

$$I_Z r_Z = (0.1)(20) = 2 \text{ mV}$$

$$V_{Z0} = 6.8 - 0.002 = 6.798 \text{ V}$$

a. $R_L = \infty$

$$I_Z = \frac{10 - 6.798}{0.5 + 0.02} \Rightarrow I_Z = 6.158 \text{ mA}$$

$$V_0 = V_Z = V_{Z0} + I_Z r_Z = 6.798 + (0.006158)(20)$$

$$\underline{V_0 = 6.921 \text{ V}}$$

b. $I = I_Z + I_L$

$$\frac{10 - V_0}{0.50} = \frac{V_0 - 6.798}{0.020} + \frac{V_0}{1}$$

$$\frac{10}{0.50} + \frac{6.798}{0.020} = V_0 \left[\frac{1}{0.50} + \frac{1}{0.020} + \frac{1}{1} \right]$$

$$359.9 = V_0(53)$$

$$V_0 = 6.791 \text{ V}$$

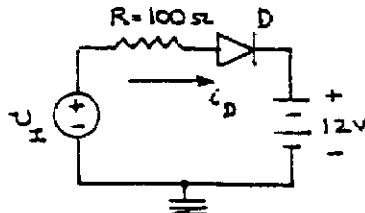
$$\Delta V_0 = 6.791 - 6.921$$

$$\underline{\Delta V_0 = -0.13 \text{ V}}$$

Chapter 2

Exercise Solutions

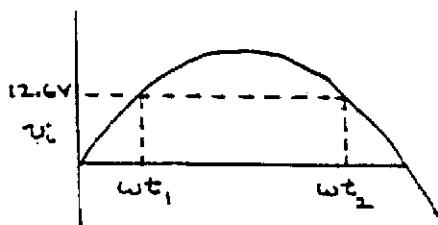
E2.1



a. $i_D(\text{peak}) = \frac{24 - 12 - 0.6}{0.10} = 114 \text{ mA}$

b. $V_R(\text{max}) = 24 + 12 = 36 \text{ V}$

c.



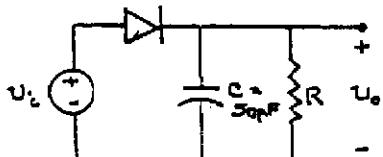
$$v_I = 24 \sin \omega t = 12.6$$

$$\omega t_1 = \sin^{-1} \left(\frac{12.6}{24} \right) = 31.7^\circ$$

$$\text{By symmetry, } \omega t_2 = 180 - 31.7 = 148.3^\circ$$

$$\% = \left(\frac{148.3 - 31.7}{360} \right) \times 100\% \Rightarrow \underline{\underline{32.4\%}}$$

E2.2



$$v_i = 75 \sin (2\pi 60t)$$

$$V_r = \frac{V_m}{fRC}$$

$$\text{or } R = \frac{V_m}{fCV_r} = \frac{75}{(60)(50 \times 10^{-6})(4)} =$$

$$\underline{\underline{R = 6.25 \text{ k}\Omega}}$$

E2.3

$$v_i = 120 \sin (2\pi 60t), V_r = 0.7, R = 2.5 \text{ k}\Omega$$

Full-wave rectifier

$$\text{Turns ratio } 1 : 2 \Rightarrow v_S = v_i$$

$$V_M = 120 - 0.7 = 119.3 \text{ V}$$

$$V_r = 119.3 - 100 = 19.3 \text{ V}$$

$$\text{So } C = \frac{V_m}{2fRV_r} = \frac{119.3}{2(60)(2.5 \times 10^3)(19.3)}$$

$$C = 2.06 \times 10^{-5} = 20.6 \times 10^{-6} \Rightarrow C = 20.6 \mu\text{F}$$

E2.4

$$v_i = 50 \sin (2\pi 60t), V_r = 0.7, R = 10 \text{ k}\Omega$$

Full-wave rectifier

$$C = \frac{V_m}{2fRV_r} = \frac{(50 - 1.4)}{2(60)(10 \times 10^3)(2)}$$

$$C = 2.025 \times 10^{-5} = 20.25 \times 10^{-6} \Rightarrow C = 20.3 \mu\text{F}$$

E2.5

Using Eq. (2-10)

a. $\omega \Delta t = \sqrt{\frac{2V_r}{V_M}} = \sqrt{\frac{2(4)}{75}} = 0.327$

$$\% = \left(\frac{0.327}{2\pi} \right) \times 100\% = \underline{\underline{5.2\%}}$$

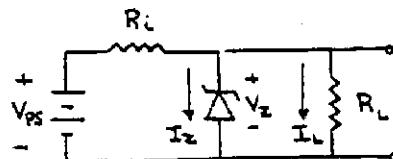
b. $\omega \Delta t = \sqrt{\frac{2V_r}{V_M}} = \sqrt{\frac{2(19.3)}{119.3}} = 0.569$

$$\% = \left(\frac{0.569}{\pi} \right) \times 100\% = \underline{\underline{18.1\%}}$$

c. $\omega \Delta t = \sqrt{\frac{2V_r}{V_M}} = \sqrt{\frac{2(2)}{48.6}} = 0.287$

$$\% = \left(\frac{0.287}{\pi} \right) \times 100\% = \underline{\underline{9.14\%}}$$

E2.6



$$10 \leq V_{PS} \leq 14 \text{ V}, V_Z = 5.6$$

$$20 \leq R_L \leq 100$$

$$I_L(\text{max}) = \frac{5.6}{20} = 0.28 \text{ A}, I_L(\text{min}) = \frac{5.6}{100} = 0.056 \text{ A}$$

$I_z(\text{max})$

$$\begin{aligned} &= \frac{[V_{PS}(\text{max}) - V_Z]I_L(\text{max})}{V_{PS}(\text{min}) - 0.9V_Z - 0.1V_{PS}(\text{max})} \\ &\quad - \frac{[V_{PS}(\text{min}) - V_Z]I_L(\text{min})}{V_{PS}(\text{min}) - 0.9V_Z - 0.1V_{PS}(\text{max})} \\ &= \frac{[14 - 5.6](280) - [10 - 5.6](56)}{10 - (0.9)(5.6) - (0.1)(14)} \\ &= \frac{2352 - 246.4}{3.56} \end{aligned}$$

$I_z(\text{max}) = 591.5 \text{ mA}$

$\text{Power(min)} = I_z(\text{max}) \cdot V_Z = (0.5915)(5.6)$

$\text{Power} = 3.31 \text{ W}$

$$R_i = \frac{V_{PS}(\text{max}) - V_Z}{I_z(\text{max}) + I_L(\text{min})} = \frac{14 - 5.6}{0.5915 + 0.056}$$

$$= \frac{8.4}{0.6475}$$

$R_i \approx 13\Omega$

E2.7

$I_z = \frac{V_{PS} - V_Z}{R_i} - I_L$

For $V_{PS}(\text{min})$ and $I_L(\text{max})$, then

$I_z(\text{min}) = \frac{11 - 9}{20} - 0 = 0$

(Minimum Zener current is zero.)

For $V_{PS}(\text{max})$ and $I_L(\text{min})$, then

$I_z(\text{max}) = \frac{13.6 - 9}{20} - 0 \Rightarrow 230 \text{ mA}$

The characteristic of the minimum Zener current being one-tenth of the maximum value is violated. The proper circuit operation is questionable.

E2.8

$I_z(\text{min}) = \frac{V_{PS}(\text{min}) - V_Z}{R_i} - I_L(\text{max})$

so

$30 = \frac{10 - 9}{0.0153} - I_L(\text{max})$

Or

$I_L(\text{max}) = 35.4 \text{ mA}$

E2.9

$\% \text{ Regulation} = \frac{V_L(\text{max}) - V_L(\text{min})}{V_L(\text{nominal})}$

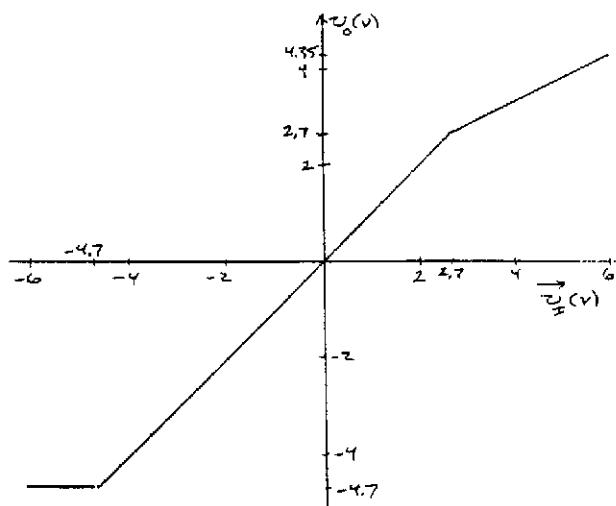
$V_L(\text{nominal}) = 5.6$

$V_L(\text{max}) = V_L(\text{nominal}) + I_z(\text{max})r_Z$

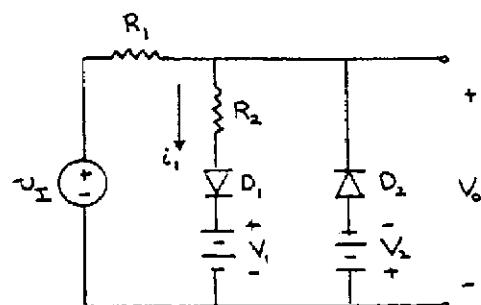
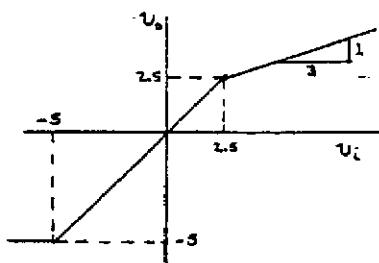
$= 5.6 + (0.5915)(1.5) = 6.487$

$$\begin{aligned} V_L(\text{min}) &= V_L(\text{nominal}) + I_z(\text{min})r_Z \\ &= 5.6 + (I_z(\text{min}))(1.5) \\ I_z(\text{min}) &= \frac{10 - 5.6}{13} = 0.280 \\ &= 0.0585 \text{ A} \\ V_L(\text{min}) &= 5.6 + (0.0585)(1.5) = 5.688 \\ \% \text{ Reg} &= \frac{6.487 - 5.688}{5.6} = 0.143 \Rightarrow 14.3\% \end{aligned}$$

E2.10



E2.11



$$V_T = 0.7 \text{ V}$$

$$\text{For } v_I < 3, D_2 \text{ on} \Rightarrow V_0 = -5 \text{ V} \Rightarrow V_2 = 4.3 \text{ V}$$

$$D_1 \text{ turns on when } v_I = 2.5 \Rightarrow V_1 = 1.8 \text{ V}$$

$$\text{For } v_I > 2.5, \frac{\Delta v_0}{\Delta v_I} = \frac{1}{3} \Rightarrow \frac{R_2}{R_1 + R_2} = \frac{1}{3}$$

$$\Rightarrow R_1 = 2R_2$$

E2.12

$$\text{For } V_s = 0, v_o(\text{max}) = -2 \text{ V}$$

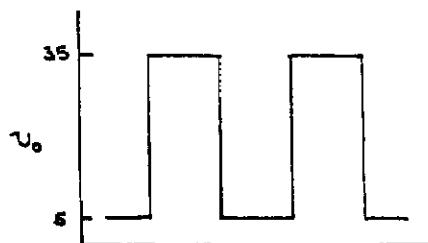
Now, $\Delta v_o = 8 \text{ V}$, so that

$$v_o(\text{min}) = -10 \text{ V}$$

E2.13

As v_S goes negative, D turns on and $v_0 = +5 \text{ V}$.

As v_S goes positive, D turns off.



Output, a square wave oscillating between +5 and +35 volts.

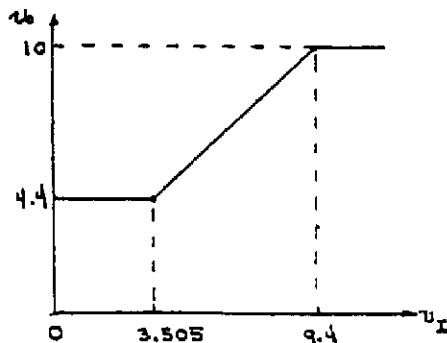
E2.14

$$v_0 = 4.4, I = \frac{10 - 4.4}{9.5} = 0.5895 \text{ mA}$$

Set $I = I_{D1}$,

$$v_I = 4.4 - 0.6 - (0.5895)(0.3)$$

$$v_I = 3.505$$



Summary: $0 \leq v_I \leq 3.5, v_0 = 4.4$

For $v_I > 3.5, D_2$ turns off and when $v_I \geq 9.4$,

$$v_0 = 10$$

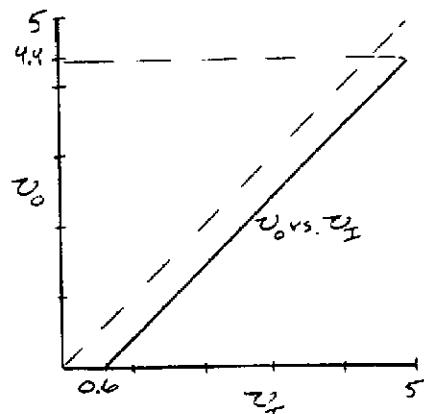
E2.15

$$V_0 = -0.6 \text{ V}, I_{D1} = 0,$$

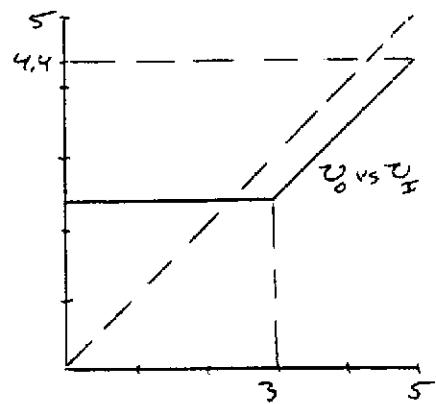
$$I_{D2} = I = \frac{-0.6 - (-10)}{2.2} \Rightarrow I_{D2} = I = 4.27 \text{ mA}$$

E2.16

(a)



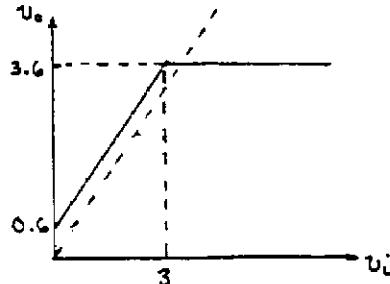
(b)



E2.17

a. $V_0 = 0.6 \text{ V}$ for all V_I

b.



E2.18

a. $I_{Ph} = \eta e \Phi A$

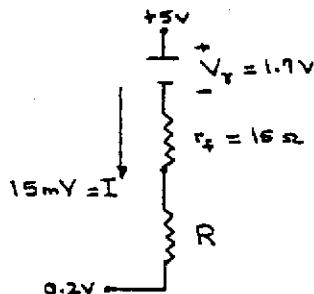
$$I_{Ph} = (0.8)(1.6 \times 10^{-19}) \left[\frac{6.4 \times 10^{-2}}{(2)(1.6 \times 10^{-19})} \right] (0.5)$$

$$\underline{I_{Ph} = 12.8 \text{ mA}}$$

- b. We have $v_0 = (12.8)(1) = 12.8 \text{ volts}$.
 The diode must be reverse biased so that
 $V_{PS} > 12.8 \text{ volts}$.

E2.19

The equivalent circuit is



$$I = \frac{5 - 1.7 - 0.2}{r_f + R} = 15 \text{ mA}$$

$$r_f + R = \frac{5 - 1.7 - 0.2}{15} = \frac{3.1}{15} = 0.207 \text{ k}\Omega$$

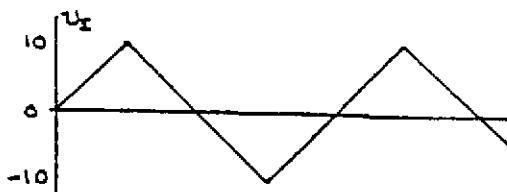
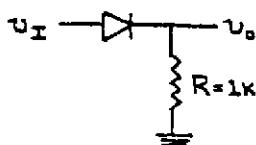
$$= 207\Omega$$

$$R = 207 - 15 \Rightarrow \underline{R = 192\Omega}$$

Chapter 2

Problem Solutions

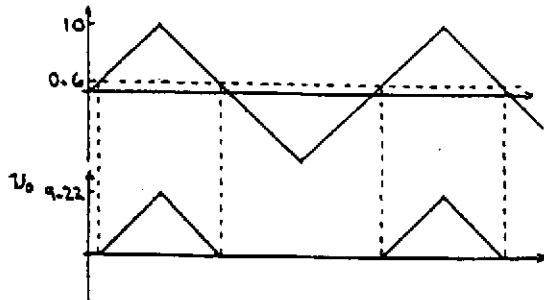
2.1



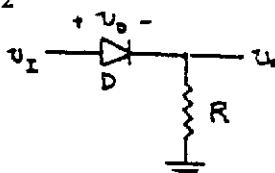
$$V_\gamma = 0.6 \text{ V}, r_f = 20 \Omega$$

$$\begin{aligned} \text{For } v_I = 10 \text{ V}, v_o &= \left(\frac{R}{R + r_f} \right) (10 - 0.6) \\ &= \left(\frac{1}{1 + 0.02} \right) (9.4) \end{aligned}$$

$$v_o = 9.22$$



2.2



$$v_o = v_I - v_D$$

$$v_D = V_T \ln \left(\frac{i_D}{I_S} \right) \text{ and } i_D = \frac{v_o}{R}$$

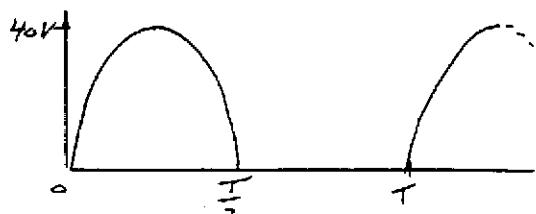
$$\underline{\underline{v_o = v_I - V_T \ln \left(\frac{v_o}{I_S R} \right)}}$$

2.3

$$(a) v_s(\max) = \frac{160}{4} = 40 \text{ V}$$

$$(b) PIV = |v_s(\max)| = 40 \text{ V}$$

(c)



$$\begin{aligned} v_o(\text{avg}) &= \frac{1}{T_0 - T_0} \int_{T_0}^{T_0} v_o(t) dt = \frac{1}{2\pi} \int_0^\pi 40 \sin x dx \\ &= \frac{40}{2\pi} [-\cos x]_0^\pi = \frac{40}{2\pi} [-(1-1)] = \frac{40}{\pi} \end{aligned}$$

or

$$\underline{\underline{v_o(\text{avg}) = 12.7 \text{ V}}}$$

(d) 50%

2.4

$$v_o = v_s - 2V_\gamma \Rightarrow v_s(\max) = v_o(\max) + 2V_\gamma$$

$$\begin{aligned} a. \quad \text{For } v_o(\max) = 25 \text{ V} &\Rightarrow v_s(\max) = 25 + 2(0.7) \\ &= 26.4 \text{ V} \end{aligned}$$

$$\frac{N_1}{N_2} = \frac{160}{26.4} \Rightarrow \underline{\underline{\frac{N_1}{N_2} = 6.06}}$$

$$b. \quad \text{For } v_o(\max) = 100 \text{ V} \Rightarrow v_s(\max) = 101.4 \text{ V}$$

$$\frac{N_1}{N_2} = \frac{160}{101.4} \Rightarrow \underline{\underline{\frac{N_1}{N_2} = 1.58}}$$

From part (a)

$$PIV = 2v_s(\max) - V_\gamma = 2(26.4) - 0.7$$

or

$$\underline{\underline{PIV = 52.1 \text{ V}}}$$

or, from part (b)

$$PIV = 2(101.4) - 0.7$$

or

$$\underline{\underline{PIV = 202.1 \text{ V}}}$$

2.5

a. $v_o(\max) = 24 \text{ V} \Rightarrow v_s(\max) = 24 + 2(0.7)$

$$v_s(\max) = 25.4 \text{ V}$$

$$v_s(\text{rms}) = \frac{25.4}{\sqrt{2}} \Rightarrow v_s(\text{rms}) = 17.96 \text{ V}$$

b. $V_r = \frac{V_M}{2fRC} \Rightarrow C = \frac{V_M}{2fV_r R}$

$$C = \frac{24}{2(60)(0.5)(150)} \Rightarrow C = 2667 \mu\text{F}$$

c. $i_{D,\max} = \frac{V_m}{R} \left(1 + 2\pi \sqrt{\frac{V_m}{2V_r}} \right)$

$$i_{D,\max} = \frac{24}{150} \left(1 + 2\pi \sqrt{\frac{24}{2(0.5)}} \right)$$

$$i_{D,\max} = 5.08 \text{ A}$$

2.6

(a) $v_s(\max) = 24 + 0.7 = 24.7 \text{ V}$

$$v_s(\text{rms}) = \frac{v_s(\max)}{\sqrt{2}} \Rightarrow v_s(\text{rms}) = 17.5 \text{ V}$$

(b) $V_r = \frac{V_m}{fRC} \Rightarrow C = \frac{V_m}{fRV_r} = \frac{24}{(60)(150)(0.5)}$

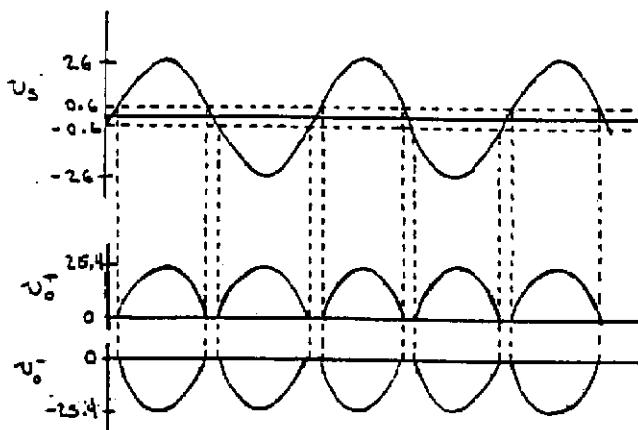
or
 $C = 5333 \mu\text{F}$

(c) For the half-wave rectifier

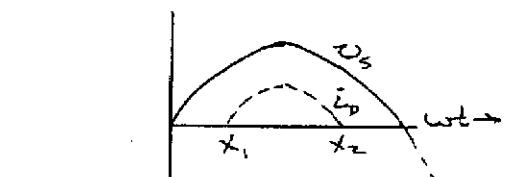
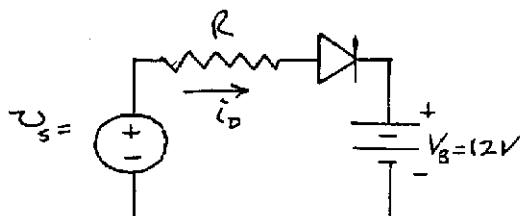
$$i_{D,\max} = \frac{V_m}{R} \left(1 + 4\pi \sqrt{\frac{V_m}{2V_r}} \right) = \frac{24}{150} \left(1 + 4\pi \sqrt{\frac{24}{2(0.5)}} \right)$$

or
 $i_{D,\max} = 10.0 \text{ A}$

2.7



2.8



$$v_s(t) = 24 \sin \omega t$$

Now

$$i_D(\text{avg}) = \frac{1}{T} \int_0^T i_D(t) dt$$

We have for $x_1 \leq \omega t \leq x_2$

$$i_D = \frac{24 \sin x - 12.7}{R}$$

To find x_1 and x_2 ,

$$24 \sin x_1 = 12.7$$

$$x_1 = 0.558 \text{ rad}$$

$$x_2 = \pi - 0.558 = 2.584 \text{ rad}$$

Then

$$i_D(\text{avg}) = 2 = \frac{1}{2\pi} \int_{x_1}^{x_2} \left[\frac{24 \sin x - 12.7}{R} \right] dx$$

$$= \frac{1}{2\pi} \left(\frac{24}{R} \right) (-\cos x) \Big|_{x_1}^{x_2} - \frac{1}{2\pi} \left(\frac{12.7}{R} \right) x \Big|_{x_1}^{x_2}$$

or

$$2 = \frac{6.482}{R} - \frac{4.095}{R} \Rightarrow R = 1.19 \Omega$$

Fraction of time diode is conducting

$$= \frac{x_2 - x_1}{2\pi} \times 100\% = \frac{2.584 - 0.558}{2\pi} \times 100\%$$

or

$$\underline{\text{Fraction} = 32.2\%}$$

Power rating

$$\begin{aligned} P_{\text{avg}} &= R \cdot i_{\text{rms}}^2 = \frac{R}{T} \int_0^T i_D^2 dt = \frac{R}{2\pi} \int_{x_1}^{x_2} \left[\frac{24 \sin x - 12.7}{R} \right]^2 dx \\ &= \frac{1}{2\pi R} \int_{x_1}^{x_2} [(24)^2 \sin^2 x - 2(12.7)(24) \sin x + (12.7)^2] dx \\ &= \frac{1}{2\pi R} \left[(24)^2 \left[\frac{x}{2} - \frac{\sin 2x}{4} \right] \Big|_{x_1}^{x_2} - 2(12.7)(24)(-\cos x) \Big|_{x_1}^{x_2} + (12.7)^2 x \Big|_{x_1}^{x_2} \right] \end{aligned}$$

For $R = 1.19 \Omega$, then

$$P_{\text{avg}} = 17.9 \text{ W}$$

2.9

$$R = \frac{15}{0.1} = 150 \Omega$$

$$v_s(\text{max}) = v_o(\text{max}) + V_r = 15 + 0.7$$

or

$$v_s(\text{max}) = 15.7 \text{ V}$$

Then

$$v_s(\text{rms}) = \frac{15.7}{\sqrt{2}} = 11.1 \text{ V}$$

Now

$$\frac{N_1}{N_2} = \frac{120}{11.1} \Rightarrow \frac{N_1}{N_2} = 10.8$$

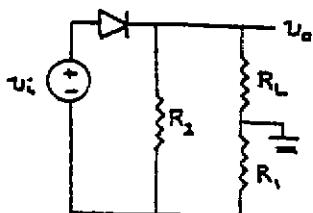
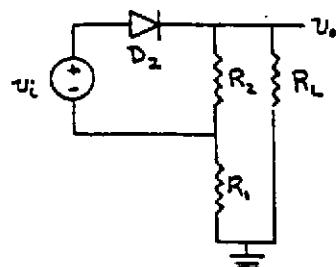
$$V_r = \frac{V_M}{2fRC} \Rightarrow C = \frac{V_M}{2fRV_r} = \frac{15}{2(60)(150)(0.4)}$$

$$\text{or } C = 2083 \mu\text{F}$$

$$PIV = 2v_s(\text{max}) - V_r = 2(15.7) - 0.7$$

$$\text{or } PIV = 30.7 \text{ V}$$

2.10

 For $v_i > 0$


$$V_r = 0$$

$$\text{Voltage across } R_L + R_1 = v_i$$

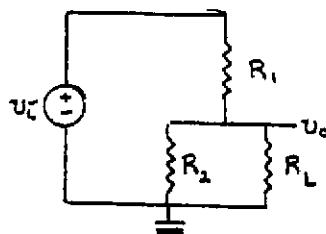
$$\text{Voltage Divider} \Rightarrow v_0 = \left(\frac{R_L}{R_L + R_1} \right) v_i = \frac{1}{2} v_i$$



2.11

 For $v_i > 0$, ($V_T = 0$)

a.



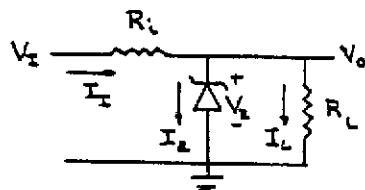
$$v_0 = \left(\frac{R_2 \parallel R_L}{R_2 \parallel R_L + R_1} \right) v_i$$

$$R_2 \parallel R_L = 2.2 \parallel 5.8 = 1.66 \text{ k}\Omega$$

$$v_0 = \left(\frac{1.66}{1.66 + 2.2} \right) v_i = 0.43 v_i$$



2.12



$$V_I = 6.3 \text{ V}, R_1 = 12 \Omega, V_Z = 4.8$$

$$\text{a. } I_I = \frac{6.3 - 4.8}{12} = 125 \text{ mA}$$

$$I_L = I_I - I_Z = 125 - I_Z$$

$$25 < I_L < 120 \text{ mA} \Rightarrow 40 < R_L < 192 \Omega$$

$$\text{b. } P_Z = I_Z V_Z = (100)(48) \Rightarrow P_Z = 480 \text{ mW}$$

$$P_L = I_L V_0 = (120)(4.8) = P_L = 576 \text{ mW}$$

2.13

$$\text{a. } I_I = \frac{20 - 10}{222} = 45.0 \text{ mA}$$

$$I_L = \frac{10}{380} \Rightarrow I_L = 26.3 \text{ mA}$$

$$I_Z = I_I - I_L \Rightarrow I_Z = 18.7 \text{ mA}$$

b. $P_Z(\max) = 400 \text{ mW} \Rightarrow I_Z(\max) = \frac{400}{10} = 40 \text{ mA}$
 $\Rightarrow I_L(\min) = I_T - I_Z(\max) = 45 - 40$
 $\Rightarrow I_L(\min) = 5 \text{ mA} = \frac{10}{R_L}$
 $\Rightarrow R_L = 2 \text{ k}\Omega$

For $R_i = 175\Omega$

$$I_I = 57.1 \text{ mA} \quad I_L = 26.3 \text{ mA} \quad I_Z = 30.8 \text{ mA}$$

$$I_Z(\max) = 40 \text{ mA} \Rightarrow I_L(\min) = 57.1 - 40 = 17.1 \text{ mA}$$

$$R_L = \frac{10}{17.1} \Rightarrow R_L = 585\Omega$$

2.14

a. From Eq. (2-23)

$$I_Z(\max) = \frac{500[20 - 10] - 50[15 - 10]}{15 - (0.9)(10) - (0.1)(20)} = \frac{5000 - 250}{4} = 1187.5 \text{ A}$$

$$I_Z(\max) = 1.1875 \text{ A}$$

$$I_Z(\min) = 0.11875 \text{ A}$$

From Eq. (2-21(b))

$$R_i = \frac{20 - 10}{1187.5 + 50} \Rightarrow R_i = 8.08\Omega$$

b. $P_Z = (1.1875)(10) \Rightarrow P_Z = 11.9 \text{ W}$
 $P_L = I_L(\max)V_0 = (0.5)(10) \Rightarrow P_L = 5 \text{ W}$

2.15

(a) As approximation, assume $I_Z(\max)$ and $I_Z(\min)$ are the same as in problem 2-14.

$$V_0(\max) = V_0(\text{nom}) + I_Z(\max)r_z = 20 + (0.453)(2) = 20.906$$

$$V_0(\min) = V_0(\text{nom}) + I_Z(\min)r_z = 20 + (0.0453)(2) = 20.0906$$

b. $\% \text{ Reg} = \frac{20.906 - 20.0906}{20} \times 100\% \Rightarrow \% \text{ Reg} = 4.08\%$

2.16

$$\% \text{ Reg} = \frac{V_L(\max) - V_L(\min)}{V_L(\text{nom})} \times 100\% = \frac{V_L(\text{nom}) + I_Z(\max)r_z - (V_L(\text{nom}) + I_Z(\min)r_z)}{V_L(\text{nom})} = \frac{[I_Z(\max) - I_Z(\min)](3)}{6} = 0.05$$

So

$$I_Z(\max) - I_Z(\min) = 0.1 \text{ A}$$

Now

$$I_L(\max) = \frac{6}{500} = 0.012 \text{ A}, \quad I_L(\min) = \frac{6}{1000} = 0.006 \text{ A}$$

Now

$$R_i = \frac{V_{PS}(\min) - V_z}{I_Z(\min) + I_L(\max)}$$

or

$$280 = \frac{15 - 6}{I_Z(\min) + 0.012} \Rightarrow I_Z(\min) = 0.020 \text{ A}$$

Then

$$I_Z(\max) = 0.1 + 0.02 = 0.12 \text{ A}$$

and

$$R_i = \frac{V_{PS}(\max) - V_z}{I_Z(\max) + I_L(\min)}$$

or

$$280 = \frac{V_{PS}(\max) - 6}{0.12 + 0.006} \Rightarrow V_{PS}(\max) = 41.3 \text{ V}$$

2.17

Using Figure 2.17

a. $V_{PS} = 20 \pm 25\% \Rightarrow 15 \leq V_{PS} \leq 25 \text{ V}$

For $V_{PS}(\min)$:

$$I_I = I_Z(\min) + I_L(\max) = 5 + 20 = 25 \text{ mA}$$

$$R_i = \frac{V_{PS}(\min) - V_z}{I_I} = \frac{15 - 10}{25} \Rightarrow R_i = 200\Omega$$

b. For $V_{PS}(\max)$

$$\Rightarrow I_I(\max) = \frac{25 - 10}{R_i} \Rightarrow I_I(\max) = 75 \text{ mA}$$

For $I_L(\min) = 0 \Rightarrow I_Z(\max) = 75 \text{ mA}$

$$V_{Z0} = V_z - I_Z r_z = 10 - (0.025)(5) = 9.875 \text{ V}$$

$$V_0(\max) = 9.875 + (0.075)(5) = 10.25$$

$$V_0(\min) = 9.875 + (0.005)(5) = 9.90$$

$$\Delta V_0 = 0.35 \text{ V}$$

c. $\% \text{ Reg} = \frac{\Delta V_0}{V_0(\text{nom})} \times 100\% \Rightarrow \% \text{ Reg} = 3.5\%$

2.18

From Equation (2.21(a))

$$R_i = \frac{V_{PS}(\min) - V_z}{I_Z(\min) + I_L(\max)} = \frac{24 - 16}{40 + 400}$$

or

$$R_i = 18.2 \Omega$$

Also

$$V_r = \frac{V_M}{2fRC} \Rightarrow C = \frac{V_M}{2fRV_r}$$

$$R \approx R_i + r_s = 18.2 + 2 = 20.2 \Omega$$

Then

$$C = \frac{24}{2(60)(1)(20.2)} \Rightarrow C = 9901 \mu\text{F}$$

2.19

$$V_Z = V_{Z0} + I_Z r_Z \quad V_Z(\text{nom}) = 8 \text{ V}$$

$$8 = V_{Z0} + (0.1)(0.5) \Rightarrow V_{Z0} = 7.95 \text{ V}$$

$$I_i = \frac{V_S(\text{max}) - V_Z(\text{nom})}{R_i} = \frac{12 - 8}{3} = 1.333 \text{ A}$$

$$\text{For } I_L = 0.2 \text{ A} \Rightarrow I_Z = 1.133 \text{ A}$$

$$\text{For } I_L = 1 \text{ A} \Rightarrow I_Z = 0.333 \text{ A}$$

$$V_L(\text{max}) = V_{Z0} + I_Z(\text{max})r_Z$$

$$= 7.95 + (1.133)(0.5) = 8.5165$$

$$V_L(\text{min}) = V_{Z0} + I_Z(\text{min})r_Z$$

$$= 7.95 + (0.333)(0.5) = 8.1165$$

$$\Delta V_L = 0.4 \text{ V}$$

$$\% \text{ Reg} = \frac{\Delta V_L}{V_0(\text{nom})} = \frac{0.4}{8} \Rightarrow \% \text{ Reg} = 5.0\%$$

$$V_r = \frac{V_M}{2fRC} \Rightarrow C = \frac{V_M}{2fRV_r}$$

$$R = R_i + r_i = 3 + 0.5 = 3.5 \Omega$$

Then

$$C = \frac{8}{2(60)(3.5)(0.8)} \Rightarrow C = 0.0238 \text{ F}$$

2.20

(a) For $-10 \leq v_I \leq 0$, both diodes are conducting $\Rightarrow v_o = 0$

For $0 \leq v_I \leq 3$, Zener not in breakdown, so

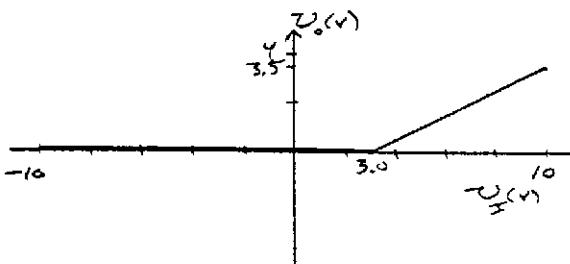
$$i_I = 0, \quad v_o = 0$$

For $v_I > 3$

$$i_I = \frac{v_I - 3}{20} \text{ mA}$$

$$v_o = \left(\frac{v_I - 3}{20} \right)(10) = \frac{1}{2}v_I - 1.5$$

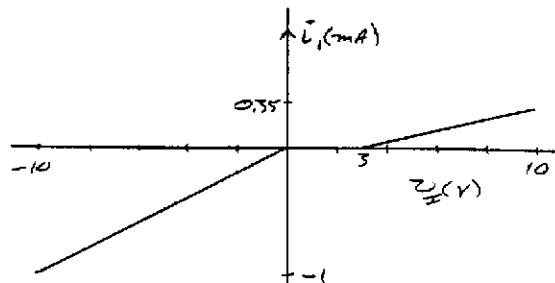
$$\text{At } v_I = 10 \text{ V}, \quad v_o = 3.5 \text{ V}, \quad i_I = 0.35 \text{ mA}$$



(b) For $v_I < 0$, both diodes forward biased

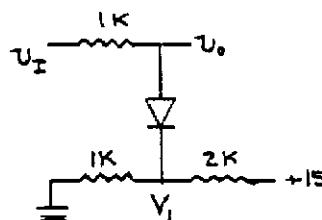
$$-i_I = \frac{0 - v_I}{10}. \quad \text{At } v_I = -10 \text{ V}, \quad i_I = -1 \text{ mA}$$

$$\text{For } v_I > 3, \quad i_I = \frac{v_I - 3}{20}. \quad \text{At } v_I = 10 \text{ V}, \quad i_I = 0.35 \text{ mA}$$



2.21

(a)



$$V_1 = \frac{1}{3} \times 15 = 5 \text{ V} \Rightarrow \text{for } v_I \leq 5.7, \quad v_o = v_I$$

$$\frac{v_I - (V_1 + 0.7)}{1} + \frac{15 - V_1}{2} = \frac{V_1}{1}, \quad v_o = V_1 + 0.7$$

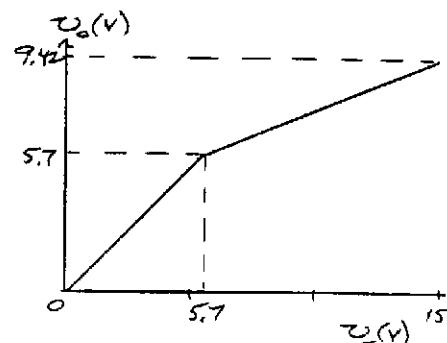
$$\frac{v_I - v_o}{1} + \frac{15 - (v_o - 0.7)}{2} = \frac{v_o - 0.7}{1}$$

$$\frac{v_I}{1} + \frac{15.7}{2} + \frac{0.7}{1} = v_o \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{1} \right) = v_o(2.5)$$

$$v_I + 8.35 = v_o(2.5) \Rightarrow v_o = \frac{1}{2.5}v_I + 3.42$$

$$v_I = 5.7 \Rightarrow v_o = 5.7$$

$$v_I = 15 \Rightarrow v_o = 9.42$$



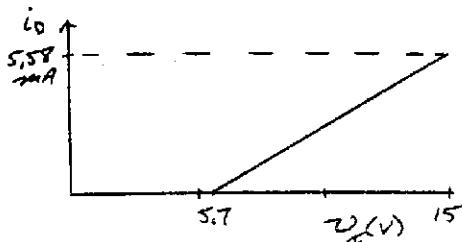
(b) $i_D = 0$ for $0 \leq v_I \leq 5.7$

Then

$$i_D = \frac{v_I - v_o}{1} = \frac{v_I - \left(\frac{v_I}{2.5} + 3.42 \right)}{1}$$

or

$$i_D = \frac{0.6v_I - 3.42}{1} \quad \text{For } v_I = 15, \quad i_D = 5.58 \text{ mA}$$



$$\text{For } v_I = 30 \text{ V, } i = \frac{30 - 10.7}{100 + 10} = 0.175 \text{ A}$$

$$v_o = i(10) + 10.7 = 12.5 \text{ V}$$

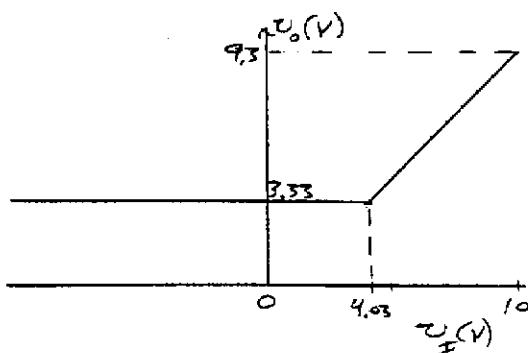
2.22

$$(a) \text{ For D off, } v_o = \left(\frac{20}{30}\right)(20) - 10 = 3.33 \text{ V}$$

Then for $v_i \leq 3.33 + 0.7 = 4.03 \text{ V} \Rightarrow v_o = 3.33 \text{ V}$

For $v_i > 4.03$, $v_o = v_i - 0.7$;

For $v_i = 10$, $v_o = 9.3$

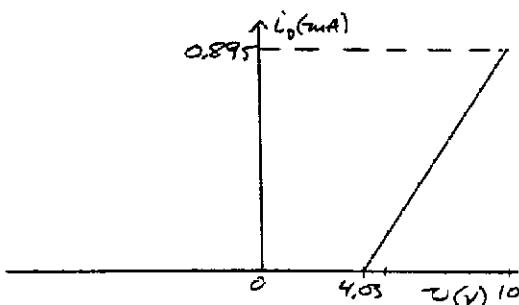


(b) For $v_i \leq 4.03 \text{ V}$, $i_D = 0$

$$\text{For } v_i > 4.03, i_D + \frac{10 - v_o}{10} = \frac{v_o - (-10)}{20}$$

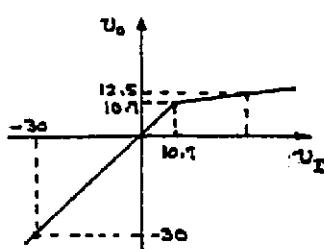
$$\text{Which yields } i_D = \frac{3}{20}v_i - 0.605$$

$$\text{For } v_i = 10, i_D = 0.895 \text{ mA}$$

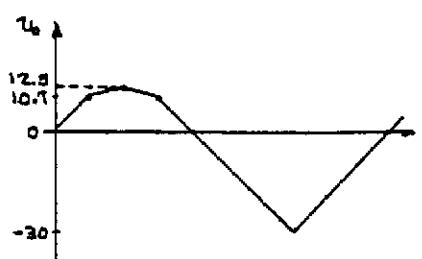


2.23

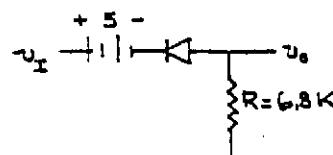
a.



b.

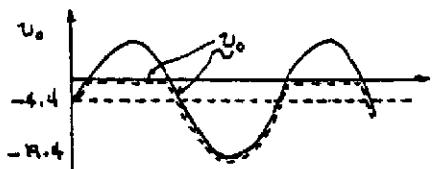


2.24

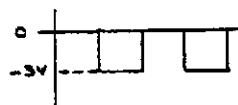


$$V_T = 0.6 \text{ V}$$

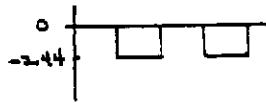
$$v_I = 15 \sin \omega t$$

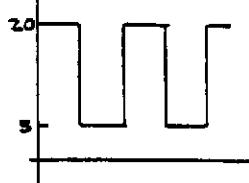


2.25

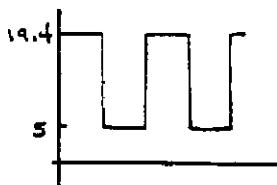
a. $V_T = 0$ 

$$V_T = 0.6$$



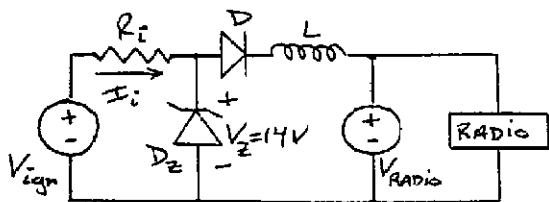
b. $V_T = 0$


$$V_T = 0.6$$



2.26

One possible example is shown.

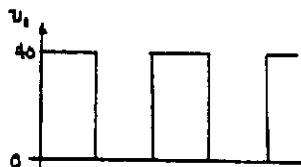


L will tend to block the transient signals
 D_z will limit the voltage to +14 V and -0.7 V.
 Power ratings depends on number of pulses per second and duration of pulse.

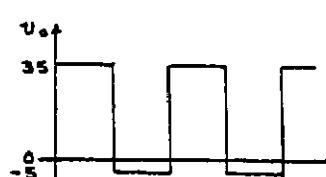
2.27

$$V_T = 0$$

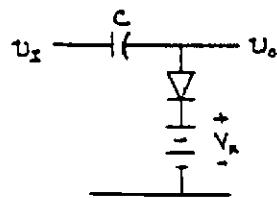
a.



b.

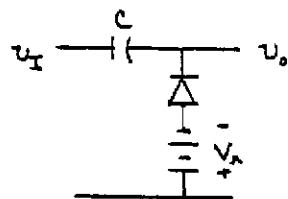


2.28



- a. For $V_T = 0 \Rightarrow V_x = 2.7 \text{ V}$
 b. For $V_T = 0.7 \text{ V} \Rightarrow V_x = 2.0 \text{ V}$

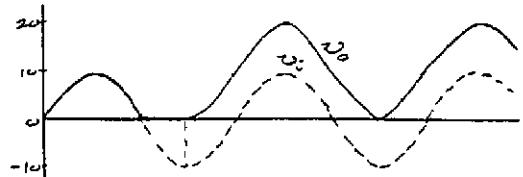
2.29



$$\text{For } V_T = 0; \quad V_x = 10 \text{ V}$$

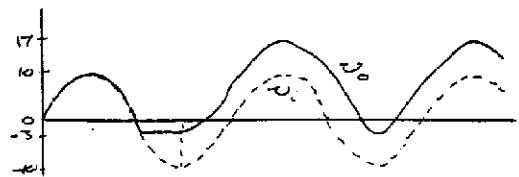
2.30

For circuit in Figure P2.27(a)

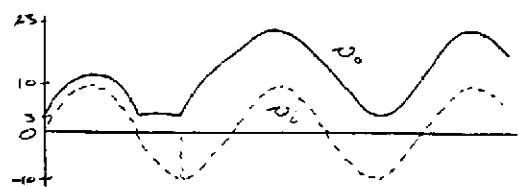


For circuit in Figure P2.27(b)

$$(i) \quad \text{For } V_B = +3 \text{ V}$$

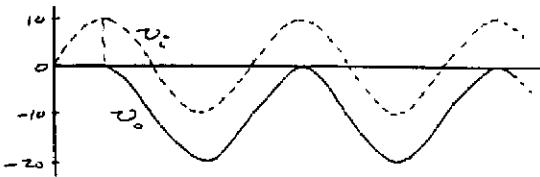


$$(ii) \quad \text{For } V_B = -3 \text{ V}$$



2.31

For Figure P2.27(a)



2.32

a. $I_{D1} = \frac{10 - 0.6}{9.5 + 0.5} \Rightarrow I_{D1} = 0.94 \text{ mA}$ $I_{D2} = 0$

$V_0 = I_{D1}(9.5) \Rightarrow V_0 = 8.93 \text{ V}$

b. $I_{D1} = \frac{5 - 0.6}{9.5 + 0.5} \Rightarrow I_{D1} = 0.44 \text{ mA}$ $I_{D2} = 0$

$V_0 = I_{D1}(9.5) \Rightarrow V_0 = 4.18 \text{ V}$

c. Same as (a)

d. $10 = \frac{(I)}{2}(0.5) + 0.6 + I(9.5) \Rightarrow I = 0.964 \text{ mA}$

$V_0 = I(9.5) \Rightarrow V_0 = 9.16 \text{ V}$

$I_{D1} = I_{D2} = \frac{I}{2} \Rightarrow I_{D1} = I_{D2} = 0.482 \text{ mA}$

2.33

a. $I = I_{D1} = I_{D2} = 0$ $V_0 = 10$

b. $10 = I(9.5) + 0.6 + I(0.5) \Rightarrow$

$I = I_{D2} = 0.94 \text{ mA}$ $I_{D1} = 0$

$V_0 = 10 - I(9.5) \Rightarrow V_0 = 1.07 \text{ V}$

c. $10 = I(9.5) + 0.6 + I(0.5) + 5 \Rightarrow$

$I = I_{D2} = 0.44 \text{ mA}$ $I_{D1} = 0$

$V_0 = 10 - I(9.5) \Rightarrow V_0 = 5.82 \text{ V}$

d. $10 = I(9.5) + 0.6 + \frac{I}{2}(0.5) \Rightarrow I = 0.964 \text{ mA}$

$I_{D1} = I_{D2} = \frac{I}{2} \Rightarrow I_{D1} = I_{D2} = 0.482 \text{ mA}$

$V_0 = 10 - I(9.5) \Rightarrow V_0 = 0.842 \text{ V}$

2.34

a. $V_1 = V_2 = 0 \Rightarrow D_1, D_2, D_3, \text{ on}$ $V_0 = 4.4 \text{ V}$

$I = \frac{10 - 4.4}{9.5} \Rightarrow I = 0.589 \text{ mA}$

$I_{D1} = I_{D2} = \frac{4.4 - 0.6}{0.5} \Rightarrow I_{D1} = I_{D2} = 7.6 \text{ mA}$

$I_{D3} = I_{D1} + I_{D2} - I = 2(7.6) - 0.589 \Rightarrow$

$I_{D3} = 14.6 \text{ mA}$

b. $V_1 = V_2 = 5 \text{ V}$ $D_1 \text{ and } D_2 \text{ on, } D_3 \text{ off}$

$10 = I(9.5) + 0.6 + \frac{I}{2}(0.5) + 5 \Rightarrow I = 0.451 \text{ mA}$

$I_{D1} = I_{D2} = \frac{I}{2} \Rightarrow I_{D1} = I_{D2} = 0.226 \text{ mA}$

$I_{D3} = 0$

$V_0 = 10 - I(9.5) = 10 - (0.451)(9.5) \Rightarrow$

$V_0 = 5.72 \text{ V}$

c. $V_1 = 5 \text{ V}, V_2 = 0$ $D_1 \text{ off, } D_2, D_3 \text{ on}$
 $V_0 = 4.4 \text{ V}$

$I = \frac{10 - 4.4}{9.5} \Rightarrow I = 0.589 \text{ mA}$

$I_{D2} = \frac{4.4 - 0.6}{0.5} \Rightarrow I_{D2} = 7.6 \text{ mA}$

$I_{D1} = 0$

$I_{D3} = I_{D2} - I = 7.6 - 0.589 \Rightarrow I_{D3} = 7.01 \text{ mA}$

d. $V_1 = 5 \text{ V}, V_2 = 2 \text{ V}$ $D_1 \text{ off, } D_2, D_3 \text{ on}$
 $V_0 = 4.4 \text{ V}$

$I = \frac{10 - 4.4}{9.5} \Rightarrow I = 0.589 \text{ mA}$

$I_{D2} = \frac{4.4 - 0.6 - 2}{0.5} \Rightarrow I_{D2} = 3.6 \text{ mA}$

$I_{D1} = 0$

$I_{D3} = I_{D2} - I = 3.6 - 0.589 \Rightarrow I_{D3} = 3.01 \text{ mA}$

2.35

(a) $D_1 \text{ on, } D_2 \text{ off, } D_3 \text{ on}$ So $I_{D2} = 0$

$\text{Now } V_1 = -0.6 \text{ V}, I_{D1} = \frac{10 - 0.6 - (-0.6)}{R_1 + R_2} = \frac{10}{2+6} \Rightarrow$

$I_{D1} = 1.25 \text{ mA}$

$V_1 = 10 - 0.6 - (1.25)(2) \Rightarrow V_1 = 6.9 \text{ V}$

$I_{R3} = \frac{-0.6 - (-5)}{2} = 2.2 \text{ mA}$

$I_{D3} = I_{R3} - I_{D1} = 2.2 - 1.25 \Rightarrow I_{D3} = 0.95 \text{ mA}$

(b) $D_1 \text{ on, } D_2 \text{ on, } D_3 \text{ off}$ So $I_{D3} = 0$

$V_1 = 4.4 \text{ V}, I_{D1} = \frac{10 - 0.6 - 4.4}{R_1} = \frac{5}{6}$

or

$I_{D1} = 0.833 \text{ mA}$

$I_{R2} = \frac{4.4 - (-5)}{R_2 + R_3} = \frac{9.4}{10} = 0.94 \text{ mA}$

$I_{D2} = I_{R2} - I_{D1} = 0.94 - 0.833 \Rightarrow I_{D2} = 0.107 \text{ mA}$

$V_2 = I_{R2}R_3 - 5 = (0.94)(5) - 5 \Rightarrow V_2 = -0.3 \text{ V}$

(c) All diodes are on

$$V_1 = 4.4 \text{ V}, \quad V_2 = -0.6 \text{ V}$$

$$I_{D_1} = 0.5 \text{ mA} = \frac{10 - 0.6 - 4.4}{R_1} \Rightarrow R_1 = 10 \text{ k}\Omega$$

$$I_{R_2} = 0.5 + 0.5 = 1 \text{ mA} = \frac{4.4 - (-0.6)}{R_2} \Rightarrow R_2 = 5 \text{ k}\Omega$$

$$I_{R_3} = 1.5 \text{ mA} = \frac{-0.6 - (-5)}{R_3} \Rightarrow R_3 = 2.93 \text{ k}\Omega$$

2.36

For v_I small, both diodes off

$$v_o = \left(\frac{0.5}{0.5+5} \right) v_I = 0.0909 v_I$$

When $v_I - v_o = 0.6$, D_1 turns on. So we have
 $v_I - 0.0909 v_I = 0.6 \Rightarrow v_I = 0.66, \quad v_o = 0.06$

For D_1 on

$$\frac{v_I - 0.6 - v_o}{5} + \frac{v_I - v_o}{5} = \frac{v_o}{0.5} \text{ which yields}$$

$$v_o = \frac{2v_I - 0.6}{12}$$

When $v_o = 0.6$, D_2 turns on. Then

$$0.6 = \frac{2v_I - 0.6}{12} \Rightarrow v_I = 3.9 \text{ V}$$

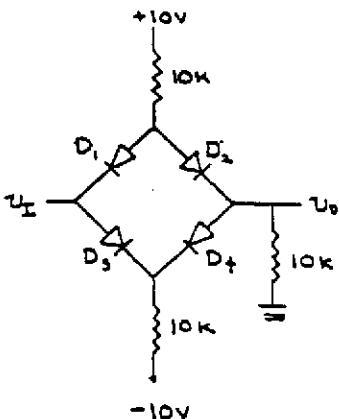
Now for $v_I > 3.9$

$$\frac{v_I - 0.6 - v_o}{5} + \frac{v_I - v_o}{5} = \frac{v_o}{0.5} + \frac{v_o - 0.6}{0.5}$$

Which yields

$$v_o = \frac{2v_I + 5.4}{22}; \text{ For } v_I = 10 \Rightarrow v_o = 1.15 \text{ V}$$

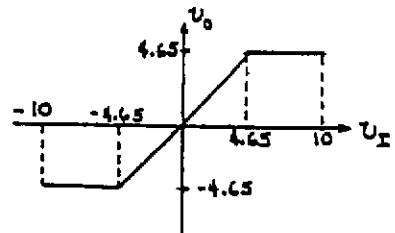
2.37



For $v_I > 0$, when D_1 turns off

$$I = \frac{10 - 0.7}{20} = 0.465 \text{ mA}$$

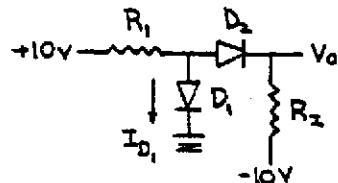
$$v_o = I(10 \text{ k}\Omega) = 4.65 \text{ V}$$



$$v_o = v_I \text{ for } -4.65 \leq v_I \leq 4.65$$

2.38

a.



$$R_1 = 5 \text{ k}\Omega, \quad R_2 = 10 \text{ k}\Omega$$

D_1 and D_2 on $\Rightarrow v_o = 0$

$$I_{D_1} = \frac{10 - 0.7}{5} - \frac{0 - (-10)}{10} = 1.86 - 1.0$$

$$I_{D_1} = 0.86 \text{ mA}$$

b. $R_1 = 10 \text{ k}\Omega, \quad R_2 = 5 \text{ k}\Omega, \quad D_1 \text{ off}, \quad D_2 \text{ on}$

$$I_{D_2} = 0$$

$$I = \frac{10 - 0.7 - (-10)}{15} = 1.287$$

$$V_o = IR_2 - 10 \Rightarrow v_o = -3.57 \text{ V}$$

2.39

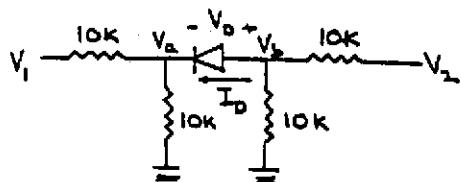
$$\frac{15 - (V_0 + 0.7)}{10} = \frac{V_0 + 0.7}{20} + \frac{V_0}{20}$$

$$\frac{15}{10} - \frac{0.7}{10} - \frac{0.7}{20} = V_0 \left(\frac{1}{10} + \frac{1}{20} + \frac{1}{20} \right) = V_0 \left(\frac{4.0}{20} \right)$$

$$V_0 = 6.975 \text{ V}$$

$$I_D = \frac{V_0}{20} \Rightarrow I_D = 0.349 \text{ mA}$$

2.40



a. $V_1 = 15 \text{ V}, V_2 = 10 \text{ V}$ Diode off

$$V_a = 7.5 \text{ V}, V_b = 5 \text{ V} \Rightarrow V_D = -2.5 \text{ V}$$

$$\underline{I_D = 0}$$

b. $V_1 = 10 \text{ V}, V_2 = 15 \text{ V}$ Diode on

$$\frac{V_2 - V_b}{10} = \frac{V_b}{10} + \frac{V_a}{10} + \frac{V_a - V_1}{10} \Rightarrow V_a = V_b - 0.6$$

$$\frac{15}{10} + \frac{10}{10} = V_b \left(\frac{1}{10} + \frac{1}{10} \right) + V_b \left(\frac{1}{10} + \frac{1}{10} \right)$$

$$-0.6 \left(\frac{1}{10} + \frac{1}{10} \right)$$

$$2.62 = V_b \left(\frac{4}{10} \right) \Rightarrow V_b = 6.55 \text{ V}$$

$$I_D = \frac{15 - 6.55}{10} - \frac{6.55}{10} \Rightarrow I_D = 0.19 \text{ mA}$$

$$\underline{V_D = 0.6 \text{ V}}$$

2.41

$v_I = 0, D_1 \text{ off}, D_2 \text{ on}$

$$I = \frac{10 - 2.5}{15} = 0.5 \text{ mA}$$

$$v_0 = 10 - (0.5)(5) \Rightarrow v_0 = 7.5 \text{ V for } 0 \leq v_I \leq 7.5$$

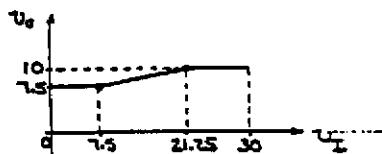
For $v_I = 30 \text{ V}, D_2 \text{ off}, v_0 = 10 \text{ V}$

Determine v_I when $V_x = 10$

$$I = \frac{v_I - 2.5}{25}$$

$$V_x = 10 = I(10) + 2.5 \Rightarrow I = 0.75 \text{ mA}$$

$$v_I = (0.75)(25) + 2.5 = 21.25$$



2.42

a. $V_{01} = V_{02} = 0$

b. $\underline{V_{01} = 4.4 \text{ V}, V_{02} = 3.8 \text{ V}}$

c. $\underline{V_{01} = 4.4 \text{ V}, V_{02} = 3.8 \text{ V}}$

Logic "1" level degrades as it goes through additional logic gates.

2.43

a. $\underline{V_{01} = V_{02} = 5 \text{ V}}$

b. $\underline{V_{01} = 0.6 \text{ V}, V_{02} = 1.2 \text{ V}}$

c. $\underline{V_{01} = 0.6 \text{ V}, V_{02} = 1.2 \text{ V}}$

Logic "0" signal degrades as it goes through additional logic gates.

2.44

$$(V_1 \text{ AND } V_2) \text{ OR } (V_1 \text{ AND } V_4)$$

2.45

$$I = \frac{10 - 1.5 - 0.2}{R + 10} = 12 \text{ mA} = 0.012$$

$$R + 10 = \frac{8.3}{0.012} = 691.7 \Omega$$

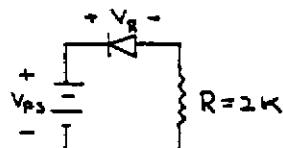
$$\underline{R = 681.7 \Omega}$$

2.46

$$I = \frac{10 - 1.7 - V_I}{0.75} = 8$$

$$V_I = 10 - 1.7 - 8(0.75) \Rightarrow \underline{V_I = 2.3 \text{ V}}$$

2.47



$$V_R = 1 \text{ V}, I = 0.8 \text{ mA}$$

$$V_{PS} = 1 + (0.8)(2)$$

$$\underline{V_{PS} = 2.6 \text{ V}}$$

2.48

$$I_{Ph} = ne\Phi A$$

$$0.6 \times 10^{-3} = (1)(1.6 \times 10^{-19})(10^{17})A$$

$$\underline{A = 3.75 \times 10^{-2} \text{ cm}^2}$$

Chapter 3

Exercise Solutions

E3.1

$$\beta = \frac{\alpha}{1 - \alpha}$$

$$\text{For } \alpha = 0.980, \beta = \frac{0.980}{1 - 0.980} = 49$$

$$\text{For } \alpha = 0.995, \beta = \frac{0.995}{1 - 0.995} = 199$$

$$\underline{49 \leq \beta \leq 199}$$

E3.2

$$\alpha = \frac{\beta}{1 + \beta} = \frac{75}{76} = 0.9868$$

$$\alpha = \frac{125}{126} = 0.9921$$

E3.3

$$I_E = (1 + \beta)I_B$$

$$\text{So } (1 + \beta) = \frac{I_E}{I_B} = \frac{0.780}{0.00960} = 81.25 \Rightarrow \underline{\beta = 80.3}$$

$$\alpha = \frac{\beta}{1 + \beta} = \frac{80.3}{81.3} \Rightarrow \underline{\alpha = 0.9877}$$

$$I_C = \beta I_B = (80.3)(9.60 \mu\text{A}) \Rightarrow \underline{I_C = 0.771 \text{ mA}}$$

E3.4

$$\beta = \frac{\alpha}{1 - \alpha} = \frac{0.990}{1 - 0.990} \Rightarrow \underline{\beta = 99}$$

$$I_B = \frac{I_E}{(1 + \beta)} = \frac{2.150}{100} \Rightarrow \underline{I_B = 21.50 \mu\text{A}}$$

$$I_C = \alpha I_E = (0.990)(2.150) \Rightarrow \underline{I_C = 2.13 \text{ mA}}$$

E3.5

$$r_0 = \frac{V_A}{I_C} = \frac{150}{I_C}$$

$$I_C = 0.1 \text{ mA} \Rightarrow r_0 = 1.5 \text{ M}\Omega$$

$$I_C = 1.0 \text{ mA} \Rightarrow r_0 = 150 \text{ k}\Omega$$

$$I_C = 10 \text{ mA} \Rightarrow r_0 = 15 \text{ k}\Omega$$

E3.6

$$I_C = I_0 \left(1 + \frac{V_{CE}}{V_A} \right)$$

$$\text{At } V_{CE} = 1, I_C = 1$$

$$\text{a. } V_A = 75$$

$$I_C = 1 = I_0 \left(1 + \frac{1}{75} \right) \Rightarrow I_0 = 0.9868 \text{ mA}$$

$$\text{At } V_{CE} = 10, I_C = (0.9868) \left(1 + \frac{10}{75} \right) \Rightarrow$$

$$\underline{I_C = 1.12 \text{ mA}}$$

b. $V_A = 150$

$$I_C = 1 = I_0 \left(1 + \frac{1}{150} \right) \Rightarrow I_0 = 0.9934 \text{ mA}$$

$$\text{At } V_{CE} = 10, I_C = (0.9934) \left(1 + \frac{10}{150} \right) \Rightarrow$$

$I_C = 1.06 \text{ mA}$

E3.7

$$BV_{CE0} = \frac{BV_{CB0}}{\sqrt{\beta}} = \frac{200}{\sqrt[3]{120}} = 40.5 \text{ volts}$$

E3.8

$$BV_{CE0} = \frac{BV_{CB0}}{\sqrt{\beta}}$$

$$BV_{CE0} = (\sqrt[3]{100})(30) = 139 \text{ V}$$

E3.9

a. $V_i = 0.2 < V_{BE}(\text{on}) \Rightarrow I_B = I_C = 0, V_C = 5 \text{ V}$
 $P = 0$

b. $V_i = 3.6$ Transistor is driven into saturation.

$$I_B = \frac{3.6 - 0.7}{0.64} \Rightarrow \underline{I_B = 4.53 \text{ mA}}$$

$$I_C = \frac{5 - V_{CE}(\text{sat})}{R_C} = \frac{5 - 0.2}{0.44} \Rightarrow \underline{I_C = 10.9 \text{ mA}}$$

Note that $\frac{I_C}{I_B} = \frac{10.9}{4.53} = 2.41 < \beta$ which shows

that the transistor is indeed in saturation.

$$P = I_C V_{CE} + I_B V_{BE} = (10.9)(0.2) + (4.53)(0.7) \\ = 2.18 + 3.17$$

$$P = 5.35 \text{ mW}$$

E3.10

For $V_{BC} = 0 \Rightarrow V_0 = 0.7 \text{ V}$

Then $I_C = \frac{5 - 0.7}{0.44} \Rightarrow I_C = 9.77 \text{ mA}$

and $I_B = \frac{I_C}{\beta} = \frac{9.77}{50} = 0.195 \text{ mA}$

$$V_I = I_B R_B + V_{BE}(\text{on}) = (0.195)(0.64) + 0.7 \\ \Rightarrow V_I = 0.825 \text{ V}$$

$$\text{Power} = I_C V_{CE} + I_B V_{BE} \\ = (9.77)(0.7) + (0.195)(0.7)$$

$$\text{Power} = 6.98 \text{ mW}$$

E3.11

For $V_C = 4 \text{ V}$ and $I_{CQ} = 1.5 \text{ mA}$

$$R_C = \frac{10 - 4}{1.5} \Rightarrow R_C = 4 \text{ k}\Omega$$

$$I_E = \frac{-V_{BE}(\text{on}) - (-10)}{R_E}$$

$$I_E = \left(\frac{101}{100} \right) I_C = 1.515 \text{ mA}$$

$$R_E = \frac{10 - 0.70}{1.515} \Rightarrow R_E = 6.14 \text{ k}\Omega$$

E3.12

$$I_C = \frac{10 - V_C}{R_C} = \frac{10 - 6.34}{4} \Rightarrow I_C = 0.915 \text{ mA}$$

$$I_E = \frac{-V_{BE}(\text{on}) - (-10)}{R_E} = \frac{10 - 0.7}{10} \Rightarrow$$

$$I_E = 0.930 \text{ mA}$$

$$I_C = \alpha I_E \Rightarrow \alpha = \frac{I_C}{I_E} = \frac{0.915}{0.930} \Rightarrow \underline{\alpha = 0.9839}$$

$$I_B = I_E - I_C = 0.930 - 0.915 \Rightarrow I_B = 0.0150 \text{ mA}$$

$$\beta = \frac{I_C}{I_B} = \frac{0.915}{0.015} \Rightarrow \underline{\beta = 61}$$

$$V_{CE} = V_C - V_E = 6.34 - (-0.70) \Rightarrow$$

$$V_{CE} = 7.04 \text{ V}$$

E3.13

$$I_E = \frac{10 - V_{EB}(\text{on})}{R_E} = \frac{10 - 0.7}{8} \Rightarrow I_E = 1.16 \text{ mA}$$

$$I_B = \frac{I_E}{1 + \beta} = \frac{1.16}{51} \Rightarrow I_B = 2.27 \mu\text{A}$$

$$I_C = \frac{\beta}{1 + \beta} I_E = \frac{50}{51}(1.16) \Rightarrow I_C = 1.14 \text{ mA}$$

$$V_C = I_C R_C - 10 = (1.14)(4) - 10 = -5.44$$

$$V_{EC} = 0.7 - (-5.44) \Rightarrow V_{EC} = 6.14 \text{ V}$$

E3.14

$$I_E = \frac{V_{BB} - V_{EB}(\text{on})}{R_E}$$

$$\Rightarrow R_E = \frac{4 - 0.7}{1.0} \Rightarrow R_E = 3.3 \text{ k}\Omega$$

$$I_C = \alpha I_E = (0.9920)(1.0) \Rightarrow I_C = 0.992 \text{ mA}$$

$$I_B = I_E - I_C = 1.0 - 0.9920 \Rightarrow I_B = 0.0080 \text{ mA}$$

$$V_{CB} = -V_{BC} = I_C R_C - V_{CC} = (0.992)(1) - 5$$

$$\Rightarrow V_{BC} = 4.01 \text{ V}$$

E3.15

$$V_{BB} = I_B R_B + V_{BE}(\text{on}) + I_E R_E$$

$$= I_B R_B + V_{BE}(\text{on}) + (1 + \beta) I_B R_E$$

$$I_B = \frac{V_{BB} - V_{BE}}{R_B + (1 + \beta) R_E} = \frac{2 - 0.7}{10 + (76)(1)}$$

$$\Rightarrow I_B = 15.1 \mu\text{A}$$

$$I_C = \beta I_B = (75)(15.1 \mu\text{A}) \Rightarrow I_C = 1.13 \text{ mA}$$

$$I_E = (1 + \beta) I_B = (76)(15.1 \mu\text{A}) \Rightarrow I_E = 1.15 \text{ mA}$$

$$V_{CE} = V_{CC} + V_{BB} - I_C R_C - I_E R_E$$

$$= 8 + 2 - (1.13)(2.5) - (1.15)(1)$$

$$V_{CE} = 6.03 \text{ V}$$

E3.16

$$V_{CE} = 2.5 \Rightarrow V_E = 2.5 \text{ V} = I_E R_E$$

$$V_{BB} = I_B R_B + V_{BE}(\text{on}) + V_E$$

$$I_B = \frac{V_{BB} - V_{BE}(\text{on}) - V_E}{R_B} = \frac{5 - 0.7 - 2.5}{10}$$

$$I_B = 0.18 \text{ mA} \Rightarrow I_E = (101)(0.18)$$

$$\Rightarrow I_E = 18.18 \text{ mA}$$

$$\text{So } R_E = \frac{2.5}{18.18} \Rightarrow R_E = 0.138 \text{ k}\Omega = 138 \Omega$$

E3.17

$$V_{BB} = I_E R_E + V_{EB}(\text{on}) + I_B R_B$$

$$I_E = 2.2 \text{ mA} \Rightarrow I_B = \frac{2.2}{51} = 0.0431 \text{ mA}$$

$$I_C = \left(\frac{\beta}{1 + \beta} \right) I_E = \left(\frac{50}{51} \right) (2.2) \Rightarrow I_C = 2.16 \text{ mA}$$

$$V_{BB} = (2.2)(1) + 0.7 + (0.0431)(50)$$

$$\Rightarrow V_{BB} = 5.06 \text{ V}$$

$$V_{EC} = 5 - I_E R_E = 5 - (2.2)(1)$$

$$\Rightarrow V_{EC} = 2.8 \text{ V}$$

E3.18

$$(1) 6 = I_B R_B + V_{EB}(\text{on}) + I_E R_E$$

$$(2) 5 = I_C R_C + V_{CE}(\text{sat}) + I_E R_E$$

$$I_E = I_B + I_C$$

$$(1) 6 = 10I_B + 0.7 + (I_B + I_C)(1)$$

$$(2) 5 = 4I_C + 0.2 + (I_B + I_C)(1)$$

$$(1) [5.3 = I_C + 11I_B] \times 5 \rightarrow 26.5 = 5I_C + 55I_B$$

$$(2) 4.8 = 5I_C + I_B \quad \underline{4.8 = 5I_C + I_B} \\ 21.7 = 54I_B$$

$$\Rightarrow I_B = 0.402 \text{ mA}$$

$$\text{From (1), } I_C = 5.3 - 11I_B \Rightarrow I_C = 0.880 \text{ mA}$$

$$I_E = 1.28 \text{ mA}, \quad V_{CE} = V_{CE}(\text{sat}) = 0.2 \text{ V}$$

E3.19

$$V_{EC} = V_{EC}(\text{sat}) = 0.2 \text{ V}$$

$$I_C = \frac{-0.2 - (-5)}{R_C} = \frac{5 - 0.2}{10} \Rightarrow I_C = 0.48 \text{ mA}$$

$$I_B = \frac{I_C}{2} = \frac{0.48}{2} = 0.24 = I_B$$

$$V_I + I_B R_B + V_{EB}(\text{on}) = 0$$

$$\Rightarrow V_I = -(0.24)(20) - 0.7 \Rightarrow V_I = -5.5 \text{ V}$$

Test Your Understanding

Chapter 3: Exercise Solutions

E3.20

- a. $V_I = -4.5 \text{ V} \Rightarrow V_{BE} < V_{BE}(\text{on}) \Rightarrow$ Transistor is cutoff. $I_B = I_C = I_E = 0, V_{CE} = 10 \text{ V}$
- b. $V_I = -3.5 \text{ V}$ Transistor is active.

$$V_I = I_B R_B + V_{BE}(\text{on}) + I_E R_E - 5$$

$$5 - 3.5 = I_B(10) + 0.7 + (76)I_B(4)$$

$$I_B = \frac{5 - 3.5 - 0.7}{10 + (76)(4)} = 0.00255 \text{ mA}$$

$$\Rightarrow I_B = 2.55 \mu\text{A}$$

$$I_C = \beta I_B = (75)(2.55 \mu\text{A}) \Rightarrow I_C = 0.191 \text{ mA}$$

$$I_E = (1 + \beta)I_B = (76)(2.55 \mu\text{A})$$

$$\Rightarrow I_E = 0.194 \text{ mA}$$

$$V_{CE} = 10 - I_C R_C - I_E R_E$$

$$= 10 - (0.191)(2) - (0.194)(4)$$

$$V_{CE} = 8.84 \text{ V}$$

- c. $V_I = +3.5 \text{ V}$ Transistor is in saturation.

$$(1) 3.5 = I_B R_B + V_{BE}(\text{on}) + I_E R_E - 5$$

$$(2) 5 = I_C R_C + V_{CE}(\text{sat}) + I_E R_E - 5$$

$$(3) I_E = I_B + I_C$$

$$(1) 3.5 + 5 - 0.7 = 10I_B + 4(I_B + I_C)$$

$$(2) 5 + 5 - 0.2 = 2I_C + 4(I_B + I_C)$$

$$(1) 7.8 = 14I_B + 4I_C$$

$$(2) 9.6 = 4I_B + 6I_C$$

$$3 \times (1) \Rightarrow 23.4 = 42I_B + 12I_C$$

$$2 \times (2) \Rightarrow 19.6 = 8I_B + 12I_C$$

$$3.8 = 34I_B$$

$$\Rightarrow I_B = 0.112 \text{ mA}$$

$$7.8 = 14(0.112) + 4I_C = 1.568 + 4I_C$$

$$\Rightarrow I_C = 1.56 \text{ mA}$$

$$\frac{I_C}{I_B} = 13.9 < \beta \Rightarrow \text{In saturation.}$$

$$I_E = I_B + I_C \Rightarrow I_E = 1.67 \text{ mA}$$

$$V_{CE} = V_{CE}(\text{sat}) = 0.2$$

E3.21

$$I_c(\text{sat}) = \frac{5 - 1.5 - 0.2}{R} = 15 \Rightarrow R = 0.220 \text{ k}\Omega$$

$$I_B = \frac{I_C}{20} = \frac{15}{20} = 0.75 \text{ mA} = \frac{5 - 0.8}{R_B}$$

or

$$R_B = 5.6 \text{ k}\Omega$$

E3.22

- a. $V_1 = V_2 = 0, I_{B1} = I_{B2} = I_{C1} = I_{C2} = I_R = 0$
 $V_0 = 5 \text{ V}$

- b. $V_1 = 5 \text{ V}, V_2 = 0, I_{B2} = I_{C2} = 0$

$$I_{B1} = \frac{5 - 0.7}{0.95} \Rightarrow I_{B1} = 4.53 \text{ mA}$$

$$I_{C1} = \frac{5 - 0.2}{0.6} \Rightarrow I_{C1} = I_R = 8 \text{ mA}$$

$$V_0 = 0.2 \text{ V}$$

- c. $V_1 = V_2 = 5 \text{ V}, I_{B1} = I_{B2} = 4.53 \text{ mA}$

$$I_R = 8 \text{ mA}, I_{C1} = I_{C2} = 4 \text{ mA}, V_0 = 0.2 \text{ V}$$

E3.23

$$v_o = 5 - i_C R_C = 5 - \beta I_B R_C \text{ and}$$

$$i_B = \frac{V_{BB} + \Delta v_I - V_{BE}(\text{on})}{R_B}$$

Then

$$\Delta v_o = \frac{-\beta R_C \Delta v_I}{R_B}$$

or

$$\frac{\Delta v_o}{\Delta v_I} = \frac{-\beta R_C}{R_B}$$

$$\text{Let } \beta = 100, R_C = 5 \text{ k}\Omega, R_B = 100 \text{ k}\Omega$$

Then

$$\frac{\Delta v_o}{\Delta v_I} = \frac{-(100)(5)}{100} = -5$$

Want Q-point to be

$$v_o(Q - p) = 2.5 = 5 - (100)I_{BQ}(5)$$

Then

$$I_{BQ} = 0.005 \text{ mA}, I_{BQ} = 0.005 = \frac{V_{BB} - 0.7}{100}$$

or

$$V_{BB} = 12 \text{ V}$$

$$\text{Also } I_{CQ} = \beta I_{BQ} = (100)(0.005)$$

Or

$$I_{CQ} = 0.5 \text{ mA}$$

E3.24

- a. For $V_{CEQ} = 2.5 \text{ V} \Rightarrow I_{CQ} = \frac{5 - 2.5}{2} \Rightarrow I_{CQ} = 1.25 \text{ mA}$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{1.25}{100} = \frac{1.25}{100} \Rightarrow I_{BQ} = 12.5 \mu\text{A}$$

$$\text{Then } R_B = \frac{5 - 0.7}{0.0125} \Rightarrow R_B = 344 \text{ k}\Omega$$

- b. I_{BQ} is independent of β .

$$\text{For } V_{CEQ} = 1 \text{ V}, I_C = \frac{5 - 1}{2} = 2 \text{ mA}$$

$$\beta = \frac{I_C}{I_B} = \frac{2}{0.0125} \Rightarrow \beta = 160$$

$$\text{For } V_{CEQ} = 4 \text{ V, } I_C = \frac{5 - 4}{2} = 0.5 \text{ mA}$$

$$\beta = \frac{I_C}{I_B} = \frac{0.5}{0.0125} \Rightarrow \beta = 40$$

$$\text{So } 40 \leq \beta \leq 160$$

E3.25

$$I_{BQ} = \frac{5 - 0.7}{800} \Rightarrow I_{BQ} = 0.005375 \text{ mA}$$

$$\beta = 75 \Rightarrow I_{CQ} = (75)(0.005375) = 0.403 \text{ mA}$$

$$\beta = 150 \Rightarrow I_{CQ} = (150)(0.005375) = 0.806 \text{ mA}$$

Largest $I_{CQ} \Rightarrow$ Smallest V_{CEQ}

$$\beta = 150 \Rightarrow R_C = \frac{5 - 1}{0.806} = 4.96 \text{ k}\Omega$$

$$\beta = 75 \Rightarrow R_C = \frac{5 - 4}{0.403} = 2.48 \text{ k}\Omega$$

For a nominal $I_C = 0.604 \text{ mA}$ and $V_{CEQ} = 2.5$

$$R_C = \frac{5 - 2.5}{0.604} = 4.14 = R_C$$

For $I_{CQ} = 0.403$,

$$V_{CEQ} = 5 - (0.403)(4.14) = 3.33 \text{ V}$$

For $I_{CQ} = 0.806$,

$$V_{CEQ} = 5 - (0.806)(4.14) = 1.66 \text{ V}$$

So for $R_C = 4.14$, $1.66 \text{ V} \leq V_{CEQ} \leq 3.33 \text{ V}$

E3.26

$$\text{a. } R_{TH} = R_1 \parallel R_2 = 9 \parallel 2.25 \Rightarrow R_{TH} = 1.8 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{2.25}{9 + 2.25} \right) (5)$$

$$\Rightarrow V_{TH} = 1.0 \text{ V}$$

$$\text{b. } I_{BQ} = \frac{V_{TH} - V_{BE(\text{on})}}{R_{TH} + (1 + \beta)R_E} = \frac{1 - 0.7}{1.8 + (151)(0.2)}$$

$$\Rightarrow I_{BQ} = 9.375 \mu\text{A}$$

$$I_{CQ} = \beta I_{BQ} = (150)(9.375 \mu\text{A})$$

$$\Rightarrow I_{CQ} = 1.41 \text{ mA}$$

$$I_{EQ} = (1 + \beta)I_{BQ} \Rightarrow I_{EQ} = 1.42 \text{ mA}$$

$$V_{CEQ} = 5 - I_{CQ}R_C - I_{EQ}R_E$$

$$= 5 - (1.41)(1) - (1.42)(0.2)$$

$$V_{CEQ} = 3.31 \text{ V}$$

c. For $\beta = 75$

$$I_{BQ} = \frac{1 - 0.7}{1.8 + (76)(0.2)} \Rightarrow I_{BQ} = 17.6 \mu\text{A}$$

$$I_{CQ} = \beta I_{BQ} = (75)(17.6 \mu\text{A})$$

$$\Rightarrow I_{CQ} = 1.32 \text{ mA}$$

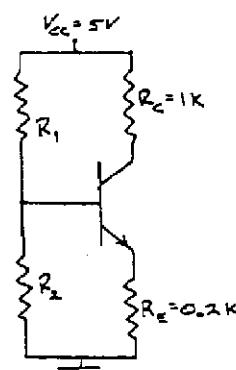
$$I_{EQ} = (1 + \beta)I_{BQ} = (76)(17.6 \mu\text{A})$$

$$\Rightarrow I_{EQ} = 1.34 \text{ mA}$$

$$V_{CEQ} = 5 - (1.32)(1) - (1.34)(0.2)$$

$$\Rightarrow V_{CEQ} = 3.41 \text{ V}$$

E3.27



$$R_1 + R_2 = 11.25 \text{ k}\Omega, \beta = 150$$

$$I_C \approx I_E, V_{CEQ} = 2.5 \text{ V}$$

$$\text{So } I_{CQ} \approx I_{EQ} = \frac{5 - 2.5}{1 + 0.2} = 2.081 \text{ mA}$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{2.08}{150} = 13.9 \mu\text{A}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE(\text{on})}}{R_{TH} + (1 + \beta)R_E}$$

$$0.0139 = \frac{\left(\frac{R_2}{R_1 + R_2} \right) V_{CC} - V_{BE(\text{on})}}{\frac{R_1 R_2}{R_1 + R_2} + (1 + \beta)R_E}$$

$$= \frac{\left(\frac{R_2}{11.25} \right) (5) - 0.7}{\frac{R_1 R_2}{11.25} + (151)(0.2)}$$

$$R_2 = 11.25 - R_1, \text{ so}$$

$$0.0139[R_1(11.25 - R_1) + (151)(0.2)(11.25)]$$

$$= 5R_2 - (0.7)(11.25)$$

$$= 5(11.25 - R_1) - (0.7)(11.25)$$

$$0.156R_1 - 0.0139R_1^2 + 4.72 = 56.25 - 5R_1 - 7.875$$

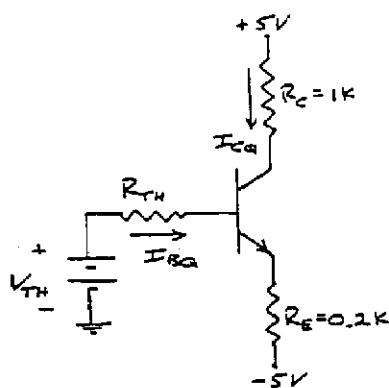
$$0.0139R_1^2 - 5.156R_1 + 43.66 = 0$$

$$R_1 = \frac{5.156 \pm \sqrt{(5.156)^2 - 4(0.0139)(43.66)}}{2(0.0139)}$$

$$\Rightarrow R_1 = 8.67 \text{ k}\Omega \text{ and } R_2 = 2.58 \text{ k}\Omega$$

E3.28

dc equivalent circuit:



$$\beta = 150, R_{TH} = R_1 \parallel R_2,$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5$$

$$I_{CQ} = \frac{5 - 0}{1} = 5 \text{ mA}$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{5}{150} = 0.0333 \text{ mA}$$

$$I_{BQ} = \frac{V_{TH} + 5 - 0.7}{R_{TH} + (1 + \beta)R_E}$$

$$\text{Set } R_{TH} = (0.1)(1 + \beta)R_E$$

$$I_{BQ} = \frac{V_{TH} + 4.3}{(1.1)(1 + \beta)R_E}$$

$$\Rightarrow 0.0333 = \frac{\left(\frac{R_2}{R_1 + R_2} \right) (10) - 5 + 4.3}{(1.1)(151)(0.2)}$$

$$(0.0333)(1.1)(151)(0.2) = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 0.7$$

$$\text{So } \left(\frac{R_2}{R_1 + R_2} \right) = 0.1806$$

$$R_{TH} = \frac{R_1 R_2}{R_1 + R_2} = (0.1)(151)(0.2) = 3.02 \text{ k}\Omega$$

$$\text{Then } R_1(0.1806) = 3.02 \Rightarrow R_1 = 16.7 \text{ k}\Omega$$

$$R_2 = (0.1806)(16.7 + R_2) \Rightarrow 0.8194 R_2 = 3.02$$

$$\Rightarrow R_2 = 3.68 \text{ k}\Omega$$

E3.29

$$\beta = 120, V_{CEQ} = 5 \text{ V}$$

$$R_{TH} = R_1 \parallel R_2, V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5$$

$$I_{CQ} \approx I_{EQ}$$

$$\text{So } I_{CQ} = \frac{10 - V_{CEQ}}{R_C + R_E}$$

$$I_{CQ} = \frac{10 - 5}{1.2 + 0.3} \Rightarrow I_{CQ} = 3.33 \text{ mA}$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{3.33}{120} \Rightarrow I_{BQ} = 0.0278 \text{ mA}$$

$$I_{BQ} = \frac{V_{TH} + 5 - 0.7}{R_{TH} + (1 + \beta)R_E}$$

$$\text{Set } R_{TH} = (0.1)(1 + \beta)R_E$$

$$I_{BQ} = 0.0278 = \frac{\left(\frac{R_2}{R_1 + R_2} \right) (10) - 5 + 5 - 0.7}{(1.1)(121)(0.3)}$$

$$(0.0278)(1.1)(121)(0.3) = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 0.7$$

$$\left(\frac{R_2}{R_1 + R_2} \right) = 0.181$$

$$R_{TH} = \frac{R_1 R_2}{R_1 + R_2} = R_1(0.181) = (0.1)(121)(0.3)$$

$$\Rightarrow R_1 = 20.1 \text{ k}\Omega$$

$$R_2 = (0.181)(20.1 + R_2) \Rightarrow 0.819 R_2 = 3.63 \text{ k}\Omega$$

$$\Rightarrow R_2 = 4.44 \text{ k}\Omega$$

E3.30

$$\text{a. } I_{CQ} = \left(\frac{\beta}{1 + \beta} \right) I_{EQ} = \left(\frac{100}{101} \right) (1) = 0.99 \text{ mA}$$

$$I_{BQ} = \frac{1 \text{ mA}}{1 + \beta} = \frac{1}{101} \Rightarrow 9.90 \mu\text{A}$$

$$V_B = -I_{BQ} R_B = -(0.0099)(50)$$

$$\Rightarrow V_B = -0.495 \text{ V}$$

$$V_{BE} = V_T \ln \left(\frac{I_{CQ}}{I_S} \right) = (0.026) \ln \left(\frac{0.99 \times 10^{-3}}{3 \times 10^{-14}} \right)$$

$$\Rightarrow V_{BE} = 0.630 \text{ V}$$

$$V_E = V_B - V_{BE} = -0.495 - 0.630$$

$$\Rightarrow V_E = -1.13 \text{ V}$$

$$V_C = 10 - (0.99)(5) = 5.05 \text{ V}$$

$$V_{CEQ} = V_C - V_E = 5.05 - (-1.13)$$

$$\Rightarrow V_{CEQ} = 6.18 \text{ V}$$

$$\text{b. } I_{EQ} = 1 \text{ mA}, I_B = \frac{1}{51} = 0.0196 \text{ mA}$$

$$V_B = -(0.0196)(50) = -0.98 \text{ V} = V_B$$

$$I_{CQ} = \left(\frac{50}{51} \right) (1) = 0.98 \text{ mA}$$

$$V_{BE} = (0.026) \ln \left(\frac{0.98 \times 10^{-3}}{3 \times 10^{-14}} \right) = 0.629 \text{ V}$$

$$V_E = -0.98 - 0.629 = -1.61$$

$$V_C = 10 - (0.98)(5) = 5.1 \text{ V}$$

$$V_{CEQ} = 5.1 - (-1.61) \Rightarrow V_{CEQ} = 6.71 \text{ V}$$

E3.31

$$I_B = \frac{I_Q}{1 + \beta} = \frac{I_Q}{121}, V_B = \left(\frac{I_Q}{121}\right)(20) = I_Q(0.165)$$

$$V_E = I_Q(0.165) + 0.7$$

$$I_{CQ} = \left(\frac{\beta}{1 + \beta}\right)I_{EQ} = \left(\frac{120}{121}\right)I_Q = (0.992)I_Q$$

$$V_C = I_{CQ}R_C - 5 = (0.992)I_Q(4) - 5$$

$$= 3.97I_Q - 5$$

$$V_{ECQ} = V_E - V_C$$

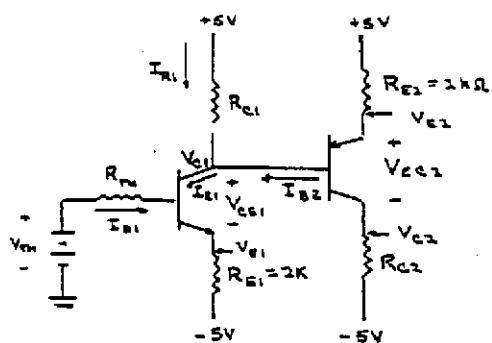
$$= [I_Q(0.165) + 0.7] - [3.97I_Q - 5]$$

$$= -3.805I_Q + 5.7$$

$$-3.805I_Q + 5.7 = 3$$

$$\Rightarrow I_Q = 0.710 \text{ mA}$$

E3.32



$$R_{TH} = 50 \parallel 100 = 33.3 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{50}{150}\right)(10) - 5 = -1.67 \text{ V}$$

$$I_{B1} = \frac{5 - 1.67 - 0.7}{33.3 + (101)(2)} = \frac{2.63}{235} = 11.2 \mu\text{A}$$

$$I_{C1} = 1.12 \text{ mA}, I_{E1} = 1.13 \text{ mA}$$

$$V_{E1} = I_{E1}R_{E1} - 5 = (1.13)(2) - 5 = -2.74 \text{ V}$$

$$V_{CE1} = 3.25 \text{ V} \Rightarrow V_{C1} = 0.51 \text{ V}$$

$$\Rightarrow V_{E2} = 0.51 + 0.7 = 1.21 \text{ V}$$

$$I_{E2} = \frac{5 - 1.21}{2} = 1.90 \text{ mA} \Rightarrow I_{B2} = 18.8 \mu\text{A}$$

$$I_{C2} = 1.88 \text{ mA}$$

$$I_{R1} = I_{C1} - I_{B2} = 1.12 - 0.0188 = 1.10 \text{ mA}$$

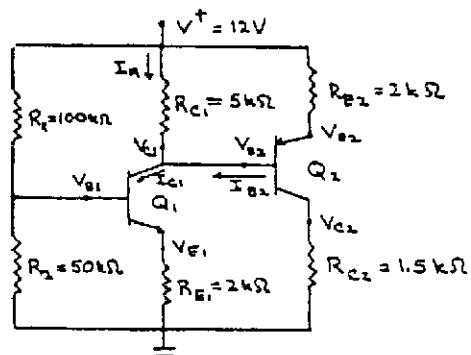
$$R_{C1} = \frac{5 - 0.51}{1.10} \Rightarrow R_{C1} = 4.08 \text{ k}\Omega$$

$$V_{EC1} = 2.5 \Rightarrow V_{C2} = V_{E2} - V_{EC2}$$

$$= 1.21 - 2.5 = -1.29$$

$$R_{C2} = \frac{-1.29 - (-5)}{1.88} \Rightarrow R_{C2} = 1.97 \text{ k}\Omega$$

E3.33



$$I_{B1} = \frac{V_{TH} - V_{BE(\text{on})}}{R_{TH} + (1 + \beta)R_{E1}} = \frac{4 - 0.7}{33.3 + (101)(2)}$$

$$I_{B1} = 14 \mu\text{A}, I_{C1} = 1.40 \text{ mA}, I_{E1} = 1.42 \text{ mA}$$

$$V_{B1} = 4 - (0.014)(33.3)$$

$$\Rightarrow V_{B1} = 3.53 \text{ V}, V_{E1} = 2.83 \text{ V}$$

$$I_R + I_{B2} = I_{C1}$$

$$\Rightarrow \frac{12 - V_{C1}}{5} + \frac{12 - (V_{C1} + 0.7)}{(101)(2)} = 1.40$$

$$\frac{12}{5} + \frac{(12 - 0.7)}{(101)(2)} - 1.40 = \frac{V_{C1}}{5} + \frac{V_{C1}}{(101)(2)}$$

$$2.4 + 0.0559 - 1.40 = V_{C1}(0.2 + 0.00495)$$

$$V_{C1} = V_{B1} = 3.15 \text{ V}, V_{E2} = 3.85$$

$$I_{E2} = \frac{12 - 3.85}{2} \Rightarrow I_{E2} = 3.08 \text{ mA}$$

$$I_{C2} = 3.04 \text{ mA}, I_{B2} = 30.4 \mu\text{A}$$

$$V_{C2} = (3.04)(1.5) \Rightarrow V_{C2} = 4.56 \text{ V}$$

Chapter 3

Problem Solutions

3.1

$$(a) \beta_F = \frac{i_C}{i_B} = \frac{510}{6} \Rightarrow \underline{\beta_F = 85}$$

$$\alpha_F = \frac{\beta_F}{1 + \beta_F} = \frac{85}{86} \Rightarrow \underline{\alpha_F = 0.9884}$$

$$i_E = (1 + \beta_F)i_B = (86)(6) \Rightarrow \underline{i_E = 516 \mu A}$$

$$(b) \beta_F = \frac{2.65}{0.050} \Rightarrow \underline{\beta_F = 53}$$

$$\alpha_F = \frac{53}{54} \Rightarrow \underline{\alpha_F = 0.9815}$$

$$i_E = (1 + \beta_F)i_B = (54)(0.050) \Rightarrow \underline{i_E = 2.70 mA}$$

3.2

(a)

For $\beta = 110$:

$$\alpha = \frac{\beta}{1 + \beta} = \frac{110}{111} = 0.99099$$

For $\beta = 180$:

$$\alpha = \frac{180}{181} = 0.99448$$

$$\underline{0.99099 \leq \alpha \leq 0.99448}$$

$$(b) I_C = \beta I_B = 110(50 \mu A) \Rightarrow \underline{I_C = 5.50 mA}$$

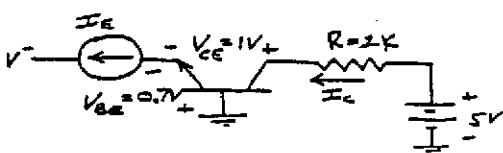
or

$$I_C = 180(50 \mu A) \Rightarrow \underline{I_C = 9.00 mA}$$

so

$$\underline{5.50 \leq I_C \leq 9.0 mA}$$

3.3



$$5 = I_C R + V_{CE} - V_{BE} = I_C(2) + 1 - 0.7$$

$$\Rightarrow I_C = 2.35 mA$$

$$I_E = \frac{I_C}{\alpha} = \frac{2.35}{0.982} \Rightarrow \underline{I_E = 2.39 mA}$$

3.4

Same Figure as Problem 3.3

$$v_C = -0.7 + 2 = 1.3 V ; i_C = \frac{5-1.3}{2} \Rightarrow \underline{i_C = 1.85 mA}$$

$$\text{For } \beta_F = 120, i_B = \frac{i_C}{\beta_F} = \frac{1.85}{120} \Rightarrow \underline{i_B = 15.4 \mu A}$$

$$i_E = \left(\frac{1 + \beta_F}{\beta_F} \right) i_C = \left(\frac{121}{120} \right)(1.85) \Rightarrow \underline{i_E = 1.865 mA}$$

3.5

$$\alpha = \frac{\beta}{1 + \beta} = \frac{60}{61} \Rightarrow \underline{\alpha = 0.9836}$$

$$I_E = \frac{i_C}{\alpha} = \frac{0.85}{0.9836} \Rightarrow \underline{I_E = 0.864 mA}$$

$$I_B = \frac{i_C}{\beta} = \frac{0.85}{60} \Rightarrow \underline{I_B = 14.2 \mu A}$$

3.6

$$i_E = I_S e^{v_{BE}/V_T} = (10^{-13}) e^{0.685/0.026} \Rightarrow \underline{i_E = 27.7 mA}$$

$$i_C = \left(\frac{90}{91} \right) (27.7) \Rightarrow \underline{i_C = 27.4 mA}$$

$$i_B = \frac{i_E}{1 + \beta} = \frac{27.7}{91} \Rightarrow \underline{i_B = 0.304 mA}$$

3.7

$$\text{Device 1: } i_E = I_{S1} e^{v_{BE}/V_T} \Rightarrow 0.5 \times 10^{-3} = I_{S1} e^{0.650/0.026}$$

So that

$$\underline{I_{S1} = 6.94 \times 10^{-15} A}$$

$$\text{Device 2: } 12.2 \times 10^{-3} = I_{S2} e^{0.650/0.026}$$

Or

$$\underline{I_{S2} = 1.69 \times 10^{-13} A}$$

$$\text{Ratio of areas} = \frac{I_{S2}}{I_{S1}} = \frac{1.69 \times 10^{-13}}{6.94 \times 10^{-15}} \Rightarrow \text{Ratio} = 24.4$$

3.8

$$(a) r_o = \frac{V_A}{I_C} = \frac{250}{1} \Rightarrow \underline{r_o = 250 k\Omega}$$

$$(b) r_o = \frac{V_A}{I_C} = \frac{250}{0.1} \Rightarrow \underline{r_o = 2.50 M\Omega}$$

3.9

$$BV_{CE0} = \frac{BV_{CE0}}{\sqrt{\beta}} = \frac{60}{\sqrt{100}}$$

$$\underline{BV_{CE0} = 12.9 V}$$

3.10

$$BV_{CE0} = \frac{BV_{CB0}}{\sqrt[3]{\beta}}$$

$$56 = \frac{220}{\sqrt[3]{\beta}} \Rightarrow \sqrt[3]{\beta} = \frac{220}{56} = 3.93$$

$$\beta = 60.6$$

3.11

$$BV_{CE0} = \frac{BV_{CB0}}{\sqrt[3]{\beta}}$$

$$BV_{CB0} = (BV_{CE0}) \sqrt[3]{\beta} = (50) \sqrt[3]{50}$$

$$\underline{BV_{CB0} = 184 \text{ V}}$$

3.12

$$(a) I_E = \frac{12 - 0.7}{10} \Rightarrow \underline{I_E = 1.13 \text{ mA}}$$

$$I_C = \left(\frac{75}{76}\right)(1.13) = 1.12 \text{ mA}$$

$$V_{CE} = 24 - (1.13)(10) - (1.12)R_C = 6$$

so that

$$R_C = 5.98 \text{ k}\Omega$$

$$(b) I_B = \frac{1}{76} = 0.0132 \text{ mA}$$

$$V_E = -I_B R_E = -(0.0132)(50) \Rightarrow \underline{V_E = -0.658 \text{ V}}$$

$$I_C = \left(\frac{75}{76}\right)(1) = 0.987 \text{ mA}$$

$$R_C = \frac{5 - 2}{0.987} \Rightarrow \underline{R_C = 3.04 \text{ k}\Omega}$$

$$c. I_B = \frac{8 - 0.7 - (-2)}{10 + (76)(10)} \Rightarrow I_B = 12.1 \mu\text{A}$$

$$\underline{I_C = 0.906 \text{ mA}}$$

$$V_E = 0.7 + (0.0121)(10) - 2$$

$$V_E = -1.18 \text{ V}$$

$$V_C = I_C R_C - 8 = (0.906)(3) - 8$$

$$\Rightarrow V_C = -5.28 \text{ V}$$

$$V_{EC} = V_E - V_C = -1.18 - (-5.28)$$

$$\Rightarrow \underline{V_{EC} = 4.1 \text{ V}}$$

$$d. 5 = (1 + \beta)I_B(10) + I_B(20) + 0.7 + (1 + \beta)I_B(2)$$

$$5 = I_B[760 + 20 + 152] + 0.7 \Rightarrow \underline{I_B = 4.61 \mu\text{A}}$$

$$I_C = \beta I_B = (75)(4.61) \Rightarrow I_C = 0.346 \text{ mA}$$

$$V_C = 5 - (1 + \beta)I_B R_C = 5 - (76)(0.00461)(10)$$

$$\Rightarrow \underline{V_C = 1.50}$$

3.13

(a) Figure P3.12(c)

$$8 = (76)I_E(10) + 0.7 + I_E(10) - 2$$

$$I_E = \frac{10 - 0.7}{10 + (76)(10)} = 0.01208 \text{ mA}$$

Then

$$I_C = (75)I_E \Rightarrow \underline{I_C = 0.906 \text{ mA}}$$

$$\text{and } \underline{I_E = 0.918 \text{ mA}}$$

$$V_{EC} = 8 - I_E R_E - I_C R_C - (-8)$$

$$V_{EC} = 16 - (0.918)(10) - (0.906)(R_C)$$

$$V_{EC} = 6.82 - (0.906)R_C$$

$$R_C = 3 \text{ k}\Omega \pm 5\% \Rightarrow 2.85 \leq R_C \leq 3.15 \text{ k}\Omega$$

Then

$$3.97 \leq V_{EC} \leq 4.24 \text{ V}$$

(b) Figure P3.12(d)

$$5 = (1 + \beta)I_E R_C + I_E(20) + 0.7 + (1 + \beta)I_E(2)$$

$$5 = (76)I_E R_C + I_E(20) + 0.7 + (76)I_E(2)$$

$$\text{Now } R_C = 10 \text{ k}\Omega \pm 5\% \Rightarrow 9.5 \leq R_C \leq 10.5 \text{ k}\Omega$$

$$\text{Then } 0.00443 \leq I_E \leq 0.00481 \text{ mA}$$

$$\text{And } V_C = 5 - (1 + \beta)I_E R_C$$

$$\text{So that } \underline{1.46 \leq V_C \leq 1.53 \text{ V}}$$

3.14

$$R_E = \frac{V_{BE} - V_{EB}}{I_E} = \frac{2.5 - 0.7}{0.015} \Rightarrow \underline{R_E = 120 \text{ k}\Omega}$$

$$I_{CQ} = (70)(15 \mu\text{A}) \Rightarrow 1.05 \text{ mA}$$

$$R_C = \frac{V_{CC} - V_{ECQ}}{I_{CQ}} = \frac{5 - 2.5}{1.05} \Rightarrow \underline{R_C = 2.38 \text{ k}\Omega}$$

3.15

$$a. V_B = -I_B R_B \Rightarrow I_B = \frac{-V_B}{R_B} = \frac{-(1)}{500} = \frac{(-1)}{500}$$

$$I_B = 2.0 \mu\text{A}$$

$$V_E = -1 - 0.7 = -1.7 \text{ V}$$

$$I_E = \frac{V_E - (-3)}{R_E} = \frac{-1.7 + 3}{4.8} = 0.2708 \text{ mA}$$

$$\frac{I_E}{I_B} = (1 + \beta) = \frac{0.2708}{0.002} = 135.4 \Rightarrow \underline{\beta = 134.4}$$

$$\alpha = \frac{\beta}{1 + \beta} \Rightarrow \underline{\alpha = 0.9926}$$

$$I_C = \beta I_B \Rightarrow \underline{I_C = 0.269 \text{ mA}}$$

$$V_{CE} = 3 - V_E = 3 - (-1.7) \Rightarrow \underline{V_{CE} = 4.7 \text{ V}}$$

b. $I_E = \frac{5 - 4}{2} \Rightarrow I_E = 0.5 \text{ mA}$

$$4 = 0.7 + I_B R_B + (I_B + I_C) R_C - 5$$

$$I_B + I_C = I_E$$

$$4 = 0.7 + I_B(100) + (0.5)(8) - 5$$

$$I_B = 0.043 \Rightarrow \frac{I_E}{I_B} = (1 + \beta) = \frac{0.5}{0.043} = 11.63$$

$$\beta = 10.63, \alpha = \frac{\beta}{1 + \beta} \Rightarrow \alpha = 0.9140$$

3.16

a. $V_B = 0 \Rightarrow \text{Cutoff} \Rightarrow I_E = 0, V_C = 6 \text{ V}$

b. $V_B = 1 \text{ V}, I_E = \frac{1 - 0.7}{1} \Rightarrow I_E = 0.3 \text{ mA}$

$$I_C \approx I_E \Rightarrow V_C = 6 - (0.3)(10) \Rightarrow V_C = 3 \text{ V}$$

c. $V_B = 2 \text{ V}$. Assume active-mode

$$I_E = \frac{2 - 0.7}{1} = I_E = 1.3 \text{ mA} \approx I_C$$

$$V_C = 6 - (1.3)(10) = -7 \text{ V}$$

Transistor in saturation

$$I_E = \frac{2 - 0.7}{1} \Rightarrow I_E = 1.3 \text{ mA}$$

$$V_E = 1.3 \text{ V}, V_{CE}(\text{sat}) = 0.2 \text{ V}$$

$$V_C = V_E + V_{CE}(\text{sat}) = 1.3 + 0.2$$

$$\Rightarrow V_C = 1.5 \text{ V}$$

3.17

a. $V_{BB} = 0$.

$$\text{Cutoff } V_0 = \left(\frac{R_L}{R_C + R_L} \right) V_{CC} = \left(\frac{10}{10 + 5} \right) (5)$$

$$V_0 = 3.33 \text{ V}$$

b. $V_{BB} = 1 \text{ V}$

$$I_B = \frac{1 - 0.7}{50} \Rightarrow 6 \mu\text{A}$$

$$I_C = \beta I_B = (75)(6) \Rightarrow I_C = 0.45 \text{ mA}$$

$$\frac{5 - V_0}{5} = I_C + \frac{V_0}{10}$$

$$1 - 0.45 = V_0 \left(\frac{1}{5} + \frac{1}{10} \right) \Rightarrow V_0 = 1.83 \text{ V}$$

c. Transistor in saturation

$$V_0 = V_{CE}(\text{sat}) = 0.2 \text{ V}$$

3.18

(a) $\beta_F = 100$

$$(i) I_Q = 0.1 \text{ mA} \quad I_C = \left(\frac{100}{101} \right) (0.1) = 0.0990 \text{ mA}$$

$$V_o = 5 - (0.099)(5) \Rightarrow V_o = 4.505 \text{ V}$$

$$(ii) I_Q = 0.5 \text{ mA} \quad I_C = \left(\frac{100}{101} \right) (0.5) = 0.495 \text{ mA}$$

$$V_o = 5 - (0.495)(5) \Rightarrow V_o = 2.525 \text{ V}$$

(iii) $I_Q = 2 \text{ mA}$ Transistor is in saturation

$$V_o = -V_{BE}(\text{sat}) + V_{CE}(\text{sat}) = -0.8 + 0.2 \Rightarrow V_o = -0.6 \text{ V}$$

(b) $\beta_F = 150$

$$(i) I_Q = 0.1 \text{ mA} \quad I_C = \left(\frac{150}{151} \right) (0.1) = 0.09934 \text{ mA}$$

$$V_o = 5 - (0.09934)(5) \Rightarrow V_o = 4.503 \text{ V}$$

$$\% \text{ change} = \frac{4.503 - 4.505}{4.503} \times 100\% = -0.044\%$$

$$(ii) I_Q = 0.5 \text{ mA} \quad I_C = \left(\frac{150}{151} \right) (0.5) = 0.4967 \text{ mA}$$

$$V_o = 5 - (0.4967)(5) \Rightarrow V_o = 2.517 \text{ V}$$

$$\% \text{ change} = \frac{2.517 - 2.525}{2.525} \times 100\% = -0.32\%$$

(iii) $I_Q = 2 \text{ mA}$ Transistor in saturation

$$V_o = -0.6 \text{ V} \quad \text{No change}$$

3.19

$$I_E = \frac{V_B - 0.7}{1}$$

$$I_C = \left(\frac{\beta}{1 + \beta} \right) I_E = \left(\frac{50}{51} \right) (V_B - 0.7) = \frac{6 - V_C}{10}$$

$$\text{and } V_C = V_B$$

$$\left(\frac{50}{51} \right) (V_B - 0.7) = \frac{6 - V_B}{10}$$

$$9.80(V_B - 0.7) = 6 - V_B$$

$$10.80V_B = 6 + (0.7)(9.80) \Rightarrow V_B = 1.19 \text{ V}$$

$$I_E = \frac{1.19 - 0.7}{1} \Rightarrow I_E = 0.49 \text{ mA}$$

3.20

$$V_{CB} = 0.5 \text{ V} \Rightarrow V_o = 0.5 \text{ V}, I_C = \frac{5 - 0.5}{5} = 0.90 \text{ mA}$$

$$I_Q = \left(\frac{101}{100} \right) (0.90) \Rightarrow I_Q = 0.909 \text{ mA}$$

3.21

$$I_E = \frac{10 - V_E}{10} = \frac{10 - 2}{10} \Rightarrow I_E = 0.80 \text{ mA}$$

$$V_B = V_E - 0.7 = 2 - 0.7 = 1.3 \text{ V}$$

$$I_B = \frac{V_B}{R_B} = \frac{1.3}{50} \Rightarrow I_B = 0.026 \text{ mA}$$

$$I_C = I_E - I_B = 0.80 - 0.026 \Rightarrow I_C = 0.774 \text{ mA}$$

$$\beta = \frac{I_C}{I_B} = \frac{0.774}{0.026} \Rightarrow \beta = 29.77$$

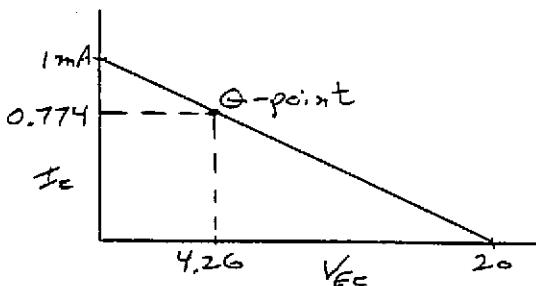
$$\alpha = \frac{\beta}{1 + \beta} = \frac{29.77}{30.77} \Rightarrow \alpha = 0.9675$$

$$V_{EC} = V_E - V_C = V_E - (I_C R_C - 10)$$

$$= 2 - [(0.774)(10) - 10]$$

$$V_{EC} = 4.26 \text{ V}$$

Load line developed assuming the V_B voltage can change and the R_B resistor is removed.



3.22

$$I_C = \left(\frac{50}{51}\right)(1) = 0.98 \text{ mA}$$

$$V_C = I_C R_C - 9 = (0.98)(4.7) - 9$$

$$\text{or } V_C = -4.39 \text{ V}$$

$$I_B = \frac{1}{51} = 0.0196 \text{ mA}$$

$$V_E = I_B R_B + V_{EB}(\text{on}) = (0.0196)(50) + 0.7$$

$$\text{or } V_E = 1.68 \text{ V}$$

3.23

$$I_C = \left(\frac{50}{51}\right)(0.5) = 0.49 \text{ mA}, \quad I_B = \frac{0.5}{51} = 0.0098 \text{ mA}$$

$$V_E = I_B R_B + V_{EB}(\text{on}) = (0.0098)(50) + 0.7$$

$$\text{or } V_E = 1.19 \text{ V}$$

$$V_C = I_C R_C - 9 = (0.49)(4.7) - 9 = -6.70 \text{ V}$$

$$\text{Then } V_{EC} = V_E - V_C = 1.19 - (-6.7) = 7.89 \text{ V}$$

$$P_Q = I_C V_{EC} + I_B V_{EB} = (0.49)(7.89) + (0.0098)(0.7)$$

$$\text{or } P_Q = 3.87 \text{ mW}$$

$$\text{Power Supplied} = P_s = I_Q(9 - V_E) = (0.5)(9 - 1.19)$$

$$\text{Or } P_s = 3.91 \text{ mW}$$

3.24

$$\text{For } I_Q = 0, \text{ then } P_Q = 0$$

$$\text{For } I_Q = 0.5 \text{ mA}, \quad I_C = \left(\frac{50}{51}\right)(0.5) = 0.49 \text{ mA}$$

$$I_B = \frac{0.5}{51} = 0.0098 \text{ mA}, \quad V_B = 0.490 \text{ V}, \quad V_E = 1.19 \text{ V}$$

$$V_C = (0.49)(4.7) - 9 = -6.70 \text{ V} \Rightarrow V_{EC} = 7.89 \text{ V}$$

$$P \cong I_C V_{EC} = (0.49)(7.89) \Rightarrow P = 3.87 \text{ mW}$$

For $I_Q = 1.0 \text{ mA}$, Using the same calculations as above, we find $P = 5.95 \text{ mW}$

$$\text{For } I_Q = 1.5 \text{ mA}, \quad P = 6.26 \text{ mW}$$

$$\text{For } I_Q = 2 \text{ mA}, \quad P = 4.80 \text{ mW}$$

$$\text{For } I_Q = 2.5 \text{ mA}, \quad P = 1.57 \text{ mW}$$

For $I_Q = 3 \text{ mA}$, Transistor is in saturation.

$$0.7 + I_B(50) = 0.2 + I_C(4.7) - 9$$

$$I_E = I_Q = I_B + I_C \Rightarrow I_B = 3 - I_C$$

$$\text{Then, } 0.7 + (3 - I_C)(50) = 0.2 + I_C(4.7) - 9$$

$$\text{Which yields } I_C = 2.916 \text{ mA and } I_B = 0.084 \text{ mA}$$

$$P = I_B V_{EB} + I_C V_{EC} = (0.084)(0.7) + (2.916)(0.2)$$

$$\text{or } P = 0.642 \text{ mW}$$

3.25

$$I_{E1} = I_{E2} = \frac{I}{2} \Rightarrow I_{E1} = I_{E2} = 0.5 \text{ mA}$$

$$I_{C1} = I_{C2} \approx 0.5 \text{ mA}$$

$$V_{C1} = V_{C2} = 5 - (0.5)(4) \Rightarrow V_{C1} = V_{C2} = 3 \text{ V}$$

3.26

$$\text{a. } I_{BQ} = \frac{V_{CC} - V_{BE}(\text{on})}{R_B}$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{2}{60} = 0.0333 \text{ mA}$$

$$R_B = \frac{24 - 0.7}{0.0333} \Rightarrow R_B = 699 \text{ k}\Omega$$

$$I_{CQ} = \frac{V_{CC} - V_{CEQ}}{R_C} \Rightarrow R_C = \frac{24 - 12}{2}$$

$$\Rightarrow R_C = 6 \text{ k}\Omega$$

$$\text{b. } I_{BQ} = \frac{V_{CC} - V_{BE}(\text{on})}{R_B} = \frac{24 - 0.7}{699}$$

$$= 0.0333 \text{ mA (Unchanged)}$$

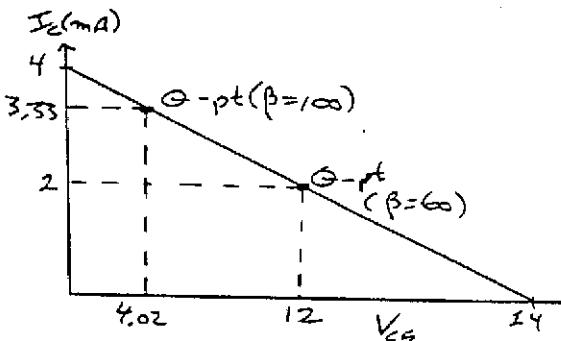
$$I_{CQ} = \beta I_{BQ} = (100)(0.0333)$$

$$\Rightarrow I_{CQ} = 3.33 \text{ mA}$$

$$V_{CEQ} = V_{CC} - I_{CQ} R_C = 24 - (3.33)(6)$$

$$\Rightarrow V_{CEQ} = 4.02 \text{ V}$$

$$(c) V_{CE} = V_{CC} - I_C R_C = 24 - I_C(6)$$



3.27

$$I_E = \frac{V_{EE} - V_{EB}(\text{on})}{R_E} = \frac{9 - 0.7}{4} \Rightarrow I_E = 2.075 \text{ mA}$$

$$I_C = \alpha I_E = (0.9920)(2.075)$$

$$\Rightarrow I_C = 2.06 \text{ mA}$$

$$V_{BC} + I_C R_C = V_{CC}$$

$$V_{BC} = 9 - (2.06)(2.2) \Rightarrow V_{BC} = 4.47 \text{ V}$$

3.28

$$I_{CQ} = \frac{V_{CC} - V_{CEQ}}{R_C} = \frac{12 - 6}{2.2} = 2.73 \text{ mA}$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{2.73}{30} \Rightarrow I_{BQ} = 0.091 \text{ mA}$$

$$I_{R2} = \frac{0.7 - (-12)}{100} = 0.127 \text{ mA}$$

$$I_{R1} = I_{R2} + I_{BQ} = 0.127 + 0.091 = 0.218 \text{ mA}$$

$$V_1 = I_{R1} R_1 + 0.7 = (0.218)(15) + 0.7$$

$$\Rightarrow V_1 = 3.97 \text{ V}$$

3.29

 For $V_{CE} = 4.5$

$$I_{CQ} = \frac{5 - 4.5}{1} = 0.5 \text{ mA}$$

$$I_{BQ} = \frac{0.5}{25} = 0.02 \text{ mA}$$

$$I_{R2} = \frac{0.7 - (-5)}{100} = 0.057 \text{ mA}$$

$$I_{R1} = I_{R2} + I_{BQ} = 0.057 + 0.02 = 0.077 \text{ mA}$$

$$V_1 = I_{R1} R_1 + V_{BE}(\text{on}) = (0.077)(15) + 0.7$$

$$= 1.86 \text{ V}$$

 For $V_{CE} = 1.0$

$$I_{CQ} = \frac{5 - 1}{1} = 4 \text{ mA}$$

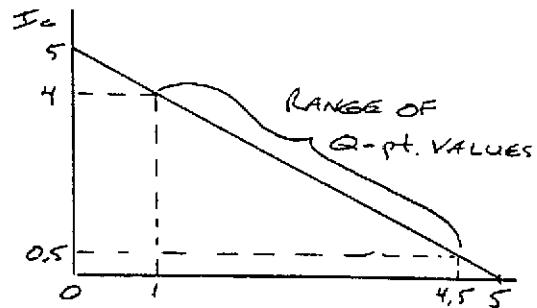
$$I_{BQ} = \frac{4}{25} = 0.16 \text{ mA}$$

$$I_{R2} = 0.057 \text{ mA}$$

$$I_{R1} = I_{R2} + I_{BQ} = 0.057 + 0.16 = 0.217 \text{ mA}$$

$$V_1 = (0.217)(15) + 0.7 \Rightarrow 3.96 \text{ V}$$

$$\text{So } 1.86 \leq V_1 \leq 3.96 \text{ V}$$



3.30

$$R_{TH} = R_1 \parallel R_2 = 33 \parallel 10 = 7.67 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{10}{10 + 33} \right) (18) = 4.19 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1 + \beta)R_E} = \frac{4.19 - 0.7}{7.67 + (51)(1)}$$

$$I_{BQ} = 0.0595 \text{ mA}$$

$$I_{CQ} = \beta I_{BQ} \Rightarrow I_{CQ} = 2.97 \text{ mA}$$

$$I_{BQ} = 3.03 \text{ mA}$$

$$V_{CEQ} = V_{CC} - I_{CQ} R_C - I_{EQ} R_E$$

$$= 18 - (2.97)(2.2) - (3.03)(1)$$

$$\Rightarrow V_{CEQ} = 8.44 \text{ V}$$

3.31

$$I_{CQ} = 12 \text{ mA}, V_{CEQ} = 9 \text{ V}, R_{TH} = 50 \text{ k}\Omega$$

$$\text{Also } I_s = \frac{1.2}{80} = 0.015 \text{ mA}$$

$$V_{TH} = I_{sQ} R_{TH} + V_{BE}(\text{on}) + (1 + \beta) I_{sQ} R_E$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (50)(18)$$

Then

$$\frac{1}{R_1} (50)(18) = (0.015)(50) + 0.7 + (81)(0.015)(1)$$

$$\text{or } R_1 = 338 \text{ k}\Omega. \text{ Then } \frac{338 R_2}{338 + R_2} = 50 \Rightarrow$$

$$R_2 = 58.7 \text{ k}\Omega \quad I_{EQ} = \left(\frac{81}{80} \right) (1.2) = 1.215 \text{ mA}$$

$$18 = I_{CQ} R_C + V_{CEQ} + I_{sQ} R_E$$

$$18 = (1.2) R_C + 9 + (1.215)(1) \Rightarrow R_C = 6.49 \text{ k}\Omega$$

3.32

$$R_{TH} = R_1 \parallel R_2 = 20 \parallel 15 = 8.57 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{15}{15+20} \right) (10) = 4.29 \text{ V}$$

$$V_{CC} = I_{EQ} R_E + V_{BE}(\text{on}) + \frac{I_{EQ}}{1+\beta} \cdot R_{TH} + V_{TH}$$

$$10 = I_{EQ}(1) + 0.7 + I_{EQ} \left(\frac{8.57}{101} \right) + 4.29$$

Then

$$I_{EQ} = \frac{10 - 0.7 - 4.29}{1 + \frac{8.57}{101}} = \frac{5.01}{1.085} \Rightarrow I_{EQ} = 4.62 \text{ mA}$$

$$V_B = \frac{I_{EQ}}{1+\beta} \cdot R_{TH} + V_{TH} = \left(\frac{4.62}{101} \right) (8.57) + 4.29$$

or
 $V_B = 4.68 \text{ V}$

3.33

a. $R_{TH} = R_1 \parallel R_2 = 58 \parallel 42 = 24.36 \text{ k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{42}{42 + 58} \right) (24)$$

$$= 10.1 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1+\beta)R_E} = \frac{10.1 - 0.7}{24.36 + (126)(10)}$$

$$I_{BQ} = 0.00732 \text{ mA}$$

$$I_{CQ} = \beta I_{BQ} = (125)(0.00732)$$

$$\Rightarrow I_{CQ} = 0.915 \text{ mA}$$

$$I_{EQ} = 0.922 \Rightarrow V_{CEQ} = 24 - (0.922)(10)$$

$$\Rightarrow V_{CEQ} = 14.8 \text{ V}$$

b. Let

$$R_2 = 42 \text{ k}\Omega + 5\% = 44.1 \text{ k}\Omega$$

$$R_1 = 58 \text{ k}\Omega - 5\% = 55.1 \text{ k}\Omega$$

$$R_E = 10 \text{ k}\Omega - 5\% = 9.5 \text{ k}\Omega$$

$$R_{TH} = R_1 \parallel R_2 = 24.5 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{44.1}{44.1 + 55.1} \right) (24) = 10.7 \text{ V}$$

$$I_{BQ} = \frac{10.7 - 0.7}{24.5 + (126)(9.5)} = 0.00819$$

$$I_{CQ} = 1.02 \text{ mA}, I_{EQ} = 1.03 \text{ mA}$$

$$V_{CEQ} = 24 - (1.03)(9.5) \Rightarrow V_{CEQ} = 14.2 \text{ V}$$

Let

$$R_2 = 42 \text{ k}\Omega - 5\% = 39.9 \text{ k}\Omega$$

$$R_1 = 58 \text{ k}\Omega + 5\% = 60.9 \text{ k}\Omega$$

$$R_E = 10 \text{ k}\Omega + 5\% = 10.5 \text{ k}\Omega$$

$$R_{TH} = R_1 \parallel R_2 = 24.1 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{39.9}{39.9 + 60.9} \right) (24) = 9.5 \text{ V}$$

$$I_{BQ} = \frac{9.5 - 0.7}{24.1 + (126)(10.5)} = 0.00653 \text{ mA}$$

$$I_{CQ} = 0.817 \text{ mA}, I_{EQ} = 0.823 \text{ mA}$$

$$V_{CEQ} = 24 - (0.823)(10.5) \Rightarrow V_{CEQ} = 15.4 \text{ V}$$

So $0.817 \leq I_{CQ} \leq 1.02 \text{ mA}$ and
 $14.2 \leq V_{CEQ} \leq 15.4 \text{ V}$.

3.34

a. $R_{TH} = R_1 \parallel R_2 = 25 \parallel 8 = 6.06 \text{ k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{8}{8 + 25} \right) (24) = 5.82 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1+\beta)R_E} = \frac{5.82 - 0.7}{6.06 + (76)(1)}$$

$$I_{BQ} = 0.0624 \text{ mA}, I_{CQ} = 4.66 \text{ mA}$$

$$I_{EQ} = 4.74$$

$$V_{CEQ} = V_{CC} - I_{CQ} R_C - I_{EQ} R_E = 24 - (4.68)(3) - (4.74)(1)$$

$$V_{CEQ} = 5.22 \text{ V}$$

b. $I_{BQ} = \frac{5.82 - 0.7}{6.06 + (151)(1)} \Rightarrow I_{BQ} = 0.0326 \text{ mA}$

$$I_{CQ} = 4.89 \text{ mA}$$

$$I_{EQ} = 4.92$$

$$V_{CEQ} = 24 - (4.89)(3) - (4.92)(1)$$

$$V_{CEQ} = 4.41 \text{ V}$$

3.35

(a) $I_{CQ} \equiv I_{EQ} = 0.4 \text{ mA}$

$$R_C = \frac{3}{0.4} \Rightarrow R_C = 7.5 \text{ k}\Omega; R_E = \frac{3}{0.4} \Rightarrow R_E = 7.5 \text{ k}\Omega$$

$$R_1 + R_2 \equiv \frac{9}{(0.2)(0.4)} = 112.5 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = I_{BQ} R_{TH} + V_{BE}(\text{on}) + (1+\beta) I_{BQ} R_E$$

$$R_{TH} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(112.5 - R_1) R_1}{112.5},$$

$$I_{BQ} = \frac{0.4}{100} = 0.004 \text{ mA}$$

$$R_1 \left(\frac{9}{112.5} \right) = (0.004) \left[\frac{(112.5 - R_1) R_1}{112.5} \right] + 0.7 + (101)(0.004)(7.5)$$

We obtain

$$R_1(0.08) = 0.004 R_1 - 3.56 \times 10^{-3} R_1^2 + 3.73$$

From this quadratic, we find

$$R_1 = 48 \text{ k}\Omega \Rightarrow R_1 = 64.5 \text{ k}\Omega$$

(b) Standard resistor values:

Set $R_E = R_C = 7.5 \text{ k}\Omega$ and

$$R_1 = 62 \text{ k}\Omega, \quad R_2 = 47 \text{ k}\Omega$$

$$\text{Now } R_{TH} = R_1 \parallel R_2 = 62 \parallel 47 = 26.7 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{47}{47+62} \right) (9) = 3.88 \text{ V}$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE}(\text{on}) + (1+\beta) I_{BQ} R_E$$

So

$$I_{BQ} = \frac{3.88 - 0.7}{26.7 + (101)(7.5)} = 0.00406 \text{ mA}$$

Then

$$I_{CQ} = 0.406 \text{ mA}$$

$$V_{RC} = V_{RE} = (0.406)(7.5) = 3.05 \text{ V}$$

3.36

$$a. \quad R_{TH} = R_1 \parallel R_2 = 12 \parallel 2 = 1.71 \text{ k}\Omega = R_{TH}$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5 = \left(\frac{2}{12+2} \right) (10) - 5 = -3.57 \text{ V} = V_{TH}$$

b.

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on}) - (-5)}{R_{TH} + (1+\beta)R_E} = \frac{-3.57 - 0.7 + 5}{1.71 + (101)(0.5)} = \frac{0.73}{52.2}$$

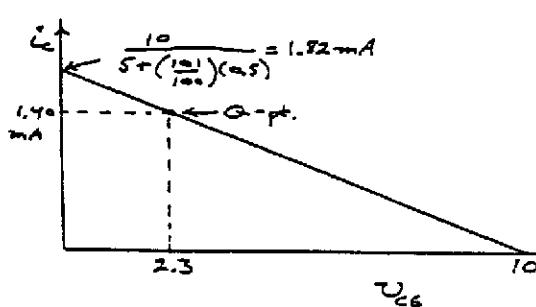
$$\Rightarrow I_{BQ} = 0.0140 \text{ mA}$$

$$I_{CQ} = 1.40 \text{ mA}, \quad I_{EQ} = 1.41 \text{ mA}$$

$$V_{CEQ} = 10 - I_{CQ} R_C - I_{EQ} R_E = 10 - (1.40)(5) - (1.41)(0.5)$$

$$V_{CEQ} = 2.30 \text{ V}$$

c.



d. For

$$R_2 = 2 \text{ k}\Omega + 5\% = 2.1 \text{ k}\Omega$$

$$R_1 = 12 \text{ k}\Omega - 5\% = 11.4 \text{ k}\Omega$$

$$R_E = 0.5 \text{ k}\Omega - 5\% = 0.475 \text{ k}\Omega$$

$$R_{TH} = R_1 \parallel R_2 = 1.77 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{2.1}{2.1 + 1.4} \right) (10) - 5 = -3.44 \text{ V}$$

$$I_{BQ} = \frac{-3.44 - 0.7 + 5}{1.77 + (101)(0.475)} = \frac{0.86}{49.7} = 0.0173 \text{ mA}$$

$$I_{CQ} = 1.73 \text{ mA}, \quad I_{EQ} = 1.75 \text{ mA}$$

$$\text{For } R_C = 5 \text{ k}\Omega + 5\% = 5.25 \text{ k}\Omega$$

$$V_{CEQ} = 10 - (1.73)(5.25) - (1.75)(0.475)$$

$$\Rightarrow V_{CEQ} = 0.0863 \text{ V} \quad (\text{Saturation})$$

For

$$R_2 = 2 \text{ k}\Omega - 5\% = 1.9 \text{ k}\Omega$$

$$R_1 = 12 \text{ k}\Omega + 5\% = 12.6 \text{ k}\Omega$$

$$R_E = 0.5 \text{ k}\Omega + 5\% = 0.525 \text{ k}\Omega$$

$$R_{TH} = R_1 \parallel R_2 = 1.65 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{1.9}{12.6 + 1.9} \right) (10) - 5 = -3.69 \text{ V}$$

$$I_{BQ} = \frac{-3.69 - 0.7 + 5}{1.65 + (101)(0.525)} = \frac{0.61}{54.7} = 0.0112 \text{ mA}$$

$$I_{CQ} = 1.12 \text{ mA}, \quad I_{EQ} = 1.13 \text{ mA}$$

$$\text{For } R_C = 5 \text{ k}\Omega - 5\% = 4.75 \text{ k}\Omega$$

$$V_{CEQ} = 10 - (1.12)(4.75) - (1.13)(0.525)$$

$$\Rightarrow V_{CEQ} = 4.09 \text{ V}$$

$$\text{So } 1.12 \leq I_{CQ} \leq 1.73 \text{ mA and } 0.0863 \leq V_{CEQ} \leq 4.09 \text{ V}.$$

Saturation

3.37

$$R_{TH} = R_1 \parallel R_2 = 9 \parallel 1 = 0.90 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (-12) = \left(\frac{1}{1+9} \right) (-12) = -1.2 \text{ V}$$

$$I_{BQ} R_E + V_{EB}(\text{on}) + I_{BQ} R_{TH} + V_{TH} = 0$$

$$I_{BQ} = \frac{-V_{TH} - V_{EB}(\text{on})}{R_{TH} + (1+\beta)R_E} = \frac{1.2 - 0.7}{0.90 + (76)(0.1)}$$

$$I_{BQ} = 0.0588, \quad I_{CQ} = 4.41 \text{ mA}$$

$$I_{EQ} = 4.47 \text{ mA}$$

$$\text{Center of load line} \Rightarrow V_{ECQ} = 6 \text{ V}$$

$$I_{EQ} R_E + V_{ECQ} + I_{CQ} R_C - 12 = 0$$

$$(4.47)(0.1) + 6 + (4.41)R_C = 12$$

$$\Rightarrow R_C = 1.26 \text{ k}\Omega$$

3.38

$$(a) R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(101)(0.5) = 5.05 \text{ k}\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.8}{100} = 0.008 \text{ mA}$$

Then

$$\frac{1}{R_1}(5.05)(10) = (0.008)(5.05) + 0.7 + (101)(0.008)(0.5)$$

or

$$R_1 = 44.1 \text{ k}\Omega, \quad \frac{44.1R_2}{44.1 + R_2} = 5.05 \Rightarrow R_2 = 5.70 \text{ k}\Omega$$

$$\text{Now } I_{EQ} = \left(\frac{101}{100} \right)(0.8) = 0.808 \text{ mA}$$

$$V_{CC} = I_{CQ}R_C + V_{CEQ} + I_{EQ}R_E$$

$$10 = (0.8)R_C + 5 + (0.808)(0.5)$$

$$R_C = 5.75 \text{ k}\Omega$$

(b) For $75 \leq \beta \leq 150$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (V_{CC}) = \left(\frac{5.7}{5.7 + 44.1} \right) (10) = 1.145 \text{ V}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E$$

$$\text{For } \beta = 75, \quad I_{BQ} = \frac{1.145 - 0.7}{5.05 + (76)(0.5)} = 0.0103 \text{ mA}$$

$$\text{Then } I_{CQ} = (75)(0.0103) = 0.775 \text{ mA}$$

$$\text{For } \beta = 150, \quad I_{BQ} = \frac{1.145 - 0.7}{5.05 + (151)(0.5)} = 0.00552 \text{ mA}$$

$$\text{Then } I_{CQ} = 0.829 \text{ mA}$$

$$\% \text{ Change} = \frac{\Delta I_{CQ}}{I_{CQ}} = \frac{0.829 - 0.775}{0.80} \times 100\% \Rightarrow$$

$$\% \text{ Change} = 6.75\%$$

(c) For $R_E = 1 \text{ k}\Omega$

$$R_{TH} = (0.1)(101)(1) = 10.1 \text{ k}\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (10.1)(10) =$$

$$(0.008)(10.1) + 0.7 + (101)(0.008)(1)$$

which yields $R_1 = 63.6 \text{ k}\Omega$

$$\text{And } \frac{63.6R_2}{63.6 + R_2} = 10.1 \Rightarrow R_2 = 12.0 \text{ k}\Omega$$

$$\text{Now } V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (V_{CC}) = \left(\frac{12}{12 + 63.6} \right) (10) = 1.587 \text{ V}$$

$$\text{For } \beta = 75, \quad I_{BQ} = \frac{1.587 - 0.7}{10.1 + (76)(1)} = 0.0103 \text{ mA}$$

$$\text{So } I_{CQ} = 0.773 \text{ mA}$$

$$\text{For } \beta = 150, \quad I_{BQ} = \frac{1.587 - 0.7}{10.1 + (151)(1)} = 0.00551 \text{ mA}$$

$$\text{Then } I_{CQ} = 0.826 \text{ mA}$$

$$\% \text{ Change} = \frac{\Delta I_{CQ}}{I_{CQ}} = \frac{0.826 - 0.773}{0.8} \times 100\% \Rightarrow$$

$$\% \text{ Change} = 6.63\%$$

3.39

$$V_{CC} \equiv I_{CQ}(R_C + R_E) + V_{CEQ}$$

$$10 = (0.8)(R_C + R_E) + 5 \Rightarrow R_C + R_E = 6.25 \text{ k}\Omega$$

$$\text{Let } R_E = 1 \text{ k}\Omega$$

$$\text{Then, for bias stable } R_{TH} = (0.1)(121)(1) = 12.1 \text{ k}\Omega$$

$$I_{BQ} = \frac{0.8}{120} = 0.00667 \text{ mA}$$

$$\frac{1}{R_1}(12.1)(10) = (0.00667)(12.1) + 0.7$$

$$+ (12.1)(0.00667)(1)$$

$$\text{So } R_1 = 76.2 \text{ k}\Omega \text{ and } \frac{76.2R_2}{76.2 + R_2} = 12.1 \Rightarrow$$

$$R_2 = 14.4 \text{ k}\Omega$$

$$\text{Then } I_R \equiv \frac{10}{76.2 + 14.4} = 0.110 \text{ mA}$$

This is close to the design specification.

3.40

$$I_{CQ} \approx I_{EQ} \Rightarrow V_{CEQ} = V_{CC} - I_{CQ}(R_C + R_E)$$

$$6 = 12 - I_{CQ}(2 + 0.2)$$

$$I_{CQ} = 2.73 \text{ mA}, \quad I_{BQ} = 0.0218 \text{ mA}$$

$$V_{CEQ} = 6 \text{ V}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E - 6$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (12) - 6, \quad R_{TH} = R_1 \parallel R_2$$

Bias stable \Rightarrow

$$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(126)(0.2) = 2.52 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{1}{R_1} \right) (R_{TH})(12) - 6$$

$$\frac{1}{R_1}(2.52)(12) - 6 = (0.0218)(2.52) + 0.7$$

$$+ (126)(0.0218)(0.2) - 6$$

$$\frac{1}{R_1}(30.24) = 0.7549 + 0.5494$$

$$R_1 = 23.2 \text{ k}\Omega, \quad \frac{23.2R_2}{23.2 + R_2} = 2.52$$

$$R_2 = 2.83 \text{ k}\Omega$$

3.41

$$\begin{aligned}
 \text{a. } I_{CQ} &= 1 \text{ mA}, \quad I_{EQ} = \left(\frac{80}{81}\right)(1) = 1.01 \text{ mA} \\
 V_{CEQ} &= 12 - (1)(2) - (1.01)(0.2) \Rightarrow V_{CEQ} = 9.80 \text{ V} \\
 I_{BQ} &= \frac{1}{80} = 0.0125 \text{ mA} \\
 R_{TH} &= +(0.1)(1 + \beta)R_E = (0.1)(81)(0.2) = 1.62 \text{ k}\Omega \\
 V_{TH} &= \left(\frac{R_2}{R_1 + R_2}\right)(12) - 6 = \frac{1}{R_1}(R_{TH})(12) - 6 \\
 &= \frac{1}{R_1}(19.44) - 6 \\
 V_{TH} &= I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E - 6 \\
 \frac{1}{R_1}(19.44) - 6 &= (0.0125)(1.62) + 0.7 \\
 &\quad + (81)(0.0125)(0.2) - 6 \\
 \frac{1}{R_1}(19.44) &= 0.923 \\
 R_1 &= 21.1 \text{ k}\Omega, \quad \frac{21.1R_2}{21.1 + R_2} = 1.62 \\
 R_2 &= 1.75 \text{ k}\Omega
 \end{aligned}$$

b.

$$\begin{aligned}
 R_1 &= 22.2 \text{ k}\Omega \text{ or } R_1 = 20.0 \text{ k}\Omega \\
 R_2 &= 1.84 \text{ k}\Omega \text{ or } R_2 = 1.66 \text{ k}\Omega \\
 R_E &= 0.21 \text{ k}\Omega \text{ or } R_E = 0.19 \text{ k}\Omega \\
 R_C &= 2.1 \text{ k}\Omega \text{ or } R_C = 1.9 \text{ k}\Omega \\
 R_2(\text{max}), R_1(\text{min}), R_E(\text{min}) \\
 R_{TH} &= (1.84)\|(20.0) = 1.685 \text{ k}\Omega
 \end{aligned}$$

$$\begin{aligned}
 V_{TH} &= \left(\frac{1.84}{1.84 + 20.0}\right)(12) - 6 = -4.99 \text{ V} \\
 I_{BQ} &= \frac{-4.99 - 0.7 - (-6)}{1.685 + (81)(0.19)} = \frac{0.31}{17.08} = 0.0182 \text{ mA} \\
 I_{CQ} &= 1.45 \text{ mA}
 \end{aligned}$$

 For max. $R_C \Rightarrow$

$$\begin{aligned}
 V_{CE} &= 12 - (1.45)(2.1) - (1.47)(0.19) \\
 V_{CE} &= 8.68 \text{ V}
 \end{aligned}$$

 $R_2(\text{min}), R_1(\text{max}), R_E(\text{max})$

$$R_{TH} = (1.66)\|(22.2) = 1.547 \text{ k}\Omega$$

$$\begin{aligned}
 V_{TH} &= \left(\frac{1.66}{1.66 + 22.2}\right)(12) - 6 = -5.165 \text{ V} \\
 I_{BQ} &= \frac{-5.165 - 0.7 + 6}{1.547 + (81)(0.21)} = \frac{0.135}{18.56} = 0.00727 \text{ mA}
 \end{aligned}$$

 For min. $R_C \Rightarrow I_{CQ} = 0.582 \text{ mA}, I_E = 0.589$

$$V_{CEQ} = 12 - (0.582)(1.9) - (0.589)(0.21)$$

$V_{CEQ} = 10.77 \text{ V}$

 So $0.582 \leq I_C \leq 1.45 \text{ mA}$

$$8.68 \leq V_{CEQ} \leq 10.77 \text{ V}$$

3.42

$$\begin{aligned}
 V_{CEQ} &\equiv V_{CC} - I_{CQ}(R_C + R_E) \\
 5 &= 12 - 3(R_C + R_E) \Rightarrow R_C + R_E = 2.33 \text{ k}\Omega \\
 \text{Let } R_E &= 0.33 \text{ k}\Omega \text{ and } R_C = 2 \text{ k}\Omega \\
 \text{Nominal value of } \beta &= 100 \\
 R_{TH} &= (0.1)(1 + \beta)R_E = (0.1)(101)(0.33) = 3.33 \text{ k}\Omega \\
 I_{BQ} &= \frac{3}{100} = 0.03 \text{ mA} \\
 V_{TH} &= \frac{1}{R_1} \cdot R_{TH} \cdot (12) - 6 = \frac{1}{R_1}(3.33)(12) - 6 \\
 \text{Then } V_{TH} &= I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E - 6 \\
 \frac{1}{R_1}(3.33)(12) - 6 &= (0.03)(3.33) + 0.7 + (101)(0.03)(0.33) - 6 \\
 \text{which yields } R_1 &= 22.2 \text{ k}\Omega \text{ and } R_2 = 3.92 \text{ k}\Omega \\
 \text{Now } V_{TH} &= \left(\frac{R_2}{R_1 + R_2}\right)(12) - 6 = \left(\frac{3.92}{3.92 + 22.2}\right)(12) - 6 \\
 \text{or } V_{TH} &= -4.20 \text{ V} \\
 \text{For } \beta = 75, \quad V_{TH} &= I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E - 6 \\
 I_{BQ} &= \frac{V_{TH} + 6 - 0.7}{R_{TH} + (1 + \beta)R_E} = \frac{-4.2 + 6 - 0.7}{3.33 + (76)(0.33)} \\
 &= 0.0387 \text{ mA} \Rightarrow I_C = 2.90 \text{ mA} \\
 \text{For } \beta = 150, \quad I_{BQ} &= \frac{-4.2 + 6 - 0.7}{3.33 + (151)(0.33)} = 0.0207 \text{ mA} \\
 \text{Then } I_C &= 3.10 \text{ mA} \\
 \text{Specifications are met.}
 \end{aligned}$$

3.43

$$\begin{aligned}
 R_{TH} &= R_1\|R_2 = 3\|2 = 2.4 \text{ k}\Omega \\
 V_{TH} &= \left(\frac{R_2}{R_1 + R_2}\right)V_{CC} = \left(\frac{12}{12 + 3}\right)(20) = 16 \text{ V} \\
 \text{(a) For } \beta = 75 \\
 20 &= (1 + \beta)I_{BQ}R_E + V_{BE}(\text{on}) + I_{BQ}R_{TH} + V_{TH} \\
 20 - 0.7 - 16 &= I_{BQ}[(76)(2) + 2.4] \\
 \text{So } I_{BQ} &= 0.0214 \text{ mA}, \quad I_{CQ} = 1.60 \text{ mA}, \quad I_{EQ} = 1.62 \text{ mA} \\
 V_{ECQ} &= 20 - (1.6)(1) - (1.62)(2) \\
 \text{or } V_{ECQ} &= 15.16 \text{ V}
 \end{aligned}$$

 (b) For $\beta = 100$, we find

$$I_{BQ} = 0.0161 \text{ mA}, \quad I_{CQ} = 1.61 \text{ mA}, \quad V_{ECQ} = 15.13 \text{ V}$$

3.44

$$I_{CQ} = 4.8 \text{ mA} \rightarrow I_{EQ} = 4.84 \text{ mA}$$

$$V_{CEQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E$$

$$6 = 18 - (4.8)(2) - (4.84)R_E \Rightarrow R_E = 0.496 \text{ k}\Omega$$

$$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(121)(0.496) = 6.0 \text{ k}\Omega$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE(on)} + (1 + \beta)I_{BQ}R_E$$

$$I_{BQ} = 0.040 \text{ mA}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(6.0)(18)$$

$$\frac{1}{R_1}(6.0)(18) = (0.04)(6.0) + 0.70$$

$$+ (121)(0.04)(0.496)$$

$$\frac{1}{R_1}(108) = 3.34$$

$$R_1 = 32.3 \text{ k}\Omega, \quad \frac{32.3R_2}{32.3 + R_2} = 6.0$$

$$R_2 = 7.37 \text{ k}\Omega$$

3.45

For nominal $\beta = 70$

$$I_{BQ} = \frac{2}{70} = 0.0286 \text{ mA} \rightarrow I_{EQ} = 2.03 \text{ mA}$$

$$V_{CEQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E$$

$$10 = 20 - (2)(4) - (2.03)R_E \Rightarrow R_E = 0.985 \text{ K}$$

$$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(71)(0.985) = 6.99 \text{ K}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE(on)} + I_{EQ}R_E$$

$$\frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = I_{BQ}R_{TH} + V_{BE(on)} + I_{EQ}R_E$$

$$\frac{1}{R_1}(6.99)(20) = (0.0286)(6.99) + 0.70$$

$$+ (2.03)(0.985)$$

$$\frac{1}{R_1}(139.8) = 2.90$$

$$R_1 = 48.2 \text{ K}, \quad \frac{48.2R_2}{48.2 + R_2} = 6.99$$

$$R_2 = 8.18 \text{ K}$$

Check: For $\beta = 50$

$$V_{TH} = \left(\frac{8.18}{8.18 + 48.2} \right)(20) = 2.90$$

$$I_{BQ} = \frac{V_{TH} - V_{BE(on)}}{R_{TH} + (1 + \beta)R_E} = \frac{2.90 - 0.7}{6.99 + (51)(0.985)} = 0.0384 \text{ mA}$$

$$I_{CQ} = 1.92 \text{ mA}$$

For $\beta = 90$

$$I_{BQ} = \frac{2.90 - 0.7}{6.99 + (91)(0.985)} = 0.0228 \text{ mA}$$

$$I_{CQ} = 2.05 \text{ mA}$$

Design criterion is satisfied.

3.46

$$I_{CQ} = 1 \text{ mA} \rightarrow I_{EQ} = 1.02 \text{ mA}$$

$$V_{CEQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E$$

$$5 = 15 - (1)(5) - (1.02)R_E \Rightarrow R_E = 4.90 \text{ k}\Omega$$

Bias stable:

$$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(61)(4.9) = 29.9 \text{ k}\Omega$$

$$I_{BQ} = \frac{1}{60} = 0.0167 \text{ mA}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = I_{BQ}R_{TH} + V_{BE(on)}$$

$$+ I_{EQ}R_E$$

$$\frac{1}{R_1}(29.9)(15) = (0.0167)(29.9) + 0.70$$

$$+ (1.02)(4.90)$$

$$\frac{1}{R_1}(448.5) = 6.197$$

$$R_1 = 72.4 \text{ k}\Omega, \quad \frac{72.4R_2}{72.4 + R_2} = 29.9$$

$$R_2 = 50.9 \text{ k}\Omega$$

Check: For $\beta = 45$

$$V_{TH} = \left(\frac{50.9}{50.9 + 72.4} \right)(15) = 6.19 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE(on)}}{R_{TH} + (1 + \beta)R_E} = \frac{6.19 - 0.7}{29.9 + (46)(4.90)} = 0.0215 \text{ mA}$$

$$I_{CQ} = 0.968 \text{ mA}, \quad \frac{\Delta I_C}{I_C} = 3.2\%$$

Check: For $\beta = 75$

$$I_{BQ} = \frac{6.19 - 0.7}{29.9 + (75)(4.90)} = 0.0136 \text{ mA}$$

$$I_{CQ} = 1.02 \text{ mA}, \quad \frac{\Delta I_C}{I_C} = 2.0\%$$

Design criterion is satisfied.

3.47

$$(a) V_{CC} \equiv I_{CQ}(R_C + R_E) + V_{CEQ}$$

$$3 = (0.1)(5R_E + R_E) + 1.4 \Rightarrow R_E = 2.67 \text{ k}\Omega$$

$$R_C = 13.3 \text{ k}\Omega, \quad I_{BQ} = \frac{100}{120} = 0.833 \mu\text{A}$$

$$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(121)(2.67) = 32.3 \text{ k}\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(32.3)(3)$$

$$= I_{BQ}R_{TH} + V_{BE(on)} + (1 + \beta)I_{BQ}R_E$$

$$= (0.000833)(32.3) + 0.7 + (121)(0.000833)(2.67)$$

$$\text{which gives } R_1 = 97.3 \text{ k}\Omega, \text{ and } R_2 = 48.4 \text{ k}\Omega$$

$$(b) I_R = \frac{3}{R_1 + R_2} = \frac{3}{97.3 + 48.4} \Rightarrow 20.6 \mu A$$

$$I_{CQ} = 100 \mu A$$

$$P = (I_{CQ} + I_R)V_{CC} = (100 + 20.6)(3)$$

$$\text{or } P = 362 \mu W$$

3.48

$$I_E = \frac{5 - V_E}{R_E} = \frac{5}{3} = 1.67 \text{ mA}$$

$$R_{TH} = R_1 \parallel R_2 = (0.1)(1 + \beta)R_E$$

$$= (0.1)(101)(3) = 30.3 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right)(4) - 2 = \frac{1}{R_1} \cdot R_{TH} \cdot (4) - 2$$

$$I_{BQ} = \frac{I_{EQ}}{1 + \beta} = 0.0165 \text{ mA}$$

$$5 = I_{BQ}R_E + V_{BE}(\text{on}) + I_B R_{TH} + V_{TH}$$

$$5 = (1.67)(3) + 0.7 + (0.0165)(30.3)$$

$$+ \frac{1}{R_1}(30.3)(4) - 2$$

$$0.80 = \frac{1}{R_1}(30.3)(4) \Rightarrow R_1 = 152 \text{ k}\Omega$$

$$\frac{152R_2}{152 + R_2} = 30.3 \Rightarrow R_2 = 37.8 \text{ k}\Omega$$

3.49

$$a. R_{TH} = R_1 \parallel R_2 = 10 \parallel 20 \Rightarrow R_{TH} = 6.67 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right)(10) - 5 = \left(\frac{20}{20 + 10} \right)(10) - 5$$

$$\Rightarrow V_{TH} = 1.67 \text{ V}$$

$$b. 10 = (1 + \beta)I_{BQ}R_E + V_{BE}(\text{on}) + I_{BQ}R_{TH} + V_{TH}$$

$$I_{BQ} = \frac{10 - 0.7 - 1.67}{6.67 + (61)(2)} = \frac{7.63}{128.7}$$

$$\Rightarrow I_{BQ} = 0.0593 \text{ mA}$$

$$I_{CQ} = 3.65 \text{ mA}, \quad I_{EQ} = 3.62 \text{ mA}$$

$$V_E = 10 - I_{EQ}R_E = 10 - (3.62)(2)$$

$$V_E = 2.76 \text{ V}$$

$$V_C = I_{CQ}R_C - 10 = (3.65)(2.2) - 10$$

$$V_C = -2.17 \text{ V}$$

3.50

$$V^+ - V^- \equiv I_{CQ}(R_C + R_E) + V_{CEQ}$$

$$20 = (0.5)(R_C + R_E) + 8 \Rightarrow (R_C + R_E) = 24 \text{ k}\Omega$$

$$\text{Let } R_E = 10 \text{ k}\Omega \text{ then } R_C = 14 \text{ k}\Omega$$

$$\text{Let } \beta = 60 \text{ from previous problem.}$$

$$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(61)(10)$$

$$\text{Or } R_{TH} = 61 \text{ k}\Omega$$

$$I_{BQ} = \frac{0.5}{60} = 0.00833 \text{ mA}$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right)(10) - 5 = \frac{1}{R_1} \cdot R_{TH} \cdot 10 - 5$$

Now

$$10 = (1 + \beta)I_{BQ}R_E + V_{EB}(\text{on}) + I_{BQ}R_{TH} + V_{TH}$$

$$10 = (61)(0.00833)(10) + 0.7 + (0.00833)(61)$$

$$+ \frac{1}{R_1}(61)(10) - 5$$

$$\text{Then } R_1 = 70.0 \text{ k}\Omega \text{ and } R_2 = 474 \text{ k}\Omega$$

$$I_R \equiv \frac{10}{R_1 + R_2} = \frac{10}{70 + 474} \Rightarrow 18.4 \mu A$$

 So the 40 μA current limit is met.

3.51

$$a. R_{TH} = R_1 \parallel R_2 = 35 \parallel 20 \Rightarrow R_{TH} = 12.7 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right)(7) - 5 = \left(\frac{20}{20 + 35} \right)(7) - 5$$

$$\Rightarrow V_{TH} = -2.45 \text{ V}$$

b.

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on}) - (-10)}{R_{TH} + (1 + \beta)R_E}$$

$$= \frac{-2.45 - 0.7 + 10}{12.7 + (76)(0.5)}$$

$$\Rightarrow I_{BQ} = 0.135 \text{ mA}$$

$$I_{CQ} = 10.1 \text{ mA}, \quad I_{EQ} = 10.3 \text{ mA}$$

$$V_{CEQ} = 20 - I_{CQ}R_C - I_{EQ}R_E$$

$$= 20 - (10.1)(0.8) - (10.3)(0.5)$$

$$V_{CEQ} = 6.77 \text{ V}$$

c.

$$R_2 = 20 + 5\% = 21 \text{ k}\Omega$$

$$R_1 = 35 - 5\% = 33.25 \text{ k}\Omega$$

$$R_E = 0.5 - 5\% = 0.475 \text{ k}\Omega$$

$$R_{TH} = R_1 \parallel R_2 = 21 \parallel 33.25 = 12.9 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right)(7) - 5$$

$$= \left(\frac{21}{21 + 33.25} \right)(7) - 5 = -2.29 \text{ V}$$

$$I_{BQ} = \frac{-2.29 - 0.7 - (-10)}{12.9 + (76)(0.475)} = 0.143 \text{ mA}$$

$$I_{CQ} = 10.7 \text{ mA}, \quad I_{EQ} = 10.9 \text{ mA}$$

 For $R_C = 0.8 + 5\% = 0.84 \text{ k}\Omega$

$$V_{CEQ} = 20 - (10.7)(0.84) - (10.9)(0.475)$$

$$\Rightarrow V_{CEQ} = 5.83 \text{ V}$$

For $R_C = 0.8 - 5\% = 0.76 \text{ k}\Omega$

$$V_{CEQ} = 20 - (10.7)(0.76) - (10.9)(0.475)$$

$$\Rightarrow V_{CEQ} = 6.69 \text{ V}$$

$$R_1 = 20 - 5\% = 19 \text{ k}\Omega$$

$$R_2 = 35 + 5\% = 36.75 \text{ k}\Omega$$

$$R_E = 0.5 + 5\% = 0.525 \text{ k}\Omega$$

$$R_{TH} = R_1 \parallel R_2 = 19 \parallel 36.75 = 12.5 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{19}{19 + 36.75} \right)(7) - 5 = -2.61 \text{ V}$$

$$I_{BQ} = \frac{-2.61 - 0.7 - (-10)}{12.5 + (76)(0.525)} = 0.128 \text{ mA}$$

$$I_{CQ} = 9.58 \text{ mA}, I_{EQ} = 9.70 \text{ mA}$$

For $R_C = 0.84 \text{ k}\Omega$

$$V_{CEQ} = 20 - (9.58)(0.84) - (9.70)(0.525)$$

$$\Rightarrow V_{CEQ} = 6.86 \text{ V}$$

For $R_C = 0.76 \text{ k}\Omega$

$$V_{CEQ} = 20 - (9.58)(0.76) - (9.70)(0.525)$$

$$\Rightarrow V_{CEQ} = 7.63 \text{ V}$$

$$\text{So } 9.58 \leq I_{CQ} \leq 10.7 \text{ mA}$$

and

$$5.83 \leq V_{CEQ} \leq 7.63 \text{ V}$$

3.52

$$1. R_{TH} = 500 \text{ k}\Omega \parallel 500 \text{ k}\Omega \parallel 70 \text{ k}\Omega = 250 \text{ k}\Omega \parallel 70 \text{ k}\Omega$$

$$\Rightarrow R_{TH} = 54.7 \text{ k}\Omega$$

$$\frac{5 - V_{TH}}{500} + \frac{3 - V_{TH}}{500} = \frac{V_{TH} - (-5)}{70}$$

$$\frac{5}{500} + \frac{3}{500} - \frac{5}{70} = V_{TH} \left(\frac{1}{500} + \frac{1}{500} + \frac{1}{70} \right)$$

$$- 0.0554 = V_{TH}(0.0183)$$

$$V_{TH} = -3.03 \text{ V}$$

b.

$$I_{BQ} = \frac{V_{TH} - V_{BE(\text{on})} - (-5)}{R_{TH}} = \frac{1}{(1 + \beta)R_E}$$

$$= \frac{-3.03 - 0.7 + 5}{54.7 + (101)(5)}$$

$$I_{BQ} = 0.00227 \text{ mA}$$

$$I_{CQ} = 0.227 \text{ mA}, I_{EQ} = 0.229$$

$$V_{CEQ} = 20 - (0.227)(50) - (0.229)(5)$$

$$V_{CEQ} = 7.51 \text{ V}$$

3.53

$$R_{TH} = 30 \parallel 60 \parallel 20 \Rightarrow R_{TH} = 10 \text{ k}\Omega$$

$$\frac{5 - V_{TH}}{30} + \frac{5 - V_{TH}}{60} = \frac{V_{TH}}{20}$$

$$\left(\frac{5}{30} + \frac{5}{60} \right) = V_{TH} \left(\frac{1}{30} + \frac{1}{60} + \frac{1}{20} \right)$$

$$V_{TH} = 2.5 \text{ V}$$

For $\beta = 100$

$$I_{BQ} = \frac{V_{TH} - V_{BE(\text{on})} - (-5)}{R_{TH} + (1 + \beta)R_E}$$

$$= \frac{2.5 - 0.7 + 5}{10 + (101)(0.2)}$$

$$I_{BQ} = 0.225 \text{ mA}$$

$$I_{CQ} = 22.5 \text{ mA}, I_{EQ} = 22.7 \text{ mA}$$

$$V_{CEQ} = 15 - (22.5)(0.5) - (22.7)(0.2)$$

$$V_{CEQ} = -0.79 \text{ V} \quad \text{In saturation}$$

$$\Rightarrow V_{CEQ} = 0.2 \text{ V}$$

$$V_E = V_{TH} - I_{BQ}R_{TH} - V_{BE(\text{on})}$$

$$= 2.5 - (0.225)(10) - 0.7$$

$$V_E = -0.45 \text{ V} \Rightarrow V_C = -0.45 + 0.2 = -0.25 \text{ V}$$

$$I_{CQ} = \frac{10 - (-0.25)}{0.5} \Rightarrow I_{CQ} = 20.5 \text{ mA}$$

3.54

$$R_{TH} = R_1 \parallel R_2 = 100 \parallel 40 = 28.6 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right)(10) = \left(\frac{40}{40 + 100} \right)(10) = 2.86 \text{ V}$$

$$I_{B1} = \frac{V_{TH} - V_{BE(\text{on})}}{R_{TH} + (1 + \beta)R_{E1}} = \frac{2.86 - 0.7}{28.6 + (121)(1)}$$

$$I_{B1} = 0.0144 \text{ mA}$$

$$I_{C1} = 1.73 \text{ mA}, I_{E1} = 1.75 \text{ mA}$$

$$\frac{10 - V_{B2}}{3} = I_{C1} + I_{B2}$$

$$I_{E2} = \frac{V_{B2} - V_{BE(\text{on})} - (-10)}{5}$$

$$\frac{10 - V_{B2}}{3} = I_{C1} + \frac{V_{B2} - 0.7 + 10}{(121)(5)}$$

$$\frac{10}{3} - 1.73 - \frac{9.3}{605} = V_{B2} \left(\frac{1}{3} + \frac{1}{(121)(5)} \right)$$

$$1.59 = V_{B2}(0.335) \Rightarrow V_{B2} = 4.75 \text{ V}$$

$$I_{E2} = \frac{4.75 - 0.7 - (-10)}{5} \Rightarrow I_{E2} = 2.61 \text{ mA}$$

$$I_{B2} = 0.0232 \text{ mA}$$

$$I_{C2} = 2.79 \text{ mA}$$

$$V_{CEQ1} = 4.75 - (1.75)(1) \Rightarrow V_{CEQ1} = 3.0 \text{ V}$$

$$V_{CEQ2} = 10 - (4.75 - 0.7) \Rightarrow V_{CEQ2} = 5.95 \text{ V}$$

Chapter 3: Problem Solutions

3.55

$$V_{E1} = -0.7$$

$$I_{R1} = \frac{-0.7 - (-5)}{20} = 0.215 \text{ mA}$$

$$V_{E2} = -0.7 - 0.7 = -1.4$$

$$I_{E2} = \frac{-1.4 - (-5)}{1} \Rightarrow I_{E2} = 3.6 \text{ mA}$$

$$I_{B2} = 0.0444 \text{ mA}$$

$$I_{C2} = 3.56 \text{ mA}$$

$$I_{E1} = I_{R1} + I_{B2} = 0.215 + 0.0444$$

$$I_{E1} = 0.259 \text{ mA}$$

$$I_{B1} = 0.00320 \text{ mA}$$

$$I_{C1} = 0.256 \text{ mA}$$

3.56

Current through V^- source = $I_{E1} + I_{E2}$ and

$$I_{E1} = I_{E2} = (1 + \beta)I_{B1} = (51)(8.26) \mu\text{A}$$

So total current = $2(51)(8.26) \mu\text{A} = 843 \mu\text{A}$

$$P^- = I \cdot |V^-| = (0.843)(5) \Rightarrow P^- = 4.22 \text{ mW}$$

(From V^- source)

From Example 3.15, $I_Q = 0.413 \text{ mA}$

$$\text{So } I_{C0} = \left(\frac{50}{51}\right)(0.413) = 0.405 \text{ mA}$$

$$P^+ = I \cdot V^+ = (0.405)(5) \Rightarrow P^+ = 2.03 \text{ mW}$$

(From V^+ source)

3.57

$$R_{TH} = R_1 \parallel R_2 = 50 \parallel 100 = 33.3 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(10) - 5$$

$$= \left(\frac{100}{100 + 50}\right)(10) - 5 = 1.67 \text{ V}$$

$$5 = I_{E1}R_{E1} + V_{EB}(\text{on}) + I_{B1}R_{TH} + V_{TH}$$

$$I_{E1} = \left(\frac{101}{100}\right)(0.8) = 0.808 \text{ mA}$$

$$I_{B1} = 0.008 \text{ mA}$$

$$5 = (0.808)R_{E1} + 0.7 + (0.008)(33.3) + 1.67$$

$$R_{E1} = 2.93 \text{ k}\Omega$$

$$V_{E1} = 5 - (0.808)(2.93) = 2.63 \text{ V}$$

$$V_{C1} = V_{E1} - V_{CEQ1} = 2.63 - 3.5 = -0.87 \text{ V}$$

$$V_{E2} = -0.87 - 0.70 = -1.57 \text{ V}$$

$$I_{E2} = \frac{-1.57 - (-5)}{R_{E2}} = 0.808 \Rightarrow R_{E2} = 4.25 \text{ k}\Omega$$

$$V_{CEQ2} = 4 \Rightarrow V_{C2} = -1.57 + 4 = 2.43 \text{ V}$$

$$R_{C2} = \frac{5 - 2.43}{0.8} \Rightarrow R_{C2} = 3.21 \text{ k}\Omega$$

$$I_{RC1} = I_{C1} - I_{B2} = 0.8 - 0.008 = 0.792 \text{ mA}$$

$$R_{C1} = \frac{-0.87 - (-5)}{0.792} \Rightarrow R_{C1} = 5.21 \text{ k}\Omega$$

Chapter 4

Exercise Solutions

E4.1

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.25}{0.026} \Rightarrow g_m = 9.62 \text{ mA/V}$$

$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{(0.0026)(120)}{0.25} \Rightarrow r_\pi = 12.5 \text{ k}\Omega$$

$$r_0 = \frac{V_A}{I_{CQ}} = \frac{150}{0.25} \Rightarrow r_0 = 600 \text{ k}\Omega$$

E4.2

$$r_0 = \frac{V_A}{I_{CQ}} \Rightarrow I_{CQ} = \frac{V_A}{r_0} = \frac{75}{200 \text{ k}\Omega}$$

$$\Rightarrow I_{CQ} = 0.375 \text{ mA}$$

E4.3

$$I_{BQ} = \frac{V_{BB} - V_{BE(\text{on})}}{R_B} = \frac{0.92 - 0.7}{100}$$

$$\Rightarrow I_{BQ} = 0.0022 \text{ mA}$$

$$I_{CQ} = (150)(0.0022) = 0.33 \text{ mA}$$

a. $g_m = \frac{I_{CQ}}{V_T} = \frac{0.33}{0.026} \Rightarrow g_m = 12.7 \text{ mA/V}$

$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{(0.0026)(150)}{0.33} \Rightarrow r_\pi = 11.8 \text{ k}\Omega$$

$$r_0 = \frac{V_A}{I_{CQ}} = \frac{200}{0.33} \Rightarrow r_0 = 606 \text{ k}\Omega$$

b. $v_0 = -g_m v_\pi (r_0 \parallel R_C), \quad v_\pi = \left(\frac{r_\pi}{r_\pi + R_B} \right) v_S$

$$A_v = \frac{v_0}{v_S} = -g_m \left(\frac{r_\pi}{r_\pi + R_B} \right) (r_0 \parallel R_C)$$

$$= -(12.7) \left(\frac{11.8}{11.8 + 100} \right) (606 \parallel 15)$$

$$= -(12.7)(0.1055)(14.64)$$

$$\Rightarrow A_v = -19.6$$

E4.4

$$g_m = \frac{I_{CQ}}{V_T}$$

a. $I_{BQ} = \frac{V_{BB} - V_{EB(\text{on})}}{R_B} = \frac{1.145 - 0.7}{50}$

$$\Rightarrow I_{BQ} = 0.0089 \text{ mA}$$

$$I_{CQ} = (90)(0.0089) \Rightarrow I_{CQ} = 0.801 \text{ mA}$$

$$g_m = \frac{0.801}{0.026} \Rightarrow g_m = 30.8 \text{ mA/V}$$

$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{(0.0026)(90)}{0.801} \Rightarrow r_\pi = 2.92 \text{ k}\Omega$$

$$r_0 = \frac{V_A}{I_{CQ}} = \frac{120}{0.801} \Rightarrow r_0 = 150 \text{ k}\Omega$$

b. $v_0 = g_m v_\pi (r_0 \parallel R_C), \quad v_\pi = -\left(\frac{r_\pi}{r_\pi + R_B} \right) v_S$

$$A_v = \frac{v_0}{v_S} = -g_m \left(\frac{r_\pi}{r_\pi + R_B} \right) (r_0 \parallel R_C)$$

$$= -(30.8) \left(\frac{2.92}{2.92 + 50} \right) (150 \parallel 2.5)$$

$$= -(30.8)(0.055)(2.46)$$

$$\Rightarrow A_v = -4.17$$

E4.5

$$R_{TH} = 250 \parallel 75 = 57.7 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_1}{R_1 + R_2} \right) (5) = \left(\frac{75}{75 + 250} \right) (5) = 1.154$$

$$I_{BQ} = \frac{1.154 - 0.7}{57.7 + (121)(0.6)} = 3.48 \mu\text{A}$$

$$I_{CQ} = 0.418 \text{ mA}$$

a. $r_\pi = \frac{(120)(0.026)}{0.418} = 7.46 \text{ k}\Omega$

$$g_m = \frac{0.418}{0.026} = 16.08 \text{ mA/V}$$

$$V_0 = -g_m V_\pi R_C$$

$$R_{ib} = r_\pi + (1 + \beta) R_E = 7.46 + (121)(0.6) = 80.1 \text{ k}\Omega$$

$$R_1 \parallel R_2 = 250 \parallel 75 = 57.7 \text{ k}\Omega$$

$$R_1 \parallel R_2 \parallel R_{ib} = 57.7 \parallel 80.1 = 33.54 \text{ k}\Omega$$

$$V'_S = \left(\frac{R_1 \parallel R_2 \parallel R_{ib}}{R_1 \parallel R_2 \parallel R_{ib} + R_S} \right) V_S = \left(\frac{33.54}{33.54 + 0.5} \right) V_S$$

$$V'_S = (0.985)V_S$$

$$V'_S = V_S \left[1 + \left(\frac{1 + \beta}{r_\pi} \right) R_E \right]$$

Then

$$A_v = (0.985)(-8.39) = -8.27$$

b. $R_{ib} = r_\pi + (1 + \beta) R_E = 7.46 + (121)(0.6)$
 $\Rightarrow R_{ib} = 80.1 \text{ k}\Omega$

E4.6

As a first approximation,

$$A_v \approx -\frac{R_C}{R_E}$$

Resulting gain is always smaller than this value.
The effect of R_S is very small.

$$\text{Set } \frac{R_C}{R_E} = 10$$

$$5 \equiv I_C(R_C + R_E) + V_{CEQ}$$

$$5 = 0.5(R_C + R_E) + 2.5$$

So that $R_E = 0.454 \text{ k}\Omega$ and $R_C = 4.54 \text{ k}\Omega$

$$I_{BQ} = \frac{0.5}{100} = 0.005 \text{ mA}$$

$$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(101)(0.454) = 4.59 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(4.59)(5)$$

$$\text{or } V_{TH} = \frac{23}{R_1}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE(on)} + (1 + \beta)I_{BQ}R_E$$

$$\frac{23}{R_1} = (0.005)(4.59) + 0.7 + (101)(0.005)(0.454)$$

which yields, $R_1 = 24.1 \text{ k}\Omega$ and $R_2 = 5.67 \text{ k}\Omega$

E4.7

dc analysis

$$R_{TH} = R_1 \parallel R_2 = 15 \parallel 85 = 12.75$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{85}{85 + 15} \right) (12)$$

$$V_{TH} = 10.2 \text{ V}$$

$$I_{BQ} = \frac{12 - 0.7 - V_{TH}}{R_{TH} + (1 + \beta)R_E}$$

$$= \frac{12 - 0.7 - 10.2}{12.75 + (101)(0.5)} = \frac{1.1}{63.25} = 0.0174$$

$$I_{CQ} = 1.74 \text{ mA}$$

ac analysis

$$V_0 = h_{fe}I_b(R_C \parallel R_L)$$

$$I_b = \frac{-V_S}{h_{ie} + (1 + h_{fe})R_E}$$

$$A_v = \frac{-h_{fe}(R_C \parallel R_L)}{h_{ie} + (1 + h_{fe})R_E}$$

$$\text{For } I_{CQ} = 1.7 \text{ mA}$$

$$h_{fe}(\max) = 110 \quad h_{fe}(\min) = 70$$

$$h_{ie}(\max) = 2 \text{ k}\Omega \quad h_{ie}(\min) = 1.1 \text{ k}\Omega$$

$$A_v(\max) = \frac{-110(4\parallel 2)}{2 + (111)(0.5)} = -2.54$$

$$A_v(\min) = \frac{-70(4\parallel 2)}{1.1 + (71)(0.5)} = -2.54$$

E4.8

First approximation, $A_v = -\frac{R_C}{R_E}$ which predicts a

low value. Set $\frac{R_C}{R_E} = 9$. Now

$$V_{CC} \equiv I_{CQ}(R_C + R_E) + V_{ECQ}$$

$$7.5 = (0.6)(9R_E + R_E) + 3.75$$

$$\text{So } R_E = 0.625 \text{ k}\Omega \text{ and } R_C = 5.62 \text{ k}\Omega$$

$$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(101)(0.625) = 6.31 \text{ k}\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(6.31)(7.5)$$

$$V_{CC} = (1 + \beta)I_{BQ}R_E + V_{BE(on)} + I_{BQ}R_{TH} + V_{TH}$$

$$I_{BQ} = \frac{0.6}{100} = 0.006 \text{ mA}$$

$$7.5 = (101)(0.006)(0.625) + 0.7 + (0.006)(6.31)$$

$$+ \frac{1}{R_1}(6.31)(7.5)$$

Then $R_1 = 7.40 \text{ k}\Omega$ and $R_2 = 42.8 \text{ k}\Omega$

E4.9

dc analysis

$$I_{BQ} = \frac{10 - 0.7}{100 + (101)(20)} = 0.00439 \text{ mA}$$

$$I_{CQ} = 0.439 \text{ mA}, I_{EQ} = 0.443 \text{ mA}$$

$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{(0.026)(100)}{0.439} = 5.92 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.439}{0.026} = 16.88 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{0.439} = 228 \text{ k}\Omega$$

$$(a) V_a = -g_m V_s (r_o \parallel R_C)$$

$$V_s = V'_s = \left(\frac{R_B \parallel r_\pi}{R_B \parallel r_\pi + R_S} \right) \cdot V_i$$

$$R_B \parallel r_\pi = 100 \parallel 5.92 = 559 \text{ k}\Omega$$

Then

$$V_s = \left(\frac{5.59}{5.59 + 0.5} \right) \cdot V_i = 0.918V_i$$

Then

$$A_v = -(161.7)(0.918) = -148$$

$$b. R_{in} = R_B \parallel r_\pi = (100) \parallel (5.92)$$

$$\Rightarrow R_{in} = 5.59 \text{ k}\Omega$$

$$R_o = R_C \parallel r_o = 10 \parallel 228 \Rightarrow R_o = 9.58 \text{ k}\Omega$$

E4.10

$$A_v = \frac{-\beta R_C}{r_\pi + (1 + \beta)R_E} = -(0.95) \left(\frac{R_C}{R_E} \right) = -(0.95) \left(\frac{2}{0.4} \right)$$

$$\text{or } A_v = -4.75$$

Assume $r_\pi = 1.2 \text{ k}\Omega$ from Example 4.5. Then

$$\frac{-\beta(2)}{1.2 + (1 + \beta)(0.4)} = -4.75$$

$$\text{which yields } \beta = 76$$

E4.11

dc analysis: $V_{TH} = 0$, $R_{TH} = R_1 \parallel R_2 = 10 \text{ k}\Omega$

$$I_{BQ} = \frac{5 - 0.7}{10 + (126)(5)} = 0.00672$$

$$I_{CQ} = 0.84 \text{ mA}$$

$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{(0.026)(125)}{0.84} = 3.87 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.84}{0.026} = 32.3 \text{ mA/V}$$

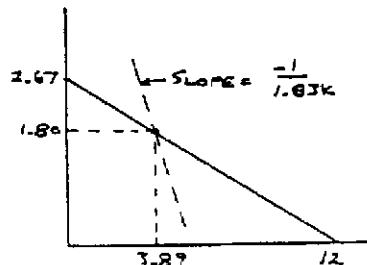
$$r_0 = \frac{V_A}{I_{CQ}} = \frac{200}{0.84} = 238 \text{ k}\Omega$$

a. $v_o = -g_m v_\pi (r_0 \parallel R_C \parallel R_L)$, $v_\pi = v_s$

$$A_v = -g_m (r_0 \parallel R_C \parallel R_L) = -(32.3)(238 \parallel 2.3 \parallel 5)$$

$$A_v = -(32.3)(1.56) \Rightarrow A_v = -50.4$$

b. $R_o = r_0 \parallel R_C \Rightarrow R_o = 2.28 \text{ k}\Omega$



$$I_{BQ} = \frac{12 - 0.7 - 10.2}{12.75 + (121)(0.5)} = \frac{1.1}{73.25}$$

$$I_{BQ} = 0.0150$$

$$I_{CQ} = 1.80, I_{EQ} = 1.82$$

$$\Rightarrow V_{ECQ} = 3.89$$

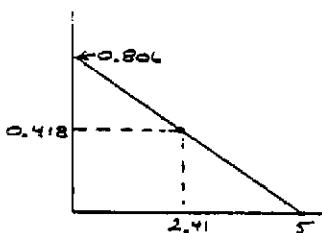
b. For $\Delta i_C = 1.8 \Rightarrow \Delta v_{EC} = (1.8)(1.83) = 3.29$

$$\text{For } \Delta v_{EC} = -3.29 \Rightarrow \Delta v_{CE} = 3.89 - 3.29 = 0.6$$

\Rightarrow Max. symmetrical swing

$$= 2 \times (3.29) = 6.58 \text{ V peak-to-peak}$$

E4.12



E4.14

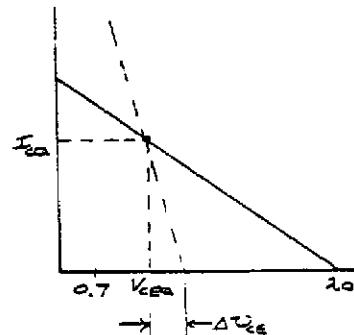
dc load line:

$$V_{CEQ} \approx (10 + 10) - I_C(R_C + R_E)$$

$$V_{CEQ} = 20 - I_C(10 + R_E)$$

$$v_{ce} = -i_C R_C = -i_C(10)$$

ac load line:



E4.13

a. dc load line:

$$V_{EC} = V_{CC} - I_E R_E - I_C R_C$$

$$V_{EC} \approx 12 - I_C(R_E + R_C) = 12 - I_C(4.5)$$

ac load line:

$$v_{ce} \approx -i_C(R_E + R_C \parallel R_L)$$

$$v_{ce} = -i_C(0.5 + 4 \parallel 2) = -i_C(1.83)$$

$$\Delta v_{ce} = V_{CEQ} - 0.7 = \Delta i_C(10) = I_{CQ}(10)$$

$$\text{So } V_{CEQ} - 0.7 = I_{CQ}(10)$$

We have

$$V_{CEQ} = 20 - I_{CQ}(10 + R_E)$$

$$I_{CQ}(10) + 0.7 = 20 - I_{CQ}(10 + R_E) \quad (1)$$

$$I_{BQ} = \frac{10 - 0.7}{100 + (101)R_E} \Rightarrow I_{CQ} = \frac{(100)(9.3)}{100 + (101)R_E} \quad (2)$$

From (1)

$$I_{CQ}[10 + R_E] = 20 - 0.7$$

Substitute (2)

$$\left[\frac{(100)(9.3)}{100 + (101)R_E} \right] (20 + R_E) = 19.3$$

$$930(20 + R_E) = 19.3[100 + (101)R_E]$$

$$18,600 + 930R_E = 1930 + 1949.3R_E$$

$$16,670 = 1019.3R_E \Rightarrow R_E = 16.35 \text{ k}\Omega$$

So

$$I_{CQ} = \frac{(100)(9.3)}{100 + (101)(16.35)} \Rightarrow I_{CQ} = 0.531 \text{ mA}$$

$$V_{CEQ} = 20 - (0.531)(10 + 16.35) \Rightarrow V_{CEQ} = 6.0 \text{ V}$$

$$\Delta v_{CE} = V_{CEQ} - 0.7 = 6 - 0.7 = 5.3$$

Max symmetrical swing

$$2 \times (5.3) = 10.6 \text{ V peak-to-peak}$$

E4.15

$$a. \quad I_{BQ} = \frac{5 - 0.7}{10 + (126)(5)} = 0.00672$$

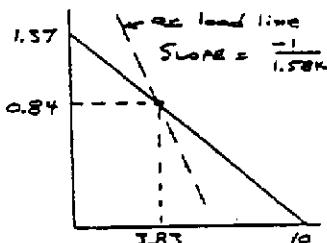
$$I_{CQ} = 0.84 \text{ mA}, \quad I_{EQ} = 0.847 \text{ mA}$$

$$r_0 = \frac{V_A}{I_{CQ}} = \frac{200}{0.84} = 238 \text{ k}\Omega$$

$$V_{CEQ} = 10 - (0.84)(2.3) - (0.847)(5)$$

$$= 10 - 1.932 - 4.235$$

$$V_{CEQ} = 3.83 \text{ V}$$



dc load line

$$V_{CE} \approx 10 - I_C(7.3)$$

ac load line

$$v_{ce} = -i_C(R_C \parallel R_L) = -i_C(2.3 \parallel 5) = -i_C(1.58)$$

(neglecting r_0)

$$b. \quad \Delta i_C = 0.84$$

$$\Rightarrow \Delta v_{CE} = (0.84)(1.58) = 1.33 \text{ V}$$

$$V_{CE}(\min) = 3.83 - 1.33 = 2.5 \text{ V}$$

$$V_{CE}(\max) = 3.83 + 1.33 = 5.16 \text{ V}$$

So max symmetrical swing

$$= 2 \times (1.33) = 2.66 \text{ V peak-to-peak}$$

E4.16

$$a. \quad V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (12), \quad V_{TH} = \frac{1}{R_1}(R_{TH})(12)$$

$$R_{TH} = (0.1)(1 + J)R_E = (0.1)(121)R_E$$

$$= 12.1R_E = 12.1 \text{ k}\Omega$$

$$I_{BQ} = \frac{12 - 0.7 - V_{TH}}{R_{TH} + (1 + J)R_E} = \frac{11.3 - \frac{1}{R_1}(12.1)(12)}{12.1 + (121)(1)}$$

$$I_{CQ} = 1.6 \Rightarrow I_{BQ} = \frac{1.6}{120} = 0.01333 \text{ mA}$$

$$0.01333 = \frac{11.3 - \frac{1}{R_1}(145.2)}{133.1}$$

$$\frac{1}{R_1}(145.2) = 11.3 - (0.01333)(133.1)$$

$$\Rightarrow R_1 = 15.24 \text{ k}\Omega$$

$$\frac{R_1 R_2}{R_1 + R_2} = 12.1 = \frac{15.24 R_2}{15.24 + R_2}$$

$$(12.1)(15.24) = (15.24 - 12.1)R_2$$

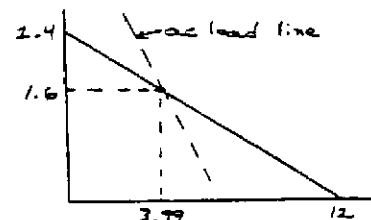
$$\Rightarrow R_2 = 58.7 \text{ k}\Omega$$

$$V_{ECC} = 12 - (16)(4) - (1.61)(11) \Rightarrow V_{ECC} = 3.99 \text{ V}$$

$$b. \quad v_o = g_m v_{re} (R_C \parallel R_L) = -g_m (R_C \parallel R_L) = -v_{ce}$$

$$i_C = g_m v_{re} = -g_m v_{re}$$

$$\text{or } -v_{ce} = i_C (R_C \parallel R_L)$$



$$\text{Want } \Delta i_C = 1.6 - 0.1 = 1.5$$

$$\Delta v_{ce} = 3.99 - 0.5 = 3.49$$

$$\frac{\Delta v_{ce}}{\Delta i_C} = \frac{3.49}{1.5} = 2.327 \text{ k}\Omega = R_C \parallel R_L$$

$$\frac{R_C R_L}{R_C + R_L} = \frac{4 R_L}{4 + R_L} = 2.327$$

$$(4 - 2.327)R_L = (4)(2.327) \Rightarrow R_L = 5.56 \text{ k}\Omega$$

E4.17

$$R_{TH} = R_1 \parallel R_2 = 25 \parallel 50 = 16.7 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{50}{50 + 25} \right) (5)$$

$$V_{TH} = 3.33 \text{ V}$$

$$I_{BQ} = \frac{V_{BB} - V_{BE(\text{on})}}{R_{TH} + (1 + J)R_E} = \frac{3.33 - 0.70}{16.7 + (121)(1)}$$

$$= \frac{2.63}{137.7}$$

$$\Rightarrow I_{BQ} = 0.0191$$

$$I_{CQ} = 2.29 \text{ mA}$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{2.29}{0.026} = 88.1 \text{ mA/V}$$

$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{(0.026)(120)}{2.29} = 1.36 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{2.29} = 43.7 \text{ k}\Omega$$

a.

$$V'_s = \left(\frac{R_1 \| R_2 \| R_{ib}}{R_1 \| R_2 \| R_{ib} + R_s} \right) \cdot V_s$$

$$R_{ib} = r_\pi + (1 + \beta)(R_E \| r_o) = (1.36) + (121)(43.7) \Rightarrow$$

$$R_{ib} = 120 \text{ k}\Omega \text{ and } R_1 \| R_2 = 16.7 \text{ k}\Omega$$

$$\text{Then } R_1 \| R_2 \| R_{ib} = 16.7 \| 120 = 14.7 \text{ k}\Omega$$

Then

$$V'_s = \left(\frac{14.7}{14.7 + 0.5} \right) \cdot V_s = (0.967)V_s$$

Now

$$V_o = \left(\frac{V_x + g_m V_x}{r_\pi} \right) (R_E \| r_o) = V_x \left(\frac{1 + \beta}{r_\pi} \right) R_E \| r_o$$

$$V_x = \frac{V'_s}{1 + \left(\frac{1 + \beta}{r_\pi} \right) R_E \| r_o} = \frac{(0.967)V_s}{1 + \left(\frac{1 + \beta}{r_\pi} \right) R_E \| r_o}$$

So

$$\frac{V_o}{V_s} = \frac{(0.967) \left(\frac{1 + \beta}{r_\pi} \right) R_E \| r_o}{1 + \left(\frac{1 + \beta}{r_\pi} \right) R_E \| r_o} = \frac{(0.967)(1 + \beta)R_E \| r_o}{r_\pi + (1 + \beta)R_E \| r_o}$$

$$R_E \| r_o = 43.7 = 0.978 \text{ k}\Omega$$

Then

$$A_v = \frac{(0.967)(121)(0.978)}{1.36 + (121)(0.978)} \Rightarrow A_v = 0.956$$

$$\text{b. } R_{ib} = r_\pi + (1 + \beta)R_E \| r_o = 1.36 + (121)(0.978)$$

$$\Rightarrow R_{ib} = 120 \text{ k}\Omega$$

(c)

$$R_o = R_E \| r_o \left| \frac{r_\pi + R_1 \| R_2 \| R_s}{1 + \beta} \right. = 43.7 \left| \frac{1.36 + 16.7}{121} \right| 0.5$$

which yields

$$R_o = 15.1 \text{ }\Omega$$

E4.18

$$V_{CEQ} = 5 \text{ V} \Rightarrow I_{EQ} = \frac{5}{2} = 2.5 \text{ mA}$$

$$I_{BQ} = \frac{2.5}{101} = 0.0248 \text{ mA}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE(\text{on})} + (1 + \beta)I_{BQ}R_E$$

$$R_{in} = R_{TH} \parallel [r_\pi + (1 + \beta)R_E]$$

$$r_\pi = 1.05 \text{ k}\Omega$$

$$65 = R_{TH} \parallel 203 = \frac{R_{TH} \cdot 203}{R_{TH} + 203}$$

$$\Rightarrow R_{TH} = 95.6 \text{ k}\Omega$$

$$\frac{1}{R_1} (95.6)(10) = (0.0248)(95.6) + 0.7 + 2.5(2) \\ = 8.07$$

$$R_1 = 118 \text{ k}\Omega, \quad \frac{118R_2}{118 + R_2} = 95.6$$

$$R_2 = 504 \text{ k}\Omega$$

$$R_m = 65 \text{ k}\Omega$$

$$V'_s = \left(\frac{R_m}{R_m + R_s} \right) \cdot V_s = \left(\frac{65}{65 + 0.5} \right) \cdot V_s = 0.992V_s$$

Then

$$A_v = \frac{(0.992)(1 + \beta)R_E}{r_\pi + (1 + \beta)R_E} = \frac{(0.992)(101)(2)}{1.05 + (101)(2)} \Rightarrow$$

$$A_v = 0.987$$

Neglecting R_s , $A_v = 0.995$

$$R_o = R_E \left| \frac{r_\pi + R_1 \| R_2 \| R_s}{1 + \beta} \right. = 2 \left| \frac{1.05 + 95.6}{101} \right| 0.5$$

or

$$R_o = 15.2 \text{ }\Omega$$

$$\text{Neglecting } R_s, R_o = 2 \left| \frac{1.05}{102} \right. \Rightarrow R_o = 10.3 \text{ }\Omega$$

E4.19

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5$$

$$\beta = 100, V_A = 125 \text{ V}, V_{BE(\text{on})} = 0.7 \text{ V}$$

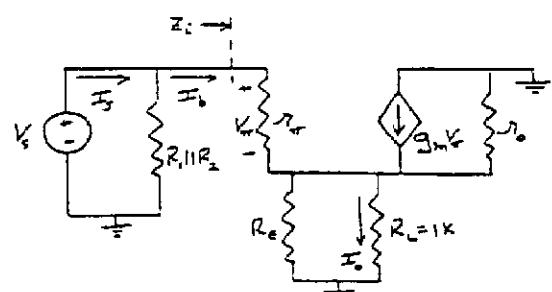
$$I_{CQ} = 0.75 \text{ mA}$$

$$\text{Then } r_o = \frac{125}{0.75} = 167 \text{ k}\Omega$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.75} \Rightarrow r_\pi = 3.47 \text{ k}\Omega$$

$$I_{BQ} = \frac{V_{TH} - V_{BE(\text{on})} - (-5)}{R_{TH} + (1 + \beta)R_E}$$

$$I_{CQ} = \beta I_{BQ}$$



$$Z_i = r_o + (1 + \beta)(R_E \parallel R_L \parallel r_o)$$

$$I_0 = \left(\frac{R_E \parallel r_o}{R_E \parallel r_o + R_L} \right) (1 + \beta) I_b$$

$$I_b = \left(\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + Z_i} \right) I_S$$

$$A_I = \frac{I_0}{I_S}$$

$$= \left(\frac{R_E \parallel r_o}{R_E \parallel r_o + R_L} \right) (1 + \beta) \left(\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + Z_i} \right)$$

Assume that $R_E \parallel r_o \approx R_E$

$$R_1 \parallel R_2 = (0.1)(1 + \beta)R_E$$

$$= (0.1)(101)R_E = 10.1R_E$$

Assume $R_L = 1\text{k}\Omega$

Then

$$A_I = 15 = \left(\frac{R_E}{R_E + 1} \right) (101) \times$$

$$\left(\frac{10.1R_E}{10.1R_E + 3.47 + (101)(R_E \parallel 1\text{k}\Omega)} \right)$$

where $R_E \parallel R_L \parallel r_o \approx R_E \parallel R_L = R_E \parallel 1\text{k}\Omega$

$$15 = \frac{(101)(10.1)R_E^2}{R_E + 1} \times$$

$$\frac{1}{\left[10.1R_E + 3.47 + \frac{101R_E}{1 + R_E} \right]}$$

$$15 = \frac{(101)(10.1)R_E^2}{R_E + 1} \times$$

$$\frac{1}{\left(1 + R_E \right) \left[10.1R_E + 3.47 + 101R_E \right]}$$

$$\frac{1}{(101)(10.1)R_E^2}$$

$$15 = \frac{10.1R_E + 3.47 + 10.1R_E^2 + 3.47R_E + 101R_E}{(101)(10.1)R_E^2}$$

$$15 = \frac{(101)(10.1)R_E^2}{10.1R_E^2 + 114.57R_E + 3.47}$$

$$(101)(10.1)R_E^2 = 15[10.1R_E^2 + 114.57R_E + 3.47]$$

$$1020.1R_E^2 = 151.5R_E^2 + 1718.55R_E + 52.05$$

$$868.6R_E^2 - 1718.55R_E - 52.05 = 0$$

$$16.7R_E^2 - 33.0R_E - 1 = 0$$

$$R_E = \frac{33 \pm \sqrt{(33)^2 + 4(16.7)}}{2(16.7)}$$

Must use + sign $\Rightarrow R_E = 2.0\text{k}\Omega$

Then $R_1 \parallel R_2 = 10.1R_E = 20.2\text{k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5$$

$$= \frac{1}{R_1} \left(\frac{R_1 R_2}{R_1 + R_2} \right) (10) - 5$$

$$= \frac{1}{R_1} (20.2)(10) - 5$$

$$I_{CQ} = 0.75$$

$$= (100) \left\{ \frac{\frac{1}{R_1} (20.2)(10) - 5 - 0.7 + 5}{20.2 + (101)(2)} \right\}$$

$$1.67 = \frac{1}{R_1} (20.2) - 0.7 \Rightarrow R_1 = 85.2\text{k}\Omega$$

$$\frac{R_1 R_2}{R_1 + R_2} = 20.2 = \frac{85.2 R_2}{85.2 + R_2} \Rightarrow R_2 = 26.5\text{k}\Omega$$

E4.20

$$\text{a. } \beta = 100, V_{BE(\text{on})} = 0.7, I_{CQ} = 1.25\text{mA}$$

$$I_{EQ} = 1.26\text{mA}, I_{BQ} = 0.0125\text{mA}$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5$$

$$V_{TH} = \frac{1}{R_1} (R_1 \parallel R_2) (10) - 5$$

$$I_{BQ} = \frac{V_{TH} - 0.7 - (-5)}{R_{TH} + (1 + \beta)R_E}$$

$$V_{CEQ} = 10 - I_{EQ}R_E = 4$$

$$I_{EQ}R_E = 6 \Rightarrow I_{EQ} = \frac{6}{R_E} \Rightarrow R_E = \frac{6}{1.26} = 4.76\text{k}\Omega$$

$$\Rightarrow R_E = 4.76\text{k}\Omega$$

$$R_{TH} = (0.1)(1 + \beta)R_E = 10.1R_E$$

Then

$$I_{BQ} = \frac{I_{EQ}}{101} = \frac{\frac{1}{R_1} (101)R_E (10) - 5 - 0.7 + 5}{10.1R_E + (101)R_E}$$

$$0.0125 = \frac{\frac{1}{R_1} (101)(4.76) - 0.7}{(111.1)(4.76)} \Rightarrow R_1 = 65.8\text{k}\Omega$$

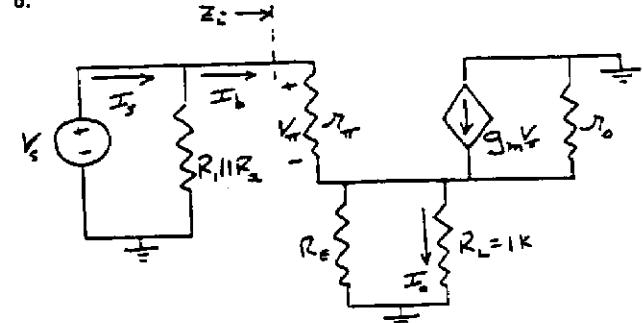
$$\frac{R_1 R_2}{R_1 + R_2} = (10.1)R_E = (10.1)(4.76) = 48.1\text{k}\Omega$$

$$(65.8)R_2 = (48.1)(65.8) + (48.1)R_2$$

$$(65.8 - 48.1)R_2 = (48.1)(65.8)$$

$$\Rightarrow R_2 = 178.8\text{k}\Omega$$

b.



$$r_0 = \frac{V_A}{I_{CQ}} = \frac{125}{1.25} = 100 \text{ k}\Omega$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1.25} = 2.08 \text{ k}\Omega$$

$$g_m V_\pi = g_m (i_b r_\pi) = \beta I_b$$

$$Z_s = r_\pi + (1 + \beta)[R_E \parallel R_L \parallel r_0]$$

$$= 2.08 + (101)[4.76 \parallel 1 \parallel 100]$$

$$= 2.08 + (101)[0.826 \parallel 100]$$

$$= 2.08 + (101)(0.819)$$

$$Z_s = 84.8 \text{ k}\Omega$$

Assume $R_L = 1 \text{ k}\Omega$

$$I_C = \left(\frac{R_E \parallel r_0}{R_E \parallel r_0 + R_L} \right) (1 + \beta) I_B$$

$$I_B = \left(\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + Z_s} \right) I_S$$

$$A_I = \frac{I_C}{I_S} = \left(\frac{R_E \parallel r_0}{R_E \parallel r_0 + R_L} \right) (1 + \beta) \left(\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + Z_s} \right)$$

$$R_E \parallel r_0 = 4.76 \parallel 100 = 4.34$$

$$A_I = \left(\frac{4.54}{4.54 + 1} \right) (101) \left(\frac{48.1}{48.1 + 84.8} \right)$$

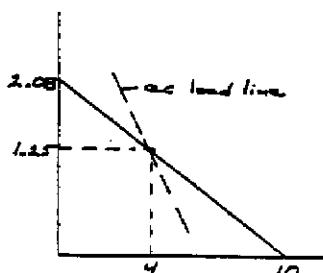
$$= \left(\frac{4.54}{5.54} \right) (101) \left(\frac{48.1}{132.9} \right)$$

$$\Rightarrow A_I = 30.0$$

$$c. \quad R_o = \frac{r_\pi}{1 + \beta} \parallel R_E \parallel r_0 = \frac{2.08}{101} \parallel 4.76 \parallel 100$$

$$\Rightarrow R_o = 20.4 \Omega$$

d.



$$v_{ce} = v_{ce} = (1 + \beta)i_b(R_E \parallel R_L \parallel r_0)$$

$$i_b = \frac{i_C}{\beta}$$

$$v_{ce} = i_C \left(\frac{1 + \beta}{\beta} \right) (R_E \parallel R_L \parallel r_0)$$

$$= i_C \left(\frac{101}{100} \right) (4.76 \parallel 1 \parallel 100)$$

$$= i_C (0.828)$$

$$\text{If } \Delta i_C = 1.25 \text{ mA, } \rightarrow \Delta v_{ce} = 1.035 \text{ V}$$

Maximum symmetrical swing in output voltage

$$is = 2\Delta v_{ce} = 2.07 \text{ V peak-to-peak}$$

E4.21

For $\beta = 130$

$$I_{BQ} = \frac{10 - 0.7}{100 + (131)(10)} \Rightarrow 6.596 \mu\text{A}$$

$$I_{CQ} = 0.857 \text{ mA}$$

From Figure 4.21

$$3 < h_{re} < 5 \text{ k}\Omega \quad \text{Let } h_{re} = 0$$

$$98 < h_{fe} < 170$$

$$8 < h_{oe} < 16 \mu\text{S}$$

$$h_{re} = 4 \text{ k}\Omega$$

$$h_{fe} = 134$$

$$h_{oe} = 12 \mu\text{S} \Rightarrow \frac{1}{h_{oe}} = 83.3 \text{ k}\Omega$$

$$a. \quad R_s = R_L = 10 \text{ k}\Omega$$

$$R_{ib} = h_{re} + (1 + h_{fe}) \left(R_E \parallel R_L \parallel \frac{1}{h_{oe}} \right)$$

$$= 4 + (135)(10 \parallel 10 \parallel 83.3)$$

$$\Rightarrow R_{ib} = 641 \text{ k}\Omega$$

$$A_v = \left(\frac{R_E \parallel R_{ib}}{R_s + R_E \parallel R_{ib}} \right) \times$$

$$\left(\frac{(1 + h_{fe})(R_E \parallel R_L \parallel \frac{1}{h_{oe}})}{h_{re} + (1 + h_{fe})(R_E \parallel R_L \parallel \frac{1}{h_{oe}})} \right)$$

$$A_v = \left(\frac{100 \parallel 641}{10 + 100 \parallel 641} \right) \left[\frac{(135)(10 \parallel 10 \parallel 83.3)}{4 + (135)(10 \parallel 10 \parallel 83.3)} \right]$$

$$= \left(\frac{86.5}{10 + 86.5} \right) \left[\frac{637}{641} \right]$$

$$\Rightarrow A_v = 0.891$$

$$A_i = \left(\frac{R_E \parallel \frac{1}{h_{oe}}}{R_E \parallel \frac{1}{h_{oe}} + R_L} \right) (1 + h_{fe}) \left(\frac{R_B}{R_B + R_{ib}} \right)$$

$$= \left(\frac{10 \parallel 83.3}{10 \parallel 83.3 + 10} \right) (135) \left(\frac{100}{100 + 641} \right)$$

$$\Rightarrow A_i = 8.59$$

$$R_o = R_E \parallel \frac{1}{h_{oe}} \parallel \frac{h_{re} + R_{ib} \parallel R_B}{1 + h_{fe}}$$

$$= 10 \parallel 83.3 \parallel \frac{4 + 10 \parallel 100}{135} = 8.93 \parallel 0.0970$$

$$\Rightarrow R_o = 96.0 \Omega$$

$$b. \quad R_s = 1 \text{ k}\Omega, \quad R_{ib} = 641 \text{ k}\Omega, \quad A_i = 8.59$$

$$A_v = \left(\frac{86.5}{1 + 86.5} \right) \left(\frac{637}{641} \right) \Rightarrow A_v = 0.982$$

$$R_o = 10 \parallel 8.33 \parallel \left[\frac{4 + 1 \parallel 100}{135} \right]$$

$$= 8.93 \parallel 0.03696 \Rightarrow R_o = 36.8 \Omega$$

E4.22

$$V_{TH} = 2.5 \text{ V}, R_{TH} = 25 \text{ k}\Omega$$

$$I_{BQ} = \frac{5 - 0.7 - 2.5}{25 + (101)(2)} = \frac{1.8}{227} = 0.00793 \text{ mA}$$

$$I_{CQ} = 0.793 \text{ mA}$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.793}{0.026} = 30.5 \text{ mA/V}$$

$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{(0.0026)(100)}{0.793} = 3.28 \text{ k}\Omega$$

$$r_0 = \frac{V_A}{I_{CQ}} = \frac{125}{0.793} = 158 \text{ k}\Omega$$

$$\text{a. } R_E \parallel R_L \parallel r_0 = 2 \parallel 0.5 \parallel 158 = 0.4 \parallel 158 \approx 0.4$$

$$A_v = \frac{(1 + \beta)(R_E \parallel R_L \parallel r_0)}{r_\pi + (1 + \beta)(R_E \parallel R_L \parallel r_0)}$$

$$= \frac{(101)(0.4)}{3.28 + (101)(0.4)} \Rightarrow A_v = 0.925$$

$$\text{b. } R_{tb} = r_\pi + (1 + \beta)(R_E \parallel R_L \parallel r_0)$$

$$R_{tb} = 3.28 + (101)(0.4)$$

$$\Rightarrow R_{tb} = 43.7 \text{ k}\Omega$$

$$R_0 = \frac{r_\pi}{1 + \beta} \parallel R_E \parallel r_0 = \frac{3.28}{101} \parallel 2 \parallel 98.3$$

$$\Rightarrow R_0 = 32.0 \text{ }\Omega$$

$$\text{c. } I_B(\text{max}) \rightarrow$$

$$R_E(\text{min}) = 1.9 \text{ k}\Omega$$

$$R_2(\text{min}) = 47.5 \text{ k}\Omega$$

$$R_1(\text{max}) = 52.5 \text{ k}\Omega$$

$$R_{TH} = 24.9 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{47.5}{100} \right) (5) = 2.375$$

$$I_{BQ} = \frac{5 - 0.7 - 2.375}{24.9 + (101)(1.9)} = \frac{1.925}{216.8}$$

$$I_{CQ} = 0.888 \text{ mA}$$

$$R_E(\text{max}) = 2.1 \text{ k}\Omega$$

$$R_2(\text{max}) = 52.5 \text{ k}\Omega$$

$$R_1(\text{min}) = 47.5 \text{ k}\Omega$$

$$R_{TH} = 24.9 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{52.5}{100} \right) (5) = 2.625$$

$$I_{CQ} = (100) \left[\frac{5 - 0.7 - 2.625}{24.9 + (101)(2.1)} \right] = \frac{(100)(1.675)}{237}$$

$$I_{CQ} = 0.707 \text{ mA}$$

$$r_\pi(\text{max}) = \frac{(100)(0.026)}{0.707} = 3.68 \text{ k}\Omega$$

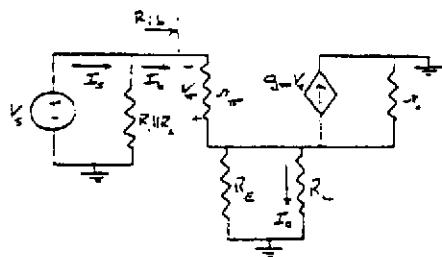
$$R_0 = 1.96 \parallel \frac{3.68}{101} = 1.96 \parallel 0.0364 = 35.7 \text{ }\Omega$$

$$r_\pi(\text{min}) = 2.93 \text{ k}\Omega$$

$$R_0 = 1.96 \parallel \frac{2.93}{101} = 1.96 \parallel 0.0290 = 28.6 \text{ }\Omega$$

$$\Rightarrow 28.6 \leq R_0 \leq 35.7 \text{ }\Omega$$

E4.23

For $V_{ECQ} = 2.5 \text{ V}$,

$$I_{BQ} = \frac{5 - 2.5}{R_E} = \frac{5 - 2.5}{0.5} = 5 \text{ mA}$$

$$I_{CQ} = \left(\frac{75}{76} \right) (5) = 4.93 \text{ mA} \Rightarrow I_{BQ} = 0.0658 \text{ mA}$$

$$V_\pi = -I_b r_\pi$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(75)(0.026)}{4.93} = 0.396 \text{ k}\Omega$$

$$r_0 = \frac{V_A}{I_{CQ}} = \frac{75}{4.93} = 15.2 \text{ k}\Omega$$

$$g_m V_\pi = g_m (-I_b r_\pi) = -\beta I_b$$

$$I_0 = \left(\frac{R_E \parallel r_0}{R_E \parallel r_0 + R_L} \right) \times (1 + \beta) I_b$$

$$I_b = \left(\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + Z_{tb}} \right) I_S$$

$$A_I = \frac{I_0}{I_S} = \left(\frac{R_E \parallel r_0}{R_E \parallel r_0 + R_L} \right) (1 + \beta) \left(\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + Z_{tb}} \right)$$

$$\text{a. } R_E = R_L = 0.5 \text{ k}\Omega$$

$$R_{tb} = r_\pi + (1 + \beta)(R_E \parallel R_L \parallel r_0)$$

$$= 0.396 + (76)(0.5 \parallel 0.5 \parallel 15.2)$$

$$= 0.396 + (76)(0.246)$$

$$\Rightarrow R_{tb} = 19.1 \text{ k}\Omega$$

$$R_E \parallel r_0 = 0.5 \parallel 15.2 = 0.484 \text{ k}\Omega$$

$$A_I = 10 = \left(\frac{0.484}{0.484 + 0.5} \right) (76) \left(\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + 19.1} \right)$$

$$10 = 37.38 \left(\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + 19.1} \right)$$

$$0.2675(R_1 \parallel R_2 + 19.1) = R_1 \parallel R_2$$

$$R_1 \parallel R_2 = 6.973 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \frac{1}{R_1} (R_1 \parallel R_2) V_{CC}$$

$$= \frac{1}{R_1} (6.973)(5)$$

$$I_{BQ} = \frac{5 - V_{EB}(\text{on}) - V_{TH}}{R_{TH} + (1 + \beta)R_E}$$

$$0.0658 = \frac{5 - 0.7 - V_{TH}}{6.975 + (76)(0.5)}$$

$$2.96 = 4.3 - V_{TH} \Rightarrow V_{TH} = 1.34 = \frac{1}{R_1}(6.975)(5)$$

$$\Rightarrow R_1 = 26.0 \text{ k}\Omega$$

$$\frac{R_1 R_2}{R_1 + R_2} = 6.975 = \frac{26 R_2}{26 + R_2}$$

$$6.975(26 + R_2) = 26 R_2$$

$$\Rightarrow R_2 = 9.53 \text{ k}\Omega$$

$$\text{b. For } R_E = 4R_L = 4(0.5) \Rightarrow R_E = 2 \text{ k}\Omega$$

$$I_{EQ} = \frac{5 - 2.5}{2} = 1.25 \text{ mA} \Rightarrow I_{CQ} = 1.23 \text{ mA}$$

$$\rightarrow I_{BQ} = 0.0164 \text{ mA}$$

$$r_\pi = \frac{(75)(0.026)}{1.23} = 1.59 \text{ k}\Omega$$

$$r_0 = \frac{75}{1.23} = 60.9 \text{ k}\Omega$$

$$Z_{\text{b}} = r_\pi + (1 + \beta)[R_E \parallel R_L \parallel r_0]$$

$$= 1.59 + (76)[2 \parallel 0.5 \parallel 60.9]$$

$$\Rightarrow Z_{\text{b}} = 31.8 \text{ k}\Omega$$

$$R_E \parallel r_0 = 2 \parallel 60.9 = 1.94 \text{ k}\Omega$$

$$A_I = \left(\frac{1.94}{1.94 + 0.5} \right)(76) \left(\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + 31.8} \right) = 10$$

$$10 = 60.4 \left(\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + 31.8} \right)$$

$$0.166(R_1 \parallel R_2 + 31.8) = R_1 \parallel R_2$$

$$R_1 \parallel R_2 = 6.33 \text{ k}\Omega$$

$$\text{Then } I_{BQ} = 0.0164 = \frac{4.3 - V_{TH}}{6.33 + (76)(2)}$$

$$V_{TH} = 1.70 = \frac{1}{R_1}(R_1 \parallel R_2)V_{CC} = \frac{1}{R_1}(6.33)(5)$$

$$\Rightarrow R_1 = 18.6 \text{ k}\Omega$$

$$\frac{R_1 R_2}{R_1 + R_2} = 6.33 = \frac{(18.6)R_2}{18.6 + R_2}$$

$$6.33(18.6 + R_2) = (18.6)R_2$$

$$\Rightarrow R_2 = 9.6 \text{ k}\Omega$$

E4.24

$$\text{a. } I_{EQ} = \frac{10 - 0.7}{10} = 0.93 \text{ mA}$$

$$I_{CQ} = \left(\frac{\beta}{1 + \beta} \right) I_{EQ} = \left(\frac{100}{101} \right)(0.93)$$

$$\Rightarrow I_{CQ} = 0.921 \text{ mA}$$

$$V_{ECQ} = 10 + 10 - I_{CQ}R_C - I_{EQ}R_E$$

$$V_{ECQ} = 20 - (0.921)(5) - (0.93)(10)$$

$$\Rightarrow V_{ECQ} = 6.1 \text{ V}$$

$$\text{b. } r_\pi = \frac{3V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.921} = 2.82 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.921}{0.026} = 35.42 \text{ mA/V}$$

$$i_0 = g_m V_\pi \text{ and } V_\pi = v_s$$

$$i_i = \frac{v_s}{R_E \parallel r_\pi} + g_m V_\pi = v_s \left(\frac{1}{R_E \parallel r_\pi} + g_m \right)$$

$$A_I = \frac{i_0}{i_i} = \frac{g_m v_s}{v_s \left(\frac{1}{R_E \parallel r_\pi} + g_m \right)} = \frac{g_m (R_E \parallel r_\pi)}{1 + g_m (R_E \parallel r_\pi)}$$

$$= \frac{(35.42)(10 \text{ k}\Omega \parallel 2.82 \text{ k}\Omega)}{1 + (35.42)(10 \text{ k}\Omega \parallel 2.82 \text{ k}\Omega)}$$

$$\Rightarrow A_I = 0.987$$

$$A_V = \frac{v_o}{v_s} \text{ and } v_o = g_m V_\pi R_C = g_m v_s R_C$$

$$A_V = g_m R_C = (35.42)(5) \Rightarrow A_V = 177.1$$

$$\text{c. } V_{ECQ} = 6.1 \text{ V} \Rightarrow V_{ECQ} = V_E - V_C$$

$$V_C = V_E - V_{ECQ} = 0.7 - 6.1 = -5.4 \text{ V}$$

$$v_C = V_C + i_0 R_C$$

$$\text{For } v_{EC} = 0.5 \Rightarrow v_C = +0.2$$

$$+ 0.2 = -5.4 + i_0 R_C$$

$$i_0 = \frac{0.2 + 5.4}{5} = 1.12 \text{ mA}$$

\Rightarrow Current limited

$$i_0(\text{max}) = 0.921$$

$$\Rightarrow v_o(\text{peak}) = (0.921)(5) = 4.61$$

$\Rightarrow 4.61 \text{ V peak-to-peak}$

E4.25

$$\text{a. } I_{BQ} = \frac{V_{EE} - V_{BE}(\text{on})}{R_B + (1 + \beta)R_E} = \frac{10 - 0.7}{100 + (101)(10)}$$

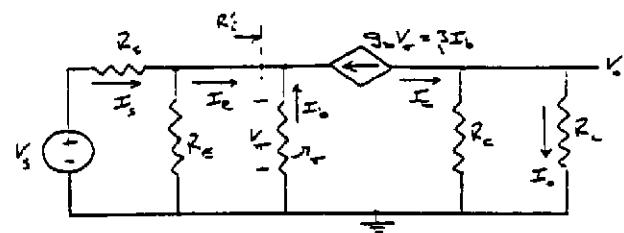
$$\Rightarrow I_{BQ} = 8.38 \mu\text{A}, I_{CQ} = 0.838 \text{ mA}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.838} \Rightarrow r_\pi = 3.10 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.838}{0.026} \Rightarrow g_m = 32.23 \text{ mA/V}$$

$$r_0 = \frac{V_A}{I_{CQ}} = \frac{\infty}{0.838} \Rightarrow r_0 = \infty$$

b.



$$g_m V_\pi + \frac{V_\pi}{r_\pi} = \left(\frac{-V_\pi}{R_E} \right) + \frac{(-V_\pi - V_S)}{R_S}$$

$$V_\pi \left[\left(\frac{1+\beta}{r_\pi} \right) + \frac{1}{R_E} + \frac{1}{R_S} \right] = -\frac{V_S}{R_S}$$

$$V_\pi = -\frac{V_S}{R_S} \left[\left(\frac{1+\beta}{r_\pi} \right) \parallel R_E \parallel R_S \right]$$

$$V_0 = -g_m V_\pi (R_C \parallel R_L)$$

$$A_v = \frac{V_0}{V_S} = g_m \frac{(R_C \parallel R_L)}{R_S} \left[\left(\frac{r_\pi}{1+\beta} \right) \parallel R_E \parallel R_S \right]$$

$$= \frac{(32.23)[10 \parallel 1]}{(1)} \left\{ \frac{3.10}{101} \parallel 10 \parallel 1 \right\}$$

$$= (32.23)(0.909)\{0.0297\} \Rightarrow A_v = 0.870$$

$$R'_e = \frac{r_\pi}{1+\beta} = \frac{3.10}{101} = 0.0307 \text{ k}\Omega$$

$$I_e = \left(\frac{R_E}{R_E + R'_e} \right) I_S \approx I_S$$

$$I_C = \left(\frac{\beta}{1+\beta} \right) I_e, \quad I_O = \left(\frac{R_C}{R_C + R_L} \right) I_C$$

$$I_e = \left(\frac{R_C}{R_C + R_L} \right) \left(\frac{\beta}{1+\beta} \right) I_S$$

$$A_I = \frac{I_O}{I_S} = \left(\frac{10}{10+1} \right) \left(\frac{100}{101} \right) \Rightarrow A_I = 0.900$$

c. $R_i = R_E \parallel R'_e = 10 \parallel 0.0307$

$$\Rightarrow R_i \approx 30.7 \Omega$$

$$R_o = R_C = 10 \text{ k}\Omega$$

E4.26

$$5 = I_B R_B + V_{BE(on)} + I_E R_E$$

$$I_B = \frac{5 - 0.7}{R_B + (101)R_E} = \frac{4.3}{R_B + (101)R_E}$$

$$I_C = \frac{(100)(4.3)}{R_B + (101)R_E}$$

$$5 = I_C R_C + V_{CE} + I_E R_E - 5$$

$$V_{CE} = 10 - I_C \left(R_C + \left(\frac{101}{100} \right) R_E \right)$$

ac analysis

$$V_0 = -g_m V_\pi (R_C \parallel R_L)$$

$$V_S = -V_\pi - \frac{V_\pi}{r_\pi} \cdot R_B = -V_\pi \left(1 + \frac{R_B}{r_\pi} \right)$$

$$\text{or } V_\pi = -\left(\frac{r_\pi}{r_\pi + R_B} \right) V_S$$

$$\frac{V_0}{V_S} = -g_m \left[-\left(\frac{r_\pi}{r_\pi + R_B} \right) \right] (R_C \parallel R_L)$$

$$A_v = \frac{V_0}{V_S} = \frac{\beta}{r_\pi + R_B} (R_C \parallel R_L)$$

For $I_C = 1 \text{ mA}$,

$$r_\pi = \frac{\beta V_T}{I_C} = \frac{(100)(0.026)}{1} = 2.6 \text{ k}\Omega$$

$$A_v = 20 = \frac{(100)(1)}{2.6 + R_B}$$

$$R_B = \frac{(100)(1)}{20} - 2.6 \Rightarrow R_B = 2.4 \text{ k}\Omega$$

$$I_C = 1 = \frac{(100)(4.3)}{2.4 + (101)R_E}$$

$$(100)(4.3) = 2.4$$

$$R_E = \frac{1}{(101)} \Rightarrow R_E = 4.23 \text{ k}\Omega$$

E4.27

a. $R_{TH} = 70 \parallel 6 = 5.526 \text{ k}\Omega$

$$V_{TH} = \left(\frac{6}{6+70} \right) (10) - 5 = -4.21 \text{ V}$$

$$I_{B1} = \frac{5 - 4.21 - 0.7}{5.526 + (126)(0.2)} = \frac{0.090}{30.726}$$

$$\Rightarrow I_{B1} = 2.93 \mu\text{A}, \quad I_{CQ1} = 0.366 \text{ mA}$$

$$\frac{5 - V_{C1}}{5} = I_{C1} + \frac{(V_{C1} - 0.7) - (-5)}{(1+\beta)1.5}$$

$$\frac{6 - V_{C1}}{5} = 0.366 + \frac{V_{C1}}{(126)(1.5)} + \frac{4.3}{(126)(1.5)}$$

$$1 - 0.366 - \frac{4.3}{(126)(1.5)} = \frac{V_{C1}}{5} + \frac{V_{S1}}{(126)(1.5)}$$

$$0.6112 = V_{C1}(0.2053) \Rightarrow V_{C1} = 2.977 \text{ V}$$

$$I_{E1} = 0.369 \text{ mA}$$

$$V_{E1} = (0.369)(0.2) - 5 \Rightarrow V_{E1} = -4.926 \text{ V}$$

$$\Rightarrow V_{CE1} = V_{C1} - V_{E1} = 2.977 - (-4.926)$$

$$\Rightarrow V_{CE1} = 7.90 \text{ V}$$

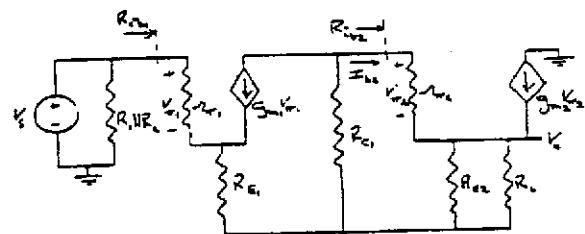
$$I_{EQ2} = \frac{(V_{C1} - 0.7) - (-5)}{1.5} = \frac{5 + 2.98 - 0.7}{1.5}$$

$$= 4.85 \text{ mA}$$

$$I_{CQ2} = \left(\frac{\beta}{1+\beta} \right) I_{EQ2} \Rightarrow I_{CQ2} = 4.81 \text{ mA}$$

$$V_{E2} = V_{C1} - 0.7 = 2.98 - 0.7 = 2.28$$

$$V_{CEQ2} = 5 - V_{E2} = 5 - 2.28 \Rightarrow V_{CEQ2} = 2.72 \text{ V}$$



$$r_{\pi 1} = \frac{\beta V_T}{I_{CQ1}} = \frac{(125)(0.026)}{0.366} = 8.86 \text{ k}\Omega$$

$$r_{\pi 2} = \frac{\beta V_T}{I_{CQ2}} = \frac{(125)(0.026)}{4.81} = 0.676 \text{ k}\Omega$$

$$g_{m1} = \frac{I_{CQ1}}{V_T} = \frac{0.366}{0.026} = 14.1 \text{ mA/V}$$

$$g_{m2} = \frac{I_{CQ2}}{V_T} = \frac{4.81}{0.026} = 185 \text{ mA/V}$$

$$R_{b1} = r_{\pi 1} + (1 + \beta)R_{S1} = 8.88 + (126)(0.2) \\ = 34.08 \text{ k}\Omega$$

$$R_{b2} = r_{\pi 2} + (1 + \beta)(R_{E2} \parallel R_L) \\ = 0.676 + (126)(1.5 \parallel 10) = 165 \text{ k}\Omega$$

$$V_0 = (1 + \beta)I_{b2}(R_{E2} \parallel R_L)$$

$$I_{b2} = \left(\frac{R_{C1}}{R_{C1} + R_{b2}} \right) (-g_{m1} V_{\pi 1})$$

$$V_{\pi 1} = \frac{V_S}{Z_1} \cdot r_{\pi 1}$$

$$A_v = (1 + \beta)(R_{E2} \parallel R_L) \left(\frac{R_{C1}}{R_{C1} + R_{b2}} \right) \left(\frac{-g_{m1} r_{\pi 1}}{R_{b1}} \right)$$

$$A_v = \frac{V_0}{V_S}$$

$$= -(126)(125)(1.5 \parallel 10) \left(\frac{5}{5 + 165} \right) \left(\frac{1}{34.08} \right) \\ = -(126)(125)(1.30) \left(\frac{5}{170} \right) \left(\frac{1}{34.08} \right)$$

$$A_v = -17.7$$

$$c. R_i = R_1 \parallel R_2 \parallel R_{b1} = (5.53) \parallel (34.1)$$

$$\Rightarrow R_i = 4.76 \text{ k}\Omega$$

$$R_o = \left(\frac{r_{\pi 2} + R_{C1}}{1 + \beta} \right) \parallel R_{E2} = \left(\frac{0.676 + 5}{126} \right) \parallel 1.5 \\ = 0.0450 \parallel 1.5 \\ \Rightarrow R_o = 43.7 \Omega$$

E4.28

$$a. I_{CQ2} = \left(\frac{100}{101} \right) (1 \text{ mA}) \Rightarrow I_{CQ2} = 0.990 \text{ mA}$$

$$I_{EQ1} = \frac{I_{EQ2}}{1 + \beta} = \frac{1}{101} \Rightarrow I_{EQ1} = 0.0099 \text{ mA}$$

$$I_{BQ1} = \frac{I_{EQ1}}{1 + \beta} = \frac{0.0099}{101}$$

$$\Rightarrow I_{BQ1} = 0.000098 \text{ mA}, I_{CQ1} = 0.0098 \text{ mA}$$

$$V_{B1} = -I_{BQ1} R_B = -(0.000098)(10)$$

$$= -0.00098 \text{ V} \approx 0$$

$$V_{E1} = -0.7 \text{ V}, V_{E2} = -1.4 \text{ V}$$

$$I_1 = I_{CQ2} + I_{CQ1} = 0.990 + 0.0098$$

$$I_1 \approx 1 \text{ mA} \Rightarrow V_0 = 5 - (1)(4) = 1 \text{ V}$$

$$V_{CEQ2} = 1 - (-1.4) = 2.4$$

$$\Rightarrow V_{CEQ2} = 2.4 \text{ V}, I_{CQ2} = 0.990 \text{ mA}$$

$$V_{CEQ1} = 1 - (-0.7) = 1.7$$

$$\Rightarrow V_{CEQ1} = 1.7 \text{ V}, I_{CQ1} = 0.0098 \text{ mA}$$

$$b. r_{\pi} = \frac{\beta V_T}{I_{CQ}} \text{ and } g_m = \frac{I_{CQ}}{V_T}$$

For Q_1 :

$$r_{\pi 1} = \frac{(100)(0.026)}{0.0098} \Rightarrow r_{\pi 1} = 265 \text{ k}\Omega$$

$$g_{m1} = \frac{0.0098}{0.026} \Rightarrow g_{m1} = 0.377 \text{ mA/V}$$

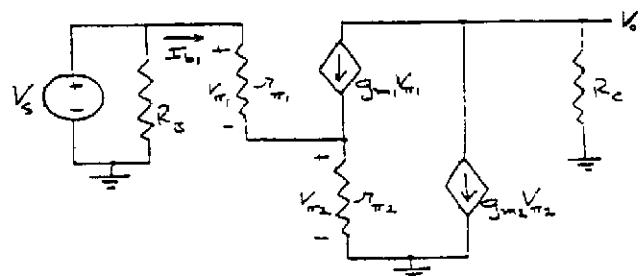
For Q_2 :

$$r_{\pi 2} = \frac{(100)(0.026)}{0.990} \Rightarrow r_{\pi 2} = 2.63 \text{ k}\Omega$$

$$g_{m2} = \frac{0.99}{0.026} \Rightarrow g_{m2} = 38.1 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \infty$$

c.



$$V_0 = -(g_{m1} V_{\pi 1} + g_{m2} V_{\pi 2}) R_C$$

$$V_S = V_{\pi 1} + V_{\pi 2}$$

$$V_{\pi 2} = \left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} \right) r_{\pi 2} = \frac{(1 + \beta)}{r_{\pi 1}} V_{\pi 1} r_{\pi 2}$$

Then

$$V_0 = - \left[g_{m1} V_{\pi 1} + g_{m2} \left(\frac{(1 + \beta) r_{\pi 2}}{r_{\pi 1}} \right) V_{\pi 1} \right] \cdot R_C$$

Also

$$V_S = V_{\pi 1} + \left(\frac{1 + \beta}{r_{\pi 1}} \right) V_{\pi 1} r_{\pi 2}$$

$$= V_{\pi 1} \left[1 + (1 + \beta) \left(\frac{r_{\pi 2}}{r_{\pi 1}} \right) \right]$$

$$V_{\pi 1} = \frac{V_S}{1 + (1 + \beta) \left(\frac{r_{\pi 2}}{r_{\pi 1}} \right)}$$

$$V_0 = - \left[g_{m1} + g_{m2} (1 + \beta) \left(\frac{r_{\pi 2}}{r_{\pi 1}} \right) \right] R_C$$

$$+ \frac{V_S}{1 + (1 + \beta) \left(\frac{r_{\pi 2}}{r_{\pi 1}} \right)}$$

$$A_v = - \frac{\left[g_{m1} + g_{m2} (1 + \beta) \left(\frac{r_{\pi 2}}{r_{\pi 1}} \right) \right] R_C}{1 + (1 + \beta) \left(\frac{r_{\pi 2}}{r_{\pi 1}} \right)}$$

$$A_v = - \frac{\left[0.377 + (38.08)(101) \left(\frac{2.626}{265.3} \right) \right] (4)}{1 + (101) \left(\frac{2.626}{265.3} \right)}$$

$$A_v = - \frac{153.8}{1.9997} \Rightarrow A_v = -76.9$$

d. $R_i = r_{\pi 1} + (1 + \beta)r_{\pi 2}$

$$R_i = 265.3 + (101)(2.626)$$

$$\Rightarrow R_i = 531 \text{ k}\Omega$$

E4.29

$$a. I_{E1} = \frac{V_{E1} - (-10)}{20} + \frac{(V_{E1} - 0.7) - (-10)}{(1 + \beta)(10)}$$

$$- I_{B1} R_B - V_{BE(\text{on})} = V_{E1}$$

So

$$(1 + \beta)I_{B1} = \frac{10 - I_{B1}R_B - 0.7}{20} + \frac{10 - 0.7 - I_{B1}R_B - 0.7}{(101)(10)}$$

$$(101)I_{B1} + I_{B1} \left(\frac{20}{20} \right) + I_{B1} \cdot \frac{20}{(101)(10)}$$

$$= \frac{9.3}{20} + \frac{8.6}{(101)(10)}$$

$$(102)I_{B1} = 0.465 + 0.00851$$

$$I_{B1} = 0.00464 \text{ mA} \Rightarrow V_{B1} = -0.09281$$

$$\Rightarrow V_{E1} = -0.793 \text{ V} \Rightarrow V_{E2} = -1.493 \text{ V}$$

$$I_{C1} = 0.464 \text{ mA}, I_{E1} = 0.469 \text{ mA}$$

$$I_1 = \frac{10 - 0.793}{20} = 0.46035 \text{ mA}$$

$$\Rightarrow I_{B2} = I_{E1} - I_1$$

$$I_{B2} = 0.00855 \text{ mA} \Rightarrow I_{C2} = 0.865 \text{ mA}$$

$$\text{or } I_{E2} = \frac{10 - 1.493}{10} = 0.851 \Rightarrow I_{C2} = 0.842 \text{ mA}$$

$$I_C = I_{C1} + I_{C2} = 0.464 + 0.842$$

$$= 1.306 \text{ mA}$$

$$V_0 = 10 - (1.306)(2) = 7.39 \text{ V}$$

$$V_{CEQ2} = 7.39 - (-1.493)$$

$$\Rightarrow V_{CEQ2} = 8.88 \text{ V}, I_{CQ2} = 0.842 \text{ mA}$$

$$V_{CEQ1} = 7.39 - (-0.793)$$

$$\Rightarrow V_{CEQ1} = 8.18 \text{ V}, I_{CQ1} = 0.464 \text{ mA}$$

b. $r_w = \frac{3V_T}{I_{CQ}}, g_m = \frac{I_{CQ}}{V_T}$

For Q₁:

$$r_{\pi 1} = \frac{(100)(0.026)}{0.464} \Rightarrow r_{\pi 1} = 5.60 \text{ k}\Omega$$

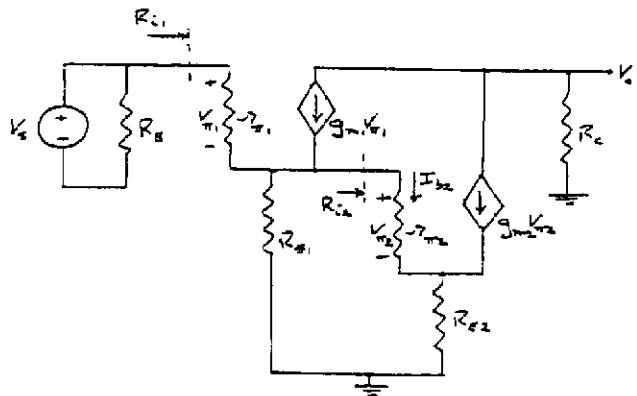
$$g_{m1} = \frac{0.464}{0.026} \Rightarrow g_{m1} = 17.8 \text{ mA/V}$$

$$r_{\pi 2} = \frac{(100)(0.026)}{0.842} \Rightarrow r_{\pi 2} = 3.09 \text{ k}\Omega$$

$$g_{m2} = \frac{0.842}{0.026} \Rightarrow g_{m2} = 32.4 \text{ mA/V}$$

$$r_{01} = r_{02} = \infty$$

c.



$$R_{i2} = r_{\pi 2} + (1 + \beta)R_{E2} = 3.09 + (101)(10)$$

$$= 1013.1 \text{ k}\Omega$$

$$R_{i1} = r_{\pi 1} + (1 + \beta)[R_{E1} \parallel R_{i2}]$$

$$= 5.60 + (101)[20 \parallel 1013.1]$$

d. $R_{i1} = 1.986 \text{ M}\Omega$

Note that $g_{m1}V_{\pi 1} = \beta I_{b1}$ and $g_{m2}V_{\pi 2} = \beta I_{b2}$

$$\text{So } V_0 = -[\beta I_{b1} + \beta I_{b2}]R_C$$

$$I_{b1} = \frac{V_0}{R_{i1}} \text{ and } I_{b2} = \left(\frac{R_{E1}}{R_{E1} + R_{i2}} \right) (1 + \beta) I_{b1}$$

So

$$V_0 = -[I_{b1} + I_{b2}] \beta R_C$$

$$= - \left\{ I_{b1} + \left(\frac{R_{E1}}{R_{E1} + R_{i2}} \right) (1 + \beta) I_{b1} \right\} \beta R_C$$

$$A_v = \frac{V_0}{V_S} = - \left[1 + \left(\frac{R_{E1}}{R_{E1} + R_{i2}} \right) (1 + \beta) \right] \frac{\beta R_C}{R_{i1}}$$

$$= - \left[1 + \left(\frac{20}{20 + 1013.1} \right) (101) \right] \frac{(100)(2)}{1986}$$

$$A_v = -0.298$$

E4.30

a. dc analysis

$$\beta = 100, V_{BE(\text{on})} = 0.7 \text{ V}, V_A = \infty$$

$$\text{Want: } I_{CQ2} = 0.5 \text{ mA}, V_{CE1} = V_{CE2} = 4 \text{ V}$$

$$R_1 + R_2 + R_3 = 100 \text{ k}\Omega$$

Neglecting base currents:

$$I_1 = \frac{12}{100} = 0.12 \text{ mA}$$

$$V_{E1} = I_{CQ1} R_E = (0.5)(0.5) = 0.25 \text{ V}$$

$$V_{C1} = V_{CEQ1} + V_{E1} = 4 + 0.25 = 4.25 \text{ V}$$

So

$$V_{C2} = V_{C1} + V_{CEQ2} = 4.25 + 4 = 8.25 \text{ V}$$

$$R_C = \frac{V_{CC} - V_{C2}}{I_{CQ}} = \frac{12 - 8.25}{0.5} \Rightarrow R_C = 7.5 \text{ k}\Omega$$

$$V_{B1} = V_{E1} + 0.7 = 0.25 + 0.7 = 0.95 \text{ V}$$

$$V_{B1} = \left(\frac{R_3}{R_1 + R_2 + R_3} \right) (12) \Rightarrow 0.95 = \frac{R_3}{100} (12)$$

$$\Rightarrow R_3 = 7.92 \text{ k}\Omega$$

$$V_{B2} = V_{C1} + 0.7 = 4.25 + 0.7 = 4.95 \text{ V}$$

$$V_{B2} = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3} \right) (12)$$

$$\Rightarrow 4.95 = \frac{R_2 + 7.92}{100} (12)$$

$$R_2 = \frac{(4.95)(100)}{12} - 7.92 \Rightarrow R_2 = 33.3 \text{ k}\Omega$$

$$R_1 = 100 - R_2 - R_3 = 100 - 33.3 - 7.92$$

$$\Rightarrow R_1 = 58.8 \text{ k}\Omega$$

b. For both Q_1 and Q_2

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.5}$$

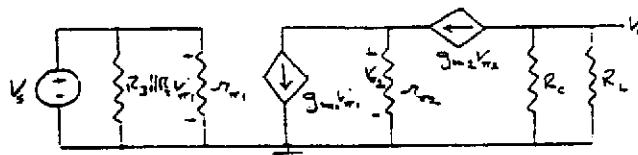
$$\Rightarrow r_{\pi1} = r_{\pi2} = 5.2 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.5}{0.026}$$

$$\Rightarrow g_{m1} = g_{m2} = 19.23 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \infty$$

c.



$$V_o = -g_{m2} V_{\pi2} (R_C \parallel R_L)$$

$$\frac{V_{\pi2}}{r_{\pi2}} + g_{m2} V_{\pi2} = g_{m1} V_{\pi1}$$

$$V_{\pi2} \left(\frac{1 + \beta}{r_{\pi2}} \right) = g_{m1} V_{\pi1}$$

$$V_{\pi2} = g_{m1} V_{\pi1} \left(\frac{r_{\pi2}}{1 + \beta} \right) \text{ and } V_{\pi1} = v_s$$

So

$$A_v = \frac{V_o}{V_s} = -g_{m2} (R_C \parallel R_L) \cdot g_{m1} \left(\frac{r_{\pi2}}{1 + \beta} \right)$$

$$= -g_{m2} (R_C \parallel R_L) \left(\frac{\beta}{1 + \beta} \right)$$

$$A_v = -\left(\frac{100}{101} \right) (19.23) (7.5 \parallel 2) \Rightarrow A_v = -30.1$$

E4.31

a. dc analysis

$$\beta = 80, V_{BE(\text{on})} = 0.7, V_A = \infty$$

$$I_{BQ} = \frac{2.32 - 0.7}{24.2 + (81)(0.5)} = \frac{1.62}{64.7}$$

$$I_{BQ} = 0.0250 \text{ mA}, I_{CQ} = 2.00 \text{ mA}$$

$$\text{Power dissipated in } R_C = I_{CQ}^2 R_C = (1.0)^2 (2)$$

$$\Rightarrow P_C = 8.0 \text{ mW}$$

$$\text{Power dissipated in } R_L = 0, P_L = 0$$

$$V_{CE} = V_{CC} - I_C \left[R_C + \left(\frac{1 + \beta}{\beta} \right) R_E \right]$$

$$= 12 - 2 \left[2 + \left(\frac{81}{80} \right) (0.5) \right]$$

$$V_{CE} = 6.99 \text{ V}$$

$$P_T = I_B V_{BE} + I_C V_{CE} = (0.0250)(0.7) + (2)(6.99)$$

$$\Rightarrow P_T = 14.0 \text{ mW}$$

$$\text{b. } r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(80)(0.026)}{2.0} \Rightarrow r_\pi = 1.04 \text{ k}\Omega$$

For $v_s = 18 \cos \omega t \text{ mV}$

From the text, power dissipation in the transistor

$$P_T = V_{CEQ} I_{CQ} - \left(\frac{\beta}{r_\pi} \right)^2 \left(\frac{V_P}{\sqrt{2}} \right)^2 (R_C \parallel R_L)$$

$$= (6.99)(2 \times 10^{-3})$$

$$- \left(\frac{80}{1.04 \times 10^3} \right)^2 \left(\frac{0.018}{\sqrt{2}} \right)^2 (2 \times 10^3 \parallel 2 \times 10^3)$$

$$P_T = (14 - 0.96) \text{ mW} \Rightarrow P_T = 13.0 \text{ mW}$$

From notes

$$|v_{ce}| = \frac{\beta}{r_\pi} (R_C \parallel R_L) V_P \cos \omega t$$

Power dissipated in R_L

$$P_L = \frac{|v_{ce}|^2}{R_L} |_{rms} = \left[\frac{\beta}{r_\pi} (R_C \parallel R_L) \right]^2 \times \frac{1}{R_L} \times \frac{V_P^2}{2}$$

$$= \left[\frac{80}{1.04} (1.0) \right]^2 \times \frac{1}{2 \times 10^3} \times \left(\frac{0.018}{2} \right)^2$$

$$\Rightarrow P_L = 0.479 \text{ mW}$$

$$R_C = 2 \text{ k}\Omega \text{ also so } P_C = 8.0 + 0.479$$

$$\Rightarrow P_C = 8.48 \text{ mW}$$

E4.32

$$\beta = 100, V_{BE}(\text{on}) = 0.7 \text{ V}, V_A = \infty$$

a. $R_{TH} = R_1 \parallel R_2 = 10 \parallel 53.8 = 8.43 \text{ k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (5) = \left(\frac{10}{10 + 53.8} \right) (5)$$

$$V_{TH} = 0.7837$$

$$I_{BQ} = \frac{0.7837 - 0.7}{8.43} = 0.00993 \text{ mA}$$

$$I_{CQ} = 0.993 \text{ mA}$$

$$V_{CEQ} = V_{CC} - I_{CQ}R_C$$

$$2.5 = 5 - (0.993)R_C \Rightarrow R_C = 2.52 \text{ k}\Omega$$

b. Power in $R_C = P_R = I_C^2 R_C = (0.993)^2 (2.52)$

$$\Rightarrow P_R = 2.48 \text{ mW}$$

$$\text{Power in } Q \equiv P_Q \approx I_{CQ}V_{CEQ} = (0.993)(2.5)$$

$$\Rightarrow P_Q = 2.48 \text{ mW}$$

c. $i_C = 0.993 \cos \omega t$

$$\text{ac power} = \frac{1}{2} \times (0.993)^2 \times R_C = 1.24 \text{ mW}$$

in R_C

$$\frac{1.24}{2.48 + 2.48} = 0.25$$

Chapter 4

Problem Solutions

4.1

$$\text{a. } g_m = \frac{I_{CQ}}{V_T} = \frac{2}{0.026} \Rightarrow g_m = 76.9 \text{ mA/V}$$

$$r_x = \frac{\beta V_T}{I_{CQ}} = \frac{(180)(0.026)}{2} \Rightarrow r_x = 2.34 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{150}{2} \Rightarrow r_o = 75 \text{ k}\Omega$$

$$\text{b. } g_m = \frac{0.5}{0.026} \Rightarrow g_m = 19.2 \text{ mA/V}$$

$$r_x = \frac{(180)(0.026)}{0.5} \Rightarrow r_x = 9.36 \text{ k}\Omega$$

$$r_o = \frac{150}{0.5} \Rightarrow r_o = 300 \text{ k}\Omega$$

4.2

$$g_m = \frac{I_{CQ}}{V_T} \Rightarrow 200 = \frac{I_{CQ}}{0.026} \Rightarrow I_{CQ} = 5.2 \text{ mA}$$

$$r_x = \frac{\beta V_T}{I_{CQ}} = \frac{(125)(0.026)}{5.2} \Rightarrow r_x = 0.625 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{200}{5.2} \Rightarrow r_o = 38.5 \text{ k}\Omega$$

4.3

$$\text{(a) } I_{BQ} = \frac{2 - 0.7}{250} = 0.0052 \text{ mA}$$

$$I_C = (120)(0.0052) = 0.624 \text{ mA}$$

$$g_m = \frac{0.624}{0.026} \Rightarrow g_m = 24 \text{ mA/V}$$

$$r_x = \frac{(120)(0.026)}{0.624} \Rightarrow r_x = 5 \text{ k}\Omega$$

$$r_o = \infty$$

$$\text{(b) } A_v = -g_m R_C \left(\frac{r_x}{r_x + R_B} \right) = -(24)(4) \left(\frac{5}{5+250} \right) \Rightarrow$$

$$A_v = -1.88$$

$$\text{(c) } v_s = \frac{v_o}{A_v} = \frac{v_o}{-1.88} \Rightarrow$$

$$v_s = -0.426 \sin 100t \text{ V}$$

4.4

$$g_m = \frac{I_{CQ}}{V_T}, \quad 1.08 \leq I_{CQ} \leq 1.32 \text{ mA}$$

$$\frac{1.08}{0.026} \leq g_m \leq \frac{1.32}{0.026} \Rightarrow \frac{41.5}{0.026} \leq g_m \leq 50.8 \text{ mA/V}$$

$$r_x = \frac{\beta V_T}{I_{CQ}}, \quad r_x(\max) = \frac{(120)(0.026)}{1.08} = 2.89 \text{ k}\Omega$$

$$r_x(\min) = \frac{(80)(0.026)}{1.32} = 1.58 \text{ k}\Omega$$

$$1.58 < r_x < 2.89 \text{ k}\Omega$$

4.5

$$\text{a. } r_x = 5.4 = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{I_{CQ}}$$

$$\Rightarrow I_{CQ} = 0.578 \text{ mA}$$

$$V_{CEQ} = \frac{1}{2} V_{CC} = \frac{1}{2}(5) = 2.5 \text{ V}$$

$$V_{CEQ} = V_{CC} - I_{CQ} R_C \Rightarrow 2.5 = 5.0 - (0.578) R_C$$

$$\Rightarrow R_C = 4.33 \text{ k}\Omega$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.578}{120} = 0.00482 \text{ mA}$$

$$V_{BB} = I_{BQ} R_B + V_{BE(\text{on})}$$

$$= (0.00482)(25) + 0.70$$

$$\Rightarrow V_{BB} = 0.821$$

$$\text{b. } r_x = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{0.578} = 5.40 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.578}{0.026} = 22.2 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{0.578} = 173 \text{ k}\Omega$$

$$V_0 = -g_m(r_o || R_C)V_{BB}, \quad V_x = \left(\frac{r_x}{r_x + R_B} \right) V_S$$

$$A_v = -g_m \left(\frac{r_x}{r_x + R_B} \right) (r_o || R_C) = -\frac{\beta(r_o || R_C)}{r_x + R_B}$$

$$A_v = -\frac{(120)[173][4.33]}{5.40 + 25} = -\frac{(120)(4.22)}{30.4}$$

$$\Rightarrow A_v = -16.7$$

4.6

$$\text{a. } V_{ECQ} = \frac{1}{2} V_{CC} = 5 \text{ V}$$

$$V_{ECQ} = 10 - I_{CQ} R_C \Rightarrow 5 = 10 - (0.5) R_C$$

$$\Rightarrow R_C = 10 \text{ k}\Omega$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.5}{100} = 0.005$$

$$V_{EB(\text{on})} + I_{BQ} R_B = V_{BB} = (0.70) + (0.005)(50)$$

$$\Rightarrow V_{BB} = 0.95 \text{ V}$$

$$\text{b. } g_m = \frac{I_{CQ}}{V_T} = \frac{0.5}{0.026} \Rightarrow g_m = 19.2 \text{ mA/V}$$

$$r_x = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.5} \Rightarrow r_x = 5.2 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{\infty}{0.5} \Rightarrow r_o = \infty$$

$$\text{c. } A_v = -\frac{\beta R_C}{r_x + R_B} = -\frac{(100)(10)}{5.2 + 50} \Rightarrow A_v = -18.1$$

4.7

a. $I_E = 0.35 \text{ mA}$, $I_B = \frac{0.35}{101} = 0.00347 \text{ mA}$

$$V_B = -I_B R_B = -(0.00347)(10)$$

$$\Rightarrow V_B = -0.0347 \text{ V}$$

$$V_E = V_B - V_{BE(\text{on})} \Rightarrow V_E = -0.735 \text{ V}$$

b. $V_C = V_{CEQ} + V_E = 3.5 - 0.735 = 2.77 \text{ V}$

$$I_C = \left(\frac{\beta}{1+\beta} \right) I_E = \left(\frac{100}{101} \right) (0.35) = 0.347 \text{ mA}$$

$$R_C = \frac{V^+ - V_C}{I_C} = \frac{5 - 2.77}{0.347} \Rightarrow R_C = 6.43 \text{ k}\Omega$$

(c) $A_v = -g_m \left(\frac{R_B \| r_\pi}{R_B \| r_\pi + R_s} \right) (R_C \| r_o)$

$$g_m = \frac{0.347}{0.026} = 13.3 \text{ mA/V}, r_o = \frac{100}{0.347} = 288 \text{ k}\Omega$$

$$r_\pi = \frac{(100)(0.026)}{0.347} = 7.49 \text{ k}\Omega$$

$$R_B \| r_\pi = 10 \| 7.49 = 4.28 \text{ k}\Omega$$

$$A_v = -(13.3) \left(\frac{4.28}{4.28 + 0.1} \right) (6.43 \| 288) \Rightarrow$$

$$A_v = -81.7$$

d. $A_v = -g_m \left(\frac{R_B \| r_\pi}{R_B \| r_\pi + R_s} \right) (R_C \| r_o)$

$$R_B \| r_\pi = 10 \| 7.49 = 4.28 \text{ k}\Omega$$

$$A_v = -(13.3) \left(\frac{4.28}{4.28 + 0.5} \right) (6.43 \| 288)$$

$$\Rightarrow A_v = -74.9$$

4.8

a. $R_{TH} = R_1 \| R_2 = 6 \| 1.5 = 1.2 \text{ k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V^+ = \left(\frac{1.5}{1.5 + 6} \right) (5) = 1.0 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE(\text{on})}}{R_{TH} + (1+\beta)R_E} = \frac{1.0 - 0.7}{1.2 + (181)(0.1)} = 0.0155 \text{ mA}$$

$$I_{CQ} = 2.80 \text{ mA}, I_{BQ} = 2.81$$

$$V_{CEQ} = V^+ - I_{CQ} R_C - I_{BQ} R_E$$

$$= 5 - (2.8)(1) - (2.81)(0.1)$$

$$\Rightarrow V_{CEQ} = 1.92 \text{ V}$$

b. $r_\pi = \frac{(180)(0.026)}{2.80} \Rightarrow r_\pi = 1.67 \text{ k}\Omega$

$$g_m = \frac{2.80}{0.026} \Rightarrow g_m = 108 \text{ mA/V}, r_o = \infty$$

(c) $A_v = -g_m \left(\frac{R_1 \| R_2 \| r_\pi}{R_1 \| R_2 \| r_\pi + R_s} \right) (R_C \| R_L)$

$$R_1 \| R_2 \| r_\pi = 6 \| 1.5 \| 1.67 = 0.698 \text{ k}\Omega$$

$$A_v = -(108) \left(\frac{0.698}{0.698 + 0.2} \right) (1 \| 1.2) \Rightarrow$$

$$A_v = -45.8$$

4.9

a. $I_{CQ} \approx I_{EQ}$

$$V_{CEQ} = 5 = 10 - I_{CQ}(R_C + R_E)$$

$$= 10 - I_{CQ}(1.2 + 0.2)$$

$$I_{CQ} = 3.57 \text{ mA}$$

$$I_{BQ} = \frac{3.57}{150} = 0.0238 \text{ mA}$$

$$R_1 \| R_2 = R_{TH} = (0.1)(1 + \beta)R_E$$

$$= (0.1)(151)(0.2) = 3.02 \text{ k}\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot (10) - 5$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE(\text{on})} + (1 + \beta) I_{BQ} R_E - 5$$

$$\frac{1}{R_1} (3.02)(10) - 5 = (0.0238)(3.02) + 0.7$$

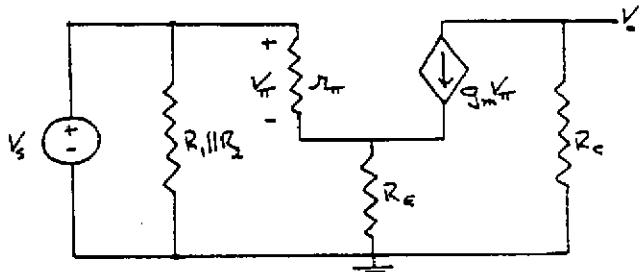
$$+ (151)(0.0238)(0.2) - 5$$

$$\frac{1}{R_1} (30.2) = 1.49 \Rightarrow R_1 = 20.3 \text{ k}\Omega$$

$$\frac{20.3 R_2}{20.3 + R_2} = 3.02 \Rightarrow R_2 = 3.55 \text{ k}\Omega$$

b. $r_\pi = \frac{(150)(0.026)}{3.57} = 1.09 \text{ k}\Omega$

$$g_m = \frac{3.57}{0.026} = 137 \text{ mA/V}$$



$$A_v = \frac{-\beta R_C}{r_\pi + (1 + \beta)R_E} = -\frac{(150)(1.2)}{1.09 + (151)(0.2)}$$

$$\Rightarrow A_v = -5.75$$

4.10

$$4. \quad V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{50}{50 + 10} \right) (12) = 10 \text{ V}$$

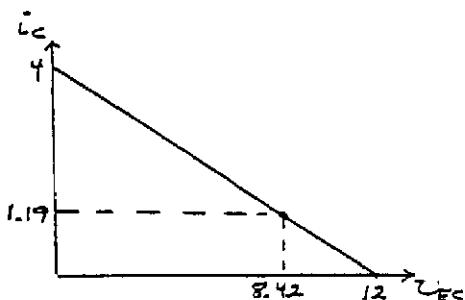
$$R_{TH} = R_1 || R_2 = 50 || 10 = 8.33 \text{ k}\Omega$$

$$I_{BQ} = \frac{12 - 0.7 - 10}{8.33 + (101)(1)} = 0.0119 \text{ mA}$$

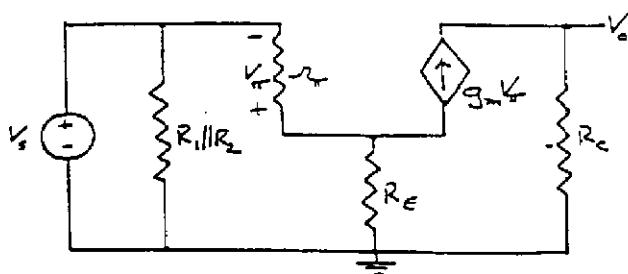
$$I_{CQ} = 1.19 \text{ mA}, \quad I_{EQ} = 1.20 \text{ mA}$$

$$V_{ECQ} = 12 - (1.20)(1) - (1.19)(2)$$

$$\underline{V_{ECQ} = 8.42 \text{ V}}$$



b.



$$r_\pi = \frac{(100)(0.026)}{1.19} = 2.18 \text{ k}\Omega$$

$$V_0 = g_m V_\pi R_C$$

$$V_S = -V_\pi - \left(\frac{V_\pi}{r_\pi} + g_m V_\pi \right) R_E$$

$$= -V_\pi \left[\frac{r_\pi + (1 + \beta) R_E}{r_\pi} \right]$$

$$A_v = \frac{-\beta R_C}{r_\pi + (1 + \beta) R_E} = \frac{-(100)(2)}{2.18 + (101)(1)}$$

$$\Rightarrow \underline{A_v = -1.94}$$

 c. Approximation: Assume r_π does not vary significantly.

$$R_C = 2 \text{ k}\Omega \pm 5\% = 2.1 \text{ k}\Omega \text{ or } 1.9 \text{ k}\Omega$$

$$R_E = 1 \text{ k}\Omega \pm 5\% = 1.05 \text{ k}\Omega \text{ or } 0.95 \text{ k}\Omega$$

 For $R_C(\max) = 2.1 \text{ k}\Omega$ and $R_E(\min)$

$$A_v = \frac{-(100)(2.1)}{2.18 + (101)(0.95)} = -2.14$$

 For $R_C(\min) = 1.9 \text{ k}\Omega$ and $R_E(\max) = 1.05 \text{ k}\Omega$

$$A_v = \frac{-(100)(1.9)}{2.18 + (101)(1.05)} = -1.76$$

$$\text{So } \underline{1.76 \leq |A_v| \leq 2.14}$$

4.11

$$(a) \quad V_{CC} = \left(\frac{1 + \beta}{\beta} \right) I_{CQ} R_E + V_{ECQ} + I_{CQ} R_C$$

$$12 = \left(\frac{101}{100} \right) I_{CQ}(1) + 6 + I_{CQ}(2)$$

$$\text{so that } \underline{I_{CQ} = 1.99 \text{ mA}}$$

$$I_{BQ} = \frac{1.99}{100} = 0.0199 \text{ mA}$$

$$R_{TH} = (0.1)(1 + \beta) R_E = (0.1)(101)(1) = 10.1 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (10.1)(12)$$

$$V_{CC} = (1 + \beta) I_{BQ} R_E + V_{EB}(\text{on}) + I_{BQ} R_{TH} + V_{TH}$$

$$12 = (101)(0.0199)(1) + 0.7 + (0.0199)(10.1) + \frac{121.2}{R_1}$$

$$\text{which yields } \underline{R_1 = 13.3 \text{ k}\Omega} \text{ and } \underline{R_2 = 42 \text{ k}\Omega}$$

$$(b) \quad A_v = \frac{-\beta R_C}{r_\pi + (1 + \beta) R_E} = \frac{-(100)(2)}{1.31 + (101)(1)} \Rightarrow \\ \underline{A_v = -1.95}$$

4.12

$$I_{CQ} = 0.25 \text{ mA}, \quad I_{EQ} = 0.2525 \text{ mA}$$

$$I_{BQ} = 0.0025 \text{ mA}$$

$$I_{BQ} R_B + V_{BE}(\text{on}) + I_{EQ}(R_S + R_E) - 5 = 0$$

$$(0.0025)(50) + 0.7 + (0.2525)(0.1 + R_E) = 5$$

$$\underline{R_E = 16.4 \text{ k}\Omega}$$

$$V_E = -(0.0025)(50) - 0.7 = -0.825 \text{ V}$$

$$V_C = V_{CEQ} + V_E = 3 - 0.825 = 2.175 \text{ V}$$

$$R_C = \frac{5 - 2.175}{0.25} \Rightarrow \underline{R_C = 11.3 \text{ k}\Omega}$$

$$A_v = \frac{-\beta R_C}{r_\pi + (1 + \beta) R_E}$$

$$r_\pi = \frac{(100)(0.026)}{0.25} = 10.4 \text{ k}\Omega$$

$$A_v = \frac{-(100)(11.3)}{10.4 + (101)(0.1)} \Rightarrow \underline{A_v = -55.1}$$

$$R_i = R_B \parallel [r_\pi + (1 + \beta) R_S]$$

$$= 50 \parallel [10.4 + (101)(0.1)]$$

$$R_i = 50 \parallel 20.5 \Rightarrow \underline{R_i = 14.5 \text{ k}\Omega}$$

4.13

$$a. \quad 9 = I_{EQ}R_E + V_{EB(\text{on})} + I_{BQ}R_S$$

$$I_{EQ} = 0.75 \text{ mA}, \quad I_{BQ} = \frac{0.75}{81} = 0.00926 \text{ mA}$$

$$I_{CQ} = 0.741 \text{ mA}$$

$$9 = (0.75)R_E + 0.7 + (0.00926)(2)$$

$$\Rightarrow R_E = 11.0 \text{ k}\Omega$$

$$b. \quad V_E = 9 - (0.75)(11) = 0.75 \text{ V}$$

$$V_C = V_E - V_{ECQ} = 0.75 - 7 = -6.25 \text{ V}$$

$$R_C = \frac{V_C - (-9)}{I_{CQ}} = \frac{9 - 6.25}{0.741} \Rightarrow R_C = 3.71 \text{ k}\Omega$$

$$c. \quad A_v = -g_m \left(\frac{r_\pi}{r_\pi + R_S} \right) (R_C \parallel R_L \parallel r_o)$$

$$r_\pi = \frac{(80)(0.026)}{0.741} = 2.81 \text{ k}\Omega$$

$$r_o = \frac{80}{0.741} = 108 \text{ k}\Omega$$

$$A_v = \frac{-80}{2.81 + 2} (3.71 \parallel 10 \parallel 108)$$

$$A_v = -43.9$$

$$d. \quad R_i = R_S + r_\pi = 2 + 2.81 \Rightarrow R_i = 4.81 \text{ k}\Omega$$

4.14

$$(a) \quad V_{CC} \equiv I_{CQ}(R_C + R_E) + V_{CEQ}$$

$$9 = I_{CQ}(2.2 + 2) + 3.75 \text{ So that}$$

$$I_{CQ} = 1.25 \text{ mA}$$

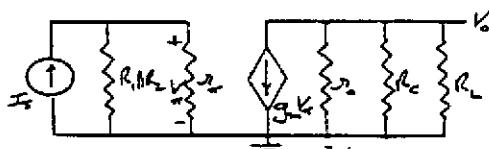
$$(b) \quad g_m = \frac{1.25}{0.026} = 48.1 \text{ mA/V}$$

$$r_\pi = \frac{(120)(0.026)}{1.25} = 250 \text{ k}\Omega$$

$$r_o = \frac{100}{1.25} = 80 \text{ k}\Omega$$

Assume circuit is to be designed to be bias stable.

$$R_{TH} = R_1 \parallel R_2 = (0.1)(1 + \beta)R_E = (0.1)(121)(2) = 24.2 \Omega$$



$$V_o = -g_m V_\pi (r_o \parallel R_C \parallel R_L)$$

$$V_\pi = I_S (R_1 \parallel R_2 \parallel r_\pi)$$

Then

$$R_n = \frac{V_o}{I_s} = -g_m (R_1 \parallel R_2 \parallel r_\pi) (r_o \parallel R_C \parallel R_L)$$

$$R_n = -48.1(24.2 \parallel 2.5) (80 \parallel 2.2 \parallel 1) = -48.1(2.27)(0.682)$$

or

$$R_n = \frac{V_o}{I_s} = -74.5 \text{ k}\Omega = -74.5 \text{ V/mA}$$

4.15

$$a. \quad I_{EQ} = 0.80 \text{ mA}, \quad I_{BQ} = \frac{0.80}{66} = 0.0121 \text{ mA}$$

$$I_{CQ} = 0.788 \text{ mA}$$

$$V_B = I_{BQ}R_B \Rightarrow R_B = \frac{0.3}{0.0121} \Rightarrow R_B = 24.8 \text{ k}\Omega$$

$$R_C = \frac{V_C - (-5)}{I_{CQ}} = \frac{5 - 3}{0.788} \Rightarrow R_C = 2.54 \text{ k}\Omega$$

$$b. \quad g_m = \frac{0.788}{0.026} = 30.3 \text{ mA/V}$$

$$r_\pi = \frac{(65)(0.026)}{0.788} = 2.14 \text{ k}\Omega$$

$$r_o = \frac{75}{0.788} = 95.2 \text{ k}\Omega$$

$$i_0 = \left(\frac{R_C \parallel r_o}{R_C \parallel r_o + R_L} \right) g_m V_\pi, \quad V_\pi = -v_s$$

$$G_f = \frac{i_0}{v_s} = -g_m \left(\frac{R_C \parallel r_o}{R_C \parallel r_o + R_L} \right)$$

$$= -(30.3) \left(\frac{2.54 \parallel 95.2}{2.54 \parallel 95.2 + 4} \right)$$

$$G_f = -11.6 \text{ mA/V}$$

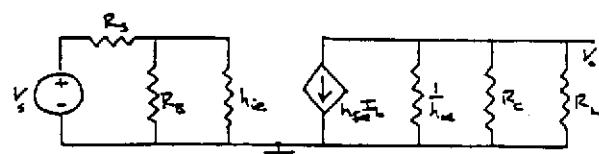
4.16

$$I_{BQ} = \frac{4 - 0.7}{5 + (101)(5)} = 0.00647$$

$$I_{CQ} = 0.647 \text{ mA}$$

$$a. \quad 80 \leq h_{fe} \leq 120, \quad 10 \leq h_{oe} \leq 20 \mu\text{s}$$

$$\underbrace{2.45 \text{ k}\Omega}_{\text{low gain}} \leq h_{ie} \leq \underbrace{3.7 \text{ k}\Omega}_{\text{high gain}}$$



$$V_o = -h_{fe} I_b \left(\frac{1}{h_{oe}} \parallel R_C \parallel R_L \right)$$

$$I_b = \frac{R_B}{R_{TH} + h_{ie}} \cdot V_s$$

$$R_{TH} = R_B \parallel R_S = 5 \parallel 1 = 0.833 \text{ k}\Omega$$

High-gain

$$I_b = \frac{\left(\frac{5}{5+1}\right)V_s}{0.833 + 3.7} = 0.1838V_s$$

Low-gain

$$I_b = \frac{\left(\frac{5}{5+1}\right)V_s}{0.833 + 2.45} = 0.2538V_s$$

For

$$\begin{aligned} h_{oe} &= 10 \Rightarrow \frac{1}{h_{oe}} \parallel R_C \parallel R_L = \frac{1}{0.010} \parallel 4 \parallel 4 \\ &= 100 \parallel 2 = 1.96 \text{ k}\Omega \end{aligned}$$

For

$$h_{oe} = 20 \Rightarrow \frac{1}{0.020} \parallel 4 \parallel 4 = 50 \parallel 2 = 1.92 \text{ k}\Omega$$

$$|A_v|_{\max} = (120)(0.1838)(1.96) = 43.2$$

$$|A_v|_{\min} = (80)(0.2538)(1.92) = 39.0$$

$$39.0 \leq |A_v| \leq 43.2$$

$$b. R_e = R_B \parallel h_{ie} = 5 \parallel 3.7 = 2.13 \text{ k}\Omega$$

$$\text{or } R_e = 5 \parallel 2.45 = 1.64 \text{ k}\Omega$$

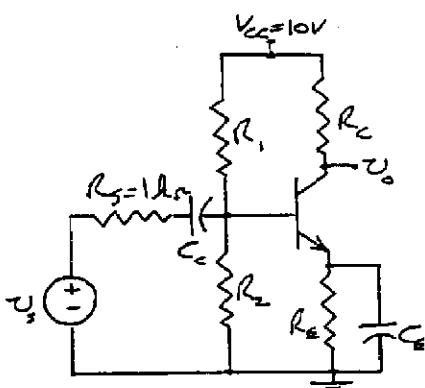
$$1.64 < R_e < 2.13 \text{ k}\Omega$$

$$R_o = \frac{1}{h_{oe}} \parallel R_C = \frac{1}{0.010} \parallel 4 = 100 \parallel 4 = 3.85 \text{ k}\Omega$$

$$\text{or } R_o = \frac{1}{0.020} \parallel 4 = 50 \parallel 4 = 3.70 \text{ k}\Omega$$

$$3.70 \leq R_o \leq 3.85 \text{ k}\Omega$$

4.17



Assume an npn transistor with $\beta = 100$ and $V_A = \infty$. Let $V_{cc} = 10V$.

$$|A_v| = \frac{0.5}{0.01} = 50$$

Bias at $I_{cq} = 1 \text{ mA}$ and let $R_E = 1 \text{ k}\Omega$

For a bias stable circuit

$$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(101)(1) = 10.1 \text{ k}\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{cc} = \frac{1}{R_1}(10.1)(10) = \frac{101}{R_1}$$

$$I_{sq} = \frac{1}{100} = 0.01 \text{ mA}$$

$$V_{TH} = I_{sq}R_{TH} + V_{BE}(\text{on}) + (1+\beta)I_{sq}R_E$$

$$\frac{101}{R_1} = (0.01)(10.1) + 0.7 + (101)(0.01)(1)$$

$$\text{which yields } R_1 = 55.8 \text{ k}\Omega \text{ and } R_2 = 12.3 \text{ k}\Omega$$

Now

$$r_\pi = \frac{(100)(0.026)}{1} = 2.6 \text{ k}\Omega$$

$$g_m = \frac{1}{0.026} = 38.46 \text{ mA/V}$$

$$V_a = -g_m V_\pi R_C$$

where

$$V_\pi = \left(\frac{R_1 \parallel R_2 \parallel r_\pi}{R_1 \parallel R_2 \parallel r_\pi + R_S} \right) \cdot V_s = \left(\frac{10.1 \parallel 2.6}{10.1 \parallel 2.6 + 1} \right) \cdot V_s$$

or

$$V_\pi = 0.674 V_s$$

Then

$$A_v = \frac{V_a}{V_s} = -(0.674)g_m R_C = -(0.674)(38.46)R_C = -50$$

$$\text{which yields } R_C = 1.93 \text{ k}\Omega$$

With this R_C , the dc bias is OK.

4.18

$$a. I_{BQ} = \frac{6 - 0.7}{10 + (101)(3)} = 0.0169 \text{ mA}$$

$$I_{CQ} = 1.69 \text{ mA}, I_{EQ} = 1.71 \text{ mA}$$

$$V_{CEQ} = (16 + 6) - (1.69)(6.8) - (1.71)(3)$$

$$V_{CEQ} = 5.38 \text{ V}$$

$$b. g_m = \frac{1.69}{0.026} \Rightarrow g_m = 65 \text{ mA/V}$$

$$r_\pi = \frac{(100)(0.026)}{1.69} \Rightarrow r_\pi = 1.54 \text{ k}\Omega, r_0 = \infty$$

$$(c) A_v = \frac{-\beta(R_C \parallel R_L)}{r_\pi + (1+\beta)R_E} \cdot \frac{R_B \parallel R_b}{R_b \parallel R_b + R_S}$$

$$R_b = r_\pi + (1+\beta)R_E = 1.54 + (101)(3) = 304.5 \text{ k}\Omega$$

$$R_b \parallel R_b = 10 \parallel 304.5 = 9.68 \text{ k}\Omega$$

Then

$$A_v = \frac{-(100)(6.8 \parallel 6.8)}{1.54 + (101)(3)} \cdot \left(\frac{9.68}{9.68 + 0.5} \right) \Rightarrow$$

$$A_v = -1.06$$

$$i_0 = \left(\frac{R_C}{R_C + R_L} \right) (-\beta i_b)$$

$$i_b = \left(\frac{R_B}{R_B + r_\pi + (1+\beta)R_E} \right) i_S$$

$$A_v = -(\beta) \left(\frac{R_C}{R_C + R_L} \right) \left(\frac{R_B}{R_B + r_\pi + (1+\beta)R_E} \right)$$

$$= -100 \left(\frac{6.8}{6.8 + 6.8} \right) \left(\frac{10}{10 + 1.54 + (101)(3)} \right)$$

$$\Rightarrow A_v = -1.59$$

(d) $R_u = R_s + R_b \parallel R_b = 0.5 + 10 \parallel 304.5 = 10.2 \text{ k}\Omega$

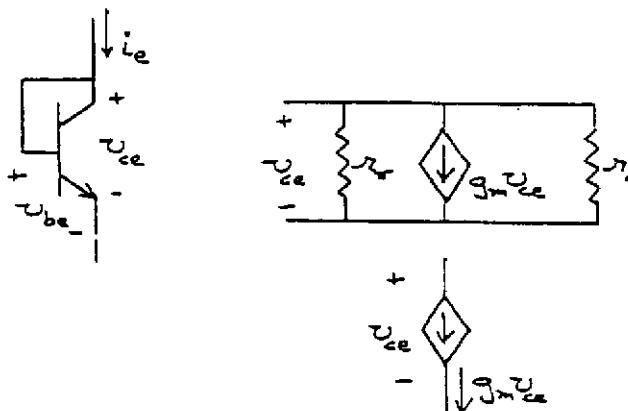
(e) $A_v = \frac{-\beta(R_C \parallel R_L)}{r_x + (1+\beta)R_E} \cdot \frac{R_b \parallel R_b}{R_b \parallel R_b + R_s}$

$$A_v = \frac{-(100)(6.8 \parallel 6.8)}{1.54 + (101)(3)} \cdot \left(\frac{9.68}{9.68 + 1} \right) \Rightarrow$$

$$A_v = -1.01$$

A_v = same as (c) $\Rightarrow A_v = -1.59$

4.19

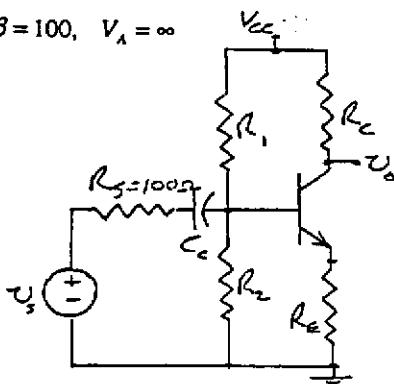


$$\tau = \frac{V_{Ce}}{g_m V_{Ce}} = \frac{1}{g_m}$$

$$\text{So } r_e = r_\pi \parallel \left(\frac{1}{g_m} \right) \parallel r_o$$

4.20

Let $\beta = 100$, $V_A = \infty$



Let $V_{CC} = 2.5V$

$$P = (I_R + I_C)V_{CC} \Rightarrow 0.12 = (I_R + I_C)(2.5) \Rightarrow$$

$$I_R + I_C = 48 \mu A, \text{ Let } I_R = 8 \mu A, I_C = 40 \mu A$$

$$R_1 + R_2 = \frac{V_{CC}}{I_R} = \frac{2.5}{8} \Rightarrow 312.5 \text{ k}\Omega$$

$$I_{BQ} = \frac{40}{100} = 0.4 \mu A$$

Let $R_E = 2 \text{ k}\Omega$. For a bias stable circuit

$$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(101)(2) = 20.2 \text{ k}\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = I_{BQ} R_{TH} + V_{BE(on)} + (1+\beta)I_{BQ} R_E$$

$$\frac{1}{R_1} (20.2)(2.5) = (0.0004)(20.2) + 0.7$$

$$+ (101)(0.0004)(2)$$

which yields $R_1 = 64 \text{ k}\Omega$ and $R_2 = 29.5 \text{ k}\Omega$

$$r_\pi = \frac{(100)(0.026)}{0.04} = 65 \text{ k}\Omega \quad \text{Neglect } R_S$$

$$A_v = \frac{V_o}{V_i} \equiv \frac{-\beta R_C}{r_\pi + (1+\beta)R_E}$$

$$-10 = \frac{-100 R_C}{65 + (101)(2)} \Rightarrow R_C = 26.7 \text{ k}\Omega$$

With this R_C , dc biasing is OK.

4.21

Need a voltage gain of $\frac{100}{5} = 20$.

Assume a sign inversion from a common-emitter is not important. Use the configuration for Figure 4.28. Need an input resistance of

$$R_i = \frac{5 \times 10^3}{0.2 \times 10^{-6}} = 25 \times 10^3 = 25 \text{ k}\Omega$$

$$R_i = R_{TH} \parallel R_b. \text{ Let } R_{TH} = 50 \text{ k}\Omega, R_b = 50 \text{ k}\Omega$$

$$R_b = r_\pi + (1+\beta)R_E \equiv (1+\beta)R_E$$

$$\text{For } \beta = 100, R_E = \frac{R_b}{1+\beta} = \frac{50}{101} = 0.495 \text{ k}\Omega$$

$$\text{Let } R_E = 0.5 \text{ k}\Omega, V_{CC} = 10 \text{ V}, I_{CQ} = 0.2 \text{ mA}$$

$$\text{Then } I_{BQ} = \frac{0.2}{100} = 0.002 \text{ mA}$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE(on)} + (1+\beta)I_{BQ} R_E$$

$$\frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (50)(10) = (0.002)(50) + 0.7$$

$$+ (101)(0.002)(0.5)$$

which yields $R_1 = 555 \text{ k}\Omega$ and $R_2 = 55 \text{ k}\Omega$

Now

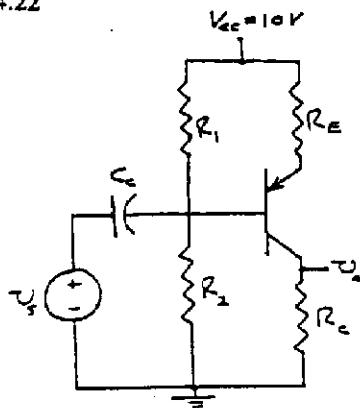
$$A_v = \frac{-\beta R_C}{r_\pi + (1+\beta)R_E}, \quad r_\pi = \frac{(100)(0.026)}{0.2} = 13 \text{ k}\Omega$$

so

$$-20 = \frac{-(100)R_C}{13 + (101)(0.5)} \Rightarrow R_C = 12.7 \text{ k}\Omega$$

[Note: $I_{CQ} R_C = (0.2)(12.7) = 2.54 \text{ V}$. So dc biasing is OK.]

4.22



$$\beta = 80, \quad A_v = \frac{-\beta R_C}{r_\pi + (1 + \beta) R_E}$$

First approximation:

$$(A_v) \approx \frac{R_C}{R_E} = 10 \Rightarrow R_C = 10 R_E$$

$$\text{Set } R_C = 12 R_E$$

$$V_{EC} \approx V_{CC} - I_C(R_C + R_E) = 10 - I_C(13R_E)$$

$$\text{For } V_{EC} = \frac{1}{2}V_{CC} = 5$$

$$5 = 10 - I_C(13R_E)$$

$$\text{For } I_C = 0.7 \text{ mA}$$

$$I_E = 0.709, \quad I_B = 0.00875 \text{ mA}$$

$$\Rightarrow R_E = 0.55 \text{ k}\Omega \rightarrow R_C = 6.6 \text{ k}\Omega$$

 Bias stable \Rightarrow

$$R_1 \parallel R_2 = R_{TH} = (0.1)(1 + \beta)R_E$$

$$= (0.1)(81)(0.55) = 4.46 \text{ k}\Omega$$

$$10 = (0.709)(0.55) + 0.7 + (0.00875)(4.46)$$

$$+ \frac{1}{R_1}(4.46)(10)$$

$$8.87 = \frac{1}{R_1}(4.46) \Rightarrow R_1 = 5.03 \text{ k}\Omega$$

$$\frac{5.03R_2}{5.03 + R_2} = 4.46 \Rightarrow R_2 = 39.4 \text{ k}\Omega$$

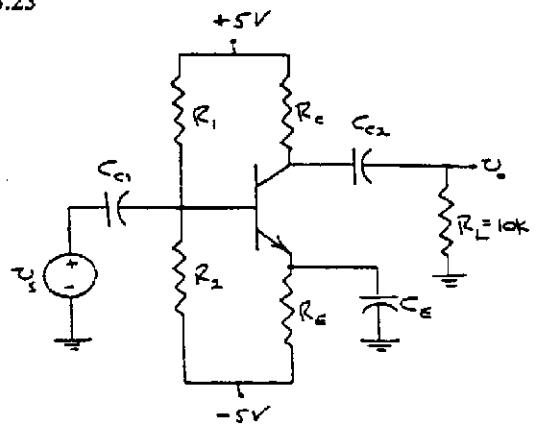
$$\frac{10}{R_1 + R_2} = \frac{10}{5.03 + 39.4} = 0.225 \text{ mA}$$

 0.7 + 0.225 = 0.925 mA from V_{CC} source.

$$\text{Now } r_\pi = \frac{(80)(0.026)}{0.7} = 2.97 \text{ k}\Omega$$

$$|A_v| = \frac{(80)(6.6)}{2.97 + (81)(0.55)} = 11.1$$

4.23



$$\beta = 120$$

$$\text{Let } I_{CQ} = 0.35 \text{ mA}, \quad I_{EQ} = 0.353 \text{ mA}$$

$$I_{BQ} = 0.00292 \text{ mA}$$

$$\text{Let } R_E = 2 \text{ k}\Omega. \text{ For } V_{CEQ} = 4 \text{ V} \Rightarrow$$

$$10 = 4 + (0.35)R_C + (0.353)(2)$$

$$R_C = 15.1 \text{ k}\Omega, \quad r_\pi = \frac{(120)(0.026)}{0.35} = 8.91 \text{ k}\Omega$$

$$A_v = \frac{-\beta(R_C \parallel R_L)}{r_\pi} = -\frac{(120)(15.1) \parallel 10}{8.91}$$

$$A_v = -81.0$$

For bias stable circuit:

$$R_1 \parallel R_2 = R_{TH} = (0.1)(1 + \beta)R_E$$

$$= (0.1)(121)(2) = 24.2 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5 = \frac{1}{R_1} \cdot R_{TH} \cdot (10) - 5$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE(\text{on})} + (1 + \beta) I_{BQ} R_E - 5$$

$$\frac{1}{R_1}(24.2)(10) - 5 = (0.00292)(24.2) + 0.7$$

$$+ (121)(0.00292)(2) - 5$$

$$\frac{1}{R_1}(242) = 1.477, \quad R_1 = 164 \text{ k}\Omega$$

$$\frac{164R_2}{164 + R_2} = 24.2 \Rightarrow R_2 = 28.4 \text{ k}\Omega$$

$$\frac{10}{164 + 28.4} = 0.052, \quad 0.35 + 0.052 = 0.402 \text{ mA}$$

4.24

From Prob. 4-10:

$$R_{TH} = R_1 \parallel R_2 = 10 \parallel 50 = 8.33 \text{ k}\Omega$$

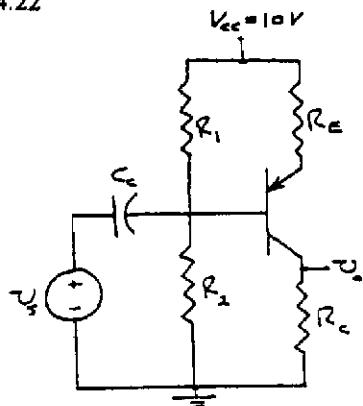
$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (12) = \left(\frac{50}{50 + 10} \right) (12) = 10 \text{ V}$$

$$I_{BQ} = \frac{12 - 0.7 - 10}{8.33 + (101)(1)} = 0.0119 \text{ mA}$$

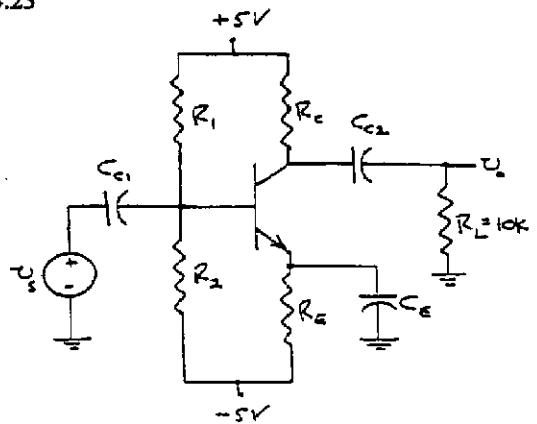
$$I_{CQ} = 1.19 \text{ mA}, \quad I_{EQ} = 1.20 \text{ mA}$$

$$V_{ECQ} = 12 - (1.19)(2) - (120)(1) = 8.42 \text{ V}$$

4.22



4.23



$$\beta = 80, A_v = \frac{-\beta R_C}{r_\pi + (1 + \beta)R_E}$$

First approximation:

$$(A_v) \approx \frac{R_C}{R_E} = 10 \Rightarrow R_C = 10R_E$$

 Set $R_C = 12R_E$

$$V_{EC} \approx V_{CC} - I_C(R_C + R_E) = 10 - I_C(13R_E)$$

$$\text{For } V_{EC} = \frac{1}{2}V_{CC} = 5$$

$$5 = 10 - I_C(13R_E)$$

 For $I_C = 0.7 \text{ mA}$

$$I_E = 0.709, I_B = 0.00875 \text{ mA}$$

$$\Rightarrow R_E = 0.55 \text{ k}\Omega \rightarrow R_C = 6.6 \text{ k}\Omega$$

 Bias stable \Rightarrow

$$R_1 \parallel R_2 = R_{TH} = (0.1)(1 + \beta)R_E$$

$$= (0.1)(81)(0.55) = 4.46 \text{ k}\Omega$$

$$10 = (0.709)(0.55) + 0.7 + (0.00875)(4.46)$$

$$+ \frac{1}{R_1}(4.46)(10)$$

$$8.87 = \frac{1}{R_1}(4.46) \Rightarrow R_1 = 5.03 \text{ k}\Omega$$

$$\frac{5.03R_2}{5.03 + R_2} = 4.46 \Rightarrow R_2 = 39.4 \text{ k}\Omega$$

$$\frac{10}{R_1 + R_2} = \frac{10}{5.03 + 39.4} = 0.225 \text{ mA}$$

 0.7 + 0.225 $\approx 0.925 \text{ mA}$ from V_{CC} source.

$$\text{Now } r_\pi = \frac{(80)(0.026)}{0.7} = 2.97 \text{ k}\Omega$$

$$|A_v| = \frac{(80)(6.6)}{2.97 + (81)(0.55)} = 11.1$$

$$\beta = 120$$

$$\text{Let } I_{CQ} = 0.35 \text{ mA}, I_{EQ} = 0.353 \text{ mA}$$

$$I_{BQ} = 0.00292 \text{ mA}$$

$$\text{Let } R_E = 2 \text{ k}\Omega. \text{ For } V_{CEQ} = 4 \text{ V} \Rightarrow$$

$$10 = 4 + (0.35)R_C + (0.353)(2)$$

$$R_C = 15.1 \text{ k}\Omega, r_\pi = \frac{(120)(0.026)}{0.35} = 8.91 \text{ k}\Omega$$

$$A_v = \frac{-\beta(R_C || R_L)}{r_\pi} = -\frac{(120)(15.1)(10)}{8.91}$$

$$A_v = -81.0$$

For bias stable circuit:

$$R_1 \parallel R_2 = R_{TH} = (0.1)(1 + \beta)R_E$$

$$= (0.1)(121)(2) = 24.2 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right)(10) - 5 = \frac{1}{R_1} \cdot R_{TH} \cdot (10) - 5$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE(\text{on})} + (1 + \beta)I_{BQ}R_E - 5$$

$$\frac{1}{R_1}(24.2)(10) - 5 = (0.00292)(24.2) + 0.7$$

$$+ (121)(0.00292)(2) - 5$$

$$\frac{1}{R_1}(242) = 1.477, R_1 = 164 \text{ k}\Omega$$

$$\frac{164R_2}{164 + R_2} = 24.2 \Rightarrow R_2 = 28.4 \text{ k}\Omega$$

$$\frac{10}{164 + 28.4} = 0.052, 0.35 + 0.052 = 0.402 \text{ mA}$$

4.24

From Prob. 4-10:

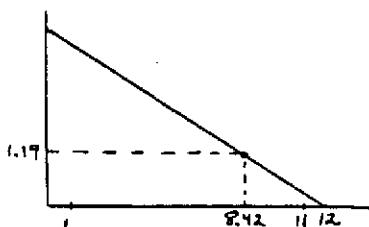
$$R_{TH} = R_1 \parallel R_2 = 10 \parallel 50 = 8.33 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right)(12) = \left(\frac{50}{50 + 10} \right)(12) = 10 \text{ V}$$

$$I_{BQ} = \frac{12 - 0.7 - 10}{8.33 + (101)(1)} = 0.0119 \text{ mA}$$

$$I_{CQ} = 1.19 \text{ mA}, I_{EQ} = 1.20 \text{ mA}$$

$$V_{ECQ} = 12 - (1.19)(2) - (120)(1) = 8.42 \text{ V}$$



4.27

$$I_{EQ} = 0.80 \text{ mA}, I_{CQ} = 0.792 \text{ mA}$$

$$I_{BQ} = 0.008 \text{ mA}$$

$$V_E = 0.7 + (0.008)(10) = 0.78 \text{ V}$$

$$V_C = I_{CQ}R_C - 5 = (0.792)(4) - 5 = -1.83 \text{ V}$$

$$V_{CEQ} = 0.78 - (-1.83) = 2.61 \text{ V}$$

Load line: Assume V_E remains constant at $\approx 0.78 \text{ V}$

For $1 \leq V_{EC} \leq 11$

$$\Delta V_{EC} = 11 - 8.42 = 2.58$$

\Rightarrow Output voltage swing = 5.16 V

(peak-to-peak)

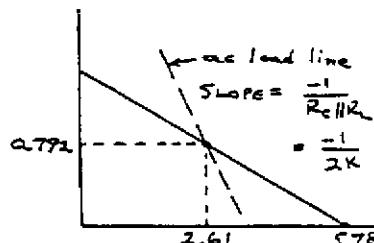
4.25

$$I_{BQ} = \frac{5 - 0.7}{50 + (101)(0.1 + 12.9)} = 0.00315 \text{ mA}$$

$$I_{CQ} = 0.315 \text{ mA}, I_{EQ} = 0.319 \text{ mA}$$

$$V_{CEQ} = (5 + 5) - (0.315)(6) - (0.319)(13)$$

$$V_{CEQ} = 3.96 \text{ V}$$



$$\Delta i_C = \frac{-1}{2 \text{ k}\Omega} \cdot \Delta V_{EC}$$

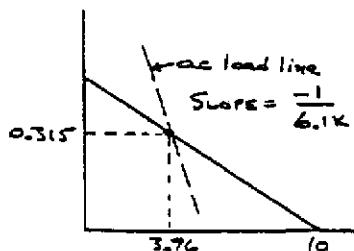
$$\text{Collector current swing} = 0.792 - 0.08 \\ = 0.712 \text{ mA}$$

$$|\Delta V_{EC}| = (0.712)(2) = 1.42 \text{ V}$$

Output swing determined by current.

Max. output swing = 2.84 V peak-to-peak

$$\text{Swing in } i_C \text{ current} = \frac{2.84}{4} \\ = 0.71 \text{ mA peak-to-peak}$$



$$\Delta i_C = -\frac{1}{6.1} \Delta V_{EC}$$

$$\text{For } \Delta i_C = 0.315 - 0.05 = 0.265$$

$$\Rightarrow |\Delta V_{EC}| = 1.62$$

$$V_{EC}(\min) = 3.96 - 1.62 = 2.34$$

Output signal swing determined by current:

Max. output swing = 3.24 V peak-to-peak

4.26

$$\text{For } R_C = 6 \text{ k}\Omega, V_C = 5 - \left(\frac{100}{101} \right)(0.35)(6) = 2.92 \text{ V}$$

$$V_E = -I_{EQ}R_S - V_{BE}(\text{on}) = -\frac{0.35}{100}(10) - 0.7 = -0.735 \text{ V}$$

$$\text{Then } V_{CE} = V_C - V_E = 2.92 - (-0.735) = 3.66 \text{ V}$$

$$\Delta V_{CE} = \Delta i_C \cdot R_C \Rightarrow (4.5 - 3.66) = \Delta i_C(6)$$

so that $\Delta i_C = 0.14 \text{ mA}$

(a) Total $\Delta V_{CE} = 2(4.5 - 3.66) = 1.68 \text{ V}$ peak-to-peak

(b) Total $\Delta i_C = 2(0.14) = 0.28 \text{ mA}$ peak-to-peak

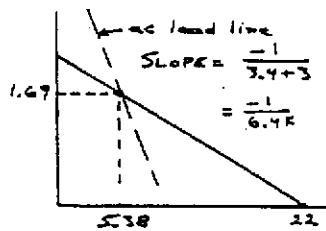
4.28

$$I_{BQ} = \frac{6 - 0.7}{10 + (101)(3)} = 0.0169 \text{ mA}$$

$$I_{CQ} = 1.69 \text{ mA}, I_{EQ} = 1.71 \text{ mA}$$

$$V_{CEQ} = (16 + 6) - (1.69)(6.8) - (1.71)(3)$$

$$V_{CEQ} = 5.38 \text{ V}$$



$$\Delta i_C = -\frac{1}{6.4} \Delta v_{ce}$$

For $v_{ce}(\min) = 1$ V, $\Delta v_{ce} = 5.38 - 1 = 4.38$ V

$$\Rightarrow |\Delta i_C| = \frac{4.38}{6.4} = 0.684 \text{ mA}$$

Output swing limited by voltage:

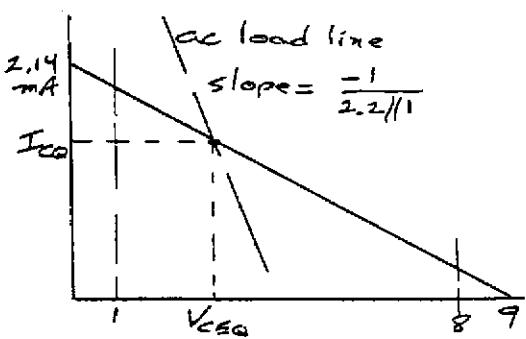
Δv_{ce} = Max. swing in output voltage

= 8.76 V peak-to-peak

$$\Delta i_0 = \frac{1}{2} \Delta i_C \Rightarrow \underline{\Delta i_0 = 0.342 \text{ mA}}$$

(peak-to-peak)

4.29



$$\Delta v_{ce}(\max) = V_{ceQ} - 1, \quad \Delta i_c(\max) = I_{co}$$

$$\Delta v_{ce} = \Delta i_c(0.6875). \text{ So}$$

$$V_{ceQ} - 1 = I_{co}(0.6875) \text{ and}$$

$$V_{ceQ} = V_{cc} - I_{co}(R_c + R_E) \Rightarrow V_{ceQ} = 9 - I_{co}(4.2)$$

Then

$$9 - I_{co}(4.2) - 1 = I_{co}(0.6875)$$

$$\text{So } \underline{I_{co} = 1.64 \text{ mA}} \text{ and } \underline{V_{ceQ} = 2.11 \text{ V}}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{cc} \text{ and } R_{TH} = (0.1)(1 + \beta)R_E$$

$$R_{TH} = (0.1)(151)(2) = 30.2 \text{ k}\Omega$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE(on)} + (1 + \beta)I_{BQ}R_E$$

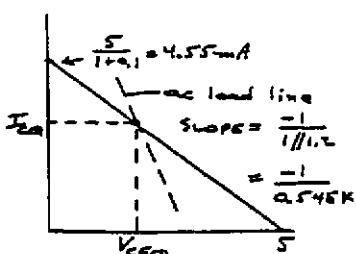
$$I_{BQ} = \frac{1.64}{150} = 0.0109 \text{ mA}$$

$$\frac{1}{R_1}(30.2)(9) = (0.0109)(30.2) + 0.7 + (151)(0.0109)(2)$$

$$\text{which yields } \underline{R_1 = 62.9 \text{ k}\Omega} \text{ and } \underline{R_2 = 58.1 \text{ k}\Omega}$$

4.30

dc load line



For maximum symmetrical swing

$$\Delta i_C = I_{CQ} - 0.25$$

$$\Delta v_{CE} = V_{CEQ} - 0.5$$

$$\text{and } |\Delta i_C| = \frac{1}{0.545 \text{ k}\Omega} \cdot |\Delta v_{CE}|$$

$$I_{CQ} - 0.25 = \frac{V_{CEQ} - 0.5}{0.545}$$

$$V_{CEQ} = 5 - I_{CQ}(1.1)$$

$$0.545(I_{CQ} - 0.25) = [5 - I_{CQ}(1.1)] - 0.5$$

$$(0.545 + 1.1)I_{CQ} = 5 - 0.5 + 0.136$$

$$\underline{I_{CQ} = 2.82 \text{ mA}}, \quad \underline{I_{BQ} = 0.0157 \text{ mA}}$$

$$R_{TH} = R_1 \parallel R_2 = (0.1)(1 + \beta)R_E$$

$$= (0.1)(181)(0.1) = 1.81 \text{ k}\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V^+ = I_{BQ}R_{TH} + V_{BE(on)}$$

$$+ (1 + \beta)I_{BQ}R_E$$

$$\frac{1}{R_1}(1.81)(5) = (0.0157)(1.81) + 0.7$$

$$+ (181)(0.0157)(0.1)$$

$$\frac{1}{R_1}(9.05) = 1.01 \Rightarrow \underline{R_1 = 8.96 \text{ k}\Omega}$$

$$\frac{8.96 R_2}{8.96 + R_2} = 1.81 \Rightarrow \underline{R_2 = 2.27 \text{ k}\Omega}$$

4.31

$$I_{CQ} = 0.647 \text{ mA}, \quad V_{CEQ} = 10 - (0.647)(9) = 4.18 \text{ V}$$

$$\Delta i_C = I_{CQ} = 0.647 \text{ mA}$$

$$\text{So } \Delta v_{ce} = \Delta i_c(4/4) = (0.647)(2) = 1.294 \text{ V}$$

Voltage swing is well within the voltage specification. Then

$$\underline{\Delta v_{ce} = 2(1.294) = 2.59 \text{ V peak-to-peak}}$$

4.32

$$\text{a. } R_{TH} = R_1 \parallel R_2 = 10 \parallel 10 = 5 \text{ k}\Omega$$

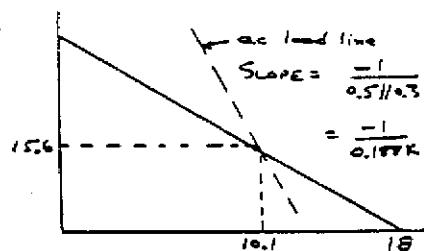
$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (18) - 9 = \left(\frac{10}{10 + 10} \right) (18) - 9 = 0$$

$$I_{BQ} = \frac{0 - 0.7 - (-9)}{5 + (181)(0.5)} = 0.0869 \text{ mA}$$

$$I_{CQ} = 15.6 \text{ mA}, \quad I_{BQ} = 15.7 \text{ mA}$$

$$V_{CEQ} = 18 - (15.7)(0.5) \Rightarrow \underline{V_{CEQ} = 10.1 \text{ V}}$$

b.



c. $r_\pi = \frac{(180)(0.026)}{15.6} = 0.30 \text{ k}\Omega$

$$A_v = \frac{(1+\beta)(R_E \| R_L)}{r_\pi + (1+\beta)(R_E \| R_L)} \cdot \left(\frac{R_1 \| R_2 \| R_b}{R_1 \| R_2 \| R_b + R_s} \right)$$

$$R_{ab} = r_\pi + (1+\beta)(R_E \| R_L) = 0.30 + (181)(0.5 \| 0.3)$$

or $R_{ab} = 34.2 \text{ k}\Omega$

$$R_1 \| R_2 \| R_b = 5 \| 34.2 = 4.36 \text{ k}\Omega$$

$$A_v = \frac{(181)(0.5 \| 0.3)}{0.3 + (181)(0.5 \| 0.3)} \cdot \left(\frac{4.36}{4.36+1} \right) \Rightarrow A_v = 0.806$$

d. $R_{ib} = r_\pi + (1+\beta)(R_E \| R_L)$

$$R_{ib} = 0.30 + (181)(0.188) \Rightarrow R_{ib} = 34.3 \text{ k}\Omega$$

$$R_o = R_E \left| \frac{r_\pi + R_1 \| R_2 \| R_f}{1+\beta} \right| = 0.5 \left| \frac{0.3 + 5 \| 1}{181} \right| \Rightarrow$$

$$R_o = 6.18 \Omega$$

4.33

a. $R_{TH} = R_1 \| R_2 = 10 \| 10 = 5 \text{ k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (-10) = -5 \text{ V}$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE(\text{on})} + (1+\beta) I_{BQ} R_E - 10$$

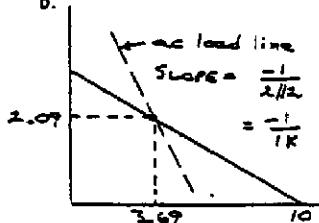
$$I_{BQ} = \frac{-5 - 0.7 - (-10)}{5 + (121)(2)} = 0.0174 \text{ mA}$$

$$I_{CQ} = 2.09 \text{ mA}, I_{EQ} = 2.11 \text{ mA}$$

$$V_{CEQ} = 10 - (2.09)(1) - (2.11)(2)$$

$$\Rightarrow V_{CEQ} = 3.69 \text{ V}$$

b.



c. $r_\pi = \frac{(120)(0.026)}{2.09} = 1.49 \text{ k}\Omega$

$$A_v = \frac{(1+\beta)(R_E \| R_L)}{r_\pi + (1+\beta)(R_E \| R_L)} \cdot \left(\frac{R_1 \| R_2 \| R_b}{R_1 \| R_2 \| R_b + R_s} \right)$$

$$R_{ab} = r_\pi + (1+\beta)(R_E \| R_L) = 1.49 + (121)(2 \| 2)$$

$$R_{ab} = 122.5 \text{ k}\Omega, R_1 \| R_2 \| R_b = 5 \| 122.5 = 4.80 \text{ k}\Omega$$

$$A_v = \frac{(121)(2 \| 2)}{1.49 + (121)(2 \| 2)} \cdot \left(\frac{4.80}{4.80+5} \right) \Rightarrow$$

A_v = 0.484

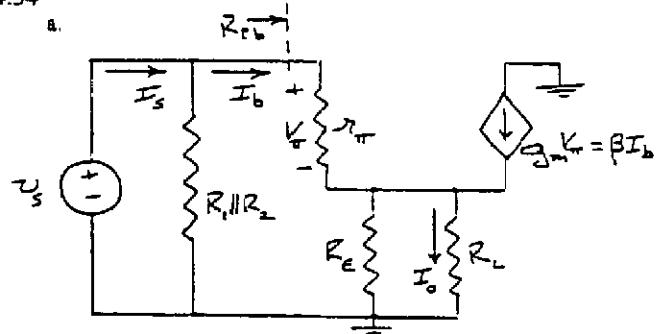
d. $R_{ib} = r_\pi + (1+\beta)(R_E \| R_L)$

$$R_{ib} = 1.49 + (121)(2 \| 2) \Rightarrow R_{ib} = 122 \text{ k}\Omega$$

$$R_o = R_E \left| \frac{r_\pi + R_1 \| R_2 \| R_s}{1+\beta} \right| = 2 \left| \frac{1.49 + 5 \| 1}{121} \right| \Rightarrow$$

$$R_o = 32.5 \Omega$$

4.34



$$I_0 = (1+\beta) I_b \left(\frac{R_E}{R_E + R_L} \right)$$

$$I_b = I_S \left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_{ab}} \right)$$

$$R_{ib} = r_\pi + (1+\beta)(R_E \| R_L)$$

$$V_{CC} = 10 \text{ V, For } V_{CEQ} = 5 \text{ V}$$

$$5 = 10 - \left(\frac{1+\beta}{\beta} \right) I_{CQ} R_E$$

$$5 = 80, \text{ For } R_E = 0.5 \text{ k}\Omega$$

$$I_{CQ} = 9.88 \text{ mA}, I_{EQ} = 10 \text{ mA}, I_{BQ} = 0.123 \text{ mA}$$

$$r_\pi = \frac{(80)(0.026)}{9.88} = 0.211 \text{ k}\Omega$$

$$R_{ib} = 0.211 + (81)(0.5 \| 0.5) \Rightarrow R_{ib} = 20.46 \text{ k}\Omega$$

$$A_v = \frac{I_0}{I_S} = (1+\beta) \left(\frac{R_E}{R_E + R_L} \right) \left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_{ab}} \right)$$

$$8 = (81) \left(\frac{1}{2} \right) \left(\frac{R_1 \| R_2}{R_1 \| R_2 + 20.46} \right)$$

$$0.1975(R_1 \| R_2 + 20.46) = R_1 \| R_2$$

$$R_1 \| R_2 \Rightarrow 5.04 \text{ k}\Omega$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE(\text{on})} + (1+\beta) I_{BQ} R_E$$

$$\frac{1}{R_1}(3.04)(10) = (0.123)(5.04) + 0.7 + (10)(0.5)$$

$$\Rightarrow R_1 = 7.97 \text{ k}\Omega$$

$$\frac{7.97 R_2}{7.97 + R_2} = 5.04 \Rightarrow R_2 = 13.7 \text{ k}\Omega$$

$$(b) R_{ib} = 0.211 + (81)(0.5)[2] = 32.6 \text{ k}\Omega$$

$$A_i = (81) \left(\frac{0.5}{0.5+2} \right) \left(\frac{5.04}{5.04+32.6} \right) = (81)(0.2)(0.134)$$

$$\underline{A_i = 2.17}$$

4.35

$$R_i = R_{TH} \| R_{ib} \text{ where } R_{ib} = r_\pi + (1+\beta)R_E$$

$$V_{CEQ} = 3.5, I_{CQ} = \frac{5-3.5}{2} = 0.75 \text{ mA}$$

$$r_\pi = \frac{(120)(0.026)}{0.75} = 4.16 \text{ k}\Omega$$

$$R_{ib} = 4.16 + (121)(2) = 246 \text{ k}\Omega$$

$$\text{Then } R_i = 120 = R_{TH} \| 246 \Rightarrow R_{TH} = 235 \text{ k}\Omega$$

$$I_{SQ} = \frac{0.75}{120} = 0.00625 \text{ mA}$$

$$V_{TH} = I_{SQ} R_{TH} + V_{BE}(\text{on}) + (1+\beta) I_{SQ} R_E$$

$$\frac{1}{R_i} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_i} (235)(5) = (0.00625)(235) \\ + 0.7 + (121)(0.00625)(2)$$

$$\text{which yields } \underline{R_1 = 319 \text{ k}\Omega} \text{ and } \underline{R_2 = 892 \text{ k}\Omega}$$

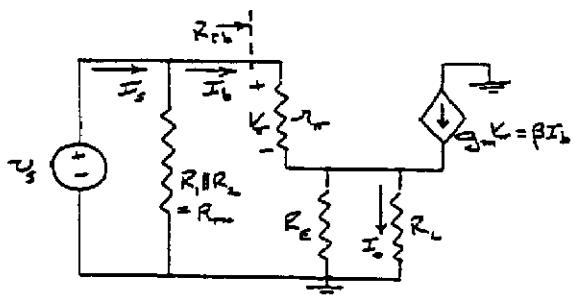
4.36

$$a. \text{ Let } R_E = 24 \Omega \text{ and } V_{CEQ} = \frac{1}{2} V_{CC} = 12 \text{ V}$$

$$\Rightarrow I_{SQ} = \frac{12}{24} = 0.5 \text{ A}$$

$$I_{CQ} = 0.493 \text{ A}, I_{BQ} = 6.58 \text{ mA}$$

$$r_\pi = \frac{(75)(0.026)}{0.493} = 3.96 \Omega$$



$$I_0 = (1+\beta) I_b \left(\frac{R_E}{R_E + R_L} \right)$$

$$I_b = I_S \left(\frac{R_{TH}}{R_{TH} + R_{ib}} \right)$$

$$R_{ib} = r_\pi + (1+\beta)(R_E || R_L)$$

$$= 3.96 + (76)(24 || 8) \Rightarrow R_{ib} = 460 \Omega$$

$$A_i = \frac{I_0}{I_S} = (1+\beta) \left(\frac{R_E}{R_E + R_L} \right) \left(\frac{R_{TH}}{R_{TH} + R_{ib}} \right)$$

$$8 = (76) \left(\frac{24}{24+8} \right) \left(\frac{R_{TH}}{R_{TH} + 460} \right)$$

$$0.140 = \frac{R_{TH}}{R_{TH} + 460}$$

$$\Rightarrow \underline{R_{TH} = 74.9 \Omega} \text{ (Minimum value)}$$

dc analysis:

$$V_{TH} = \frac{1}{R_i} \cdot R_{TH} \cdot V_{CC}$$

$$= I_{BQ} R_{TH} + V_{BE}(\text{on}) + I_{EQ} R_E$$

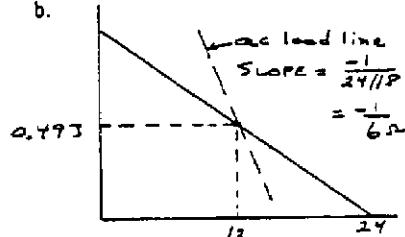
$$\frac{1}{R_i} (74.9)(24) = (0.00658)(74.9) + 0.70$$

$$+ (0.5)(24)$$

$$= 12.75$$

$$\underline{R_1 = 136 \Omega}, \frac{136 R_2}{136 + R_2} = 74.9 \Rightarrow \underline{R_2 = 157 \Omega}$$

b.



$$\Delta i_C = -\frac{1}{6} \Delta v_{ce}$$

 For $\Delta i_C = 0.493$

$$\Rightarrow |\Delta v_{ce}| = (0.493)(6)$$

 ⇒ Max. swing in output voltage for this design
 $= 5.92 \text{ V peak-to-peak}$

$$c. R_0 = \frac{r_\pi}{1+\beta} \parallel R_E = \frac{3.96}{76} \parallel 24 = 0.0521 \parallel 24$$

$$\Rightarrow \underline{R_0 = 52 \text{ m}\Omega}$$

4.37

$$a. R_{TH} = R_1 \| R_2 = 60 \| 40 = 24 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{40}{40+60} \right)(5) = 2 \text{ V}$$

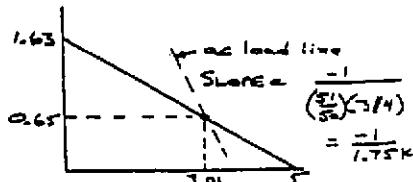
$$I_{BQ} = \frac{5 - 0.7 - 2}{24 + (51)(3)} = 0.0130 \text{ mA}$$

$$I_{CQ} = 0.650 \text{ mA}, I_{EQ} = 0.663 \text{ mA}$$

$$V_{ECQ} = 5 - I_{EQ} R_E = 5 - (0.663)(3)$$

$$\Rightarrow \underline{V_{CEQ} = 3.01 \text{ V}}$$

b.

For $R_E(\max)$, $R_L(\min)$, $R_{TH}(\max)$

$$A_i = (51) \left(\frac{3.07}{3.07 + 3.8} \right) \left(\frac{25.2}{25.2 + 88.6} \right)$$

$$\Rightarrow A_i = 5.05$$

4.38

The output of the emitter follower is

$$c. r_\pi = \frac{(50)(0.026)}{0.650} = 2 \text{ k}\Omega, r_o = \frac{80}{0.65} = 123 \text{ k}\Omega$$

Define $R'_L = R_E \| R_L \| r_o = 3 \| 4 \| 123 = 1.69 \text{ k}\Omega$

$$A_v = \frac{(1+\beta)R'_L}{r_\pi + (1+\beta)R'_L} = \frac{(51)(1.69)}{2 + (51)(1.69)}$$

$$\Rightarrow A_v = 0.977$$

$$A_i = (1+\beta)I_b \left(\frac{R_E \| r_o}{R_E \| r_o + R_L} \right)$$

$$I_b = I_s \left(\frac{R_{TH}}{R_{TH} + R_{iB}} \right)$$

$$R_{iB} = r_\pi + (1+\beta)R'_L = 2 + (51)(1.69) = 88.2$$

$$R_E \| r_o = 3 \| r_o = 3 \| 123 = 2.93$$

$$A_i = (51) \left(\frac{2.93}{2.93 + 4} \right) \left(\frac{24}{24 + 88.2} \right)$$

$$\Rightarrow A_i = 4.61$$

$$d. R_{iB} = r_\pi + (1+\beta)R_E \| R_L \| r_o = 2 + (51)(1.69)$$

$$\Rightarrow R_{iB} = 88.2 \text{ k}\Omega$$

$$R_o = \frac{r_\pi}{1+\beta} \| R_E = \left(\frac{2}{51} \right) \| 3 = 0.0392 \| 3$$

$$R_o = 38.7 \Omega$$

e. Assume variations in r_π and r_o have negligible effects

$$R_1 = 60 \pm 5\% \quad R_1 = 63 \text{ k}\Omega, \quad R_1 = 57 \text{ k}\Omega$$

$$R_2 = 40 \pm 5\% \quad R_2 = 42 \text{ k}\Omega, \quad R_2 = 38 \text{ k}\Omega$$

$$R_E = 3 \pm 5\% \quad R_E = 3.15 \text{ k}\Omega, \quad R_E = 2.85 \text{ k}\Omega$$

$$R_L = 4 \pm 5\% \quad R_L = 4.2 \text{ k}\Omega, \quad R_L = 3.8 \text{ k}\Omega$$

$$A_i = (1+\beta) \left(\frac{R_E \| r_o}{R_E \| r_o + R_L} \right) \left(\frac{R_{TH}}{R_{TH} + R_{iB}} \right)$$

$$R_{iB} = r_\pi + (1+\beta)(R_E \| R_L \| r_o)$$

$$R_{TH}(\max) = 25.2 \text{ k}\Omega, \quad R_{TH}(\min) = 22.8 \text{ k}\Omega$$

$$R_{iB}(\max) = 92.5 \text{ k}\Omega, \quad R_{iB}(\min) = 84.0 \text{ k}\Omega$$

$$R_E(\max), \quad R_L(\min), \quad R_{iB} = 88.6 \text{ k}\Omega$$

$$R_E(\min), \quad R_L(\max), \quad R_{iB} = 87.4 \text{ k}\Omega$$

$$R_E(\max) \| r_o = 3.07 \text{ k}\Omega$$

$$R_E(\min) \| r_o = 2.79 \text{ k}\Omega$$

For $R_E(\min)$, $R_L(\max)$, $R_{TH}(\min)$

$$A_i = (51) \left(\frac{2.79}{2.79 + 4.2} \right) \left(\frac{22.8}{22.8 + 87.4} \right)$$

$$\Rightarrow A_i = 4.21$$

For $R_E(\max)$, $R_L(\min)$, $R_{TH}(\max)$

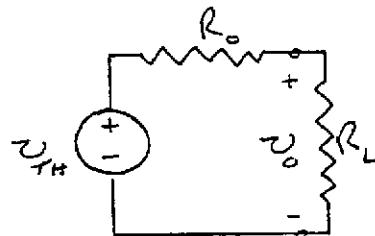
$$A_i = (51) \left(\frac{3.07}{3.07 + 3.8} \right) \left(\frac{25.2}{25.2 + 88.6} \right)$$

$$\Rightarrow A_i = 5.05$$

4.38

The output of the emitter follower is

$$v_o = \left(\frac{R_L}{R_L + R_o} \right) \cdot v_{TH}$$

For v_o to be within 5% for a range of R_L , we have

$$\frac{R_L(\min)}{R_L(\min) + R_o} = (0.95) \frac{R_L(\max)}{R_L(\max) + R_o}$$

$$\frac{4}{4 + R_o} = (0.95) \frac{10}{10 + R_o} \quad \text{which yields}$$

$$R_o = 0.364 \text{ k}\Omega$$

$$\text{We have } R_o = \left(\frac{r_\pi + R_i \| R_2 \| R_s}{1 + \beta} \right) \| R_E \| r_o$$

The first term dominates

Let $R_i \| R_2 \| R_s \equiv R_s$, then

$$R_o \equiv \frac{r_\pi + R_s}{1 + \beta} \Rightarrow 0.364 = \frac{r_\pi + 4}{1 + \beta}$$

or

$$0.364 = \frac{r_\pi}{1 + \beta} + \frac{4}{1 + \beta} = \frac{\beta V_T}{I_{CQ}(1 + \beta)} + \frac{4}{1 + \beta}$$

$$0.364 \equiv \frac{V_T}{I_{CQ}} + \frac{4}{1 + \beta}$$

The factor $\frac{4}{1 + \beta}$ is in the range of $\frac{4}{91} = 0.044$ to

$$\frac{4}{131} = 0.0305. \quad \text{We can set } R_o \equiv 0.32 = \frac{V_T}{I_{CQ}}$$

Or $I_{CQ} = 0.08125 \text{ mA}$. To take into account other factors, set $I_{CQ} = 0.15 \text{ mA}$,

$$I_{BQ} = \frac{0.15}{110} = 0.00136 \text{ mA}$$

$$\text{For } V_{CEQ} \equiv 5V, \text{ set } R_E = \frac{5}{0.15} = 33.3 \text{ k}\Omega$$

Design a bias stable circuit.

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5 = \frac{1}{R_1} (R_{TH}) (10) - 5$$

$$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(111)(33.3) = 370 \text{ k}\Omega$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E - 5$$

$$\text{So } \frac{1}{R_1}(370)(10) - 5 = (0.00136)(370) + 0.7$$

$$+ (111)(0.00136)(33.3) - 5$$

which yields $R_1 = 594 \text{ k}\Omega$ and $R_2 = 981 \text{ k}\Omega$

Now

$$A_v = \frac{(1 + \beta)(R_E \| R_L)}{r_e + (1 + \beta)(R_E \| R_L)} \cdot \left(\frac{R_{TH} \| R_{ib}}{R_{TH} \| R_{ib} + R_S} \right)$$

$$R_{ib} = r_e + (1 + \beta)(R_E \| R_L) \text{ and } r_e = \frac{\beta V_T}{I_{CQ}}$$

For $\beta = 90$, $R_L = 4 \text{ k}\Omega$,

$$r_e = 15.6 \text{ k}\Omega, R_{ib} = 340.6 \text{ k}\Omega$$

$$A_v = \frac{(91)(33.3 \| 4)}{15.6 + (91)(33.3 \| 4)} \cdot \frac{370 \| 340.6}{370 \| 340.6 + 4} \Rightarrow$$

$$A_v = 0.9332$$

For $\beta = 90$, $R_L = 10 \text{ k}\Omega$

$$R_{ib} = 715.4 \text{ k}\Omega$$

$$A_v = \frac{(91)(33.3 \| 10)}{15.6 + (91)(33.3 \| 10)} \cdot \frac{370 \| 715.4}{370 \| 715.4 + 4} \Rightarrow$$

$$A_v = 0.9625$$

For $\beta = 130$, $R_L = 4 \text{ k}\Omega$

$$r_e = 22.5 \text{ k}\Omega, R_{ib} = 490 \text{ k}\Omega$$

$$A_v = \frac{(131)(33.3 \| 4)}{22.5 + (131)(33.3 \| 4)} \cdot \frac{370 \| 490}{370 \| 490 + 4} \Rightarrow$$

$$A_v = 0.9360$$

For $\beta = 130$, $R_L = 10 \text{ k}\Omega$

$$R_{ib} = 1030 \text{ k}\Omega$$

$$A_v = \frac{(131)(33.3 \| 10)}{22.5 + (131)(33.3 \| 10)} \cdot \frac{370 \| 1030}{370 \| 1030 + 4} \Rightarrow$$

$$A_v = 0.9645$$

Now $v_o(\text{min}) = |A_v(\text{min})| \cdot v_s = 3.73 \sin \omega t$

$$v_o(\text{max}) = |A_v(\text{max})| \cdot v_s = 3.86 \sin \omega t$$

$$\frac{\Delta v_o}{v_o} = 3.5\%$$

4.39

$$\text{a. } R_{TH} = R_1 \| R_2 = 40 \| 60 = 24 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{60}{60 + 40} \right) (10) = 6 \text{ V}$$

$$\beta = 75$$

$$I_{BQ} = \frac{6 - 0.7}{24 + (76)(5)} = 0.0131 \text{ mA}$$

$$I_{CQ} = 0.984 \text{ mA}$$

$$\beta = 150$$

$$I_{BQ} = \frac{6 - 0.7}{24 + (151)(5)} = 0.00680 \text{ mA}$$

$$I_{CQ} = 1.02 \text{ mA}$$

$$\beta = 75$$

$$r_\pi = \frac{(75)(0.026)}{0.984} = 1.98 \text{ k}\Omega$$

$$\beta = 150$$

$$r_\pi = 3.82 \text{ k}\Omega$$

$$\beta = 75$$

$$R_{sb} = r_\pi + (1 + \beta)(R_E \| R_L) = 65.3 \text{ k}\Omega$$

$$\beta = 150$$

$$R_{sb} = 130 \text{ k}\Omega$$

$$A_v = \frac{(1 + \beta)(R_E \| R_L)}{r_\pi + (1 + \beta)(R_E \| R_L)} \cdot \frac{R_1 \| R_2 \| R_{ib}}{R_1 \| R_2 \| R_{ib} + R_S}$$

$$\text{For } \beta = 75, R_1 \| R_2 \| R_{ib} = 40 \| 60 \| 65.3 = 175 \text{ k}\Omega$$

$$A_v = \frac{(76)(0.833)}{1.98 + (76)(0.833)} \cdot \frac{17.5}{17.5 + 4} \Rightarrow$$

$$A_v = 0.789$$

$$\text{For } \beta = 150, R_1 \| R_2 \| R_{ib} = 40 \| 60 \| 130 = 20.3 \text{ k}\Omega$$

$$A_v = \frac{(151)(0.833)}{3.82 + (151)(0.833)} \cdot \frac{20.3}{20.3 + 4} \Rightarrow$$

$$A_v = 0.811$$

$$A_v = (1 + \beta) \left(\frac{R_E}{R_E + R_L} \right) \left(\frac{R_{TH}}{R_{TH} + R_{ib}} \right)$$

$$\beta = 75$$

$$A_v = (76) \left(\frac{5}{5 + 1} \right) \left(\frac{24}{24 + 65.3} \right) \Rightarrow A_v = 17.0$$

$$\beta = 150$$

$$A_v = (151) \left(\frac{5}{6} \right) \left(\frac{24}{24 + 130} \right) \Rightarrow A_v = 19.6$$

$$17.0 < A_v < 19.6$$

b. Current gain is the same as part (a)

(b) For $R_f = 5 \text{ k}\Omega$

$$\beta = 75 \Rightarrow A_v = 0.754$$

$$\beta = 150 \Rightarrow A_v = 0.779$$

4.40

$$\text{(a) } I_{BQ} = \frac{0.5}{81} = 0.00617 \text{ mA}$$

$$V_B = I_{BQ}R_B = (0.00617)(10) \Rightarrow V_B = 0.0617 \text{ V}$$

$$V_E = V_B + 0.7 \Rightarrow V_E = 0.7617 \text{ V}$$

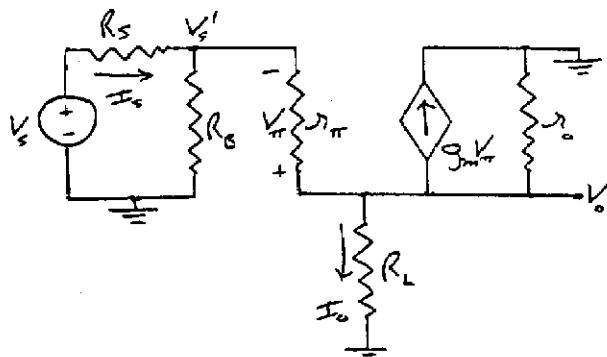
$$(b) I_{CQ} = (0.5) \left(\frac{80}{81} \right) = 0.494 \text{ mA}$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.494}{0.026} \Rightarrow g_m = 19 \text{ mA/V}$$

$$r_x = \frac{\beta V_T}{I_{CQ}} = \frac{(80)(0.026)}{0.494} \Rightarrow r_x = 4.21 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{150}{0.494} \Rightarrow r_o = 304 \text{ k}\Omega$$

(c)

For $R_s = 0$

$$V_o = -\left(\frac{V_x + g_m V_x}{r_x} \right) (R_L \| r_o)$$

so that

$$V_x = \frac{-V_o}{\left(\frac{1+\beta}{r_x} \right) (R_L \| r_o)}$$

Now

$$V_i + V_x = V_o$$

or

$$V_i = V_o - V_x = V_o + \frac{V_o}{\left(\frac{1+\beta}{r_x} \right) (R_L \| r_o)}$$

We find

$$\begin{aligned} A_v &= \frac{V_o}{V_i} = \frac{(1+\beta)(R_L \| r_o)}{r_x + (1+\beta)(R_L \| r_o)} = \frac{(81)(0.5 \| 304)}{4.21 + (81)(0.5 \| 304)} \\ &\equiv \frac{(81)(0.5)}{4.21 + (81)(0.5)} \Rightarrow A_v = 0.906 \end{aligned}$$

$$R_b = r_x + (1+\beta)(R_L \| r_o) \approx 4.21 + (81)(0.5) = 44.7 \text{ k}\Omega$$

$$I_b = \left(\frac{R_s}{R_s + R_b} \right) \cdot I_i \text{ and } I_o = \left(\frac{r_o}{r_o + R_L} \right) (1+\beta) I_b$$

Then

$$A_i = \frac{I_o}{I_s} = (1+\beta) \left(\frac{R_s}{R_s + R_b} \right) \left(\frac{r_o}{r_o + R_L} \right)$$

$$A_i \approx (81) \left(\frac{10}{10+44.7} \right) (1) \Rightarrow A_i = 14.8$$

(d)

$$V'_s = \left(\frac{R_s \| R_b}{R_s \| R_b + R_s} \right) \cdot V_s = \left(\frac{10 \| 44.7}{10 \| 44.7 + 2} \right) \cdot V_s = (0.803) V_s$$

Then

$$\begin{aligned} A_v &= (0.803)(0.906) \Rightarrow A_v = 0.728 \\ A_i &= 14.8 \text{ (Unchanged)} \end{aligned}$$

4.41

$$V_o = (1+\beta) I_b R_L$$

$$I_b = \frac{V_o}{r_x + (1+\beta) R_L}$$

$$\text{so } A_v = \frac{(1+\beta) R_L}{r_x + (1+\beta) R_L}$$

For $\beta = 100$, $R_L = 0.5 \text{ k}\Omega$

$$r_x = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$\text{Then } A_v(\min) = \frac{(101)(0.5)}{5.2 + (101)(0.5)} = 0.9066$$

For $\beta = 180$, $R_L = 500 \text{ k}\Omega$

$$r_x = \frac{(180)(0.026)}{0.5} = 9.36 \text{ k}\Omega$$

$$\text{Then } A_v(\max) = \frac{(181)(500)}{9.36 + (181)(500)} = 0.9999$$

4.42

$$a. \quad I_{EQ} = 1 \text{ mA}, \quad V_{CEQ} = V_{CC} - I_{EQ} R_E$$

$$5 = 10 - (1)(R_E) \Rightarrow R_E = 5 \text{ k}\Omega$$

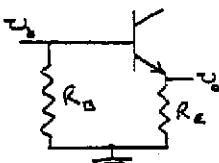
$$I_{BQ} = \frac{1}{101} = 0.0099$$

$$10 = I_{BQ} R_B + V_{BE(\text{on})} + I_{EQ} R_E$$

$$10 = (0.0099) R_B + 0.7 + (1)(5)$$

$$\Rightarrow R_B = 434 \text{ k}\Omega$$

b.

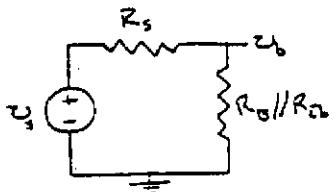


$$r_x = \frac{(100)(0.026)}{0.99} = 2.63 \text{ k}\Omega$$

$$\frac{v_o}{v_b} = \frac{(1+\beta) R_E}{r_x + (1+\beta) R_E} = \frac{(101)(5)}{2.63 + (101)(5)} = 0.995$$

$$\Rightarrow v_o = \frac{v_b}{0.995} = \frac{4}{0.995}$$

 $\Rightarrow v_o = 4.02 \text{ V peak-to-peak at base}$



$$R_{ib} = r_\pi + (1 + \beta)R_E = 508 \text{ k}\Omega$$

$$R_B \parallel R_{ib} = 434 \parallel 508 = 234 \text{ k}\Omega$$

$$v_b = \frac{R_B \parallel R_{ib}}{R_B \parallel R_{ib} + R_S} \cdot v_s = \frac{234 v_s}{234 + 0.7} = \frac{234}{234.7} v_s$$

$$v_b = 0.997 v_s$$

$$\Rightarrow v_s = \frac{4.02}{0.997} \Rightarrow v_s = 4.03 \text{ V peak-to-peak}$$

c. $R_{ib} = r_\pi + (1 + \beta)(R_E \parallel R_L)$

$$R_{ib} = 2.63 + (101)(5 \parallel 1) = 86.8 \text{ k}\Omega$$

$$R_B \parallel R_{ib} = 434 \parallel 86.8 = 72.3 \text{ k}\Omega$$

$$v_b = \left(\frac{72.3}{72.3 + 0.7} \right) v_s = 0.99 v_s = (0.99)(4.03)$$

$$v_b = 3.99 \text{ V peak-to-peak}$$

$$v_o = \frac{(1 + \beta)(R_E \parallel R_L)}{r_\pi + (1 + \beta)(R_E \parallel R_L)} \cdot v_b$$

$$= \frac{(101)(0.833)}{2.63 + (101)(0.833)} (3.99)$$

$$v_o = 3.87 \text{ V peak-to-peak}$$

4.43

$$P_{AVG} = i_L^2(rms)R_L \Rightarrow 1 = i_L^2(rms)(12)$$

$$\text{so } i_L(rms) = 0.289 \text{ A} \Rightarrow i_L(\text{peak}) = \sqrt{2}(0.289)$$

$$i_L(\text{peak}) = 0.409 \text{ A}$$

$$v_L(\text{peak}) = i_L(\text{peak}) \cdot R_L = (0.409)(12) = 4.91 \text{ V}$$

$$\text{Need a gain of } \frac{4.91}{5} = 0.982$$

With $R_S = 10 \text{ k}\Omega$, we will not be able to meet this voltage gain requirement. Need to insert a buffer or an op-amp voltage follower (see Chapter 9) between R_S and C_C .

$$\text{Set } I_{EQ} = 0.5 \text{ A}, \quad V_{CEQ} = \frac{1}{3}(12 - (-12)) = 8 \text{ V}$$

$$24 = I_{SQ}R_E + V_{CEQ} = (0.5)R_E + 8 \Rightarrow R_E = 32 \Omega$$

$$\text{Let } \beta = 50, \quad I_{CQ} = \frac{50}{51}(0.5) = 0.49 \text{ A}$$

$$r_x = \frac{\beta V_T}{I_{CQ}} = \frac{(50)(0.026)}{0.49} = 2.65 \Omega$$

$$R_{ib} = r_x + (1 + \beta)(R_E \parallel R_L) = 2.65 + (51)(32 \parallel 12)$$

$$R_{ib} = 448 \Omega$$

$$I_{SQ} = \frac{0.49}{50} = 0.0098 \text{ A} = 9.8 \text{ mA}$$

$$\text{Let } I_R = \frac{24}{R_1 + R_2} = 10I_E = 98 \text{ mA}$$

$$\text{So that } R_1 + R_2 = 245 \Omega$$

$$V_{TH} = \frac{R_2}{R_1 + R_2} (24) - 12 = I_{SQ}R_{TH} + V_{BE}(\text{on}) + I_{EQ}R_E - 12$$

$$\left(\frac{R_2}{245} \right) (24) = \frac{(0.0098)R_1R_2}{245} + 0.7 + (0.5)(32)$$

$$\text{Now } R_1 = 245 - R_2$$

So we obtain

$$4 \times 10^{-5} R_2^2 + 0.0882 R_2 - 16.7 = 0$$

which yields $R_2 = 175 \Omega$ and $R_1 = 70 \Omega$

4.44

$$(a) \quad R_{TH} = R_1 \parallel R_2 = 25.6 \parallel 10.4 = 7.40 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (V_{CC}) = \left(\frac{10.4}{10.4 + 25.6} \right) (18) = 5.2 \text{ V}$$

$$V_{TH} = I_{SQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{SQ}R_E$$

$$I_{SQ} = \frac{5.2 - 0.7}{7.40 + (126)(3)} = 0.0117 \text{ mA}$$

Then $I_{CQ} = 1.46 \text{ mA}$ and $I_{EQ} = 1.47 \text{ mA}$

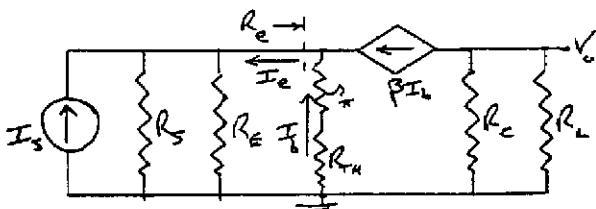
$$V_{CEQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E$$

$$V_{CEQ} = 18 - (1.46)(4) - (1.47)(3) \Rightarrow V_{CEQ} = 7.75 \text{ V}$$

(b)

$$r_x = \frac{(125)(0.026)}{1.46} = 2.23 \text{ k}\Omega$$

$$g_m = \frac{1.46}{0.026} = 56.2 \text{ mA/V}$$



$$R_e = \frac{r_x + R_{TH}}{1 + \beta} = \frac{2.23 + 7.40}{126} = 0.0764 \text{ k}\Omega$$

$$I_e = \frac{-(R_s \parallel R_E)}{(R_s \parallel R_E) + R_e} \cdot I_i = \frac{-(100 \parallel 3)}{(100 \parallel 3) + 0.0764} \cdot I_i$$

$$\text{or } I_e = -(0.974)I_i$$

$$V_o = -I_e(R_C \parallel R_L) = -\left(\frac{\beta}{1 + \beta} \right) I_i (R_C \parallel R_L)$$

Then

$$\frac{V_o}{I_s} = -\left(\frac{\beta}{1+\beta}\right)(-0.974)(R_C \parallel R_L) = \left(\frac{125}{126}\right)(0.974)(4 \parallel 4)$$

Then

$$R_m = \frac{V_o}{I_s} = 1.93 \text{ k}\Omega = 1.93 \text{ V / mA}$$

(c)

$$V_i = I_s(R_s \parallel R_E \parallel R_e) = I_s(100 \parallel 3 \parallel 0.0764) = I_s(0.0744)$$

$$\text{or } I_s = \frac{V_i}{0.0744}$$

$$\text{which yields } \frac{V_o}{I_s} = \frac{V_o}{V_i}(0.0744) = 1.93$$

$$\text{or } A_v = \frac{V_o}{V_i} = 25.9$$

4.45

$$(a) A_v = \frac{\beta(R_C \parallel R_L)}{r_e + R_1 \parallel R_2}, \quad R_L = 12 \text{ k}\Omega, \quad \beta = 100$$

$$\text{Let } R_1 \parallel R_2 = 50 \text{ k}\Omega, \quad I_{CQ} = 0.5 \text{ mA}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1+\beta)I_{BQ}R_E$$

$$I_{BQ} = \frac{0.5}{100} = 0.005 \text{ mA}, \quad r_e = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$\frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(50)(12) = (0.005)(50) + 0.7 \\ + (101)(0.005)(0.5)$$

$$\text{which yields } R_1 = 500 \text{ k}\Omega \text{ and } R_2 = 55.6 \text{ k}\Omega$$

$$A_v = \frac{(100)(12 \parallel 12)}{5.2 + 50} = 10.7, \quad \text{Design criterion is met.}$$

(b)

$$I_{CQ} = 0.5 \text{ mA}, \quad I_{EQ} = 0.505 \text{ mA}$$

$$V_{CEQ} = 12 - (0.5)(12) - (0.505)(0.5) \Rightarrow$$

$$V_{CEQ} = 5.75 \text{ V}$$

$$A_v = g_m(R_C \parallel R_L), \quad g_m = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$A_v = (19.23)(12 \parallel 12) \Rightarrow A_v = 115$$

4.46

$$i_s(\text{peak}) = 2.5 \mu\text{A}, \quad v_o(\text{peak}) = 5 \text{ mV}$$

$$\text{So we need } R_m = \frac{v_o}{i_s} = \frac{5 \times 10^{-3}}{2.5 \times 10^{-6}} = 2 \times 10^3 = 2 \text{ k}\Omega$$

From Problem 4.44

$$\frac{V_o}{I_s} = \left(\frac{\beta}{1+\beta}\right)(R_C \parallel R_L) \left(\frac{R_s \parallel R_E}{R_s \parallel R_E + R_e}\right)$$

$$\text{Let } R_C = 4 \text{ k}\Omega, R_L = 5 \text{ k}\Omega, R_E = 2 \text{ k}\Omega$$

Now $\beta = 120$, so we have

$$2 = \left(\frac{120}{121}\right)(4 \parallel 5) \left(\frac{R_s \parallel R_E}{R_s \parallel R_E + R_e}\right) = 2.20 \left(\frac{R_s \parallel R_E}{R_s \parallel R_E + R_e}\right)$$

$$\text{Then } \frac{R_s \parallel R_E}{R_s \parallel R_E + R_e} = 0.909$$

$$R_s \parallel R_E = 50 \parallel 2 = 1.92 \text{ k}\Omega, \text{ so that } R_e = 0.192 \text{ k}\Omega$$

$$\text{Assume } V_{CEQ} = 6 \text{ V}$$

$$V_{CC} \equiv I_{CQ}(R_C + R_E) + V_{CEQ}$$

$$12 = I_{CQ}(4 + 2) + 6 \Rightarrow I_{CQ} = 1 \text{ mA}$$

$$r_e = \frac{(120)(0.026)}{1} = 3.12 \text{ k}\Omega$$

$$R_e = \frac{r_e + R_{TH}}{1 + \beta} \Rightarrow 0.192 = \frac{3.12 + R_{TH}}{121}$$

$$\text{which yields } R_{TH} = 20.1 \text{ k}\Omega$$

$$\text{Now } V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + I_{EQ}R_E$$

$$I_{BQ} = \frac{1}{120} = 0.00833 \text{ mA}, \quad I_{EQ} = \left(\frac{121}{120}\right)(1) = 1.008 \text{ mA}$$

$$V_{TH} = \frac{1}{R_e} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_e}(20.1)(5) = (0.00833)(20.1) \\ + 0.7 + (1.008)(2)$$

$$\text{which yields } R_1 = 34.9 \text{ k}\Omega \text{ and } R_2 = 47.4 \text{ k}\Omega$$

4.47

a. Emitter current

$$I_{EQ} = I_{CC} = 0.5 \text{ mA}$$

$$I_{BQ} = \frac{0.5}{101} = 0.00495 \text{ mA}$$

$$V_E = I_{EQ}R_E = (0.5)(1) \Rightarrow V_E = 0.5 \text{ V}$$

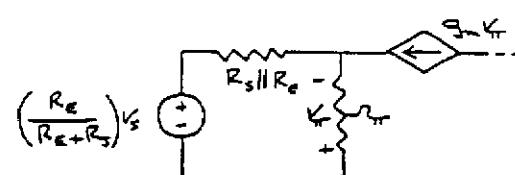
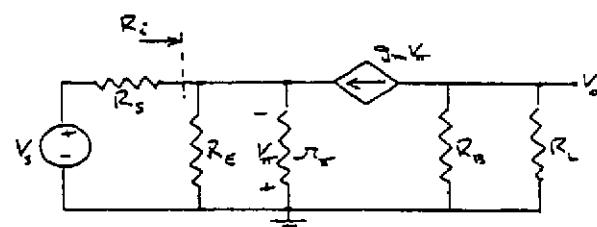
$$V_B = V_E + V_{BE}(\text{on}) = 0.5 + 0.7 \Rightarrow V_B = 1.20 \text{ V}$$

$$V_C = V_B + I_{BQ}R_B = 1.20 + (0.00495)(100)$$

$$\Rightarrow V_C = 1.7 \text{ V}$$

$$\text{b. } r_e = \frac{(100)(0.026)}{(100)(0.00495)} = 5.25 \text{ k}\Omega$$

$$g_m = \frac{(100)(0.00495)}{0.026} = 19.0 \text{ mA/V}$$



$$V_0 = -g_m V_\pi (R_E \parallel R_L)$$

$$g_m V_\pi + \frac{V_\pi}{r_\pi} + \frac{\left(\frac{R_E}{R_E + R_S}\right) V_\pi - (-V_\pi)}{R_S \parallel R_E} = 0$$

$$V_\pi \left[g_m + \frac{1}{r_\pi} + \frac{1}{R_S \parallel R_E} \right] = -\left(\frac{R_E}{R_E + R_S}\right) V_S$$

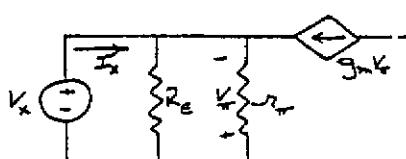
$$V_\pi \left[19.0 + \frac{1}{5.25} + \frac{1}{0.05 \parallel 1} \right] = -\left(\frac{1}{1 + 0.05}\right) V_S$$

$$V_\pi (40.19) = -20V_S \Rightarrow V_\pi = -(0.4976)V_S$$

$$V_0 = (19)(0.4976)V_S(100 \parallel 1)$$

$$\underline{A_V = 9.36}$$

c.



$$I_X = \frac{V_X}{R_E} + \frac{V_X}{r_\pi} - g_m V_\pi, \quad V_\pi = -V_X$$

$$\frac{I_X}{V_X} = \frac{1}{R_E} = \frac{1}{R_E} + \frac{1}{r_\pi} + g_m$$

$$\text{or } R_i = R_E \parallel r_\pi \parallel \frac{1}{g_m} = 1 \parallel 5.25 \parallel \frac{1}{19}$$

$$R_i = 0.84 \parallel 0.0526$$

$$\Rightarrow \underline{R_i = 49.5 \Omega}$$

4.48

$$\text{a. } I_{EQ} = \frac{20 - 0.7}{10} = 1.93 \text{ mA}$$

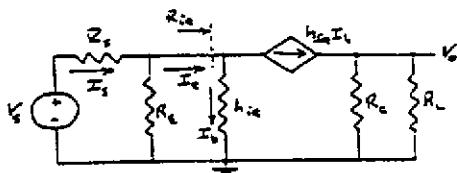
$$\underline{I_{EQ} = 1.91 \text{ mA}}$$

$$V_{ECQ} = V_{CC} + V_{EB(\text{on})} - I_C R_C$$

$$= 25 + 0.7 - (1.91)(6.5)$$

$$\Rightarrow \underline{V_{ECQ} = 13.3 \text{ V}}$$

b.



Neglect effect of h_{re}

From Problem 4-16, assume

$$2.45 \leq h_{ie} \leq 3.7 \text{ k}\Omega$$

$$80 \leq h_{fe} \leq 120$$

$$V_0 = (h_{fe} I_b)(R_C \parallel R_L)$$

$$R_{ie} = \frac{h_{ie}}{1 + h_{ie}}, \quad I_e = \left(\frac{R_E}{R_E + R_{ie}}\right) I_S$$

$$I_b = \left(\frac{I_e}{1 + h_{fe}}\right), \quad I_S = \frac{V_S}{R_S + R_E \parallel R_{ie}}$$

$$A_v = \left(\frac{h_{fe}}{1 + h_{fe}}\right) (R_C \parallel R_L) \left(\frac{R_E}{R_E + R_{ie}}\right) \times \left(\frac{1}{R_S + R_E \parallel R_{ie}}\right)$$

High gain device: $h_{ie} = 3.7 \text{ k}\Omega, h_{fe} = 120$

$$R_{ie} = \frac{3.7}{121} = 0.0306 \text{ k}\Omega$$

$$R_E \parallel R_{ie} = 10 \parallel 0.0306 = 0.0305$$

$$A_v = \left(\frac{120}{121}\right) (6.5 \parallel 5) \left(\frac{10}{10 + 0.0306}\right) \left(\frac{1}{1 + 0.0305}\right)$$

$$\Rightarrow \underline{A_v = 2.71}$$

Low gain device: $h_{ie} = 2.45 \text{ k}\Omega, h_{fe} = 80$

$$R_{ie} = \frac{2.45}{81} = 0.03025 \text{ k}\Omega$$

$$R_E \parallel R_{ie} = 10 \parallel 0.03025 = 0.0302$$

$$A_v = \left(\frac{80}{81}\right) (6.5 \parallel 5) \left(\frac{10}{10 + 0.03025}\right) \left(\frac{1}{1 + 0.0302}\right)$$

$$\Rightarrow \underline{A_v = 2.70} \text{ So } A_v \approx \text{constant}$$

$$\underline{2.70 \leq A_v \leq 2.71}$$

$$\text{c. } R_i = R_E \parallel R_{ie}$$

$$\text{We found } \underline{0.0302 \leq R_i \leq 0.0305 \text{ k}\Omega}$$

$$\text{Neglecting } h_{re}, \quad R_o = R_C = 6.5 \text{ k}\Omega$$

4.49

a. Small-signal voltage gain

$$A_v = g_m (R_C \parallel R_L) \Rightarrow 25 = g_m (R_C \parallel 1)$$

$$\text{For } V_{ECQ} = 3 \text{ V}$$

$$\Rightarrow V_C = -V_{ECQ} + V_{EB(\text{on})} = -3 + 0.7$$

$$\Rightarrow V_C = -2.3$$

$$V_{CC} - I_{CQ} R_C + V_C = 0$$

$$\Rightarrow I_{CQ} = \frac{5 - 2.3}{R_C} = \frac{2.7}{R_C} = I_{CQ}$$

$$\text{For } I_{CQ} = 1 \text{ mA, } \underline{R_C = 2.7 \text{ k}\Omega}$$

$$g_m = \frac{1}{0.026} = 38.5 \text{ mA/V}$$

$$A_v = (38.5)(2.7 \parallel 1) = 28.1$$

Design criterion satisfied and V_{ECQ} satisfied.

$$I_E = \left(\frac{101}{100} \right) (1) = 1.01 \text{ mA}$$

$$V_{EB} = I_E R_E + V_{EB}(\text{on})$$

$$\Rightarrow R_E = \frac{5 - 0.7}{1.01} \Rightarrow R_E = 4.26 \text{ k}\Omega$$

$$\text{b. } r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1}$$

$$\Rightarrow r_\pi = 2.6 \text{ k}\Omega, g_m = 38.5 \text{ mA/V}, r_0 = \infty$$

4.50

$$\text{a. } V_{TH1} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{20}{20 + 80} \right) (10)$$

$$\Rightarrow V_{TH1} = 2.0 \text{ V}$$

$$R_{TH1} = R_1 \| R_2 = 20 \| 80 = 16 \text{ k}\Omega$$

$$I_{B1} = \frac{2 - 0.7}{16 + (101)(1)} = 0.0111 \text{ mA}$$

$$I_{C1} = 1.11 \text{ mA}$$

$$\Rightarrow g_{m1} = \frac{1.11}{0.026} \Rightarrow g_{m1} = 42.7 \text{ mA/V}$$

$$r_{\pi1} = \frac{(100)(0.026)}{1.11} \Rightarrow r_{\pi1} = 2.34 \text{ k}\Omega$$

$$r_{o1} = \frac{\infty}{1.11} \Rightarrow r_{o1} = \infty$$

$$V_{TH2} = \left(\frac{R_4}{R_3 + R_4} \right) V_{CC} = \left(\frac{15}{15 + 85} \right) (10) = 1.50 \text{ V}$$

$$R_{TH2} = R_3 \| R_4 = 15 \| 85 = 12.75 \text{ k}\Omega$$

$$I_{B2} = \frac{1.50 - 0.70}{12.75 + (101)(0.5)} = 0.0126 \text{ mA}$$

$$I_{C2} = 1.26 \text{ mA}$$

$$\Rightarrow g_{m2} = \frac{1.26}{0.026} \Rightarrow g_{m2} = 48.5 \text{ mA/V}$$

$$r_{\pi2} = \frac{(100)(0.026)}{1.26} \Rightarrow r_{\pi2} = 2.06 \text{ k}\Omega$$

$$r_{o2} = \infty$$

$$\text{b. } A_{v1} = -g_{m1} R_{C1} = -(42.7)(2)$$

$$\Rightarrow A_{v1} = -85.4$$

$$A_{v2} = -g_{m2}(R_{C2} \| R_L) = -(48.5)(4 \| 4)$$

$$\Rightarrow A_{v2} = -97$$

c. Input resistance of 2nd stage

$$R_{i2} = R_3 \| R_4 \| r_{\pi2} = 15 \| 85 \| 2.06$$

$$= 12.75 \| 2.06 \Rightarrow R_{i2} = 1.77 \text{ k}\Omega$$

$$A'_{v1} = -g_{m1}(R_{C1} \| R_{i2}) = -(42.7)(2 \| 1.77)$$

$$A'_{v1} = -40.1$$

Overall gain: $A_v = (-40.1)(-97) \Rightarrow A_v = 3890$ If we had $A_{v1} \cdot A_{v2} = (-85.4)(-97) = 8284$

Loading effect reduces overall gain

4.51

$$\text{a. } V_{TH1} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{12.7}{12.7 + 67.3} \right) (12)$$

$$\Rightarrow V_{TH1} = 1.905 \text{ V}$$

$$R_{TH1} = R_1 \| R_2 = 12.7 \| 67.3 = 10.68 \text{ k}\Omega$$

$$I_{B1} = \frac{1.905 - 0.70}{10.68 + (121)(2)} = 0.00477 \text{ mA}$$

$$I_{C1} = 0.572 \text{ mA}$$

$$g_{m1} = \frac{0.572}{0.026} \Rightarrow g_{m1} = 22 \text{ mA/V}$$

$$r_{\pi1} = \frac{(120)(0.026)}{0.572} \Rightarrow r_{\pi1} = 5.45 \text{ k}\Omega$$

$$r_{o1} = \frac{\infty}{0.572} \Rightarrow r_{o1} = \infty$$

$$V_{TH2} = \left(\frac{R_4}{R_3 + R_4} \right) V_{CC} = \left(\frac{45}{45 + 15} \right) (12)$$

$$\Rightarrow V_{TH2} = 9.0 \text{ V}$$

$$R_{TH2} = R_3 \| R_4 = 15 \| 45 = 11.25 \text{ k}\Omega$$

$$I_{B2} = \frac{9.0 - 0.70}{11.25 + (121)(1.6)} = 0.0405 \text{ mA}$$

$$I_{C2} = 4.86 \text{ mA}$$

$$g_{m2} = \frac{4.86}{0.026} \Rightarrow g_{m2} = 187 \text{ mA/V}$$

$$r_{\pi2} = \frac{(120)(0.026)}{4.86} \Rightarrow r_{\pi2} = 0.642 \text{ k}\Omega$$

$$r_{o2} = \infty$$

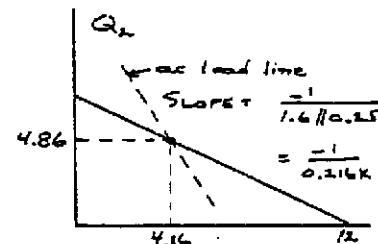
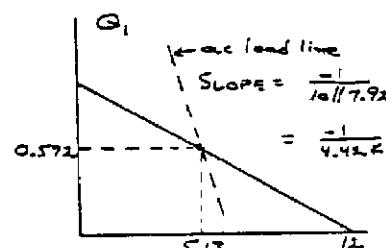
$$\text{b. } I_{E1} = 0.577 \text{ mA}$$

$$V_{CEQ1} = 12 - (0.572)(10) - (0.577)(2)$$

$$\Rightarrow V_{CEQ1} = 5.13 \text{ V}$$

$$I_{E2} = 4.90$$

$$V_{CEQ2} = 12 - (4.90)(1.6) \Rightarrow V_{CEQ2} = 4.16 \text{ V}$$



$$R_{i2} = R_3 \parallel R_4 \parallel R_{i3}$$

$$R_{i3} = r_{\pi 2} + (1 + \beta)(R_{E2} \parallel R_L)$$

$$= 0.642 + (121)(1.6 \parallel 0.25)$$

$$R_{i3} = 26.8$$

$$R_{i2} = 15 \parallel 45 \parallel 26.8$$

$$R_{i2} = 7.92 \text{ k}\Omega$$

$$c. A_{v1} = -g_{m1}(R_{C1} \parallel R_{i2}) = -(22)(10 \parallel 7.92)$$

$$\Rightarrow A_{v2} = -97.2$$

$$A_{v2} = \frac{(1 + \beta)(R_{E2} \parallel R_L)}{r_{\pi 2} + (1 + \beta)(R_{E2} \parallel R_L)}$$

$$= \frac{(121)(0.216)}{0.642 + (121)(0.216)} = 0.976$$

$$\text{Overall gain} = (-97.2)(0.976) = \underline{-94.9}$$

$$d. R_{i5} = R_1 \parallel R_2 \parallel r_{\pi 1} = 67.3 \parallel 12.7 \parallel 5.45$$

$$\Rightarrow \underline{R_{i5} = 3.61 \text{ k}\Omega}$$

$$R_0 = \frac{r_{\pi 2} + R_S}{1 + \beta} \parallel R_{E2} \text{ where}$$

$$R_S = R_3 \parallel R_4 \parallel R_{C1}$$

$$= 15 \parallel 45 \parallel 10 \Rightarrow R_S = 5.29 \text{ k}\Omega$$

$$R_0 = \frac{0.642 + 5.29}{121} \parallel 1.6 \Rightarrow 0.049 \parallel 1.6$$

$$\Rightarrow \underline{R_0 = 47.5 \Omega}$$

$$e. \Delta i_C = \frac{-1}{0.216 \text{ k}\Omega} \cdot \Delta v_{ce}. \Delta i_C = 4.86$$

$$|\Delta v_{ce}| = (4.86)(0.216) = 1.05 \text{ V}$$

$$\begin{aligned} \text{Max. output voltage swing} \\ = \underline{2.10 \text{ V peak-to-peak}} \end{aligned}$$

4.52

$$(a) I_{R1} = \frac{5 - 2(0.7)}{0.050} = 72 \text{ mA}$$

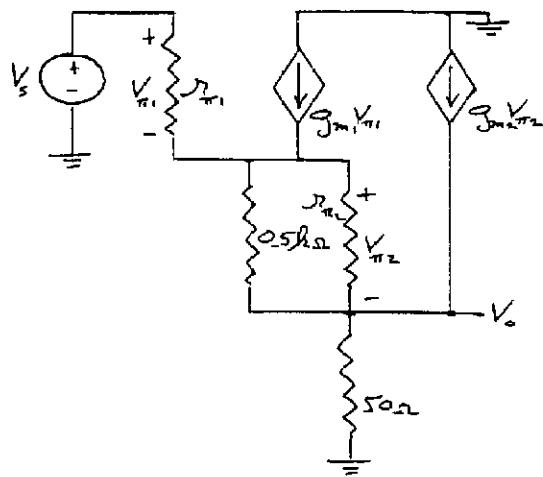
$$I_{R2} = \frac{0.7}{0.5} = 1.4 \text{ mA}$$

$$I_{C2} = \left(\frac{\beta}{1 + \beta} \right) (72 - 1.4) \Rightarrow \underline{I_{C2} = 69.9 \text{ mA}}$$

$$I_{S1} = \frac{69.9}{100} = 0.699 \text{ mA}$$

$$I_{C1} = \left(\frac{\beta}{1 + \beta} \right) (1.4 + 0.699) \Rightarrow \underline{I_{C1} = 2.08 \text{ mA}}$$

(b)



$$V_s = V_{\pi 1} + V_{\pi 2} + V_o$$

$$(1) V_o = \left(\frac{V_{\pi 2}}{0.5} + \frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2} V_{\pi 2} \right) (0.05)$$

$$r_{\pi 2} = \frac{(100)(0.026)}{69.9} = 0.0372 \text{ k}\Omega$$

$$g_{m2} = \frac{69.9}{0.026} = 2688 \text{ mA/V}$$

$$V_o = V_{\pi 2} \left(\frac{1}{0.05} + \frac{1}{0.0372} + 2688 \right) (0.05)$$

$$\text{so that (1)} V_{\pi 2} = \frac{V_o}{136.7}$$

$$(2) \frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} = \frac{V_{\pi 2}}{0.5} + \frac{V_{\pi 2}}{r_{\pi 2}}$$

$$r_{\pi 1} = \frac{(100)(0.026)}{2.08} = 1.25 \text{ k}\Omega$$

$$g_{m1} = \frac{2.08}{0.026} = 80 \text{ mA/V}$$

$$V_{\pi 1} \left(\frac{1}{1.25} + 80 \right) = V_{\pi 2} \left(\frac{1}{0.5} + \frac{1}{0.0372} \right)$$

$$V_{\pi 1}(80.8) = V_{\pi 2}(28.88) = \left(\frac{V_o}{136.7} \right) (28.88)$$

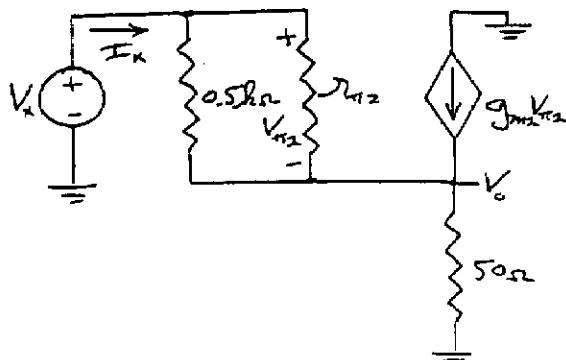
$$\text{or (2)} V_{\pi 1} = V_o (0.00261)$$

Then

$$V_s = V_o (0.00261) + \frac{V_o}{136.7} + V_o = V_o (1.00993)$$

$$\text{or } A_v = \frac{V_o}{V_s} = 0.990$$

$$(c) \underline{R_{i3} = r_{\pi 1} + (1 + \beta)[R_i]}$$



$$I_x = \frac{V_{x1}}{0.5} + \frac{V_{x2}}{r_{x2}} = V_{x1} \left(\frac{1}{0.5} + \frac{1}{r_{x2}} \right)$$

$$\frac{V_o}{0.05} = \frac{V_x - V_{x2}}{0.05} = I_x + g_{m2}V_{x2}$$

$$\frac{V_x}{0.05} - I_x = V_{x2} \left(\frac{1}{0.05} + g_{m2} \right) = \frac{I_x \left(\frac{1}{0.05} + g_{m2} \right)}{\left(\frac{1}{0.05} + g_{m2} \right)}$$

We find $\frac{V_x}{I_x} = R_x = 2.89\text{k}\Omega$

Then $R_{ab} = 1.25 + (101)(2.89) \Rightarrow R_{ab} = 293\text{k}\Omega$

$$\text{Now } I_x = \frac{V_x}{0.05} - V_{x2} \left(g_{m2} + \frac{1}{0.05|r_{x2}|} \right)$$

So that

$$I_x = \frac{V_x}{0.05} + (0.633)V_x \left[2688 + \frac{1}{0.05|0.0372|} \right]$$

which yields

$$R_o = \frac{V_x}{I_x} = 0.583\Omega$$

4.53

$$\text{a. } R_{TH} = R_1 \parallel R_2 = 335 \parallel 125 = 91.0\text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC}$$

$$= \left(\frac{125}{125 + 335} \right) (10) = 2.717\text{ V}$$

$$V_{TH} = I_{B1} R_{TH} + V_{BE1} + V_{BE2} + I_{E2} R_{E2}$$

$$I_{E2} = (1 + \beta) I_{E1} = (1 + \beta)^2 I_{B1}$$

$$I_{B1} = \frac{2.717 - 1.40}{91.0 + (101)^2(1)} \Rightarrow I_{B1} = 0.128\text{ }\mu\text{A}$$

$$I_{C1} = 12.8\text{ }\mu\text{A}$$

$$I_{C2} = \beta I_{E1} = \beta(1 + \beta) I_{B1} = (100)(101)(0.128\text{ }\mu\text{A})$$

$$I_{C2} = 1.29\text{ mA}, \quad I_{E2} = 1.31\text{ mA}$$

$$I_{RC} = I_{C2} + I_{C1} = 1.29 + 0.0128 = 1.31\text{ mA}$$

$$V_C = 10 - I_{RC} R_C = 10 - (1.31)(2.2) = 7.12\text{ V}$$

$$V_E = I_{E2} R_{E2} = (1.31)(1) = 1.31\text{ V}$$

$$V_{CE2} = 7.12 - 1.31 = 5.81\text{ V}$$

$$V_{CE1} = V_{CE2} - V_{BE2} = 5.81 - 0.7$$

$$V_{CE1} = 5.11\text{ V}$$

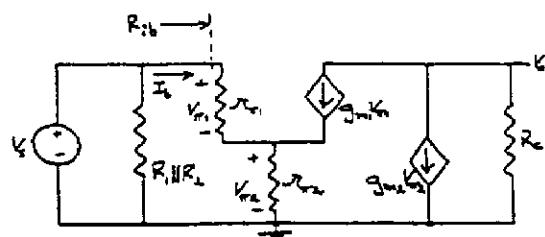
Summary:

$$I_{C1} = 12.8\text{ }\mu\text{A} \quad I_{C2} = 1.29\text{ mA}$$

$$V_{CE1} = 5.11\text{ V} \quad V_{CE2} = 5.81\text{ V}$$

$$\text{b. } g_{m1} = \frac{0.0128}{0.026} = 0.492\text{ mA/V}$$

$$g_{m2} = \frac{1.29}{0.026} = 49.6\text{ mA/V}$$



To find R_o :

$$(1) \quad I_x = \frac{V_x}{0.05} - g_{m2}V_{x2} - \frac{V}{0.05|r_{x2}|}$$

$$(2) \quad V_{x2} = \left(\frac{V_{x1}}{r_{x1}} + g_{m1}V_{x1} \right) \left(0.05|r_{x2}| \right)$$

$$= V_{x1} \left(\frac{1}{125} + 80 \right) \left(0.05 |0.0372| \right)$$

$$\text{or } V_{x2} = (1.72)V_{x1}$$

$$(3) \quad V_{x1} + V_{x2} + V_x = 0 \Rightarrow V_{x1} + (1.72)V_{x1} + V_x = 0$$

$$\text{so that } V_{x1} = -(0.368)V_x$$

$$\text{and } V_{x2} = (1.72)[-(0.368)V_x] = -(0.633)V_x$$

$$V_0 = -(g_{m1}V_{\pi 1} + g_{m2}V_{\pi 2})R_C$$

$$V_S = V_{\pi 1} + V_{\pi 2}, \quad V_{\pi 1} = V_S - V_{\pi 2}$$

$$V_{\pi 2} = \left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} \right) r_{\pi 2}$$

$$V_{\pi 2} = V_{\pi 1} \left(\frac{1 + \beta}{r_{\pi 1}} \right) r_{\pi 2}$$

$$r_{\pi 1} = \frac{(100)(0.026)}{0.0128} = 203 \text{ k}\Omega$$

$$r_{\pi 2} = \frac{(100)(0.026)}{1.29} = 2.02 \text{ k}\Omega$$

$$V_0 = -[g_{m1}(V_S - V_{\pi 2}) + g_{m2}V_{\pi 2}]R_C$$

$$V_0 = -[g_{m1}V_S + (g_{m2} - g_{m1})V_{\pi 2}]R_C$$

$$V_{\pi 2} = (V_S - V_{\pi 2})(1 + \beta) \left(\frac{r_{\pi 2}}{r_{\pi 1}} \right)$$

$$V_{\pi 2} \left[1 + (1 + \beta) \left(\frac{r_{\pi 2}}{r_{\pi 1}} \right) \right] = V_S(1 + \beta) \left(\frac{r_{\pi 2}}{r_{\pi 1}} \right)$$

$$V_0 = - \left\{ g_{m1}V_S + (g_{m2} - g_{m1}) \cdot \frac{V_S(1 + \beta) \left(\frac{r_{\pi 2}}{r_{\pi 1}} \right)}{1 + (1 + \beta) \left(\frac{r_{\pi 2}}{r_{\pi 1}} \right)} \right\} R_C$$

$$A_v = \frac{V_0}{V_S}$$

$$= - \left\{ (0.492) + \frac{(49.6 - 0.492)(101) \left(\frac{2.02}{203} \right)}{1 + (101) \left(\frac{2.02}{203} \right)} \right\} 2.2$$

$$A_v = -55.2$$

$$\text{c. } R_{i_s} = R_1 \parallel R_2 \parallel R_{i_b}$$

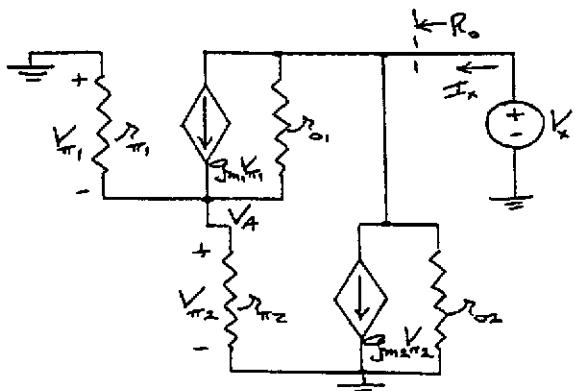
$$R_{i_b} = r_{\pi 1} + (1 + \beta)r_{\pi 2}$$

$$= 203 + (101)(2.02) = 407 \text{ k}\Omega$$

$$R_{i_s} = 91 \parallel 407 = \underline{74.4 \text{ k}\Omega} = R_{i_s}$$

$$R_0 = R_C = 2.2 \text{ k}\Omega$$

4.54



$$(1) I_x = g_{m2}V_{\pi 2} + \frac{V_x}{r_{\pi 2}} + \frac{V_x - V_A}{r_{o1}} + g_{m1}V_{\pi 1}$$

$$(2) \frac{V_x - V_A}{r_{o1}} + g_{m1}V_{\pi 1} = \frac{V_A}{r_{\pi 1} \parallel r_{\pi 2}}$$

$$(3) V_{\pi 2} = V_A = -V_{\pi 1}$$

Then from (2)

$$\frac{V_x}{r_{o1}} = V_A \left(\frac{1}{r_{\pi 1}} + g_{m1} + \frac{1}{r_{\pi 1} \parallel r_{\pi 2}} \right)$$

$$(1) I_x = g_{m2}V_A + \frac{V_x}{r_{\pi 2}} + \frac{V_x - V_A}{r_{o1}} - g_{m1}V_A$$

$$\text{or } I_x = V_x \left(\frac{1}{r_{\pi 1}} + \frac{1}{r_{\pi 2}} \right) + V_A \left(g_{m2} - \frac{1}{r_{\pi 1}} - g_{m1} \right)$$

Solving for V_A from Equation (2) and substituting into Equation (1), we find

$$R_o = \frac{V_x}{I_x} = \frac{\frac{1}{r_{\pi 1}} + g_{m1} + \frac{1}{r_{\pi 1} \parallel r_{\pi 2}}}{\frac{1}{r_{\pi 2}} \left(\frac{1}{r_{\pi 1}} + g_{m1} + \frac{1}{r_{\pi 1} \parallel r_{\pi 2}} \right) + \frac{1}{r_{\pi 1}} \left(\frac{1}{r_{\pi 1} \parallel r_{\pi 2}} + g_{m2} \right)}$$

For $\beta = 100$, $V_A = 100 \text{ V}$, $I_{C1} = I_{B1w} = 1 \text{ mA}$

$$r_{\pi 1} = r_{\pi 2} = \frac{100}{1} = 100 \text{ k}\Omega$$

$$r_{\pi 1} = r_{\pi 2} = \frac{(100)(0.026)}{1} = 2.6 \text{ k}\Omega$$

$$g_{m1} = g_{m2} = \frac{1}{0.026} = 38.46 \text{ mA/V}$$

Then

$$R_o = \frac{\frac{1}{100} + 38.46 + \frac{1}{2.6 \parallel 2.6}}{\frac{1}{100} \left(\frac{1}{100} + 38.46 + \frac{1}{2.6 \parallel 2.6} \right) + \frac{1}{100} \left(\frac{1}{2.6 \parallel 2.6} + 38.46 \right)}$$

or

$$R_o = 50.0 \text{ k}\Omega$$

Now

$$I_{C2} = 1 \text{ mA}, I_{B1w} = 0$$

$$\text{Replace } I_{B1w} \text{ by } \frac{I_{C2}}{\beta} \cdot \frac{\beta}{1 + \beta} = \frac{I_{C2}}{1 + \beta}, \quad I_{C1} \approx 0.01 \text{ mA}$$

$$r_{\pi 2} = \frac{100}{1} = 100 \text{ k}\Omega, r_{\pi 1} = \frac{100}{0.01} = 10,000 \text{ k}\Omega$$

$$g_{m1} = \frac{1}{0.026} = 38.46 \text{ mA/V}, g_{m2} = 0.3846 \text{ mA/V}$$

$$r_{\pi 2} = \frac{(100)(0.026)}{1} = 2.6 \text{ k}\Omega, r_{\pi 1} = 260 \text{ k}\Omega$$

Then

$$R_o = 66.4 \text{ k}\Omega$$

4.55

$$a. \quad R_{TH} = R_1 \parallel R_2 = 93.7 \parallel 6.3 = 5.90 \text{ k}\Omega$$

$$\begin{aligned} V_{TH} &= \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} \\ &= \left(\frac{6.3}{6.3 + 93.7} \right) (12) = 0.756 \text{ V} \\ I_{BQ} &= \frac{0.756 - 0.70}{5.90} = 0.00949 \text{ mA} \\ I_{CQ} &= 0.949 \text{ mA} \\ V_{CEQ} &= 12 - (0.949)(6) \Rightarrow V_{CEQ} = 6.31 \text{ V} \end{aligned}$$

Transistor:

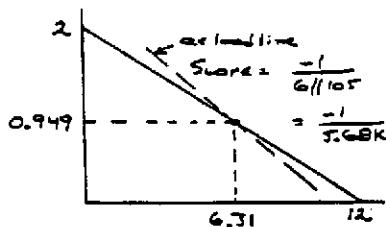
$$P_Q \approx I_{CQ} V_{CEQ} = (0.949)(6.31)$$

$$\Rightarrow P_Q = 5.99 \text{ mW}$$

$$R_C : \quad P_R = I_{CQ}^2 R_C = (0.949)^2 (6)$$

$$\Rightarrow P_R = 5.40 \text{ mW}$$

b.



$$\tau_0 = \frac{100}{0.949} = 105 \text{ k}\Omega$$

Peak signal current = 0.949 mA

$$|V_0(\max)| = (5.68)(0.949) = 5.39 \text{ V}$$

$$\begin{aligned} P_{RC} &= \frac{1}{2} \cdot \frac{V_0^2(\max)}{R_C} = \frac{1}{2} \left[\frac{(5.39)^2}{6} \right] \\ \Rightarrow P_{RC} &= 2.42 \text{ mW} \end{aligned}$$

4.56

$$(a) \quad 10 = I_{SQ} R_S + V_{BE(on)} + (1+\beta) I_{SQ} R_E$$

$$I_{SQ} = \frac{10 - 0.7}{100 + (12)(20)} = 0.00369 \text{ mA}$$

$$I_{CQ} = 0.443 \text{ mA}, \quad I_{EQ} = 0.447 \text{ mA}$$

$$\text{For } R_C: \quad P_{RC} = (0.443)^2 (10) \Rightarrow P_{RC} = 1.96 \text{ mW}$$

$$\text{For } R_E: \quad P_{RE} = (0.447)^2 (20) \Rightarrow P_{RE} = 4.0 \text{ mW}$$

(b)

$$\Delta i_C = 0.443 \text{ mA}, \quad \Delta v_{CE} = (0.443)(10) = 4.43 \text{ V}$$

Then

$$\bar{P}_{RC} = \frac{1}{2} (\Delta i_C)^2 R_C = \frac{1}{2} (0.443)^2 (10)$$

$$\bar{P}_{RC} = 0.981 \text{ mW}$$

4.57

$$a. \quad I_{BQ} = \frac{10 - 0.7}{50 + (151)(10)} = 0.00596 \text{ mA}$$

$$I_{CQ} = 0.894 \text{ mA}, \quad I_{EQ} = 0.90 \text{ mA}$$

$$V_{ECQ} = 20 - (0.894)(5) - (0.90)(10)$$

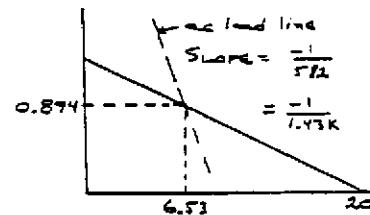
$$\Rightarrow V_{ECQ} = 6.53 \text{ V}$$

$$P_Q \approx I_{CQ} V_{CEQ} = (0.894)(6.53) \Rightarrow P_Q = 5.84 \text{ mW}$$

$$P_{RC} \approx I_{CQ}^2 R_C = (0.894)^2 (5) \Rightarrow P_{RC} = 4.0 \text{ mW}$$

$$P_{RE} \approx I_{EQ}^2 R_E = (0.90)^2 (10) \Rightarrow P_{RE} = 8.1 \text{ mW}$$

b.



$$\Delta i_C = \frac{-1}{1.43 \text{ k}\Omega} \cdot \Delta v_{CE}$$

$$\Delta i_C = 0.894 \Rightarrow |\Delta v_{CE}| = (0.894)(1.43) = 1.28 \text{ V}$$

$$\Delta i_O = \left(\frac{5}{5+2} \right) \Delta i_C = 0.639 \text{ mA}$$

$$\bar{P}_{RL} = \frac{1}{2} (0.639)^2 (2) \Rightarrow \bar{P}_{RL} = 0.408 \text{ mW}$$

$$\bar{P}_{RC} = \frac{1}{2} (0.894 - 0.639)^2 (5) \Rightarrow \bar{P}_{RC} = 0.163 \text{ mW}$$

$$\bar{P}_{RE} = 0$$

$$\bar{P}_Q = 5.84 - 0.408 - 0.163 \Rightarrow \bar{P}_Q = 5.27 \text{ mW}$$

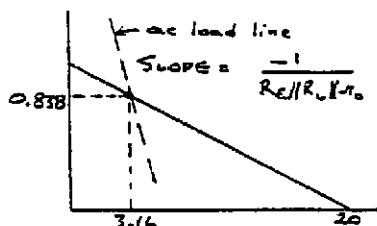
4.58

$$I_{BQ} = \frac{10 - 0.70}{100 + (101)(10)} = 0.00838 \text{ mA}$$

$$I_{CQ} = 0.838 \text{ mA}, \quad I_{EQ} = 0.846 \text{ mA}$$

$$V_{CEQ} = 20 - (0.838)(10) - (0.846)(10)$$

$$\Rightarrow V_{CEQ} = 3.16 \text{ V}$$



$$r_0 = \frac{100}{0.838} = 119 \text{ k}\Omega$$

Neglecting base currents:

a. $R_L = 1 \text{ k}\Omega$

$$\text{slope} = \frac{-1}{10 \parallel 1 \parallel 119} = \frac{-1}{0.902 \text{ k}\Omega}$$

$$\Delta i_C = \frac{-1}{0.902 \text{ k}\Omega} \cdot \Delta V_{ce}$$

$$\Delta i_C = 0.838 \Rightarrow |\Delta V_{ce}| = (0.902)(0.838)$$

$$= 0.756 \text{ V}$$

$$\overline{P_{RL}} = \frac{1}{2} \frac{(0.756)^2}{1} \Rightarrow \overline{P_{RL}} = 0.285 \text{ mW}$$

b. $R_L = 10 \text{ k}\Omega$

$$\text{slope} = \frac{-1}{10 \parallel 10 \parallel 119} = \frac{-1}{4.80}$$

For

$$\Delta i_C = 0.838 \Rightarrow |\Delta V_{ce}| = (0.838)(4.80) = 4.02$$

Max. swing determined by voltage

$$\overline{P_{RL}} = \frac{1}{2} \frac{(3.16)^2}{10} \Rightarrow \overline{P_{RL}} = 0.499 \text{ mW}$$

4.59

a. $I_{BQ} = \frac{10 - 0.7}{100 + (101)(10)} = 0.00838 \text{ mA}$

$$I_{CQ} = 0.838 \text{ mA}, \quad I_{EQ} = 0.846 \text{ mA}$$

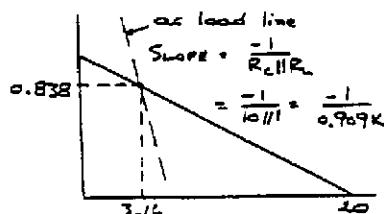
$$V_{CEQ} = 20 - (0.838)(10) - (0.846)(10)$$

$$\Rightarrow V_{CEQ} = 3.16 \text{ V}$$

$$P_Q \cong I_{CQ} V_{CEQ} = (0.838)(3.16) \Rightarrow P_Q = 2.65 \text{ mW}$$

$$P_{RC} \cong I_{CQ}^2 R_C = (0.838)^2 (10) \Rightarrow P_{RC} = 7.02 \text{ mW}$$

b.



$$\Delta i_C = \frac{-1}{0.909 \text{ k}\Omega} \cdot \Delta V_{ce}$$

For

$$\Delta i_C = 0.838 \Rightarrow |\Delta V_{ce}| = (0.909)(0.838) = 0.762 \text{ V}$$

$$\Delta i_0 = \left(\frac{R_C}{R_C + R_L} \right) \Delta i_C = \left(\frac{10}{10 + 1} \right) \Delta i_C = 0.762 \text{ mA}$$

$$\overline{P_{RL}} = \frac{1}{2} (0.762)^2 (1) \Rightarrow \overline{P_{RL}} = 0.290 \text{ mW}$$

$$\overline{P_{RC}} = \frac{1}{2} \cdot (0.838 - 0.762)^2 (10) \Rightarrow \overline{P_{RC}} = 0.0289 \text{ mW}$$

$$\overline{P_Q} = 2.65 - 0.290 - 0.0289 \Rightarrow \overline{P_Q} = 2.33 \text{ mW}$$

Chapter 5

Exercise Solutions

E5.1

(a) $V_{TN} = 1.2 \text{ V}$, $V_{GS} = 2 \text{ V}$

$V_{DS}(sat) = V_{GS} - V_{TN} = 2 - 1.2 = 0.8 \text{ V}$

(i) $V_{DS} = 0.4 \Rightarrow$ Nonsaturation(ii) $V_{DS} = 1 \Rightarrow$ Saturation(iii) $V_{DS} = 5 \Rightarrow$ Saturation

(b) $V_{TN} = -1.2 \text{ V}$, $V_{GS} = 2 \text{ V}$

$V_{DS}(sat) = V_{GS} - V_{TN} = 2 - (-1.2) = 3.2 \text{ V}$

(i) $V_{DS} = 0.4 \Rightarrow$ Nonsaturation(ii) $V_{DS} = 1 \Rightarrow$ Nonsaturation(iii) $V_{DS} = 5 \Rightarrow$ Saturation

E5.2

(a) $K_n = \frac{W\mu_n C_{ox}}{2L}$

$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{450 \times 10^{-8}} = 7.67 \times 10^{-8} \text{ F/cm}^2$

$K_n = \frac{(100)(500)(7.67 \times 10^{-8})}{2(7)} \Rightarrow K_n = 0.274 \text{ mA/V}^2$

(b) $V_{TN} = 1.2 \text{ V}$, $V_{GS} = 2 \text{ V}$

(i) $V_{DS} = 0.4 \text{ V} \Rightarrow$ Nonsaturation

$I_D = (0.274)[2(2-1.2)(0.4)-(0.4)^2] \Rightarrow I_D = 0.132 \text{ mA}$

(ii) $V_{DS} = 1 \text{ V} \Rightarrow$ Saturation

$I_D = (0.274)(2-1.2)^2 \Rightarrow I_D = 0.175 \text{ mA}$

(iii) $V_{DS} = 5 \text{ V} \Rightarrow$ Saturation

$I_D = (0.274)(2-1.2)^2 \Rightarrow I_D = 0.175 \text{ mA}$

$V_{TN} = -1.2 \text{ V}$, $V_{GS} = 2 \text{ V}$

(i) $V_{DS} = 0.4 \text{ V} \Rightarrow$ Nonsaturation

$I_D = (0.274)[2(2+1.2)(0.4)-(0.4)^2] \Rightarrow I_D = 0.658 \text{ mA}$

(ii) $V_{DS} = 1 \text{ V} \Rightarrow$ Nonsaturation

$I_D = (0.274)[2(2+1.2)(1)-(1)^2] \Rightarrow I_D = 1.48 \text{ mA}$

(iii) $V_{DS} = 5 \text{ V} \Rightarrow$ Saturation

$I_D = (0.274)(2+1.2)^2 \Rightarrow I_D = 2.81 \text{ mA}$

E5.3

$V_{TN} = 1 \text{ V}$, $V_{GS} = 3 \text{ V}$, $V_{DS} = 4.5 \text{ V}$

$V_{DS} = 4.5 > V_{DS}(sat) = V_{GS} - V_{TN} = 3 - 1 = 2 \text{ V}$

Transistor biased in the saturation region

$I_D = K_n(V_{GS} - V_{TN})^2 \Rightarrow 0.8 = K_n(3-1)^2 \Rightarrow$

$K_n = 0.2 \text{ mA/V}^2$

(a) $V_{GS} = 2 \text{ V}$, $V_{DS} = 4.5 \text{ V}$

Saturation region:

$I_D = (0.2)(2-1)^2 \Rightarrow I_D = 0.2 \text{ mA}$

(b) $V_{GS} = 3 \text{ V}$, $V_{DS} = 1 \text{ V}$

Nonsaturation region:

$I_D = (0.2)[2(3-1)(1)-(1)^2] \Rightarrow I_D = 0.6 \text{ mA}$

E5.4

(a) $V_{TP} = -2 \text{ V}$, $V_{SG} = 3 \text{ V}$

$V_{SD}(sat) = V_{SG} + V_{TP} = 3 - 2 = 1 \text{ V}$

(i) $V_{SD} = 0.5 \text{ V} \Rightarrow$ Nonsaturation(ii) $V_{SD} = 2 \text{ V} \Rightarrow$ Saturation(iii) $V_{SD} = 5 \text{ V} \Rightarrow$ Saturation

(b) $V_{TP} = 0.5 \text{ V}$, $V_{SG} = 3 \text{ V}$

$V_{SD}(sat) = V_{SG} + V_{TP} = 3 + 0.5 = 3.5 \text{ V}$

(i) $V_{SD} = 0.5 \text{ V} \Rightarrow$ Nonsaturation(ii) $V_{SD} = 2 \text{ V} \Rightarrow$ Nonsaturation(iii) $V_{SD} = 5 \text{ V} \Rightarrow$ Saturation

E5.5

(a) $\lambda = 0$, $V_{DS}(sat) = 2.5 - 0.8 = 1.7 \text{ V}$

For $V_{DS} = 2 \text{ V}$, $V_{DS} = 10 \text{ V} \Rightarrow$ Saturation Region

$I_D = (0.1)(2.5-0.8)^2 \Rightarrow I_D = 0.289 \text{ mA}$

(b) $\lambda = 0.02 \text{ V}^{-1}$

$I_D = K_n(V_{GS} - V_{TN})^2 (1 + \lambda V_{DS})$

For $V_{DS} = 2 \text{ V}$

$I_D = (0.1)(2.5-0.8)^2 [1 + (0.02)(2)] \Rightarrow I_D = 0.300 \text{ mA}$

$V_{DS} = 10 \text{ V}$

$I_D = (0.1)[(2.5-0.8)^2 (1 + (0.02)(10))] \Rightarrow I_D = 0.347 \text{ mA}$

(c) For part (a), $\lambda = 0 \Rightarrow r_o = \infty$

For part (b), $\lambda = 0.02 \text{ V}^{-1}$,

$r_o = [\lambda K_n (V_{GS} - V_{TN})^2]^{-1} = [(0.02)(0.1)(2.5-0.8)^2]^{-1}$

or $r_o = 173 \text{ k}\Omega$

E5.6

$$V_{TN} = V_{TNO} + 2 \left[\sqrt{2\phi_f + V_{SS}} - \sqrt{2\phi_f} \right]$$

$$2\phi_f = 0.70 \text{ V}, \quad V_{TNO} = 1 \text{ V}$$

$$(a) \quad V_{SS} = 0 \Rightarrow V_{TN} = 1 \text{ V}$$

$$(b) \quad V_{SS} = 1 \text{ V},$$

$$V_{TN} = 1 + (0.35) \left[\sqrt{0.7+1} - \sqrt{0.7} \right] \Rightarrow V_{TN} = 1.16 \text{ V}$$

$$(c) \quad V_{SS} = 4 \text{ V},$$

$$V_{TN} = 1 + (0.35) \left[\sqrt{0.7+4} - \sqrt{0.7} \right] \Rightarrow V_{TN} = 1.47 \text{ V}$$

E5.7

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5 = \left(\frac{40}{40+60} \right) (10) - 5$$

$$V_G = -1 \text{ V}$$

$$V_S = I_D R_S - 5$$

Then

$$V_{GS} = V_G - V_S = -1 - (I_D R_S - 5) = 4 - I_D R_S$$

Assume transistor is biased in saturation region

$$I_D = K_n (V_{GS} - V_{TN})^2 = \frac{4 - V_{GS}}{R_S}$$

$$4 - V_{GS} = (0.5)(0.1)[V_{GS} - 1]^2 \Rightarrow$$

$$0.5V_{GS}^2 - 3.5 = 0 \Rightarrow V_{GS} = 2.65 \text{ V}$$

$$I_D = (0.5)(2.65 - 1)^2 \Rightarrow I_D = 1.36 \text{ mA}$$

$$V_{DS} = 10 - I_D (R_S + R_D) = 10 - (1.36)(1+2) \Rightarrow$$

$$V_{DS} = 5.92 \text{ V}$$

$$V_{GS} > V_{GS}(\text{sat}), \text{ Yes}$$

E5.8

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5$$

$$= \left(\frac{200}{350} \right) (10) - 5 = 0.714 \text{ V}$$

$$V_S = 5 - I_D R_S = 5 - (1.2)I_D$$

So

$$V_{SG} = V_S - V_G = 5 - (1.2)I_D - 0.714 \\ = 4.286 - (1.2)I_D$$

$$I_D = \frac{4.286 - V_{SG}}{1.2}$$

$$I_D = K_p (V_{SG} + V_{TP})^2$$

$$4.286 - V_{SG} = (1.2)(0.25) \times \\ (V_{SG}^2 - 2V_{SG}(-1) + (-1)^2)$$

$$4.286 - V_{SG} = (0.3)V_{SG}^2 - 0.6V_{SG} + 0.3$$

$$0.3V_{SG}^2 + 0.4V_{SG} - 3.986 = 0$$

$$V_{SG} = \frac{-0.4 \pm \sqrt{(0.4)^2 + 4(0.3)(3.986)}}{2(0.3)}$$

$$\text{Must use + sign} \Rightarrow V_{SG} = 3.04 \text{ V}$$

$$I_D = (0.25)(3.04 - 1)^2 \Rightarrow I_D = 1.04 \text{ mA}$$

$$V_{SD} = 10 - I_D(R_S + R_D) = 10 - (1.04)(1.2 + 4)$$

$$\Rightarrow V_{SD} = 4.59 \text{ V}$$

$$V_{SD} > V_{SD}(\text{sat}), \text{ Yes}$$

E5.9

$$I_D = K_n (V_{GS} - V_{TN})^2$$

$$0.4 = 0.25(V_{GS} - 0.8)^2 \Rightarrow V_{GS} = 2.06 \text{ V}$$

$$V_{GS} = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD}$$

$$2.06 = \left(\frac{R_2}{250} \right) (7.5) \Rightarrow R_2 = 68.7 \text{ k}\Omega$$

$$R_1 = 181.3 \text{ k}\Omega$$

$$V_{DS} = 4 = V_{DD} - I_D R_D$$

$$R_D = \frac{7.5 - 4}{0.4} \Rightarrow R_D = 8.75 \text{ k}\Omega$$

$$V_{DS} > V_{DS}(\text{sat}), \text{ Yes}$$

E5.10

$$I_D = \frac{V_S - (-5)}{R_S} \text{ and } V_S = -V_{GS}$$

$$\text{So } R_S = \frac{5 - V_{GS}}{0.1}$$

$$I_D = K_n (V_{GS} - V_{TN})^2$$

$$0.1 = (0.080)(V_{GS} - 1.2)^2 \Rightarrow V_{GS} = 2.32 \text{ V}$$

$$\text{So } R_S = \frac{5 - 2.32}{0.1} \Rightarrow R_S = 26.8 \text{ k}\Omega$$

$$V_{DS} = V_D - V_S \Rightarrow V_D = V_{DS} + V_S = 4.5 - 2.32$$

$$V_D = 2.18$$

$$R_D = \frac{5 - V_D}{I_D} = \frac{5 - 2.18}{0.1} \Rightarrow R_D = 28.2 \text{ k}\Omega$$

$$V_{DS} > V_{DS}(\text{sat}), \text{ Yes}$$

E5.11

$$I_D = \frac{10 - V_{SG}}{R_S} \text{ and } I_D = K_p (V_{SG} + V_{TP})^2$$

$$0.12 = (0.050)(V_{SG} - 0.8)^2$$

$$V_{SG} = 2.35 \text{ V}$$

$$R_S = \frac{10 - 2.35}{0.12} \Rightarrow R_S = 63.75 \text{ k}\Omega$$

$$V_{SD} = 8 = 20 - I_D(R_S + R_D)$$

$$8 = 20 - (0.12)(63.75) - (0.12)R_D$$

$$R_D = \frac{20 - (0.12)(63.75) - 8}{0.12}$$

$$\Rightarrow R_D = 36.25 \text{ k}\Omega$$

E5.12

$$I_D = \frac{V_{DD} - V_{GS}}{R_s}, \quad I_D = K_n(V_{GS} - V_{TN})^2$$

$$10 - V_{GS} = (10)(0.2)(V_{GS}^2 - 2V_{GS}V_{TN} + V_{TN}^2)$$

$$10 - V_{GS} = 2V_{GS}^2 - 8V_{GS} + 8$$

$$2V_{GS}^2 - 7V_{GS} - 2 = 0$$

$$V_{GS} = \frac{7 \pm \sqrt{(7)^2 + 4(2)2}}{2(2)}$$

$$\text{Use } + \text{ sign: } V_{GS} = V_{DS} = 3.77 \text{ V}$$

$$I_D = \frac{10 - 3.77}{10} \Rightarrow I_D = 0.623 \text{ mA}$$

$$\text{Power} = I_D V_{DS} = (0.623)(3.77)$$

$$\Rightarrow \text{Power} = 2.35 \text{ mW}$$

E5.13

$$\text{For } V_{DS} = 2.2 \text{ V}$$

$$I_D = \frac{5-2.2}{5} \Rightarrow I_D = 0.56 \text{ mA}$$

$$I_D = K_n(V_{GS} - V_{TN})^2$$

$$0.56 = K_n(2.2 - 1)^2$$

$$K_n = 0.389 \text{ mA/V} = \frac{W}{L} \cdot \frac{\mu_n C_{ox}}{2}$$

$$\frac{W}{L} = \frac{(389)(2)}{(40)} \Rightarrow \frac{W}{L} = 19.5$$

E5.14

(a) The transition point is

$$V_h = \frac{V_{DD} - V_{TN} + V_{TN}(1 + \sqrt{K_{nD}/K_{nL}})}{1 + \sqrt{K_{nD}/K_{nL}}} \\ = \frac{5 - 1 + 1(1 + \sqrt{0.05/0.01})}{1 + \sqrt{0.05/0.01}} \\ = \frac{7.236}{3.236} \Rightarrow V_{ht} = 2.24 \text{ V}$$

$$V_{ot} = V_h - V_{TN} = 2.24 - 1 \Rightarrow V_{ot} = 1.24 \text{ V}$$

(b) We may write

$$I_D = K_{nD}(V_{GS} - V_{TN})^2 = (0.05)(2.24 - 1)^2 \\ \Rightarrow I_D = 76.9 \mu\text{A}$$

E5.15

$$V_h = \frac{V_{DD} - V_{TN} + V_{TN}(1 + \sqrt{K_{nD}/K_{nL}})}{1 + \sqrt{K_{nD}/K_{nL}}} \\ 2.5 = \frac{5 - 1 + 1(1 + \sqrt{K_{nD}/K_{nL}})}{1 + \sqrt{K_{nD}/K_{nL}}}$$

$$2.5 + 2.5\sqrt{K_{nD}/K_{nL}} = 5 + \sqrt{K_{nD}/K_{nL}} \Rightarrow$$

$$\sqrt{K_{nD}/K_{nL}} = \frac{5 - 2.5}{1.5} = 1.67 \Rightarrow$$

$$\underline{K_{nD}/K_{nL} = 2.78}$$

b. For $V_I = 5$, driver in nonsaturated region.

$$I_{DD} = I_{DL}$$

$$K_{nD}[2(V_I - V_{TN})V_O - V_O^2] = K_{nL}(V_{DSL} - V_{TNL})^2$$

$$\frac{K_{nD}}{K_{nL}}[2(V_I - V_{TN})V_O - V_O^2] = [V_{DD} - V_O - V_{TNL}]^2$$

$$2.78[2(5 - 1)V_O - V_O^2] = [5 - V_O - 1]^2$$

$$22.24V_O - 2.78V_O^2 = (4 - V_O)^2$$

$$= 16 - 8V_O + V_O^2$$

$$3.78V_O^2 - 30.24V_O + 16 = 0$$

$$V_O = \frac{30.24 \pm \sqrt{(30.24)^2 - 4(3.78)(16)}}{2(3.78)}$$

$$\Rightarrow \underline{V_O = 0.57 \text{ V}}$$

E5.16

If the transistor is biased in the saturation region

$$I_D = K_n(V_{GS} - V_{TN})^2 = K_n(-V_{TN})^2$$

$$I_D = (0.25)(2.5)^2 \Rightarrow \underline{I_D = 1.56 \text{ mA}}$$

$$V_{DS} = V_{DD} - I_D R_S = 10 - (1.56)(4)$$

$$\Rightarrow \underline{V_{DS} = 3.76}$$

$$V_{DS} > V_{GS} - V_{TN} = -V_{TN}$$

$$3.76 > -(-2.5)$$

Yes — biased in the saturation region

$$\text{Power} = I_D V_{DS} = (1.56)(3.76)$$

$$\Rightarrow \underline{\text{Power} = 5.87 \text{ mW}}$$

E5.17

We have $V_{DS} = 1.2 \text{ V} < V_{GS} - V_{TN} = -V_{TN} = 1.8 \text{ V}$

Transistor is biased in the nonsaturation region.

$$I_D = K_n[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$$

and

$$I_D = \frac{V_{DD} - V_{DS}}{R_s} = \frac{5 - 1.2}{8} \Rightarrow$$

$$\underline{I_D = 0.475 \text{ mA}}$$

$$0.475 = K_n[2(0 - (-1.8))(1.2) - (1.2)^2]$$

$$0.475 = K_n(2.88) \Rightarrow K_n = 0.165 \text{ mA/V}^2$$

$$K_n = \frac{W}{L} \cdot \frac{\mu_n C_{ox}}{2}$$

$$\frac{W}{L} = \frac{(165)(2)}{35} \Rightarrow \underline{\frac{W}{L} = 9.43}$$

E5.18

- (a) Transition point for the load transistor - Driver is in the saturation region.

$$I_{DD} = I_{DL}$$

$$K_{nD}(V_{GSD} - V_{TN})^2 = K_{nL}(V_{GSL} - V_{TN})^2$$

$$V_{DSL}(\text{sat}) = V_{GSL} - V_{TN} = -V_{TN}$$

$$\Rightarrow V_{DSL} = V_{DD} - V_{Oz} = 2 \text{ V}$$

$$\text{Then } V_{Oz} = 5 - 2 = 3 \text{ V}, \quad V_{Oz} = 3 \text{ V}$$

$$\sqrt{\frac{K_{nD}}{K_{nL}}}(V_h - 1) = (-V_{TN})$$

$$\sqrt{\frac{0.08}{0.01}}(V_h - 1) = 2 \Rightarrow V_h = 1.89 \text{ V}$$

- (b) For the driver:

$$V_{Oz} = V_h - V_{TN}$$

$$V_h = 1.89 \text{ V}, \quad V_{Oz} = 0.89 \text{ V}$$

E5.19

- (a) For $V_i = 5 \text{ V}$, Load in saturation and driver in nonsaturation.

$$I_{DD} = I_{DL}$$

$$K_{nD}[2(V_i - V_{TN})V_O - V_O^2] = K_{nL}(-V_{TN})^2$$

$$\frac{K_{nD}}{K_{nL}}[2(5-1)(0.25) - (0.25)^2] = 4 \Rightarrow$$

$$\frac{K_{nD}}{K_{nL}} = 2.06$$

$$(b) I_{DL} = K_{nL}(-V_{TN})^2 \Rightarrow 0.2 = K_{nL}[-(-2)]^2$$

$$K_{nL} = 50 \mu\text{A/V}^2 \text{ and } K_{nD} = 103 \mu\text{A/V}^2$$

E5.20

$$(a) I_{REF} = K_{n3}(V_{GS3} - V_{TN})^2 = K_{n4}(V_{GS4} - V_{TN})^2$$

$$V_{GS3} = 2 \text{ V} \Rightarrow V_{GS4} = 3 \text{ V}$$

$$(2-1)^2 = \frac{K_{n4}}{K_{n3}}(3-1)^2 \Rightarrow \frac{K_{n4}}{K_{n3}} = \frac{1}{4}$$

$$(b) I_Q = K_{n2}(V_{GS2} - V_{TN})^2$$

$$\text{But } V_{GS2} = V_{GS3} = 2 \text{ V}$$

$$0.1 = K_{n2}(2-1)^2 \Rightarrow K_{n2} = 0.1 \text{ mA/V}^2$$

$$(c) 0.2 = K_{n3}(2-1)^2 \Rightarrow K_{n3} = 0.2 \text{ mA/V}^2$$

$$0.2 = K_{n4}(3-1)^2 \Rightarrow K_{n4} = 0.05 \text{ mA/V}^2$$

E5.21

For $R_D = 10 \text{ k}\Omega$, $V_{DD} = 5 \text{ V}$, and $V_o = 1 \text{ V}$

$$I_D = \frac{5-1}{10} = 0.4 \text{ mA}$$

$$I_D = K_n[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$$

$$I_D = 0.4 = K_n[2(5-1)(1) - (1)^2] \Rightarrow$$

$$K_n = 0.057 \text{ mA/V}^2$$

$$P = I_D \cdot V_{DS} = (0.4)(1) \Rightarrow P = 0.4 \text{ mW}$$

E5.22

$$I_D = K_n[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$$

$$= (0.050)[2(10 - 0.7)(0.35) - (0.35)^2]$$

$$I_D = 0.319 \text{ mA}$$

$$R_D = \frac{V_{DD} - V_0}{I_D} = \frac{10 - 0.35}{0.319}$$

$$\Rightarrow R_D = 30.3 \text{ k}\Omega$$

E5.23

- (a) Transistor biased in the nonsaturation region

$$I_D = \frac{5-1.5-V_{DS}}{R} = 12$$

$$I_D = K_n[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$$

$$12 = 4[2(5-0.8)V_{DS} - V_{DS}^2]$$

$$4V_{DS}^2 - 33.6V_{DS} + 12 = 0 \Rightarrow V_{DS} = 0.374 \text{ V}$$

Then

$$R = \frac{5-1.5-0.374}{12} \Rightarrow R = 261 \Omega$$

E5.24

$$a. \quad V_1 = 5 \text{ V}, \quad V_2 = 0, \quad M_2 \text{ cutoff} \Rightarrow I_{D2} = 0$$

$$I_D = K_n[2(V_i - V_{TN})V_O - V_O^2] = \frac{5-V_O}{R_D}$$

$$(0.05)(30)[2(5-1)V_0 - V_0^2] = 5 - V_0$$

$$1.5V_0^2 - 13V_0 + 5 = 0$$

$$V_0 = \frac{13 \pm \sqrt{(13)^2 - 4(1.5)(5)}}{2(1.5)} \Rightarrow V_0 = 0.40 \text{ V}$$

$$I_R = I_{D1} = \frac{5-0.40}{30} \Rightarrow I_R = I_{D1} = 0.153 \text{ mA}$$

$$b. \quad V_1 = V_2 = 5 \text{ V}$$

$$\frac{5-V_O}{R_D} = 2\{K_n[2(V_i - V_{TN})V_O - V_O^2]\}$$

$$5 - V_0 = 2(0.05)(30)[2(5-1)V_0 - V_0^2]$$

$$3V_0^2 - 25V_0 + 5 = 0$$

$$V_0 = \frac{25 \pm \sqrt{(25)^2 - 4(3)(5)}}{2(3)} \Rightarrow V_0 = 0.205 \text{ V}$$

$$I_R = \frac{5 - 0.205}{30} \Rightarrow I_R = 0.160 \text{ mA}$$

$$I_{D1} = I_{D2} = 0.080 \text{ mA}$$

E5.25

$$(a) I_D = \frac{5 - V_0}{R_D} = K_n [2(V_2 - V_{TN})V_0 - V_0^2]$$

$$\frac{5 - (0.10)}{25} = K_n [2(5 - 1)(0.10) - (0.10)^2] \Rightarrow$$

$$K_n = 0.248 \text{ mA/V}^2$$

$$b. \frac{5 - V_0}{25} = 2(0.248)[2(5 - 1)V_0 - V_0^2]$$

$$5 - V_0 = 12.4[8V_0 - V_0^2]$$

$$12.4V_0^2 - 100.2V_0 + 5 = 0$$

$$V_0 = \frac{100.2 \pm \sqrt{(100.2)^2 - 4(12.4)(5)}}{2(12.4)}$$

$$\Rightarrow V_0 = 0.0502 \text{ V}$$

E5.26

$$V_{DS(\text{sat})} = V_{GS} - V_P = -1.2 - (-4.5)$$

$$\Rightarrow V_{DS(\text{sat})} = 3.3 \text{ V}$$

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2 = 12 \left(1 - \frac{(-1.2)}{(-4.5)}\right)^2$$

$$\Rightarrow I_D = 6.45 \text{ mA}$$

E5.27

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$1.2 = 2 \left(1 - \frac{V_{GS}}{(-2.5)}\right)^2 \Rightarrow V_{GS} = -0.564 \text{ V}$$

E5.28

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$3 = I_{DSS} \left(1 - \frac{0.8}{3.8}\right)^2 \Rightarrow I_{DSS} = 4.81 \text{ mA}$$

$$V_{SD(\text{sat})} = V_P - V_{GS} = 3.8 - 0.8$$

$$V_{SD(\text{sat})} = 3.0 \text{ V}$$

E5.29

$$I_D = K(V_{GS} - V_{TN})^2$$

- $V_{GS} = 0.35 \Rightarrow I_D = 25(0.35 - 0.25)^2$
 $\Rightarrow I_D = 0.25 \mu\text{A}$
- $V_{GS} = 0.50 \Rightarrow I_D = 25(0.50 - 0.25)^2$
 $\Rightarrow I_D = 1.56 \mu\text{A}$

E5.30

Assume the transistor is biased in the saturation region.

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$8 = 18 \left(1 - \frac{V_{GS}}{(-3.5)}\right)^2 \Rightarrow V_{GS} = -1.17 \text{ V}$$

$$\Rightarrow V_S = -V_{GS} = 1.17$$

$$V_D = 15 - (8)(0.8) = 8.6$$

$$V_{DS} = 8.6 - (1.17) = 7.43 \text{ V}$$

$$V_{DS} = 7.43 > V_{GS} - V_P = -1.17 - (-3.5)$$

$$= 2.33$$

Yes, the transistor is biased in the saturation region.

E5.31

$$I_D = 2.5 \text{ mA}$$

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$2.5 = 6 \left(1 - \frac{V_{GS}}{(-4)}\right)^2 \Rightarrow V_{GS} = -1.42 \text{ V}$$

$$V_S = I_D R_S - 5 = (2.5)(0.25) - 5$$

$$V_S = -4.375$$

$$V_{DS} = 6 \Rightarrow V_D = 6 - 4.375 = 1.625$$

$$R_D = \frac{5 - 1.625}{2.5} \Rightarrow R_D = 1.35 \text{ k}\Omega$$

$$\frac{(20)^2}{R_1 + R_2} = 2 \Rightarrow R_1 + R_2 = 200 \text{ k}\Omega$$

$$V_G = V_{GS} + V_S = -1.42 - 4.375 = -5.795$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)(20) - 10$$

$$-5.795 = \left(\frac{R_2}{200}\right)(20) - 10 \Rightarrow$$

$$R_2 = 42.05 \text{ k}\Omega \rightarrow 42 \text{ k}\Omega$$

$$R_1 = 157.95 \text{ k}\Omega \rightarrow 158 \text{ k}\Omega$$

E5.32

$$V_S = -V_{GS}, \quad I_D = \frac{0 - V_S}{R_S} = \frac{V_{GS}}{R_S}$$

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$\frac{V_{GS}}{1} = 6 \left(1 - \frac{V_{GS}}{4}\right)^2 = 6 \left(1 - \frac{V_{GS}}{2} + \frac{V_{GS}^2}{16}\right)$$

$$0.375V_{GS}^2 - 4V_{GS} + 6 = 0$$

$$V_{GS} = \frac{4 \pm \sqrt{16 - 4(0.375)(6)}}{2(0.375)}$$

$$V_{GS} = 8.86 \text{ or } V_{GS} = 1.81 \text{ V}$$

impossible

$$I_D = \frac{V_{GS}}{R_S} = 1.81 \text{ mA}$$

$$V_D = I_D R_D - 5 = (1.81)(0.4) - 5 = -4.276$$

$$V_{SD} = V_S - V_D = -1.81 - (-4.276)$$

$$\Rightarrow V_{SD} = 2.47 \text{ V}$$

$$V_{SD}(\text{sat}) = V_P - V_{GS} = 4 - 1.81 = 2.19$$

So $V_{SD} > V_{SD}(\text{sat})$

E5.33

$$R_{in} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = 100 \text{ k}\Omega$$

$$I_{DQ} = 5 \text{ mA}, \quad V_S = -I_{DQ} R_S = -(5)(1.2) = -6 \text{ V}$$

$$V_{SDQ} = 12 \text{ V} \Rightarrow V_D = V_S - V_{SDQ}$$

$$= -6 - 12 = -18 \text{ V}$$

$$R_D = \frac{-18 - (-20)}{5} \Rightarrow R_D = 0.4 \text{ k}\Omega$$

$$I_{DQ} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2 \Rightarrow 5 = 8 \left(1 - \frac{V_{GS}}{4}\right)^2$$

$$V_{GS} = 0.838 \text{ V}$$

$$V_G = V_{GS} + V_S = 0.838 - 6 = -5.162$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)(-20)$$

$$-5.162 = \frac{1}{R_1}(100)(-20) \Rightarrow R_1 = 387 \text{ k}\Omega$$

$$\frac{R_1 R_2}{R_1 + R_2} = 100 \Rightarrow (387)R_2 = 100(387) + 100R_2$$

$$(387 - 100)R_2 = (100)(387)$$

$$\Rightarrow R_2 = 135 \text{ k}\Omega$$

E5.34

$$I_{DQ} = K(V_{GS} - V_{TN})^2 \Rightarrow 5 = 50(V_{GS} - 0.15)^2$$

$$\Rightarrow V_{GS} = 0.466 \text{ V}$$

$$V_S = (0.005)(10) = 0.050 \text{ V}$$

$$\Rightarrow V_{GG} = V_{GS} + V_S = 0.466 + 0.050$$

$$\Rightarrow V_{GG} = 0.516 \text{ V}$$

$$V_D = 5 - (0.005)(100) \Rightarrow V_D = 4.5 \text{ V}$$

$$V_{DS} = V_D - V_S = 4.5 - 0.050$$

$$\Rightarrow V_{DS} = 4.45 \text{ V}$$

E5.35

$$I_D = K[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$$

$$= 100[2(0.7 - 0.2)(0.1) - (0.1)^2]$$

$$I_D = 9 \mu\text{A}$$

$$R_D = \frac{2.5 - 0.1}{0.009} \Rightarrow R_D = 267 \text{ k}\Omega$$

Chapter 5

Problem Solutions

5.1

(a) $V_{DS} = 6 \text{ V} > V_{GS} - V_{TN} = 5 - 1.5 = 3.5 \text{ V}$

Biased in the saturation region

$$I_D = K_n(V_{GS} - V_{TN})^2 = (0.25)(5 - 1.5)^2 \Rightarrow I_D = 3.06 \text{ mA}$$

(b) $V_{DS} = 2.5 \text{ V} < V_{DS(sat)} = 3.5 \text{ V}$

Biased in nonsaturation region

$$I_D = K_n[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2] \\ I_D = (0.25)[2(5 - 1.5)(2.5) - (2.5)^2] \Rightarrow I_D = 2.81 \text{ mA}$$

5.2

(a) $I_D = \frac{k'_n}{2} \cdot \frac{W}{L} (V_{GS} - V_{TN})^2$
 $0.5 = \left(\frac{0.080}{2}\right)(5)(V_{GS} - 0.8)^2$
 $\sqrt{\frac{0.5}{0.2}} + 0.8 = V_{GS} \Rightarrow V_{GS} = 2.38 \text{ V}$

$V_{DS(sat)} = V_{GS} - V_{TN} = 2.38 - 0.8 \Rightarrow$

$V_{DS(sat)} = 1.58 \text{ V}$

(b) $1.5 = \left(\frac{0.080}{2}\right)(5)(V_{GS} - 0.8)^2$
 $\sqrt{\frac{1.5}{0.2}} + 0.8 = V_{GS} \Rightarrow V_{GS} = 3.54 \text{ V}$
 $V_{DS(sat)} = 3.54 - 0.8 \Rightarrow V_{DS(sat)} = 2.74 \text{ V}$

5.3

a. $V_{GS} = 0$

$V_{DS(sat)} = V_{GS} - V_{TN} = 0 - (-2.5) = 2.5 \text{ V}$

i. $V_{DS} = 0.5 \text{ V} \Rightarrow$ Biased in nonsaturation

$I_D = (1.1)[2(0 - (-2.5))(0.5) - (0.5)^2] \Rightarrow I_D = 2.48 \text{ mA}$

ii. $V_{DS} = 2.5 \text{ V} \Rightarrow$ Biased in saturation

$I_D = (1.1)(0 - (-2.5))^2 \Rightarrow I_D = 6.88 \text{ mA}$

iii. $V_{DS} = 5 \text{ V}$ Same as (ii) $\Rightarrow I_D = 6.88 \text{ mA}$

b. $V_{GS} = 2 \text{ V}$

$V_{DS(sat)} = 2 - (-2.5) = 4.5 \text{ V}$

i. $V_{DS} = 0.5 \text{ V} \Rightarrow$ Nonsaturation

$I_D = (1.1)[2(2 - (-2.5))(0.5) - (0.5)^2]$

$\Rightarrow I_D = 4.68 \text{ mA}$

ii. $V_{DS} = 2.5 \text{ V} \Rightarrow$ Nonsaturation

$I_D = (1.1)[2(2 - (-2.5))(2.5) - (2.5)^2]$

$\Rightarrow I_D = 17.9 \text{ mA}$

iii. $V_{DS} = 5 \text{ V} \Rightarrow$ Saturation

$I_D = (1.1)(2 - (-2.5))^2$

$\Rightarrow I_D = 22.3 \text{ mA}$

5.4

$V_{DS} > V_{GS} - V_{TN} = 0 - (-2) = 2 \text{ V}$

Biased in the saturation region

$I_D = \frac{k'_n}{2} \cdot \frac{W}{L} (V_{GS} - V_{TN})^2 \\ 1.5 = \left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)[0 - (-2)]^2 \Rightarrow \frac{W}{L} = 9.375$

5.5

a. $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{450 \times 10^{-8}}$

$\Rightarrow \frac{\epsilon_{ox}}{t_{ox}} = 7.67 \times 10^{-8} \text{ F/cm}^2$

$K_n = \frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L} \\ = \frac{1}{2}(650)(7.67 \times 10^{-8})\left(\frac{64}{4}\right)$

$K_n = 0.399 \text{ mA/V}^2$

b. $V_{GS} = V_{DS} = 3 \text{ V} \Rightarrow$ Saturation

$I_D = K_n(V_{GS} - V_{TN})^2 = (0.399)(3 - 0.8)^2$

$\Rightarrow I_D = 1.93 \text{ mA}$

5.6

a. $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{600 \times 10^{-8}}$

$\Rightarrow \frac{\epsilon_{ox}}{t_{ox}} = 5.75 \times 10^{-8} \text{ F/cm}^2$

$K_n = \frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L} \\ = \frac{1}{2}(500)(5.75 \times 10^{-8})\left(\frac{100}{5}\right) \Rightarrow$

$K_n = 0.288 \text{ mA/V}^2$

b. i. $V_{GS} = 0, V_{DS} = 5 \text{ V}$

$$V_{DS}(\text{sat}) = 0 - (-2) = 2 \text{ V}$$

Biased in saturation

$$I_D = (0.288)(0 - (-2))^2 \Rightarrow I_D = 1.15 \text{ mA}$$

ii. $V_{GS} = 2 \text{ V}, V_{DS} = 1 \text{ V}$

$$V_{DS}(\text{sat}) = 2 - (-2) = 4 \text{ V}$$

Nonsaturation

$$I_D = (0.288)[2(2 - (-2))(1) - (1)^2]$$

$$\Rightarrow I_D = 2.02 \text{ mA}$$

5.7

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{400 \times 10^{-8}} = 8.63 \times 10^{-8} \text{ F/cm}^2$$

$$K_n = \frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L} = \frac{1}{2}(600)(8.63 \times 10^{-8})\left(\frac{W}{2.5}\right)$$

$$K_n = (1.04 \times 10^{-5})W$$

$$I_D = K_n(V_{GS} - V_{TR})^2$$

$$1.2 \times 10^{-3} = (1.04 \times 10^{-5})W(5 - 1)^2$$

$$\Rightarrow W = 7.21 \mu\text{m}$$

5.8

Biased in the saturation region in both cases.

$$I_D = \frac{k'_p}{2} \cdot \frac{W}{L} (V_{SG} + V_{TP})^2$$

$$(1) 0.225 = \left(\frac{0.040}{2}\right)\left(\frac{W}{L}\right)(3 + V_{TP})^2$$

$$(2) 1.40 = \left(\frac{0.040}{2}\right)\left(\frac{W}{L}\right)(4 + V_{TP})^2$$

Take ratio of (2) to (1):

$$\frac{1.40}{0.225} = 6.222 = \frac{(4 + V_{TP})^2}{(3 + V_{TP})^2}$$

$$\sqrt{6.222} = 2.49 = \frac{4 + V_{TP}}{3 + V_{TP}} \Rightarrow V_{TP} = -2.33 \text{ V}$$

Then

$$0.225 = \left(\frac{0.040}{2}\right)\left(\frac{W}{L}\right)(3 - 2.33)^2 \Rightarrow \frac{W}{L} = 25.1$$

5.9

$$V_S = 5 \text{ V}, V_G = 0 \Rightarrow V_{SG} = 5 \text{ V}$$

$$V_{TP} = -0.5 \text{ V}$$

$$\Rightarrow V_{SD}(\text{sat}) = V_{SG} + V_{TP} = 5 - 0.5 = 4.5 \text{ V}$$

i. $V_D = 0 \Rightarrow V_{SD} = 5 \text{ V}$

\Rightarrow Biased in saturation

$$I_D = 2(5 - 0.5)^2 \Rightarrow I_D = 40.5 \text{ mA}$$

ii. $V_D = 2 \text{ V} \Rightarrow V_{SD} = 3 \text{ V}$

\Rightarrow Nonsaturation

$$I_D = 2[2(5 - 0.5)(3) - (3)^2]$$

$$\Rightarrow I_D = 36 \text{ mA}$$

iii. $V_D = 4 \text{ V} \Rightarrow V_{SD} = 1 \text{ V}$

\Rightarrow Nonsaturation

$$I_D = 2[2(5 - 0.5)(1) - (1)^2]$$

$$\Rightarrow I_D = 16 \text{ mA}$$

iv. $V_D = 5 \text{ V} \Rightarrow V_{SD} = 0 \Rightarrow I_D = 0$

5.10

$$V_{SD}(\text{sat}) = V_{SG} + V_{TP}$$

$$(a) V_{SD}(\text{sat}) = -1 + 2 \Rightarrow V_{SD}(\text{sat}) = 1 \text{ V}$$

$$(b) V_{SD}(\text{sat}) = 0 + 2 \Rightarrow V_{SD}(\text{sat}) = 2 \text{ V}$$

$$(c) V_{SD}(\text{sat}) = 1 + 2 \Rightarrow V_{SD}(\text{sat}) = 3 \text{ V}$$

$$I_D = \frac{k'_p}{2} \cdot \frac{W}{L} (V_{SG} + V_{TP})^2 = \frac{k'_p}{2} \cdot \frac{W}{L} [V_{SD}(\text{sat})]^2$$

$$(a) I_D = \left(\frac{0.040}{2}\right)(6)(1)^2 \Rightarrow I_D = 0.12 \text{ mA}$$

$$(b) I_D = \left(\frac{0.040}{2}\right)(6)(2)^2 \Rightarrow I_D = 0.48 \text{ mA}$$

$$(c) I_D = \left(\frac{0.040}{2}\right)(6)(3)^2 \Rightarrow I_D = 1.08 \text{ mA}$$

5.11

$$V_{SD}(\text{sat}) = V_{SG} + V_{TP} = 0 + 2 = 2 \text{ V}$$

$$(a) V_{SD} = 1 \text{ V, Nonsaturation}$$

$$I_D = K_p [2(V_{SG} + V_{TP})V_{SD} - V_{SD}^2]$$

$$I_D = (0.5)[2(0 + 2)(1) - (1)^2] \Rightarrow I_D = 1.5 \text{ mA}$$

$$(b) V_{SD} = 2 \text{ V, Saturation}$$

$$I_D = K_p (V_{SG} + V_{TP})^2 = (0.5)(0 + 2)^2 \Rightarrow$$

$$I_D = 2 \text{ mA}$$

$$(c) V_{SD} = 3 \text{ V, Saturation}$$

$$\text{Same as (b), } I_D = 2 \text{ mA}$$

5.12

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{500 \times 10^{-8}} = 6.90 \times 10^{-4} \text{ F/cm}^2$$

$$k'_n = (\mu_n C_{ox}) = (675)(6.90 \times 10^{-4}) \Rightarrow 46.6 \mu\text{A/V}^2$$

$$k'_p = (\mu_p C_{ox}) = (375)(6.90 \times 10^{-4}) \Rightarrow 25.9 \mu\text{A/V}^2$$

PMOS:

$$I_D = \frac{k'_p}{2} \left(\frac{W}{L} \right)_p (V_{SG} + V_{TP})^2$$

$$0.8 = \left(\frac{0.0259}{2} \right) \left(\frac{W}{L} \right)_p (5 - 0.6)^2$$

$$\Rightarrow \left(\frac{W}{L} \right)_p = 3.19$$

$$L = 4 \mu\text{m} \Rightarrow W_p = 12.8 \mu\text{m}$$

$$K_p = \left(\frac{0.0259}{2} \right) (3.19) \Rightarrow K_p = 41.3 \mu\text{A/V}^2 = K_n$$

 Want $K_n = K_p$

$$\frac{k'_n}{2} \left(\frac{W}{L} \right)_n = \frac{k'_p}{2} \left(\frac{W}{L} \right)_p = 41.3$$

$$\left(\frac{46.6}{2} \right) \left(\frac{W}{L} \right)_n = 41.3 \Rightarrow \left(\frac{W}{L} \right)_n = 1.77$$

$$L = 4 \mu\text{m} \Rightarrow W_n = 7.08 \mu\text{m}$$

5.13

$$V_{GS} = 2 \text{ V}, I_D = (0.2)(2 - 1.2)^2 = 0.128 \text{ mA}$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.01)(0.128)} \Rightarrow r_o = 781 \text{ k}\Omega$$

$$V_{GS} = 4 \text{ V}, I_D = (0.2)(4 - 1.2)^2 = 1.57 \text{ mA}$$

$$r_o = \frac{1}{(0.01)(1.57)} \Rightarrow r_o = 63.7 \text{ k}\Omega$$

$$V_A = \frac{1}{\lambda} = \frac{1}{(0.01)} \Rightarrow V_A = 100 \text{ V}$$

5.14

$$I_D = \left(\frac{0.080}{2} \right) (4)(3 - 0.8)^2 = (0.16)(3 - 0.8)^2 \Rightarrow$$

$$I_D = 0.774 \text{ mA}$$

$$r_o = \frac{1}{\lambda I_D} \Rightarrow \lambda = \frac{1}{r_o I_D} = \frac{1}{(200)(0.774)} \Rightarrow$$

$$\lambda(\max) = 0.00646 \text{ V}^{-1}$$

$$V_A(\min) = \frac{1}{\lambda(\max)} = \frac{1}{0.00646} \Rightarrow V_A(\min) = 155 \text{ V}$$

5.15

$$V_{TN} = V_{TNO} + \gamma \left[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f} \right]$$

$$\Delta V_{TN} = 2 = (0.8) \left[\sqrt{2(0.35) + V_{SB}} - \sqrt{2(0.35)} \right]$$

$$2.5 + 0.837 = \sqrt{2(0.35) + V_{SB}} \Rightarrow V_{SB} = 10.4 \text{ V}$$

5.16

$$\Delta V_{TN} = \gamma \left[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f} \right]$$

$$1.2 = \gamma \left[\sqrt{2(0.37) + 10} - \sqrt{2(0.37)} \right] = \gamma(2.42)$$

$$\text{Then } \gamma = 0.496 \text{ V}^{1/2}$$

5.17

$$\text{a. } V_G = \epsilon_{ox} t_{ox} = (6 \times 10^6)(275 \times 10^{-3})$$

$$V_G = 16.5 \text{ V}$$

$$\text{b. } V_G = \frac{16.5}{3} \Rightarrow V_G = 5.5 \text{ V}$$

5.18

$$\text{Want } V_G = (3)(24) = \epsilon_{ox} t_{ox} = (6 \times 10^6)t_{ox}$$

$$t_{ox} = 1.2 \times 10^{-5} \text{ cm} = 1200 \text{ Angstroms}$$

5.19

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \left(\frac{18}{18 + 32} \right) (10) = 3.6 \text{ V}$$

Assume transistor biased in saturation region

$$I_D = \frac{V_S}{R_S} = \frac{V_G - V_{GS}}{R_S} = K_n (V_{GS} - V_{TN})^2$$

$$3.6 - V_{GS} = (0.5)(2)(V_{GS} - 0.8)^2$$

$$= V_{GS}^2 - 1.6V_{GS} + 0.64$$

$$V_{GS}^2 - 0.6V_{GS} - 2.96 = 0$$

$$V_{GS} = \frac{0.6 \pm \sqrt{(0.6)^2 + 4(2.96)}}{2}$$

$$\Rightarrow V_{GS} = 2.05 \text{ V}$$

$$I_D = \frac{V_G - V_{GS}}{R_S} = \frac{3.6 - 2.05}{2} \Rightarrow I_D = 0.775 \text{ mA}$$

$$V_{DS} = V_{DD} - I_D(R_D + R_S)$$

$$= 10 - (0.775)(4 + 2)$$

$$\Rightarrow V_{DS} = 5.35 \text{ V}$$

$$V_{DS} > V_{DS(\text{sat})}$$

5.20

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) (20) - 10 = \left(\frac{6}{14+6} \right) (20) - 10 \Rightarrow$$

$$V_G = -4 \text{ V}$$

Assume transistor is biased in saturation region

$$I_D = \frac{V_S - (-10)}{R_S} = \frac{V_G - V_{GS} + 10}{R_S} = K_n (V_{GS} - V_{TN})^2$$

$$K_n = \left(\frac{0.060}{2} \right) (60) \Rightarrow 1.8 \text{ mA/V}^2$$

$$-4 - V_{GS} + 10 = (1.8)(0.5)(V_{GS} - 2)^2 \\ = 0.9V_{GS}^2 - 3.6V_{GS} + 3.6$$

$$\text{Then } 0.9V_{GS}^2 - 2.6V_{GS} - 2.4 = 0$$

$$V_{GS} = \frac{2.6 \pm \sqrt{(2.6)^2 + 4(0.9)(2.4)}}{2(0.9)} \Rightarrow V_{GS} = 3.62 \text{ V}$$

$$I_D = \frac{V_G - V_{GS} + 10}{R_S} = \frac{-4 - 3.62 + 10}{0.5} \Rightarrow$$

$$I_D = 4.76 \text{ mA}$$

$$V_{DS} = 20 - I_D(R_D + R_S) = 20 - (4.76)(1.2 + 0.5) \Rightarrow$$

$$V_{DS} = 11.9 \text{ V}$$

$$\underline{V_{DS} = 11.9 \text{ V} > V_{GS} - V_{TN} = 3.62 - 2 = 1.62 \text{ V}}$$

5.21

$$I_D = \frac{10 - V_S}{R_S} = K_p (V_{SG} + V_{TP})^2$$

Assume transistor biased in saturation region

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) (20) - 10$$

$$= \left(\frac{22}{8+22} \right) (20) - 10$$

$$\Rightarrow V_G = 4.67 \text{ V}$$

$$V_S = V_G + V_{SG}$$

$$10 - (4.67 + V_{SG}) = (1)(0.5)(V_{SG} - 2)^2$$

$$5.33 - V_{SG} = 0.5(V_{SG}^2 - 4V_{SG} + 4)$$

$$0.5V_{SG}^2 - V_{SG} - 3.33 = 0$$

$$V_{SG} = \frac{1 \pm \sqrt{(1)^2 + 4(0.5)(3.33)}}{2(0.5)}$$

$$\Rightarrow \underline{V_{SG} = 3.77 \text{ V}}$$

$$I_D = \frac{10 - (4.67 + 3.77)}{0.5} \Rightarrow \underline{I_D = 3.12 \text{ mA}}$$

$$V_{SD} = 20 - I_D(R_S + R_D)$$

$$= 20 - (3.12)(0.5 + 2)$$

$$\Rightarrow \underline{V_{SD} = 12.3 \text{ V}}$$

$$\underline{V_{SD} > V_{SD}(\text{sat})}$$

5.22

$$V_G = 0, \quad V_{SG} = V_S$$

Assume saturation region

$$I_D = 0.4 = K_p (V_{SG} + V_{TP})^2$$

$$0.4 = (0.2)(V_S - 0.8)^2$$

$$V_S = \sqrt{\frac{0.4}{0.2}} + 0.8 \Rightarrow \underline{V_S = 2.21 \text{ V}}$$

$$V_D = I_D R_D - 5 = (0.4)(5) - 5 = -3 \text{ V}$$

$$V_{SD} = V_S - V_D = 2.21 - (-3)$$

$$\Rightarrow \underline{V_{SD} = 5.21 \text{ V}}$$

$$\underline{V_{SD} > V_{SD}(\text{sat})}$$

5.23

$$V_{DD} = I_{DQ} R_D + V_{DSQ} + I_{DQ} R_S$$

$$(1) \quad 10 = I_{DQ}(5) + 5 + V_{DS} \quad \text{and}$$

$$I_{DQ} = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) (V_{GS} - V_{TN})^2$$

$$\text{or (2)} \quad I_{DQ} = \left(\frac{0.060}{2} \right) \left(\frac{W}{L} \right) (V_{GS} - 1.2)^2$$

$$\text{Let } \underline{V_{GS} = 2.5 \text{ V}}$$

$$\text{Then from (1), } 10 = I_{DQ}(5) + 5 + 2.5 \Rightarrow I_D = 0.5 \text{ mA}$$

$$\text{Then from (2), } 0.5 = \left(\frac{0.060}{2} \right) \left(\frac{W}{L} \right) (2.5 - 1.2)^2 \Rightarrow$$

$$\underline{\frac{W}{L} = 9.86}$$

$$I_{DQ} R_S = V_{GS} \Rightarrow R_S = \frac{V_{GS}}{I_{DQ}} = \frac{2.5}{0.5} \Rightarrow \underline{R_S = 5 \text{ k}\Omega}$$

$$I_R = \frac{10}{R_1 + R_2} = (0.5)(0.05) = 0.025 \text{ mA}$$

$$\text{Then } R_1 + R_2 = \frac{10}{0.025} = 400 \text{ k}\Omega$$

$$\left(\frac{R_2}{R_1 + R_2} \right) (V_{DD}) = 2V_{GS} \Rightarrow \left(\frac{R_2}{400} \right) (10) = 2(2.5) \Rightarrow$$

$$\underline{R_1 = R_2 = 200 \text{ k}\Omega}$$

5.24

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5$$

$$= \left(\frac{5.5}{14.5 + 5.5} \right) (10) - 5 = -2.25 \text{ V}$$

$$I_D = \frac{V_G - V_{GS} - (-5)}{R_S} = K_n (V_{GS} - V_{TN})^2$$

Assume transistor biased in saturation region

$$-2.25 - V_{GS} + 5 = (0.5)(0.6)(V_{GS} - (-1))^2$$

$$2.75 - V_{GS} = (0.3)(V_{GS}^2 + 2V_{GS} + 1)$$

$$0.3V_{GS}^2 + 1.6V_{GS} - 2.45 = 0$$

$$V_{GS} = \frac{-1.6 \pm \sqrt{(1.6)^2 + 4(0.3)(2.45)}}{2(0.3)}$$

$$\Rightarrow V_{GS} = 1.24 \text{ V}$$

$$I_D = \frac{-2.25 - 1.24 + 5}{0.6} \Rightarrow I_D = 2.52 \text{ mA}$$

$$V_{DS} = 10 - I_D(R_s + R_D)$$

$$= 10 - (2.52)(0.6 + 0.8)$$

$$\Rightarrow V_{DS} = 6.47 \text{ V}$$

$$V_{DS} > V_{DS(\text{sat})}$$

5.25

$$20 = I_{DQ}R_s + V_{SDQ} + I_{DQ}R_D$$

$$(1) 20 = V_{SG} + 10 + I_{DQ}R_D$$

$$I_{DQ} = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)(V_{SG} + V_{TP})^2$$

$$(2) I_{DQ} = \left(\frac{0.040}{2}\right)\left(\frac{W}{L}\right)(V_{SG} - 2)^2$$

For example, let $I_{DQ} = 0.8 \text{ mA}$ and $V_{SG} = 4 \text{ V}$

$$\text{Then } 0.8 = \left(\frac{0.040}{2}\right)\left(\frac{W}{L}\right)(4 - 2)^2 \Rightarrow \frac{W}{L} = 10$$

$$I_{DQ}R_s = V_{SG} \Rightarrow (0.8)R_s = 4 \Rightarrow R_s = 5 \text{ k}\Omega$$

$$\text{From (1)} 20 = 4 + 10 + (0.8)R_D \Rightarrow R_D = 75 \text{ k}\Omega$$

$$I_s = \frac{20}{R_1 + R_2} = (0.8)(0.1) \Rightarrow R_1 + R_2 = 250 \text{ k}\Omega$$

$$\left(\frac{R_1}{R_1 + R_2}\right)(20) = 2V_{SG} = (2)(4)$$

$$\frac{R_1}{250}(20) = 8 \Rightarrow R_1 = 100 \text{ k}\Omega, R_2 = 150 \text{ k}\Omega$$

5.26

$$(a) (i) I_Q = 50 = 500(V_{GS} - 1.2)^2 \Rightarrow V_{GS} = 1516 \text{ V}$$

$$V_{DS} = 5 - (-1516) \Rightarrow V_{DS} = 6516 \text{ V}$$

$$(iv) I_Q = 1 = (0.5)(V_{GS} - 1.2)^2 \Rightarrow V_{GS} = 2.61 \text{ V}$$

$$V_{DS} = 5 - (-2.61) \Rightarrow V_{DS} = 7.61 \text{ V}$$

$$(b) (i) \text{ Same as (a)} \quad V_{GS} = V_{DS} = 1516 \text{ V}$$

$$(iv) \quad V_{GS} = V_{DS} = 2.61 \text{ V}$$

5.27

$$I_D = K_p(V_{GS} - V_{TN})^2$$

$$0.25 = (0.2)(V_{GS} - 0.6)^2$$

$$V_{GS} = \sqrt{\frac{0.25}{0.2}} + 0.6 \Rightarrow V_{GS} = 1.72 \text{ V}$$

$$\Rightarrow V_S = -1.72 \text{ V}$$

$$V_D = 9 - (0.25)(24) \Rightarrow V_D = 3 \text{ V}$$

5.28

$$I_D = \frac{5 - V_D}{R_D} \Rightarrow 0.8 = \frac{5 - 1}{R_D} \Rightarrow R_D = 5 \text{ k}\Omega$$

$$V_G = 0$$

$$I_D = K_n(V_{GS} - V_{TN})^2 \Rightarrow 0.8 = (0.4)(V_{GS} - 1.7)^2$$

$$V_{GS} = \sqrt{\frac{0.8}{0.4}} + 1.7 \Rightarrow V_{GS} = 3.11 \text{ V}$$

$$\Rightarrow V_S = -3.11 \text{ V}$$

$$I_D = 0.8 = \frac{-3.11 - (-5)}{R_S} \Rightarrow R_S = 2.36 \text{ k}\Omega$$

5.29

$$V_{DD} = V_{SD} + I_{DQ}R$$

$$9 = 2.5 + (0.1)R \Rightarrow R = 65 \text{ k}\Omega$$

$$I_{DQ} = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)(V_{SG} + V_{TP})^2$$

$$(0.1) = \left(\frac{0.025}{2}\right)\left(\frac{W}{L}\right)(2.5 - 15)^2 \Rightarrow \frac{W}{L} = 8$$

Then for $L = \mu\text{m}$, $W = 32 \mu\text{m}$

5.30

$$5 = I_{DQ}R_s + V_{SDQ} = I_{DQ}(2) + 2.5$$

$$I_{DQ} = 1.25 \text{ mA}$$

$$I_R = \frac{10}{R_1 + R_2} = (1.25)(0.1) \Rightarrow R_1 + R_2 = 80 \text{ k}\Omega$$

$$I_{DQ} = K_p(V_{SG} + V_{TP})^2$$

$$1.25 = 0.5(V_{SG} + 1.5)^2 \Rightarrow \sqrt{\frac{1.25}{0.5}} - 1.5 = V_{SG}$$

$$V_{SG} = 0.0811 \text{ V}$$

$$V_G = V_S - V_{SG} = 2.5 - 0.0811 = 2.42 \text{ V}$$

$$V_G = \left(\frac{R_1}{R_1 + R_2}\right)(10) - 5$$

$$2.42 = \left(\frac{R_1}{80}\right)(10) - 5 \Rightarrow$$

$$R_1 = 59.4 \text{ k}\Omega, R_2 = 20.6 \text{ k}\Omega$$

5.31

$$K_p = \left(\frac{0.030}{2}\right)(20) \Rightarrow 0.30 \text{ mA/V}^2$$

$$I_D = K_p(V_{SG} + V_{TP})^2$$

$$0.5 = 0.30(V_{SG} - 1.2)^2 \Rightarrow V_{SG} = 2.49 \text{ V}$$

$$V_S = V_{SG} = 2.49 \text{ V}$$

$$I_D = \frac{5 - V_S}{R_S} \Rightarrow R_S = \frac{5 - 2.49}{0.5} \Rightarrow R_S = 5.02 \text{ k}\Omega$$

$$R_D = \frac{V_D - (-5)}{I_D} = \frac{5 - 3}{0.5} \Rightarrow R_D = 4 \text{ k}\Omega$$

5.32

$$I_D = \frac{-V_{SD} - (-10)}{R_D} \Rightarrow 5 = \frac{-6 + 10}{R_D}$$

$$\Rightarrow R_D = 0.8 \text{ k}\Omega$$

$$I_D = K_p (V_{SG} + V_{TP})^2 \Rightarrow 5 = 3(V_{SG} - 1.75)^2$$

$$V_{SG} = \sqrt{\frac{5}{3}} + 1.75 = 3.04 \text{ V} \Rightarrow V_G = -3.04$$

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5 = -3.04$$

$$R_{in} = R_1 \parallel R_2 = 80 \text{ k}\Omega$$

$$\frac{1}{R_1} \cdot (80)(10) = 5 - 3.04 \Rightarrow R_1 = 408 \text{ k}\Omega$$

$$\frac{408R_2}{408 + R_2} = 80 \Rightarrow R_2 = 99.5 \text{ k}\Omega$$

5.33

$$(a) K_{n1} = \left(\frac{60}{2} \right) (4) = 120 \mu\text{A/V}^2$$

$$K_{n2} = \left(\frac{60}{2} \right) (1) = 30 \mu\text{A/V}^2$$

For $v_i = 1 \text{ V}$, M_1 Sat. region, M_2 Non-sat region.

$$I_{D2} = I_{D1}$$

$$30[2(-V_{TML})(5-v_o) - (5-v_o)^2] = 120(1-0.8)^2$$

$$\text{We find } v_o^2 - 6.4v_o + 7.16 = 0 \Rightarrow v_o = 4.955 \text{ V}$$

(b) For $v_i = 3 \text{ V}$, M_1 Non-sat region, M_2 Sat. region. $I_{D2} = I_{D1}$

$$30[-(-1.8)]^2 = 120[2(3-0.8)v_o - v_o^2]$$

$$\text{We find } 4v_o^2 - 17.6v_o + 3.24 = 0 \Rightarrow v_o = 0.193 \text{ V}$$

(c) For $v_i = 5 \text{ V}$, biasing same as (b)

$$30[-(-1.8)]^2 = 120[2(5-0.8)v_o - v_o^2]$$

$$\text{We find } 4v_o^2 - 33.6v_o + 3.24 = 0 \Rightarrow v_o = 0.0976 \text{ V}$$

5.34

For $v_i = 5 \text{ V}$, M_1 Non-sat region, M_2 Sat. region.

$$I_{D1} = I_{D2}$$

$$\left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right)_1 [2(V_{GS1} - V_{TML})V_{DS1} - V_{DS1}^2] = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right)_2 (V_{GS2} - V_{TML})^2$$

$$\left(\frac{W}{L} \right)_1 [2(5-0.8)(0.15) - (0.15)^2] = (1)[0 - (-2)]$$

$$\text{which yields } \left(\frac{W}{L} \right)_1 = 3.23$$

5.35

a. M_1 and M_2 in saturation

$$K_{n1}(V_{GS1} - V_{TML})^2 = K_{n2}(V_{GS2} - V_{TML})^2$$

$$K_{n1} = K_{n2}, \quad V_{TML} = V_{TML}$$

$$\Rightarrow V_{GS1} = V_{GS2} = 2.5 \text{ V}, \quad V_0 = 2.5 \text{ V}$$

$$I_D = (15)(40)(2.5 - 0.8)^2 \Rightarrow I_D = 1.73 \text{ mA}$$

$$b. \quad \left(\frac{W}{L} \right)_1 > \left(\frac{W}{L} \right)_2 \Rightarrow V_{GS1} < V_{GS2}$$

$$40(V_{GS1} - 0.8)^2 = (15)(V_{GS2} - 0.8)^2$$

$$V_{GS2} = 5 - V_{GS1}$$

$$1.63(V_{GS1} - 0.8) = (5 - V_{GS1} - 0.8)$$

$$2.63V_{GS1} = 5.50 \Rightarrow V_{GS1} = 2.09 \text{ V}$$

$$V_{GS2} = 2.91 \text{ V}, \quad V_0 = V_{GS1} = 2.09 \text{ V}$$

$$I_D = (15)(15)(2.91 - 0.8)^2 \Rightarrow I_D = 1.0 \text{ mA}$$

5.36

Each transistor biased in saturation.

$$M_1: \quad V_1 = V_{GS3} = 2 \text{ V}$$

$$I_D = 0.5 = 0.018 \left(\frac{W}{L} \right)_3 (2-1)^2$$

$$\Rightarrow \left(\frac{W}{L} \right)_3 = 27.8$$

$$M_2: \quad V_{GS2} = V_2 - V_1 = 5 - 2 = 3 \text{ V}$$

$$I_D = 0.5 = 0.018 \left(\frac{W}{L} \right)_2 (3-1)^2$$

$$\Rightarrow \left(\frac{W}{L} \right)_2 = 6.94$$

$$M_1: \quad V_{GS1} = 10 - V_2 = 10 - 5 = 5 \text{ V}$$

$$I_D = 0.5 = 0.018 \left(\frac{W}{L} \right)_1 (5-1)^2$$

$$\Rightarrow \left(\frac{W}{L} \right)_1 = 1.74$$

5.37

 M_L in saturation M_D in nonsaturation

$$\left(\frac{W}{L} \right)_L (V_{GSL} - V_{TML})^2$$

$$= \left(\frac{W}{L} \right)_D [2(V_{GSD} - V_{TND})V_{DS0} - V_{DS0}^2]$$

$$(1)(5 - 0.1 - 0.8)^2$$

$$= \left(\frac{W}{L} \right)_D [2(5 - 0.8)(0.1) - (0.1)^2]$$

$$16.81 = \left(\frac{W}{L} \right)_D [0.83]$$

$$\left(\frac{W}{L} \right)_D = 20.3$$

5.38

 M_L in saturation

 M_D in nonsaturation

$$\left(\frac{W}{L}\right)_L (V_{GSL} - V_{TN})^2 = \left(\frac{W}{L}\right)_D [2(V_{GSD} - V_{TND})V_{DSD} - V_{DSD}^2]$$

$$(1)(1.8)^2 = \left(\frac{W}{L}\right)_D [2(5 - 0.8)(0.05) - (0.05)^2]$$

$$3.24 = \left(\frac{W}{L}\right)_D [0.4175]$$

$$\left(\frac{W}{L}\right)_D = 7.76$$

5.39

$$I_{REF} = K_{n4}(V_{GS4} - V_{TN})^2 = K_{n3}(V_{GS3} - V_{TN})^2$$

$$V_{GS4} = 5 - V_{GS3}$$

$$\sqrt{\frac{400}{200}}(5 - V_{GS3} - 1) = (V_{GS3} - 1)$$

$$2.41V_{GS3} = 6.66 \Rightarrow V_{GS3} = 2.76 \text{ V}$$

$$V_{GS4} = 2.24 \text{ V}, \quad V_{GS2} = V_{GS3} = 2.76 \text{ V}$$

$$I_{REF} = K_{n1}(V_{GS1} - V_{TN})^2 = (0.2)(2.76 - 1)^2$$

$$I_{REF} = 0.620 \text{ mA}$$

$$I_Q = K_{n2}(V_{GS2} - V_{TN})^2 = (0.1)(2.76 - 1)^2$$

$$I_Q = 0.310 \text{ mA}$$

$$I_Q = K_{n1}(V_{GS1} - V_{TN})^2$$

$$\Rightarrow 0.310 = (0.08)(V_{GS1} - 1)^2$$

$$\text{Then } V_{GS1} = \sqrt{\frac{0.310}{0.08}} + 1 \Rightarrow V_{GS1} = 2.97 \text{ V}$$

5.40

$$I_D = \frac{V_{DD} - V_0}{R_D} = \frac{5 - 0.1}{10} = 0.49 \text{ mA}$$

Transistor biased in nonsaturation

$$I_D = 0.49$$

$$= (0.015)\left(\frac{W}{L}\right)[2(4.2 - 0.8)(0.1) - (0.1)^2]$$

$$0.49 = \left(\frac{W}{L}\right)[0.67] \Rightarrow \frac{W}{L} = 0.731$$

5.41

$$S = I_D R_D + V_r + V_{DS}$$

$$S = (12)R_D + 1.6 + 0.2 \Rightarrow R_D = 267 \Omega$$

$$I_D = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2$$

$$12 = \left(\frac{0.040}{2}\right)\left(\frac{W}{L}\right)(5 - 0.8)^2 \Rightarrow \frac{W}{L} = 34$$

5.42

$$S = V_{SD} + I_D R_D + V_r$$

$$S = 0.15 + (15)R_D + 1.6 \Rightarrow R_D = 217 \Omega$$

$$I_D = \left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)(V_{SG} + V_{TP})^2$$

$$15 = \left(\frac{0.020}{2}\right)\left(\frac{W}{L}\right)(5 - 0.8)^2 \Rightarrow \frac{W}{L} = 85$$

5.43

$$V_{DS}(\text{sat}) = V_{GS} - V_P$$

$$\text{So } V_{DS} > V_{DS}(\text{sat}) = -V_P, \quad I_D = I_{DSS}$$

5.44

$$V_{DS}(\text{sat}) = V_{GS} - V_P = V_{GS} + 3 = V_{DS}(\text{sat})$$

$$\text{a. } V_{GS} = 0 \Rightarrow I_D = I_{DSS} = 6 \text{ mA}$$

$$\text{b. } I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2 = 6 \left(1 - \frac{-1}{-3}\right)^2$$

$$\Rightarrow I_D = 2.67 \text{ mA}$$

$$\text{c. } I_D = 6 \left(1 - \frac{-2}{-3}\right)^2 \Rightarrow I_D = 0.667 \text{ mA}$$

$$\text{d. } I_D = 0$$

5.45

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$2.8 = I_{DSS} \left(1 - \frac{1}{V_P}\right)^2$$

$$0.30 = I_{DSS} \left(1 - \frac{3}{V_P}\right)^2$$

$$\frac{2.8}{0.30} = \frac{\left(1 - \frac{1}{V_P}\right)^2}{\left(1 - \frac{3}{V_P}\right)^2} = 9.33$$

$$\frac{\left(1 - \frac{1}{V_P}\right)}{\left(1 - \frac{3}{V_P}\right)} = 3.055$$

$$1 - \frac{1}{V_P} = 3.055 - \frac{9.165}{V_P}$$

$$\frac{8.165}{V_P} = 2.055 \Rightarrow V_P = 3.97 \text{ V}$$

$$2.8 = I_{DSS} \left(1 - \frac{1}{3.97}\right)^2 = I_{DSS}(0.560)$$

$$\Rightarrow I_{DSS} = 5.0 \text{ mA}$$

5.46

$$V_S = -V_{GS}, \quad V_{SD} = V_S - V_{DD}$$

$$\text{Want } V_{SD} \geq V_{SD(\text{sat})} = V_P - V_{GS}$$

$$V_S - V_{DD} \geq V_P - V_{GS}$$

$$-V_{GS} - V_{DD} \geq V_P - V_{GS} \Rightarrow V_{DD} \leq -V_P$$

$$\text{So } V_{DD} \leq -2.5 \text{ V}$$

$$I_D = 2 = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$2 = 6 \left(1 - \frac{V_{GS}}{2.5}\right)^2 \Rightarrow V_{GS} = 1.06 \text{ V}$$

$$\Rightarrow V_S = -1.06 \text{ V}$$

5.47

$$I_D = K_a (V_{GS} - V_{TN})^2$$

$$18.5 = K_a (0.35 - V_{TN})^2$$

$$86.2 = K_a (0.5 - V_{TN})^2$$

Then

$$\frac{18.5}{86.2} = 0.2146 = \frac{(0.35 - V_{TN})^2}{(0.50 - V_{TN})^2} \Rightarrow V_{TN} = 0.221 \text{ V}$$

$$18.5 = K_a (0.35 - 0.221)^2 \Rightarrow K_a = 1.11 \text{ mA/V}^2$$

5.48

$$I_D = K(V_{GS} - V_{TN})^2$$

$$250 = K(0.75 - 0.24)^2 \Rightarrow K = 0.961 \text{ mA/V}^2$$

5.49

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2 = \frac{V_S}{R_S} = -\frac{V_{GS}}{R_S}$$

$$10 \left(1 - \frac{V_{GS}}{-5}\right)^2 = -\frac{V_{GS}}{0.2}$$

$$2 \left(1 + \frac{2V_{GS}}{5} + \frac{V_{GS}^2}{25}\right) = -V_{GS}$$

$$\frac{2}{25} V_{GS}^2 + \frac{9}{5} V_{GS} + 2 = 0$$

$$2V_{GS}^2 + 45V_{GS} + 50 = 0$$

$$V_{GS} = \frac{-45 \pm \sqrt{(45)^2 - 4(2)(50)}}{2(2)}$$

$$\Rightarrow V_{GS} = -1.17 \text{ V}$$

$$I_D = -\frac{V_{GS}}{R_S} = \frac{1.17}{0.2} \Rightarrow I_D = 5.85 \text{ mA}$$

$$V_D = 20 - (5.85)(2) = 8.3 \text{ V}$$

$$V_{DS} = V_D - V_S = 8.3 - 1.17 \Rightarrow V_{DS} = 7.13 \text{ V}$$

5.50

$$V_{DS} = V_{DD} - V_S$$

$$8 = 10 - V_S \Rightarrow V_S = 2 \text{ V} = I_D R_S = (5)R_S$$

$$\Rightarrow R_S = 0.4 \text{ k}\Omega$$

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$5 = I_{DSS} \left(1 - \frac{-1}{V_P}\right)^2 \text{ Let } I_{DSS} = 10 \text{ mA}$$

$$5 = 10 \left(1 - \frac{-1}{V_P}\right)^2 \Rightarrow V_P = -3.41 \text{ V}$$

$$V_G = V_{GS} + V_S = -1 + 2 = 1 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$1 = \frac{1}{R_1} (500)(10) \Rightarrow R_1 = 5 \text{ M}\Omega$$

$$\frac{5R_2}{5+R_2} = 0.5 \Rightarrow R_2 = 0.556 \text{ M}\Omega$$

5.51

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$5 = 8 \left(1 - \frac{V_{GS}}{4}\right)^2 \Rightarrow V_{GS} = 0.838 \text{ V}$$

$$V_{SD} = V_{DD} - I_D (R_S + R_D)$$

$$= 20 - (5)(0.5 + 2) \Rightarrow V_{SD} = 7.5 \text{ V}$$

$$V_S = 20 - (5)(0.5) = 17.5 \text{ V}$$

$$V_G = V_S + V_{GS} = 17.5 + 0.838 = 18.3 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$18.3 = \frac{1}{R_1} (100)(20) \Rightarrow R_1 = 109 \text{ k}\Omega$$

$$\frac{109R_2}{109 + R_2} = 100 \Rightarrow R_2 = 1.21 \text{ M}\Omega$$

5.52

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$6 = 7 \left(1 - \frac{V_{GS}}{3}\right)^2 \Rightarrow V_{GS} = 0.465 \text{ V}$$

$$V_{SD} = V_{DD} - I_D (R_S + R_D)$$

$$6 = 12 - (5)(0.3 + R_D) \Rightarrow R_D = 0.9 \text{ k}\Omega$$

$$V_S = 12 - (5)(0.3) = 10.5 \text{ V}$$

$$V_G = V_S + V_{GS} = 10.5 + 0.465 = 10.965 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right) V_{DD}$$

$$10.965 = \left(\frac{R_2}{100}\right) (12) \Rightarrow R_2 = 91.4 \text{ k}\Omega$$

$$\Rightarrow R_1 = 8.6 \text{ k}\Omega$$

5.53

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \left(\frac{60}{140 + 60} \right) (20) \\ \Rightarrow \underline{V_G = 6 \text{ V}}$$

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2 = \frac{V_S}{R_S} = \frac{V_G - V_{GS}}{R_S}$$

$$(8)(2) \left(1 - \frac{V_{GS}}{(-4)} \right)^2 = 6 - V_{GS}$$

$$16 \left(1 + \frac{V_{GS}}{2} + \frac{V_{GS}^2}{16} \right) = 6 - V_{GS}$$

$$V_{GS}^2 + 9V_{GS} + 10 = 0$$

$$V_{GS} = \frac{-9 \pm \sqrt{(9)^2 - 4(10)}}{2} \Rightarrow \underline{V_{GS} = -1.30}$$

$$I_D = 8 \left(1 - \frac{(-1.30)}{(-4)} \right)^2 \Rightarrow \underline{I_D = 3.65 \text{ mA}}$$

$$V_{DS} = V_{DD} - I_D(R_S + R_D) \\ = 20 - (3.65)(2 + 2.7)$$

$$V_{DS} = 2.85 \text{ V}$$

$$V_{DS} > V_{DS(\text{sat})} = V_{GS} - V_P \\ = -1.30 - (-4) \\ = 2.7 \text{ V} \quad (\text{Yes})$$

5.54

$$V_{DS} = V_{DD} - I_D(R_S + R_D)$$

$$5 = 12 - I_D(0.5 + 1) \Rightarrow \underline{I_D = 4.67 \text{ mA}}$$

$$V_S = I_D R_S = (4.67)(0.5) \Rightarrow \underline{V_S = 2.33 \text{ V}}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \left(\frac{20}{450 + 20} \right) (12) \\ \Rightarrow \underline{V_G = 0.511 \text{ V}}$$

$$V_{GS} = V_G - V_S = 0.511 - 2.33$$

$$\Rightarrow \underline{V_{GS} = -1.82 \text{ V}}$$

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2$$

$$4.67 = 10 \left(1 - \frac{(-1.82)}{V_P} \right)^2 \Rightarrow \underline{V_P = -6.75 \text{ V}}$$

5.55

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2, \quad V_{GS} = 0$$

$$\underline{I_D = I_{DSS} = 4 \text{ mA}}$$

$$R_D = \frac{V_{DD} - V_{DS}}{I_D} = \frac{10 - 3}{4} \Rightarrow \underline{R_D = 1.75 \text{ k}\Omega}$$

5.56

$$V_{SD} = V_{DD} - I_D R_S$$

$$10 = 20 - (1)R_S \Rightarrow \underline{R_S = 10 \text{ k}\Omega}$$

$$R_1 + R_2 = \frac{V_{DD}}{I} = \frac{20}{0.1} = 200 \text{ k}\Omega$$

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2$$

$$1 = 2 \left(1 - \frac{V_{GS}}{2} \right)^2 \Rightarrow \underline{V_{GS} = 0.586 \text{ V}}$$

$$V_G = V_S + V_{GS} = 10 + 0.586 = 10.586$$

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD}$$

$$10.586 = \left(\frac{R_2}{200} \right) (20) \Rightarrow \underline{R_2 = 106 \text{ k}\Omega}$$

$$\underline{R_1 = 94 \text{ k}\Omega}$$

5.57

$$V_{DS} = V_{DD} - I_D(R_S + R_D)$$

$$2 = 3 - (0.040)(10 + R_D) \Rightarrow \underline{R_D = 15 \text{ k}\Omega}$$

$$I_D = K(V_{GS} - V_{TN})^2$$

$$40 = 250(V_{GS} - 0.20)^2 \Rightarrow \underline{V_{GS} = 0.60 \text{ V}}$$

$$V_G = V_{GS} + V_S = 0.60 + (0.040)(10) = 1.0 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD}$$

$$1 = \left(\frac{R_2}{150} \right) (3) \Rightarrow \underline{R_2 = 50 \text{ k}\Omega}$$

$$\underline{R_1 = 100 \text{ k}\Omega}$$

5.58

$$\text{For } V_o = 0.70 \text{ V} \Rightarrow$$

$$V_{DS} = 0.70 > V_{DS(\text{sat})} = V_{GS} - V_{TN}$$

$$0.75 - 0.15 = 0.6$$

Biased in the saturation region

$$I_D = \frac{V_{DD} - V_{DS}}{R_D} = \frac{3 - 0.7}{50} \Rightarrow \underline{I_D = 46 \mu\text{A}}$$

$$I_D = K(V_{GS} - V_{TN})^2 \Rightarrow 46 = K(0.75 - 0.15)^2 \Rightarrow \\ K = 128 \mu\text{A/V}^2$$

Chapter 6

Exercise Solutions

E6.1

$$g_m = 2K_n(V_{GS} - V_{TN}) \text{ and}$$

$$I_D = K_n(V_{GS} - V_{TN})^2$$

$$0.75 = 0.5(V_{GS} - 0.8)^2 \Rightarrow V_{GS} = 2.025 \text{ V}$$

$$g_m = 2(0.5)(2.025 - 0.8) \Rightarrow g_m = 1.22 \text{ mA/V}$$

$$r_o = [\lambda K_n(V_{GS} - V_{TN})^2]^{-1}$$

$$= [(0.01)(0.5)(2.025 - 0.8)]^{-1} \Rightarrow$$

$$r_o = 133 \text{ k}\Omega$$

E6.2

$$g_m = 2K_n(V_{GS} - V_{TN}) \text{ and } I_D = K_n(V_{GS} - V_{TN})^2$$

$$\Rightarrow V_{GS} - V_{TN} = \sqrt{\frac{I_{DQ}}{K_n}} \text{ and}$$

$$g_m = 2K_n\sqrt{\frac{I_{DQ}}{K_n}} = 2\sqrt{K_n I_{DQ}}$$

$$K_n = \frac{g_m^2}{4I_{DQ}} = \frac{(3.4)^2}{4(2)} = 1.45 \text{ mA/V}$$

$$K_n = \frac{\mu_n C_{ox} \cdot W}{2L}$$

$$1.45 = (0.018)\left(\frac{W}{L}\right) \Rightarrow \frac{W}{L} = 80.6$$

$$r_o = [\lambda K_n(V_{GS} - V_{TN})^2]^{-1} = [\lambda I_{DQ}]^{-1}$$

$$r_o = [(0.015)(2)]^{-1} \Rightarrow r_o = 33.3 \text{ k}\Omega$$

E6.3

$$a. \quad I_{DQ} = K_n(V_{GS} - V_{TN})^2$$

$$0.4 = 0.5(V_{GS} - 2)^2 \Rightarrow V_{GS} = 2.89 \text{ V}$$

$$V_{DSQ} = V_{DD} - I_{DQ}R_D = 10 - (0.4)(10)$$

$$\Rightarrow V_{DSQ} = 6 \text{ V}$$

$$b. \quad g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.5)(2.89 - 2)$$

$$\Rightarrow g_m = 0.89 \text{ mA/V}$$

$$r_o = [\lambda I_{DQ}]^{-1}, \quad \lambda = 0 \Rightarrow r_o = \infty$$

$$A_v = \frac{v_o}{v_i} = -g_m R_D = -(0.89)(10)$$

$$\Rightarrow A_v = -8.9$$

$$c. \quad v_i = 0.4 \sin \omega t \Rightarrow v_{ds} = -(8.9)(0.4) \sin \omega t$$

$$v_{ds} = -3.56 \sin \omega t$$

$$\cdot \text{At } V_{DS1} = 6 - 3.56 = 2.44$$

$$V_{GS1} = 2.89 + 0.4 = 3.29$$

$$V_{GS1} - V_{TN} = 3.29 - 2 = 1.29$$

So $V_{DS1} > V_{GS1} - V_{TN} \Rightarrow$ Biased in saturation region

E6.4

$$a. \quad V_{SDQ} = V_{DD} - I_{DQ}R_D$$

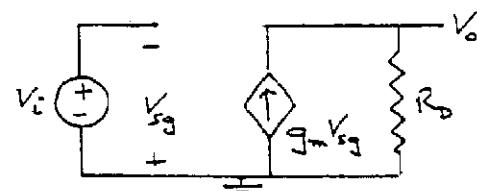
$$7 = 12 - I_{DQ}(6) \Rightarrow I_{DQ} = 0.833 \text{ mA}$$

$$I_{DQ} = K_p(V_{SG} - |V_{TP}|)^2$$

$$0.833 = 2(V_{SG} - 1)^2 \Rightarrow V_{SG} = 1.65 \text{ V}$$

$$b. \quad g_m = 2K_p(V_{SG} - |V_{TP}|) = 2(2)(1.65 - 1)$$

$$\Rightarrow g_m = 2.6 \text{ mA/V}, \quad r_o = \infty$$



$$A_v = \frac{v_o}{v_i} = -g_m R_D = -(2.6)(6)$$

$$\Rightarrow A_v = -15.6$$

E6.5

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2 \Rightarrow V_{GS} - V_{TN} = \sqrt{\frac{I_{DQ}}{K_n}}$$

$$g_m = 2K_n(V_{GS} - V_{TN}) = 2K_n\sqrt{\frac{I_{DQ}}{K_n}}$$

$$\text{So } g_m = 2\sqrt{K_n I_{DQ}}$$

E6.6

$$\eta = \frac{\gamma}{2\sqrt{2\phi_f + v_{ss}}}$$

$$(a) \quad \eta = \frac{0.40}{2\sqrt{2(0.35)+1}} \Rightarrow \eta = 0.153$$

$$(b) \eta = \frac{0.40}{2\sqrt{2(0.35)+3}} \Rightarrow \underline{\eta = 0.104}$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.75)} = 1.22 \text{ mA/V}$$

$$\text{For (a), } g_{mb} = g_m \eta = (1.22)(0.153) \Rightarrow$$

$$g_{mb} = 0.187 \text{ mA/V}$$

$$\text{For (b), } g_{mb} = (1.22)(0.104) \Rightarrow \underline{g_{mb} = 0.127 \text{ mA/V}}$$

E6.7

$$a. \quad I_{DQ} = K_n (V_{GS} - V_{TN})^2 = (0.25)(2 - 0.8)^2$$

$$\Rightarrow \underline{I_{DQ} = 0.36 \text{ mA}}$$

$$V_{DSQ} = V_{DD} - I_{DQ} R_D = 5 - (0.36)(5)$$

$$\Rightarrow \underline{V_{DSQ} = 3.2 \text{ V}}$$

$$b. \quad g_m = 2K_n (V_{GS} - V_{TN}) = 2(0.25)(2 - 0.8)$$

$$\Rightarrow \underline{g_m = 0.6 \text{ mA/V}, \quad r_o = \infty}$$

$$c. \quad A_v = \frac{v_o}{v_i} = -g_m R_D = -(0.6)(5)$$

$$\Rightarrow \underline{A_v = -3.0}$$

E6.8

$$v_i = v_{gs} = 0.1 \sin \omega t$$

$$i_d = g_m v_{gs} = (0.6)(0.1) \sin \omega t$$

$$i_d = 0.06 \sin \omega t \text{ mA}$$

$$v_{ds} = (-3)(0.1) \sin \omega t = -0.3 \sin \omega t$$

$$\text{Then } i_D = I_{DQ} + i_d = 0.36 + 0.06 \sin \omega t \\ = i_D \text{ mA}$$

$$v_{DS} = V_{DSQ} + v_{ds} = 3.2 - 0.3 \sin \omega t = v_{DS}$$

E6.9

$$V_{SDQ} = 3 \text{ V and } I_{DQ} = 0.5 \text{ mA}$$

$$\Rightarrow R_D = \frac{5 - 3}{0.5} \Rightarrow \underline{R_D = 4 \text{ k}\Omega}$$

$$I_{DQ} = K_p (V_{SG} - |V_{TP}|)^2$$

$$0.5 = 1(V_{SG} - 1)^2 \Rightarrow V_{SG} = 1.71 \text{ V}$$

$$\Rightarrow V_{GG} = 5 - 1.71 \Rightarrow \underline{V_{GG} = 3.29 \text{ V}}$$

$$A_v = -g_m R_D$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(1)(0.5)}$$

$$g_m = 1.41 \text{ mA/V}$$

$$A_v = -(1.41)(4) \Rightarrow \underline{A_v = -5.64}$$

$$A_v = \frac{v_o}{v_i} = \frac{-v_{sd}}{v_i} = -\frac{0.46 \sin \omega t}{v_i} = -5.64$$

$$\Rightarrow \underline{v_i = 0.0816 \sin \omega t}$$

E6.10

$$a. \quad V_{SG} = 9 - I_{DQ} R_S, \quad I_{DQ} = K_p (V_{SG} - |V_{TP}|)^2$$

$$V_{SG} = 9 - (2)(1.2)(V_{SG} - 2)^2$$

$$= 9 - 2.4(V_{SG}^2 - 4V_{SG} + 4)$$

$$2.4V_{SG}^2 - 8.6V_{SG} + 0.6 = 0$$

$$V_{SG} = \frac{8.6 \pm \sqrt{(8.6)^2 - 4(2.4)(0.6)}}{2(2.4)}$$

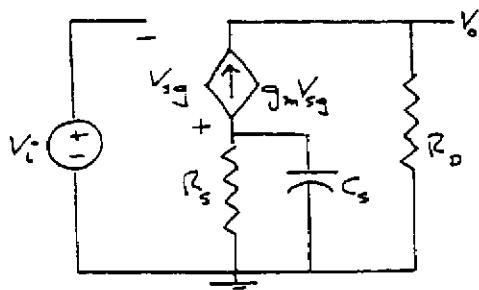
$$V_{SG} = 3.51 \text{ V}, \quad I_{DQ} = 2(3.51 - 2)^2$$

$$\Rightarrow \underline{I_{DQ} = 4.56 \text{ mA}}$$

$$V_{SDQ} = 9 + 9 - I_{DQ}(1.2 + 1) = 18 - (4.56)(2.2)$$

$$\Rightarrow \underline{V_{SDQ} = 7.97 \text{ V}}$$

b.



$$V_0 = g_m V_{SG} R_D$$

$$A_v = -g_m R_D = -(6.04)(1) \Rightarrow \underline{A_v = -6.04}$$

E6.11

$$I_{DQ} = I_Q = 0.5 \text{ mA}$$

$$\text{Let } \frac{W}{L} = 25$$

$$K_p = (20)(25) = 500 \mu\text{A/V}^2$$

$$V_{GS} = \sqrt{\frac{0.5}{0.5}} + 1.5 = 2.5 \text{ V} \Rightarrow V_S = -2.5 \text{ V}$$

$$A_v = -g_m R_D$$

$$g_m = 2(0.5)(2.5 - 1.5) = 1 \text{ mA/V}$$

$$\text{For } A_v = -4.0 \Rightarrow R_D = 4 \text{ k}\Omega$$

$$V_D = 5 - (0.5)(4) = 3 \text{ V}$$

$$\Rightarrow V_{DSQ} = 3 - (-2.5) = 5.5 \text{ V}$$

E6.12

a. With $R_G \Rightarrow V_{GS} = V_{DS} \Rightarrow$ transistor biased in sat. region

$$I_D = K_p (V_{GS} - V_{TN})^2 = K_p (V_{DS} - V_{TN})^2$$

$$V_{DS} = V_{DD} - I_D R_D$$

$$= V_{DD} - K_p R_D (V_{DS} - V_{TN})^2$$

$$\begin{aligned}
 V_{DS} &= 15 - (0.15)(10)(V_{DS} - 1.8)^2 \\
 &= 15 - 1.5(V_{DS}^2 - 3.6V_{DS} + 3.24) \\
 1.5V_{DS}^2 - 4.4V_{DS} - 10.14 &= 0 \\
 V_{DS} &= \frac{4.4 \pm \sqrt{(4.4)^2 + (4)(1.5)(10.14)}}{2(1.5)} \\
 \Rightarrow V_{DSQ} &= 4.45 \text{ V} \\
 I_{DQ} &= (0.15)(4.45 - 1.8)^2 \Rightarrow I_{DQ} = 1.05 \text{ mA}
 \end{aligned}$$

b. Neglecting effect of R_G :

$$\begin{aligned}
 A_V &= -g_m(R_D \parallel R_L) \\
 g_m &= 2K_p(V_{GS} - V_{TN}) = 2(0.15)(4.45 - 1.8) \\
 \Rightarrow g_m &= 0.795 \text{ mA/V} \\
 A_V &= -(0.795)(10 \parallel 5) \Rightarrow A_V = -2.65
 \end{aligned}$$

c. $R_G \Rightarrow$ establishes $V_{GS} = V_{DS} \Rightarrow$ essentially no effect on small-signal voltage gain.

E6.13

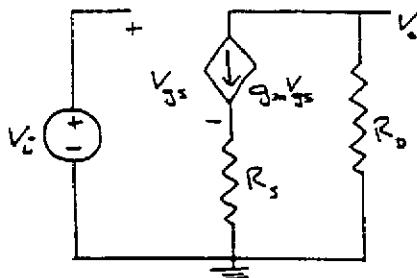
$$\begin{aligned}
 \text{a. } I_{DQ} &= K_p(V_{GS} - V_{TN})^2 \\
 I_{DQ} &= 0.8(2 - V_{SG})^2 = \frac{V_{SG}}{R_S} = \frac{V_{SG}}{4} \\
 3.2(4 - 4V_{SG} + V_{SG}^2) &= V_{SG} \\
 3.2V_{SG}^2 - 13.8V_{SG} + 12.8 &= 0 \\
 V_{SG} &= \frac{13.8 \pm \sqrt{(13.8)^2 - 4(3.2)(12.8)}}{2(3.2)} \\
 V_{SG} &= 1.35 \text{ V} \Rightarrow I_{DQ} = 0.8(2 - 1.35)^2 \\
 \Rightarrow I_{DQ} &= 0.338 \text{ mA}
 \end{aligned}$$

$$\text{b. } V_{DSQ} = V_{DD} - I_{DQ}(R_D + R_S)$$

$$6 = 10 - (0.338)(R_D + 4)$$

$$R_D = \frac{10 - (0.338)(4) - 6}{0.338} \Rightarrow R_D = 7.83 \text{ k}\Omega$$

c.

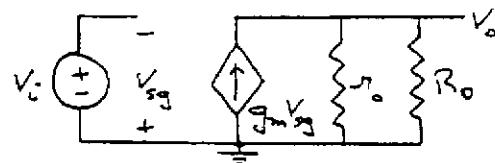


$$\begin{aligned}
 V_i &= V_{gs} + g_m V_{gs} R_S \Rightarrow V_{gs} = \frac{V_i}{1 + g_m R_S} \\
 V_0 &= -g_m V_{gs} R_D \\
 g_m &= 2K_p(V_{GS} - V_{TN}) = 2(0.8)(-1.35 + 2) \\
 &= 1.04 \text{ mA/V} \\
 A_V &= \frac{V_0}{V_i} = \frac{-g_m R_D}{1 + g_m R_S} = \frac{-(1.04)(7.83)}{1 + (1.04)(4)} \\
 \Rightarrow A_V &= -1.58
 \end{aligned}$$

E6.14

$$\begin{aligned}
 \text{a. } S &= I_{DQ} R_S + V_{SG} \text{ and} \\
 I_{DQ} &= K_p(V_{SG} + V_{TP})^2 \\
 0.8 &= 0.5(V_{SG} + 0.8)^2 \Rightarrow V_{SG} = 0.465 \text{ V} \\
 5 &= (0.8)R_S + 0.465 \Rightarrow R_S = 5.67 \text{ k}\Omega \\
 V_{SDQ} &= 10 - I_{DQ}(R_S + R_D) \\
 3 &= 10 - (0.8)(5.67 + R_D) \\
 R_D &= \frac{10 - (0.8)(5.67) - 3}{0.8} \Rightarrow R_D = 3.08 \text{ k}\Omega
 \end{aligned}$$

b.

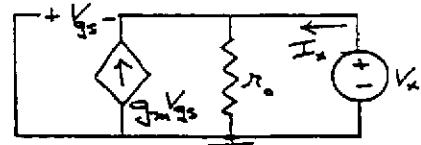


$$\begin{aligned}
 V_0 &= g_m V_{sg} (R_D \parallel r_o) = -g_m V_i (R_D \parallel r_o) \\
 A_V &= \frac{V_0}{V_i} = -g_m (R_D \parallel r_o) \\
 g_m &= 2K_p(V_{SG} + V_{TP}) = 2(0.5)(0.465 + 0.8) \\
 &= 1.27 \text{ mA/V}
 \end{aligned}$$

$$\begin{aligned}
 r_o &= \frac{1}{\lambda I_0} = \frac{1}{(0.02)(0.8)} = 62.5 \text{ k}\Omega \\
 A_V &= -(1.27)(3.08 \parallel 62.5) \Rightarrow A_V = -3.73
 \end{aligned}$$

E6.15

$$\begin{aligned}
 V_0 &= g_m V_{gs} r_o \\
 V_i &= V_{gs} + V_0 \Rightarrow V_{gs} = V_i - V_0 \\
 \text{So } V_0 &= g_m r_o (V_i - V_0) \\
 A_V &= \frac{V_0}{V_i} = \frac{g_m r_o}{1 + g_m r_o} = \frac{(4)(50)}{1 + (4)(50)} \\
 \Rightarrow A_V &= 0.995
 \end{aligned}$$



$$I_s + g_m V_{gs} = \frac{V_s}{r_0} \text{ and } V_{gs} = -V_s$$

$$I_s = g_m V_s + \frac{V_s}{r_0} \Rightarrow R_0 = r_0 \parallel \frac{1}{g_m} = 50 \parallel \frac{1}{4}$$

$$\Rightarrow R_0 \approx 0.25 \text{ k}\Omega$$

With $R_S = 4 \text{ k}\Omega \Rightarrow A_v = \frac{g_m(r_0 \parallel R_S)}{1 + g_m(r_0 \parallel R_S)}$

$$r_0 \parallel R_S = 50 \parallel 4 = 3.7 \text{ k}\Omega \Rightarrow A_v = \frac{(4)(3.7)}{1 + (4)(3.7)}$$

$$\Rightarrow A_v = 0.937$$

E6.16

$$V_{DSQ} = V_{DD} - I_{DQ}R_S$$

$$5 = 10 - (1.5)R_S \Rightarrow R_S = 3.33 \text{ k}\Omega$$

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2 \Rightarrow 1.5 = (1)(V_{GS} - 0.8)^2$$

$$V_{GS} = 2.02 \text{ V} = V_G - V_S = V_G - 5$$

$$\Rightarrow V_G = 7.02 \text{ V} = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \frac{R_2}{400} \cdot 10$$

So $R_2 = 280.8 \text{ k}\Omega$, $R_1 = 119.2 \text{ k}\Omega$

Neglecting R_{SI} ,

$$A_v = \frac{g_m(R_S \parallel r_0)}{1 + g_m(R_S \parallel r_0)}$$

$$r_0 = [\lambda I_{DQ}]^{-1} = [(0.015)(1.5)]^{-1} = 44.4 \text{ k}\Omega$$

$$R_S \parallel r_0 = 3.33 \parallel 44.4 = 3.1 \text{ k}\Omega$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(1)(1.5)} = 2.45 \text{ mA/V}$$

$$A_v = \frac{(2.45)(3.1)}{1 + (2.45)(3.1)} \Rightarrow A_v = 0.884$$

$$R_0 = \frac{1}{g_m} \parallel R_S \parallel r_0 = \frac{1}{2.45} \parallel 3.33 \parallel 44.4$$

$$= 0.408 \parallel 3.1$$

$$\Rightarrow R_0 = 0.36 \text{ k}\Omega$$

E6.17

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \left(\frac{9.3}{70.7 + 9.3} \right) (5)$$

$$= 0.581 \text{ V}$$

$$I_{DQ} = K_p(V_{SG} - |V_{TP}|)^2 = K_p(V_s - V_G - |V_{TP}|)^2$$

$$= \frac{5 - V_s}{R_S}$$

Then $(0.4)(5)(V_s - 0.581 - 0.8)^2 = 5 - V_s$

$$2(V_s - 1.381)^2 = 5 - V_s$$

$$2(V_s^2 - 2.762V_s + 1.907) = 5 - V_s$$

$$2V_s^2 - 4.52V_s - 1.19 = 0$$

$$V_s = \frac{4.52 \pm \sqrt{(4.52)^2 + 4(2)(1.19)}}{2(2)}$$

$$V_s = 2.50 \text{ V} \Rightarrow I_{DQ} = \frac{5 - 2.5}{5} = 0.5 \text{ mA}$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.4)(0.5)} = 0.894 \text{ mA/V}$$

$$A_v = \frac{g_m R_S}{1 + g_m R_S} \cdot \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{SI}}$$

$$= \frac{(0.894)(5)}{1 + (0.894)(5)} \cdot \frac{70.7 \parallel 9.3}{70.7 \parallel 9.3 + 0.5} \Rightarrow A_v = 0.770$$

Neglecting R_{SI} , $A_v = 0.817$

$$R_0 = R_S \parallel \frac{1}{g_m} = 5 \parallel \frac{1}{0.894} = 5 \parallel 1.12$$

$$\Rightarrow R_0 = 0.915 \text{ k}\Omega$$

E6.18

(a) $g_m = 2\sqrt{K_n I_{DQ}} = 2 = 2\sqrt{K_n(0.8)} \Rightarrow$
 $K_n = 1.25 \text{ mA/V}^2$
 $K_n = \frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L} \Rightarrow 1.25 = (0.020)\left(\frac{W}{L}\right)$
 So $\frac{W}{L} = 62.5$

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2 \Rightarrow 0.8 = 1.25(V_{GS} - 2)^2$$

$$\Rightarrow V_{GS} = 2.8 \text{ V}$$

b. $r_0 = [\lambda I_{DQ}]^{-1} = [(0.01)(0.5)]^{-1} = 125 \text{ k}\Omega$
 $A_v = \frac{g_m(r_0 \parallel R_L)}{1 + g_m(r_0 \parallel R_L)}$
 $r_0 \parallel R_L = 125 \parallel 4 = 3.88$
 $A_v = \frac{(2)(3.88)}{1 + (2)(3.88)} \Rightarrow A_v = 0.886$
 $R_0 = \frac{1}{g_m} \parallel r_0 = \frac{1}{2} \parallel 125 \Rightarrow R_0 \approx 0.5 \text{ k}\Omega$

E6.19

$$I_{DQ} = K_p(V_{SG} - |V_{TP}|)^2$$

$$3 = 2(V_{SG} - 2)^2 \Rightarrow V_{SG} = 3.22 \text{ V}$$

$$I_{DQ} = \frac{5 - V_{SG}}{R_S} \Rightarrow 3 = \frac{5 - 3.22}{R_S}$$

$$\Rightarrow R_S = 0.593 \text{ k}\Omega$$

$$r_0 = [\lambda I_{DQ}]^{-1} = [(0.02)(3)]^{-1} = 16.7 \text{ k}\Omega$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(2)(3)} = 4.9 \text{ mA/V}$$

For $R_L = \infty$, $A_v = \frac{g_m(r_0 \parallel R_S)}{1 + g_m(r_0 \parallel R_S)}$
 $r_0 \parallel R_S = 16.7 \parallel 0.593 = 0.573 \text{ k}\Omega$
 $A_v = \frac{(4.9)(0.573)}{1 + (4.9)(0.573)} \Rightarrow A_v = 0.737$

If A_v is reduced by 10%

$$\Rightarrow A_v = 0.737 - 0.0737 = 0.663$$

$$A_v = \frac{g_m(r_0 \parallel R_S \parallel R_L)}{1 + g_m(r_0 \parallel R_S \parallel R_L)}$$

Let $r_0 \parallel R_S \parallel R_L = z$

$$0.663 = \frac{(4.9)z}{1 + (4.9)z} \Rightarrow 0.663 = 4.9z(1 - 0.663)$$

$$z = 0.402 = 0.573 \parallel R_L$$

$$\frac{0.573 R_L}{R_L + 0.573} = 0.402$$

$$\Rightarrow (0.573 - 0.402)R_L = (0.402)(0.573)$$

$$\Rightarrow R_L = 1.35 \text{ k}\Omega$$

E6.20

$$R_{in} = \frac{1}{g_m} = 0.35 \text{ k}\Omega \Rightarrow g_m = 2.86 \text{ mA/V}$$

$$\frac{V_D}{I_s} = R_D \parallel R_L = 2.4 = R_D \parallel 4 = \frac{4 R_D}{4 + R_D}$$

$$(4 - 2.4)R_D = (2.4)(4) \Rightarrow R_D = 6 \text{ k}\Omega$$

$$g_m = 2\sqrt{K_n I_{DQ}}$$

$$2.86 = 2\sqrt{K_n(0.5)} \Rightarrow K_n = 4.09 \text{ mA/V}^2$$

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2$$

$$0.5 = 4.09(V_{GS} - 1)^2 \Rightarrow V_{GS} = 1.35 \text{ V}$$

$$\Rightarrow V_S = -1.35 \text{ V}, \quad V_D = 5 - (0.5)(6) = 2 \text{ V}$$

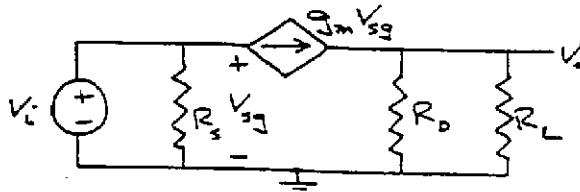
$$V_{DS} = V_D - V_S = 2 - (-1.35) = 3.35 \text{ V}$$

We have

$$V_{DS} = 3.35 > V_{GS} - V_{TN} = 1.35 - 1 = 0.35 \text{ V}$$

\Rightarrow Biased in the saturation region

E6.21



$$V_0 = g_m V_{sg} (R_D \parallel R_L) \text{ and } V_{sg} = V_i$$

$$A_v = g_m (R_D \parallel R_L)$$

$$I_{DQ} = \frac{5 - V_{SG}}{R_s} = K_p (V_{SG} - |V_{TN}|)^2$$

$$5 - V_{SG} = (1)(4)(V_{SG} - 0.8)^2$$

$$5 - V_{SG} = 4(V_{SG}^2 - 1.6V_{SG} + 0.64)$$

$$4V_{SG}^2 - 5.4V_{SG} + 2.44 = 0$$

$$V_{SG} = \frac{5.4 \pm \sqrt{(5.4)^2 + (4)(4)(2.44)}}{2(4)}$$

$$V_{SG} = 1.71 \text{ V}$$

$$I_{DQ} = \frac{5 - 1.71}{4} = 0.822 \text{ mA}$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(1)(0.822)} = 1.81 \text{ mA/V}$$

$$A_v = (1.81)(2/4) = (1.81)(1.33) \Rightarrow A_v = 2.41$$

$$R_{in} = R_S \parallel \frac{1}{g_m} = 4 \parallel \frac{1}{1.81} = 4 \parallel 0.552$$

$$\Rightarrow R_{in} = 0.485 \text{ k}\Omega$$

E6.22

$$K_{n1} = \frac{\mu_n C_{ox}}{2} \cdot \left(\frac{W}{L} \right)_1 = (0.020)(80) = 1.6 \text{ mA/V}^2$$

$$K_{n2} = \frac{\mu_n C_{ox}}{2} \cdot \left(\frac{W}{L} \right)_2 = (0.020)(1) = 0.020 \text{ mA/V}^2$$

$$A_v = -\sqrt{\frac{K_{n1}}{K_{n2}}} = -\sqrt{\frac{1.6}{0.020}} \Rightarrow A_v = -8.94$$

The transition point is determined from

$$v_{GS1} - V_{TN1} = V_{DD} - V_{TN2} - \sqrt{\frac{K_{n1}}{K_{n2}}(v_{GS1} - V_{TN1})}$$

$$v_{GS1} - 0.8 = (5 - 0.8) - (8.94)(v_{GS1} - 0.8)$$

$$v_{GS1} = \frac{(5 - 0.8) + (8.94)(0.8)}{1 + 8.94}$$

$$v_{GS1} = 1.22 \text{ V}$$

For Q-point in middle of saturation region

$$V_{GS} = \frac{1.22 - 0.8}{2} + 0.8 \Rightarrow V_{GS} = 1.01 \text{ V}$$

E6.23

$$K_{n2} = \frac{\mu_n C_{ox}}{2} \cdot \left(\frac{W}{L} \right)_2 = (0.015)(2) = 0.030 \text{ mA/V}^2$$

$$A_v = -\sqrt{\frac{K_{n1}}{K_{n2}}} = -6 \Rightarrow \frac{K_{n1}}{K_{n2}} = 36$$

$$K_{n1} = (36)(0.030) = 1.08 \text{ mA/V}^2$$

$$1.08 = (0.015) \left(\frac{W}{L} \right)_1 \Rightarrow \left(\frac{W}{L} \right)_1 = 72$$

The transition point is found from

$$v_{GS1} - 1 = (10 - 1) - (6)(v_{GS1} - 1)$$

$$v_{GS1} = \frac{10 - 1 + 6 + 1}{1 + 6} = 2.29 \text{ V}$$

For Q-point in middle of saturation region

$$V_{GS} = \frac{2.29 - 1}{2} + 1 \Rightarrow V_{GS} = 1.645 \text{ V}$$

E6.24

(a) Transition points:

$$\text{For } M_2: v_{GS} = V_{DD} - |V_{TN}| = 5 - 1.2 = 3.8 \text{ V}$$

For M_1 :

$$K_{n1} [(v_{GS})^2 (1 + \lambda v_{GS})]$$

$$= K_{n2} [(V_{TNL})^2 (1 + \lambda_2 [V_{DD} - v_{GS}])]$$

$$250[v_{GS}^2 + (0.01)v_{GS}^3]$$

$$= 25[(1.2)^2 (1 + (0.01)(5) - (0.01)v_{GS})]$$

$$10[v_{OA}^2 + (0.01)v_{OA}^3] = 1512 - 0.0144v_{OA}$$

$$(0.01)v_{OA}^3 + v_{OA}^2 + 0.00144v_{OA} - 0.512 = 0$$

which yields $v_{OA} \approx 0.388 V$

Then middle of saturation region

$$v_{DQ} = \frac{3.8 - 0.388}{2} + 0.388 \Rightarrow V_{DSQ1} = 2.094 V$$

$$K_{n1}[(V_{GS1} - V_{TN1})^2(1 + \lambda_1 v_o)]$$

$$= K_{n1}[(V_{TN1})^2(1 + \lambda_2 [V_{DD} - v_o])]$$

$$250[(V_{GS1} - 0.8)^2(1 + [0.01][2.094])]$$

$$= 25[(1.2)^2(1 + [0.01][5 - 2.094])]$$

$$10[(V_{GS1} - 0.8)^2(1.0209)] = 1.482$$

$$(V_{GS1} - 0.8)^2 = 0.145 \Rightarrow V_{GS1} = 1.18 V$$

b. $I_{DQ} = K_{n1}[(V_{GS1} - 0.8)^2(1 + (0.01)(2.094))]$

$$I_{DQ} = (0.25)[(0.145)^2(1.02094)]$$

$$\Rightarrow I_{DQ} = 37.0 \mu A$$

c. $A_v = \frac{-g_{m1}}{I_{DQ}(\lambda_1 + \lambda_2)} = -g_{m1}(r_{o1} \parallel r_{o2})$

$$g_{m1} = 2K_{n1}(V_{GS1} - V_{TN1})$$

$$= 2(0.25)(1.18 - 0.8) = 0.19 mA/V$$

$$A_v = \frac{-0.19}{(0.037)(0.01 + 0.01)} \Rightarrow A_v = -257$$

E6.25

$$R_o = R_{S2} \parallel \frac{1}{g_{m2}}$$

$$g_{m2} = 0.632 mA/V, R_{S2} = 8 k\Omega$$

$$R_o = 8 \parallel \frac{1}{0.632} = 8 \parallel 1.58 \Rightarrow R_o = 1.32 k\Omega$$

E6.26

a. $I_{DSQ2} = 2mA, V_{SDQ2} = 10 V$

$$I_{DSQ2} \cdot R_{S2} = 10 = 2R_{S2} \Rightarrow R_{S2} = 5 k\Omega$$

$$I_{DSQ2} = K_{n2}(V_{GS2} - V_{TN2})^2$$

$$2 = 1(V_{GS2} - 2)^2 \Rightarrow V_{GS2} = 3.41 V$$

$$\Rightarrow V_{G2} = 3.41 V$$

Then $R_{D1} = \frac{10 - 3.41}{2} \Rightarrow R_{D1} = 3.3 k\Omega$

For $V_{DSQ1} = 10 V \Rightarrow V_{S1} = 3.41 - 10 = -6.59 V$

Then $R_{S1} = \frac{-6.59 - (-10)}{2} \Rightarrow R_{S1} = 1.71 k\Omega$

$$I_{D1} = K_{n1}(V_{GS1} - V_{TN1})^2$$

$$2 = 1(V_{GS1} - 2)^2 \Rightarrow V_{GS1} = 3.41 V$$

$$V_{GS1} = \left(\frac{R_2}{R_1 + R_2} \right) (20) - I_{DQ1} R_{S1}$$

$$\frac{R_2}{R_1 + R_2} = \frac{1}{R_1} \cdot R_{in}$$

$$3.41 = \frac{1}{R_1} (200)(20) - (2)(1.71)$$

$$\Rightarrow \underline{R_1 = 586 k\Omega}$$

$$\frac{586 R_2}{586 + R_2} = 200 \Rightarrow (586 - 200)R_2 = (200)(586)$$

$$\Rightarrow \underline{R_2 = 304 k\Omega}$$

b. $g_{m1} = 2\sqrt{K_{n1} I_{DQ1}} = 2\sqrt{(1)(2)}$

$$\Rightarrow g_{m1} = g_{m2} = 2.83 mA/V$$

From Example 6-16

$$A_v = \frac{-g_{m1} g_{m2} R_{D1} (R_{S2} \parallel R_L)}{1 + g_{m2} (R_{S2} \parallel R_L)}$$

$$R_{S2} \parallel R_L = 5 \parallel 4 = 2.22 k\Omega$$

$$A_v = \frac{-(2.83)(2.83)(3.3)(2.22)}{1 + (2.83)(2.22)}$$

$$\Rightarrow \underline{A_v = -8.06}$$

$$R_o = \frac{1}{g_{m2}} \parallel R_{S2} = \frac{1}{2.83} \parallel 5 = 0.353 \parallel 5$$

$$\Rightarrow \underline{R_o = 0.330 k\Omega}$$

E6.27

a. $I_{DQ1} = K_{n1}(V_{GS1} - V_{TN1})^2$

$$1 = 1.2(V_{GS1} - 2)^2 \Rightarrow V_{GS1} = V_{GS2} = 2.91 V$$

$$R_S = 10 k\Omega \Rightarrow V_{S1} = I_{DQ1} R_S - 10$$

$$= (1)(10) - 10 = 0$$

$$V_{G1} = 2.91 = \left(\frac{R_3}{R_1 + R_2 + R_3} \right) (10)$$

$$= \left(\frac{R_3}{500} \right) (10)$$

$$\Rightarrow \underline{R_3 = 145.5 k\Omega}$$

$$V_{DSQ1} = 3.5 \Rightarrow V_{S2} = 3.5 V \Rightarrow 3.5 + 2.91$$

$$\Rightarrow V_{G2} = 6.41$$

$$V_{G2} = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3} \right) (10) = 6.41$$

$$= \left(\frac{R_2 + R_3}{500} \right) (10)$$

$$R_2 + R_3 = 320.5 = R_2 + 145.5 \Rightarrow \underline{R_2 = 175 k\Omega}$$

$$\text{Then } R_1 + R_2 + R_3 = 500 = R_1 + 175 + 145.5$$

$$\Rightarrow \underline{R_1 = 179.5 k\Omega}$$

$$\text{Now } V_{S2} = 3.5 \Rightarrow V_{D2} = V_{S2} + V_{SDQ2}$$

$$= 3.5 + 3.5 = 7 V$$

$$\text{So } R_D = \frac{10 - 7}{1} \Rightarrow R_D = 3 \text{ k}\Omega$$

b. From Example 6-18:

$$A_v = -g_m R_D$$

$$g_m = 2\sqrt{K_{n1} I_{DQ}} = 2\sqrt{(1.2)(1)} = 2.19 \text{ mA/V}$$

$$A_v = -(2.19)(3) \Rightarrow A_v = -6.57$$

E6.28

From Example 6-18:

$$g_m = 2.98 \text{ mA/V}, \quad r_o = 42.1 \text{ k}\Omega$$

$$R_1 \parallel R_2 = 420 \parallel 180 = 126 \text{ k}\Omega$$

$$V_{gs} = \left(\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_s} \right) V_i$$

$$= \left(\frac{126}{126 + 20} \right) V_i = 0.863 V_i$$

$$A_v = \frac{-g_m V_{gs} (r_o \parallel R_D \parallel R_L)}{V_i}$$

$$= -(2.98)(0.863)(42.1 \parallel 2.7 \parallel 4)$$

$$= -(2.57)(42.1 \parallel 1.61) = -(2.57)(1.55)$$

$$\Rightarrow A_v = -3.98$$

E6.29

$$V_S = I_{DQ} R_S = (1.2)(2.7) = 3.24 \text{ V}$$

$$V_D = V_S + V_{DSQ} = 3.24 + 12 = 15.24$$

$$R_D = \frac{20 - 15.24}{1.2} \Rightarrow R_D = 3.97 \text{ k}\Omega$$

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2$$

$$1.2 = 4 \left(1 - \frac{V_{GS}}{V_P} \right)^2 \Rightarrow \frac{V_{GS}}{V_P} = 0.452$$

$$V_{GS} = (0.452)(-3) = -1.356$$

$$V_G = V_S + V_{GS} = 3.24 - 1.356 = 1.88 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) (20) = \left(\frac{R_2}{500} \right) (20) = 1.88$$

$$\Rightarrow R_2 = 47 \text{ k}\Omega, \quad R_1 = 453 \text{ k}\Omega$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.005)(1.2)} = 167 \text{ k}\Omega$$

$$g_m = \frac{2I_{DSS}}{(-V_P)} \left(1 - \frac{V_{GS}}{V_P} \right) = \frac{2(4)}{3} \left(1 - \frac{1.356}{3} \right) \\ = 1.46 \text{ mA/V}$$

$$A_v = -g_m (r_o \parallel R_D \parallel R_L) = -(146)(167 \parallel 3.97 \parallel 4)$$

$$\Rightarrow A_v = -2.87$$

E6.30

$$\text{a. } V_{G1} = \left(\frac{R_2}{R_1 + R_2} \right) (V_{DD})$$

$$V_{G1} = \left(\frac{430}{430 + 70} \right) (20) = 17.2 \text{ V}$$

$$I_{DQ1} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2 = 6 \left(1 - \frac{V_{G1} - V_{S1}}{2} \right)^2$$

$$= 6 \left(1 - \frac{17.2}{2} + \frac{V_{S1}}{2} \right)^2 = 6 \left(\frac{V_{S1}}{2} - 7.6 \right)^2$$

$$\text{and } I_{DQ1} = \frac{20 - V_{S1}}{4}$$

$$\text{Then } \frac{20 - V_{S1}}{4} = 6 \left(\frac{V_{S1}}{2} - 7.6 \right)^2$$

$$20 - V_{S1} = 24 \left(\frac{V_{S1}^2}{4} - 7.6 V_{S1} + 57.76 \right)$$

$$= 6V_{S1}^2 - 182.4V_{S1} + 1386.24$$

$$6V_{S1}^2 - 181.4V_{S1} + 1366.24 = 0$$

$$V_{S1} = \frac{181.4 \pm \sqrt{(181.4)^2 - 4(6)(1366.24)}}{2(6)}$$

$$V_{S1} = 14.2 \text{ V} \Rightarrow V_{GS1} = 17.2 - 14.2$$

$$= 3 \text{ V} > V_P$$

$$\text{So } V_{S1} = 16.0 \Rightarrow V_{GS1} = 17.2 - 16$$

$$= 1.2 < V_P - Q$$

$$\text{on } I_{DQ1} = \frac{20 - 16}{4} \Rightarrow I_{DQ1} = 1 \text{ mA}$$

$$V_{SDQ1} = 20 - I_{DQ1}(R_{S1} + R_{D1})$$

$$= 20 - (1)(8)$$

$$\Rightarrow V_{SDQ1} = 12 \text{ V}$$

$$V_{D1} = I_{DQ1} R_{D1} = (1)(4) = 4 \text{ V} = V_{G2}$$

$$I_{DQ2} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2 = 6 \left(1 - \frac{V_{G2} - V_{S2}}{(-2)} \right)^2$$

$$= 6 \left(1 + \frac{4}{2} - \frac{V_{S2}}{2} \right)^2 = 6 \left(3 - \frac{V_{S2}}{2} \right)^2$$

$$\text{and } I_{DQ2} = \frac{V_{S2}}{R_{S2}} = \frac{V_{S2}}{4}$$

Then

$$\frac{V_{S2}}{4} = 6 \left(3 - \frac{V_{S2}}{2} \right)^2$$

$$V_{S2} = 24 \left(9 - 3V_{S2} + \frac{V_{S2}^2}{4} \right)$$

$$6V_{S2}^2 - 73V_{S2} + 216 = 0$$

$$V_{S2} = \frac{73 \pm \sqrt{(73)^2 - 4(6)(216)}}{2(6)}$$

$$\Rightarrow V_{S2} = 7.09 \text{ V or } = 5.08 \text{ V}$$

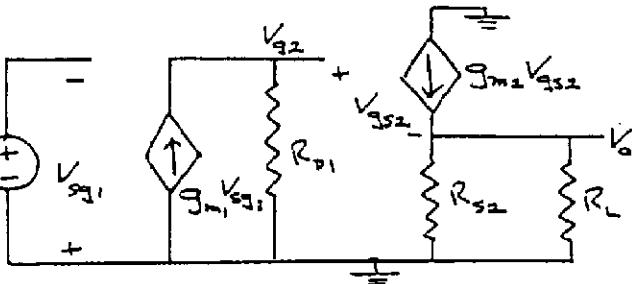
$$\text{For } V_{S2} = 5.08 \text{ V}$$

$$\Rightarrow V_{GS2} = 4 - 5.08 = -1.08 \text{ V transistor biased ON}$$

$$I_{DQ2} = \frac{5.08}{4} \Rightarrow I_{DQ1} = 1.27 \text{ mA}$$

$$V_{DS2} = 20 - V_{S2} = 20 - 5.08 \Rightarrow V_{DS2} = 14.9 \text{ V}$$

b.



$$V_{g2} = g_{m1} V_{gs1} R_{D1} = -g_{m1} V_i R_{D1}$$

$$V_0 = g_{m2} V_{g2} (R_{S2} \parallel R_L)$$

$$V_{g2} = V_{gs2} + V_0 \Rightarrow V_{gs2} = \frac{V_{g2}}{1 + g_{m2}(R_{S2} \parallel R_L)}$$

$$A_v = \frac{V_0}{V_i} = \frac{-g_{m1} R_{D1}}{1 + g_{m2}(R_{S2} \parallel R_L)}$$

$$g_{m1} = \frac{2I_{DSS}}{|V_P|} \left(1 - \frac{V_{GS}}{V_P}\right)$$

$$= \frac{2(6)}{2} \left(1 - \frac{1.2}{2}\right) = 2.4 \text{ mA/V}$$

$$g_{m2} = \frac{2(6)}{2} \left(1 - \frac{1.08}{2}\right) = 2.76 \text{ mA/V}$$

$$\text{Then } A_v = \frac{-(2.4)(4)}{1 + (2.76)(4)(2)} = -2.05 = A_v$$

E6.31

$$a. \quad I_{DQ} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

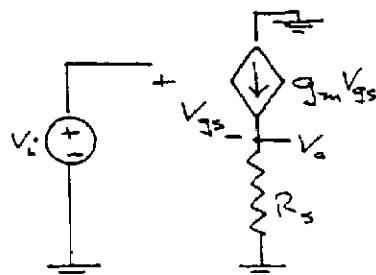
$$2 = 8 \left(1 - \frac{V_{GS}}{V_P}\right)^2 \Rightarrow \frac{V_{GS}}{V_P} = 0.5$$

$$V_{GS} = (0.5)(-3.5) \Rightarrow V_{GS} = -1.75$$

$$\text{Also } I_{DQ} = \frac{-V_{GS} - (-10)}{R_s}$$

$$2 = \frac{1.75 + 10}{R_s} \Rightarrow R_s = 5.88 \text{ k}\Omega$$

b.



$$g_m = \frac{2I_{DSS}}{|V_P|} \left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(8)}{3.5} \left(1 - \frac{1.75}{3.5}\right)$$

$$= 2.29 \text{ mA/V}$$

$$r_0 = \frac{1}{(0.01)(2)} = 50 \text{ k}\Omega$$

$$V_i = V_{gs} + g_m R_s V_{gs} \Rightarrow V_{gs} = \frac{V_i}{1 + g_m R_s}$$

$$A_v = \frac{V_0}{V_i} = \frac{g_m R_s \| r_0}{1 + g_m R_s \| r_0} = \frac{(2.29)(5.88 \| 50)}{1 + (2.29)(5.88 \| 50)}$$

$$\Rightarrow A_v = 0.923$$

$$c. \quad A_v = \frac{g_m (R_s \| R_L)}{1 + g_m (R_s \| R_L)} = 0.931 - 0.186 = 0.745$$

$$\frac{(2.29)(R_s \| R_L)}{1 + (2.29)(R_s \| R_L)} = 0.745$$

$$(2.29)(R_s \| R_L)(1 - 0.745) = 0.745$$

$$\Rightarrow R_s \| R_L = 1.28 \text{ k}\Omega$$

$$\frac{5.88 R_L}{5.88 + R_L} = 1.28$$

$$\Rightarrow (5.88 - 1.28) R_L = (1.28)(5.88)$$

$$\Rightarrow R_L = 1.54 \text{ k}\Omega$$

Chapter 6

Problem Solutions

6.1

(a) $g_m = 2\sqrt{K_n I_D} = 2\sqrt{\left(\frac{\mu_n C_{ox}}{2}\right)\left(\frac{W}{L}\right)I_D}$
 $0.5 = 2\sqrt{(0.020)\left(\frac{W}{L}\right)(0.5)} \Rightarrow \frac{W}{L} = 12.5$

(b) $I_D = \left(\frac{\mu_n C_{ox}}{2}\right)\left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2$
 $0.5 = (0.02)(12.5)(V_{GS} - 0.8)^2$
 $\Rightarrow V_{GS} = 2.21 \text{ V}$

6.2

(a) $g_m = 2\sqrt{K_p I_D} = 2\sqrt{\left(\frac{\mu_p C_{ox}}{2}\right)\left(\frac{W}{L}\right)I_D}$
 $\left(\frac{50}{2}\right)^2 = (10)\left(\frac{W}{L}\right)(100) \Rightarrow \frac{W}{L} = 0.625$

(b) $I_D = \left(\frac{\mu_p C_{ox}}{2}\right)\left(\frac{W}{L}\right)(V_{SG} + V_{TP})^2$
 $100 = (10)(0.625)(V_{SG} - 1.2)^2$
 $\Rightarrow V_{SG} = 4.2 \text{ V}$

6.3

$I_D = K_n(V_{GS} - V_{TN})^2(1 + \lambda V_{DS})$
 $\frac{I_{D1}}{I_{D2}} = \frac{1 + \lambda V_{DS1}}{1 + \lambda V_{DS2}} \Rightarrow \frac{3.4}{3.0} = \frac{1 + \lambda(10)}{1 + \lambda(5)}$
 $3.4[1 + 5\lambda] = 3.0[1 + 10\lambda]$

$3.4 - 3.0 = \lambda(3 \cdot 10 - (3.4) \cdot 5)$

$\Rightarrow \lambda = 0.0308$

$r_0 \approx \frac{1}{\lambda I_D} = \frac{1}{(0.0308)(3)} \Rightarrow r_0 = 10.8 \text{ k}\Omega$

6.4

$r_0 = \frac{1}{\lambda I_D}$
 $I_D = \frac{1}{\lambda r_0} = \frac{1}{(0.012)(100)} \Rightarrow I_D = 0.833 \text{ mA}$

6.5

$A_v = -g_m(r_0 \| R_D) = -(1)(50 \| 10)$
 $\Rightarrow A_v = -8.33$

6.6

a. $R_D = \frac{V_{DD} - V_{DSQ}}{I_{DQ}} = \frac{10 - 6}{0.5} \Rightarrow R_D = 8 \text{ k}\Omega$

For $V_{GSQ} = 2 \text{ V}$, for example,

$$I_{DQ} = \left(\frac{\mu_n C_{ox}}{2}\right)\left(\frac{W}{L}\right)(V_{GSQ} - V_{TN})^2$$

$$0.5 = (0.030)\left(\frac{W}{L}\right)(2 - 0.8)^2$$

$$\Rightarrow \frac{W}{L} = 11.6$$

b. $g_m = 2\sqrt{K_n I_{DQ}} = 2K_n(V_{GSQ} - V_{TN})$

$$g_m = 2(0.030)(11.6)(2 - 0.8)$$

$$\Rightarrow g_m = 0.835 \text{ mA/V}$$

$I_{DQ} = (0.030)(11.6)(2 - 0.8)^2 = 0.501 \text{ mA}$

$$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.015)(0.501)} \Rightarrow r_0 = 133 \text{ k}\Omega$$

c. $A_v = -g_m(r_0 \| R_D) = -(0.835)(133 \| 8)$

$\Rightarrow A_v = -6.30$

6.7

$K_n v_{et}^2 = K_n [V_{et} \sin \omega t]^2 = K_n V_{et}^2 \sin^2 \omega t$

$\sin^2 \omega t = \frac{1}{2}[1 - \cos 2\omega t]$

So $K_n v_{et}^2 = \frac{K_n V_{et}^2}{2}[1 - \cos 2\omega t]$

Ratio of signal at 2ω to that at ω :

$$\frac{\frac{K_n V_{et}^2}{2} \cdot \cos 2\omega t}{2K_n(V_{GSQ} - V_{TN})V_{et} \cdot \sin \omega t}$$

The coefficient of this expression is then:

$$\frac{V_{et}}{4(V_{GSQ} - V_{TN})}$$

6.8

$$0.01 = \frac{V_{et}}{4(V_{GSQ} - V_{TN})}$$

So $V_{et} = (0.01)(4)(3 - 1)$
 $\Rightarrow V_{et} = 0.08 \text{ V}$

6.9

$V_s = -g_m V_{et} (r_o \| R_D)$

$$V_s = \frac{R_1 \| R_2}{R_1 \| R_2 + R_{Si}} \cdot V_i = \left(\frac{50}{50+2}\right) V_i = (0.962) V_i$$

$A_v = -g_m (0.962) (r_o \| R_D) = -(1)(0.962)(50 \| 10) \Rightarrow$
 $A_v = -8.02$

6.10

$$\begin{aligned}A_v &= -g_m(r_o \parallel R_D) \\-10 &= -g_m(100 \parallel 5) \\&\Rightarrow g_m = 2.1 \text{ mA/V}\end{aligned}$$

6.11

$$\begin{aligned}\text{a. } V_G &= \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} \\V_G &= \left(\frac{200}{200 + 300}\right)(12) = 4.8 \text{ V} \\I_D &= \frac{V_G - V_{GS}}{R_s} = K_n(V_{GS} - V_{TN})^2 \\4.8 - V_{GS} &= (1)(2)(V_{GS}^2 - 4V_{GS} + 4)\end{aligned}$$

$$\begin{aligned}2V_{GS}^2 - 7V_{GS} + 3.2 &= 0 \\V_{GS} &= \frac{7 \pm \sqrt{(7)^2 - 4(2)(3.2)}}{2(2)} = 2.96 \text{ V}\end{aligned}$$

$$\begin{aligned}I_D &= (1)(2.96 - 2)^2 \Rightarrow I_D = 0.920 \text{ mA} \\V_{DS} &= V_{DD} - I_D(R_D + R_S)\end{aligned}$$

$$\begin{aligned}&= 12 - (0.92)(3 + 2) \\&\Rightarrow V_{DS} = 7.4 \text{ V}\end{aligned}$$

$$\text{(b) } V_o = \frac{-g_m V_{DS} (R_D \parallel R_L)}{1 + g_m R_S}$$

$$\text{where } V_{DS} = \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{SD}} \cdot V_i = \frac{300 \parallel 200}{300 \parallel 200 + 2} \cdot V_i = \frac{120}{120 + 2} \cdot V_i = (0.984)V_i$$

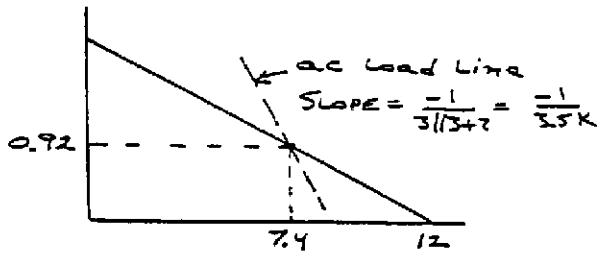
Then

$$A_v = \frac{-g_m(0.984)(R_D \parallel R_L)}{1 + g_m R_S}$$

$$g_m = 2(1)(2.96 - 2) = 1.92 \text{ mA/V}$$

$$\text{So } A_v = \frac{-(1.92)(0.984)(3 \parallel 3)}{1 + (1.92)(2)} \Rightarrow A_v = -0.586$$

c.



$$\Delta i_D = \frac{-1}{3.5 \text{ k}\Omega} \cdot \Delta v_{ds}$$

$$\Delta i_D = 0.92 \text{ mA}$$

$$\Rightarrow |\Delta v_{ds}| = (0.92)(3.5) = 3.22$$

$$\Rightarrow 6.44 \text{ V peak-to-peak}$$

6.12

$$\begin{aligned}\text{a. } I_{DQ} &= 3 \text{ mA} \Rightarrow V_S = I_{DQ} R_S = (3)(0.5) = 1.5 \text{ V} \\I_{DQ} &= K_n(V_{GS} - V_{TN})^2 \\3 &= (2)(V_{GS} - 2)^2 \Rightarrow V_{GS} = 3.22 \text{ V} \\V_G &= V_{GS} + V_S = 3.22 + 1.5 = 4.72 \text{ V} \\V_G &= \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD} \\4.72 &= \frac{1}{R_1}(200)(15) \Rightarrow R_1 = 636 \text{ k}\Omega \\&\frac{636 R_2}{636 + R_2} = 200 \Rightarrow R_2 = 292 \text{ k}\Omega\end{aligned}$$

$$\begin{aligned}\text{b. } A_v &= \frac{-g_m(R_D \parallel R_L)}{1 + g_m R_S} \\g_m &= 2(2)(3.22 - 2) = 4.88 \text{ mA/V} \\A_v &= \frac{-(4.88)(2 \parallel 5)}{1 + (4.88)(0.5)} \Rightarrow A_v = -2.03\end{aligned}$$

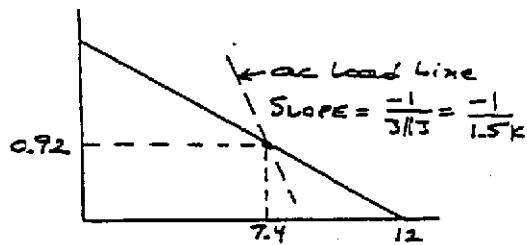
6.13

From Problem 6-11: $I_D = 0.920 \text{ mA}$

$$V_{DS} = 7.4 \text{ V}$$

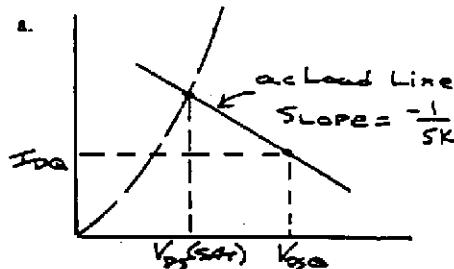
$$g_m = 1.92 \text{ mA/V}$$

$$\begin{aligned}A_v &= -g_m(R_D \parallel R_L) \left(\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{SD}} \right) \\&= -(1.92)(3 \parallel 3) \left(\frac{200 \parallel 300}{200 \parallel 300 + 2} \right) = -(2.88)(0.984) \\A_v &= -2.83\end{aligned}$$



$$\begin{aligned}\Delta i_D &= \frac{-1}{1.5 \text{ k}\Omega} \cdot \Delta v_{ds}, \quad \Delta i_D = 0.92 \text{ mA} \\&\Rightarrow |\Delta v_{ds}| = (0.92)(1.5) = 1.38 \\&\Rightarrow 2.76 \text{ V peak-to-peak output voltage swing}\end{aligned}$$

6.14



$$V_{DSQ} = V^+ - I_{DQ}R_D - (-V_{GSQ})$$

$$\text{Output Voltage Swing} = V_{DSQ} - V_{DS(\text{sat})}$$

$$= [V^+ - I_{DQ}R_D + V_{GSQ}] - (V_{GSQ} - V_{TN})$$

$$= V^+ - I_{DQ}R_D + V_{TN}$$

$$\text{So } |\Delta I_D| = I_{DQ} = \frac{1}{5\text{k}\Omega} \cdot |\Delta V_{DS}|$$

$$= \frac{1}{5\text{k}\Omega} [V^+ - I_{DQ}R_D + V_{TN}]$$

$$I_{DQ} = \frac{1}{5\text{k}\Omega} [5 - I_{DQ}(10) + 1]$$

$$= 1.2 - 2I_{DQ} = I_{DQ}$$

$$\Rightarrow I_{DQ} = 0.4 \text{ mA} = I_Q$$

$$\text{b. } g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.4)} = 0.894 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(0.4)} = 250 \text{ k}\Omega$$

$$A_v = -g_m(R_D \parallel R_L \parallel r_o)$$

$$= -(0.894)(10 \parallel 10 \parallel 250)$$

$$\Rightarrow A_v = -4.38$$

6.15

$$\text{a. } I_D = K_n(V_{GS} - V_{TN})^2$$

$$2 = 4(V_{GS} - (-1))^2$$

$$V_{GS} = -0.293 \text{ V}$$

$$\Rightarrow V_S = 0.293 \text{ V} = I_D R_S = (2)R_S$$

$$\Rightarrow R_S = 0.146 \text{ k}\Omega$$

$$V_D = V_{DS} + V_S = 6 + 0.293 = 6.293$$

$$R_D = \frac{10 - 6.293}{2} \Rightarrow R_D = 1.85 \text{ k}\Omega$$

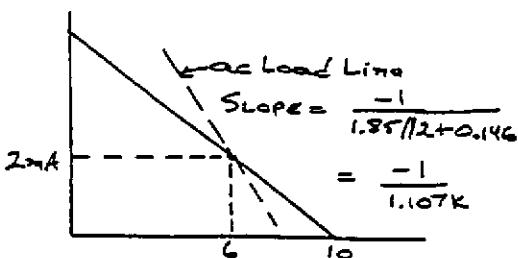
$$\text{b. } A_v = \frac{-g_m(R_D \parallel R_L)}{1 + g_m R_S}$$

$$g_m = 2K_n(V_{GS} - V_{TN})$$

$$g_m = 2(4)(-0.293 + 1) = 5.66 \text{ mA/V}$$

$$A_v = \frac{-(5.66)(1.85||2)}{1 + (5.66)(0.146)} \Rightarrow A_v = -2.98$$

c.



$$\Delta v_o = (\Delta i_D)(1.85||2) = (2)(1.85||2)$$

$$= 1.92 \text{ V}$$

$$\Delta v_i = \frac{1.92}{2.98} = 0.645 \Rightarrow V_i = 0.645 \text{ V}$$

6.16

$$\text{a. } V_{DSQ} = V_{DD} - I_{DQ}(R_D + R_S)$$

$$2.5 = 5 - I_{DQ}(2 + R_S)$$

$$I_{DQ} = \frac{2.5}{2 + R_S}$$

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2 = \frac{-V_{GS}}{R_S}$$

$$\Rightarrow V_{GS} = -I_{DQ}R_S = \frac{-2.5R_S}{2 + R_S}$$

$$K_n \left[\frac{-2.5R_S}{2 + R_S} - V_{TN} \right]^2 = \frac{2.5}{2 + R_S}$$

$$4 \left[1 - \frac{2.5R_S}{2 + R_S} \right]^2 = \frac{2.5}{2 + R_S}$$

$$4 \left[\frac{2 + R_S - 2.5R_S}{2 + R_S} \right]^2 = \frac{2.5}{2 + R_S}$$

$$4 \frac{(2 - 1.5R_S)^2}{2 + R_S} = 2.5$$

$$4(4 - 6R_S + 2.25R_S^2) = 2.5(2 + R_S)$$

$$9R_S^2 - 26.5R_S + 11 = 0$$

$$R_S = \frac{26.5 \pm \sqrt{(26.5)^2 - 4(9)(11)}}{2(9)}$$

$$R_S = 0.5 \text{ k}\Omega \text{ or } 2.44 \text{ k}\Omega$$

But $R_S = 2.44 \text{ k}\Omega \Rightarrow V_{GS} = -1.37 \text{ Cut off.}$

$$\Rightarrow R_S = 0.5 \text{ k}\Omega, \quad I_{DQ} = 1.0 \text{ mA}$$

$$\text{b. } A_v = \frac{-g_m(R_D \parallel R_L)}{1 + g_m R_S}$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(4)(1)} = 4 \text{ mA/V}$$

$$A_v = \frac{-(4)(2||2)}{1 + (4)(0.5)} \Rightarrow A_v = -1.33$$

6.17

$$\text{a. } 5 = I_{DQ}R_S + V_{SDQ} + I_{DQ}R_D - 5$$

$$5 = I_{DQ}R_S + 6 + I_{DQ}(10) - 5$$

$$1. \quad I_{DQ} = \frac{4}{R_S + 10}$$

$$V_S = V_{SDQ} + I_{DQ}R_D - 5 = V_{SGQ}$$

$$2. \quad 1 + I_{DQ}(10) = V_{SGQ}$$

$$3. \quad I_{DQ} = K_p(V_{SGQ} - 2)^2$$

Choose $R_S = 10 \text{ k}\Omega \Rightarrow I_{DQ} = \frac{4}{20} = 0.20 \text{ mA}$

$$V_{SGQ} = 1 + (0.2)(10) = 3 \text{ V}$$

$$0.20 = K_p(3-2)^2 \Rightarrow K_p = 0.20 \text{ mA/V}^2$$

b. $I_{DQ} = (0.20)(3-2)^2 = 0.20 \text{ mA}$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.2)(0.2)} = 0.4 \text{ mA/V}$$

$$A_v = -g_m(R_D \parallel R_L) = -(0.4)(10 \parallel 10)$$

$$\Rightarrow A_v = -2.0$$

c. Choose $R_S = 20 \text{ k}\Omega \Rightarrow I_{DQ} = \frac{4}{30} = 0.133 \text{ mA}$

$$V_{SGQ} = 1 + (0.133)(10) = 2.33 \text{ V}$$

$$0.133 = K_p(2.33-2)^2 \Rightarrow K_p = 1.22 \text{ mA/V}^2$$

$$g_m = 2\sqrt{(1.22)(0.133)} = 0.806 \text{ mA/V}$$

$$A_v = -(0.806)(10 \parallel 10) \Rightarrow A_v = -4.03$$

A larger gain can be achieved.

6.18

a. $I_{DQ} = 1 = K_p(V_{SGQ} + V_{TP})^2$

$$1 = 5(V_{SGQ} - 1.5)^2 \Rightarrow V_{SGQ} = 1.95 \text{ V}$$

$$R_S = \frac{5 - 1.95}{1} \Rightarrow R_S = 3.05 \text{ k}\Omega$$

$$V_{SDQ} = 10 - I_{DQ}(R_S + R_D)$$

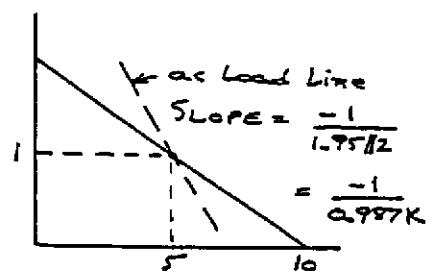
$$5 = 10 - (1)(3.05 + R_D) \Rightarrow R_D = 1.95 \text{ k}\Omega$$

b. $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(5)(1)} = 4.47 \text{ mA/V}$

$$A_v = -g_m(R_D \parallel R_L) = -(4.47)(1.95 \parallel 2)$$

$$\Rightarrow A_v = -4.41$$

c.



$$\Delta i_D = -\frac{1}{0.987 \text{ k}\Omega} \cdot \Delta v_{DS}$$

$$\Rightarrow |\Delta v_{DS}| = (1)(0.987) = 0.987 \text{ V}$$

Swing in output voltage = 1.97 V peak-to-peak

6.19

$$I_{DQ} = K_p(V_{SGQ} - V_{TN})^2$$

$$g_m = 2\sqrt{K_p I_{DQ}}$$

$$2.2 = 2\sqrt{K_p(6)} \Rightarrow K_p = 0.202 \text{ mA/V}^2$$

$$6 = 0.202(2.8 - V_{TN})^2 \Rightarrow V_{TN} = -2.65 \text{ V}$$

$$V_{DSQ} = 18 - I_{DQ}(R_S + R_D)$$

$$R_S + R_D = \frac{18 - 10}{6} = 1.33 \text{ k}\Omega \Rightarrow R_S = 1.33 - R_D$$

$$A_v = -\frac{g_m(R_D \parallel R_L)}{1 + g_m R_S}$$

$$-1 = \frac{-2.2\left(\frac{R_D + 1}{R_D + 1}\right)}{1 + (2.2)(1.33 - R_D)}$$

$$1 + 2.93 - 2.2R_D = \frac{2.2R_D}{1 + R_D}$$

$$(3.93 - 2.2R_D)(1 + R_D) = 2.2R_D$$

$$3.93 + 1.73R_D - 2.2R_D^2 = 2.2R_D$$

$$2.2R_D^2 + 0.47R_D - 3.93 = 0$$

$$R_D = \frac{-0.47 + \sqrt{(0.47)^2 + 4(2.2)(3.93)}}{2(2.2)}$$

$$\Rightarrow R_D = 1.23 \text{ k}\Omega, \quad R_S = 0.10 \text{ k}\Omega$$

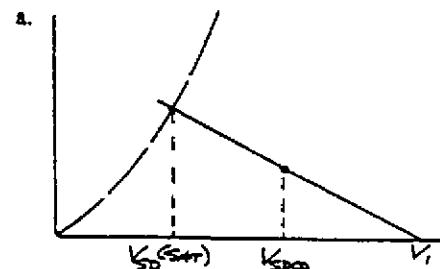
$$V_G = V_{GS} + V_S = 2.8 + (6)(0.1) = 3.4 \text{ V}$$

$$V_G = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD} = \frac{1}{R_1}(100)(18) = 3.4$$

$$\Rightarrow R_1 = 529 \text{ k}\Omega$$

$$\frac{529 R_2}{529 + R_2} = 100 \Rightarrow R_2 = 123 \text{ k}\Omega$$

6.20



$$V_i = 9 + V_{SG}, \quad V_{SD}(sat) = V_{SG} + V_{TP}$$

$$V_{SDQ} = \frac{V_i - V_{SD}(sat)}{2} + V_{SD}(sat)$$

$$= \frac{(9 + V_{SG}) - (V_{SG} + V_{TP})}{2} + (V_{SG} + V_{TP})$$

$$= \frac{9 + 1.5}{2} + V_{SG} - 1.5$$

$$V_{SDQ} = 3.75 + V_{SG} = 9 + V_{SG} - I_{DQ} R_D$$

$$I_{DQ} = K_p(V_{SG} + V_{TP})^2$$

Set $R_D = 0.1 R_L = 2 \text{ k}\Omega$

$$3.75 = 9 - I_{DQ}(2) \Rightarrow I_{DQ} = 2.625 \text{ mA}$$

$$\text{b. } g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(2)(2.625)} = 4.58 \text{ mA/V}$$

$$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(2.625)} = 38.1 \text{ k}\Omega$$

Open circuit.

$$A_v = -g_m(R_D \parallel r_0)$$

$$A_v = -4.58(2 \parallel 38.1) \Rightarrow A_v = -8.70$$

c. With R_L

$$A_v = -4.58(2 \parallel 20 \parallel 38.1) \Rightarrow A_v = -7.95$$

⇒ Change = 8.62%

6.21

$$\text{a. } I_{DQ} = K_p(V_{SG} + V_{TP})^2$$

$$2 = (0.5)(V_{SG} + 2)^2 \Rightarrow V_{SG} = 0 \text{ V}$$

$$R_S = \frac{10 - 0}{2} \Rightarrow R_S = 5 \text{ k}\Omega$$

$$V_D = 0 - V_{SDQ} = 0 - 6 \Rightarrow R_D = \frac{-6 - (-10)}{2}$$

$$\Rightarrow R_D = 2 \text{ k}\Omega$$

$$A_v = -g_m R_D$$

$$\text{b. } g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.5)(2)} = 2 \text{ mA/V}$$

$$A_v = -(2)(2) \Rightarrow A_v = -4.0$$

6.22

$$A_v = -g_m(R_D \parallel R_L)$$

$$V_{DSQ} = V_{DD} - I_{DQ}(R_S + R_D)$$

$$10 = 20 - (1)(R_S + R_D) \Rightarrow R_S + R_D = 10 \text{ k}\Omega$$

$$\text{Let } R_D = 8 \text{ k}\Omega. \quad R_S = 2 \text{ k}\Omega$$

$$A_v = -10 = -g_m(8 \parallel 20)$$

$$g_m = 1.75 \text{ mA/V} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{K_n(1)} \Rightarrow$$

$$K_n = 0.766 \text{ mA/V}^2$$

$$V_S = I_{DQ} R_S = (1)(2) = 2 \text{ V}$$

$$I_{DQ} = K_n(V_{GS} - V_{TP})^2 \Rightarrow I = 0.766(V_{GS} - 2)^2$$

$$\Rightarrow V_{GS} = 3.31 \text{ V}$$

$$V_G = V_{GS} + V_S = 3.31 + 2 = 5.31$$

$$V_G = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$\Rightarrow \frac{1}{R_1}(200)(20) = 5.31 \Rightarrow R_1 = 753 \text{ k}\Omega$$

$$\frac{753 R_2}{753 + R_2} = 200 \Rightarrow R_2 = 272 \text{ k}\Omega$$

6.23

$$A_v = \frac{g_m r_0}{1 + g_m r_0} = \frac{(4)(50)}{1 + (4)(50)} \Rightarrow A_v = 0.995$$

$$R_0 = \frac{1}{g_m} \parallel r_0 = \frac{1}{4} \parallel 50 \Rightarrow R_0 = 0.249 \text{ k}\Omega$$

$$A_v = \frac{g_m(r_0 \parallel R_S)}{1 + g_m(r_0 \parallel R_S)} = \frac{4(50 \parallel 2.5)}{1 + 4(50 \parallel 2.5)}$$

$$= \frac{4(2.38)}{1 + 4(2.38)} \Rightarrow A_v = 0.905$$

$$R_0 = \frac{1}{g_m} \parallel r_0 \parallel R_S = \frac{1}{4} \parallel 50 \parallel 2.5$$

$$\Rightarrow R_0 = 0.226 \text{ k}\Omega$$

6.24

$$\text{a. } V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \left(\frac{396}{396 + 1240} \right) (10)$$

$$\Rightarrow V_G = 2.42 \text{ V}$$

$$I_{DQ} = \frac{10 - (V_{SG} + 2.42)}{R_S} = K_p(V_{SG} + V_{TP})^2$$

$$7.58 - V_{SG} = (2)(4)(V_{SG}^2 - 4V_{SG} + 4)$$

$$8V_{SG}^2 - 31V_{SG} + 24.4 = 0$$

$$V_{SG} = \frac{31 \pm \sqrt{(31)^2 - 4(8)(24.4)}}{2(8)}$$

$$\Rightarrow V_{SG} = 2.78 \text{ V}$$

$$I_{DQ} = (2)(2.78 - 2)^2 \Rightarrow I_{DQ} = 1.21 \text{ mA}$$

$$V_{SDQ} = 10 - I_{DQ} R_S = 10 - (1.21)(4)$$

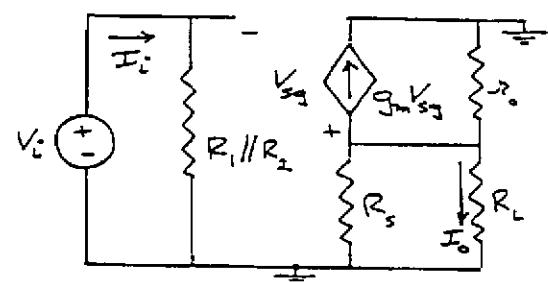
$$\Rightarrow V_{SDQ} = 5.16 \text{ V}$$

$$\text{b. } g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(2)(1.21)} = 3.11 \text{ mA/V}$$

$$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(1.21)} = 41.3 \text{ k}\Omega$$

$$A_v = \frac{g_m(R_S \parallel R_L \parallel r_0)}{1 + g_m(R_S \parallel R_L \parallel r_0)}$$

$$= \frac{3.11(4 \parallel 4 \parallel 41.3)}{1 + (3.11)(4 \parallel 4 \parallel 41.3)} \Rightarrow A_v = 0.886$$



$$I_0 = -(g_m V_{sg}) \left(\frac{R_S \parallel r_0}{R_S \parallel r_0 + R_L} \right)$$

$$V_{sg} = \frac{-V_i}{1 + g_m(R_S \parallel R_L \parallel r_0)}$$

$$V_i = I_0(R_1 \parallel R_2)$$

$$A_v = \frac{I_o}{I_i} = \frac{g_m(R_1||R_2)}{1 + g_m(R_S||R_L||r_o)} \cdot \frac{R_S||r_o}{R_S||r_o + R_L}$$

$$= \frac{(3.11)(396||1240)}{1 + (3.11)(4||4||41.3)} \cdot \frac{4||41.3}{4||41.3 + 4}$$

$$= \frac{(3.11)(300)}{1 + (3.11)(1.908)} \cdot \frac{3.647}{3.647 + 4}$$

$$\Rightarrow A_v = 64.2$$

$$R_o = \frac{1}{g_m} \parallel R_S \parallel R_L \parallel r_o = \frac{1}{3.11} \parallel 4 \parallel 4 \parallel 41.3$$

$$\Rightarrow R_o = 0.275 \text{ k}\Omega$$

6.25

$$g_m = 2\sqrt{K_n I_Q} = 2\sqrt{(1)(1)} = 2 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_Q} = \frac{1}{(0.01)(1)} = 100 \text{ k}\Omega$$

$$A_v = \frac{g_m(r_o||R_L)}{1 + g_m(r_o||R_L)} = \frac{2(100||4)}{1 + 2(100||4)}$$

$$\Rightarrow A_v = 0.885$$

$$R_o = \frac{1}{g_m} \parallel r_o = \frac{1}{2} \parallel 100 \Rightarrow R_o = 0.490 \text{ k}\Omega$$

6.26

a. $A_v = \frac{g_m R_L}{1 + g_m R_L} \Rightarrow 0.95 = \frac{g_m(4)}{1 + g_m(4)}$

$$0.95 = 4(1 - 0.95)g_m \Rightarrow g_m = 4.75 \text{ mA/V}$$

$$g_m = 2\sqrt{\left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)I_Q}$$

$$4.75 = 2\sqrt{(0.030)\left(\frac{W}{L}\right)(4)} \Rightarrow \frac{W}{L} = 47.0$$

b. $g_m = 2\sqrt{\left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)I_Q}$

$$4.75 = 2\sqrt{(0.030)(60)I_Q} \Rightarrow I_Q = 3.13 \text{ mA}$$

6.27

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2$$

$$5 = 5(V_{GS} + 2)^2 \Rightarrow V_{GS} = -1 \text{ V}$$

$$\Rightarrow V_S = -V_{GS} = 1 \text{ V}$$

$$I_{DQ} = \frac{V_S - (-5)}{R_S} \Rightarrow R_S = \frac{1+5}{5} \Rightarrow R_S = 1.2 \text{ k}\Omega$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(5)(5)} = 10 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(5)} = 20 \text{ k}\Omega$$

$$A_v = \frac{g_m(r_o||R_S||R_L)}{1 + g_m(r_o||R_S||R_L)}$$

$$= \frac{(10)(20||1.2||2)}{1 + (10)(20||1.2||2)} \Rightarrow A_v = 0.878$$

$$R_o = \frac{1}{g_m} \parallel r_o \parallel R_S = \frac{1}{10} \parallel 20 \parallel 1.2$$

$$\Rightarrow R_o = 91.9 \text{ }\Omega$$

6.28

$$A_v = \frac{g_m R_S}{1 + g_m R_S} \Rightarrow 0.90 = \frac{g_m(10)}{1 + g_m(10)}$$

$$0.90 = 10(1 - 0.90)g_m \Rightarrow g_m = 0.90 \text{ mA/V}$$

$$R_o = \frac{1}{g_m} \parallel R_S = \frac{1}{0.90} \parallel 10 \Rightarrow R_o = 1 \text{ k}\Omega$$

With R_L :

$$A_v = \frac{g_m(R_S||R_L)}{1 + g_m(R_S||R_L)} = \frac{(0.90)(10||2)}{1 + (0.90)(10||2)}$$

$$\Rightarrow A_v = 0.60$$

6.29

$$R_o = \frac{1}{g_m} \parallel R_S$$

Output resistance determined primarily by g_m .

$$\text{Set } \frac{1}{g_m} = 0.2 \text{ k}\Omega \Rightarrow g_m = 5 \text{ mA/V}$$

$$g_m = 2\sqrt{K_n I_{DQ}}$$

$$\Rightarrow 5 = 2\sqrt{(4)I_{DQ}} \Rightarrow I_{DQ} = 1.56 \text{ mA}$$

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2$$

$$1.56 = 4(V_{GS} + 2)^2$$

$$V_{GS} = -1.38 \text{ V}, \quad V_S = -V_{GS} = 1.38 \text{ V}$$

$$R_S = \frac{1.38 - (-5)}{1.56} \Rightarrow R_S = 4.09 \text{ k}\Omega$$

$$A_v = \frac{g_m(R_S||R_L)}{1 + g_m(R_S||R_L)} = \frac{5(4.09||2)}{1 + 5(4.09||2)}$$

$$\Rightarrow A_v = 0.870$$

6.30

a. $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(5)(5)} = 10 \text{ mA/V}$

$$R_o = \frac{1}{g_m} = \frac{1}{10} \Rightarrow R_o = 100 \text{ }\Omega$$

b. Open-circuit gain

$$A_v = \frac{g_m r_o}{1 + g_m r_o} \quad \text{But } r_o = \infty \text{ so } A_v = 1.0$$

With R_L :

$$A_v = \frac{g_m R_L}{1 + g_m R_L}$$

$$0.50 = \frac{10 R_L}{1 + 10 R_L} \Rightarrow 0.50 = 10(1 - 0.5)R_L$$

$$\Rightarrow R_L = 0.1 \text{ k}\Omega$$

6.31

$$|\Delta i_D| = I_{DQ} = \frac{-1}{R_S \parallel R_L} \cdot \Delta v_0$$

$$\Delta v_0 = -I_{DQ} \cdot R_S \parallel R_L = -\frac{I_{DQ} \cdot R_S R_L}{R_S + R_L}$$

$$v_0(\text{min}) = -\frac{I_{DQ} R_S}{1 + \frac{R_S}{R_L}}$$

$$A_v = \frac{g_m(R_S \parallel R_L)}{1 + g_m(R_S \parallel R_L)} = \frac{v_0}{v_i}$$

$$v_i = \frac{-I_{DQ}(R_S \parallel R_L)[1 + g_m(R_S \parallel R_L)]}{g_m(R_S \parallel R_L)}$$

$$v_i(\text{min}) = -\frac{I_{DQ}}{g_m}[1 + g_m(R_S \parallel R_L)]$$

6.32

$$a. V_{DSQ} = V_{DD} - I_{DQ} R_S$$

$$3 = 5 - (1.7)R_S \Rightarrow R_S = 1.18 \text{ k}\Omega$$

$$I_{DQ} = K_s(V_{GS} - V_{TH})^2$$

$$1.7 = (1)(V_{GS} - 1)^2 \Rightarrow V_{GS} = 2.30 \text{ V}$$

$$V_S = V_{DD} - V_{DSQ} = 5 - 3 = 2 \text{ V}$$

$$V_G = V_S + V_{GS} = 2 + 2.30 = 4.30 \text{ V}$$

$$V_G = \frac{1}{R_1} \cdot R_i \cdot V_{DD} = \frac{1}{R_1}(300)(5) = 4.30$$

$$\Rightarrow R_1 = 349 \text{ k}\Omega$$

$$\frac{349 R_2}{349 + R_2} = 300 \Rightarrow R_2 = 2.14 \text{ M}\Omega$$

$$b. g_m = 2\sqrt{K_s I_{DQ}} = 2\sqrt{(1)(1.7)} = 2.61 \text{ mA/V}$$

$$A_v = \frac{g_m R_S}{1 + g_m R_S} = \frac{2.61(1.18)}{1 + (2.61)(1.18)}$$

$$\Rightarrow A_v = 0.755$$

$$R_d = \frac{1}{g_m} \parallel R_S = \frac{1}{2.61} \parallel 1.18 = 0.383 \parallel 1.18$$

$$\Rightarrow R_d = 0.289 \text{ k}\Omega$$

6.33

$$a. V_{GS} + I_{DQ} R_S = 5$$

$$I_{DQ} = \frac{5 - V_{GS}}{R_S} = K_s(V_{GS} - V_{TH})^2$$

$$5 - V_{GS} = (10)(3)(V_{GS}^2 - 2V_{GS} + 1)$$

$$30V_{GS}^2 - 59V_{GS} + 25 = 0$$

$$V_{GS} = \frac{59 \pm \sqrt{(59)^2 - 4(30)(25)}}{2(30)} \Rightarrow V_{GS} = 1.35 \text{ V}$$

$$I_{DQ} = (3)(1.35 - 1)^2 \Rightarrow I_{DQ} = 0.365 \text{ mA}$$

$$V_{DSQ} = 10 - (0.365)(5 + 10) \Rightarrow V_{DSQ} = 4.53 \text{ V}$$

$$b. g_m = 2\sqrt{K_s I_{DQ}} = 2\sqrt{(3)(0.365)} \Rightarrow$$

$$g_m = 2.09 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0)(0.365)} \Rightarrow r_o = \infty$$

$$c. A_v = g_m(R_D \parallel R_L) = (2.09)(5 \parallel 4)$$

$$\Rightarrow A_v = 4.64$$

6.34

$$a. I_{DQ} = K_s(V_{SG} + V_{TH})^2$$

$$0.75 = (0.5)(V_{SG} - 1)^2 \Rightarrow V_{SG} = 2.22 \text{ V}$$

$$5 = I_{DQ} R_S + V_{SG} \Rightarrow R_S = \frac{5 - 2.22}{0.75}$$

$$\Rightarrow R_S = 3.71 \text{ k}\Omega$$

$$V_{SDQ} = 10 - I_{DQ}(R_S + R_D)$$

$$6 = 10 - (0.75)(3.71 + R_D) \Rightarrow R_D = 1.62 \text{ k}\Omega$$

$$b. R_i = \frac{1}{g_m}$$

$$g_m = 2\sqrt{K_s I_{DQ}} = 2\sqrt{(0.5)(0.75)} = 1.22 \text{ mA/V}$$

$$R_i = \frac{1}{1.22} \Rightarrow R_i = 0.816 \text{ k}\Omega$$

$$R_d = R_D \Rightarrow R_d = 1.62 \text{ k}\Omega$$

$$c. i_0 = \left(\frac{R_D}{R_D + R_L} \right) \left(\frac{R_S}{R_S + [1/g_m]} \right) \cdot i_i$$

$$i_0 = \left(\frac{1.62}{1.62 + 2} \right) \left(\frac{3.71}{3.71 + 0.816} \right) i_i$$

$$i_0 = 0.367 i_i \Rightarrow i_0 = 1.84 \sin \omega t (\mu\text{A})$$

$$v_0 = i_0 R_L = (1.84)(2) \sin \omega t$$

$$\Rightarrow v_0 = 3.68 \sin \omega t (\text{mV})$$

6.35

$$a. I_{DQ} = K_s(V_{GS} - V_{TH})^2$$

$$5 = 4(V_{GS} - 2)^2 \Rightarrow V_{GS} = 3.12 \text{ V}$$

$$V^+ = I_{DQ} R_D + V_{DSQ} - V_{GS}$$

$$10 = (5)R_D + 12 - 3.12 \Rightarrow R_D = 0.224 \text{ k}\Omega$$

$$b. g_m = 2\sqrt{K_s I_{DQ}} = 2\sqrt{(4)(5)} \Rightarrow g_m = 8.94 \text{ mA/V}$$

$$R_i = \frac{1}{g_m} = \frac{1}{8.94} \Rightarrow R_i = 0.112 \text{ k}\Omega$$

$$c. A_v = g_m(R_D \parallel R_L) = (8.94)(0.224 \parallel 2)$$

$$\Rightarrow A_v = 1.80$$

6.36

$$a. \quad I_{DQ} = K_p (V_{SG} + V_{TP})^2$$

$$3 = 2(V_{SG} - 2)^2 \Rightarrow V_{SG} = 3.22 \text{ V}$$

$$V^+ = I_{DQ} R_S + V_{SG}$$

$$R_S = \frac{10 - 3.22}{3} \Rightarrow R_S = 2.26 \text{ k}\Omega$$

$$V_{SDQ} = 20 - I_{DQ}(R_S + R_D)$$

$$10 = 20 - (3)(2.26 + R_D) \Rightarrow R_D = 1.07 \text{ k}\Omega$$

$$b. \quad A_v = g_m(R_D \parallel R_L)$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(2)(3)} = 4.90 \text{ mA/V}$$

$$A_v = (4.90)(1.07 \parallel 10) \Rightarrow A_v = 4.74$$

6.37

$$a. \quad |A_v| = \sqrt{\frac{(W/L)_D}{(W/L)_L}} = 5$$

$$\Rightarrow (W/L)_D = 25(0.5) \Rightarrow (W/L)_D = 12.5$$

$$K_D = \frac{1}{2} \mu_s C_{ox} \left(\frac{W}{L} \right)_D = (30)(12.5) = 375 \mu A/V^2$$

$$K_L = (30)(0.5) = 15 \mu A/V^2$$

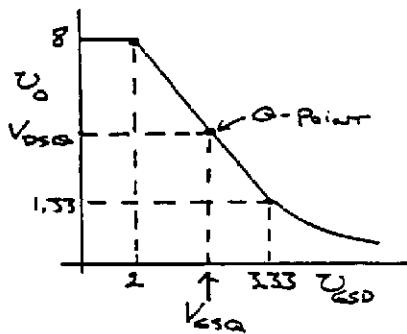
Transition point:

$$V_{GSD} - V_{TND} = (V_{DD} - V_{TND}) - \sqrt{\frac{K_D}{K_L}} (V_{GSD} - V_{TND})$$

$$V_{GSD} - 2 = (10 - 2) - \sqrt{\frac{375}{15}} (V_{GSD} - 2)$$

$$V_{GSD}(1 + 5) = (10 - 2) + 2 + 5(2)$$

$$V_{GSD} = 3.33 \text{ V and } V_{DSQ} = 1.33 \text{ V}$$



$$V_{GSQ} = \frac{3.33 - 2}{2} + 2 \Rightarrow V_{GSQ} = 2.67 \text{ V}$$

$$b. \quad I_{DQ} = K_D (V_{GSQ} - V_{TND})^2$$

$$I_{DQ} = 0.375(2.67 - 2)^2 \Rightarrow I_{DQ} = 0.167 \text{ mA}$$

$$V_{DSQ} = \frac{8 - 1.33}{2} + 1.33 \Rightarrow V_{DSQ} = 4.67 \text{ V}$$

6.38

(a) Transition point: Load:

$$V_{DS} = V_{DD} - |V_{TND}| = 10 - 2 = 8 \text{ V}$$

Driver:

$$K_D \left[(V_{OA})^2 (1 + \lambda_D V_{OA}) \right] \\ = K_L \left[(-V_{TND})^2 (1 + \lambda_L (V_{DD} - V_{OA})) \right]$$

$$0.5 [v_{0tA}^2 + (0.02)v_{0tA}^3] \\ = 0.1 [(4)(1 + 0.02(10 - v_{0tA}))]$$

We have

$$0.01v_{0tA}^3 + 0.5v_{0tA}^2 + 0.008v_{0tA} - 0.48 = 0$$

$$\text{Therefore } v_{0tA} = 0.963 \text{ V}$$

Now

$$v_{0Q} = \frac{8 - 0.963}{2} + 0.963 = 4.48 \text{ V} = V_{DSQ}$$

Then

$$K_D \left[(V_{GSD} - V_{TND})^2 (1 + \lambda_D V_{OQ}) \right] \\ = K_L \left[(-V_{TND})^2 (1 + \lambda_L (V_{DD} - V_{OQ})) \right]$$

$$0.5 [(V_{GSD} - 1.2)^2 (1 + (0.02)(4.48))] \\ = 0.1 [(4)(1 + 0.02(10 - 4.48))]$$

which yields $V_{GSD} = 2.103 \text{ V}$

$$b. \quad I_{DQ} = K_D \left[(V_{GSD} - V_{TND})^2 (1 + \lambda_D V_{OQ}) \right]$$

$$I_{DQ} = 0.5 [(2.103 - 1.2)^2 (1 + (0.02)(4.48))]$$

So $I_{DQ} = 0.444 \text{ mA}$

$$c. \quad r_{oD} = r_{oL} = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.444)} = 113 \text{ k}\Omega$$

$$g_{mD} = 2K_D(V_{GSD} - V_{TND}) = 2(0.5)(2.103 - 1.2) \Rightarrow$$

$$g_{mD} = 0.903 \text{ mA/V}$$

Then

$$A_v = -g_{mD}(r_{oD} \parallel r_{oL}) = -(0.903)(113 \parallel 113)$$

or $A_v = -51.0$

6.39

$$I_D = K_n(V_{GS} - V_{TN})^2, \quad V_{DS} = V_{GS}$$

$I_D = 0$ when $V_{DS} = 1.5V \Rightarrow V_{TN} = 1.5V$

 $0.8 = K_n(3 - 1.5)^2 \Rightarrow K_n = 0.356 \text{ mA/V}^2$
 $g_m = \frac{dI_D}{dV_{DS}} = \frac{1}{R_o} = 2K_n(V_{DS} - V_{TN})$
 $= 2(0.356)(3 - 1.5)$
 $\Rightarrow R_o = 0.936 \text{ k}\Omega$

6.40

a.

 $I_{DQ} = K_{nD}(V_{GS} - V_{TND})^2 = (0.5)(0 - (-1))^2$
 $I_{DQ} = 0.5 \text{ mA}$
 $I_{DQ} = K_{nL}(V_{GSL} - V_{TNL})^2 = K_{nL}(V_{DD} - V_o - V_{TNL})^2$
 $0.5 = 0.030(10 - V_o - 1)^2$
 $\sqrt{\frac{0.5}{0.030}} = 9 - V_o \Rightarrow V_o = 4.92 \text{ V}$

b.

 $I_{DD} = I_{DL}$
 $K_{nD}(V_i - V_{TND})^2 = K_{nL}(V_{DD} - V_o - V_{TNL})^2$
 $\sqrt{\frac{K_{nD}}{K_{nL}}}(V_i - V_{TND}) = V_{DD} - V_o - V_{TNL}$
 $V_o = V_{DD} - V_{TNL} - \sqrt{\frac{K_{nD}}{K_{nL}}}(V_i - V_{TND})$
 $A_v = \frac{dV_o}{dV_i} = -\sqrt{\frac{K_{nD}}{K_{nL}}} = -\sqrt{\frac{(W/L)_D}{(W/L)_L}}$
 $A_v = -\sqrt{\frac{500}{30}} \Rightarrow A_v = -4.08$

6.41

(a) $I_{DQ} = K_L(V_{GSL} - V_{TNL})^2 = K_L(V_{DSL} - V_{TNL})^2$

 $I_D = (0.1)(4 - 1)^2 = 0.9 \text{ mA}$
 $I_{DQ} = K_D(V_{GSD} - V_{TND})^2$
 $0.9 = (1)(V_{GSD} - 1)^2 \Rightarrow V_{GSD} = 1.95 \text{ V}$
 $V_{GG} = V_{GSD} + V_{DSL} = 1.95 + 4$
 $\Rightarrow V_{GG} = 5.95 \text{ V}$

b. $I_{DD} = I_{DL}$

$K_D(V_{GSD} - V_{TND})^2 = K_L(V_{GSL} - V_{TNL})^2$
 $\sqrt{\frac{K_D}{K_L}}(V_{GG} + V_i - V_o - V_{TND}) = V_o - V_{TNL}$
 $V_o \left(1 + \sqrt{\frac{K_D}{K_L}} \right) = \sqrt{\frac{K_D}{K_L}}(V_{GG} + V_i - V_{TND}) + V_{TNL}$
 $A_v = \frac{dV_o}{dV_i} = \frac{\sqrt{K_D/K_L}}{1 + \sqrt{K_D/K_L}} \Rightarrow A_v = \frac{1}{1 + \sqrt{K_L/K_D}}$

(c) From Problem 6.39:

$R_{LD} = \frac{1}{2K_L(V_{DSL} - V_{TNL})}$
 $= \frac{1}{2(0.1)(4 - 1)} = 1.67 \text{ k}\Omega$
 $g_m = 2\sqrt{K_D I_{DQ}} = 2\sqrt{(1)(0.9)} = 1.90 \text{ mA/V}$
 $A_v = \frac{g_m(R_{LD} \| R_L)}{1 + g_m(R_{LD} \| R_L)} = \frac{(1.90)(1.67 \| 4)}{1 + (1.90)(1.67 \| 4)}$
 $\Rightarrow A_v = 0.691$

6.42

a. From Problem 6-41:

$A_v = \frac{g_m(R_{LD} \| R_L)}{1 + g_m(R_{LD} \| R_L)} = \frac{(1.90)(1.67 \| 10)}{1 + (1.90)(1.67 \| 10)}$
 $A_v = 0.731$

b. $R_0 = \frac{1}{g_m} \| R_{LD} = \frac{1}{1.90} \| 1.67 = 0.526 \| 1.67$

 $R_0 = 0.40 \text{ k}\Omega$

6.43

a. $I_{D1} = K_1(V_{GS1} - V_{TN1})^2$

 $0.4 = 0.1(V_{GS1} - 2)^2 \Rightarrow V_{GS1} = 4 \text{ V}$
 $V_{S1} = I_{D1}R_{S1} = (0.4)(4) = 1.6 \text{ V}$
 $V_{G1} = V_{S1} + V_{GS1} = 1.6 + 4 = 5.6 \text{ V}$
 $V_{G1} = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$

$5.6 = \frac{1}{R_1}(200)(10) \Rightarrow R_1 = 357 \text{ k}\Omega$

$\frac{357R_2}{357 + R_2} = 200 \Rightarrow R_2 = 455 \text{ k}\Omega$

$V_{DS1} = V_{DD} - I_{D1}(R_{S1} + R_{D1})$

$4 = 10 - (0.4)(4 + R_{D1}) \Rightarrow R_{D1} = 11 \text{ k}\Omega$

$V_{D1} = 10 - (0.4)(11) = 5.6 \text{ V}$

$I_{D2} = K_2(V_{SG2} + V_{TP2})^2$

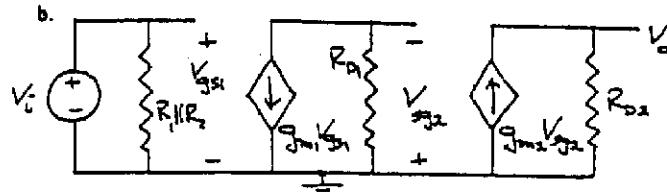
$2 = 1(V_{SG2} - 2)^2 \Rightarrow V_{SG2} = 3.41 \text{ V}$

$V_{S2} = V_{D1} + V_{SG2} = 5.6 + 3.41 = 9.01$

$R_{S2} = \frac{10 - 9.01}{2} \Rightarrow R_{S2} = 0.495 \text{ k}\Omega$

$V_{SD2} = V_{DD} - I_{D2}(R_{S2} + R_{D2})$

$5 = 10 - (2)(0.495 + R_{D2}) \Rightarrow R_{D2} = 2.01 \text{ k}\Omega$



$$V_o = g_{m2} V_{sg2} R_{D2} = (g_{m2} R_{D2})(g_{m1} V_{gs1} R_{D1})$$

$$V_{gs1} = V_i$$

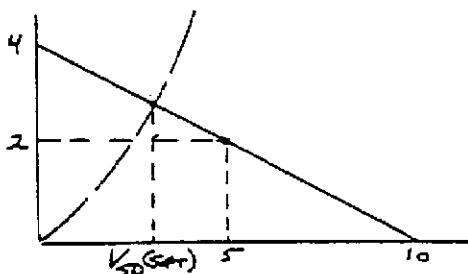
$$A_v = \frac{V_o}{V_i} = g_{m1} g_{m2} R_{D1} R_{D2}$$

$$g_{m1} = 2\sqrt{K_1 I_{D1}} = 2\sqrt{(0.1)(0.4)} = 0.4 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{K_2 I_{D2}} = 2\sqrt{(1)(2)} = 2.83 \text{ mA/V}$$

$$A_v = (0.4)(2.83)(11)(2.01) \Rightarrow A_v = 25.0$$

c.



$$\begin{aligned} V_{SD}(\text{sat}) &= V_{SG} + V_{Th} \\ &= V_{DD} - I_{D1}(R_{D2} + R_{S2}) \\ &= V_{DD} - k_{p2}(R_{D2} + R_{S2})V_{SD}^2(\text{sat}) \end{aligned}$$

So

$$(1)(2.01 + 0.495)V_{SD}^2(\text{sat}) + V_{SD}(\text{sat}) - V_{DD} = 0$$

$$2.505V_{SD}^2(\text{sat}) + V_{SD}(\text{sat}) - 10 = 0$$

$$V_{SD}(\text{sat}) = \frac{-1 \pm \sqrt{1 + 4(2.505)(10)}}{2(2.505)}$$

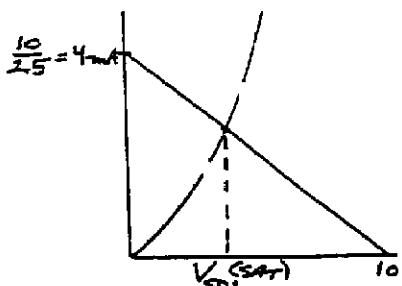
$$V_{SD}(\text{sat}) = 1.81 \text{ V}$$

$$5 - 1.81 = 3.19$$

\Rightarrow Max. output swing = 6.38 V peak-to-peak

6.44

a.



$$\begin{aligned} V_{SD1}(\text{sat}) &= V_{DD} - I_{D1}(R_{D2} + R_{S2}) \\ &= V_{DD} - K_{p2}(R_{D2} + R_{S2})V_{SD1}^2(\text{sat}) \end{aligned}$$

$$(1)(2 + 0.5)V_{SD1}^2(\text{sat}) + V_{SD1}(\text{sat}) - 10 = 0$$

$$2.5V_{SD1}^2(\text{sat}) + V_{SD1}(\text{sat}) - 10 = 0$$

$$V_{SD1}(\text{sat}) = \frac{-1 \pm \sqrt{1 + 4(2.5)(10)}}{2(2.5)} = 1.81 \text{ V}$$

$$V_{SDQ2} = \frac{10 - 1.81}{2} + 1.81 \Rightarrow V_{SDQ2} = 5.91 \text{ V}$$

$$V_{SDQ2} = V_{DD} - I_{DQ2}(R_{D2} + R_{S2})$$

$$5.91 = 10 - I_{DQ2}(2 + 0.5) \Rightarrow I_{DQ2} = 1.64 \text{ mA}$$

$$V_{S2} = 10 - (1.64)(0.5) = 9.18 \text{ V}$$

$$I_{DQ2} = K_{p2}(V_{SG2} + V_{TP2})^2$$

$$1.64 = (1)(V_{SG2} - 2)^2 \Rightarrow V_{SG2} = 3.28 \text{ V}$$

$$V_{D1} = V_{S2} - V_{SG2} = 9.18 - 3.28 = 5.90 \text{ V}$$

$$R_{D1} = \frac{10 - 5.90}{0.4} \Rightarrow R_{D1} = 10.25 \text{ k}\Omega$$

$$I_{DQ1} = K_{n1}(V_{GS1} - V_{TN1})^2$$

$$0.4 = (0.1)(V_{GS1} - 2)^2 \Rightarrow V_{GS1} = 4 \text{ V}$$

$$V_{S1} = (0.4)(1) = 0.4 \text{ V}$$

$$V_{G1} = V_{S1} + V_{GS1} = 0.4 + 4 = 4.4 \text{ V}$$

$$V_{G1} = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$4.4 = \frac{1}{R_1} \cdot (200)(10) \Rightarrow R_1 = 455 \text{ k}\Omega$$

$$\frac{455 R_2}{455 + R_2} = 200 \Rightarrow R_2 = 357 \text{ k}\Omega$$

$$b. I_{DQ2} = 1.64 \text{ mA}$$

$$V_{SDQ2} = 10 - (1.64)(2 + 0.5) \Rightarrow V_{SDQ2} = 5.90 \text{ V}$$

$$V_{DSQ1} = 10 - (0.4)(10.25 + 1) \Rightarrow V_{DSQ1} = 5.50 \text{ V}$$

$$(c) g_{m1} = 2\sqrt{K_{n1} I_{DQ1}} = 2\sqrt{(0.1)(0.4)} = 0.4 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{K_{p2} I_{DQ2}} = 2\sqrt{(1)(1.64)} = 2.56 \text{ mA/V}$$

$$A_v = g_{m1} g_{m2} R_{D1} R_{D2} = (0.4)(2.56)(10.25)(2)$$

$$\Rightarrow A_v = 21.0$$

6.45

$$a. V_{DSQ2} = 7 = V_{DD} - I_{DQ2} R_{S2} = 10 - I_{DQ2}(6)$$

$$I_{DQ2} = 0.5 \text{ mA}$$

$$I_{DQ2} = K_2(V_{GS2} - V_{TN2})^2$$

$$0.5 = (0.2)(V_{GS2} - 0.8)^2 \Rightarrow V_{GS2} = 2.38 \text{ V}$$

$$V_{S1} = V_{S2} + V_{GS2} = 3 + 2.38 = 5.38$$

$$I_{DQ1} = \frac{V_{S1}}{R_{S1}} = \frac{5.38}{20} = 0.269 \text{ mA}$$

$$I_{DQ1} = K_1(V_{GS1} - V_{TN1})^2$$

$$0.269 = (0.2)(V_{GS1} - 0.8)^2 \Rightarrow V_{GS1} = 1.96 \text{ V}$$

$$V_{G1} = V_{S1} + V_{GS1} = 5.38 + 1.96 = 7.34 \text{ V}$$

$$V_{G1} = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$7.34 = \frac{1}{R_1} (400)(10) \Rightarrow R_1 = 545 \text{ k}\Omega$$

$$\frac{545 R_2}{545 + R_2} = 400 \Rightarrow R_2 = 1.50 \text{ M}\Omega$$

b. $I_{DQ1} = 0.269 \text{ mA}, I_{DQ2} = 0.5 \text{ mA}$

$$V_{DSQ1} = 10 - (0.269)(20) \Rightarrow V_{DSQ1} = 4.62 \text{ V}$$

c. $A_v = \frac{g_{m1} R_{S1}}{1 + g_{m1} R_{S1}} \cdot \frac{g_{m2} R_{S2}}{1 + g_{m2} R_{S2}}$

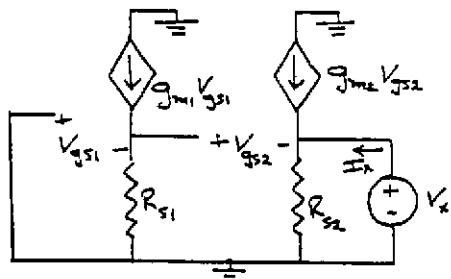
$$g_{m1} = 2\sqrt{K_t I_{DQ1}} = 2\sqrt{(0.2)(0.269)} = 0.464 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{K_t I_{DQ2}} = 2\sqrt{(0.2)(0.5)} = 0.632 \text{ mA/V}$$

$$A_v = \frac{(0.464)(20)}{1 + (0.464)(20)} \cdot \frac{(0.632)(6)}{1 + (0.632)(6)}$$

$$= (0.9027)(0.7913)$$

$$= A_v = 0.714$$



$$R_o = \frac{1}{g_{m2}} \parallel R_{S2} = \frac{1}{0.632} \parallel 6 = 1.582 \parallel 6$$

$$\Rightarrow R_o = 1.23 \text{ k}\Omega$$

6.46

(a) $I_{DQ1} = \frac{10 - V_{GS1}}{R_{S1}} = K_{n1}(V_{GS1} - V_{TN1})^2$

$$10 - V_{GS1} = (4)(10)(V_{GS1}^2 - 4V_{GS1} + 4)$$

$$40V_{GS1}^2 - 159V_{GS1} + 150 = 0$$

$$V_{GS1} = \frac{159 \pm \sqrt{(159)^2 - 4(40)(150)}}{2(40)}$$

$$\Rightarrow V_{GS1} = 2.435 \text{ V}$$

$$I_{DQ1} = (4)(2.435 - 2)^2 \Rightarrow I_{DQ1} = 0.757 \text{ mA}$$

$$V_{DSQ1} = 20 - (0.757)(10) \Rightarrow V_{DSQ1} = 12.4 \text{ V}$$

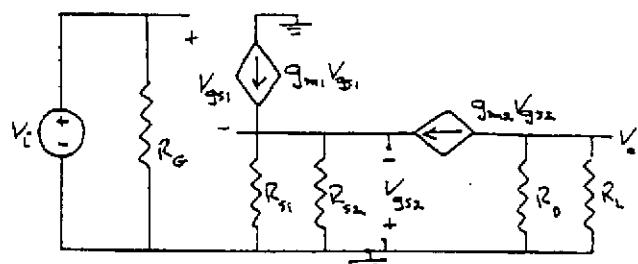
Also $I_{DQ2} = 0.757 \text{ mA}$

$$V_{DSQ2} = 20 - (0.757)(10 + 5) \Rightarrow V_{DSQ2} = 8.65 \text{ V}$$

(b) $g_{m1} = g_{m2} = 2\sqrt{KI_{DQ2}} = 2\sqrt{(4)(0.757)} \Rightarrow$

$$g_{m1} = g_{m2} = 3.48 \text{ mA/V}$$

c.



$$V_0 = -(g_{m2} V_{gs2})(R_D \parallel R_L)$$

$$V_{gs2} = (-g_{m1} V_{gs1} - g_{m2} V_{gs2})(R_{S1} \parallel R_{S2})$$

$$V_i = V_{gs1} - V_{gs2} \Rightarrow V_{gs1} = V_i + V_{gs2}$$

$$V_{gs2} + g_{m2} V_{gs2} (R_{S1} \parallel R_{S2})$$

$$= -g_{m1} (V_i + V_{gs2})(R_{S1} \parallel R_{S2})$$

$$V_{gs2} + g_{m2} V_{gs2} (R_{S1} \parallel R_{S2}) + g_{m1} V_{gs2} (R_{S1} \parallel R_{S2})$$

$$= -g_{m1} V_i (R_{S1} \parallel R_{S2})$$

$$V_{gs2} = \frac{-g_{m1} V_i (R_{S1} \parallel R_{S2})}{1 + g_{m2} (R_{S1} \parallel R_{S2}) + g_{m1} (R_{S1} \parallel R_{S2})}$$

$$A_v = \frac{V_0}{V_i} = \frac{g_{m1} g_{m2} (R_{S1} \parallel R_{S2}) (R_D \parallel R_L)}{1 + (g_{m1} + g_{m2})(R_{S1} \parallel R_{S2})}$$

$$A_v = \frac{(3.48)^2 (10 \parallel 10) (5 \parallel 2)}{1 + (3.48 + 3.48)(10 \parallel 10)}$$

$$\Rightarrow A_v = 2.42$$

6.47

a. $I_{DQ} = 3 \text{ mA}$

$$V_{S1} = I_{DQ} R_S - 5 = (3)(1.2) - 5 = -1.4 \text{ V}$$

$$I_{DQ} = K_1 (V_{GS} - V_{TN})^2$$

$$3 = 2(V_{GS} - 1)^2 \Rightarrow V_{GS} = 2.22 \text{ V}$$

$$V_{G1} = V_{GS} + V_{S1} = 2.22 - 1.4 = 0.82 \text{ V}$$

$$V_{G1} = \left(\frac{R_3}{R_1 + R_2 + R_3} \right) (5) \Rightarrow 0.82 = \left(\frac{R_3}{500} \right) (5)$$

$$\Rightarrow R_3 = 82 \text{ k}\Omega$$

$$V_{D1} = V_{S1} + V_{DSQ1} = -1.4 + 2.5 = 1.1 \text{ V}$$

$$V_{G2} = V_{D1} + V_{GS} = 1.1 + 2.22 = 3.32 \text{ V}$$

$$V_{G2} = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3} \right) (5)$$

$$\Rightarrow 3.32 = \left(\frac{R_2 + R_3}{500} \right) (5)$$

$$R_2 + R_3 = 332 \Rightarrow R_2 = 250 \text{ k}\Omega$$

$$R_1 = 500 - 250 - 62 \Rightarrow R_1 = 168 \text{ k}\Omega$$

$$V_{D2} = V_{D1} + V_{DSQ1} = 1.1 + 2.5 = 3.6 \text{ V}$$

$$R_D = \frac{5 - 3.6}{3} \Rightarrow R_D = 0.467 \text{ k}\Omega$$

b. $A_v = -g_m R_D$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(2)(3)} = 4.90 \text{ mA/V}$$

$$A_v = -(4.90)(0.467) \Rightarrow A_v = -2.29$$

6.48

a. $V_{S1} = I_{DQ} R_S - 10 = (5)(2) - 10 \Rightarrow V_{S1} = 0$

$$I_{DQ} = K_i(V_{GS1} - V_{TN})^2$$

$$5 = 4(V_{GS1} - 1.5)^2 \Rightarrow V_{GS1} = 2.618 \text{ V}$$

$$V_{G1} = V_{GS1} + V_{S1} = 2.618 \text{ V} = I R_3 = (0.1) R_3$$

$$\Rightarrow R_3 = 26.2 \text{ k}\Omega$$

$$V_{D1} = V_{S1} + V_{DSQ1} = 0 + 3.5 = 3.5 \text{ V}$$

$$V_{G2} = V_{D1} + V_{GS} = 3.5 + 2.62 = 6.12 \text{ V}$$

$$= (0.1)(R_2 + R_3)$$

$$R_2 + R_3 = 61.2 \text{ k}\Omega \Rightarrow R_2 = 35 \text{ k}\Omega$$

$$V_{D2} = V_{D1} + V_{DSQ2} = 3.5 + 3.5 = 7.0 \text{ V}$$

$$R_D = \frac{10 - 7}{5} \Rightarrow R_D = 0.6 \text{ k}\Omega$$

$$R_1 = \frac{10 - 6.12}{0.1} \Rightarrow R_1 = 38.8 \text{ k}\Omega$$

b. $A_v = -g_m R_D$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(4)(5)} = 8.94 \text{ mA/V}$$

$$A_v = -(8.94)(0.6) \Rightarrow A_v = -5.36$$

6.49

a. $I_{DQ} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$

$$4 = 6 \left(1 - \frac{V_{GS}}{(-3)}\right)^2$$

$$V_{GS} = (-3) \left[1 - \sqrt{\frac{4}{6}}\right] \Rightarrow V_{GS} = -0.551 \text{ V}$$

$$V_{DSQ} = V_{DD} - I_{DQ} R_D$$

$$6 = 10 - (4)R_D \Rightarrow R_D = 1 \text{ k}\Omega$$

b. $g_m = \frac{2I_{DSS}}{(-V_P)} \left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(6)}{3} \left(1 - \frac{-0.551}{-3}\right)$

$$\Rightarrow g_m = 3.27 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(4)} \Rightarrow r_o = 25 \text{ k}\Omega$$

c. $A_v = -g_m(r_o \parallel R_D) = -(3.27)(25 \parallel 1)$

$$\Rightarrow A_v = -3.14$$

6.50

$$V_{GS} + I_{DQ}(R_{S1} + R_{S2}) = 0$$

$$I_{DQ} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$V_{GS} + I_{DSS}(R_{S1} + R_{S2}) \left(1 - \frac{V_{GS}}{V_P}\right)^2 = 0$$

$$V_{GS} + (2)(0.1 + 0.25) \left(1 - \frac{V_{GS}}{V_P}\right)^2 = 0$$

$$V_{GS} + 0.7 \left(1 - \frac{2V_{GS}}{(-2)} + \frac{V_{GS}^2}{(-2)^2}\right) = 0$$

$$0.175V_{GS}^2 + 1.7V_{GS} + 0.7 = 0$$

$$V_{GS} = \frac{-1.7 \pm \sqrt{(1.7)^2 - 4(0.175)(0.7)}}{2(0.175)}$$

$$\Rightarrow V_{GS} = -0.431 \text{ V}$$

$$g_m = \frac{2I_{DSS}}{-V_P} \left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(2)}{2} \left(1 - \frac{-0.431}{-2}\right)$$

$$\Rightarrow g_m = 1.57 \text{ mA/V}$$

$$A_v = \frac{-g_m(R_D \parallel R_L)}{1 + g_m R_{S1}} = \frac{-(1.57)(8 \parallel 4)}{1 + (1.57)(0.1)}$$

$$\Rightarrow A_v = -3.62$$

$$A_v = \frac{i_o}{i_i} = \frac{(v_o/R_L)}{(v_i/R_G)} = \frac{v_o}{v_i} \cdot \frac{R_G}{R_L} = (-3.62) \left(\frac{50}{4}\right)$$

$$\Rightarrow A_v = -45.3$$

6.51

$$I_{DQ} = \frac{I_{DSS}}{2} = 4 \text{ mA}$$

$$V_{DSQ} = \frac{V_{DD}}{2} = 10 \text{ V}$$

$$V_{DSQ} = V_{DD} - I_{DQ}(R_S + R_D)$$

$$10 = 20 - (4)(R_S + R_D) \Rightarrow R_S + R_D = 2.5 \text{ k}\Omega$$

$$V_S = 2 \text{ V} = I_{DQ} R_S = 4R_S$$

$$\Rightarrow R_S = 0.5 \text{ k}\Omega, \quad R_D = 2.0 \text{ k}\Omega$$

$$I_{DQ} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$4 = 8 \left(1 - \frac{V_{GS}}{(-4.2)}\right)^2 \Rightarrow V_{GS} = (-4.2) \left(1 - \sqrt{\frac{4}{8}}\right)$$

$$\Rightarrow V_{GS} = -1.23 \text{ V}$$

$$V_G = V_S + V_{GS} = 2 - 1.23$$

$$V_G = 0.77 \text{ V} = \left(\frac{R_2}{R_1 + R_2}\right)(20) = \left(\frac{R_2}{100}\right)(20)$$

$$\Rightarrow R_2 = 3.85 \text{ k}\Omega, \quad R_1 = 95.2 \text{ k}\Omega$$

6.52

$$a. \quad I_{DQ} = \frac{I_{DSS}}{2} = 5 \text{ mA}$$

$$V_{DSQ} = \frac{V_{DD}}{2} = \frac{12}{2} = 6 \text{ V}$$

$$R_S = \frac{12 - 6}{5} \Rightarrow R_S = 1.2 \text{ k}\Omega$$

$$I_{DQ} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2$$

$$5 = 10 \left(1 - \frac{V_{GS}}{(-5)} \right)^2 \Rightarrow V_{GS} = (-5) \left(1 - \sqrt{\frac{5}{10}} \right)$$

$$\Rightarrow V_{GS} = -1.46 \text{ V}$$

$$V_G = V_S + V_{GS} = 6 - 1.46 = 4.54 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$4.54 = \frac{1}{R_1} (100)(12) \Rightarrow R_1 = 264 \text{ k}\Omega$$

$$\frac{264R_2}{264 + R_2} = 100 \Rightarrow R_2 = 161 \text{ k}\Omega$$

$$b. \quad g_m = \frac{2I_{DSS}}{(-V_P)} \left(1 - \frac{V_{GS}}{V_P} \right) = \frac{2(10)}{5} \left(1 - \frac{-1.46}{-5} \right)$$

$$\Rightarrow g_m = 2.83 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(5)} = 20 \text{ k}\Omega$$

$$A_v = \frac{g_m(r_o \parallel R_S \parallel R_L)}{1 + g_m(r_o \parallel R_S \parallel R_L)}$$

$$A_v = \frac{(2.83)(20 \parallel 1.2 \parallel 0.5)}{1 + (2.83)(20 \parallel 1.2 \parallel 0.5)}$$

$$\Rightarrow A_v = 0.495$$

$$R_o = \frac{1}{g_m} \parallel R_S = \frac{1}{2.83} \parallel 1.2 = 0.353 \parallel 1.2$$

$$\Rightarrow R_o = 0.273 \text{ k}\Omega$$

$$V_{GS} = \frac{12.43 \pm \sqrt{(12.43)^2 - 4(3.265)(5.5)}}{2(3.265)}$$

$$\Rightarrow V_{GS} = 0.511 \text{ V}$$

$$I_{DQ} = (2) \left(1 - \frac{0.511}{1.75} \right)^2 \Rightarrow I_{DQ} = 1.42 \text{ mA}$$

$$V_{SDQ} = 10 - (1.42)(5) \Rightarrow V_{SDQ} = 2.9 \text{ V}$$

$$b. \quad g_m = \frac{2I_{DSS}}{V_P} \left(1 - \frac{V_{GS}}{V_P} \right) = \frac{2(2)}{1.75} \left(1 - \frac{0.511}{1.75} \right)$$

$$\Rightarrow g_m = 1.62 \text{ mA/V}$$

$$A_v = \frac{g_m(R_S \parallel R_L)}{1 + g_m(R_S \parallel R_L)} = \frac{(1.62)(5 \parallel 10)}{1 + (1.62)(5 \parallel 10)}$$

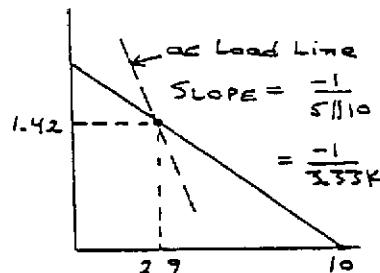
$$\Rightarrow A_v = 0.844$$

$$A_i = \frac{i_o}{i_i} = \frac{(\nu_o / R_L)}{(\nu_i / R_i)} = A_v \cdot \left(\frac{R_i}{R_L} \right)$$

$$R_i = R_1 \parallel R_2 = 90 \parallel 110 = 49.5 \text{ k}\Omega$$

$$A_i = (0.844) \left(\frac{49.5}{10} \right) \Rightarrow A_i = 4.18$$

c.



Maximum swing in output voltage
= 5.8 V peak-to-peak

6.54

$$I_{DQ} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2$$

$$4 = 8 \left(1 - \frac{V_{GS}}{4} \right)^2 \Rightarrow V_{GS} = 4 \left(1 - \sqrt{\frac{4}{8}} \right)$$

$$\Rightarrow V_{GS} = 1.17 \text{ V}$$

$$V_{SDQ} = V_{DD} - I_{DQ}(R_S + R_D)$$

$$7.5 = 20 - 4(R_S + R_D) \Rightarrow R_S + R_D = 3.125 \text{ k}\Omega$$

$$g_m = \frac{2I_{DSS}}{V_P} \left(1 - \frac{V_{GS}}{V_P} \right) = \frac{2(8)}{4} \left(1 - \frac{1.17}{4} \right)$$

$$\Rightarrow g_m = 2.83 \text{ mA/V}$$

$$R_S = 3.125 - R_D$$

6.53

$$a. \quad V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \left(\frac{110}{110 + 90} \right) (10) = 5.5 \text{ V}$$

$$I_{DQ} = \frac{10 - (V_G - V_{GS})}{R_S} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2$$

$$10 - 5.5 + V_{GS} = (2)(5) \left(1 - \frac{V_{GS}}{1.75} \right)^2$$

$$4.5 + V_{GS} = 10(1 - 1.143V_{GS} + 0.3265V_{GS}^2)$$

$$3.265V_{GS}^2 - 12.43V_{GS} + 5.5 = 0$$

$$A_v = \frac{-g_m R_D}{1 + g_m R_S}$$

$$-3(1 + g_m R_S) = -g_m R_D$$

$$3[1 + (2.83)(3.125 - R_D)] = (2.83)R_D$$

$$9.844 - 2.83R_D = 0.7075R_D \Rightarrow R_D = 2.78 \text{ k}\Omega$$

$$R_S = 0.345 \text{ k}\Omega$$

$$V_S = 20 - (4)(0.345) \Rightarrow V_S = 18.6 \text{ V}$$

$$V_G = V_S - V_{GS} = 18.6 - 1.17 = 17.4 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \left(\frac{R_2}{400} \right) (20)$$

$$\Rightarrow R_2 = 348 \text{ k}\Omega, \quad R_1 = 52 \text{ k}\Omega$$

Chapter 7

Exercise Solutions

E7.1

a. $R_S = R_P = 4 \text{ k}\Omega$

$\omega = \frac{1}{r_s} = \frac{1}{(R_S + R_P)C_S}$

$C_S = \frac{1}{2\pi f(R_S + R_P)} = \frac{1}{2\pi(20)(4+4) \times 10^3}$

$C_S = 0.995 \mu\text{F}$

b. $|T(j\omega)| = \left(\frac{R_P}{R_S + R_P} \right) \frac{\omega r_s}{\sqrt{1 + \omega^2 r_s^2}}$

$r_s = (R_S + R_P)C_S = 7.96 \times 10^{-3}$

$\frac{R_P}{R_S + R_P} = \frac{4}{4+4} = 0.5$

$f = 40 \text{ Hz}$

$|T(j\omega)| = \frac{(0.5)(2\pi)(40)(7.96 \times 10^{-3})}{\sqrt{1 + [2\pi(40)(7.96 \times 10^{-3})]^2}}$

$|T(j\omega)| = 0.447$

$f = 80 \text{ Hz}$

$|T(j\omega)| = \frac{(0.5)(2\pi)(80)(7.96 \times 10^{-3})}{\sqrt{1 + [2\pi(80)(7.96 \times 10^{-3})]^2}}$

$|T(j\omega)| = 0.485$

$f = 200 \text{ Hz}$

$|T(j\omega)| = \frac{(0.5)(2\pi)(200)(7.96 \times 10^{-3})}{\sqrt{1 + [2\pi(200)(7.96 \times 10^{-3})]^2}}$

$|T(j\omega)| = 0.498$

E7.2

$\omega = \frac{1}{r_p} = \frac{1}{(R_S \parallel R_P)C_P}$

$C_P = \frac{1}{2\pi f(R_S \parallel R_P)}$

$= \frac{1}{2\pi(500 \times 10^3)(10 \parallel 10) \times 10^3}$

$C_P = 63.7 \mu\text{F}$

E7.3

a. $V_0 = -(g_m V_\pi) R_L$

$V_\pi = \frac{r_\pi}{r_\pi + \frac{1}{sC_C} + R_S} \times V_i$

$T(s) = \frac{V_0(s)}{V_i(s)} = \frac{-g_m r_\pi R_L}{r_\pi + R_S + (1/sC_C)}$
 $= \frac{-g_m r_\pi R_L (sC_C)}{1 + s(r_\pi + R_S)C_C}$

$g_m r_\pi = \beta$

$T(s) = \frac{-\beta R_L}{r_\pi + R_S} \times \left(\frac{s(r_\pi + R_S)C_C}{1 + s(r_\pi + R_S)C_C} \right)$

b. $f_{3-\text{dB}} = \frac{1}{2\pi(r_\pi + R_S)C_C}$

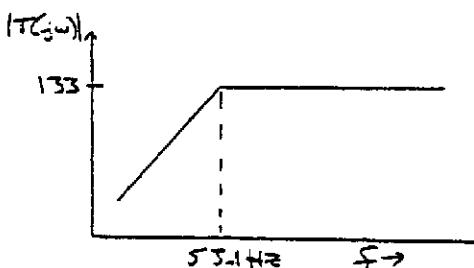
$f_{3-\text{dB}} = \frac{1}{2\pi[2 \times 10^3 + 1 \times 10^3][10^{-6}]}$

$\Rightarrow f_{3-\text{dB}} = 53.1 \text{ Hz}$

$|T(j\omega)|_{\max} = \frac{r_\pi g_m R_L}{r_\pi + R_S} = \frac{(2)(50)(4)}{2+1}$

$|T(j\omega)|_{\max} = 133$

c.



E7.4

a. $V_0 = -g_m V_\pi \left(R_L \parallel \frac{1}{sC_L} \right)$

$V_\pi = \left(\frac{r_\pi}{r_\pi + R_S} \right) \times V_i$

$T(s) = \frac{V_0(s)}{V_i(s)} = -g_m \frac{r_\pi}{r_\pi + R_S} \left(\frac{R_L \times \frac{1}{sC_L}}{R_L + \frac{1}{sC_L}} \right)$

$T(s) = \frac{-\beta R_L}{r_\pi + R_S} \times \left(\frac{1}{1 + sR_L C_L} \right)$

b. $f_{3-\text{dB}} = \frac{1}{2\pi R_L C_L} = \frac{1}{2\pi(5 \times 10^3)(10 \times 10^{-12})}$

$$\Rightarrow f_{3dB} = 3.18 \text{ MHz}$$

$$|T(j\omega)| = \frac{g_m r_\pi R_L}{r_\pi + R_S} = \frac{(75)(1.5)(5)}{1.5 + 0.5}$$

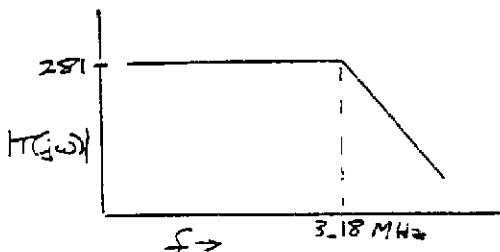
$$|T(j\omega)|_{\max} = 281$$

$$A_V = \frac{V_o}{V_i} = \frac{-g_m r_\pi R_L}{r_\pi + R_S}$$

$$= \frac{-(65)(2)(4)}{2 + 0.25}$$

$$A_V = -231$$

c.



E7.5

$$a. 20 \log_{10} \left(\frac{R_P}{R_P + R_S} \right) = -1$$

$$\Rightarrow \frac{R_P}{R_P + R_S} = 0.891 = \frac{R_P}{R_P + 1}$$

$$\Rightarrow (1 - 0.891)R_P = 0.891 \Rightarrow R_P = 8.17 \text{ k}\Omega$$

$$f_L = \frac{1}{2\pi(R_S + R_P)C_S}$$

$$\Rightarrow C_S = \frac{1}{2\pi(100)(1 + 8.17) \times 10^{-6}}$$

$$C_S = 0.174 \mu\text{F}$$

$$f_H = \frac{1}{2\pi(R_S \parallel R_P)C_P}$$

$$\Rightarrow C_P = \frac{1}{2\pi(10^6)(1 \parallel 8.17) \times 10^{-6}}$$

$$C_P = 179 \text{ pF}$$

$$b. r_s = (R_S + R_P)C_S$$

$$r_s = (1 \times 10^3 + 8.17 \times 10^3)(0.174 \times 10^{-6})$$

$$r_s = 1.60 \text{ ms open-circuit time-constant}$$

$$r_P = (R_S \parallel R_P)C_P$$

$$r_P = (1 \parallel 8.17) \times 10^3 (179 \times 10^{-12})$$

$$r_P = 0.160 \mu\text{s short-circuit time-constant}$$

$$c. f_L = \frac{1}{2\pi r_s} = \frac{1}{2\pi(4.5 \times 10^{-3})} \Rightarrow f_L = 35.4 \text{ Hz}$$

$$f_H = \frac{1}{2\pi r_P} = \frac{1}{2\pi(0.2 \times 10^{-6})} \Rightarrow f_H = 0.796 \text{ MHz}$$

E7.7

$$a. r_s = (R_s + R_E)C_C$$

$$b. f = \frac{1}{2\pi r_s}$$

$$R_{TH} = R_1 \parallel R_2 = 2.2 \parallel 20 = 1.98 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{2.2}{2.2 + 20} \right) (10)$$

$$= 0.991 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE(\text{on})}}{R_{TH} + (1 + \beta)R_E} = \frac{0.991 - 0.7}{1.98 + (201)(0.1)}$$

$$= 0.0132 \text{ mA}$$

$$I_{CQ} = 2.64 \text{ mA}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(200)(0.026)}{2.64} = 1.97 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{2.64}{0.026} = 102 \text{ mA/V}$$

$$R_{ib} = r_\pi + (1 + \beta)R_E = 1.97 + (201)(0.1)$$

$$= 22.1 \text{ k}\Omega$$

$$R_B = R_1 \parallel R_2 = 1.98 \text{ k}\Omega$$

$$R_i = R_S \parallel R_{ib} = 1.98 \parallel 22.1 = 1.82 \text{ k}\Omega$$

$$r_s = (R_s + R_E)C_C$$

$$= (1.82 + 0.1)(\times 10^3)(47 \times 10^{-6})$$

$$= 90.24 \text{ ms}$$

$$f = \frac{1}{2\pi(90.24 \times 10^{-3})} \Rightarrow f = 1.76 \text{ Hz}$$

E7.6

a. Open-circuit time constant ($C_L \rightarrow \text{open}$)

$$r_s = (R_S + r_\pi)C_C$$

$$= (0.25 + 2) \times 10^3 (2 \times 10^{-6}) = 4.5 \text{ ms}$$

Short-circuit time constant ($C_C \rightarrow \text{short}$)

$$r_P = R_L C_L = (4 \times 10^3)(50 \times 10^{-12})$$

$$r_P = 0.2 \mu\text{s}$$

b. Midband gain

$$V_o = -g_m V_\pi R_L, \quad V_\pi = \left(\frac{r_\pi}{r_\pi + R_S} \right) V_i$$

Midband Gain

$$A_V = \frac{-\beta R_C}{r_\pi + (1 + \beta)R_E} \cdot \frac{R_i}{R_i + R_S}$$

$$= \frac{-(200)(2)}{1.97 + (201)(0.1)} \cdot \frac{1.82}{1.82 + 0.1}$$

$$A_V = -17.2$$

E7.8

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2$$

a. $\sqrt{\frac{0.8}{0.5}} + 2 = V_{GS} \Rightarrow V_{GS} = 3.26 \text{ V}$

$$V_S = -3.26 \Rightarrow I_{DQ} = \frac{V_S - (-5)}{R_S}$$

$$R_S = \frac{-3.26 + 5}{0.8} \Rightarrow R_S = 2.18 \text{ k}\Omega$$

$$V_D = 0 \Rightarrow R_D = \frac{5}{0.8} \Rightarrow R_D = 6.25 \text{ k}\Omega$$

b. $r_s = (R_D + R_L)C_C = (10 + 6.25) \times 10^3 \times C_C$

$$f = \frac{1}{2\pi r_s} \Rightarrow C_C = \frac{1}{2\pi f(16.25 \times 10^3)}$$

$$C_C = \frac{1}{2\pi(20)(16.25 \times 10^3)}$$

$$\Rightarrow C_C = 0.49 \mu\text{F}$$

E7.9

$$r_s = (R_L + R_E \| R_0)C_{C2}$$

$$f = \frac{1}{2\pi r_s} \Rightarrow C_{C2} = \frac{1}{2\pi f(R_L + R_E \| R_0)}$$

$$R_0 = r_0 \left| \left\{ \frac{r_\pi + (R_S \| R_B)}{1 + \beta} \right\} \right.$$

From Example 7-5, $R_0 = 35.6 \Omega$

$$R_0 \| R_B = 0.0356 \| 10 \approx 0.0356 \text{ k}\Omega$$

$$C_{C2} = \frac{1}{2\pi(10)[10 \times 10^3 + 35.6]}$$

$$C_{C2} = 1.59 \mu\text{F}$$

E7.10

a. $I_{DQ} = K_p(V_{SG} + V_{TP})^2$

$$\sqrt{\frac{1}{0.5} - (-2)} = V_{SG} \Rightarrow V_{SG} = 3.41 \text{ V}$$

$$V_S = 3.41$$

$$R_S = \frac{5 - 3.41}{1} \Rightarrow R_S = 1.59 \text{ k}\Omega$$

For $V_{SDG} = V_{SGQ} \Rightarrow V_D = 0$

$$\Rightarrow R_D = \frac{5}{1} \Rightarrow R_D = 5 \text{ k}\Omega$$

b. $r_P = (R_D \| R_L)C_L$

$$f = \frac{1}{2\pi r_P} \Rightarrow C_L = \frac{1}{2\pi f(R_D \| R_L)}$$

$$C_L = \frac{1}{2\pi(10^6)(5 \| 10) \times 10^3}$$

$$\Rightarrow C_L = 47.7 \text{ pF}$$

E7.11

a. $I_{BQ} = \frac{0 - 0.7 - (-10)}{0.5 + (101)(4)} = 0.0230 \text{ mA}$

$$I_{CQ} = 2.30 \text{ mA}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{2.30} = 1.13 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{2.30}{0.026} = 88.5 \text{ mA/V}$$

$$\tau_B = \frac{R_E(R_S + r_\pi)C_E}{R_S + r_\pi + (1 + \beta)R_E}$$

$$= \frac{(4 \times 10^3)(0.5 + 1.13)C_E}{0.5 + 1.13 + (101)(4)}$$

$$\tau_B = \frac{1}{2\pi f_B} = \frac{1}{2\pi(200)} = 0.796 \text{ ms}$$

$$\tau_B = 16.07C_E \Rightarrow C_E = \frac{0.796 \times 10^{-3}}{16.07}$$

$$\Rightarrow C_E = 49.5 \mu\text{F}$$

b. $\tau_A = R_E C_E = (4 \times 10^3)(49.5 \times 10^{-6})$

$$\Rightarrow \tau_A = 0.195 \text{ s}$$

$$f_A = \frac{1}{2\pi\tau_A} = \frac{1}{2\pi(0.195)} \Rightarrow f_A = 0.80 \text{ Hz}$$

E7.14

$$r_\pi = \frac{\beta_0 V_T}{I_{CQ}} = \frac{(150)(0.026)}{0.5} = 7.8 \text{ k}\Omega$$

$$f_B = \frac{1}{2\pi r_\pi(C_\pi + C_\mu)}$$

$$= \frac{1}{2\pi(7.8 \times 10^3)(2 + 0.3) \times 10^{-12}}$$

$$\Rightarrow f_B = 8.87 \text{ MHz}$$

E7.15

$$r_\pi = \frac{\beta_0 V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.25} = 10.4 \text{ k}\Omega$$

$$f_B = \frac{1}{2\pi r_\pi(C_\pi + C_\mu)}$$

$$C_\pi + C_\mu = \frac{1}{2\pi f_B r_\pi} = \frac{1}{2\pi(11.5 \times 10^6)(10.4 \times 10^3)}$$

$$C_\pi + C_\mu = 1.33 \text{ pF}$$

$$C_\mu = 0.1 \text{ pF}$$

$$\Rightarrow C_\pi = 1.23 \text{ pF}$$

E7.16

$$h_{fe} = \frac{\beta_0}{1 + j(f/f_B)}$$

$$f_B = 5 \text{ MHz}, \beta_0 = 100$$

At $f = 50 \text{ MHz}$

$$|h_{fe}| = \frac{100}{\sqrt{1 + \left(\frac{50}{5}\right)^2}} \Rightarrow |h_{fe}| = 9.95$$

$$\text{Phase} = -\tan^{-1} \left(\frac{50}{5} \right) \Rightarrow \text{Phase} = -84.3^\circ$$

E7.17

$$f_B = \frac{f_T}{\beta_0} = \frac{500}{120} \Rightarrow f_B = 4.17 \text{ MHz}$$

$$f_B = \frac{1}{2\pi r_\pi(C_\pi + C_\mu)}$$

$$C_\pi + C_\mu = \frac{1}{2\pi f_B r_\pi} = \frac{1}{2\pi(4.17 \times 10^6)(5 \times 10^{-12})}$$

$$C_\pi + C_\mu = 7.63 \text{ pF}$$

$$C_\mu = 0.2 \text{ pF}$$

$$\Rightarrow C_\pi = 7.43 \text{ pF}$$

E7.18

$$r_\pi = \frac{\beta_0 V_T}{I_{CQ}} = \frac{(150)(0.026)}{1} \Rightarrow r_\pi = 3.9 \text{ k}\Omega$$

$$f_B = \frac{1}{2\pi r_\pi(C_\pi + C_\mu)}$$

$$= \frac{1}{2\pi(3.9 \times 10^3)(4 + 0.5)(10^{-12})}$$

$$\Rightarrow f_B = 9.07 \text{ MHz}$$

$$f_T = \beta_0 f_B = (150)(9.07)$$

$$\Rightarrow f_T = 1.36 \text{ GHz}$$

E7.19

$$R_{TH} = R_1 \parallel R_2 = 200 \parallel 220 = 105 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{220}{200 + 220} \right)(5) = 2.62 \text{ V}$$

$$I_{BQ} = \frac{2.62 - 0.7}{105 + (101)(1)} = 0.00932 \text{ mA}$$

$$I_{CQ} = 0.932 \text{ mA}$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.932}{0.026} \Rightarrow g_m = 35.8 \text{ mA/V}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.932} \Rightarrow r_\pi = 2.79 \text{ k}\Omega$$

a. $C_M = C_\mu [1 + g_m(R_C \parallel R_L)]$

$$C_M = (2)[1 + (35.8)(2.2 \parallel 4.7)]$$

$$\Rightarrow C_M = 109 \text{ pF}$$

b. $R_B = r_s \parallel R_1 \parallel R_2 = 100 \parallel 200 \parallel 220 = 51.2 \text{ k}\Omega$

$$f_{3dB} = \frac{1}{2\pi(R_B \parallel r_\pi)(C_\pi + C_\mu)}$$

$$= \frac{1}{2\pi[51.2 \parallel 2.79] \times 10^3 \times (10 + 109) \times 10^{-12}}$$

$$\Rightarrow f_{3dB} = 0.505 \text{ MHz}$$

E7.20

$$(a) g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.4)(3 - 1)$$

$$\Rightarrow g_m = 1.6 \text{ mA/V}$$

$$g'_m = 80\% \text{ of } g_m = 1.28 \text{ mA/V}$$

$$g'_m = \frac{g_m}{1 + g_m r_s}$$

$$1 + g_m r_s = \frac{g_m}{g'_m}$$

$$r_s = \frac{1}{g_m} \left(\frac{g_m}{g'_m} - 1 \right) = \frac{1}{1.6} \left(\frac{1.6}{1.28} - 1 \right)$$

$$r_s = 0.156 \text{ k}\Omega \Rightarrow r_s = 156 \text{ ohms}$$

$$(b) g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.4)(5 - 1)$$

$$\Rightarrow g_m = 3.2 \text{ mA/V}$$

$$g'_m = \frac{g_m}{1 + g_m r_s} = \frac{3.2}{1 + (3.2)(0.156)} = 2.13$$

$$\frac{\Delta g_m}{g_m} = \frac{3.2 - 2.13}{3.2} \Rightarrow \text{A 33.4\% reduction}$$

E7.21

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2$$

$$0.4 = 0.2(V_{GS} - 1)^2 \Rightarrow V_{GS} = 2.41 \text{ V}$$

$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.2)(2.41 - 1) = 0.564 \text{ mA/V}$$

$$f_T = \frac{0.564 \times 10^{-3}}{2\pi(0.25 + 0.02) \times 10^{-12}} \Rightarrow$$

$$f_T = 332 \text{ MHz}$$

E7.22

$$f_T = \frac{g_m}{2\pi(C_{gsT} + C_{gdT})}$$

$$= \frac{g_m}{2\pi(C_{gs} + C_{gsp} + C_{gdp})}$$

$$C_{gs} = \frac{g_m}{2\pi f_T} - C_{gsp} - C_{gdp}$$

$$= \frac{0.5 \times 10^{-3}}{2\pi(500 \times 10^6)} - (0.01 + 0.01) \times 10^{-12}$$

$$\Rightarrow C_{gs} = 0.139 \text{ pF}$$

E7.23

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gsp} + C_{gd})}$$

$$C_{gsp} = C_{gd}$$

$$2C_{gsp} = \frac{g_m}{2\pi f_T} - C_{gs} = \frac{1 \times 10^{-3}}{2\pi(350 \times 10^6)} - 0.4 \times 10^{-12}$$

$$2C_{gsp} = 0.0547 \text{ pF}$$

$$\Rightarrow C_{gsp} = C_{gd} = 0.0274 \text{ pF}$$

E7.24

dc analysis

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \left(\frac{166}{166 + 234} \right)(10) = 4.15 \text{ V}$$

$$I_D = \frac{V_S}{R_S} \text{ and } V_S = V_G - V_{GS}$$

$$K_n(V_{GS} - V_{TN})^2 = \frac{V_G - V_{GS}}{R_S}$$

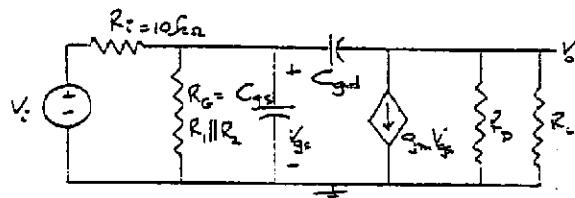
$$(0.5)(0.5)(V_{GS}^2 - 4V_{GS} + 4) = 4.15 - V_{GS}$$

$$0.25V_{GS}^2 - 3.15 = 0 \Rightarrow V_{GS} = 3.55 \text{ V}$$

$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.5)(2.55 - 2)$$

$$= 1.55 \text{ mA/V}$$

Small-signal equivalent circuit.



$$a. \quad C_M = C_{ss}(1 + g_m(R_D || R_L))$$

$$C_M = (0.1)[1 + (1.55)(4/20)]$$

$$\Rightarrow C_M = 0.617 \text{ pF}$$

$$b. \quad f_H = \frac{1}{2\pi r_P}$$

$$r_P = (R_g || R_s)(C_{ss} + C_M)$$

$$R_g = R_1 || R_2 = 234 || 166 = 97.1 \text{ k}\Omega$$

$$R_g || R_s = 97.1 || 10 = 9.07 \text{ k}\Omega$$

$$r_P = (9.07 \times 10^3)(1 + 0.617) \times 10^{-12} = 14.7 \text{ ns}$$

$$f_H = \frac{1}{2\pi(14.7 \times 10^{-9})} \Rightarrow f_H = 10.9 \text{ MHz}$$

E7.25

dc analysis

$$V_{TH} = 0, \quad R_{TH} = 10 \text{ k}\Omega$$

$$I_{BQ} = \frac{0 - 0.7 - (-5)}{10 + (126)(5)} = 0.00672 \text{ mA}$$

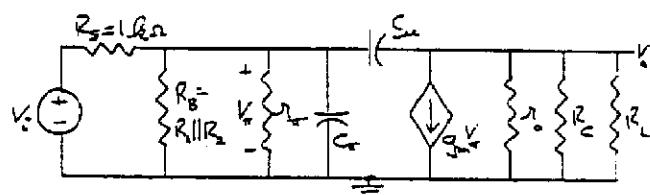
$$I_{CQ} = 0.840 \text{ mA}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(125)(0.026)}{0.840} = 3.87 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.840}{0.026} = 32.3 \text{ mA/V}$$

$$r_0 = \frac{V_A}{I_{CQ}} = \frac{200}{0.84} = 238 \text{ k}\Omega$$

High-frequency equivalent circuit



a. Miller capacitance

$$C_M = C_{ss}(1 + g_m R'_L)$$

$$R'_L = r_0 || R_C || R_L$$

$$R'_L = 238 || 2.3 || 5 = 1.57 \text{ k}\Omega$$

$$C_M = (3)[1 + (32.3)(1.57)] \Rightarrow C_M = 155 \text{ pF}$$

$$b. \quad R_{eq} = R_S || R_B || r_\pi = R_S || R_1 || R_2 || r_\pi$$

$$R_{eq} = 1 || 20 || 20 || 3.87 = 0.736 \text{ k}\Omega$$

$$r_P = R_{eq}(C_\pi + C_M)$$

$$= (0.736 \times 10^3)(24 + 155) \times 10^{-12}$$

$$= 1.32 \times 10^{-7}$$

$$f_H = \frac{1}{2\pi(1.32 \times 10^{-7})} \Rightarrow f_H = 1.21 \text{ MHz}$$

$$c. \quad (A_V)_M = -g_m R'_L \left[\frac{R_B || r_\pi}{R_B || r_\pi + R_S} \right]$$

$$(A_V)_M = -(32.3)(1.57) \left[\frac{10 || 3.87}{10 || 3.87 + 1} \right]$$

$$\Rightarrow (A_V)_M = -37.3$$

E7.26

dc analysis

$$V_G = \left(\frac{50}{50+150} \right) (10) - 5 = -2.5$$

$$V_S = V_G - V_{GS}, \quad I_D = \frac{V_S - (-5)}{R_S}$$

$$K_n(V_{GS} - V_{TN})^2 = \frac{V_G - V_{GS} + 5}{R_S}$$

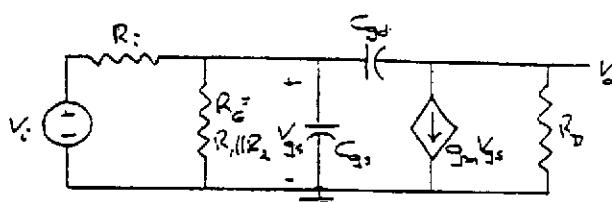
$$(1)(2)[V_{GS}^2 - 1.6V_{GS} + 0.64] = -2.5 - V_{GS} + 5 \\ 2V_{GS}^2 - 2.2V_{GS} - 1.22 = 0$$

$$V_{GS} = \frac{2.2 \pm \sqrt{(2.2)^2 + 4(2)(1.22)}}{2(2)}$$

$$\Rightarrow V_{GS} = 1.51 \text{ V}$$

$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(1)(1.51 - 0.8) \\ = 1.42 \text{ mA/V}$$

Equivalent circuit



$$(a) C_M = C_{gs}(1 + g_m R_D) = (0.2)[1 + (1.42)(5)] \Rightarrow \\ C_M = 1.62 \text{ pF}$$

$$(b) \tau_P = (R_S \| R_G)(C_{gs} + C_M)$$

$$\tau_P = [20 \| 50 \| 150] \times 10^3 \times (2 + 1.62) \times 10^{-12} \\ = (13 \times 10^3)(3.62 \times 10^{-12}) = 4.71 \times 10^{-9} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau_P} = \frac{1}{2\pi(4.71 \times 10^{-9})} \\ \Rightarrow f_H = 3.38 \text{ MHz}$$

$$c. \quad (A_V)_M = -g_m R_D \left(\frac{R_G}{R_G + R_S} \right)$$

$$(A_V)_M = -(1.42)(5) \left(\frac{37.5}{37.5 + 20} \right)$$

$$\Rightarrow (A_V)_M = -4.63$$

E7.27

The dc analysis

$$I_{BQ} = \frac{10 - 0.7}{100 + (101)(10)} = 0.00838 \text{ mA}$$

$$I_{CQ} = 0.838 \text{ mA}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.838} = 3.10 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = 32.2 \text{ mA/V}$$

For the input

$$\begin{aligned} \tau_{P\pi} &= \left[\left(\frac{r_\pi}{1 + \beta} \right) \| R_E \| R_S \right] C_\pi \\ &= \left[\frac{3.10}{101} \| 10 \| 1 \right] \times 10^3 \times 24 \times 10^{-12} \\ &= 7.13 \times 10^{-10} \text{ s} \end{aligned}$$

$$f_{H\pi} = \frac{1}{2\pi\tau_{P\pi}} = \frac{1}{2\pi(7.13 \times 10^{-10})} \\ \Rightarrow f_{H\pi} = 223 \text{ MHz}$$

For the output

$$\begin{aligned} \tau_{P\mu} &= [R_C \| R_L] C_\mu = (10 \| 1) \times 10^3 \times 3 \times 10^{-12} \\ &= 2.73 \times 10^{-9} \end{aligned}$$

$$\begin{aligned} f_{H\mu} &= \frac{1}{2\pi\tau_{P\mu}} = \frac{1}{2\pi(2.73 \times 10^{-9})} \\ \Rightarrow f_{H\mu} &= 58.3 \text{ MHz} \end{aligned}$$

$$(A_V)_M = g_m (R_C \| R_L) \left[\frac{R_E \| \left(\frac{r_\pi}{1 + \beta} \right)}{R_E \| \left(\frac{r_\pi}{1 + \beta} \right) + R_S} \right]$$

$$= (32.2)(10 \| 1) \left[\frac{10 \| \left(\frac{3.1}{101} \right)}{10 \| \left(\frac{3.1}{101} \right) + 1} \right]$$

$$\Rightarrow (A_V)_M = 0.869$$

Chapter 7

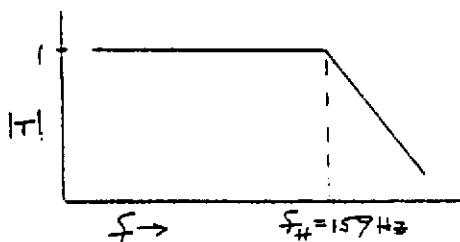
Problem Solutions

7.1

a. $T(s) = \frac{V_o(s)}{V_i(s)} = \frac{1/(sC_1)}{[1/(sC_1)] + R_1}$

$$\underline{T(s) = \frac{1}{1 + sR_1C_1}}$$

b.



$$f_H = \frac{1}{2\pi R_1 C_1} = \frac{1}{2\pi(10^3)(10^{-6})}$$

$$\Rightarrow \underline{f_H = 159 \text{ Hz}}$$

c. $V_o(s) = V_i(s) \cdot \frac{1}{1 + sR_1C_1}$

For a step function $V_i(s) = \frac{1}{s}$

$$V_o(s) = \frac{1}{s} \cdot \frac{1}{1 + sR_1C_1} = \frac{K_1}{s} + \frac{K_2}{1 + sR_1C_1}$$

$$= \frac{K_1(1 + sR_1C_1) + K_2s}{s(1 + sR_1C_1)}$$

$$= \frac{K_1 + s(K_1R_1C_1 + K_2)}{s(1 + sR_1C_1)}$$

$K_2 = -K_1R_1C_1$ and $K_1 = 1$

$$V_o(s) = \frac{1}{s} + \frac{-R_1C_1}{1 + sR_1C_1}$$

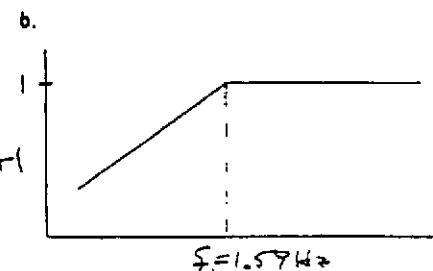
$$= \frac{1}{s} - \frac{1}{\frac{R_1C_1}{s} + 1}$$

$$\underline{v_o(t) = 1 - e^{-t/R_1C_1}}$$

7.2

a. $T(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_2 + [1/(sC_2)]}$

$$\underline{T(s) = \frac{sR_2C_2}{1 + sR_2C_2}}$$



$$f_L = \frac{1}{2\pi R_2 C_2} = \frac{1}{2\pi(10^4)(10 \times 10^{-6})}$$

$$\Rightarrow \underline{f_L = 1.59 \text{ Hz}}$$

c. $V_o(s) = V_i(s) \cdot \frac{sR_2C_2}{1 + sR_2C_2}$

$$V_i(s) = \frac{1}{s}$$

$$V_o(s) = \frac{R_2C_2}{1 + sR_2C_2} = \frac{1}{s + \frac{1}{R_2C_2}}$$

$$\underline{v_o(t) = e^{-t/R_2C_2}}$$

7.3

a. $T(s) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{R_P}{sC_P}}{R_P \parallel \frac{1}{sC_P} + \left(R_S + \frac{1}{sC_S} \right)}$

$$R_P \parallel \frac{1}{sC_P} = \frac{R_P \cdot \frac{1}{sC_P}}{R_P + \frac{1}{sC_P}} = \frac{R_P}{1 + sR_P C_P}$$

Then

$$T(s) = \frac{R_P}{R_P + \left(R_S + \frac{1}{sC_S} \right) (1 + sR_P C_P)}$$

$$= \frac{R_P}{R_P + R_S + \frac{R_P C_P}{C_S} + \frac{1}{sC_S} + sR_S R_P C_P}$$

$$T(s) = \left(\frac{R_P}{R_P + R_S} \right) \times \left(1 / \left[1 + \frac{R_P}{R_P + R_S} \cdot \frac{C_P}{C_S} + \frac{1}{s(R_S + R_P)C_S} + \frac{sR_P R_S}{R_S + R_P} \cdot C_P \right] \right)$$

b.

$$\begin{aligned} T(s) &= \left(\frac{10}{10+10} \right) \\ &\times \left(1 + \frac{10}{20} \cdot \frac{10^{-11}}{10^{-6}} + \frac{1}{s(2 \times 10^4) \cdot 10^{-6}} \right. \\ &\quad \left. + s(5 \times 10^3) \cdot 10^{-11} \right) \\ &\equiv \frac{1}{2} \cdot \frac{1}{1 + \frac{1}{s(0.02)} + s(5 \times 10^{-8})} \end{aligned}$$

$$s = j\omega$$

$$T(j\omega) = \frac{1}{2} \cdot \frac{1}{1 + j \left[\omega(5 \times 10^{-8}) - \frac{1}{\omega(0.02)} \right]}$$

$$\text{For } \omega_L = \frac{1}{(R_S + R_P)C_S} = \frac{1}{(2 \times 10^4)(10^{-6})} = 50$$

$$\begin{aligned} T(j\omega) &\approx \frac{1}{2} \cdot \frac{1}{1 + j \left[(50)(5 \times 10^{-8}) - \frac{1}{(50)(0.02)} \right]} \\ &\approx \frac{1}{2} \cdot \frac{1}{1 - j} \Rightarrow |T(j\omega)| = \frac{1}{2} \cdot \frac{1}{\sqrt{2}} \end{aligned}$$

For

$$\omega_H = \frac{1}{(R_S || R_P)C_P} = \frac{1}{(5 \times 10^3)(10^{-11})} = 2 \times 10^7$$

$$T(j\omega)$$

$$= \frac{1}{2} \cdot \frac{1}{1 + j \left[(2 \times 10^7)(5 \times 10^{-8}) - \frac{1}{(2 \times 10^7)(0.02)} \right]}$$

$$T(j\omega) \approx \frac{1}{2} \cdot \frac{1}{1 + j} \Rightarrow |T(j\omega)| = \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$

$$\text{In each case, } |T(j\omega)| = \frac{1}{\sqrt{2}} \cdot \frac{R_P}{R_P + R_S}$$

$$c. \quad R_S = R_P = 10 \text{ k}\Omega, \quad C_S = C_P = 0.1 \mu\text{F}$$

$$T(s) = \frac{1}{2} \cdot \left(1 / \left[1 + \frac{1}{2} \cdot \frac{1}{1} + \frac{1}{s(2 \times 10^4)(10^{-7})} \right] \right. \\ \left. + s(5 \times 10^3)(10^{-11}) \right)$$

$$s = j\omega$$

$$T(j\omega) = \frac{1}{2} \cdot \frac{1}{1 + \frac{1}{2} + j \left[\omega(5 \times 10^{-4}) - \frac{1}{\omega(2 \times 10^4)} \right]}$$

$$\text{For } \omega = \frac{1}{(2 \times 10^4)(10^{-7})} = 500$$

$$T(j\omega)$$

$$= \frac{1}{2} \cdot \frac{1}{1.5 + j \left[(500)(5 \times 10^{-4}) - \frac{1}{(500)(2 \times 10^4)} \right]}$$

$$= \frac{1}{2} \cdot \frac{1}{1.5 - j(0.75)} \Rightarrow |T(j\omega)| = 0.298$$

$$\text{For } \omega = \frac{1}{(5 \times 10^3)(10^{-7})} = 2 \times 10^3$$

$$\begin{aligned} T(j\omega) &= \frac{1}{2} \cdot \left\{ 1 / \left(1.5 + j \left[(2 \times 10^3)(5 \times 10^{-4}) \right. \right. \right. \\ &\quad \left. \left. \left. - \frac{1}{(2 \times 10^3)(2 \times 10^{-3})} \right] \right) \right\} \\ &= \frac{1}{2} \cdot \frac{1}{1.5 + j(0.75)} \Rightarrow |T(j\omega)| = 0.298 \end{aligned}$$

$$\text{In each case, } |T(j\omega)| < \frac{1}{\sqrt{2}} \cdot \frac{R_P}{R_P + R_S}$$

7.4

Circuit (a):

$$\begin{aligned} T = \frac{V_o}{V_i} &= \frac{R_2}{R_2 + R_1 \parallel \frac{1}{sC_1}} = \frac{R_2}{R_2 + \frac{R_1(1/sC_1)}{R_1 + (1/sC_1)}} \\ &= \frac{R_2}{R_2 + \frac{R_1}{1+sR_1C_1}} = \frac{R_2(1+sR_1C_1)}{R_2 + sR_1R_2C_1 + R_1} \end{aligned}$$

or

$$\frac{V_o}{V_i} = \frac{R_2}{R_1 + R_2} \cdot \frac{(1+sR_1C_1)}{1+sR_1 \parallel R_2C_1}$$

Low frequency:

$$\left| \frac{V_o}{V_i} \right| = \frac{R_2}{R_1 + R_2} = \frac{20}{10+20} = \frac{2}{3}$$

High frequency:

$$\left| \frac{V_o}{V_i} \right| = 1$$

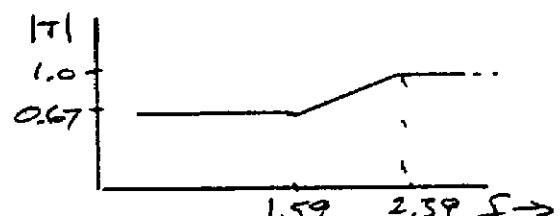
$$\tau_1 = R_1C_1 = (10^4)(10 \times 10^{-6}) = 0.10 \Rightarrow$$

$$f_1 = \frac{1}{2\pi\tau_1} = 1.59 \text{ Hz}$$

$$\tau_2 = (R_1 \parallel R_2)C_1 = (10 \parallel 20) \times 10^3 \times (10 \times 10^{-6}) \Rightarrow$$

$$\tau_2 = 0.0667 \Rightarrow$$

$$f_2 = \frac{1}{2\pi\tau_2} = 2.39 \text{ Hz}$$



Circuit (b):

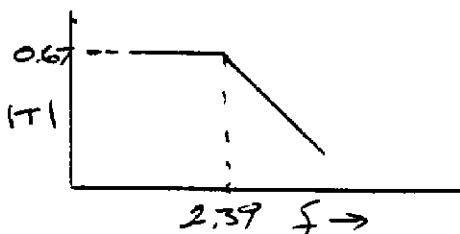
$$T = \frac{V_o}{V_i} = \frac{\frac{R_2}{sC_2}}{\frac{R_1}{sC_2} + R_1} = \frac{\frac{R_2}{1+sR_2C_2}}{\frac{R_1}{1+sR_2C_2} + R_1} = \left(\frac{R_2}{R_1 + R_2} \right) \left(\frac{1}{1+s(R_1R_2)C_2} \right)$$

Low frequency:

$$\left| \frac{V_o}{V_i} \right| = \frac{R_2}{R_1 + R_2} = \frac{20}{20+10} = \frac{2}{3}$$

$$\tau = (R_1R_2)C_2 = (10)(20) \times 10^3 \times 10 \times 10^{-6} = 0.0667$$

$$f = \frac{1}{2\pi\tau} = 2.39 \text{ Hz}$$



7.5

$$a. \quad r_s = (R_i + R_P)C_S = [30 + 10] \times 10^3 \times 10 \times 10^{-6}$$

$$\Rightarrow r_s = 0.40 \text{ s}$$

$$r_P = (R_i \parallel R_P)C_P = [30 \parallel 10] \times 10^3 \times 50 \times 10^{-12}$$

$$\Rightarrow r_P = 0.375 \mu\text{s}$$

$$b. \quad f_L = \frac{1}{2\pi r_s} = \frac{1}{2\pi(0.4)} \Rightarrow f_L = 0.398 \text{ Hz}$$

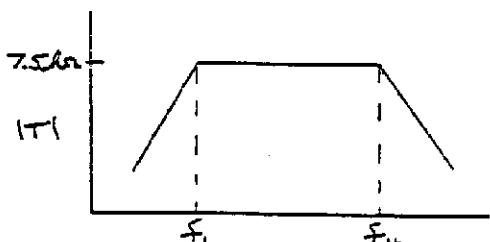
$$f_H = \frac{1}{2\pi r_P} = \frac{1}{2\pi(0.375 \times 10^{-9})} \Rightarrow f_H = 424 \text{ kHz}$$

At midband, $C_S \rightarrow \text{short}$, $C_P \rightarrow \text{open}$

$$V_o = I_o(R_i \parallel R_P)$$

$$T(s) = R_i \parallel R_P = 30 \parallel 10 \Rightarrow T(s) = 7.5 \text{ k}\Omega$$

c.



7.6

$$(a) \quad T = \frac{1}{(1+j2\pi f\tau)^2} \Rightarrow$$

$$|T| = \frac{1}{(\sqrt{1+(2\pi f\tau)^2})^2} = \frac{1}{1+(2\pi f\tau)^2}$$

$$|T|_{\max} = 1$$

$$\text{At } f = \frac{1}{2\pi\tau} \Rightarrow |T| = \frac{1}{1+(1)^2} = \frac{1}{2}$$

$$|T|_{dB} = 20 \log_{10} \left(\frac{1}{2} \right) \Rightarrow |T|_{dB} \equiv -6 \text{ dB}$$

$$\text{Phase} = 2 \tan^{-1}(2\pi f\tau) = -2 \tan^{-1}(1) = -2(45^\circ) \Rightarrow$$

$$\text{Phase} = -90^\circ$$

(b) Slope

$$= -2(6 \text{ dB/oct}) =$$

$$-12 \text{ dB/oct} = -40 \text{ dB/decade}$$

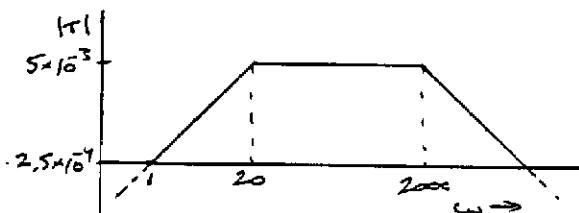
$$\text{Phase} = -2(90^\circ) \Rightarrow \text{Phase} = -180^\circ$$

7.7

$$(a) \quad T(j\omega) = \frac{-10(j\omega)}{20 \left(1 + \frac{j\omega}{20} \right) (2000) \left(1 + \frac{j\omega}{2000} \right)}$$

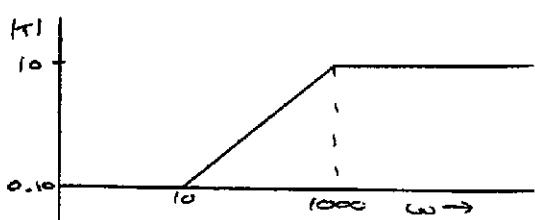
$$= \frac{2.5 \times 10^{-4}(j\omega)}{\left(1 + \frac{j\omega}{20} \right) \left(1 + \frac{j\omega}{2000} \right)}$$

$$|T| = \frac{2.5 \times 10^{-4}(\omega)}{\sqrt{1 + \left(\frac{\omega}{20} \right)^2} \cdot \sqrt{1 + \left(\frac{\omega}{2000} \right)^2}}$$



$$(b) \quad T(j\omega) = \frac{(10)(10) \left(1 + \frac{j\omega}{10} \right)}{1000 \left(1 + \frac{j\omega}{1000} \right)}$$

$$|T| = \frac{(0.10) \sqrt{1 + \left(\frac{\omega}{10} \right)^2}}{\sqrt{1 + \left(\frac{\omega}{1000} \right)^2}}$$



7.8

$$a. \quad V_0 = -g_m V_\pi R_L \quad V_\pi = \left(\frac{r_\pi}{r_\pi + R_S} \right) V_i$$

$$|T| = g_m R_L \left(\frac{r_\pi}{r_\pi + R_S} \right) = (29)(6) \left(\frac{5.2}{5.2 + 0.5} \right)$$

$$|T_{\text{midband}}| = 159$$

$$b. \quad r_s = (R_S + r_\pi) C_C$$

$$f_L = \frac{1}{2\pi r_s} \Rightarrow r_s = \frac{1}{2\pi f_L} = \frac{1}{2\pi(30)}$$

$\Rightarrow r_s = 5.31 \text{ ms}$ Open-circuit

$$r_P = \frac{1}{2\pi f_H} = \frac{1}{2\pi(480 \times 10^3)}$$

$\Rightarrow r_P = 0.332 \mu\text{s}$ Short-circuit

$$c. \quad C_C = \frac{r_s}{(R_S + r_\pi)} = \frac{5.31 \times 10^{-3}}{(0.5 + 5.2) \times 10^3}$$

$$\Rightarrow C_C = 0.932 \mu\text{F}$$

$$r_P = R_L C_L$$

$$C_L = \frac{r_P}{R_L} = \frac{0.332 \times 10^{-6}}{6 \times 10^3} \Rightarrow C_L = 55.3 \text{ pF}$$

7.11

$$a. \quad R_{TH} = R_1 \parallel R_2 = 10 \parallel 1.5 = 1.30 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{1.5}{1.5 + 10} \right) (12) \\ = 1.565 \text{ V}$$

$$I_{BQ} = \frac{1.565 - 0.7}{1.30 + (101)(0.1)} = 0.0759 \text{ mA}$$

$$I_{CQ} = 7.59 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{7.59} = 0.343 \text{ k}\Omega$$

$$g_m = \frac{7.59}{0.026} = 292 \text{ mA/V}$$

$$R_i = R_1 \parallel R_2 \parallel [r_\pi + (1 + \beta) R_E]$$

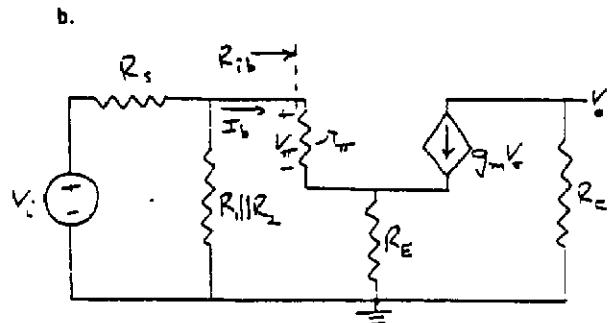
$$= 10 \parallel 1.5 \parallel [0.343 + (101)(0.1)]$$

$$= 1.30 \parallel 10.4 \Rightarrow R_i = 1.16 \text{ k}\Omega$$

$$\tau = (R_S + R_i) C_C = [0.5 + 1.16] \times 10^3 \times 0.1 \times 10^{-6}$$

$$\tau = 1.66 \times 10^{-4}$$

$$f_L = \frac{1}{2\pi\tau} = \frac{1}{2\pi(1.66 \times 10^{-4})} \Rightarrow f_L = 959 \text{ Hz}$$



$$V_0 = -(\beta I_b) R_C$$

$$R_{ib} = r_\pi + (1 + \beta) R_E$$

$$= 0.343 + (101)(0.1) = 10.4 \text{ k}\Omega$$

$$I_b = \left(\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{ib}} \right) I_i$$

$$= \left(\frac{1.30}{1.30 + 10.4} \right) I_i = (0.111) I_i$$

$$I_i = \frac{V_i}{R_S + R_1 \parallel R_2 \parallel R_{ib}}$$

$$= \frac{V_i}{0.5 + (1.3) \parallel (10.4)}$$

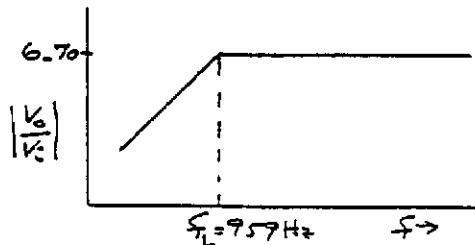
$$I_i = \frac{V_i}{1.656}$$

$$\left| \frac{V_0}{V_i} \right| = \frac{\beta R_C (0.111)}{1.656}$$

$$\Rightarrow \left| \frac{V_0}{V_i} \right|_{\text{midband}} = \frac{(100)(1)(0.111)}{1.656}$$

$$\Rightarrow \left| \frac{V_0}{V_i} \right|_{\text{midband}} = 6.70$$

c.



7.12

$$I_{DQ} = 0.5 \text{ mA} \Rightarrow V_S = (0.5)(0.5) = 0.25 \text{ V}$$

$$I_{DQ} = K_n (V_{GS} - V_{TN})^2$$

$$\Rightarrow V_{GS} = \sqrt{\frac{0.5}{0.2}} + 1.5 = 3.08 \text{ V}$$

$$V_G = V_{GS} + V_S = 3.08 + 0.25 \Rightarrow V_G = 3.33 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} \Rightarrow 3.33 = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$3.33 = \frac{1}{R_1} (200)(9) \Rightarrow R_1 = 541 \text{ k}\Omega$$

$$\frac{541 R_2}{541 + R_2} = 200 \Rightarrow R_2 = 317 \text{ k}\Omega$$

$$V_D = V_{DSQ} + V_S = 4.5 + 0.25 = 4.75$$

$$R_D = \frac{9 - 4.75}{0.5} \Rightarrow R_D = 8.5 \text{ k}\Omega$$

$$f_L = \frac{1}{2\pi r_L} \Rightarrow r_L = \frac{1}{2\pi f_L} = \frac{1}{2\pi(20)} = 7.96 \text{ ms}$$

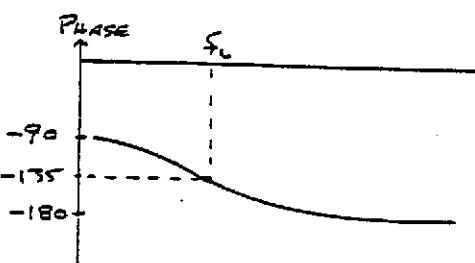
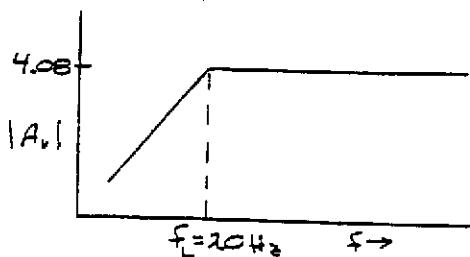
$$r_L = R_{in} \cdot C_C \Rightarrow C_C = \frac{r_L}{R_{in}} = \frac{7.96 \times 10^{-3}}{200 \times 10^3}$$

$$\Rightarrow C_C = 0.0398 \mu\text{F}$$

$$g_m = 2(0.2)(3.08 - 1.5) = 0.632 \text{ mA/V}$$

$$|A_v|_{midband} = \frac{g_m R_D}{1 + g_m R_S} = \frac{(0.632)(8.5)}{1 + (0.632)(0.5)}$$

$$\Rightarrow |A_v| = 4.08$$



7.13

$$I_{DQ} = K_p (V_{GS} - V_{TN})^2$$

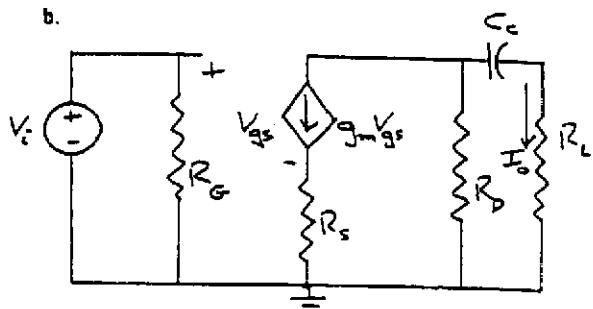
$$\Rightarrow V_{GS} = \sqrt{\frac{I_{DQ}}{K_p}} + V_{TN} = \sqrt{\frac{1}{0.5}} + 1 = 2.41 \text{ V}$$

$$V_S = -2.41 \text{ V}$$

$$R_S = \frac{-2.41 - (-5)}{1} \Rightarrow R_S = 2.59 \text{ k}\Omega$$

$$V_D = V_{DSQ} + V_S = 3 - 2.41 = 0.59 \text{ V}$$

$$R_D = \frac{5 - 0.59}{1} \Rightarrow R_D = 4.4 \text{ k}\Omega$$



$$I_0 = -(g_m V_{gs}) \left(\frac{R_D}{R_D + R_L + \frac{1}{sC_C}} \right)$$

$$V_{gs} = \frac{V_i}{1 + g_m R_S}$$

$$\frac{I_0(s)}{V_i(s)} = \frac{-g_m}{1 + g_m R_S} \cdot R_D \left[\frac{sC_C}{1 + s(R_D + R_L)C_C} \right]$$

$$T(s) = \frac{I_0(s)}{V_i(s)}$$

$$= \frac{-g_m R_D}{1 + g_m R_S} \cdot \frac{1}{R_D + R_L} \cdot \frac{s(R_D + R_L)C_C}{1 + s(R_D + R_L)C_C}$$

$$\text{c. } f_L = \frac{1}{2\pi r_L} \rightarrow r_L = \frac{1}{2\pi f_L} = \frac{1}{2\pi(10)} = 15.9 \text{ ms}$$

$$r_L = (R_D + R_L)C_C$$

$$\Rightarrow C_C = \frac{r_L}{R_D + R_L} = \frac{15.9 \times 10^{-3}}{(4.41 + 4) \times 10^3}$$

$$\Rightarrow C_C = 1.89 \mu\text{F}$$

7.14

$$\text{a. } \frac{9 - V_{SG}}{R_S} = I_D = K_p (V_{SG} + V_{TP})^2$$

$$9 - V_{SG} = (0.5)(12)(V_{SG}^2 - 4V_{SG} + 4)$$

$$6V_{SG}^2 - 23V_{SG} + 15 = 0$$

$$V_{SG} = \frac{23 \pm \sqrt{(23)^2 - 4(6)(15)}}{2(6)} \Rightarrow V_{SG} = 3 \text{ V}$$

$$g_m = 2K_p(V_{SG} + V_{TP}) = 2(0.5)(3 - 2)$$

$$\Rightarrow g_m = 1 \text{ mA/V}$$

$$R_0 = \frac{1}{g_m} \parallel R_S = 1 \parallel 12 \Rightarrow R_0 = 0.923 \text{ k}\Omega$$

$$\text{b. } \tau = (R_0 + R_L)C_C$$

$$\text{c. } f_L = \frac{1}{2\pi\tau} \Rightarrow \tau = \frac{1}{2\pi f_L} = \frac{1}{2\pi(20)} = 7.96 \text{ ms}$$

$$C_C = \frac{\tau}{R_0 + R_L} = \frac{7.96 \times 10^{-3}}{(0.923 + 10) \times 10^3}$$

$$\Rightarrow C_C = 0.729 \mu\text{F}$$

7.15

a. $I_{CQ} = 1 \text{ mA}, I_{BQ} = \frac{1}{120} = 0.00833 \text{ mA}$

$$R_1 \parallel R_2 = (0.1)(1 + \beta)(R_E)$$

$$= (0.1)(121)(4) = 48.4 \text{ k}\Omega$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE(\text{on})} + (1 + \beta)I_{BQ}R_E$$

$$\frac{1}{R_1} \cdot R_{TH} \cdot V_{CC}$$

$$= (0.00833)(48.4) + 0.7 + (121)(0.00833)(4)$$

$$\frac{1}{R_1}(48.4)(12) = 5.13$$

$$R_1 = 113 \text{ k}\Omega$$

$$\frac{113R_2}{113 + R_2} = 48.4 \Rightarrow R_2 = 84.7 \text{ k}\Omega$$

b. $R_o = \frac{r_\pi}{1 + \beta} \parallel R_E \parallel r_0$

$$r_\pi = \frac{(120)(0.026)}{1} = 3.12 \text{ k}\Omega$$

$$r_0 = \frac{80}{1} = 80 \text{ k}\Omega$$

$$R_o = \frac{3.12}{121} \parallel 4 \parallel 80 = 0.0256 \parallel 4 \parallel 80$$

$$\Rightarrow R_o = 25.6 \Omega$$

c. $\tau = (R_o + R_L)C_{C2}$

$$\tau = (0.0256 + 4) \times 10^3 \times 2 \times 10^{-6} = 8.05 \times 10^{-3} \text{ s}$$

$$f = \frac{1}{2\pi\tau} = \frac{1}{2\pi(8.05 \times 10^{-3})} \Rightarrow f = 19.8 \text{ Hz}$$

7.16

(a) $\frac{5 - V_{SG}}{R_1} = K_p(V_{SG} + V_{TP})^2$

$$5 - V_{SG} = (1)(1.2)(V_{SG} + 1.5)^2 = (1.2)(V_{SG}^2 + 3V_{SG} + 2.25)$$

$$1.2V_{SG}^2 + 2.6V_{SG} + 2.3 = 0 \Rightarrow V_{SG} = 2.84 \text{ V}$$

$$I_{DQ} = 1.8 \text{ mA}$$

$$\frac{V_{SDQ}}{V_{SDQ}} = 10 - (1.8)(1.2 + 1.2) \Rightarrow V_{SDQ} = 5.68 \text{ V}$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(1)(1.8)} = 2.68 \text{ mA/V}$$

$$r_o = \infty$$

(b) $R_u = \frac{1}{g_m} = \frac{1}{2.68} = 0.373 \text{ k}\Omega$

$$R_i = 1.2 \parallel 0.373 = 0.285 \text{ k}\Omega$$

$$\text{For } C_{C1}, \tau_{A1} = (285 + 200)(4.7 \times 10^{-6}) = 2.28 \text{ ms}$$

$$\text{For } C_{C2}, \tau_{A2} = (1.2 \times 10^3 + 50 \times 10^3)(10^{-6}) = 51.2 \text{ ms}$$

(c) C_{C2} dominates.

$$f_{3-dB} = \frac{1}{2\pi\tau_{A2}} = \frac{1}{2\pi(51.2 \times 10^{-3})} = 3.1 \text{ Hz}$$

7.17

Assume $V_{TN} = 1 \text{ V}, k'_n = 80 \mu\text{A}/\text{V}^2, \lambda = 0$

Neglecting $R_{SI} = 200 \Omega$, Midband gain is:

$$|A_v| = g_m R_D$$

$$\text{Let } I_{DQ} = 0.2 \text{ mA}, V_{DSQ} = 5 \text{ V}$$

$$\text{Then } R_D = \frac{9 - 5}{0.2} \Rightarrow R_D = 20 \text{ k}\Omega$$

We need

$$g_m = \frac{|A_v|}{R_D} = \frac{10}{20} = 0.5 \text{ mA/V}^2$$

and

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)I_{DQ}}$$

or

$$0.5 = 2\sqrt{\left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)(0.2)} \Rightarrow \frac{W}{L} = 7.81$$

$$\text{Let } R_1 + R_2 = \frac{9}{(0.2)I_{DQ}} = \frac{9}{(0.2)(0.2)} = 225 \text{ k}\Omega$$

$$I_{DQ} = 0.2 = \left(\frac{0.080}{2}\right)(7.81)(V_{GS} - 1)^2$$

$$\Rightarrow V_{GS} = 1.80 = \left(\frac{R_2}{R_1 + R_2}\right)(9) = \left(\frac{R_2}{225}\right)(9) \Rightarrow$$

$$R_2 = 45 \text{ k}\Omega, R_1 = 180 \text{ k}\Omega$$

$$R_{TH} = R_1 \parallel R_2 = 180 \parallel 45 = 36 \text{ k}\Omega$$

$$\tau_1 = \frac{1}{2\pi f_1} = \frac{1}{2\pi(200)} = 7.96 \times 10^{-5} \text{ s} = (R_S + R_{TH})C_C$$

or

$$C_C = \frac{7.96 \times 10^{-4}}{(200 + 36 \times 10^3)} \Rightarrow C_C = 220 \mu\text{F}$$

$$\tau_2 = \frac{1}{2\pi f_2} = \frac{1}{2\pi(3 \times 10^3)} = 5.31 \times 10^{-5} \text{ s} = R_D C_L$$

or

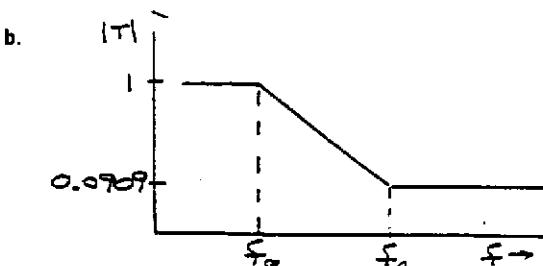
$$C_L = \frac{5.31 \times 10^{-5}}{20 \times 10^3} \Rightarrow C_L = 2.66 \text{ nF}$$

7.18

a. $T(s) = \frac{R_2 + (1/sC)}{R_2 + (1/sC) + R_1}$

$$T(s) = \frac{1 + sR_2 C}{1 + s(R_1 + R_2)C}$$

$$\tau_A = R_2 C, \tau_B = (R_1 + R_2)C$$



c. $\tau_A = R_2 C = (10^3)(100 \times 10^{-12}) = 10^{-7} \text{ s} = \tau_A$

$$R_{eq} = 12.6 \Omega \Rightarrow C_{C2} = \frac{0.00637}{12.6} \Rightarrow C_{C2} = 506 \mu\text{F}$$

$$\tau_B = (R_1 + R_2)C = [10 + 1] \times 10^3 \times 100 \times 10^{-12} \\ = 1.1 \times 10^{-6} \text{ s} = \tau_B$$

$$f_A \approx \frac{1}{2\pi\tau_A} = \frac{1}{2\pi(10^{-7})} \Rightarrow f_A = 1.59 \text{ MHz}$$

$$f_B \approx \frac{1}{2\pi\tau_B} = \frac{1}{2\pi(1.1 \times 10^{-6})} \\ \Rightarrow f_B = 0.145 \text{ MHz}$$

7.19

$$I_{BQ} = \frac{10 - 0.7}{430 + (201)(2.5)} = 0.00997 \text{ mA}$$

$$I_{CQ} = (200)I_{BQ} = 1.99 \text{ mA}$$

$$r_s = \frac{(200)(0.026)}{1.99} = 2.61 \text{ k}\Omega$$

$$R_{ib} = 2.61 + (201)(2.5) = 505 \text{ k}\Omega$$

$$\tau_s = \frac{1}{2\pi f_L} = \frac{1}{2\pi(15)} = 0.0106 \text{ s}$$

$$= R_{eq} C_C = (0.5 + 505)[430] \times 10^3 C_C = 232.7 \times 10^3 C_C$$

Or

$$C_C = 4.56 \times 10^{-8} \text{ F} \Rightarrow 45.6 \text{ nF}$$

7.20

$$R_{TH} = R_1 \| R_2 = 1.2 \| 1.2 = 0.6 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (V_{CC}) = \left(\frac{1.2}{1.2 + 1.2} \right) (5) = 2.5 \text{ V}$$

$$I_{BQ} = \frac{2.5 - 0.7}{0.6 + (101)(0.05)} = 0.319 \text{ mA}$$

$$I_{CQ} = 31.9 \text{ mA}$$

$$r_s = \frac{(100)(0.026)}{31.9} = 0.0815 \text{ k}\Omega$$

$$\tau_{C_{C1}} \gg \tau_{C_{C2}} \text{ and } f = \frac{1}{2\pi\tau} \text{ so that}$$

$$f_{3-dB}(C_{C1}) \ll f_{3-dB}(C_{C2})$$

Then, for $f_{3-dB}(C_{C1}) \Rightarrow C_{C2}$ acts as an open and for

$f_{3-dB}(C_{C2}) \Rightarrow C_{C1}$ acts as a short circuit.

$$f_{3-dB}(C_{C2}) = 25 \text{ Hz} = \frac{1}{2\pi\tau_2}, \text{ so that}$$

$$\tau_2 = \frac{1}{2\pi(25)} = 0.00637 \text{ s} = R_{eq} C_{C2}$$

$$\text{where } R_{eq} = R_L + R_E \left(\frac{r_s + R_1 \| R_2 \| R_S}{1 + \beta} \right)$$

$$= 10 + 50 \left(\frac{81.5 + 600 \| 300}{101} \right) = 10 + 50 \| 2.79 \Rightarrow$$

$$R_{ib} = r_s + (1 + \beta)R_E \text{ Assume } C_{C2} \text{ an open}$$

$$R_{ib} = 81.5 + (101)(50) = 5132 \Omega$$

$$\tau_1 = (100)\tau_2 = (100)(0.00637) = 0.637 \text{ s} = R_{eq} C_{C1}$$

$$R_{eq1} = R_S + R_{TH} \| R_{ib} = 300 + 600 \| 5132 = 837 \Omega$$

$$\text{So } C_{C1} = \frac{0.637}{837} \Rightarrow C_{C1} = 761 \mu\text{F}$$

7.21

$$\text{a. } I_D = K_n(V_{GS} - V_{TN})^2$$

$$V_{GS} = \sqrt{\frac{I_D}{K_n}} + V_{TN} = \sqrt{\frac{0.5}{0.5}} + 0.8 = 1.8 \text{ V}$$

$$R_S = \frac{-V_{GS} - (-5)}{0.5} = \frac{5 - 1.8}{0.5} \Rightarrow R_S = 6.4 \text{ k}\Omega$$

$$V_D = V_{DSQ} + V_S = 4 - 1.8 = 2.2 \text{ V}$$

$$R_D = \frac{5 - 2.2}{0.5} \Rightarrow R_D = 5.6 \text{ k}\Omega$$

$$\text{(b) } g_m = 2\sqrt{K_n I_D} = 2\sqrt{(0.5)(0.5)} = 1 \text{ mA/V}$$

From Problem 7.20,

$$\tau_A = R_S C_S = (6.4 \times 10^3)(5 \times 10^{-6})$$

$$= 3.2 \times 10^{-2} \text{ s}$$

$$\tau_A = \frac{1}{2\pi\tau_A} = \frac{1}{2\pi(3.2 \times 10^{-2})} \Rightarrow f_A = 4.97 \text{ Hz}$$

$$\tau_B = \left(\frac{R_S}{1 + g_m R_S} \right) C_S = \left[\frac{6.4 \times 10^3}{1 + (1)(6.4)} \right] (5 \times 10^{-6}) \\ = 4.32 \times 10^{-3} \text{ s}$$

$$f_B = \frac{1}{2\pi\tau_B} = \frac{1}{2\pi(4.32 \times 10^{-3})} \Rightarrow f_B = 36.8 \text{ Hz}$$

c.

$$|A_v| = \frac{g_m R_D (1 + s R_S C_S)}{(1 + g_m R_S) \left[1 + s \left(\frac{R_S}{1 + g_m R_S} \right) C_S \right]}$$

 As R_S becomes large

$$|A_v| \rightarrow \frac{g_m R_D (s R_S C_S)}{(g_m R_S) \left[1 + s \left(\frac{R_S}{g_m R_S} \right) C_S \right]}$$

$$A_v = \frac{(g_m R_D) \left[s \left(\frac{1}{g_m} \right) C_S \right]}{1 + s \left(\frac{1}{g_m} \right) C_S}$$

The corner frequency $f_B = \frac{1}{2\pi(1/g_m)C_S}$ and the corresponding $f_A \rightarrow 0$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{(0.5)(0.5)} = 1 \text{ mA/V}$$

$$f_B = \frac{1}{2\pi \left(\frac{1}{10^{-3}} \right) (5 \times 10^{-6})} \Rightarrow f_B = 31.8 \text{ Hz}$$

7.22

$$\text{a. } f_B = \frac{1}{2\pi r_B}$$

$$\text{and } r_B = \frac{R_E(R_S + r_\pi)C_E}{R_S + r_\pi + (1 + \beta)R_E}$$

$$\text{For } R_S = 0 \quad r_B = \frac{R_E r_\pi C_E}{r_\pi + (1 + \beta)R_E}$$

$$I_{EQ} = \frac{-0.7 - (-10)}{5} = 1.86 \text{ mA}$$

$$\beta = 75 \Rightarrow I_{CQ} = 1.84 \text{ mA}$$

$$\beta = 125 \Rightarrow I_{CQ} = 1.85 \text{ mA}$$

For $f_B \leq 200 \text{ Hz}$

$$\Rightarrow r_B \geq \frac{1}{2\pi(200)} = 0.796 \text{ ms}$$

 $r_\pi \alpha \beta$ so smallest r_B will occur for smallest β .

$$\beta = 75; \quad r_\pi = \frac{(75)(0.026)}{1.84} = 1.06 \text{ k}\Omega$$

$$0.796 \times 10^{-3} = \frac{(5 \times 10^3)(1.06)C_E}{1.06 + (75)(5)}$$

$$\Rightarrow C_E = 57.2 \mu\text{F}$$

$$\text{b. For } \beta = 125; \quad r_\pi = \frac{(125)(0.026)}{1.85} = 1.76 \text{ k}\Omega$$

$$r_B = \frac{(5 \times 10^3)(1.76)(57.2 \times 10^{-6})}{1.76 + (125)(5)} = 0.797 \text{ ms}$$

$$f_B = \frac{1}{2\pi r_B} = \frac{1}{2\pi(0.797 \times 10^{-3})}$$

 $\Rightarrow f_B = 199.7 \text{ Hz}$ Essentially independent of β .

$$r_A = R_E C_E = (5 \times 10^3)(57.2 \times 10^{-6}) = 0.286 \text{ sec}$$

$$f_A = \frac{1}{2\pi r_A} = \frac{1}{2\pi(0.286)}$$

 $\Rightarrow f_A = 0.556 \text{ Hz}$ Independent of β .

7.23

a. Expression for the voltage gain is the same as Equation (7.58) with $R_s = 0$.

$$\text{b. } r_A = R_E C_E$$

$$r_B = \frac{R_E r_\pi C_E}{r_\pi + (1 + \beta)R_E}$$

7.24

$$r_H = (R_L \parallel R_C)C_L = (10 \parallel 5) \times 10^3 \times 15 \times 10^{-12}$$

$$r_H = 5 \times 10^{-8} \text{ s}$$

$$f_H = \frac{1}{2\pi r_H} = \frac{1}{2\pi(5 \times 10^{-8})} \Rightarrow f_H = 3.18 \text{ MHz}$$

$$I_{EQ} = \frac{10 - 0.7}{10} = 0.93 \text{ mA}, \quad I_{CQ} = 0.921 \text{ mA}$$

$$g_m = \frac{0.921}{0.026} = 35.4 \text{ mA/V}$$

$$A_V = g_m (R_C \parallel R_L) = 35.4(5 \parallel 10) \Rightarrow A_V = 118$$

7.25

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \left(\frac{166}{166 + 234} \right) (10) = 4.15 \text{ V}$$

$$I_D = \frac{V_G - V_{GS}}{R_S} = K_n (V_{GS} - V_{TN})^2$$

$$4.15 - V_{GS} = (0.5)(0.5)(V_{GS}^2 - 4V_{GS} + 4)$$

$$0.25V_{GS}^2 - 3.15 = 0 \Rightarrow V_{GS} = 3.55 \text{ V}$$

$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.5)(3.55 - 2)$$

$$g_m = 1.55 \text{ mA/V}$$

$$R_0 = R_S \parallel \frac{1}{g_m} = 0.5 \parallel \frac{1}{1.55} = 0.5 \parallel 0.645$$

$$R_0 = 0.282 \text{ k}\Omega$$

$$\tau = (R_0 \parallel R_L)C_L \text{ and } f_H = \frac{1}{2\pi\tau}$$

$$\beta\omega \approx f_H = 5 \text{ MHz}$$

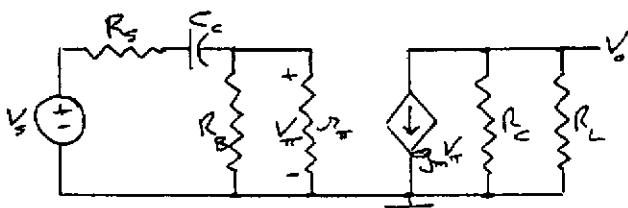
$$\Rightarrow \tau = \frac{1}{2\pi(5 \times 10^6)} = 3.18 \times 10^{-8} \text{ s}$$

$$C_L = \frac{\tau}{R_0 \parallel R_L} = \frac{3.18 \times 10^{-8}}{(0.282 \parallel 4) \times 10^3}$$

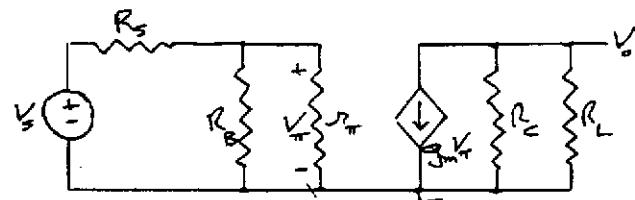
$$\Rightarrow C_L = 121 \text{ pF}$$

7.26

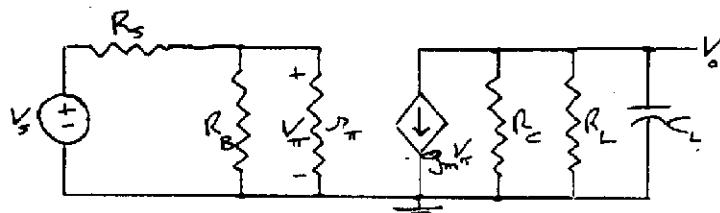
(a) Low-frequency



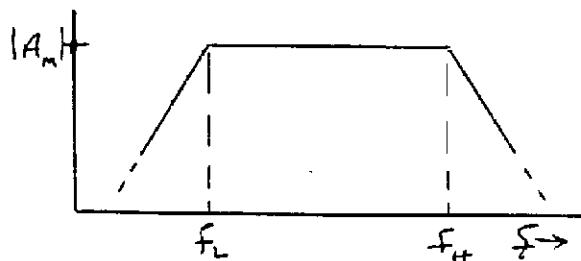
Mid-Band



High-frequency



(b)



$$(c) I_{BQ} = \frac{12 - 0.7}{1 M\Omega} = 11.3 \mu A$$

$$I_{CQ} = 1.13 mA$$

$$r_x = \frac{(100)(0.026)}{1.13} = 2.3 k\Omega$$

$$g_m = \frac{1.13}{0.026} = 43.46 mA/V$$

$$A_m = \frac{V_o}{V_i} (\text{midband}) = -g_m (R_C \| R_L) \left(\frac{R_B \| r_x}{R_B \| r_x + R_s} \right)$$

$$= -(43.46)(5.1 \| 500) \left(\frac{1000 \| 2.3}{1000 \| 2.3 + 1} \right)$$

$$= -(43.46)(5.05) \left(\frac{2.29}{2.29 + 1} \right) \Rightarrow |A_m| = 153$$

$$|A_m|_{dB} = 43.7 dB$$

$$f_L = \frac{1}{2\pi\tau_L}, \quad \tau_L = (R_s + R_B \| r_x) C_C$$

or

$$\tau_L = (1 + 1000 \| 2.3) \times 10^3 (10 \times 10^{-6}) \Rightarrow$$

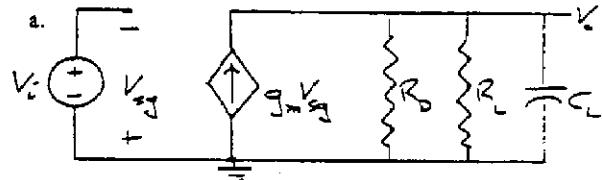
$$\tau_L = 3.29 \times 10^{-2} s \Rightarrow f_L = 4.84 Hz$$

$$f_H = \frac{1}{2\pi\tau_H}, \quad \tau_H = (R_C \| R_L) C_L \Rightarrow$$

$$\tau_L = (5.1 \| 500) \times 10^3 (10 \times 10^{-12}) = 5.05 \times 10^{-4} s$$

$$\Rightarrow f_H = 3.15 MHz$$

7.27



$$V_0 = (g_m V_{sg}) \left(R_D \| R_L \| \frac{1}{sC_L} \right)$$

$$V_{sg} = -V_i$$

$$A_v(s) = \frac{V_0(s)}{V_i(s)} = -g_m \left(R_D \| R_L \| \frac{1}{sC_L} \right)$$

$$= -g_m \left[\frac{R_D \| R_L \cdot \frac{1}{sC_L}}{R_D \| R_L + \frac{1}{sC_L}} \right]$$

$$A_v(s) = -g_m (R_D \| R_L) \cdot \frac{1}{1 + s(R_D \| R_L) C_L}$$

$$b. \quad \tau = (R_D \| R_L) C_L$$

$$c. \quad \tau = (10 \| 20) \times 10^3 \times 10 \times 10^{-12}$$

$$\Rightarrow \tau = 6.67 \times 10^{-8} s$$

$$f_H = \frac{1}{2\pi\tau} = \frac{1}{2\pi(6.67 \times 10^{-8})}$$

$$\Rightarrow f_H = 2.39 MHz$$

 From Example 7.6, $g_m = 0.705 mA/V$

$$|A_v| = g_m (R_D \| R_L) = (0.705)(10 \| 20)$$

$$\Rightarrow |A_v| = 4.7$$

7.31

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1}{0.026} = 38.5 mA/V$$

$$f_T = \frac{38.5 \times 10^{-3}}{2\pi(10 + 2) \times 10^{-12}}$$

$$f_T = 511 MHz$$

$$f_B = \frac{f_T}{\beta} = \frac{511}{120} \Rightarrow f_B = 4.26 MHz$$

7.32

$$f_B = \frac{f_T}{\beta} = \frac{5000 \text{ MHz}}{150} \Rightarrow f_B = 33.3 \text{ MHz}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$g_m = \frac{0.5}{0.026} = 19.2 \text{ mA/V}$$

$$5 \times 10^9 = \frac{19.2 \times 10^3}{2\pi(C_\pi + 0.15) \times 10^{-12}}$$

$$C_\pi + 0.15 = \frac{19.2 \times 10^3}{2\pi(10^{-12})(5 \times 10^9)} = 0.611 \text{ pF}$$

$$C_\pi = 0.461 \text{ pF}$$

7.33

$$a. \quad f_B = \frac{f_T}{\beta} = \frac{2000 \text{ MHz}}{150} = 13.3 \text{ MHz} = f_B$$

$$b. \quad h_{FE} = \frac{150}{1 + j(f/f_B)}$$

$$|h_{FE}| = \frac{150}{\sqrt{1 + (f/f_B)^2}} = 10$$

$$1 + \left(\frac{f}{f_B}\right)^2 = \left(\frac{150}{10}\right)^2 = 225$$

$$f = f_B \cdot \sqrt{225} = (13.3)\sqrt{225}$$

$$\Rightarrow f = 199 \text{ MHz}$$

7.34

$$(a) V_o = -g_m V_x R_L \text{ where}$$

$$\begin{aligned} V_x &= \frac{\frac{1}{sC_1}}{\frac{1}{sC_1} + r_b} \cdot V_i = \frac{\frac{r_x}{sC_1}}{1 + s r_x C_1} \cdot V_i \\ &= \frac{r_x}{r_x + r_b + s r_b r_x C_1} \cdot V_i = \left(\frac{r_x}{r_x + r_b} \right) \left(\frac{1}{1 + s(r_b \| r_x) C_1} \right) \cdot V_i \end{aligned}$$

So

$$A_v(s) = \frac{V_o(s)}{V_i(s)} = -g_m R_L \left(\frac{r_x}{r_x + r_b} \right) \left(\frac{1}{1 + s(r_b \| r_x) C_1} \right)$$

(b) Midband gain:

$$r_x = \frac{(100)(0.026)}{1} = 2.6 \text{ k}\Omega ,$$

$$g_m = \frac{1}{0.026} = 38.46 \text{ mA/V}$$

(i) For $r_b = 100 \Omega$

$$A_{v1} = -(38.46)(4) \left(\frac{2.6}{2.6 + 0.1} \right) \Rightarrow A_{v1} = -148.1$$

(ii) For $r_b = 500 \Omega$

$$A_{v2} = -(38.46)(4) \left(\frac{2.6}{2.6 + 0.5} \right) \Rightarrow A_{v2} = -129.0$$

$$(c) f_{3-dB} = \frac{1}{2\pi\tau}, \quad \tau = (r_b \| r_x) C_1$$

(i) For $r_b = 100 \Omega$

$$\tau_1 = (0.1 \| 2.6) \times 10^3 (2.2 \times 10^{-12}) = 2.12 \times 10^{-10} \text{ s}$$

$$\Rightarrow f_{3-dB} = 751 \text{ MHz}$$

(ii) For $r_b = 500 \Omega$

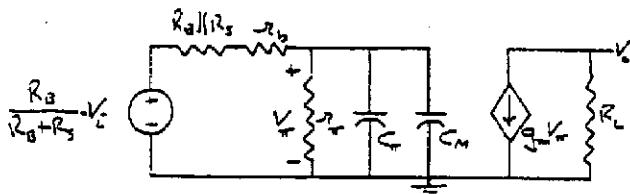
$$\tau_2 = (0.5 \| 2.3) \times 10^3 (2.2 \times 10^{-12}) = 9.04 \times 10^{-10} \text{ s}$$

$$f_{3-dB} = 176 \text{ MHz}$$

7.35

$$a. \quad C_M = C_\mu (1 + g_m R_L)$$

b.



$$V_0 = -g_m V_x R_L \quad \text{Let } C_\pi + C_M = C_1$$

$$V_x = \frac{\frac{r_x}{sC_1}}{\frac{r_x}{sC_1} + R_B \| R_S + r_b} \cdot \left(\frac{R_B}{R_B + R_S} \right) V_i$$

$$A_v(s) = \frac{V_o(s)}{V_i(s)}$$

$$= -g_m R_L \left(\frac{R_B}{R_B + R_S} \right) \left[\frac{\frac{r_x}{sC_1}}{\frac{r_x}{sC_1} + \frac{1}{sC_1}} \right]$$

$$= -g_m R_L \left(\frac{R_B}{R_B + R_S} \right)$$

$$\times \left[\frac{\frac{r_x}{sC_1}}{\frac{r_x}{sC_1} + (1 + s r_x C_1)(R_B \| R_S + r_b)} \right]$$

$$\text{Let } R_{eq} = (R_B \| R_S + r_b)$$

$$A_v(s) = -\beta R_L \left(\frac{R_B}{R_B + R_S} \right)$$

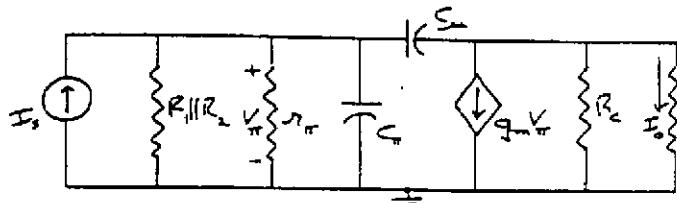
$$\times \left[\frac{1}{(r_x + R_{eq})(1 + s(r_x \| R_{eq}) C_1)} \right]$$

$$A_v(s) = \frac{-\beta R_L}{r_x + R_{eq}} \cdot \left(\frac{R_B}{R_B + R_S} \right) \cdot \frac{1}{1 + s(r_x \| R_{eq}) C_1}$$

$$c. \quad f_H = \frac{1}{2\pi(r_x \| R_{eq}) C_1}$$

7.36

High Freq. $\Rightarrow C_{C1}, C_{C2}, C_B \rightarrow$ short circuits



$$g_m = \frac{I_{CQ}}{V_T} = \frac{5}{0.026} = 192 \text{ mA/V}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} \Rightarrow 250 \times 10^6 = \frac{192 \times 10^{-3}}{2\pi(C_\pi + C_\mu)}$$

$$C_\pi + C_\mu = 122 \text{ pF} \Rightarrow C_\mu = 5 \text{ pF}, C_\pi = 117 \text{ pF}$$

$$C_M = C_\mu(1 + g_m(R_C \parallel R_L))$$

$$= 5[1 + (192)(1 \parallel 1)] \Rightarrow C_M = 485 \text{ pF}$$

$$C_i = C_\pi + C_M = 117 + 485 = 602 \text{ pF}$$

$$r_\pi = \frac{(200)(0.026)}{5} = 1.04 \text{ k}\Omega$$

$$R_{eq} = R_1 \parallel R_2 \parallel r_\pi = 5 \parallel 1.04 = 0.861 \text{ k}\Omega$$

$$\tau = R_{eq} \cdot C_i = (0.861 \times 10^3)(602 \times 10^{-12}) = 5.18 \times 10^{-7} \text{ s}$$

$$f = \frac{1}{2\pi\tau} = \frac{1}{2\pi(5.18 \times 10^{-7})} \Rightarrow f = 307 \text{ kHz}$$

7.37

$$R_{TH} = R_1 \parallel R_2 = 60 \parallel 5.5 = 5.04 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{5.5}{5.5 + 60} \right) (15) = 1.26 \text{ V}$$

$$I_{BQ} = \frac{1.26 - 0.7}{5.04 + (101)(0.2)} = 0.0222 \text{ mA}$$

$$I_{CQ} = 2.22 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{2.22} = 1.17 \text{ k}\Omega$$

$$g_m = \frac{2.22}{0.026} = 85.4 \text{ mA/V}$$

Lower 3 - dB frequency:

$$r_L = R_{eq} \cdot C_{C1}$$

$$R_{eq} = R_S + R_1 \parallel R_2 \parallel r_\pi$$

$$= 2 + 60 \parallel 5.5 \parallel 1.17 = 2.95 \text{ k}\Omega$$

$$r_L = (2.95 \times 10^3)(0.1 \times 10^{-6}) = 2.95 \times 10^{-4} \text{ s}$$

$$f_L = \frac{1}{2\pi r_L} = \frac{1}{2\pi(2.95 \times 10^{-4})} \Rightarrow f_L = 540 \text{ Hz}$$

Upper 3 - dB frequency:

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} \Rightarrow 400 \times 10^6 = \frac{85.4 \times 10^{-3}}{2\pi(C_\pi + C_\mu)}$$

$$C_\pi + C_\mu = 34 \text{ pF}, C_\mu = 2 \text{ pF} \Rightarrow C_\pi = 32 \text{ pF}$$

$$C_M = C_\mu(1 + g_m R_C) = 2(1 + (85.4)(4))$$

$$\Rightarrow C_M = 685 \text{ pF}$$

$$C_i = C_\pi + C_M = 32 + 685 = 717 \text{ pF}$$

$$R_{eq} = R_S \parallel R_1 \parallel R_2 \parallel r_\pi = 2 \parallel 60 \parallel 5.5 \parallel 1.17$$

$$= 0.644 \text{ k}\Omega$$

$$\tau = R_{eq} \cdot C_i = (0.644 \times 10^3)(717 \times 10^{-12})$$

$$= 4.62 \times 10^{-7} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau} \Rightarrow f_H = 344 \text{ kHz}$$

7.38

$$f_T = \frac{g_m}{2\pi(C_{ss} + C_{gd})}$$

$$g_m = 2\sqrt{K_n I_D}, K_n = (15)\left(\frac{40}{10}\right) = 60 \mu\text{A/V}^2$$

$$g_m = 2\sqrt{(60)(100)} = 155 \mu\text{A/V}^2$$

$$f_T = \frac{155 \times 10^{-6}}{2\pi(0.5 + 0.05) \times 10^{-12}} \Rightarrow f_T = 44.9 \text{ MHz}$$

7.39

$$a. \quad C_M = C_{gdT} (1 + g_m(r_0 \parallel R_D))$$

$$C_M = 5[1 + (3)(15 \parallel 10)] \Rightarrow C_M = 95 \text{ pF}$$

$$b. \quad \tau = (r_0)(C_{gsT} + C_M)$$

$$\tau = (10 \times 10^3)(50 + 95) \times 10^{-12} = 1.45 \times 10^{-6} \text{ s}$$

$$f = \frac{1}{2\pi\tau} = \frac{1}{2\pi(1.45 \times 10^{-6})}$$

$$\Rightarrow f = 110 \text{ kHz}$$

7.40

$$f_T = \frac{g_m}{2\pi(C_{gsT} + C_{gdT})} \quad (\text{Eq. 7.90})$$

$$\text{Let } C_{gdT} = 0 \text{ and } C_{gsT} = \left(\frac{2}{3}\right)(WLC_{ox})$$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{\left(\frac{\mu_n C_{ox}}{2}\right) \left(\frac{W}{L}\right) I_D}$$

$$\text{So } f_T = \frac{2\sqrt{\left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)I_D}}{2\pi\left(\frac{2}{3}\right)(WLC_{ox})}$$

$$= \frac{3}{2\pi L} \cdot \frac{\sqrt{\left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)I_D}}{WC_{ox}}$$

$$f_T = \frac{3}{2\pi L} \cdot \sqrt{\frac{\mu_n I_D}{2WC_{ox} L}}$$

7.41

$$(a) g'_m = \frac{g_m}{1 + g_m r_s}$$

$$g_m = 2K_n(V_{GS} - V_{TN})$$

$$K_n = \left(\frac{\mu_n C_{ox}}{2} \right) \left(\frac{W}{L} \right) = \left(\frac{(400)(7.25 \times 10^{-8})}{2} \right) (10)$$

$$K_n = 1.45 \times 10^{-4} \text{ mA/V}^2$$

For $V_{GS} = 5 \text{ V}$

$$g_m = 2(1.45 \times 10^{-4})(5 - 0.65) = 1.26 \times 10^{-3}$$

$$g'_m = (0.80)g_m = 1.01 \times 10^{-3}$$

$$1.01 \times 10^{-3} = \frac{1.26 \times 10^{-3}}{1 + (1.26 \times 10^{-3})r_s}$$

$$1 + (1.26 \times 10^{-3})r_s = 1.25$$

$$\Rightarrow r_s = 198 \Omega$$

b. For $V_{GS} = 3 \text{ V}$

$$g_m = 2(1.45 \times 10^{-4})(3 - 0.65) = 0.6815 \times 10^{-3}$$

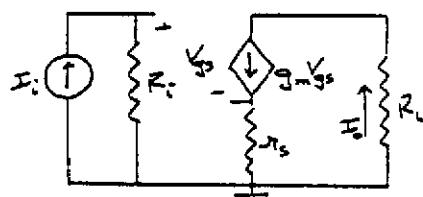
$$g'_m = \frac{0.6815 \times 10^{-3}}{1 + (0.6815 \times 10^{-3})(198)}$$

$$g'_m = 0.60 \times 10^{-3} \text{ A/V}$$

Reduced by $\approx 12\%$

7.42

a.

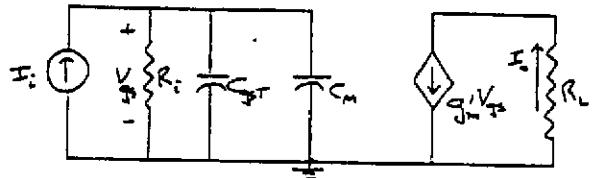


$$I_0 = g_m V_{gs} \text{ and } V_{gs} = I_i R_i - g_m V_{gs} r_s$$

$$\text{so } V_{gs} = \frac{I_i R_i}{1 + g_m r_s}$$

$$\text{Then } A_v = \frac{I_0}{I_i} = \frac{g_m R_i}{1 + g_m r_s}$$

b. As an approximation, consider



In this case

$$A_v = \frac{I_0}{I_i} = g'_m R_s \cdot \frac{1}{1 + g'_m R_s (C_{gsT} + C_M)}$$

$$\text{where } C_M = C_{gdT}(1 + g'_m R_L) \text{ and } g'_m = \frac{g_m}{1 + g_m r_s}$$

c. As r_s increases, C_M decreases, so the bandwidth increases, but the current gain magnitude decreases.

7.43

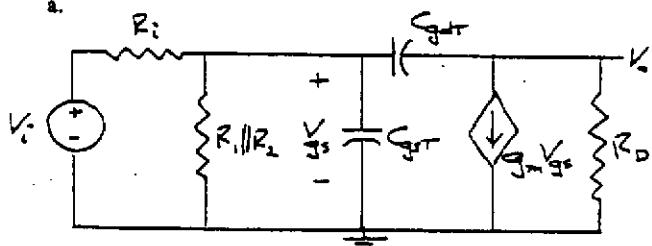
$$V_{GS} = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \left(\frac{225}{225 + 500} \right) (10)$$

$$V_{GS} = 3.10 \text{ V}$$

$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(1)(3.10 - 2)$$

$$g_m = 2.2 \text{ mA/V}$$

a.



$$C_M = C_{gdT}(1 + g_m R_D) = (1)[1 + (2.2)(5)]$$

$$C_M = 12 \text{ pF}$$

$$c. \tau = (R_i || R_1 || R_2)(C_{gsT} + C_M)$$

$$R_i || R_1 || R_2 = 1 || 500 || 225 = 1 || 155 = 0.994 \text{ k}\Omega$$

$$\tau = (0.994 \times 10^3)(5 + 12) \times 10^{-12} = 1.69 \times 10^{-8} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau} = \frac{1}{2\pi(1.69 \times 10^{-8})} \Rightarrow f_H = 9.42 \text{ MHz}$$

$$A_v = \frac{-g_m V_{gs} R_D}{V_i} \text{ and}$$

$$V_{gs} = \frac{R_1 || R_2}{R_1 || R_2 + R_i} \cdot V_i = \frac{155}{155 + 1} \cdot V_i = 0.994 V_i$$

$$A_v = -(2.2)(5)(0.994) \Rightarrow A_v = -10.9$$

7.44

$$R_{TH} = R_1 \parallel R_2 = 33 \parallel 22 = 13.2 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (5) = \left(\frac{22}{22 + 33} \right) (5) = 2 \text{ V}$$

$$I_{BQ} = \frac{2 - 0.7}{13.2 + (121)(4)} = 0.00261 \text{ mA}$$

$$I_{CQ} = 0.314 \text{ mA}$$

$$r_\pi = \frac{(120)(0.026)}{0.314} = 9.94 \text{ k}\Omega$$

$$g_m = \frac{0.314}{0.026} = 12.1 \text{ mA/V}$$

$$r_o = \frac{100}{0.314} = 318 \text{ k}\Omega$$

$$r_\pi = \frac{(120)(0.026)}{0.758} = 4.12 \text{ k}\Omega$$

$$g_m = \frac{0.758}{0.026} = 29.2 \text{ mA/V}$$

$$r_o = \infty$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{29.2 \times 10^{-3}}{2\pi(250 \times 10^6)}$$

$$C_\pi + C_\mu = 18.6 \text{ pF}; C_\mu = 3 \text{ pF} \Rightarrow C_\pi = 15.6 \text{ pF}$$

$$\text{a. } f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{12.1 \times 10^{-3}}{2\pi(600 \times 10^6)}$$

$$C_\pi + C_\mu = 3.21 \text{ pF}; C_\mu = 1 \text{ pF} \Rightarrow C_\pi = 2.21 \text{ pF}$$

$$C_M = C_\mu[1 + g_m(r_o \parallel R_C \parallel R_L)]$$

$$= (1)[1 + (12.1)(318 \parallel 4 \parallel 5)]$$

$$C_M = 27.7 \text{ pF}$$

$$\text{b. } \tau = R_{eq}(C_\pi + C_M)$$

$$R_{eq} = R_1 \parallel R_2 \parallel R_S \parallel r_\pi = 33 \parallel 22 \parallel 2 \parallel r_\pi$$

$$= 1.74 \parallel 9.94 \text{ k}\Omega \Rightarrow R_{eq} = 1.48 \text{ k}\Omega$$

$$\tau = (1.48 \times 10^3)(2.21 + 27.7) \times 10^{-12}$$

$$\tau = 4.43 \times 10^{-8} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau} = \frac{1}{2\pi(4.43 \times 10^{-8})}$$

$$\Rightarrow f_H = 3.59 \text{ MHz}$$

$$V_0 = -g_m V_\pi (r_o \parallel R_C \parallel R_L)$$

$$V_\pi = \left(\frac{R_1 \parallel R_2 \parallel r_\pi}{R_1 \parallel R_2 \parallel r_\pi + R_S} \right) V_i$$

$$R_1 \parallel R_2 \parallel r_\pi = 33 \parallel 22 \parallel 9.94 = 5.67 \text{ k}\Omega$$

$$V_\pi = \left(\frac{5.67}{5.67 + 2} \right) V_i = 0.739 V_i$$

$$r_o \parallel R_C \parallel R_L = 318 \parallel 4 \parallel 5 = 2.18 \text{ k}\Omega$$

$$A_v = -(12.1)(0.739)(2.18)$$

$$\underline{A_v = -19.5}$$

$$\tau_H = (0.405 \times 10^3)(15.6 + 149) \times 10^{-12}$$

$$= 6.67 \times 10^{-8} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau_H} \Rightarrow f_H = 2.39 \text{ MHz}$$

For lower frequency:

$$r_L = R_{eq} C_{C1}$$

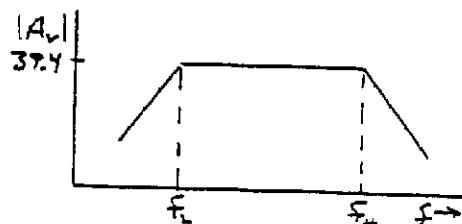
$$R_{eq} = R_S + R_1 \parallel R_2 \parallel r_\pi = 0.5 + 40 \parallel 5 \parallel 4.12$$

$$R_{eq} = 2.64 \text{ k}\Omega$$

$$\tau_L = (2.64 \times 10^3)(4.7 \times 10^{-6}) = 1.24 \times 10^{-2} \text{ s}$$

$$f_L = \frac{1}{2\pi\tau_L} \Rightarrow f_L = 12.8 \text{ Hz}$$

b.



$$V_0 = -g_m V_\pi (R_C \parallel R_L)$$

$$V_\pi = \left(\frac{R_1 \parallel R_2 \parallel r_\pi}{R_1 \parallel R_2 \parallel r_\pi + R_S} \right) V_i$$

$$V_\pi = \left(\frac{2.14}{2.14 + 0.5} \right) V_i = 0.8106 V_i$$

$$|A_v| = (29.2)(0.8106)(5 \parallel 2.5)$$

$$\underline{|A_v| = 39.4}$$

7.45

$$R_{TH} = R_1 \parallel R_2 = 40 \parallel 5 = 4.44 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{5}{5 + 40} \right) (10) = 1.11 \text{ V}$$

$$I_{BQ} = \frac{1.11 - 0.7}{4.44 + (121)(0.5)} = 0.00631 \text{ mA}$$

$$I_{CQ} = 0.758 \text{ mA}$$

7.46

$$I_D = K_p(V_{SG} + V_{TP})^2 = \frac{9 - V_{SG}}{R_s}$$

$$(2)(1.2)(V_{SG}^2 - 4V_{SG} + 4) = 9 - V_{SG}$$

$$2.4V_{SG}^2 - 8.6V_{SG} + 0.6 = 0$$

$$V_{SG} = \frac{8.6 \pm \sqrt{(8.6)^2 - 4(2.4)(0.6)}}{2(2.4)}$$

$$V_{SG} = 3.51 \text{ V}$$

$$g_m = 2K_p(V_{SG} + V_{TP}) = 2(2)(3.51 - 2)$$

$$g_m = 6.04 \text{ mA/V}$$

$$I_D = (2)(3.51 - 2)^2 = 4.56 \text{ mA}$$

$$r_o = \frac{1}{\lambda I_0} = \frac{1}{(0.01)(4.56)} \Rightarrow r_o = 21.9 \text{ k}\Omega$$

$$\text{a. } C_M = C_{gdT}(1 + g_m(r_o \| R_D))$$

$$C_M = (1)[1 + (6.04)(21.9 \| 1)] \Rightarrow C_M = 6.78 \text{ pF}$$

$$\text{b. } \tau_H = (R_s \| R_O)(C_{gsT} + C_M)$$

$$\tau_H = (2 \| 100) \times 10^3 (10 + 6.78) \times 10^{-12}$$

$$\tau_H = 3.29 \times 10^{-8} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau_H} \rightarrow f_H = 4.84 \text{ MHz}$$

$$V_0 = -g_m(r_o \| R_D) \cdot V_{gs}$$

$$V_{gs} = \left(\frac{R_G}{R_G + R_i} \right) V_i = \left(\frac{100}{102} \right) V_i$$

$$A_v = -(6.04) \left(\frac{100}{102} \right) (21.9 \| 1)$$

$$A_v = -5.66$$

7.47

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) (20) - 10 = \left(\frac{22}{22 + 8} \right) (20) - 10$$

$$V_G = 4.67 \text{ V}$$

$$I_D = \frac{10 - V_{SG} - 4.67}{R_s} = K_p(V_{SG} + V_{TP})^2$$

$$5.33 - V_{SG} = (1)(0.5)(V_{SG}^2 - 4V_{SG} + 4)$$

$$0.5V_{SG}^2 - V_{SG} - 3.33 = 0$$

$$V_{SG} = \frac{1 \pm \sqrt{1 + 4(0.5)(3.33)}}{2(0.5)} \Rightarrow V_{SG} = 3.77 \text{ V}$$

$$g_m = 2K_p(V_{SG} + V_{TP}) = 2(1)(3.77 - 2)$$

$$g_m = 3.54 \text{ mA/V}$$

$$\text{b. } C_M = C_{gdT}(1 + g_m(R_D \| R_L))$$

$$C_M = (3)[1 + (3.54)(2 \| 5)] \Rightarrow C_M = 18.2 \text{ pF}$$

$$\text{a. } \tau = R_{eq}(C_{gsT} + C_M)$$

$$R_{eq} = R_s \| R_1 \| R_2 = 0.5 \| 8 \| 22 = 0.461 \text{ k}\Omega$$

$$\begin{aligned} \tau &= (0.461 \times 10^3)(15 + 18.2) \times 10^{-12} \\ &= 1.53 \times 10^{-8} \text{ s} \end{aligned}$$

$$f_H = \frac{1}{2\pi\tau} \Rightarrow f_H = 10.4 \text{ MHz}$$

$$\text{c. } V_0 = -g_m V_{gs} (R_D \| R_L)$$

$$V_{gs} = \left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_i} \right) V_i = \left(\frac{5.87}{5.87 + 0.5} \right) V_i$$

$$\Rightarrow V_{gs} = (0.9215) V_i$$

$$\begin{aligned} A_v &= -(3.54)(0.9215)(2 \| 5) \\ &\Rightarrow A_v = -4.66 \end{aligned}$$

7.48

$$I_E = 0.5 \text{ mA} \Rightarrow I_{CQ} = \left(\frac{100}{101} \right) (0.5) = 0.495 \text{ mA}$$

$$g_m = \frac{0.495}{0.026} = 19.0 \text{ mA/V}$$

$$\tau_\pi = \frac{(100)(0.026)}{0.495} = 5.25 \text{ k}\Omega$$

a. Input: From Eq. 7.107b

$$\begin{aligned} r_{P\pi} &= \left[\frac{r_\pi}{1 + \beta} \| R_E \| R_S \right] C_\pi \\ &= \left[\frac{5.25}{101} \| 0.5 \| 0.05 \right] \times 10^3 \times 10 \times 10^{-12} \\ &= 2.43 \times 10^{-10} \text{ s} \end{aligned}$$

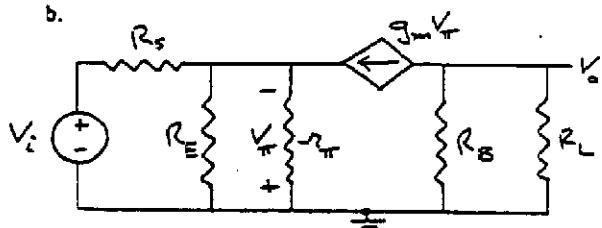
$$f_{H\pi} = \frac{1}{2\pi r_{P\pi}} \Rightarrow f_{H\pi} = 656 \text{ MHz}$$

Output: From Eq. 7.108b

$$\begin{aligned} r_{P\mu} &= (R_B \| R_L) C_\mu = (100 \| 1) \times 10^3 \times 10^{-12} \\ &= 9.90 \times 10^{-10} \text{ s} \end{aligned}$$

$$f_{H\mu} = \frac{1}{2\pi r_{P\mu}} \Rightarrow f_{H\mu} = 161 \text{ MHz}$$

b.



$$V_0 = -g_m V_\pi (R_B \parallel R_L)$$

$$g_m V_\pi + \frac{V_\pi}{r_\pi} + \frac{V_\pi}{R_E} + \frac{V_i - (-V_\pi)}{R_S} = 0$$

$$V_\pi \left[g_m + \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{R_S} \right] = \frac{-V_i}{R_S}$$

$$V_\pi \left[19 + \frac{1}{5.25} + \frac{1}{0.5} + \frac{1}{0.05} \right] = \frac{-V_i}{0.05}$$

$$V_\pi(41.19) = -V_i(20)$$

$$V_\pi = -(0.4856)V_i$$

$$\frac{V_0}{V_i} = -(19)(-0.4856)(100 \parallel 1)$$

$$A_v = 9.14$$

c. $\tau = C_L(R_L \parallel R_C) = (15 \times 10^{-12})(6.5 \parallel 5) \times 10^3$

$$\tau = 1.485 \times 10^{-8} \text{ s}$$

$$f = \frac{1}{2\pi\tau} = f = 10.7 \text{ MHz}$$

Since $f < f_{P\mu} \Rightarrow 3dB$ freq. dominated by C_L .

7.49

$$I_{EQ} = \frac{20 - 0.7}{10} = 1.93 \text{ mA}$$

$$I_{CQ} = \left(\frac{100}{101} \right) (1.93) = 1.91 \text{ mA}$$

$$g_m = \frac{1.91}{0.026} = 73.5 \text{ mA/V}$$

$$r_\pi = \frac{(100)(0.026)}{1.91} = 1.36 \text{ k}\Omega$$

a. Input:

$$\tau_{P\pi} = \left[\frac{r_\pi}{1+\beta} \parallel R_E \parallel R_S \right] \cdot C_\pi$$

$$= \left[\frac{1.36}{101} \parallel 10 \parallel 1 \right] \times 10^3 \times 10 \times 10^{-12}$$

$$\tau_{P\pi} = 1.327 \times 10^{-10} \text{ s}$$

$$f_{P\pi} = \frac{1}{2\pi\tau_{P\pi}} \Rightarrow f_{P\pi} = 1.20 \text{ GHz}$$

Output:

$$\tau_{P\mu} = (R_C \parallel R_L) C_\mu = (6.5 \parallel 5) \times 10^3 \times 10^{-12}$$

$$\tau_{P\mu} = 2.826 \times 10^{-9} \text{ s}$$

$$f_{P\mu} = \frac{1}{2\pi\tau_{P\mu}} \Rightarrow f_{P\mu} = 56.3 \text{ MHz}$$

b. $V_0 = -g_m V_\pi (R_C \parallel R_L)$

$$g_m V_\pi + \frac{V_\pi}{r_\pi} + \frac{V_\pi}{R_E} + \frac{V_i - (-V_\pi)}{R_S} = 0$$

$$V_\pi \left(g_m + \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{R_S} \right) = -\frac{V_i}{R_S}$$

$$V_\pi \left[73.5 + \frac{1}{1.36} + \frac{1}{10} + \frac{1}{1} \right] = \frac{-V_i}{(1)}$$

$$V_\pi(75.34) = -V_i \Rightarrow V_\pi = -(0.01327)V_i$$

$$V_0 = -(73.5)(-0.01327)(6.5 \parallel 5)V_i$$

$$A_v = 2.76$$

c. $\tau = C_L(R_L \parallel R_C) = (15 \times 10^{-12})(6.5 \parallel 5) \times 10^3$

$$\tau = 4.24 \times 10^{-8} \text{ s}$$

$$f = \frac{1}{2\pi\tau} \rightarrow f = 3.75 \text{ MHz}$$

Since $f < f_{P\mu}$, 3dB frequency is dominated by C_L .

7.50

$$V_{GS} + I_D R_S = 5$$

$$I_D = \frac{5 - V_{GS}}{R_S} = K_n (V_{GS} - V_{TN})^2$$

$$5 - V_{GS} = (3)(10)(V_{GS}^2 - 2V_{GS} + 1)$$

$$30V_{GS}^2 - 59V_{GS} + 25 = 0$$

$$V_{GS} = \frac{59 \pm \sqrt{(59)^2 - 4(30)(25)}}{2(30)} \Rightarrow V_{GS} = 1.35 \text{ V}$$

$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(3)(1.35 - 1)$$

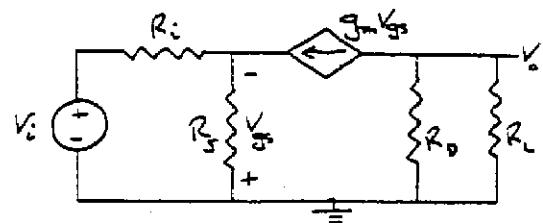
$$g_m = 2.1 \text{ mA/V}$$

On the output:

$$\tau_{P\mu} = (R_D \parallel R_L) C_{gdT} = (5 \parallel 4) \times 10^3 \times 4 \times 10^{-12}$$

$$\tau_{P\mu} = 8.89 \times 10^{-9} \text{ s}$$

$$f_{P\mu} = \frac{1}{2\pi\tau_{P\mu}} \Rightarrow f_{P\mu} = 17.9 \text{ MHz}$$



$$V_0 = -g_m V_{gs} (R_D \parallel R_L)$$

$$g_m V_{gs} + \frac{V_{gs}}{R_S} + \frac{V_i - (-V_{gs})}{R_S} = 0$$

$$V_{gs} \left(g_m + \frac{1}{R_S} + \frac{1}{R_i} \right) = -\frac{V_i}{R_S}$$

$$V_{gs} \left(2.1 + \frac{1}{10} + \frac{1}{2} \right) = -\frac{V_i}{2}$$

$$V_{gs} = -(0.185)V_i$$

$$A_v = \frac{V_0}{V_i} = (2.1)(0.185)(5 \parallel 4)$$

$$A_v = 0.863$$

7.51

dc analysis

$$I_D = \frac{V^+ - V_{SG}}{R_S} = K_p(V_{SG} + V_{TP})^2$$

$$5 - V_{SG} = (1)(4)(V_{SG} - 0.8)^2$$

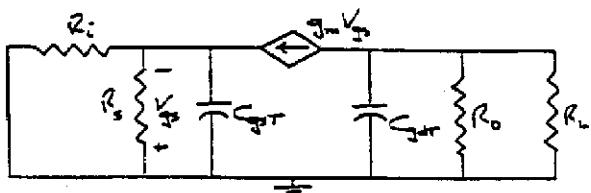
$$= 4(V_{SG}^2 - 1.6V_{SG} + 0.64)$$

$$4V_{SG}^2 - 5.4V_{SG} - 2.44 = 0$$

$$V_{SG} = \frac{5.4 \pm \sqrt{(5.4)^2 + 4(4)(2.44)}}{2(4)} = 1.707$$

$$g_m = 2K_p(V_{SG} + V_{TP}) = 2(1)(1.707 - 0.8)$$

$$g_m = 1.81 \text{ mA/V}$$



$$3 \text{-dB frequency due to } C_{gsT}: R_{eq} = \frac{1}{g_m} \parallel R_S \parallel R_i$$

$$f_A = \frac{1}{2\pi R_{eq} \cdot C_{gsT}}$$

$$R_{eq} = \frac{1}{1.81} \parallel 4 \parallel 0.5 = 0.246 \text{ k}\Omega$$

$$f_A = \frac{1}{2\pi(246)(4 \times 10^{-12})} = 162 \text{ MHz}$$

3-dB frequency due to C_{gsT}

$$f_B = \frac{1}{2\pi(R_D \parallel R_L)C_{gsT}}$$

$$= \frac{1}{2\pi(2 \parallel 4) \times 10^3 \times 10^{-12}}$$

$$f_B = 119 \text{ MHz}$$

Midband gain

$$V_{gs} = \frac{-\frac{1}{g_m} \parallel R_S}{\frac{1}{g_m} \parallel R_S + R_i} \cdot V_i = \frac{-\frac{1}{1.81} \parallel 4}{\frac{1}{1.81} \parallel 4 + 0.5} \cdot V_i$$

$$= -0.492V_i$$

$$V_0 = -g_m V_{gs} (R_D \parallel R_L)$$

$$A_v = (0.492)(1.81)(4) \parallel 2 \Rightarrow A_v = 1.19$$

7.52

$$\tau_\pi = \frac{(120)(0.026)}{1.02} = 3.06 \text{ k}\Omega$$

$$g_m = 39.2 \text{ mA/V}$$

$$\text{a. Input: } f_{H\pi} = \frac{1}{2\pi\tau_\pi}$$

$$\tau_\pi = [R_S \parallel R_2 \parallel R_3] \parallel [r_\pi + 2C_\mu]$$

$$= 0.1 \parallel 20.5 \parallel 28.3 \parallel 3.06 = 0.096 \text{ k}\Omega$$

$$\tau_\pi = (96)(12 + 2(2)) \times 10^{-12} = 1.536 \times 10^{-9} \text{ s}$$

$$f_{H\pi} = \frac{1}{2\pi(1.536 \times 10^{-9})} = 103.6 \text{ MHz}$$

$$\text{Output: } f_{H\mu} = \frac{1}{2\pi\tau_\mu}$$

$$\tau_\mu = (R_C \parallel R_L)C_\mu$$

$$= (5 \parallel 10) \times 10^3 \times 2 \times 10^{-12}$$

$$= 6.67 \times 10^{-9}$$

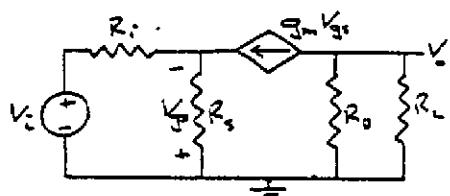
$$f_{H\mu} = \frac{1}{2\pi(6.67 \times 10^{-9})} = 23.9 \text{ MHz}$$

$$\text{b. } A = g_m(R_C \parallel R_L) \left[\frac{R_2 \parallel R_3 \parallel r_\pi}{R_2 \parallel R_3 \parallel r_\pi + R_S} \right]$$

$$R_2 \parallel R_3 \parallel r_\pi = 20.5 \parallel 28.3 \parallel 3.06 = 2.43 \text{ k}\Omega$$

$$A = (39.2)(5 \parallel 10) \left[\frac{2.43}{2.43 + 0.1} \right] \Rightarrow A = 125.5$$

c. $C_L = 15 \text{ pF} > C_\mu \Rightarrow C_L \text{ dominates frequency response.}$



Chapter 8

Exercise Solutions

E8.1

For $V_{DS} = 0$, $I_D(\max) = \frac{24}{20} = 1.2 \text{ A} = I_D(\max)$

For $I_D = 0 \Rightarrow V_{DS}(\max) = 24 \text{ V}$

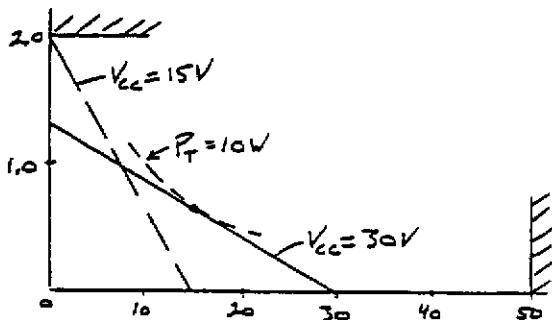
Maximum power when

$$V_{DS} = \frac{V_{DS}(\max)}{2} = 12 \text{ V} \text{ and}$$

$$I_D = \frac{I_D(\max)}{2} = 0.6 \text{ A}$$

$$\Rightarrow P_D(\max) = (12)(0.6) = 7.2 \text{ Watts}$$

E8.2



a. $V_{CC} = 30 \text{ V}$, $V_{CE} = 30 - I_C R_C$, $I_C V_{CE} = 10$

Maximum power at $V_{CE} = \frac{1}{2} V_{CC} = 15$

$$I_C = \frac{10}{V_{CE}} = \frac{10}{15} = \frac{2}{3}$$

$$\text{So } 15 = 30 - \frac{2}{3} R_L \Rightarrow R_L = 22.5 \Omega$$

$$\Rightarrow \underline{\text{Maximum Power} = 10 \text{ W}}$$

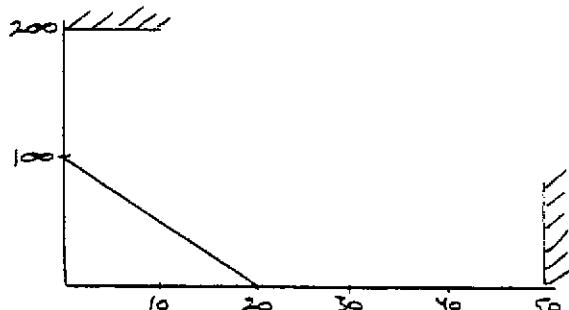
b. $V_{CC} = 15 \text{ V}$, $I_{C,\max} = 2 \text{ A}$

$$V_{CE} = 15 - I_C R_L$$

$$0 = 15 - 2R_L \Rightarrow R_L = 7.5 \Omega$$

$$\Rightarrow \underline{\text{Maximum Power} = (1)(7.5) = 7.5 \text{ W}}$$

E8.3



Maximum power at center of load line

$$P_{\max} = (0.05)(10) \Rightarrow \underline{P_{\max} = 0.5 \text{ W}}$$

E8.4

$$\text{Power} = i_D \cdot V_{DS} = (1)(12) = 12 \text{ watts}$$

c. $T_{sink} = T_{amb} + P \cdot \theta_{sink-amb}$

$$T_{sink} = 25 + (12)(4) \Rightarrow \underline{T_{sink} = 73^\circ\text{C}}$$

b. $T_{case} = T_{sink} + P \cdot \theta_{case-sink}$

$$T_{case} = 73 + (12)(1) \Rightarrow \underline{T_{case} = 85^\circ\text{C}}$$

d. $T_{dev} = T_{case} + P \cdot \theta_{dev-case}$

$$T_{dev} = 85 + (12)(3) \Rightarrow \underline{T_{dev} = 121^\circ\text{C}}$$

E8.5

$$\theta_{dev-case} = \frac{T_{j,max} - T_{amb}}{P_{D,\text{rated}}} = \frac{200 - 25}{50} = 3.5^\circ\text{C/W}$$

$$\begin{aligned} P_{D,max} &= \frac{T_{j,max} - T_{amb}}{\theta_{dev-case} + \theta_{case-sink} + \theta_{sink-amb}} \\ &= \frac{200 - 25}{3.5 + 0.5 + 2} \\ &\Rightarrow \underline{P_{D,max} = 29.2 \text{ W}} \end{aligned}$$

$$T_{case} = T_{amb} + P_{D,max}(\theta_{case-sink} + \theta_{sink-amb})$$

$$= 25 + (29.2)(0.5 + 2)$$

$$\Rightarrow \underline{T_{case} = 98^\circ\text{C}}$$

E8.6

a. $P_Q = V_{CEQ} \cdot I_{CQ} = (7.5)(7.5)$

$P_Q = 56.3 \text{ mW}$

b. $\overline{P_L} = \frac{1}{2} \cdot \frac{V_P^2}{R_L} = \frac{1}{2} \cdot \frac{(6.5)^2}{1} \Rightarrow \overline{P_L} = 21.1 \text{ mW}$

$\overline{P_S} = (15)(7.5) \Rightarrow \overline{P_S} = 113 \text{ mW}$

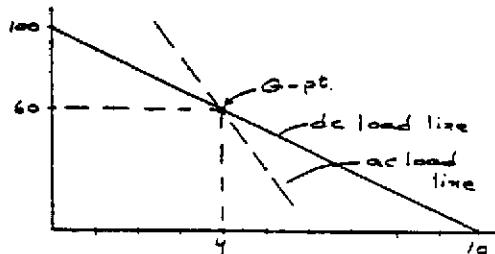
$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{21.1}{113} \Rightarrow \eta = 18.7\%$

$P_Q = 56.3 - 21.1 = 35.2 \text{ mW}$

E8.7

a. $I_{DQ} = \frac{10 - 4}{0.1} \Rightarrow I_{DQ} = 60 \text{ mA}$

b.



$v_{ds} = -\left(\frac{9}{10}\right)(60)(0.050) = -2.7 \text{ V}$

$\Rightarrow v_{DS}(\min) = 4 - 2.7 = 1.3 \text{ V}$

So maximum swing is determined by drain-to-source voltage.

$V_{PP} = 2 \times (2.5) = 5.0 \text{ V}$

c. $\overline{P_L} = \frac{1}{2} \cdot \frac{V_P^2}{R_L} = \frac{1}{2} \cdot \frac{(2.5)^2}{0.1} \Rightarrow \overline{P_L} = 31.25 \text{ mW}$

$\overline{P_S} = V_{DD} \cdot I_{DQ} = (10)(60) = 600 \text{ mW}$

$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{31.25}{600} \Rightarrow \eta = 3.2\%$

E8.8

a. $\overline{P_L} = \frac{1}{2} \cdot \frac{V_P^2}{R_L}$

$\Rightarrow V_P = \sqrt{2R_L \overline{P_L}} = \sqrt{2(8)(25)} \Rightarrow V_P = 20 \text{ V}$

$\Rightarrow V_{CC} = \frac{20}{0.8} \Rightarrow V_{CC} = 25 \text{ V}$

b. $I_P = \frac{V_P}{R_L} = \frac{20}{8} \Rightarrow I_P = 2.5 \text{ A}$

c. $\overline{P_Q} = \frac{V_{CC}V_P}{\pi R_L} - \frac{V_P^2}{4R_L}$

$$\begin{aligned} \overline{P_Q} &= \frac{(25)(20)}{\pi(8)} - \frac{(20)^2}{4(8)} = 19.9 - 12.5 \\ &\Rightarrow \overline{P_Q} = 7.4 \text{ W} \end{aligned}$$

d. $\eta = \frac{\pi V_P}{4V_{CC}} = \frac{\pi}{4} \cdot \frac{20}{25} \Rightarrow \eta = 62.8\%$

E8.9

a. $\overline{P_L} = \frac{1}{2} \cdot \frac{V_P^2}{R_L} = \frac{(4)^2}{2(0.1)} \Rightarrow \overline{P_L} = 80 \text{ mW}$

b. $I_P = \frac{V_P}{R_L} = \frac{4}{0.1} \Rightarrow I_P = 40 \text{ mA}$

c. $\overline{P_Q} = \frac{V_{CC}V_P}{\pi R_L} - \frac{V_P^2}{4R_L}$

$$\begin{aligned} \overline{P_Q} &= \frac{(5)(4)}{\pi(0.1)} - \frac{(4)^2}{4(0.1)} = 63.7 - 40 \\ &\Rightarrow \overline{P_Q} = 23.7 \text{ mW} \end{aligned}$$

d. $\eta = \frac{\pi V_P}{4V_{CC}} = \frac{\pi}{4} \cdot \frac{4}{5} \Rightarrow \eta = 62.8\%$

E8.10

a. $v_I = v_0 + v_{GSn} - \frac{V_{BB}}{2}$

$$\frac{dv_I}{dv_0} = 1 + \frac{dv_{GSn}}{dv_0}$$

$i_{Dn} = K_n(v_{GSn} - V_{TN})^2$

$$v_{GSn} = \sqrt{\frac{i_{Dn}}{K_n}} + V_{TN}$$

$$\frac{dv_{GSn}}{dv_o} = \frac{dv_{GSn}}{di_{Dn}} \cdot \frac{di_{Dn}}{dv_o}$$

$$\text{So } \frac{dv_{GSn}}{di_{Dn}} = \frac{1}{\sqrt{K_n}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{i_{Dn}}}$$

At $v_0 = 0$, $i_{Dn} = 0.050 \text{ A}$

$$\text{So } \frac{dv_{GSn}}{di_{Dn}} = \frac{1}{\sqrt{0.2}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{0.050}} = 5$$

$i_{Dn} = i_L + i_{Dp}$

For a small change in $v_0 - \Delta v_0 = \Delta i_L = \Delta i_{Dn} - (-\Delta i_{Dp})$

So $\Delta i_{Dn} = \frac{1}{2} \Delta i_L$

or

$$\frac{di_{Dn}}{dv_0} = \frac{1}{2} \cdot \frac{di_L}{dv_0} = \frac{1}{2} \cdot \frac{1}{R_L} = \frac{1}{2} \cdot \frac{1}{20} = 0.025$$

Then $\frac{dv_{GSn}}{dv_0} = (5)(0.025) = 0.125$

Then $\frac{dv_I}{dv_0} = 1 + 0.125 = 1.125$

and $A_v = \frac{dv_0}{dv_I} = \frac{1}{1.125} \Rightarrow A_v = 0.889$

b. For $v_0 = 5 \text{ V}$, $i_L = 0.25 \text{ A} = i_{Dn}$, and $i_{Dp} = 0$

$$\frac{dv_{GSn}}{dv_0} = \frac{dv_{GSn}}{di_{Dn}} \cdot \frac{di_{Dn}}{dv_0}$$

$$\frac{dv_{GSn}}{di_{Dn}} = \frac{1}{\sqrt{K_n}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{i_{Dn}}} = \frac{1}{\sqrt{0.2}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{0.25}} = 2.24$$

$$\frac{di_{Dn}}{dv_0} = \frac{di_L}{dv_0} = \frac{1}{20} = 0.05$$

$$\frac{dv_{GSn}}{dv_0} = (2.24)(0.05) = 0.112$$

$$\frac{dv_I}{dv_0} = 1 + 0.112 = 1.112$$

$$A_v = \frac{dv_0}{dv_I} = \frac{1}{1.112} \Rightarrow A_v = 0.899$$

E8.11

$$a. I_{CQ} \approx \frac{1}{2} \cdot \left(\frac{2V_{CC}}{R_L} \right) = \frac{V_{CC}}{R_L} = \frac{12}{1.5} = 8 \text{ mA}$$

$$R_{TH} = R_1 \parallel R_2$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC}$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{8}{75} = 0.107 \text{ mA} = \frac{V_{TH} - V_{BE}}{R_{TH} + (1 + \beta)R_E}$$

$$\text{Let } R_{TH} = (1 + \beta)R_E = (76)(0.1) = 7.6 \text{ k}\Omega$$

$$0.107 = \frac{\frac{1}{R_1} \cdot (7.6)(12) - 0.7}{7.6 + 7.6}$$

$$\frac{1}{R_1} \cdot (91.2) = 2.33 \Rightarrow R_1 = 39.1 \text{ k}\Omega$$

$$\frac{39.1R_2}{39.1 + R_2} = 7.6 \Rightarrow (39.1 - 7.6)R_2 = (7.6)(39.1)$$

$$\Rightarrow R_2 = 9.43 \text{ k}\Omega$$

$$b. \overline{P_L} = \frac{1}{2} \cdot (0.9I_{CQ})^2 R_L = \frac{1}{2}[(0.9)(8)]^2 (1.5) \\ \Rightarrow \overline{P_L} = 38.9 \text{ mW}$$

$$\overline{P_S} = V_{CC}I_{CQ} = (12)(8) = 96 \text{ mW}$$

$$\overline{P_Q} = \overline{P_S} - \overline{P_L} = 96 - 38.9 \Rightarrow \overline{P_Q} = 57.1 \text{ mW}$$

$$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{38.9}{96} \Rightarrow \eta = 40.5\%$$

E8.12

$$a. R_b = r_\pi + (1 + \beta)R'_E$$

$$\text{and } R'_E = a^2 R_L = (10)^2 (8) = 800 \text{ }\Omega$$

$$R_b = 1.5 \text{ k}\Omega = R_{TH} \parallel R_b$$

$$I_Q = \frac{V_{CC}}{a^2 R_L} = \frac{18}{(10)^2 (8)} = 22.5 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{22.5} = 0.116 \text{ k}\Omega$$

$$R_b = 0.116 + (101)(0.8) = 80.9 \text{ k}\Omega$$

$$1.5 = R_{TH} \parallel 80.9 = \frac{R_{TH}(80.9)}{R_{TH} + (80.9)}$$

$$\Rightarrow (80.9 - 1.5)R_{TH} = (1.5)(80.9)$$

$$\Rightarrow R_{TH} = 1.53 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC}$$

$$I_{BQ} = \frac{I_Q}{3} = \frac{22.5}{100} = 0.225 \text{ mA}$$

$$I_{BQ} = \frac{V_{TH} - 0.7}{R_{TH}}$$

$$\Rightarrow \frac{1}{R_1}(1.53)(18) = (0.225)(1.53) + 0.7$$

$$\Rightarrow R_1 = 26.4 \text{ k}\Omega$$

$$\frac{26.4R_2}{26.4 + R_2} = 1.53$$

$$(26.4 - 1.53)R_2 = (1.53)(26.4)$$

$$\Rightarrow R_2 = 1.62 \text{ k}\Omega$$

$$b. v_E = 0.9V_{CC} = (0.9)(18) = 16.2 \text{ V}$$

$$i_E = 0.9I_{CQ} = (0.9)(22.5) = 20.25 \text{ mA}$$

$$v_0 = \frac{v_E}{a} = \frac{16.2}{10} \Rightarrow V_P = 1.62 \text{ V}$$

$$i_0 = a i_E = (10)(20.25) \Rightarrow I_P = 203 \text{ mA}$$

$$\overline{P_L} = \frac{1}{2}(1.62)(0.203) \Rightarrow \overline{P_L} = 0.164 \text{ W}$$

E8.13

$$a. I_C = I_{SQ} \exp \left(\frac{V_{BE}}{V_T} \right) \Rightarrow V_{BE} = V_T \ln \left(\frac{I_C}{I_{SQ}} \right)$$

$$V_{BE} = (0.026) \ln \left(\frac{5 \times 10^{-3}}{2 \times 10^{-13}} \right) = 0.6225 \text{ V}$$

$$\Rightarrow V_{D1} = V_{D2} = 0.6225$$

$$I_{Bias} = I_D = I_{SD} \exp \left(\frac{0.6225}{0.026} \right)$$

$$= 5 \times 10^{-13} \exp \left(\frac{0.6225}{0.026} \right)$$

$$I_{Bias} = 12.5 \text{ mA}$$

$$b. \quad V_0 = 2 \text{ V}, \quad i_L = \frac{2}{0.075} = 26.7 \text{ mA}$$

1st approximation:

$$i_{Cn} \approx 26.7 \text{ mA}, \quad i_{Bn} = 0.444 \text{ mA}$$

$$V_{BE} = (0.026) \ln \left(\frac{26.7 \times 10^{-3}}{2 \times 10^{-13}} \right) = 0.6661$$

$$I_D = 12.5 - 0.444 = 12.056 \text{ mA}$$

$$V_D = (0.026) \ln \left(\frac{12.056 \times 10^{-3}}{5 \times 10^{-13}} \right) = 0.6216$$

$$2V_D = 1.243 \text{ V}$$

$$V_{EB} = 2V_D - V_{BE} = 0.5769$$

$$i_{Cp} = 2 \times 10^{-13} \exp \left(\frac{0.5769}{0.026} \right) = 0.866 \text{ mA}$$

2nd approximation:

$$i_{En} = i_L + i_{Cp} = 26.7 + 0.866 \approx 27.6 \text{ mA} = i_{E1}$$

$$i_{Cn} = \left(\frac{60}{61} \right) (27.6) \Rightarrow i_{Cn} = 27.1 \text{ mA}$$

$$i_{Bn} = 0.452 \text{ mA}$$

$$I_D = 12.5 - 0.452 \Rightarrow I_D = 12.05 \text{ mA}$$

$$V_{BEn} = (0.026) \ln \left(\frac{27.1 \times 10^{-3}}{2 \times 10^{-13}} \right) \Rightarrow V_{BEn} = 0.6664 \text{ V}$$

$$V_D = (0.026) \ln \left(\frac{12.05 \times 10^{-3}}{5 \times 10^{-13}} \right) = 0.6215 \text{ V}$$

$$2V_{DD} = 1.243 \text{ V}$$

$$V_{EB} = 1.243 - 0.6664 \Rightarrow V_{EBp} = 0.5766 \text{ V}$$

$$i_{Cp} = 2 \times 10^{-13} \exp \left(\frac{0.5766}{0.026} \right) \Rightarrow i_{Cp} = 0.856 \text{ mA}$$

$$c. \quad V_0 = 10 \text{ V}, \quad i_L = \frac{10}{0.075} = 133 \text{ mA}$$

$$i_{En} \approx i_L = 133 \text{ mA} \Rightarrow i_{Cn} = 131 \text{ mA}$$

$$i_{Bn} = 2.18 \text{ mA} \Rightarrow I_D = 12.5 - 2.18$$

$$\Rightarrow I_D = 10.3 \text{ mA}$$

$$V_D = (0.026) \ln \left(\frac{10.3 \times 10^{-3}}{5 \times 10^{-13}} \right) = 0.6175$$

$$2V_{DD} = 1.235 \text{ V}$$

$$V_{BEn} = (0.026) \ln \left(\frac{131 \times 10^{-3}}{2 \times 10^{-13}} \right) \Rightarrow V_{BEn} = 0.7074 \text{ V}$$

$$V_{EBp} = 1.235 - 0.7074 \Rightarrow V_{EBp} = 0.5276 \text{ V}$$

$$i_{Cp} = 2 \times 10^{-13} \exp \left(\frac{0.5276}{0.026} \right)$$

$$\Rightarrow i_{Cp} = 0.130 \text{ mA}$$

E8.14

$$a. \quad v_I = 0 = V_0, \quad v_{B3} = 0.7 \text{ V}$$

$$I_{R1} = \frac{12 - 0.7}{R_1} = \frac{11.3}{0.25} \Rightarrow I_{R1} = 45.2 \text{ mA}$$

If transistors are matched, then

$$i_{E1} = i_{E3}$$

$$i_{R1} = i_{E1} + i_{B3} = i_{E1} + \frac{i_{E3}}{1+\beta}$$

$$i_{R1} = i_{E1} \left(1 + \frac{1}{1+\beta} \right) = i_{E1} \left(1 + \frac{1}{41} \right)$$

$$i_{E1} = \frac{45.2}{1.024} \Rightarrow i_{E1} = i_{E2} = 44.1 \text{ mA}$$

$$i_{B1} = i_{B2} = \frac{i_{E1}}{1+\beta} = \frac{44.1}{41} \Rightarrow i_{B1} = i_{B2} = 1.08 \text{ mA}$$

$$b. \quad \text{For } v_I = 5 \text{ V} \Rightarrow V_0 = 5 \text{ V}$$

$$i_0 = \frac{5}{8} \Rightarrow i_0 = 0.625 \text{ A}$$

$$i_{E1} \approx 0.625 \text{ A}, \quad i_{E3} = \frac{0.625}{41} \Rightarrow i_{E3} = 15.2 \text{ mA}$$

$$v_{B3} = 5.7 \text{ V} \Rightarrow i_{R1} = \frac{12 - 5.7}{0.25} = 25.2 \text{ mA}$$

$$i_{E1} = 25.2 - 15.2 \Rightarrow i_{E1} = 10.0 \text{ mA}$$

$$\Rightarrow i_{B1} = \frac{10}{41} = 0.244 \text{ mA}$$

$$v_{B4} = 5 - 0.7 = 4.3 \text{ V}$$

$$I_{R2} = \frac{4.3 - (-12)}{0.25} = 65.2 \text{ mA} \approx i_{E2}$$

$$i_{B2} = \frac{65.2}{41} = 1.59 \text{ mA}$$

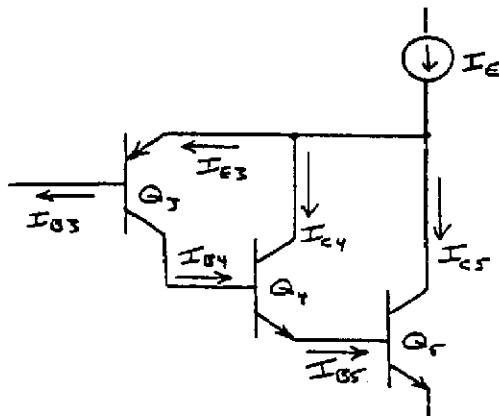
$$i_I = i_{B2} - i_{B1} = 1.59 - 0.244 \Rightarrow i_I = 1.35 \text{ mA}$$

$$c. \quad A_I = \frac{i_0}{i_I} = \frac{625}{1.35} \Rightarrow A_I = 463$$

From Equation (8.54)

$$A_I = \frac{(1+\beta)R}{2R_L} = \frac{(41)(250)}{2(8)} = 641$$

E8.15



$$\begin{aligned}
 I_E &= I_{E3} + I_{C4} + I_{C5} \\
 &= I_{E3} + I_{C4} + \beta_5 I_{B5} \\
 &= I_{E3} + \beta_4 I_{B4} + \beta_5 (1 + \beta_4) I_{B4} \\
 I_E &= (1 + \beta_3) I_{B3} + \beta_4 \beta_3 I_{B3} + \beta_5 (1 + \beta_4) \beta_3 I_{B3}
 \end{aligned}$$

If β_4 and β_5 are large, then

$$I_E \approx \beta_3 \beta_4 \beta_5 I_{B3}$$

So that composite current gain is

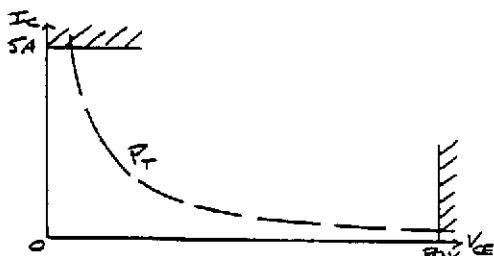
$$\underline{\underline{\beta}} = \underline{\underline{\beta_3 \beta_4 \beta_5}}$$

Chapter 8

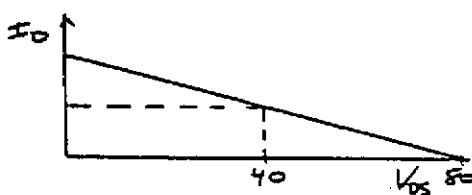
Problem Solutions

8.1

a.



b. $I_C = V_{DD} = 80 \text{ V}$

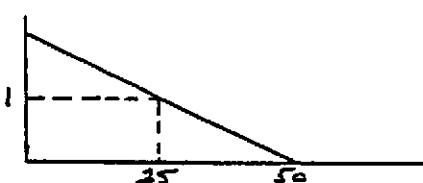


Maximum power at $V_{DS} = \frac{V_{DD}}{2} = 40 \text{ V}$

$$I_D = \frac{P_T}{V_{DS}} = \frac{25}{40} = 0.625 \text{ A}$$

$$R_D = \frac{80 - 40}{0.625} \Rightarrow R_D = 64 \Omega$$

c. $V_{DD} = 50 \text{ V}$



Maximum power at $V_{DS} = \frac{V_{DD}}{2} = 25 \text{ V}$

$$I_D = \frac{P_T}{V_{DS}} = \frac{25}{25} = 1 \text{ A}$$

$$R_D = \frac{50 - 25}{1} \Rightarrow R_D = 25 \Omega$$

8.2

a. $P_Q(\max) = I_{CQ} \cdot \frac{V_{CC}}{2}$

$$\text{So } I_{CQ} = \frac{2P_Q(\max)}{V_{CC}} = \frac{2(20)}{24} = 1.67 \text{ A}$$

$$R_L = \frac{V_{CC} - (V_{CC}/2)}{I_{CQ}} = \frac{24 - 12}{1.67} \Rightarrow R_L = 7.2 \Omega$$

$$I_B = \frac{I_{CQ}}{\beta} = \frac{1.67}{80} \Rightarrow 20.8 \text{ mA}$$

$$R_B = \frac{24 - 0.7}{20.8} \Rightarrow R_B = 1.12 \text{ k}\Omega$$

b. $|A_V| = g_m R_L = \frac{I_{CQ} \cdot R_L}{V_T} = \frac{(1.67)(7.2)}{0.026} = 462$

$$V_o(\max) = 12 \text{ V} \Rightarrow V_P = \frac{V_o(\max)}{|A_V|} = \frac{12}{462} \Rightarrow V_P \approx 26 \text{ mV}$$

8.3

a. For maximum power delivered to the load, set

$$V_{CEQ} = \frac{V_{CC}}{2}$$

Set $V_{CC} = 25 \text{ V} = V_{CE(\text{max})}$

$$\text{Then } I_{Cm} = \frac{V_{CC}}{R_L} = \frac{25}{0.1}$$

$$I_{Cm} = 250 \text{ mA} < I_{C,\max}$$

$$I_{CQ} = \frac{25 - 12.5}{0.1} = 125 \text{ mA}$$

$$P_Q(\max) = I_{CQ} \cdot \frac{V_{CC}}{2} = (0.125)(12.5) = 1.56 \text{ W} < P_{D,\max}$$

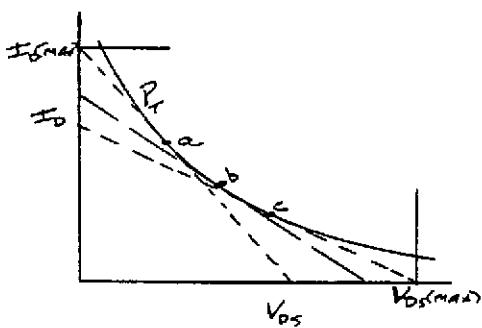
$$I_{BQ} = \frac{125}{100} = 1.25 \text{ mA}$$

$$R_B = \frac{25 - 0.7}{1.25} \Rightarrow R_B = 19.4 \text{ k}\Omega$$

b. $P_L(\max) = \frac{1}{2} \cdot I_{CQ}^2 \cdot R_L = \frac{1}{2}(0.125)^2(100)$

$$\Rightarrow P_L(\max) = 0.781 \text{ W}$$

8.4



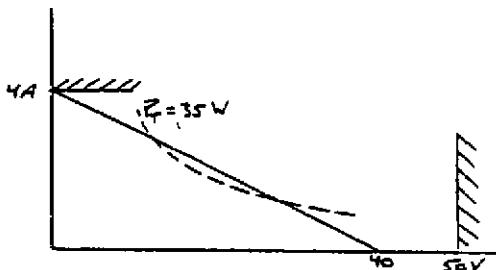
Point (b): Maximum power delivered to load.

Point (a): Will obtain maximum signal current output.

Point (c): Will obtain maximum signal voltage output.

8.5

a.



b. $V_{GG} = 5 \text{ V}, I_D = 0.25(5 - 4)^2 = 0.25 \text{ A},$

$$V_{DS} = 37.5 \text{ V}, P = 9.375 \text{ W}$$

$V_{GG} = 6 \text{ V}, I_D = 0.25(6 - 4)^2 = 1.0 \text{ A},$

$$V_{DS} = 30 \text{ V}, P = 30 \text{ W}$$

$V_{GG} = 7 \text{ V}, I_D = 0.25(7 - 4)^2 = 2.25 \text{ A},$

$$V_{DS} = 17.5 \text{ V}, P = 39.375 \text{ W}$$

$V_{GG} = 8 \text{ V}, I_D = 0.25[2(8 - 4)V_{DS} - V_{DS}^2]$

$$= \frac{40 - V_{DS}}{10} \Rightarrow V_{DS} = 2.92$$

$$I_D = 3.71 \text{ A}, P = 10.8 \text{ W}$$

$V_{GG} = 9 \text{ V}, I_D = 0.25[2(9 - 4)V_{DS} - V_{DS}^2]$

$$= \frac{40 - V_{DS}}{10} \Rightarrow V_{DS} = 1.88 \text{ V}$$

$$I_D = 3.81 \text{ A}, P = 7.16 \text{ W}$$

c. Yes, at $V_{GG} = 7 \text{ V}, P = 39.375 \text{ W} > P_{D,\max} = 35 \text{ W}$

8.6

a. Set $V_{DSQ} = \frac{V_{DD}}{2} = 25 \text{ V}$

$$I_{DQ} = \frac{50 - 25}{20} = 1.25 \text{ A}$$

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2$$

$$\sqrt{\frac{1.25}{0.2}} + 4 = V_{GS} = 6.5 \text{ V}$$

$$V_{GS} = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD}$$

$$\text{Let } R_1 + R_2 = 100 \text{ k}\Omega$$

$$6.5 = \left(\frac{R_2}{100} \right) (50) \Rightarrow R_2 = 13 \text{ k}\Omega$$

$$R_1 = 87 \text{ k}\Omega$$

b. $P_D = I_{DQ}V_{DSQ} = (1.25)(25) \Rightarrow P_D = 31.25 \text{ W}$

c. $I_{D,\max} = 2I_{DQ} \Rightarrow I_{D,\max} = 2.5 \text{ A}$

$$V_{DS,\max} = V_{DD} \Rightarrow V_{DS,\max} = 50 \text{ V}$$

$$P_{D,\max} = 31.25 \text{ W}$$

d. $\left| \frac{V_o}{V_i} \right| = g_m R_L$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.2)(1.25)} = 1 \text{ A/V}$$

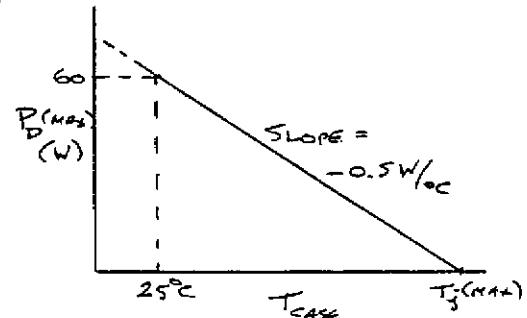
$$|V_o| = (1)(20)(0.5) = 10 \text{ V}$$

$$\overline{P}_L = \frac{1}{2} \cdot \frac{V_o^2}{R_L} = \frac{1}{2} \cdot \frac{(10)^2}{20} \Rightarrow \overline{P}_L = 2.5 \text{ W}$$

$$\overline{P}_Q = 31.25 - 2.5 \Rightarrow \overline{P}_Q = 28.75 \text{ W}$$

8.7

(a)



(b) $P_D = P_{D,\max} - (\text{Slope})(T_j - 25)$

$$\text{At } P_D = 0, T_{j,\max} = \frac{60}{0.5} + 25 \Rightarrow T_{j,\max} = 145^\circ C$$

(c) $P_{D,\max} = \frac{T_{j,\max} - T_{case}}{\theta_{dev-amb}}$

or

$$\theta_{dev-amb} = \frac{145 - 25}{60} \Rightarrow \theta_{dev-amb} = 2^\circ C/W$$

8.8

$$P_{D,\text{rated}} = \frac{T_{j,\max} - T_{amb}}{\theta_{dev-case}}$$

or

$$\theta_{dev-case} = \frac{T_{j,\max} - T_{amb}}{P_{D,\text{rated}}}$$

$$= \frac{150 - 25}{50} = 2.5^\circ C/W$$

Then

$$T_{dev} - T_{amb} = P_D(\theta_{dev-case} + \theta_{case-amb})$$

$$150 - 25 = P_D(2.5 + \theta_{case-amb})$$

$$\Rightarrow 125 = P_D(2.5 + \theta_{case-amb})$$

8.9

$$P_D = I_D \cdot V_{DS} = (4)(5) = 20 \text{ W}$$

$$T_{dev} - T_{amb} = P_D(\theta_{dev-case} + \theta_{case-sink} + \theta_{sink-amb})$$

$$T_{dev} - 25 = 20(1.75 + 0.8 + 3) = 111$$

$$\Rightarrow T_{dev} = 136^\circ\text{C}$$

$$T_{dev} - T_{case} = P_D \cdot \theta_{dev-case} = (20)(1.75) = 35$$

$$T_{case} = T_{dev} - 35 = 136 - 35 \Rightarrow T_{case} = 101^\circ\text{C}$$

$$T_{case} - T_{sink} = P_D \cdot \theta_{case-sink} = (20)(0.8) = 16^\circ\text{C}$$

$$T_{sink} = T_{case} - 16 = 101 - 16 \Rightarrow T_{sink} = 85^\circ\text{C}$$

8.10

$$T_{dev} - T_{amb} = P_D(\theta_{dev-case} + \theta_{case-amb})$$

$$200 - 25 = 25(3 + \theta_{case-amb})$$

$$\Rightarrow \theta_{case-amb} = 4^\circ\text{C/W}$$

8.11

$$\theta_{dev-case} = \frac{T_{j,max} - T_{amb}}{P_{D,rated}} = \frac{175 - 25}{15} = 10^\circ\text{C/W}$$

$$P_D = \frac{T_{j,max} - T_{amb}}{\theta_{dev-case} + \theta_{case-sink} + \theta_{sink-amb}}$$

$$= \frac{175 - 25}{10 + 1 + 4} \Rightarrow P_D = 10 \text{ W}$$

8.12

$$\eta = \frac{\overline{P}_L}{\overline{P}_S}$$

$$\overline{P}_S = V_{CC} \cdot I_Q$$

$$\overline{P}_L = V_P \cdot I_P = \left(\frac{V_{CC}}{2}\right)(I_Q)$$

$$\eta = \frac{\frac{1}{2} \cdot V_{CC} \cdot I_Q}{V_{CC} \cdot I_Q} \Rightarrow \eta = 50\%$$

8.13

a. Neglect base currents.

$$V_o(\max) = V^+ - V_{CE}(\text{sat}) = 10 - 0.2 = 9.8 \text{ V}$$

$$i_L(\max) = I_Q = \frac{9.8}{R_L} = \frac{9.8}{1} \Rightarrow I_Q = 9.8 \text{ mA}$$

$$R = \frac{0 - 0.7 - (-10)}{9.8} \Rightarrow R = 949 \Omega$$

$$i_{E1}(\max) = 2I_Q \Rightarrow i_{E1}(\max) = 19.6 \text{ mA}$$

$$i_{E1}(\min) = 0$$

$$i_L(\max) = I_Q = 9.8 \text{ mA}$$

$$i_L(\min) = -I_Q = -9.8 \text{ mA}$$

$$\text{b. } \overline{P}_L = \frac{1}{2}(i_L(\max))^2 R_L = \frac{1}{2}(9.8)^2(1)$$

$$\Rightarrow \overline{P}_L = 48.02 \text{ mW}$$

$$\overline{P}_S = I_Q(V^+ - V^-) + I_Q(0 - V^-)$$

$$= 9.8(20) + 9.8(10) \Rightarrow \overline{P}_S = 294 \text{ mW}$$

$$\eta = \frac{\overline{P}_L}{\overline{P}_S} = \frac{48.02}{294} \Rightarrow \eta = 16.3\%$$

8.14

$$\text{a. } I_Q(\min) = \frac{V_o(\max)}{R_L} = \frac{10}{0.1} \Rightarrow I_Q(\min) = 100 \text{ mA}$$

$$R = \frac{0 - 0.7 - (-12)}{100} \Rightarrow R = 113 \Omega$$

$$\text{b. } P_{Q1} = I_Q \cdot V_{CE1} = (100)(12) \Rightarrow P_{Q1} = 1.2 \text{ W}$$

$$P(\text{source}) = 2I_Q(12) = 2.4 \text{ W}$$

$$\text{c. } \overline{P}_L = \frac{1}{2} \cdot \frac{V_P^2}{R_L} = \frac{(10)^2}{2(100)} = 0.5 \text{ W}$$

$$\overline{P}_S = 1.2 + 2.4 = 3.6 \text{ W}$$

$$\eta = \frac{\overline{P}_L}{\overline{P}_S} = \frac{0.5}{3.6} \Rightarrow \eta = 13.9\%$$

8.15

$$\overline{P}_L = \frac{V_P^2}{R_L} = \frac{(V^+)^2}{R_L}$$

$$\overline{P}_S = \frac{1}{2} \cdot \frac{(V^+)^2}{R_L} + \frac{1}{2} \cdot \frac{(V^-)^2}{R_L}, \quad V^- = -V^+$$

$$\text{So } \overline{P}_S = \frac{(V^+)^2}{R_L}$$

$$\eta = \frac{\overline{P}_L}{\overline{P}_S} \Rightarrow \eta = 100\%$$

8.16

$$\text{(a) } V_{DS} \geq V_{DS}(\text{sat}) = V_{GS} - V_{TN} = V_{GS}$$

$$V_{DS} = 10 - V_o(\max) \text{ and } I_D = I_L = K_n(V_{GS})^2$$

$$\frac{V_o(\max)}{R_L} = K_n(V_{GS})^2$$

$$V_{GS} = \sqrt{\frac{V_o(\max)}{R_L \cdot K_n}}$$

So

$$10 - V_o(\max) = \sqrt{\frac{V_o(\max)}{R_L \cdot K_n}} = \sqrt{\frac{V_o(\max)}{(5)(0.4)}}$$

$$[10 - V_o(\max)]^2 = \frac{V_o(\max)}{2}$$

$$100 - 20V_o(\max) + V_o^2(\max) = \frac{V_o(\max)}{2}$$

$$V_0^2(\max) - 20.5V_0(\max) + 100 = 0$$

$$V_0(\max) = \frac{20.5 \pm \sqrt{(20.5)^2 - 4(100)}}{2}$$

$$\Rightarrow V_0(\max) = 8 \text{ V}$$

$$i_L = \frac{8}{5} \Rightarrow i_L = 1.6 \text{ mA}$$

$$V_{GS} = \sqrt{\frac{i_L}{K_n}} = \sqrt{\frac{1.6}{0.4}} = 2 \text{ V}$$

$$\Rightarrow v_I = 10 \text{ V}$$

b. $P_L = \frac{1}{2} \cdot \frac{(8)^2}{5} = 6.4 \text{ mW}$

$$\overline{P_S} = \frac{20(1.6)}{\pi} = 10.2 \text{ mW}$$

$$\eta = \frac{P_L}{\overline{P_S}} = \frac{6.4}{10.2} \Rightarrow \eta = 62.7\%$$

8.17

$$v_o = i_L R_L \text{ and } i_L = i_D = K_n(v_{GS} - V_{TN})^2$$

$$\text{or } i_L = K_n(v_{GS})^2 \text{ and } v_{GS} = v_I - v_o$$

Then

$$v_o = K_n R_L (v_I - v_o)^2 \text{ or } v_o = 2(v_I - v_o)^2$$

$$\frac{dv_o}{dv_I} = 2.2(v_I - v_o) \left(1 - \frac{dv_o}{dv_I}\right)$$

$$\frac{dv_o}{dv_I} [1 + 4(v_I - v_o)] = 4(v_I - v_o)$$

$$\text{or } \frac{dv_o}{dv_I} = \frac{4(v_I - v_o)}{1 + 4(v_I - v_o)}$$

$$\text{For } v_I = 10 \text{ V}, v_o = 8 \text{ V} \Rightarrow \frac{dv_o}{dv_I} = \frac{4(10 - 8)}{1 + 4(10 - 8)}$$

$$\Rightarrow \frac{dv_o}{dv_I} = 0.889$$

$$\text{At } v_I = 0, v_o = 0 \Rightarrow \frac{dv_o}{dv_I} = 0$$

$$\text{At } v_I = 1, v_o = 0.5 \Rightarrow \frac{dv_o}{dv_I} = 0.667$$

8.18

a. $V_{BE} = V_T \ln \left(\frac{i_C}{I_S} \right) = (0.026) \ln \left(\frac{5 \times 10^{-3}}{5 \times 10^{-13}} \right)$

$$V_{BE} = \frac{V_{BB}}{2} = 0.5987 \text{ V}$$

$$\Rightarrow V_{BB} = 1.1973 \text{ V}$$

$$P_Q = i_C \cdot v_{CE} = (5)(10) \Rightarrow P_Q = 50 \text{ mW}$$

b. $v_o = -8 \text{ V}$

$$i_L = \frac{-8}{0.1} \Rightarrow i_L = -80 \text{ mA}$$

$$i_{CP} \approx 80 \text{ mA}$$

$$v_{EB} = V_T \ln \left(\frac{i_{CP}}{I_S} \right) = (0.026) \ln \left(\frac{80 \times 10^{-3}}{5 \times 10^{-13}} \right)$$

$$v_{EB} = 0.6708 \text{ V}$$

$$v_I = \frac{V_{BB}}{2} - v_{EB} + v_o = 0.5987 - 0.6708 - 8$$

$$\Rightarrow v_I = -8.072 \text{ V}$$

$$v_{BE} = V_{BB} - v_{EB} = 1.1973 - 0.6708 = 0.5265 \text{ V}$$

$$i_{Cn} = I_S \exp \left(\frac{v_{BE}}{V_T} \right) = 5 \times 10^{-13} \exp \left(\frac{0.5265}{0.026} \right)$$

$$\Rightarrow i_{Cn} = 0.311 \text{ mA}$$

$$P_L = i_L^2 R_L = (80)^2 (0.1) \Rightarrow P_L = 640 \text{ mW}$$

$$P_{Qn} = i_{Cn} \cdot v_{CE} = (0.311)(10 - (-8))$$

$$\Rightarrow P_{Qn} = 5.60 \text{ mW}$$

$$P_{Qp} = i_{CP} \cdot v_{CE} = (80)(2) \Rightarrow P_{Qp} = 160 \text{ mW}$$

8.19

(a) $i_{Dn} = K_n(v_{GSn} - V_{TN})^2$

$$\sqrt{\frac{0.5}{2}} + 2 = v_{GSn} = 2.5 \text{ V} = \frac{V_{BB}}{2}$$

$$\Rightarrow V_{BB} = 5.0 \text{ V}$$

$$P_n = (0.5)(10) \Rightarrow P_n = P_p = 5 \text{ mW}$$

(b) $V_{DS} = V_{GS} - V_{TN} \Rightarrow V_{DS} = V_{GS} - 2$

$$V_{DS} = 10 - v_o(\max)$$

and

$$V_{GS} = \sqrt{\frac{i_L}{K_n}} + V_{TN} = \sqrt{\frac{v_o(\max)}{R_L K_n}} + 2$$

$$= \sqrt{\frac{v_o(\max)}{(2)(1)}} + 2$$

so

$$10 - v_o(\max) = \sqrt{\frac{v_o(\max)}{2}} + 2 - 2 = \sqrt{\frac{v_o(\max)}{2}}$$

$$\text{so } v_o(\max) = 8 \text{ V}$$

$$i_{Dn} = i_L = \frac{8}{1} \Rightarrow i_{Dn} = i_L = 8 \text{ mA}$$

$$V_{GS} = \sqrt{\frac{8}{2}} + 2 \Rightarrow V_{GS} = 4 \text{ V}$$

$$\text{Then } v_I = v_o + V_{GS} - \frac{V_{BB}}{2} = 8 + 4 - 2.5$$

$$\Rightarrow v_I = 9.5 \text{ V}$$

$$v_{SGp} = v_o - \left(v_I - \frac{V_{BB}}{2} \right) = 8 - (9.5 - 2.5)$$

$$v_{SGp} = 1 \text{ V} \Rightarrow M_p \text{ cutoff} \Rightarrow i_{Dp} = 0$$

$$P_L = i_L^2 R_L = (8)^2 (1) \Rightarrow P_L = 64 \text{ mW}$$

$$P_{Mn} = i_{Dn} \cdot v_{DS} = (8)(10 - 8) \Rightarrow P_{Mn} = 16 \text{ mW}$$

$$P_{Mp} = i_{Dp} \cdot v_{SD} \Rightarrow P_{Mp} = 0$$

8.20

$$\text{a. } v_0 = 24 \text{ V} \Rightarrow i_L = \frac{24}{8} \Rightarrow i_L \approx i_N = 3 \text{ A}$$

$$i_{Bn} = \frac{3}{41} \Rightarrow i_{Bn} = 73.2 \text{ mA}$$

$$\text{For } i_D = 25 \text{ mA} \Rightarrow i_{R1} = 25 + 73.2 = 98.2 \text{ mA}$$

$$V_{BE} = V_T \ln \left(\frac{i_N}{I_S} \right) = (0.026) \ln \left(\frac{3}{6 \times 10^{-12}} \right) \\ = 0.7004 \text{ V}$$

$$\text{Then } 98.2 = \frac{30 - (24 + 0.7)}{R_1} \Rightarrow R_1 = \frac{5.3}{98.2} \\ \Rightarrow R_1 = 53.97 \Omega$$

$$V_D = (0.026) \ln \left(\frac{25 \times 10^{-3}}{6 \times 10^{-12}} \right) = 0.5759 \text{ V}$$

$$V_{EB} = 2V_D - V_{BE} = 2(0.5759) - 0.7004 \\ = 0.4514 \text{ V}$$

$$i_P = I_S \exp \left(\frac{V_{EB}}{V_T} \right) = (6 \times 10^{-12}) \exp \left(\frac{0.4514}{0.026} \right) \\ \Rightarrow i_P = 0.208 \text{ mA}$$

b. Neglecting base current

$$i_D \approx \frac{30 - 0.6}{R_1} = \frac{30 - 0.6}{53.97} \Rightarrow i_D \approx 545 \text{ mA}$$

$$V_D = (0.026) \ln \left(\frac{0.545}{6 \times 10^{-12}} \right) = 0.656 \text{ V}$$

 Approximation for i_D is okay.

 Diodes and transistors matched $\Rightarrow i_N = i_P = 545 \text{ mA}$

8.21

$$(a) I_{D1} = K_1(V_{GS1} - V_{TN})^2$$

$$V_{GS1} = \sqrt{\frac{5}{5}} + 1 = 2 \text{ V}$$

$$I_{D1} = K_1(V_{GS1} - V_{TN})^2$$

$$200 = K_1(2 - 1)^2 \Rightarrow K_1 = K_{p4} = 200 \mu\text{A/V}^2$$

$$(b) v_I + V_{SG4} + V_{GS3} - V_{GS1} = v_o$$

$$\text{For } v_o \text{ large, } i_L = i_1 = K_{n1}(V_{GS1} - V_{TN})^2$$

$$V_{GS1} = \sqrt{\frac{i_L}{K_{n1}}} + V_{TN} = \sqrt{\frac{v_o}{R_L K_{n1}}} + V_{TN}$$

$$\text{So } v_I + 2 + 2 - \left(\sqrt{\frac{v_o}{(0.5)(5)}} + 1 \right) = v_o$$

$$v_I = v_o + \sqrt{\frac{v_o}{2.5}} - 3$$

$$\frac{dv_I}{dv_I} = 1 = \frac{dv_o}{dv_I} + \frac{1}{2} \cdot \frac{1}{\sqrt{2.5v_o}} \cdot \frac{dv_o}{dv_I}$$

$$1 = \frac{dv_o}{dv_I} \left[1 + \frac{1}{2\sqrt{2.5v_o}} \right]$$

 For $v_o = 5 \text{ V}$:

$$1 = \frac{dv_o}{dv_I} \left[1 + \frac{1}{2\sqrt{2.5(5)}} \right] = \frac{dv_o}{dv_I} (1.1414) \\ \Rightarrow \frac{dv_o}{dv_I} = 0.876$$

8.22

$$v_o = v_I + \frac{V_{BB}}{2} - V_{TN} \text{ and } V_{GS} = \sqrt{\frac{I_{DQ}}{K_n}} + V_{TN}$$

$$\text{For } v_o = 0, I_{Dn} = I_{DQ} + i_L = I_{DQ} + \frac{v_o}{R_L}$$

Then

$$v_o = v_I + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ} + (v_o/R_L)}{K_n}}$$

or

$$v_o = v_I + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ}}{K_n}} \cdot \sqrt{1 + \frac{v_o}{I_{DQ} R_L}}$$

 For v_o small,

$$v_o \equiv v_I + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ}}{K_n}} \cdot \left(1 + \frac{1}{2} \cdot \frac{v_o}{I_{DQ} R_L} \right)$$

$$v_o \left[1 + \frac{1}{2} \cdot \sqrt{\frac{I_{DQ}}{K_n}} \cdot \frac{1}{I_{DQ} R_L} \right]$$

$$= v_I + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ}}{K_n}}$$

Now

$$\frac{dv_o}{dv_I} = \frac{1}{\left[1 + \frac{1}{2} \cdot \sqrt{\frac{I_{DQ}}{K_n}} \cdot \frac{1}{I_{DQ} R_L} \right]} = 0.95$$

$$\text{So } \frac{1}{2} \cdot \sqrt{\frac{I_{DQ}}{K_n}} \cdot \frac{1}{I_{DQ} R_L} = \frac{1}{0.95} - 1 = 0.0526$$

$$\text{For } R_L = 0.1 \text{ k}\Omega, \text{ then } \frac{1}{\sqrt{K_n I_{DQ}}} = 0.01052$$

$$\text{Or } \sqrt{K_n I_{DQ}} = 95.1$$

We can write

$$g_m = 2\sqrt{K_n I_{DQ}} = 190 \text{ mA/V}$$

This is the required transconductance for the output transistor. This implies a very large transistor.

8.23

$$A_v = -g_m R_L$$

$$\text{So } -12 = -g_m(2) \Rightarrow g_m = 6 \text{ mA/V} = \frac{I_{CQ}}{V_T}$$

$$I_{CQ} = (6)(0.026) \Rightarrow I_{CQ} = 0.156 \text{ mA}$$

But for maximum symmetrical swing, set

$$I_{CQ} = \frac{V_{CC}}{R_L} = \frac{10}{2} = 5 \text{ mA} \Rightarrow |A_v| > 12$$

Maximum power to the load:

$$\overline{P_L}(\text{max}) = \frac{1}{2} \cdot \frac{V_{CC}^2}{R_L} = \frac{(10)^2}{2(2)} \Rightarrow \overline{P_L}(\text{max}) = 25 \text{ mW}$$

$$\overline{P_S} = V_{CC} \cdot I_{CQ} = (10)(5) = 50 \text{ mW}$$

$$\text{So } \eta = 50\%$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{5}{180} = 0.0278 \text{ mA}$$

$$R_i = R_{TH} = 6 \text{ k}\Omega$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE(\text{on})} + (1 + \beta) I_{BQ} R_E$$

$$\text{Set } R_E = 20 \text{ }\Omega$$

$$V_{TH} = (0.0278)(6) + 0.7 + (181)(0.0278)(0.020)$$

$$V_{TH} = 0.967 \text{ V}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC}$$

$$0.967 = \frac{1}{R_1}(6)(10) \Rightarrow R_1 = 62.0 \text{ k}\Omega$$

$$R_2 = 6.64 \text{ k}\Omega$$

8.24

$$I_{CQ} = \frac{V_{CC}}{R_L} = \frac{15}{1} = 15 \text{ mA}$$

$$I_{BQ} = \frac{15}{100} = 0.15 \text{ mA}$$

$$\overline{P_L}(\text{max}) = \frac{1}{2} \cdot \frac{V_{CC}^2}{R_L} = \frac{(15)^2}{2(1)} \Rightarrow \overline{P_L}(\text{max}) = 112.5 \text{ mW}$$

$$\text{Let } R_{TH} = 10 \text{ k}\Omega$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE} + (1 + \beta) I_{BQ} R_E$$

$$= (0.15)(10) + 0.7 + (101)(0.15)(0.1)$$

$$V_{TH} = 3.715 = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} \cdot (10)(15)$$

$$R_1 = 40.4 \text{ k}\Omega$$

$$R_2 = 13.3 \text{ k}\Omega$$

8.25

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{1.55}{1.55 + 0.73} \right) (10) \\ = 6.80 \text{ V}$$

$$R_{TH} = R_1 \| R_2 = 0.73 \| 1.55 = 0.496 \text{ k}\Omega$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}}{R_{TH} + (1 + \beta) R_E} = \frac{6.80 - 0.70}{0.496 + (26)(0.02)}$$

$$I_{BQ} = 5.0 \text{ mA}, I_{CQ} = 150 \text{ mA}$$

$$A_v = -g_m R'_L \text{ and } R'_L = \alpha^2 R_L = (3)^2 (8) = 72 \text{ }\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{150}{0.026} \Rightarrow 5.77 \text{ A/V}$$

$$A_v = -(5.77)(72) = -415$$

$$|V_0'|' = |A_v| \cdot V_i = (415)(0.017) = 7.06 \text{ V}$$

$$V_0 = \frac{7.06}{3} = 2.35 \text{ V}$$

$$\overline{P_L} = \frac{1}{2} \cdot \frac{V_0^2}{R_L} = \frac{(2.35)^2}{2(8)} \Rightarrow \overline{P_L} = 345 \text{ mW}$$

$$\overline{P_S} = I_{CQ} \cdot V_{CC} = (0.15)(10) = 1.5 \text{ W}$$

$$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{0.345}{1.5} \Rightarrow \eta = 23\%$$

8.26

a. Assuming the maximum power is being delivered, then

$$V_0'(\text{peak}) = 36 \text{ V} \Rightarrow V_0 = \frac{36}{4} = 9 \text{ V}$$

$$\Rightarrow V_{\text{rms}} = \frac{9}{\sqrt{2}} \Rightarrow V_{\text{rms}} = 6.36 \text{ V}$$

$$\text{b. } V_0 = \frac{36}{\sqrt{2}} \Rightarrow V_0 = 25.5 \text{ V}$$

$$\text{c. Secondary } I_{\text{rms}} = \frac{R_L}{V_{\text{rms}}} = \frac{2}{6.36} \Rightarrow I_{\text{rms}} = 0.314 \text{ A}$$

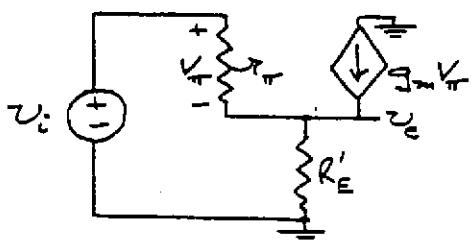
$$\text{Primary } I_P = \frac{0.314}{4} \Rightarrow I_P = 78.6 \text{ mA}$$

$$\text{d. } \overline{P_S} = I_{CQ} \cdot V_{CC} = (0.15)(36) = 5.4 \text{ W}$$

$$\eta = \frac{2}{5.4} \Rightarrow \eta = 37\%$$

8.27

a.



$$\begin{aligned} v_e &= \left(\frac{V_\pi}{r_\pi} + g_m V_\pi \right) R'_E = V_\pi \left(\frac{1}{r_\pi} + g_m \right) R'_E \\ &= V_\pi \left(\frac{1+\beta}{r_\pi} \right) R'_E \end{aligned}$$

$$v_i = V_\pi + v_e \Rightarrow V_\pi = v_i - v_e$$

$$v_e = (v_i - v_e) \left(\frac{1+\beta}{r_\pi} \right) R'_E$$

$$\frac{v_e}{v_i} = \frac{\frac{1+\beta}{r_\pi} \cdot R'_E}{1 + \frac{1+\beta}{r_\pi} \cdot R'_E} = \frac{(1+\beta)R'_E}{r_\pi + (1+\beta)R'_E} = \frac{v_e}{v_i}$$

where $R'_E = \left(\frac{n_1}{n_2} \right)^2 R_L$

$$v_0 = \frac{v_e}{\left(\frac{n_1}{n_2} \right)} \text{ so } v_e = v_0 \left(\frac{n_1}{n_2} \right)$$

$$\text{so } \frac{v_0}{v_i} = \frac{1}{\left(\frac{n_1}{n_2} \right)} \cdot \frac{(1+\beta)R'_E}{r_\pi + (1+\beta)R'_E}$$

b. $\overline{P_L} = \frac{1}{2} \cdot I_P^2 R_L, a = \frac{n_1}{n_2}, I_{CQ} = \frac{I_P}{a}$

$$\text{so } \overline{P_L} = \frac{1}{2} \cdot a^2 I_{CQ}^2 R_L$$

$$\overline{P_S} = I_{CQ} \cdot V_{CC}$$

For $\eta = 50\%$:

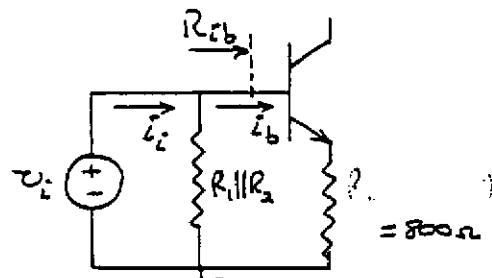
$$\frac{\overline{P_L}}{\overline{P_S}} = 0.5 = \frac{\frac{1}{2} \cdot a^2 I_{CQ}^2 R_L}{I_{CQ} \cdot V_{CC}} = \frac{a^2 I_{CQ} R_L}{2 V_{CC}}$$

$$\text{so } a^2 = \frac{V_{CC}}{I_{CQ} \cdot R_L} = \frac{V_{CC}}{(0.1)(50)} \Rightarrow a^2 = \frac{V_{CC}}{5}$$

c. $R_o = \frac{r_\pi}{1+\beta} = \frac{\beta V_T}{(1+\beta) I_{CQ}} = \frac{49(0.026)}{(50)(0.1)}$
 $\Rightarrow R_o = 0.255 \Omega$

8.28

- a. With a 10:1 transformer ratio, we need a current gain of 8 through the transistor.



$$i_c = (1+\beta)i_b \text{ and } i_b = \left(\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{ib}} \right) i, \text{ so we need}$$

$$\frac{i_c}{i} = 8 = (1+\beta) \left(\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{ib}} \right)$$

where

$$\begin{aligned} R_{ib} &= r_\pi + (1+\beta)R_L \approx (1+\beta)R_L \\ &= (101)(0.8) = 80.8 \end{aligned}$$

$$\text{Then } 8 = (101) \left(\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + 80.8} \right)$$

$$\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + 80.8} = 0.0792 \text{ or } R_1 \parallel R_2 = 6.95 \text{ k}\Omega$$

Set

$$\frac{2V_{CC}}{2I_{CQ}} = R'_L \Rightarrow I_{CQ} = \frac{V_{CC}}{R'_L} = \frac{12}{0.8} = 15 \text{ mA}$$

$$I_{BQ} = \frac{15}{100} = 0.15 \text{ mA}$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE}$$

$$\frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = I_{BQ} R_{TH} + V_{BE}$$

$$\frac{1}{R_1} (6.95)(12) = (0.15)(6.95) + 0.7$$

$$\Rightarrow R_1 = 47.9 \text{ k}\Omega \text{ then } R_2 = 8.13 \text{ k}\Omega$$

b. $I_e = 0.9 I_{CQ} = 13.5 \text{ mA} = \frac{I_L}{a} \Rightarrow I_L = 135 \text{ mA}$

$$\overline{P_L} = \frac{1}{2} (0.135)^2 (8) \Rightarrow \overline{P_L} = 72.9 \text{ mW}$$

$$\overline{P_S} = V_{CC} I_{CQ} = (12)(15) \Rightarrow \overline{P_S} = 180 \text{ mW}$$

$$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \eta = 40.5\%$$

8.29

a. $V_P = \sqrt{2R_L P_L}$

$V_P = \sqrt{2(8)(1)} = 5.66 \text{ V} = \text{peak output voltage}$

$I_P = \frac{V_P}{R_L} = \frac{5.66}{8} = 0.708 \text{ A} = \text{peak output current}$

Set $V_e = 0.9V_{CC} = aV_P$ to minimize distortion

Then $a = \frac{(0.9)(18)}{5.66} \Rightarrow a = 2.86$

b. Now

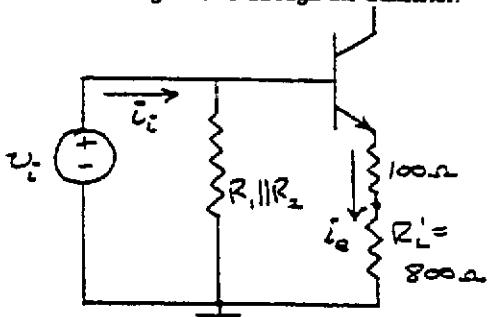
$I_{CQ} = \frac{1}{0.9} \left(\frac{I_P}{a} \right) = \frac{1}{0.9} \left(\frac{0.708}{2.86} \right) \Rightarrow I_{CQ} = 0.275 \text{ A}$

Then $P_Q = V_{CC} I_{CQ} = (18)(0.275)$

$\Rightarrow P_Q = 4.95 \text{ W}$ Power rating of transistor

8.30

a. Need a current gain of 8 through the transistor.



$$\frac{i_C}{i_I} = 8 = (1 + \beta) \left(\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{ib}} \right)$$

where $R_{ib} \approx (1 + \beta)(0.9) = 90.9 \text{ k}\Omega$

$$\frac{8}{101} = \left(\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + 90.9} \right) = 0.0792$$

or $R_1 \parallel R_2 = 7.82 \text{ k}\Omega$

Set

$$\frac{2V_{CC}}{2I_{CQ}} = 0.9 \text{ k}\Omega \Rightarrow I_{CQ} = \frac{12}{0.9} = 13.3 \text{ mA}$$

$$I_{BQ} = \frac{13.3}{100} = 0.133 \text{ mA}$$

Then

$$\frac{1}{R_1} (7.82)(12) = (0.133)(7.82) + 0.7$$

$\Rightarrow R_1 = 53.9 \text{ k}\Omega$ and $R_2 = 9.15 \text{ k}\Omega$

b. $I_e = (0.9)I_{CQ} = 12 \text{ mA} = \frac{I_L}{a} \Rightarrow I_L = 120 \text{ mA}$

$$\overline{P_L} = \frac{1}{2}(0.12)^2(8) \Rightarrow \overline{P_L} = 57.6 \text{ mW}$$

$$\overline{P_S} = V_{CC} I_{CQ} = (12)(13.3) \Rightarrow \overline{P_S} = 159.6 \text{ mW}$$

$$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{57.6}{159.6} \Rightarrow \eta = 36.1\%$$

8.31

a. All transistors are matched.

$$3 \text{ mA} = i_{E1} + i_{B3} = \left(\frac{1 + \beta}{\beta} \right) i_C + \frac{i_C}{\beta}$$

$$3 = \left(\frac{61}{60} + \frac{1}{60} \right) i_C \Rightarrow i_C = 2.90 \text{ mA}$$

b. For $v_0 = 6 \text{ V}$, let $R_L = 200 \Omega$.

$$i_0 = \frac{6}{200} = 0.03 \text{ A} = 30 \text{ mA} \approx i_{E3}$$

$$i_{B3} = \frac{30}{61} = 0.492 \text{ mA}$$

$$i_{E1} = 3 - 0.492 = 2.508 \text{ mA}$$

$$i_{B1} = \frac{2.508}{61} \Rightarrow i_{B1} = 41.11 \mu\text{A}$$

$$i_{E2} \approx 3 \text{ mA} \Rightarrow i_{B2} = \frac{3}{61} \Rightarrow 49.18 \mu\text{A}$$

$$i_I = i_{B2} - i_{B1} = 49.18 - 41.11 \Rightarrow i_I = 8.07 \mu\text{A}$$

Current gain

$$A_i = \frac{30 \times 10^{-3}}{8.07 \times 10^{-6}} \Rightarrow A_i = 3.72 \times 10^3$$

$$V_{BE3} = V_T \ln \left(\frac{i_{E3}}{I_S} \right) = (0.026) \ln \left(\frac{30 \times 10^{-3}}{5 \times 10^{-13}} \right)$$

$$V_{BE3} = 0.6453 \text{ V}$$

$$V_{EB1} = V_T \ln \left(\frac{i_{E1}}{I_S} \right) = (0.026) \ln \left(\frac{2.508 \times 10^{-3}}{5 \times 10^{-13}} \right)$$

$$V_{EB1} = 0.5807 \text{ V}$$

$$v_I = v_0 + V_{BE3} - V_{EB1} = 6 + 0.6453 - 0.5807$$

$$v_I = 6.0646 \text{ V}$$

Voltage gain

$$A_v = \frac{v_0}{v_I} = \frac{6}{6.0646} \Rightarrow A_v = 0.989$$

8.32

a. For $i_0 = 1 \text{ A}$, $I_{B3} \approx \frac{1}{50} \Rightarrow 20 \text{ mA}$

We can then write

$$\frac{10 - V_{EB1}}{R_1} = 2 \left[\frac{10 - (\nu_{0,\max} + V_{BE3})}{R_1} - 20 \right]$$

If, for simplicity, we assume $V_{EB1} = V_{BE3} = 0.7 \text{ V}$, then

$$\frac{10 - V_{BE}}{R_1} = \frac{2\nu_{0,\max}}{R_1} + 40$$

If we assume $\nu_{0,\max} = 4 \text{ V}$, then

$$\frac{9.3}{R_1} = \frac{2(4)}{R_1} + 40$$

which yields $R_1 = R_2 = 32.5 \Omega$

b. For $\nu_I = 0$,

$$I_{E1} = \frac{9.3}{32.5} \Rightarrow I_{E1} = 0.286 \text{ A} = I_{E2}$$

Since $I_{S3,4} = 10I_{S1,2}$, then

$$I_{E3} = I_{E4} = 2.86 \text{ A}$$

c. We can write

$$R_0 = \frac{1}{2} \left\{ \frac{r_{\pi 3} + R_1 \parallel \frac{r_{\pi 1}}{1 + \beta_1}}{1 + \beta_3} \right\}$$

$$\text{Now } r_{\pi 3} = \frac{\beta_3 V_T}{I_{C3}} = \frac{(50)(0.026)}{2.86} = 0.4545 \Omega$$

$$r_{\pi 1} = \frac{\beta_1 V_T}{I_{C1}} = \frac{(120)(0.026)}{0.286} = 10.91 \Omega$$

So

$$R_0 = \frac{1}{2} \left\{ \frac{0.4545 + 32.5 \parallel \frac{10.91}{121}}{51} \right\}$$

$$32.5 \parallel \frac{10.91}{121} = 32.5 \parallel 0.0902 = 0.0900$$

Then

$$R_0 = \frac{1}{2} \left\{ \frac{0.4545 + 0.0900}{51} \right\} \text{ or } R_0 = 0.00534 \Omega$$

8.33

$$R_i = \frac{1}{2} \{ r_{\pi 1} + (1 + \beta)[R_1 \parallel (r_{\pi 3} + (1 + \beta)R_L)] \}$$

$i_{C1} \approx 7.2 \text{ mA}$ and $i_{C3} \approx 7.2 \text{ mA}$

$$\text{Then } r_{\pi} = \frac{(60)(0.026)}{7.2} = 0.217 \text{ k}\Omega$$

So

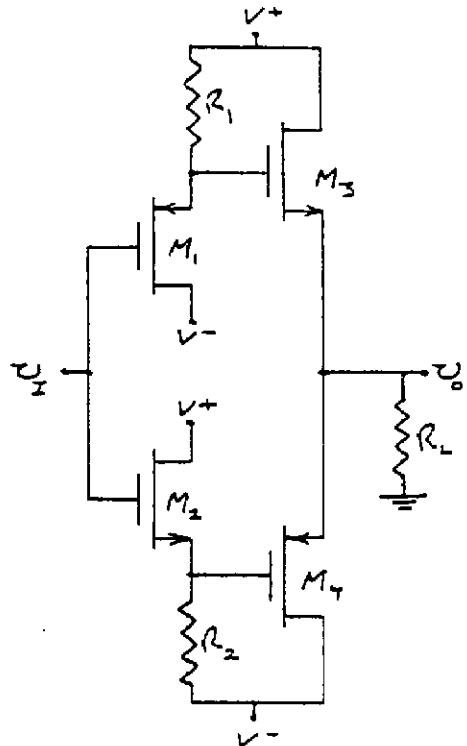
$$\begin{aligned} R_i &= \frac{1}{2} \{ 0.217 + (61)[2 \parallel (0.217 + (61)(0.1))] \} \\ &= \frac{1}{2} \{ 0.217 + 61[2 \parallel 6.32] \} \end{aligned}$$

or

$$R_i = 46.4 \text{ k}\Omega$$

8.34

a.



b. $I_1 = K_1(V_{SG} + V_{TP})^2 = \frac{V^+ - V_{SG}}{R_1}$

$$5 = 10(V_{SG} - 2)^2 \Rightarrow V_{SG} = 2.707 \text{ V}$$

$$5 = \frac{10 - 2.707}{R_1} \Rightarrow R_1 = R_2 = 1.46 \text{ k}\Omega$$

c. $R_L = 100 \Omega$ For a sinusoidal output signal:

$$\overline{P}_L = \frac{1}{2} \cdot \frac{(v_0)^2}{R_L} = \frac{1}{2} \cdot \frac{(5)^2}{0.1} \Rightarrow \overline{P}_L = 125 \text{ mW}$$

$$i_{D3} \approx \frac{(v_0)}{R_L} = \frac{(5)}{0.1} \Rightarrow i_{D3} = 50 \text{ mA}$$

$$V_{GS3} = \sqrt{\frac{50}{10}} + 2 = 4.236 \text{ V}$$

$$I_1 = \frac{10 - (4.236 + 5)}{1.46} \Rightarrow I_{D1} = 0.523 \text{ mA}$$

$$V_{SG1} = \sqrt{\frac{0.523}{10}} + 2 = 2.229 \text{ V}$$

$$v_I = 5 + 4.236 - 2.229 \Rightarrow v_I = 7.007 \text{ V}$$

$$I_{D2} = \frac{(v_I - V_{SG}) - (-10)}{1.46} = 10(V_{SG} - 2)^2$$

$$\frac{17.007 - V_{SG}}{1.46} = 10(V_{SG}^2 - 4V_{SG} + 4)$$

$$14.6V_{SG}^2 - 57.4V_{SG} + 41.4 = 0$$

$$V_{SG} = \frac{57.4 \pm \sqrt{(57.4)^2 - 4(14.6)(41.4)}}{2(14.6)}$$

$$V_{SG1} = 2.98 \text{ V}$$

$$I_{D2} = 10(2.98 - 2)^2 \Rightarrow I_{D2} = 9.60 \text{ mA}$$

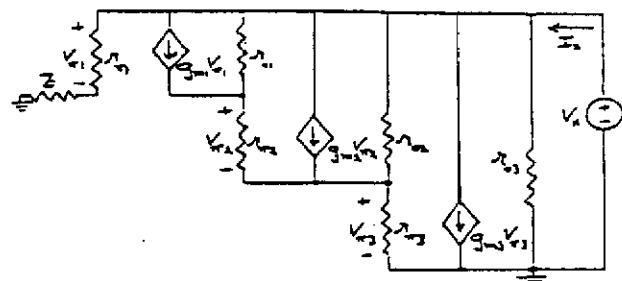
$$V_{G4} = v_I - V_{GS2} = 7 - 2.98 = 4.02 \text{ V}$$

$$V_{SG4} = 5 - 4.02 = 0.98 \text{ V} \Rightarrow I_{D4} = 0$$

$$I_{E1} = 1.692 \mu\text{A} \Rightarrow I_{C1} = 1.534 \mu\text{A}$$

$$I_{C2} = (50) \left(\frac{10}{11} \right) (1.692) \Rightarrow I_{C2} = 76.9 \mu\text{A}$$

$$I_{C3} = (51)(50) \left(\frac{10}{11} \right) (1.692) \Rightarrow I_{C3} = 3.92 \text{ mA}$$



Because of $r_{\pi 1}$ and Z , neglect effect of r_o . Then neglecting r_{o1} , r_{o2} , and r_{o3} , we find

$$I_X = g_{m3} V_{\pi 3} + g_{m2} V_{\pi 2} + g_{m1} V_{\pi 1} + \frac{V_X}{r_{\pi 1} + Z}$$

Now

$$V_{\pi 1} = \left(\frac{r_{\pi 1}}{r_{\pi 1} + Z} \right) V_X, \quad V_{\pi 2} = g_{m1} V_{\pi 1} r_{\pi 2}$$

and

$$V_{\pi 3} = (g_{m1} V_{\pi 1} + g_{m2} V_{\pi 2}) r_{\pi 3} \\ = [g_{m1} V_{\pi 1} + g_{m2} (g_{m1} V_{\pi 1} r_{\pi 2})] r_{\pi 3}$$

$$V_{\pi 3} = \left(\frac{r_{\pi 1}}{r_{\pi 1} + Z} \right) [g_{m1} + g_{m1} g_{m2} r_{\pi 2}] r_{\pi 3} \cdot V_X$$

$$V_{\pi 3} = \frac{(\beta_1 + \beta_1 \beta_2) r_{\pi 3}}{r_{\pi 1} + Z} \cdot V_X$$

and

$$V_{\pi 2} = g_{m1} \left(\frac{r_{\pi 1}}{r_{\pi 1} + Z} \right) r_{\pi 2} V_X = \left(\frac{\beta_1 r_{\pi 2}}{r_{\pi 1} + Z} \right) V_X$$

Then

$$I_X = \frac{(\beta_1 + \beta_1 \beta_2) \beta_3}{r_{\pi 1} + Z} \cdot V_X + \frac{\beta_1 \beta_2}{r_{\pi 1} + Z} \cdot V_X \\ + \frac{\beta_1}{r_{\pi 1} + Z} \cdot V_X + \frac{V_X}{r_{\pi 1} + Z}$$

Then

$$R_o = \frac{V_X}{I_X} = \frac{r_{\pi 1} + Z}{1 + \beta_1 + \beta_1 \beta_2 + (\beta_1 + \beta_1 \beta_2) \beta_3}$$

$$r_{\pi 1} = \frac{(10)(0.026)}{1.534} = 0.169 \text{ M}\Omega$$

$$Z = 25 \text{ k}\Omega$$

8.35

For $v_0 = 0$

$$I_Q = I_{C3} + I_{C2} + I_{E1}$$

$$I_{B3} = I_{E2} = \left(\frac{1 + \beta_n}{\beta_n} \right) I_{C2} = \frac{I_{C2}}{\beta_n}$$

$$I_{C3} = (1 + \beta_n) I_{C2}$$

$$I_{B2} = I_{C1} = \left(\frac{\beta_p}{1 + \beta_p} \right) I_{E1} = \frac{I_{C2}}{\beta_p}$$

$$I_{C2} = \beta_n \left(\frac{\beta_p}{1 + \beta_p} \right) I_{E1}$$

$$I_{C3} = (1 + \beta_n) \beta_n \left(\frac{\beta_p}{1 + \beta_p} \right) I_{E1}$$

$$I_Q = (1 + \beta_n) \beta_n \left(\frac{\beta_p}{1 + \beta_p} \right) I_{E1} \\ + \beta_n \left(\frac{\beta_p}{1 + \beta_p} \right) I_{E1} + I_{E1}$$

$$= (51)(50) \left(\frac{10}{11} \right) I_{E1} + (50) \left(\frac{10}{11} \right) I_{E1} + I_{E1}$$

$$I_Q = 2318.18 I_{E1} + 45.45 I_{E1} + I_{E1}$$

Then

$$R_o = \frac{169 + 25}{1 + (10) + (10)(50) + [10 + (10)(50)](50)} = 194 \text{ k}\Omega$$

$$R_o = \frac{194}{26,011} = 0.00746 \text{ k}\Omega$$

$$\text{or } R_o = 7.46 \Omega$$

8.36

a. Neglect base currents.

$$\begin{aligned} V_{BB} &= 2V_D = 2V_T \ln\left(\frac{I_{B_{max}}}{I_S}\right) \\ &= 2(0.026) \ln\left(\frac{5 \times 10^{-3}}{10^{-13}}\right) \\ &\Rightarrow \underline{V_{BB} = 1.281 \text{ V}} \end{aligned}$$

$$V_{BE1} + V_{EB3} = V_{BB}$$

$$I_{E1} = I_{E3} + I_{C2}$$

$$I_{E2} = I_{C3} = \left(\frac{\beta_p}{1 + \beta_p}\right) I_{E3}$$

$$I_{C2} = \beta_n I_{B2} = \beta_n \left(\frac{\beta_p}{1 + \beta_p}\right) I_{E3}$$

$$I_{E1} = I_{E3} + \beta_n \left(\frac{\beta_p}{1 + \beta_p}\right) I_{E3}$$

$$I_{E1} = I_{E3} \left[1 + \beta_n \left(\frac{\beta_p}{1 + \beta_p}\right)\right]$$

$$\left(\frac{1 + \beta_n}{\beta_n}\right) I_{C1} = \left(\frac{1 + \beta_p}{\beta_p}\right) I_{C3} \left[1 + \beta_n \left(\frac{\beta_p}{1 + \beta_p}\right)\right]$$

$$V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_S}\right), \quad V_{EB3} = V_T \ln\left(\frac{I_{C3}}{I_S}\right)$$

$$(1.01) I_{C1} = \left(\frac{21}{20}\right) I_{C3} \left[1 + (100) \left(\frac{20}{21}\right)\right]$$

$$= I_{C1} \left[\frac{21}{20} + 100\right] = 101.05 I_{C3}$$

$$I_{C1} = 100.05 I_{C3}$$

$$V_T \ln\left(\frac{100.05 I_{C3}}{I_S}\right) + V_T \ln\left(\frac{I_{C3}}{I_S}\right) = V_{BB}$$

$$V_T \ln\left(\frac{100.05 I_{C3}^2}{I_S^2}\right) = V_{BB}$$

$$\frac{100.05 I_{C3}^2}{I_S^2} = \exp\left(\frac{V_{BB}}{V_T}\right)$$

$$I_{C3} = \frac{I_S}{\sqrt{100.05}} \sqrt{\exp\left(\frac{V_{BB}}{V_T}\right)} = \underline{0.4997 \text{ mA} = I_{C3}}$$

$$\text{Then } I_{E3} = 0.5247 \text{ mA}$$

Now

$$I_{C1} = 100.05 I_{C3} = 50 \text{ mA} = I_{C1}$$

$$I_{C2} = (100) \left(\frac{20}{21}\right) (0.5247) = 49.97 \text{ mA} = I_{C2}$$

$$\text{b. } v_0 = 10 \text{ V} \Rightarrow i_{E1} \approx \frac{10}{100} = 0.10 \text{ A} = i_{C1}$$

$$i_{B1} = \frac{100}{100} = 1 \text{ mA}$$

$$V_{BB} = 2(0.026) \ln\left(\frac{4 \times 10^{-3}}{10^{-13}}\right) = 1.269 \text{ V}$$

$$V_{BE1} = (0.026) \ln\left(\frac{0.1}{10^{-13}}\right) = 0.7184$$

$$V_{EB3} = 1.269 - 0.7184 = 0.5506 \text{ V}$$

$$I_{C3} = 10^{-13} \exp\left(\frac{0.5506}{0.026}\right) = 0.157 \text{ mA}$$

$$\overline{P_L} = \frac{v_0^2}{R_L} = \frac{(10)^2}{100} \Rightarrow \overline{P_L} = 1 \text{ W}$$

$$P_{Q1} = i_{C1} \cdot v_{CE1} = (0.1)(12 - 10) \Rightarrow \underline{P_{Q1} = 0.2 \text{ W}}$$

$$P_{Q3} = i_{C3} \cdot v_{CE3} = (0.157)(10 - [0.7 - 12])$$

$$\Rightarrow \underline{P_{Q3} = 3.34 \text{ mW}}$$

$$i_{C2} = (100)(i_{C3}) = (100)(0.157) = 15.7 \text{ mA}$$

$$\begin{aligned} P_{Q2} &= i_{C2} \cdot v_{CE2} = (15.7)(10 - [-12]) \\ &\Rightarrow \underline{P_{Q2} = 0.345 \text{ W}} \end{aligned}$$

8.37

$$\text{a. } V_{BB} = 3(0.026) \ln\left(\frac{10 \times 10^{-3}}{2 \times 10^{-12}}\right) \Rightarrow \underline{V_{BB} = 1.74195 \text{ V}}$$

$$V_{BE1} + V_{BE2} + V_{EB3} = V_{BB}$$

$$I_{C1} \approx \frac{I_{C2}}{\beta_n}, \quad I_{C3} \approx \frac{I_{C2}}{\beta_n}$$

$$V_T \ln\left(\frac{I_{C1}}{I_S}\right) + V_T \ln\left(\frac{I_{C2}}{I_S}\right) + V_T \ln\left(\frac{I_{C3}}{I_S}\right) = V_{BB}$$

$$V_T \ln\left[\frac{I_{C2}^3}{\beta_n^3 I_S^3}\right] = V_{BB}$$

$$I_{C2} = \beta_n I_S \sqrt[3]{\exp\left(\frac{V_{BB}}{V_T}\right)}$$

$$= (20)(2 \times 10^{-12}) \sqrt[3]{\exp\left(\frac{1.74195}{0.026}\right)}$$

$$I_{C2} = 0.20 \text{ A}, \quad I_{C1} \approx 10 \text{ mA}, \quad I_{C3} \approx 0.5 \text{ mA}$$

$$V_{BE1} = (0.026) \ln\left(\frac{10 \times 10^{-3}}{2 \times 10^{-12}}\right)$$

$$\Rightarrow \underline{V_{BE1} = 0.58065 \text{ V}}$$

$$V_{BE2} = (0.026) \ln \left(\frac{0.2}{2 \times 10^{-12}} \right)$$

$$\Rightarrow V_{BE2} = 0.6585 \text{ V}$$

$$V_{EB3} = (0.026) \ln \left(\frac{0.5 \times 10^{-3}}{2 \times 10^{-12}} \right)$$

$$\Rightarrow V_{EB3} = 0.50276 \text{ V}$$

b. $\overline{P_L} = 10 \text{ W} = \frac{1}{2} \cdot \frac{V_0^2}{R_L} = \frac{1}{2} \cdot \frac{V_0^2}{20}$
 $\Rightarrow V_0(\max) = 20 \text{ V}$

For $V_0(\max)$:

$$P_L = \frac{V_0^2}{R_L} = \frac{(20)^2}{20} \Rightarrow P_L = 20 \text{ W}$$

$$i_0(\max) = -\frac{20}{20} = -1 \text{ A}$$

$$i_{C3} + i_{C4} + i_{E3} = -i_0(\max) = 1 \text{ A}$$

$$i_{C3} + \frac{i_{C3}}{\beta_n} \cdot \left(\frac{1 + \beta_p}{\beta_p} \right) + \frac{i_{C4}}{\beta_n} \cdot \left(\frac{1 + \beta_p}{\beta_p} \right) = 1$$

$$i_{C3} \left[1 + \frac{1}{\beta_n} \cdot \left(\frac{1 + \beta_p}{\beta_p} \right) + \left\{ \frac{1}{\beta_n} \cdot \left(\frac{1 + \beta_p}{\beta_p} \right) \right\}^2 \right] = 1$$

$$i_{C3} \left[1 + \frac{1}{20} \cdot \left(\frac{6}{5} \right) + \left(\frac{1}{20} \cdot \frac{6}{5} \right)^2 \right]$$

$$i_{C3}[1 + 0.06 + 0.0036] = 1 \Rightarrow i_{C3} = 0.940 \text{ A}$$

$$i_{C4} = 0.0564 \text{ A}$$

$$i_{E3} = 3.38 \text{ mA}$$

$$i_{C2} = 2.82 \text{ mA}$$

$$V_{EB3} = (0.026) \ln \left(\frac{2.82 \times 10^{-3}}{2 \times 10^{-12}} \right) = 0.5477 \text{ V}$$

$$V_{BE1} + V_{BE2} = 1.74195 - 0.5477 = 1.1942$$

$$V_T \ln \left(\frac{I_{C2}}{\beta_n I_S} \right) + V_T \ln \left(\frac{I_{C2}}{I_S} \right) = 1.1942$$

$$I_{C2} = \sqrt{\beta_n} \cdot I_S \sqrt{\exp \left(\frac{1.1942}{0.026} \right)}$$

$$= \sqrt{20}(18.83) \text{ mA}$$

$$I_{C2} = 84.2 \text{ mA}$$

Chapter 9

Exercise Solutions

E9.1

$$A_{CL} = -\frac{R_2}{R_1} = \frac{-100 \text{ k}\Omega}{10 \text{ k}\Omega} \Rightarrow A_{CL} = -10$$

$v_I = 0.25 \text{ V} \Rightarrow v_0 = -2.5 \text{ V}$

 $i_1 = \frac{v_I}{R_1} = \frac{0.25}{10 \text{ k}\Omega} = 0.025 \text{ mA} \Rightarrow i_1 = 25 \mu\text{A}$
 $i_2 = i_1 = 25 \mu\text{A}$
 $R_s = R_1 = 10 \text{ k}\Omega$

E9.2

$$A_{CL} = -\frac{R_2}{R_1} = -15$$
 $R_1 = R_2 = 20 \text{ k}\Omega \Rightarrow R_2 = (15)(20 \text{ k}\Omega)$
 $R_2 = 300 \text{ k}\Omega$

E9.3

(a) $A_v = \frac{-R_2}{R_1 + R_s}$

 $A_v(\min) = \frac{-100}{19 + 1.3} = -4.926$
 $A_v(\max) = \frac{-100}{19 + 0.7} = -5.076$

so $4.926 \leq |A_v| \leq 5.076$

(b) $i_1(\max) = \frac{0.1}{19 + 0.7} = 5.076 \mu\text{A}$

 $i_1(\min) = \frac{0.1}{19 + 1.3} = 4.926 \mu\text{A}$

so $4.926 \leq i_1 \leq 5.076 \mu\text{A}$

(c) Maximum current specification is violated.

E9.4

We can write

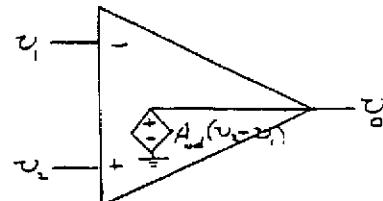
$$A_{CL} = -\frac{R_2}{R_1} \left(1 + \frac{R_3}{R_4}\right) - \frac{R_3}{R_1}$$
 $R_1 = R_2 = 10 \text{ k}\Omega$

Want $A_{CL} = -50$ Set $R_2 = R_3 = 50 \text{ k}\Omega$

 $A_{CL} = -50 = -5 \left(1 + \frac{R_3}{R_4}\right) - 5$
 $1 + \frac{R_3}{R_4} = 9 \Rightarrow \frac{R_3}{R_4} = 8 = \frac{50}{R_4}$
 $R_4 = 6.25 \text{ k}\Omega$

E9.5

$v_0 = A_d(v_2 - v_1) \quad A_d = 10^3$



a. $v_2 = 0, v_0 = 5$

$v_1 = -\frac{v_0}{A_d} = -\frac{5}{10^3} \Rightarrow v_1 = -5 \text{ mV}$

b. $v_1 = 5, v_0 = -10$

$\frac{v_0}{A_d} = v_2 - v_1$
 $\frac{-10}{10^3} = v_2 - 5 \Rightarrow v_2 = 4.99 \text{ V}$

c. $v_1 = 0.001, v_2 = -0.001$

$v_0 = 10^3(-0.001 - 0.001)$
 $v_0 = -2 \text{ V}$

d. $v_2 = 3, v_0 = 3$

$v_0 = A_d(v_2 - v_1)$

$\frac{v_0}{A_d} = v_2 - v_1$
 $\frac{3}{10^3} = 3 - v_1 \Rightarrow v_1 = 2.997 \text{ V}$

E9.6

We have

$$A_{CL} = -\frac{R_2}{R_1} \cdot \frac{1}{\left[1 + \frac{1}{A_d} \left(1 + \frac{R_2}{R_1}\right)\right]}$$
 $R_1 = R_2 = 25 \text{ k}\Omega \quad \text{Let } \frac{R_2}{R_1} = z$
 $-12 = -\frac{z}{1 + \frac{1}{5 \times 10^3}(1+z)}$
 $= \frac{-z}{1.0002 + \frac{z}{5 \times 10^3}}$

$12 \left(1.0002 + \frac{z}{5 \times 10^3}\right) = z$

$12.0024 = z - (2.4 \times 10^{-3})z$

$z = \frac{12.0024}{0.9976} = 12.0313 = \frac{R_2}{25 \text{ k}\Omega}$

$R_2 = 300.78 \text{ k}\Omega$

E9.7

$$v_0 = - \left(\frac{R_1}{R_1} v_{I1} + \frac{R_2}{R_2} v_{I2} + \frac{R_3}{R_3} v_{I3} \right)$$

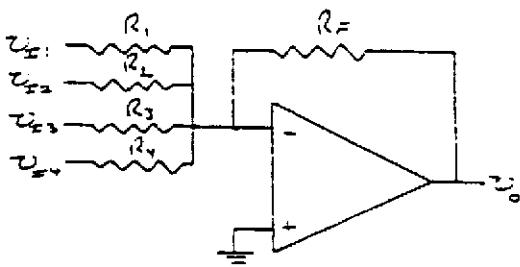
$$v_0 = - \left[\left(\frac{40}{10} \right) (250) + \left(\frac{40}{20} \right) (200) + \left(\frac{40}{30} \right) (75) \right]$$

$$v_0 = - [1000 + 400 + 100]$$

$$v_0 = -1500 \mu V = -1.5 \text{ mV}$$

E9.8

$$v_0 = - \left(\frac{R_F}{R_1} v_{I1} + \frac{R_F}{R_2} v_{I2} + \frac{R_F}{R_3} v_{I3} + \frac{R_F}{R_4} v_{I4} \right)$$



We need

$$\frac{R_F}{R_1} = 7, \quad \frac{R_F}{R_2} = 14, \quad \frac{R_F}{R_3} = 3.5, \quad \frac{R_F}{R_4} = 10$$

 Set $R_F = 280 \text{ k}\Omega$

$$\text{Then } R_1 = \frac{280}{7} = 40 \text{ k}\Omega$$

$$R_2 = \frac{280}{14} = 20 \text{ k}\Omega$$

$$R_3 = \frac{280}{3.5} = 80 \text{ k}\Omega$$

$$R_4 = \frac{280}{10} = 28 \text{ k}\Omega$$

E9.9

$$|v_o| = \frac{v_{I1} + v_{I2} + v_{I3}}{3} = \frac{R_F}{R} (v_{I1} + v_{I2} + v_{I3})$$

$$\frac{R_F}{R} = \frac{1}{3} \Rightarrow R_1 = R_2 = R_3 = R = 1 M\Omega$$

$$\text{Then } R_F = \frac{1}{3} M\Omega = 333 k\Omega$$

E9.10

$$A_v = \frac{v_0}{v_I} = \left(1 + \frac{R_2}{R_1} \right) = 5$$

$$\text{so that } \frac{R_2}{R_1} = 4$$

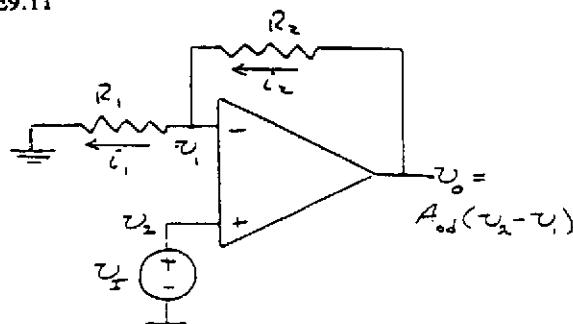
$$\text{For } v_0 = 10 \text{ V, } v_I = 2 \text{ V}$$

$$\text{Then } i_1 = \frac{2}{R_1} = 50 \mu A \Rightarrow R_1 = 40 \text{ k}\Omega$$

 Then $R_2 = 160 \text{ k}\Omega$ we find

$$i_2 = \frac{v_0 - v_I}{R_2} = \frac{10 - 2}{160} = 50 \mu A$$

E9.11



$$v_0 = A_{od}(v_2 - v_1) = A_{od}(v_I - v_1)$$

$$\frac{v_0}{A_{od}} - v_I = -v_1 \text{ or } v_1 = v_I - \frac{v_0}{A_{od}}$$

$$i_1 = \frac{v_1}{R_1} = i_2 \text{ and } i_2 = \frac{v_0 - v_1}{R_2}$$

$$\text{Then } v_1 \left(\frac{1}{R_1} \right) = \frac{v_0 - v_1}{R_2}$$

$$v_1 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{v_0}{R_2}$$

$$v_0 \left(1 + \frac{R_2}{R_1} \right) v_1 = \left(1 + \frac{R_2}{R_1} \right) \left(v_I - \frac{v_0}{A_{od}} \right)$$

$$\text{So } A_v = \frac{v_0}{v_I} = \frac{\left(1 + \frac{R_2}{R_1} \right)}{1 + \frac{1}{A_{od}} \left(1 + \frac{R_2}{R_1} \right)}$$

E9.12

$$\text{For } v_{I2} = 0, \quad v_2 = \left(\frac{R_b}{R_b + R_a} \right) v_{I1} \text{ and}$$

$$v_0(v_{I1}) = \left(1 + \frac{R_2}{R_1} \right) \left(\frac{R_a}{R_b + R_a} \right) v_{I1}$$

$$= \left(1 + \frac{70}{5} \right) \left(\frac{50}{50 + 25} \right) v_{I1}$$

$$= 10v_{I1}$$

 For $v_{I1} = 0,$

$$v_2 = \left(\frac{R_a}{R_b + R_a} \right) v_{I2}$$

$$v_0(v_{I2}) = \left(1 + \frac{R_2}{R_1} \right) \left(\frac{R_a}{R_b + R_a} \right) v_{I2}$$

$$= \left(1 + \frac{70}{5} \right) \left(\frac{25}{25 + 50} \right) v_{I2}$$

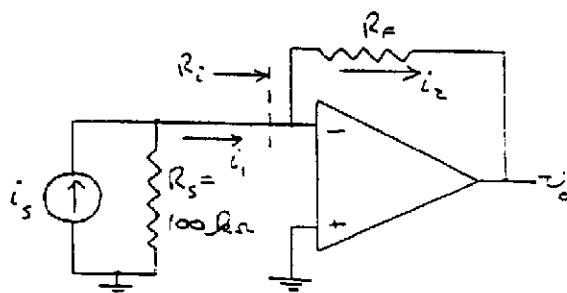
$$= 5v_{I2}$$

Then

$$v_0 = v_0(v_{I1}) + v_0(v_{I2})$$

$$v_0 = 10v_{I1} + 5v_{I2}$$

E9.13



$$R_s \gg R_i \text{ so } i_1 = i_2 = i_s = 100 \mu\text{A}$$

$$v_o = -i_2 R_F$$

$$\text{We want } -10 = -(100 \times 10^{-6}) R_F$$

$$\Rightarrow R_F = 100 \text{ k}\Omega$$

E9.14

We may note that:

$$\frac{R_3}{R_2} = \frac{3}{1.5} = 2 \text{ and } \frac{R_F}{R_1} = \frac{20}{10} = 2$$

so that:

$$\frac{R_3}{R_2} = \frac{R_F}{R_1}$$

Then:

$$i_L = \frac{-v_I}{R_2} = \frac{-(-3)}{1.5 \text{ k}\Omega}$$

$$\Rightarrow i_L = 2 \text{ mA}$$

$$v_L = i_L Z_L = (2 \times 10^{-3})(200) = 0.4 \text{ V}$$

$$i_4 = \frac{v_L}{R_2} = \frac{0.4}{1.5 \text{ k}\Omega} = 0.267 \text{ mA}$$

$$i_3 = i_4 + i_L = 0.267 + 2 = 2.267 \text{ mA}$$

$$v_o = i_3 R_3 + v_L = (2.267 \times 10^{-3})(3 \times 10^3) - 0.4$$

$$\Rightarrow v_o = 7.2 \text{ V}$$

E9.15

We want $i_L = 1 \text{ mA}$ when $v_I = -5 \text{ V}$

$$i_L = \frac{-v_I}{R_2} \Rightarrow R_2 = \frac{-v_I}{i_L} = \frac{-(-5)}{10^{-3}}$$

$$\Rightarrow R_2 = 3 \text{ k}\Omega$$

$$v_L = i_L Z_L = (10^{-3})(500)$$

$$\Rightarrow v_L = 0.5 \text{ V}$$

$$i_4 = \frac{v_L}{R_2} = \frac{0.5}{3 \text{ k}\Omega} \Rightarrow i_4 = 0.1 \text{ mA}$$

$$i_3 = i_4 + i_L = 0.1 + 1 = 1.1 \text{ mA}$$

If op-amp is biased at $\pm 10 \text{ V}$, output must be limited to $\approx 8 \text{ V}$.

$$\text{So } v_o = i_3 R_3 + v_L$$

$$8 = (1.1 \times 10^{-3}) R_3 + 0.5$$

$$\Rightarrow R_3 = 6.82 \text{ k}\Omega$$

$$\text{Let } R_3 = 7.0 \text{ k}\Omega$$

Then we want:

$$\frac{R_3}{R_2} = \frac{R_F}{R_1} = \frac{7}{5} = 1.4$$

Can choose $R_1 = 10 \text{ k}\Omega$ and $R_F = 14 \text{ k}\Omega$

E9.16

Refer to Fig. 9.24

$$R_1 = 2 R_3 = 5 \text{ k}\Omega$$

$$\text{Let } R_1 = R_3 = 2.5 \text{ k}\Omega$$

$$\text{Set } R_2 = R_4$$

$$\text{Differential Gain} = \frac{v_o}{v_i} = \frac{R_2}{R_1} = 100 = \frac{R_2}{2.5 \text{ k}\Omega}$$

$$\Rightarrow R_2 = R_4 = 250 \text{ k}\Omega$$

E9.17

We have the general relation that:

$$v_o = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{[R_4/R_3]}{1 + [R_4/R_3]} \right) v_{I2} - \frac{R_2}{R_1} v_{I1}$$

$$R_1 = R_3 = 10 \text{ k}\Omega, R_2 = 20 \text{ k}\Omega, R_4 = 21 \text{ k}\Omega$$

$$v_o = \left(1 + \frac{20}{10}\right) \left(\frac{[21/10]}{1 + [21/10]} \right) v_{I2} - \left(\frac{20}{10}\right) v_{I1}$$

$$v_o = 2.0323 v_{I2} - 2.0 v_{I1}$$

$$\text{a. } v_{I1} = 1, v_{I2} = -1$$

$$v_o = -2.0323 - 2.0 \Rightarrow v_o = -4.032 \text{ V}$$

$$\text{b. } v_{I1} = v_{I2} = 1 \text{ V}$$

$$v_o = 2.0323 - 2.0 \Rightarrow v_o = 0.0323 \text{ V}$$

$$\text{c. } v_{cm} = v_{I1} = v_{I2} \text{ so common-mode gain}$$

$$A_{cm} = \frac{v_o}{v_{cm}} = 0.0323$$

$$\text{d. } CMRR_{dB} = 20 \log_{10} \left(\frac{A_d}{A_{cm}} \right)$$

$$A_d = \frac{2.0323}{2} - (2.0) \left(-\frac{1}{2} \right) = 2.016$$

$$CMRR_{dB} = 20 \log_{10} \left(\frac{2.016}{0.0323} \right) = 35.9 \text{ dB}$$

E9.18

$$\nu_0 = -\frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1}\right) (\nu_{I1} - \nu_{I2})$$

$$\text{Differential gain (magnitude)} = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1}\right)$$

Minimum Gain \Rightarrow Maximum $R_1 = 1 + 50 = 51 \text{ k}\Omega$

$$\text{So } A_d = \frac{20}{20} \left(1 + \frac{2(100)}{51}\right) \Rightarrow A_d = 4.92$$

Maximum Gain \Rightarrow Minimum $R_1 = 1 \text{ k}\Omega$

$$A_d = \frac{20}{20} \left(1 + \frac{2(100)}{1}\right) \Rightarrow A_d = 201$$

Range of Differential Gain = 4.92–201

E9.19

$$\text{a. } i_1 = \frac{\nu_{I1} - \nu_{I2}}{R_1}$$

$$\nu_{01} = \nu_{I1} + i_1 R'_2, \quad \nu_{02} = \nu_{I2} - i_1 R_2 \text{ and}$$

$$\nu_0 = \frac{R_4}{R_3} (\nu_{02} - \nu_{01})$$

$$\nu_0 = \frac{R_4}{R_3} [\nu_{I2} - i_1 R_2 - \nu_{I1} - i_1 R'_2]$$

$$\nu_0 = \frac{R_4}{R_3} [(\nu_{I2} - \nu_{I1}) - i_1 (R_2 + R'_2)]$$

$$\nu_0 = \frac{R_4}{R_3} \left[(\nu_{I2} - \nu_{I1}) - \left(\frac{\nu_{I2} - \nu_{I1}}{R_1} \right) (R_2 + R'_2) \right]$$

For common-mode input $\nu_{I2} = \nu_{I1}$ $\Rightarrow \nu_0 = 0 \Rightarrow \text{Common Gain} = 0, \text{ CMRR} = \infty$ b. $A_d(\min) \Rightarrow R'_2 \text{ min, } R_1 \text{ max}$

$$A_d = \left(\frac{20}{20}\right) \left[1 + \frac{100 + 95}{51}\right] = 4.82$$

$$A_d(\max) = \left(\frac{20}{20}\right) \left[1 + \frac{100 + 105}{1}\right] = 206$$

$$\text{c. } \text{CMRR} = \left| \frac{A_d}{A_{cm}} \right|$$

 $A_{cm} = 0 \Rightarrow \text{CMRR} = \infty$

E9.20

$$\text{Differential Gain} = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1}\right)$$

Let $R_3 = R_4$ so the difference amplifier gain is unity.Minimum Gain \Rightarrow Maximum R_1

$$\text{So } \left(1 + \frac{2R_2}{R_1(\max)}\right) = 2$$

We want $2R_2 = R_1(\max)$ Maximum Gain \Rightarrow Minimum R_1

$$\text{So } \left(1 + \frac{2R_2}{R_1(\min)}\right) = 1000 \text{ or } 2R_2 = 999R_1(\min)$$

If $R_2 = 50 \text{ k}\Omega$, let $R_1(\min) = 100 \Omega$ fixed resistor and let $R_1(\max) = \underbrace{100 \text{ k}\Omega}_{\text{pot}} + 100 \Omega = 100.1 \Omega$ Then actual differential gain is in the range of $1.999 - 1001$

E9.21

$$\text{Time constant} = \tau = R_1 C_2 = (10^4)(0.1 \times 10^{-6})$$

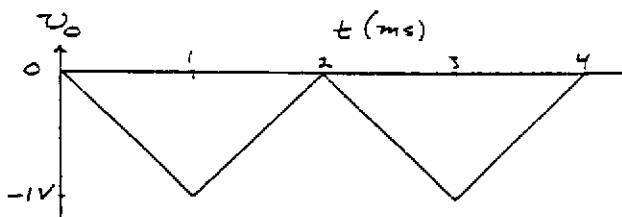
$$= 1 \text{ m sec}$$

$$0 \leq t \leq 1 \Rightarrow \nu_0 = \frac{-1}{R_1 C_2} \times t$$

$$\text{At } t = 1 \text{ m sec} \Rightarrow \nu_0 = -1 \text{ V}$$

$$0 \leq t \leq 2 \Rightarrow \nu_0 = -1 + \frac{1}{R_1 C_2} \times (t - 1)$$

$$\text{At } t = 2 \text{ m sec} \Rightarrow \nu_0 = -1 + \frac{(2 - 1)}{1} = 0$$



E9.22

End of 1st pulse:

$$\nu_0 = \frac{-1}{\tau} \times t \Big|_0^{10 \mu\text{sec}} = \frac{-10 \times 10^{-6}}{\tau}$$

After 10 pulses:

$$\nu_0 = -5 = \frac{-(10)(10 \times 10^{-6})}{\tau}$$

$$\text{So } \tau = \frac{100 \times 10^{-6}}{5} = 20 \mu\text{sec} = \tau$$

$$\tau = R_1 C_2 = 20 \mu\text{sec} = 20 \times 10^{-6}$$

For example,

$$C_2 = 0.01 \times 10^{-6} = 0.01 \mu\text{F} \Rightarrow R_1 = 2 \text{ k}\Omega$$

E9.23

$$\nu_0 = \nu_{I1} + 10\nu_{I2} - 25\nu_{I3} - 80\nu_{I4}$$

From Figure 9.37, ν_{I3} input to R_1 , ν_{I4} input to R_2 , ν_{I1} input to R_A , and ν_{I2} input to R_B .

From Equation (9.87)

$$\frac{R_F}{R_1} = 25 \text{ and } \frac{R_F}{R_2} = 80$$

Set $R_F = 500 \text{ k}\Omega$, then $R_1 = 20 \text{ k}\Omega$, and $R_2 = 6.25 \text{ k}\Omega$.

$$\text{Also } \left(1 + \frac{R_F}{R_N}\right) \left(\frac{R_P}{R_A}\right) = 1$$

$$\text{and } \left(1 + \frac{R_F}{R_N}\right) \left(\frac{R_P}{R_B}\right) = 10$$

$$\text{where } R_N = R_1 \parallel R_2 = 20 \parallel 6.25 = 4.76 \text{ k}\Omega$$

$$\text{and } R_P = R_A \parallel R_B \parallel R_C$$

$$\text{We find that } \frac{R_A}{R_B} = 10$$

$$\text{Let } R_A = 200 \text{ k}\Omega, R_B = 20 \text{ k}\Omega$$

$$\text{Now } \left(1 + \frac{500}{4.76}\right) \left(\frac{R_P}{R_A}\right) = 1 = (106) \left(\frac{R_P}{200}\right)$$

$$\text{Then } R_P = 1.89 \text{ k}\Omega$$

$$R_A \parallel R_B = 200 \parallel 20 = 18.2 \text{ k}\Omega$$

$$\text{So } R_P = 1.89 = \frac{18.2 R_C}{18.2 + R_C} \Rightarrow R_C = 2.11 \text{ k}\Omega$$

E9.25

$$\begin{aligned} \nu_{01} &= \left[\frac{R - \Delta R}{(R - \Delta R) + (R + \Delta R)} - \frac{R + \Delta R}{(R + \Delta R) + (R - \Delta R)} \right] V^+ \\ &= \left[\frac{R - \Delta R}{2R} - \frac{R + \Delta R}{2R} \right] V^+ \\ &= \left(\frac{R - \Delta R - R - \Delta R}{2R} \right) V^+ \end{aligned}$$

$$\nu_{01} = -\left(\frac{\Delta R}{R}\right) V^+$$

$$\text{For } V^+ = 3.5 \text{ V}, \Delta R = 50, R = 10 \times 10^3$$

$$\nu_{01} = -\left(\frac{50}{10^4}\right)(3.5) = -1.75 \times 10^{-2}$$

Need an amplifier with a gain of

$$A_d = \frac{\nu_0}{\nu_i} = \frac{5}{-1.75 \times 10^{-2}} \Rightarrow A_d = -285.7$$

Use instrumentation amplifier, Fig. 9.25.

Connect ν_{01} to ν_{I1} and $(-\nu_{01})$ to ν_{I2} .

$$|A_d| = \left(\frac{R_4}{R_3}\right) \left(1 + \frac{2R_2}{R_1}\right) = 285.7$$

$$\text{Let } R_4 = 150 \text{ k}\Omega, R_3 = 10 \text{ k}\Omega \text{ Then } \frac{R_2}{R_1} = 9.02$$

$$\text{Let } R_2 = 100 \text{ k}\Omega, R_1 = 11.1 \text{ k}\Omega$$

E9.26

$$\begin{aligned} \nu_{01} &= \left[\frac{1}{2} - \frac{R}{R + R(1 + \delta)} \right] V^+ \\ &= \left[\frac{R + R(1 + \delta) - 2R}{2(R + R(1 + \delta))} \right] V^+ \\ &= \frac{R\delta}{2R(2 + \delta)} \times V^+ \end{aligned}$$

$$\nu_{01} \approx \left(\frac{\delta}{4}\right) V^+$$

$$V^+ = 5 \text{ For } \delta = 0.01$$

$$\nu_{01} = \left(\frac{0.01}{4}\right)(5) = 0.0125$$

Need a gain

$$A_d = \frac{\nu_0}{\nu_{01}} = \frac{5}{0.0125} = 400$$

Use an instrumentation amplifier

$$A_d = 400 = \left(\frac{R_4}{R_3}\right) \left(1 + \frac{2R_2}{R_1}\right)$$

$$\text{Let } R_4 = 150 \text{ k}\Omega, R_3 = 10 \text{ k}\Omega, \text{ then } \frac{R_2}{R_1} = 12.8$$

$$\text{Let } R_2 = 150 \text{ k}\Omega, R_1 = 11.7 \text{ k}\Omega$$

Chapter 9

Problem Solutions

9.1

$$\left. \begin{array}{l} A_v = -\frac{200}{20} = -10 \\ \text{and} \\ R_i = 20 \text{ k}\Omega \end{array} \right\} \text{for each case}$$

9.2

a. $A_v = -\frac{100}{10} = -10$

$R_i = R_1 = 10 \text{ k}\Omega$

b. $A_v = -\frac{100||100}{10} = -5$

$R_i = R_1 = 10 \text{ k}\Omega$

c. $A_v = -\frac{100}{10||10} = -20$

$R_i = 10||10 = 5 \text{ k}\Omega$

9.3

$A_v = -\frac{R_2}{R_1} = -12 \Rightarrow R_2 = 12R_1$

$R_i = R_1 = 25 \text{ k}\Omega$

$\Rightarrow R_2 = (12)(25) = 300 \text{ k}\Omega$

9.4

$A_v = -\frac{R_2}{R_1} = -8 \Rightarrow R_2 = 8R_1$

For $v_i = -1$, $i_1 = \frac{1}{R_1} = 15 \mu\text{A} \Rightarrow R_1 = 66.7 \text{ k}\Omega$

$\Rightarrow R_2 = 533.3 \text{ k}\Omega$

9.5

$A_v = -\frac{R_2}{R_1} = -30 \Rightarrow R_2 = 30R_1$

Set $R_2 = 1 \text{ M}\Omega$

$\Rightarrow R_1 = 33.3 \text{ k}\Omega$

9.6

a. $A_v = \frac{R_2}{R_1} \Rightarrow \frac{1.05R_2}{0.95R_1} = 1.105 \left(\frac{R_2}{R_1} \right)$

$\frac{0.95R_2}{1.05R_1} = 0.905 \left(\frac{R_2}{R_1} \right)$

Deviation in gain is +10.5% and -9.5%

b. $A_v \Rightarrow \frac{1.01R_2}{0.99R_1} = 1.02 \left(\frac{R_2}{R_1} \right)$

$\Rightarrow \frac{0.99R_2}{1.01R_1} = 0.98 \left(\frac{R_2}{R_1} \right)$

Deviation in gain = ±2%

9.7

(a) $A_v = \frac{v_o}{v_i} = \frac{-15}{1} = -15$

$v_o = -15v_i \Rightarrow v_o = -150 \sin \omega t \text{ (mV)}$

(b) $i_2 = i_1 = \frac{v_i}{R_1} = 10 \sin \omega t \text{ (\mu A)}$

$i_L = \frac{v_o}{R_L} \Rightarrow i_L = -37.5 \sin \omega t \text{ (\mu A)}$

$i_o = i_L - i_2$

$i_o = -47.5 \sin \omega t \text{ (\mu A)}$

9.8

$A_v = -\frac{R_2}{R_1 + R_s}$

$A_v = -30 \pm 2.5\% \Rightarrow 29.25 \leq |A_v| \leq 30.75$

So $\frac{R_2}{R_1 + 2} = 29.25$ and $\frac{R_2}{R_1 + 1} = 30.75$

We have $29.25(R_1 + 2) = 30.75(R_1 + 1)$

Which yields $R_1 = 18.5 \text{ k}\Omega$ and $R_2 = 599.6 \text{ k}\Omega$

For $v_i = 25 \text{ mV}$, then

$0.731 \leq |v_o| \leq 0.769 \text{ V}$

9.9

$v_{o1} = -\left(\frac{50}{10}\right)v_i = (-5)(0.15) \Rightarrow v_{o1} = -0.75 \text{ V}$

$v_0 = -\frac{150}{25} \cdot v_{o1} = (-6)(-0.75) \Rightarrow v_0 = 4.5 \text{ V}$

$i_1 = i_2 = \frac{0.15}{10} \Rightarrow i_1 = i_2 = 15 \mu\text{A}$

$i_3 = i_4 = \frac{v_{o1}}{R_3} = -\frac{0.75}{25} \Rightarrow i_3 = i_4 = -30 \mu\text{A}$

First op-amp must sink $15 + 30 = 45 \mu\text{A}$

9.10

$$(a) A_v = -\frac{R_2}{R_1} = -\frac{22}{1} \Rightarrow A_v = -22$$

$$(b) A_v = -\frac{R_2}{R_1} \cdot \frac{1}{1 + \frac{1}{A_{ad}} \left(1 + \frac{R_2}{R_1} \right)}$$

$$= -\frac{22}{1} \cdot \frac{1}{1 + \frac{1}{1.5 \times 10^5} \left(1 + \frac{22}{1} \right)} = -\frac{22}{1} \cdot \frac{1}{1.000153} \Rightarrow$$

$$A_v = -21.99663$$

$$(c) |A_v| = 22 - 1\% = 22 - 0.22 = 21.78$$

$$\text{Then } 21.78 = \frac{22}{1} \cdot \frac{1}{1 + \frac{1}{A_{ad}} (23)} \Rightarrow A_{ad} = 2277$$

9.11

$$A_v = -\frac{R_2}{R_1} \left(1 + \frac{R_3}{R_4} + \frac{R_3}{R_2} \right)$$

$$a. -10 = -\frac{R_2}{100} \left(1 + \frac{100}{100} + \frac{100}{R_2} \right)$$

$$10 = \frac{2R_2}{100} + 1 \Rightarrow R_2 = 450 \text{ k}\Omega$$

$$b. 100 = \frac{2R_2}{100} + 1 \Rightarrow R_2 = 4.95 \text{ M}\Omega$$

9.12

$$a. A_v = -\frac{R_2}{R_1} \left(1 + \frac{R_3}{R_4} + \frac{R_3}{R_2} \right)$$

$$R_1 = 500 \text{ k}\Omega$$

$$80 = \frac{R_2}{500} \left(1 + \frac{R_3}{R_4} + \frac{R_3}{R_2} \right)$$

$$\text{Set } R_2 = R_3 = 500 \text{ k}\Omega$$

$$80 = 1 \left(1 + \frac{500}{R_4} + 1 \right) = 2 + \frac{500}{R_4}$$

$$\Rightarrow R_4 = 6.41 \text{ k}\Omega$$

$$b. \text{ For } v_I = -0.05 \text{ V}$$

$$i_1 = i_2 = \frac{-0.05}{500 \text{ k}\Omega} \Rightarrow i_1 = i_2 = -0.1 \mu\text{A}$$

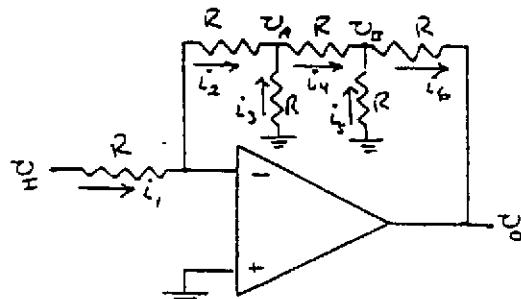
$$v_X = -i_2 R_2 = -(-0.1 \times 10^{-6}) (500 \times 10^3)$$

$$= 0.05$$

$$i_4 = -\frac{v_X}{R_4} = -\frac{0.05}{6.41} \Rightarrow i_4 = -7.80 \mu\text{A}$$

$$i_3 = i_2 + i_4 = -0.1 - 7.80 \Rightarrow i_3 = -7.90 \mu\text{A}$$

9.13



$$i_1 = \frac{v_I}{R} = i_2$$

$$v_A = -i_2 R = -\left(\frac{v_I}{R}\right) R = -v_I$$

$$i_3 = -\frac{v_A}{R} = \frac{v_I}{R}$$

$$i_4 = i_2 + i_3 = -\frac{v_A}{R} - \frac{v_A}{R} = -\frac{2v_A}{R} = \frac{2v_I}{R}$$

$$v_B = v_A - i_4 R = -v_I - \left(\frac{2v_I}{R}\right)(R) = -3v_I$$

$$i_5 = -\frac{v_B}{R} = -\frac{(-3v_I)}{R} = \frac{3v_I}{R}$$

$$i_6 = i_4 + i_5 = \frac{2v_I}{R} + \frac{3v_I}{R} = \frac{5v_I}{R}$$

$$v_0 = v_B - i_6 R = -3v_I - \left(\frac{5v_I}{R}\right)R$$

$$\Rightarrow \frac{v_0}{v_I} = -8$$

From Figure 9.11 $\Rightarrow A_v = -3$

9.14

$$(a) A_v = -\frac{R_2}{R_1} \cdot \frac{1}{1 + \frac{1}{A_{ad}} \left(1 + \frac{R_2}{R_1} \right)}$$

$$= -\frac{50}{10} \cdot \frac{1}{1 + \frac{1}{2 \times 10^5} \left(1 + \frac{50}{10} \right)} \Rightarrow A_v = -4.99985$$

$$(b) v_o = -(4.99985)(100 \times 10^{-3}) \Rightarrow$$

$$v_o = -499.985 \text{ mV}$$

$$(c) \text{ Error} = \frac{0.5 - 0.499985}{0.5} \times 100\% \Rightarrow 0.003\%$$

9.15

a. From Equation (9.23)

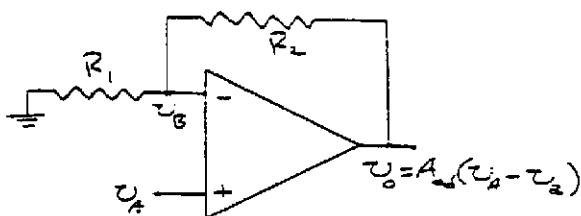
$$A_v = -\frac{R_2}{R_1} \cdot \frac{1}{1 + \frac{1}{A_{od}} \left(1 + \frac{R_2}{R_1} \right)}$$

$$= -\frac{100}{100} \cdot \frac{1}{1 + \frac{1}{10^3} \left(1 + \frac{100}{100} \right)} = -0.9980$$

$$\text{Then } v_o = A_v \cdot v_I = (-0.9980)(2)$$

$$\Rightarrow v_o = \underline{-1.9960}$$

b.



$$v_o = A_{od}(v_A - v_B)$$

$$\frac{v_B}{R_1} = \frac{v_o - v_B}{R_2} \Rightarrow v_B \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{v_o}{R_2}$$

$$v_B = \frac{v_o}{\left(1 + \frac{R_2}{R_1} \right)}$$

$$\text{Then } v_o = A_{od}v_A - \frac{A_{od}v_o}{\left(1 + \frac{R_2}{R_1} \right)}$$

$$v_o \left[1 + \frac{A_{od}}{\left(1 + \frac{R_2}{R_1} \right)} \right] = A_{od}v_A$$

$$v_o \left[\frac{\left(1 + \frac{R_2}{R_1} \right) + A_{od}}{\left(1 + \frac{R_2}{R_1} \right)} \right] = A_{od}v_A$$

$$v_o = \frac{A_{od} \left(1 + \frac{R_2}{R_1} \right) v_A}{A_{od} + \left(1 + \frac{R_2}{R_1} \right)}$$

$$v_o = \frac{\left(1 + \frac{R_2}{R_1} \right) v_A}{1 + \frac{1}{A_{od}} \left(1 + \frac{R_2}{R_1} \right)}$$

So

$$v_o = \frac{\left(1 + \frac{10}{10} \right) \left(\frac{v_I}{2} \right)}{1 + \frac{1}{10^3} \left(1 + \frac{10}{10} \right)} = 0.9980v_I$$

$$\text{For } v_I = 2 \text{ V}$$

$$v_o = \underline{1.9960 \text{ V}}$$

9.16

$$(a) i_i = \frac{v_I}{R_1} = i_2 = -\frac{v_o}{R_2} \Rightarrow \frac{v_o}{v_I} = -\frac{R_2}{R_1}$$

$$(b) i_2 = i_1 = \frac{v_I}{R_1} = i_3 + \frac{v_o}{R_L} = i_3 + \frac{1}{R_L} \left(-\frac{R_2}{R_1} \cdot v_I \right)$$

$$\text{Then } i_3 = \frac{v_I}{R_1} \left(1 + \frac{R_2}{R_L} \right)$$

9.17

$$V_{x,\max} = \left(\frac{R_3 \parallel R_1}{R_3 \parallel R_1 + R_4} \right) \cdot V^+ = \left(\frac{0.1 \parallel 1}{0.1 \parallel 1 + 10} \right) (10) \Rightarrow$$

$$V_{x,\max} = 0.09008 \text{ V}$$

$$|v_o| = \frac{R_2}{R_1} \cdot V_{x,\max}$$

$$10 = \frac{R_2}{R_1} (0.09008) \Rightarrow \frac{R_2}{R_1} = 111$$

$$\text{So } \underline{R_2 = 111 \text{ k}\Omega}$$

9.18

$$v_o = A_{oL}(v_2 - v_1)$$

$$a. A_{oL} = \frac{1}{[1 - (-1)] \times 10^{-3}} \Rightarrow \underline{A_{oL} = 500}$$

$$b. 1 = 500(v_2 - 1 \times 10^{-3}) \Rightarrow \underline{v_2 = 3 \text{ mV}}$$

$$c. 5 = 500(1 - v_1) \Rightarrow \underline{v_1 = 0.99 \text{ V}}$$

$$d. v_o = 500(-1 - (-1)) \Rightarrow \underline{v_o = 0}$$

$$e. -3 = 500(v_2 - (-0.5)) \Rightarrow \underline{v_2 = -0.506 \text{ V}}$$

9.19

$$a. v_o = -\frac{R_F}{R_1} \cdot v_{I1} - \frac{R_F}{R_2} \cdot v_{I2} - \frac{R_F}{R_3} \cdot v_{I3}$$

$$v_o = -\frac{80}{20}(0.5) - \frac{80}{40}(-1) - \frac{80}{60}(2)$$

$$= -4(0.5) - 2(-1) - 1.33(2)$$

$$v_o = \underline{-2.667 \text{ V}}$$

$$b. -5.2 = -4(1) - 2(0.25) - 1.33v_{I3}$$

$$v_{I3} = \underline{0.525}$$

9.20

$$\begin{aligned} v_0 &= -8v_{I1} - 2v_{I2} - 5v_{I3} \\ &= -\frac{R_F}{R_1}v_{I1} - \frac{R_F}{R_2}v_{I2} - \frac{R_F}{R_3}v_{I3} \\ \frac{R_F}{R_1} &= 8 \quad \frac{R_F}{R_2} = 2 \quad \frac{R_F}{R_3} = 5 \end{aligned}$$

Let $R_F = 500 \text{ k}\Omega \Rightarrow R_1 = 62.5 \text{ k}\Omega$

$$R_2 = 250 \text{ k}\Omega$$

$$R_3 = 100 \text{ k}\Omega$$

9.21

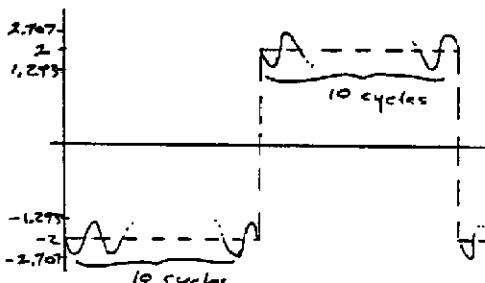
$$\begin{aligned} v_0 &= -4v_{I1} - 0.5v_{I2} = -\frac{R_F}{R_1}v_{I1} - \frac{R_F}{R_2}v_{I2} \\ \frac{R_F}{R_1} &= 4 \quad \frac{R_F}{R_2} = 0.5 \end{aligned}$$

$\Rightarrow R_1$ is the smallest resistor

$$\begin{aligned} |i| &= 100 \mu\text{A} = \frac{v_I}{R_1} = \frac{2}{R_1} \Rightarrow R_1 = 20 \text{ k}\Omega \\ &\Rightarrow R_F = 80 \text{ k}\Omega \\ &\Rightarrow R_2 = 160 \text{ k}\Omega \end{aligned}$$

9.22

$$\begin{aligned} v_{I1} &= (0.05)\sqrt{2} \sin(2\pi ft) = 0.0707 \sin(2\pi ft) \\ f &= 1 \text{ kHz} \Rightarrow T = \frac{1}{10^3} \Rightarrow 1 \text{ ms} \\ v_{I2} &\Rightarrow T_2 = \frac{1}{100} \Rightarrow 10 \text{ ms} \\ v_0 &= -\frac{R_F}{R_1} \cdot v_{I1} - \frac{R_F}{R_2} \cdot v_{I2} = -\frac{10}{1} \cdot v_{I1} - \frac{10}{5} \cdot v_{I2} \\ v_0 &= -(10)(0.0707 \sin(2\pi ft)) - (2)(\pm 1 \text{ V}) \\ v_0 &= -0.707 \sin(2\pi ft) - (\pm 2 \text{ V}) \end{aligned}$$



9.23

$$\begin{aligned} v_0 &= -\frac{R_F}{R_1} \cdot v_{I1} - \frac{R_F}{R_2} \cdot v_{I2} - \frac{R_F}{R_3} \cdot v_{I3} \\ v_0 &= -\frac{20}{10} \cdot v_{I1} - \frac{20}{5} \cdot v_{I2} - \frac{20}{2} \cdot v_{I3} \\ K \sin \alpha x &= -2v_{I1} - 4[2 + 100 \sin \alpha x] - 0 \\ \text{Set } v_{I1} &= -4 \end{aligned}$$

9.24

a.

$$\begin{aligned} v_0 &= -\frac{R_F}{R_3} \cdot a_3(-5) - \frac{R_F}{R_2} \cdot a_2(-5) - \frac{R_F}{R_1} \cdot a_1(-5) \\ &\quad - \frac{R_F}{R_0} \cdot a_0(-5) \end{aligned}$$

$$\text{So } v_0 = \frac{R_F}{10} \left[\frac{a_3}{2} + \frac{a_2}{4} + \frac{a_1}{8} + \frac{a_0}{16} \right] (5)$$

$$\text{b. } v_0 = 2.5 = \frac{R_F}{10} \cdot \frac{1}{2} \cdot 5 \Rightarrow R_F = 10 \text{ k}\Omega$$

$$\begin{aligned} \text{c. i. } v_0 &= \frac{10}{10} \cdot \frac{1}{16} \cdot 5 \Rightarrow v_0 = 0.3125 \text{ V} \\ \text{ii. } v_0 &= \frac{10}{10} \left[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \right] (5) \\ &\Rightarrow v_0 = 4.6875 \text{ V} \end{aligned}$$

9.25

$$\text{(a) } v_{o1} = -\frac{10}{1} \cdot v_{I1}$$

$$\begin{aligned} v_o &= -\frac{20}{1} \cdot v_{o1} - \frac{20}{1} \cdot v_{I2} = -(20)(-10)v_{I1} - (20)v_{I2} \\ v_o &= 200v_{I1} - 20v_{I2} \end{aligned}$$

$$\text{(b) } v_{I1} = 1 + 2 \sin \alpha x \text{ (mV)}$$

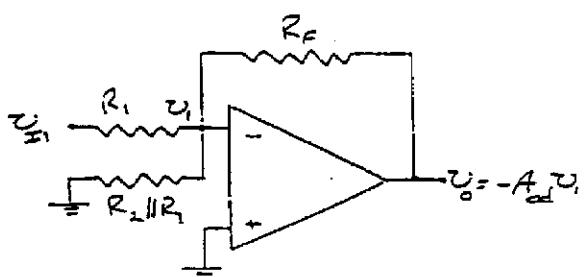
$$v_{I2} = -10 \text{ mV}$$

$$\text{Then } v_o = 200(1 + 2 \sin \alpha x) - 20(-10)$$

$$\text{So } v_o = 0.4 + 0.4 \sin \alpha x \text{ (V)}$$

9.26

For one-input



$$v_1 = -\frac{v_0}{A_{od}}$$

$$\frac{v_{I1} - v_1}{R_1} = \frac{v_1}{R_2 \parallel R_3} + \frac{v_1 - v_0}{R_F}$$

$$\frac{v_{I1}}{R_1} = v_1 \left[\frac{1}{R_1} + \frac{1}{R_2 \parallel R_3} + \frac{1}{R_F} \right] - \frac{v_0}{R_F}$$

$$= -\frac{v_0}{A_{od}} \left[\frac{1}{R_1} + \frac{1}{R_2 \parallel R_3} + \frac{1}{R_F} \right] - \frac{v_0}{R_F}$$

$$= -v_0 \left\{ \frac{1}{A_{od} R_F} + \frac{1}{R_F} + \frac{1}{A_{od}} \left(\frac{1}{R_1} + \frac{1}{R_2 \parallel R_3} \right) \right\}$$

$$= -\frac{v_0}{R_F} \left\{ \frac{1}{A_{od}} + 1 + \frac{1}{A_{od}} \cdot \frac{R_F}{R_1 \parallel R_2 \parallel R_3} \right\}$$

$$\nu_0 = -\frac{R_F}{R_1} \cdot \nu_{I1} \cdot \left\{ \frac{1}{1 + \frac{1}{A_{ad}} \left(1 + \frac{R_F}{R_P} \right)} \right\}$$

where $R_P = R_1 \parallel R_2 \parallel R_3$

Therefore, for three-inputs

$$\begin{aligned} \nu_0 &= \frac{-1}{1 + \frac{1}{A_{ad}} \left(1 + \frac{R_F}{R_P} \right)} \\ &\quad \times \left(\frac{R_F}{R_1} \cdot \nu_{I1} + \frac{R_F}{R_2} \cdot \nu_{I2} + \frac{R_F}{R_3} \cdot \nu_{I3} \right) \end{aligned}$$

9.27

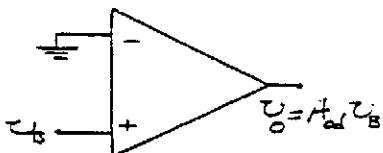
$$A_v = \left(1 + \frac{R_2}{R_1} \right) = 10$$

$$\frac{R_2}{R_1} = 9$$

$$|\nu| = \frac{\nu_I}{R_1} = \frac{0.8}{R_1} = 100 \mu A$$

$$\Rightarrow R_1 = 8 k\Omega, R_2 = 72 k\Omega$$

9.28



$$\nu_B = \left(\frac{1}{1 + 300} \right) \nu_I, \quad \nu_0 = A_{ad} \left(\frac{1}{501} \right) \nu_I$$

$$a. \quad 2.5 = A_{ad} \left(\frac{1}{501} \right) (5) \Rightarrow A_{ad} = 250.5$$

$$b. \quad \nu_0 = 5000 \left(\frac{1}{501} \right) (5) \Rightarrow \nu_0 = 49.9 V$$

9.29

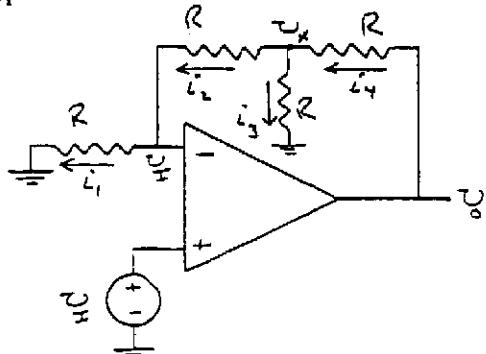
$$\nu_0 = \left(1 + \frac{50}{50} \right) \left[\left(\frac{20}{20+40} \right) \nu_{I2} + \left(\frac{40}{20+40} \right) \nu_{I1} \right]$$

$$\nu_0 = 1.33 \nu_{I1} + 0.667 \nu_{I2}$$

9.30

$$\begin{aligned} \nu_0 &= \left(1 + \frac{100}{50} \right) \\ &\quad \times \left[\left(\frac{10||40}{10||40+20} \right) \nu_{I1} + \left(\frac{10||20}{10||20+40} \right) \nu_{I2} \right] \\ \nu_0 &= 3 \left[\left(\frac{8}{8+20} \right) \nu_{I1} + \left(\frac{6.67}{6.67+40} \right) \nu_{I2} \right] \\ \nu_0 &= 0.657 \nu_{I1} + 0.429 \nu_{I2} \end{aligned}$$

9.31



$$i_1 = \frac{\nu_I}{R} = i_2$$

$$\nu_X = i_2 R + \nu_I = \left(\frac{\nu_I}{R} \right) R + \nu_I = 2\nu_I$$

$$i_3 = \frac{\nu_X}{R} = \frac{2\nu_I}{R}$$

$$i_4 = i_2 + i_3 = \frac{\nu_I}{R} + \frac{2\nu_I}{R} = \frac{3\nu_I}{R}$$

$$\nu_0 = i_4 R + \nu_X = \left(\frac{3\nu_I}{R} \right) R + 2\nu_I$$

$$\frac{\nu_0}{\nu_I} = 5$$

9.32

$$(a) \quad \frac{\nu_0}{\nu_I} = 1$$

(b) From Exercise 9.11

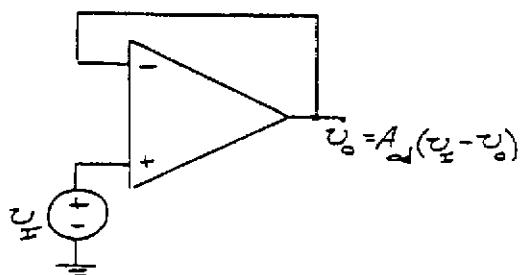
$$\frac{\nu_0}{\nu_I} = \frac{\left(1 + \frac{R_2}{R_1} \right)}{\left[1 + \frac{1}{A_{ad}} \left(1 + \frac{R_2}{R_1} \right) \right]}$$

But $R_2 = 0, R_1 = \infty$

$$\frac{\nu_0}{\nu_I} = \frac{1}{1 + \frac{1}{A_{ad}}} = \frac{1}{1 + \frac{1}{1.5 \times 10^5}} \Rightarrow \frac{\nu_0}{\nu_I} = 0.99999$$

$$(b) \text{ Want } \frac{\nu_0}{\nu_I} = 0.990 = \frac{1}{1 + \frac{1}{A_{ad}}} \Rightarrow \underline{A_{ad} = 99}$$

9.33



$$v_0 = A_{ad}(v_I - v_0)$$

$$\left(\frac{1}{A_{ad}} + 1\right)v_0 = v_I$$

$$v_0 = \frac{v_I}{\left(1 + \frac{1}{A_{ad}}\right)}$$

$$A_{ad} = 10^4; \frac{v_0}{v_I} = 0.99990$$

$$A_{ad} = 10^3; \frac{v_0}{v_I} = 0.9990$$

$$A_{ad} = 10^2; \frac{v_0}{v_I} = 0.990$$

$$A_{ad} = 10; \frac{v_0}{v_I} = 0.909$$

9.34

$$v_{o1} = \left(1 + \frac{R_2}{R_1}\right)v_I$$

$$v_{o2} = \left(1 + \frac{R_2}{R_1}\right)v_I, \quad v_{o2} = -\left(1 + \frac{R_2}{R_1}\right)v_I$$

$$\text{So } v_{o1} = -v_{o2}$$

9.35

$$(a) i_L = \frac{v_I}{R_L}$$

$$(b) v_{o1} = i_L R_L + v_I = i_L R_L + i_L R_i$$

$$v_{o1}(\max) \approx 8V = i_L(1+9) = 10i_L$$

$$\text{So } i_L(\max) \approx 0.8mA$$

$$\text{Then } v_I(\max) \equiv i_L R_i = (0.8)(9) \Rightarrow v_I(\max) \approx 7.2V$$

9.36

$$(a) v_x = \left(\frac{20}{20+40}\right) \cdot v_I = \left(\frac{20}{60}\right)(6) = 2$$

$$v_o = 2V$$

(b) Same as (a)

$$(c) v_x = \left(\frac{6}{6+48}\right)(6) = 0.666V$$

$$v_o = \left(1 + \frac{10}{10}\right) \cdot v_x \Rightarrow v_o = 1.33V$$

9.37

$$a. \quad R_{in} = \frac{v_1}{i_1} \text{ and } \frac{v_1 - v_0}{R_F} = i_1 \text{ and } v_0 = -A_{ad}v_1$$

$$\text{So } i_1 = \frac{v_1 - (-A_{ad}v_1)}{R_F} = \frac{v_1(1+A_{ad})}{R_F}$$

$$\text{Then } R_{in} = \frac{v_1}{i_1} = \frac{R_F}{1+A_{ad}}$$

$$b. \quad i_1 = \left(\frac{R_S}{R_S + R_{in}}\right)i_S \text{ and } v_0 = -A_{ad} \cdot \frac{R_F}{1+A_{ad}} \cdot i_1$$

$$\text{So } v_0 = -R_F \left(\frac{A_{ad}}{1+A_{ad}}\right) \left(\frac{R_S}{R_S + R_{in}}\right) i_S$$

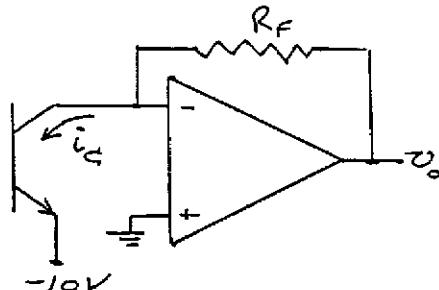
$$R_{in} = \frac{R_F}{1+A_{ad}} = \frac{10}{1001} = 0.009990$$

$$v_0 = -R_F \left(\frac{1000}{1001}\right) \left(\frac{R_S}{R_S + 0.009990}\right) i_S$$

$$\text{Want } \left(\frac{1000}{1001}\right) \left(\frac{R_S}{R_S + 0.009990}\right) \geq 0.990$$

 which yields $R_S > 1.099 k\Omega$

9.38



$$v_o = i_C R_F, \quad 0 \leq i_C \leq 8mA$$

 For $v_o(\max) = 8V$, Then

$$R_F = 1k\Omega$$

9.39

$$i = \frac{v_I}{R} \text{ so } 1 = \frac{10}{R} \Rightarrow R = 10k\Omega$$

 In the ideal op-amp, R_1 has no influence.

$$\text{Output voltage: } v_0 = \left(1 + \frac{R_2}{R}\right) v_I$$

v_0 must remain within the bias voltages of the op-amp; the larger the R_2 , the smaller the range of input voltage v_I in which the output is valid.

9.40

$$i_L = \frac{-v_I}{R_2} \Rightarrow 10 = -\frac{(-10)}{R_2} \Rightarrow R_2 = 1 \text{ k}\Omega$$

$$i_4 = \frac{v_L}{R_2} \text{ and } v_L = i_L Z_L = (0.010)(100) = 1 \text{ V}$$

$$i_4 = \frac{1}{1 \text{ k}\Omega} = 1 \text{ mA}$$

$$i_3 = i_4 + i_L = 1 + 10 = 11 \text{ mA}$$

$$\text{For } v_o(\text{max}) \approx 12 \text{ V} = i_3 R_3 + v_L = (11)R_3 + 1 \\ \Rightarrow R_3 = 1 \text{ k}\Omega$$

We need

$$\frac{R_3}{R_2} = \frac{R_F}{R_1} = \frac{1 \text{ k}\Omega}{1 \text{ k}\Omega} \Rightarrow R_1 = R_F$$

9.41

$$(a) i_1 = i_2 \text{ and } i_1 = \frac{v_x}{R_2} + i_D, \quad v_x = -i_2 R_F$$

$$\text{Then } i_1 = -i_1 \left(\frac{R_F}{R_2} \right) + i_D$$

$$\text{Or } i_D = i_1 \left(1 + \frac{R_F}{R_2} \right)$$

$$(b) R_1 = \frac{v_L}{i_1} = \frac{5}{1} \Rightarrow R_1 = 5 \text{ k}\Omega$$

$$12 = (1) \left(1 + \frac{R_F}{R_2} \right) \Rightarrow \frac{R_F}{R_2} = 11$$

$$\text{For example, } R_2 = 5 \text{ k}\Omega, \quad R_F = 55 \text{ k}\Omega$$

9.42

$$(1) I_x = \frac{V_x}{R_2} + \frac{V_x - v_o}{R_3}$$

$$(2) \frac{V_x}{R_1} + \frac{V_x - v_o}{R_F} = 0$$

$$\text{From (2)} \quad v_o = V_x \left(1 + \frac{R_F}{R_1} \right)$$

$$\text{Then (1)} \quad I_x = V_x \left(\frac{1}{R_2} + \frac{1}{R_3} \right) - \frac{1}{R_3} \cdot V_x \left(1 + \frac{R_F}{R_1} \right)$$

$$\begin{aligned} \frac{I_x}{V_x} &= \frac{1}{R_2} = \frac{1}{R_2} + \frac{1}{R_3} - \frac{1}{R_3} - \frac{R_F}{R_1 R_3} = \frac{1}{R_2} - \frac{R_F}{R_1 R_3} \\ &= \frac{R_1 R_2 - R_2 R_F}{R_1 R_2 R_3} \end{aligned}$$

or

$$R_s = \frac{R_1 R_2 R_3}{R_1 R_3 - R_2 R_F}$$

$$\text{Note: If } \frac{R_F}{R_1 R_3} = \frac{1}{R_2} \Rightarrow R_2 R_F = R_1 R_3$$

then $R_s = \infty$, which corresponds to an ideal current source.

9.43

$$A_d = \frac{R_2}{R_1} = \frac{R_4}{R_3} = 5$$

Minimum resistance seen by v_{T1} is R_1 .

Set $R_1 = R_3 = 25 \text{ k}\Omega$ Then $R_2 = R_4 = 125 \text{ k}\Omega$

$$i_L = \frac{v_o}{R_L} \Rightarrow v_o = i_L R_L = (0.5)(5) = 2.5 \text{ V}$$

$$v_o = 5(v_{T2} - v_{T1})$$

$$2.5 = 5(v_{T2} - 2) \Rightarrow v_{T2} = 2.5 \text{ V}$$

9.44

a. From superposition:

$$v_{o1} = -\frac{R_2}{R_1} \cdot v_{T1}$$

$$v_{o2} = \left(1 + \frac{R_2}{R_1} \right) \left(\frac{R_4}{R_3 + R_4} \right) v_{T2}$$

Setting $v_{T1} = v_{T2} = v_{cm}$

$$v_o = v_{o1} + v_{o2} = \left[\left(1 + \frac{R_2}{R_1} \right) \left(\frac{1}{1 + \frac{R_3}{R_4}} \right) - \frac{R_2}{R_1} \right] v_{cm}$$

$$\begin{aligned} A_{cm} &= \frac{v_o}{v_{cm}} = \frac{R_4}{R_3} \cdot \left(1 + \frac{R_2}{R_1} \right) \left(\frac{1}{1 + \frac{R_3}{R_4}} \right) - \frac{R_2}{R_1} \\ &= \frac{\frac{R_4}{R_3} \left(1 + \frac{R_2}{R_1} \right) - \frac{R_2}{R_1} \left(1 + \frac{R_4}{R_3} \right)}{\left(1 + \frac{R_4}{R_3} \right)} \end{aligned}$$

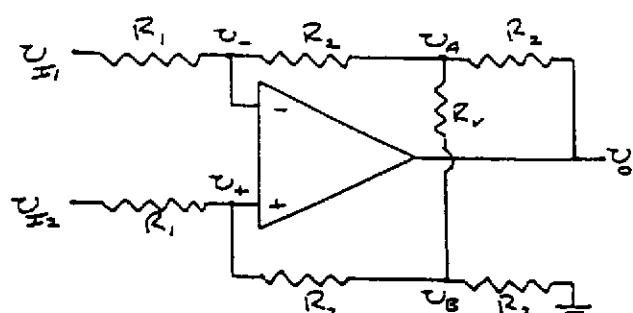
$$A_{cm} = \frac{\frac{R_4}{R_3} - \frac{R_2}{R_1}}{\left(1 + \frac{R_4}{R_3} \right)}$$

b. Max. $|A_{cm}| \Rightarrow \text{Min. } \frac{R_4}{R_3}$ and Max. $\frac{R_2}{R_1}$

$$\text{Max. } |A_{cm}| = \frac{\frac{47.5}{10.5} - \frac{52.5}{9.5}}{1 + \frac{47.5}{10.5}} = \frac{4.5238 - 5.5263}{1 + 4.5238}$$

$$\Rightarrow |A_{cm}|_{\max} = 0.1815$$

9.45



$$\frac{\nu_{I1} - \nu_A}{R_1 + R_2} = \frac{\nu_A - \nu_B}{R_V} + \frac{\nu_A - \nu_0}{R_2} \quad (1)$$

$$\frac{\nu_{I2} - \nu_B}{R_1 + R_2} = \frac{\nu_B - \nu_A}{R_V} + \frac{\nu_B}{R_2} \quad (2)$$

$$\nu_- = \left(\frac{R_1}{R_1 + R_2} \right) \nu_A + \left(\frac{R_2}{R_1 + R_2} \right) \nu_{I1} \quad (3)$$

$$\nu_+ = \left(\frac{R_1}{R_1 + R_2} \right) \nu_B + \left(\frac{R_2}{R_1 + R_2} \right) \nu_{I2} \quad (4)$$

$$\text{Now } \nu_- = \nu_+ \Rightarrow R_1 \nu_A + R_2 \nu_{I1} = R_1 \nu_B + R_2 \nu_{I2}$$

$$\text{So that } \nu_A = \nu_B + \frac{R_2}{R_1} (\nu_{I2} - \nu_{I1})$$

$$\frac{\nu_{I1}}{R_1 + R_2} = \nu_A \left(\frac{1}{R_1 + R_2} + \frac{1}{R_V} + \frac{1}{R_2} \right) - \frac{\nu_B}{R_V} - \frac{\nu_0}{R_2} \quad (1)$$

$$\frac{\nu_{I2}}{R_1 + R_2} = \nu_B \left(\frac{1}{R_1 + R_2} + \frac{1}{R_V} + \frac{1}{R_2} \right) - \frac{\nu_A}{R_V} \quad (2)$$

Then

$$\begin{aligned} \frac{\nu_{I1}}{R_1 + R_2} &= \nu_B \left(\frac{1}{R_1 + R_2} + \frac{1}{R_V} + \frac{1}{R_2} \right) - \frac{\nu_B}{R_V} - \frac{\nu_0}{R_2} \quad (1) \\ &+ \left(\frac{R_2}{R_1} \right) \left(\frac{1}{R_1 + R_2} + \frac{1}{R_V} + \frac{1}{R_2} \right) (\nu_{I2} - \nu_{I1}) \end{aligned}$$

$$\begin{aligned} \frac{\nu_{I2}}{R_1 + R_2} &= \nu_B \left(\frac{1}{R_1 + R_2} + \frac{1}{R_V} + \frac{1}{R_2} \right) \quad (2) \\ &- \frac{1}{R_V} \left[\nu_B + \frac{R_2}{R_1} (\nu_{I2} - \nu_{I1}) \right] \end{aligned}$$

Substitute (1)-(2)

$$\begin{aligned} \frac{1}{R_1 + R_2} (\nu_{I1} - \nu_{I2}) &= \left(\frac{R_2}{R_1} \right) \left(\frac{1}{R_1 + R_2} + \frac{1}{R_V} + \frac{1}{R_2} \right) (\nu_{I2} - \nu_{I1}) \\ &- \frac{\nu_0}{R_2} + \frac{1}{R_V} \cdot \frac{R_2}{R_1} (\nu_{I2} - \nu_{I1}) \end{aligned}$$

$$\begin{aligned} \frac{\nu_0}{R_2} &= (\nu_{I2} - \nu_{I1}) \left\{ \left(\frac{R_2}{R_1} \right) \left(\frac{1}{R_1 + R_2} + \frac{1}{R_V} + \frac{1}{R_2} \right) \right. \\ &\quad \left. + \frac{1}{R_1 + R_2} + \frac{1}{R_V} \cdot \frac{R_2}{R_1} \right\} \end{aligned}$$

$$\begin{aligned} \nu_0 &= (\nu_{I2} - \nu_{I1}) \left(\frac{R_2}{R_1} \right) \left\{ \frac{R_2}{R_1 + R_2} + \frac{R_2}{R_V} + 1 \right. \\ &\quad \left. + \frac{R_1}{R_1 + R_2} + \frac{R_2}{R_V} \right\} \end{aligned}$$

$$\nu_0 = \frac{2R_2}{R_1} \left(1 + \frac{R_2}{R_V} \right) (\nu_{I2} - \nu_{I1})$$

9.46

$$\begin{aligned} \nu_{01} &= \left(1 + \frac{R_2}{R_1} \right) \nu_{I1} - \frac{R_2}{R_1} \cdot \nu_{I2} \\ &= \left(1 + \frac{50}{10} \right) (-25 \sin \omega t) - \frac{50}{10} (25 \sin \omega t) \\ \underline{\nu_{01}} &= -275 \sin \omega t \text{ mV} \end{aligned}$$

$$\begin{aligned} \nu_{02} &= \left(1 + \frac{R_2}{R_1} \right) \nu_{I2} - \frac{R_2}{R_1} \cdot \nu_{I1} \\ &= \left(1 + \frac{50}{10} \right) (25 \sin \omega t) - \frac{50}{10} (-25 \sin \omega t) \\ \underline{\nu_{02}} &= 275 \sin \omega t \text{ mV} \end{aligned}$$

$$\begin{aligned} \nu_0 &= \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1} \right) (\nu_{I2} - \nu_{I1}) \\ &= \frac{30}{20} \left(1 + 2 \left[\frac{50}{10} \right] \right) (25 - [-25]) \sin \omega t \\ \underline{\nu_0} &= 825 \sin \omega t \text{ mV} \end{aligned}$$

Current in R_1 and R_2 :

$$i_1 = \frac{\nu_{I1} - \nu_{I2}}{R_1} = \frac{(-25 - 25) \sin \omega t \text{ mV}}{10 \text{ k}\Omega}$$

$$|i_1| = 5 \sin \omega t \mu\text{A}$$

Current in bottom R_3 and R_4 :

$$i_3 = \frac{\nu_{02}}{R_3 + R_4} = \frac{275 \sin \omega t \text{ mV}}{(20 + 30) \text{ k}\Omega}$$

$$|i_3| = 5.5 \sin \omega t \mu\text{A}$$

Current in top R_3 and R_4 :

$$\begin{aligned} i'_3 &= \frac{\nu_{01} - \left(\frac{R_4}{R_3 + R_4} \right) \nu_{02}}{R_3} \\ &= \frac{[-275 - \left(\frac{30}{30 + 20} \right) (275)] \sin \omega t \text{ mV}}{20 \text{ k}\Omega} \\ |i'_3| &= 22 \sin \omega t \mu\text{A} \end{aligned}$$

9.47

$$\begin{aligned} \nu_0 &= \frac{30}{20} \left(1 + \frac{2(50)}{R_1} \right) (25 - (-25)) \sin \omega t \text{ mV} \\ |\nu_0| &= (1.5)(50) \left(1 + \frac{100}{R_1} \right) \text{ mV} \end{aligned}$$

For

$$|\nu_0| = 0.1 \text{ V} = (1.5)(0.050) \left(1 + \frac{100}{R_1} \right)$$

$$\Rightarrow \underline{R_1 = 300 \text{ k}\Omega}$$

For

$$|\nu_0| = 5 \text{ V} = (1.5)(0.050) \left(1 + \frac{100}{R_1}\right)$$

$$\Rightarrow R_1 = 1.52 \text{ k}\Omega$$

$$\text{So } R_{1f} = 1.52 \text{ k}\Omega \Rightarrow \text{Potentiometer} \approx 300 \text{ k}\Omega$$

9.48

$$A_d = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1}\right)$$

$$\text{For } A_d(\text{max}), R_1 = R_1(\text{min}) = R_{1f}$$

$$200 = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_{1f}}\right)$$

$$\text{For } A_d(\text{min}), R_1 = R_1(\text{max}) \approx 50 \text{ k}\Omega$$

$$0.5 = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{50}\right)$$

$$\text{Let } \frac{R_4}{R_3} = 0.4$$

$$\text{Then } 0.5 = 0.4 \left(1 + \frac{2R_2}{50}\right) \Rightarrow R_2 = 6.25 \text{ k}\Omega$$

and

$$200 = (0.4) \left(1 + \frac{2(6.25)}{R_{1f}}\right) \Rightarrow R_{1f} = 25.05 \Omega$$

9.49

$$\text{For a common-mode gain, } \nu_{cm} = \nu_{I1} = \nu_{I2}$$

Then

$$\nu_{01} = \left(1 + \frac{R_2}{R_1}\right) \nu_{cm} - \frac{R_2}{R_1} \nu_{cm} = \nu_{cm}$$

$$\nu_{02} = \left(1 + \frac{R_2}{R_1}\right) \nu_{cm} - \frac{R_2}{R_1} \nu_{cm} = \nu_{cm}$$

From Problem 9.41, we can write

$$A_{cm} = \frac{\frac{R_4}{R_3} - \frac{R_4}{R'_3}}{\left(1 + \frac{R_4}{R_3}\right)}$$

$$R_3 = R_4 = 20 \text{ k}\Omega, R'_3 = 20 \text{ k}\Omega \pm 5\%$$

$$A_{cm} = \frac{1 - \frac{20}{R'_3}}{(1 + 1)} = \frac{1}{2} \left(1 - \frac{20}{R'_3}\right)$$

$$\text{For } R'_3 = 20 \text{ k}\Omega - 5\% = 19 \text{ k}\Omega$$

$$A_{cm} = \frac{1}{2} \left(1 - \frac{20}{19}\right) = -0.0263$$

$$\text{For } R'_3 = 20 \text{ k}\Omega + 5\% = 21 \text{ k}\Omega$$

$$A_{cm} = \frac{1}{2} \left(1 - \frac{20}{21}\right) = 0.0238$$

$$\text{So } |A_{cm}|_{max} = 0.0263$$

9.50

$$\text{a. } \nu_0 = \frac{1}{R_1 C_2} \cdot \int \nu_I(t') dt'$$

$$\int 0.5 \sin \omega t dt = -\frac{0.5}{\omega} \cos \omega t$$

$$|\nu_0| = 0.5 = \frac{1}{R_1 C_2} \cdot \frac{(0.5)}{\omega} = \frac{0.5}{2\pi R_1 C_2 f}$$

$$f = \frac{1}{2\pi R_1 C_2} = \frac{1}{2\pi(50 \times 10^3)(0.1 \times 10^{-6})}$$

$$\Rightarrow f = 31.8 \text{ Hz}$$

Output signal lags input signal by 90°

$$\text{b. i. } f = \frac{0.5}{2\pi(50 \times 10^3)(0.1 \times 10^{-6})} \Rightarrow f = 15.9 \text{ Hz}$$

$$\text{ii. } f = \frac{0.5}{(0.1)(2\pi)(50 \times 10^3)(0.1 \times 10^{-6})}$$

$$\Rightarrow f = 159 \text{ Hz}$$

9.51

$$\text{a. } \nu_o = -\frac{1}{RC} \int \nu_i(t') dt'$$

$$\nu_o = -\frac{1}{0.2}(0.5)(2) \Rightarrow \nu_o = -5 \text{ V}$$

$$\text{b. } -15 = -\frac{1}{0.2}(0.5)t \Rightarrow t = 6 \text{ s}$$

9.52

$$\text{a. } \frac{\nu_0}{\nu_I} = \frac{-R_2 \parallel \frac{1}{j\omega C_2}}{R_1} = -\frac{R_2 \cdot \frac{1}{j\omega C_2}}{R_1 \left(R_2 + \frac{1}{j\omega C_2}\right)}$$

$$\frac{\nu_0}{\nu_I} = -\frac{R_2}{R_1} \cdot \frac{1}{1 + j\omega R_2 C_2}$$

$$\text{b. } \frac{\nu_0}{\nu_I} = -\frac{R_2}{R_1}$$

$$\text{c. } f = \frac{1}{2\pi R_2 C_2}$$

9.53

$$\text{a. } \frac{\nu_0}{\nu_I} = \frac{-R_2}{R_1 + \frac{1}{j\omega C_1}} = -\frac{R_2(j\omega C_1)}{1 + j\omega R_1 C_1}$$

$$\frac{\nu_0}{\nu_I} = -\frac{R_2}{R_1} \cdot \frac{j\omega R_1 C_1}{1 + j\omega R_1 C_1}$$

$$\text{b. } \frac{\nu_0}{\nu_I} = -\frac{R_2}{R_1}$$

$$\text{c. } f = \frac{1}{2\pi R_1 C_1}$$

9.54

Assuming the Zener diode is in breakdown,

$$v_o = -\frac{R_2}{R_1} \cdot V_z = -\frac{1}{1}(6.8) \Rightarrow v_o = -6.8 V$$

$$i_2 = \frac{0 - v_o}{R_2} = \frac{0 - (-6.8)}{1} \Rightarrow i_2 = 6.8 mA$$

$$i_z = \frac{10 - V_z}{R_s} - i_2 = \frac{10 - 6.8}{5.6} - 6.8 \Rightarrow i_z = -6.2 mA!!!$$

Circuit is not in breakdown. Now

$$\frac{10 - 0}{R_s + R_1} = i_2 = \frac{10}{5.6 + 1} \Rightarrow i_2 = 1.52 mA$$

$$v_o = -i_2 R_2 = -(1.52)(1) \Rightarrow v_o = -1.52 V$$

$$i_z = 0$$

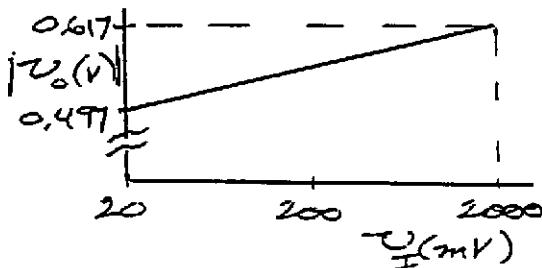
9.55

$$v_o = -V_T \ln\left(\frac{v_i}{I_s R_t}\right) = -(0.026) \ln\left[\frac{v_i}{(10^{-14})(10^4)}\right] \Rightarrow$$

$$v_o = -0.026 \ln\left(\frac{v_i}{10^{-10}}\right)$$

$$\text{For } v_i = 20 mV, |v_o| = 0.497 V$$

$$\text{For } v_i = 2 V, |v_o| = 0.617 V$$



9.56

$$v_o = \left(\frac{333}{20}\right)(v_{o1} - v_{o2}) = 16.65(v_{o1} - v_{o2})$$

$$v_{o1} = -v_B s_1 = -V_T \ln\left(\frac{i_{C1}}{I_S}\right)$$

$$v_{o2} = -v_B s_2 = -V_T \ln\left(\frac{i_{C2}}{I_S}\right)$$

$$v_{o1} - v_{o2} = -V_T \ln\left(\frac{i_{C1}}{i_{C2}}\right) = V_T \ln\left(\frac{i_{C2}}{i_{C1}}\right)$$

$$i_{C2} = \frac{v_2}{R_2}, \quad i_{C1} = \frac{v_1}{R_1}$$

$$\text{So } v_{o1} - v_{o2} = V_T \ln\left(\frac{v_2}{R_2} \cdot \frac{R_1}{v_1}\right)$$

Then

$$v_o = (16.65)(0.026) \ln\left(\frac{v_2}{v_1} \cdot \frac{R_1}{R_2}\right)$$

$$v_o = 0.4329 \ln\left(\frac{v_2}{v_1} \cdot \frac{R_1}{R_2}\right)$$

$$\ln(x) = \log_e(x) = [\log_{10}(x)] \cdot [\log_e(10)] \\ = 2.3026 \log_{10}(x)$$

Then

$$v_o = (1.0) \log_{10}\left(\frac{v_2}{v_1} \cdot \frac{R_1}{R_2}\right)$$

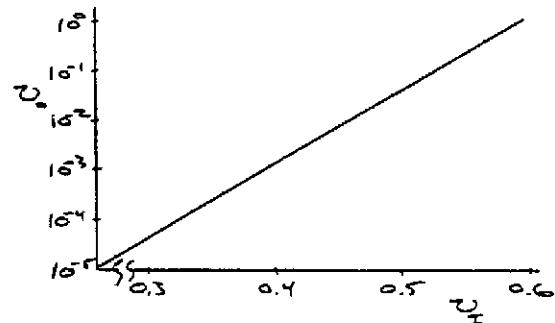
9.57

$$v_o = -I_s R (e^{v_i / N_T}) = -(10^{-14})(10^4) e^{v_i / 0.026}$$

$$|v_o| = (10^{-10}) e^{v_i / 0.026}$$

$$\text{For } v_i = 0.30 V, |v_o| = 1.03 \times 10^{-5} V$$

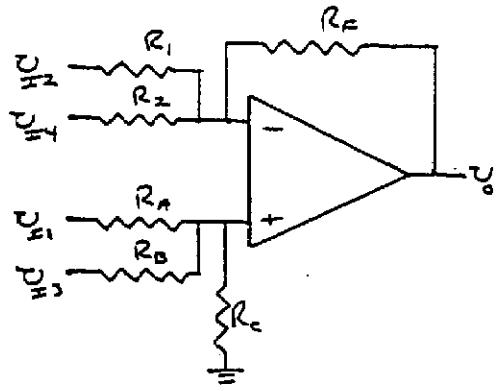
$$\text{For } v_i = 0.60 V, |v_o| = 1.05 V$$



9.58

$$v_o = 2v_{i1} - 10v_{i2} + 3v_{i3} - v_{i4}$$

From Figure 9.37



From Equation (9.110), we can write

$$v_o = -\frac{R_F}{R_1} \cdot v_{i2} - \frac{R_F}{R_2} \cdot v_{i4} + \left(1 + \frac{R_F}{R_N}\right) \left[\frac{R_P}{R_A} \cdot v_{i1} + \frac{R_P}{R_B} \cdot v_{i3} \right]$$

$$\text{where } R_N = R_1 \parallel R_2, \quad R_P = R_A \parallel R_B \parallel R_C$$

$$\text{Then } \frac{R_F}{R_1} = 10; \quad \frac{R_F}{R_2} = 1$$

Set $R_F = 500 \text{ k}\Omega$, then $R_1 = 50 \text{ k}\Omega$, $R_2 = 500 \text{ k}\Omega$

Now $R_N = R_1 \parallel R_2 = 50 \parallel 500 = 45.45 \text{ k}\Omega$

$$\left(1 + \frac{R_F}{R_N}\right) = \left(1 + \frac{500}{45.45}\right) = 12$$

$$\text{Then } (12) \left(\frac{R_P}{R_A}\right) = 2; \quad (12) \left(\frac{R_P}{R_B}\right) = 3$$

$$\text{Now } \frac{12(R_P/R_A)}{12(R_P/R_B)} = \frac{2}{3} = \frac{R_B}{R_A}$$

R_A is the largest resistor

Set $R_A = 500 \text{ k}\Omega$, then $R_B = 333.3 \text{ k}\Omega$

$$\text{Then } \frac{12R_P}{R_A} = 2 = \frac{12R_P}{500} = 2 \Rightarrow R_P = 83.33 \text{ k}\Omega$$

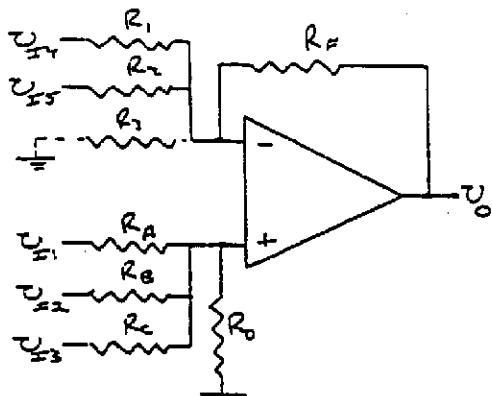
$$R_P = R_A \parallel R_B \parallel R_C \text{ and } R_A \parallel R_B = 500 \parallel 333.3 = 200 \text{ k}\Omega$$

Then $200 \parallel R_C = 83.33$ so $R_C = 142.8 \text{ k}\Omega$

9.59

$$v_o = 6v_{11} + 3v_{12} + 5v_{13} - v_{14} - 2v_{15}$$

From Figure 9.37



$$v_o = -\frac{R_F}{R_1} \cdot v_{14} - \frac{R_F}{R_2} \cdot v_{15} + \left(1 + \frac{R_F}{R_N}\right) \left[\frac{R_P}{R_A} \cdot v_{11} + \frac{R_P}{R_B} \cdot v_{12} + \frac{R_P}{R_C} \cdot v_{13} \right]$$

where

$$R_N = R_1 \parallel R_2, \quad R_P = R_A \parallel R_B \parallel R_C \parallel R_D$$

$$\text{Then } \frac{R_F}{R_1} = 1; \quad \frac{R_F}{R_2} = 2$$

Let $R_F = 250 \text{ k}\Omega$, then $R_1 = 250 \text{ k}\Omega$, $R_2 = 125 \text{ k}\Omega$

Then $R_N = R_1 \parallel R_2 = 250 \parallel 125 = 83.33 \text{ k}\Omega$

$$\left(1 + \frac{R_F}{R_N}\right) = \left(1 + \frac{250}{83.33}\right) = 4$$

$$\text{Now } 4 \left(\frac{R_P}{R_A}\right) = 6; \quad 4 \left(\frac{R_P}{R_B}\right) = 3; \quad 4 \left(\frac{R_P}{R_C}\right) = 5$$

$$\frac{4(R_P/R_A)}{4(R_P/R_B)} = \frac{6}{3} = 2 = \frac{R_B}{R_A}$$

$$\frac{4(R_P/R_C)}{4(R_P/R_B)} = \frac{5}{3} = 1.667 = \frac{R_B}{R_C}$$

Set $R_B = 250 \text{ k}\Omega$, then

$$R_A = 125 \text{ k}\Omega, \quad R_C = 150 \text{ k}\Omega$$

$$\text{Then } \frac{4R_P}{R_A} = 6 = \frac{4R_P}{125} = 6 \Rightarrow R_P = 187.5 \text{ k}\Omega$$

⇒ won't work since

$$R_P = R_A \parallel R_B \parallel R_C \parallel R_D > R_A \text{ and } R_C$$

Add a resistor R_3 in parallel with R_1 and R_2 to decrease R_N (but with zero input to R_3).

Set $R_D = \infty \Rightarrow R_P = R_A \parallel R_B \parallel R_C = 53.57 \text{ k}\Omega$

Then

$$\left(1 + \frac{R_F}{R_N}\right) \cdot \frac{R_P}{R_A} = 6 = \left(1 + \frac{R_F}{R_N}\right) \cdot \left(\frac{53.57}{125}\right)$$

$$\Rightarrow \frac{R_F}{R_N} = 13.0$$

So

$$R_N = \frac{250}{13} = 19.23 = R_1 \parallel R_2 \parallel R_3 = 83.33 \parallel R_3$$

$$\text{So } R_3 = 25 \text{ k}\Omega$$

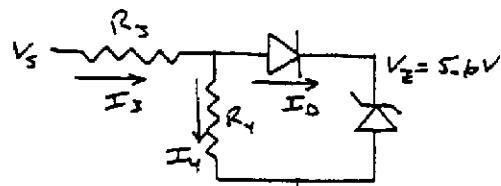
9.60

$$\frac{V_o}{V_z} = \left(1 + \frac{R_2}{R_1}\right)$$

$$\frac{9}{5.6} = 1 + \frac{R_2}{R_1} \Rightarrow \frac{R_2}{R_1} = 0.607$$

$$I_F = \frac{V_o - V_z}{R_F}$$

$$\text{Set } I_F = 0.8 \text{ mA} = \frac{9 - 5.6}{R_F} \Rightarrow R_F = 4.25 \text{ k}\Omega$$



$$V'_z = 5.6 + 0.7 = 6.3 \text{ V}$$

$$I_4 = \frac{V'_z}{R_4} = \frac{6.3}{R_4}, \quad I_3 = \frac{V_s - V'_z}{R_3}$$

$$\text{If } V_s = 10 \text{ V}, \quad I_3 = \frac{10 - 6.3}{R_3} = \frac{3.7}{R_3}$$

Want $I_{D1} = 0.1 \text{ mA}$; if we set $I_4 = 0.1 \text{ mA} = \frac{6.3}{R_4}$

$$\Rightarrow R_4 = 63 \text{ k}\Omega$$

$$\text{Then } I_3 = 0.2 \text{ mA} = \frac{3.7}{R_3} \Rightarrow R_3 = 18.5 \text{ k}\Omega$$

9.61

For $I_Z = 1 \text{ mA}$

$$I_Z = \frac{V_0 - V_Z}{R_1} = \frac{10 - 5.6}{R_1} = 1 \text{ mA} \Rightarrow R_1 = 4.4 \text{ k}\Omega$$

$$\begin{aligned} V_Z &= \left(\frac{R_3}{R_2 + R_3} \right) \cdot V_0 \Rightarrow 5.6 = \frac{R_3}{R_2 + R_3} \cdot 10 \\ &= \frac{10}{\left(1 + \frac{R_2}{R_3} \right)} \end{aligned}$$

$$1 + \frac{R_2}{R_3} = \frac{10}{5.6} \Rightarrow \frac{R_2}{R_3} = 0.786$$

$$\begin{aligned} \text{Let } \frac{V_0}{R_2 + R_3} &= 1 \text{ mA} = \frac{10}{R_2 + R_3} = 1 \text{ mA} \\ \Rightarrow R_2 + R_3 &= 10 \text{ k}\Omega \end{aligned}$$

$$R_3 = 0.786 R_2 \Rightarrow 0.786 R_3 + R_3 = 10$$

$$\Rightarrow R_3 = 5.6 \text{ k}\Omega, R_2 = 4.4 \text{ k}\Omega$$

$$\text{Let } \frac{V_{in} - V_0}{R_4} = 2 \text{ mA} = \frac{12 - 10}{R_4} \Rightarrow R_4 = 1 \text{ k}\Omega$$

In this case, the op-amp must supply the load current.

9.62

For $v_{o1} - v_{o2} = 0$ at $T = 250^\circ \text{ K}$, set

$$R_1 = R_2 = R_3 = R_T @ 250^\circ \text{ K} = 12 \text{ k}\Omega$$

Assuming v_{o1} and v_{o2} look into an open circuit.

$$v_{o2} = \left(\frac{R_T}{R_T + R_2} \right) V^+ \text{ and}$$

$$v_{o1} = \left(\frac{R_3}{R_1 + R_3} \right) \cdot V^+ = \frac{V^+}{2}$$

As temperature increases, R_T decreases. Let $R_T = 12(1 - \delta)$ where δ is positive.

$$R_T = 10 \text{ k}\Omega @ 300^\circ \Rightarrow \delta = 0.1667$$

$$v_{o2} = \left(\frac{12(1 - \delta)}{12(1 - \delta) + 12} \right) V^+$$

Consider

$$\begin{aligned} v_{o2} - v_{o1} &= \left(\frac{12(1 - \delta)}{12(1 - \delta) + 12} - \frac{1}{2} \right) (10) \\ &= \left(\frac{12(1 - \delta) - 6[(1 - \delta) + 1]}{12[(1 - \delta) + 1]} \right) (10) \\ &= \left(\frac{6(1 - \delta) - 6}{12[(1 - \delta) + 1]} \right) (10) = \frac{60\delta}{12[(1 - \delta) + 1]} \end{aligned}$$

$$\text{Now consider } v_{o1} - v_{o2} = \frac{5\delta}{2 - \delta} \approx \frac{5}{2} \cdot \delta$$

Connect v_{o1} to the v_{T2} terminal of the instrumentation amplifier and v_{o2} to the v_{T1} terminal.

$$\text{Now } v_0 = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1} \right) \left(\frac{5}{2} \right) \cdot \delta$$

$$\text{For } \delta = 0.1667, v_0 = 5$$

$$\begin{aligned} \frac{5}{0.1667} &= 30 = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1} \right) \left(\frac{5}{2} \right) \text{ or} \\ \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1} \right) &= 12 \end{aligned}$$

$$\text{Set } \frac{R_4}{R_3} = 4 \text{ Then } \frac{2R_2}{R_1} = 3 \text{ If } R_2 = 30 \text{ k}\Omega$$

$$R_1 = 20 \text{ k}\Omega$$

Resistor R_1 can be a fixed resistor in series with a potentiometer for more precise control.

9.63

Using the bridge circuit shown in Figure 9.44

$$\begin{aligned} v_{o1} &= \left[\frac{1}{2} - \frac{R}{R + R(1 + \delta)} \right] V^+ = \left[\frac{1}{2} - \frac{1}{2 + \delta} \right] (10) \\ &= \left(\frac{2 + \delta - 2}{2(2 + \delta)} \right) (10) \\ &\Rightarrow v_{o1} \approx \frac{10\delta}{4} = 2.5\delta \end{aligned}$$

Connect v_{o1} to an instrumentation amplifier.

$$v_0 = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1} \right) (2.5\delta)$$

$$\text{When } \delta = 0.02, v_0 = 5$$

$$\frac{5}{(2.5)(0.02)} = 100 = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1} \right)$$

$$\text{Set } \frac{R_4}{R_3} = 10 \text{ Then } \frac{2R_2}{R_1} = 9$$

Chapter 10

Exercise Solutions

E10.1

$$I_{REF} = \frac{V^+ - V_{BE}(\text{on})}{R_1} = \frac{10 - 0.7}{15}$$

$$\underline{I_{REF} = 0.62 \text{ mA}}$$

$$I_0 = \frac{I_{REF}}{1 + \frac{2}{\beta}} = \frac{0.62}{1 + \frac{2}{75}}$$

$$\underline{I_0 = 0.604 \text{ mA}}$$

E10.2

For $I_0 = 0.75 \text{ mA}$

$$I_{REF} = I_0 \left(1 + \frac{2}{\beta}\right) = (0.75) \left(1 + \frac{2}{100}\right)$$

$$\underline{I_{REF} = 0.765 \text{ mA}}$$

$$I_{REF} = \frac{V^+ - V_{BE}(\text{on}) - V^-}{R_1}$$

$$R_1 = \frac{5 - 0.7 - (-5)}{0.765}$$

$$\underline{R_1 = 12.2 \text{ k}\Omega}$$

E10.3

$$I_{REF} = \frac{V^+ - V_{BE}(\text{on}) - V^-}{R_1} = \frac{5 - 0.7 - (-5)}{12}$$

$$\underline{I_{REF} = 0.775 \text{ mA}}$$

$$I_0 = \frac{I_{REF}}{1 + \frac{2}{\beta}} = \frac{0.775}{1 + \frac{2}{75}} = 0.754 \text{ mA}$$

$$\Delta I_0 = (0.02)(0.754) = 0.0151 \text{ mA}$$

$$\text{and } \Delta I_0 = \frac{1}{r_0} \Delta V_{CE2} \Rightarrow r_0 = \frac{\Delta V_{CE2}}{\Delta I_0}$$

$$r_0 = \frac{4}{0.0151} = 265 \text{ k}\Omega = \frac{V_A}{I_0}$$

$$\Rightarrow V_A = (265)(0.754) \Rightarrow \underline{V_A \approx 200 \text{ V}}$$

E10.4

$$I_{REF} = \frac{V^+ - 2V_{BE}(\text{on})}{R_1} = \frac{9 - 2(0.7)}{12}$$

$$\underline{I_{REF} = 0.6333 \text{ mA}}$$

$$I_0 = \frac{I_{REF}}{1 + \frac{2}{\beta(1 + \beta)}} = \frac{0.6333}{1 + \frac{2}{75(76)}} = 0.6331 \text{ mA}$$

$$\underline{I_0 = 0.6331 \text{ mA} = I_{C1}}$$

$$I_{B1} = I_{B2} = \frac{I_0}{\beta} \Rightarrow \underline{I_{B1} = I_{B2} = 8.44 \mu\text{A}}$$

$$I_{E3} = I_{B1} + I_{B2} \Rightarrow \underline{I_{E3} = 16.88 \mu\text{A}}$$

$$I_{B3} = \frac{I_{E3}}{1 + \beta} \Rightarrow \underline{I_{B3} = 0.222 \mu\text{A}}$$

E10.5

$$I_{REF} = \frac{10 - (0.7)(2)}{12} = 0.717 \text{ mA}$$

$$I_0 \approx I_{REF} = 0.717$$

$$r_0 = \frac{V_A}{I_0} = \frac{100}{0.717} \Rightarrow \underline{r_0 = 139 \text{ k}\Omega}$$

$$\Delta I_0 = \frac{1}{r_0} \Delta V_{CE2} = \frac{4}{139}$$

$$\Rightarrow \underline{\Delta I_0 = 0.0288 \text{ mA}}$$

E10.6

$$I_0 = I_{REF} \cdot \frac{1}{\left(1 + \frac{2}{\beta(1 + \beta)}\right)} = \frac{0.50}{\left(1 + \frac{2}{50(51)}\right)}$$

$$\Rightarrow \underline{I_0 = 0.4996 \text{ mA}}$$

$$I_{B3} = \frac{I_0}{\beta} \Rightarrow \underline{I_{B3} = 9.99 \mu\text{A}}$$

$$I_{B3} = \left(\frac{1 + \beta}{\beta}\right) I_{C3} \Rightarrow \underline{I_{E3} = 0.5096 \text{ mA}}$$

$$I_{C2} = \frac{I_{B3}}{\left(1 + \frac{2}{\beta}\right)} = \frac{0.5096}{\left(1 + \frac{2}{50}\right)}$$

$$\Rightarrow \underline{I_{C2} = 0.490 \text{ mA} = I_{C1}}$$

$$I_{B1} = I_{B2} = \frac{I_{C2}}{\beta} \Rightarrow \underline{I_{B1} = I_{B2} = 9.80 \mu\text{A}}$$

E10.7

$$I_0 R_E = V_T \ln \left(\frac{I_{REF}}{I_0} \right)$$

$$R_E = \frac{V_T}{I_0} \ln \left(\frac{I_{REF}}{I_0} \right) = \frac{0.026}{0.025} \ln \left(\frac{0.75}{0.025} \right)$$

$$\Rightarrow \underline{R_E = 3.54 \text{ k}\Omega}$$

$$R_1 = \frac{5 - 0.7}{0.75} \Rightarrow \underline{R_1 = 5.73 \text{ k}\Omega}$$

$$V_{BE1} - V_{BE2} = I_0 R_E = (0.025)(3.54)$$

$$\Rightarrow \underline{V_{BE1} - V_{BE2} = 88.5 \text{ mV}}$$

E10.8

$$I_{REF} = \frac{5 - 0.7 - (-5)}{12} \Rightarrow \underline{I_{REF} = 0.775 \text{ mA}}$$

$$I_0 R_E = V_T \ln \left(\frac{I_{REF}}{I_0} \right)$$

$$I_0(6) = (0.026) \ln \left(\frac{0.775}{I_0} \right)$$

$$\Rightarrow \underline{I_0 \approx 16.6 \mu\text{A}}$$

E10.9

$$I_0 R_E = V_T \ln \left(\frac{I_{REF}}{I_0} \right)$$

$$R_E = \frac{0.026}{0.025} \ln \left(\frac{0.70}{0.025} \right) \Rightarrow R_E = 3.47 \text{ k}\Omega$$

$$g_{m2} = \frac{I_0}{V_T} = \frac{0.025}{0.026} \Rightarrow g_{m2} = 0.962 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta V_T}{I_0} = \frac{(150)(0.026)}{0.025} = 156 \text{ k}\Omega$$

$$r_{02} = \frac{V_A}{I_0} = \frac{100}{0.025} = 4000 \text{ k}\Omega$$

$$R'_E = R_E \| r_{\pi 2} = 3.47 \| 156 = 3.39 \text{ k}\Omega$$

$$R_0 = r_{02}(1 + g_{m2} R'_E) = 4000[1 + (0.962)(3.39)]$$

$$R_0 = 17,045 \text{ k}\Omega$$

$$dI_0 = \frac{1}{R_0} \cdot dV_{C2} = \frac{3}{17,045}$$

$$\Rightarrow \underline{dI_0 = 0.176 \mu\text{A}}$$

E10.10

$$I_{REF} = I_R + I_{BR} + I_{B1} + \dots + I_{BN}$$

$$I_R = I_{01} = I_{02} = \dots = I_{0N}$$

$$\text{and } I_{BR} = I_{B1} = I_{B2} = \dots = I_{BN} = \frac{I_{01}}{\beta}$$

$$I_{REF} = I_{01} + (N+1) \left(\frac{I_{01}}{\beta} \right) = I_{01} \left(1 + \frac{N+1}{\beta} \right)$$

$$\text{So } I_{01} = I_{02} = \dots = I_{0N} = \frac{I_{REF}}{1 + \frac{N+1}{\beta}}$$

$$\frac{I_{01}}{I_{REF}} = 0.90 = \frac{1}{1 + \frac{N+1}{50}}$$

$$1 + \frac{N+1}{50} = \frac{1}{0.9}$$

$$N+1 = \left(\frac{1}{0.9} - 1 \right)(50)$$

$$N = \left(\frac{1}{0.9} - 1 \right)(50) - 1$$

$$N = 4.55 \Rightarrow \underline{N = 4}$$

E10.11

a. From Equation (10.52),

$$V_{GS1} = \frac{\sqrt{\frac{3}{12}}}{1 + \sqrt{\frac{3}{12}}} \times 10 + \left(\frac{1 - \sqrt{\frac{3}{12}}}{1 + \sqrt{\frac{3}{12}}} \right) \times (1.8)$$

$$V_{GS1} = \left(\frac{0.5}{1 + 0.5} \right)(10) + \left(\frac{1 - 0.5}{1 + 0.5} \right) \times (1.8)$$

$$V_{GS1} = 3.93 \text{ V also } V_{DS1} = 3.93 \text{ V}$$

$$I_{REF} = (12)(0.020)[3.93 - 1.8]^2[1 + (0.01)(3.93)]$$

$$\Rightarrow \underline{I_{REF} = 1.13 \text{ mA}}$$

$$\text{b. } I_0 = I_{REF} \times \frac{(W/L)_2}{(W/L)_1} \times \frac{(1 + \lambda V_{DS2})}{(1 + \lambda V_{DS1})}$$

$$I_0 = (1.13) \times \left(\frac{6}{12} \right) \times \frac{[1 + (0.01)(2)]}{[1 + (0.01)(3.93)]} \\ \Rightarrow \underline{I_0 = 0.555 \text{ mA}}$$

c. For $V_{DS2} = 6 \text{ V}$

$$\Rightarrow \underline{I_0 = 0.576 \text{ mA}}$$

E10.12

$$K_{n1}(V_{GS1} - V_{TN})^2 = K_{n3}(V_{GS3} - V_{TN})^2$$

$$V_{GS1} - 2 = \left(\sqrt{\frac{0.10}{0.25}} \right) (V_{GS3} - 2)$$

$$V_{GS1} - 2 = (0.632)(V_{GS3} - 2)$$

$$V_{GS3} = 10 - V_{GS1}$$

$$V_{GS1} - 2 = (0.632)(10 - V_{GS1}) - (0.632)(2)$$

$$1.632V_{GS1} = 7.056 \Rightarrow V_{GS1} = 4.32 \text{ V}$$

$$I_{REF} = K_{n1}(V_{GS1} - V_{TN})^2 = (0.25)(4.32 - 2)^2 \Rightarrow \\ \underline{I_{REF} = 1.35 \text{ mA}}$$

$$I_0 = 3K_{n2}(V_{GS1} - V_{TN})^2 = 3(0.25)(4.32 - 2)^2$$

$$I_0 = 3I_{REF} \Rightarrow \underline{I_0 = 4.04 \text{ mA}}$$

E10.13

$$V_{DS}(\text{sat}) = 1 \text{ V} = V_{GS2} - V_{TN} = V_{GS2} - 2$$

$$\Rightarrow V_{GS2} = 3 \text{ V}$$

$$I_0 = K_{n2}(V_{GS1} - V_{TN})^2 = \left(\frac{\mu_n C_{ox}}{2} \right) \left(\frac{W}{L} \right)_2 (V_{GS2} - V_{TN})^2$$

$$0.20 = (0.020) \left(\frac{W}{L} \right)_2 (3-2)^2 \Rightarrow \left(\frac{W}{L} \right)_2 = 10$$

$$I_{REF} = \left(\frac{\mu_n C_{ox}}{2} \right) \left(\frac{W}{L} \right)_1 (V_{GS1} - V_{TN})^2$$

$$V_{GS1} = V_{GS2}$$

$$0.5 = (0.020) \left(\frac{W}{L} \right)_1 (3-2)^2 \Rightarrow \left(\frac{W}{L} \right)_1 = 25$$

$$V_{GS3} = V^+ - V_{GS1} = 10 - 3 = 7 \text{ V}$$

$$I_{REF} = \left(\frac{\mu_n C_{ox}}{2} \right) \left(\frac{W}{L} \right)_3 (V_{GS3} - V_{TN})^2$$

$$0.5 = (0.020) \left(\frac{W}{L} \right)_3 (7-2)^2 \Rightarrow \left(\frac{W}{L} \right)_3 = 1$$

E10.14

$$\text{a. } I_{REF} = K_n(V_{GS} - V_{TN})^2$$

$$0.020 = 0.080(V_{GS} - 1)^2$$

$$\underline{V_{GS} = 1.5 \text{ V all transistors}}$$

$$b. \quad V_{G4} = V_{GS3} + V_{GS4} + V^- = 1.5 + 1.5 - 5 = -2 \text{ V}$$

$$V_{S4} = V_{G4} - V_{DS4} = -2 - 1.5 = -3.5 \text{ V}$$

$$V_{D4}(\min) = V_{S4} + V_{DS4}(\text{sat})$$

$$\text{and } V_{DS4}(\text{sat}) = V_{GS4} - V_{TN} = 1.5 - 1 = 0.5 \text{ V}$$

$$\text{So } V_{D4}(\min) = -3.5 + 0.5$$

$$\Rightarrow \underline{V_{D4}(\min) = -3.0 \text{ V}}$$

$$c. \quad R_o = r_{o4} + r_{o2}(1 + g_m r_{o4})$$

$$r_{o2} = r_{o4} = \frac{1}{\lambda I_0} = \frac{1}{(0.02)(0.020)} = 2500 \text{ k}\Omega$$

$$g_m = 2K_a(V_{GS} - V_{TN}) = 2(0.080)(1.5 - 1) \Rightarrow$$

$$g_m = 0.080 \text{ mA/V}$$

$$R_o = 2500 + 2500(1 + (0.080)(2500))$$

$$\Rightarrow \underline{R_o = 505 \text{ M}\Omega}$$

E10.15

$$I_{REF} = 0.20 = K_{n1}(V_{GS1} - V_{TN})^2 = 0.15(V_{GS1} - 1)^2$$

$$\Rightarrow \underline{V_{GS1} = V_{GS2} = 2.15 \text{ V}}$$

$$I_o = K_{n2}(V_{GS2} - V_{TN})^2 = \frac{0.15}{2}(2.15 - 1)^2 \Rightarrow$$

$$\underline{I_o = 0.10 \text{ mA}}$$

$$I_o = K_{n3}(V_{GS3} - V_{TN})^2$$

$$0.10 = 0.15(V_{GS3} - 1)^2 \Rightarrow \underline{V_{GS3} = 1.82 \text{ V}}$$

E10.16

All transistors are identical

$$\Rightarrow \underline{I_o = I_{REF} = 250 \mu\text{A}}$$

$$I_{REF} = K_a(V_{GS} - V_{TN})^2$$

$$0.25 = 0.20(V_{GS} - 1)^2$$

$$\Rightarrow \underline{V_{GS} = 2.12 \text{ V}}$$

E10.17

$$\text{For } Q_2 : \nu_{DS}(\min) = |V_P| = 2 \text{ V}$$

$$\Rightarrow V_S(\min) = \nu_{DS}(\min) - 5 = 2 - 5$$

$$\Rightarrow \underline{V_S(\min) = -3 \text{ V}}$$

$$I_o = I_{DS2}(1 + \lambda \nu_{DS2}) = 0.5(1 + (0.15)(2))$$

$$\Rightarrow \underline{I_o = 0.65 \text{ mA}}$$

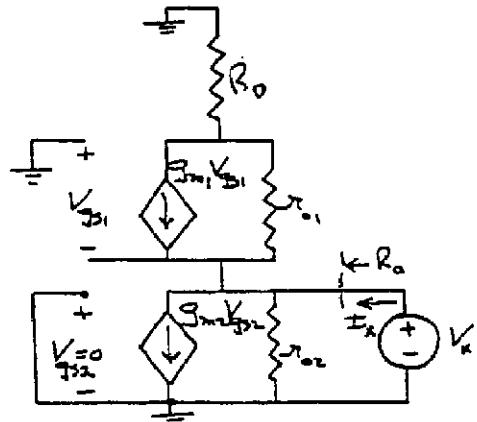
$$I_o = I_{DS1} \left(1 - \frac{\nu_{GS1}}{V_{P1}}\right)^2$$

$$0.65 = 0.80 \left(1 - \frac{\nu_{GS1}}{-2}\right)^2$$

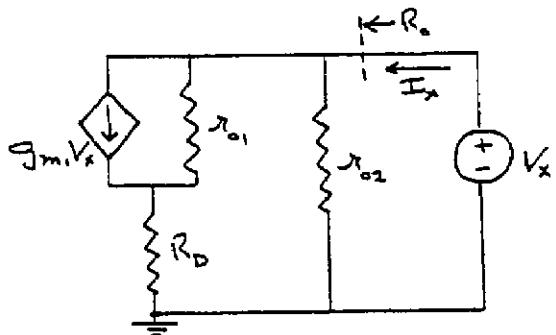
$$\frac{\nu_{GS1}}{-2} = 0.0986 \Rightarrow \nu_{GS1} = -0.197 \text{ V}$$

$$\nu_{GS1} = V_I - V_S$$

$$-0.197 = V_I - (-3) \Rightarrow \underline{V_I(\min) = -3.2 \text{ V}}$$



$$V_{gs2} = 0, \quad V_{gs1} = -V_X$$



$$I_X = \frac{V_X}{r_{o2}} + \frac{V_X - V_1}{r_{o1}} + g_{m1} V_X \quad (1)$$

$$\frac{V_1}{R_D} + \frac{V_1 - V_X}{r_{o1}} = g_{m1} V_X \quad (2)$$

$$V_1 = \frac{V_X \left(\frac{1}{r_{o1}} + g_{m1} \right)}{\frac{1}{R_D} + \frac{1}{r_{o1}}}$$

$$\begin{aligned} \frac{I_X}{V_X} &= \frac{1}{R_D} = \frac{1}{r_{o2}} + \frac{1}{r_{o1}} + g_{m1} - \frac{\frac{1}{r_{o1}} \left(\frac{1}{r_{o1}} + g_{m1} \right)}{\frac{1}{R_D} + \frac{1}{r_{o1}}} \\ &= \frac{1}{r_{o2}} + \left(\frac{1}{r_{o1}} + g_{m1} \right) \left[1 - \frac{\frac{1}{r_{o1}}}{\frac{1}{R_D} + \frac{1}{r_{o1}}} \right] \\ &= \frac{1}{r_{o2}} + \left(\frac{1}{r_{o1}} + g_{m1} \right) \left(\frac{\frac{1}{R_D}}{\frac{1}{R_D} + \frac{1}{r_{o1}}} \right) \end{aligned}$$

For $R_D \ll r_{01}$

$$\Rightarrow \frac{1}{R_0} \approx \frac{1}{r_{02}} + \left(\frac{1}{r_{01}} + g_{m1} \right)$$

For Q_1 :

$$g_{m1} = \frac{2I_{DSS1}}{|V_P|} \left(1 - \frac{V_{GS1}}{V_P} \right) = \frac{2(0.8)}{2} \left(1 - \frac{-0.197}{-2} \right)$$

$$g_{m1} = 0.721 \text{ mA/V}$$

$$r_0 = \frac{1}{\lambda I_0} = \frac{1}{(0.15)(0.65)} = 10.3 \text{ k}\Omega$$

$$\frac{1}{R_0} = \frac{1}{10.3} + \frac{1}{10.3} + 0.721 = 0.915$$

$$\Rightarrow R_0 = 1.09 \text{ k}\Omega$$

E10.18

$$\text{For } Q_1: i_D = I_{DSS1}(1 + \lambda \nu_{DS1})$$

$$\text{For } Q_2: i_D = I_{DSS1} \left(1 - \frac{\nu_{GS2}}{V_P} \right)^2 (1 + \lambda \nu_{DS2})$$

$$\nu_{GS2} = -\nu_{DS1}$$

$$\text{and } \nu_{DS2} = V_{DS} - \nu_{DS1}$$

So

$$I_{DSS1}(1 + \lambda \nu_{DS1}) \\ = I_{DSS1} \left[1 - \frac{-\nu_{DS1}}{V_P} \right]^2 [1 + \lambda(V_{DS} - \nu_{DS1})]$$

$$I_{DSS1} = I_{DSS2}$$

$$[1 + (0.1)\nu_{DS1}]$$

$$= \left[1 - \frac{\nu_{DS1}}{2} \right]^2 [1 + (0.1)(3) - (0.1)\nu_{DS1}]$$

$$1 + 0.1\nu_{DS1} = (1 - \nu_{DS1} + 0.25\nu_{DS1}^2)(1.3 - 0.1\nu_{DS1})$$

This becomes

$$0.025\nu_{DS1}^3 - 0.425\nu_{DS1}^2 + 1.5\nu_{DS1} - 0.3 = 0$$

$$\text{We find } \nu_{DS1} = 0.212 \text{ V}, \nu_{DS2} = 2.79 \text{ V}$$

$$\nu_{GS2} = -0.212 \text{ V}$$

$$i_D = I_{DSS1}(1 + \lambda \nu_{DS1}) = 2[1 + (0.1)(0.212)]$$

$$i_D = 2.04 \text{ mA}$$

$$R_0 = r_{02} + r_{01}(1 + g_{m2}r_{02})$$

$$g_m = \frac{2I_{DSS}}{-V_P} \left(1 - \frac{\nu_{GS2}}{V_P} \right) = \frac{2(2)}{2} \left(1 - \frac{-0.212}{-2} \right)$$

$$g_m = 1.79 \text{ mA/V}$$

$$r_{02} = r_{04} = \frac{1}{\lambda I_{DSS}} = \frac{1}{(0.1)(2)} = 5 \text{ k}\Omega$$

$$R_0 = 5 + 5[1 + (1.79)(5)]$$

$$\Rightarrow R_0 = 54.8 \text{ k}\Omega$$

E10.19

$$\text{a. } I_{REF} = I_S \exp \left(\frac{V_{EB2}}{V_T} \right)$$

$$V_{EB2} = V_T \ln \left(\frac{I_{REF}}{I_S} \right) = (0.026) \ln \left(\frac{0.5 \times 10^{-3}}{10^{-12}} \right) \\ \Rightarrow V_{EB2} = 0.521 \text{ V}$$

$$\text{b. } R_1 = \frac{5 - 0.521}{0.5} \Rightarrow R_1 = 8.96 \text{ k}\Omega$$

c. From Equation (10.72)

$$I_{S0} \left[\exp \left(\frac{V_I}{V_T} \right) \right] \left(1 + \frac{V_{CE0}}{V_{AN}} \right) = I_{REF} \times \frac{\left(1 + \frac{V_{EC2}}{V_{AP}} \right)}{\left(1 + \frac{V_{EB2}}{V_{AP}} \right)} \\ 10^{-12} \left[\exp \left(\frac{V_I}{V_T} \right) \right] \left(1 + \frac{2.5}{100} \right) \\ = (0.5 \times 10^{-3}) \frac{\left(1 + \frac{2.5}{100} \right)}{\left(1 + \frac{0.521}{100} \right)}$$

$$1.03 \times 10^{-12} \exp \left(\frac{V_I}{V_T} \right) = 5.125 \times 10^{-4}$$

$$\exp \left(\frac{V_I}{V_T} \right) = 4.976 \times 10^8$$

$$\Rightarrow V_I = 0.521 \text{ V}$$

d.

$$A_\nu = \frac{-\left(\frac{1}{V_T} \right)}{\frac{1}{V_{AN}} + \frac{1}{V_{AP}}} = \frac{-\frac{1}{0.026}}{\frac{1}{100} + \frac{1}{100}} = \frac{-38.46}{0.01 + 0.01} \\ \Rightarrow A_\nu = -1923$$

E10.20

$$\text{a. } V_{EB2} = (0.026) \ln \left(\frac{0.1 \times 10^{-3}}{5 \times 10^{-14}} \right)$$

$$\Rightarrow V_{EB2} = 0.557 \text{ V}$$

$$\text{b. } R_1 = \frac{5 - 0.557}{0.1} \Rightarrow R_1 = 44.4 \text{ k}\Omega$$

$$\text{c. } I_{S0} \left[\exp \left(\frac{V_I}{V_T} \right) \right] \left(1 + \frac{V_{CE0}}{V_{AN}} \right) \\ = I_{REF} \times \left(\frac{1 + \frac{V_{EC1}}{V_{AP}}}{1 + \frac{V_{EB1}}{V_{AP}}} \right)$$

$$\begin{aligned} & 5 \times 10^{-14} \left[\exp \left(\frac{V_I}{V_T} \right) \right] \left(1 + \frac{2.5}{100} \right) \\ &= (0.1 \times 10^{-3}) \left(\frac{1 + \frac{2.5}{100}}{1 + \frac{0.557}{100}} \right) \\ (5.125 \times 10^{-14}) \exp \left(\frac{V_I}{V_T} \right) &= 1.019 \times 10^{-4} \\ \exp \left(\frac{V_I}{V_T} \right) &= 1.988 \times 10^9 \\ \Rightarrow V_I &= 0.557 \text{ V} \end{aligned}$$

d. $A_V = \frac{-\frac{1}{0.026}}{\frac{1}{100} + \frac{1}{100}} \Rightarrow A_V = -1923$

E10.21

a. $I_{REF} = K_{p1}(V_{SG} + V_{TP})^2$

$$0.25 = 0.20(V_{SG} - 1)^2$$

$$\Rightarrow V_{SG} = 2.12 \text{ V}$$

b. From Equation (10.89)

$$V_{DSO} = V_o = \frac{\left[1 + \lambda_p (V^+ - V_{SG}) \right]}{\lambda_n + \lambda_p} - \frac{K_n (V_I - V_{TN})^2}{I_{REF} (\lambda_n + \lambda_p)}$$

$$5 = \frac{1 + (0.015)(10 - 2.12)}{0.030} - \frac{(0.2)(V_I - 1)^2}{0.25(0.030)}$$

$$0.15 = 1.12 - 0.8(V_I - 1)^2$$

$$\Rightarrow V_I = 2.10 \text{ V}$$

c. $A_V = \frac{-2K_n(V_I - V_{TN})}{I_{REF}(\lambda_n + \lambda_p)}$

$$A_V = -\frac{2(0.2)(2.10 - 1.0)}{0.25(0.030)}$$

$$\Rightarrow A_V = -58.7$$

E10.22

(a) $I_{REF} = K_{p1}(V_{SG} + V_{TP})^2$

$$80 = 50(V_{SG} - 1)^2 \Rightarrow V_{SG} = 2.26 \text{ V}$$

(b) $V_{DSO} = V_o = \frac{\left[1 + \lambda_p (V^+ - V_{SG}) \right]}{\lambda_n + \lambda_p} - \frac{K_n (V_I - V_{TN})^2}{I_{REF} (\lambda_n + \lambda_p)}$

$$5 = \frac{[1 + (0.015)(10 - 2.26)]}{0.030} - \frac{(50)(V_I - 1)^2}{(80)(0.030)}$$

$$20.83(V_I - 1)^2 = 32.2 \Rightarrow V_I = 2.24 \text{ V}$$

(c) $A_V = \frac{-2K_n(V_I - V_{TN})}{I_{REF}(\lambda_n + \lambda_p)} = \frac{-2(50)(2.24 - 1)}{(80)(0.030)} \Rightarrow A_V = -51.7$

E10.23

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.8}{0.026} = 30.8 \text{ mA/V}$$

$$r_0 = r_{02} = \frac{V_A}{I_{CQ}} = \frac{80}{0.8} = 100 \text{ k}\Omega$$

a. $V_0 = -g_m V_{x1} (r_0 \| r_{02}), \quad V_{x1} = V_i$

$$A_V = -g_m (r_0 \| r_{02}) = -(30.8)[100 \| 100]$$

$$\Rightarrow A_V = -1540$$

b. $A_V = -g_m (r_0 \| r_{02} \| R_L)$

$$A_V = -\frac{1540}{2} = -770$$

$$-770 = -(30.8)(50 \| R_L) \Rightarrow (50 \| R_L) = 25$$

$$\Rightarrow R_L = 50 \text{ k}\Omega$$

E10.24

a. $g_m = \frac{I_{CQ}}{V_T} = \frac{0.5}{0.026} \Rightarrow g_m = 19.2 \text{ mA/V}$

$$r_0 = \frac{V_{AN}}{I_{CQ}} = \frac{120}{0.5} \Rightarrow r_0 = 240 \text{ k}\Omega$$

$$r_{02} = \frac{V_{AP}}{I_{CQ}} = \frac{80}{0.5} \Rightarrow r_{02} = 160 \text{ k}\Omega$$

b. $A_V = -g_m (r_0 \| r_{02} \| R_L) = -(19.2)[240 \| 160 \| 50]$

$$\Rightarrow A_V = -631$$

E10.25

(a) Neglecting effect of λ and R_L

$$I_o = I_{REF} = K_n (V_{IQ} - V_{TN})^2$$

$$0.40 = 0.25(V_{IQ} - 1)^2$$

$$\text{Then } \underline{V_{IQ} = 2.26 \text{ V}}$$

$$\text{b. } r_0 = r_{02} = \frac{1}{\lambda I_0} = \frac{1}{(0.02)(0.4)} = 125 \text{ k}\Omega$$

$$g_m = 2K_n (V_{IQ} - V_{TN}) = 2(0.25)(2.26 - 1)$$

$$= 0.63 \text{ mA/V}$$

$$A_v = -g_m(r_0 \parallel r_{02}) = -(0.63)(125 \parallel 125)$$

$$\Rightarrow \underline{A_v = -39.4}$$

$$\text{c. } A_v = -g_m(r_0 \parallel r_{01} \parallel R_L)$$

$$-\frac{39.4}{2} = -(0.63)(62.5 \parallel R_L)$$

$$\Rightarrow 62.5 \parallel R_L = 31.25 \Rightarrow \underline{R_L = 62.5 \text{ k}\Omega}$$

E10.26

 M_1 and M_2 identical $\Rightarrow I_0 = I_{REF}$

$$\text{a. } I_o = K_n (V_I - V_{TN})^2$$

$$0.25 = 0.2(V_I - 1)^2$$

$$V_I = 2.12 \text{ V}$$

$$g_m = 2K_n (V_I - V_{TN}) = 2(0.2)(2.12 - 1)$$

$$\Rightarrow \underline{g_m = 0.448 \text{ mA/V}}$$

$$r_{on} = \frac{1}{\lambda_n I_0} = \frac{1}{(0.01)(0.25)} \Rightarrow \underline{r_{on} = 400 \text{ k}\Omega}$$

$$r_{op} = \frac{1}{\lambda_p I_0} = \frac{1}{(0.02)(0.25)} \Rightarrow \underline{r_{op} = 200 \text{ k}\Omega}$$

$$\text{b. } A_v = -g_m(r_0 \parallel r_{02} \parallel R_L)$$

$$A_v = -(0.448)[400 \parallel 200 \parallel 100]$$

$$\Rightarrow \underline{A_v = -25.6}$$

Chapter 10

Problem Solutions

10.1

a. $I_1 = I_2 = \frac{0 - 2V_T - V^-}{R_1 + R_2}$

$$2V_T + I_2 R_2 = V_{BE} + I_C R_3$$

$$2V_T + \frac{R_2}{R_1 + R_2} (-2V_T - V^-) = V_{BE} + I_C R_3$$

$$I_C = \frac{1}{R_3} \left\{ 2V_T - (2V_T + V^-) \left(\frac{R_2}{R_1 + R_2} \right) - V_{BE} \right\}$$

b. $V_T = V_{BE}$ and $R_1 = R_2$

$$I_C = \frac{1}{R_3} \left\{ 2V_T - \frac{1}{2} (2V_T + V^-) - V_{BE} \right\}$$

or $I_C = \frac{-V^-}{2R_3}$

c. $I_C = 2 \text{ mA} = \frac{-(-10)}{2R_3} \Rightarrow R_3 = 2.5 \text{ k}\Omega$

$$I_1 = I_2 = 2 \text{ mA} = \frac{-2(0.7) - (-10)}{R_1 + R_2}$$

$$\Rightarrow R_1 + R_2 = 4.3 \text{ k}\Omega$$

$$\Rightarrow R_1 = R_2 = 2.15 \text{ k}\Omega$$

10.2

$$I_{C2} = \frac{I_{REF}}{1 + \frac{2}{\beta}} = \frac{1}{1 + \frac{2}{50}}$$

$$I_{C1} = I_{C2} = 0.962 \text{ mA}$$

$$I_{B1} = I_{B2} = \frac{I_{C2}}{\beta} \Rightarrow I_{B1} = I_{B2} = 0.0192 \text{ mA}$$

10.3

(a) $I_{REF} = \frac{V^+ - V_{BE}(\text{on}) - V^-}{R_1}$

or

$$R_1 = \frac{15 - 0.7 - (-15)}{0.5} \Rightarrow R_1 = 58.6 \text{ k}\Omega$$

(b) $R_1 = \frac{V^+ - V_{BE}(\text{on}) - V^-}{I_{REF}} = \frac{0 - 0.7 - (-15)}{0.5} \Rightarrow$

$$R_1 = 28.6 \text{ k}\Omega$$

Advantage: Requires smaller resistance.

(c) For part (a):

$$I_o(\text{max}) = \frac{29.3}{(58.6)(0.95)} = 0.526 \text{ mA}$$

$$I_o(\text{min}) = \frac{29.3}{(58.6)(1.05)} = 0.476 \text{ mA}$$

$$\Delta I_o = 0.526 - 0.476 = 0.05 \text{ mA} \Rightarrow \pm 5\%$$

For part (b): $I_o(\text{max}) = \frac{14.3}{(28.6)(0.95)} = 0.526 \text{ mA}$

$$I_o(\text{min}) = \frac{14.3}{(28.6)(1.05)} = 0.476 \text{ mA}$$

$$\Delta I_o = 0.05 \text{ mA} \Rightarrow \underline{\pm 5\%}$$

10.4

a. $I_{REF} = I_0 \left(1 + \frac{2}{\beta} \right) = 2 \left(1 + \frac{2}{100} \right)$

or $I_{REF} = 2.04 \text{ mA}$

$$R_1 = \frac{15 - 0.7}{2.04} \Rightarrow R_1 = 7.01 \text{ k}\Omega$$

b. $r_0 = \frac{V_A}{I_0} = \frac{80}{2} = 40 \text{ k}\Omega$

$$\frac{\Delta I_0}{\Delta V_{CE}} = \frac{1}{r_0} \Rightarrow \Delta I_0 = \left(\frac{1}{40} \right) (9.3) = 0.2325 \text{ mA}$$

$$\frac{\Delta I_0}{I_0} = \frac{0.2325}{2} \Rightarrow \frac{\Delta I_0}{I_0} = 11.6\%$$

10.5

$I_{REF} = I_0 \left(1 + \frac{2}{\beta} \right) = (0.5) \left(1 + \frac{2}{25} \right)$

$$= 0.54 \text{ mA} = I_{REF}$$

$$R_1 = \frac{5 - 0.7}{0.54} \Rightarrow R_1 = 7.96 \text{ k}\Omega$$

10.6

a. $I_{REF} = \frac{5 - 0.7}{18} = 0.239 \text{ mA}$

$$I_0 = \frac{0.239}{1 + \frac{2}{50}} \Rightarrow I_0 = 0.230 \text{ mA}$$

b. $r_0 = \frac{V_A}{I_0} = \frac{50}{0.230} = 217 \text{ k}\Omega$

$$\Delta I_0 = \frac{1}{r_0} \cdot \Delta V_{EC} = \left(\frac{1}{217} \right) (1.3) = 0.00599 \text{ mA}$$

$$\Rightarrow I_0 = 0.236 \text{ mA}$$

c. $\Delta I_0 = \left(\frac{1}{217} \right) (3.3) = 0.0152 \text{ mA}$

$$\Rightarrow I_0 = 0.245 \text{ mA}$$

10.7

$$a. \quad I_{REF} = 1 = \frac{5 - 0.7 - (-5)}{R_1}$$

$$\Rightarrow R_1 = 9.3 \text{ k}\Omega$$

$$b. \quad I_0 = 2I_{REF} \Rightarrow I_0 = 2 \text{ mA}$$

$$c. \quad \text{For } V_{BE1}(\text{min}) = 0.7 \Rightarrow R_{C1} = \frac{5 - 0.7}{2}$$

$$\Rightarrow R_{C1} = 2.15 \text{ k}\Omega$$

10.8

$$I_0 = nI_{C1}$$

$$I_{REF} = I_{C1} + I_{B1} + I_{B2} = I_{C1} + \frac{I_{C1}}{\beta} + \frac{I_0}{\beta}$$

$$I_{REF} = I_{C1} \left(1 + \frac{1}{\beta} + \frac{n}{\beta} \right) = I_{C1} \left(1 + \frac{1+n}{\beta} \right)$$

$$= \frac{I_0}{n} \left(1 + \frac{1+n}{\beta} \right)$$

$$\text{or } I_0 = \frac{nI_{REF}}{\left(1 + \frac{1+n}{\beta} \right)}$$

10.9

Using the results of Problem 10.8.

$$2 = \frac{2I_{REF}}{1 + \frac{3}{50}} \Rightarrow I_{REF} = 1.06 \text{ mA}$$

$$R_1 = \frac{5 - 0.7}{1.06} \Rightarrow R_1 = 4.06 \text{ k}\Omega$$

10.10

First approximation - BE area of Q_2 is 3 times that of Q_1 .

$$R_1 \equiv \frac{5 - 0.7 - (-5)}{0.5} \Rightarrow R_1 = 18.6 \text{ k}\Omega$$

Second approximation - take into account I_C vs V_{BE} variation.For Q_1 :

$$\frac{I_{C1}}{I_{C2}} = \exp\left(\frac{V_{BE1} - V_{BE2}}{V_T}\right)$$

or

$$V_{BE1} - V_{BE2} = V_T \ln\left(\frac{I_{C1}}{I_{C2}}\right)$$

$$0.7 - V_{BE2} = 0.026 \ln\left(\frac{1}{0.5}\right) \Rightarrow V_{BE} = 0.682 \text{ V for}$$

$$I_C = 0.5 \text{ mA}$$

Then

$$R_1 = \frac{5 - 0.682 - (-5)}{0.5} \Rightarrow R_1 = 18.64 \text{ k}\Omega$$

Now

$$I_{S1} = \frac{I_C}{\exp\left(\frac{V_{BE}}{V_T}\right)} = \frac{1 \times 10^{-3}}{\exp\left(\frac{0.7}{0.026}\right)} \Rightarrow I_{S1} = 2.03 \times 10^{-15} \text{ A}$$

For Q_2 :

$$I_{S2} = \frac{I_{C2}}{\exp\left(\frac{V_{BE2}}{V_T}\right)} = \frac{1.5 \times 10^{-3}}{\exp\left(\frac{0.682}{0.026}\right)} = 6.09 \times 10^{-15} \text{ A}$$

10.11

$$I_2 = 2I_1 \text{ and } I_3 = 3I_1$$

$$(a) \quad I_2 = 1.0 \text{ mA}, I_3 = 1.5 \text{ mA}$$

$$(b) \quad I_1 = 0.25 \text{ mA}, I_3 = 0.75 \text{ mA}$$

$$(c) \quad I_1 = 0.167 \text{ mA}, I_2 = 0.333 \text{ mA}$$

10.12

$$a. \quad I_0 = I_{C1} \text{ and } I_{REF} = I_{C1} + I_{BS} = I_{C1} + \frac{I_{ES}}{1 + \beta}$$

$$I_{ES} = I_{B1} + I_{B2} + \frac{V_{BE}}{R_2} = \frac{2I_{C1}}{\beta} + \frac{V_{BE}}{R_2}$$

$$I_{REF} = I_{C1} + \frac{2I_{C1}}{\beta(1 + \beta)} + \frac{V_{BE}}{(1 + \beta)R_2}$$

$$I_{REF} - \frac{V_{BE}}{(1 + \beta)R_2} = I_0 \left(1 + \frac{2}{\beta(1 + \beta)} \right)$$

$$I_0 = \frac{I_{REF} - \frac{V_{BE}}{(1 + \beta)R_2}}{\left(1 + \frac{2}{\beta(1 + \beta)} \right)}$$

$$b. \quad I_{REF} = (0.70) \left(1 + \frac{2}{(80)(81)} \right) + \frac{0.7}{(81)(10)}$$

$$I_{REF} = 0.700216 + 0.000864$$

$$I_{REF} = 0.7011 \text{ mA} = \frac{10 - 2(0.7)}{R_1}$$

$$\Rightarrow R_1 = 12.27 \text{ k}\Omega$$

10.13

$$a. \quad I_{0i} = I_{CR} \text{ and } I_{REF} = I_{CR} + I_{BS} = I_{CR} + \frac{I_{ES}}{1 + \beta}$$

$$I_{ES} = I_{BR} + I_{B1} + I_{B2} + \dots + I_{BN} = (1 + N)I_{BR}$$

$$= \frac{(1 + N)I_{CR}}{\beta}$$

$$\text{Then } I_{REF} = I_{CR} + \frac{(1 + N)I_{CR}}{\beta(1 + \beta)}$$

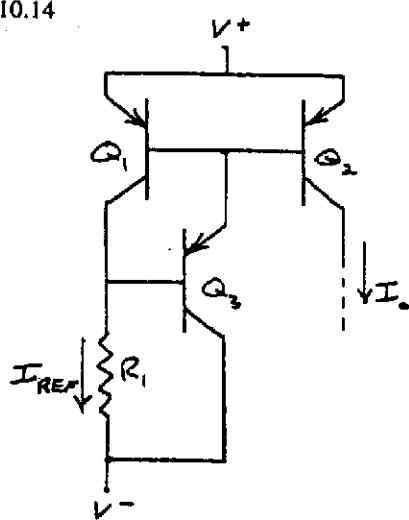
$$\text{or } I_{0i} = \frac{I_{REF}}{\left(1 + \frac{(1 + N)}{\beta(1 + \beta)} \right)}$$

$$b. \quad I_{REF} = (0.5) \left[1 + \frac{6}{(50)(51)} \right] = 0.5012 \text{ mA}$$

$$R_1 = \frac{5 - 2(0.7) - (-5)}{0.5012}$$

$$\Rightarrow R_1 = 17.16 \text{ k}\Omega$$

10.14

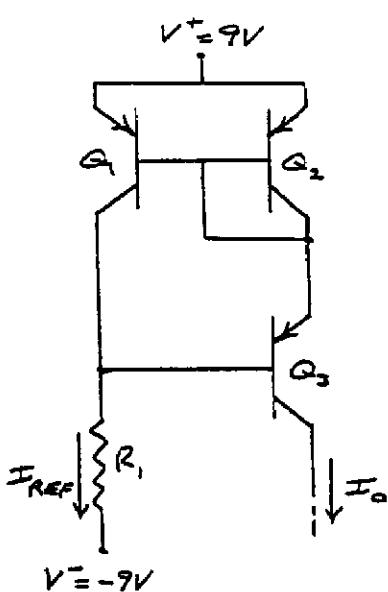


$$I_{REF} = I_0 \left(1 + \frac{2}{\beta(1+\beta)}\right) = (0.5) \left[1 + \frac{2}{(50)(51)}\right]$$

$$\Rightarrow I_{REF} = 0.5004 \text{ mA}$$

$$R_1 = \frac{5 - 2(0.7) - (-5)}{0.5004} \Rightarrow R_1 = 17.19 \text{ k}\Omega$$

10.15



$$I_0 = I_{REF} \cdot \frac{1}{\left(1 + \frac{2}{\beta(2+\beta)}\right)}$$

$$\text{For } I_0 = 0.8 \text{ mA}$$

$$I_{REF} = (0.8) \left(1 + \frac{2}{25(27)}\right)$$

$$\Rightarrow I_{REF} = 0.8024 \text{ mA}$$

$$R_1 = \frac{16 - 2(0.7)}{0.8024} \Rightarrow R_1 = 20.69 \text{ k}\Omega$$

10.16

The analysis is exactly the same as in the text. We have

$$I_0 = I_{REF} \cdot \frac{1}{\left(1 + \frac{2}{\beta(2+\beta)}\right)}$$

10.17

$$I_0 = 2 \text{ mA}, I_{B2} = \frac{2}{75} = 0.0267 \text{ mA}$$

$$I_{C1} = 1 \text{ mA}, I_{B1} = \frac{1}{75} = 0.0133 \text{ mA}$$

$$I_{E3} = I_{B1} + I_{B2} = 0.0133 + 0.0267 = 0.04 \text{ mA}$$

$$I_{B3} = \frac{I_{E3}}{1+\beta} = \frac{0.04}{76} = 0.000526 \text{ mA}$$

$$I_{REF} = I_{C1} + I_{B3} \Rightarrow I_{REF} = 1.000526 \approx 1 \text{ mA}$$

$$R_1 = \frac{10 - 2(0.7)}{I_{REF}} = \frac{8.6}{1} \Rightarrow R_1 = 8.6 \text{ k}\Omega$$

10.18

a. We have

$$R_o \approx \frac{\beta r_{03}}{2}$$

$$r_{03} = \frac{V_A}{I_0} \approx \frac{V_A}{I_{REF}} = \frac{80}{0.5} = 160 \text{ k}\Omega$$

Then

$$R_o \approx \frac{(80)(160)}{2} \Rightarrow R_o \approx 5.4 \text{ M}\Omega$$

$$\text{b. } \Delta I_0 = \frac{1}{R_o} \cdot \Delta V_C = \frac{5}{6.4} \Rightarrow \Delta I_0 = 0.781 \mu\text{A}$$

10.19

$$V_{BE} = V_T \ln \left(\frac{I_{REF}}{I_S} \right)$$

$$0.7 = (0.026) \ln \left(\frac{10^{-3}}{I_S} \right) \Rightarrow I_S = 2.03 \times 10^{-15} \text{ A}$$

$$\text{At } 2 \text{ mA, } V_{BE} = (0.026) \ln \left(\frac{2 \times 10^{-3}}{2.03 \times 10^{-15}} \right) \\ = 0.718 \text{ V}$$

$$R_1 = \frac{15 - 0.718}{2} \Rightarrow R_1 = 7.14 \text{ k}\Omega$$

$$R_E = \frac{V_T}{I_0} \ln \left(\frac{I_{REF}}{I_0} \right) = \frac{0.026}{0.050} \cdot \ln \left(\frac{2}{0.050} \right) \\ \Rightarrow R_E = 1.92 \text{ k}\Omega$$

10.20

$$a. \quad I_{REF} \approx \frac{10 - 0.7}{20} = 0.465 \text{ mA}$$

 Let $V^- = 0$

$$V_{BE} \approx V_T \ln \left(\frac{I_{REF}}{I_S} \right)$$

$$0.7 = (0.026) \ln \left(\frac{10^{-3}}{I_S} \right) \Rightarrow I_S = 2.03 \times 10^{-15} \text{ A}$$

Then

$$V_{BE} \approx (0.026) \ln \left(\frac{0.465 \times 10^{-3}}{2.03 \times 10^{-15}} \right) = 0.680 \text{ V}$$

Then

$$I_{REF} \approx \frac{10 - 0.680}{20} \Rightarrow I_{REF} = 0.466 \text{ mA}$$

$$b. \quad R_E = \frac{V_T}{I_o} \ln \left(\frac{I_{REF}}{I_o} \right) = \frac{0.026}{0.10} \cdot \ln \left(\frac{0.466}{0.10} \right)$$

$$\Rightarrow R_E = 400\Omega$$

10.21

$$(a) \quad I_{REF} = \frac{V^+ - V_{BE1} - V^-}{R_1} = \frac{5 - 0.7 - (-5)}{100} \Rightarrow$$

$$I_{REF} = 93 \mu\text{A}$$

$$I_o R_E = V_T \ln \left(\frac{I_{REF}}{I_o} \right) \Rightarrow I_o (10) = 0.026 \ln \left(\frac{93 \times 10^{-3} \text{ mA}}{I_o} \right)$$

 By trial and error, $I_o \approx 68 \mu\text{A}$

$$R_o = r_{o2}(1 + g_{m2} R'_E)$$

Now

$$r_{o2} = \frac{30}{6.8} = 4.41 \text{ M}\Omega$$

$$g_{m2} = \frac{0.0068}{0.026} = 0.262 \text{ mA/V}$$

$$r_{x2} = \frac{(100)(0.026)}{0.0068} = 382 \text{ k}\Omega$$

So

$$R'_E = r_{x2} \| R_E = 382 \| 10 = 9.74 \text{ k}\Omega$$

Then

$$R_o = 4.41[1 + (0.262)(9.74)] \Rightarrow R_o = 15.7 \text{ M}\Omega$$

$$(d) \quad V_{BE1} - V_{BE2} = I_o R_E = (0.0068)(10) \Rightarrow$$

$$V_{BE1} - V_{BE2} = 0.068 \text{ V}$$

10.22

$$\Delta I_o = \frac{1}{R_0} \cdot \Delta V_C$$

$$R_0 = r_{o2}(1 + g_{m2} R'_E)$$

$$r_{o2} = \frac{V_A}{I_0} = \frac{80}{17.4} = 4.6 \text{ M}\Omega$$

$$g_{m2} = \frac{I_0}{V_T} = \frac{0.0174}{0.026} = 0.669 \text{ mA/V}$$

$$r_{x2} = \frac{(80)(0.026)}{0.0174} = 119.5 \text{ k}\Omega$$

$$R'_E = R_E \| r_{x2} = 7 \| 119.5$$

$$R'_E = 6.61 \text{ k}\Omega$$

$$R_0 = (4.6)[1 + (0.669)(6.61)] \Rightarrow R_0 = 24.9 \text{ M}\Omega$$

Now

$$\Delta I_o = \left(\frac{1}{24.9} \right) (5) \Rightarrow \Delta I_o = 0.201 \mu\text{A}$$

10.23

$$R_0 = r_{o2}(1 + g_{m2} R'_E) \text{ where } R'_E = R_E \| r_{x2}$$

$$r_{o2} = \frac{V_A}{I_0} = \frac{75}{25} = 3 \text{ M}\Omega$$

$$g_{m2} = \frac{I_0}{V_T} = \frac{0.025}{0.026} = 0.962 \text{ mA/V}$$

$$r_{x2} = \frac{\beta V_T}{I_0} = \frac{(80)(0.026)}{0.025} = 83.2 \text{ k}\Omega$$

$$R_E = \frac{V_T}{I_0} \ln \left(\frac{I_{REF}}{I_o} \right) = \frac{0.026}{0.025} \cdot \ln \left(\frac{0.75}{0.025} \right) = 3.54 \text{ k}\Omega$$

$$R'_E = 3.54 \| 83.2 = 3.40 \text{ k}\Omega$$

$$R_0 = 3[1 + (0.962)(3.4)] = 12.8 \text{ M}\Omega$$

$$\Delta I_o = \frac{1}{R_0} \cdot \Delta V_{C2} = \frac{3}{12.8} = 0.234 \mu\text{A}$$

So

$$\frac{\Delta I_o}{I_0} = \frac{0.234}{25} \Rightarrow 0.936\%$$

10.24

 Let $R_1 = 5 \text{ k}\Omega$, Then

$$I_{REF} = \frac{12 - 0.7 - (-12)}{5} \Rightarrow I_{REF} = 4.66 \text{ mA}$$

Now

$$I_o R_E = V_T \ln \left(\frac{I_{REF}}{I_o} \right) \Rightarrow$$

$$R_E = \frac{0.026}{0.10} \ln \left(\frac{4.66}{0.10} \right) \Rightarrow R_E \approx 1 \text{ k}\Omega$$

10.25

$$I_{REF} \approx \frac{10 - 0.7 - (-10)}{40} = 0.4825 \text{ mA}$$

$$V_{BE} \equiv V_T \ln \left(\frac{I_{REF}}{I_S} \right)$$

$$0.7 = (0.026) \ln \left(\frac{10^{-3}}{I_S} \right) \Rightarrow I_S = 2.03 \times 10^{-15} \text{ A}$$

Now

$$V_{BE} = (0.026) \ln \left(\frac{0.4825 \times 10^{-3}}{2.03 \times 10^{-15}} \right) = 0.681 \text{ V}$$

$$V_{BE1} = 0.681 \text{ V}$$

So

$$I_{REF} \approx \frac{10 - 0.681 - (-10)}{40}$$

$$\Rightarrow I_{REF} = 0.483 \text{ mA}$$

$$I_0 R_E = V_T \ln \left(\frac{I_{REF}}{I_0} \right)$$

$$I_0 (12) = (0.026) \ln \left(\frac{0.483}{I_0} \right)$$

By trial and error,

$$\Rightarrow I_0 \approx 8.7 \mu\text{A}$$

$$V_{BE2} = V_{BE1} - I_0 R_E = 0.681 - (0.0087)(12)$$

$$\Rightarrow V_{BE2} = 0.5766 \text{ V}$$

10.26

$$V_{BE1} + I_{REF} R_{E1} = V_{BE2} + I_0 R_{E2}$$

$$V_{BE1} - V_{BE2} = I_0 R_{E2} - I_{REF} R_{E1}$$

For matched transistors

$$V_{BE1} = V_T \ln \left(\frac{I_{REF}}{I_S} \right)$$

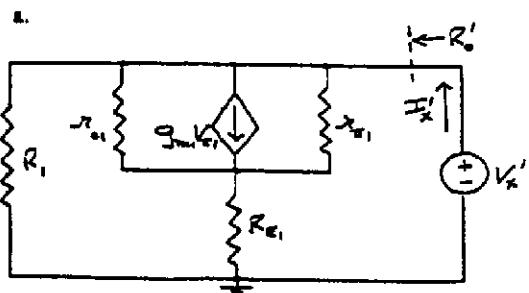
$$V_{BE2} = V_T \ln \left(\frac{I_0}{I_S} \right)$$

Then

$$V_T \ln \left(\frac{I_{REF}}{I_0} \right) = I_0 R_{E2} - I_{REF} R_{E1}$$

 Output resistance looking into the collector of Q_2 is increased.

10.27



$$I'_X = \frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} + \frac{V_{\pi 1}}{r_{01}} + \frac{V'_X}{R'_1} \quad (1)$$

$$V'_X = V_{\pi 1} + \left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} + \frac{V_{\pi 1}}{r_{01}} \right) R_{E1} \quad (2)$$

$$V'_X = V_{\pi 1} \left[1 + \left(\frac{1}{r_{\pi 1}} + g_{m1} + \frac{1}{r_{01}} \right) R_{E1} \right]$$

$$I_{REF} \approx \frac{10 - 0.7}{13.6 + 5} = 0.5 \text{ mA}$$

$$r_{\pi 1} = \frac{(50)(0.026)}{0.5} = 2.6 \text{ k}\Omega$$

$$r_{01} = \frac{75}{0.5} = 150 \text{ k}\Omega$$

$$g_{m1} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

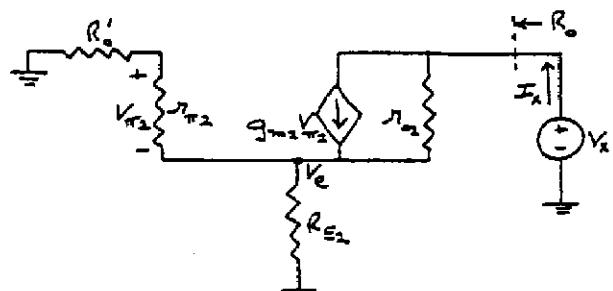
$$V'_X = V_{\pi 1} \left[1 + \left(\frac{1}{2.6} + 19.23 + \frac{1}{150} \right) (5) \right]$$

$$\Rightarrow V_{\pi 1} = V'_X (0.01009)$$

$$\text{Then } I'_X = V'_X (0.01009) \left[\frac{1}{2.6} + 19.23 + \frac{1}{150} \right] + \frac{V'_X}{13.6}$$

$$I'_X = V'_X (0.1980) + V'_X (0.0735)$$

$$\Rightarrow R'_0 = \frac{V'_X}{I'_X} = 3.683 \text{ k}\Omega$$



$$I_X = \frac{V_X - V_e}{r_{02}} + g_{m2} V_{\pi 2}$$

$$V_e = I_X [R_{E2} \parallel (r_{\pi 2} + R'_0)]$$

$$V_{\pi 2} = - \left(\frac{r_{\pi 2}}{r_{\pi 2} + R'_0} \right) V_e$$

Then

$$I_X = \frac{V_X}{r_{02}} - \frac{I_X}{r_{02}} [R_{E2} \parallel (r_{\pi 2} + R'_0)] - g_{m2} \left(\frac{r_{\pi 2}}{r_{\pi 2} + R'_0} \right) (I_X) [R_{E2} \parallel (r_{\pi 2} + R'_0)]$$

$$R_{E1} = R_{E2} \Rightarrow I_{REF} = I_0 \Rightarrow r_{\pi 2} = 2.6 \text{ k}\Omega$$

$$r_{02} = 150 \text{ k}\Omega$$

$$g_{m2} = 19.23 \text{ mA/V}$$

$$I_X = \frac{V_X}{150} - \frac{I_X}{150}[5||(2.6 + 3.68)] \\ - (19.23) \left(\frac{2.6}{2.6 + 3.68} \right) (I_X)[5||(2.6 + 3.68)]$$

$$I_X = V_X(0.00666) - I_X(0.01853) - I_X(22.13)$$

$$I_X(23.148) = V_X(0.00666)$$

$$\Rightarrow R_o = \frac{V_X}{I_X} = 3.48 \text{ M}\Omega$$

b. When $R_{E1} = R_{E2} = 0$

$$R_o \approx r_{o2} = 150 \text{ k}\Omega$$

10.28

Assume all transistors are matched.

a.

$$2V_{BE1} = V_{BE3} + I_o R_E$$

$$V_{BE1} = V_T \ln \left(\frac{I_{REF}}{I_S} \right)$$

$$V_{BE3} = V_T \ln \left(\frac{I_o}{I_S} \right)$$

$$2V_T \ln \left(\frac{I_{REF}}{I_S} \right) - V_T \ln \left(\frac{I_o}{I_S} \right) = I_o R_E$$

$$V_T \left[\ln \left(\frac{I_{REF}}{I_S} \right)^2 - \ln \left(\frac{I_o}{I_S} \right) \right] = I_o R_E$$

$$V_T \ln \left(\frac{I_{REF}^2}{I_o I_S} \right) = I_o R_E$$

$$\text{b. } V_{BE} = 0.7 \text{ V at } 1 \text{ mA} \Rightarrow 10^{-3} = I_S \exp \left(\frac{0.7}{0.026} \right)$$

$$\text{or } I_S = 2.03 \times 10^{-15} \text{ A}$$

$$V_{BE} \text{ at } 0.1 \text{ mA}$$

$$\Rightarrow V_{BE} = (0.026) \ln \left(\frac{0.1 \times 10^{-3}}{2.03 \times 10^{-15}} \right) = 0.640 \text{ V}$$

Since $I_o = I_{REF}$, then

$$V_{BE} = I_o R_E \Rightarrow R_E = \frac{0.640}{0.1}$$

$$\text{or } R_E = 6.4 \text{ k}\Omega$$

10.29

$$I_{REF} = \frac{10 - 0.7}{R_1} = 0.5 \Rightarrow R_1 = 18.6 \text{ k}\Omega$$

$$I_{o1} R_{E2} = V_T \ln \left(\frac{I_{REF}}{I_{o1}} \right)$$

$$R_{E2} = \frac{0.026}{0.010} \cdot \ln \left(\frac{0.50}{0.01} \right) \Rightarrow R_{E2} = 10.17 \text{ k}\Omega$$

$$R_{E3} = \frac{0.026}{0.030} \cdot \ln \left(\frac{0.50}{0.03} \right) \Rightarrow R_{E3} = 2.438 \text{ k}\Omega$$

$$V_{BE2} = 0.7 - I_{o2} R_{E2} = 0.7 - (0.01)(10.17)$$

$$\Rightarrow V_{BE2} = 0.598 \text{ V}$$

$$V_{BE3} = 0.7 - I_{o3} R_{E3} = 0.7 - (0.03)(2.438)$$

$$\Rightarrow V_{BE3} = 0.627 \text{ V}$$

10.30

$$(a) V_{BE1} = V_{BE2}$$

$$I_{REF} = \frac{V^+ - 2V_{BE1} - V^-}{R_1 + R_2}$$

Now

$$2V_{BE1} + I_{REF} R_2 = V_{BE3} + I_o R_E$$

or

$$I_o R_E = 2V_{BE1} - V_{BE3} + I_{REF} R_2$$

We have

$$V_{BE1} = V_T \ln \left(\frac{I_{REF}}{I_S} \right) \text{ and } V_{BE3} = V_T \ln \left(\frac{I_o}{I_S} \right)$$

$$(b) \text{ Let } R_1 = R_2 \text{ and } I_o = I_{REF} \Rightarrow V_{BE1} = V_{BE3} \equiv V_{BE}$$

Then

$$V_{BE} = I_o R_E - I_{REF} R_2 = I_o (R_E - R_2)$$

so

$$I_{REF} = I_o = \frac{V^+ - V^- - 2I_o(R_E - R_2)}{2R_2} \\ = \frac{V^+ - V^-}{2R_2} - I_o \left(\frac{R_E}{R_2} \right) + I_o$$

Then

$$I_o = \frac{V^+ - V^-}{2R_E}$$

(c) Want $I_o = 0.5 \text{ mA}$

$$\text{So } R_E = \frac{5 - (-5)}{2(0.5)} \Rightarrow R_E = 10 \text{ k}\Omega$$

$$2R_2 = \frac{5 - 2(0.7) - (-5)}{0.5} = 17.2 \text{ k}\Omega$$

$$\text{Then } R_1 = R_2 = 8.6 \text{ k}\Omega$$

10.31

$$\text{a. } I_{REF} = \frac{20 - 0.7 - 0.7}{12} = 1.55 \text{ mA}$$

$$I_{o1} = 2I_{REF} = 3.1 \text{ mA}$$

$$I_{o2} = I_{REF} = 1.55 \text{ mA}$$

$$I_{o3} = 3I_{REF} = 4.65 \text{ mA}$$

$$\text{b. } V_{CE1} = -I_{o1} R_{C1} - (-10) = -(3.1)(2) + 10$$

$$\Rightarrow V_{CE1} = 3.8 \text{ V}$$

$$V_{EC2} = 10 - I_{o2} R_{C2} = 10 - (1.55)(3)$$

$$\Rightarrow V_{EC2} = 5.35 \text{ V}$$

$$V_{EC3} = 10 - I_{o3} R_{C3} = 10 - (4.65)(1)$$

$$\Rightarrow V_{EC3} = 5.35 \text{ V}$$

10.32

a. 1st approximation

$$I_{REF} \approx \frac{20 - 1.4}{8} = 2.325 \text{ mA}$$

$$\text{Now } V_{BS} - 0.7 = (0.026) \ln \left(\frac{2.32}{1} \right)$$

$$\Rightarrow V_{BS} = V_{EB} = 0.722 \text{ V}$$

Then 2nd approximation

$$I_{REF} \approx \frac{20 - 2(0.722)}{8} = 2.32 \text{ mA}$$

$$I_{01} = 2I_{REF} = 4.64 \text{ mA}$$

$$I_{02} = I_{REF} = 2.32 \text{ mA}$$

$$I_{03} = 3I_{REF} = 6.96 \text{ mA}$$

 b. At the edge of saturation, $V_{CS} = V_{BE} = 0.722 \text{ V}$

$$R_{C1} = \frac{0 - 0.722 - (-10)}{4.64} \Rightarrow R_{C1} = 2.0 \text{ k}\Omega$$

$$R_{C2} = \frac{10 - 0.722}{2.32} \Rightarrow R_{C2} = 4.0 \text{ k}\Omega$$

$$R_{C3} = \frac{10 - 0.722}{6.96} \Rightarrow R_{C3} = 1.33 \text{ k}\Omega$$

b. $V_{DS1} = 4 \text{ V}$

$$I_0 = (200) \left[\frac{1 + (0.02)(4)}{1 + (0.02)(1.89)} \right] \Rightarrow I_0 \approx 208 \mu\text{A}$$

c. $V_{DS2} = 6 \text{ V}$

$$I_0 = (200) \left[\frac{1 + (0.02)(6)}{1 + (0.02)(1.89)} \right] \Rightarrow I_0 \approx 216 \mu\text{A}$$

10.35

$$(a) V_{GS} = V_{TN1} + \sqrt{\frac{I_{REF}}{K_{n1}}} = 1 + \sqrt{\frac{0.5}{0.5}} = 2 \text{ V}$$

$$I_0 = K_{n2} \left(\sqrt{\frac{I_{REF}}{K_{n1}}} \right)^2 = K_{n2} \left(\frac{I_{REF}}{K_{n1}} \right)$$

$$I_0(\max) = (0.5)(1.05) \left(\frac{0.5}{0.5} \right) \Rightarrow I_0(\max) = 0.525 \text{ mA}$$

$$I_0(\min) = (0.5)(0.95) \left(\frac{0.5}{0.5} \right) \Rightarrow I_0(\min) = 0.475 \text{ mA}$$

So

$$0.475 \leq I_0 \leq 0.525 \text{ mA}$$

10.33

1st approximation

$$I_{REF} = \frac{10 - 0.7}{6.3 + 3} = 1 \text{ mA}$$

 $\Rightarrow V_B = 0.7 \text{ V}$ as assumed

$$V_{REF} = I_{REF} \cdot R_{ER} = (1)(3) = 3 \text{ V}$$

$$V_{RE1} = 3 \text{ V} \Rightarrow R_{E1} = \frac{V_{RE1}}{I_{01}} = \frac{3}{1} \Rightarrow R_{E1} = 3 \text{ k}\Omega$$

$$V_{RE2} = 3 \text{ V} \Rightarrow R_{E2} = \frac{V_{RE2}}{I_{02}} = \frac{3}{2} \Rightarrow R_{E2} = 1.5 \text{ k}\Omega$$

$$V_{RE3} = 3 \text{ V} \Rightarrow R_{E3} = \frac{V_{RE3}}{I_{03}} = \frac{3}{4} \Rightarrow R_{E3} = 0.75 \text{ k}\Omega$$

$$I_{01} = 1 \text{ mA}$$

$$I_{02} = 2 \text{ mA}$$

$$I_{03} = 4 \text{ mA}$$

$$(b) I_0 = K_{n2} \left[\sqrt{\frac{I_{REF}}{K_{n1}}} + V_{TN1} - V_{TN2} \right]^2$$

$$I_0(\min) = (0.5) \left[\sqrt{\frac{0.5}{0.5}} + 1 - 1.05 \right]^2$$

$$\Rightarrow I_0(\min) = 0.451 \text{ mA}$$

$$I_0(\max) = (0.5) \left[\sqrt{\frac{0.5}{0.5}} + 1 - 0.95 \right]^2$$

$$\Rightarrow I_0(\max) = 0.551 \text{ mA}$$

So

$$0.451 \leq I_0 \leq 0.551 \text{ mA}$$

10.34

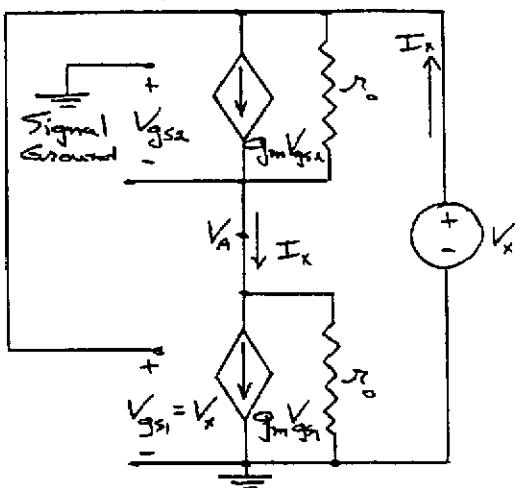
$$V_{GS} = V_{TN1} + \sqrt{\frac{I_{REF}}{K_{n1}}} = 1 + \sqrt{\frac{200}{250}} = 1.89 \text{ V} = V_{DS1}$$

$$\frac{I_0}{I_{REF}} = \frac{1 + \lambda V_{DS2}}{1 + \lambda V_{DS1}}$$

$$a. V_{DS2} = 2 \text{ V}$$

$$I_0 = (200) \left[\frac{1 + (0.02)(2)}{1 + (0.02)(1.89)} \right] \Rightarrow I_0 \approx 200 \mu\text{A}$$

10.36



10.37

$$(a) V_{DS(sat)} = V_{GS} - V_{TN}$$

$$\text{or } V_{GS} = V_{DS(sat)} + V_{TN} = 0.2 + 0.8 = 1.0$$

$$I_D = \frac{k_n'}{2} \left(\frac{W}{L} \right) (V_{GS} - V_{TN})^2$$

$$50 = 48 \left(\frac{W}{L} \right) (0.2)^2 \Rightarrow \left(\frac{W}{L} \right) = 26$$

$$(b) V_{GSs} - V_{TN} = 2(V_{GS} - V_{TN})$$

$$V_{GSs} = 0.8 + 2(0.2) \Rightarrow \underline{\underline{V_{GSs} = 1.2 \text{ V}}}$$

$$(c) V_{Dl(\min)} = 2V_{DS(sat)} = 2(0.2) \Rightarrow$$

$$\underline{\underline{V_{Dl(\min)} = 0.4 \text{ V}}}$$

10.38

$$(1) I_x = \frac{V_x - V_A}{r_o} + g_m V_{gs2}$$

$$(2) I_x = \frac{V_A}{r_o} + g_m V_{gs1}$$

$$V_{gs1} = V_x, \quad V_{gs2} = -V_A$$

So

$$(1) I_x = \frac{V_x}{r_o} - V_A \left(\frac{1}{r_o} + g_m \right)$$

$$(2) I_x = \frac{V_A}{r_o} + g_m V_x \Rightarrow V_A = r_o [I_x - g_m V_x]$$

Then

$$I_x = \frac{V_x}{r_o} - r_o (I_x - g_m V_x) \left(\frac{1}{r_o} + g_m \right)$$

$$I_x = \frac{V_x}{r_o} - r_o \left[\frac{I_x}{r_o} + g_m I_x - \frac{g_m}{r_o} \cdot V_x - g_m^2 V_x \right]$$

$$I_x = \frac{V_x}{r_o} - I_x - g_m r_o I_x + g_m V_x + g_m^2 r_o V_x$$

$$I_x [2 + g_m r_o] = V_x \left[\frac{1}{r_o} + g_m + g_m^2 r_o \right]$$

$$\text{Since } g_m \gg \frac{1}{r_o}$$

$$I_x [2 + g_m r_o] \approx V_x (g_m) (1 + g_m r_o)$$

Then

$$\frac{V_x}{I_x} = R_o = \frac{2 + g_m r_o}{g_m (1 + g_m r_o)}$$

Usually, $g_m r_o \gg 2$, so that

$$R_o \equiv \frac{1}{g_m}$$

$$V_{DS2(sat)} = 2 \text{ V} = V_{GS2} - V_{TN2} = V_{GS2} - 1.5 \Rightarrow$$

$$V_{GS2} = 3.5 \text{ V}$$

$$I_D = \left(\frac{1}{2} \mu_n C_{ox} \right) \left(\frac{W}{L} \right)_2 (V_{GS2} - V_{TN2})^2$$

$$250 = (20) \left(\frac{W}{L} \right)_2 (3.5 - 1.5)^2 \Rightarrow$$

$$\underline{\underline{\left(\frac{W}{L} \right)_2 = 3.125}}$$

$$I_{REF} = \left(\frac{1}{2} \mu_n C_{ox} \right) \left(\frac{W}{L} \right)_1 (V_{GS2} - V_{TN1})$$

$$100 = (20) \left(\frac{W}{L} \right)_1 (3.5 - 1.5)^2 \Rightarrow$$

$$\underline{\underline{\left(\frac{W}{L} \right)_1 = 1.25}}$$

$$\text{Now } V_{GS3} = 10 - V_{GS2} = 10 - 3.5 = 6.5 \text{ V}$$

$$\text{So } 100 = (20) \left(\frac{W}{L} \right)_3 (6.5 - 1.5)^2 \Rightarrow$$

$$\underline{\underline{\left(\frac{W}{L} \right)_3 = 0.2}}$$

10.39

a. From Equation (10.50).

$$V_{GS1} = V_{GS2} = \left(\frac{\sqrt{\frac{5}{25}}}{1 + \sqrt{\frac{5}{25}}} \right) (5) + \left(\frac{1 - \sqrt{\frac{5}{25}}}{1 + \sqrt{\frac{5}{25}}} \right) (0.5)$$

$$= \left(\frac{0.447}{1 + 0.447} \right) (5) + \left(\frac{1 - 0.447}{1 + 0.447} \right) (0.5)$$

$$V_{GS1} = V_{GS2} = 1.74 \text{ V}$$

$$I_{REF} = K_n(V_{GS1} - V_{TN})^2 = (18)(25)(1.74 - 0.5)^2 \Rightarrow I_{REF} = 0.692 \text{ mA}$$

$$\text{b. } I_o = \left(\frac{1}{2} \mu_n C_{ox} \right) \left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TN})^2 (1 + \lambda V_{DS2})$$

$$I_o = (18)(15)(1.74 - 0.5)^2 [1 + (0.02)(2)] \\ = (415)(104) \Rightarrow I_o = 0.432 \text{ mA}$$

$$\text{c. } I_o = (415)[1 + (0.02)(4)]$$

$$\Rightarrow I_o = 0.448 \text{ mA}$$

10.40

$$\text{(a) } I_{REF} = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_1 (V_{SG1} + V_{TP})^2 \\ = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_3 (V_{SG3} + V_{TP})^2$$

$$\text{But } V_{SG3} = 3 - V_{SG1}$$

So

$$25(V_{SG1} - 0.4)^2 = 5(3 - V_{SG1} - 0.4)^2$$

$$\text{which yields } V_{SG1} = 1.08 \text{ V and } V_{SG3} = 1.92 \text{ V}$$

$$I_{REF} = 20(25)(1.08 - 0.4)^2 \Rightarrow I_{REF} = 231 \mu\text{A}$$

$$\frac{I_o}{I_{REF}} = \frac{(W/L)_2}{(W/L)_1} = \frac{15}{25} = 0.6$$

$$\text{Then } I_o = (0.6)(231) = 139 \mu\text{A}$$

$$\text{(b) } V_{DS2}(\text{sat}) = 1.08 - 0.4 = 0.68 \text{ V}$$

$$V_R = 3 - 0.68 = 2.32 = I_o R$$

then

$$R = \frac{2.32}{0.139} \Rightarrow R = 16.7 \text{ k}\Omega$$

10.41

$$V_{SD1}(\text{sat}) = 0.25 = V_{SG} + V_{TP} = V_{SG} - 0.4 \Rightarrow$$

$$V_{SG1} = 0.65 \text{ V}$$

$$I_o = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_2 (V_{SG2} + V_{TP})^2$$

$$25 = \frac{40}{2} \left(\frac{W}{L}\right)_2 (0.65 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 20$$

$$I_{REF} = 75 \mu\text{A} = \frac{(W/L)_1}{(W/L)_2} \cdot I_o \Rightarrow \left(\frac{W}{L}\right)_1 = 60$$

$$I_{REF} = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_3 (V_{SG3} + V_{TP})^2$$

$$V_{SG3} = 3 - 0.65 = 2.35 \text{ V}$$

Then

$$75 = \frac{40}{2} \left(\frac{W}{L}\right)_3 (2.35 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 0.986$$

10.42

$$\text{a. } I_{REF} = K_n(V_{GS} - V_{TN})^2$$

$$100 = 100(V_{GS} - 2)^2 \Rightarrow V_{GS} = 3 \text{ V}$$

For $V_{D4} = -3 \text{ V}$, $I_0 \approx 100 \mu\text{A}$

$$\text{b. } R_o = r_{o4} + r_{o2}(1 + g_m r_{o4})$$

$$r_{o2} = r_{o4} = \frac{1}{\lambda I_0} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

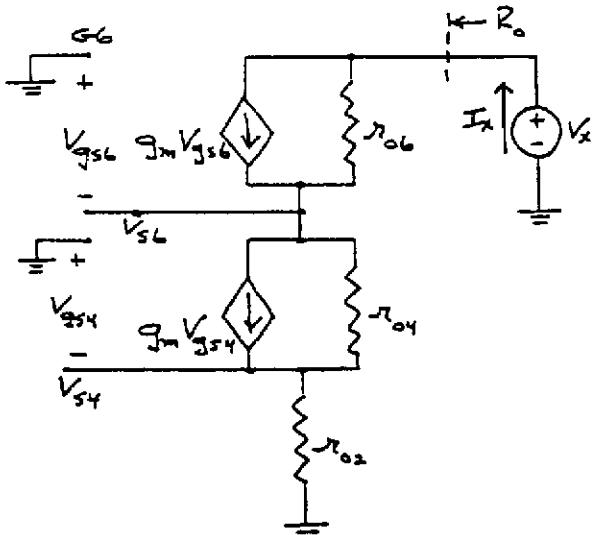
$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.1)(3 - 2) = 0.2 \text{ mA/V}$$

$$R_o = 500 + 500[1 + (0.2)(500)]$$

$$R_o = 51 \text{ M}\Omega$$

$$\Delta I_o = \frac{1}{R_o} \cdot \Delta V_{D4} = \frac{6}{51} \Rightarrow \Delta I_o = 0.118 \mu\text{A}$$

10.43



$$V_{GS4} = -I_X r_{o2}$$

$$V_{S6} = (I_X - g_m V_{GS4}) r_{o4} + I_X r_{o2}$$

$$= (I_X + g_m I_X r_{o2}) r_{o4} + I_X r_{o2}$$

$$V_{S6} = I_X [r_{o2} + (1 + g_m r_{o2}) r_{o4}] = -V_{GS6}$$

$$I_X = g_m V_{GS6} + \frac{V_X - V_{SG}}{r_{o6}} = \frac{V_X}{r_{o6}} - V_{S6} \left(g_m + \frac{1}{r_{o6}} \right)$$

$$I_X = \frac{V_X}{r_{o6}} - I_X \left(g_m + \frac{1}{r_{o6}} \right) [r_{o2} + (1 + g_m r_{o2}) r_{o4}]$$

$$I_X \left\{ 1 + \left(g_m + \frac{1}{r_{o6}} \right) [r_{o2} + (1 + g_m r_{o2}) r_{o4}] \right\} = \frac{V_X}{r_{o6}}$$

$$\frac{V_X}{I_X} = R_o = r_{o6} + (1 + g_m r_{o6}) [r_{o2} + (1 + g_m r_{o2}) r_{o4}]$$

$$I_0 \approx I_{REF} = 0.2 \text{ mA} = 0.2(V_{GS} - 1)^2$$

$$V_{GS} = 2 \text{ V}$$

$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.2)(2 - 1) = 0.4 \text{ mA/V}$$

$$r_{DS} = r_{DS} = \frac{1}{\lambda I_D} = \frac{1}{(0.02)(0.2)} = 250 \text{ k}\Omega$$

$$R_O = 250 + [1 + (0.4)(250)] \times [250 + [1 + (0.4)(250)](250)]$$

$$R_O = 2575750 \text{ k}\Omega$$

$$\Rightarrow R_O = 2.58 \times 10^9 \Omega$$

10.44

$$\frac{k'_n}{2} \left(\frac{W}{L} \right)_1 (V_{GS1} - V_{TN})^2 = \frac{k'_n}{2} \left(\frac{W}{L} \right)_3 (V_{GS3} - V_{TN})^2$$

$$= \frac{k'_n}{2} \left(\frac{W}{L} \right)_4 (V_{GS4} + V_{TP})^2$$

$$(1) 50(20)(V_{GS1} - 0.5)^2 = 50(5)(V_{GS3} - 0.5)^2$$

$$(2) 50(20)(V_{GS1} - 0.5)^2 = 20(10)(V_{GS4} - 0.5)^2$$

$$(3) V_{GS4} + V_{GS3} + V_{GS1} = 6$$

$$\text{From (1)} 4(V_{GS1} - 0.5)^2 = (V_{GS3} - 0.5)^2 \Rightarrow$$

$$V_{GS3} = 2(V_{GS1} - 0.5) + 0.5$$

$$\text{From (2)} 5(V_{GS1} - 0.5)^2 = (V_{GS4} - 0.5)^2 \Rightarrow$$

$$V_{GS4} = \sqrt{5}(V_{GS1} - 0.5) + 0.5$$

Then (3) becomes

$$\sqrt{5}(V_{GS1} - 0.5) + 0.5 + 2(V_{GS1} - 0.5) + 0.5 + V_{GS1} = 6$$

which yields $V_{GS1} = 1.36 \text{ V}$ and

$$V_{GS3} = 2.22 \text{ V}, \quad V_{GS4} = 2.42 \text{ V}$$

Then

$$I_{REF} = \frac{k'_n}{2} \left(\frac{W}{L} \right)_1 (V_{GS1} - V_{TN})^2 = 50(20)(1.36 - 0.5)^2$$

$$\text{or } I_{REF} = I_0 = 0.740 \text{ mA}$$

$$V_{GS1} = V_{GS2} = 1.36 \text{ V}$$

$$V_{DS2}(\text{sat}) = V_{GS2} - V_{TN} = 1.36 - 0.5 \Rightarrow$$

$$V_{DS2}(\text{sat}) = 0.86 \text{ V}$$

10.45

$$V_{DS2}(\text{sat}) = 0.5 \text{ V} = V_{GS2} - V_{TN} = V_{GS2} - 0.5 \Rightarrow$$

$$V_{GS2} = 1 \text{ V}$$

$$I_0 = 50 \mu\text{A} = \frac{k'_n}{2} \left(\frac{W}{L} \right)_2 (V_{GS2} - V_{TN})^2$$

$$= 50 \left(\frac{W}{L} \right)_2 (1 - 0.5)^2 \Rightarrow \left(\frac{W}{L} \right)_2 = 4$$

$$V_{GS1} = V_{GS2} = 1 \text{ V} \Rightarrow$$

$$I_{REF} = 150 = \frac{k'_n}{2} \left(\frac{W}{L} \right)_1 (V_{GS1} - V_{TN})^2$$

$$= 50 \left(\frac{W}{L} \right)_1 (1 - 0.5)^2 \Rightarrow \left(\frac{W}{L} \right)_1 = 12$$

$$V_{GS3} + V_{GS4} + V_{GS1} = 6$$

$$2V_{GS3} = 6 - 1 = 5 \text{ V} \Rightarrow V_{GS3} = 2.5 \text{ V}$$

$$I_{REF} = 150 = 50 \left(\frac{W}{L} \right)_3 (2.5 - 0.5)^2 \Rightarrow \left(\frac{W}{L} \right)_3 = 0.75$$

$$I_{REF} = \frac{k'_p}{2} \left(\frac{W}{L} \right)_4 (V_{GS4} + V_{TP})^2$$

$$150 = 20 \left(\frac{W}{L} \right)_4 (2.5 - 0.5)^2 \Rightarrow \left(\frac{W}{L} \right)_4 = 1.88$$

10.46

a. As a first approximation

$$I_{REF} = 80 = 80(V_{GS1} - 1)^2 \Rightarrow V_{GS1} = 2 \text{ V}$$

Then $V_{DS1} \approx 2(2) = 4 \text{ V}$

The second approximation

$$80 = 80(V_{GS1} - 1)^2 [1 + (0.02)(4)]$$

$$\text{Or } \frac{80}{86.4} = (V_{GS1} - 1)^2 \Rightarrow V_{GS1} = 1.962$$

Then

$$I_0 = K_n(V_{GS1} - V_{TN})^2 (1 + \lambda_n V_{GS1})$$

$$= 80(1.962 - 1)^2 [1 + (0.02)(1.962)]$$

$$\text{Or } I_0 = 76.94 \mu\text{A}$$

b. From a PSpice analysis, $I_0 = 77.09 \mu\text{A}$ for $V_{D3} = -1 \text{ V}$ and $I_0 = 77.14 \mu\text{A}$ for $V_{D3} = 3 \text{ V}$. The change is $\Delta I_0 \approx 0.05 \mu\text{A}$ or 0.065%.

10.47

a. For a first approximation,

$$I_{REF} = 80 = 80(V_{GS4} - 1)^2 \Rightarrow V_{GS4} = 2 \text{ V}$$

As a second approximation

$$I_{REF} = 80 = 80(V_{GS4} - 1)^2 [1 + (0.02)(2)]$$

$$\text{Or } V_{GS4} = 1.98 \text{ V} = V_{GS1}$$

$$I_0 = K_n(V_{GS2} - V_{TN})^2 (1 + \lambda V_{GS2})$$

To a very good approximation

$$I_0 = 80 \mu\text{A}$$

b. From a PSpice analysis, $I_0 = 80.00 \mu\text{A}$ for $V_{D3} = -1 \text{ V}$ and the output resistance is $R_o = 76.9 \text{ M}\Omega$. Then

$$\Delta I_0 = \frac{1}{R_o} \cdot V_{D3} = \frac{4}{76.9} = 0.052 \mu\text{A}$$

or a change of 0.065%.

10.48

$$(a) K_{nl} = \frac{k'}{2} \left(\frac{W}{L} \right)_1 = 50(5) = 250 \mu A/V^2$$

$$R = \frac{1}{\sqrt{K_{nl} I_{Dl}}} \left(1 - \sqrt{\frac{(W/L)_1}{(W/L)_2}} \right)$$

$$= \frac{1}{\sqrt{(0.25)(0.05)}} \left(1 - \sqrt{\frac{5}{50}} \right) = (8.94)(0.684)$$

$$R = 6.11 k\Omega$$

$$(b) V^+ - V^- = V_{SD}(sat) + V_{GS1}$$

$$V_{SD}(sat) = V_{SG3} + V_{TP}$$

$$I_{Dl} = 50 = 20(5)(V_{SG3} - 0.5)^2 \Rightarrow V_{SG3} = 1.21 V$$

Then

$$V_{SD}(sat) = 1.21 - 0.5 = 0.71 V$$

Also

$$I_{Dl} = 50 = 50(5)(V_{GS1} - 0.5)^2 \Rightarrow V_{GS1} = 0.947 V$$

Then

$$(V^+ - V^-)_{min} = 0.71 + 0.947 = 1.66 V$$

$$(c) I_{o1} = 25 = 50 \left(\frac{W}{L} \right)_3 (0.947 - 0.5)^2 \Rightarrow \left(\frac{W}{L} \right)_3 = 2.5$$

$$I_{o2} = 75 = 20 \left(\frac{W}{L} \right)_6 (1.21 - 0.5)^2 \Rightarrow \left(\frac{W}{L} \right)_6 = 7.44$$

10.49

$$V_{GS3} = \frac{1}{3}(5) = 1.667 V$$

$$I_{REF} = \left(\frac{1}{2} \mu_n C_{ox} \right) \left(\frac{W}{L} \right)_3 (V_{GS3} - V_{TN})^2$$

$$100 = (20) \left(\frac{W}{L} \right)_3 (1.667 - 1)^2$$

$$\Rightarrow \left(\frac{W}{L} \right)_3 = \left(\frac{W}{L} \right)_4 = \left(\frac{W}{L} \right)_5 = 11.2$$

$$I_{o1} = \left(\frac{1}{2} \mu_n C_{ox} \right) \left(\frac{W}{L} \right)_1 (V_{GS3} - V_{TN})^2$$

$$\text{Or } \frac{I_{REF}}{I_{o1}} = \frac{\left(\frac{W}{L} \right)_3}{\left(\frac{W}{L} \right)_1}$$

$$\left(\frac{W}{L} \right)_1 = \left(\frac{I_{o1}}{I_{REF}} \right) \left(\frac{W}{L} \right)_3 = \left(\frac{0.2}{0.1} \right) (11.2)$$

$$\Rightarrow \left(\frac{W}{L} \right)_1 = 22.4$$

$$\text{And } \left(\frac{W}{L} \right)_2 = \left(\frac{I_{o2}}{I_{REF}} \right) \left(\frac{W}{L} \right)_3 = \left(\frac{0.3}{0.1} \right) (11.2)$$

$$\Rightarrow \left(\frac{W}{L} \right)_2 = 33.6$$

10.50

$$I_{REF} = \frac{24 - V_{SGP} - V_{GSN}}{R}$$

Also

$$I_{REF} = 40(1)(V_{GSN} - 1.2)^2$$

$$I_{REF} = 18(1)(V_{SGP} - 1.2)^2$$

Then

$$\sqrt{40(V_{GSN} - 1.2)} = \sqrt{18(V_{SGP} - 1.2)}$$

which yields

$$V_{SGP} = \frac{6.325}{4.243} (V_{GSN} - 1.2) + 1.2$$

Then

$$[0.040(V_{GSN} - 1.2)^2] \cdot R = 24 - V_{GSN} - 1.49(V_{GSN} - 1.2) - 1.2$$

which yields

$$V_{GSN} = 2.69 V \text{ and } V_{SGP} = 3.42 V$$

Now

$$I_{REF} = \frac{24 - 3.42 - 2.69}{200} \Rightarrow I_{REF} = 89.5 \mu A$$

$$I_1 = \frac{89.5}{5} = 17.9 \mu A$$

$$I_2 = (1.25)(89.5) = 112 \mu A$$

$$I_3 = (0.8)(89.5) = 71.6 \mu A$$

$$I_4 = 4(89.5) = 358 \mu A$$

10.51

$$\text{We have } V_{GSN} = 2.69 V \text{ and } V_{SGP} = 3.42 V$$

So

$$I_{REF} = \frac{10 - 2.69 - 3.42}{R} = \frac{3.89}{200} \Rightarrow I_{REF} = 19.45 \mu A$$

Then

$$I_1 = (0.2)(19.45) = 3.89 \mu A$$

$$I_2 = (1.25)(19.45) = 24.3 \mu A$$

$$I_3 = (0.8)(19.45) = 15.56 \mu A$$

$$I_4 = 4(19.45) = 77.8 \mu A$$

10.52

$$\text{For } v_{GS} = 0, i_D = I_{DSS}(1 + \lambda v_{DS})$$

$$a. \quad V_D = -5 V, v_{DS} = 5$$

$$i_D = (2)[1 + (0.05)(5)]$$

$$\Rightarrow i_D = 2.5 \text{ mA}$$

$$b. \quad V_D = 0, v_{DS} = 10$$

$$i_D = (2)[1 + (0.05)(10)]$$

$$\Rightarrow i_D = 3 \text{ mA}$$

$$c. \quad V_D = 5 V, v_{DS} = 15 V$$

$$i_D = (2)[1 + (0.05)(15)]$$

$$\Rightarrow i_D = 3.5 \text{ mA}$$

10.54

$$I_0 = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$2 = 4 \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$\frac{V_{GS}}{V_P} = 1 - \sqrt{\frac{2}{4}} = 0.293$$

$$\text{So } V_{GS} = (0.293)(-4) = -1.17 \text{ V}$$

Then $I_0 = \frac{V_S}{R}$ and $V_S = -V_{GS}$

$$R = \frac{-V_{GS}}{I_0} = -\frac{(-1.17)}{2} \Rightarrow R = 0.585 \text{ k}\Omega$$

10.55

$$\text{a. } I_{REF} = I_{S1} \exp\left(\frac{V_{EB1}}{V_T}\right)$$

$$\text{or } V_{EB1} = V_T \ln\left(\frac{I_{REF}}{I_{S1}}\right) = (0.026) \ln\left(\frac{1 \times 10^{-3}}{5 \times 10^{-13}}\right)$$

$$\Rightarrow V_{EB1} = 0.5568$$

$$\text{b. } R_1 = \frac{5 - 0.5568}{1} \Rightarrow R_1 = 4.44 \text{ k}\Omega$$

c. From Equation (10.72) and letting $V_{CE0} = V_{EC2} = 2.5 \text{ V}$

$$10^{-12} \exp\left(\frac{V_I}{V_T}\right) \left[1 + \frac{2.5}{120}\right] = 10^{-3} \left(\frac{1 + \frac{2.5}{80}}{1 + \frac{0.5568}{80}}\right)$$

$$1.0208 \times 10^{-12} \exp\left(\frac{V_I}{V_T}\right) = (10^{-3}) \left(\frac{103125}{1.00696}\right)$$

Then

$$V_I = 0.026 \ln(1.003613 \times 10^9)$$

$$\text{So } V_I = 0.5389 \text{ V}$$

$$\text{d. } A_V = \frac{-(1/V_T)}{(1/V_{AN}) + (1/V_{AP})}$$

$$A_V = \frac{-\frac{1}{0.026}}{\frac{1}{120} + \frac{1}{80}} = \frac{-38.46}{0.00833 + 0.0125}$$

$$A_V = -1846$$

10.56

$$\text{a. } V_{BE} = V_T \ln\left(\frac{I_{REF}}{I_{S1}}\right) = (0.026) \ln\left(\frac{0.5 \times 10^{-3}}{10^{-12}}\right)$$

$$\Rightarrow V_{BE} = 0.5208$$

$$\text{b. } R_1 = \frac{5 - 0.5208}{0.5} \Rightarrow R_1 = 8.96 \text{ k}\Omega$$

c. From Equation (10.72) applies with slight modifications

$$I_{S0} \exp\left(\frac{V_{EB0}}{V_T}\right) \left[1 + \frac{V_{EC0}}{V_{AP}}\right] = I_{REF} \cdot \left(\frac{1 + \frac{V_{CE2}}{V_{AN}}}{1 + \frac{V_{BE2}}{V_{AN}}}\right)$$

$$(5 \times 10^{-13}) \left[\exp\left(\frac{V_{EB0}}{V_T}\right)\right] \left(1 + \frac{2.5}{80}\right)$$

$$= (0.5 \times 10^{-3}) \cdot \left(\frac{1 + \frac{2.5}{120}}{1 + \frac{0.5208}{120}}\right)$$

$$5.15625 \times 10^{-13} \exp\left(\frac{V_{EB0}}{V_T}\right) = (0.5 \times 10^{-3}) \cdot \frac{1.02083}{1.00434}$$

$$V_{EB0} = 0.5384 \Rightarrow V_I = 5 - 0.5384$$

$$\Rightarrow V_I = 4.462 \text{ V}$$

$$\text{d. } A_V = \frac{-(1/V_T)}{(1/V_{AN}) + (1/V_{AP})}$$

$$A_V = \frac{-\frac{1}{0.026}}{\frac{1}{120} + \frac{1}{80}} = \frac{-38.46}{0.00833 + 0.0125}$$

$$A_V = -1846$$

10.57

Ignore $(W/L)_3 = 5$ specification

a. M_1 and M_2 matched, so we must have

$$V_{SD2} = V_{SG} = V_{SG3} = V_{DS0} = 2.5 \text{ V}$$

For M_1 and M_3 :

$$I_{REF} = \left(\frac{1}{2} \mu_n C_{ox}\right) \left(\frac{W}{L}\right)_1 (V_{SG} + V_{TN})^2 (1 + \lambda_p V_{SD})$$

$$100 = 10 \left(\frac{W}{L}\right)_1 (2.5 - 1)^2 [1 + (0.02)(2.5)]$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 4.23 = \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_2$$

For M_0 :

$$I_0 = \left(\frac{1}{2} \mu_n C_{ox}\right) \left(\frac{W}{L}\right)_0 (V_{GS} - V_{TN})^2 (1 + \lambda_s V_{DS})$$

$$100 = 20 \left(\frac{W}{L}\right)_0 (2 - 1)^2 [1 + (0.02)(2.5)]$$

$$\Rightarrow \left(\frac{W}{L}\right)_0 = 4.76$$

b. $r_{on} = r_{op} = \frac{1}{\lambda I_0} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$

$$g_m = 2\sqrt{K_n I_o} = 2\sqrt{\left(\frac{1}{2} \mu_n C_{os} \right) \left(\frac{W}{L}\right)_o I_o}$$

$$= 2\sqrt{(0.02)(4.76)(0.1)}$$

$$g_m = 0.195 \text{ mA/V}$$

$$A_v = -g_m(r_{on} \parallel r_{op}) = -(0.195)(500 \parallel 500)$$

$$\Rightarrow A_v = -48.75$$

10.58

a. $I_{REF} = K_{p1}(V_{SG} + V_{TP})^2$

$$100 = 100(V_{SG} - 1)^2 \Rightarrow V_{SG} = 2 \text{ V}$$

b. $V_{DS0} = V_{DS2} = 5 \text{ V}$

From Equation (10.87)

$$100(V_I - 1)^2 = 100[1 + (0.02)(5)] \times [1 - (0.02)(2)][1 - (0.02)(5)]$$

$$(V_I - 1)^2 = (1.1)(0.96)(0.90) = 0.9504$$

$$\Rightarrow V_I = 1.975 \text{ V}$$

c. $A_v = -g_m(r_{on} \parallel r_{op})$

$$r_{on} = r_{op} = \frac{1}{\lambda I_{REF}} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$g_m = 2\sqrt{K_n I_{REF}} = 2\sqrt{(0.1)(0.1)} = 0.2 \text{ mA/V}$$

$$A_v = -(0.2)(500 \parallel 500)$$

$$\Rightarrow A_v = -50$$

10.59

- a. Using the results of problem 10.27, we find the resistance looking into the collector of Q_2 to be

$$R_0 = r_{02} \left[1 + \frac{R_E \parallel (r_{n2} + R'_0)}{r_{02}} + g_{m2} \left(\frac{r_{n2}}{r_{n2} + R'_0} \right) [R_E \parallel (r_{n2} + R'_0)] \right]$$

where R'_0 is the resistance from the base of Q_2 toward Q_1 . We found

$$\frac{1}{R'_0} = \frac{1}{R_1} + \frac{\frac{1}{r_{n1}} + g_{m1} + \frac{1}{r_{01}}}{1 + \left(\frac{1}{r_{n1}} + g_{m1} + \frac{1}{r_{01}} \right) (R_E)}$$

b. $A_v = -g_{m0}(r_0 \parallel R_L \parallel R_0)$

10.60

Output resistance of Wilson source

$$R_0 \approx \frac{\beta r_{03}}{2}$$

Then

$$A_v = -g_m(r_0 \parallel R_0)$$

$$r_{03} = \frac{V_{AP}}{I_{REF}} = \frac{80}{0.2} = 400 \text{ k}\Omega$$

$$r_0 = \frac{V_{AN}}{I_{REF}} = \frac{120}{0.2} = 600 \text{ k}\Omega$$

$$g_m = \frac{I_{REF}}{V_T} = \frac{0.2}{0.026} = 7.69 \text{ mA/V}$$

$$A_v = -7.69 \left[600 \parallel \frac{(80)(400)}{2} \right] = -7.69[600 \parallel 16,000]$$

$$\Rightarrow A_v = -4447$$

10.61

a. $g_m(M_o) = 2\sqrt{K_n I_{REF}}$

$$g_m(M_o) = 2\sqrt{(0.25)(0.2)}$$

$$\Rightarrow g_m(M_o) = 0.447 \text{ mA/V}$$

$$r_{on} = \frac{1}{\lambda_n I_{REF}} = \frac{1}{(0.02)(0.2)} \Rightarrow r_{on} = 250 \text{ k}\Omega$$

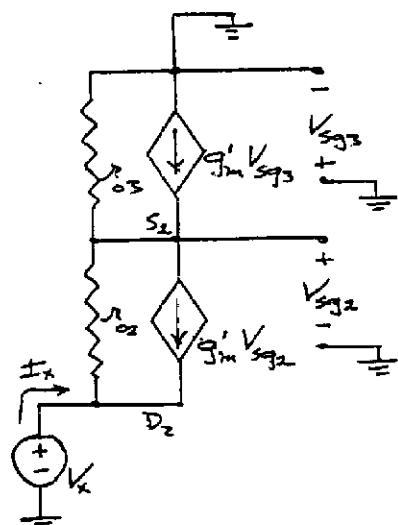
$$r_{op} = \frac{1}{\lambda_p I_{REF}} = \frac{1}{(0.03)(0.2)} \Rightarrow r_{op} = 167 \text{ k}\Omega$$

b. $A_v = -g_m(r_{on} \parallel r_{op}) = -(0.447)(250 \parallel 167)$

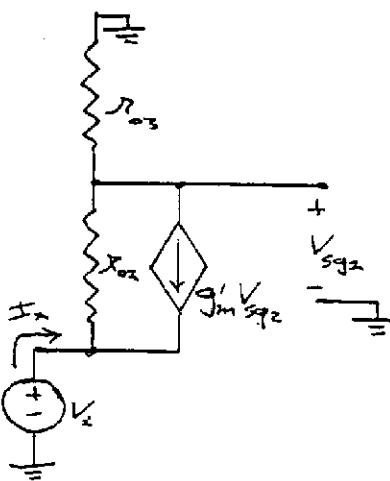
$$\Rightarrow A_v = -44.8$$

c. $R_L = 250 \parallel 167 = r_{on} \parallel r_{op}$ or $R_L = 100 \text{ k}\Omega$

10.62



Since $V_{sg3} = 0$, the circuit becomes



10.63

$$A_v = -g_m (R_{o2} \parallel R_{o3})$$

From the results of JFETs:

$$R_{o2} = r_{o1} + r_{o2}(1 + g'_m r_{o1})$$

From results of Problem 10.62

$$R_{o3} = r_{o3} + r_{o4}(1 + g'_m r_{o3})$$

We find

$$g_m = 2\sqrt{(0.05)(20)(0.08)} = 0.566 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.02)(0.08)} = 625 \text{ k}\Omega$$

$$r_{o3} = r_{o4} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.02)(0.08)} = 625 \text{ k}\Omega$$

$$g'_m = 2\sqrt{(0.02)(40)(0.08)} = 0.506 \text{ mA/V}$$

Then

$$R_{o2} = 625 + 625[1 + (0.506)(625)] \Rightarrow 199 \text{ M}\Omega$$

$$R_{o3} = 625 + 625[1 + (0.506)(625)] \Rightarrow 199 \text{ M}\Omega$$

Then

$$A_v = -(0.566)[199000 \parallel 199000] \Rightarrow \underline{A_v = -56,317}$$

$$I_x = -g'_m V_{sg2} + \frac{V_x - V_{sg2}}{r_{o2}} \quad \text{and} \quad V_{sg2} = I_x r_{o3}$$

Then

$$I_x \left(1 + g'_m r_{o3} + \frac{r_{o3}}{r_{o2}} \right) = \frac{V_x}{r_{o2}}$$

so that

$$\frac{V_x}{I_x} = R_o = r_{o2} \left(1 + g'_m r_{o3} + \frac{r_{o3}}{r_{o2}} \right)$$

or

$$R_o = r_{o2} + r_{o3}(1 + g'_m r_{o2})$$

$$A_v = \frac{V_o}{V_i} = -g_m (r_{o1} \parallel R_o)$$

Now

$$g_m = 2\sqrt{(0.05)(20)(0.10)} = 0.632 \text{ mA/V}$$

$$r_{o1} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.02)(0.10)} = 500 \text{ k}\Omega$$

$$g'_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.020)(80)(0.1)} = 0.80 \text{ mA/V}$$

$$r_{o2} = r_{o3} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.020)(0.1)} = 500 \text{ k}\Omega$$

Then

$$R_o = 500 + 500[1 + (0.8)(500)] \Rightarrow 201 \text{ M}\Omega$$

$$A_v = -(0.632)(500 \parallel 201000) \Rightarrow \underline{A_v = -315}$$

Chapter 11

Exercise Solutions

E11.1

$$\begin{aligned}
 V_d &= V_1 - V_2 \\
 &= 2 + 0.005 \sin \omega t - (0.5 - 0.005 \sin \omega t) \\
 \Rightarrow V_d &= 1.5 + 0.010 \sin \omega t \text{ (V)} \\
 V_{cm} &= \frac{V_1 + V_2}{2} \\
 &= \frac{2 + 0.005 \sin \omega t + 0.5 - 0.005 \sin \omega t}{2} \\
 \Rightarrow V_{cm} &= 1.25 \text{ V}
 \end{aligned}$$

E11.2

$$\begin{aligned}
 v_E &= -V_{BE(\text{on})} \Rightarrow v_E = -0.7 \text{ V} \\
 I_{C1} &= I_{C2} = 0.5 \text{ mA} \\
 v_{C1} &= v_{C2} = 10 - (0.5)(10) \\
 \Rightarrow v_{C1} &= v_{C2} = 5 \text{ V}
 \end{aligned}$$

E11.3

$$\begin{aligned}
 \text{For } v_1 = v_2 = +4 \text{ V} \\
 \Rightarrow \text{Minimum } v_{C1} = v_{C2} = 4 \text{ V} \\
 I_{C1} = I_{C2} = \frac{I_Q}{2} = 1 \text{ mA} \\
 R_C = \frac{10 - 4}{1} \Rightarrow R_C = 6 \text{ k}\Omega
 \end{aligned}$$

E11.4

$$\begin{aligned}
 \frac{i_{C2}}{I_Q} &= \frac{1}{1 + \exp\left(\frac{v_d}{V_T}\right)} = 0.99 \\
 1 + \exp\left(\frac{v_d}{V_T}\right) &= \frac{1}{0.99} \\
 \exp\left(\frac{v_d}{V_T}\right) &= \frac{1}{0.99} - 1 \\
 v_d &= V_T \ln \left[\frac{1}{0.99} - 1 \right] \\
 \Rightarrow v_d &= -119.5 \text{ mV}
 \end{aligned}$$

E11.6

$$\begin{aligned}
 \text{a. } v_1 = v_2 = 0 \Rightarrow v_E = 0.7 \text{ V} \\
 \Delta V_{RC} = (0.25)(8) = 2 \text{ V} \\
 \Rightarrow v_{C1} = v_{C2} = -3 \text{ V} \\
 \Rightarrow v_{EC1} = 3.7 \text{ V}
 \end{aligned}$$

b. $v_1 = v_2 = 2.5 \text{ V} \Rightarrow v_E = 3.2 \text{ V}$

$\Rightarrow v_{EC1} = 6.2 \text{ V}$

c. $v_1 = v_2 = -2.5 \text{ V} \Rightarrow v_E = -1.8 \text{ V}$

$\Rightarrow v_{EC1} = 1.2 \text{ V}$

E11.7

Let $I_Q = 1 \text{ mA}$, then $I_{CQ1} = I_{CQ2} = 0.5 \text{ mA}$

$$g_{m1} = g_{m2} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

At v_{C2} ,

$$A_d = \frac{v_{c2}}{v_d} = \frac{1}{2} g_m R_{C2}$$

$$\text{So, } 150 = \frac{1}{2}(19.23)R_{C2} \Rightarrow R_{C2} = 15.6 \text{ k}\Omega$$

At v_{C1} ,

$$A_d = \frac{v_{c1}}{v_d} = -\frac{1}{2} g_m R_{C1}$$

$$\text{So, } -100 = -\frac{1}{2}(19.23)R_{C1} \Rightarrow R_{C1} = 10.4 \text{ k}\Omega$$

If $V^+ = +10 \text{ V}$ and $V^- = -10 \text{ V}$, dc biasing is OK.

E11.8

a. Diff. Gain $A_d = \frac{I_Q R_C}{4V_T}$

For $v_1 = v_2 = 5 \text{ V} \Rightarrow$ Minimum collector voltage
 $v_{C2} = 5 \text{ V}$

$$\Rightarrow \frac{I_Q}{2} \cdot R_C = 15 - 5 = 10 \text{ V}$$

or $I_Q R_C = 20 \text{ V}$ for max. A_d

Then

$$A_d = \frac{20}{2(0.026)} \Rightarrow A_d(\text{max}) = 192$$

b. If $I_Q = 0.5 \text{ mA}$, $R_C = 40 \text{ k}\Omega$

$$A_{cm} = \frac{-\left(\frac{I_Q R_C}{2V_T}\right)}{\left[1 + \frac{(1+\beta)I_Q R_Q}{V_T \beta}\right]}$$

$$\text{Then } A_{cm} = \frac{-\left(\frac{20}{2(0.026)}\right)}{\left[1 + \frac{(201)(0.5)(100)}{(0.026)(200)}\right]}$$

$\Rightarrow A_{cm} = -0.199$

and $C M R R_{dB} = 20 \log_{10} \left(\frac{192}{0.199} \right)$

$\Rightarrow C M R R_{dB} = 59.7 \text{ dB}$

E11.9

For $v_1 = v_2 = 5 \text{ V} \Rightarrow \min v_{C1} = v_{C2} = 5 \text{ V}$

So $I_{C1}R_C = 10 - 5 = 0.25R_C \Rightarrow R_C = 20 \text{ k}\Omega$

$$A_d = \frac{I_Q R_C}{4V_T} = \frac{(0.5)(20)}{4(0.026)} \Rightarrow A_d = 96.2 \quad \text{Let } I_Q = 0.5 \text{ mA}$$

$CMR_{dB} = 95 \text{ dB} \Rightarrow CMRR = 5.62 \times 10^4$

$$\Rightarrow A_{cm} = \frac{96.2}{5.62 \times 10^4} \Rightarrow |A_{cm}| = 1.71 \times 10^{-3}$$

$$|A_{cm}| = \frac{\left(\frac{I_Q R_C}{2V_T}\right)}{\left[1 + \frac{(1+\beta)I_Q R_0}{V_T \beta}\right]} = 1.71 \times 10^{-3}$$

$$\frac{\left[\frac{(0.5)(20)}{2(0.026)}\right]}{\left[1 + \frac{(201)(0.5)R_0}{(0.026)(200)}\right]} = 1.71 \times 10^{-3}$$

$$1 + 19.3R_0 = 1.12 \times 10^5 \Rightarrow R_0 = 5.83 \times 10^3 \text{ k}\Omega$$

$$= 5.83 \text{ M}\Omega$$

We have $R_0 = r_{o4}[1 + g_{m2}(R_2 \parallel r_{\pi2})]$

$$r_{o4} = \frac{V_A}{I_Q} = \frac{125}{0.5} = 250 \text{ k}\Omega$$

$$g_{m2} = \frac{I_Q}{V_T} = \frac{0.5}{0.026} = 19.2 \text{ mA/V}$$

$$r_{\pi2} = \frac{\beta V_T}{I_Q} = \frac{(200)(0.026)}{0.5} = 10.4 \text{ k}\Omega$$

$$5830 = 250[1 + g_{m2}(R_2 \parallel r_{\pi2})]$$

$$19.2(R_2 \parallel r_{\pi2}) = 22.3$$

$$(R_2 \parallel r_{\pi2}) = 1.16 \text{ k}\Omega$$

$$\frac{R_2(10.4)}{R_2 + 10.4} = 1.16$$

$$R_2(10.4 - 1.16) = (1.16)(10.4)$$

$$\Rightarrow R_2 = 1.31 \text{ k}\Omega$$

$$I_Q R_2 - I_1 R_3 = V_T \ln \left(\frac{I_1}{I_Q} \right) \quad \text{Let } I_1 = 1 \text{ mA}$$

$$(0.5)(1.31) - (1)R_3 = (0.026) \ln \left(\frac{1}{0.5} \right)$$

$$\Rightarrow R_3 = 0.637 \text{ k}\Omega$$

$$\text{If } V_{BE}(Q_3) \approx 0.7 \text{ V}$$

$$R_1 + R_3 = \frac{10 - 0.7 - (-10)}{1} = 19.3$$

$$\Rightarrow R_1 \approx 18.7 \text{ k}\Omega$$

E11.10

a. $v_0 = A_d v_d + A_{cm} v_{cm}$

$$v_d = v_1 - v_2 = 0.505 \sin \omega t - 0.495 \sin \omega t$$

$$= 0.01 \sin \omega t$$

$$v_{cm} = \frac{v_1 + v_2}{2} = \frac{0.505 \sin \omega t + 0.495 \sin \omega t}{2}$$

$$= 0.50 \sin \omega t$$

$$v_0 = (60)(0.01 \sin \omega t) + (0.5)(0.5 \sin \omega t)$$

$$\Rightarrow v_0 = 0.85 \sin \omega t \text{ (V)}$$

b.

$$v_d = v_1 - v_2$$

$$= 0.5 + 0.005 \sin \omega t - (0.5 - 0.005 \sin \omega t)$$

$$= 0.01 \sin \omega t$$

$$v_{cm} = \frac{v_1 + v_2}{2}$$

$$= \frac{0.5 + 0.005 \sin \omega t + 0.5 - 0.005 \sin \omega t}{2}$$

$$= 0.5$$

$$v_0 = (60)(0.01 \sin \omega t) + (0.5)(0.5)$$

$$\Rightarrow v_0 = 0.25 + 0.6 \sin \omega t \text{ (V)}$$

E11.11

a. $I_{B1} = I_{B2} = \frac{I_Q/2}{(1+\beta)} = \frac{1}{151}$

$$\Rightarrow I_{B1} = I_{B2} = 6.62 \mu\text{A}$$

b. $r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(150)(0.026)}{1} = 3.9 \text{ k}\Omega$

$$R_{id} = 2r_\pi = 2(3.9) = 7.8 \text{ k}\Omega$$

$$I_b = \frac{V_d}{R_{id}} = \frac{10 \sin \omega t \text{ (mV)}}{7.8 \text{ k}\Omega}$$

$$\Rightarrow I_b = 1.28 \sin \omega t \text{ (\mu A)}$$

c. $R_{cm} \approx 2(1+\beta)R_0 = 2(151)(50) \Rightarrow 15.1 \text{ M}\Omega$

$$I_b = \frac{V_{cm}}{R_{cm}} = \frac{3 \sin \omega t}{15.1 \text{ M}\Omega} \Rightarrow I_b = 0.199 \sin \omega t \text{ (\mu A)}$$

E11.12

$$\frac{i_{D1}}{I_Q} = \frac{1}{2} + \sqrt{\frac{K_n}{2I_Q} \cdot v_d} \sqrt{1 - \left(\frac{K_n}{2I_Q} \right) v_d^2}$$

Using the parameters in Example 11.11, $K_n = 0.5 \text{ mA/V}^2$, $I_Q = 1 \text{ mA}$, then

$$\frac{i_{D1}}{I_Q} = 0.90 = \frac{1}{2} + \sqrt{\frac{0.5}{2(1)} \cdot v_d} \sqrt{1 - \left(\frac{0.5}{2(1)} \right) v_d^2}$$

By trial and error,

$$v_d = 0.894 \text{ V}$$

E11.13

$$I_1 = \frac{10 - V_{GS4}}{R_4} = K_{n3}(V_{GS4} - V_{th})^2$$

$$10 - V_{GS4} = (0.1)(80)(V_{GS4} - 0.8)^2$$

$$10 - V_{GS4} = 8(V_{GS4}^2 - 1.6V_{GS4} + 0.64)$$

$$8V_{GS4}^2 - 11.8V_{GS4} - 4.88 = 0$$

$$V_{GS4} = \frac{11.8 \pm \sqrt{(11.8)^2 + 4(8)(4.88)}}{2(8)} = 1.81 \text{ V}$$

$$I_1 = I_Q = \frac{10 - 1.81}{80} = 0.102 \text{ mA}$$

$$I_{D1} = I_{D2} = \frac{0.102}{2} = 0.051 \text{ mA}$$

$$= K_n(V_{GS1} - V_{TN})^2$$

$$0.051 = 0.050(V_{GS1} - 0.8)^2 \Rightarrow V_{GS1} = 1.81 \text{ V}$$

$$v_{o1} = v_{o2} = 5 - (0.051)(40) = 2.96 \text{ V}$$

$$\text{Max } v_{cm}: V_{DS1}(\text{sat}) = V_{GS1} - V_{TN}$$

$$= 1.81 - 0.8 = 1.01 \text{ V}$$

$$v_{cm}(\text{max}) = v_{o1} - V_{DS1}(\text{sat}) + V_{GS1}$$

$$= 2.96 - 1.01 + 1.81$$

$$v_{cm}(\text{max}) = 3.76 \text{ V}$$

$$\text{Min } v_{cm}: V_{DS4}(\text{sat}) = V_{GS4} - V_{TN}$$

$$= 1.81 - 0.8 = 1.01 \text{ V}$$

$$v_{cm}(\text{min}) = V_{GS1} + V_{DS4}(\text{sat}) - 5$$

$$= 1.81 + 1.01 - 5$$

$$v_{cm}(\text{min}) = -2.18 \text{ V}$$

$$\underline{-2.18 \leq v_{cm} \leq 3.76 \text{ V}}$$

E11.14

$$g_f(\text{max}) = \sqrt{\frac{K_n I_D}{2}} = \sqrt{\frac{(1)(2)}{(2)}} \Rightarrow$$

$$g_f(\text{max}) = 1 \text{ mA/V}$$

$$A_d = g_f R_D = (1)(5) \Rightarrow \underline{A_d = 5}$$

E11.15

$$A_d = g_f R_D$$

$$8 = g_f(4) \Rightarrow \underline{g_f(\text{max}) = 2 \text{ mA/V}}$$

$$g_f(\text{max}) = \sqrt{\frac{K_n I_D}{2}}$$

$$(2)^2 = K_n \left(\frac{4}{2}\right) \Rightarrow \underline{K_n = 2 \text{ mA/V}^2}$$

E11.16

From Example 11-10, $I_Q = 0.588 \text{ mA}$

$$A_d = \sqrt{\frac{K_n I_D}{2}} \cdot R_D = \sqrt{\frac{(0.1)(0.588)}{2}} \cdot (16)$$

$$\Rightarrow \underline{A_d = 2.74}$$

$$\text{For } M_4, R_o = \frac{1}{\lambda_4 I_Q} = \frac{1}{(0.02)(0.588)} \Rightarrow R_o = 85 \text{ k}\Omega$$

$$g_m = 2K_n(V_{GS2} - V_{TN}) = 2(0.1)(2.71 - 1)$$

$$= 0.342 \text{ mA/V}$$

$$A_{cm} = \frac{-g_m R_D}{1 + 2g_m R_o} = \frac{-(0.342)(16)}{1 + 2(0.342)(85)}$$

$$\Rightarrow \underline{A_{cm} = -0.0925}$$

$$CMRR_{dB} = 20 \log_{10} \left(\frac{2.74}{0.0925} \right)$$

$$\Rightarrow \underline{CMRR_{dB} = 29.4 \text{ dB}}$$

E11.17

$$CMRR = \frac{1}{2} [1 + 2\sqrt{2K_n I_Q} \cdot R_o]$$

$$CMRR_{dB} = 60 \text{ dB} \Rightarrow CMRR = 1000$$

$$1000 = \frac{1}{2} [1 + 2\sqrt{2(0.1)(0.2)} \cdot R_o]$$

$$2000 = 1 + 0.4R_o$$

$$\Rightarrow \underline{R_o \geq 5 \text{ M}\Omega}$$

E11.18

$$R_o = r_{o4} + r_{o2}(1 + g_{m4}r_{o4})$$

$$\text{Assume } I_{REF} = I_o = 100 \mu A \text{ and } \lambda = 0.01 V^{-1}$$

$$r_{o2} = r_{o4} = \frac{1}{\lambda I_D} = \frac{1}{(0.01)(0.1)} \Rightarrow 1 \text{ M}\Omega$$

$$\text{Let } K_n \text{ (all devices)} = 0.1 \text{ mA/V}^2$$

$$\text{Then}$$

$$g_{m4} = 2\sqrt{K_n I_D} = 2\sqrt{(0.1)(0.1)} = 0.2 \text{ mA/V}$$

$$R_o = 1000 + 1000(1 + (0.2)(1000)) \Rightarrow 202 \text{ M}\Omega$$

$$\text{Now}$$

$$V_{GS1} = V_{GS2} = \sqrt{\frac{I_D}{K_n}} + V_{TN} = \sqrt{\frac{0.05}{0.1}} + 1 = 1.71 \text{ V}$$

$$V_{DS1}(\text{sat}) = V_{GS1} - V_{TN} = 1.71 - 1 = 0.71 \text{ V}$$

So

$$v_{o1}(\text{min}) = +4 - V_{GS1} + V_{DS1}(\text{sat}) = 4 - 1.71 + 0.71$$

$$v_{o1}(\text{min}) = 3 \text{ V} = 10 - I_D R_D = 10 - (0.05)R_D \Rightarrow$$

$$R_D = 140 \text{ k}\Omega$$

For a one-sided output, the differential gain is:

$$A_d = \frac{1}{2} g_{m1} R_D \text{ where } g_{m1} = 2\sqrt{K_n I_D}$$

$$= 2\sqrt{(0.1)(0.05)} = 0.141 \text{ mA/V}$$

$$A_d = \frac{1}{2}(0.141)(140) \Rightarrow \underline{A_d = 9.87}$$

The common-mode gain is:

$$A_{cm} = \frac{\sqrt{2K_n I_D} \cdot R_D}{1 + 2\sqrt{2K_n I_D} \cdot R_o} = \frac{\sqrt{2(0.1)(0.1)} \cdot (140)}{1 + 2\sqrt{2(0.1)(0.1)} \cdot (202000)}$$

$$\Rightarrow \underline{A_{cm} = 0.000347}$$

Then

$$CMRR_{dB} = 20 \log_{10} \left| \frac{A_d}{A_{cm}} \right| \Rightarrow \underline{CMRR_{dB} = 89.1 \text{ dB}}$$

E11.19

$$\text{a. } I_{BS} = \frac{I_Q}{\beta(1+\beta)} = \frac{0.5}{(180)(181)} \Rightarrow 15.3 \text{ nA}$$

So $I_0 = 15.3 \text{ nA}$

b. For a balanced condition

$$V_{EC4} = V_{EC3} = V_{EB3} \Rightarrow V_{EC4} = 0.7 \text{ V}$$

$$V_{CE2} = V_{C2} - V_{E2} = (10 - 0.7) - (-0.7)$$

$$\Rightarrow V_{CE2} = 10 \text{ V}$$

$$r_{o2} = \frac{V_{A2}}{I_{C2}} = \frac{125}{0.05} \Rightarrow 2.5 \text{ M}\Omega$$

$$r_{o4} = \frac{V_{A4}}{I_{C4}} = \frac{85}{0.05} \Rightarrow 1.7 \text{ M}\Omega$$

$$g_m = \frac{0.05}{0.026} = 1.923 \text{ mA/V}$$

$$A_d = g_m(r_{o2} \parallel r_{o4} \parallel R_L) = (1.923)(2500 \parallel 1700 \parallel 90) \Rightarrow$$

$$\underline{A_d = 159}$$

E11.20

$$\text{a. } g_f = \frac{I_Q}{4V_T} = \frac{0.5}{4(0.026)} = 4.81 \text{ mA/V}$$

$$r_{o2} = \frac{V_{A2}}{I_{C2}} = \frac{125}{0.25} = 500 \text{ k}\Omega$$

$$r_{o4} = \frac{V_{A4}}{I_{C4}} = \frac{85}{0.25} = 340 \text{ k}\Omega$$

$$A_d = 2g_f(r_{o2} \parallel r_{o4}) = 2(4.81)(500 \parallel 340)$$

$$\Rightarrow \underline{A_d = 1947}$$

$$\text{b. } A_d = 2g_f(r_{o2} \parallel r_{o4} \parallel R_L)$$

$$A_d = 2(4.81)(500 \parallel 340 \parallel 100)$$

$$\Rightarrow \underline{A_d = 644}$$

$$\text{c. } r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(150)(0.026)}{0.25} = 15.6 \text{ k}\Omega$$

$$R_{id} = 2r_\pi \Rightarrow \underline{R_{id} = 31.2 \text{ k}\Omega}$$

$$\text{d. } R_o = r_{o2} \parallel r_{o4} = 500 \parallel 340 \Rightarrow \underline{R_o = 202 \text{ k}\Omega}$$

E11.21

$$A_d = 2g_f(r_{o2} \parallel r_{o4})$$

$$g_f = \frac{I_Q}{4V_T} = \frac{0.2}{4(0.026)} = 1.92 \text{ mA/V}$$

$$r_{o2} = \frac{V_{A2}}{I_{C2}} = \frac{120}{0.1} = 1200 \text{ k}\Omega$$

$$r_{o4} = \frac{V_{A4}}{I_{C4}} = \frac{80}{0.1} = 800 \text{ k}\Omega$$

$$A_d = 2(1.92)(1200 \parallel 800) \Rightarrow \underline{A_d = 1843}$$

E11.22

$$P = (I_Q + I_{REF})(5 - (-5))$$

$$10 = (0.1 + I_{REF})(10) \Rightarrow I_{REF} = 0.9 \text{ mA}$$

$$R_i = \frac{5 - 0.7 - (-5)}{I_{REF}} = \frac{9.3}{0.9} \Rightarrow \underline{R_i = 10.3 \text{ k}\Omega}$$

$$I_Q R_g = V_T \ln \left(\frac{I_{REF}}{I_Q} \right)$$

$$R_g = \frac{0.026}{0.1} \ln \left(\frac{0.9}{0.1} \right) \Rightarrow \underline{R_g = 0.571 \text{ k}\Omega}$$

E11.23

$$\text{a. } R_o = r_{o2} \parallel r_{o4}$$

$$r_{o2} = \frac{120}{0.1} = 1.2 \text{ M}\Omega$$

$$r_{o4} = \frac{80}{0.1} = 0.8 \text{ M}\Omega$$

$$R_o = 1.2 \parallel 0.8 \Rightarrow \underline{R_o = 0.48 \text{ M}\Omega}$$

$$\text{b. } A_d(\text{open circuit}) = 2g_f(r_{o2} \parallel r_{o4})$$

$$A_d(\text{with load}) = 2g_f(r_{o2} \parallel r_{o4} \parallel R_L)$$

$$\text{For } A_d(\text{with load}) = \frac{1}{2} A_d(\text{open circuit})$$

$$\Rightarrow R_L = (r_{o2} \parallel r_{o4}) \Rightarrow \underline{R_L = 0.48 \text{ M}\Omega}$$

E11.24

$$A_d = 2 \sqrt{\frac{2K_m}{I_Q} \cdot \frac{1}{(\lambda_2 + \lambda_4)}}$$

$$= 2 \sqrt{\frac{2(0.1)}{0.1} \cdot \frac{1}{(0.01 + 0.015)}} \Rightarrow$$

$$\underline{A_d = 113}$$

E11.25

For the MOSFET, $I_D = 25 \mu\text{A}$

$$25 = 20(V_{GS} - 1)^2 \Rightarrow V_{GS} = 2.12 \text{ V}$$

$$g_{m1} = 2K_m(V_{GS} - V_{TN}) = 2(20)(2.12 - 1)$$

$$\Rightarrow \underline{g_{m1} = 44.8 \mu\text{A/V}}, \quad r_0 = \infty$$

For the Bipolar, $I_Q = 100 - 25 = 75 \mu\text{A}$

$$r_{\pi2} = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.075} \Rightarrow \underline{r_{\pi2} = 34.7 \text{ k}\Omega}$$

$$g_{m2} = \frac{I_{CQ}}{V_T} = \frac{75}{0.026}$$

$$\Rightarrow \underline{g_{m2} = 2.88 \text{ mA/V}}, \quad \underline{r_{o2} = \infty}$$

$$g_m^C = \frac{g_{m1}(1 + g_{m2}r_\pi)}{(1 + g_{m1}r_\pi)}$$

$$= \frac{(44.8)[1 + (2.88)(34.7)]}{1 + (0.0448)(34.7)}$$

$$= \frac{(44.8)(100.9)}{2.55}$$

$$\Rightarrow \underline{g_m^C = 1.77 \text{ mA/V}}$$

E11.26

From Figure 11.41

$$r_{\pi 4} = \frac{80}{0.5} = 160 \text{ k}\Omega$$

$$R_0 \approx \beta r_{\pi 4} = (150)(160) \text{ k}\Omega \Rightarrow R_0 = 24 \text{ M}\Omega$$

From Figure 11.42

$$r_{\pi 6} = \frac{1}{\lambda I_D} = \frac{1}{(0.0125)(0.5)} \Rightarrow r_{\pi 6} = 160 \text{ k}\Omega$$

$$0.5 = 0.5(V_{GS} - 1)^2 \Rightarrow V_{GS} = 2 \text{ V}$$

$$g_{m6} = 2K_n(V_{GS} - V_{TN}) = 2(0.5)(2 - 1) = 1 \text{ mA/V}$$

$$r_{\pi 6} = 160 \text{ k}\Omega$$

$$R_0 = (g_{m6})(r_{\pi 6})(\beta r_{\pi 4}) = (1)(160)(150)(160)$$

$$\Rightarrow R_0 = 3,840 \text{ M}\Omega$$

E11.27

From Equation (11.103)

$$R_i = \frac{2(1+\beta)\beta V_T}{I_Q} = \frac{2(121)(120)(0.026)}{0.5}$$

$$\Rightarrow R_i = 1.51 \text{ M}\Omega$$

$$r_{\pi 11} = \frac{\beta V_T}{I_Q} = \frac{(120)(0.026)}{0.5} = 6.24 \text{ k}\Omega$$

$$R'_E = r_{\pi 11} \| R_3 = 6.24 \| 0.1 = 0.0984 \text{ k}\Omega$$

$$g_{m11} = \frac{I_Q}{V_T} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$r_{011} = \frac{V_A}{I_Q} = \frac{120}{0.5} = 240 \text{ k}\Omega$$

Then

$$\begin{aligned} R_{C11} &= r_{011}(1 + g_{m11}R'_E) \\ &= 240[1 + (19.23)(0.0984)] \\ &= 694 \text{ k}\Omega \end{aligned}$$

$$r_{\pi 6} = \frac{\beta V_T}{I_{C6}} = \frac{(120)(0.026)}{2} = 1.56 \text{ k}\Omega$$

$$\begin{aligned} R_{bs} &= r_{\pi 6} + (1 + \beta)R_4 = 1.56 + (121)(5) \\ &= 607 \text{ k}\Omega \end{aligned}$$

Then

$$R_{L7} = R_{C11} \| R_{bs} = 694 \| 607 = 324 \text{ k}\Omega$$

Then

$$\begin{aligned} A_v &= \left(\frac{I_Q}{2V_T} \right) R_{L7} = \left[\frac{0.5}{2(0.026)} \right] (324) \\ &\Rightarrow A_v = 3115 \end{aligned} \quad (324)$$

$$R_0 = R_4 \parallel \left(\frac{r_{\pi 6} + Z}{1 + \beta} \right)$$

$$Z = R_{C11} \| R_{C7}$$

$$R_{C7} = \frac{V_A}{I_Q} = \frac{120}{0.5} = 240 \text{ k}\Omega$$

$$Z = 694 \| 240 = 178 \text{ k}\Omega$$

$$\begin{aligned} R_0 &= 5 \parallel \left(\frac{1.56 + 178}{121} \right) = 5 \| 1.48 \\ &\Rightarrow R_0 = 1.14 \text{ k}\Omega \end{aligned}$$

E11.28

$$A_v = \left(\frac{I_Q}{2V_T} \right) R_{L7}$$

$$10^3 = \left(\frac{0.5}{2(0.026)} \right) R_{L7}$$

$$\Rightarrow R_{L7} = 104 \text{ k}\Omega$$

E11.29

a. $R_1 = \frac{10 - 0.7 - (-10)}{I_1} = \frac{19.3}{0.6} \Rightarrow R_1 = 32.2 \text{ k}\Omega$

$I_{C1} = I_{C2} = 0.1 \text{ mA} \Rightarrow I_Q \approx 0.2 \text{ mA}$

$R_2 = \frac{V_T}{I_Q} \cdot \ln\left(\frac{I_1}{I_Q}\right) = \frac{0.026}{0.2} \cdot \ln\left(\frac{0.6}{0.2}\right)$
 $\Rightarrow R_2 = 143 \text{ }\Omega$

$I_{R8} = I_1 = 0.6 \text{ mA} \Rightarrow R_3 = 0$

$V_{O2} = V_{CE2} + V_B = 4 - 0.7 = 3.3 \text{ V}$

$R_C = \frac{10 - 3.3}{I_{C2}} = \frac{6.7}{0.1} \Rightarrow R_C = 67 \text{ k}\Omega$

$V_{E4} = V_{O2} - 2V_{BE} = 3.3 - 2(0.7) = 1.9 \text{ V}$

$R_4 = \frac{1.9}{I_{R4}} = \frac{1.9}{0.6} \Rightarrow R_4 = 3.17 \text{ k}\Omega$

$V_{O3} = V_{CE4} + V_{E4} = 3 + 1.9 = 4.9$

$R_5 = \frac{10 - 4.9}{I_{R4}} = \frac{5.1}{0.6} \Rightarrow R_5 = 8.5 \text{ k}\Omega$

$V_{E5} = V_{O3} - V_{BE} = 4.9 - 0.7 = 4.2$

$R_6 = \frac{4.2 - 0.7}{I_{R6}} = \frac{3.5}{0.6} \Rightarrow R_6 = 5.83 \text{ k}\Omega$

$R_7 = \frac{0 - (-10)}{I_{R7}} = \frac{10}{5} \Rightarrow R_7 = 2 \text{ k}\Omega$

b. $R_{12} = r_{\pi 3} + (1 + \beta)r_{\pi 4}$

$r_{\pi 4} = \frac{\beta V_T}{I_{R4}} = \frac{(100)(0.026)}{0.6} = 4.33 \text{ k}\Omega$

$r_{\pi 3} \approx \frac{\beta^2 V_T}{I_{R4}} = \frac{(100)^2(0.026)}{0.6} = 433 \text{ k}\Omega$

$R_{12} = 433 + (101)(4.33) \Rightarrow R_{12} = 870 \text{ k}\Omega$

$R_{13} = r_{\pi 5} + (1 + \beta)[R_4 + r_{\pi 6} + (1 + \beta)R_7]$

$r_{\pi 5} = \frac{\beta V_T}{I_{R5}} = \frac{(100)(0.026)}{0.6} = 4.33 \text{ k}\Omega$

$r_{\pi 6} = \frac{\beta V_T}{I_{R7}} = \frac{(100)(0.026)}{5} = 0.52 \text{ k}\Omega$

$R_{13} = 4.33 + (101)[5.83 + 0.52 + (101)(2)]$

$\Rightarrow R_{13} = 21.0 \text{ M}\Omega$

c. $A_d = A_{d1} \cdot A_2 \cdot A_3$

$A_{d1} = g_f(R_C \parallel R_{12})$

$g_f = \frac{I_Q}{4V_T} = \frac{0.2}{4(0.026)} = 1.92 \text{ mA/V}$

$A_{d1} = (1.92)(67 \parallel 870) = 119$

$A_2 = \left(\frac{I_{R4}}{2V_T}\right) R_5 = \frac{0.6}{2(0.026)}(8.5) = 98.1$

$A_3 \approx 1$

$A_d = (119)(98.1)(1)$

$\Rightarrow A_d = 11,674$

Chapter 11

Problem Solutions

11.1

a. $I_E = I_{C1} + I_{C2} = 4 \text{ mA} = \frac{-0.7 - (-8.7)}{R_E}$

So $R_E = \frac{8.0}{4} = 2 \text{ k}\Omega = R_E$

Then

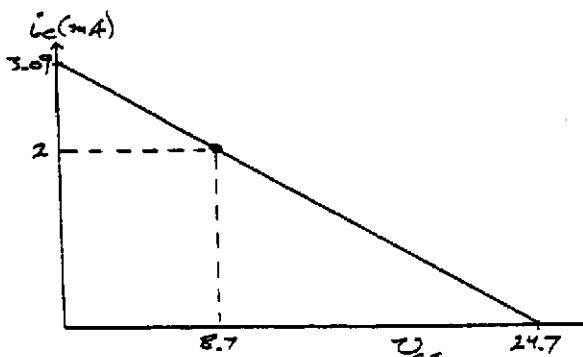
$$R_C = \frac{16 - V_{B2}}{I_{C2}} = \frac{16 - 8}{2} \Rightarrow R_C = 4 \text{ k}\Omega$$

b. Neglecting base currents

$$16 = I_C R_C + V_{CE2} + 2I_C R_E - 8.7$$

$$V_{CE2} = 24.7 - I_C(R_C + 2R_E) = 24.7 - I_C(8)$$

For $I_C = 2 \text{ mA}$, $V_{CEQ} = 8.7 \text{ V}$



c. $v_{cm}(\max)$ for $V_{CB} = 0 \Rightarrow v_{cm}(\max) = 8 \text{ V}$

$v_{cm}(\min)$ for Q_1 and Q_2 at edge of cutoff

$$\Rightarrow v_{cm}(\min) = -8 \text{ V}$$

11.2

$$P = (I_1 + I_{C4})(V^+ - V^-)$$

$$I_1 \cong I_{C4} \text{ so}$$

$$1.2 = 2I_1(6) \Rightarrow I_1 = I_{C4} = 0.1 \text{ mA}$$

$$R_i = \frac{3 - 0.7 - (-3)}{0.1} \Rightarrow R_i = 53 \text{ k}\Omega$$

For $v_{CM} = +1 \text{ V} \Rightarrow V_{C1} = V_{C2} = 1 \text{ V} \Rightarrow$

$$R_C = \frac{3 - 1}{0.05} \Rightarrow R_C = 40 \text{ k}\Omega$$

One-sided output

$$A_d = \frac{1}{2} g_m R_C \text{ where } g_m = \frac{0.05}{0.026} = 1.923 \text{ mA/V}$$

Then

$$A_d = \frac{1}{2}(1.923)(40) \Rightarrow A_d = 38.5$$

11.3

a. $I_1 = \frac{10 - 2(0.7)}{8.5} \Rightarrow I_1 = 1.01 \text{ mA}$

$$I_{C2} = \frac{I_1}{1 + \frac{2}{\beta(1+\beta)}} = \frac{1.01}{1 + \frac{2}{(100)(101)}} \Rightarrow I_{C2} \cong 1.01 \text{ mA}$$

$$I_{C4} = \left(\frac{100}{101}\right)\left(\frac{1.01}{2}\right) \Rightarrow I_{C4} \cong 0.50 \text{ mA}$$

$$V_{CE2} = (0 - 0.7) - (-5) \Rightarrow V_{CE2} = 4.3 \text{ V}$$

$$V_{CE4} = [5 - (0.5)(2)] - (-0.7) \Rightarrow V_{CE4} = 4.7 \text{ V}$$

b. For $V_{CE4} = 2.5 \text{ V} \Rightarrow V_{C4} = -0.7 + 2.5 = 1.8 \text{ V}$

$$I_{C4} = \frac{5 - 1.8}{2} \Rightarrow I_{C4} = 1.6 \text{ mA}$$

$$I_{C2} + \left(\frac{1+\beta}{\beta}\right)(2I_{C4}) = \left(\frac{101}{100}\right)(2)(1.6)$$

$$\Rightarrow I_{C2} = 3.23 \text{ mA}$$

$$I_1 \approx I_{C2} = 3.23 \text{ mA}$$

$$R_i = \frac{10 - 2(0.7)}{3.23} \Rightarrow R_i = 2.66 \text{ k}\Omega$$

11.4

a. $0 = 0.7 + \frac{I_E}{2}(2) + I_E(85) - 5$

$$I_E = \frac{5 - 0.7}{85 + 1} \Rightarrow I_E = 0.050 \text{ mA}$$

$$I_{C1} = I_{C2} = \left(\frac{\beta}{1+\beta}\right)\left(\frac{I_E}{2}\right) = \left(\frac{100}{101}\right)\left(\frac{0.050}{2}\right)$$

$$\text{Or } I_{C1} = I_{C2} = 0.0248 \text{ mA}$$

$$V_{CE1} = V_{CE2} = [5 - I_{C1}(100)] - (-0.7)$$

$$\text{So } V_{CE1} = V_{CE2} = 3.22 \text{ V}$$

b. $v_{cm}(\max)$ for $V_{CB} = 0$ and

$$V_C = 5 - I_{C1}(100) = 2.52 \text{ V}$$

$$\text{So } v_{cm}(\max) = 2.52 \text{ V}$$

$v_{cm}(\min)$ for Q_1 and Q_2 at the edge of cutoff

$$\Rightarrow v_{cm}(\min) = -4.3 \text{ V}$$

(c) Differential-mode half circuits

$$-\frac{v_d}{2} = V_x + \left(\frac{V_x}{r_x} + g_m V_x\right) \cdot R_E \\ = V_x \left[1 + \frac{(1+\beta)}{r_x} R_E \right]$$

Then

$$V_s = \frac{-(v_d/2)}{1 + \frac{(1+\beta)}{r_s} R_E}$$

$$v_o = -g_m V_s R_C \Rightarrow A_d = \frac{1}{2} \cdot \frac{\beta R_C}{r_s + (1+\beta) R_E}$$

$$r_s = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.0248} = 105 \text{ k}\Omega$$

Then

$$A_d = \frac{1}{2} \cdot \frac{(100)(100)}{105 + (101)(2)} \Rightarrow A_d = 16.3$$

11.5

a. i. $v_{o1} - v_{o2} = 0$

ii. $I_{C1} = I_{C2} = 1 \text{ mA}$

$$\begin{aligned} v_{o1} - v_{o2} &= [V^+ - I_{C1} R_{C1}] - [V^+ - I_{C2} R_{C2}] \\ &= I_C (R_{C2} - R_{C1}) = (1)(7.9 - 8) \end{aligned}$$

$$\Rightarrow v_{o1} - v_{o2} = -0.1 \text{ V}$$

b. $I_0 = (I_{S1} + I_{S2}) \exp\left(\frac{v_{BE}}{V_T}\right)$

$$\begin{aligned} \text{So } \exp\left(\frac{v_{BE}}{V_T}\right) &= \frac{2 \times 10^{-3}}{10^{-13} + 1.1 \times 10^{-13}} \\ &= 9.524 \times 10^9 \end{aligned}$$

$$I_{C1} = I_{S1} \exp\left(\frac{v_{BE}}{V_T}\right) = (10^{-13})(9.524 \times 10^9)$$

$$\Rightarrow I_{C1} = 0.952 \text{ mA}$$

$$I_{C2} = (1.1 \times 10^{-13})(9.524 \times 10^9)$$

$$\Rightarrow I_{C2} = 1.048 \text{ mA}$$

i. $v_{o1} - v_{o2} = I_{C1} R_{C2} - I_{C2} R_{C1}$

$$\Rightarrow v_{o1} - v_{o2} = (1.048 - 0.952)(8)$$

$$\Rightarrow v_{o1} - v_{o2} = 0.768 \text{ V}$$

ii. $v_{o1} - v_{o2} = (1.048)(7.9) - (0.952)(8)$

$$v_{o1} - v_{o2} = 8.279 - 7.616$$

$$\Rightarrow v_{o1} - v_{o2} = 0.663 \text{ V}$$

11.6

From Equation (11.12(b))

$$i_{C2} = \frac{I_Q}{1 + e^{v_d/V_T}}$$

$$0.90 = \frac{1}{1 + e^{v_d/V_T}}$$

$$\text{So } e^{v_d/V_T} = \frac{1}{0.90} - 1 = 0.111$$

$$v_d = V_T \ln(0.111) = (0.026) \ln(0.111)$$

$$\Rightarrow v_d = -0.0571 \text{ V}$$

11.7

For $v_{CM} = 3.5 \text{ V}$ and a maximum peak-to-peak swing in the output voltage of 2 V , we need the quiescent collector voltage to be

$$V_c = 3.5 + 1 = 4.5 \text{ V}$$

Assume the bias is $\pm 10 \text{ V}$, and $I_Q = 0.5 \text{ mA}$.

Then $I_C = 0.25 \text{ mA}$

$$\text{Now } R_C = \frac{10 - 4.5}{0.25} \Rightarrow R_C = 22 \text{ k}\Omega$$

$$\text{In this case, } r_s = \frac{(100)(0.026)}{0.25} = 10.4 \text{ k}\Omega$$

Then

$$A_d = \frac{(100)(22)}{2(10.4 + 0.5)} = 101 \text{ So gain specification is met.}$$

For $CMRR_{dB} = 80 \text{ dB} \Rightarrow$

$$\begin{aligned} CMRR = 10^4 &= \frac{1}{2} \left[1 + \frac{(1+\beta)I_Q R_o}{V_T \beta} \right] \\ &= \frac{1}{2} \left[1 + \frac{(101)(0.5)R_o}{(0.026)(100)} \right] \Rightarrow \end{aligned}$$

$$R_o = 1.03 \text{ M}\Omega$$

Need to use a Modified Widlar current source.

$$R_o = r_s \left[1 + g_m (R_{E1} \| r_s) \right]$$

$$\text{If } V_A = 100 \text{ V, then } r_s = \frac{100}{0.5} = 200 \text{ k}\Omega$$

$$r_s = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$g_m = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

Then

$$1030 = 200 \left[1 + (19.23)(R_{E1} \| r_s) \right] \Rightarrow$$

$$R_{E1} \| r_s = 0.216 \text{ k}\Omega = R_{E1} \| 5.2 \Rightarrow$$

$$R_{E1} = 225 \text{ }\Omega$$

Also let $R_{E2} = 225 \text{ }\Omega$ and $I_{REF} = 0.5 \text{ mA}$

11.8

- a. For $v_1 = v_2 = 0$ and neglecting base currents

$$R_E = \frac{-0.7 - (-10)}{0.15} \Rightarrow R_E = 62 \text{ k}\Omega$$

- b. Using the small-signal equivalent circuit shown in Figure 11.8, but including the R_B resistors, we have

$$\frac{V_{\pi 1}}{r_\pi} + g_m V_{\pi 1} + g_m V_{\pi 2} + \frac{V_{\pi 2}}{r_\pi} = \frac{V_e}{R_E}$$

$$(V_{\pi 1} + V_{\pi 2}) \left(\frac{1 + \beta}{r_\pi} \right) = \frac{V_e}{R_E}$$

$$\text{Now } \frac{v_1 - V_e}{R_B + r_\pi} = \frac{V_{\pi 1}}{r_\pi} \text{ and } \frac{v_2 - V_e}{R_B + r_\pi} = \frac{V_{\pi 2}}{r_\pi}$$

Then

$$V_{\pi 1} = \left(\frac{r_\pi}{R_B + r_\pi} \right) (v_1 - V_e)$$

$$\text{and } V_{\pi 2} = \left(\frac{r_\pi}{R_B + r_\pi} \right) (v_2 - V_e)$$

Substituting, we find

$$(v_1 + v_2 - 2V_e) \left(\frac{r_\pi}{R_B + r_\pi} \right) \left(\frac{1 + \beta}{r_\pi} \right) = \frac{V_e}{R_E}$$

or

$$(v_1 + v_2 - 2V_e) \left(\frac{1 + \beta}{R_B + r_\pi} \right) = \frac{V_e}{R_E}$$

Solving for V_e ,

$$V_e = \frac{v_1 + v_2}{2 + \frac{R_B + r_\pi}{(1 + \beta)R_E}}$$

$$\text{Now } v_{02} = -g_m V_{\pi 2} R_C$$

$$= -g_m R_C \left(\frac{r_\pi}{R_B + r_\pi} \right) (v_2 - V_e)$$

Substituting for V_e ,

$$\begin{aligned} v_{02} &= \frac{-\beta R_C}{R_B + r_\pi} \left[v_2 - \frac{v_1 + v_2}{2 + \frac{R_B + r_\pi}{(1 + \beta)R_E}} \right] \\ &= \frac{-\beta R_C}{R_B + r_\pi} \left[\frac{v_2 \left(1 + \frac{R_B + r_\pi}{(1 + \beta)R_E} \right) - v_1}{2 + \frac{R_B + r_\pi}{(1 + \beta)R_E}} \right] \end{aligned}$$

$$\text{Now } v_1 = v_{cm} + \frac{v_d}{2}$$

$$v_2 = v_{cm} - \frac{v_d}{2}$$

Substituting and rearranging terms, we obtain

$$v_{02} = \frac{-\beta R_C}{R_B + r_\pi} \left\{ \frac{v_d}{2} - v_{cm} \left[\frac{1}{1 + \frac{2R_E(1 + \beta)}{R_B + r_\pi}} \right] \right\}$$

$$A_d = \frac{v_{02}}{v_d} = \frac{\beta R_C}{2(r_\pi + R_B)}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.075} = 34.7 \text{ k}\Omega$$

$$A_d = \frac{(100)(50)}{2(34.7 + 0.5)} \Rightarrow A_d = 71.0$$

$$A_{cm} = -\frac{\beta R_C}{r_\pi + R_B} \left[\frac{1}{1 + \frac{2R_E(1 + \beta)}{r_\pi + R_B}} \right]$$

$$= -\frac{(100)(50)}{34.7 + 0.5} \left[\frac{1}{1 + \frac{2(62)(101)}{34.7 + 0.5}} \right]$$

$$\Rightarrow A_{cm} = -0.398$$

$$CMRR_{dB} = 20 \log_{10} \left| \frac{71.0}{0.398} \right| \Rightarrow CMRR_{dB} = 45.0 \text{ dB}$$

$$c. R_{id} = 2(r_\pi + R_B)$$

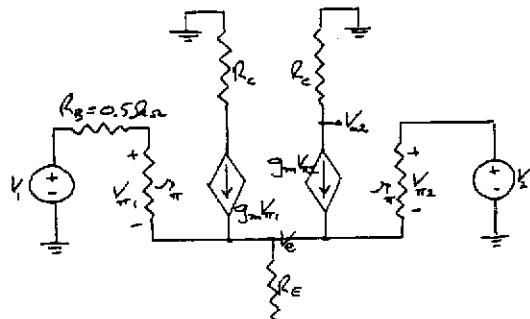
$$R_{id} = 2(34.7 + 0.5) \Rightarrow R_{id} = 70.4 \text{ k}\Omega$$

Common-mode input resistance

$$\begin{aligned} R_{icm} &= \frac{1}{2} [r_\pi + R_B + 2(1 + \beta)R_E] \\ &= \frac{1}{2} [34.7 + 0.5 + 2(101)(62)] \\ &\Rightarrow R_{icm} = 6.28 \text{ M}\Omega \end{aligned}$$

11.9

$$(a) R_E = \frac{-0.7 - (-10)}{0.25} \Rightarrow R_E = 372 \text{ k}\Omega$$



(b)

$$\frac{V_{\pi 1}}{r_\pi} + g_m V_{\pi 1} + \frac{V_{\pi 2}}{r_\pi} + g_m V_{\pi 2} = \frac{V_s}{R_E}$$

$$\text{or (1)} \left(\frac{1 + \beta}{r_\pi} \right) (V_{\pi 1} + V_{\pi 2}) = \frac{V_s}{R_E}$$

$$\frac{V_{\pi 1}}{r_\pi} = \frac{V_1 - V_s}{R_E + r_\pi} \Rightarrow V_{\pi 1} = \left(\frac{r_\pi}{r_\pi + R_E} \right) (V_1 - V_s)$$

$$V_{o2} = V_2 - V_s$$

Then

$$(1) \left(\frac{1+\beta}{r_\pi} \right) \left[\frac{r_\pi}{r_\pi + R_B} (V_1 - V_s) + (V_2 - V_s) \right] = \frac{V_s}{R_g}$$

From this, we find

$$V_s = \frac{V_1 + \frac{r_\pi + R_B}{r_\pi} \cdot V_2}{\frac{r_\pi + R_B}{R_g(1+\beta)} + 1 + \frac{r_\pi + R_B}{r_\pi}}$$

Now

$$V_o = -g_m V_{o2} R_C = -g_m R_C (V_2 - V_s)$$

We have

$$r_\pi = \frac{(120)(0.026)}{0.125} \approx 25 \text{ k}\Omega, \quad g_m = \frac{0.125}{0.026} = 4.81 \text{ mA/V}$$

$$(i) \quad \text{Set } V_1 = \frac{V_d}{2} \text{ and } V_2 = -\frac{V_d}{2}$$

Then

$$V_s = \frac{\frac{V_d}{2} \left(1 - \left(\frac{25+0.5}{25} \right) \right)}{\frac{25+0.5}{(37.2)(121)} + 1 + \frac{25+0.5}{25}} = \frac{\frac{V_d}{2} (-0.02)}{2.026}$$

So

$$V_o = -0.00494 V_d$$

Now

$$V_o = -(4.81)(50) \left(-\frac{V_d}{2} - (-0.00494) V_d \right) \Rightarrow$$

$$A_d = \frac{V_o}{V_d} = 119$$

$$(ii) \quad \text{Set } V_1 = V_2 = V_{cm}$$

Then

$$V_s = \frac{\frac{V_{cm}}{2} \left(1 + \frac{25+0.5}{25} \right)}{\frac{25+0.5}{(37.2)(121)} + 1 + \frac{25+0.5}{25}} = \frac{V_{cm}(2.02)}{2.02567}$$

$$V_o = V_{cm}(0.9972)$$

Then

$$V_o = -(4.81)(50)[V_{cm} - V_{cm}(0.9972)]$$

or

$$A_{cm} = \frac{V_o}{V_{cm}} = -0.673$$

11.10

a. Neglecting base currents

$$I_1 = I_3 = 400 \mu\text{A} \Rightarrow R_1 = \frac{30 - 0.7}{0.4}$$

$$\Rightarrow R_1 = 73.25 \text{ k}\Omega$$

$$V_{CE1} = 10 \text{ V} \Rightarrow V_{C1} = 9.3 \text{ V}$$

$$R_C = \frac{15 - 9.3}{0.2} \Rightarrow R_C = 28.5 \text{ k}\Omega$$

$$b. \quad r_\pi = \frac{(100)(0.026)}{0.2} = 13 \text{ k}\Omega$$

$$r_0(Q_3) = \frac{50}{0.4} = 125 \text{ k}\Omega$$

Using the results from problem 11.9, we have

$$A_d = \frac{\beta R_C}{2(r_\pi + R_B)} = \frac{(100)(28.5)}{2(13 + 10)} \Rightarrow A_d = 62$$

$$A_{cm} = -\frac{\beta R_C}{r_\pi + R_B} \left\{ \frac{1}{1 + \frac{2r_0(1+\beta)}{r_\pi + R_B}} \right\}$$

$$= -\frac{(100)(28.5)}{13 + 10} \left\{ \frac{1}{1 + \frac{2(125)(101)}{13 + 10}} \right\}$$

$$\Rightarrow A_{cm} = -0.113$$

$$CMRR_{dB} = 20 \log_{10} \left(\frac{62}{0.113} \right)$$

$$\Rightarrow CMRR_{dB} = 54.8 \text{ dB}$$

$$c. \quad R_{id} = 2(r_\pi + R_B) = 2(13 + 10) \Rightarrow R_{id} = 46 \text{ k}\Omega$$

$$R_{icm} = \frac{1}{2}[r_\pi + R_B + 2(1+\beta)r_0]$$

$$= \frac{1}{2}[13 + 10 + 2(101)(125)]$$

$$\Rightarrow R_{icm} = 12.6 \text{ M}\Omega$$

11.11

From Equation (11.18)

$$v_0 = v_{C2} - v_{C1} = g_m R_C v_d$$

$$g_m = \frac{I_{CQ}}{V_T}$$

$$\text{For } I_Q = 2 \text{ mA, } I_{CQ} = 1 \text{ mA}$$

$$\text{Then } g_m = \frac{1}{0.026} = 38.46 \text{ mA/V}$$

Now

$$2 = (38.46)R_C(0.015)$$

$$\text{So } R_C = 3.47 \text{ k}\Omega$$

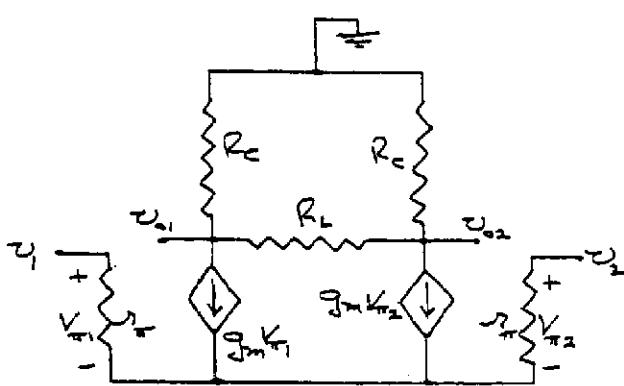
$$\text{Now } V_C = V^+ - I_C R_C = 10 - (1)(3.47)$$

$$= 6.53 \text{ V}$$

$$\text{For } V_{CB} = 0 \Rightarrow v_{cm}(\text{max}) = 6.53 \text{ V}$$

11.12

The small-signal equivalent circuit is



$$A \text{ KVL equation: } v_1 = V_{\pi 1} - V_{\pi 2} + v_2$$

$$v_1 - v_2 = V_{\pi 1} - V_{\pi 2}$$

A KCL equation

$$\frac{V_{\pi 1}}{r_\pi} + g_m V_{\pi 1} + \frac{V_{\pi 2}}{r_\pi} + g_m V_{\pi 2} = 0$$

$$(V_{\pi 1} + V_{\pi 2}) \left(\frac{1}{r_\pi} + g_m \right) = 0 \Rightarrow V_{\pi 1} = -V_{\pi 2}$$

$$\text{Then } v_1 - v_2 = 2V_{\pi 1} \Rightarrow V_{\pi 1} = \frac{1}{2}(v_1 - v_2)$$

$$\text{and } V_{\pi 2} = -\frac{1}{2}(v_1 - v_2)$$

At the v_{01} node:

$$\frac{v_{01}}{R_C} + \frac{v_{01} - v_{02}}{R_L} + g_m V_{\pi 1} = 0$$

$$v_{01} \left(\frac{1}{R_C} + \frac{1}{R_L} \right) - v_{02} \left(\frac{1}{R_L} \right) = \frac{1}{2} g_m (v_2 - v_1) \quad (1)$$

At the v_{02} node:

$$\frac{v_{02}}{R_C} + \frac{v_{02} - v_{01}}{R_L} + g_m V_{\pi 2} = 0$$

$$v_{02} \left(\frac{1}{R_C} + \frac{1}{R_L} \right) - v_{01} \left(\frac{1}{R_L} \right) = \frac{1}{2} g_m (v_1 - v_2) \quad (2)$$

From (1) :

$$v_{02} = v_{01} \left(1 + \frac{R_L}{R_C} \right) - \frac{1}{2} g_m R_L (v_2 - v_1)$$

Substituting into (2)

$$\begin{aligned} v_{01} \left(1 + \frac{R_L}{R_C} \right) \left(\frac{1}{R_C} + \frac{1}{R_L} \right) \\ - \frac{1}{2} g_m R_L (v_2 - v_1) \left(\frac{1}{R_C} + \frac{1}{R_L} \right) \\ - v_{01} \left(\frac{1}{R_L} \right) = \frac{1}{2} g_m (v_1 - v_2) \end{aligned}$$

$$\begin{aligned} v_{01} \left(\frac{1}{R_C} + \frac{R_L}{R_C^2} + \frac{1}{R_C} \right) \\ = \frac{1}{2} g_m (v_1 - v_2) \left[1 - \left(\frac{R_L}{R_C} + 1 \right) \right] \\ \frac{v_{01}}{R_C} \left(2 + \frac{R_L}{R_C} \right) = -\frac{1}{2} g_m \left(\frac{R_L}{R_C} \right) (v_1 - v_2) \end{aligned}$$

For $v_1 - v_2 = v_d$

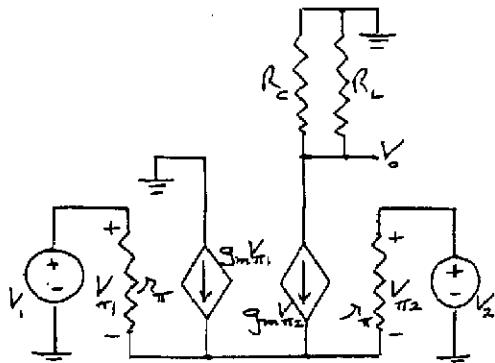
$$A_{v1} = \frac{v_{01}}{v_d} = \frac{-\frac{1}{2} g_m R_L}{\left(2 + \frac{R_L}{R_C} \right)}$$

$$\text{From symmetry: } A_{v2} = \frac{v_{02}}{v_d} = \frac{\frac{1}{2} g_m R_L}{\left(2 + \frac{R_L}{R_C} \right)}$$

$$\text{Then } A_v = \frac{v_{02} - v_{01}}{v_d} = \frac{g_m R_L}{\left(2 + \frac{R_L}{R_C} \right)}$$

11.13

The small-signal equivalent circuit is



$$\text{KVL equation: } v_1 = V_{\pi 1} - V_{\pi 2} + v_2 \text{ or}$$

$$v_1 - v_2 = V_{\pi 1} - V_{\pi 2}$$

KCL equation:

$$\frac{V_{\pi 1}}{r_\pi} + g_m V_{\pi 1} + g_m V_{\pi 2} + \frac{V_{\pi 2}}{r_\pi} = 0$$

$$(V_{\pi 1} + V_{\pi 2}) \left(\frac{1}{r_\pi} + g_m \right) = 0 \Rightarrow V_{\pi 1} = -V_{\pi 2}$$

$$\text{Then } v_1 - v_2 = -2V_{\pi 2} \text{ or } V_{\pi 2} = -\frac{1}{2}(v_1 - v_2)$$

$$\text{Now } v_0 = -g_m V_{\pi 2} (R_C \parallel R_L)$$

$$= \frac{1}{2} g_m (R_C \parallel R_L) (v_1 - v_2)$$

For $v_1 - v_2 \equiv v_d$

$$\Rightarrow A_d = \frac{v_0}{v_d} = \frac{1}{2} g_m (R_C \parallel R_L)$$

11.14

We have

$$V_{C2} = -g_m V_{s2} R_C = -g_m (V_{b2} - V_s) R_C$$

and

$$V_{C1} = -g_m V_{s1} R_C = -g_m (V_{b1} - V_s) R_C$$

Then

$$\begin{aligned} V_0 &= V_{C2} - V_{C1} \\ &= -g_m (V_{b2} - V_s) R_C - [-g_m (V_{b1} - V_s) R_C] \\ &= g_m R_C (V_{b1} - V_{b2}) \end{aligned}$$

Differential gain

$$A_d = \frac{V_0}{V_{b1} - V_{b2}} = g_m R_C$$

Common-mode gain

$$A_{cm} = 0$$

11.15

$$(a) v_{cm} = 3V \Rightarrow V_{c1} = V_{c2} = 3V$$

$$\text{Then } R_C = \frac{10 - 3}{0.1} \Rightarrow R_C = 70 k\Omega$$

$$(b) CMRR_{dB} = 75 dB \Rightarrow CMRR = 5623$$

Now

$$CMRR = \frac{1}{2} \left[1 + \frac{(1+\delta)I_Q R_o}{\beta V_T} \right]$$

$$5623 = \frac{1}{2} \left[1 + \frac{(151)(0.2)R_o}{(150)(0.026)} \right] \Rightarrow R_o = 1.45 M\Omega$$

Use a Widlar current source.

$$R_o = r_s [1 + g_m R'_s]$$

Let V_A of current source transistor be 100V.

Then

$$r_s = \frac{100}{0.2} = 500 k\Omega, g_m = \frac{0.2}{0.026} = 7.69 mA/V$$

$$r_s = \frac{(150)(0.026)}{0.2} = 19.5 k\Omega$$

So

$$1450 = 500 [1 + (7.69)R'_s] \Rightarrow R'_s = 0.247 k\Omega$$

Now

$$R'_s = R_s \| r_s \Rightarrow 0.247 = R_s \| 19.5 \Rightarrow R_s = 250 \Omega$$

Then

$$I_Q R_s = V_T \ln \left(\frac{I_{REF}}{I_Q} \right)$$

$$(0.2)(0.250) = (0.026) \ln \left(\frac{I_{REF}}{(0.2)} \right) \Rightarrow I_{REF} = 1.37 mA$$

Then

$$R_i = \frac{10 - 0.7 - (-10)}{1.37} \Rightarrow R_i = 14.1 k\Omega$$

11.16

$$A_d = 180, CMRR_{dB} = 85 dB$$

$$CMRR = 17,783 = \left| \frac{A_d}{A_{cm}} \right| = \frac{180}{A_{cm}}$$

$$\Rightarrow |A_{cm}| = 0.01012$$

Assume the common-mode gain is negative.

$$v_0 = A_d v_d + A_{cm} v_{cm}$$

$$= 180 v_d - 0.01012 v_{cm}$$

$$v_d = 180(2 \sin \omega t) mV - (0.01012)(2 \sin \omega t) V$$

$$v_0 = 0.36 \sin \omega t - 0.02024 \sin \omega t$$

$$\text{Ideal Output: } v_0 = 0.360 \sin \omega t (V)$$

$$\text{Actual Output: } v_0 = 0.340 \sin \omega t (V)$$

11.17

At terminal A.

$$R_{THA} = R_A \| R = \frac{R(1+\delta) \cdot R}{R(1+\delta) + R} = \frac{R(1+\delta)}{2+\delta} \approx \frac{R}{2} = 5 k\Omega$$

Variation in R_{TH} is not significant

$$V_{THA} = \left(\frac{R_A}{R_A + R} \right) V^+ = \frac{R(1+\delta)(5)}{R(1+\delta) + R} = \frac{5(1+\delta)}{2+\delta}$$

At terminal B.

$$R_{THB} = R \| R = \frac{R}{2} = 5 k\Omega$$

$$V_{THB} = \left(\frac{R}{R + R} \right) V^+ = 2.5V$$

From Eq. (11.27)

$$V_o = \frac{-\beta R_c (V_2 - V_1)}{2(r_s + R_s)} \text{ where } V_2 = V_{THB} \text{ and } V_1 = V_{THA}$$

$$R_s = 5 k\Omega, r_s = \frac{(120)(0.026)}{0.25} = 12.5 k\Omega$$

So

$$V_o = \frac{-(120)(3)(V_2 - V_1)}{2(12.5 + 5)} = -10.3(V_2 - V_1)$$

We can find $V_2 - V_1 = V_{THB} - V_{THA}$

$$V_{THB} - V_{THA} = 2.5 - \left[\frac{5(1+\delta)}{2+\delta} \right]$$

$$= \frac{2.5(2+\delta) - 5(1+\delta)}{2+\delta} = \frac{2.5\delta - 5\delta}{2+\delta}$$

$$\approx \frac{-2.5\delta}{2} = -1.25\delta$$

Then

$$V_o = -(10.3)(-1.25)\delta = 12.9\delta$$

So for $-0.01 \leq \delta \leq 0.01$

We have

$$-0.129 \leq V_o \leq 0.129 V$$

11.18

a. $R_{id} = 2r_\pi$

$$r_\pi = \frac{(180)(0.026)}{0.2} = 23.4 \text{ k}\Omega$$

So

$$\underline{R_{id} = 46.8 \text{ k}\Omega}$$

b. Assuming $r_\mu \rightarrow \infty$, then

$$R_{icm} \approx [(1 + \beta)R_0] \parallel \left[(1 + \beta) \left(\frac{r_0}{2} \right) \right]$$

$$r_0 = \frac{125}{0.2} = 625 \text{ k}\Omega$$

$$R_{icm} = [(181)(1)] \parallel [(181)(0.3125)] \\ = 181 \parallel 56.56$$

$$\Rightarrow \underline{R_{icm} = 43.1 \text{ M}\Omega}$$

11.19

a. For $I_1 = 1 \text{ mA}$, $V_{BE3} = 0.7 \text{ V}$

$$R_1 = \frac{20 - 0.7}{1} \Rightarrow \underline{R_1 = 19.3 \text{ k}\Omega}$$

$$R_2 = \frac{V_T}{I_Q} \cdot \ln \left(\frac{I_1}{I_Q} \right) = \frac{0.026}{0.1} \cdot \ln \left(\frac{1}{0.1} \right) \\ \Rightarrow \underline{R_2 = 0.599 \text{ k}\Omega}$$

b. $r_{\pi 1} = \frac{(180)(0.026)}{0.1} = 46.8 \text{ k}\Omega$

$$g_m = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$r_{o1} = \frac{100}{0.1} \Rightarrow 1 \text{ M}\Omega$$

From Chapter 10

$$R_0 = r_{o1}[1 + g_m(R_E || r_{\pi 1})]$$

$$R_E || r_{\pi 1} = 0.599 || 46.8 = 0.591$$

$$R_0 = (1)[1 + (3.846)(0.591)] = 3.27 \text{ M}\Omega$$

$$r_{o1} = \frac{100}{0.05} \Rightarrow 2 \text{ M}\Omega$$

$$R_{icm} \approx [(1 + \beta)R_0] \parallel \left[(1 + \beta) \left(\frac{r_{o1}}{2} \right) \right]$$

$$= [(181)(3.27)] \parallel [(181)(1)]$$

$$= 592 \parallel 181$$

$$\Rightarrow \underline{R_{icm} = 139 \text{ M}\Omega}$$

(c) From Eq. (11.32(b))

$$A_{cm} = \frac{-g_m R_c}{1 + \frac{2(1+\beta)R_o}{r_s + R_s}}$$

$$g_m = \frac{0.05}{0.026} = 1.923 \text{ mA/V}$$

$$r_s = \frac{(180)(0.026)}{0.05} = 93.6 \text{ k}\Omega$$

$$R_s = 0$$

Then

$$A_{cm} = \frac{-(1.923)(50)}{1 + \frac{2(181)(3270)}{93.6}} \Rightarrow \underline{A_{cm} = -0.00760}$$

11.20

$$A_{d1} = g_{m1}(R_1 || r_{s3})$$

$$g_{m1} = \frac{I_{Q1}/2}{V_T} = 19.23 I_{Q1}$$

$$r_{s3} = \frac{\beta V_T}{I_{Q2}/2} = \frac{2(100)(0.026)}{I_{Q2}} = \frac{5.2}{I_{Q2}}$$

$$A_{d2} = \frac{g_{m3} R_2}{2}, \quad g_{m3} = \frac{I_{Q2}/2}{V_T} = 19.23 I_{Q2}$$

Then

$$30 = \frac{(19.23)I_{Q2}}{2} \cdot R_2 \Rightarrow I_{Q2}R_2 = 3.12 \text{ V}$$

Maximum $v_{o2} - v_{o1} = \pm 18 \text{ mV}$ for linearity

$$v_{o3}(\max) = (\pm 18)(30) \text{ mV} \Rightarrow \pm 0.54 \text{ V}$$

so $I_{Q2}R_2 = 3.12 \text{ V}$ is OK.

From A_{d1} :

$$20 = 19.23 I_{Q1}(R_1 || r_{s3})$$

$$= 19.23 I_{Q1} \left(\frac{R_1 \left(\frac{5.2}{I_{Q2}} \right)}{R_1 + \left(\frac{5.2}{I_{Q2}} \right)} \right)$$

$$20 = \frac{19.23 I_{Q1} R_1 (5.2)}{I_{Q2} R_1 + 5.2}$$

$$\text{Let } \frac{I_{Q1}}{2} \cdot R_1 = 5 \text{ V} \Rightarrow I_{Q1} R_1 = 10 \text{ V}$$

Then

$$20 = \frac{19.23(10)(5.2)}{I_{Q2} R_1 + 5.2} \Rightarrow I_{Q2} R_1 = 44.8 \text{ V}$$

Now

$$I_{Q1} R_1 = 10 \Rightarrow R_1 = \frac{10}{I_{Q1}}$$

So

$$I_{Q2} \left(\frac{10}{I_{Q1}} \right) = 44.8 \Rightarrow \frac{I_{Q2}}{I_{Q1}} = 4.48$$

$$\text{Let } I_{Q1} = 100 \mu\text{A}, I_{Q2} = 448 \mu\text{A}$$

Then

$$I_{Q2} R_1 = 3.12 \Rightarrow R_1 = 6.96 \text{ k}\Omega$$

$$I_{Q1} R_1 = 10 \Rightarrow \underline{R_1 = 100 \text{ k}\Omega}$$

11.21

a. $I_1 = \frac{20 - V_{GS3}}{50} = 0.25(V_{GS3} - 2)^2$

$$20 - V_{GS3} = 12.5(V_{GS3}^2 - 4V_{GS3} + 4)$$

$$12.5V_{GS3}^2 - 49V_{GS3} + 30 = 0$$

$$V_{GS3} = \frac{49 \pm \sqrt{(49)^2 - 4(12.5)(30)}}{2(12.5)}$$

$$\Rightarrow V_{GS3} = 3.16 \text{ V}$$

$$I_1 = \frac{20 - 3.16}{50} \Rightarrow I_1 = I_Q = 0.337 \text{ mA}$$

$$I_{D1} = \frac{I_Q}{2} \Rightarrow I_{D1} = 0.168 \text{ mA}$$

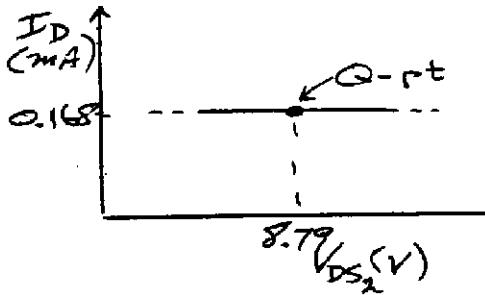
$$0.168 = 0.25(V_{GS1} - 2)^2 \Rightarrow V_{GS1} = 2.82 \text{ V}$$

$$V_{DS4} = -2.82 - (-10) \Rightarrow V_{DS4} = 7.18 \text{ V}$$

$$V_{D1} = 10 - (0.168)(24) = 5.97 \text{ V}$$

$$V_{DS1} = 5.97 - (-2.82) \Rightarrow V_{DS1} = 8.79 \text{ V}$$

(b)



(c) Max $v_{cm} \Rightarrow V_{DS1} = V_{DS2} = V_{DS}(sat) = V_{as1} - V_{TN}$
 $2.82 - 2 = 0.82 \text{ V}$

Now $V_{D1} = 10 - (0.168)(24) = 5.97 \text{ V}$

$$V_s(\max) = 5.97 - V_{DS1}(sat) = 5.97 - 0.82$$

$$V_s(\max) = 5.15 \text{ V}$$

$$v_{cm}(\max) = V_s(\max) + V_{as1} = 5.15 + 2.82$$

$$v_{cm}(\max) = 7.97 \text{ V}$$

$$v_{cm}(\min) = V^- + V_{DS4}(sat) + V_{as1}$$

$$V_{DS4}(sat) = V_{as4} - V_{TN} = 3.16 - 2 = 1.16 \text{ V}$$

Then

$$v_{cm}(\min) = -10 + 1.16 + 2.82 \Rightarrow$$

$$v_{cm}(\min) = -6.02 \text{ V}$$

11.22

a. $I_{D1} = I_{D2} = 120 \mu\text{A} = 100(V_{GS1} - 1.2)^2$

$$\Rightarrow V_{GS1} = V_{GS2} = 2.30 \text{ V}$$

For $v_1 = v_2 = -5.4 \text{ V}$ and $V_{DS1} = V_{DS2} = 12 \text{ V}$

$$\Rightarrow V_0 = -5.4 - 2.30 + 12 = 4.3 \text{ V}$$

$$R_D = \frac{10 - 4.3}{0.12} \Rightarrow R_D = 47.5 \text{ k}\Omega$$

$I_Q = I_{D1} + I_{D2} \Rightarrow I_Q = I_1 = 240 \mu\text{A}$

$$I_1 = 240 = 200(V_{GS3} - 1.2)^2 \Rightarrow V_{GS3} = 2.30 \text{ V}$$

$$R_1 = \frac{20 - 2.3}{0.24} \Rightarrow R_1 = 73.75 \text{ k}\Omega$$

b. $r_{04} = \frac{1}{M_Q} = \frac{1}{(0.01)(0.24)} = 416.7 \text{ k}\Omega$

$$\Delta I_Q = \frac{1}{r_{04}} \cdot \Delta V_{DS} = \frac{5.4}{416.7} \Rightarrow \Delta I_Q \approx 13 \mu\text{A}$$

11.23

a. $R_D = \frac{10 - 7}{0.5} \Rightarrow R_D = 6 \text{ k}\Omega$

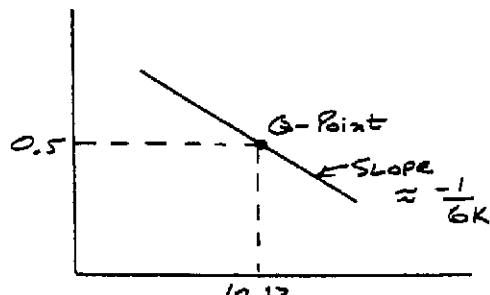
$$I_Q = I_{D1} + I_{D2} \Rightarrow I_Q = 1 \text{ mA}$$

b. $10 = I_D(6) + V_{DS} - V_{GS}$

and $V_{as} = \sqrt{\frac{I_D}{K_n} + V_{TN}}$

For $I_D = 0.5 \text{ mA}$, $V_{GS} = \sqrt{\frac{0.5}{0.4}} + 2 = 3.12 \text{ V}$

and $V_{DS} = 10.12$



Load line is actually nonlinear.

c. Maximum common-mode voltage when M_1 and M_2 reach the transition point, or

$$V_{DS}(sat) = V_{as} - V_{TN} = 3.12 - 2 = 1.12 \text{ V}$$

Then

$$v_{cm} = v_{02} - v_{DS}(sat) + V_{GS} = 7 - 1.12 + 3.12$$

$$\text{Or } v_{cm}(\max) = 9 \text{ V}$$

Minimum common-mode voltage, voltage across I_Q becomes zero.

$$\text{So } v_{cm}(\min) = -10 + 3.12$$

$$\Rightarrow v_{cm}(\min) = -6.88 \text{ V}$$

11.24

a. $I_{D1} = I_{D2} = 0.5 \text{ mA}$

$$v_{01} - v_{02} = [V^+ - I_{D1}R_{D1}] - [V^+ - I_{D2}R_{D2}]$$

$$v_{01} - v_{02} = I_{D2}R_{D2} - I_{D1}R_{D1} = I_D(R_{D2} - R_{D1})$$

i. $R_{D1} = R_{D2} = 6 \text{ k}\Omega, v_{o1} - v_{o2} = 0$

ii. $R_{D1} = 6 \text{ k}\Omega, R_{D2} = 5.9 \text{ k}\Omega$

$$v_{o1} - v_{o2} = (0.5)(5.9 - 6)$$

$$\Rightarrow v_{o1} - v_{o2} = -0.05 \text{ V}$$

b. $K_m = 0.4 \text{ mA/V}^2, K_{m2} = 0.44 \text{ mA/V}^2$

$$V_{GS1} = V_{GS2}$$

$$I_Q = (K_m + K_{m2})(V_{GS} - V_{TH})^2$$

$$1 = (0.4 + 0.44)(V_{GS} - V_{TH})^2$$

$$\Rightarrow (V_{GS} - V_{TH})^2 = 1.19$$

$$I_{D1} = (0.4)(1.19) = 0.476 \text{ mA}$$

$$I_{D2} = (0.44)(1.19) = 0.524 \text{ mA}$$

i. $R_{D1} = R_{D2} = 6 \text{ k}\Omega$

$$v_{o1} - v_{o2} = (0.524 - 0.476)(6)$$

$$\Rightarrow v_{o1} - v_{o2} = 0.288 \text{ V}$$

ii. $R_{D1} = 6 \text{ k}\Omega, R_{D2} = 5.9 \text{ k}\Omega$

$$v_{o1} - v_{o2} = (0.524)(5.9) - (0.476)(6)$$

$$= 3.0916 - 2.856$$

$$\Rightarrow v_{o1} - v_{o2} = 0.236 \text{ V}$$

11.25

(a) From Equation (11.51)

$$\frac{i_{D2}}{I_Q} = \frac{1}{2} - \sqrt{\frac{K_m}{2I_Q} \cdot v_d} \sqrt{1 - \left(\frac{K_m}{2I_Q}\right)v_d^2}$$

$$0.90 = 0.50 - \sqrt{\frac{0.1}{2(0.25)} \cdot v_d} \sqrt{1 - \left[\frac{0.1}{2(0.25)}\right]v_d^2}$$

$$-0.40 = -(0.4472)v_d \sqrt{1 - (0.2)v_d^2}$$

$$0.8945 = v_d \sqrt{1 - (0.2)v_d^2}$$

Square both sides

$$0.80 = v_d^2(1 - [0.2]v_d^2)$$

$$(0.2)(v_d^2)^2 - v_d^2 + 0.80 = 0$$

$$v_d^2 = \frac{1 \pm \sqrt{1 - 4(0.2)(0.80)}}{2(0.2)} = 4V^2 \text{ or } 1V^2$$

Then $v_d = 2 \text{ V or } 1 \text{ V}$

$$\text{But } |v_d|_{\max} = \sqrt{\frac{I_Q}{k_m}} = \sqrt{\frac{0.25}{0.1}} = 1.58$$

So $v_d = 1 \text{ V}$

b. From part (a), $v_{d,\max} = 1.58 \text{ V}$

11.26

$$A_d = \frac{g_m R_D}{2}$$

For $v_{CM} = 2.5 \text{ V}$

$$I_{D1} = I_{D2} = \frac{I_Q}{2} = 0.25 \text{ mA}$$

Let $V_{D1} = V_{D2} = 3 \text{ V}$, then $R_D = \frac{10 - 3}{0.25} \Rightarrow$

$$R_D = 28 \text{ k}\Omega$$

Then $100 = \frac{g_m(28)}{2} \Rightarrow g_m = 7.14 \text{ mA/V}$

$$\text{And } g_m = 2\sqrt{\frac{k'_m}{2} \left(\frac{W}{L}\right) I_D}$$

$$7.14 = 2\sqrt{\left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)(0.25)} \Rightarrow$$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 1274 \text{ (Extremely large transistors to meet the gain requirement.)}$$

Need $|A_{CM}| = 0.10$

From Eq.(11.64(b))

$$|A_{CM}| = \frac{g_m R_o}{1 + 2g_m R_o}$$

$$\text{So } 0.10 = \frac{(7.14)(28)}{1 + 2(7.14)R_o} \Rightarrow R_o = 140 \text{ k}\Omega$$

For the basic 2-transistor current source

$$R_o = r_o = \frac{1}{M_Q} = \frac{1}{(0.01)(0.5)} = 200 \text{ k}\Omega$$

This current source is adequate to meet common-mode gain requirement.

11.27

a. $I_S = \frac{-V_{GS1} - (-5)}{R_S}$

$$\text{and } I_S = 2I_D = 2K_m(V_{GS1} - V_{TH})^2$$

$$\frac{5 - V_{GS1}}{20} = 2(0.050)(V_{GS1} - 1)^2$$

$$5 - V_{GS1} = 2(V_{GS1}^2 - 2V_{GS1} + 1)$$

$$2V_{GS1}^2 - 3V_{GS1} - 3 = 0$$

$$V_{GS1} = \frac{3 \pm \sqrt{(3)^2 + 4(2)(3)}}{2(2)} \Rightarrow V_{GS1} = 2.186 \text{ V}$$

$$I_S = \frac{5 - 2.186}{20} \Rightarrow I_S = 0.141 \text{ mA}$$

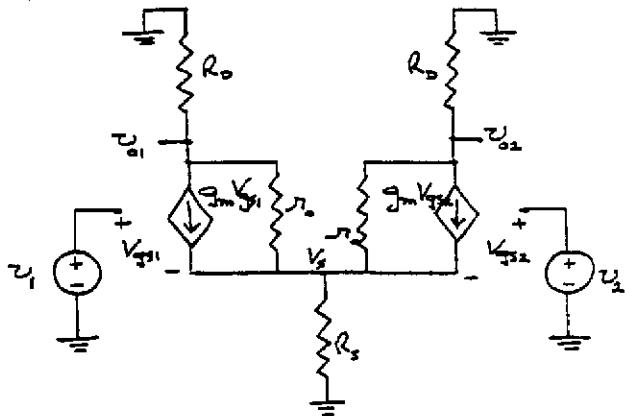
$$I_{D1} = I_{D2} = \frac{I_S}{2} \Rightarrow I_{D1} = I_{D2} = 0.0704 \text{ mA}$$

$$v_{o2} = 5 - (0.0704)(25) \Rightarrow v_{o2} = 3.24 \text{ V}$$

$$b. \quad g_m = 2K_n(V_{GS} - V_{TH}) = 2(0.05)(2.186 - 1)$$

$$g_m = 0.119 \text{ mA/V}$$

$$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.0704)} = 710 \text{ k}\Omega$$



$$V_{gs1} = \nu_1 - V_S, \quad V_{gs2} = \nu_2 - V_S$$

$$\frac{\nu_{01}}{R_D} + g_m V_{gs1} + \frac{\nu_{01} - V_S}{r_0} = 0$$

$$\nu_{01} \left(\frac{1}{R_D} + \frac{1}{r_0} \right) + g_m (\nu_1 - V_S) - \frac{V_S}{r_0} = 0 \quad (1)$$

$$\frac{\nu_{02}}{R_D} + g_m V_{gs2} + \frac{\nu_{02} - V_S}{r_0} = 0$$

$$\nu_{02} \left(\frac{1}{R_D} + \frac{1}{r_0} \right) + g_m (\nu_2 - V_S) - \frac{V_S}{r_0} = 0 \quad (2)$$

$$g_m V_{gs1} + \frac{\nu_{01} - V_S}{r_0} + \frac{\nu_{02} - V_S}{r_0} + g_m V_{gs2} = \frac{V_S}{R_S}$$

$$g_m (\nu_1 - V_S) + \frac{\nu_{01}}{r_0} + \frac{\nu_{02}}{r_0} - \frac{2V_S}{r_0} + g_m (\nu_2 - V_S) = \frac{V_S}{R_S}$$

$$g_m (\nu_1 + \nu_2) + \frac{\nu_{01}}{r_0} + \frac{\nu_{02}}{r_0} = V_S \left\{ 2g_m + \frac{2}{r_0} + \frac{1}{R_S} \right\} \quad (3)$$

From (1)

$$\nu_{01} = \frac{V_S \left(g_m + \frac{1}{r_0} \right) - g_m \nu_1}{\left(\frac{1}{R_D} + \frac{1}{r_0} \right)}$$

Then

$$g_m (\nu_1 + \nu_2) + \frac{V_S \left(g_m + \frac{1}{r_0} \right) - g_m \nu_1}{r_0 \left(\frac{1}{R_D} + \frac{1}{r_0} \right)} + \frac{\nu_{02}}{r_0} = V_S \left\{ 2g_m + \frac{2}{r_0} + \frac{1}{R_S} \right\} \quad (3)$$

$$g_m (\nu_1 + \nu_2) r_0 \left(\frac{1}{R_D} + \frac{1}{r_0} \right) + V_S \left(g_m + \frac{1}{r_0} \right) \\ - g_m \nu_1 + \nu_{02} \left(\frac{1}{R_D} + \frac{1}{r_0} \right) \\ = V_S \left\{ 2g_m + \frac{2}{r_0} + \frac{1}{R_S} \right\} \cdot r_0 \left(\frac{1}{R_D} + \frac{1}{r_0} \right)$$

$$g_m (\nu_1 + \nu_2) \left(1 + \frac{r_0}{R_D} \right) - g_m \nu_1 + \nu_{02} \left(\frac{1}{R_D} + \frac{1}{r_0} \right) \\ = V_S \left\{ \left(2g_m + \frac{2}{r_0} + \frac{1}{R_S} \right) \left(1 + \frac{r_0}{R_D} \right) - \left(g_m + \frac{1}{r_0} \right) \right\}$$

$$g_m \left(\nu_1 \cdot \frac{r_0}{R_D} + \nu_2 + \nu_2 \cdot \frac{r_0}{R_D} \right) + \nu_{02} \left(\frac{1}{R_D} + \frac{1}{r_0} \right) \\ = V_S \left\{ 2g_m + \frac{2}{r_0} + \frac{1}{R_S} + 2g_m \cdot \frac{r_0}{R_D} + \frac{2}{R_D} \right. \\ \left. + \frac{r_0}{R_S R_D} - g_m - \frac{1}{r_0} \right\}$$

$$g_m \left(\nu_1 \cdot \frac{r_0}{R_D} + \nu_2 + \nu_2 \cdot \frac{r_0}{R_D} \right) + \nu_{02} \left(\frac{1}{R_D} + \frac{1}{r_0} \right) \\ = V_S \left\{ 2g_m + \frac{1}{r_0} + \frac{1}{R_S} \left(1 + \frac{r_0}{R_D} \right) \right. \\ \left. + \frac{2}{R_D} (1 + g_m r_0) \right\} \quad (4)$$

Then substituting into (2),

$$\nu_{02} \left(\frac{1}{R_D} + \frac{1}{r_0} \right) + g_m \nu_2 = V_S \left(g_m + \frac{1}{r_0} \right)$$

Substitute numbers:

$$(0.119) \left[\nu_1 \frac{710}{25} + \nu_2 + \nu_2 \frac{710}{25} \right] + \nu_{02} \left[\frac{1}{25} + \frac{1}{710} \right] \\ = V_S \left\{ 0.119 + \frac{1}{710} + \frac{1}{20} \left(1 + \frac{710}{25} \right) \right. \\ \left. + \frac{2}{25} [1 + (0.119)(710)] \right\} \quad (4)$$

$$(0.119)[28.4\nu_1 + 29.4\nu_2] + (0.0414)\nu_{02}$$

$$= V_S \{0.1204 + 1.470 + 6.8392\}$$

$$= V_S (8.4296)$$

or

$$V_S = 0.4010\nu_1 + 0.4150\nu_2 + 0.00491\nu_{02}$$

Then

$$\nu_{02} \left(\frac{1}{25} + \frac{1}{710} \right) + (0.119)\nu_2 = V_S \left(0.119 + \frac{1}{710} \right) \quad (2)$$

$$\nu_{02}(0.0414) + \nu_2(0.119)$$

$$= (0.1204)[0.401\nu_1 + 0.4150\nu_2 + 0.00491\nu_{02}]$$

$$\nu_{02}(0.0408) = (0.04828)\nu_1 - (0.0690)\nu_2$$

$$\nu_{02} = (1.183)\nu_1 - (1.691)\nu_2$$

$$\text{Now } \nu_1 = \nu_{cm} + \frac{\nu_d}{2}$$

$$\nu_2 = \nu_{cm} - \frac{\nu_d}{2}$$

So

$$\nu_{02} = (1.183) \left(\nu_{cm} + \frac{\nu_d}{2} \right) - (1.691) \left(\nu_{cm} - \frac{\nu_d}{2} \right)$$

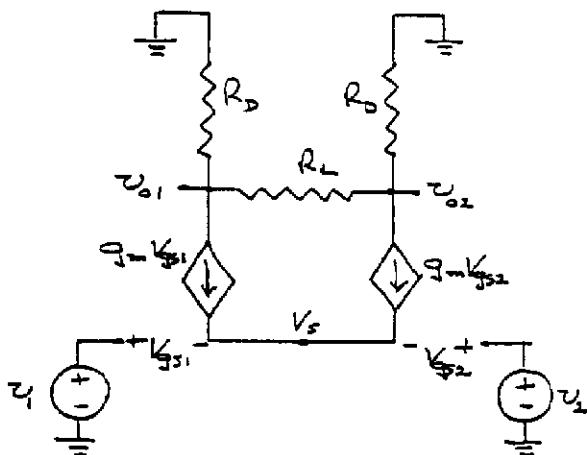
$$\text{Or } \nu_{02} = 1.437\nu_d - 0.508\nu_{cm}$$

$$\Rightarrow A_d = 1.437, \quad A_{cm} = -0.508$$

$$CMRR_{dB} = 20 \log_{10} \left(\frac{1.437}{0.508} \right)$$

$$\Rightarrow CMRR_{dB} = 9.03 \text{ dB}$$

11.28



$$\text{From (1): } \nu_{01} = \nu_{02} \left(1 + \frac{R_L}{R_D} \right) + g_m R_L V_{gs2}$$

Substitute into (2):

$$\begin{aligned} -g_m V_{gs1} &= \nu_{02} \left(1 + \frac{R_L}{R_D} \right) \left(\frac{1}{R_D} + \frac{1}{R_L} \right) \\ &\quad + g_m R_L \left(\frac{1}{R_D} + \frac{1}{R_L} \right) V_{gs2} - \frac{\nu_{02}}{R_L} \\ -g_m (\nu_1 - \nu_2) &+ g_m \left(1 + \frac{R_L}{R_D} \right) \left(\frac{1}{2} \right) (\nu_1 - \nu_2) \\ &= \nu_{02} \left(\frac{1}{R_D} + \frac{R_L}{R_D^2} + \frac{1}{R_D} \right) \end{aligned}$$

$$\frac{1}{2} g_m \left(\frac{R_L}{R_D} \right) (\nu_1 - \nu_2) = \frac{\nu_{02}}{R_D} \left(2 + \frac{R_L}{R_D} \right)$$

$$\Rightarrow A_{d2} = \frac{\nu_{02}}{\nu_1 - \nu_2} = \frac{\frac{1}{2} g_m R_L}{\left(2 + \frac{R_L}{R_D} \right)}$$

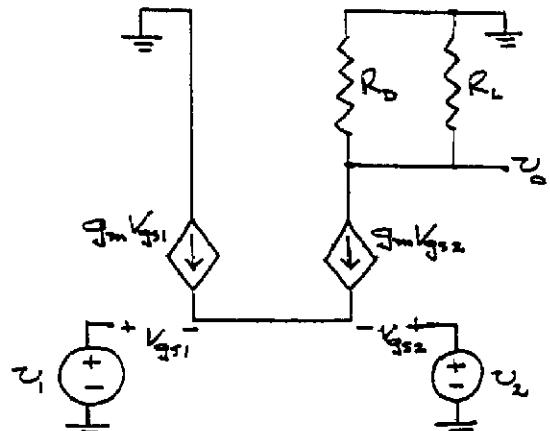
From symmetry

$$A_{d1} = \frac{\nu_{01}}{\nu_1 - \nu_2} = \frac{-\frac{1}{2} g_m R_L}{\left(2 + \frac{R_L}{R_D} \right)}$$

Then

$$A_v = \frac{\nu_{02} - \nu_{01}}{\nu_1 - \nu_2} = \frac{g_m R_L}{\left(2 + \frac{R_L}{R_D} \right)}$$

11.29



KVL:

$$\nu_1 = V_{gs1} - V_{gs2} + \nu_2$$

$$\text{So } \nu_1 - \nu_2 = V_{gs1} - V_{gs2}$$

KCL:

$$g_m V_{gs1} + g_m V_{gs2} = 0 \Rightarrow V_{gs1} = -V_{gs2}$$

$$\text{So } V_{gs1} = \frac{1}{2}(\nu_1 - \nu_2), \quad V_{gs2} = -\frac{1}{2}(\nu_1 - \nu_2)$$

Now

$$\begin{aligned} \frac{\nu_{02}}{R_D} + \frac{\nu_{02} - \nu_{01}}{R_L} &= -g_m V_{gs2} \\ &= \nu_{02} \left(\frac{1}{R_D} + \frac{1}{R_L} \right) - \frac{\nu_{01}}{R_L} \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{\nu_{01}}{R_D} + \frac{\nu_{01} - \nu_{02}}{R_L} &= -g_m V_{gs1} \\ &= \nu_{01} \left(\frac{1}{R_D} + \frac{1}{R_L} \right) - \frac{\nu_{02}}{R_L} \end{aligned} \quad (2)$$

$$\nu_1 - \nu_2 = V_{gs1} - V_{gs2}$$

$$\text{and } g_m V_{gs1} + g_m V_{gs2} = 0 \Rightarrow V_{gs1} = -V_{gs2}$$

$$\text{Then } \nu_1 - \nu_2 = -2V_{gs2}$$

$$\text{Or } V_{gs2} = -\frac{1}{2}(\nu_1 - \nu_2)$$

$$\nu_0 = -g_m V_{gs2} (R_D \| R_L) = \frac{g_m}{2} (R_D \| R_L) (\nu_1 - \nu_2)$$

$$\text{Or } A_d = \frac{g_m}{2} (R_D \| R_L)$$

11.30

$$\text{From Equation (11.64(a)), } A_d = \sqrt{\frac{K_n I_Q}{2}} \cdot R_D$$

$$\text{We need } A_d = \frac{2}{0.2} = 10$$

$$\text{Then } 10 = \sqrt{\frac{K_n(0.5)}{2}} \cdot R_D \text{ or } \sqrt{K_n} \cdot R_D = 20$$

$$\text{If we set } R_D = 20 \text{ k}\Omega, \text{ then } K_n = 1 \text{ mA/V}^2$$

$$\text{For this case } V_D = 10 - (0.25)(20) = 5 \text{ V}$$

$$V_{GS} = \sqrt{\frac{0.25}{1}} + 1 = 1.5 \text{ V}$$

$$V_{DS}(\text{sat}) = V_{GS} - V_{DV} = 1.5 - 1 = 0.5 \text{ V}$$

$$\begin{aligned} \text{Then } \nu_{cm}(\text{max}) &= V_D - V_{DS}(\text{sat}) + V_{GS} \\ &= 5 - 0.5 + 1.5 \end{aligned}$$

$$\text{Or } \nu_{cm}(\text{max}) = 6 \text{ V}$$

11.31

$$V_{d1} = -g_m V_{gs1} R_D = -g_m R_D (V_1 - V_s)$$

$$V_{d2} = -g_m V_{gs2} R_D = -g_m R_D (V_2 - V_s)$$

Now

$$V_o = V_{d2} - V_{d1} = -g_m R_D (V_2 - V_s) - (-g_m R_D (V_1 - V_s))$$

$$V_o = g_m R_D (V_1 - V_2)$$

$$\text{Define } V_1 - V_2 = V_d$$

Then

$$A_d = \frac{V_d}{V_s} = g_m R_D$$

and

$$A_{cm} = 0$$

11.32

$$(a) K_{n1} = K_{n2} = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right) = \left(\frac{0.080}{2}\right) (10) = 0.40 \text{ mA/V}^2$$

$$V_{GS1} = V_{GS2} = \sqrt{\frac{I_D}{K_n}} + V_{TN} = \sqrt{\frac{0.1}{0.4}} + 1 = 1.5 \text{ V}$$

$$V_{DS1}(\text{sat}) = 1.5 - 1 = 0.5 \text{ V}$$

$$\begin{aligned} \text{For } \nu_{CM} = +3 \text{ V} \Rightarrow V_{D1} = V_{D2} = \nu_{CM} - V_{GS1} + V_{DS1}(\text{sat}) \\ = 3 - 1.5 + 0.5 \Rightarrow V_{D1} = V_{D2} = 2 \text{ V} \end{aligned}$$

$$R_D = \frac{10 - 2}{0.1} \Rightarrow R_D = 80 \text{ k}\Omega$$

$$(b) A_d = \frac{1}{2} g_m R_D \text{ and } g_m = 2\sqrt{(0.4)(0.1)} = 0.4 \text{ mA/V}$$

$$\text{Then } A_d = \frac{1}{2} (0.4)(80) = 16$$

$$CMRR_{dB} = 45 \Rightarrow CMRR = 177.8 = \frac{16}{A_{cm}}$$

$$\text{So } |A_{cm}| = 0.090$$

$$|A_{cm}| = \frac{g_m R_D}{1 + 2g_m R_o}$$

$$0.090 = \frac{(0.4)(80)}{1 + 2(0.4)R_o} \Rightarrow R_o = 443 \text{ k}\Omega$$

If we assume $\lambda = 0.01 \text{ V}^{-1}$ for the current source transistor, then

$$r_o = \frac{1}{\lambda I_Q} = \frac{1}{(0.01)(0.2)} = 500 \text{ k}\Omega$$

So the CMRR specification can be met by a 2-transistor current source.

$$\text{Let } \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 1$$

$$\text{Then } K_{n3} = K_{n4} = \left(\frac{0.080}{2}\right) (1) = 0.040 \text{ mA/V}^2$$

$$\text{and } V_{GS3} = \sqrt{\frac{I_Q}{K_{n3}}} + V_{TN} = \sqrt{\frac{0.2}{0.04}} + 1 = 3.24 \text{ V}$$

$$\text{For } \nu_{CM} = -3 \text{ V}, V_{D3} = -3 - V_{GS1} = -3 - 1.5 = -4.5 \text{ V}$$

$$\Rightarrow V_{DS3}(\text{min}) = -4.5 - (-10) = 5.5 \text{ V} > V_{DS3}(\text{sat})$$

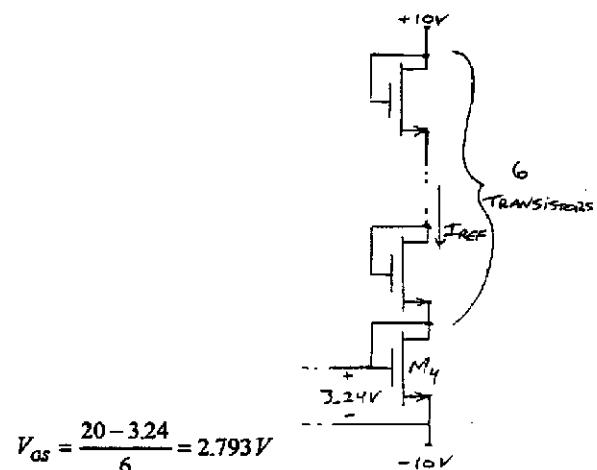
So design is OK.

$$\text{On reference side: For } \left(\frac{W}{L}\right) \geq 1, V_{GS}(\text{max}) = 3.24 \text{ V}$$

$$20 - V_{GS3} = 20 - 3.24 = 16.76 \text{ V}$$

Then

$$\frac{16.67}{3.24} = 5.17 \Rightarrow \text{We need six transistors in series.}$$



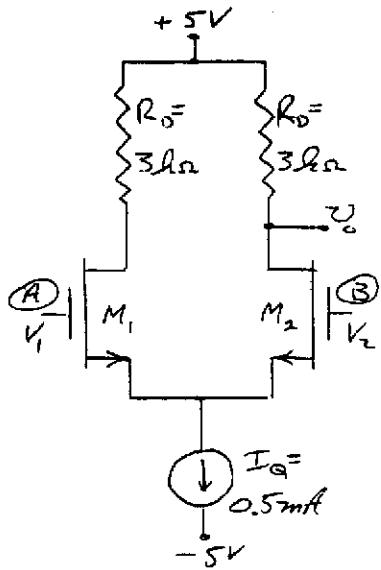
$$V_{GS} = \frac{20 - 3.24}{6} = 2.793 \text{ V}$$

$$I_{REF} = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right) (V_{GS} - V_{TN})^2$$

$$0.2 = \left(\frac{0.080}{2}\right) \left(\frac{W}{L}\right) (2.793 - 1)^2 \Rightarrow$$

$$\left(\frac{W}{L}\right) = 1.56 \text{ for each of the 6 transistors.}$$

11.33



$$2\left(\frac{0.060}{2}\right)\left(\frac{W}{L}\right)_1(0.1) = \left(\frac{2(30)}{50}\right)^2 \Rightarrow \\ \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 240$$

11.35

$$\begin{aligned} a. \quad i_{D1} &= I_{DSS} \left(1 - \frac{v_{GS1}}{V_P}\right)^2 \\ i_{D2} &= I_{DSS} \left(1 - \frac{v_{GS2}}{V_P}\right)^2 \\ \sqrt{i_{D1}} - \sqrt{i_{D2}} &= \sqrt{I_{DSS}} \left(1 - \frac{v_{GS1}}{V_P}\right) - \sqrt{I_{DSS}} \left(1 - \frac{v_{GS2}}{V_P}\right) \\ &= \frac{\sqrt{I_{DSS}}}{V_P} (v_{GS2} - v_{GS1}) \\ &= -\frac{\sqrt{I_{DSS}}}{V_P} \cdot v_d = \frac{\sqrt{I_{DSS}}}{(-V_P)} \cdot v_d \end{aligned}$$

$$A_d = \frac{1}{2} g_m R_o$$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{(0.25)(0.25)} = 0.50 \text{ mA/V}$$

$$A_d = \frac{1}{2}(0.50)(3) = 0.75$$

From Problem 11.17

$$V_1 = V_A = \frac{5(1+\delta)}{2+\delta}, \quad V_2 = V_B = 2.5V$$

$$\text{and } V_1 - V_2 = 1.25\delta$$

Then

$$V_{o2} = A_d \cdot (V_1 - V_2) = (0.75)(1.25\delta) = 0.9375\delta$$

 So for $-0.01 \leq \delta \leq 0.01$

$$-9.375 \leq V_{o2} \leq 9.375 \text{ mV}$$

11.34

From previous results

$$A_{d1} = \frac{v_{o2} - v_{o1}}{v_1 - v_2} = g_m R_1 = \sqrt{2K_n I_{Q1}} \cdot R_1 = 20$$

and

$$A_{d2} = \frac{v_{o2}}{v_{o2} - v_{o1}} = \frac{1}{2} g_m R_2 = \frac{1}{2} \sqrt{2K_n I_{Q2}} \cdot R_2 = 30$$

$$\text{Set } \frac{I_{Q1} R_1}{2} = 5V \quad \text{and} \quad \frac{I_{Q2} R_2}{2} = 25V$$

$$\text{Let } I_{Q1} = I_{Q2} = 0.1 \text{ mA}$$

$$\text{Then } R_1 = 100 \text{ k}\Omega, \quad R_2 = 50 \text{ k}\Omega$$

Then

$$2\left(\frac{0.06}{2}\right)\left(\frac{W}{L}\right)_1(0.1) = \left(\frac{20}{100}\right)^2 \Rightarrow$$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 6.67$$

and

$$i_{D1} + i_{D2} = I_Q \Rightarrow i_{D2} = I_Q - i_{D1}$$

$$\left(\sqrt{i_{D1}} - \sqrt{i_{D2} - i_{D1}}\right)^2 = \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2$$

$$i_{D1} - 2\sqrt{i_{D1}(I_Q - i_{D1})} + (I_Q - i_{D1}) = \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2$$

Then

$$\sqrt{i_{D1}(I_Q - i_{D1})} = \frac{1}{2} \left[I_Q - \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2 \right]$$

Square both sides

$$i_{D1}^2 - i_{D1} I_Q + \frac{1}{4} \left[I_Q - \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2 \right]^2 = 0$$

$$i_{D1} = \frac{I_Q \pm \sqrt{I_Q^2 - 4 \left(\frac{1}{4} \right) \left[I_Q - \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2 \right]^2}}{2}$$

$$i_{D1} =$$

$$\frac{I_Q}{2} \pm \frac{1}{2} \sqrt{I_Q^2 - \left[I_Q^2 - \frac{2I_Q I_{DSS} v_d^2}{(-V_P)^2} + \left(\frac{I_{DSS} v_d^2}{(-V_P)^2} \right)^2 \right]}$$

Use + sign

$$i_{D1} = \frac{I_Q}{2} + \frac{1}{2} \sqrt{\frac{2I_Q I_{DSS}}{(-V_P)^2} \cdot v_d^2 - \left(\frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2 \right)^2}$$

$$i_{D1} = \frac{I_Q}{2} + \frac{1}{2} \frac{I_Q}{(-V_P)} v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q} \right)^2 \left(\frac{v_d}{V_P} \right)^2}$$

Or

$$\frac{i_{D1}}{I_Q} = \frac{1}{2} + \left(\frac{1}{-2V_P} \right) \cdot v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q} \right)^2 \left(\frac{v_d}{V_P} \right)^2}$$

We had

$$i_{D2} = I_Q - i_{D1}$$

Then

$$\frac{i_{D2}}{I_Q} = \frac{1}{2} - \left(\frac{1}{-2V_P} \right) \cdot v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q} \right)^2 \left(\frac{v_d}{V_P} \right)^2}$$

b. If $i_{D1} = I_Q$, then

$$1 = \frac{1}{2} + \left(\frac{1}{-2V_P} \right) \cdot v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q} \right)^2 \left(\frac{v_d}{V_P} \right)^2}$$

$$|V_P| = v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q} \right)^2 \left(\frac{v_d}{V_P} \right)^2}$$

Square both sides

$$|V_P|^2 = v_d^2 \left[\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q} \right)^2 \left(\frac{v_d}{V_P} \right)^2 \right]$$

$$\left(\frac{I_{DSS}}{I_Q} \right)^2 \left(\frac{1}{V_P} \right)^2 (v_d^2)^2 - \frac{2I_{DSS}}{I_Q} \cdot v_d^2 + |V_P|^2 = 0$$

$$v_d^2 =$$

$$\frac{\frac{2I_{DSS}}{I_Q} \pm \sqrt{\left(\frac{2I_{DSS}}{I_Q} \right)^2 - 4 \left(\frac{I_{DSS}}{I_Q} \right)^2 \left(\frac{1}{V_P} \right)^2 (V_P)^2}}{2 \left(\frac{2I_{DSS}}{I_Q} \right)^2 \left(\frac{1}{V_P} \right)^2}$$

$$v_d^2 = (V_P)^2 \left(\frac{I_Q}{I_{DSS}} \right)$$

$$\text{Or } v_d = |V_P| \left(\frac{I_Q}{I_{DSS}} \right)^{1/2}$$

c. For v_d small,

$$i_{D1} \approx \frac{I_Q}{2} + \frac{1}{2} \cdot \frac{I_Q}{(-V_P)} \cdot v_d \sqrt{\frac{2I_{DSS}}{I_Q}}$$

$$g_f = \frac{di_{D1}}{dv_d} \Big|_{v_d=0} = \frac{1}{2} \cdot \frac{I_Q}{(-V_P)} \cdot \sqrt{\frac{2I_{DSS}}{I_Q}}$$

Or

$$\Rightarrow g_f(\max) = \left(\frac{1}{-V_P} \right) \sqrt{\frac{I_Q I_{DSS}}{2}}$$

11.36

$$\text{a. } I_Q = I_{D1} + I_{D2} \Rightarrow I_Q = 1 \text{ mA}$$

$$v_0 = 7 = 10 - (0.5)R_D \Rightarrow R_D = 6 \text{ k}\Omega$$

$$\text{b. } g_f(\max) = \left(\frac{1}{-V_P} \right) \sqrt{\frac{I_Q \cdot I_{DSS}}{2}}$$

$$g_f(\max) = \left(\frac{1}{4} \right) \sqrt{\frac{(1)(2)}{2}}$$

$$\Rightarrow g_f(\max) = 0.25 \text{ mA/V}$$

$$\text{c. } A_d = \frac{g_m R_D}{2} = g_f(\max) \cdot R_D$$

$$A_d = (0.25)(6) \Rightarrow A_d = 1.5$$

11.37

$$\text{a. } I_S = \frac{-V_{GS} - (-5)}{R_S} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2$$

$$5 - V_{GS} = (0.8)(20) \left(1 - \frac{V_{GS}}{(-2)} \right)^2$$

$$5 - V_{GS} = 16 \left(1 + V_{GS} + \frac{1}{4} V_{GS}^2 \right)$$

$$4V_{GS}^2 + 17V_{GS} + 11 = 0$$

$$V_{GS} = \frac{-17 \pm \sqrt{(17)^2 - 4(4)(11)}}{2(4)}$$

$$V_{GS} = -0.796 \text{ V}$$

$$I_S = \frac{5 - (-0.796)}{20} \Rightarrow I_S = 0.290 \text{ mA}$$

$$I_{D1} = I_{D2} = \frac{I_S}{2} \Rightarrow I_{D1} = I_{D2} = 0.145 \text{ mA}$$

$$v_{02} = 5 - (0.145)(25) \Rightarrow v_{02} = 1.375 \text{ V}$$

b. Taking into account the r_o parameters of Q_1 and Q_2 , the analysis is identical to that in problem 11.34.

11.38

Equivalent circuit and analysis is identical to that in problem 11.36.

$$A_{d2} = \frac{\frac{1}{2} \cdot g_m R_L}{\left(2 + \frac{R_L}{R_D} \right)}$$

$$A_{d1} = \frac{-\frac{1}{2} \cdot g_m R_L}{\left(2 + \frac{R_L}{R_D} \right)}$$

$$A_v = \frac{v_{02} - v_{01}}{v_d} = \frac{g_m R_L}{\left(2 + \frac{R_L}{R_D} \right)}$$

11.39

a. Using the results of problem 10.59, the resistance from the base of Q_4 looking toward Q_3 :

$$\frac{1}{R'_0} = \frac{1}{r_{o1}} + \left[1 + \left(\frac{1}{r_{o3}} + g_{m3} + \frac{1}{r_{o3}} \right) R_E \right]$$

$$r_{o1} = \frac{120}{0.1} = 1200 \text{ k}\Omega, r_{o3} = \frac{80}{0.1} = 800 \text{ k}\Omega$$

Assume $\beta = 100$

$$r_{o3} = \frac{(100)(0.026)}{0.1} = 26 \text{ k}\Omega$$

$$g_{m3} = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$\begin{aligned} \frac{1}{R_0} &= \frac{1}{1200} + \left[\frac{\left(\frac{1}{26} + 3.846 + \frac{1}{800} \right)}{1 + \left(\frac{1}{26} + 3.846 + \frac{1}{800} \right)(1)} \right] \\ &= \frac{1}{1200} + \frac{3.886}{1 + (3.886)(1)} \Rightarrow R'_0 = 1.256 \text{ k}\Omega \\ R_0 &= r_{02} \left[1 + \frac{R_E \parallel (r_{\pi 2} + R'_0)}{r_{02}} \right. \\ &\quad \left. + g_m r_{\pi 2} \left(\frac{r_{\pi 2}}{r_{\pi 2} + R'_0} \right) \left(R_E \parallel (r_{\pi 2} + R'_0) \right) \right] \\ R_E \parallel (r_{\pi 2} + R'_0) &= (1) \parallel (26 + 1.256) \\ &= (1) \parallel (27.256) \\ &= 0.965 \text{ k}\Omega \end{aligned}$$

$$R_0 = 1200 \left[1 + \frac{0.965}{1200} + (3.846) \left(\frac{26}{26 + 1.256} \right) (0.965) \right] \\ \Rightarrow R_0 = 5.45 \text{ M}\Omega$$

Then

$$A_v = -g_m (r_{02} \parallel R_0) \\ r_{02} = \frac{120}{0.1} = 1200 \text{ k}\Omega \\ g_m = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$A_v = -(3.846)[1200 \parallel 5450] \\ \Rightarrow A_v = -3782$$

$$\text{b. For } R = 0, \quad r_{04} = \frac{80}{0.1} = 800 \text{ k}\Omega$$

$$A_v = -g_m (r_{02} \parallel r_{04}) \\ = -(3.846)[1200 \parallel 800] \\ \Rightarrow A_v = -1846$$

$$\text{(c) For part (a), } R_o = (5.45 \parallel 1.2) = 0.983 \text{ M}\Omega$$

$$\text{For part (b), } R_o = (1.2 \parallel 0.8) = 0.48 \text{ M}\Omega$$

11.40

$$I_{BS} = \frac{I_{E3}}{1 + \beta} = \frac{I_{B3} + I_{B4}}{1 + \beta} = \frac{I_{C3} + I_{C4}}{\beta(1 + \beta)}$$

$$\text{Now } I_{C3} + I_{C4} \approx I_Q$$

$$\text{So } I_{BS} \approx \frac{I_Q}{\beta(1 + \beta)}$$

$$I_{BS} = \frac{I_{E3}}{1 + \beta} = \frac{I_{Q1}}{\beta(1 + \beta)}$$

For balance, we want $I_{BS} = I_{B3}$

So that $\underline{I_{Q1} = I_Q}$

11.41

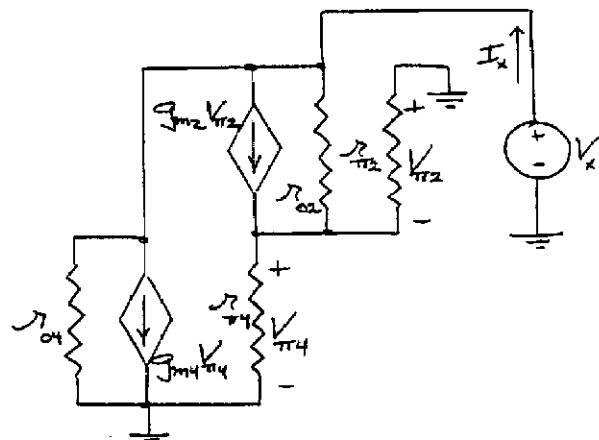
$$\text{a. } A_d = g_m (r_{02} \parallel r_{04}) \\ r_{02} = \frac{V_{A2}}{I_{C2}} = \frac{150}{0.4} = 375 \text{ k}\Omega \\ r_{04} = \frac{V_{A4}}{I_{C4}} = \frac{100}{0.4} = 250 \text{ k}\Omega \\ g_m = \frac{I_{C2}}{V_T} = \frac{0.4}{0.026} = 15.38 \text{ mA/V}$$

$$A_d = (15.38)(375 \parallel 250) \\ \Rightarrow A_d = 2307$$

$$\text{b. } R_L = r_{02} \parallel r_{04} = 375 \parallel 250 \\ \Rightarrow R_L = 150 \text{ k}\Omega$$

11.42

(a) For Q_2, Q_4



$$(1) I_x = \frac{V_x - V_{\pi 4}}{r_{\pi 2}} + g_{m2} V_{\pi 2} + g_{m4} V_{\pi 4} + \frac{V_x}{r_{\pi 4}}$$

$$(2) g_{m2} V_{\pi 2} + \frac{V_x - V_{\pi 4}}{r_{\pi 2}} = \frac{V_{\pi 4}}{r_{\pi 4} \parallel r_{\pi 2}}$$

$$(3) V_{\pi 4} = -V_{\pi 2}$$

From (2)

$$\frac{V_x}{r_{\pi 2}} = V_{\pi 4} \left[\frac{1}{r_{\pi 4} \parallel r_{\pi 2}} + \frac{1}{r_{\pi 2}} + g_{m2} \right]$$

Now

$$I_{C4} = \left(\frac{\beta}{1 + \beta} \right) \left(\frac{I_Q}{2} \right) = \left(\frac{120}{121} \right) (0.5) = 0.496 \text{ mA}$$

$$I_{C2} = \left(\frac{I_Q}{2} \right) \left(\frac{1}{1 + \beta} \right) \left(\frac{\beta}{1 + \beta} \right) = (0.5) \left(\frac{120}{(121)^2} \right) \Rightarrow$$

$$I_{C2} = 0.0041 \text{ mA}$$

So

$$r_{o1} = \frac{(120)(0.026)}{0.0041} = 761 \text{ k}\Omega$$

$$g_{m1} = \frac{0.0041}{0.026} = 0.158 \text{ mA/V}$$

$$r_{o2} = \frac{100}{0.0041} \Rightarrow 24.4 \text{ M}\Omega$$

$$r_{o3} = \frac{(120)(0.026)}{0.496} = 6.29 \text{ k}\Omega$$

$$g_{m2} = \frac{0.496}{0.026} = 19.08 \text{ mA/V}$$

$$r_{o4} = \frac{100}{0.496} = 202 \text{ k}\Omega$$

Now

$$\frac{V_x}{r_{o2}} = V_{o4} \left[\frac{1}{6.29 \parallel 761} + \frac{1}{24400} + 0.158 \right] \Rightarrow$$

which yields

$$V_{o4} = \frac{V_x}{(0.318)r_{o2}}$$

From (1),

$$I_x = \frac{V_x}{r_{o2}} + \frac{V_x}{r_{o4}} + V_{o4} \left(g_{m1} - g_{m2} - \frac{1}{r_{o2}} \right)$$

$$\frac{I_x}{V_x} = \left[\frac{1}{24400} + \frac{1}{202} + \frac{\left(19.08 - 0.158 - \frac{1}{24400} \right)}{(0.318)(24400)} \right]$$

which yields

$$R_{o2} = \frac{V_x}{I_x} = 135 \text{ k}\Omega$$

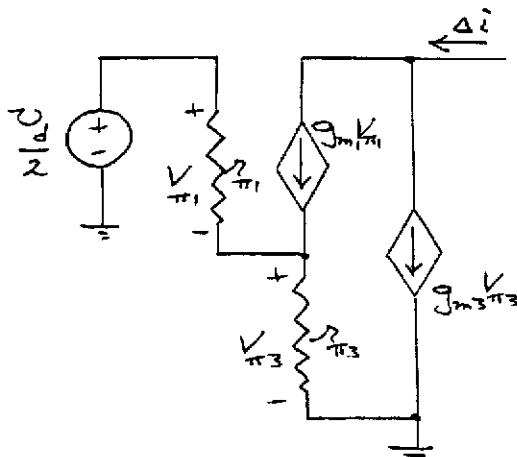
Now

$$r_{o\infty} = \frac{80}{0.5} = 160 \text{ k}\Omega$$

Then

$$R_o = R_{o2} \parallel r_{o\infty} = 135 \parallel 160 \Rightarrow R_o = 73.2 \text{ k}\Omega$$

$$(b) A_d = g_m^s R_o \text{ where } g_m^s = \frac{\Delta i}{v_d/2}$$



$$\Delta i = g_{m1}V_{x1} + g_{m2}V_{x2} \text{ and } V_{x1} + V_{x2} = \frac{v_d}{2}$$

$$\text{Also } \left(\frac{V_{x1}}{r_{o1}} + g_{m1}V_{x1} \right) r_{o3} = V_{x3}$$

$$\text{So } V_{x1} \left(\frac{1+\beta}{r_{o1}} \right) r_{o3} = V_{x3}$$

Or

$$V_{x1} \left(\frac{121}{761} \right) (6.29) = V_{x3} \equiv V_{x1}$$

$$\text{Then } 2V_{x1} = \frac{v_d}{2} \Rightarrow V_{x1} = \frac{v_d}{4}$$

So

$$\Delta i = (g_{m1} + g_{m2})V_{x1} = (0.158 + 19.08) \left(\frac{v_d}{4} \right) = 9.62 \left(\frac{v_d}{2} \right)$$

So

$$g_m^s = \frac{\Delta i}{v_d/2} = 9.62 \Rightarrow A_d = (9.62)(73.2) \Rightarrow$$

$$A_d = 704$$

Now

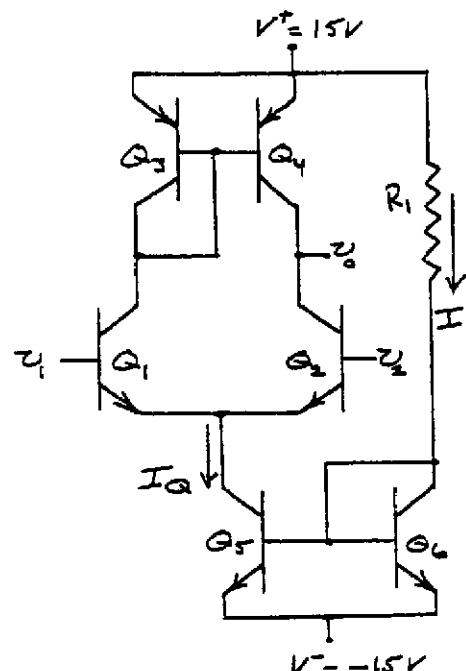
$$R_{id} = 2R_i \text{ where } R_i = r_{o1} + (1+\beta)r_{o3}$$

$$R_i = 761 + (121)(6.29) = 1522 \text{ k}\Omega$$

$$\text{Then}$$

$$R_{id} = 3.044 \text{ M}\Omega$$

11.43



$$a. \quad g_f = \frac{I_Q}{4V_T} \Rightarrow I_Q = g_f(4V_T) = (8)(4)(0.026)$$

$$\Rightarrow I_Q = 0.832 \text{ mA}$$

Neglecting base currents,

$$R_1 = \frac{30 - 0.7}{0.832} \Rightarrow R_1 = 35.2 \text{ k}\Omega$$

$$b. \quad r_{o4} = r_{o2} = \frac{V_A}{I_{CQ}} = \frac{100}{0.416} = 240 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.416}{0.026} = 16 \text{ mA/V}$$

$$A_d = g_m(r_{o2} \parallel r_{o4}) = 16(240 \parallel 240)$$

$$\Rightarrow A_d = 1920$$

$$R_{id} = 2r_x, \quad r_x = \frac{(180)(0.026)}{0.416} = 11.25 \text{ k}\Omega$$

$$\Rightarrow R_{id} = 22.5 \text{ k}\Omega$$

$$R_0 = r_{o2} \parallel r_{o4} \Rightarrow R_0 = 120 \text{ k}\Omega$$

c. Max. common-mode voltage when

$$V_{CB} = 0 \text{ for } Q_1 \text{ and } Q_2.$$

Therefore

$$v_{cm}(\max) = V^+ - V_{EB}(Q_3) = 15 - 0.7$$

$$v_{cm}(\max) = 14.3 \text{ V}$$

Min. common-mode voltage when

$$V_{CB} = 0 \text{ for } Q_3.$$

Therefore

$$v_{cm}(\min) = 0.7 + 0.7 + (-15) = -13.6 \text{ V}$$

$$\text{So } -13.6 \leq v_{cm} \leq 14.3 \text{ V}$$

$$R_{icm} \approx \frac{1}{2}(1 + \beta)(2R_0)$$

$$R_0 = \frac{V_A}{I_Q} = \frac{100}{0.832} = 120 \text{ k}\Omega$$

$$R_{icm} = (181)(120) \Rightarrow R_{icm} = 21.7 \text{ M}\Omega$$

11.44

$$a. \quad I_0 = I_{B3} + I_{B4} \approx 2 \left(\frac{I_Q}{2} \right) \left(\frac{1}{\beta} \right)$$

$$I_0 = \frac{I_Q}{\beta} = \frac{0.2}{100} \Rightarrow I_0 = 2 \mu\text{A}$$

$$b. \quad r_{o2} = r_{o4} = \frac{V_A}{I_{CQ}} = \frac{100}{0.1} = 1000 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$A_d = g_m(r_{o2} \parallel r_{o4}) = (3.846)(1000 \parallel 1000)$$

$$\Rightarrow A_d = 1923$$

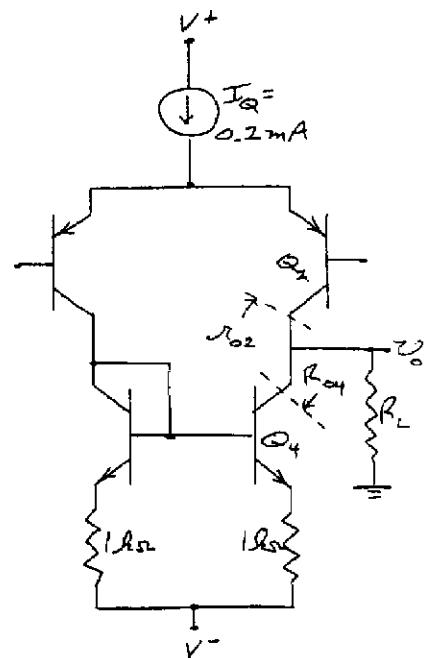
$$c. \quad A_d = g_m(r_{o2} \parallel r_{o4} \parallel R_L)$$

$$A_d = (3.846)(1000 \parallel 1000 \parallel 250)$$

$$\Rightarrow A_d = 641$$

11.45

$$\text{Let } \beta = 100, \quad V_A = 100 \text{ V}$$



$$r_{o2} = \frac{V_A}{I_{CQ}} = \frac{100}{0.1} = 1000 \text{ k}\Omega$$

$$R_{o4} = r_{o4}[1 + g_m R'_E] \text{ where } R'_E = r_x \parallel R_E$$

Now

$$r_x = \frac{(100)(0.026)}{0.1} = 26 \text{ k}\Omega$$

$$g_m = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$R'_E = 26 \parallel 1 = 0.963 \text{ k}\Omega$$

Then

$$R_{o4} = 1000[1 + (3.846)(0.963)] = 4704 \text{ k}\Omega$$

$$A_d = g_m(r_{o2} \parallel R_{o4}) = 3.846(1000 \parallel 4704) \Rightarrow$$

$$A_d = 3172$$

11.46

$$a. \quad A_d = g_m(r_{o2} \parallel r_{o4} \parallel R_L)$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{I_Q}{2V_T}$$

$$r_{o2} = \frac{V_{A2}}{I_{CQ}} = \frac{125}{I_{CQ}}$$

$$r_{o4} = \frac{V_{A4}}{I_{CQ}} = \frac{80}{I_{CQ}}$$

If $I_Q = 2 \text{ mA}$, then $g_m = 38.46 \text{ mA/V}$

$$r_{o2} = 125 \text{ k}\Omega, r_{o4} = 80 \text{ k}\Omega$$

$$\text{So } A_d = 38.46[125 \parallel 80 \parallel 200]$$

$$\text{Or } A_d = 1508$$

For each gain of 1000, lower the current level

For $I_Q = 0.60 \text{ mA}$, $I_{CQ} = 0.30 \text{ mA}$

$$g_m = \frac{0.3}{0.026} = 11.54 \text{ mA/V}$$

$$r_{o2} = \frac{125}{0.3} = 417 \text{ k}\Omega$$

$$r_{o4} = \frac{80}{0.3} = 267 \text{ k}\Omega$$

$$A_d = 11.54[417 \parallel 267 \parallel 200] = 1036$$

So $I_Q = 0.60 \text{ mA}$ is adequate

b. For $V^+ = 10 \text{ V}$, $V_{BS} = V_{EB} = 0.6 \text{ V}$

For $V_{CB} = 0$,

$$v_{cm}(\max) = V^+ - 2V_{BB} = 10 - 2(0.6)$$

$$\text{Or } v_{cm}(\max) = 8.8 \text{ V}$$

11.48

a. From symmetry,

$$V_{GS3} = V_{GS4} = V_{DS3} = V_{DS4} = \sqrt{\frac{0.1}{0.1}} + 1$$

$$\text{Or } V_{DS3} = V_{DS4} = 2 \text{ V}$$

$$V_{SG1} = V_{SG2} = \sqrt{\frac{0.1}{0.1}} + 1 = 2 \text{ V}$$

$$V_{SD1} = V_{SD2} = V_{SG1} - (V_{DS3} - 10)$$

$$= 2 - (2 - 10)$$

$$\text{Or } V_{SD1} = V_{SD2} = 10 \text{ V}$$

$$\text{b. } r_{o\alpha} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.01)(0.1)} \Rightarrow 1 \text{ M}\Omega$$

$$r_{o\beta} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.015)(0.1)} \Rightarrow 0.667 \text{ M}\Omega$$

$$g_m = 2K_p(V_{SD} + V_{TP})$$

$$= 2(0.1)(2 - 1) = 0.2 \text{ mA/V}$$

$$A_d = g_m(r_{o\alpha} \parallel r_{o\beta}) = (0.2)(1000 \parallel 667)$$

$$\Rightarrow A_d = 80$$

$$(c) I_{D2} = I_{D1} = \frac{I_Q}{2} = 0.1 \text{ mA}$$

$$r_{o2} = \frac{1}{\lambda_n I_{D2}} = \frac{1}{(0.01)(0.1)} = 1000 \text{ k}\Omega$$

$$r_{o4} = \frac{1}{\lambda_p I_{D2}} = \frac{1}{(0.015)(0.1)} = 667 \text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 667 \parallel 1000 = 400 \text{ k}\Omega$$

11.49

$$A_d = g_m(r_{o2} \parallel r_{o4})$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.12)(0.075)}$$

$$= 0.1897 \text{ mA/V}$$

$$r_{o2} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.015)(0.075)} = 889 \text{ k}\Omega$$

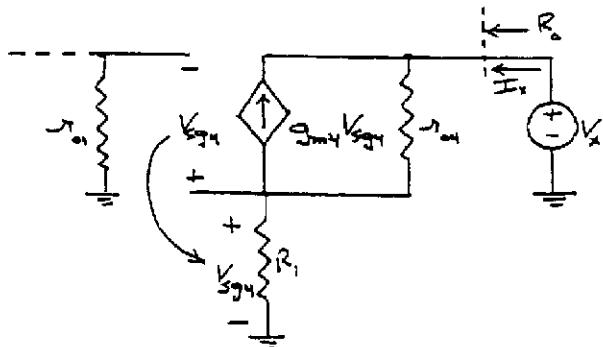
$$r_{o4} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.02)(0.075)} = 667 \text{ k}\Omega$$

$$A_d = (0.1897)(889 \parallel 667)$$

$$\Rightarrow A_d = 72.3$$

11.50

Resistance looking into drain of M_4 .



$$V_{sg4} = I_X R_1$$

$$I_X = g_{m4} V_{sg4} = \frac{V_X - V_{sg4}}{r_{o4}}$$

$$I_X \left[1 + g_{m4} R_1 + \frac{R_1}{r_{o4}} \right] = \frac{V_X}{r_{o4}}$$

$$\text{Or } R_0 = r_{o4} \left[1 + g_{m4} R_1 + \frac{R_1}{r_{o4}} \right]$$

$$\text{a. } A_d = g_{m2}(r_{o2} \parallel R_o)$$

$$g_{m2} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.080)(0.1)} = 0.179 \text{ mA/V}$$

$$r_{o2} = \frac{1}{\lambda_n I_{D2}} = \frac{1}{(0.015)(0.1)} = 667 \text{ k}\Omega$$

$$g_{m4} = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.080)(0.1)} = 0.179 \text{ mA/V}$$

$$r_{o4} = \frac{1}{\lambda_p I_{D2}} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$R_o = 500 \left[1 + (0.179)(1) + \frac{1}{500} \right] = 590.5 \text{ k}\Omega$$

$$A_d = (0.179)(667 \parallel 590.5)$$

$$\Rightarrow A_d = 56.06$$

b. When $R_1 = 0$, $R_0 = r_{o4} = 500 \text{ k}\Omega$

$$A_d = (0.179)[667 \parallel 500]$$

$$\Rightarrow A_d = 51.15$$

(c) For part (a), $R_o = r_{o2} \parallel R_o = 667 \parallel 590.5 \Rightarrow$
 $\underline{R_o = 313 \text{ k}\Omega}$

For part (b), $R_o = r_{o2} \parallel r_{o4} = 667 \parallel 500 \Rightarrow$
 $\underline{R_o = 286 \text{ k}\Omega}$

11.51

$$(a) A_d = 100 = g_m(r_{o2} \parallel r_{o4})$$

Let $I_Q = 0.5 \text{ mA}$

$$r_{o2} = \frac{1}{\lambda_n I_D} = \frac{1}{(0.02)(0.25)} = 200 \text{ k}\Omega$$

$$r_{o4} = \frac{1}{\lambda_p I_D} = \frac{1}{(0.025)(0.25)} = 160 \text{ k}\Omega$$

Then

$$100 = g_m(200 \parallel 160) \Rightarrow g_m = 1.125 \text{ mA/V}$$

$$g_m = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_1} I_D$$

$$1.125 = 2\sqrt{\left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)_1(0.25)} \Rightarrow \left(\frac{W}{L}\right)_1 = 31.6$$

Now $\left(\frac{W}{L}\right)_2$ somewhat arbitrary. Let $\left(\frac{W}{L}\right)_2 = 31.6$

11.52

$$A_d = g_m(r_{o2} \parallel r_{o4})$$

$$P = (I_Q + I_{REF})(V^+ - V^-)$$

Let $I_Q = I_{REF}$

Then $0.5 = 2I_Q(3 - (-3)) \Rightarrow I_Q = I_{REF} = 0.0417 \text{ mA}$

$$r_{o2} = \frac{1}{\lambda_n I_D} = \frac{1}{(0.015)(0.0208)} = 3205 \text{ k}\Omega$$

$$r_{o4} = \frac{1}{\lambda_p I_D} = \frac{1}{(0.02)(0.0208)} = 2404 \text{ k}\Omega$$

Then

$$A_d = 80 = g_m(3205 \parallel 2404) \Rightarrow g_m = 0.0582 \text{ mA/V}$$

$$g_m = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_1} I_D$$

$$0.0582 = 2\sqrt{\left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)_1(0.0208)} \Rightarrow$$

$$\left(\frac{W}{L}\right)_1 = 1.02$$

11.53

$$A_d = g_m(r_{o2} \parallel R_o)$$

Want $A_d = 400$

From Example 11.15, $r_{o2} = 1 \text{ M}\Omega$

Assuming that $g_m = 0.283 \text{ mA/V}$ for the PMOS from Example 11.15, then $R_o = 285 \text{ M}\Omega$.

So

$$400 = g_m(1000 \parallel 285000) \Rightarrow$$

$$g_m = 0.4014 \text{ mA/V} = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_1} I_D$$

$$0.04028 = \left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)_1(0.1) \Rightarrow$$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 10.1$$

11.54

$$A_d = g_m(R_{o4} \parallel R_{o6})$$

where

$$R_{o4} = r_{o4} + r_{o2}[1 + g_{m4}r_{o4}]$$

$$R_{o6} = r_{o6} + r_{o4}[1 + g_{m6}r_{o6}]$$

We have

$$r_{o2} = r_{o4} = \frac{1}{(0.015)(0.040)} = 1667 \text{ k}\Omega$$

$$r_{o6} = r_{o4} = \frac{1}{(0.02)(0.040)} = 1250 \text{ k}\Omega$$

$$g_{m4} = 2\sqrt{\left(\frac{0.060}{2}\right)(15)(0.040)} = 0.268 \text{ mA/V}$$

$$g_{m6} = 2\sqrt{\left(\frac{0.025}{2}\right)(10)(0.040)} = 0.141 \text{ mA/V}$$

Then

$$R_{o4} = 1667 + 1667[1 + (0.268)(1667)] = 748 \text{ M}\Omega$$

$$R_{o6} = 1250 + 1250[1 + (0.141)(1250)] = 222.8 \text{ M}\Omega$$

(a)

$$R_o = R_{o4} \parallel R_{o6} = 748 \parallel 222.8 \Rightarrow \underline{R_o = 172 \text{ M}\Omega}$$

(b)

$$A_d = g_m(R_{o4} \parallel R_{o6}) = (0.268)(172000) \Rightarrow$$

$$\underline{A_d = 46096}$$

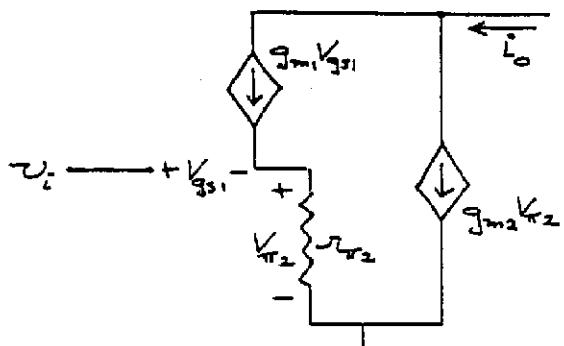
11.55

$$g_{m1} = 2\sqrt{K_n I_{BLUR}} = 2\sqrt{(0.2)(0.25)}$$

$$= 0.447 \text{ mA/V}$$

$$g_{m2} = \frac{I_{CQ}}{V_T} = \frac{0.75}{0.026} = 28.85 \text{ mA/V}$$

$$r_{\pi2} = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{0.75} = 4.16 \text{ k}\Omega$$



$$i_o = g_{m1} V_{gs1} + g_{m2} V_{gs2}$$

$$V_{gs2} = g_{m1} V_{gs1} r_{\pi 2} \text{ and } v_i = V_{gs1} + V_{gs2}$$

$$i_o = V_{gs1} (g_{m1} + g_{m2} \cdot g_{m1} r_{\pi 2})$$

$$v_i = V_{gs1} + g_{m1} V_{gs1} r_{\pi 2}$$

$$\text{and } V_{gs1} = \frac{v_i}{1 + g_{m1} r_{\pi 2}}$$

$$i_o = v_i \cdot \frac{g_{m1}(1 + \beta)}{1 + g_{m1} r_{\pi 2}}$$

$$g_m^C = \frac{i_o}{v_i} = \frac{g_{m1}(1 + \beta)}{1 + g_{m1} r_{\pi 2}} \\ = \frac{(0.447)(121)}{1 + (0.447)(4.16)}$$

$$\Rightarrow g_m^C = 18.9 \text{ mA/V}$$

11.56

$$r_0(M_2) = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.01)(0.2)} = 500 \text{ k}\Omega$$

$$r_0(Q_2) = \frac{V_A}{I_{CQ}} = \frac{80}{0.2} = 400 \text{ k}\Omega$$

$$g_m(M_2) = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.2)(0.2)} \\ = 0.4 \text{ mA/V}$$

$$A_d = g_m(M_2)[r_0(M_2) \parallel r_0(Q_2)]$$

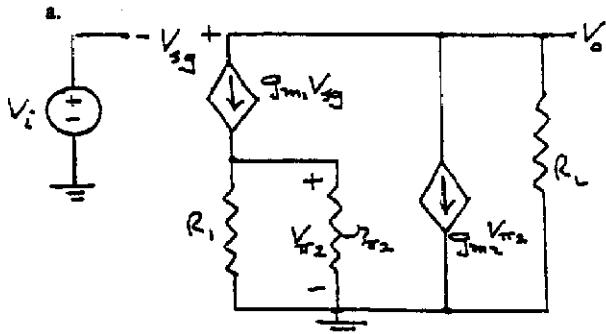
$$= 0.4[500 \parallel 400]$$

$$\Rightarrow A_d = 88.9$$

If the I_Q current source is ideal,

$$A_{cm} = 0 \text{ and } CMRR_{dB} = \infty$$

11.57

b. Assume R_L is capacitively coupled. Then

$$I_{CQ} + I_{DQ} = I_Q$$

$$I_{DQ} = \frac{V_{BE}}{R_1} = \frac{0.7}{8} = 0.0875 \text{ mA}$$

$$I_{CQ} = 0.9 - 0.0875 = 0.8125 \text{ mA}$$

$$g_{m1} = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(1)(0.0875)}$$

$$\Rightarrow g_{m1} = 0.592 \text{ mA/V}$$

$$g_{m2} = \frac{I_{CQ}}{V_T} = \frac{0.8125}{0.026} \Rightarrow g_{m2} = 31.25 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.8125} \Rightarrow r_{\pi 2} = 3.2 \text{ k}\Omega$$

$$c. V_0 = (-g_{m1} V_{sg} - g_{m2} V_{\pi 2}) R_L$$

$$V_i + V_{sg} = V_0 \Rightarrow V_{sg} = V_0 - V_i$$

$$V_{\pi 2} = (g_{m1} V_{sg})(R_1 \parallel r_{\pi 2})$$

$$V_0 = -[g_{m1} V_{sg} + g_{m2} g_{m1} V_{sg} (R_1 \parallel r_{\pi 2})] R_L$$

$$V_0 = -(V_0 - V_i)(g_{m1} + g_{m2} g_{m1} (R_1 \parallel r_{\pi 2})) R_L$$

$$A_v = \frac{V_0}{V_i} = \frac{[g_{m1} + g_{m2} g_{m1} (R_1 \parallel r_{\pi 2})] R_L}{1 + [g_{m1} + g_{m2} g_{m1} (R_1 \parallel r_{\pi 2})] R_L}$$

We find

$$g_{m1} + g_{m2} g_{m1} (R_1 \parallel r_{\pi 2})$$

$$= 0.592 + (31.25)(0.592)(8 \parallel 3.2)$$

$$= 42.88$$

$$\text{Then } A_v = \frac{(42.88)(R_L)}{1 + (42.88)(R_L)}$$

11.58

a. Assume R_L is capacitively coupled.

$$I_{DQ} = \frac{0.7}{8} = 0.0875 \text{ mA}$$

$$I_{CQ} = 1.2 - 0.0875 = 1.11 \text{ mA}$$

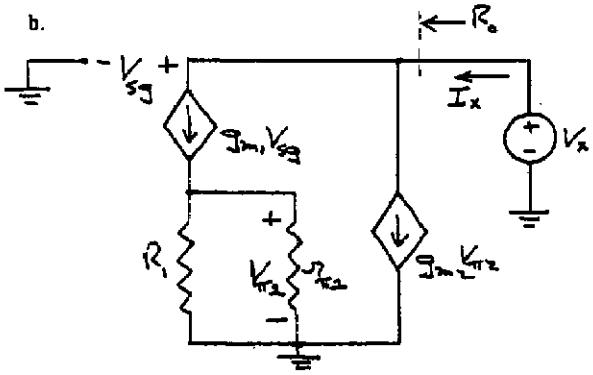
$$g_{m1} = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(1)(0.0875)}$$

$$\Rightarrow g_{m1} = 0.592 \text{ mA/V}$$

$$g_{m2} = \frac{I_{CQ}}{V_T} = \frac{1.11}{0.026} \Rightarrow g_{m2} = 42.7 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1.11} \Rightarrow r_{\pi 2} = 2.34 \text{ k}\Omega$$

b.



$$V_{sg} = V_X$$

$$I_X = g_{m2}V_{sg} + g_{m1}V_{sg}$$

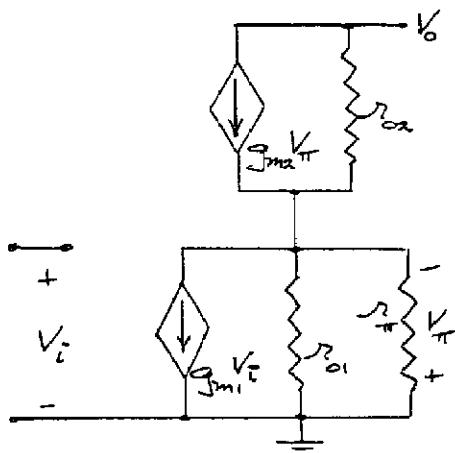
$$(g_{m1}V_{sg})(R_1 \parallel r_{n2}) = V_{sg}$$

$$I_X = V_X [g_{m1} + g_{m2}g_{m1}(R_1 \parallel r_{n2})]$$

$$\begin{aligned} R_o &= \frac{V_X}{I_X} = \frac{1}{g_{m1} + g_{m2}g_{m1}(R_1 \parallel r_{n2})} \\ &= \frac{1}{0.592 + (0.592)(42.7)(8 \parallel 2.34)} \\ \Rightarrow R_o &= 21.6 \Omega \end{aligned}$$

11.59

(a)



$$(1) g_{m2}V_{pi} + \frac{V_o - (-V_i)}{r_{o2}} = 0$$

$$(2) g_{m2}V_{pi} + \frac{V_o - (-V_i)}{r_{o2}} = g_{m1}V_i + \frac{-V_x}{r_{o1}} + \frac{-V_x}{r_s}$$

or

$$0 = g_{m1}V_i - V_x \left(\frac{1}{r_{o1}} + \frac{1}{r_s} \right)$$

Then

$$V_x = \frac{g_{m1}V_i}{\left(\frac{1}{r_{o1}} + \frac{1}{r_s} \right)}$$

From (1)

$$\left(g_{m2} + \frac{1}{r_{o2}} \right) V_x + \frac{V_o}{r_{o2}} = 0$$

$$V_o = -r_{o2} \left(g_{m2} + \frac{1}{r_{o2}} \right) V_x = -r_{o2} g_{m1} V_i \frac{\left(g_{m2} + \frac{1}{r_{o2}} \right)}{\left(\frac{1}{r_{o1}} + \frac{1}{r_s} \right)}$$

$$A_v = \frac{V_o}{V_i} = \frac{-g_{m1}r_{o2} \left(g_{m2} + \frac{1}{r_{o2}} \right)}{\left(\frac{1}{r_{o1}} + \frac{1}{r_s} \right)}$$

Now

$$g_{m1} = 2\sqrt{K_n I_Q} = 2\sqrt{(0.25)(0.025)} = 0.158 \text{ mA/V}$$

$$g_{m2} = \frac{I_Q}{V_T} = \frac{0.025}{0.026} = 0.9615 \text{ mA/V}$$

$$r_{o1} = \frac{1}{A_v} = \frac{1}{(0.02)(0.025)} = 2000 \text{ k}\Omega$$

$$r_{o2} = \frac{V_A}{I_Q} = \frac{50}{0.025} = 2000 \text{ k}\Omega$$

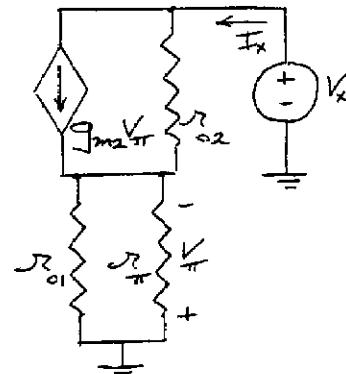
$$r_s = \frac{\beta V_T}{I_Q} = \frac{(100)(0.026)}{0.025} = 104 \text{ k}\Omega$$

Then

$$A_v = \frac{-(0.158)(2000) \left(0.9615 + \frac{1}{2000} \right)}{\left(\frac{1}{2000} + \frac{1}{104} \right)} =$$

$$A_v = -30039$$

To find R_o ; set $V_i = 0 \Rightarrow g_{m1}V_i = 0$



$$I_x = g_{m2}V_{pi} + \frac{V_x - (-V_x)}{r_{o2}}$$

$$V_x = -I_x(r_{o1} \parallel r_s)$$

Then

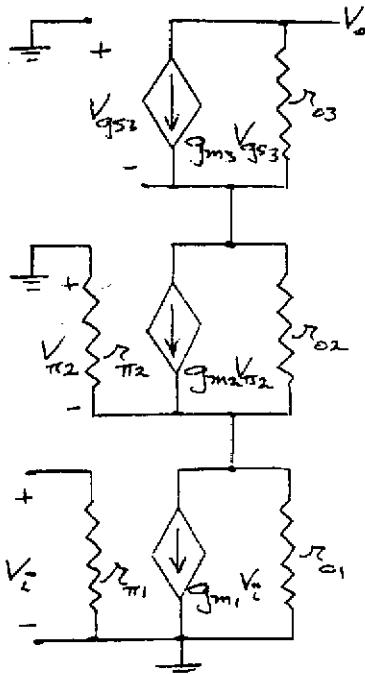
$$I_x = \left(g_{m2} + \frac{1}{r_{o2}} \right) (-I_x)(r_{o1} \parallel r_s) + \frac{V_x}{r_{o2}}$$

Combining terms,

$$\begin{aligned} R_o &= \frac{V_x}{I_x} = r_{o2} \left[1 + (r_{o1} \parallel r_s) \left(g_{m2} + \frac{1}{r_{o2}} \right) \right] \\ &= 2000 \left[1 + (2000 \parallel 104) \left(0.9615 + \frac{1}{2000} \right) \right] \end{aligned}$$

$$R_o = 192.2 \text{ M}\Omega$$

(b)



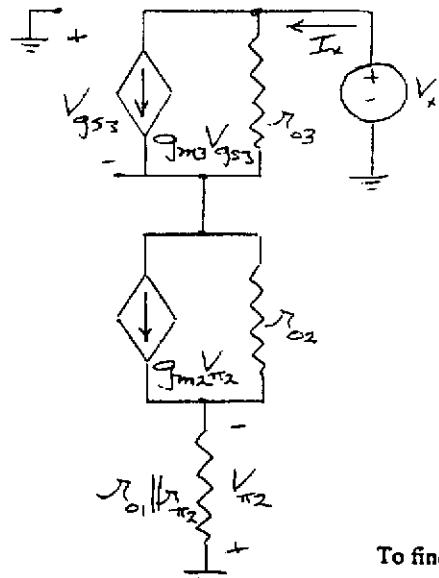
From (1),

$$\left(g_{m3} + \frac{1}{r_{o3}} \right) V_{pi3} = \frac{-V_o}{r_{o3}}$$

or

$$V_o = -2000 \left(0.158 + \frac{1}{2000} \right) (1.83 \times 10^5) V_i$$

$$A_v = \frac{V_o}{V_i} = -5.80 \times 10^7$$

To find R_o

$$(1) I_x = g_{m3} V_{pi3} + \frac{V_x - (-V_{pi3})}{r_{o3}}$$

$$(2) g_{m3} V_{pi3} + \frac{V_x - (-V_{pi3})}{r_{o3}} = g_{m2} V_{pi2} + \frac{-V_{pi3} - (-V_{pi2})}{r_{o2}}$$

$$(3) V_{pi2} = -I_x (r_{o1} \parallel r_{o2})$$

$$\text{From (1)} I_x = V_{pi3} \left(g_{m3} + \frac{1}{r_{o3}} \right) + \frac{V_x}{r_{o3}}$$

$$I_x = V_{pi3} \left(0.158 + \frac{1}{2000} \right) + \frac{V_x}{2000}$$

So

$$V_{pi3} = \frac{I_x - \frac{V_x}{2000}}{0.1585}$$

From (2),

$$V_{pi3} \left[g_{m3} + \frac{1}{r_{o3}} + \frac{1}{r_{o2}} \right] + \frac{V_x}{r_{o3}} = V_{pi2} \left(g_{m2} + \frac{1}{r_{o2}} \right)$$

$$V_{pi3} \left[0.158 + \frac{1}{2000} + \frac{1}{2000} \right] + \frac{V_x}{2000}$$

$$= V_{pi2} \left(0.9615 + \frac{1}{2000} \right)$$

Then

$$\left[\frac{I_x - V_x / 2000}{0.1585} \right] (0.159) + \frac{V_x}{2000} = -I_x (2000 \parallel 104)(0.962)$$

We find

$$R_o = \frac{V_x}{I_x} = 6.09 \times 10^{10} \Omega$$

Then $V_{pi3} = 1.83 \times 10^5 V_i$

11.60

Assume emitter of Q_1 is capacitively coupled to signal ground.

$$I_{CQ} = 0.2 \left(\frac{80}{81} \right) = 0.1975 \text{ mA}$$

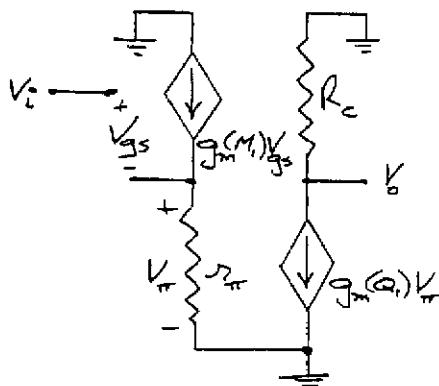
$$I_{DQ} = \frac{0.2}{81} = 0.00247 \text{ mA}$$

$$r_s = \frac{(80)(0.026)}{0.1975} = 10.5 \text{ k}\Omega$$

$$g_m(Q_1) = \frac{0.1975}{0.026} = 7.60 \text{ mA/V}$$

$$g_m(M_1) = 2\sqrt{K_n I_D} = 2\sqrt{(0.2)(0.00247)}$$

$$g_m(M_1) = 0.0445 \text{ mA/V}$$



$$V_i = V_{gs} + V_\pi \text{ and } V_\pi = g_m(M_1)V_{gs}r_\pi$$

or

$$V_\pi = \frac{V_\pi}{g_m(M_1)r_\pi}$$

Then

$$V_i = V_\pi \left(1 + \frac{1}{g_m(M_1)r_\pi} \right)$$

or

$$V_\pi = \frac{V_i}{\left(1 + \frac{1}{g_m(M_1)r_\pi} \right)}$$

$$V_\pi = -g_m(Q_1)V_\pi R_c \Rightarrow$$

$$A_v = \frac{V_o}{V_i} = \frac{-g_m(Q_1)R_c}{\left(1 + \frac{1}{g_m(M_1)r_\pi} \right)}$$

Then

$$A_v = \frac{-(7.60)(20)}{\left(1 + \frac{1}{(0.0445)(10.5)} \right)} \Rightarrow$$

$$\underline{A_v = -48.4}$$

11.61

Using the results from Chapter 4 for the emitter-follower:

$$R_0 = R_4 \parallel \left[\frac{r_{\pi 3} + r_{07} + R_{011}}{1 + \beta} \right]$$

$$r_{\pi 3} = \frac{\beta V_T}{I_{C8}} = \frac{(100)(0.026)}{1} = 2.6 \text{ k}\Omega$$

$$I_{C8} \approx \frac{I_{C8}}{\beta} = \frac{1}{100} = 0.01 \text{ mA}$$

$$r_{\pi 3} = \frac{(100)(0.026)}{0.01} = 260 \text{ k}\Omega$$

$$r_{07} = \frac{V_A}{I_Q} = \frac{100}{0.2} = 500 \text{ k}\Omega$$

$$r_{011} = \frac{V_A}{I_Q} = \frac{100}{0.2} = 500 \text{ k}\Omega$$

$$R_{011} = r_{011}[1 + g_m R'_S], \quad g_m = \frac{0.2}{0.026} = 7.69$$

$$r_{\pi 11} = \frac{(100)(0.026)}{0.2} = 13 \text{ k}\Omega$$

$$R'_S = 0.2 \parallel 13 = 0.197 \text{ k}\Omega$$

$$R_{011} = 500[1 + (7.69)(0.197)] = 1257 \text{ k}\Omega$$

Then

$$R_0 = 5 \parallel \left[\frac{2.6 + \frac{260 + 500 + 1257}{101}}{101} \right] \\ = 5 \parallel 0.223 \\ \Rightarrow \underline{R_0 = 0.213 \text{ k}\Omega}$$

11.62

$$R_i = r_{\pi 1} + (1 + \beta)r_{\pi 2}$$

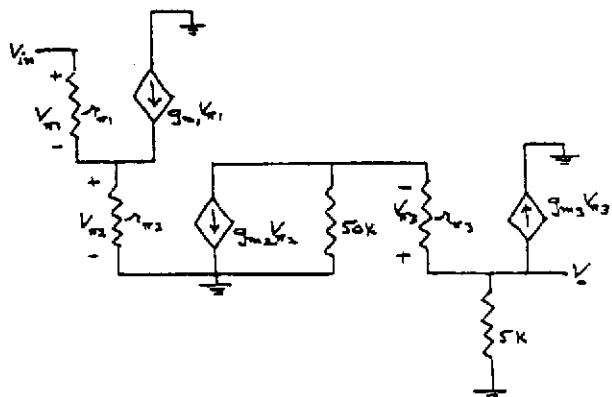
$$r_{\pi 2} = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$r_{\pi 1} = \frac{(100)(0.026)}{(0.5/100)} = \frac{(100)^2(0.026)}{0.5} = 520 \text{ k}\Omega$$

$$R_i = 520 + (101)(5.2) \Rightarrow \underline{R_i = 1.05 \text{ M}\Omega}$$

$$R_0 = 5 \parallel \frac{r_{\pi 3} + 50}{101}, \quad r_{\pi 3} = \frac{(100)(0.026)}{1} = 2.6 \text{ k}\Omega$$

$$R_0 = 5 \parallel \frac{2.6 + 50}{101} = 5 \parallel 0.521 \Rightarrow \underline{R_0 = 0.472 \text{ k}\Omega}$$



$$V_0 = -\left(\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3} V_{\pi 3}\right) \quad (5)$$

$$V_0 = -V_{\pi 3} \left(\frac{1+\beta}{r_{\pi 3}}\right) \quad (5)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} = g_{m2} V_{\pi 2} + \frac{(V_0 - V_{\pi 3})}{50}$$

$$g_{m2} V_{\pi 2} = V_{\pi 3} \left(\frac{1}{r_{\pi 3}} + \frac{1}{50}\right) - \frac{V_0}{50} \quad (2)$$

$$V_{\pi 2} = \left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1}\right) r_{\pi 2} \quad (3)$$

$$= V_{\pi 1} \left(\frac{1+\beta}{r_{\pi 1}}\right) r_{\pi 2}$$

and

$$V_{in} = V_{\pi 1} + V_{\pi 2} \quad (4)$$

$$g_{m2} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

Then

$$V_0 = -V_{\pi 3} \left(\frac{101}{2.6}\right) \quad (5)$$

$$\Rightarrow V_{\pi 3} = -V_0 (0.005149)$$

And

$$19.23 V_{\pi 2} = -V_0 (0.005149) \left(\frac{1}{2.6} + \frac{1}{50}\right) - \frac{V_0}{50} \quad (2)$$

$$= -V_0 (0.02208)$$

$$\text{Or } V_{\pi 2} = -V_0 (0.001148)$$

And

$$V_{\pi 1} = V_{in} - V_{\pi 2} = V_{in} + V_0 (0.001148) \quad (4)$$

So

$$-V_0 (0.001148)$$

$$= [V_{in} + V_0 (0.001148)] \left(\frac{101}{520}\right) \quad (5.2)$$

$$-V_0 (0.001148) - V_0 (0.001159) = V_{in} (1.01)$$

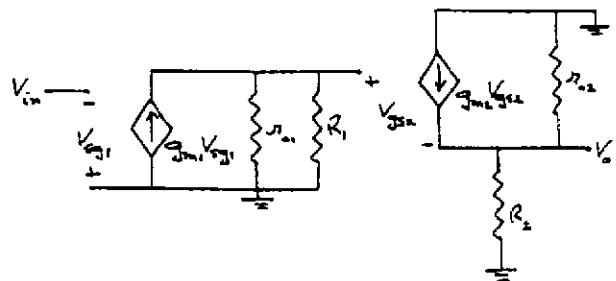
$$\Rightarrow A_v = \frac{V_0}{V_{in}} = -438$$

11.63

$$I_2 = \frac{5}{5} = 1 \text{ mA}$$

$$V_{GS2} = \sqrt{\frac{1}{0.5}} + 0.8 = 2.21 \text{ V}$$

$$I_1 = \frac{2.21 - (-5)}{35} = 0.206 \text{ mA}$$



$$V_0 = (g_{m2} V_{gs2})(R_2 \| r_{o2})$$

$$V_{gs2} = (g_{m1} V_{gs1})(r_{o1} \| R_1) - V_0$$

$$\text{and } V_{gs1} = -V_{in}$$

So

$$V_{gs2} = -(g_{m1} V_{in})(r_{o1} \| R_1) - V_0$$

Then

$$V_0 = g_{m2} (R_2 \| r_{o2}) [-(g_{m1} V_{in})(r_{o1} \| R_1) - V_0]$$

$$A_v = \frac{V_0}{V_{in}} = \frac{-g_{m2} (R_2 \| r_{o2}) g_{m1} (r_{o1} \| R_1)}{1 + g_{m2} (R_2 \| r_{o2})}$$

$$g_{m2} = 2\sqrt{K_{p2} I_{D2}} = 2\sqrt{(0.5)(1)} = 1.414 \text{ mA/V}$$

$$g_{m1} = 2\sqrt{K_{p1} I_{D1}} = 2\sqrt{(0.2)(0.206)} = 0.406 \text{ mA/V}$$

$$r_{o1} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.01)(0.206)} = 485 \text{ k}\Omega$$

$$r_{o2} = \frac{1}{\lambda_2 I_{D2}} = \frac{1}{(0.01)(1)} = 100 \text{ k}\Omega$$

$$R_2 \| r_{o2} = 5 \| 100 = 4.76 \text{ k}\Omega$$

$$R_1 \| r_{o1} = 35 \| 485 = 32.6 \text{ k}\Omega$$

Then

$$A_v = \frac{-(1.414)(4.76)(0.406)(32.6)}{1 + (1.414)(4.76)}$$

So

$$\Rightarrow A_V = -11.5$$

Output Resistance—From the results for a source follower in Chapter 6.

$$R_o = \frac{1}{g_{m2}} \parallel R_2 \parallel r_{o2} = \frac{1}{1.414} \parallel 5 \parallel 100 \\ = 0.707 \parallel 4.76$$

$$\text{So } R_o = 0.616 \text{ k}\Omega$$

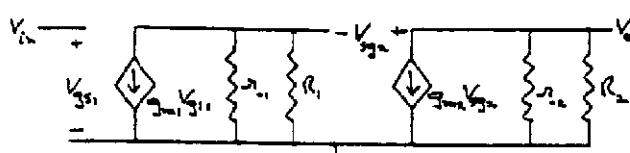
11.64

$$\text{a. } R_2 = \frac{5}{0.5} \Rightarrow R_2 = 10 \text{ k}\Omega$$

$$V_{SG2} = \sqrt{\frac{I_{D2}}{K_{p2}}} - V_{TP2} = \sqrt{\frac{0.5}{0.25}} + 1 = 2.41V$$

$$R_1 = \frac{5 - (-2.41)}{0.1} \Rightarrow R_1 = 74.1 \text{ k}\Omega$$

b.



$$V_0 = -(g_{m2} V_{sg2})(r_{o2} \parallel R_2)$$

$$V_{sg2} = V_0 - [-(g_{m1} V_{gs1})(r_{o1} \parallel R_1)]$$

$$\text{and } V_{gs1} = V_{in}$$

$$A_V = \frac{V_0}{V_{in}} = \frac{-(g_{m2})(r_{o2} \parallel R_2)(g_{m1})(r_{o1} \parallel R_1)}{1 + (g_{m2})(r_{o2} \parallel R_2)}$$

$$g_{m1} = 2\sqrt{K_{n1} I_{D1}} = 2\sqrt{(0.1)(0.1)} = 0.2 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{K_{p2} I_{D2}} = 2\sqrt{(0.25)(0.5)} = 0.707 \text{ mA/V}$$

$$r_{o1} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.01)(0.1)} = 1000 \text{ k}\Omega$$

$$r_{o2} = \frac{1}{\lambda_2 I_{D2}} = \frac{1}{(0.01)(0.5)} = 200 \text{ k}\Omega$$

$$r_{o2} \parallel R_2 = 200 \parallel 10 = 9.52 \text{ k}\Omega$$

$$r_{o1} \parallel R_1 = 1000 \parallel 74.1 = 69.0 \text{ k}\Omega$$

Then

$$A_V = \frac{-(0.707)(9.52)(0.2)(69)}{1 + (0.707)(9.52)}$$

So

$$\Rightarrow A_V = -12.0$$

$$R_o = \frac{1}{g_{m2}} \parallel R_2 \parallel r_{o2} = \frac{1}{0.707} \parallel 10 \parallel 200 \\ = 1.414 \parallel 9.52$$

$$\text{Or } R_o = 1.23 \text{ k}\Omega$$

11.65

$$\text{a. } I_{C2} = 0.25 \text{ mA}$$

$$R = \frac{5 - 2}{0.25} \Rightarrow R = 12 \text{ k}\Omega$$

$$I_{C3} = \frac{V_{D2} - V_{BE}(\text{on})}{R_{E1}} \Rightarrow R_{E1} = \frac{2 - 0.7}{0.5}$$

$$\Rightarrow R_{E1} = 2.6 \text{ k}\Omega$$

$$R_C = \frac{5 - V_{D3}}{I_{C3}} = \frac{5 - 3}{0.5} \Rightarrow R_C = 4 \text{ k}\Omega$$

$$I_{C4} = \frac{[V_{D3} - V_{BE}(\text{on})] - (-5)}{R_{E2}}$$

$$R_{E2} = \frac{3 - 0.7 + 5}{3} \Rightarrow R_{E2} = 2.43 \text{ k}\Omega$$

b. Input resistance to base of Q_3 .

$$R_{i3} = r_{\pi3} + (1 + \beta)R_{E1}$$

$$r_{\pi3} = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$R_{i3} = 5.2 + (101)(2.6) = 267.8 \text{ k}\Omega$$

$$A_{d1} = \frac{V_{D2}}{V_d} = \frac{1}{2} g_{m2} (R \parallel R_{i3})$$

$$g_{m2} = \frac{0.25}{0.026} = 9.62 \text{ mA/V}$$

$$A_{d1} = \frac{1}{2}(9.62)(12 \parallel 267.8) \Rightarrow A_{d1} = 55.2$$

$$\text{Now } \frac{V_{D3}}{V_{D2}} = \frac{-\beta(R_C \parallel R_{i4})}{r_{\pi3} + (1 + \beta)R_{E1}}$$

$$\text{where } R_{i4} = r_{\pi4} + (1 + \beta)R_{E2}$$

$$\text{and } \frac{V_0}{V_{D3}} = \frac{(1 + \beta)R_{E2}}{r_{\pi4} + (1 + \beta)R_{E2}}$$

$$r_{\pi4} = \frac{(100)(0.026)}{3} = 0.867 \text{ k}\Omega$$

$$\frac{V_0}{V_{D3}} = \frac{(101)(2.43)}{0.867 + (101)(2.43)} = 0.9965$$

$$R_{i4} = 0.867 + (101)(2.43) = 246.3 \text{ k}\Omega$$

$$r_{\pi3} = 5.2 \text{ k}\Omega$$

So

$$\frac{V_{D3}}{V_{D2}} = \frac{-(100)(4 \parallel 246.3)}{5.2 + (101)(2.6)} = -1.47$$

So

$$A_d = \frac{V_0}{V_d} = (55.2)(0.9965)(-1.47) \\ \Rightarrow A_d = -80.9$$

c. Using Equation (11.32b)

$$A_{cm1} = \frac{-g_{m2}(R \parallel R_{i3})}{1 + \frac{2(1 + \beta)R_0}{r_{\pi2}}}$$

$$r_{\pi2} = \frac{(100)(0.026)}{0.25} = 10.4 \text{ k}\Omega$$

$$A_{cm1} = \frac{-(9.62)(12 \parallel 267.8)}{1 + \frac{2(101)(100)}{10.4}} = -0.0569 = A_{cm1}$$

Then

$$\begin{aligned} A_{cm} &= \left(\frac{\nu_0}{\nu_{03}} \right) \left(\frac{\nu_{03}}{\nu_{02}} \right) \cdot A_{cm1} \\ &= (0.9965)(-1.47)(-0.0569) \\ \Rightarrow A_{cm} &= 0.08335 \end{aligned}$$

$$\begin{aligned} CMRR_{dB} &= 20 \log_{10} \left(\frac{80.9}{0.08335} \right) \\ \Rightarrow CMRR_{dB} &= 59.7 \text{ dB} \end{aligned}$$

11.66

a. $R_{C1} = \frac{10 - \nu_{01}}{I_{C1}} = \frac{10 - 2}{0.1} \Rightarrow R_{C1} = 80 \text{ k}\Omega$

$$R_{C2} = \frac{10 - \nu_{04}}{I_{C4}} = \frac{10 - 6}{0.2} \Rightarrow R_{C2} = 20 \text{ k}\Omega$$

b. $A_{d1} = \frac{\nu_{01} - \nu_{02}}{\nu_d} = -g_{m1}(R_{C1} \parallel r_{\pi3})$

$$g_{m1} = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$r_{\pi3} = \frac{(180)(0.026)}{0.2} = 23.4 \text{ k}\Omega$$

$$A_{d1} = -(3.846)(80 \parallel 23.4) \Rightarrow A_{d1} = -69.6$$

$$A_{d2} = \frac{\nu_{04}}{\nu_{01} - \nu_{02}} = \frac{1}{2} g_{m4} R_{C2}$$

$$g_{m4} = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$$

$$A_{d2} = \frac{1}{2}(7.692)(20) = 76.9$$

Then $A_d = (76.9)(-69.6)$

$$\Rightarrow A_d = -5352$$

11.67

- a. Neglect the effect of r_0 in determining the differential-mode gain.

$$A_{d1} = \frac{\nu_{02}}{\nu_d} = \frac{1}{2} g_{m2}(R_C \parallel R_{i3})$$

where $R_{i3} = r_{\pi3} + (1 + \beta)R_E$

$$A_{d2} = \frac{-\beta R_{C2}}{r_{\pi3} + (1 + \beta)R_E}$$

$$I_1 = \frac{12 - 0.7 - (-12)}{R_1} = \frac{23.3}{12} = 1.94 \text{ mA} \approx I_{C5}$$

$$g_{m2} = \frac{\frac{1}{2} \cdot (1.94)}{0.026} = 37.3 \text{ mA/V}$$

$$r_{\pi3} = \frac{(200)(0.026)}{I_{C3}}$$

$$\nu_{02} = 12 - \frac{1}{2}(1.94)(8) = 4.24 \text{ V}$$

$$I_{C3} = \frac{4.24 - 0.7}{3.3} = 1.07 \text{ mA}$$

$$r_{\pi3} = \frac{(200)(0.026)}{1.07} = 4.86 \text{ k}\Omega$$

$$R_{i3} = 4.86 + (201)(3.3) = 668 \text{ k}\Omega$$

$$A_{d1} = \frac{1}{2}(37.3)[8 \parallel 668] = 147.4$$

$$A_2 = \frac{-(200)(4)}{4.86 + (201)(3.3)} = -1.197$$

Then

$$A_d = A_{d1} \cdot A_2 = (147.4)(-1.197) \Rightarrow A_d = -176$$

$$R_0 = r_{05} = \frac{V_A}{I_{C5}} = \frac{80}{1.94} = 41.2 \text{ k}\Omega$$

$$A_{cm1} = \frac{-g_{m2}(R_C \parallel R_{i3})}{1 + \frac{2(1 + \beta)R_0}{r_{\pi2}}}$$

$$r_{\pi2} = \frac{(200)(0.026)}{\frac{1}{2} \cdot (1.94)} = 5.36 \text{ k}\Omega$$

$$A_{cm1} = \frac{-(37.3)(8 \parallel 668)}{1 + \frac{2(201)(41.2)}{5.36}} = -0.09539$$

$$A_2 = -1.197$$

$$A_{cm} = (-0.09539)(-1.197) \Rightarrow A_{cm} = 0.114$$

b. $\nu_d = \nu_1 - \nu_2 = 2.015 \sin \omega t - 1.985 \sin \omega t$

$$\nu_d = 0.03 \sin \omega t (\text{V})$$

$$\nu_{cm} = \frac{\nu_1 + \nu_2}{2} = 2.0 \sin \omega t$$

$$\nu_{03} = A_d \nu_d + A_{cm} \nu_{cm}$$

$$= (-176)(0.03) + (0.114)(2)$$

Or

$$\nu_{03} = -5.052 \sin \omega t$$

Ideal, $A_{cm} = 0$

So

$$\nu_{03} = A_d \nu_d = (-176)(0.03)$$

$$\nu_{03} = -5.28 \sin \omega t$$

c. $R_{id} = 2r_{\pi2} = 2(5.36) \Rightarrow R_{id} = 10.72 \text{ k}\Omega$

$$2R_{icm} \approx 2(1 + \beta)R_0 \parallel (1 + \beta)r_0$$

$$r_0 = \frac{V_A}{I_{C2}} = \frac{80}{\frac{1}{2} \cdot (1.94)} = 82.5 \text{ k}\Omega$$

$$2R_{icm} = [2(201)(41.2)] \parallel [(201)(82.5)]$$

$$= 16.6 \text{ M}\Omega \parallel 16.6 \text{ M}\Omega$$

So

$$\Rightarrow R_{icm} = 4.15 \text{ M}\Omega$$

11.68

$$I_1 = \frac{24 - V_{GS4}}{R_1} = k_n(V_{GS4} - V_{Th})^2$$

$$24 - V_{GS4} = (55)(0.2)(V_{GS4} - 2)^2$$

$$24 - V_{GS4} = 11(V_{GS4}^2 - 4V_{GS4} + 4)$$

$$11V_{GS4}^2 - 43V_{GS4} + 20 = 0$$

$$V_{GS4} = \frac{43 \pm \sqrt{(43)^2 - 4(11)(20)}}{2(11)} = 3.37 \text{ V}$$

$$I_1 = \frac{24 - 3.37}{55} = 0.375 \text{ mA} = I_Q$$

$$\nu_{02} = 12 - \left(\frac{0.375}{2}\right)(40) = 4.5 \text{ V}$$

$$\frac{\nu_{02} - V_{GS3}}{R_5} = I_{D3} = k_n(V_{GS3} - V_{Th})^2$$

$$4.5 - V_{GS3} = (0.2)(6)(V_{GS3}^2 - 4V_{GS3} + 4)$$

$$1.2V_{GS3}^2 - 3.8V_{GS3} + 0.3 = 0$$

$$V_{GS3} = \frac{3.8 \pm \sqrt{(3.8)^2 - 4(1.2)(0.3)}}{2(1.2)} = 3.09 \text{ V}$$

$$I_{D3} = \frac{4.5 - 3.09}{6} = 0.235 \text{ mA}$$

$$g_{m2} = 2\sqrt{K_s I_{D2}} = 2\sqrt{(0.2)\left(\frac{0.375}{2}\right)} \\ = 0.387 \text{ mA/V}$$

$$A_{d1} = \frac{1}{2}g_{m2}R_D = \frac{1}{2}(0.387)(40)$$

$$\Rightarrow A_{d1} = 7.74$$

$$A_2 = \frac{-g_{m3}R_{D3}}{1 + g_{m3}R_5}$$

$$g_{m3} = 2\sqrt{K_s I_{D3}} = 2\sqrt{(0.2)(0.235)} \\ = 0.434 \text{ mA/V}$$

$$A_2 = \frac{-(0.434)(4)}{1 + (0.434)(6)} = -0.482$$

$$\text{So } A_d = A_{d1} \cdot A_2 = (7.74)(-0.482)$$

$$\Rightarrow A_d = -3.73$$

$$R_0 = r_{os} = \frac{1}{\lambda I_Q} = \frac{1}{(0.02)(0.375)} = 133 \text{ k}\Omega$$

$$A_{cm1} = \frac{-g_{m2}R_D}{1 + 2g_{m2}R_0} = \frac{-(0.387)(40)}{1 + 2(0.387)(133)} \\ = -0.149$$

$$A_{cm} = (-0.149)(-0.482) \Rightarrow A_{cm} = 0.0718$$

$$\text{b. } \nu_d = \nu_1 - \nu_2 = 0.3 \sin \omega t$$

$$\nu_{cm} = \frac{\nu_1 + \nu_2}{2} = 2 \sin \omega t$$

$$\nu_{03} = A_d \nu_d + A_{cm} \nu_{cm}$$

$$= (-3.73)(0.3) + (0.0718)(2)$$

$$\Rightarrow \nu_{03} = -0.975 \sin \omega t (\text{V})$$

 Ideal, $A_{cm} = 0$

$$\nu_{03} = A_d \nu_d = (-3.73)(0.3)$$

Or

$$\Rightarrow \nu_{03} = -1.12 \sin \omega t (\text{V})$$

11.69

The low-frequency, one-sided differential gain is

$$A_{\nu 2} = \frac{\nu_{02}}{\nu_d} = \frac{1}{2}g_m R_C \left(\frac{r_\pi}{r_\pi + R_B} \right)$$

$$= \frac{\frac{1}{2}\beta R_C}{r_\pi + R_B}$$

$$r_\pi = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$A_{\nu 2} = \frac{\frac{1}{2}(100)(10)}{5.2 + 0.5} \Rightarrow A_{\nu 2} = 87.7$$

$$C_M = C_\mu(1 + g_m R_C)$$

$$g_m = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$C_M = 2[1 + (19.23)(10)] \Rightarrow C_M = 387 \text{ pF}$$

$$f_H = \frac{1}{2\pi[r_\pi \parallel R_B](C_\pi + C_M)}$$

$$= \frac{1}{2\pi[5.2 \parallel 0.5] \times 10^3 \times (8 + 387) \times 10^{-12}}$$

So

$$\Rightarrow f_H = 883 \text{ kHz}$$

11.70

a. From Equation (11.117),

$$f_Z = \frac{1}{2\pi R_Q C_0} = \frac{1}{2\pi(5 \times 10^6)(0.8 \times 10^{-12})}$$

$$\text{Or } f_Z = 39.8 \text{ kHz}$$

 b. From Problem 11.69, $f_H = 883 \text{ kHz}$. From Equation (11.116(b)), the low-frequency common-mode gain is

$$A_{cm} = \frac{-g_m R_C}{\left[\left(1 + \frac{R_B}{r_\pi} \right) + \frac{2(1 + \beta)R_0}{r_\pi} \right]}$$

$$r_\pi = 5.2 \text{ k}\Omega, g_m = 19.23 \text{ mA/V}$$

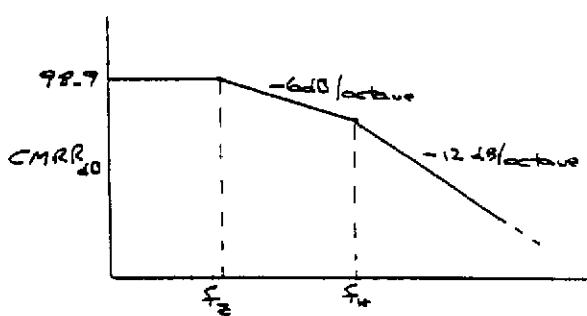
So

$$A_{cm} = \frac{-(19.23)(10)}{\left[\left(1 + \frac{0.5}{5.2} \right) + \frac{2(101)(5 \times 10^6)}{5.2 \times 10^3} \right]} \\ = -9.9 \times 10^{-4}$$

$$CMRR_{dB} = 20 \log_{10} \left(\frac{87.7}{9.9 \times 10^{-4}} \right) = 98.9 \text{ dB}$$

11.72

The differential-mode half circuit is:



11.71

a. From Equation (7.72),

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$g_m = \frac{1}{0.026} = 38.46 \text{ mA/V}$$

Then

$$800 \times 10^6 = \frac{38.46 \times 10^{-3}}{2\pi(C_\pi + C_\mu)}$$

Or

$$C_\pi + C_\mu = 7.65 \times 10^{-12} \text{ F} = 7.65 \text{ pF}$$

And $C_\pi = 6.65 \text{ pF}$

$$C_M = C_\mu(1 + g_m R_C) = 1[1 + (38.46)(10)] \\ = 386 \text{ pF}$$

$$f_H = \frac{1}{2\pi[r_\pi || R_B](C_\pi + C_M)}$$

$$r_\pi = \frac{(120)(0.026)}{1} = 3.12 \text{ k}\Omega$$

$$f_H = \frac{1}{2\pi[3.12 || 1] \times 10^3 \times (6.65 + 386) \times 10^{-12}}$$

Or

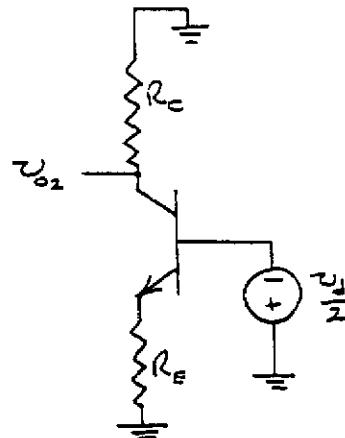
$$\underline{f_H = 535 \text{ kHz}}$$

b. From Equation (11.117),

$$f_z = \frac{1}{2\pi R_0 C_0} = \frac{1}{2\pi(10 \times 10^6)(10^{-12})}$$

Or

$$\underline{f_z = 15.9 \text{ kHz}}$$



$$v_{o2} = \frac{g_m \left(\frac{V_d}{2} \right) R_C}{1 + \left(\frac{1 + \beta}{r_\pi} \right) R_E} \text{ or } A_v = \frac{\left(\frac{1}{2} \right) \beta R_C}{r_\pi + (1 + \beta) R_E}$$

$$r_\pi = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$A_v = \frac{\left(\frac{1}{2} \right) (100)(10)}{5.2 + (101)R_E} = \frac{500}{5.2 + (101)R_E}$$

- a. For $R_E = 0.1 \text{ k}\Omega$: $\underline{A_v = 32.7}$
 b. For $R_E = 0.25 \text{ k}\Omega$: $\underline{A_v = 16.4}$

Chapter 12

Exercise Solutions

E12.1

$$a. \quad A_f = \frac{A}{1 + A\beta}$$

$$1 + A\beta = \frac{A}{A_f} \Rightarrow A\beta = \frac{A}{A_f} - 1$$

$$\beta = \frac{1}{A_f} - \frac{1}{A} = \frac{1}{20} - \frac{1}{10^4} = 0.05 - 0.0001 \\ \Rightarrow \underline{\beta = 0.0499}$$

$$b. \quad \frac{A_f}{(1/\beta)} = \frac{20}{(1/0.0499)} = \frac{20}{20.040} = \underline{0.998}$$

E12.2

$$A_f = \frac{A}{1 + A\beta}$$

$$A_f + A_f A\beta = A$$

$$A_f = A(1 - A_f \beta)$$

$$A = \frac{A_f}{1 - A_f \beta} = \frac{80}{1 - (80)(0.0120)} \\ \underline{A = 2000}$$

E12.3

$$A_f = \frac{A}{1 + A\beta}$$

$$\beta = \frac{1}{A_f} - \frac{1}{A} = \frac{1}{100} - \frac{1}{10^4} = 0.01 - 10^{-6}$$

$$\beta = 0.009999$$

$$\frac{dA_f}{A_f} = \frac{1}{(1 + \beta A)} \cdot \frac{dA}{A} = \left(\frac{A_f}{A}\right) \cdot \frac{dA}{A}$$

$$\frac{dA_f}{A_f} = \left(\frac{100}{10^4}\right) (20)\% \\ \Rightarrow \underline{\frac{dA_f}{A_f} = 0.002\%}$$

E12.4

$$\frac{dA_f}{A_f} = \frac{1}{(1 + \beta A)} \cdot \frac{dA}{A} = \left(\frac{A_f}{A}\right) \cdot \frac{dA}{A}$$

$$\frac{dA}{A} = \frac{dA_f}{A_f} \cdot \left(\frac{A}{A_f}\right) = (0.001) \left(\frac{5 \times 10^5}{100}\right)$$

$$\Rightarrow \underline{\frac{dA}{A} = \pm 5\%}$$

E12.5

$$\text{Bandwidth} = \omega_H(1 + \beta A_0)$$

$$= \omega_H \left(\frac{A_0}{A_f}\right) = (2\pi)(10) \left(\frac{10^5}{100}\right)$$

$$\underline{\omega = (2\pi)(10^4) \text{ rad/sec} \Rightarrow f = 10 \text{ kHz}}$$

E12.6

$$A_f \cdot f_H = A_0 \cdot f_1$$

$$A_f = \frac{A_0 \cdot f_1}{f_H} = \frac{(10^4)(8)}{250 \times 10^3} \\ \Rightarrow \underline{A_f(0) = 32}$$

E12.7

$$a. \quad V_e = V_S - V_{fb} = 100 - 99 = 1 \text{ mV}$$

$$V_0 = A_v V_e \Rightarrow A_v = \frac{5}{0.001} \Rightarrow \underline{A_v = 5000 \text{ V/V}}$$

$$V_{fb} = \beta V_0 \Rightarrow \beta = \frac{V_{fb}}{V_0} = \frac{0.099}{5} \Rightarrow \underline{\beta = 0.0198 \text{ V/V}}$$

$$A_{vf} = \frac{A_v}{1 + \beta A_v} = \frac{5000}{1 + (0.0198)(5000)} \\ \Rightarrow \underline{A_{vf} = 50 \text{ V/V}}$$

$$b. \quad R_{if} = R_i(1 + \beta A_v) = (5)[1 + (0.0198)(5000)] \\ \Rightarrow \underline{R_{if} = 500 \text{ k}\Omega}$$

$$R_{of} = \frac{R_o}{1 + \beta A_v} = \frac{4}{1 + (0.0198)(5000)} \\ \Rightarrow \underline{R_{of} = 40 \Omega}$$

E12.8

$$a. \quad I_e = I_S - I_{fb} = 100 - 99 = 1 \mu\text{A}$$

$$A_i = \frac{I_o}{I_e} = \frac{5}{0.001} \Rightarrow \underline{A_i = 5000 \text{ A/A}}$$

$$\beta = \frac{I_{fb}}{I_o} = \frac{0.099}{5} \Rightarrow \underline{\beta = 0.0198 \text{ A/A}}$$

$$A_{if} = \frac{A_i}{1 + A_i \beta} = \frac{5000}{1 + (5000)(0.0198)} \\ \Rightarrow \underline{A_{if} = 50 \text{ A/A}}$$

$$b. \quad R_{if} = \frac{R_i}{1 + \beta A_i} = \frac{5}{1 + (0.0198)(5000)} \\ \Rightarrow \underline{R_{if} = 50 \Omega}$$

$$R_{of} = (1 + \beta A_i) R_o = [1 + (0.0198)(5000)][4] \\ \Rightarrow \underline{R_{of} = 400 \text{ k}\Omega}$$

E12.9

$$V_e = V_S - V_{fb} = 100 - 99 = 1 \text{ mV}$$

$$A_g = \frac{I_o}{V_e} = \frac{5 \text{ mA}}{1 \text{ mV}} \Rightarrow \underline{A_g = 5 \text{ A/V}}$$

$$\beta = \frac{V_{fb}}{I_o} = \frac{99 \text{ mV}}{5 \text{ mA}} \Rightarrow \underline{\beta = 19.8 \text{ V/A}}$$

$$A_{gf} = \frac{A_g}{1 + \beta A_g} = \frac{5}{1 + (19.8)(5)} \\ \Rightarrow \underline{A_{gf} = 0.05 \text{ A/V} = 50 \text{ mA/V}}$$

E12.10

$$I_e = I_S - I_{fb} = 100 - 99 = 1 \mu\text{A}$$

$$A_v = \frac{V_o}{I_e} = \frac{5 \text{ V}}{1 \mu\text{A}} \Rightarrow A_v = 5 \times 10^6 \text{ V/A}$$

$$\beta = \frac{I_{fb}}{V_0} = \frac{99 \mu\text{A}}{5 \text{ V}} \Rightarrow \beta = 1.98 \times 10^{-5} \text{ A/V}$$

$$A_{vf} = \frac{A_v}{1 + \beta A_v} = \frac{5 \times 10^6}{1 + (1.98 \times 10^{-5})(5 \times 10^6)} \Rightarrow A_{vf} = 5 \times 10^4 \text{ V/A} = 50 \text{ V/mA}$$

E12.11

$$A_{vf} = \frac{A_v}{1 + \frac{A_v}{1 + (R_2/R_1)}} = \frac{10^4}{1 + \frac{10^4}{1 + (30/10)}} \Rightarrow A_{vf} = 3.9984$$

$$A_{vf} = \frac{10^5}{1 + \frac{10^5}{1 + (30/10)}} = 3.99984$$

$$\frac{3.99984 - 3.9984}{3.9984} \times 100\% \Rightarrow 0.0360\%$$

E12.12

a. $r_\pi = \frac{h_{FE} V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$A_{vf} = \frac{\left(\frac{1}{r_\pi} + g_m\right) R_E}{1 + \left(\frac{1}{r_\pi} + g_m\right) R_E} = \frac{\left(\frac{1}{5.2} + 19.23\right)(2)}{1 + \left(\frac{1}{5.2} + 19.23\right)(2)}$$

$$= \frac{(19.42)(2)}{1 + (19.42)(2)} \Rightarrow A_{vf} = 0.97490$$

$$R_{if} = r_\pi + (1 + h_{FE}) R_E = 5.2 + (101)(2)$$

$$\Rightarrow R_{if} = 207.2 \text{ k}\Omega$$

$$R_{of} = R_E \parallel \frac{r_\pi}{1 + h_{FE}} = 2 \parallel \frac{5.2}{101}$$

$$\Rightarrow R_{of} = 0.0502 \text{ k}\Omega \Rightarrow 50.2 \text{ }\Omega$$

b. $h_{FE} = 150 \Rightarrow r_\pi = 7.8 \text{ k}\Omega, g_m = 19.23 \text{ mA/V}$

$$A_{vf} = \frac{\left(\frac{1}{7.8} + 19.23\right)(2)}{1 + \left(\frac{1}{7.8} + 19.23\right)(2)} = \frac{(19.36)(2)}{1 + (19.36)(2)}$$

$$A_{vf} = 0.97482 \Rightarrow 0.0082\% \text{ change in } A_{vf}$$

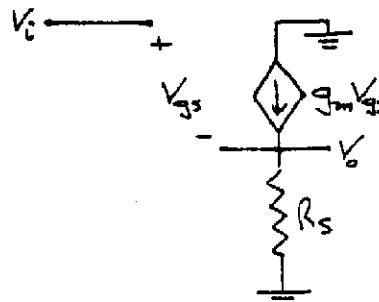
$$R_{if} = 7.8 + (101)(2) = 209.8 \text{ k}\Omega$$

$$\Rightarrow 1.25\% \text{ change in } R_{if}$$

$$R_{of} = R_E \parallel \frac{r_\pi}{1 + h_{FE}} = 2 \parallel \frac{7.8}{151} = 2 \parallel 0.0517$$

$$R_{of} = 50.4 \Omega \Rightarrow 0.397\% \text{ change in } R_{of}$$

E12.13



$$V_o = (g_m V_{TS}) R_S$$

$$V_i = V_{TS} + g_m R_S V_{TS}$$

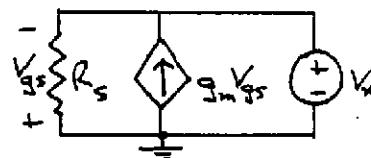
$$V_{TS} = \frac{V_i}{1 + g_m R_S}$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.2)(0.25)} = 0.447 \text{ mA/V}$$

$$A_{vf} = \frac{V_o}{V_i} = \frac{g_m R_S}{1 + g_m R_S}$$

$$A_{vf} = \frac{(0.447)(5)}{1 + (0.447)(5)} \Rightarrow A_{vf} = 0.691$$

$$R_{if} = \infty$$



$$I_X = g_m V_{TS} = \frac{V_X}{R_S}$$

$$V_{TS} = -V_X$$

$$I_X = V_X \left(g_m + \frac{1}{R_S} \right)$$

$$R_{of} = \frac{1}{g_m} \parallel R_S = \frac{1}{0.447} \parallel 5$$

$$R_{of} = 1.55 \text{ k}\Omega$$

E12.15

$$i_o = \left(\frac{h_{FE}}{1 + h_{FE}} \right) \left(\frac{R_E}{R_E + \frac{r_\pi}{1 + h_{FE}}} \right) i_i$$

$$r_\pi = \frac{(80)(0.026)}{0.5} = 4.16 \text{ k}\Omega$$

Then

$$\frac{r_\pi}{1 + h_{FE}} = \frac{4.16}{81} = 0.0514 \text{ k}\Omega$$

Then we want

$$\frac{I_o}{I_i} = 0.95 = \left(\frac{80}{81} \right) \left(\frac{R_E}{R_E + 0.0514} \right)$$

or

$$\left(\frac{R_E}{R_E + 0.0514} \right) = 0.9619$$

which yields

$$R_E (\text{min}) = 1.30 \text{ k}\Omega$$

and

$$V^+ = I_g R_E + 0.7 = \left(\frac{81}{80} \right) (0.5)(1.3) + 0.7 \Rightarrow$$

$$\underline{V^+ (\text{min}) = 1.36 V}$$

E12.19

$$\text{a. } A_{gf} = \frac{h_{FE} \cdot A_g}{1 + (h_{FE} A_g) R_S} = \frac{(200)(10^3)}{1 + (200)(10^3)(10^3)}$$

$$\Rightarrow \underline{A_{gf} = 10^{-3} \text{ A/V} = 1 \text{ mA/V}}$$

$$\text{b. } A_{gf} = \frac{(200)(10^4)}{1 + (200)(10^4)(10^3)} \Rightarrow \underline{A_{gf} \approx 1 \text{ mA/V}}$$

Percent change is negligible. ($4.5 \times 10^{-7}\%$)

$$\frac{(200)(10^4)}{1 + (200)(10^4)(10^3)} - \frac{(200)(10^3)}{1 + (200)(10^3)(10^3)}$$

$$= \frac{(200)(10^4)[1 + (200)(10^3)(10^3)]}{(10^{-3})(2 \times 10^9)(2 \times 10^4)} - \frac{(200)(10^3)[1 + (200)(10^4)(10^3)]}{(10^{-3})(2 \times 10^9)(2 \times 10^8)}$$

$$= \frac{200 \times 10^4 - 200 \times 10^3}{(10^{-3})(2 \times 10^9)(2 \times 10^8)} = 4.5 \times 10^{-9}$$

E12.20

$$\text{a. } V_G = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5 = \left(\frac{20}{20 + 30} \right) (10) - 5$$

$$V_G = -1 \text{ V}$$

$$V_S = -1 - V_{GS}$$

$$I_D = \frac{V_S - (-5)}{R_S} = K_n (V_{GS} - V_{TN})^2$$

$$1 - V_{GS} + 5 = (1.5)(0.4)(V_{GS} - 2)^2$$

$$4 - V_{GS} = 0.6(V_{GS}^2 - 4V_{GS} + 4)$$

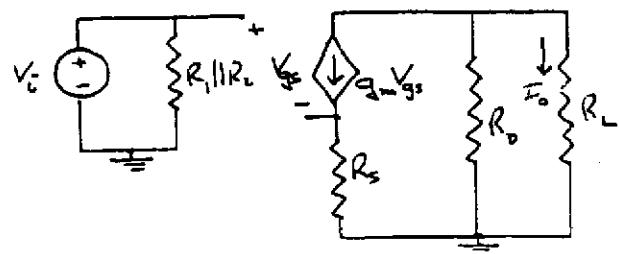
$$0.6V_{GS}^2 - 1.4V_{GS} - 1.6 = 0$$

$$V_{GS} = \frac{1.4 \pm \sqrt{(1.4)^2 + 4(0.6)(1.6)}}{2(0.6)}$$

$$V_{GS} = 3.17 \text{ V}$$

$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(1.5)(3.17 - 2)$$

$$g_m = 3.51 \text{ mA/V}$$



$$I_0 = -\left(\frac{R_D}{R_D + R_L} \right) (g_m V_{gs}) = -\left(\frac{2}{2+2} \right) (3.51) V_{gs}$$

$$I_0 = -1.76 V_{gs}$$

$$V_i = V_{gs} + g_m V_{gs} R_S \Rightarrow V_{gs} = \frac{V_i}{1 + g_m R_S}$$

$$V_{gs} = \frac{V_i}{1 + (3.51)(0.4)} = (0.416)V_i$$

$$I_0 = -(1.76)(0.416)V_i$$

$$\Rightarrow \underline{A_{gf} = \frac{I_0}{V_i} = -0.732 \text{ mA/V}}$$

$$\text{b. For } K_n = 1 \text{ mA/V}^2$$

From dc analysis:

$$4 - V_{GS} = (1)(0.4)(V_{GS} - 2)^2$$

$$4 - V_{GS} = 0.4(V_{GS}^2 - 4V_{GS} + 4)$$

$$0.4V_{GS}^2 - 0.6V_{GS} - 2.4 = 0$$

$$V_{GS} = \frac{0.6 \pm \sqrt{(0.6)^2 + 4(0.4)(2.4)}}{2(0.4)}$$

$$V_{GS} = 3.31 \text{ V}$$

$$g_m = 2(1)(3.31 - 2) = 2.62 \text{ mA/V}$$

$$I_0 = -\left(\frac{2}{2+2} \right) (2.62) V_{gs} = -1.31 V_{gs}$$

$$V_{gs} = \frac{V_i}{1 + (2.62)(0.4)} = V_i(0.488)$$

$$I_0 = -(1.31)(0.488)V_i$$

$$A_{gf} = \frac{I_0}{V_i} = -0.639 \text{ mA/V}$$

$$\% \text{ change} = \frac{0.732 - 0.639}{0.732} \Rightarrow \underline{12.7\%}$$

E12.22

dc analysis:

$$\frac{10 - V_0}{4.7} = I_D + \frac{V_0}{47 + 20} \quad (1)$$

$$I_D = K_n(V_{GS} - V_{TN})^2 \quad (2)$$

$$V_{GS} = \left(\frac{20}{20 + 47} \right) V_0 = 0.2985 V_0 \quad (3)$$

$$2.13 - V_0(0.213) = I_D + (0.0149)V_0 \quad (1)$$

$$I_D = 2.13 - V_0(0.2279)$$

From (2):

$$2.13 - V_0(0.2279) = 1[(0.2985V_0) - 1.5]^2$$

$$2.13 - V_0(0.2279) = 0.0891V_0^2 - 0.8955V_0 + 2.25$$

$$0.0891V_0^2 - 0.6676V_0 + 0.12 = 0$$

$$V_0 = \frac{0.6676 \pm \sqrt{(0.6676)^2 - 4(0.0891)(0.12)}}{2(0.0891)}$$

$$V_0 = 7.31 \text{ V}$$

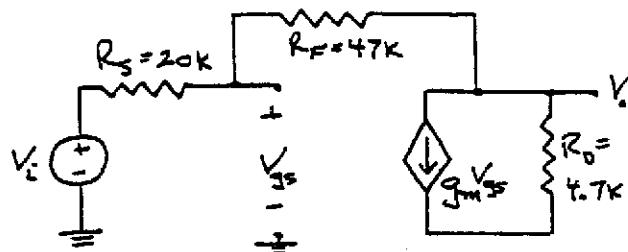
$$I_D = \frac{10 - 7.31}{4.7} - \frac{7.31}{67} = 0.572 - 0.109$$

$$I_D = 0.463 \text{ mA}$$

$$V_{GS} = \sqrt{\frac{0.463}{1}} + 1.5 = 2.18$$

$$a. \quad g_m = 2K_n(V_{GS} - V_{TN}) = 2(1)(2.18 - 1.5)$$

$$\Rightarrow g_m = 1.36 \text{ mA/V}$$



$$\frac{V_0}{R_D} + g_m V_{GS} + \frac{V_0 - V_{GS}}{R_F} = 0 \quad (1)$$

$$\frac{V_{GS} - V_i}{R_S} + \frac{V_{GS} - V_0}{R_F} = 0 \quad (2)$$

$$V_{GS} \left(\frac{1}{R_S} + \frac{1}{R_F} \right) = \frac{V_0}{R_F} + \frac{V_i}{R_S}$$

$$V_{GS} \left(\frac{1}{20} + \frac{1}{47} \right) = \frac{V_0}{47} + \frac{V_i}{20}$$

$$V_{GS}(0.0713) = V_0(0.0213) + V_i(0.050)$$

$$V_{GS} = V_0(0.299) + V_i(0.701)$$

From (1):

$$\frac{V_0}{4.7} + (1.36)V_{GS} + \frac{V_0}{47} - \frac{V_{GS}}{47} = 0$$

$$V_0(0.213) + (1.36)V_{GS}$$

$$+ V_0(0.0213) - V_{GS}(0.0213) = 0$$

$$V_0(0.234) + V_{GS}(1.34) = 0$$

$$V_0(0.234) + (1.34)[(V_0)(0.299) + (V_i)(0.701)] = 0$$

$$V_0(0.635) + V_i(0.939) = 0$$

$$\Rightarrow A_{v,f} = \frac{V_0}{V_i} = -1.48$$

b. For $K_n = 1.5 \text{ mA/V}^2$

From dc analysis:

$$2.13 - V_0(0.2279) = 1.5[(0.2985V_0) - 1.5]^2$$

$$= 1.5[0.0891V_0^2 - 0.8955V_0 + 2.25]$$

$$= 0.1337V_0^2 - 1.343V_0 + 3.375$$

$$0.1337V_0^2 - 1.115V_0 + 1.245 = 0$$

$$V_0 = \frac{1.115 \pm \sqrt{(1.115)^2 - 4(0.1337)(1.245)}}{2(0.1337)}$$

$$V_0 = \frac{1.115 \pm 0.7597}{2(0.1337)} \Rightarrow V_0 = 7.01 \text{ V}$$

$$I_D = \frac{10 - 7.01}{4.7} - \frac{7.01}{67} = 0.636 - 0.105$$

$$I_D = 0.531 \text{ mA}$$

$$V_{GS} = \sqrt{\frac{0.531}{1.5}} + 1.5 = 2.09$$

$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(1.5)(2.09 - 1.5)$$

$$= 1.77 \text{ mA/V}$$

From ac analysis:

$$\frac{V_0}{4.7} + (1.77)V_{GS} + \frac{V_0}{47} - \frac{V_{GS}}{47} = 0$$

$$V_0(0.213) + (1.77)V_{GS}$$

$$+ V_0(0.0213) - V_{GS}(0.0213) = 0$$

$$V_0(0.234) + V_{GS}(1.75) = 0$$

$$V_0(0.234) + (1.75)[V_0(0.299) + V_i(0.701)] = 0$$

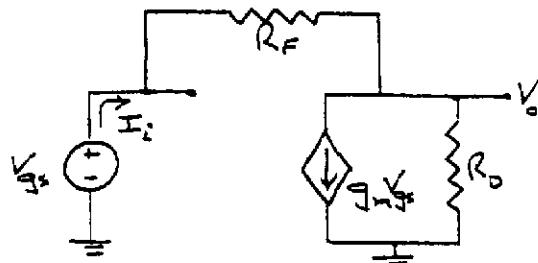
$$V_0(0.757) + V_i(1.23) = 0$$

$$\Rightarrow A_{v,f} = \frac{V_0}{V_i} = -1.62$$

$$\% \text{ change} = \frac{1.62 - 1.48}{1.48} \Rightarrow 9.46\%$$

E12.23

a. Input resistance.



$$\frac{V_0}{R_D} + g_m V_{gs} + \frac{V_0 - V_{gs}}{R_F} = 0 \quad (1)$$

$$I_i = \frac{V_{gs} - V_0}{R_F} \quad (2)$$

$$\text{So } V_0 = V_{gs} - I_i R_F$$

$$V_0 \left(\frac{1}{R_D} + \frac{1}{R_F} \right) + V_{gs} \left(g_m - \frac{1}{R_F} \right) = 0 \quad (1)$$

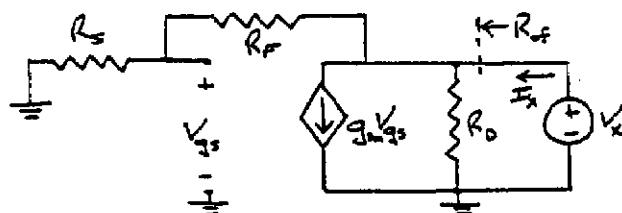
$$[V_{gs} - I_i(47)] \left(\frac{1}{4.7} + \frac{1}{47} \right) + V_{gs} \left(1.36 - \frac{1}{47} \right) = 0$$

$$[V_{gs} - I_i(47)](0.234) + V_{gs}(1.34) = 0$$

$$V_{gs}(1.57) = I_i(11.0)$$

$$\Rightarrow R_{if} = \frac{V_{gs}}{I_i} = 7.0 \text{ k}\Omega$$

Output Resistance.



$$I_X = \frac{V_X}{R_D} + g_m V_{gs} + \frac{V_X}{R_S + R_F}$$

$$V_{gs} = \left(\frac{R_S}{R_S + R_F} \right) V_X = \left(\frac{20}{20 + 47} \right) V_X = 0.2985 V_X$$

$$I_X = \frac{V_X}{4.7} + (1.36)(0.2985) V_X + \frac{V_X}{20 + 47}$$

$$I_X = V_X [0.213 + 0.406 + 0.0149]$$

$$R_{of} = \frac{V_X}{I_X} = 1.58 \text{ k}\Omega$$

b. From part (a)

$$[V_{gs} - I_i(47)](0.234) + V_{gs} \left(1.77 - \frac{1}{47} \right) = 0$$

$$V_{gs}(1.98) = I_i(11)$$

$$\Rightarrow R_{if} = \frac{V_{gs}}{I_i} = 5.56 \text{ k}\Omega$$

$$I_X = \frac{V_X}{4.7} + (1.77)(0.2985) V_X + \frac{V_X}{20 + 47}$$

$$I_X = V_X [0.213 + 0.528 + 0.0149]$$

$$\Rightarrow R_{of} = \frac{V_X}{I_X} = 1.32 \text{ k}\Omega$$

E12.25

$$V_{TH} = \left(\frac{5.5}{5.5 + 51} \right) (10) = 0.973 \text{ V}$$

$$R_{TH} = 5.5 \parallel 51 = 4.96 \text{ k}\Omega$$

$$I_{BQ} = \frac{0.973 - 0.7}{4.96 + (121)(1)} = 0.00217 \text{ mA}$$

$$I_{CQ} = 0.260 \text{ mA}$$

$$r_\pi = 12 \text{ k}\Omega, g_m = 10 \text{ mA/V}$$

$$R_{eq} = R_S \parallel R_1 \parallel R_2 \parallel r_\pi = 10,000 \parallel 51 \parallel 5.5 \parallel 12 \\ = 3.51 \text{ k}\Omega$$

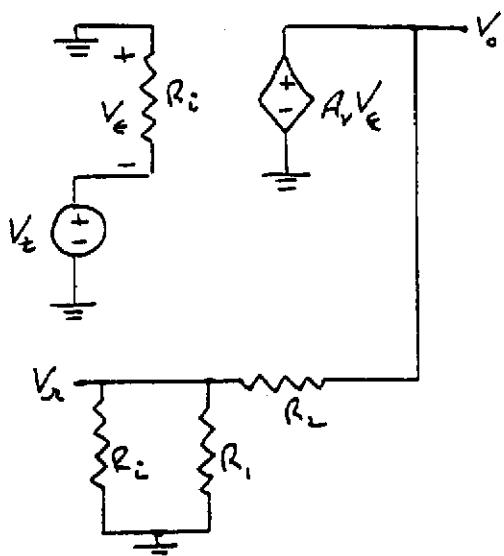
From Equation (12.99(b)):

$$T = (g_m R_C) \left(\frac{R_{eq}}{R_C + R_F + R_{eq}} \right)$$

$$= (10)(10) \left(\frac{3.51}{10 + 82 + 3.51} \right)$$

$$\Rightarrow T = 3.68$$

E12.26



$$V_t = -V_c, \quad V_0 = A_v V_c = -A_v V_t$$

$$V_r = \left(\frac{R_1 \| R_i}{R_1 \| R_i + R_2} \right) V_0 = - \left(\frac{R_1 \| R_i}{R_2 \| R_i + R_2} \right) (A_v V_t)$$

$$T = -\frac{V_r}{V_t} = A_v \left(\frac{R_1 \| R_i}{R_1 \| R_i + R_2} \right)$$

or

$$T = \frac{A_v}{1 + \frac{R_2}{R_1 \| R_i}}$$

E12.27

$$T = A_v \beta = \frac{A_v \beta}{\left(1 + j \cdot \frac{f}{f_1} \right)} = \frac{(10^5)(0.01)}{1 + j \cdot \left(\frac{f}{10} \right)}$$

$$|T(f_1)| = 1 = \frac{10^3}{\sqrt{1 + \left(\frac{f_E}{10} \right)^2}}$$

$$1 + \left(\frac{f_E}{10} \right)^2 = 10^6$$

$$f_E = 10\sqrt{10^6 - 1} \Rightarrow f_E \approx 10^4 \text{ Hz}$$

$$\text{Phase} = \phi = -\tan^{-1} \left(\frac{f_E}{10} \right) = -\tan^{-1} \left(\frac{10^4}{10} \right) = -\tan^{-1} (10^3)$$

$$\phi \approx 90^\circ$$

$$\text{Phase Margin} = 180 - 90 \Rightarrow \text{Phase Margin} = 90^\circ$$

E12.28

$$T = A_v \beta = \frac{A_v \beta}{\left(1 + j \cdot \frac{f}{f_1} \right) \left(1 + j \cdot \frac{f}{f_2} \right)}$$

$$\text{Phase} = - \left[\tan^{-1} \left(\frac{f}{f_1} \right) + \tan^{-1} \left(\frac{f}{f_2} \right) \right]$$

$$\text{Phase Margin} = 60^\circ \Rightarrow \text{Phase} = -120^\circ$$

$$-120^\circ = - \left[\tan^{-1} \left(\frac{f}{10^4} \right) + \tan^{-1} \left(\frac{f}{10^5} \right) \right]$$

$$\text{At } f' = 7.66 \times 10^4 \text{ Hz,}$$

$$\begin{aligned} \text{Phase} &= -[\tan^{-1}(7.66) + \tan^{-1}(0.766)] \\ &= -[82.56 + 37.45] \\ &= -120^\circ \end{aligned}$$

$$\begin{aligned} |T(f')| &= 1 = \frac{(10^5)\beta}{\sqrt{1 + (7.66)^2} \times \sqrt{1 + (0.766)^2}} \\ 1 &= \frac{(10^5)\beta}{(7.725)(1.26)} \Rightarrow \underline{\beta = 9.73 \times 10^{-5}} \end{aligned}$$

E12.29

$$\text{Phase} = -180^\circ = -3 \tan^{-1} \left(\frac{f'}{10^5} \right)$$

$$\text{or } \tan^{-1} \left(\frac{f'}{10^5} \right) = 60^\circ \Rightarrow f' = 1.732 \times 10^5 \text{ Hz}$$

$$\begin{aligned} |T(f')| &= 1 = \frac{\beta(100)}{\left[\sqrt{1 + \left(\frac{f'}{10^5} \right)^2} \right]^3} \\ &= \frac{\beta(100)}{\left[\sqrt{1 + (1.732)^2} \right]^3} \\ &\Rightarrow \underline{\beta = 0.08} \end{aligned}$$

E12.30

$$\text{Phase Margin} = 60^\circ \Rightarrow \text{Phase} = -120^\circ$$

$$\text{Phase} = -120^\circ = -3 \tan^{-1} \left(\frac{f'}{10^5} \right)$$

$$\tan^{-1} \left(\frac{f'}{10^5} \right) = 40^\circ \Rightarrow f' = 0.839 \times 10^5 \text{ Hz}$$

$$\begin{aligned} |T(f')| &= 1 = \frac{\beta(100)}{\left[\sqrt{1 + \left(\frac{f'}{10^5} \right)^2} \right]^3} \\ &= \frac{\beta(100)}{\left[\sqrt{1 + (0.839)^2} \right]^3} \\ &\Rightarrow \underline{\beta = 0.0222} \end{aligned}$$

E12.31

The new loop gain function is

$$T'(f) = \frac{10^5}{\left(1 + j \cdot \frac{f}{f_{PD}}\right)\left(1 + j \cdot \frac{f}{5 \times 10^5}\right)} \\ \times \frac{1}{\left(1 + j \cdot \frac{f}{10^7}\right)\left(1 + j \cdot \frac{f}{5 \times 10^8}\right)}$$

$$\text{Phase} = -\left\{\tan^{-1}\left(\frac{f}{f_{PD}}\right) + \tan^{-1}\left(\frac{f}{5 \times 10^5}\right) + \tan^{-1}\left(\frac{f}{10^7}\right) + \tan^{-1}\left(\frac{f}{5 \times 10^8}\right)\right\}$$

For a phase margin $45^\circ \Rightarrow \text{Phase} = -135^\circ$, the poles are far apart so this will occur at approximately $f' = 5 \times 10^5$ Hz. Then

$$|T'(f)| = 1 = \frac{10^5}{\sqrt{1 + \left(\frac{5 \times 10^5}{f_{PD}}\right)^2} \times \sqrt{1+1} \times \sqrt{1} \times \sqrt{1}}$$

$$1 = \frac{10^5}{(1.414)\sqrt{1 + \left(\frac{5 \times 10^5}{f_{PD}}\right)^2}}$$

$$1 + \left(\frac{5 \times 10^5}{f_{PD}}\right)^2 = 5 \times 10^9$$

$$f_{PD} \approx \frac{5 \times 10^5}{\sqrt{5 \times 10^9}} \Rightarrow f_{PD} = 7.07 \text{ Hz}$$

E12.32

Phase Margin = $45^\circ \Rightarrow \text{Phase} = -135^\circ$

This will occur at approximately $f' = 10^7$ Hz

$$|T'(f)| = 1 = \frac{10^5}{\sqrt{1 + \left(\frac{10^7}{f_{PD}}\right)^2} \times \sqrt{2} \times \sqrt{1}} \\ 1 + \left(\frac{10^7}{f_{PD}}\right)^2 = 5 \times 10^9 \\ f_{PD} \approx \frac{10^7}{\sqrt{5 \times 10^9}} \Rightarrow f_{PD} = 141 \text{ Hz}$$

E12.33

$$A_f(0) = \frac{A_0}{1 + \beta A_0} = \frac{2 \times 10^5}{1 + (0.05)(2 \times 10^5)} \\ \Rightarrow A_f(0) \approx 20$$

$$f_C = f_{PD}(1 + \beta A_0) = 100[1 + (0.05)(2 \times 10^5)] \\ \Rightarrow f_C \approx 1 \text{ MHz}$$

Chapter 12

Problem Solutions

12.1

a. $A_f = \frac{A}{1 + A\beta}$

$$80 = \frac{10^5}{1 + (10^5)\beta} \Rightarrow 1 + (10^5)\beta = \frac{10^5}{80}$$

$$\Rightarrow \beta = \frac{\frac{10^5}{80} - 1}{10^5} \Rightarrow \beta = 0.01249$$

b. $\frac{dA_f}{A_f} = \left(\frac{A_f}{A} \right) \cdot \frac{dA}{A} = \frac{80}{10^5}(-20)$

or

$$\frac{dA_f}{A_f} = -0.016\%$$

$$A_f = 80 - (0.00016)80 \Rightarrow A_f \approx 79.99$$

c. $80 = \frac{10^3}{1 + (10^3)\beta}$

$$\beta = \frac{\frac{10^3}{80} - 1}{10^3} \Rightarrow \beta = 0.0115$$

$$\frac{dA_f}{A_f} = \left(\frac{80}{10^3} \right)(-20) \Rightarrow \frac{dA_f}{A_f} = -1.6\%$$

$$A_f = 80 - (0.016)(80) \Rightarrow A_f = 78.72$$

12.2

a. $A_f = \frac{(A)^3}{1 + (A)^3\beta}$

$$100 = \frac{1000}{1 + (1000)\beta} \Rightarrow \beta = \frac{\frac{1000}{100} - 1}{1000} \Rightarrow \beta = 0.009$$

b. A goes from 10 to 11 so

$$A_f = \frac{(11)^3}{1 + (11)^3(0.009)} = \frac{1331}{1 + (1331)(0.009)}$$

or

$$A_f = 102.55$$

so

$$\frac{\Delta A_f}{A_f} = \frac{2.55}{100} \Rightarrow 2.55\% \text{ change}$$

12.3

(a) $V_o = (-10)(-15)(-20)V_s = -3000V_s$

$$V_s = \beta V_o + V_s$$

$$\text{So } V_o = -3000(\beta V_o + V_s)$$

We find

$$A_f = \frac{V_o}{V_s} = \frac{-3000}{1 + 3000\beta}$$

For $A_f = -120 = \frac{-3000}{1 + 3000\beta} \Rightarrow \beta = 0.008$

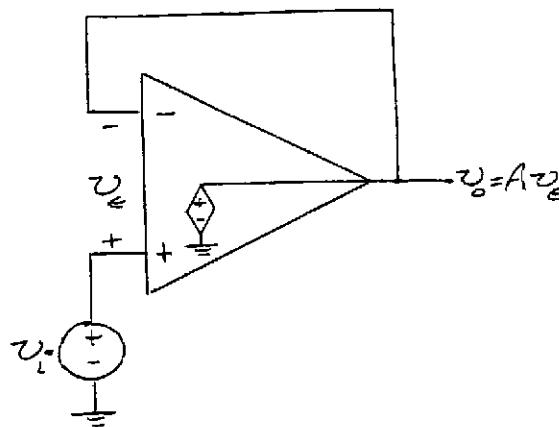
(b) Now $V_o = (-9)(-13.5)(-18)V_s = -2187V_s$

Then

$$A_f = \frac{-2187}{1 + 2187\beta} = \frac{-2187}{1 + 2187(0.008)} = -118.24$$

$$\% \text{ change} = \frac{120 - 118.24}{120} \times 100 \Rightarrow 1.47\% \text{ change}$$

12.4



$$v_o = v_i - v_s \Rightarrow v_o = v_i - v_o$$

$$\text{Then } v_o = A(v_i - v_o) = Av_i - Av_o$$

And

$$v_o(1 + A) = Av_i$$

so

$$\frac{v_o}{v_i} = \frac{A}{1 + A} = 0.9998 \Rightarrow A = 4999$$

12.5

$$(10^5)(4) = (50)f_B \Rightarrow f_B = 8 \text{ kHz}$$

12.6

(a) $(50)f_{1-\text{dB}} = (10^5)(4) \Rightarrow f_{1-\text{dB}} = 8 \text{ kHz}$

(b) $(10)f_{1-\text{dB}} = (10^5)(4) \Rightarrow f_{1-\text{dB}} = 40 \text{ kHz}$

12.7

$$(50)(20 \times 10^3) = 5A_0 \text{ so } A_0 = 2 \times 10^5$$

12.8

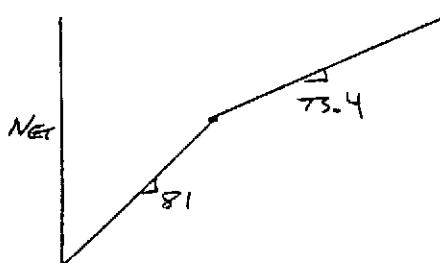
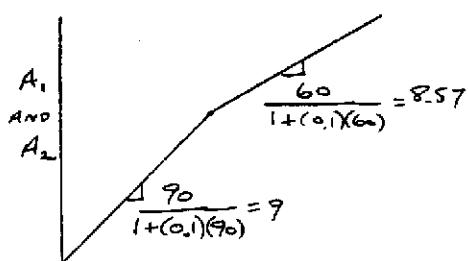
$$v_0 = A_1 A_2 v_i + A_1 v_n$$

$$v_0 = (100)v_i + (1)v_n = (100)(10) + (1)(1)$$

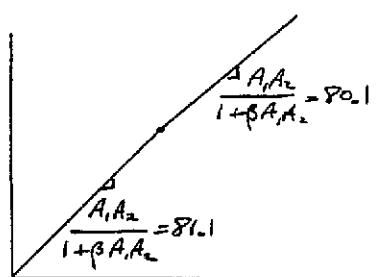
$$\Rightarrow \frac{S_0}{N_0} = \frac{1000}{1} = 1000$$

12.9

(a)



(b)



Circuit (b) - less distortion

12.10

(a) Low input $R \Rightarrow$ Shunt input
 Low output $R \Rightarrow$ Shunt output
 Or a Shunt-Shunt circuit

(b) High input $R \Rightarrow$ Series input
 High output $R \Rightarrow$ Series output
 Or a Series-Series circuit

(c) Shunt-Series circuit

(d) Series-Shunt circuit

12.11

(a) $R_i(\max) = R_i(1+T) = 10(1+10^4) \Rightarrow$
 $R_i(\max) \approx 10^5 \text{ k}\Omega$

$$R_i(\min) = \frac{R_i}{1+T} = \frac{10}{1+10^4} \approx 10^{-3} \text{ k}\Omega$$

Or $R_i(\min) = 1\Omega$

(b) $R_o(\max) = R_o(1+T) = 1(1+10^4) \Rightarrow$
 $R_o(\max) \approx 10^4 \text{ k}\Omega$

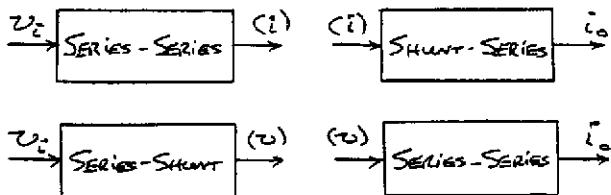
$$R_o(\min) = \frac{R_o}{1+T} = \frac{1}{1+10^4} \approx 10^{-4} \text{ k}\Omega$$

Or $R_o(\min) = 0.1\Omega$

12.12

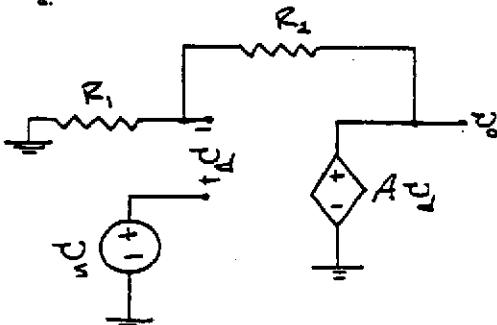
Overall Transconductance Amplifier, $A_g = \frac{i_o}{v_i}$

Series output = current signal and Shunt input = current signal. Also, Shunt output = voltage signal and Series input = voltage signal. Two possible solutions are shown.



12.13

a.



$$\begin{aligned} \frac{v_s - v_d}{R_1} &= \frac{v_0 - (v_s - v_d)}{R_2} \text{ and } v_d = \frac{v_0}{A} \\ \frac{v_s}{R_1} + \frac{v_s}{R_2} &= \frac{v_0}{R_2} + v_d \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \\ &= \frac{v_0}{R_2} + \frac{v_0}{A} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \end{aligned}$$

$$\nu_s \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{\nu_0}{R_2} \left[1 + \frac{1}{A} \left(1 + \frac{R_2}{R_1} \right) \right]$$

$$\frac{\nu_0}{\nu_s} = \frac{\left(1 + \frac{R_2}{R_1} \right)}{1 + \frac{1}{A} \left(1 + \frac{R_2}{R_1} \right)}$$

which can be written as

$$A_{vf} = \frac{\nu_0}{\nu_s} = \frac{A}{1 + \left[A / \left(1 + \frac{R_2}{R_1} \right) \right]}$$

b. $\beta = \frac{1}{1 + \frac{R_2}{R_1}}$

c. $20 = \frac{10^5}{1 + (10^5)\beta}$

So $\beta = \frac{\frac{10^5}{20} - 1}{10^5} \Rightarrow \underline{\beta = 0.04999}$

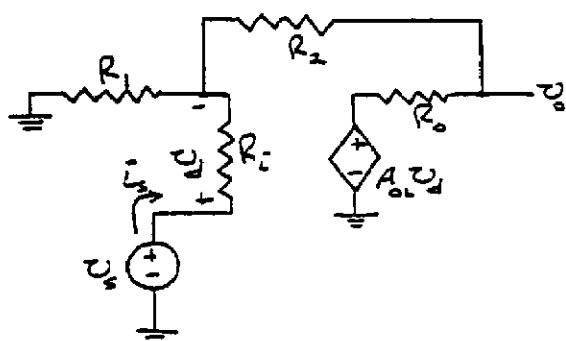
Then $\frac{R_2}{R_1} = \frac{1}{\beta} - 1 = \frac{1}{0.04999} - 1$
 $\Rightarrow \underline{\frac{R_2}{R_1} = 19.004}$

d. $A \rightarrow 8 \times 10^4$

$$A_f = \frac{8 \times 10^4}{1 + (8 \times 10^4)(0.04999)} = 19.999$$

$$\frac{\Delta A_f}{A_f} = -\frac{0.001}{20} \Rightarrow \underline{\frac{\Delta A_f}{A_f} = -0.005\%}$$

12.14



$$A_{vf} \approx \left(1 + \frac{R_2}{R_1} \right) = 20 \Rightarrow \frac{R_2}{R_1} = 19$$

$$v_d = i_s R_i$$

$$i_s = \frac{\nu_s - v_d}{R_1} + \frac{(\nu_s - v_d) - \nu_0}{R_2} \quad (1)$$

$$\frac{\nu_0 - A_{OL}v_d}{R_0} + \frac{\nu_0 - (\nu_s - v_d)}{R_2} = 0 \quad (2)$$

$$\begin{aligned} \nu_0 \left(\frac{1}{R_0} + \frac{1}{R_2} \right) &= \frac{A_{OL}v_d}{R_0} + \frac{(\nu_s - v_d)}{R_2} \\ \nu_0 &= \frac{\frac{A_{OL}v_d}{R_0} + \frac{(\nu_s - v_d)}{R_2}}{\left(\frac{1}{R_0} + \frac{1}{R_2} \right)} \end{aligned}$$

From (1):

$$i_s = \frac{\nu_s - v_d}{R_1} + \frac{\nu_s - v_d}{R_2} - \frac{\frac{1}{R_2} \cdot \left[\frac{A_{OL}v_d}{R_0} + \frac{(\nu_s - v_d)}{R_2} \right]}{\left(\frac{1}{R_0} + \frac{1}{R_2} \right)}$$

$$\begin{aligned} i_s &= \nu_s \left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{\frac{1}{R_2}}{1 + \frac{R_2}{R_0}} \right) \\ &\quad - v_d \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{\frac{A_{OL}}{R_0} - \frac{1}{R_2}}{1 + \frac{R_2}{R_0}} \right) \end{aligned}$$

$$v_d = i_s R_i$$

$$\begin{aligned} i_s &= \nu_s \left\{ 1 + \frac{\frac{R_i}{\left(\frac{1}{R_1} + \frac{1}{R_2} \right) \left(1 + \frac{R_2}{R_0} \right) + \frac{A_{OL}}{R_0} - \frac{1}{R_2}}}{1 + \frac{R_2}{R_0}} \right\} \\ &= \nu_s \left[\frac{\left(\frac{1}{R_1} + \frac{1}{R_2} \right) \left(1 + \frac{R_2}{R_0} \right) - \frac{1}{R_2}}{1 + \frac{R_2}{R_0}} \right] \end{aligned}$$

$$\begin{aligned} i_s &= \nu_s \left\{ 1 + \frac{R_2}{R_0} + R_i \left[\frac{1}{R_1} + \frac{R_2}{R_1} \cdot \frac{1}{R_0} + \frac{1}{R_0} + \frac{A_{OL}}{R_0} \right] \right\} \\ &= \nu_s \left[\frac{1}{R_1} + \frac{R_2}{R_1} \cdot \frac{1}{R_0} + \frac{1}{R_0} \right] \end{aligned}$$

$$\begin{aligned} i_s &= \nu_s \left\{ R_0 + R_2 + R_i \left[\frac{R_0}{R_1} + \left(1 + \frac{R_2}{R_1} \right) + A_{OL} \right] \right\} \\ &= \nu_s \left[\frac{R_0}{R_1} + \left(1 + \frac{R_2}{R_1} \right) \right] \end{aligned} \quad (1)$$

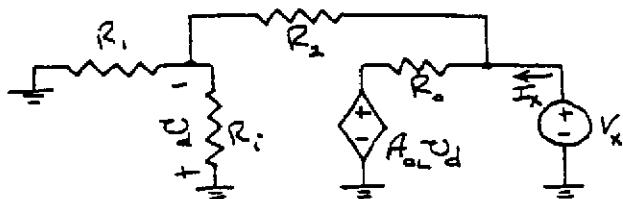
$$\text{Let } R_2 = 190 \text{ k}\Omega, R_i = 10 \text{ k}\Omega$$

$$\begin{aligned} i_s &= \nu_s \left\{ 0.1 + 190 + 100 \cdot \left[\frac{0.1}{10} + 20 + 10^5 \right] \right\} \\ &= \nu_s \left[\frac{0.1}{10} + 20 \right] \end{aligned}$$

$$i_s (1.000219 \times 10^7) = \nu_s (20.01)$$

$$R_{if} = \frac{\nu_s}{i_s} \approx 5 \times 10^5 \text{ k}\Omega \Rightarrow \underline{R_{if} \approx 500 \text{ M}\Omega}$$

Output Resistance



$$I_x = \frac{V_x - A_{oi} I_x}{R_3} + \frac{V_x}{R_2 + R_1 \| R_i}$$

$$I_x = \frac{-R_1 \| R_i}{R_1 \| R_i + R_2} \cdot V_x$$

$$\frac{I_x}{V_x} = \frac{1}{R_{o_f}} = \frac{1}{R_3} + \frac{A_{oi} \cdot R_1 \| R_i}{R_0(R_1 \| R_i + R_2)} + \frac{1}{R_2 + R_1 \| R_i}$$

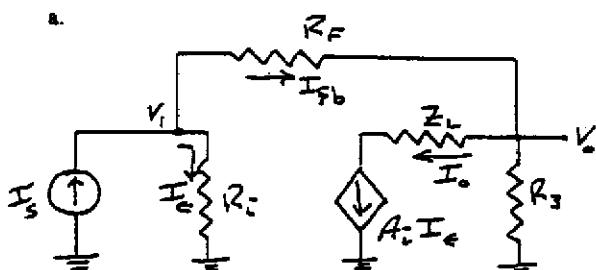
$$R_1 \| R_i = 10 \| 100 = 9.09$$

$$\frac{1}{R_{o_f}} = \frac{1}{0.1} + \frac{10^5}{0.1} \cdot \left(\frac{9.09}{9.09 + 190} \right) + \frac{1}{190 + 9.09}$$

$$= 10 + 4.566 \times 10^4 + 0.00502$$

$$R_{o_f} = 2.19 \times 10^{-5} \text{ k}\Omega \Rightarrow R_{o_f} = 0.0219 \Omega$$

12.15



Assume that V_1 is at virtual ground.

$$V_0 = -I_{fb} R_F$$

Now

$$I_{fb} = I_0 + \frac{V_0}{R_3} = I_0 - \frac{I_{fb} R_F}{R_3}$$

$$I_{fb} = I_S - I_e$$

and

$$I_0 = A_i I_e \Rightarrow I_e = \frac{I_0}{A_i}$$

so

$$I_{fb} = I_S - \frac{I_0}{A_i}$$

From above

$$I_{fb} \left(1 + \frac{R_F}{R_3} \right) = I_0$$

$$\left(I_S - \frac{I_0}{A_i} \right) \left(1 + \frac{R_F}{R_3} \right) = I_0$$

$$I_S \left(1 + \frac{R_F}{R_3} \right) = I_0 \left[1 + \frac{1}{A_i} \left(1 + \frac{R_F}{R_3} \right) \right]$$

or

$$A_{if} = \frac{I_0}{I_S} = \frac{\left(1 + \frac{R_F}{R_3} \right)}{\left[1 + \frac{1}{A_i} \left(1 + \frac{R_F}{R_3} \right) \right]}$$

$$= \frac{A_i}{1 + \frac{A_i}{\left(1 + \frac{R_F}{R_3} \right)}} = A_{if}$$

$$b. \quad \beta_i = \frac{1}{\left(1 + \frac{R_F}{R_3} \right)}$$

$$c. \quad 25 = \frac{10^5}{1 + (10^5) \beta_i}$$

$$\text{so } \beta_i = \frac{25}{10^5} - 1 \Rightarrow \beta_i = 0.03999$$

$$\text{so } \frac{R_F}{R_3} = \frac{1}{\beta_i} - 1 = \frac{1}{0.03999} - 1 \Rightarrow \frac{R_F}{R_3} = 24.0$$

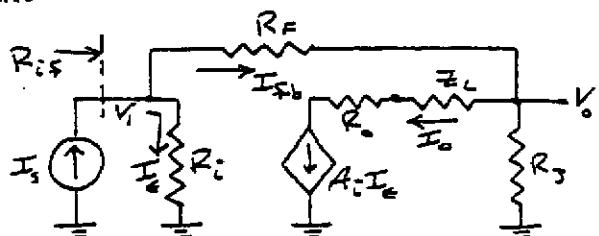
$$d. \quad A_i = 10^5 - (0.15)(10^4) = 8.5 \times 10^4$$

$$\text{so } A_{if} = \frac{8.5 \times 10^4}{1 + (8.5 \times 10^4)(0.03999)} = 24.9989$$

$$\text{so } \frac{\Delta A_{if}}{A_{if}} = -\frac{1.10 \times 10^{-3}}{25} = -4.41 \times 10^{-5}$$

$$\Rightarrow -4.41 \times 10^{-3} \%$$

12.16



$$I_S = I_e + I_{fb}, \quad V_1 = I_e R_i$$

$$I_{fb} = I_0 + \frac{V_0}{R_3} \text{ and } V_0 = V_1 - I_{fb} R_F$$

$$I_0 = A_i I_e \Rightarrow I_e = \frac{I_0}{A_i}$$

Now

$$I_{fb} = A_i I_e + \frac{1}{R_3} (V_t - I_{fb} R_F)$$

$$I_{fb} \left[1 + \frac{R_F}{R_3} \right] = A_i I_e + \frac{V_t}{R_3}$$

$$I_{fb} = I_S - I_e$$

$$(I_S - I_e) \left[1 + \frac{R_F}{R_3} \right] = A_i I_e + \frac{V_t}{R_3}$$

$$I_S \left[1 + \frac{R_F}{R_3} \right] = I_e \left[\left(1 + \frac{R_F}{R_3} \right) + A_i \right] + \frac{V_t}{R_3}$$

$$I_e = \frac{V_t}{R_i}$$

$$I_S \left[1 + \frac{R_F}{R_3} \right] = V_t \left\{ \frac{1}{R_i} \cdot \left[\left(1 + \frac{R_F}{R_3} \right) + A_i \right] + \frac{1}{R_3} \right\}$$

$$R_{if} = \frac{V_t}{I_S} = \frac{\left(1 + \frac{R_F}{R_3} \right)}{\left\{ \frac{1}{R_i} \cdot \left[\left(1 + \frac{R_F}{R_3} \right) + A_i \right] + \frac{1}{R_3} \right\}}$$

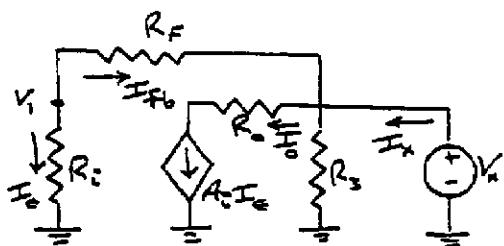
The $1/R_3$ term in the denominator will be negligible.

Using the results of Problem 12.15:

$$R_{if} = \frac{25}{\left\{ \frac{1}{2} [(25) + 10^5] \right\}}$$

$$R_{if} = 5 \times 10^{-4} \text{ k}\Omega \Rightarrow R_{if} = 0.5 \Omega$$

Output Resistance (Let $Z_L = 0$)



$$I_X = \frac{V_X}{R_3} + A_i I_e + \frac{V_X}{R_F + R_i}$$

$$I_e = \frac{V_X}{R_F + R_i}$$

so

$$\frac{I_X}{V_X} = \frac{1}{R_{of}} = \frac{1}{R_3} + \frac{A_i + 1}{R_F + R_i}, \quad \frac{R_F}{R_3} = 24$$

Let $R_F = 240 \text{ k}\Omega$, $R_3 = 10 \text{ k}\Omega$

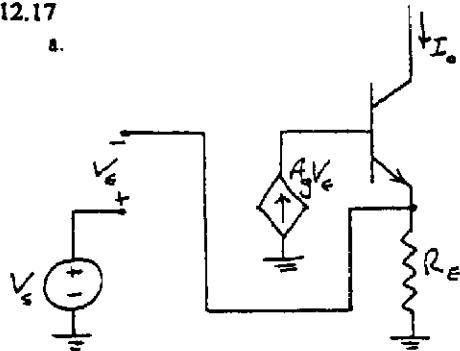
$$\frac{1}{R_{of}} = \frac{1}{10} + \frac{10^5 + 1}{240 + 2}$$

$$\text{so } R_{of} \approx \frac{R_F + R_i}{A_i + 1} = \frac{240 + 2}{10^5 + 1}$$

$$\Rightarrow R_{of} \approx 2.42 \times 10^{-3} \text{ k}\Omega \text{ or } R_{of} \approx 2.42 \Omega$$

12.17

a.



$$I_E = \frac{(1 + h_{FE})}{h_{FE}} \cdot I_0 = \frac{V_S - V_t}{R_E}$$

$$\text{Also } I_0 = h_{FE} (A_g V_t) \text{ so } V_t = \frac{I_0}{h_{FE} A_g}$$

Then

$$\frac{1 + h_{FE}}{h_{FE}} \cdot I_0 = \frac{V_S}{R_E} - \frac{I_0}{h_{FE} A_g R_E}$$

$$\left[\frac{1 + h_{FE}}{h_{FE}} + \frac{1}{h_{FE} A_g R_E} \right] I_0 = \frac{V_S}{R_E}$$

$$\left[\frac{A_g (1 + h_{FE}) R_E + 1}{h_{FE} A_g R_E} \right] I_0 = \frac{V_S}{R_E}$$

$$\frac{I_0}{V_S} = \frac{1}{R_E} \cdot \left[\frac{h_{FE} A_g R_E}{1 + A_g (1 + h_{FE}) R_E} \right]$$

$$\Rightarrow \frac{I_0}{V_S} \approx \frac{h_{FE} A_g}{1 + (h_{FE} A_g) R_E}$$

b. $\beta_x = R_E$

c. $10 = \frac{5 \times 10^4}{1 + (5 \times 10^5) \beta_x}$

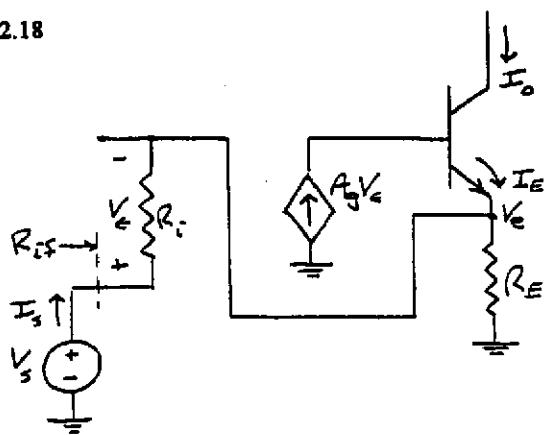
$$\beta_x = \frac{5 \times 10^5}{5 \times 10^4 - 1} \Rightarrow \beta_x = R_E = 0.099998 \text{ k}\Omega$$

d. If $A_g \rightarrow 5.5 \times 10^5$ then

$$A_{gf} = \frac{5.5 \times 10^5}{1 + (5.5 \times 10^5)(0.099998)} = 10.0000182$$

$$\frac{\Delta A_{gf}}{A_{gf}} = \frac{1.82 \times 10^{-5}}{10} \Rightarrow 1.82 \times 10^{-4} \%$$

12.18



$$I_E = (1 + h_{FE})A_g V_e, \quad I_E = \frac{V_e}{R_E} - I_S \text{ and } V_e = I_S R_i,$$

$$V_e = V_s - V_t = V_s - I_S R_i$$

$$\text{Now } (1 + h_{FE})A_g I_S R_i = \frac{1}{R_E} \cdot (V_s - I_S R_i) - I_S$$

$$\left[(1 + h_{FE})A_g R_i + \frac{R_i}{R_E} + 1 \right] I_S = \frac{V_s}{R_E}$$

$$R_{if} = \frac{V_s}{I_S} = R_E \left[(1 + h_{FE})A_g R_i + \frac{R_i}{R_E} + 1 \right]$$

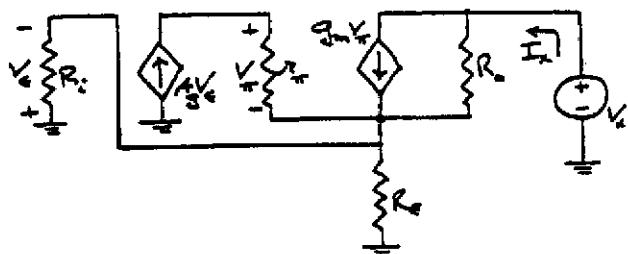
From Problem 12.16:

$$(1 + h_{FE})A_g \approx h_{FE} A_g = 5 \times 10^5 \text{ mS}$$

$$R_E \approx 0.1 \text{ k}\Omega$$

$$\text{so } R_{if} = (0.1) \left[(5 \times 10^5)(20) + \frac{20}{0.1} + 1 \right]$$

$$\text{or } R_{if} = 10^6 \text{ k}\Omega$$



$$\frac{V_e}{r_\pi} = A_g V_e$$

$$I_X = g_m V_\pi + \frac{V_X - (-V_t)}{R_0} \quad (1)$$

$$V_e = -(I_X + A_g V_e)(R_E \parallel R_i) \quad (2)$$

$$\text{or } V_t = [1 + A_g(R_E \parallel R_i)] = -I_X(R_E \parallel R_i)$$

Now

$$I_X = g_m A_g r_\pi V_\pi + \frac{V_X}{R_0} + \frac{V_t}{R_0} \quad (1)$$

$$I_X = \left(g_m A_g r_\pi + \frac{1}{R_0} \right) \left[\frac{-I_X(R_E \parallel R_i)}{1 + A_g(R_E \parallel R_i)} \right] + \frac{V_X}{R_0}$$

$$R_{if} = \frac{V_X}{I_X}$$

$$= R_0 \left\{ 1 + \left(g_m A_g r_\pi + \frac{1}{R_0} \right) \left[\frac{(R_E \parallel R_i)}{1 + A_g(R_E \parallel R_i)} \right] \right\}$$

$$g_m r_\pi A_g = h_{FE} A_g = 5 \times 10^5 \text{ mS}$$

$$\text{Let } h_{FE} = 100 \text{ so } A_g = 5 \times 10^3 \text{ mS}$$

$$R_E \parallel R_i = 0.1 \parallel 20 \approx 0.1 \text{ k}\Omega$$

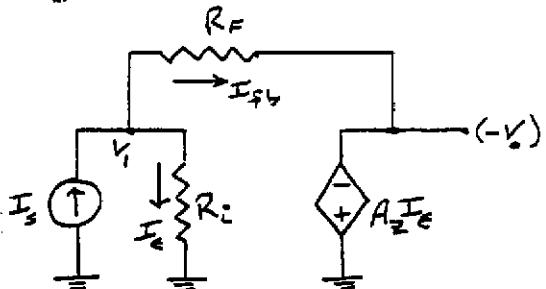
Then

$$R_{if} = 50 \left\{ 1 + \left(5 \times 10^5 + \frac{1}{50} \right) \left[\frac{0.1}{1 + (5 \times 10^3)(0.1)} \right] \right\}$$

$$\text{or } R_{if} = 5.04 \text{ M}\Omega$$

12.19

a.

Assuming V_t is at virtual ground

$$(-V_0) = -I_{fb} R_F \text{ and } (-V_0) = -A_z I_e \Rightarrow I_e = \frac{V_0}{A_z}$$

$$I_{fb} = I_S - I_e$$

$$\text{So } V_0 = (I_S - I_e) R_F = I_S R_F - \left(\frac{V_0}{A_z} \right) R_F$$

$$V_0 \left[1 + \frac{R_F}{A_z} \right] = I_S R_F$$

$$\text{so } A_{if} = \frac{V_0}{I_S} = \frac{R_F}{\left[1 + \frac{R_F}{A_z} \right]} = \frac{A_z R_F}{A_z + R_F}$$

$$\text{or } A_{if} = \frac{A_z}{1 + A_z \left(\frac{1}{R_F} \right)} = \frac{A_z}{1 + A_z \beta_g}$$

$$\text{b. } \beta_g = \frac{1}{R_F}$$

$$c. \quad 5 \times 10^4 = \frac{5 \times 10^6}{1 + (5 \times 10^4)\beta_g}$$

$$\beta_g = \frac{\frac{5 \times 10^6}{5 \times 10^4} - 1}{5 \times 10^4} \Rightarrow \beta_g = 1.98 \times 10^{-5}$$

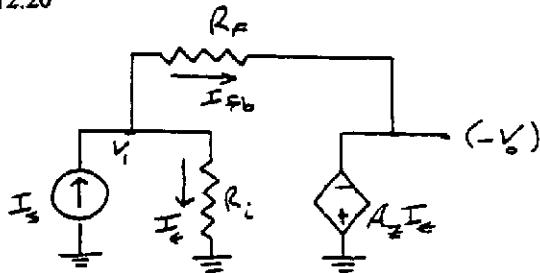
$$R_F = \frac{1}{\beta_g} \Rightarrow R_F = 50.5 \text{ k}\Omega$$

$$d. \quad A_{zf} = (0.9)(5 \times 10^6) = 4.5 \times 10^6$$

$$A_{zf} = \frac{4.5 \times 10^6}{1 + (4.5 \times 10^6)(1.98 \times 10^{-5})} = 4.994 \times 10^4$$

$$\Delta A_{zf} = -\frac{55.4939}{5 \times 10^4} = -1.11 \times 10^{-3} \Rightarrow -0.111\%$$

12.20



$$V_1 = I_s R_i, \quad -V_o = -A_z I_s \Rightarrow V_o = A_z I_s$$

$$I_{fb} = I_s - I_s \text{ and } -V_o = V_1 - I_{fb} R_F$$

$$-A_z I_s = V_1 - (I_s - I_s) R_F$$

$$-A_z \left(\frac{V_1}{R_i} \right) = V_1 - I_s R_F + \left(\frac{V_1}{R_i} \right) R_F$$

$$I_s R_F = V_1 \left[1 + \frac{A_z}{R_i} + \frac{R_F}{R_i} \right]$$

$$R_{sf} = \frac{V_1}{I_s} = \frac{R_F}{\left[1 + \frac{A_z}{R_i} + \frac{R_F}{R_i} \right]}$$

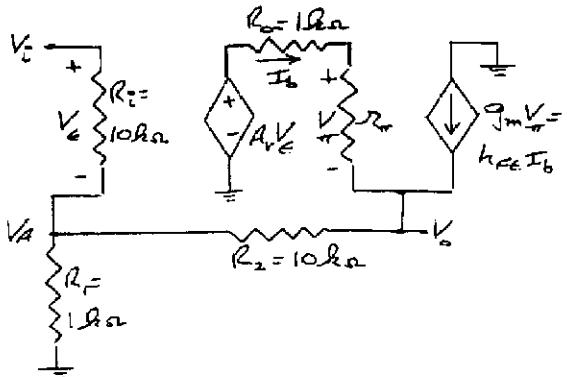
Or, using the results from Problem 12.18.

$$R_{sf} = \frac{50.5 \times 10^3}{\left[1 + \frac{5 \times 10^6}{10 \times 10^3} + \frac{50.5 \times 10^3}{10 \times 10^3} \right]} \\ = \frac{50.5 \times 10^3}{[1 + 500 + 5.05]} \\ \Rightarrow R_{sf} = 99.79 \Omega$$

12.21

Assume $I_{CG} = 0.2 \text{ mA}$

$$\text{Then } r_s = \frac{(100)(0.026)}{0.2} = 13 \text{ k}\Omega$$



$$(1) \frac{V_i - V_A}{R_i} = \frac{V_A}{R_1} + \frac{V_A - V_o}{R_2} \Rightarrow$$

$$\frac{V_i}{R_i} + \frac{V_o}{R_2} = V_A \left(\frac{1}{R_i} + \frac{1}{R_1} + \frac{1}{R_2} \right)$$

Now

$$\frac{V_i}{10} + \frac{V_o}{10} = V_A \left(\frac{1}{10} + \frac{1}{1} + \frac{1}{10} \right) \Rightarrow V_i + V_o = V_A (12)$$

$$\text{or } V_A = \frac{1}{12} (V_i + V_o)$$

$$(2) \left(\frac{A_v V_i - V_o}{R_o + r_s} \right) (1 + h_{FE}) = \frac{V_o - V_A}{R_2}$$

$$\text{where } V_e = V_i - V_A$$

Then

$$\left(\frac{A_v (V_i - V_A) - V_o}{R_o + r_s} \right) (1 + h_{FE}) = \frac{V_o - V_A}{R_2}$$

we find

$$\frac{A_v V_i (1 + h_{FE})}{R_o + r_s} - \frac{V_o (1 + h_{FE})}{R_o + r_s} - \frac{V_o}{R_2} = \frac{A_v V_i (1 + h_{FE})}{R_o + r_s} - \frac{V_A}{R_2}$$

Then

$$\frac{(5 \times 10^3)(101)V_i}{14} - \frac{V_o(101)}{14} - \frac{V_o}{10}$$

$$= \left(\frac{(5 \times 10^3)(101)}{14} - \frac{1}{10} \right) V_A$$

Rearranging terms, we find

$$A_{vf} = \frac{V_o}{V_i} = \frac{10.97}{1}$$

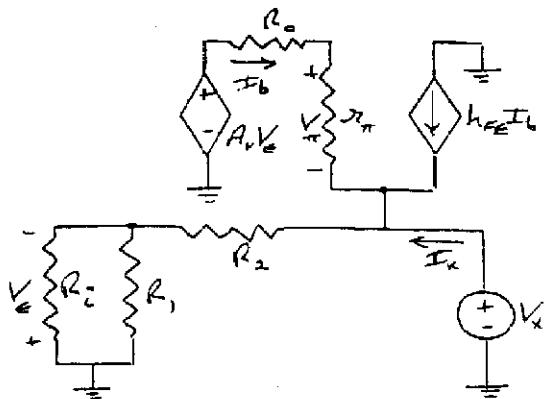
$$R_V = \frac{V_i}{I_i} = \frac{V_i}{\left(\frac{V_i - V_A}{R_i} \right)} = \left(\frac{V_i}{V_i - V_A} \right) R_i$$

$$V_A = \frac{1}{12} (V_i + V_o) = \frac{1}{12} (V_i + 10.97 V_i) = 0.9975 V_i$$

Then

$$R_V = \left(\frac{1}{1 - 0.9975} \right) (10 \text{ k}\Omega) \Rightarrow R_V = 4 \text{ M}\Omega$$

To find the output resistance:



$$I_x + \frac{(A_v V_x - V_x)(1+h_{FE})}{R_o + r_{\pi}} = \frac{V_x}{R_2 + R_1 \| R_i}$$

$$V_x = - \left(\frac{R_i \| R_1}{R_i \| R_1 + R_2} \right) \cdot V_i$$

Now

$$R_i \| R_1 = 1 \| 10 = 0.909$$

Then

$$V_x = -0.0833 V_i$$

Now

$$I_x = V_i \left\{ \left[\frac{(5 \times 10^3)(0.0833) + 1}{1+13} \right] (101) + \frac{1}{10 + 0.909} \right\}$$

$$= V_i \{ 3.012 \times 10^3 + 0.0917 \}$$

Or

$$\frac{V_i}{I_x} = R_{of} = 332 \times 10^{-4} \text{ k}\Omega \Rightarrow R_{of} = 0.332 \Omega$$

12.22

a. Neglecting base currents

$$I_{C2} = 0.5 \text{ mA}, V_{C2} = 12 - (0.5)(22.6) = 0.7 \text{ V}$$

$$I_{C1} = 0.5 \text{ mA}$$

$$\Rightarrow V_0 = 0$$

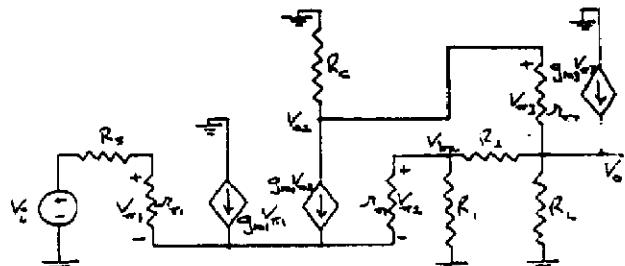
$$\text{Then } I_{C3} = 2 \text{ mA}$$

$$b. r_{\pi 1} = r_{\pi 2} = \frac{h_{FE} \cdot V_T}{I_{C1}} = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$g_{m1} = g_{m2} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(100)(0.026)}{2} = 1.3 \text{ k}\Omega$$

$$g_{m3} = \frac{2}{0.026} = 76.92 \text{ mA/V}$$



$$\begin{aligned} \frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} + g_{m1} V_{\pi 2} + \frac{V_{\pi 2}}{r_{\pi 1}} &= 0 \\ (V_{\pi 1} + V_{\pi 2}) \left(\frac{1}{r_{\pi 1}} + g_{m1} \right) &= 0 \\ \Rightarrow V_{\pi 2} &= -V_{\pi 1} \end{aligned} \quad (1)$$

$$V_i = \frac{V_{\pi 1}}{r_{\pi 1}} (R_s + r_{\pi 1}) - V_{\pi 2} + V_{b2}$$

or

$$V_i = V_{\pi 1} \left(1 + \frac{R_s}{r_{\pi 1}} \right) - V_{\pi 2} + V_{b2}$$

But $V_{\pi 2} = -V_{\pi 1}$

so

$$V_i = V_{\pi 1} \left(2 + \frac{R_s}{r_{\pi 1}} \right) + V_{b2} \quad (2)$$

$$\frac{V_{b2}}{R_C} + g_{m1} V_{\pi 2} + \frac{V_{b2} - V_0}{r_{\pi 3}} = 0 \quad (3)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3} V_{\pi 3} = \frac{V_0}{R_L} + \frac{V_0 - V_{b2}}{R_2}$$

$$V_{\pi 3} = V_{b2} - V_0$$

so

$$(V_{b2} - V_0) \left(\frac{1 + h_{FE}}{r_{\pi 3}} \right) = V_0 \left(\frac{1}{R_L} + \frac{1}{R_2} \right) - \frac{V_{b2}}{R_2} \quad (4)$$

$$\frac{V_{b2} - V_0}{R_2} + \frac{V_{b2}}{R_1} + \frac{V_{\pi 2}}{r_{\pi 1}} = 0 \quad (5)$$

Substitute numbers into (2), (3), (4) and (5):

$$V_i = -V_{\pi 2} \left(2 + \frac{1}{5.2} \right) + V_{b2}$$

$$V_i = -V_{\pi 2}(2.192) + V_{b2} \quad (2)$$

$$V_{b2} \left(\frac{1}{22.6} + \frac{1}{1.3} \right) + (19.23)V_{\pi 2} - V_0 \left(\frac{1}{1.3} \right) = 0$$

$$V_{b2}(0.8135) + (19.23)V_{\pi 2} - (0.7692)V_0 = 0 \quad (3)$$

$$V_{02} \left(\frac{101}{1.3} \right) = V_0 \left(\frac{101}{1.3} + \frac{1}{4} + \frac{1}{50} \right) - V_{02} \left(\frac{1}{50} \right)$$

$$V_{02}(77.69) = V_0(77.96) - V_{02}(0.02) \quad (4)$$

$$V_{02} \left(\frac{1}{50} + \frac{1}{10} \right) - V_0 \left(\frac{1}{50} \right) + V_{\pi 2} \left(\frac{1}{5.2} \right) = 0$$

$$V_{02}(0.120) - V_0(0.020) + V_{\pi 2}(0.1923) = 0 \quad (5)$$

From (2): $V_{02} = V_i + V_{\pi 2}(2.192)$. Substitute in (4) and (5) to obtain:

$$V_{02}(77.69) = V_0(77.96) - [V_i + V_{\pi 2}(2.192)](0.02) \quad (4)$$

$$\{V_i + V_{\pi 2}(2.192)\}(0.120) - V_0(0.020) \quad (5)$$

$$+ V_{\pi 2}(0.1923) = 0$$

So we now have the following three equations:

$$V_{02}(0.8135) + (19.23)V_{\pi 2} - (0.7692)V_0 = 0 \quad (3)$$

$$V_{02}(77.69) \quad (4)$$

$$= V_0(77.96) - V_i(0.02) - V_{\pi 2}(0.04384)$$

$$(0.120)V_i + V_{\pi 2}(0.4553) - V_0(0.020) = 0 \quad (5)$$

From (3): $V_{02} = V_0(0.9455) - V_{\pi 2}(23.64)$. Substitute for V_{02} in (4) to obtain:

$$(77.69)[V_0(0.9455) - V_{\pi 2}(23.64)]$$

$$= V_0(77.96) - V_i(0.02) - V_{\pi 2}(0.04384)$$

or

$$0 = V_0(4.504) - V_i(0.02) + V_{\pi 2}(1836.5)$$

Next, solve (5') for $V_{\pi 2}$:

$$(0.120)V_i + V_{\pi 2}(0.4553) - V_0(0.020) = 0$$

$$V_{\pi 2} = V_0(0.04393) - V_i(0.2636)$$

Finally,

$$v = V_0(4.504) - V_i(0.02)$$

$$+ (1836.5)[V_0(0.04393) - V_i(0.2636)]$$

$$0 = V_0(85.18) - V_i(484.12)$$

So

$$A_{v_f} = \frac{V_0}{V_i} = \frac{484.12}{85.18} \Rightarrow A_{v_f} = 5.68$$

12.23

$$a. R_{TH} = R_1 \parallel R_2 = 400 \parallel 75 = 63.2 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{75}{75 + 400} \right)(10)$$

$$= 1.579 \text{ V}$$

$$I_{BQ1} = \frac{1.579 - 0.7}{63.2 + (121)(0.5)} = 0.007106 \text{ mA}$$

$$I_{CQ1} = 0.853 \text{ mA}$$

$$V_{C1} = 10 - (0.853)(8.8) = 2.49 \text{ V}$$

$$I_{C2} \approx \frac{2.49 - 0.7}{3.6} = 0.497 \text{ mA}$$

$$V_{C2} = 10 - (0.497)(13) = 3.54 \text{ V}$$

$$I_{C3} \approx \frac{3.54 - 0.7}{1.4} = 2.03 \text{ mA}$$

Then

$$r_{\pi 1} = \frac{(120)(0.026)}{0.853} = 3.66 \text{ k}\Omega$$

$$g_{m1} = \frac{0.853}{0.026} = 32.81 \text{ mA/V}$$

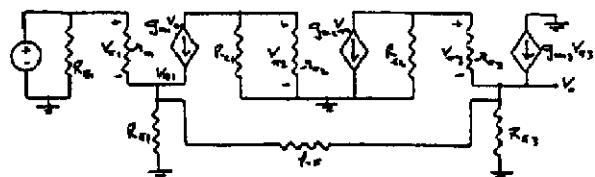
$$r_{\pi 2} = \frac{(120)(0.026)}{0.497} = 6.28 \text{ k}\Omega$$

$$g_{m2} = \frac{0.497}{0.026} = 19.12 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(120)(0.026)}{2.03} = 1.54 \text{ k}\Omega$$

$$g_{m3} = \frac{2.03}{0.026} = 78.08 \text{ mA/V}$$

b.



$$V_i = V_{\pi 1} + V_{e1} \Rightarrow V_{e1} = V_i - V_{\pi 1} \quad (1)$$

$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} = \frac{V_{e1}}{R_{E1}} + \frac{V_{e1} - V_0}{R_F} \quad (2)$$

$$V_{\pi 2} = -(g_{m1}V_{\pi 1})(R_{C1} \parallel r_{\pi 2}) \quad (3)$$

$$g_{m2}V_{\pi 2} + \frac{V_{\pi 2} + V_0}{R_{C2}} + \frac{V_{\pi 3}}{r_{\pi 3}} = 0 \quad (4)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3}V_{\pi 3} = \frac{V_0}{R_{E3}} + \frac{V_0 - V_{e1}}{R_F} \quad (5)$$

Substitute numbers in (2), (3), (4) and (5):

$$V_{\pi 1} \left(\frac{1}{3.66} + 32.81 \right) = (V_i - V_{\pi 1}) \left(\frac{1}{0.5} + \frac{1}{10} \right) - \frac{V_0}{10}$$

or $V_{\pi 1}(35.18) = V_i(2.10) - V_0(0.10)$ (2)

$$V_{\pi 2} = -(32.81)V_{\pi 1}(88 \parallel 6.28)$$

or $V_{\pi 2} = -V_{\pi 1}(120.2)$ (3)

$$(19.12)V_{\pi 2} + \frac{V_{\pi 2}}{13} + \frac{V_0}{13} + \frac{V_{\pi 3}}{1.54} = 0$$

or

$$V_{\pi 2}(19.12) + V_{\pi 3}(0.7263) + V_0(0.07692) = 0$$
 (4)

$$V_{\pi 3} \left(\frac{1}{1.54} + 78.08 \right) = V_0 \left(\frac{1}{1.4} + \frac{1}{10} \right) - \frac{V_i - V_{\pi 1}}{10}$$

or

$$V_{\pi 3}(78.73) = V_0(0.8143) - V_i(0.10) + V_{\pi 1}(0.10)$$
 (5)

Now substituting $V_{\pi 2} = -V_{\pi 1}(120.2)$ in (4):

$$(19.12)[-V_{\pi 1}(120.2)] + V_{\pi 3}(0.7263) + V_0(0.07692) = 0$$

or

$$-V_{\pi 1}(2298.2) + V_{\pi 3}(0.7263) + V_0(0.07692) = 0$$

Then

$$V_{\pi 3} = V_{\pi 1}(3164.3) - V_0(0.1059)$$

Substituting $V_{\pi 3} = V_{\pi 1}(3164.3) - V_0(0.1059)$ in (5):

$$(78.73)[V_{\pi 1}(3164.3) - V_0(0.1059)] = V_0(0.8143) - V_i(0.10) + V_{\pi 1}(0.10)$$

or

$$V_{\pi 1}(2.49 \times 10^5) - V_0(9.152) = -V_i(0.10)$$

Then

$$V_{\pi 1} = V_0(3.674 \times 10^{-5}) - V_i(4.014 \times 10^{-7})$$

Now substituting $V_{\pi 1} = V_0(3.674 \times 10^{-5}) - V_i(4.014 \times 10^{-7})$ in (2):

$$(35.18)[V_0(3.674 \times 10^{-5}) - V_i(4.014 \times 10^{-7})] = V_i(2.10) - V_0(0.10)$$

or $V_0(0.1013) = V_i(2.10)$

So $\frac{V_0}{V_i} = 20.7$

c. $R_{if} = \frac{V_i}{I_i}$ and $I_i = I_{RB1} + I_{b1}$

$$I_{RB1} = \frac{V_i}{R_{B1}}$$

$$I_{b1} = \frac{V_{\pi 1}}{r_{\pi 1}}$$

Now

$$V_{\pi 1} = (20.7V_i)(3.674 \times 10^{-5}) - V_i(4.014 \times 10^{-7})$$

$$V_{\pi 1} = V_i(7.60 \times 10^{-4})$$

Then

$$R_{if} = \frac{\frac{V_i}{V_i(7.60 \times 10^{-4})}}{\frac{63.2}{3.66} + \frac{1}{0.01582 + 2.077 \times 10^{-4}}}$$

or $R_{if} = 62.4 \text{ k}\Omega$

d. To determine R_{of} :

Equation (1) is modified to $V_{\pi 1} + V_{e1} = 0$ ($V_i = 0$)

Equation (5) is modified to:

$$V_{\pi 3}(78.73) + I_X = V_0(0.8143) + V_{\pi 1}(0.10) \quad (5)$$

Now

$$V_{\pi 1}(35.18) = -V_0(0.10) \quad (2)$$

$$V_{\pi 2} = -V_{\pi 1}(120.2) \quad (3)$$

$$V_{\pi 2}(19.12) + V_{\pi 3}(0.7263) + V_0(0.07692) = 0 \quad (4)$$

Now

$$V_{\pi 1} = -V_0(0.002843)$$

so

$$V_{\pi 2} = -(-V_0)(0.002843)(120.2)$$

$$V_{\pi 2} = V_0(0.3417)$$

Then

$$V_0(0.3417)(19.12) + V_{\pi 3}(0.7263) + V_0(0.07692) = 0$$

or

$$V_{\pi 3} = -V_0(9.101) \quad (4)$$

So then

$$-V_0(9.101)(78.73) + I_X = V_0(0.8143) + (0.10)(-V_0)(0.002843)$$

or

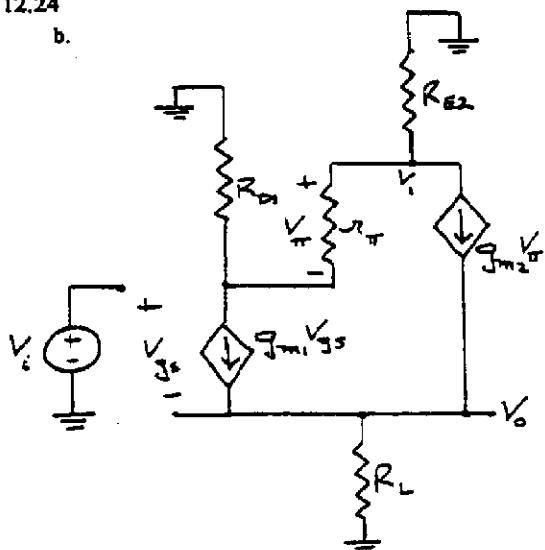
$$I_X = V_0(717.3) \quad (5)$$

or

$$R_{of} = \frac{V_0}{I_X} = 0.00139 \text{ k}\Omega \Rightarrow R_{of} = 1.39 \Omega$$

12.24

b.



$$V_0 = (g_{m1} V_{gs} + g_{m2} V_{pi}) R_L \quad (1)$$

$$V_i = V_{gs} + V_0 \quad (2)$$

$$\begin{aligned} \frac{V_{pi}}{r_{pi}} + g_{m2} V_{pi} + \frac{V_1}{R_{E2}} &= 0 \\ \Rightarrow V_{pi} \left(\frac{1 + h_{FE}}{r_{pi}} \right) + \frac{V_1}{R_{E2}} &= 0 \end{aligned} \quad (3)$$

$$\begin{aligned} \frac{V_{pi}}{r_{pi}} &= g_{m1} V_{gs} + \frac{V_1 - V_{pi}}{R_{D1}} = 0 \\ \text{or } V_1 &= R_{D1} \left[\frac{V_{pi}}{r_{pi}} + \frac{V_{pi}}{R_{D1}} - g_{m1} V_{gs} \right] \end{aligned} \quad (4)$$

Then

$$V_{pi} \left(\frac{1 + h_{FE}}{r_{pi}} \right) + \frac{R_{D1}}{R_{E2}} \left[\frac{V_{pi}}{r_{pi}} + \frac{V_{pi}}{R_{D1}} - g_{m1} V_{gs} \right] = 0 \quad (3)$$

$$V_{pi} \left\{ \left(\frac{1 + h_{FE}}{r_{pi}} \right) + \frac{R_{D1}}{R_{E2} r_{pi}} + \frac{1}{R_{E2}} \right\} = g_{m1} \left(\frac{R_{D1}}{R_{E2}} \right) V_{gs}$$

Let

$$V_{pi} \cdot \frac{1}{R_{eq}} = g_{m1} \left(\frac{R_{D1}}{R_{E2}} \right) V_{gs}$$

$$\text{so } V_{pi} = g_{m1} R_{eq} \left(\frac{R_{D1}}{R_{E2}} \right) V_{gs}$$

Then

$$V_0 = \left[g_{m1} V_{gs} + g_{m1} g_{m2} R_{eq} \left(\frac{R_{D1}}{R_{E2}} \right) V_{gs} \right] R_L \quad (1)$$

so

$$V_0 = g_{m1} R_L \left[1 + g_{m2} R_{eq} \left(\frac{R_{D1}}{R_{E2}} \right) \right] (V_i - V_0)$$

so

$$A_v = \frac{V_0}{V_i} = \frac{g_{m1} R_L \left[1 + g_{m2} R_{eq} \left(\frac{R_{D1}}{R_{E2}} \right) \right]}{1 + g_{m1} R_L \left[1 + g_{m2} R_{eq} \left(\frac{R_{D1}}{R_{E2}} \right) \right]}$$

 c. Set $V_i = 0$

$$I_X + g_{m1} V_{gs} + g_{m2} V_{pi} = \frac{V_X}{R_L}$$

$$V_{gs} = -V_X$$

From part (b), we have

$$V_{pi} = g_{m1} R_{eq} \left(\frac{R_{D1}}{R_{E2}} \right) V_{gs} = -g_{m1} R_{eq} \left(\frac{R_{D1}}{R_{E2}} \right) V_X$$

Then

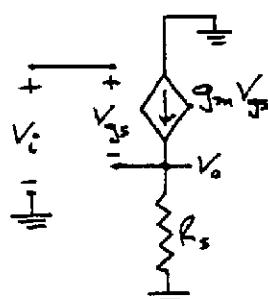
$$\frac{I_X}{V_X} = \frac{1}{R_L} = \frac{1}{R_L} + g_{m1} g_{m2} R_{eq} \left(\frac{R_{D1}}{R_{E2}} \right)$$

or

$$R_0 = R_L \parallel \frac{1}{g_{m1}} \parallel \frac{1}{g_{m1} g_{m2} R_{eq} \left(\frac{R_{D1}}{R_{E2}} \right)}$$

12.25

$$\text{a. } g_m = 2\sqrt{K_n J_{DQ}} = 2\sqrt{(0.5)(0.5)} = 1 \text{ mA/V}$$



$$V_0 = (g_m V_{gs}) R_s$$

$$V_i = V_{gs} + V_0 \text{ so } V_{gs} = V_i - V_0$$

Then

$$V_0 = g_m R_S (V_i - V_0)$$

$$A_v = \frac{g_m R_S}{1 + g_m R_S} = \frac{1(2)}{1 + (1)(2)} \Rightarrow A_v = 0.667$$

To determine R_{of} ,

$$I_X + g_m V_{gs} = \frac{V_X}{R_S} \text{ and } V_{gs} = -V_X$$

$$\frac{I_X}{V_X} = \frac{1}{R_{of}} = g_m + \frac{1}{R_S}$$

$$\text{so } R_{of} = \frac{1}{g_m} \parallel R_S = \frac{1}{1} \parallel 2 \Rightarrow R_{of} = 0.667 \text{ k}\Omega$$

b. For $K_n = 0.8 \text{ mA/V}^2$

$$g_m = 2\sqrt{(0.8)(0.5)} = 1.265 \text{ mA/V}$$

$$A_v = \frac{(1.265)(2)}{1 + (1.265)(2)} = 0.7167$$

$$\frac{\Delta A_f}{A_f} = \frac{0.7167 - 0.667}{0.667} \Rightarrow 7.45\% \text{ increase}$$

$$R_{of} = \frac{1}{1.265} \parallel 2 = 0.7905 \parallel 2$$

$$R_{of} = 0.5666$$

$$\frac{\Delta R_{of}}{R_{of}} = \frac{0.5666 - 0.667}{0.667} \Rightarrow 15.05\% \text{ decrease}$$

12.26

dc analysis:

$$R_{TH1} = 150 \parallel 47 = 35.8 \text{ k}\Omega,$$

$$V_{TH1} = \left(\frac{47}{47 + 150} \right) (25) = 5.96 \text{ V}$$

$$R_{TH2} = 33 \parallel 47 = 19.4 \text{ k}\Omega,$$

$$V_{TH2} = \left(\frac{33}{33 + 47} \right) (25) = 10.3 \text{ V}$$

$$I_{B1} = \frac{5.96 - 0.7}{35.8 + (51)(4.8)} = 0.0187 \text{ mA}$$

$$I_{C1} = (50)(0.0187) = 0.935 \text{ mA}$$

$$I_{B2} = \frac{10.3 - 0.7}{19.4 + (51)(4.7)} = 0.03705 \text{ mA}$$

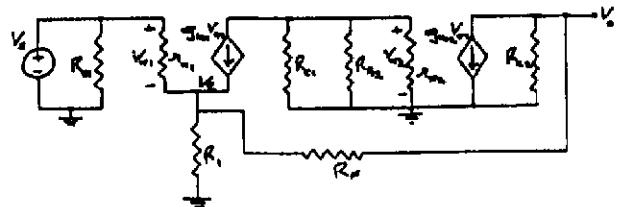
$$I_{C2} = (50)(0.03705) = 1.85 \text{ mA}$$

$$r_{\pi 1} = \frac{(50)(0.026)}{0.935} = 1.39 \text{ k}\Omega;$$

$$r_{\pi 2} = \frac{(50)(0.026)}{1.85} = 0.703 \text{ k}\Omega$$

$$g_{m1} = \frac{0.935}{0.026} = 35.96 \text{ mA/V}$$

$$g_{m2} = \frac{1.85}{0.026} = 71.15 \text{ mA/V}$$



$$V_S = V_{\pi 1} + V_o \quad (1)$$

$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} = \frac{V_o}{R_o} + \frac{V_o - V_0}{R_F} \quad (2)$$

$$g_{m1} V_{\pi 1} + \frac{V_{\pi 2}}{R_{C1}} + \frac{V_{\pi 2}}{R_{B2}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \quad (3)$$

$$g_{m2} V_{\pi 2} + \frac{V_0}{R_{C2}} + \frac{V_0 - V_o}{R_F} = 0 \quad (4)$$

Substitute numerical values in (2), (3) and (4):

$$V_o = V_S - V_{\pi 1} \quad (1)$$

$$\frac{V_{\pi 1}}{1.39} + (35.96)V_{\pi 1} = (V_S - V_{\pi 1})\left(\frac{1}{0.1} + \frac{1}{4.7}\right) - V_0\left(\frac{1}{4.7}\right)$$

or

$$V_{\pi 1}(46.89) = V_S(10.213) - V_0(0.2128) \quad (2)$$

$$(35.96)V_{\pi 1} + V_{\pi 2}\left(\frac{1}{10} + \frac{1}{19.4} + \frac{1}{0.703}\right) = 0$$

or

$$(35.96)V_{\pi 1} + V_{\pi 2}(1.374) = 0 \quad (3)$$

$$(71.15)V_{\pi 2} + V_0\left(\frac{1}{4.7} + \frac{1}{4.7}\right)$$

$$- (V_S - V_{\pi 1})\left(\frac{1}{4.7}\right) = 0$$

or

$$(71.15)V_{\pi 2} + V_0(0.4255) - V_S(0.2128) + V_{\pi 1}(0.2128) = 0 \quad (4)$$

$$\text{From (3): } V_{\pi 2} = -V_{\pi 1}(22.85)$$

Then substitute in (4):

$$-(71.15)V_{\pi 1}(22.85) + V_0(0.4255)$$

$$- V_S(0.2128) + V_{\pi 1}(0.2128) = 0$$

or

$$- V_{\pi 1}(1625.6) + V_0(0.4255) - V_S(0.2128) = 0$$

$$\text{From (2): } V_{\pi 1} = V_S(0.2178) - V_0(0.004538)$$

Then

$$-(1625.6)[V_S(0.2178) - V_0(0.004538)]$$

$$+ V_0(0.4255) - V_S(0.2128) = 0$$

or $-V_S(354.3) + V_o(7.802) = 0$

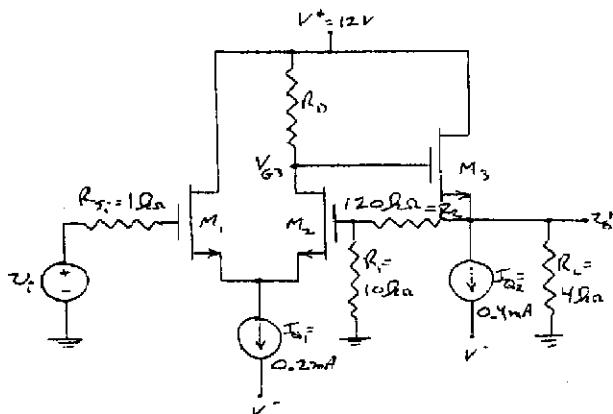
Finally

$$\Rightarrow \frac{V_o}{V_S} = 45.4$$

12.27

For example, use the circuit shown in Figure 12.23

12.28



For M_3 : $K_{n3} = \frac{k'_n}{2} \cdot \left(\frac{W}{L}\right)_3$, Let $\left(\frac{W}{L}\right)_3 = 25$

Then $K_{n3} = \left(\frac{0.080}{2}\right)(25) = 1 \text{ mA/V}^2$

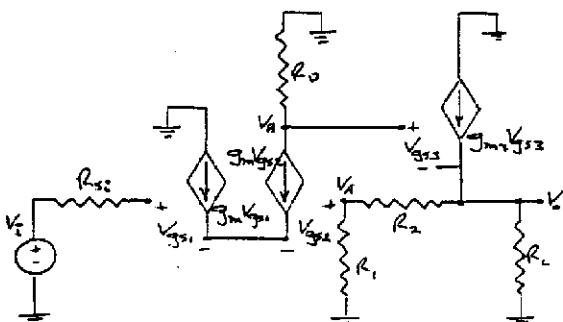
Want $v_o = 0$ for $v_i = 0$, so that

$$I_{D3} = I_{Q2} = 0.4 = 1 \cdot (V_{GS3} - V_{TH})^2$$

Then $V_{GS3} = \sqrt{\frac{0.4}{1}} + 2 = 2.63 \text{ V}$

For $V_{GS3} = 2.63 \text{ V} \Rightarrow V_{GS} = 12 - I_{D2}R_D$

Or $2.63 = 12 - (0.1)R_D \Rightarrow R_D = 93.7 \text{ kΩ}$



$$g_{m3} = 2\sqrt{K_{n3}I_{D3}} = 2\sqrt{(1)(0.4)} = 1.26 \text{ mA/V}$$

$$V_A = \left(\frac{R_1}{R_1 + R_2}\right)V_o = \left(\frac{10}{120 + 10}\right)V_o = 0.0769V_o$$

(Small amount of feedback)

$$(1) V_i = V_{gs1} - V_{gs2} + V_A$$

$$(2) g_m V_{gs1} + g_m V_{gs2} = 0 \Rightarrow V_{gs1} = -V_{gs2}$$

Then

$$V_i = -2V_{gs2} + V_A \Rightarrow V_{gs2} = \frac{1}{2}(V_A - V_i)$$

$$V_{gs2} = 0.03846V_o - 0.5V_i$$

$$(3) V_B = -g_m V_{gs2} R_D = -g_m R_D [0.03846V_o - 0.5V_i]$$

$$(4) V_{gs3} = V_B - V_o \text{ and } V_o = g_{m3} V_{gs3} [R_L (R_1 + R_2)]$$

So

$$V_o = g_{m3} [R_L (R_1 + R_2)] (V_B - V_o)$$

Then

$$V_o = g_{m3} [R_L (R_1 + R_2)] [-g_m R_D (0.03846V_o - 0.5V_i) - V_i]$$

Or

$$V_o [1 + g_{m3} [R_L (R_1 + R_2)] g_m R_D (0.03846) + 1] =$$

$$= g_{m3} [R_L (R_1 + R_2)] 0.5 g_m R_D \cdot V_i$$

Now

$$R_L (R_1 + R_2) = 4 \parallel 130 = 3.88$$

So

$$V_o [1 + (1.26)(3.88)[g_m (93.7)(0.03846) + 1]] = (1.26)(3.88)(0.5)g_m (93.7)V_i$$

Rearranging terms, we find

$$\frac{V_o}{V_i} = \frac{229g_m}{5.89 + 17.6g_m} = 10 \Rightarrow g_m = 1.11 \text{ mA/V}$$

We have

$$g_m = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)I_D} \Rightarrow$$

$$1.11 = 2\sqrt{\left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)(0.1)} \Rightarrow$$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 77$$

12.29

Assuming an ideal op-amp, then from Equation (12.58)

$$\frac{I_0}{I_S} = 1 + \frac{R_1}{R_2} = \frac{20}{0.2} = 100$$

$$\text{Then } \frac{R_1}{R_2} = 99$$

For example, set $R_2 = 5 \text{ kΩ}$ and $R_1 = 495 \text{ kΩ}$

12.30

$$(a) I_{C1} = \left(\frac{h_{FE}}{1+h_{FE}}\right)I_{E1} = \left(\frac{100}{101}\right)(0.2) = 0.198 \text{ mA}$$

$$V_{C1} = 10 - (0.198)(40) = 2.08 \text{ V}$$

$$I_{E2} = \frac{2.08 - 0.7}{1} = 1.38 \text{ mA}$$

$$I_{C2} = \left(\frac{100}{101}\right)(1.38) = 1.37 \text{ mA}$$

For Q₁:

$$r_{\pi 1} = \frac{(100)(0.026)}{0.198} = 13.1 \text{ k}\Omega$$

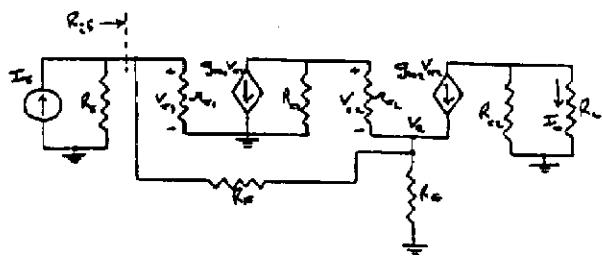
$$g_{m1} = \frac{0.198}{0.026} = 7.62 \text{ mA/V}$$

For Q₂:

$$r_{\pi 2} = \frac{(100)(0.026)}{1.37} = 1.90 \text{ k}\Omega$$

$$g_{m2} = \frac{1.37}{0.026} = 52.7 \text{ mA/V}$$

(b)



$$I_s = \frac{V_{\pi 1}}{R_s} + \frac{V_{\pi 1}}{r_{\pi 1}} + \frac{V_{\pi 1} - V_e}{R_f} \quad (1)$$

$$g_{m1}V_{\pi 1} + \frac{V_{\pi 2} + V_e}{R_{C1}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \quad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2}V_{\pi 2} = \frac{V_e}{R_f} + \frac{V_e - V_{\pi 1}}{R_f} \quad (3)$$

Substitute numerical values in (1), (2), and (3):

$$I_s = V_{\pi 1} \left(\frac{1}{10} + \frac{1}{13.1} + \frac{1}{10} \right) - V_e \left(\frac{1}{10} \right) \quad (1)$$

$$I_s = V_{\pi 1}(0.2763) - V_e(0.10) \quad (1)$$

$$(7.62)V_{\pi 1} + V_{\pi 2} \left(\frac{1}{40} + \frac{1}{1.90} \right) + V_e \left(\frac{1}{40} \right) = 0$$

$$(7.62)V_{\pi 1} + V_{\pi 2}(0.5513) + V_e(0.025) = 0 \quad (2)$$

$$V_{\pi 2} \left(\frac{1}{1.90} + 52.7 \right) = V_e \left(\frac{1}{1} + \frac{1}{10} \right) - V_{\pi 1} \left(\frac{1}{10} \right) \quad (3)$$

$$V_{\pi 2}(53.23) = V_e(1.10) - V_{\pi 1}(0.10) \quad (3)$$

From (3),

$$V_e = V_{\pi 2}(48.39) + V_{\pi 1}(0.0909)$$

Substituting into (1),

$$I_s = V_{\pi 1}(0.2763)$$

$$-(0.10)[V_{\pi 2}(48.39) + V_{\pi 1}(0.0909)]$$

or

$$I_s = V_{\pi 1}(0.2672) - V_{\pi 2}(4.839) \quad (1')$$

and substituting into (2),

$$(7.62)V_{\pi 1} + V_{\pi 2}(0.5513) + (0.025)[V_{\pi 2}(48.39) + V_{\pi 1}(0.0909)] = 0$$

or

$$(7.622)V_{\pi 1} + V_{\pi 2}(1.761) = 0 \Rightarrow V_{\pi 1} = -V_{\pi 2}(0.2310) \quad (2')$$

Then substituting (2') into (1'), we obtain

$$I_s = (0.2672)(-V_{\pi 2})(0.2310) - V_{\pi 2}(4.839)$$

or

$$I_s = -V_{\pi 2}(4.901)$$

Now

$$I_o = -g_{m2}V_{\pi 2} \left(\frac{R_{C2}}{R_{C2} + R_L} \right) = -(52.7) \left(\frac{2}{2+0.5} \right) V_{\pi 2} = -(42.16)V_{\pi 2}$$

Then

$$I_o = -(42.16) \left(\frac{-I_s}{4.901} \right)$$

or

$$A_{if} = \frac{I_o}{I_s} = 8.60$$

$$(c) R_i = \frac{V_{\pi 1}}{I_s} \text{ and } R_i = R_s \parallel R_{if}$$

We had

$$V_{\pi 1} = -V_{\pi 2}(0.2310) \text{ and } I_s = -V_{\pi 2}(4.901)$$

so

$$I_s = - \left(\frac{-V_{\pi 1}}{0.2310} \right) (4.901) = V_{\pi 1}(21.22)$$

Then

$$R_i = \frac{V_{\pi 1}}{I_s} = \frac{1}{21.22} = 0.04713$$

Finally

$$0.04713 = \frac{10R_{if}}{10 + R_{if}} \Rightarrow$$

$$R_{if} = 47.4 \Omega$$

12.31

(a) Using Figure 12.25

$$I_i = \frac{V_{\pi 1}}{R_s \parallel R_{B1} \parallel r_{\pi 1}} + \frac{V_{\pi 1} - V_{\pi 2}}{R_f} \quad (1)$$

$$g_{m1}V_{\pi 1} + \frac{V_{\pi 2}}{R_{C1} \parallel R_{B2}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \quad (2)$$

$$= g_{m1}V_{\pi 1} + \frac{V_{\pi 2}}{R_{C1} \parallel R_{B2} \parallel r_{\pi 2}}$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2} V_{\pi 2} = \frac{V_{\pi 2}}{R_{E2}} + \frac{V_{\pi 1} - V_{\pi 2}}{R_F} \quad (3)$$

$$I_o = -\left(\frac{R_{C2}}{R_{C2} + R_L}\right)(g_{m2} V_{\pi 2}) \quad (4)$$

$$I_i = \frac{V_{\pi 1}}{R_S \| R_{B1} \| r_{\pi 1} \| R_F} - \frac{V_{\pi 2}}{R_F} \quad (1')$$

so that

$$V_{\pi 2} = \left(\frac{R_F}{R_S \| R_{B1} \| r_{\pi 1} \| R_F}\right) V_{\pi 1} - R_F I_i$$

Then

$$V_{\pi 2} \left(\frac{1 + \beta_2}{r_{\pi 2}}\right) = \left(\frac{1}{R_{E2}} + \frac{1}{R_F}\right) \times \left\{\left(\frac{R_F}{R_S \| R_{B1} \| r_{\pi 1} \| R_F}\right) V_{\pi 1} - R_F I_i\right\} - \frac{V_{\pi 1}}{R_F} \quad (3')$$

From (2):

$$V_{\pi 1} = -\frac{V_{\pi 2}}{g_{m1}} \cdot \frac{1}{(R_{C1} \| R_{B2} \| r_{\pi 2})}$$

Then

$$V_{\pi 2} \left(\frac{1 + \beta_2}{r_{\pi 2}}\right) = \frac{V_{\pi 2}}{g_{m1} R_F} \cdot \frac{1}{R_{C1} \| R_{B2} \| r_{\pi 2}} \times \left\{1 - \left(1 + \frac{R_F}{R_{E2}}\right) \left(\frac{R_F}{R_S \| R_{B1} \| r_{\pi 1} \| R_F}\right)\right\} - \left(1 + \frac{R_F}{R_{E2}}\right) I_i$$

Solve for $V_{\pi 2}$ and substitute into Equation 4.

$$(b) R_{m1} = 20 \| 80 = 16 k\Omega = R_{B1}$$

$$V_{m1} = \left(\frac{20}{100}\right)(10) = 2 V$$

$$I_{m2} = \frac{2 - 0.7}{16 + (10)(1)} = 0.0111 mA \Rightarrow$$

$$I_{CQ1} = 1.11 mA$$

$$R_{m2} = 15 \| 85 = 12.75 k\Omega$$

$$V_{m2} = \left(\frac{15}{15+85}\right)(10) = 1.5 V$$

$$I_{m2} = \frac{1.5 - 0.7}{12.75 + (10)(0.5)} = 0.0126 mA \Rightarrow$$

$$I_{CQ2} = 1.26 mA$$

$$g_{m1} = \frac{1.11}{0.026} = 42.69 mA/V$$

$$g_{m2} = \frac{1.26}{0.026} = 48.46 mA/V$$

$$r_{\pi 1} = \frac{(100)(0.026)}{1.11} = 2.34 k\Omega$$

$$r_{\pi 2} = \frac{(100)(0.026)}{1.26} = 2.06 k\Omega$$

From part (a)

$$V_{\pi 2} \left(\frac{101}{2.06}\right) = \frac{V_{\pi 2}}{(42.69)(10)} \cdot \frac{1}{2 \| 12.75 \| 2.06}$$

$$x \left\{ 1 - \left(1 + \frac{10}{0.5}\right) \left(\frac{10}{10000 \| 16 \| 2.34 \| 10}\right) \right\} - \left(1 + \frac{10}{0.5}\right) I_i$$

So

$$V_{\pi 2} (49.34) = -(21) I_i$$

or

$$V_{\pi 2} = -0.4256 I_i$$

Now

$$I_o = -\left(\frac{4}{4+4}\right)(48.46)(-0.4256) I_i$$

or

$$A_i = \frac{I_o}{I_i} = 10.3$$

From Example 12.9, computer analysis showed $A_i = 9.58$. The difference in results is usually in the calculation of quiescent currents which leads to slight differences in the small-signal parameter values.

12.32

$$a. \quad R_{TH} = 13.5 \| 38.3 = 9.98 k\Omega$$

$$V_{TH} = \left(\frac{13.5}{13.5 + 38.3}\right)(10) = 2.606 V$$

$$I_{C1} = \frac{(120)(2.606 - 0.7)}{9.98 + (121)(1)} = 1.75 mA$$

$$V_{C1} = 10 - (1.75)(3) = 4.75 V$$

$$I_{C2} \approx \frac{4.75 - 0.7}{8.1} = 0.50 mA$$

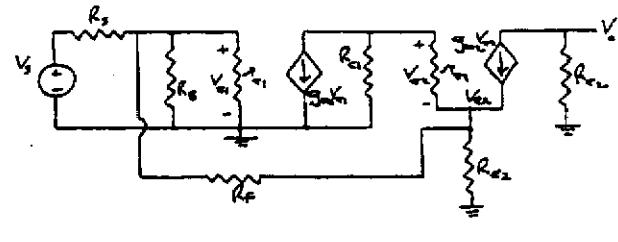
$$r_{\pi 1} = \frac{(120)(0.026)}{1.75} = 1.78 k\Omega$$

$$g_{m1} = \frac{1.75}{0.026} = 67.31 mA/V$$

$$r_{\pi 2} = \frac{(120)(0.026)}{0.50} = 6.24 k\Omega$$

$$g_{m2} = \frac{0.50}{0.026} = 19.23 mA/V$$

b.



$$\frac{V_S - V_{\pi 1}}{R_S} = \frac{V_{\pi 1}}{R_B || r_{\pi 1}} + \frac{V_{\pi 1} - V_{e2}}{R_F} \quad (1)$$

$$g_{m1} V_{\pi 1} + \frac{V_{\pi 2} + V_{e2}}{R_C 1} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \quad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2} V_{\pi 2} = \frac{V_{e2}}{R_E 2} + \frac{V_{e2} - V_{\pi 1}}{R_F} \quad (3)$$

and

$$V_0 = -(g_{m2} V_{\pi 2}) R_C 2 \quad (4)$$

Substitute numerical values in (1), (2), and (3):

$$\frac{V_S}{0.6} = V_{\pi 1} \left[\frac{1}{0.6} + \frac{V_{\pi 1}}{9.98 \parallel 1.78} + \frac{1}{1.2} \right] - \frac{V_{e2}}{1.2}$$

or

$$V_S(1.667) = V_{\pi 1}(4.011) - V_{e2}(0.8333) \quad (1)$$

$$(67.31)V_{\pi 1} + V_{\pi 2} \left(\frac{1}{3} + \frac{1}{6.24} \right) + \frac{V_{e2}}{3} = 0$$

or

$$V_{\pi 1}(67.31) + V_{\pi 2}(0.4936) + V_{e2}(0.3333) = 0 \quad (2)$$

$$V_{\pi 2} \left(\frac{1}{6.24} + 19.23 \right) = \frac{V_{e2}}{8.1} + \frac{V_{e2}}{1.2} - \frac{V_{\pi 1}}{1.2}$$

or

$$V_{\pi 2}(19.39) = V_{e2}(0.9568) - V_{\pi 1}(0.8333) \quad (3)$$

From (1)

$$V_{e2} = V_{\pi 1}(4.813) - V_S(2.00)$$

Then

$$V_{\pi 1}(67.31) + V_{\pi 2}(0.4936) + (0.3333)[V_{\pi 1}(4.813) - V_S(2.00)] = 0$$

or

$$V_{\pi 1}(68.91) + V_{\pi 2}(0.4936) - V_S(0.6666) = 0 \quad (2')$$

and

$$V_{\pi 2}(19.39) = (0.9568)[V_{\pi 1}(4.813) - V_S(2.00)] - V_{\pi 1}(0.8333)$$

or

$$V_{\pi 2}(19.39) = V_{\pi 1}(3.772) - V_S(1.914) \quad (3')$$

We find

$$V_{\pi 1} = V_S(0.009673) - V_{\pi 2}(0.007163)$$

Then

$$V_{\pi 2}(19.39) = (3.772)[V_S(0.009673) - V_{\pi 2}(0.007163)] - V_S(1.914)$$

$$V_{\pi 2}(19.42) = V_S(-1.878) \text{ or } V_{\pi 2} = -V_S(0.09670)$$

so that

$$V_0 = -(19.23)(4)(-V_S)(0.09670)$$

Then

$$\frac{V_0}{V_S} = 1.86$$

12.33

Using the circuit from Problem 12.32, we have $R_Y = \frac{V_{\pi 1}}{I_S}$

$$\text{where } I_S = \frac{V_S - V_{\pi 1}}{R_S}$$

From Problem 12.32

$$V_{\pi 1} = V_S(0.009673) - V_{\pi 2}(0.007163) \\ = V_S(0.009673) - (0.007163)(-V_S)(0.09670) \\ = V_S(0.01037)$$

So

$$R_{Y1} = \frac{V_S(0.01037) \cdot (0.6)}{V_S - V_S(0.01037)} = 0.00629 \text{ k}\Omega$$

or

$$\underline{R_{Y1} = 6.29 \Omega}$$

12.34

$$R_{TH} = 1.4 \parallel 17.9 = 1.298 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{1.4}{1.4 + 17.9} \right) (10) = 0.7254 \text{ V}$$

$$I_{B1} = \frac{0.7254 - 0.7}{1.298} = 0.0196 \text{ mA}$$

$$I_{C1} = (50)(0.0196) = 0.98 \text{ mA}$$

Neglecting dc base currents,

$$V_{B2} = 10 - (0.98)(7) = 3.14 \text{ V}$$

$$I_{E2} = \frac{3.14 - 0.7}{0.25 + 0.5} = 3.25 \text{ mA}$$

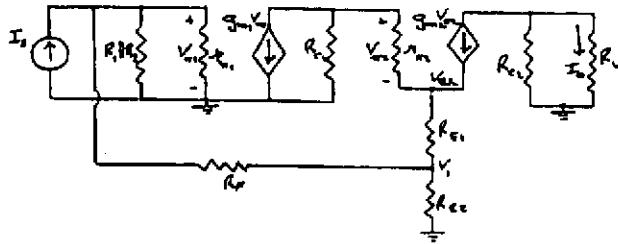
$$I_{C2} = \left(\frac{50}{51} \right) (3.25) = 3.19 \text{ mA}$$

$$r_{\pi 1} = \frac{(50)(0.026)}{0.98} = 1.33 \text{ k}\Omega$$

$$g_{m1} = \frac{0.98}{0.026} = 37.7 \text{ mA/V}$$

$$r_{\pi 2} = \frac{(50)(0.026)}{3.19} = 0.408 \text{ k}\Omega$$

$$g_{m2} = \frac{3.19}{0.026} = 123 \text{ mA/V}$$



$$I_S = \frac{V_{\pi 1}}{R_1 \| R_2 \| r_{\pi 1}} + \frac{V_{\pi 1} - V_1}{R_F} \quad (1)$$

$$g_{m1}V_{\pi 1} + \frac{V_{\pi 2}}{r_{\pi 2}} + \frac{V_{\pi 2} + V_{e2}}{R_{C1}} = 0 \quad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2}V_{\pi 2} = \frac{V_{\pi 2} - V_1}{R_{E1}} \quad (3)$$

$$\frac{V_{e2} - V_{\pi 1}}{R_{E1}} = \frac{V_1}{R_{E2}} + \frac{V_1 - V_{\pi 1}}{R_F} \quad (4)$$

Enter numerical values in (1), (2), (3) and (4):

$$I_S = \frac{V_{\pi 1}}{17.9 \| 1.4 \| 1.33} + \frac{V_{\pi 1} - V_1}{5}$$

or

$$I_S = V_{\pi 1}(1.722) - V_1(0.20) \quad (1)$$

$$(37.7)V_{\pi 1} + \frac{V_{\pi 2}}{0.408} + \frac{V_{\pi 2} + V_{e2}}{7} = 0$$

or

$$V_{\pi 1}(37.7) + V_{\pi 2}(2.594) + V_{e2}(0.1429) = 0 \quad (2)$$

$$\frac{V_{\pi 2}}{0.408} + (123)V_{\pi 2} = \frac{V_{\pi 2} - V_1}{0.25}$$

or

$$V_{\pi 2}(125.5) = V_{\pi 2}(4) - V_1(4) \quad (3)$$

$$\frac{V_{\pi 2} - V_1}{0.25} = \frac{V_1}{0.50} + \frac{V_1 - V_{\pi 1}}{5}$$

or

$$V_{\pi 2}(4) = V_1(6.20) - V_{\pi 1}(0.20) \quad (4)$$

From (4):

$$V_{e2} = V_1(1.55) - V_{\pi 1}(0.05)$$

Then substituting in (3):

$$V_{\pi 2}(125.5) = (4)[V_1(1.55) - V_{\pi 1}(0.05)] - V_1(4)$$

or

$$V_{\pi 2}(125.5) = V_1(2.20) - V_{\pi 1}(0.20) \quad (3')$$

and substituting in (2):

$$V_{\pi 1}(37.7) + V_{\pi 2}(2.594) + (0.1429)[V_1(1.55) - V_{\pi 1}(0.05)] = 0$$

or

$$V_{\pi 1}(37.69) + V_{\pi 2}(2.594) + V_1(0.2215) = 0$$

Now

$$V_1 = -V_{\pi 1}(170.16) - V_{\pi 2}(11.71)$$

Then substituting in (1):

$$I_S = V_{\pi 1}(1.722) - (0.20)[-V_{\pi 1}(170.16) - V_{\pi 2}(11.71)]$$

or

$$I_S = V_{\pi 1}(35.75) + V_{\pi 2}(2.342)$$

and substituting in (3'):

$$V_{\pi 2}(125.5) = (2.20)[-V_{\pi 1}(170.16) - V_{\pi 2}(11.71)] - V_{\pi 1}(0.20)$$

or $V_{\pi 2}(151.3) = -V_{\pi 1}(374.55)$ so that

$$V_{\pi 1} = -V_{\pi 2}(0.4040)$$

Then

$$I_S = (35.75)[-V_{\pi 2}(0.4040)] + V_{\pi 2}(2.342)$$

$$I_S = -V_{\pi 2}(12.10)$$

$$I_0 = -(g_{m2}V_{\pi 2})\left(\frac{R_{C2}}{R_{C2} + R_L}\right)$$

$$= -(123)\left(\frac{2.2}{2.2 + 2}\right)V_{\pi 2} = -(64.43)V_{\pi 2}$$

$$\text{or } V_{\pi 2} = -(0.01552)I_0$$

Then

$$\frac{I_0}{I_S} = \frac{1}{(0.01552)(12.10)} \Rightarrow \frac{I_0}{I_S} = 5.33$$

12.35

For example, use the circuit shown in Figure P12.30

12.36

$$r_{\pi 1} = 6.24 \text{ k}\Omega, r_{\pi 2} = 3.12 \text{ k}\Omega, r_{\pi 3} = 1.56 \text{ k}\Omega$$

$$g_{m1} = 19.23 \text{ mA/V}, g_{m2} = 38.46 \text{ mA/V},$$

$$g_{m3} = 76.92 \text{ mA/V}$$

$$\text{From (1)} V_{\pi 1} = V_S - V_{e1}$$

Then

$$(V_S - V_{e1})(19.39) = V_{e1}(11.25) - V_{e3}(1.25)$$

or

$$V_S(19.39) = V_{e1}(30.64) - V_{e3}(1.25) \quad (2')$$

$$V_{\pi 2} = -V_S(36.94) + V_{e1}(36.94) \quad (3')$$



$$(38.46)[-V_S(36.94) + V_{e1}(36.94)] + V_{\pi 3}(1.141) + V_{e3}(0.5) = 0 \quad (4)$$

$$\text{From (5)}: V_{\pi 3} = V_{\pi 1}(6.894) + V_{e1}(0.1111)$$

Then

$$V_S(19.39) = V_{e1}(30.64) - (1.25)[V_{\pi 3}(6.894) + V_{e1}(0.1111)]$$

or

$$V_S(19.39) = V_{e1}(30.50) - V_{\pi 3}(8.6175) \quad (2'')$$

$$V_S = V_{\pi 1} + V_{e1} \quad (1)$$

$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} = \frac{V_{e1}}{R_{e1}} + \frac{V_{e1} - V_{\pi 3}}{R_F} \quad (2)$$

$$V_{\pi 2} = -g_{m1}V_{\pi 1}(R_{C1} \parallel r_{\pi 2}) \quad (3)$$

$$g_{m2}V_{\pi 2} + \frac{V_{\pi 3} + V_{e3}}{R_{C2}} + \frac{V_{\pi 3}}{r_{\pi 3}} = 0 \quad (4)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3}V_{\pi 3} = \frac{V_{e3}}{R_{e2}} + \frac{V_{e3} - V_{e1}}{R_F} \quad (5)$$

Enter numerical values in (2)-(5):

$$\frac{V_{\pi 1}}{6.24} + (19.23)V_{\pi 1} = V_{e1}\left(\frac{1}{0.1} + \frac{1}{0.8}\right) - V_{e3}\left(\frac{1}{0.8}\right)$$

or

$$V_{\pi 1}(19.39) = V_{e1}(11.25) - V_{e3}(1.25) \quad (2)$$

$$V_{\pi 2} = -(19.23)V_{\pi 1}(5 \parallel 3.12) = -(36.94)V_{\pi 1} \quad (3)$$

$$(38.46)V_{\pi 2} + V_{\pi 3}\left(\frac{1}{2} + \frac{1}{1.56}\right) + V_{e3}\left(\frac{1}{2}\right) = 0$$

or

$$V_{\pi 2}(38.46) + V_{\pi 3}(1.141) + V_{e3}(0.5) = 0 \quad (4)$$

$$V_{\pi 3}\left(\frac{1}{1.56} + 76.92\right) = V_{e3}\left(\frac{1}{0.1} + \frac{1}{0.8}\right) - V_{e3}\left(\frac{1}{0.8}\right)$$

or

$$V_{\pi 3}(77.56) = V_{e3}(11.25) - V_{e1}(1.25) \quad (5)$$

and

$$-V_S(1420.7) + V_{e1}(1420.7) + V_{\pi 3}(1.141) + (0.5)[V_{\pi 3}(6.894) + V_{e1}(0.1111)] = 0$$

or

$$-V_S(1420.7) + V_{e1}(1420.76) + V_{\pi 3}(4.588) = 0 \quad (4'')$$

From (2''):

$$V_{e1} = V_S(0.6357) + V_{\pi 3}(0.2825)$$

Then substituting in (4''):

$$-V_S(1420.7) + (1420.76)[V_S(0.6357) + V_{\pi 3}(0.2825)] + V_{\pi 3}(4.588) = 0$$

$$-V_S(517.5) + V_{\pi 3}(405.95) = 0$$

Now

$$I_0 = g_{m3}V_{\pi 3} = 76.92V_{\pi 3} \text{ or } V_{\pi 3} = I_0(0.0130)$$

$$\text{Then } -V_S(517.5) + I_0(0.0130)(405.95) = 0$$

or

$$\underline{\underline{\frac{I_0}{V_S} = 98.06 \text{ mA/V}}}$$

12.38

$$r_{\pi 1} = r_{\pi 2} = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$g_{m1} = g_{m2} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(100)(0.026)}{2} = 1.3 \text{ k}\Omega$$

$$g_{m3} = \frac{2}{0.026} = 76.92 \text{ mA/V}$$

$$(77.71)V_{\pi 3} = (0.0989)V_s$$

or

$$V_{\pi 3} = (0.001273)V_s$$

so that

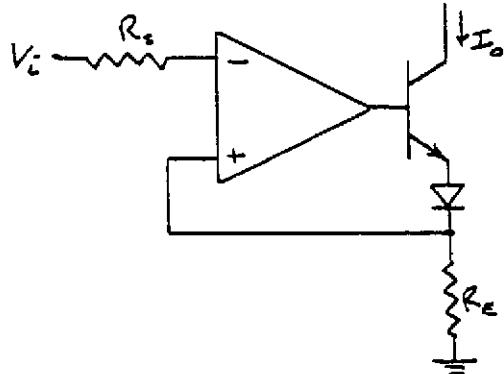
$$I_o = -(51.28)(0.001273)V_s$$

or

$$\frac{I_o}{V_s} = -(0.0653) \text{ mA/V}$$

12.39

Use the basic circuit shown in Figure 12.27.



$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} + g_{m2}V_{\pi 2} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \quad (1)$$

Since $r_{\pi 1} = r_{\pi 2}$ and $g_{m1} = g_{m2}$, then $V_{\pi 1} = -V_{\pi 2}$

$$V_s = V_{\pi 1} - V_{\pi 2} + V_{e3} = -2V_{\pi 2} + V_{e3} \quad (2)$$

$$g_{m2}V_{\pi 2} + \frac{V_{\pi 3}}{r_{\pi 3}} + \frac{V_{\pi 3} + V_{e3}}{R_{C2}} = 0 \quad (3)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3}V_{\pi 3} = \frac{V_{e3}}{R_F} + \frac{V_{\pi 2}}{r_{\pi 2}} \quad (4)$$

$$I_o = -\left(\frac{R_{C3}}{R_{C3} + R_L}\right)(g_{m3}V_{\pi 3}) \quad (5)$$

From (2): $V_{e3} = V_s + 2V_{\pi 2}$

$$(19.23)V_{\pi 2} + \frac{V_{\pi 3}}{1.3} + \frac{V_{\pi 3}}{18.6} + \frac{1}{18.6}(V_s + 2V_{\pi 2}) = 0$$

or

$$(19.23)V_{\pi 2} + (0.8230)V_{\pi 3} + (0.05376)V_s = 0 \quad (3')$$

$$V_{\pi 3}\left(\frac{1}{1.3} + 76.92\right) = \left(\frac{1}{10}\right)(V_s + 2V_{\pi 2}) + \frac{V_{\pi 2}}{5.2}$$

or

$$(77.69)V_{\pi 3} = (0.3923)V_{\pi 2} + (0.1)V_s \quad (4')$$

$$I_o = -\left(\frac{2}{2+1}\right)(76.92)V_{\pi 3} = -(51.28)V_{\pi 3} \quad (5')$$

From (3'):

$$V_{\pi 2} = -(0.04255)V_{\pi 3} - (0.002780)V_s$$

Then

$$(77.69)V_{\pi 3} = (0.3923)[-(0.04255)V_{\pi 3} - (0.002780)V_s] + (0.1)V_s$$

For the ideal case

$$\frac{I_o}{V_s} = \frac{1}{R_s}$$

we want

$$\frac{I_o}{V_s} = 10^{-3} \text{ A/V} = 1 \text{ mA/V}$$

 Set $R_s = 1 \text{ k}\Omega$

Since the op-amp has a finite gain, finite input resistance, and finite output resistance, the closed-loop gain is slightly less than the ideal. R_E will need to be slightly decreased to increase the gain.

12.40

dc analysis

$$I_E R_E + V_{BE(\text{on})} + I_B R_B + V_{CC} = ?$$

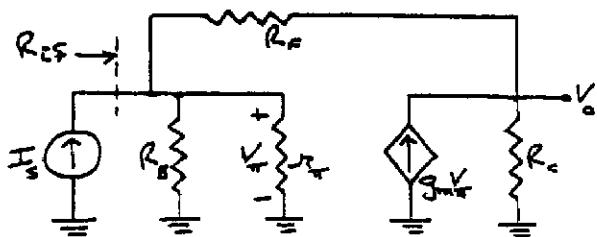
$$I_B = \frac{5 - 0.7}{100 + (51)(0.5)} = 0.0343$$

$$I_C = (50)(0.0343) = 1.71 \text{ mA}$$

$$\text{Then } r_v = \frac{(50)(0.026)}{1.71} = 0.760 \text{ k}\Omega$$

$$g_m = \frac{1.71}{0.026} = 65.77 \text{ mA/V}$$

a.

To determine R_V :

$$I_S + \frac{V_\pi}{R_B || r_\pi} + \frac{V_0 - (-V_\pi)}{R_F} = 0 \quad (1)$$

$$g_m V_\pi = \frac{V_0}{R_C} + \frac{V_0 - (-V_\pi)}{R_F} \quad (2)$$

Now from (2):

$$(65.77)V_\pi - \frac{V_\pi}{10} = V_0 \left(\frac{1}{1} + \frac{1}{10} \right)$$

$$(65.67)V_\pi = V_0(1.10)$$

or

$$V_0 = (59.7)V_\pi$$

and from (1):

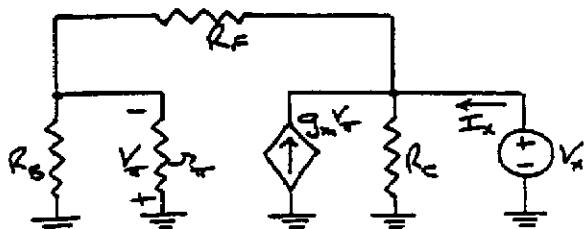
$$I_S + \frac{V_\pi}{100 || 0.760} + \frac{V_\pi}{10} + \frac{V_0}{10} = 0$$

$$I_S + V_\pi(0.8543) + (0.1)(59.7)V_\pi = 0$$

$$I_S = -V_\pi(6.824)$$

Now

$$R_{if} = \frac{(-V_\pi)}{I_S} \Rightarrow R_{if} = 147 \Omega$$

To determine R_V :

$$I_X = \frac{V_X}{R_C} + \frac{V_X}{R_F + R_B || r_\pi} - g_m V_\pi \quad (3)$$

$$V_\pi = \left(\frac{-(R_B || r_\pi)}{(R_B || r_\pi) + R_F} \right) (V_X) \quad (4)$$

Now

$$V_\pi = \left(\frac{-(100 || 0.760)}{(100 || 0.760) + 10} \right) (V_X) = -(0.07014)V_X$$

so

$$I_X = V_X \left(\frac{1}{1} + \frac{1}{10.754} + (65.77)(0.07014) \right)$$

$$R_{of} = \frac{V_X}{I_X} \Rightarrow R_{of} = 175 \Omega$$

b. From part (a), we find

$$V_\pi = -\frac{I_S}{6.824}$$

then

$$V_0 = (59.7) \left(\frac{-I_S}{6.824} \right)$$

or

$$\frac{V_0}{V_S} = -8.75 \text{ k}\Omega$$

c. If capacitance is finite, a phase shift will be introduced.

12.41

dc analysis: $V_{GS} = V_{DS}$

$$I_D = \frac{V_{DD} - V_{GS}}{R_D} = K_n(V_{GS} - V_{TN})^2$$

$$10 - V_{GS} = (0.20)(8)(V_{GS} - 2)^2$$

$$10 - V_{GS} = 1.6(V_{GS}^2 - 4V_{GS} + 4)$$

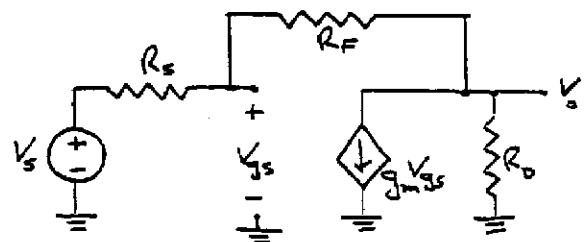
$$1.6V_{GS}^2 - 5.4V_{GS} - 3.6 = 0$$

$$V_{GS} = \frac{5.4 \pm \sqrt{(5.4)^2 + 4(1.6)(3.6)}}{2(1.6)} = 3.95 \text{ V}$$

$$I_D = \frac{10 - 3.95}{8} = 0.756 \text{ mA}$$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{0.2)(0.756)}$$

$$\Rightarrow g_m = 0.778 \text{ mA/V}$$



a.

$$\frac{V_{gs} - V_S}{R_S} + \frac{V_{gs} - V_0}{R_F} = 0$$

$$V_{gs} \left(\frac{1}{R_S} + \frac{1}{R_F} \right) = \frac{V_S}{R_S} + \frac{V_0}{R_F} \quad (1)$$

$$\frac{V_0}{R_D} + g_m V_{gs} + \frac{V_0 - V_{gs}}{R_F} = 0$$

$$V_0 \left(\frac{1}{R_D} + \frac{1}{R_F} \right) = V_{gs} \left(\frac{1}{R_F} - g_m \right) \quad (2)$$

So from (1):

$$V_{gs} \left(\frac{1}{10} + \frac{1}{100} \right) = \frac{V_S}{10} + \frac{V_0}{100}$$

or

$$V_{gs}(0.11) = V_S(0.10) + V_0(0.010)$$

$$V_{gs} = V_S(0.909) + V_0(0.0909)$$

Then from (2):

$$V_0 \left(\frac{1}{8} + \frac{1}{100} \right) = V_{gs} \left(\frac{1}{100} - 0.778 \right)$$

$$V_0(0.135) = V_{gs}(-0.768)$$

$$= (-0.768)[V_S(0.909) + V_0(0.0909)]$$

$$V_0(0.2048) = -V_S(0.6981)$$

so

$$A_v = \frac{V_0}{V_S} = -3.41$$

b. We have

$$\begin{aligned} V_{gs} &= V_S(0.909) + V_0(0.0909) \\ &= V_S(0.909) + (0.0909)(-3.41V_S) \\ &= 0.599V_S \end{aligned}$$

Now

$$A_{zf} = \frac{V_0}{I_S} = \frac{V_0}{V_S - V_{gs}} = \frac{(-3.41V_S)R_S}{V_S - 0.599V_S}$$

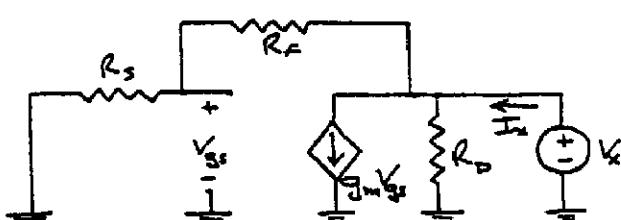
or

$$A_{zf} = \frac{(-3.41)(10)}{0.401} \Rightarrow A_{zf} = -85.0 \text{ V/mA}$$

$$\text{c. } R_{zf} = \frac{V_{gs}}{I_S} = \frac{V_{gs}}{\frac{0.401V_S}{R_S}} = \frac{0.599V_S}{0.401V_S} \cdot (10)$$

$$\Rightarrow R_{zf} = 14.9 \text{ k}\Omega$$

d.



$$\begin{aligned} I_X &= \frac{V_X}{R_D} + g_m V_{gs} + \frac{V_X}{R_S + R_F} \\ V_{gs} &= \left(\frac{R_S}{R_S + R_F} \right) V_X = \left(\frac{10}{10 + 100} \right) V_X \\ &= (0.0909)V_X \end{aligned}$$

$$\begin{aligned} I_X &= V_X \left[\frac{1}{8} + (0.778)(0.0909) + \frac{1}{10 + 100} \right] \\ \frac{I_X}{V_X} &= \frac{1}{R_D} = 0.2048 \Rightarrow R_{zf} = 4.88 \text{ k}\Omega \end{aligned}$$

12.42

$$\text{As } g_m \rightarrow \infty, \frac{V_0}{V_S} = \frac{-R_F}{R_S} = \frac{-100}{10} = -10$$

To be within 10% of ideal,

$$\frac{V_0}{V_S} = -10(0.9) = -9$$

From Problem 12.41, we had

$$\begin{aligned} V_{gs} &= V_S(0.909) + V_0(0.0909) \\ &= V_S(0.909) + (-9V_S)(0.0909) \\ &= 0.0909V_S \end{aligned}$$

Also from Problem 12.41, we had

$$V_0(0.135) = V_{gs}(0.010 - g_m)$$

or

$$(-9V_S)(0.135) = (0.0909)V_S(0.010 - g_m)$$

$$-1.215 = 0.000909 - 0.0909g_m$$

or

$$g_m = 13.36 \text{ mA/V}$$

12.43

dc analysis

$$R_{TH} = 24\parallel 150 = 20.7 \text{ k}\Omega$$

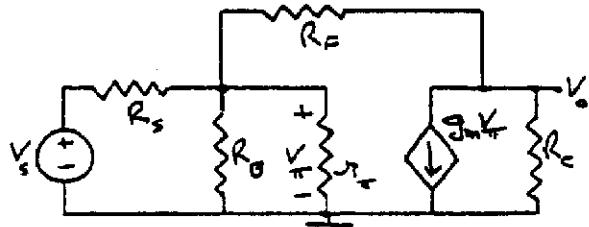
$$V_{TH} = \left(\frac{24}{24 + 150} \right) (12) = 1.655 \text{ V}$$

$$I_{BQ} = \frac{1.655 - 0.7}{20.7 + (151)(1)} = 0.00556 \text{ mA}$$

$$\text{so } I_{CQ} = 0.834 \text{ mA}$$

$$r_e = \frac{(150)(0.026)}{0.834} = 4.68 \text{ k}\Omega$$

$$g_m = \frac{0.834}{0.026} = 32.08 \text{ mA/V}$$



$$\frac{V_s - V_\pi}{R_s} = \frac{V_\pi}{R_B || r_\pi} + \frac{V_\pi - V_0}{R_F} \quad (1)$$

$$g_m V_\pi + \frac{V_0}{R_C} + \frac{V_0 - V_\pi}{R_F} = 0 \quad (2)$$

From (1):

$$\frac{V_s}{5} = V_\pi \left[\frac{1}{5} + \frac{1}{20.7 \parallel 4.68} + \frac{1}{R_F} \right] - \frac{V_0}{R_F}$$

or

$$V_s(0.20) = V_\pi \left(0.4620 + \frac{1}{R_F} \right) - \frac{V_0}{R_F}$$

From (2):

$$\left(32.08 - \frac{1}{R_F} \right) V_\pi + V_0 \left(\frac{1}{6} + \frac{1}{R_F} \right) = 0$$

so

$$V_\pi = \frac{-V_0 \left(0.1667 + \frac{1}{R_F} \right)}{\left(32.08 - \frac{1}{R_F} \right)} \quad (2)$$

Then

$$V_s(0.20)$$

$$= \left(0.4620 + \frac{1}{R_F} \right) \left[\frac{-V_0 \left(0.1667 + \frac{1}{R_F} \right)}{\left(32.08 - \frac{1}{R_F} \right)} \right] - \frac{V_0}{R_F}$$

Neglect the R_F in the denominator term. Now

$$\frac{V_0}{V_s} = -5 \Rightarrow V_s = -\frac{V_0}{5} = -V_0(0.20)$$

$$\begin{aligned} & -V_0(0.20)(0.20)R_F \\ & = (0.4620R_F + 1) \left[\frac{-V_0(0.1667R_F + 1)}{32.08R_F} \right] - V_0 \\ & -1.283R_F^2 = -(0.4620R_F + 1)(0.1667R_F + 1) \\ & \quad - 32.08R_F \end{aligned}$$

$$1.206R_F^2 - 32.71R_F - 1 = 0$$

$$R_F = \frac{32.71 \pm \sqrt{(32.71)^2 + 4(1.206)(1)}}{2(1.206)}$$

so that

$$\underline{R_F = 27.2 \text{ k}\Omega}$$

12.44

dc analysis

$$R_{TH} = 4 \parallel 15 = 3.16 \text{ k}\Omega = R_B$$

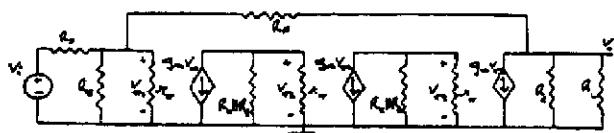
$$V_{TH} = \left(\frac{4}{4+15} \right) 12 = 2.526 \text{ V}$$

$$I_{BQ} = \frac{2.526 - 0.7}{3.16 + (181)(4)} = 0.00251$$

$$I_{CQ} = 0.452 \text{ mA}$$

$$r_\pi = \frac{(180)(0.026)}{0.452} = 10.4 \text{ k}\Omega$$

$$g_m = \frac{0.452}{0.026} = 17.4 \text{ mA/V}$$



$$\frac{V_i - V_{\pi1}}{R_s} = \frac{V_{\pi1}}{R_B \parallel r_\pi} + \frac{V_{\pi1} - V_0}{R_F} \quad (1)$$

$$g_m V_{\pi1} + \frac{V_{\pi2}}{R_C \parallel R_B \parallel r_\pi} = 0 \quad (2)$$

$$g_m V_{\pi2} + \frac{V_{\pi3}}{R_C \parallel R_B \parallel r_\pi} = 0 \quad (3)$$

$$g_m V_{\pi3} + \frac{V_0}{R_C} + \frac{V_0 - V_{\pi1}}{R_L} = 0 \quad (4)$$

Now

$$R_C \parallel R_B \parallel r_\pi = 8 \parallel 3.16 \parallel 10.4 = 1.86 \text{ k}\Omega$$

$$R_B \parallel r_\pi = 3.16 \parallel 10.4 = 2.42 \text{ k}\Omega$$

Now substituting in (2):

$$(17.4)V_{\pi1} + \frac{V_{\pi2}}{1.86} = 0 \text{ or } V_{\pi2} = -(32.36)V_{\pi1}$$

and substituting in (3):

$$(17.4)V_{\pi2} + \frac{V_{\pi3}}{1.86} = 0$$

$$(17.4)[-(32.36)V_{\pi1}] + \frac{V_{\pi3}}{1.86} = 0$$

$$\text{or } V_{\pi3} = (1047.3)V_{\pi1}$$

Substitute numerical values in (1):

$$\frac{V_i}{10} = V_{\pi 1} \left(\frac{1}{10} + \frac{1}{2.42} + \frac{1}{R_F} \right) - \frac{V_0}{R_F}$$

or

$$V_i(0.10) = V_{\pi 1} \left(0.513 + \frac{1}{R_F} \right) - \frac{V_0}{R_F}$$

Substitute numerical values in (4):

$$(17.4)(1047.3)V_{\pi 1} + V_0 \left(\frac{1}{8} + \frac{1}{4} + \frac{1}{R_F} \right) - \frac{V_{\pi 1}}{R_F} = 0$$

$$V_{\pi 1} \left(1.822 \times 10^4 - \frac{1}{R_F} \right) + V_0 \left(0.375 + \frac{1}{R_F} \right) = 0$$

$$V_{\pi 1} = \frac{-V_0 \left(0.375 + \frac{1}{R_F} \right)}{1.822 \times 10^4 - \frac{1}{R_F}}$$

so that

$$V_i(0.10)$$

$$= \left(0.513 + \frac{1}{R_F} \right) \left[\frac{-V_0 \left(0.375 + \frac{1}{R_F} \right)}{1.822 \times 10^4 - \frac{1}{R_F}} \right] - \frac{V_0}{R_F}$$

$$\text{We have } \frac{V_0}{V_i} = -80 \text{ or } V_i = -\frac{V_0}{80}$$

$$-\frac{(0.10)}{80}$$

$$= \left(0.513 + \frac{1}{R_F} \right) \left[\frac{-\left(0.375 + \frac{1}{R_F} \right)}{1.822 \times 10^4 - \frac{1}{R_F}} \right] - \frac{1}{R_F}$$

Neglect the $1/R_F$ term in the denominator.

$$\begin{aligned} -(0.00125R_F) &= -\frac{(0.513R_F + 1)(0.375R_F + 1)}{1.822 \times 10^4 R_F} - 1 \\ 22.775R_F^2 &= (0.513R_F + 1)(0.375R_F + 1) + 1.822 \times 10^4 R_F \end{aligned}$$

We find

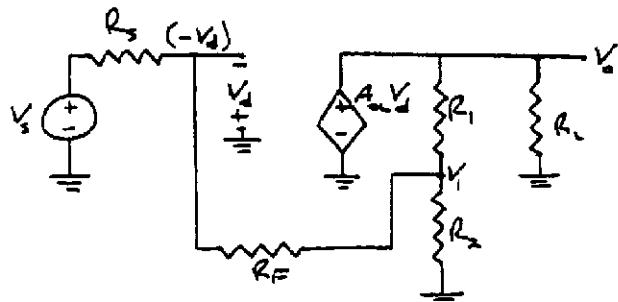
$$22.58R_F^2 - 1.822 \times 10^4 R_F - 1 = 0$$

$$R_F = \frac{1.822 \times 10^4 \pm \sqrt{(1.822 \times 10^4)^2 + 4(22.58)(1)}}{2(22.58)}$$

or

$$\underline{R_F = 0.807 \text{ M}\Omega}$$

12.45



a.

$$\frac{V_S - (-V_d)}{R_S} = \frac{-V_d - V_1}{R_F}$$

or

$$V_d \left(\frac{1}{R_S} + \frac{1}{R_F} \right) + \frac{V_S}{R_S} + \frac{V_1}{R_F} = 0 \quad (1)$$

$$\frac{V_0 - V_1}{R_1} = \frac{V_1}{R_2} + \frac{V_1 - (-V_d)}{R_F}$$

or

$$\frac{V_0}{R_1} = V_1 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_F} \right) + \frac{V_d}{R_F} \quad (2)$$

$$\text{and } V_0 = A_{vL} V_d \text{ or } V_d = \frac{V_0}{A_{vL}}$$

Substitute numerical values in (1) and (2):

$$\frac{V_0}{10^4} \cdot \left(\frac{1}{5} + \frac{1}{10} \right) + \frac{V_S}{5} + \frac{V_1}{10} = 0$$

or

$$V_0(0.3 \times 10^{-4}) + V_S(0.20) + V_1(0.10) = 0 \quad (1)$$

$$\frac{V_0}{50} = V_1 \left(\frac{1}{50} + \frac{1}{10} + \frac{1}{10} \right) + \frac{V_0}{10^4} \cdot \left(\frac{1}{10} \right)$$

or

$$V_0(0.02 - 10^{-5}) = V_1(0.22) \quad (2)$$

$$\text{Then } V_1 = V_0 \left(\frac{0.02 - 10^{-5}}{0.22} \right)$$

and

$$V_0(0.3 \times 10^{-4}) + V_S(0.20)$$

$$+ (0.10) \left[V_0 \left(\frac{0.02 - 10^{-5}}{0.22} \right) \right] = 0$$

$$V_0 [0.3 \times 10^{-4} - 0.4545 \times 10^{-5} + 0.00909] + V_S(0.20) = 0$$

$$\text{Then } \frac{V_0}{V_S} = \frac{-0.20}{9.115 \times 10^{-3}} \Rightarrow \frac{V_0}{V_S} = -21.94$$

$$\text{b. } R_{if} = \frac{-V_d}{V_S - (-V_d)} = \frac{-V_d \cdot R_S}{V_S + V_d}$$

$$\text{Now } V_d = \frac{V_0}{A_{oL}} = -\frac{21.94 V_S}{10^4}$$

$$\text{Then } R_{if} = \frac{(21.94 \times 10^{-4})(5)}{1 - 21.94 \times 10^{-4}}$$

$$\text{or } R_{if} = 1.099 \times 10^{-2} \text{ k}\Omega$$

$$\Rightarrow R_{if} = 10.99 \Omega$$

c. Because of the $A_{oL} V_d$ source,

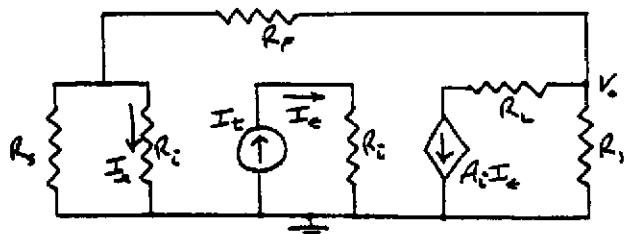
$$R_{of} = 0$$

12.46

For example, use the circuit shown in Figure 12.41

12.47

Break the loop



$$I_t = I_e$$

$$\text{Now } A_i I_t + \frac{V_0}{R_i} + \frac{V_0}{R_F + R_S \parallel R_i} = 0$$

$$I_r = \left(\frac{R_S}{R_S + R_i} \right) \cdot \frac{V_0}{R_F + R_S \parallel R_i}$$

$$\text{or } V_0 = I_r \left(\frac{R_S + R_i}{R_S} \right) \cdot (R_F + R_S \parallel R_i)$$

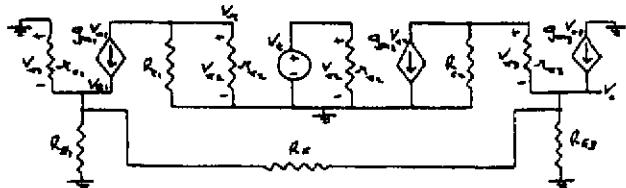
Then

$$A_i I_r + \left(\frac{1}{R_i} + \frac{1}{R_F + R_S \parallel R_i} \right) \times \left[I_r \left(\frac{R_S + R_i}{R_S} \right) (R_F + R_S \parallel R_i) \right] = 0$$

$$T = -\frac{I_r}{I_t} \Rightarrow$$

$$T = \frac{A_i}{\left[\frac{1}{R_i} + \frac{1}{R_F + R_S \parallel R_i} \right] \left(\frac{R_S + R_i}{R_S} \right) (R_F + R_S \parallel R_i)}$$

12.48



$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} = \frac{V_{e1}}{R_{E1}} + \frac{V_{e1} - V_0}{R_F} \quad (1)$$

$$g_{m1} V_{\pi 1} + \frac{V_r}{R_{C1} \parallel r_{\pi 2}} = 0 \\ \Rightarrow V_r = -(g_{m1} V_{\pi 1})(R_{C1} \parallel r_{\pi 2}) \quad (2)$$

$$V_{\pi 2} = V_r \text{ so that}$$

$$g_{m2} V_r + \frac{V_{\pi 3} + V_0}{R_{C2}} + \frac{V_{\pi 3}}{r_{\pi 3}} = 0 \quad (3)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3} V_{\pi 3} = \frac{V_0}{R_{E3}} + \frac{V_0 - V_{e1}}{R_F} \quad (4)$$

From (4):

$$V_0 \left(\frac{1}{R_{E3}} + \frac{1}{R_F} \right) = V_{\pi 3} \left(\frac{1}{r_{\pi 3}} + g_{m3} \right) + \frac{V_{e1}}{R_F}$$

$$\text{But } V_{e1} = -V_{\pi 1}$$

$$\text{so } V_0 = \frac{V_{\pi 3} \left(\frac{1}{r_{\pi 3}} + g_{m3} \right) - \frac{V_{\pi 1}}{R_F}}{\left(\frac{1}{R_{E3}} + \frac{1}{R_F} \right)}$$

Then

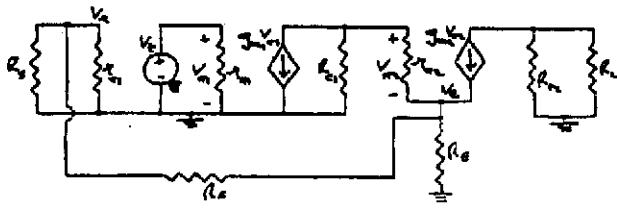
$$V_{\pi 1} \left[\left(\frac{1}{r_{\pi 1}} + g_{m1} \right) - \left(\frac{1}{R_{E1}} + \frac{1}{R_F} \right) \right] = \frac{-V_{\pi 3} \left(\frac{1}{r_{\pi 3}} + g_{m3} \right) + \frac{V_{\pi 1}}{R_F}}{R_F \cdot \left(\frac{1}{R_{E3}} + \frac{1}{R_F} \right)} \quad (1')$$

and

$$g_{m2} V_r + V_{\pi 3} \left(\frac{1}{R_{C2}} + \frac{1}{r_{\pi 3}} \right) + \frac{V_{\pi 3} \left(\frac{1}{r_{\pi 3}} + g_{m3} \right) - \frac{V_{\pi 1}}{R_F}}{R_{C2} \cdot \left(\frac{1}{R_{E3}} + \frac{1}{R_F} \right)} = 0 \quad (3')$$

From (3'), solve for $V_{\pi 3}$ and substitute into (1'). Then from (1'), solve for $V_{\pi 1}$ and substitute into (2). Then $T = -\frac{V_r}{V_t}$.

12.49



$$\frac{V_r}{R_s} + \frac{V_r}{r_{\pi 1}} + \frac{V_r - V_e}{R_F} = 0 \quad (1)$$

$$g_{m1} V_t + \frac{V_{\pi 2} + V_e}{R_{C1}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \quad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2} V_{\pi 2} = \frac{V_e}{R_B} + \frac{V_e - V_r}{R_F} \quad (3)$$

Using the parameters from Problem 12.29, we obtain

$$V_r \left(\frac{1}{10} + \frac{1}{15.8} + \frac{1}{10} \right) - \frac{V_e}{10} = 0$$

or

$$V_r(0.2633) = V_e(0.10) \quad (1)$$

$$(7.62)V_t + V_{\pi 2} \left(\frac{1}{40} + \frac{1}{2.28} \right) + \frac{V_e}{40} = 0$$

or

$$V_t(7.62) + V_{\pi 2}(0.4636) + V_e(0.025) = 0 \quad (2)$$

$$V_{\pi 2} \left(\frac{1}{2.28} + 52.7 \right) = V_e \left(\frac{1}{1} + \frac{1}{10} \right) - \frac{V_r}{10} \quad (2)$$

or

$$V_{\pi 2}(53.14) = V_e(1.10) - V_r(0.10)$$

Then

$$V_{\pi 2} = V_e(0.0207) - V_r(0.001882) \quad (3)$$

Substituting in (2):

$$V_t(7.62) + (0.4636)[V_e(0.0207) - V_r(0.001882)] + V_e(0.025) = 0$$

or

$$V_t(7.62) + V_e(0.03460) - V_r(0.0008725) = 0$$

From (1) $V_e = V_r(2.633)$

Then

$$V_t(7.62) + V_r(2.633)(0.03460) - V_r(0.0008725) = 0$$

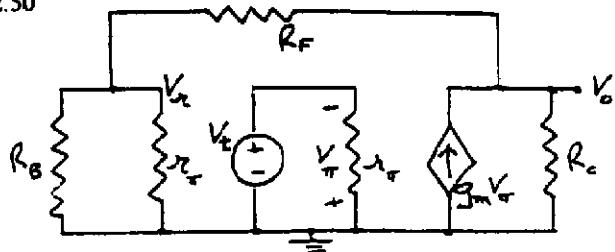
$$V_t(7.62) + V_r(0.09023) = 0$$

$$\text{or } \frac{V_r}{V_t} = -84.45$$

Now

$$T = -\frac{V_r}{V_t} \Rightarrow T = 84.45$$

12.50



$$V_\pi = -V_t$$

$$g_m V_\pi = \frac{V_0}{R_C} + \frac{V_0}{R_F + R_B \parallel r_\pi} \quad (1)$$

and

$$V_r = \left(\frac{R_B \parallel r_\pi}{R_B \parallel r_\pi + R_F} \right) V_0 \quad (2)$$

Now

$$(65.77)V_\pi = V_0 \left(\frac{1}{1} + \frac{1}{10 + 100 \parallel 0.760} \right)$$

$$\text{or } (65.77)V_\pi = V_0(1.0930)$$

and

$$V_r = \left(\frac{0.754}{10 + 0.754} \right) V_0 = (0.07011)V_0$$

$$\text{so } V_0 = (14.26)V_r$$

$$\text{Then } (65.77)(-V_t) = (14.26)V_r(1.0930)$$

$$\frac{V_r}{V_t} = -4.22 \text{ so that } T = 4.22$$

12.51

$$\text{a. } \phi = -\tan^{-1} \left(\frac{f}{5 \times 10^2} \right) - 2 \tan^{-1} \left(\frac{f}{10^4} \right)$$

or

$$-180 = -\tan^{-1} \left(\frac{f_{180}}{5 \times 10^2} \right) - 2 \tan^{-1} \left(\frac{f_{180}}{10^4} \right) \\ \Rightarrow f_{180} \approx 1.05 \times 10^4 \text{ Hz}$$

b.

$$|T(f_{180})| = 1$$

$$= \frac{\beta(10^5)}{\sqrt{1 + \left(\frac{1.05 \times 10^4}{5 \times 10^2} \right)^2} \left[1 + \left(\frac{1.05 \times 10^4}{10^4} \right)^2 \right]}$$

$$1 = \frac{\beta(10^5)}{(21.02)(2.105)}$$

or

$$\beta = 4.42 \times 10^{-4}$$

12.52

$$A = \frac{5 \times 10^3}{\left(1 + j \frac{f}{10^4}\right) \left(1 + j \frac{f}{10^5}\right)^2}$$

$$\text{Phase} = \phi = -\tan^{-1}\left(\frac{f}{10^4}\right) - 2\tan^{-1}\left(\frac{f}{10^5}\right)$$

By trial and error, when $f = 1.095 \times 10^5 \text{ Hz}$,
 $\phi \approx 180^\circ$

For $|T| = 1$ at $f = 1.095 \times 10^5 \text{ Hz}$,

$$1 = \frac{\beta(5 \times 10^3)}{\sqrt{1 + \left(\frac{f}{10^4}\right)^2} \cdot \sqrt{1 + \left(\frac{f}{10^5}\right)^2}} \Rightarrow$$

$$1 = \frac{\beta(5 \times 10^3)}{(10.996)(2.199)} \Rightarrow \beta = 4.84 \times 10^{-3}$$

12.53

$$\phi = -\tan^{-1}\left(\frac{f}{10^4}\right) - \tan^{-1}\left(\frac{f}{5 \times 10^4}\right) - \tan^{-1}\left(\frac{f}{10^5}\right)$$

At $f = 8.1 \times 10^4 \text{ Hz}$, $\phi = -180.28^\circ$

Determine $|T(f)|$ at this frequency.

 $|T|$

$$\begin{aligned} &= \beta(10^3) \times \frac{1}{\sqrt{1 + \left(\frac{8.1 \times 10^4}{10^4}\right)^2}} \\ &\times \frac{1}{\sqrt{1 + \left(\frac{8.1 \times 10^4}{5 \times 10^4}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{8.1 \times 10^4}{10^5}\right)^2}} \\ &= \frac{\beta(10^3)}{(8.161)(1.904)(1.287)} \end{aligned}$$

a. For $\beta = 0.005$

$|T(f)| = 0.250 < 1 \Rightarrow$ Stable

b. For $\beta = 0.05$

$|T(f)| = 2.50 > 1 \Rightarrow$ Unstable

12.54

(b) Phase margin = $80^\circ \Rightarrow \phi = -100^\circ$

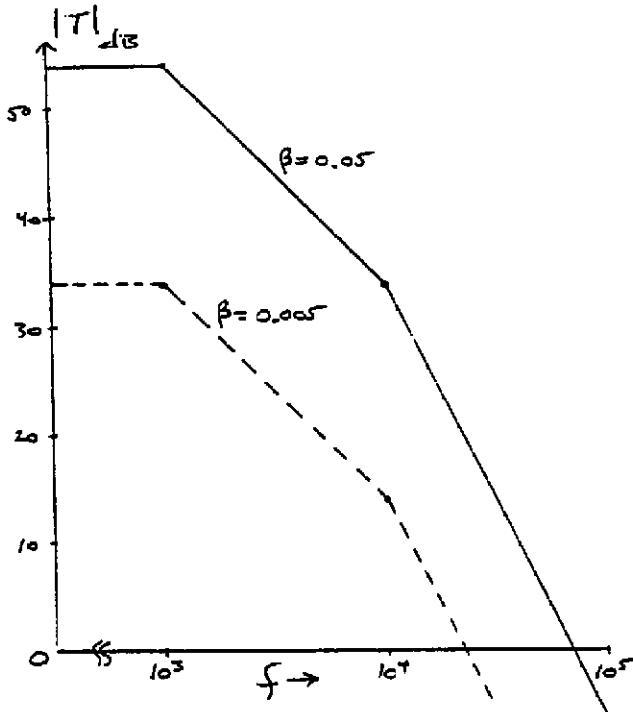
$$\phi = -100 = -2\tan^{-1}\left(\frac{f}{10^3}\right) - \tan^{-1}\left(\frac{f}{5 \times 10^4}\right)$$

By trial and error, $f = 1.16 \times 10^5 \text{ Hz}$

Then

$$\begin{aligned} |T| &= 1 = \frac{\beta(5 \times 10^3)}{\left(\sqrt{1 + \left(\frac{1.16 \times 10^5}{10^3}\right)^2}\right)^2 \cdot \sqrt{1 + \left(\frac{1.16 \times 10^5}{5 \times 10^4}\right)^2}} \\ &= \frac{\beta(5 \times 10^3)}{(2.35)(1.00)} \Rightarrow \beta = 4.7 \times 10^{-4} \end{aligned}$$

12.55

a. For $\beta = 0.005$

$|T(f)| = 0.250 < 1 \Rightarrow$ Stable

c. For $\beta = 0.005$,

$|T(f)| = 1$ (0 dB) at $f \approx 2.24 \times 10^4 \text{ Hz}$

Then

$$\begin{aligned} \phi &= -\tan^{-1}\left(\frac{2.24 \times 10^4}{10^3}\right) - \tan^{-1}\left(\frac{2.24 \times 10^4}{10^4}\right) \\ &\quad - \tan^{-1}\left(\frac{2.24 \times 10^4}{10^5}\right) \\ &= -87.44 - 65.94 - 12.63 \end{aligned}$$

or

$\phi = -166^\circ$ System is stable.

Phase margin = 14°

For $\beta = 0.05$,

$$|T(f)| = 1 \text{ (0 dB) at } f \approx 7.08 \times 10^4 \text{ Hz}$$

Then

$$\begin{aligned}\phi &= -\tan^{-1} \left(\frac{7.08 \times 10^4}{10^3} \right) - \tan^{-1} \left(\frac{7.08 \times 10^4}{10^4} \right) \\ &\quad - \tan^{-1} \left(\frac{7.08 \times 10^4}{10^5} \right)\end{aligned}$$

$$= -89.19 - 81.96 - 35.30$$

or

$$\phi = -206.45^\circ \Rightarrow \text{System is unstable.}$$

12.56

$$T = A\beta = \frac{\beta(10^5)}{\left(1 + j \frac{f}{5 \times 10^4}\right) \left(1 + j \frac{f}{10^5}\right) \left(1 + j \frac{f}{5 \times 10^5}\right)}$$

$$\text{Phase Margin} = 60^\circ \Rightarrow \phi = -120^\circ$$

So

$$-120 = -\tan^{-1} \left(\frac{f}{5 \times 10^4} \right) - \tan^{-1} \left(\frac{f}{10^5} \right) - \tan^{-1} \left(\frac{f}{5 \times 10^5} \right)$$

$$\text{By trial and error, at } f = 10^5 \text{ Hz, } \phi \equiv -120^\circ$$

Then

$$\begin{aligned}|T| = 1 &= \frac{\beta(10^5)}{\sqrt{1 + \left(\frac{10^5}{5 \times 10^4}\right)^2} \cdot \sqrt{1 + \left(\frac{10^5}{10^5}\right)^2} \cdot \sqrt{1 + \left(\frac{10^5}{5 \times 10^5}\right)^2}} \\ 1 &= \frac{\beta(10^5)}{(2.236)(1.414)(1.02)} \Rightarrow \underline{\beta = 3.22 \times 10^{-5}}$$

12.57

$$\text{a. Phase Margin} = 60^\circ \Rightarrow \phi = -120^\circ$$

Then

$$\phi = -120^\circ = -2 \tan^{-1} \left(\frac{f}{10^3} \right)$$

$$\text{or } f = 1.732 \times 10^3 \text{ Hz}$$

Then

$$|T(f)| = 1 = \frac{\beta(10^3)}{\left[1 + \left(\frac{1.732 \times 10^3}{10^3} \right)^2 \right]}$$

which yields

$$\underline{\beta = 4 \times 10^{-3}}$$

12.58

$$T(0) = A(0)\beta = (500)(0.6) = 300$$

$$T(f) = \frac{300}{\left(1 + j \frac{f}{10^4}\right) \left(1 + j \frac{f}{10^5}\right) \left(1 + j \frac{f}{10^6}\right)}$$

Find f at which $|T| = 1$

$$1 = \frac{300}{\sqrt{1 + \left(\frac{f_1}{10^4}\right)^2} \cdot \sqrt{1 + \left(\frac{f_1}{10^5}\right)^2} \cdot \sqrt{1 + \left(\frac{f_1}{10^6}\right)^2}}$$

$$\text{By trial and error, } f_1 = 5.12 \times 10^5 \text{ Hz}$$

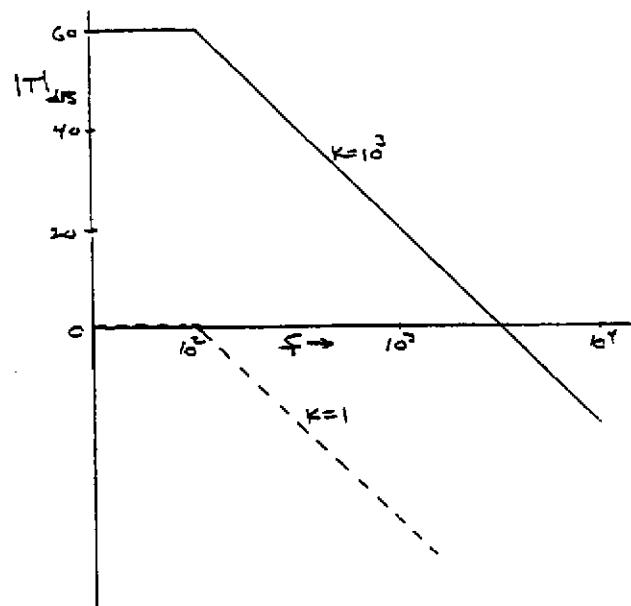
Then

$$\phi = -\tan^{-1} \left(\frac{f_1}{10^4} \right) - \tan^{-1} \left(\frac{f_1}{10^5} \right) - \tan^{-1} \left(\frac{f_1}{10^6} \right)$$

$$= -88.88 - 78.95 - 27.1 = -194.9^\circ$$

System is unstable, Phase margin is not defined.

12.59



12.60

$$\text{Phase Margin} = 45^\circ \Rightarrow \phi = -135^\circ$$

$$\phi = -135^\circ$$

$$\begin{aligned}&= -\tan^{-1} \left(\frac{f}{10^3} \right) - \tan^{-1} \left(\frac{f}{10^4} \right) - \tan^{-1} \left(\frac{f}{10^5} \right) \\ &\quad - \tan^{-1} \left(\frac{f}{10^6} \right)\end{aligned}$$

$$\text{At } f = 10^4 \text{ Hz, } \phi = -135.6^\circ$$

$$|T| = 1 = \beta(10^3) \times \frac{1}{\sqrt{1 + \left(\frac{10^4}{10^3}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{10^4}{10^4}\right)^2}} \times \\ \times \frac{1}{\sqrt{1 + \left(\frac{10^4}{10^5}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{10^4}{10^6}\right)^2}} \\ 1 = \frac{\beta(10^3)}{(10.05)(1.414)(1.005)(1.00)}$$

or

$$\underline{\beta = 0.01428}$$

12.61

$$T = 5000 \times \frac{1}{\left(1 + j\frac{f}{f_{PD}}\right)} \times \frac{1}{\left(1 + j\frac{f}{300 \times 10^3}\right)} \\ \times \frac{1}{\left(1 + j\frac{f}{2 \times 10^6}\right)} \times \frac{1}{\left(1 + j\frac{f}{25 \times 10^6}\right)}$$

$$\text{Phase Margin} = 45^\circ \Rightarrow \phi = -135^\circ \text{ at } f = 300 \text{ kHz} \\ -135^\circ = -\tan^{-1}\left(\frac{300 \times 10^3}{f_{PD}}\right) \\ -\tan^{-1}\left(\frac{300 \times 10^3}{300 \times 10^3}\right) - 0 - 0 \\ = -90^\circ - 45^\circ$$

Now

$$|T| = 1 @ f = 300 \text{ kHz}$$

$$|T| = 1 \approx \frac{5000}{\sqrt{1 + \left(\frac{300 \times 10^3}{f_{PD}}\right)^2} \cdot \sqrt{2} \cdot 1 \cdot 1}$$

$$1 + \left(\frac{300 \times 10^3}{f_{PD}}\right)^2 = \left(\frac{5000}{\sqrt{2}}\right)^2$$

$$f_{PD} \approx \frac{300 \times 10^3 \sqrt{2}}{5000} \\ \Rightarrow \underline{f_{PD} = 84.8 \text{ Hz}}$$

12.62

$$\text{a. } T(0) = 100 \text{ dB} \Rightarrow T(0) = 10^5$$

$$T(f)$$

$$= \frac{10^5}{\left(1 + j\frac{f}{10}\right)\left(1 + j\frac{f}{5 \times 10^6}\right)\left(1 + j\frac{f}{10 \times 10^6}\right)}$$

$$|T| = 1 = 10^5 \times \frac{1}{\sqrt{1 + \left(\frac{f}{10}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{f}{5 \times 10^6}\right)^2}} \\ \times \frac{1}{\sqrt{1 + \left(\frac{f}{10 \times 10^6}\right)^2}}$$

By trial and error

$$\underline{f = 0.976 \text{ MHz}}$$

$$\phi = -\tan^{-1}\left(\frac{0.976 \times 10^6}{10}\right) - \tan^{-1}\left(\frac{0.976}{5}\right) \\ - \tan^{-1}\left(\frac{0.976}{10}\right) \\ = -90^\circ - 11.05^\circ - 5.574^\circ = -106.6^\circ$$

$$\text{Phase Margin} = 180^\circ - 106.6^\circ = \underline{73.4^\circ}$$

$$\text{b. } f'_{P1} \propto \frac{1}{C_F} \text{ so } \frac{10}{f'_{P1}} = \frac{75}{20}$$

or

$$\underline{f'_{P1} = 2.67 \text{ Hz}}$$

Now

$$|T| = 1 = 10^5 \times \frac{1}{\sqrt{1 + \left(\frac{f}{2.67}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{f}{5 \times 10^6}\right)^2}} \\ \times \frac{1}{\sqrt{1 + \left(\frac{f}{10 \times 10^6}\right)^2}}$$

By trial and error

$$f \approx 2.66 \times 10^5 \text{ Hz}$$

then

$$\phi = -\tan^{-1}\left(\frac{2.66 \times 10^5}{2.67}\right) - \tan^{-1}\left(\frac{0.266}{5}\right) \\ - \tan^{-1}\left(\frac{0.266}{10}\right)$$

$$= -90^\circ - 3.045^\circ - 1.524^\circ = -94.57^\circ$$

$$\text{Phase Margin} = 180^\circ - 94.57^\circ = \underline{85.4^\circ}$$

12.63

$$(a) f_{3-dB} = \frac{1}{2\pi\tau} \text{ where } \tau = (R_{o1}\|R_{i2})C_i \\ = (500\|1000)\times 10^3 \times 2 \times 10^{-12} \Rightarrow \tau = 6.67 \times 10^{-7} \text{ s}$$

Then

$$f_{3-dB} = \frac{1}{2\pi(6.67 \times 10^{-7})} \Rightarrow f_{3-dB} = 239 \text{ kHz}$$

(b) For

$$f_{PD} = 10 \text{ Hz}, \quad \tau = \frac{1}{2\pi f_{PD}} = \frac{1}{2\pi(10)} = 0.0159 \text{ s}$$

$$\text{Then } \tau = (R_{o1}\|R_{i2})(C_i + C_M) \\ 0.0159 = (500\|1000)\times 10^3 \times (C_i + C_M)$$

or

$$(C_i + C_M) = 4.77 \times 10^{-8} = 2 \times 10^{-12} + C_M \Rightarrow \\ C_M = 477 \mu\text{F}$$

If we want a phase margin of 45° , then

$$-135^\circ \approx -90^\circ - \tan^{-1}\left(\frac{f}{10^6}\right) - \tan^{-1}\left(\frac{f}{10^7}\right)$$

By trial and error, $f \approx 0.845 \text{ MHz}$

Then

$$|T| = 1 = \frac{0.2 \times 10^4}{\sqrt{1 + \left(\frac{0.845 \times 10^6}{f_{PD}}\right)^2} \sqrt{1 + \left(\frac{0.845}{1}\right)^2}} \\ \times \frac{1}{\sqrt{1 + \left(\frac{0.845}{10}\right)^2}}$$

$$\frac{0.845 \times 10^4}{f_{PD}} \approx \frac{0.2 \times 10^4}{(1.309)(1.0036)} \\ \text{so } f_{PD} = 555 \text{ Hz}$$

12.64

Want $f_1 = 12 \text{ MHz}$ for a phase margin of 45°

$$T_{dB}(0) = 80 \text{ dB} \Rightarrow T(0) = 10^4$$

Then

$$T(f) = \frac{T(0)}{\left(1 + j \frac{f}{f_{PD}}\right) \left(1 + j \frac{f}{12 \times 10^6}\right)}$$

Set $f = f_1$ and $|T| = 1$

So

$$|T| = 1 = \frac{10^4}{\sqrt{1 + \left(\frac{12 \times 10^6}{f_{PD}}\right)^2} \cdot \sqrt{2}}$$

which yields

$$\frac{12 \times 10^6}{f_{PD}} = \frac{10^4}{\sqrt{2}} \Rightarrow f_{PD} = 1.70 \text{ kHz}$$

12.65

$$A_0 = 80 \text{ dB} \Rightarrow A_0 = 10^4$$

$$A_f(0) = \frac{A_0}{1 + \beta A_0}$$

$$\text{or } 5 = \frac{10^4}{1 + \beta(10^4)} \Rightarrow \beta \approx 0.2$$

$$\text{Then } T(0) = \beta A_0 = 0.2 \times 10^4$$

Inserting a dominate pole

$$\phi = -\tan^{-1}\left(\frac{f}{f_{PD}}\right) - \tan^{-1}\left(\frac{f}{10^6}\right) - \tan^{-1}\left(\frac{f}{10^7}\right)$$

12.66

Assuming a phase margin of 45° .

$$-135^\circ \approx -90^\circ - \tan^{-1}\left(\frac{f}{2 \times 10^6}\right) \\ - \tan^{-1}\left(\frac{f}{25 \times 10^6}\right)$$

By trial and error, $f \approx 1.74 \text{ MHz}$

Then

$$|T| = 1 \\ = 5000 \times \frac{1}{\sqrt{1 + \left(\frac{1.74 \times 10^6}{f_{PD}}\right)^2}} \\ \times \frac{1}{\sqrt{1 + \left(\frac{1.74}{2}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{1.74}{25}\right)^2}}$$

or

$$\frac{1.74 \times 10^6}{f_{PD}} \approx \frac{5000}{(1.325)(1.0024)} \\ \text{so } f_{PD} = 462 \text{ Hz}$$

Chapter 13

Exercise Solutions

E13.3

$$\begin{aligned} V_{iN}(\max) &= V^+ - V_{BE}(\text{on}) = 15 - 0.6 = 14.4 \text{ V} \\ V_{iN}(\min) &\geq 4V_{BE}(\text{on}) + V^+ \\ &= 4(0.6) - 15 = -12.6 \text{ V} \\ -12.6 \leq V_{iN}(\text{cm}) &\leq 14.4 \text{ V} \end{aligned}$$

E13.4

$$\begin{aligned} a. \quad V_0(\max) &\geq V^+ - 2V_{BE}(\text{on}) = 15 - 2(0.6) \\ V_0(\max) &= 13.8 \text{ V} \\ V_0(\min) &= 3V_{BE}(\text{on}) + V^- = 3(0.6) - 15 \\ V_0(\min) &\leq -13.2 \text{ V} \\ -13.2 \leq V_0 &\leq 13.8 \text{ V} \end{aligned}$$

$$\begin{aligned} b. \quad V_0(\max) &= 5 - 1.2 = 3.8 \text{ V} \\ V_0(\min) &\geq 3V_{BE} + V^- = 3(0.6) - 5 = -3.2 \text{ V} \\ -3.2 \leq V_0 &\leq 3.8 \text{ V} \end{aligned}$$

E13.5

$$\begin{aligned} I_{C1} = I_{C2} &\approx 9.5 \mu\text{A} \\ I_{B1} = I_{B2} &= \frac{9.5 \mu\text{A}}{200} = 0.0475 \mu\text{A} \\ \Rightarrow I_{B1} = I_{B2} &= 47.5 \text{nA} \end{aligned}$$

E13.6

$$\begin{aligned} I_{REF} &\approx \frac{15 - 2(0.6) - (-15)}{40} = 0.72 \text{ mA} \\ V_{BS} &= V_T \ln \left(\frac{I_{REF}}{I_S} \right) = (0.026) \ln \left(\frac{0.72 \times 10^{-3}}{10^{-14}} \right) \\ &= 0.650 \text{ V} \end{aligned}$$

So

$$\begin{aligned} I_{REF} &= \frac{30 - 2(0.65)}{40} \Rightarrow I_{REF} = 0.718 \text{ mA} \\ V_{BS11} &= 0.650 \text{ V} \end{aligned}$$

$$I_{C10}R_4 = V_T \ln \left(\frac{I_{REF}}{I_{C10}} \right)$$

$$I_{C10}(5) = (0.026) \ln \left(\frac{0.718}{I_{C10}} \right)$$

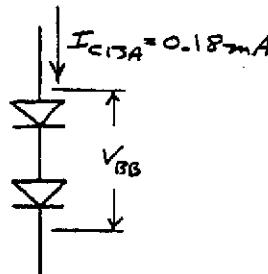
By trial and error: $I_{C10} = 18.9 \mu\text{A}$

$$\begin{aligned} V_{BS10} &= V_{BS11} - I_{C10}R_4 = 0.650 - (0.0189)(5) \\ \Rightarrow V_{BS10} &\approx 0.556 \text{ V} \end{aligned}$$

$$I_{C6} \approx \frac{I_{C10}}{2} = \frac{18.9}{2} = 9.45 \mu\text{A}$$

$$\begin{aligned} V_{BE6} &= V_T \ln \left(\frac{I_{C6}}{I_S} \right) = (0.026) \ln \left(\frac{9.45 \times 10^{-6}}{10^{-14}} \right) \\ \Rightarrow V_{BE6} &= 0.537 \text{ V} \end{aligned}$$

E13.7



$$0.18 \times 10^{-3} = 10^{-14} \exp \left(\frac{V_D}{V_T} \right)$$

$$\begin{aligned} V_D &= V_T \ln \left(\frac{0.18 \times 10^{-3}}{10^{-14}} \right) \\ &= (0.026) \ln \left(\frac{0.18 \times 10^{-3}}{10^{-14}} \right) \end{aligned}$$

$$V_D = 0.6140$$

$$V_{BB} = 2V_{DD} \approx 1.228 \text{ V}$$

$$I_{C14} = I_{C20} = I_S \exp \left(\frac{V_{BB}/2}{V_T} \right)$$

$$= 3 \times 10^{-14} \exp \left(\frac{0.6140}{0.026} \right)$$

$$I_{C14} = I_{C20} = 0.541 \text{ mA}$$

E13.8

$$\begin{aligned} I_{REF} &= \frac{10 - 0.6 - 0.6 - (-10)}{40} \\ \Rightarrow I_{REF} &= 0.47 \text{ mA} \end{aligned}$$

$$I_{C10}R_4 = V_T \ln \left(\frac{I_{REF}}{I_{C10}} \right)$$

$$I_{C10}(5) = (0.026) \ln \left(\frac{0.47}{I_{C10}} \right)$$

By trial and error:

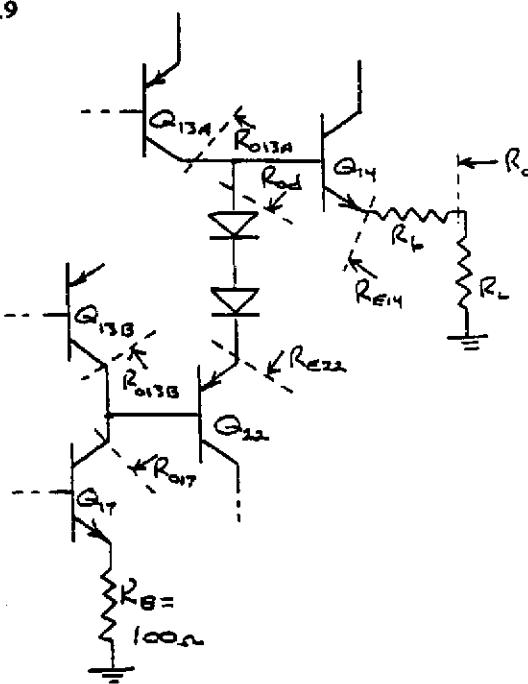
$$\Rightarrow I_{C10} \approx 17.2 \mu\text{A}$$

$$I_{C6} \approx \frac{I_{C10}}{2} \Rightarrow I_{C6} = 8.6 \mu\text{A}$$

$$I_{C13B} = (0.75)I_{REF} \Rightarrow I_{C13B} = 0.353 \text{ mA}$$

$$I_{C13A} = (0.25)I_{REF} \Rightarrow I_{C13A} = 0.118 \text{ mA}$$

E13.9



$$R_o = R_d + R_{E14}$$

$$R_{E14} = \frac{r_{\pi14} + R_{d4} \| R_{013A}}{1 + \beta_n}$$

The diode resistance can be found as

$$I_D = I_S \exp\left(\frac{V_D}{V_T}\right)$$

$$\frac{1}{r_d} = \frac{\partial I_D}{\partial V_D} = I_S \left(\frac{1}{V_T} \right) \cdot \exp\left(\frac{V_D}{V_T}\right) = \frac{I_D}{V_T}$$

or

$$r_d = \frac{V_T}{I_D} = \frac{V_T}{I_{C13A}} = \frac{0.026}{0.18} \Rightarrow 144 \Omega$$

$$R_{E22} = \frac{r_{\pi22} + R_{017} \| R_{013B}}{1 + \beta_P}$$

$$R_{013B} = r_{013B} = 92.6 \text{ k}\Omega$$

$$R_{017} = r_{017}[1 + g_{m17}(R_d \| r_{\pi17})] = 283 \text{ k}\Omega$$

From previous calculations

$$R_{E22} = 1.51 \text{ k}\Omega$$

$$R_{d4} = 2r_d + R_{E22} = 2(0.144) + 1.51 = 1.80 \text{ k}\Omega$$

$$R_{013A} = r_{013A} = 278 \text{ k}\Omega$$

$$r_{\pi14} = \frac{\beta_n V_T}{I_{C14}} = \frac{(200)(0.026)}{5} = 1.04 \text{ k}\Omega$$

$$R_{E14} = \frac{1.04 + 1.8 \| 278}{201} \Rightarrow 14.1 \Omega$$

$$R_o = R_d + R_{E14} = 27 + 14.1$$

$$\Rightarrow R_o \approx 41 \Omega$$

E13.10

For Q_6 we have $V_{SG8} = V_{SG6} = 1.06 \text{ V}$

So

$$V_{SD6}(\text{sat}) = 1.06 - 0.5 = 0.56 \text{ V}$$

For M_1 and M_2

$$I_D = \frac{I_Q}{2} = K_p(V_{SD1} + V_{TN})^2$$

$$\frac{0.0397}{2} = 0.125(V_{SG1} - 0.5)^2$$

$$\Rightarrow V_{SG1} = 0.898 \text{ V}$$

So maximum input voltage

$$= V^+ - V_{SD6}(\text{sat}) - V_{SG1}$$

$$= 5 - 0.56 - 0.898$$

$$\Rightarrow V_{IN}(\text{max}) = 3.54 \text{ V}$$

For M_3 ,

$$K_p = (6.25)(20) = 125 \mu A/V^2$$

$$I_{D3} = \frac{I_Q}{2} = \frac{39.7}{2} \mu A$$

$$\frac{39.7}{2} = 125(V_{GS3} - V_{TN})^2$$

$$V_{TN} = 0.5 \text{ V} \Rightarrow$$

$$V_{GS3} = 0.898 \text{ V}$$

$$V_{SD1}(\text{sat}) = 0.898 - 0.5 = 0.398 \text{ V}$$

$$V_{IN}(\text{min}) = V^- + V_{GS3} + V_{SD1}(\text{sat}) - V_{SG1}$$

$$= -5 + 0.898 + 0.398 - 0.898$$

$$V_{IN}(\text{min}) = -4.60 \text{ V}$$

$$-4.60 \leq V_{IN}(\text{cm}) \leq 3.54 \text{ V}$$

E13.11

$$V_0(\text{max}) = V^+ - V_{SD8}(\text{sat})$$

$$V_{SG8} = V_{SG5} = 1.06 \text{ V}$$

$$V_{SD8}(\text{sat}) = 1.06 - 0.5 = 0.56 \text{ V}$$

$$V_0(\text{max}) = 5 - 0.56 = 4.44 \text{ V}$$

$$V_0(\text{min}) = V^- + V_{DS7}(\text{sat})$$

$$V_{GS7} = 1.06 \Rightarrow V_{DS7}(\text{sat}) = 1.06 - 0.5 = 0.56$$

$$V_0(\text{min}) = -5 + 0.56 = -4.44$$

$$-4.44 \leq V_0 \leq 4.44 \text{ V}$$

E13.12

(a) For M_3 , $K_{ps} = 125 \mu A/V^2$

$$K_{ps}(V_{SG3} + V_{rr})^2 = \frac{V^+ - V^- - V_{SG3}}{R_{ss}}$$

$$0.125(V_{SG3} - 0.5)^2 = \frac{5 + 5 - V_{SG3}}{100}$$

$$12.5(V_{SG3}^2 - V_{SG3} + 0.25) = 10 - V_{SG3}$$

$$12.5V_{SG3}^2 - 11.5V_{SG3} - 6.875 = 0$$

$$V_{SG3} = \frac{11.5 \pm \sqrt{(11.5)^2 + 4(12.5)(6.875)}}{2(12.5)}$$

$$V_{SG3} = 1.33 \text{ V}$$

Then

$$I_{REF} = I_Q = I_{D6} = I_{D7} = \frac{10 - 1.33}{100} \Rightarrow 86.7 \mu A$$

$$I_{D1} = I_{D2} = I_{D3} = I_{D4} = \frac{I_Q}{2} = 43.35 \mu A$$

(b) $K_{p1} = K_{p2} = 125 \mu A/V^2$

$$r_{o2} = r_{o4} = \frac{1}{\lambda L_D} = \frac{1}{(0.02)(0.04335)} = 1153 k\Omega$$

Input stage gain

$$A_d = \sqrt{2K_{p1}I_Q} \cdot (r_{o2}\|r_{o4}) \\ = \sqrt{2(0.125)(0.0867)} \cdot (1153\|1153) \Rightarrow$$

$$A_d = 84.9$$

Transconductance of M_7 ,

$$g_{m7} = 2\sqrt{K_{m7}I_{D7}} = 2\sqrt{(0.250)(0.0867)} \\ = 0.294 mA/V$$

$$r_{o7} = r_{o8} = \frac{1}{\lambda L_{D7}} = \frac{1}{(0.02)(0.0867)} = 577 k\Omega$$

Second stage gain

$$A_{v2} = g_{m7}(r_{o7}\|r_{o8}) = (0.294)(577\|577) \Rightarrow$$

$$A_{v2} = 84.8$$

$$\text{Overall gain} = A_d \cdot A_{v2} = (84.9)(84.8) \Rightarrow$$

$$A = 7,200$$

E13.13

$$I_{D1} = I_{D2} = 25 \mu A$$

$$g_{m1} = g_{m2} = 2\sqrt{\frac{k_p}{2} \left(\frac{W}{L}\right) I_{D2}} = 2\sqrt{\left(\frac{40}{2}\right)(25)(25)} \Rightarrow$$

$$g_{m1} = g_{m2} = 224 \mu A/V$$

$$g_{m6} = 2\sqrt{\left(\frac{k_p}{2}\right)\left(\frac{W}{L}\right) I_{D2}} = 2\sqrt{\left(\frac{80}{2}\right)(25)(25)} \Rightarrow$$

$$g_{m6} = 316 \mu A/V$$

$$r_{o1} = r_{o6} = r_{o8} = r_{o10} = \frac{1}{\lambda L_D} = \frac{1}{(0.02)(25)} = 2 M\Omega$$

$$r_{o4} = \frac{1}{\lambda L_{D4}} = \frac{1}{(0.02)(50)} = 1 M\Omega$$

$$R_{os} = g_{m1}(r_{o1}r_{o10}) = (224)(2)(2) = 896 M\Omega$$

$$R_{os} = g_{m6}(r_{o6}\|r_{o1}) = 316(2)(12) = 421 M\Omega$$

Then

$$A_d = g_{m1}(R_{os}\|R_{os}) = 224(421\|896) \Rightarrow$$

$$A_d = 64,158$$

E13.14

$$(a) A_d = Bg_{m1}(r_{o6}\|r_{o8})$$

$$g_{m1} = 2\sqrt{\left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right) I_{D1}} = 2\sqrt{\left(\frac{80}{2}\right)(20)(50)}$$

$$g_{m1} = 400 \mu A/V$$

$$r_{o6} = \frac{1}{\lambda p I_{D6}} = \frac{1}{(0.02)(150)} = 0.333 M\Omega$$

$$r_{o8} = \frac{1}{\lambda n I_{D8}} = \frac{1}{(0.02)(150)} = 0.333 M\Omega$$

$$A_d = 3(400)(0.333\|0.333) \Rightarrow A_d = 200$$

$$(b) f_{PD} = \frac{1}{2\pi R_s(C_L + C_P)}$$

$$\text{where } R_s = r_{o6}\|r_{o8} = 0.333\|0.333 M\Omega$$

$$f_{PD} = \frac{1}{2\pi(0.333\|0.333) \times 10^6 \times 2 \times 10^{-12}} \Rightarrow$$

$$f_{PD} = 477 \text{ kHz}$$

$$f_{PD} \cdot A_d = (477 \times 10^3)(200) \Rightarrow 95.4 \text{ MHz}$$

E13.15

(a) From Exercise 13.14, $g_{m1} = 400 \mu A/V$

$$r_{o6} = r_{o8} = r_{o10} = r_{o12} = 0.333 M\Omega$$

$$g_{m10} = 2\sqrt{\left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right) I_{D10}} = 2\sqrt{\left(\frac{40}{2}\right)(20)(150)} \Rightarrow$$

$$g_{m10} = 490 \mu A/V$$

$$g_{m12} = 2\sqrt{\left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right) I_{D12}} = 2\sqrt{\left(\frac{80}{2}\right)(20)(150)} \Rightarrow$$

$$g_{m12} = 693 \mu A/V$$

$$R_{o10} = g_{m10}(r_{o10}r_{o6}) = (490)(0.333)(0.333) = 54.3 M\Omega$$

$$R_{o12} = g_{m12}(r_{o12}r_{o8}) = (693)(0.333)(0.333) = 76.8 M\Omega$$

$$A_d = Bg_{m1}(R_{o10}\|R_{o12}) = 3(400)(54.3\|76.8) \Rightarrow$$

$$A_d = 38,172$$

$$(b) R_o = R_{o10}\|R_{o12} = 54.3\|76.8 = 31.8 M\Omega$$

$$f_{PD} = \frac{1}{2\pi(31.8 \times 10^6)(2 \times 10^{-12})} = 2.50 \text{ kHz}$$

$$f_{PD} \cdot A_d = (2.5 \times 10^3)(38,172) \Rightarrow 95.4 \text{ MHz}$$

E13.16

$$(a) A_d = g_m (R_{os} \parallel R_{oe})$$

From Example 13.10,

$$g_m = 316 \mu A/V, R_{os} = 316 M\Omega$$

Now

$$R_{oe} = g_m (r_{oe}) (r_{oe} \parallel r_{o1})$$

$$r_{o1} = 1 M\Omega, r_{oe} = 0.5 M\Omega$$

$$g_{me} = \frac{I_{C6}}{V_T} = \frac{50}{0.026} \Rightarrow 1.923 mA/V$$

$$r_{oe} = \frac{V_{AE}}{I_{C6}} = \frac{80}{50} = 1.6 M\Omega$$

Then

$$R_{oe} = (1.923)(1600)(0.5 \parallel 1) = 1026 M\Omega$$

$$A_d = (316)(1026 \parallel 316) \Rightarrow \underline{A_d = 76,343}$$

$$(b) f_{PD} = \frac{1}{2\pi(316 \parallel 1026) \times 10^6 \times 2 \times 10^{-12}} \Rightarrow$$

$$\underline{f_{PD} = 329 \text{ Hz}}$$

$$\underline{f_{PD} \cdot A_d = (329)(76,343) \Rightarrow 25.1 \text{ MHz}}$$

E13.17

$$\begin{aligned} V^+ - V^- &= V_{EB1} + V_{EB6} + V_{BE7} + I_1 R_1 \\ &= 0.6 + 0.6 + 0.6 + (0.24)(8) = 3.72 \text{ V} \end{aligned}$$

So

$$\underline{V^+ = -V^- = 1.86 \text{ V}}$$

E13.18

For Q_7 and R_1

$$V_{SG} = V_{BB7} + I_1 R_1 = 0.6 + I_1(5)$$

For M_6 :

$$I_2 = K_p (V_{SG} + V_{\pi})^2$$

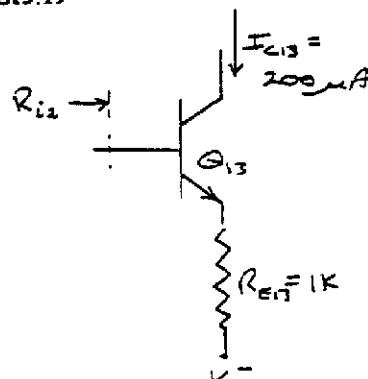
$$I_2 = 0.3(V_{SG} - 1.4)^2$$

By trial and error:

$$\underline{V_{SG} = 2.54 \text{ V}}$$

$$\underline{I_1 = I_2 = 0.388 \text{ mA}}$$

E13.19



$$r_{\pi13} = \frac{\beta V_T}{I_{C13}} = \frac{(200)(0.026)}{0.20} = 26 \text{ k}\Omega$$

$$\begin{aligned} R_{L2} &= r_{\pi13} + (1 + \beta) R_{E13} = 26 + 201(1) \\ &= 227 \text{ k}\Omega \end{aligned}$$

$$r_{010} = \frac{1}{\lambda I_{D10}} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$r_{012} = \frac{V_A}{I_{C12}} = \frac{50}{0.1} = 500 \text{ k}\Omega$$

$$g_{m12} = \frac{I_{C12}}{V_T} = \frac{0.1}{0.026} = 3.85 \text{ mA/V}$$

$$r_{\pi12} = \frac{\beta V_T}{I_{C12}} = \frac{(200)(0.026)}{0.1} = 52 \text{ k}\Omega$$

$$\begin{aligned} R_{act1} &= r_{012}[1 + g_{m12}(r_{\pi12} \parallel R_5)] \\ &= 500[1 + (3.85)(52 \parallel 0.5)] = 1453 \text{ k}\Omega \end{aligned}$$

$$\begin{aligned} A_d &= \sqrt{2K_p I_{D5} \cdot (r_{010} \parallel R_{act1} \parallel R_2)} \\ &= \sqrt{2(0.6)(0.2)} \cdot (500 \parallel 1453 \parallel 227) \\ &= (0.490)(141) \Rightarrow \underline{A_d = 69.1} \end{aligned}$$

E13.20

For J_6 biased in the saturation region

$$\Rightarrow I_{C3} = I_{DSS} = 300 \mu\text{A}$$

 Q_1, Q_2, Q_3 are matched

$$\Rightarrow \underline{I_{C1} = I_{C2} = I_{C3} = 300 \mu\text{A}}$$

Chapter 13

Problem Solutions

13.3

(a) $A_d = g_m(r_{o1} \parallel r_{o2} \parallel R_{i6})$

$$g_m = \frac{I_{C1}}{V_T} = \frac{20}{0.026} \Rightarrow 0.769 \text{ mA/V}$$

$$r_{o2} = \frac{V_{A2}}{I_{C2}} = \frac{80}{20} = 4 \text{ M}\Omega$$

$$r_{o4} = \frac{V_{A4}}{I_{C2}} = \frac{80}{20} = 4 \text{ M}\Omega$$

$$R_{i6} = r_{o6} + (1 + \beta_n)[R_1 \parallel r_{o7}]$$

$$r_{o7} = \frac{(120)(0.026)}{0.2} = 15.6 \text{ k}\Omega$$

$$I_{C6} \equiv \frac{V_{BE(on)}}{R_1} = \frac{0.6}{20} = 0.030 \text{ mA}$$

$$r_{o6} = \frac{(120)(0.026)}{0.030} = 104 \text{ k}\Omega$$

Then

$$R_{i6} = 104 + (121)[20 \parallel 15.6] \Rightarrow 1.16 \text{ M}\Omega$$

Then

$$A_d = 769(4 \parallel 1.16) \Rightarrow A_d = 565$$

Now

$$V_o = -I_{o1}r_{o1} = -(\beta_n I_{o1})r_{o1} = -\beta_n r_{o1} \left(\frac{R_1}{R_1 + r_{o7}} \right) I_{o1}$$

$$= -\beta_n (1 + \beta_n) r_{o1} \left(\frac{R_1}{R_1 + r_{o7}} \right) I_{o1} \text{ and } I_{o1} = \frac{V_o}{R_{i6}}$$

Then

$$A_{v2} = \frac{V_o}{V_{o1}} = \frac{-\beta_n (1 + \beta_n) r_{o1}}{R_{i6}} \left(\frac{R_1}{R_1 + r_{o7}} \right)$$

$$r_{o1} = \frac{V_A}{I_{C7}} = \frac{80}{0.2} = 400 \text{ k}\Omega$$

So

$$A_{v2} = \frac{-(120)(121)(400)}{1160} \left(\frac{20}{20 + 15.6} \right) \Rightarrow$$

$$A_{v2} = -2813$$

Overall gain = $A_d \cdot A_{v2} = (565)(-2813) \Rightarrow$

$$A = -1.59 \times 10^6$$

(b) $R_d = 2r_{o1}$ and $r_{o1} = \frac{(80)(0.026)}{0.020} = 104 \text{ k}\Omega$

$$R_d = 208 \text{ k}\Omega$$

(c) $f_{PD} = \frac{1}{2\pi R_{eq} C_M}$ and

$$C_M = (10)(1 + 2813) = 28,140 \text{ pF}$$

$$R_{eq} = r_{o2} \parallel r_{o4} \parallel R_{i6} = 4 \parallel 4 \parallel 1.16 = 0.734 \text{ M}\Omega$$

$$f_{PD} = \frac{1}{2\pi(0.734 \times 10^6)(28,140 \times 10^{-12})} = 7.71 \text{ Hz}$$

$$\text{Gain-Bandwidth Product} = (7.71)(1.59 \times 10^6) \Rightarrow 12.3 \text{ MHz}$$

13.4

- a. Q_3 acts as the protection device.
- b. Same as part (a).

13.5

If we assume $V_{BE(on)} = 0.7 \text{ V}$, then

$$V_{in} = 0.7 + 0.7 + 50 + 5$$

So breakdown voltage $\approx 56.4 \text{ V}$

13.6

(a) $I_{REF} = \frac{15 - 0.6 - 0.6 - (-15)}{R_s} = 0.50$
 $\Rightarrow R_s = 57.6 \text{ k}\Omega$

$$I_{C10} R_4 = V_T \ln \left(\frac{I_{REF}}{I_{C10}} \right)$$

$$R_4 = \frac{0.026}{0.030} \ln \left(\frac{0.50}{0.030} \right) \Rightarrow R_4 = 2.44 \text{ k}\Omega$$

(b) $I_{REF} = \frac{5 - 0.6 - 0.6 - (-5)}{57.6} \Rightarrow I_{REF} = 0.153 \text{ mA}$

$$I_{C10}(2.44) = (0.026) \ln \left(\frac{0.153}{I_{C10}} \right)$$

By trial and error, $I_{C10} \approx 21.1 \mu\text{A}$

13.7

(a) $I_{REF} \equiv 0.50 \text{ mA}$

$$V_{BE} = V_T \ln \left(\frac{I_{REF}}{I_s} \right) = (0.026) \ln \left(\frac{0.50 \times 10^{-3}}{10^{-14}} \right) \Rightarrow$$

$$V_{BE11} = 0.641 \text{ V} = V_{BE12}$$

Then

$$R_s = \frac{15 - 0.641 - 0.641 - (-15)}{0.50} \Rightarrow R_s = 57.4 \text{ k}\Omega$$

$$R_4 = \frac{0.026}{0.030} \ln \left(\frac{0.50}{0.030} \right) \Rightarrow R_4 = 2.44 \text{ k}\Omega$$

$$V_{BE10} = 0.026 \ln \left(\frac{0.030 \times 10^{-3}}{10^{-14}} \right) \Rightarrow V_{BE10} = 0.567 \text{ V}$$

(b) From Problem 13.6, $I_{REF} \approx 0.15 \text{ mA}$

$$V_{BB11} = V_{BB12} = 0.026 \ln \left(\frac{0.15 \times 10^{-3}}{10^{-14}} \right) = 0.609 \text{ V}$$

$$\text{Then } I_{REF} = \frac{5 - 0.609 - 0.609 - (-5)}{57.4} \Rightarrow$$

$$I_{REF} = 0.153 \text{ mA}$$

Then $I_{C10} \approx 21.1 \mu\text{A}$ from Problem 13.6

13.8

$$\text{a. } I_{REF} = \frac{5 - 0.6 - 0.6 - (-5)}{40} \Rightarrow I_{REF} = 0.22 \text{ mA}$$

$$I_{C10} R_4 = V_T \ln \left(\frac{I_{REF}}{I_{C10}} \right)$$

$$I_{C10}(5) = (0.026) \ln \left(\frac{0.22}{I_{C10}} \right)$$

By trial and error:

$$I_{C10} \approx 14.2 \mu\text{A}$$

$$I_{C6} \approx \frac{I_{C10}}{2} \Rightarrow I_{C6} = 7.10 \mu\text{A}$$

$$I_{C17} = 0.75 I_{REF} \Rightarrow I_{C17} = 0.165 \text{ mA}$$

$$I_{C13A} = 0.25 I_{REF} \Rightarrow I_{C13A} = 0.055 \text{ mA}$$

b. Using Example 13.4

$$r_{w17} = 31.5 \text{ k}\Omega$$

$$R'_E = 50 \parallel [31.5 + (201)(0.1)] = 50 \parallel 51.6 = 25.4 \text{ k}\Omega$$

$$r_{w16} = \frac{\beta_p V_T}{I_{C16}} \text{ and}$$

$$I_{C16} = \frac{0.165}{200} + \frac{(0.165)(0.1) + 0.6}{50} = 0.0132 \text{ mA}$$

$$r_{w16} = 394 \text{ k}\Omega$$

Then

$$R_{i2} = 394 + (201)(25.4) \Rightarrow 5.5 \text{ M}\Omega$$

$$r_{w6} = 732 \text{ k}\Omega$$

$$g_{ms} = \frac{0.00710}{0.026} = 0.273 \text{ mA/V}$$

$$r_{o6} = \frac{50}{0.0071} = 7.04 \text{ M}\Omega$$

Then

$$R_{act1} = 7.04[1 + (0.273)(1 \parallel 732)] = 8.96 \text{ M}\Omega$$

$$r_{o4} = \frac{50}{0.0071} = 7.04 \text{ M}\Omega$$

Then

$$A_d = - \left(\frac{7.1}{0.026} \right) (7.04 \parallel 8.96 \parallel 5.5)$$

or

$$A_d = -627 \text{ Gain of differential amp stage}$$

Using Example 13.5, and neglecting the input resistance to the output stage:

$$R_{act2} = \frac{V_A}{I_{C13B}} = \frac{50}{0.165} = 303 \text{ k}\Omega$$

$$A_{v2} = \frac{-(200)(201)(50)(303)}{(5500)[50 + 31.5 + (201)(0.1)]}$$

or

$$A_{v2} = -1090 \text{ Gain of second stage}$$

13.9

$$I_{C10} = 19 \mu\text{A}$$

From Equation (13.6)

$$I_{C10} = 2I \left[\frac{\beta_P^2 + 2\beta_P + 2}{\beta_P^2 + 3\beta_P + 2} \right] = 2I \left[\frac{(10)^2 + 2(10) + 2}{(10)^2 + 3(10) + 2} \right] = 2I \left[\frac{122}{132} \right]$$

So

$$2I = (19) \left(\frac{132}{122} \right) = 20.56 \mu\text{A}$$

$$I_{C2} = I = 10.28 \mu\text{A}$$

$$I_{C9} = \frac{2I}{\left(1 + \frac{2}{\beta_P} \right)} = \frac{20.56}{\left(1 + \frac{2}{10} \right)} \Rightarrow I_{C9} = 17.13 \mu\text{A}$$

$$I_{B9} = \frac{I_{C9}}{\beta_P} = \frac{17.13}{10} \Rightarrow I_{B9} = 1.713 \mu\text{A}$$

$$I_{B4} = \frac{I}{\left(1 + \beta_P \right)} = \frac{10.28}{11} \Rightarrow I_{B4} = 0.9345 \mu\text{A}$$

$$I_{C4} = I \left(\frac{\beta_P}{1 + \beta_P} \right) = (10.28) \left(\frac{10}{11} \right) \Rightarrow I_{C4} = 9.345 \mu\text{A}$$

13.10

$$V_{BS} - V^- = V_{BE}(\text{on}) + I_{C5}(1)$$

$$= 0.6 + (0.0095)(1) = 0.6095$$

$$I_{C7} = \frac{0.6095}{50} \Rightarrow I_{C7} = 12.2 \mu\text{A}$$

$$I_{C8} = I_{C9} = 19 \mu A$$

$$I_{REF} = 0.72 \text{ mA}$$

$$I_{E13} = I_{REF} = 0.72 \text{ mA}$$

$$I_{C14} = 138 \mu A$$

$$\begin{aligned} \text{Power} &= (V^+ - V^-)[I_{C7} + I_{C8} + I_{C9} \\ &\quad + I_{REF} + I_{E13} + I_{C14}] \\ &= 30[0.0122 + 0.019 + 0.019 \\ &\quad + 0.72 + 0.72 + 0.138] \\ &\Rightarrow \text{Power} = 48.8 \text{ mW} \end{aligned}$$

Current supplied by V^+ and V^-

$$\begin{aligned} &= I_{C7} + I_{C8} + I_{C9} + I_{REF} + I_{E13} + I_{C14} \\ &= 1.63 \text{ mA} \end{aligned}$$

13.11

$$(a) v_{cm}(\min) = -15 + 0.6 + 0.6 + 0.6 + 0.6 = -12.6 V$$

$$v_{cm}(\max) = +15 - 6 = 14.4 V$$

$$\text{So } -12.6 \leq v_{cm} \leq 14.4 V$$

$$(b) v_{cm}(\min) = -5 + 4(0.6) = -2.6 V$$

$$v_{cm}(\max) = 5 - 0.6 = 4.4 V$$

$$\text{So } -2.6 \leq v_{cm} \leq 4.4 V$$

13.12

If $v_0 = V^- = -15 \text{ V}$, the base voltage of Q_{14} is pulled low, and Q_{18} and Q_{19} are effectively cut off. As a first approximation

$$I_{C14} = \frac{0.6}{0.027} = 22.2 \text{ mA}$$

$$I_{B14} = \frac{22.2}{200} = 0.111 \text{ mA}$$

Then

$$I_{C15} = I_{C13A} - I_{B14} = 0.18 - 0.111 = 0.069 \text{ mA}$$

Now

$$\begin{aligned} V_{BE13} &= V_T \ln \left(\frac{I_{C15}}{I_S} \right) \\ &= (0.026) \ln \left(\frac{0.069 \times 10^{-3}}{10^{-14}} \right) \\ &= 0.589 \text{ V} \end{aligned}$$

As a second approximation

$$I_{C14} = \frac{0.589}{0.027} \Rightarrow I_{C14} = 21.8 \text{ mA}$$

$$I_{B14} = \frac{21.8}{200} = 0.109 \text{ mA}$$

and

$$I_{C15} = 0.18 - 0.109 \Rightarrow I_{C15} = 0.071 \text{ mA}$$

13.13

a. Neglecting base currents:

$$I_D = I_{BIAS}$$

Then

$$\begin{aligned} V_{BB} &= 2V_D = 2V_T \ln \left(\frac{I_D}{I_S} \right) \\ &= 2(0.026) \ln \left(\frac{0.25 \times 10^{-3}}{2 \times 10^{-14}} \right) \end{aligned}$$

or

$$\begin{aligned} V_{BB} &= 1.2089 \text{ V} \\ I_{CN} &= I_{CP} = I_S \exp \left(\frac{V_{BB}/2}{V_T} \right) \\ &= 5 \times 10^{-14} \exp \left(\frac{1.2089}{2(0.026)} \right) \end{aligned}$$

So

$$I_{CN} = I_{CP} = 0.625 \text{ mA}$$

b. For $v_I = 5 \text{ V}$, $v_0 \approx 5 \text{ V}$

$$i_L \approx \frac{5}{4} = 1.25 \text{ mA}$$

As a first approximation

$$I_{CN} \approx i_L = 1.25 \text{ mA}$$

$$V_{BEN} = (0.026) \ln \left(\frac{1.25 \times 10^{-3}}{5 \times 10^{-14}} \right) = 0.6225 \text{ V}$$

Neglecting base currents,

$$V_{BB} = 1.2089 \text{ V}$$

$$\text{Then } V_{EBP} = 1.2089 - 0.6225 = 0.5864 \text{ V}$$

$$I_{CP} = 5 \times 10^{-14} \exp \left(\frac{0.5864}{0.026} \right) \Rightarrow I_{CP} = 0.312 \text{ mA}$$

As a second approximation,

$$I_{CN} = i_L + I_{CP} = 1.25 + 0.31 \Rightarrow I_{CN} \approx 1.56 \text{ mA}$$

$$V_{BEN} = (0.026) \ln \left(\frac{1.56 \times 10^{-3}}{5 \times 10^{-14}} \right) = 0.62826 \text{ V}$$

$$V_{EBP} = 1.2089 - 0.62826 = 0.5806 \text{ V}$$

$$I_{CP} = 5 \times 10^{-14} \exp \left(\frac{0.5806}{0.026} \right) \Rightarrow I_{CP} = 0.25 \text{ mA}$$

13.14

$$R_1 + R_2 = \frac{V_{BB}}{(0.1)I_{BIAS}} = \frac{1.157}{0.018} = 64.28 \text{ k}\Omega$$

$$\begin{aligned} V_{BE} &= V_T \ln \left(\frac{I_C}{I_S} \right) = (0.026) \ln \left(\frac{(0.9)I_{BIAS}}{I_S} \right) \\ &= (0.026) \ln \left(\frac{0.162 \times 10^{-3}}{10^{-14}} \right) \end{aligned}$$

$$V_{BS} = 0.6112 \text{ V}$$

$$V_{BS} = \left(\frac{R_2}{R_1 + R_2} \right) V_{BB}$$

$$0.6112 = \left(\frac{R_2}{64.28} \right) (1.157)$$

So

$$\underline{R_2 = 33.96 \text{ k}\Omega}$$

Then

$$\underline{R_1 = 30.32 \text{ k}\Omega}$$

13.15

$$(a) A_d = -g_m (r_{o4} \| R_{12})$$

From example 13.4

$$g_m = \frac{9.5}{0.026} = 365 \mu\text{A/V}, r_{o4} = 5.26 M\Omega$$

Now

$$r_{o6} = r_{o4} = 5.26 M\Omega$$

Assuming $R_s = 0$, we find

$$R_{12} = r_{o16} + (1 + \beta_n) R'_s \\ = 329 + (201)(50 \| 9.63) \Rightarrow 1.95 M\Omega$$

Then

$$A_d = -(365)(5.26 \| 5.26 \| 1.95) \Rightarrow \underline{A_d = -409}$$

(b) From Equation (13.20),

$$A_{d2} = \frac{-\beta_n(1 + \beta_n) R_s (R_{o12} \| R_{13} \| R_{o17})}{R_{12} \{ R_s + [r_{o17} + (1 + \beta_n) R_s] \}}$$

For $R_s = 0$, $R_{12} = 1.95 M\Omega$

Using the results of Example 13.5

$$A_{d2} = \frac{-200(201)(50)(92.6 \| 4050 \| 92.6)}{(1950)\{50 + 9.63\}} \Rightarrow$$

$$\underline{A_{d2} = -792}$$

13.16

Let $I_{C10} = 40 \mu\text{A}$, then $I_{C1} = I_{C2} = 20 \mu\text{A}$. Using Example 13.5.

$$R_{12} = 4.07 M\Omega$$

$$r_{o6} = \frac{(200)(0.026)}{0.020} = 260 \text{ k}\Omega$$

$$g_{m6} = \frac{0.020}{0.026} = 0.769 \text{ mA/V}$$

$$r_{o6} = \frac{50}{0.02} \Rightarrow 2.5 M\Omega$$

Then

$$R_{oext1} = 2.5[1 + (0.769)(1 \| 260)] = 4.42 M\Omega$$

$$r_{o6} = \frac{50}{0.02} \Rightarrow 2.5 M\Omega$$

Then

$$A_d = - \left(\frac{I_{CQ}}{V_T} \right) (r_{o4} \| R_{oext1} \| R_{12})$$

$$= - \left(\frac{20}{0.026} \right) (2.5 \| 4.42 \| 4.07)$$

So

$$\underline{A_d = -882}$$

13.17

Now

$$R_{e14} = \frac{r_{\pi14} + R_{o1}}{1 + \beta_P} \text{ and } R_o = R_6 + R_{e14}$$

Assume series resistance of Q_{10} and Q_{19} is small. Then

$$R_{o1} = r_{013A} \| R_{e22}$$

$$\text{where } R_{e22} = \frac{r_{\pi22} + R_{o17} \| r_{013B}}{1 + \beta_P}$$

$$\text{and } R_{o17} = r_{017}[1 + g_{m17}(R_s \| r_{\pi17})]$$

Using results from Example 13.6,

$$r_{\pi17} = 9.63 \text{ k}\Omega \quad r_{\pi22} = 7.22 \text{ k}\Omega$$

$$g_{m17} = 20.8 \text{ mA/V} \quad r_{017} = 92.6 \text{ k}\Omega$$

Then

$$R_{o17} = 92.6[1 + (20.8)(0.1 \| 9.63)] = 283 \text{ k}\Omega$$

$$r_{013B} = \frac{50}{0.54} = 92.6 \text{ k}\Omega$$

Then

$$R_{e22} = \frac{7.22 + 283 \| 92.6}{51} = 1.51 \text{ k}\Omega$$

$$R_{o1} = r_{013A} \| R_{e22} = 278 \| 1.51 = 1.50 \text{ k}\Omega$$

$$r_{\pi14} = \frac{(50)(0.026)}{2} = 0.65 \text{ k}\Omega$$

Then

$$R_{e14} = \frac{0.55 + 1.50}{51} = 0.0422 \text{ k}\Omega$$

or

$$R_{e14} = 42.2 \text{ }\Omega$$

Then

$$R_o = 42.2 + 27 \Rightarrow \underline{R_o = 69.2 \Omega}$$

13.18

$$R_{id} = 2 \left[r_{n1} + (1 + \beta_n) \left(\frac{r_{n3}}{1 + \beta_P} \right) \right]$$

 Assume $\beta_n = 200$ and $\beta_P = 10$

Then

$$r_{n1} = \frac{(200)(0.026)}{0.0095} = 547 \text{ k}\Omega$$

$$r_{n3} = \frac{(10)(0.026)}{0.0095} = 27.4 \text{ k}\Omega$$

Then

$$R_{id} = 2 \left[547 + \frac{(201)(27.4)}{11} \right]$$

or

$$R_{id} = 2.095 \text{ M}\Omega$$

13.19

We can write

$$\begin{aligned} A(f) &= \frac{A_0}{\left(1 + j \frac{f}{f_{PD}}\right) \left(1 + j \frac{f}{f_1}\right)} \\ &= \frac{356,796}{\left(1 + j \frac{f}{5.43}\right) \left(1 + j \frac{f}{f_1}\right)} \end{aligned}$$

Phase:

$$\phi = -\tan^{-1} \left(\frac{f}{5.43} \right) - \tan^{-1} \left(\frac{f}{f_1} \right)$$

 For a phase margin = 70°, $\phi = -110^\circ$

So

$$-110^\circ = -\tan^{-1} \left(\frac{f}{5.43} \right) - \tan^{-1} \left(\frac{f}{f_1} \right)$$

 Assuming $f \gg 5.43$, we have

$$\tan^{-1} \left(\frac{f}{f_1} \right) = 20^\circ \Rightarrow \frac{f}{f_1} = 0.364$$

 At this frequency, $|A(f)| = 1$, so

$$\begin{aligned} 1 &= \frac{356,796}{\sqrt{1 + \left(\frac{f}{5.43} \right)^2} \cdot \sqrt{1 + (0.364)^2}} \\ &= \frac{335,275}{\sqrt{1 + \left(\frac{f}{5.43} \right)^2}} \end{aligned}$$

$$\text{or } \frac{f}{5.43} = 335,275 \Rightarrow f = 1.82 \text{ MHz}$$

Then, second pole at

$$f_1 = \frac{f}{0.364} \Rightarrow f_1 = 5 \text{ MHz}$$

13.20

 a. Original g_{m1} and g_{m2}

$$K_{p1} = K_{p2} = \left(\frac{W}{L} \right) \left(\frac{\mu_p C_{oe}}{2} \right) = (12.5)(10) \\ = 125 \mu A/V^2$$

So

$$g_{m1} = g_{m2} = 2 \sqrt{K_p \left(\frac{I_Q}{2} \right)} = 2 \sqrt{(0.125)(10)} \\ = 0.09975 \text{ mA/V}$$

 If $\left(\frac{W}{L} \right)$ is increased to 50, then

$$K_{p1} = K_{p2} = (50)(10) = 500 \mu A/V^2$$

So

$$g_{m1} = g_{m2} = 2 \sqrt{(0.5)(0.0199)} = 0.1995 \text{ mA/V}$$

b. Gain of first stage

$$A_d = g_{m1}(r_{o2} \parallel r_{o4}) = (0.1995)(5025 \parallel 5025)$$

or

$$\underline{A_d = 501}$$

Voltage gain of second stage remains the same, or

$$A_{v2} = 251$$

$$\text{Then } A_v = A_d \cdot A_{v2} = (501)(251)$$

or

$$\underline{A_v = 125,751}$$

13.22

$$\text{a. } K_p = (10)(20) = 200 \mu A/V^2 = 0.2 \text{ mA/V}^2$$

$$I_{REF} = I_{SET} = \frac{10 - V_{SG} - (-10)}{200}$$

$$= k_p(V_{SG} - 1.5)^2$$

$$20 - V_{SG} = (0.2)(200)(V_{SG}^2 - 3V_{SG} + 2.25)$$

$$40V_{SG}^2 - 119V_{SG} + 70 = 0$$

$$V_{SG} = \frac{119 \pm \sqrt{(119)^2 - 4(40)(70)}}{2(40)}$$

$$\Rightarrow \underline{V_{SG} = 2.17 \text{ V}}$$

Then

$$I_{REF} = \frac{20 - 2.17}{200} \Rightarrow \underline{I_{REF} = 89.2 \mu A}$$

 M_3, M_4, M_5 matched transistors so that

$$\underline{I_Q = I_{D7} = I_{REF} = 89.2 \mu A}$$

b. Small-signal voltage gain of input stage:

$$A_d = \sqrt{2K_{p1}I_D} \cdot (r_{o2} \parallel r_{o4})$$

$$r_{o2} = \frac{1}{\lambda_p I_D} = \frac{1}{(0.02) \left(\frac{89.2}{2} \right)} = 1.12 \text{ M}\Omega$$

$$r_{o4} = \frac{1}{\lambda_n I_D} = \frac{1}{(0.01) \left(\frac{89.2}{2} \right)} = 2.24 \text{ M}\Omega$$

Then

$$A_d = \sqrt{2(200)(89.2)} \cdot (1.12 \parallel 2.24)$$

or

$$\underline{A_d = 141}$$

Small-signal voltage gain of second stage:

$$A_{v2} = g_{m7}(r_{o7} \parallel r_{o8})$$

$$K_{n7} = (20)(20) = 400 \mu A/V^2$$

So

$$g_{m7} = 2\sqrt{K_{n7}I_{D7}} = 2\sqrt{(0.4)(0.0892)}$$

$$= 0.378 \text{ mA/V}$$

$$r_{o8} = \frac{1}{\lambda_p I_{D7}} = \frac{1}{(0.02)(0.0892)} = 561 \text{ k}\Omega$$

$$r_{o7} = \frac{1}{\lambda_n I_{D7}} = \frac{1}{(0.01)(0.0892)} = 1121 \text{ k}\Omega$$

So

$$A_{v2} = (0.378)(1121 \parallel 561) \Rightarrow \underline{A_{v2} = 141}$$

Then overall voltage gain

$$A_v = A_d \cdot A_{v2} = (141)(141) \Rightarrow \underline{A_v = 19,881}$$

13.23

Small-signal voltage gain of input stage:

$$A_d = \sqrt{2K_{p1}I_D} \cdot (r_{o2} \parallel r_{o4})$$

$$K_{p1} = (10)(10) = 100 \mu A/V^2$$

$$r_{o2} = \frac{1}{\lambda_p \left(\frac{I_D}{2} \right)} = \frac{1}{(0.01) \left(\frac{0.2}{2} \right)} = 1000 \text{ k}\Omega$$

$$r_{o4} = \frac{1}{\lambda_n \left(\frac{I_D}{2} \right)} = \frac{1}{(0.005) \left(\frac{0.2}{2} \right)} = 2000 \text{ k}\Omega$$

Then

$$A_d = \sqrt{2(0.1)(0.2)} \cdot (1000 \parallel 2000)$$

or

$$\underline{A_d = 133}$$

Small-signal voltage gain of second stage:

$$A_{v2} = g_{m7}(r_{o7} \parallel r_{o8})$$

$$K_{n7} = (20)(20) = 400 \mu A/V^2$$

So

$$g_{m7} = 2\sqrt{K_{n7}I_{D7}} = 2\sqrt{(0.4)(0.02)} \\ = 0.566 \text{ mA/V}$$

$$r_{o8} = \frac{1}{\lambda_p I_{D7}} = \frac{1}{(0.01)(0.2)} = 500 \text{ k}\Omega$$

$$r_{o7} = \frac{1}{\lambda_n I_{D7}} = \frac{1}{(0.005)(0.2)} = 1000 \text{ k}\Omega$$

So

$$A_{v2} = (0.566)(1000 \parallel 500) \Rightarrow \underline{A_{v2} = 189}$$

Then overall voltage gain is

$$A_v = A_d \cdot A_{v2} = (133)(189) \Rightarrow \underline{A_v = 25,137}$$

13.24

$$f_{PD} = \frac{1}{2\pi R_{eq}C_i}$$

where $R_{eq} = r_{o4} \parallel r_{o2}$ and $C_i = C_1(1 + |A_{v2}|)$

We can find that

$$A_{v2} = 251 \text{ and } r_{o4} = r_{o2} = 5.025 \text{ M}\Omega$$

Now

$$R_{eq} = 5.025 \parallel 5.025 = 2.51 \text{ M}\Omega$$

and

$$C_i = 12(1 + 251) = 3024 \text{ pF}$$

So

$$f_{PD} = \frac{1}{2\pi(2.51 \times 10^6)(3024 \times 10^{-12})}$$

or

$$\underline{f_{PD} = 21.0 \text{ Hz}}$$

13.25

$$f_{PD} = \frac{1}{2\pi R_{eq}C_i}$$

where $R_{eq} = r_{o4} \parallel r_{o2}$

From Problem 13.22,

$$r_{o2} = 1.12 \text{ M}\Omega, r_{o4} = 2.24 \text{ M}\Omega \text{ and } A_{v2} = 141$$

So

$$B = \frac{1}{2\pi(1.12)(2.24) \times 10^6 \times C_1}$$

or

$$C_1 = 2.66 \times 10^{-8} = C_1(1 + |A_{v2}|) = C_1(142)$$

or

$$C_1 = 188 \text{ pF}$$

13.26

$$R_o = r_{o7} \parallel r_{o8}$$

We can find that

$$r_{o7} = r_{o8} = 2.52 \text{ M}\Omega$$

Then

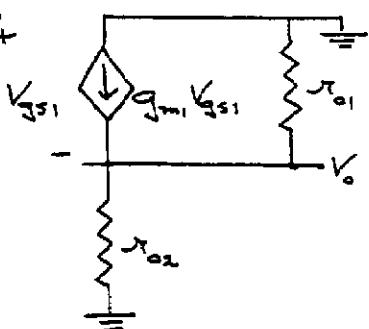
$$R_o = 2.52 \parallel 2.52$$

or

$$R_o = 1.26 \text{ M}\Omega$$

13.27

a.



$$V_0 = (g_{m1} V_{gss1}) (r_{o1} \parallel r_{o2})$$

$$V_I = V_{gss1} + V_0$$

$$\text{Then } V_0 = g_{m1} (r_{o1} \parallel r_{o2}) (V_I - V_0)$$

or

$$A_v = \frac{g_{m1} (r_{o1} \parallel r_{o2})}{1 + g_{m1} (r_{o1} \parallel r_{o2})}$$

$$\text{b. } I_X + g_{m1} V_{gss1} = \frac{V_X}{r_{o2}} + \frac{V_X}{r_{o1}} \text{ and } V_{gss1} = -V_X$$

$$R_o = \frac{1}{g_{m1}} \parallel r_{o1} \parallel r_{o2}$$

13.28

$$(a) A_d = g_{m1} (R_{o6} \parallel R_{o5})$$

$$g_{m1} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.025)} \Rightarrow 224 \mu A/V$$

$$g_{m5} = g_{m6}$$

$$g_{m5} = 2\sqrt{(0.5)(0.025)} \Rightarrow 224 \mu A/V$$

$$r_{o1} = r_{o6} = r_{o5} = r_{o10} = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.015)(25)} = 2.67 \text{ M}\Omega$$

$$r_{o4} = \frac{1}{\lambda I_{D4}} = \frac{1}{(0.015)(50)} = 1.33 \text{ M}\Omega$$

Now

$$R_{o6} = g_{m5} (r_{o5} r_{o10}) = (224)(2.67)(2.67) = 1597 \text{ M}\Omega$$

$$R_{o5} = g_{m6} (r_{o6}) (r_{o4} \parallel r_{o1}) = (224)(2.67)(2.67 \parallel 1.33) \Rightarrow$$

$$R_{o5} = 531 \text{ M}\Omega$$

Then

$$A_d = (224)(531 \parallel 1597) \Rightarrow A_d = 89,264$$

$$(b) R_o = R_{o6} \parallel R_{o5} = 531 \parallel 1597 \Rightarrow R_o = 398 \text{ M}\Omega$$

$$(c) f_{PD} = \frac{1}{2\pi R_o C_L} = \frac{1}{2\pi(398 \times 10^6)(5 \times 10^{-12})} \Rightarrow$$

$$\frac{f_{PD}}{GBW} = \frac{80 \text{ Hz}}{(89,264)(80)} \Rightarrow GBW = 7.14 \text{ MHz}$$

13.29

$$(a) r_{o1} = r_{o4} = r_{o10} = \frac{1}{\lambda_n I_D} = \frac{1}{(0.02)(25)} = 2 \text{ M}\Omega$$

$$r_{o6} = \frac{1}{\lambda_n I_D} = \frac{1}{(0.015)(25)} = 2.67 \text{ M}\Omega$$

$$r_{o4} = \frac{1}{\lambda_n I_{D4}} = \frac{1}{(0.015)(50)} = 1.33 \text{ M}\Omega$$

$$g_{m1} = 2\sqrt{\left(\frac{35}{2}\right)\left(\frac{W}{L}\right)_1}(25) = 41.8\sqrt{\left(\frac{W}{L}\right)_1} = g_{m4}$$

$$g_{m6} = 2\sqrt{\left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_4}(25) = 63.2\sqrt{\left(\frac{W}{L}\right)_4}$$

$$R_o = R_{o6} \parallel R_{o4} = [g_{m6} (r_{o6}) (r_{o4} \parallel r_{o1})] \parallel [g_{m1} (r_{o1} r_{o10})]$$

$$\text{Define } X_1 = \sqrt{\left(\frac{W}{L}\right)_1} \text{ and } X_6 = \sqrt{\left(\frac{W}{L}\right)_4}$$

$$\text{Then } R_o = [63.2 X_6 (2.67) (1.33)] \parallel [41.8 X_1 (2)(2)]$$

$$= 134.8 X_6 \parallel 167.2 X_1 = \frac{22,539 X_1 X_6}{134.8 X_6 + 167.2 X_1}$$

$$A_d = g_{m1} R_o = (41.8 X_1) \left(\frac{22,539 X_1 X_6}{134.8 X_6 + 167.2 X_1} \right) = 10,000$$

$$\text{Now } X_6 = \sqrt{\left(\frac{W}{L}\right)_4} = \sqrt{\frac{1}{2.2}\left(\frac{W}{L}\right)_1} = 0.674 X_1$$

We then find

$$X_1^2 = \left(\frac{W}{L}\right)_1 = 4.06 = \left(\frac{W}{L}\right),$$

and

$$\left(\frac{W}{L}\right)_n = 1.85$$

13.30

Let $V^+ = 5V$, $V^- = -5V$

$$P = I_T(10) = 3 \Rightarrow I_T = 0.3mA$$

$$\Rightarrow I_{REF} = 0.1mA = 100\mu A$$

$$r_{o1} = r_{os} = r_{o10} = \frac{1}{(0.02)(50)} = 1M\Omega$$

$$r_{os} = \frac{1}{(0.015)(50)} = 1.33M\Omega$$

$$r_{os} = \frac{1}{(0.015)(100)} = 0.667M\Omega$$

$$g_{m1} = 2\sqrt{\frac{35}{2}\left(\frac{W}{L}\right)(50)} = 59.2X_1 = g_{os}$$

$$\text{where } X_1 = \sqrt{\left(\frac{W}{L}\right)_1}$$

Assume all width-to-length ratios are the same.

$$g_{os} = 2\sqrt{\left(\frac{80}{2}\right)\left(\frac{W}{L}\right)(50)} = 89.4X_1$$

Now

$$\begin{aligned} R_o &= R_{os}\|R_{os} = [g_{os}(r_{os})(r_{os}r_{o1})]\|g_{os}(r_{os}r_{o10})] \\ &= [89.4X_1(1.33)(0.667)]\|59.2X_1(1)(1)] \\ &= [47.6X_1][59.2X_1] = \frac{(47.6X_1)(59.2X_1)}{47.6X_1 + 59.2X_1} \end{aligned}$$

$$\text{So } R_o = 26.4X_1$$

Now

$$A_d = g_{m1}R_o = (59.2X_1)(26.4X_1) = 25,000$$

$$\text{So that } X_1^2 = \frac{W}{L} = 16 \text{ for all transistors}$$

13.31

$$(a) A_d = Bg_{m1}(r_{os}\|r_{os})$$

$$r_{os} = r_{os} = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.015)(90)} = 0.741M\Omega$$

$$g_{m1} = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)I_{D1}} = 2\sqrt{(500)(30)} = 245\mu A/V$$

$$A_d = (3)(245)(0.741)\|0.741 \Rightarrow A_d = 272$$

$$(b) R_o = r_{os}\|r_{os} = 0.741\|0.741 \Rightarrow R_o = 371k\Omega$$

$$(c) f_{PD} = \frac{1}{2\pi R_o C} = \frac{1}{2\pi(371 \times 10^3)(5 \times 10^{-12})} \Rightarrow f_{PD} = 85.8 kHz$$

$$GBW = (272)(85.8 \times 10^3) \Rightarrow GBW = 23.3 MHz$$

13.32

$$(a) r_{os} = \frac{1}{(0.02)(2.5)(40)} = 0.5M\Omega$$

$$r_{os} = \frac{1}{(0.015)(2.5)(40)} = 0.667M\Omega$$

$$A_d = Bg_{m1}(r_{os}\|r_{os})$$

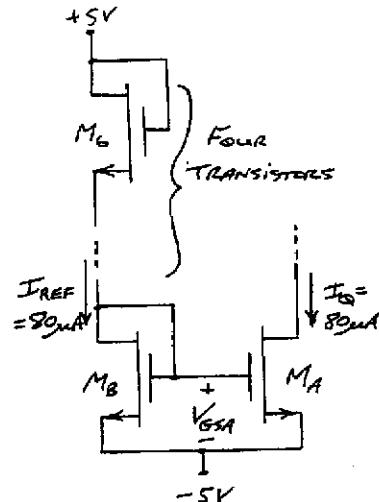
$$400 = (2.5)g_{m1}(0.5\|0.667) \Rightarrow g_{m1} = 560\mu A/V$$

$$g_{m1} = 560 = 2\sqrt{\left(\frac{80}{2}\right)\left(\frac{W}{L}\right)(40)} \Rightarrow \left(\frac{W}{L}\right) = 49$$

Assume all (W/L) ratios are the same except for

$$M_5 \text{ and } M_6. \quad \left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = 122.5$$

(b) Assume the bias voltages are
 $V^+ = 5V$, $V^- = -5V$.



$$\text{Assume } \left(\frac{W}{L}\right)_A = \left(\frac{W}{L}\right)_B = 49$$

$$I_Q = \left(\frac{80}{2}\right)(49)(V_{GS1} - 0.5)^2 = 80 \Rightarrow V_{GS1} = 0.702V$$

Then

$$I_{REF} = 80 = \left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_C(V_{GS2} - 0.5)^2$$

13.34

$$A_d = g_m(M_2) \cdot [r_{o2}(M_2) \| r_{o2}(Q_2)]$$

$$g_m(M_2) = 2\sqrt{\frac{40}{2}}(25)(100) = 447 \mu A/V$$

$$r_{o2}(M_2) = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.1)} = 500 k\Omega$$

$$r_{o2}(Q_2) = \frac{V_A}{I_{CQ}} = \frac{120}{0.1} = 1200 k\Omega$$

Then

$$A_d = 447(0.5 \| 1.2) \Rightarrow A_d = 158$$

13.35

$$A_d = g_m(M_2) \cdot [r_{o2}(M_2) \| r_{o2}(Q_2)]$$

$$g_m(M_2) = 2\sqrt{\frac{80}{2}}(25)(100) = 632 \mu A/V$$

$$r_{o2}(M_2) = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.015)(0.1)} = 667 k\Omega$$

$$r_{o2}(Q_2) = \frac{V_A}{I_{CQ}} = \frac{80}{0.1} = 800 k\Omega$$

$$A_d = (632)(0.667 \| 0.80) \Rightarrow A_d = 230$$

13.36

$$I_{REF} = 200 \mu A \quad K_n = K_p = 0.5 mA/V^2$$

$$\lambda_n = \lambda_p = 0.015 V^{-1}$$

$$A_d = g_m(R_{o6} \| R_{o4})$$

where

$$R_{o6} = g_m(r_{o6} r_{o10})$$

$$R_{o6} = g_m(r_{o6})(r_{o4} \| r_{o1})$$

Now

$$g_m = 2\sqrt{K_p I_{D1}} = 2\sqrt{(0.5)(0.1)} = 0.447 mA/V$$

$$r_{o4} = \frac{1}{\lambda_p I_{D4}} = \frac{1}{(0.015)(0.1)} = 667 k\Omega$$

$$r_{o10} = \frac{1}{\lambda_p I_{D4}} = 667 k\Omega$$

$$g_{m6} = \frac{I_{C6}}{V_T} = \frac{0.1}{0.026} = 3.846 mA/V$$

$$r_{o6} = \frac{V_A}{I_{C6}} = \frac{80}{0.1} = 800 k\Omega$$

$$r_{o4} = \frac{1}{\lambda_n I_{D4}} = \frac{1}{(0.015)(0.2)} = 333 k\Omega$$

$$r_{o1} = \frac{1}{\lambda_p I_{D1}} = \frac{1}{(0.015)(0.1)} = 667 k\Omega$$

$$g_{m1} = 2\sqrt{K_p I_{D1}} = 2\sqrt{(0.5)(0.1)} = 0.447 mA/V$$

So

$$R_{o6} = (0.447)(667)(667) \Rightarrow 198.9 M\Omega$$

$$R_{o6} = (3.846)(800)(333 \| 667) \Rightarrow 683.4 M\Omega$$

Then

$$A_d = 447(198.9 \| 683.4) \Rightarrow A_d = 68,865$$

13.37

Assume biased at $V^+ = 10 V, V^- = -10 V$

$$P = 3I_{REF}(20) = 10 \Rightarrow I_{REF} = 167 \mu A$$

$$A_d = g_m(R_{o6} \| R_{o4}) = 25,000$$

$$k'_n = 80 \mu A/V^2, k'_p = 35 \mu A/V^2$$

$$\lambda_n = 0.015 V^{-1}, \lambda_p = 0.02 V^{-1}$$

$$\text{Assume } \left(\frac{W}{L}\right)_p = 2.2 \left(\frac{W}{L}\right)_n$$

$$R_{o6} = g_m(r_{o6} r_{o10})$$

$$R_{o6} = g_m(r_{o6})(r_{o4} \| r_{o1})$$

$$r_{o6} = \frac{1}{\lambda_p I_{D6}} = \frac{1}{(0.02)(83.3)} = 0.60 M\Omega$$

$$r_{o10} = \frac{1}{\lambda_p I_{D4}} = 0.60 M\Omega$$

$$g_{m6} = 2\sqrt{\left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_1 I_{D6}} = 2\sqrt{\left(\frac{35}{2}\right)(2.2)X^2(83.3)}$$

$$= 113.3 X$$

$$\text{where } X^2 = \left(\frac{W}{L}\right)_n$$

$$r_{o4} = \frac{V_A}{I_{C6}} = \frac{80}{83.3} = 0.960 M\Omega$$

$$r_{o4} = \frac{1}{\lambda_n I_{D4}} = \frac{1}{(0.015)(167)} = 0.40 M\Omega$$

$$r_{o1} = \frac{1}{\lambda_p I_{D1}} = \frac{1}{(0.02)(83.3)} = 0.60 M\Omega$$

$$g_{m6} = \frac{I_{C6}}{V_T} = \frac{83.3}{0.026} = 3204 \mu A/V$$

$$g_{m1} = 2\sqrt{\left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_1 I_{D1}} = 2\sqrt{\left(\frac{35}{2}\right)(2.2)X^2(83.3)}$$

$$= 113.3 X$$

Now

$$R_{o6} = (3204)(0.960)[0.40 \| 0.60] = 738 M\Omega$$

$$R_{o6} = (113.3 X)(0.60)(0.60) = 40.8 X M\Omega$$

Then

$$A_d = 25,000 = (113.3 X)[738 \| 40.8 X]$$

$$= (113.3 X) \left[\frac{30,110 X}{738 + 40.8 X} \right]$$

which yields $X = 2.48$

or

$$X^2 = 6.16 = \left(\frac{W}{L}\right)_n$$

and

$$\left(\frac{W}{L}\right)_s = (2.2)(6.16) = 12.3$$

13.38

For $v_{cm}(\max)$, assume $V_{CB}(Q_5) = 0$. Then

$$V_S = 15 - 0.6 - 0.6 = 13.8 \text{ V}$$

$$I_{D9} = I_{D10} = \frac{0.236}{2} = 0.118 \text{ mA}$$

Using parameters given in Example 13.11

$$V_{so} = \sqrt{\frac{I_{D9}}{K_p}} - V_{tp} = \sqrt{\frac{0.118}{0.20}} + 1.4 = 2.17 \text{ V}$$

Then

$$v_{cm}(\max) = 13.8 - 2.17 \Rightarrow v_{cm}(\max) = 11.6 \text{ V}$$

For

$v_{cm}(\min)$, assume

$$V_{so}(M_9) = V_{so}(\text{sat}) = V_{so} + V_{tp}$$

$$= 2.17 - 1.4 = 0.77 \text{ V}$$

Now

$$V_{D10} = I_{D10}(0.5) + 0.6 + I_{D10}(0.5) - 15$$

$$= 0.118 + 0.6 - 15 \Rightarrow V_{D10} = -14.28 \text{ V}$$

Then

$$v_{cm}(\min) = -14.28 + V_{SD}(\text{sat}) - V_{SG}$$

$$= -14.28 + 0.77 - 2.17 = -15.68 \text{ V}$$

Then, common-mode voltage range

$$-15.68 \leq v_{cm} \leq 11.6$$

Or, assuming the input is limited to $\pm 15 \text{ V}$, then

$$-15 \leq v_{cm} \leq 11.6 \text{ V}$$

13.39

For $I_1 = I_2 = 300 \mu\text{A}$,

$$V_{SG} = V_{BE} + (0.3)(8) = 0.6 + 2.4 = 3.0 \text{ V}$$

Then

$$I_1 = K_p(V_{so} + V_{tp})^2$$

$$0.3 = K_p(3 - 1.4)^2$$

$$\Rightarrow K_p = 0.117 \text{ mA/V}^2$$

13.40

For $V_{CB} = 0$ for both Q_6 and Q_7 , then

$$V_S = 0.6 + 0.6 + V_{SG} + (-V_S)$$

$$\text{So } 2V_S = 1.2 + V_{SG}$$

Now

$$0.6 + I_1 R_i = V_{so} = \sqrt{\frac{I_1}{K_p}} + V_{tp} \text{ and } I_1 = I_2$$

$$\text{Also } I_1 = I_2 = K_p(V_{so} + V_{tp})^2 \text{ so}$$

$$0.6 + (0.25)(8)(V_{SG} - 1.4)^2 = V_{SG}$$

$$0.6 + 2(V_{SG}^2 - 2.8V_{SG} + 1.96) = V_{SG}$$

$$2V_{SG}^2 - 6.5V_{SG} + 4.52 = 0$$

$$V_{SG} = \frac{6.6 \pm \sqrt{(6.6)^2 - 4(2)(4.52)}}{2(2)} = 2.33 \text{ V}$$

$$\text{Then } 2V_S = 1.2 + 2.33 = 3.53 \text{ and}$$

$$V_S = 1.765 \text{ V}$$

13.41

$$I_{C3} = I_{C4} = 300 \mu\text{A}$$

Using the parameters from Examples 13.12 and 13.13, we have

$$R_{12} = r_{\pi 13} = \frac{\beta_n V_T}{I_{C13}} = \frac{(200)(0.026)}{0.3} = 17.3 \text{ k}\Omega$$

$$A_d = \sqrt{2K_p I_{C3}} \cdot (R_{12}) = \sqrt{2(0.6)(0.3)} \cdot (17.3)$$

or

$$A_d = 10.38$$

Now

$$g_{m13} = \frac{I_{C13}}{V_T} = \frac{0.3}{0.026} = 11.5 \text{ mA/V}$$

$$r_{013} = \frac{V_A}{I_{C13}} = \frac{50}{0.3} = 167 \text{ k}\Omega$$

Then

$$|A_{v2}| = g_{m13} \cdot r_{013} = (11.5)(167)$$

or

$$|A_{v2}| = 1917$$

Overall gain:

$$|A_v| = (10.38)(1917) = 19,895$$

13.42

Assuming the resistances looking into Q_4 and into the output stage are very large, we have

$$|A_{v2}| = \frac{\beta R_{013}}{r_{\pi 13} + (1 + \beta) R_{E13}}$$

$$\text{where } R_{013} = r_{013}[1 + g_{m13}(R_{E13} \| r_{\pi 13})]$$

$$I_{C13} = 300 \mu\text{A}, r_{o13} = \frac{50}{0.3} = 167 \text{ k}\Omega$$

$$g_{m13} = \frac{0.3}{0.026} = 11.5 \text{ mA/V}$$

$$r_{\pi13} = \frac{(200)(0.026)}{0.3} = 17.3 \text{ k}\Omega$$

So

$$R_{o13} = (167)[1 + (11.5)(1\|17.3)] \Rightarrow 1.98 \text{ M}\Omega$$

Then

$$|A_{v2}| = \frac{(200)(1980)}{17.3 + (201)(1)} = 1814$$

Now

$$C_i = C_1(1 + |A_{v2}|) = 12[1 + 1814] \\ \Rightarrow C_i = 21,780 \text{ pF}$$

$$f_{PD} = \frac{1}{2\pi R_{eq} C_i}$$

$$R_{eq} = R_{i2} \| r_{o12} \| r_{o10}$$

Neglecting R_3 ,

$$r_{o10} = \frac{1}{\lambda I_{D10}} = \frac{1}{(0.02)(0.15)} = 333 \text{ k}\Omega$$

Neglecting R_5 ,

$$r_{o12} = \frac{50}{0.15} = 333 \text{ k}\Omega$$

$$R_{i2} = r_{\pi13} + (1 + \beta)R_{E13} = 17.3 + (201)(1) \\ = 218 \text{ k}\Omega$$

Then

$$f_{PD} = \frac{1}{2\pi[218\|333\|333] \times 10^3 \times (21,780) \times 10^{-12}}$$

or

$$\underline{f_{PD} = 77.4 \text{ Hz}}$$

Unity-Gain Bandwidth

Gain of first stage:

$$A_d = \sqrt{2K_n I_{C3} \cdot (R_{i2} \| r_{o12} \| r_{o10})} \\ = \sqrt{2(0.6)(0.3)} \cdot (218\|333\|333) \\ = (0.6)(218\|333\|333)$$

$$\text{or } A_d = 56.6$$

Overall gain:

$$A_v = (56.6)(1814) = 102,672$$

Then unity-gain bandwidth = $(77.4)(102,672)$

$$\Rightarrow \underline{1.95 \text{ MHz}}$$

13.43

Since $V_{GS} = 0$ in J_6 , $I_{REF2} = I_{DSS}$

$$\Rightarrow \underline{I_{DSS} = 0.8 \text{ mA}}$$

13.44

$$\text{a. } R_{i2} = r_{\pi5} + (1 + \beta)[r_{\pi6} + (1 + \beta)R_E]$$

$$r_{\pi6} = \frac{(100)(0.026)}{0.2} = 13 \text{ k}\Omega$$

$$I_{C5} \approx \frac{I_{C6}}{\beta} = \frac{200 \mu\text{A}}{100} = 2 \mu\text{A}$$

So

$$r_{\pi5} = \frac{(100)(0.026)}{0.002} = 1300 \text{ k}\Omega$$

Then

$$R_{i2} = 1300 + (101)[13 + (101)(0.3)]$$

or

$$\underline{R_{i2} = 5.67 \text{ M}\Omega}$$

$$\text{b. } A_v = g_{m2}(r_{o2} \| r_{o4} \| R_{i2})$$

$$g_{m2} = \frac{2}{V_P} \cdot \sqrt{I_D \cdot I_{DSS}} = \frac{2}{3} \cdot \sqrt{(0.1)(0.2)} \\ = 0.0943 \text{ mA/V}$$

$$r_{o2} = \frac{1}{\lambda I_D} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$r_{o4} = \frac{V_A}{I_{C4}} = \frac{5.0}{0.1} = 500 \text{ k}\Omega$$

Then

$$A_v = (0.0943)[500\|500\|5670]$$

or

$$\underline{A_v = 22.6}$$

13.45

- a. Need $V_{SD}(Q_E) \geq V_{SD}(\text{sat}) = V_P$
For minimum bias $\pm 3 \text{ V}$

Set $V_P = 3 \text{ V}$ and $V_{ZK} = 3 \text{ V}$

$$I_{REF2} = \frac{V_{ZK} - V_{D1}}{R_3}$$

$$\text{so that } R_3 = \frac{3 - 0.6}{0.1} \Rightarrow \underline{R_3 = 24 \text{ k}\Omega}$$

Set bias in $Q_E = I_{REF2} + I_{Z2} = 0.1 + 0.1 = 0.2 \text{ mA}$

Therefore,

$$\underline{I_{DSS} = 0.2 \text{ mA}}$$

b. Neglecting base currents

$$I_{D1} = I_{RESF1} = 0.5 \text{ mA} = \frac{12 - 0.6}{R_4}$$

so that

$$\underline{R_4 = 22.8 \text{ k}\Omega}$$

13.46

a. We have

$$g_{m2} = \frac{2}{|V_P|} \cdot \sqrt{I_D \cdot I_{DSS}} = \frac{2}{4} \cdot \sqrt{(0.5)(1)} \\ = 0.354 \text{ mA/V}$$

$$r_{02} = \frac{1}{\lambda I_D} = \frac{1}{(0.02)(0.5)} = 100 \text{ k}\Omega$$

$$r_{04} = \frac{V_A}{I_D} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

$$g_{m4} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$r_{\pi4} = \frac{(200)(0.026)}{0.5} = 10.4 \text{ k}\Omega$$

So

$$R_{04} = r_{04}[1 + g_{m4}(r_{\pi4} \| R_2)] \\ = 200[1 + (19.23)(10.4 \| 0.5)] \\ = 2035 \text{ k}\Omega$$

$$|A_d| = g_{m2}(r_{02} \| R_{04} \| R_L)$$

For $R_L \rightarrow \infty$

$$|A_d| = 0.354(100 \| 2035) = 33.7$$

With these parameter values, gain can never reach 500.

b. Similarly for this part, gain can never reach 700.

Chapter 15

Exercise Solutions

E15.1

$$f_{3dB} = \frac{1}{2\pi RC}$$

$$RC = \frac{1}{2\pi f_{3dB}} = \frac{1}{2\pi(10^4)} = 1.59 \times 10^{-5}$$

$$\text{Let } C = 0.01 \mu\text{F} \Rightarrow R = 1.59 \text{ k}\Omega$$

Then

$$C_1 = 0.03546 \mu\text{F}$$

$$C_2 = 0.01392 \mu\text{F}$$

$$C_3 = 0.002024 \mu\text{F}$$

$$|T| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3d}}\right)^6}} = \frac{1}{\sqrt{1 + \left(\frac{20}{10}\right)^6}}$$

$$|T| = 0.124 \text{ or } |T| = -18.1 \text{ dB}$$

E15.2

$$f_{3dB} = \frac{1}{2\pi RC} \Rightarrow RC = \frac{1}{2\pi f_{3dB}}$$

$$RC = \frac{1}{2\pi(50 \times 10^3)} = 3.18 \times 10^{-6}$$

$$\text{Let } C = 0.001 \mu\text{F} = 1 \text{ nF} \Rightarrow R = 3.18 \text{ k}\Omega$$

Then

$$R_1 = 2.94 \text{ k}\Omega$$

$$R_2 = 3.44 \text{ k}\Omega$$

$$R_3 = 1.22 \text{ k}\Omega$$

$$R_4 = 8.31 \text{ k}\Omega$$

$$|T| = 0.01 = \frac{1}{\sqrt{1 + \left(\frac{f_{3dB}}{f}\right)^6}}$$

$$1 + \left(\frac{f_{3dB}}{f}\right)^6 = \left(\frac{1}{0.01}\right)^2 = 10^4$$

$$\left(\frac{f_{3dB}}{f}\right)^2 = 10 \Rightarrow f = \frac{f_{3dB}}{\sqrt{10}}$$

$$\Rightarrow f \approx 15.8 \text{ kHz}$$

E15.3

$$1\text{-pole } |T| = \frac{1}{\sqrt{1 + \left(\frac{12}{10}\right)^2}} \Rightarrow -3.87 \text{ dB}$$

$$2\text{-pole } |T| = \frac{1}{\sqrt{1 + \left(\frac{12}{10}\right)^4}} \Rightarrow -4.88 \text{ dB}$$

$$3\text{-pole } |T| = \frac{1}{\sqrt{1 + \left(\frac{12}{10}\right)^6}} \Rightarrow -6.0 \text{ dB}$$

$$4\text{-pole } |T| = \frac{1}{\sqrt{1 + \left(\frac{12}{10}\right)^8}} \Rightarrow -7.24 \text{ dB}$$

E15.4

$$R_{eq} = \frac{1}{f_C C}$$

$$\text{or } f_C C = \frac{1}{R_{eq}} = \frac{1}{5 \times 10^6} = 2 \times 10^{-7}$$

$$\text{If } C = 10 \text{ pF} \Rightarrow f_C = 20 \text{ kHz}$$

E15.5

$$\text{Low-frequency gain: } T = -\frac{C_1}{C_2} = -\frac{30}{5} = -6$$

$$f_{3dB} = \frac{f_C C_2}{2\pi C_F} = \frac{(100 \times 10^3)(5 \times 10^{-12})}{2\pi(12 \times 10^{-12})} \Rightarrow f_{3dB} = 6.63 \text{ kHz}$$

E15.6

$$f_0 = \frac{1}{2\pi\sqrt{3}RC}$$

$$RC = \frac{1}{2\pi f_0 \sqrt{3}} = \frac{1}{2\pi(15 \times 10^3)\sqrt{3}} = 6.13 \times 10^{-6}$$

$$\text{Let } C = 0.001 \mu\text{F} = 1 \text{ nF}$$

$$\text{Then } R = 6.13 \text{ k}\Omega \text{ so } R_2 = 8R = 49 \text{ k}\Omega$$

E15.7

$$f_0 = \frac{1}{2\pi\sqrt{6}RC} = \frac{1}{2\pi\sqrt{6}(10^4)(100 \times 10^{-12})} \Rightarrow f_0 \approx 65 \text{ kHz}$$

$$R_2 = 29R = 29(10^4)$$

$$\Rightarrow R_2 = 290 \text{ k}\Omega$$

E15.8

$$f_0 = \frac{1}{2\pi RC} \Rightarrow C = \frac{1}{2\pi f_0 R}$$

$$C = \frac{1}{2\pi(800)(10^4)} \Rightarrow C \approx 0.02 \mu\text{F}$$

$$R_2 = 2R_1 = 2(10) \Rightarrow R_2 = 20 \text{ k}\Omega$$

E15.9

$$f_0 = \frac{1}{2\pi\sqrt{L \cdot \left(\frac{C_1 C_2}{C_1 + C_2}\right)}} = \frac{1}{2\pi\sqrt{(10^{-6}) \left[\frac{(10^{-9})^2}{2 \times 10^{-9}}\right]}}$$

$$\Rightarrow f_0 = 7.12 \text{ MHz}$$

$$\frac{C_2}{C_1} = g_m R$$

$$g_m = \frac{C_2}{C_1} \cdot \frac{1}{R} = \frac{1}{4 \times 10^3} \Rightarrow g_m = 0.25 \text{ mA/V}$$

We have

$$g_m = 2\left(\frac{k'}{2}\right)\left(\frac{W}{L}\right)(V_{GS} - V_{TH})$$

$$k' \approx 20 \mu\text{A/V}^2, V_{GS} - V_{TH} \approx 1 \text{ V}$$

$$\text{So } \frac{W}{L} = \frac{0.25 \times 10^{-3}}{(20 \times 10^{-6})(1)} = 12.5$$

and a value of $W/L = 12.5$ is certainly reasonable.

E15.10

$$V_{TH} = \left(\frac{R_1}{R_1 + R_2}\right)V_H$$

$$2 = \left(\frac{R_1}{R_1 + 20}\right)(12)$$

$$2(R_1 + 20) = 12R_1$$

$$40 = 10R_1 \Rightarrow R_1 = 4 \text{ k}\Omega$$

E15.11

$$V_{TH} = -\left(\frac{R_1}{R_2}\right)V_L$$

$$0.10 = -\left(\frac{R_1}{R_2}\right)(-10) \Rightarrow \frac{R_1}{R_2} = 0.010$$

$$\text{Let } R_1 = 0.10 \text{ k}\Omega \text{ then } R_2 = 10 \text{ k}\Omega$$

E15.12

a. $V_S = \left(\frac{R_2}{R_1 + R_2}\right)V_{REF} = \left(\frac{10}{1+10}\right)(2)$
 $\underline{V_S = 1.82 \text{ V}}$

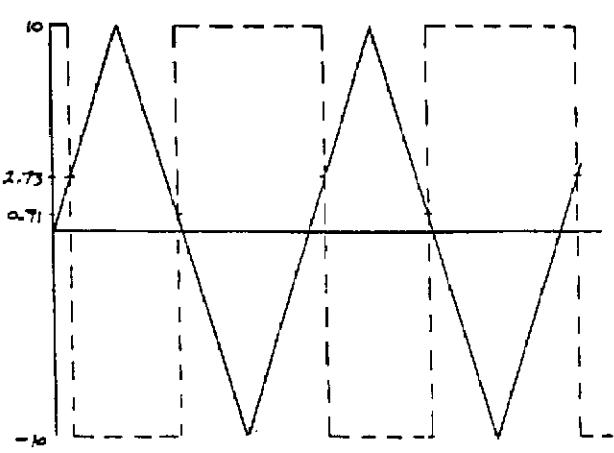
$$V_{TH} = V_S + \left(\frac{R_1}{R_1 + R_2}\right)V_H = 1.82 + \left(\frac{1}{1+10}\right)(10)$$

$$\underline{V_{TH} = 2.73 \text{ V}}$$

$$V_{TL} = V_S + \left(\frac{R_1}{R_1 + R_2}\right)V_L = 1.82 + \left(\frac{1}{1+10}\right)(-10)$$

$$\underline{V_{TL} = 0.91 \text{ V}}$$

b.



E15.13

$$V_S = \left(1 + \frac{R_1}{R_2}\right)V_{REF}$$

$$V_{TH} = V_S - \left(\frac{R_1}{R_2}\right)V_L \text{ and } V_{TL} = V_S - \left(\frac{R_1}{R_2}\right)V_H$$

$$\text{Hysteresis Width} = V_{TH} - V_{TL} = \left(\frac{R_1}{R_2}\right)(V_H - V_L)$$

$$2.5 = \left(\frac{R_1}{R_2}\right)(5 - [-5]) = 10\left(\frac{R_1}{R_2}\right)$$

$$\text{So } \underline{\frac{R_1}{R_2} = 0.25}$$

Then

$$V_S = -1 = \left(1 + \frac{R_1}{R_2}\right)V_{REF} = (1 + 0.25)V_{REF}$$

$$\Rightarrow \underline{V_{REF} = -0.8 \text{ V}}$$

Then

$$V_{TH} = -1 - (0.25)(-5) \Rightarrow \underline{V_{TH} = 0.25 \text{ V}}$$

$$V_{TL} = -1 - (0.25)(5) \Rightarrow \underline{V_{TL} = -2.25 \text{ V}}$$

E15.14

$$V_{TH} - V_{TL} = \left(\frac{R_1}{R_1 + R_2}\right)(V_H - V_L)$$

$$0.10 = \left(\frac{R_1}{R_1 + R_2}\right)(10 - [-10])$$

$$1 + \frac{R_2}{R_1} = \frac{20}{0.10} = 200 \Rightarrow \underline{\frac{R_2}{R_1} = 199}$$

$$V_S = \left(\frac{R_2}{R_1 + R_2}\right)V_{REF}$$

$$V_{REF} = \left(1 + \frac{R_1}{R_2}\right)V_S = \left(1 + \frac{1}{199}\right)(1)$$

$$\Rightarrow \underline{V_{REF} = 1.005 \text{ V}}$$

$$I = \frac{V_H - V_{BB(\text{on})} - V_t}{R + 0.1}$$

$$R + 0.1 = \frac{10 - 0.7 - 0.7}{0.2} = 43 \text{ k}\Omega$$

$$\underline{R = 42.9 \text{ k}\Omega}$$

E15.15

At $t = 0^-$, let $v_0 = -5$ so $v_X = -2.5$. For $t > 0$

$$v_X = 10 + (-2.5 - 10) \exp\left(-\frac{t}{\tau_X}\right)$$

When $v_X = 5.0$, output switches

$$5.0 = 10 - 12.5 \exp\left(-\frac{t_1}{\tau_X}\right)$$

$$\exp\left(-\frac{t_1}{\tau_X}\right) = \frac{10 - 5}{12.5} = \frac{5.0}{12.5}$$

$$\exp\left(+\frac{t_1}{\tau_X}\right) = \frac{12.5}{5.0} \Rightarrow t_1 = \tau_X \cdot \ln\left(\frac{12.5}{5.0}\right)$$

$$\Rightarrow t_1 = \tau_X(0.916)$$

During the next part of the cycle

$$v_X = -5 + (5 - [-5]) \exp\left(-\frac{t}{\tau_X}\right)$$

When $v_X = -2.5$, output switches

$$-2.5 = -5 + 10 \exp\left(-\frac{t_2}{\tau_X}\right)$$

$$\exp\left(-\frac{t_2}{\tau_X}\right) = \frac{5 - 2.5}{10} = \frac{2.5}{10}$$

$$\exp\left(+\frac{t_2}{\tau_X}\right) = \frac{10}{2.5} \Rightarrow t_2 = \tau_X \cdot \ln\left(\frac{10}{2.5}\right)$$

$$\Rightarrow t_2 = \tau_X(1.39)$$

$$\text{Period} = t_1 + t_2 = T = [(0.916) + (1.39)]\tau_X$$

$$= 2.31\tau_X$$

$$\Rightarrow \text{Frequency} = \frac{1}{2.31\tau_X}$$

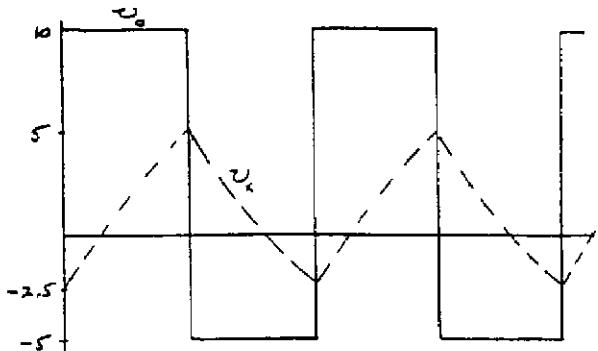
$$\tau_X = (50 \times 10^3)(0.01 \times 10^{-6}) = 5 \times 10^{-4} \text{ s}$$

$$\Rightarrow f = 866 \text{ Hz}$$

$$\text{Duty cycle} = \frac{t_1}{t_1 + t_2} \times 100\%$$

$$= \frac{(0.916)}{(0.916) + (1.39)} \times 100\%$$

$$\Rightarrow \underline{\text{Duty cycle} = 39.7\%}$$



E15.16

$$v_X = \left(\frac{R_1}{R_1 + R_2}\right)v_0 = \left(\frac{10}{10 + 20}\right)v_0 = \frac{1}{3}v_0$$

$$t = 0, v_X = -\frac{10}{3}.$$

$$v_X = 10 + \left(-\frac{10}{3} - 10\right) \exp\left(-\frac{t}{\tau_X}\right)$$

Output switches when $v_X = \frac{10}{3}$

$$\frac{10}{3} = 10 - 13.33 \exp\left(-\frac{t_1}{\tau_X}\right)$$

$$\exp\left(-\frac{t_1}{\tau_X}\right) = \frac{10 - 3.33}{13.33} = \frac{6.67}{13.33}$$

$$\exp\left(+\frac{t_1}{\tau_X}\right) = \frac{13.33}{6.67} \approx 2$$

$$t_1 = \tau_X \ln(2) = (0.693)\tau_X$$

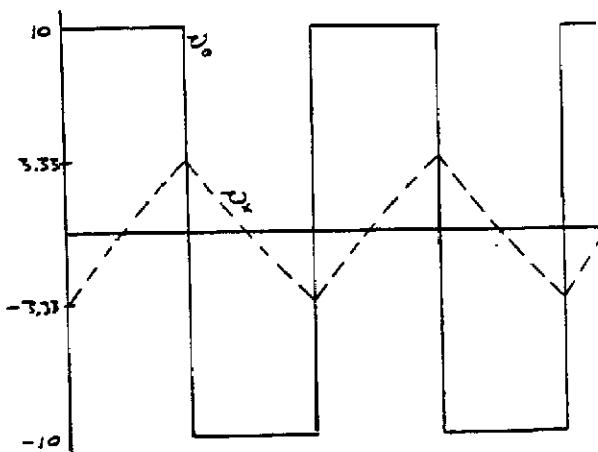
$$T = 2(0.693)\tau_X$$

$$f = \frac{1}{2(0.693)\tau_X}$$

$$\tau_X = R_X C_X = (10^4)(0.1 \times 10^{-6}) = 1 \times 10^{-3}$$

$$\Rightarrow f = 722 \text{ Hz}$$

$$\Rightarrow \underline{\text{Duty cycle} = 50\%}$$



E15.17

a. $\tau_X = R_X C_X$

$$v_T = \left(\frac{R_1}{R_1 + R_2} \right) v_0 = \left(\frac{10}{10 + 90} \right) (12) = 1.2 \text{ V}$$

$$\beta = \frac{R_1}{R_1 + R_2} = 0.10$$

$$T = \tau_X \ln \left[\frac{1 + V_T/V_P}{1 - \beta} \right] = \tau_X \ln \left[\frac{1 + \frac{0.7}{12}}{1 - (0.10)} \right]$$

$$T = 50 \times 10^{-6} = \tau_X \ln [1.18] = (0.162)\tau_X$$

$$R_X = \frac{50 \times 10^{-6}}{(0.1 \times 10^{-6})(0.162)} \Rightarrow R_X = 3.09 \text{ k}\Omega$$

b. Recovery time

$$v_X = V_P + (-1.2 - V_P) \exp \left(-\frac{t}{\tau_X} \right)$$

When $v_X = V_T$, $t = t_2$

$$0.7 = 12 + (-1.2 - 12) \exp \left(-\frac{t_2}{\tau_X} \right)$$

$$\exp \left(-\frac{t_2}{\tau_X} \right) = \frac{12 - 0.7}{13.2} = 0.856$$

$$t_2 = \tau_X \ln \left(\frac{1}{0.856} \right) = (0.155)\tau_X$$

$$\tau_X = (3.09 \times 10^3)(0.1 \times 10^{-6}) = 3.09 \times 10^{-4}$$

$$\Rightarrow t_2 = 47.9 \mu\text{s}$$

E15.18

$$\beta = \left(\frac{R_1}{R_1 + R_2} \right) = \frac{20}{20 + 40} = 0.333$$

$$\tau_X = R_X C_X = (10^4)(0.01 \times 10^{-6}) = 1 \times 10^{-4}$$

$$T = \tau_X \ln \left(\frac{1 + V_T/V_P}{1 - \beta} \right) = (1 \times 10^{-4}) \ln \left[\frac{1 + \frac{0.7}{8}}{1 - 0.333} \right]$$

$$\Rightarrow T = 48.9 \mu\text{s}$$

Recovery time

$$0.7 = 8 + (-2.66 - 8) \exp \left(-\frac{t_2}{\tau_X} \right)$$

$$\exp \left(-\frac{t_2}{\tau_X} \right) = \frac{8 - 0.7}{10.66} = 0.685$$

$$t_2 = \tau_X \ln \left(\frac{1}{0.685} \right)$$

$$\Rightarrow t_2 = 37.8 \mu\text{s}$$

E15.19

$$f = \frac{1}{0.693(R_A + 2R_B)C}$$

$$= \frac{1}{(0.693)[20 + 2(80)] \times 10^3 \times (0.01 \times 10^{-6})}$$

$$\Rightarrow f = 802 \text{ Hz}$$

$$\text{Duty cycle} = \frac{R_A + R_B}{R_A + 2R_B} \times 100\%$$

$$= \frac{20 + 80}{20 + 2(80)} \times 100\%$$

$$\Rightarrow \text{Duty cycle} = 55.6\%$$

E15.20

$$f = \frac{1}{(0.693)(R_A + R_B)C}$$

$$R_A + R_B = \frac{1}{(0.693)fC}$$

$$\text{Let } C = 0.01 \mu\text{F}, f = 1 \text{ kHz}$$

$$R_A + R_B = \frac{1}{(0.693)(10^3)(0.01 \times 10^{-6})} = 1.44 \times 10^5$$

$$\text{Duty cycle} = 55 = \frac{R_A + R_B}{R_A + 2R_B} \times 100\%$$

$$55 = \frac{(1.44 \times 10^5)(100)}{(1.44 \times 10^5) + R_B}$$

$$R_B = \frac{(1.44 \times 10^5)(100 - 55)}{55}$$

$$\Rightarrow R_B = 118 \text{ k}\Omega \text{ so } R_A = 26 \text{ k}\Omega$$

E15.21

a. $P = \frac{1}{2} \frac{V_P^2}{R_L}$

$$V_P = \sqrt{2R_L P} = \sqrt{2(8)(1)} \Rightarrow V_P = 4 \text{ V}$$

$$I_P = \frac{V_P}{R_L} = \frac{4}{8} \Rightarrow I_P = 0.5 \text{ A}$$

b. $V_{CE} = 12 - 4 = 8 \text{ V}$

$$I_C \approx 0.5 \text{ A}$$

$$\text{So } P = I_C \cdot V_{CE} = (0.5)(8) \Rightarrow P = 4 \text{ W}$$

E15.22

a. $\frac{V_{01}}{V_I} = \left(1 + \frac{R_2}{R_1} \right) = \left(1 + \frac{30}{20} \right) = 2.5$

$$\frac{V_{02}}{V_I} = -\frac{R_4}{R_3} = -\frac{50}{20} = -2.5$$

(b) $P = \frac{1}{2} \frac{V_L^2}{R_L} = \frac{1}{2} \frac{[12 - (-12)]^2}{12} = 240 \text{ mW}$

Or

$$P = 0.24 \text{ W}$$

c. $\frac{12}{2.5} = V_{pi} = 4.8 \text{ V}$

Test Your Understanding

Chapter 15: Exercise Solutions

E15.23

$$\text{Line regulation} = \frac{dV_o}{dV^+} = \frac{dV_o}{dV_Z} \cdot \frac{dV_Z}{dV^+}$$

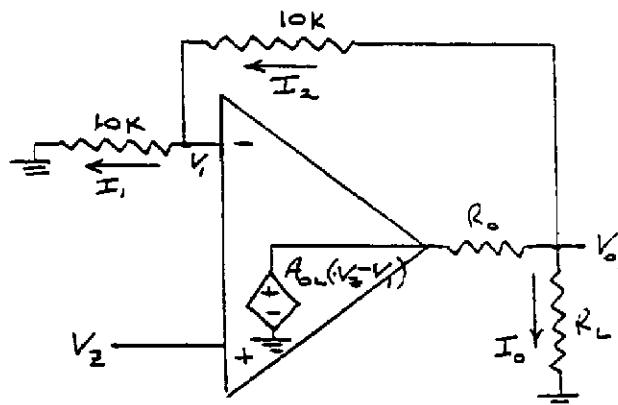
Now

$$\frac{dV_o}{dV_Z} = \left(1 + \frac{10}{10}\right) = 2$$

$$\frac{dV_Z}{dV^+} = \left(\frac{r_Z}{r_Z + R_1}\right) = \frac{10}{10 + 4400} = 0.00227$$

$$\text{So Line regulation} = (2)(0.00227) = 0.00454 \\ \underline{0.454\%}$$

E15.24



$$\frac{V_1}{10} = \frac{V_0 - V_1}{10} \Rightarrow V_1 \left(\frac{1}{10} + \frac{1}{10} \right) = \frac{V_0}{10}$$

$$V_1 \left(\frac{2}{10} \right) = \frac{V_0}{10} \Rightarrow V_0 = 2V_1 \Rightarrow V_1 = \frac{V_0}{2}$$

$$\frac{V_0 - V_1}{10} + \frac{V_0}{R_L} + \frac{V_0 - A_{OL}(V_Z - V_1)}{R_O} = 0$$

$$\frac{V_0}{10} + \frac{V_0}{R_L} + \frac{V_0}{R_O} - \frac{A_{OL}V_Z}{R_O} = \frac{V_1}{10} - \frac{A_{OL}V_1}{R_O} \\ = \frac{V_0}{2(10)} - \frac{A_{OL}V_0}{2R_O}$$

$$\frac{V_0}{10} + I_0 + \frac{V_0}{0.5} - \frac{1000(6.3)}{0.5} = \frac{V_0}{20} - \frac{(1000)V_0}{2(0.5)}$$

$$V_0[0.10 + 2.0 - 0.05 + 1000] + I_0 = 12,600$$

$$V_0(1002.05) + I_0 = 12,600$$

$$\text{For } I_0 = 1 \text{ mA} \Rightarrow V_0 = 12.5732$$

$$\text{For } I_0 = 100 \text{ mA} \Rightarrow V_0 = 12.4744$$

$$\text{Load reg} = \frac{V_0(\text{NL}) - V_0(\text{FL})}{V_0(\text{NL})} \times 100\% \\ = \frac{12.5732 - 12.4744}{12.5732} \times 100\%$$

$$\underline{\text{Load reg} = 0.786\%}$$

E15.25

$$\text{a. } I_{C3} = \frac{V_Z - 3V_{BE(\text{on})}}{R_1 + R_2 + R_3}$$

$$I_{C3} = \frac{5.6 - 3(0.6)}{3.9 + 3.4 + 0.576} = \frac{3.8}{7.88} \\ \Rightarrow \underline{I_{C3} = 0.482 \text{ mA}}$$

$$I_{C4}R_4 = V_T \ln \left(\frac{I_{C1}}{I_{C4}} \right)$$

$$I_{C4}(0.1) = (0.026) \ln \left(\frac{0.482}{I_{C4}} \right)$$

By trial and error

$$\underline{I_{C4} = 0.213 \text{ mA}}$$

$$V_{B7} = 2(0.6) + (0.482)(3.9) \\ \Rightarrow \underline{V_{B7} = 3.08 \text{ V}}$$

$$\text{b. } \left(\frac{R_{13}}{R_{13} + R_{12}} \right) V_0 = V_{B8} = V_{B7}$$

$$\left(\frac{2.23}{2.23 + R_{12}} \right)(5) = 3.08$$

$$(2.23)(5) = (3.08)(2.23) + (3.08)R_{12}$$

$$11.15 = 6.868 = 3.08R_{12}$$

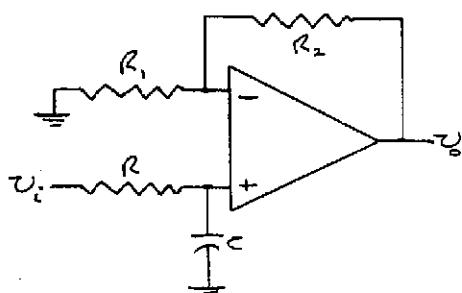
$$\Rightarrow \underline{R_{12} = 1.39 \text{ k}\Omega}$$

Chapter 15

Problem Solutions

15.1

(a) For example:

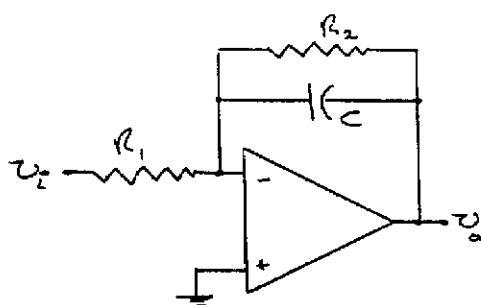


Low-Frequency: $\frac{v_o}{v_i} = \left(1 + \frac{R_2}{R_1}\right) = 10 \Rightarrow \frac{R_2}{R_1} = 9$

Corner Frequency:

$$f = \frac{1}{2\pi RC} = 5 \times 10^3 \Rightarrow RC = 3.18 \times 10^{-5}$$

(b) For Example:



$$\frac{v_o}{v_i} = \frac{-R_2}{R_1} \left| \frac{1}{j\omega C} \right| = -\frac{R_2}{R_1} \cdot \frac{1}{1 + j\omega R_2 C}$$

So, set

$$\frac{R_2}{R_1} = 15 \Rightarrow \text{For example, } R_1 = 10 \text{ k}\Omega, R_2 = 150 \text{ k}\Omega$$

$$R_2 C = \frac{1}{2\pi f_{3-dB}} = \frac{1}{2\pi(15 \times 10^3)} = 1.06 \times 10^{-5}$$

$$\text{Then } C = 70.7 \text{ pF}$$

15.2

$$(a) |A_v| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3-dB}}\right)^2}} = \frac{1}{\sqrt{1 + (2)^2}} = 0.447 \Rightarrow |A_v| = -7 \text{ dB}$$

$$(b) |A_v| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3-dB}}\right)^4}} = \frac{1}{\sqrt{1 + (2)^4}} = 0.2425 \Rightarrow |A_v| = -12.3 \text{ dB}$$

$$(c) |A_v| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3-dB}}\right)^6}} = \frac{1}{\sqrt{1 + (2)^6}} = 0.1240 \Rightarrow |A_v| = -18.1 \text{ dB}$$

15.3

Using Figure 15.9(a)

$$f_{3-dB} = \frac{1}{2\pi RC}$$

So

$$RC = \frac{1}{2\pi f_{3-dB}} = \frac{1}{2\pi(10 \times 10^3)} = 1.59 \times 10^{-5}$$

For example, $C = 0.001 \mu\text{F}$, $R = 15.9 \text{ k}\Omega$
so that $R_3 = 11.2 \text{ k}\Omega$ and $R_4 = 22.4 \text{ k}\Omega$

15.4

Use Figure 15.10(b)

$$f_{3-dB} = \frac{1}{2\pi RC}$$

or

$$RC = \frac{1}{2\pi(50 \times 10^3)} = 3.18 \times 10^{-6}$$

For example, let $C = 100 \text{ pF}$ Then $R = 31.8 \text{ k}\Omega$ And $R_1 = 8.97 \text{ k}\Omega$

$$R_2 = 22.8 \text{ k}\Omega$$

$$R_3 = 157 \text{ k}\Omega$$

From Equation (15.26)

$$|T| = \frac{1}{\sqrt{1 + \left(\frac{f_{3-dB}}{f}\right)^6}}$$

We find

| f kHz | $ T $ |
|---------|-------|
| 30 | 0.211 |
| 35 | 0.324 |
| 40 | 0.456 |
| 45 | 0.589 |

15.5

From Equation (15.7),

$$T(s) = \frac{Y_1 Y_2}{Y_1 Y_2 + Y_4(Y_1 + Y_2 + Y_3)}$$

For a high-pass filter, let $Y_1 = Y_2 = sC$,

$$Y_3 = \frac{1}{R_3}, \text{ and } Y_4 = \frac{1}{R_4}$$

Then

$$\begin{aligned} T(s) &= \frac{s^2 C^2}{s^2 C^2 + \frac{1}{R_4} \left(sC + sC + \frac{1}{R_3} \right)} \\ &= \frac{1}{1 + \frac{1}{s R_4 C} \left(2 + \frac{1}{s R_3 C} \right)} \end{aligned}$$

Define $r_3 = R_3 C$ and $r_4 = R_4 C$

$$T(s) = \frac{1}{1 + \frac{1}{s r_4} \left(2 + \frac{1}{s r_3} \right)}$$

Set $s = j\omega$

$$\begin{aligned} T(j\omega) &= \frac{1}{1 + \frac{1}{j\omega r_4} \left(2 + \frac{1}{j\omega r_3} \right)} \\ &= \frac{1}{1 - \frac{j}{\omega r_4} \left(2 - \frac{j}{\omega r_3} \right)} \\ &= \frac{1}{\left(1 - \frac{1}{\omega^2 r_3 r_4} \right) - \frac{2j}{\omega r_4}} \end{aligned}$$

$$|T(j\omega)| = \left\{ \left(1 - \frac{1}{\omega^2 r_3 r_4} \right)^2 + \frac{4}{\omega^2 r_4^2} \right\}^{-1/2}$$

For a maximally flat filter, we want

$$\frac{d|T|}{d\omega} \Big|_{\omega=\infty} = 0$$

Taking the derivative, we find

$$\begin{aligned} \frac{d|T(j\omega)|}{d\omega} &= -\frac{1}{2} \left\{ \left(1 - \frac{1}{\omega^2 r_3 r_4} \right)^2 + \frac{4}{\omega^2 r_4^2} \right\}^{-3/2} \\ &\quad \times \left[2 \left(1 - \frac{1}{\omega^2 r_3 r_4} \right) \left(\frac{2}{\omega^3 r_3 r_4} \right) + \frac{4(-2)}{\omega^3 r_4^2} \right] \end{aligned}$$

or

$$\begin{aligned} \frac{d|T(j\omega)|}{d\omega} \Big|_{\omega=\infty} &= 0 \\ &= \left[\left(\frac{4}{\omega^3 r_3 r_4} \right) \left(1 - \frac{1}{\omega^2 r_3 r_4} \right) - \frac{8}{\omega^3 r_4^2} \right] \\ &= \frac{4}{\omega^3} \left[\frac{1}{r_3 r_4} \left(1 - \frac{1}{\omega^2 r_3 r_4} \right) - \frac{2}{r_4^2} \right] \end{aligned}$$

Then

$$\left[\frac{1}{r_3 r_4} \left(1 - \frac{1}{\omega^2 r_3 r_4} \right) - \frac{2}{r_4^2} \right] \Big|_{\omega=\infty} = 0$$

$$\text{So that } \frac{1}{r_3} = \frac{2}{r_4} \Rightarrow 2r_3 = r_4$$

Then the transfer function can be written as:

$$\begin{aligned} |T(j\omega)| &= \left\{ \left[1 - \frac{1}{\omega^2 (2r_3^2)} \right]^2 + \frac{4}{\omega^2 (4r_3^2)} \right\}^{-1/2} \\ &= \left\{ 1 - \frac{1}{\omega^2 r_3^2} + \frac{1}{4(\omega^2 r_3^2)^2} + \frac{1}{\omega^2 r_3^2} \right\}^{-1/2} \\ &= \left\{ 1 + \frac{1}{4(\omega^2 r_3^2)^2} \right\}^{-1/2} \end{aligned}$$

3-dB frequency

$$2\omega^2 r_3^2 = 1 \text{ or } \omega = \frac{1}{\sqrt{2r_3}} = \frac{1}{\sqrt{2R_3 C}}$$

Define

$$\omega = \frac{1}{RC}$$

So that

$$R_3 = \frac{R}{\sqrt{2}}$$

We had $2r_3 = r_4$ or $2(R_3 C) = R_4 C \Rightarrow R_4 = 2R_3$
So that $\underline{R_4 = \sqrt{2} \cdot R}$

15.6

From Equation (15.25)

$$|T| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3-dB}} \right)^{2N}}}$$

$$-25 \text{ dB} \Rightarrow |T| = 0.0562$$

$$\frac{f}{f_{3-dB}} = \frac{20}{10} = 2$$

So

$$0.0562 = \frac{1}{\sqrt{1 + (2)^{2N}}}$$

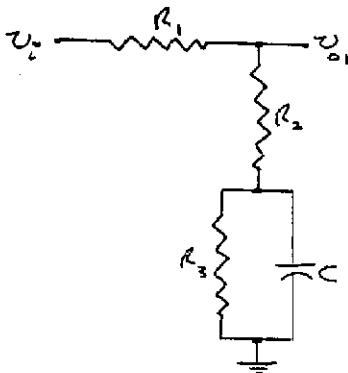
$$1 + (2)^{2N} = 316.6 \Rightarrow (2)^{2N} = 315.6$$

$$2N \cdot \ln(2) = \ln(315.6)$$

$$\Rightarrow N = 4.15 \Rightarrow \underline{N = 5} \text{ A 5-pole filter}$$

15.7

Consider

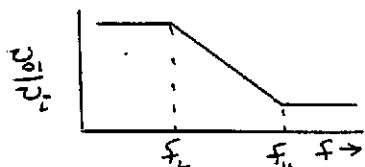


$$\text{For low-frequency: } \frac{v_o}{v_i} = \frac{R_2 + R_3}{R_1 + R_2 + R_3}$$

$$\text{For high-frequency: } \frac{v_o}{v_i} = \frac{R_2}{R_1 + R_2}$$

So we need

$$\frac{R_2 + R_3}{R_1 + R_2 + R_3} = 25 \left(\frac{R_2}{R_1 + R_2} \right)$$



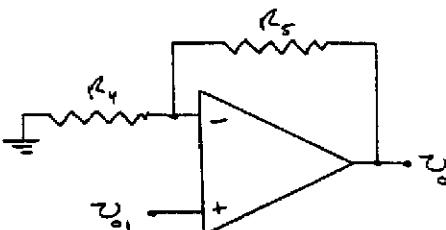
$$\text{Let } R_1 + R_2 = 50 \text{ k}\Omega \text{ and } \underline{R_2 = 15 \text{ k}\Omega} \Rightarrow$$

$$\underline{R_1 = 48.5 \text{ k}\Omega}$$

Then

$$\frac{15 + R_3}{50 + R_3} = 25 \left(\frac{15}{50} \right) \Rightarrow \underline{R_3 = 144 \text{ k}\Omega}$$

Connect the output of this circuit to a non-inverting op-amp circuit.



At low-frequency:

$$\frac{R_2 + R_3}{R_1 + R_2 + R_3} \cdot v_i = \frac{1.5 + 144}{48.5 + 1.5 + 144} \cdot v_i = 0.75 v_i$$

Need to have $v_o = 25$.

$$v_o = 25 = \left(1 + \frac{R_5}{R_4} \right) \cdot v_i = \left(1 + \frac{R_5}{R_4} \right) (0.75) v_i \Rightarrow$$

$$\frac{R_5}{R_4} = 32.3$$

To check at high-frequency,

$$\frac{R_2}{R_1 + R_2} v_i = \frac{1.5}{1.5 + 48.5} v_i = 0.03 v_i$$

$$v_o = (1 + 32.3) v_i = (33.3)(0.03) v_i = (1.0) v_i$$

which meets the design specification

Consider the frequency response.

$$\frac{v_{o1}}{v_i} = \frac{\frac{R_2 + R_3}{sC}}{\frac{R_1 + R_2 + R_3}{sC}}$$

Now

$$R_3 \parallel \frac{1}{sC} = \frac{R_3}{1 + sR_3C}$$

Then, we find

$$\frac{v_{o1}}{v_i} = \frac{R_2 + R_3 (1 + sR_3C)}{R_1 + (R_1 + R_2)(1 + sR_3C)}$$

which can be rearranged as

$$\frac{v_{o1}}{v_i} = \frac{(R_2 + R_3)(1 + s(R_1 \parallel R_2)C)}{(R_1 + R_2 + R_3)(1 + s(R_3 \parallel (R_1 + R_2))C)}$$

So

$$f_L \equiv \frac{1}{2\pi(R_1 \parallel R_2)C} = \frac{1}{2\pi(1.5 \parallel 144) \times 10^3 C} = \frac{1}{(9.33 \times 10^3)C}$$

$$f_H \equiv \frac{1}{2\pi(R_3 \parallel (R_1 + R_2))C} = \frac{1}{2\pi(144 \parallel 50) \times 10^3 C} \\ = \frac{1}{(2.33 \times 10^5)C}$$

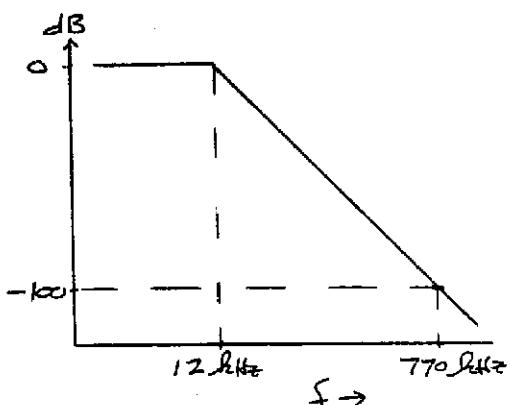
Set

$$25 \text{ kHz} = \frac{f_L + f_H}{2} = \frac{1}{2} \left[\frac{1}{(9.33 \times 10^3)C} + \frac{1}{(2.33 \times 10^5)C} \right]$$

Which yields

$$\underline{C = 2.23 \text{ nF}}$$

15.8



$$|A_v| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{-3dB}}\right)^{2N}}}$$

$$-100 \text{ dB} \Rightarrow 10^{-5}$$

So

$$10^{-5} = \frac{1}{\sqrt{1 + \left(\frac{770}{12}\right)^{2N}}}$$

or

$$1 + (64.2)^{2N} = \left(\frac{1}{10^{-5}}\right)^2 = 10^{10}$$

or

$$(64.2)^{2N} \equiv 10^{10}$$

Now

| <u>N</u> | Left Side |
|----------|---------------------|
| 1 | 4.112×10^3 |
| 2 | 1.7×10^7 |
| 3 | 7×10^{10} |

So, we need a 3rd order filter.Low-pass: $-50 \text{ dB} \Rightarrow 3.16 \times 10^{-3}$

Then

$$3.16 \times 10^{-3} = \frac{1}{\sqrt{1 + \left(\frac{f}{f_L}\right)^4}} = \frac{1}{\sqrt{1 + \left(\frac{60}{f_L}\right)^4}}$$

$$\text{We find } f_L = 3.37 \text{ Hz}$$

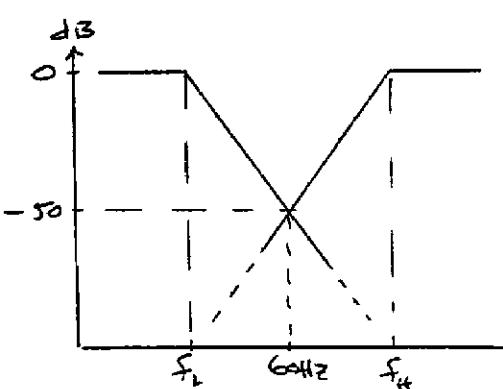
High Pass:

$$3.16 \times 10^{-3} = \frac{1}{\sqrt{1 + \left(\frac{f_H}{f}\right)^4}} = \frac{1}{\sqrt{1 + \left(\frac{f_H}{60}\right)^4}}$$

$$\text{We find } f_H = 1067 \text{ Hz}$$

$$\text{Bandwidth: } BW = f_H - f_L = 1067 - 3.37 \Rightarrow BW \approx 1064 \text{ Hz}$$

15.9



15.10

a.

$$\frac{\nu_I}{R_4} = -\frac{\nu_{02}}{R_3} - \frac{\nu_0}{R_1 \parallel \left(\frac{1}{sC}\right)} \quad (1)$$

$$\frac{\nu_0}{R_2} = -\frac{\nu_{01}}{\left(\frac{1}{sC}\right)} \quad (2)$$

$$\frac{\nu_{01}}{R_5} = -\frac{\nu_{02}}{R_4} \Rightarrow \nu_{01} = -\nu_{02} \quad (3)$$

Then

$$\frac{\nu_0}{R_2} = +\frac{\nu_{02}}{\left(\frac{1}{sC}\right)} \text{ or } \nu_{02} = \nu_0 \left(\frac{1}{sR_2 C}\right) \quad (2)$$

And

$$\begin{aligned} \frac{\nu_I}{R_4} &= -\frac{\nu_0}{R_3} \cdot \left(\frac{1}{sR_2 C}\right) - \frac{\nu_0}{R_1 \parallel \left(\frac{1}{sC}\right)} \\ &= -\nu_0 \left[\frac{1}{R_3(sR_2 C)} + \frac{1}{R_1 \cdot (1/sC)} \right] \\ &= -\nu_0 \left[\frac{1}{R_3(sR_2 C)} + \frac{1 + sR_1 C}{R_1} \right] \\ &= -\nu_0 \left[\frac{R_1 + (1 + sR_1 C)(sR_2 R_3 C)}{(sC)R_1 R_2 R_3} \right] \end{aligned} \quad (1)$$

Then

$$\frac{\nu_0}{\nu_I} = -\frac{1}{R_4} \left[\frac{(sC)(R_1 R_2 R_3)}{R_1 + sR_2 R_3 C + s^2 R_1 R_2 R_3 C^2} \right]$$

or

$$A_v(s) = \frac{\nu_0}{\nu_I} = \frac{-\frac{1}{R_4}}{\frac{1}{R_1} + sC + \frac{1}{sCR_2 R_3}}$$

b. $A_V(j\omega) = \frac{-\frac{1}{R_4}}{\frac{1}{R_1} + j\omega C + \frac{1}{j\omega C R_2 R_3}}$

or

$$A_V(j\omega) = \frac{-\frac{1}{R_4}}{\frac{1}{R_1} + j\left[\omega C - \frac{1}{\omega C R_2 R_3}\right]} \\ = -\frac{R_1}{R_4} \cdot \frac{1}{\left\{1 + j\left[\omega R_1 C - \frac{R_1}{\omega C R_2 R_3}\right]\right\}}$$

$$|A_V(j\omega)| = \frac{R_1}{R_4} \cdot \frac{1}{\left\{1 + \left[\omega R_1 C - \frac{R_1}{\omega C R_2 R_3}\right]^2\right\}^{1/2}}$$

$$|A_V|_{max} \text{ when } \left[\omega R_1 C - \frac{R_1}{\omega C R_2 R_3}\right] = 0$$

Then

$$|A_V|_{max} = \frac{R_1}{R_4} = \frac{85}{3} \Rightarrow |A_V|_{max} = 28.3$$

Now

$$\omega R_1 C \left[1 - \frac{1}{\omega^2 C^2 R_2 R_3}\right] = 0 \text{ or } \omega = \frac{1}{C \sqrt{R_2 R_3}}$$

Then

$$f = \frac{1}{2\pi C \sqrt{R_2 R_3}} = \frac{1}{2\pi(0.1 \times 10^{-6})\sqrt{(300)^2}}$$

So

$$\underline{f = 5.305 \text{ kHz}}$$

To find the two $3 - dB$ frequencies,

$$\left[\omega R_1 C - \frac{R_1}{\omega C R_2 R_3}\right] = \pm 1$$

$$\omega^2 R_1 R_2 R_3 C^2 - R_1 = \pm \omega R_2 R_3 C$$

$$\omega^2 (85 \times 10^3) (300)^2 (0.1 \times 10^{-6})^2 - 85 \times 10^3 \\ = \pm \omega (300)^2 (0.1 \times 10^{-6})$$

$$\omega^2 (7.65 \times 10^{-5}) - 85 \times 10^3 = \pm \omega (9 \times 10^{-3})$$

$$\omega^2 (7.65 \times 10^{-5}) \pm \omega (9 \times 10^{-3}) - 85 \times 10^3 = 0$$

$$\omega = \frac{\pm (9 \times 10^{-3})}{2(7.65 \times 10^{-5})} \\ \pm \frac{\sqrt{(9 \times 10^{-3})^2 + 4(7.65 \times 10^{-5})(85 \times 10^3)}}{2(7.65 \times 10^{-5})}$$

We find $f = 5.315 \text{ kHz}$ and $f = 5.296 \text{ kHz}$

15.11

a.

$$\frac{\nu_I - \nu_A}{R} = \frac{\nu_A}{\left(\frac{1}{sC}\right)} \quad (1)$$

$$\frac{\nu_I - \nu_B}{R} = \frac{\nu_B - \nu_0}{R} \quad (2)$$

$$\text{and } \nu_A = \nu_B$$

So

$$\frac{\nu_I}{R} = \nu_A \left(\frac{1}{R} + sC\right) = \nu_A \left(\frac{1 + sRC}{R}\right) \quad (1)$$

or

$$\nu_A = \frac{\nu_I}{1 + sRC}$$

Then

$$\nu_I + \nu_0 = 2\nu_B = 2\nu_A = \frac{2\nu_I}{1 + sRC} \quad (2)$$

$$\nu_0 = \nu_I \left[\frac{2}{1 + sRC} - 1\right] = \nu_I \left[\frac{1 - sRC}{1 + sRC}\right]$$

Now

$$\frac{\nu_0}{\nu_I} = A(j\omega) = \frac{1 - j\omega RC}{1 + j\omega RC}$$

$$|A| = \frac{\sqrt{1 + \omega^2 R^2 C^2}}{\sqrt{1 + \omega^2 R^2 C^2}} \Rightarrow |A| = 1$$

Phase:

$$\phi = -2 \tan^{-1} (\omega RC)$$

b. $RC = (10^4)(15.9 \times 10^{-9}) = 1.59 \times 10^{-4}$

| f | ϕ |
|-------------------------------|-------------|
| 0 | 0 |
| 10^2 | -11.4 |
| 5×10^3 | -53.1 |
| $1/2\pi RC = 10^3 \text{ Hz}$ | -90° |
| 5×10^3 | -157 |
| 10^4 | -169 |

15.12

$$a. \frac{V_i}{R_1} + \frac{V_i - V_0}{R_2 \parallel (1/sC)} = 0$$

$$\frac{V_i}{R_1} + \frac{V_i - V_0}{\frac{R_2}{1+sR_2C}} = 0$$

$$\frac{R_2}{R_1} \cdot \frac{1}{1+sR_2C} (V_i) + V_i = V_0$$

$$\frac{V_0}{V_i} = \frac{R_2 + R_1(1+sR_2C)}{R_1(1+sR_2C)}$$

$$= \frac{(R_2 + R_1)(1+s(R_1 \parallel R_2)C)}{R_1(1+sR_2C)}$$

$$\Rightarrow \frac{V_0}{V_i} = \left(1 + \frac{R_2}{R_1}\right) \left[\frac{1+s(R_1 \parallel R_2)C}{(1+sR_2C)} \right]$$

$$\Rightarrow f_{3dB_1} = \frac{1}{2\pi R_2 C}$$

$$\Rightarrow f_{3dB_2} = \frac{1}{2\pi(R_1 \parallel R_2)C}$$

$$b. \frac{V_i}{R_1 \parallel (1/sC)} + \frac{V_i - V_0}{R_2} = 0$$

$$\frac{V_i}{\left(\frac{R_1}{1+sR_1C}\right)} + \frac{V_i}{R_2} = \frac{V_0}{R_2}$$

$$V_i \left[\frac{R_2}{R_1} \cdot (1+sR_1C) + 1 \right] = V_0$$

$$\frac{V_i}{R_1} \cdot [R_2 + R_1 + sR_1R_2C] = V_0$$

$$\frac{V_0}{V_i} = \frac{R_2 + R_1}{R_1} \cdot [1 + s(R_1 \parallel R_2)C]$$

$$\Rightarrow \frac{V_0}{V_i} = \left(1 + \frac{R_2}{R_1}\right) [1 + s(R_1 \parallel R_2)C]$$

$$\Rightarrow f_{3dB} = \frac{1}{2\pi(R_1 \parallel R_2)C}$$

15.13

$$a. \frac{V_i}{R_1 + (1/sC_1)} = \frac{-V_0}{R_2 \parallel (1/sC_2)}$$

$$V_i \left(\frac{sC_1}{1+sR_1C_1} \right) = -V_0 \left(\frac{1+sR_2C_2}{sC_2} \right)$$

$$\frac{V_0}{V_i} = \frac{-sR_2C_1}{(1+sR_1C_1)(1+sR_2C_2)} \\ = \frac{-sR_2C_1}{1+sR_1C_1 + sR_2C_2 + s^2R_1R_2C_1C_2}$$

$$\frac{V_0}{V_i} = -\frac{R_2}{R_1} \times$$

$$\times \left[\frac{sC_1}{\frac{1}{R_1} + sC_1 \left(1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1} \right) + s^2R_1R_2C_1C_2} \right]$$

or

$$T(s) = \frac{V_0}{V_i}$$

$$= -\frac{R_2}{R_1} \cdot \left[\frac{1}{\frac{1}{sR_1C_1} + \left(1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1} \right) + sR_2C_2} \right]$$

b.

$$|T(j\omega)| = -\frac{R_2}{R_1} \times \frac{1}{\left\{ \left(1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1} \right)^2 + \left(\omega R_2 C_2 - \frac{1}{\omega R_1 C_1} \right)^2 \right\}^{1/2}}$$

when $\left(\omega R_2 C_2 - \frac{1}{\omega R_1 C_1} \right) = 0$, we want

$$|T(j\omega)| = 50 = \frac{R_2}{R_1} \cdot \frac{1}{\left(1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1} \right)}$$

At the $3 - dB$ frequencies, we want

$$\left(\omega R_2 C_2 - \frac{1}{\omega R_1 C_1} \right) = \pm \left(1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1} \right)$$

For $f = 5$ kHz, use + sign and for $f = 200$ Hz, use - sign.

$$\omega_1 = 2\pi(200) = 1257$$

$$\omega_2 = 2\pi(5 \times 10^3) = 3.142 \times 10^4$$

Define $r_2 = R_2C_2$ and $r_1 = R_1C_1$
Then

$$50 = \frac{R_2}{R_1} \cdot \frac{1}{1 + \frac{r_2}{r_1}} \quad (1)$$

$$\left(\omega_2 r_2 - \frac{1}{\omega_2 r_1} \right) = + \left(1 + \frac{r_2}{r_1} \right) \quad (2)$$

$$\left(\omega_1 r_2 - \frac{1}{\omega_1 r_1} \right) = - \left(1 + \frac{r_2}{r_1} \right) \quad (3)$$

From (2)

$$\frac{\omega_2^2 r_1 r_2 - 1}{\omega_2 r_1} = \frac{r_1 + r_2}{r_1}$$

or

$$\omega_2 r_1 r_2 - \frac{1}{\omega_2} = r_1 + r_2$$

$$r_1 (\omega_2 r_2 - 1) = \frac{1}{\omega_2} + r_2$$

So

$$r_1 = \frac{\frac{1}{\omega_1} + r_2}{\omega_2 r_2 - 1}$$

Substituting into (3), we find

$$\omega_1 r_2 - \frac{1}{\omega_1 \left[\frac{1}{\omega_2} + r_2 \right]} = - \left[1 + \frac{r_2(\omega_2 r_2 - 1)}{\frac{1}{\omega_2} + r_2} \right]$$

$$\begin{aligned} \omega_1 r_2 \left[\frac{1}{\omega_2} + r_2 \right] - \frac{1}{\omega_1} (\omega_2 r_2 - 1) \\ = - \left[\left(\frac{1}{\omega_2} + r_2 \right) + r_2 (\omega_2 r_2 - 1) \right] \end{aligned}$$

$$\begin{aligned} \frac{\omega_1}{\omega_2} \cdot r_2 + \omega_1 r_2^2 - \frac{\omega_2}{\omega_1} \cdot r_2 + \frac{1}{\omega_1} \\ = - \frac{1}{\omega_2} - r_2 - \omega_2 r_2^2 + r_2 \end{aligned}$$

$$(\omega_1 + \omega_2) r_2^2 + \left(\frac{\omega_1}{\omega_2} - \frac{\omega_2}{\omega_1} \right) r_2 + \left(\frac{1}{\omega_1} + \frac{1}{\omega_2} \right) = 0$$

$$(3.2677 \times 10^4) r_2^2 - 24.96 r_2 + 8.273 \times 10^{-4} = 0$$

$$\begin{aligned} r_2 = \frac{24.96}{2(3.2677 \times 10^4)} \\ \pm \frac{\sqrt{(24.96)^2 - 4(3.2677 \times 10^4)(8.273 \times 10^{-4})}}{2(3.2677 \times 10^4)} \end{aligned}$$

Since ω_2 is large, r_2 should be small so use minus sign:

$$r_2 = 3.47 \times 10^{-5}$$

Then

$$r_1 = \frac{3.18 \times 10^{-5} + 3.47 \times 10^{-5}}{9.09 \times 10^{-2}} \Rightarrow r_1 = 7.32 \times 10^{-4}$$

Now

$$50 = \frac{R_2}{R_1} \cdot \frac{1}{1 + \frac{3.47 \times 10^{-5}}{7.32 \times 10^{-4}}}$$

Then

$$\frac{R_2}{R_1} = 52.37 \text{ or } R_2 = 524 \text{ k}\Omega$$

Also

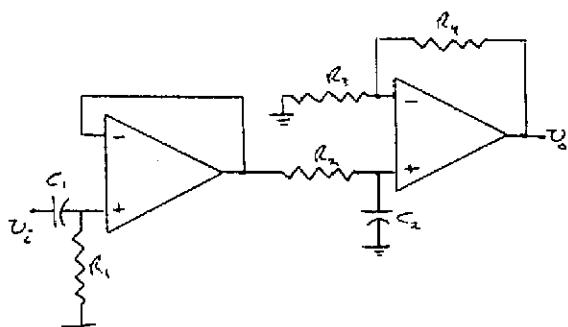
$$r_1 = R_1 C_1 \text{ so that } C_1 = 0.0732 \mu\text{F}$$

$$r_2 = R_2 C_2 \text{ so that } C_2 = 66.3 \text{ pF}$$

15.14

$$\text{Gain} = 10 \text{ dB} \Rightarrow \text{Gain} = 3.162$$

For example, we may have



$$\text{Want } \frac{R_4}{R_3} = 2.162$$

For example, let $R_3 = 50 \text{ k}\Omega$,

$$R_4 = 108 \text{ k}\Omega$$

$$f_1 = \frac{1}{2\pi R_1 C_1} = 200$$

So

$$R_1 C_1 = \frac{1}{2\pi(200)} = 0.796 \times 10^{-3}$$

For example, let $R_1 = 200 \text{ k}\Omega \Rightarrow$ A large input resistance

$$C_1 = 0.00398 \mu\text{F}$$

$$f_2 = \frac{1}{2\pi R_2 C_2} = 50 \times 10^3$$

$$\Rightarrow R_2 C_2 = \frac{1}{2\pi(50 \times 10^3)} = 3.18 \times 10^{-6}$$

For example, let

$$R_2 = 10 \text{ k}\Omega \text{ and } C_2 = 318 \text{ pF}$$

15.15

$$f_C = 100 \text{ kHz}$$

$$R_{eq} = \frac{1}{f_C C}$$

- a. For $C = 1 \text{ pF}$, $R_{eq} = 10 \text{ M}\Omega$
- b. For $C = 10 \text{ pF}$, $R_{eq} = 1 \text{ M}\Omega$
- c. For $C = 30 \text{ pF}$, $R_{eq} = 333 \text{ k}\Omega$

15.16

- a. From Equation (15.28),

$$Q = \frac{V_1 - V_2}{R_{eq}} \cdot T_C$$

and $f_C = 100 \text{ kHz}$ so that $T_C = \frac{1}{100 \times 10^3} \Rightarrow 10 \mu\text{s}$

Now

$$R_{eq} = \frac{1}{f_C C} = \frac{1}{(100 \times 10^3)(10 \times 10^{-12})} = 1 \text{ M}\Omega$$

So

$$Q = \frac{(2-1)(10 \times 10^{-6})}{10^6} = 10 \times 10^{-12} \text{ C}$$

or

$$\underline{Q = 10 \text{ pC}}$$

b. $I_{eq} = \frac{Q}{T_C} = \frac{10 \times 10^{-12}}{10 \times 10^{-6}} \text{ or } \underline{I_{eq} = 1 \mu\text{A}}$

c.

$Q = CV$ so find the time that V_0 reaches 99% of its full value.

$$V_0 = V_1 \left(1 - e^{-t/\tau}\right) \text{ where } \tau = RC$$

$$\text{Then } 0.99 = 1 - e^{-t/\tau} \text{ or } e^{-t/\tau} = 0.01$$

$$\text{or } t = \tau \ln(100)$$

$$\tau = RC = (10^3)(10 \times 10^{-12}) = 10^{-8} \text{ s}$$

Then

$$\underline{t = 4.61 \times 10^{-4} \text{ s}}$$

15.17

$$\text{Low frequency gain} = -10 \Rightarrow \frac{C_1}{C_2} = 10$$

$$f_{3dB} = 10 \times 10^3 \text{ Hz} = \frac{f_C C_2}{2\pi C_F}$$

Set

$$f_C = 10 f_{3dB} = 100 \text{ kHz}$$

Then

$$\frac{C_2}{C_F} = \frac{2\pi(10 \times 10^3)}{100 \times 10^3} = 0.628$$

The largest capacitor is C_1 , so let

$$\underline{C_1 = 30 \text{ pF}}$$

Then

$$\underline{C_2 = 3 \text{ pF}}$$

and

$$\underline{C_F = 4.78 \text{ pF}}$$

15.18

- a. Time constant = $R_{eq} \cdot C_F = \tau$ where

$$R_{eq} = \frac{1}{f_C C_1} = \frac{1}{(100 \times 10^3)(5 \times 10^{-12})} = 2 \times 10^6 \Omega$$

Then

$$\tau = (2 \times 10^6)(30 \times 10^{-12})$$

or

$$\underline{\tau = 60 \mu\text{s}}$$

b. $\nu_0 = -\frac{1}{\tau} \int \nu_I \cdot dt$

or

$$\Delta \nu_0 = \frac{(1)T_C}{\tau}, \quad T_C = \frac{1}{f_C}$$

So

$$\Delta \nu_0 = \frac{1}{(60 \times 10^{-6})(100 \times 10^3)}$$

or

$$\underline{\Delta \nu_0 = 0.167 \text{ V}}$$

c. Now $\Delta \nu_0 = 13 = N(0.167)$

or

$$\underline{N = 78 \text{ clock pulses}}$$

15.19

Using Equation (15.41)

$$f_0 = \frac{1}{2\pi\sqrt{3RC}} = \frac{1}{2\pi\sqrt{3}(4 \times 10^3)(10 \times 10^{-9})}$$

or

$$\underline{f_0 = 2.3 \text{ kHz}}$$

$$\frac{R_2}{R} = 8 \text{ so that } R_2 = 8(4 \times 10^3) \Rightarrow \underline{R_2 = 32 \text{ k}\Omega}$$

15.20

a. $\nu_1 = \frac{R}{R + (1/sC_V)} \cdot \nu_0 = \left(\frac{sRC_V}{1+sRC_V}\right) \cdot \nu_0$

$$\nu_2 = \frac{R}{R + \frac{1}{sC}} \cdot \nu_1 = \left(\frac{sRC}{1+sRC}\right) \cdot \nu_1$$

$$\nu_3 = \frac{R}{R + \frac{1}{sC}} \cdot \nu_2 = \left(\frac{sRC}{1+sRC}\right) \cdot \nu_2$$

$$\nu_0 = -\frac{R_2}{R} \cdot \nu_3$$

Chapter 15: Problem Solutions

Then

$$\nu_0 = -\frac{R_2}{R} \left(\frac{sRC}{1+sRC} \right)^2 \left(\frac{sRCV}{1+sRCV} \right) \nu_0$$

Set $s = j\omega$

$$1 = -\frac{R_2}{R} \left(\frac{-\omega^2 R^2 C^2}{1+2j\omega RC - \omega^2 R^2 C^2} \right) \left(\frac{j\omega RCV}{1+j\omega RCV} \right)$$

The real part of the denominator must be zero.

$$1 - \omega^2 R^2 C^2 - 2\omega^2 R^2 C C V = 0$$

so

$$\omega_0 = \frac{1}{R\sqrt{C(C+2CV)}}$$

b. $f_{0,\max} = \frac{1}{2\pi(10^4)\sqrt{(10^{-11})(10^{-11} + 2[10^{-11}])}}$

$$f_{0,\max} = 919 \text{ kHz}$$

$$f_{0,\min} = \frac{1}{2\pi(10^4)\sqrt{(10^{-11})(10^{-11} + 2[50 \times 10^{-12}])}}$$

$$f_{0,\min} = 480 \text{ kHz}$$

15.21

From Equation (15.46)

$$\nu_0 = \frac{1}{2\pi\sqrt{6}RC}$$

$$\text{so } R = \frac{1}{2\pi\sqrt{6}(80 \times 10^3)(100 \times 10^{-12})} \text{ or } R = 8.12 \text{ k}\Omega$$

We need

$$\frac{R_2}{R} = 29$$

so that

$$R_2 = 236 \text{ k}\Omega$$

15.22

$$\frac{\nu_0 - \nu_1}{\frac{1}{sC}} = \frac{\nu_1}{R} + \frac{\nu_1 - \nu_2}{\frac{1}{sC}} \quad (1)$$

$$\text{or } (\nu_0 - \nu_1)sC = \frac{\nu_1}{R} + (\nu_1 - \nu_2)sC$$

$$\frac{\nu_1 - \nu_2}{\frac{1}{sC}} = \frac{\nu_2}{R} + \frac{\nu_2}{\frac{1}{sC} + R} \quad (2)$$

$$\text{or } (\nu_1 - \nu_2)sC = \frac{\nu_2}{R} + \frac{\nu_2(sC)}{1+sRC}$$

$$\frac{\nu_2}{\frac{1}{sC} + R} = -\frac{\nu_0}{R_2} \quad (3)$$

$$\text{or } \frac{\nu_2 sC}{1+sRC} = -\frac{\nu_0}{R_2}$$

so

$$\nu_2 = \frac{-\nu_0}{sR_2C} (1 + sRC)$$

From (2)

$$\nu_1(sC) = \nu_2 \left[sC + \frac{1}{R} + \frac{sC}{1+sRC} \right]$$

or

$$\nu_1 = -\frac{\nu_0(1+sRC)}{sR_2C} \cdot \left[1 + \frac{1}{sRC} + \frac{1}{1+sRC} \right]$$

From (1)

$$\nu_0(sC) = \nu_1 \left[sC + \frac{1}{R} + sC \right] - \nu_2(sC)$$

Then

$$\nu_0 = \left[2 + \frac{1}{sRC} \right] \left[\frac{-\nu_0(1+sRC)}{sR_2C} \right] \times \left[\frac{1+sRC}{sRC} + \frac{1}{1+sRC} \right] + \frac{\nu_0}{sR_2C} \cdot (1+sRC)$$

$$-1 = \left[\frac{1+2sRC}{sRC} \right] \left[\frac{1+sRC}{sR_2C} \right] \left[\frac{(1+sRC)^2 + sRC}{(sRC)(1+sRC)} \right] - \frac{1+sRC}{sR_2C}$$

$$-1 = \frac{(1+2sRC)(1+2sRC + s^2 R^2 C^2 + sRC)}{(sRC)^2 (sR_2C)} - \frac{(1+sRC)(sRC)^2}{(sRC)^2 (sR_2C)}$$

Set $s = j\omega$

$$-1 = \frac{(1+2j\omega RC)(1+3j\omega RC + \omega^2 R^2 C^2)}{(-\omega^2 R^2 C^2)(j\omega R_2 C)} - \frac{(1+j\omega RC)(-\omega^2 R^2 C^2)}{(-\omega^2 R^2 C^2)(j\omega R_2 C)}$$

The real part of the numerator must be zero.

$$1 - \omega^2 R^2 C^2 - 6\omega^2 R^2 C^2 + \omega^2 R^2 C^2 = 0$$

$$6\omega^2 R^2 C^2 = 1$$

so that

$$\omega_0 = \frac{1}{\sqrt{6}RC}$$

Condition for oscillation:

$$-1 = \frac{2j\omega RC + 3j\omega RC - 2j\omega^3 R^3 C^3 + j\omega^3 R^3 C^3}{(-\omega^2 R^2 C^2)(j\omega R_2 C)}$$

$$1 = \frac{5 - \omega^2 R^2 C^2}{(\omega RC)(\omega R_2 C)}$$

But

$$\omega = \omega_0 = \frac{1}{\sqrt{6}RC}$$

Then

$$1 = \frac{5 - \frac{1}{6}}{\frac{(RC)(R_2C)}{6R^2C^2}} = \frac{\left(5 - \frac{1}{6}\right)(6R^2C^2)}{RR_2C^2}$$

$$1 = \frac{\left(\frac{29}{6}\right)(6R)}{R_2} \text{ or } \frac{R_2}{R} = 29$$

15.23

a.

$$\nu_{01} = \left(1 + \frac{R_{F1}}{R_{A1}}\right) \left(\frac{\frac{1}{sC_1}}{\frac{1}{sC_1} + R_1} \right) \cdot \nu_0 \quad (1)$$

$$\nu_{02} = \left(1 + \frac{R_{F2}}{R_{A2}}\right) \left(\frac{\frac{1}{sC_2}}{\frac{1}{sC_2} + R_2} \right) \cdot \nu_{01} \quad (2)$$

$$\nu_{03} = \left(\frac{R_{A3}}{R_{A3} \parallel \frac{1}{sC_3} + R_3} \right) \cdot \nu_{02} \quad (3)$$

$$\nu_0 = -\frac{R_{F3}}{R_{A3}} \cdot \nu_{03} \quad (4)$$

With all resistors equal and all capacitors equal, we find:

$$\nu_{01} = (2) \left(\frac{1}{1 + sRC} \right) \nu_0 \quad (1)$$

$$\nu_{02} = (2) \left(\frac{1}{1 + sRC} \right) \nu_{01} \quad (2)$$

$$\begin{aligned} \nu_{03} &= (2) \left(\frac{\frac{R}{1 + sRC}}{\frac{R}{1 + sRC} + R} \right) \nu_{02} \\ &= \left[\frac{R}{R + R(1 + sRC)} \right] \nu_{02} \\ &= \left(\frac{1}{2 + sRC} \right) \nu_{02} \end{aligned} \quad (3)$$

and

$$\nu_0 = -\nu_{03} \quad (4)$$

Then

$$\nu_0 = -\left(\frac{1}{2 + sRC} \right) (2) \left(\frac{1}{1 + sRC} \right) (2) \left(\frac{1}{1 + sRC} \right) \nu_0$$

Let $s = j\omega$

$$(2 + j\omega RC)(1 + j\omega RC)(1 + j\omega RC) = -4$$

$$(2 + j\omega RC)(1 + 2j\omega RC - \omega^2 R^2 C^2) = -4 \quad (A)$$

The imaginary term on the left must be zero.

$$4j\omega RC + j\omega RC - j\omega^3 R^3 C^3 = 0$$

$$\omega RC(5 - \omega^2 R^2 C^2) = 0$$

or

$$\omega = \frac{\sqrt{5}}{RC} \quad (\text{Not the same as in book})$$

15.24

a.

$$\frac{\nu_0 - \nu_{01}}{R} = \frac{\nu_{01}}{\left(\frac{1}{sC}\right)} + \frac{\nu_{01} - \nu_{02}}{R} \quad (1)$$

$$\frac{\nu_{01} - \nu_{02}}{R} = \frac{\nu_{02}}{\left(\frac{1}{sC}\right)} + \frac{\nu_{02} - \nu_{03}}{R} \quad (2)$$

$$\frac{\nu_{02} - \nu_{03}}{R} = \frac{\nu_{03}}{\left(\frac{1}{sC}\right)} + \frac{\nu_{03}}{R} \quad (3)$$

$$\nu_0 = -\frac{R_F}{R} \cdot \nu_{03} \quad (4)$$

We can write the equations as

$$\nu_0 - \nu_{01} = \nu_{01}(sRC) + \nu_{01} - \nu_{02} \quad (1)$$

$$\nu_{01} - \nu_{02} = \nu_{02}(sRC) + \nu_{02} - \nu_{03} \quad (2)$$

$$\nu_{02} - \nu_{03} = \nu_{03}(sRC) + \nu_{03} \quad (3)$$

and

$$\nu_0 = -\frac{R_F}{R} \cdot \nu_{03} \quad (4)$$

Combining terms, we find

$$\nu_0 = \nu_{01}(2 + sRC) - \nu_{02} \quad (1)$$

$$\nu_{01} = \nu_{02}(2 + sRC) - \nu_{03} \quad (2)$$

$$\nu_{02} = \nu_{03}(2 + sRC) \quad (3)$$

and

$$\nu_0 = -\frac{R_F}{R} \cdot \nu_{03} \quad (4)$$

Combining Equations (3) and (2)

$$\nu_{01} = \nu_{03}(2 + sRC)^2 - \nu_{03} = \nu_{03}[(2 + sRC)^2 - 1] \quad (2)$$

Then Equation (1) is

$$\begin{aligned} \nu_0 &= \nu_{03}[(2 + sRC)^2 - 1](2 + sRC) \\ &\quad - \nu_{03}(2 + sRC) \end{aligned}$$

Using Equation (4), we find

$$-\frac{R_F}{R} \cdot v_{03} = v_{03} \{ [(2 + sRC)^2 - 1](2 + sRC) \\ - (2 + sRC) \}$$

To find the frequency of oscillation, set $s = j\omega$ and set the imaginary part of the right side of the equation to zero.

We will have

$$-\frac{R_F}{R} = (2 + j\omega RC)[4 + 4j\omega RC - \omega^2 R^2 C^2 - 1 - 1]$$

Then

$$j\omega RC(2 - \omega^2 R^2 C^2) + 8j\omega RC = 0$$

or

$$j\omega RC[2 - \omega^2 R^2 C^2 + 8] = 0$$

Then the frequency of oscillation is

$$f_0 = \frac{1}{2\pi} \cdot \frac{\sqrt{10}}{RC}$$

The condition to sustain oscillations is determined from

$$-\frac{R_F}{R} = 2[2 - \omega^2 R^2 C^2] - 4\omega^2 R^2 C^2$$

or

$$-\frac{R_F}{R} = 4 - 6\omega^2 R^2 C^2$$

Setting $\omega^2 = \frac{10}{R^2 C^2}$, we have

$$-\frac{R_F}{R} = 4 - 6(10)$$

or

$$\frac{R_F}{R} = 56$$

b. For $R = 5 \text{ k}\Omega$ and $f_0 = 5 \text{ kHz}$, we find

$$C = \frac{\sqrt{10}}{2\pi(5 \times 10^3)(5 \times 10^3)} \Rightarrow C = 0.02 \mu\text{F}$$

$$\text{and } R_F = 56(5) \Rightarrow R_F = 280 \text{ k}\Omega$$

15.25

a. We can write

$$v_A = \left(\frac{R_1}{R_1 + R_2} \right) v_0 \text{ and } v_B = \left(\frac{Z_p}{Z_p + Z_s} \right) v_0$$

$$\text{where } Z_p = R_B \parallel \frac{1}{sC_B} = \frac{R_B}{1 + sR_B C_B}$$

$$\text{and } Z_s = R_A + \frac{1}{sC_A} = \frac{1 + sR_A C_A}{sC_A}$$

Setting $v_A = v_B$, we have

$$\frac{R_1}{R_1 + R_2} = \frac{\frac{R_B}{1 + sR_B C_B}}{\frac{R_B}{1 + sR_B C_B} + \frac{1 + sR_A C_A}{sC_A}}$$

$$\frac{R_1}{R_1 + R_2} = \frac{sR_B C_A}{sR_B C_A + (1 + sR_A C_A)(1 + sR_B C_B)} \quad (1)$$

To find the frequency of oscillation, set $s = j\omega$ and set the real part of the denominator on the right side of Equation (1) equal to zero.

The denominator term is

$$j\omega R_B C_A + (1 + j\omega R_A C_A)(1 + j\omega R_B C_B)$$

or

$$j\omega R_B C_A + 1 + j\omega R_A C_A + j\omega R_B C_B \\ - \omega^2 R_A R_B C_A C_B \quad (2)$$

Then from (2), we must have

$$1 - \omega_0^2 R_A R_B C_A C_B = 0$$

or

$$f_0 = \frac{1}{2\pi\sqrt{R_A R_B C_A C_B}}$$

b. To find the condition for sustained oscillation, combine Equations (1) and (2). Then

$$\frac{R_1}{R_1 + R_2} = \frac{j\omega R_B C_A}{j\omega R_B C_A + j\omega R_A C_A + j\omega R_B C_B}$$

or

$$1 + \frac{R_1}{R_1} = 1 + \frac{R_A}{R_B} + \frac{C_B}{C_A}$$

Then

$$\frac{R_2}{R_1} = \frac{R_A}{R_B} + \frac{C_B}{C_A}$$

15.26

a. We can write

$$\nu_A = \left(\frac{R_1}{R_1 + R_2} \right) \nu_0$$

and

$$\nu_B = \left(\frac{R \parallel sL}{R \parallel sL + R + sL} \right) \nu_0$$

Setting $\nu_A = \nu_B$, we have

$$\frac{R_1}{R_1 + R_2} = \left[\frac{\frac{sRL}{R + sL}}{\frac{sRL}{R + sL} + R + sL} \right] \cdot \nu_0$$

$$\frac{R_1}{R_1 + R_2} = \frac{sRL}{sRL + (R + sL)^2} \quad (1)$$

To find the frequency of oscillation, set $s = j\omega$ and set the real part of the denominator on the right side of Equation (1) equal to zero.

The denominator term is:

$$j\omega RL + (R + j\omega L)^2$$

or

$$j\omega RL + R^2 + 2j\omega RL - \omega^2 L^2 \quad (2)$$

Then

$$R^2 - \omega_0^2 L^2 = 0$$

or

$$f_0 = \frac{1}{2\pi} \cdot \sqrt{\frac{R}{L}}$$

b. To find the condition for sustained oscillations, combine Equations (1) and (2).

$$\frac{R_1}{R_1 + R_2} = \frac{j\omega RL}{j\omega RL + 2j\omega RL} = \frac{1}{3}$$

Then

$$1 + \frac{R_2}{R_1} = 3$$

so that

$$\frac{R_2}{R_1} = 2$$

15.27

From Equation (15.52(b))

$$f_0 = \frac{1}{2\pi RC}$$

or

$$RC = \frac{1}{2\pi f} = \frac{1}{2\pi(80 \times 10^3)} \\ RC \approx 2 \times 10^{-6}$$

Set $R = 20 \text{ k}\Omega$ and $C = 100 \text{ pF}$

We must have

$$\frac{R_2}{R_1} = 2$$

Set $R_2 = 40 \text{ k}\Omega$ and $R_1 = 20 \text{ k}\Omega$, for example.

15.28

From Equation (15.59)

$$f_0 = \frac{1}{2\pi \sqrt{L \left(\frac{C_1 C_2}{C_1 + C_2} \right)}}$$

and from Equation (15.61)

$$\frac{C_2}{C_1} = g_m R$$

Now,

$$g_m = 2\sqrt{k_n I_D q} = 2\sqrt{(0.5)(1)} = 1.414 \text{ mA/V}$$

We have $C_1 = 0.01 \mu\text{F}$, $R = 4 \text{ k}\Omega$, $f_0 = 400 \text{ kHz}$

So

$$C_2 = g_m R C_1 = (1.414)(4)(0.01)$$

or

$$\underline{C_2 = 0.0566 \mu\text{F}}$$

and

$$400 \times 10^3 = \frac{1}{2\pi \sqrt{L \left[\frac{(0.01)(0.0566)}{0.01 + 0.0566} \right] \times 10^{-6}}}$$

$$L(8.5 \times 10^{-9}) = \left[\frac{1}{2\pi(400 \times 10^3)} \right]^2 = 1.58 \times 10^{-13}$$

Then

$$\underline{L = 18.6 \mu\text{H}}$$

15.29

$$V_\pi = -V_0$$

$$\frac{V_0}{\left(\frac{1}{sC_2}\right)} + \frac{V_0}{R_L} + \frac{V_0 - V_1}{\left(\frac{1}{sC_1}\right)} = g_m V_\pi = -g_m V_0$$

$$V_0 \left[sC_2 + sC_1 + \frac{1}{R_L} + g_m \right] = V_1(sC_1) \quad (1)$$

$$\frac{V_1}{sL} + \frac{V_0 - V_1}{\left(\frac{1}{sC_1}\right)} + g_m V_\pi = 0 \quad (2)$$

$$V_1 \left(\frac{1}{sL} + sC_1 \right) = V_0(sC_1 + g_m)$$

$$V_1 = \frac{V_0(sC_1 + g_m)}{\left(\frac{1}{sL} + sC_1\right)}$$

Then

$$V_0 \left[s(C_1 + C_2) + \frac{1}{R_L} + g_m \right] = \frac{V_0(sC_1)(sC_1 + g_m)}{\left(\frac{1}{sL} + sC_1\right)}$$

$$\left[s(C_1 + C_2) + \frac{1}{R_L} + g_m \right] \left(\frac{1}{sL} + sC_1 \right) = sC_1(sC_1 + g_m)$$

$$\frac{C_1 + C_2}{L} + s^2 C_1(C_1 + C_2) + \frac{1}{sR_L L} + \frac{sC_1}{R_L} + sg_m C_1 + \frac{g_m}{sL} = s^2 C_1^2 + sg_m C_1$$

$$\frac{C_1 + C_2}{L} + s^2 C_1 C_2 + \frac{1}{sR_L L} + \frac{sC_1}{R_L} + \frac{g_m}{sL} = 0$$

Set $s = j\omega$

$$\frac{C_1 + C_2}{L} - \omega^2 C_1 C_2 + \frac{1}{j\omega R_L L} + \frac{j\omega C_1}{R_L} + \frac{g_m}{j\omega L} = 0$$

Then

$$\omega^2 = \frac{C_1 + C_2}{C_1 C_2 L} \Rightarrow \omega_0 = \sqrt{\frac{C_1 + C_2}{C_1 C_2 L}}$$

and

$$\frac{g_m}{\omega L} + \frac{1}{\omega R_L L} = \frac{\omega C_1}{R_L}$$

Then

$$\frac{g_m}{L} + \frac{1}{R_L L} = \frac{(C_1 + C_2)C_1}{C_1 C_2 L R_L}$$

$$g_m + \frac{1}{R_L} = \frac{C_1 + C_2}{C_2 R_L}$$

$$g_m R_L + 1 = \frac{C_1}{C_2} + 1 \text{ or } \underline{\frac{C_1}{C_2} = g_m R_L}$$

15.30

a.

$$\frac{V_0}{sL_1} + \frac{V_0}{R} + g_m V_\pi + \frac{V_0}{\frac{1}{sC} + sL_2} = 0 \quad (1)$$

$$V_\pi = \left(\frac{sL_2}{\frac{1}{sC} + sL_2} \right) V_0 \quad (2)$$

Then

$$V_0 \left\{ \frac{1}{sL_1} + \frac{1}{R} + \frac{sC}{1 + s^2 L_2 C} + \frac{g_m (s^2 L_2 C)}{1 + s^2 L_2 C} \right\} = 0$$

$$\left\{ \frac{R(1 + s^2 L_2 C) + (sL_1)(1 + s^2 L_2 C)}{(sRL_1)(1 + s^2 L_2 C)} + \frac{s^2 RL_1 C + g_m (sRL_1)(s^2 L_2 C)}{(sRL_1)(1 + s^2 L_2 C)} \right\} = 0$$

Set $s = j\omega$. Both real and imaginary parts of the numerator must be zero.

$$R(1 - \omega^2 L_2 C) + j\omega L_1(1 - \omega^2 L_2 C) - \omega^2 RL_1 C + (j\omega g_m RL_1)(-\omega^2 L_2 C) = 0$$

Real part:

$$R(1 - \omega^2 L_2 C) - \omega^2 RL_1 C = 0$$

$$R = \omega^2 RC(L_1 + L_2)$$

or

$$\omega_0 = \frac{1}{\sqrt{C(L_1 + L_2)}}$$

b. Imaginary part:

$$j\omega L_1(1 - \omega^2 L_2 C) - j\omega g_m RL_1(\omega^2 L_2 C) = 0$$

$$L_1 = \omega^2 L_1 L_2 C + g_m RL_1(\omega^2 L_2 C)$$

$$\text{Now } \omega^2 = \frac{1}{(L_1 + L_2)}$$

$$1 = \frac{1}{C(L_1 + L_2)} [L_2 C + g_m RL_2 C]$$

$$1 = \frac{L_2}{L_1 + L_2} (1 + g_m R) \Rightarrow \frac{L_1}{L_2} = (1 + g_m R) - 1$$

or

$$\underline{\frac{L_1}{L_2} = g_m R}$$

15.31

$$\omega_0 = 2\pi(800 \times 10^3) = \frac{1}{\sqrt{C(L_1 + L_2)}}$$

or

$$C(L_1 + L_2) = 3.96 \times 10^{-14}$$

Also $\frac{L_1}{L_2} = g_m R$

For example, if $R = 1 \text{ k}\Omega$, then $\frac{L_1}{L_2} = (20)(1) = 20$

So

$$L_1 = 20L_2$$

Then

$$C(21L_2) = 3.96 \times 10^{-14} \text{ or } CL_2 = 1.89 \times 10^{-15}$$

$$\text{If } C = 0.01 \mu\text{F}$$

$$\text{then } L_2 = 0.189 \mu\text{H}$$

$$\text{and } L_1 = 3.78 \mu\text{H}$$

15.32

$$\frac{\nu_0 - \nu_1}{\left(\frac{1}{sC}\right)} = \frac{\nu_1}{R} + \frac{\nu_1 - \nu_B}{R} \quad (1)$$

and

$$\frac{\nu_B}{\left(\frac{1}{sC}\right)} + \frac{\nu_B - \nu_1}{R} = 0 \quad (2)$$

or

$$\nu_B \left(sC + \frac{1}{R} \right) = \frac{\nu_1}{R} \Rightarrow \nu_1 = \nu_B(1 + sRC)$$

From (1)

$$\nu_0(sC) = \nu_1 \left(sC + \frac{2}{R} \right) - \frac{\nu_B}{R}$$

or

$$\begin{aligned} \nu_0(sRC) &= \nu_B(1 + sRC)(2 + sRC) - \nu_B \\ &= \nu_B[(1 + sRC)(2 + sRC) - 1] \end{aligned}$$

Now

$$\begin{aligned} T(s) &= \left(1 + \frac{R_2}{R_1}\right) \left[\frac{sRC}{(1 + sRC)(2 + sRC) - 1} \right] \\ &= \left(1 + \frac{R_2}{R_1}\right) \left[\frac{sRC}{2 + 3sRC + s^2R^2C^2 - 1} \right] \end{aligned}$$

or

$$T(s) = \left(1 + \frac{R_2}{R_1}\right) \left[\frac{sRC}{s^2R^2C^2 + 3sRC + 1} \right]$$

$$T(j\omega) = \left(1 + \frac{R_2}{R_1}\right) \left[\frac{j\omega RC}{1 - \omega^2 R^2 C^2 + 3j\omega RC} \right]$$

Frequency of oscillation:

$$f_0 = \frac{1}{2\pi RC}$$

Condition for oscillation:

$$1 = \left(1 + \frac{R_2}{R_1}\right) \left[\frac{j\omega RC}{3j\omega RC} \right]$$

or

$$\frac{R_2}{R_1} = 2$$

15.33

$$\frac{\nu_0 - \nu_1}{sL} = \frac{\nu_1}{R} + \frac{\nu_1 - \nu_B}{R} \quad (1)$$

$$\nu_B = \left(\frac{sL}{R + sL} \right) \nu_1 \quad (2)$$

or

$$\nu_1 = \left(\frac{R + sL}{sL} \right) \nu_B$$

Then

$$\frac{\nu_0}{sL} = \nu_1 \left(\frac{1}{sL} + \frac{2}{R} \right) - \frac{\nu_B}{R}$$

or

$$\begin{aligned} \frac{\nu_0}{sL} &= \left(\frac{R + sL}{sL} \right) \left(\frac{1}{sL} + \frac{2}{R} \right) \nu_B - \frac{\nu_B}{R} \\ &= \nu_B \left\{ \left(\frac{R + sL}{sL} \right) \left(\frac{R + 2sL}{sRL} \right) - \frac{1}{R} \right\} \end{aligned} \quad (1)$$

Then

$$\nu_B = \frac{\nu_0}{sL} \cdot \frac{1}{\left(\frac{(R + sL)(R + 2sL) - (sL)^2}{(sL)(sRL)} \right)}$$

Now

$$T(s) = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{sRL}{R^2 + 3sRL + 2s^2L^2 - s^2L^2} \right)$$

or

$$T(s) = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{sRL}{s^2L^2 + 3sRL + R^2} \right)$$

And

$$T(j\omega) = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{j\omega RL}{R^2 - \omega^2 L^2 + 3j\omega RL} \right)$$

$$\text{Frequency of oscillation: } f_0 = \frac{R}{2\pi L}$$

Condition for oscillation:

$$1 = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{1}{3} \right)$$

or

$$\frac{R_2}{R_1} = 2$$

15.34

From Equation (15.65(b)), the crossover voltage is

$$v_I = -\frac{R_2}{R_1} \cdot V_{REF}$$

Let $R_2 = R_{VAR} + R_F$ where R_{VAR} is the potentiometer and R_F is the fixed resistor.

Let $V_{REF} = -5$ V, $R_F = 10$ kΩ, and $R_{VAR} = 40$ kΩ

Then we have

$$v_I = -\frac{R_F}{R_1} \cdot V_{REF} = -\left(\frac{10}{50}\right)(-5) = 1$$

and

$$v_I = -\left(\frac{50}{50}\right)(-5) = 5$$

15.35

$$i_{max} = \frac{10}{R_1 + R_2} = 0.1 \Rightarrow R_1 + R_2 = 100$$

Hysteresis width

$$\Delta V = V_{TH} - V_{TL} = \left(\frac{R_1}{R_1 + R_2}\right)(V_H - V_L)$$

$$\text{or } 0.1 = \left(\frac{R_1}{100}\right)(20)$$

so that

$$R_1 = 0.5$$

$$R_2 = 99.5$$

15.36

$$\text{a. } V_{TH} = \left(\frac{R_1}{R_1 + R_2}\right)V_H = \left(\frac{10}{10 + 40}\right)(10)$$

$$\text{so } \underline{V_{TH} = 2 \text{ V}}$$

$$V_{TL} = \left(\frac{R_1}{R_1 + R_2}\right)V_L = \left(\frac{10}{10 + 40}\right)(-10)$$

$$\text{so } \underline{V_{TL} = -2 \text{ V}}$$

$$\text{b. } v_I = 5 \sin \omega t$$

15.37

a. Upper crossover voltage when $v_0 = +V_P$.

Now

$$v_B = \left(\frac{R_1}{R_1 + R_2}\right)(+V_P)$$

and

$$v_A = \left(\frac{R_A}{R_A + R_B}\right)V_{REF} + \left(\frac{R_B}{R_A + R_B}\right)V_{TH}$$

$v_A = v_B$ so that

$$\begin{aligned} & \left(\frac{R_1}{R_1 + R_2}\right)V_P \\ &= \left(\frac{R_A}{R_A + R_B}\right)V_{REF} + \left(\frac{R_B}{R_A + R_B}\right)V_{TH} \end{aligned}$$

or

$$V_{TH} = \left(\frac{R_A + R_B}{R_1 + R_2}\right)\left(\frac{R_1}{R_B}\right)V_P - \left(\frac{R_A}{R_B}\right)V_{REF}$$

Lower crossover voltage when $v_0 = -V_P$

So

$$V_{TL} = -\left(\frac{R_A + R_B}{R_1 + R_2}\right)\left(\frac{R_1}{R_B}\right)V_P - \left(\frac{R_A}{R_B}\right)V_{REF}$$

$$\text{b. } V_{TH} = \left(\frac{10 + 20}{5 + 20}\right)\left(\frac{5}{20}\right)(10) - \left(\frac{10}{20}\right)(2)$$

$$\text{or } \underline{V_{TH} = 2 \text{ V}}$$

and

$$V_{TL} = -\left(\frac{10 + 20}{5 + 20}\right)\left(\frac{5}{20}\right)(10) - 1 \Rightarrow \underline{V_{TL} = -4 \text{ V}}$$

15.38

$$\text{a. } \frac{v_B}{R_1} = \frac{V_{REF} - v_B}{R_3} + \frac{v_0 - v_B}{R_2}$$

$$v_B\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{V_{REF}}{R_3} + \frac{v_0}{R_2}$$

$V_{TH} = v_B$ when $v_0 = +V_P$ and $V_{TL} = v_B$ when $v_0 = -V_P$

So

$$V_{TH} = \frac{\frac{V_{REF}}{R_3} + \frac{V_P}{R_2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

and

$$V_{TL} = \frac{\frac{V_{REF}}{R_3} - \frac{V_P}{R_2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

b.

$$V_S = \frac{V_{REF}}{R_3\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

$$-5 = \frac{-10}{10\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{5} - \frac{1}{10} = 0.10$$

$$\Delta V_T = V_{TH} - V_{TL} = \frac{\frac{2V_P}{R_2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

$$0.2 = \frac{2(12)}{R_2(0.10 + 0.10)}$$

So $R_2 = 600 \text{ k}\Omega$

Then

$$\frac{1}{R_1} + \frac{1}{R_2} = 0.10$$

$$\frac{1}{R_1} + \frac{1}{600} = 0.10 \Rightarrow R_1 = 10.17 \text{ k}\Omega$$

$$\text{c. } V_{TH} = -5 + 0.1 = -4.9$$

$$V_{TL} = -5 - 0.1 = -5.1$$

15.39

- a. If the saturated output voltage is $|V_P| < 6.2 \text{ V}$, then the circuit behaves as a comparator

where $|v_0| < 6.2 \text{ V}$.

If the saturated output voltage is $|V_P| > 6.2 \text{ V}$, the output will flip to either $+V_P$ or $-V_P$ and the input has no control.

- b. Same as part (a) except the curve at $v_I \approx 0$ will have a finite slope.

c.

Circuit works as a comparator as long as $v_{o1} < 8.7 \text{ V}$ and $v_{o2} > -3.7 \text{ V}$. Otherwise the input has no control.

15.40

- a. Switching point is when $v_0 = 0$. Then

$$v_+ = v_I \equiv V_S = \left(\frac{R_2}{R_1 + R_2}\right) V_{REF}$$

V_{TH} occurs when $v_0 = V_H$, then by superposition

$$v_+ = V_{TH} = \left(\frac{R_1}{R_1 + R_2}\right) V_H + \left(\frac{R_2}{R_1 + R_2}\right) V_{REF}$$

or

$$V_{TH} = V_S + \left(\frac{R_1}{R_1 + R_2}\right) V_H$$

V_{TL} occurs when $v_0 = V_L$, then by superposition

$$v_+ = V_{TL} = \left(\frac{R_1}{R_1 + R_2}\right) V_L + \left(\frac{R_2}{R_1 + R_2}\right) V_{REF}$$

or

$$V_{TL} = V_S + \left(\frac{R_1}{R_1 + R_2}\right) V_L$$

- b. For $V_{TH} = 2 \text{ V}$ and $V_{TL} = 1 \text{ V}$, then $V_S = 1.5 \text{ V}$

Now

$$2 = 1.5 + \left(\frac{10}{10 + R_2}\right)(10)$$

$$\frac{0.5}{10} = \frac{10}{10 + R_2} \Rightarrow R_2 = 190 \text{ k}\Omega$$

$$\text{Now } V_S = 1.5 = \left(\frac{190}{10 + 190}\right) V_{REF}$$

so

$$V_{REF} = 1.579 \text{ V}$$

15.41

- a. Switching point when $v_0 = 0$.

Now

$$v_+ = V_{REF} = \left(\frac{R_2}{R_1 + R_2}\right) v_I \text{ where } v_I = V_S.$$

Then

$$V_S = \left(\frac{R_1 + R_2}{R_2}\right) V_{REF} = \left(1 + \frac{R_1}{R_2}\right) V_{REF}$$

Now upper crossover voltage for v_I occurs when $v_0 = V_L$ and $v_+ = V_{REF}$. Then

$$\frac{V_{TH} - V_{REF}}{R_1} = \frac{V_{REF} - V_L}{R_2}$$

$$\text{or } V_{TH} = -\frac{R_1}{R_2} \cdot V_L + V_{REF} \left(1 + \frac{R_1}{R_2}\right)$$

$$\text{or } V_{TH} = V_S - \frac{R_1}{R_2} \cdot V_L$$

Lower crossover voltage for v_I occurs when $v_0 = V_H$ and $v_+ = V_{REF}$. Then

$$\frac{V_H - V_{REF}}{R_1} = \frac{V_{REF} - V_{TL}}{R_2}$$

$$\text{or } V_{TL} = -\frac{R_1}{R_2} \cdot V_H + V_{REF} \left(1 + \frac{R_1}{R_2}\right)$$

$$\text{or } V_{TL} = V_S - \frac{R_1}{R_2} \cdot V_H$$

b. For $V_{TH} = -1$ and $V_{TL} = -2$, $V_S = -1.5$ V.

$$\text{Then } V_{TL} = V_S - \frac{R_1}{R_2} \cdot V_H \Rightarrow -2 = -1.5 - \frac{R_1}{20}(12)$$

so that $R_1 = 0.833 \text{ k}\Omega$

Now

$$V_S = \left(1 + \frac{R_1}{R_2}\right) V_{REF}$$

$$-1.5 = \left(1 + \frac{0.833}{20}\right) V_{REF}$$

which gives

$$V_{REF} = -1.44 \text{ V}$$

15.42

a. $V_H = 5.6 + 0.7 = 6.3$ V and $V_L = -6.3$ V

From Equation (15.72(b)),

$$V_{TH} = -\left(\frac{R_1}{R_2}\right) V_L$$

and from Equation (15.75(b)),

$$V_{TL} = -\left(\frac{R_1}{R_2}\right) V_H$$

$$V_{TH} - V_{TL} = 1 = -\left(\frac{R_1}{R_2}\right)(V_L - V_H)$$

$$= +\left(\frac{R_1}{R_2}\right)(2)(6.3)$$

$$\text{so } \frac{R_1}{R_2} = 0.07937$$

$$\text{Then } R_2 = \frac{1}{0.07937} \Rightarrow R_2 = 12.6 \text{ k}\Omega$$

Now $V_{TH} = -(0.07937)(-6.3) = 0.5$,

$$V_{TL} = -0.5$$

b. For v_0 high, we have

$$I = I_D + I_R, I \text{ is fixed for a given } R.$$

Assume v_I varies between ± 12 V

$$I_R(\max) = \frac{6.3 - (-12)}{13.6} = 1.35 \text{ mA}$$

$$I_R(\min) = \frac{6.3 - 0.5}{13.6} = 0.426 \text{ mA}$$

$$I_R(\text{avg}) = \frac{1.35 + 0.426}{2} = 0.888 \text{ mA}$$

Then we want

$$I = I_D(\text{avg}) + I_R(\text{avg}) = 1 + 0.888 = 1.888 \text{ mA}$$

Then

$$R = \frac{12 - 6.3}{1.888} \Rightarrow R = 3.02 \text{ k}\Omega$$

15.43

a. $v_0 = V_{REF} + 2V_T$

$$5 = V_{REF} + 2(0.7)$$

or

$$V_{REF} = 3.6 \text{ V}$$

b. $V_{TH} = \left(\frac{R_1}{R_1 + R_2}\right)(V_{REF} + 2V_T)$

$$0.5 = \left(\frac{R_1}{R_1 + R_2}\right)(5)$$

$$\text{or } 1 + \frac{R_2}{R_1} = 10 \Rightarrow \frac{R_2}{R_1} = 9$$

For example, let $R_2 = 90 \text{ k}\Omega$ and $R_1 = 10 \text{ k}\Omega$

c. For $v_I = 10$ V, and v_0 is in its low state. D_1 is on and D_2 is off.

$$\frac{v_I - (v_1 + 0.7)}{100} + \frac{V_{REF} - v_1}{1} = \frac{v_1 - v_0}{1}$$

For $v_1 = -0.7$, then

$$\frac{10 - 0}{100} + \frac{3.6 - (-0.7)}{1} = \frac{-0.7 - v_0}{1}$$

or

$$v_0 = -5.1 \text{ V}$$

15.44

For $v_0 = \text{High} = (V_{REF} + 2V_T)$. Then switching point is when

$$v_I = v_B = \left(\frac{R_1}{R_1 + R_2}\right) v_0$$

$$\text{or } V_{TH} = \left(\frac{R_1}{R_1 + R_2}\right)(V_{REF} + 2V_T)$$

Lower switching point is when

$$v_1 = v_B = \left(\frac{R_1}{R_1 + R_2}\right) v_0 \text{ and } v_0 = -(V_{REF} + 2V_T)$$

so

$$V_{TL} = -\left(\frac{R_1}{R_1 + R_2}\right)(V_{REF} + 2V_T)$$

15.45

By symmetry, inverting terminal switches about zero.

Now, for v_0 low, upper diode is on.

$$V_{REF} - v_1 = v_1 - v_0$$

$$v_0 = 2v_1 - V_{REF} \text{ where } v_1 = -V_T$$

so

$$v_0 = -(V_{REF} + 2V_T)$$

Similarly, in the high state

$$v_0 = (V_{REF} + 2V_T)$$

Switching occurs when non-inverting terminal is zero.

So for v_0 low,

$$\frac{V_{TH} - 0}{R_1} = \frac{0 - [-(V_{REF} + 2V_T)]}{R_2}$$

$$\text{or } V_{TH} = \frac{R_1}{R_2} \cdot (V_{REF} + 2V_T)$$

By symmetry

$$V_{TL} = -\frac{R_1}{R_2} \cdot (V_{REF} + 2V_T)$$

15.46

$f_0 = 5 \text{ kHz}$ and 50% duty cycle.

From Equation (15.88)

$$f = \frac{1}{2.2R_X C_X}$$

so

$$R_X C_X = \frac{1}{2.2(5 \times 10^3)} = 9.09 \times 10^{-4}$$

Let $C_X = 0.01 \mu\text{F}$. Then $R_X = 9.09 \text{ k}\Omega$.

Also let $R_1 = R_2 = 10 \text{ k}\Omega$

15.47

Switching point occurs when

$$v_X = \left(\frac{R_1}{R_1 + R_2} \right) V_P = \left(\frac{30}{30 + 10} \right) (10) \\ \Rightarrow v_X = \pm 7.5 \text{ V}$$

b. Duty cycle = 50%

From Equation (15.83(a)), we can write

$$v_X = V_P + \left(-\frac{3}{4}V_P - V_P \right) e^{-t_1/r_X} = \frac{2}{3}V_P$$

$$\text{At time } t = t_1 \text{ (one-half period)} \quad v_X = \frac{3}{4} \cdot V_P$$

So

$$\frac{3}{4} \cdot V_P = V_P - \frac{7}{4} \cdot V_P e^{-t_1/r_X}$$

$$1 = 7e^{-t_1/r_X}$$

$$\text{or } t_1 = r_X \ln(7)$$

One period is $T = 2r_X \ln(7) = 3.89r_X$ or the frequency is

$$f = \frac{1}{3.89R_X C_X}$$

Then

$$f = \frac{1}{(3.89)(10^4)(0.1 \times 10^{-6})} \Rightarrow f = 257 \text{ Hz}$$

15.48

Only change from Problem (15.47) is that maximum output is $\pm 15 \text{ V}$ and the v_X switching voltages are $\pm 11.25 \text{ V}$.

15.49

$$t_1 = 1.1R_X C_X = (1.1)(10^4)(0.1 \times 10^{-6})$$

$$\Rightarrow t_1 = 1.1 \text{ ms}$$

$$0 < t < t_1, \quad v_Y = 10 \left(1 - e^{-t/r_Y} \right)$$

$$r_Y = R_Y C_Y = (2 \times 10^3)(0.02 \times 10^{-6})$$

$$= 4 \times 10^{-5} \text{ s}$$

$$\text{Now } \frac{t_1}{r_Y} = 2.75$$

$\Rightarrow C_Y$ completely charges during each cycle.

15.50

a. Switching voltage

$$v_X = \left(\frac{R_1 + R_3}{R_1 + R_3 + R_2} \right) \cdot V_P = \left(\frac{10 + 10}{10 + 10 + 10} \right) (\pm 10)$$

$$\text{So } v_X = \pm 6.667 \text{ V}$$

Using Equation (15.83(b))

$$v_X = V_P + \left(-\frac{2}{3}V_P - V_P \right) e^{-t_1/r_X} = \frac{2}{3}V_P$$

$$\text{Then } 1 - \frac{5}{3} \cdot e^{-t_1/r_X} = \frac{2}{3}$$

$$\frac{1}{3} = \frac{5}{3} \cdot e^{-t_1/r_X} \text{ or } t_1 = r_X \ln(5)$$

$$t_1 = \frac{T}{2} = \frac{1}{2f} = \frac{1}{2(500)} \Rightarrow t_1 = 0.001 \text{ s}$$

$$10^{-3} = r_X \ln(5) \Rightarrow r_X = 6.21 \times 10^{-4}$$

$$= R_X (0.01 \times 10^{-6})$$

$$\text{So } R_X = 62.1 \text{ k}\Omega$$

b. Switching voltage

$$v_X = \left(\frac{R_1}{R_1 + R_3 + R_2} \right) (\pm V_P) \\ = \left(\frac{10}{10 + 10 + 10} \right) (\pm V_P) = \frac{1}{3} \cdot (\pm V_P)$$

Using Equation (15.83(b))

$$v_X = V_P + \left(-\frac{1}{3}V_P - V_P \right) e^{-t_1/r_X} = \frac{1}{3}V_P$$

$$\text{Then } 1 - \frac{4}{3} \cdot e^{-t_1/r_X} = \frac{1}{3}$$

$$\frac{2}{3} = \frac{4}{3} \cdot e^{-t_1/r_X}$$

$$t_1 = r_X \ln(2) = (6.21 \times 10^{-4}) \ln(2) = 4.30 \times 10^{-4} \text{ s}$$

$$T = 2t_1 = 8.6 \times 10^{-4} \text{ s}$$

$$f = \frac{1}{T} \Rightarrow f = 1.16 \text{ kHz}$$

Chapter 15: Problem Solutions

15.51

From Equation (15.92)

$$T = \tau_X \ln \left(\frac{1 + \left(\frac{V_Y}{V_P} \right)}{1 - \beta} \right)$$

$$\text{where } \beta = \frac{R_1}{R_1 + R_2} = \frac{10}{10 + 25} = 0.2857$$

so

$$100 = \tau_X \ln \left[\frac{1 + \frac{0.7}{5}}{1 - 0.2857} \right]$$

$$\text{so } \tau_X = 213.9 \mu\text{s} = R_X C_X$$

$$\text{For example, } R_X = 10 \text{ k}\Omega, C_X = 0.0214 \mu\text{F}$$

$$v_Y = \left(\frac{R_1}{R_1 + R_2} \right) V_P = \left(\frac{10}{10 + 25} \right) (5) = 1.43 \text{ V}$$

$$\text{and } v_X = 0.7 \text{ V}$$

To trigger the circuit, v_Y must be brought to a voltage less than v_X .

Therefore minimum triggering pulse is -0.73 V .

Using Equation (15.82) for $T < t < T'$

$$v_X = V_P + (-0.2857 V_P - V_P) e^{-t'/\tau_X}$$

Recovery period is when $v_X = V_Y = 0.7 \text{ V}$.

$$0.7 = 5 + (-6.43) e^{-t'/\tau_X}$$

$$6.43 e^{-t'/\tau_X} = 4.3$$

$$\text{or } t' = \tau_X \ln (1.495)$$

$$\tau_X = 213.9 \mu\text{s}$$

so

$$t' = T' - T = 86.1 \mu\text{s}$$

15.52

From Equation (15.92), the pulse width

$$T = \tau_X \ln \left(\frac{1 + \left(\frac{V_Y}{V_P} \right)}{1 - \beta} \right)$$

$$\text{where } \beta = \frac{R_1}{R_1 + R_2} = \frac{20}{20 + 20} = 0.5$$

$$\tau_X = R_X C_X = (50 \times 10^3)(0.1 \times 10^{-6}) = 5 \text{ ms}$$

$$\text{So } T = 5 \ln \left[\frac{1 + \frac{0.7}{10}}{1 - 0.5} \right] \Rightarrow T = 3.80 \text{ ms}$$

$$\text{Recovery time } \approx 0.4 \tau_X = 2 \text{ ms}$$

15.53

a. From Equation (15.95)

$$T = 1.1RC$$

$$\text{For } T = 60 \text{ s} = 1.1RC$$

$$\text{then } RC = 54.55 \text{ s}$$

For example, let

$$C = 50 \mu\text{F} \text{ and } R = 1.04 \text{ M}\Omega$$

b. Recovery time: capacitor is discharged by current through the discharge transistor.

$$\text{If } V^+ = 5 \text{ V}, \text{ then } I_B \approx \frac{5 - 0.7}{100} = 0.043 \text{ mA}$$

$$\text{If } \beta = 100, I_C = 4.3 \text{ mA}$$

$$V_C = \frac{1}{C} \int I_C dt = \frac{I_C}{C} \cdot t$$

Capacitor has charged to $\frac{2}{3} \cdot V^+ = 3.33 \text{ V}$

$$\text{So that } t = \frac{V_C \cdot C}{I_C} = \frac{(3.33)(50 \times 10^{-6})}{4.3 \times 10^{-3}}$$

$$\text{So recovery time } t \approx 38.7 \text{ ms}$$

15.54

$$T = 1.1RC$$

$$5 \times 10^{-6} = 1.1RC$$

$$\text{so } RC = 4.545 \times 10^{-6} \text{ s}$$

For example, let

$$C = 100 \text{ pF} \text{ and } R = 45.5 \text{ k}\Omega$$

From Problem (15.53), recovery time

$$t \approx \frac{V_C \cdot C}{I_C} = \frac{(3.33)(100 \times 10^{-12})}{4.3 \times 10^{-3}}$$

or

$$t = 77.4 \text{ ns}$$

15.55

From Equation (15.102).

$$f = \frac{1}{(0.693)(20 + 2(20)) \times 10^3 \times (0.1 \times 10^{-6})}$$

$$\text{or } f = 240.5 \text{ Hz}$$

$$\text{Duty cycle} = \frac{20 + 20}{20 + 2(20)} \times 100\% = 66.7\%$$

15.56

$$f = \frac{1}{(0.693)(R_A + 2R_B)C}$$

$$R_A = R_1 = 10 \text{ k}\Omega, R_B = R_2 + zR_3$$

$$\text{So } 10 \text{ k}\Omega \leq R_B \leq 110 \text{ k}\Omega$$

$$f_{\min} = \frac{1}{(0.693)(10 + 2(110)) \times 10^3 \times (0.01 \times 10^{-6})} \\ = 627 \text{ Hz}$$

$$f_{\max} = \frac{1}{(0.693)(10 + 2(10)) \times 10^3 \times (0.01 \times 10^{-6})} \\ = 4.81 \text{ kHz}$$

So $627 \text{ Hz} \leq f \leq 4.81 \text{ kHz}$

$$\text{Duty cycle} = \frac{R_A + R_B}{R_A + 2R_B} \times 100\%$$

Now

$$\frac{10 + 10}{10 + 2(10)} \times 100\% = 66.7\%$$

and

$$\frac{10 + 110}{10 + 2(110)} \times 100\% = 52.2\%$$

So $52.2 \leq \text{Duty cycle} \leq 66.7\%$

15.57

$$1 \text{ k}\Omega \leq R_A \leq 51 \text{ k}\Omega$$

$$1 \text{ k}\Omega \leq R_B \leq 51 \text{ k}\Omega$$

$$f_{\min} = \frac{1}{(0.693)(1 + 2(51)) \times 10^3 \times (0.01 \times 10^{-6})} \\ = 1.40 \text{ kHz}$$

$$f_{\max} = \frac{1}{(0.693)(51 + 2(1)) \times 10^3 \times (0.01 \times 10^{-6})} \\ = 2.72 \text{ kHz}$$

or $1.40 \text{ kHz} \leq f \leq 2.72 \text{ kHz}$

$$\text{Duty cycle} = \frac{R_A + R_B}{R_A + 2R_B} \times 100\%$$

$$\frac{1 + 51}{1 + 2(51)} \times 100\% = 50.5\%$$

or

$$\frac{51 + 1}{51 + 2(1)} \times 100\% = 98.1\%$$

or $50.5\% \leq \text{Duty cycle} \leq 98.1\%$

15.58

$$a. I_{E3} = I_{B4} = \frac{V^+ - 3V_{BB}}{R_{1A} + R_{1B}}$$

Assume $V_{BB} = 0.7$

$$I_{E3} = I_{B4} = \frac{22 - 3(0.7)}{25 + 25} = 0.398 \text{ mA}$$

Now

$$I_{C3} = I_{C4} = I_{C5} = I_{C6} = \left(\frac{20}{21}\right)(0.398)$$

$$I_{C3} = I_{C4} = I_{C5} = I_{C6} = 0.379 \text{ mA}$$

$$I_{C1} = I_{C2} = \frac{0.398}{21} \left(\frac{20}{21}\right) \Rightarrow I_{C1} = I_{C2} = 0.018 \text{ mA}$$

b. $I_D = 0.398 \text{ mA}$, current in D_1 and D_2

$$V_{BB} = 2V_D = 2V_T \ln \left(\frac{I_D}{I_S} \right) \\ = 2(0.026) \ln \left(\frac{0.398 \times 10^{-3}}{10^{-13}} \right)$$

$$\text{or } V_{BB} = 1.149 \text{ V} = V_{BE7} + V_{EB8}$$

Now

$$I_{C7} \approx I_{C4} + I_{C9} + I_{E8}$$

$$I_{C4} = 0.379 \text{ mA}$$

$$I_{B9} = I_{C8} = \left(\frac{20}{21}\right) I_{E8}$$

So

$$I_{E8} = 1.05 I_{B9} = 1.05 \left(\frac{I_{C8}}{100}\right)$$

$$I_{C7} = I_{C4} + \left(\frac{100}{1.05}\right) I_{E8} + I_{E8}$$

$$= I_{C4} + (96.24) \left(\frac{21}{20}\right) I_{C8}$$

$$\text{So } I_{C7} = 0.379 \text{ mA} + 101 I_{C8}$$

and

$$V_{BE7} = V_T \ln \left(\frac{I_{C7}}{I_S} \right); V_{EB8} = V_T \ln \left(\frac{I_{C8}}{I_S} \right)$$

Then

$$1.149 = 0.026 \left[\ln \left(\frac{I_{C7}}{I_S} \right) + \ln \left(\frac{I_{C8}}{I_S} \right) \right]$$

$$44.19 = \ln \left[\frac{I_{C8}(0.379 \times 10^{-3}) + 101 I_{C8}}{(10^{-13})^2} \right]$$

$$(10^{-13})^2 \exp(44.19) = 101 I_{C8}^2 + 3.79 \times 10^{-4} I_{C8}$$

$$1.554 \times 10^{-7} = 101 I_{C8}^2 + 3.79 \times 10^{-4} I_{C8}$$

$$I_{C8} = \frac{-3.79 \times 10^{-4}}{2(101)} \pm \sqrt{\frac{(3.79 \times 10^{-4})^2 + 4(101)(1.554 \times 10^{-7})}{2(101)}}$$

$$I_{C8} = 37.4 \mu\text{A}$$

$$I_{C7} = 0.379 + 101(0.0374) \Rightarrow I_{C7} = 4.16 \text{ mA}$$

$$I_{C9} = 4.16 - 0.379 - 0.0374 \left(\frac{21}{20}\right)$$

$$I_{C9} = 3.74 \text{ mA}$$

$$c. P = (0.398 + 0.398 + 4.16)(22) \Rightarrow P = 109 \text{ mW}$$

15.59

- a. From Figure 15.47, 3.7 W to the load

b. $V^+ \approx 19 \text{ V}$

c. $\bar{P} = \frac{1}{2} \frac{V_P^2}{R_L}$

or

$$V_P = \sqrt{2R_L \bar{P}} = \sqrt{2(10)(3.7)} \Rightarrow V_P = 8.6 \text{ V}$$

15.60

$$\bar{P} = \frac{1}{2} \frac{V_P^2}{R_L}$$

$$\text{so } V_P = \sqrt{2R_L \bar{P}} = \sqrt{2(10)(20)} = 20 \text{ V}$$

peak-to-peak output voltage

Maximum output voltage of each op-amp = $\pm 10 \text{ V}$. Current is $(20/10) = 2 \text{ A}$. Bias op-amps at $\pm 12 \text{ V}$.

$$\text{For } A_1, \frac{v_{o1}}{v_I} = \left(1 + \frac{R_2}{R_1}\right) = 15 \Rightarrow \frac{R_2}{R_1} = 14$$

$$\text{For } A_2, \left| \frac{v_{o2}}{v_I} \right| = \frac{R_4}{R_3} = 15$$

For example, let $R_1 = R_3 = 10 \text{ k}\Omega$, and $R_2 = 140 \text{ k}\Omega$ and $R_4 = 150 \text{ k}\Omega$.

15.61

a. $v_{o1} = iR_2 + v_I$ where $i = \frac{v_I}{R_1}$

Then

$$v_{o1} = v_I \left(1 + \frac{R_2}{R_1}\right)$$

Now

$$v_{o2} = -iR_3 = -v_I \left(\frac{R_3}{R_1}\right)$$

So

$$v_L = v_{o1} - v_{o2} = v_I \left(1 + \frac{R_2}{R_1}\right) - \left[-v_I \left(\frac{R_3}{R_1}\right)\right]$$

$$A_v = \frac{v_L}{v_I} = 1 + \frac{R_2}{R_1} + \frac{R_3}{R_1}$$

b. Want $A_v = 10 \Rightarrow \frac{R_2}{R_1} + \frac{R_3}{R_1} = 9$

Also want $\left(1 + \frac{R_2}{R_1}\right) = \frac{R_3}{R_1}$

Then $\frac{R_2}{R_1} + \left(1 + \frac{R_2}{R_1}\right) = 9$ so $\frac{R_2}{R_1} = 4$

For $R_1 = 50 \text{ k}\Omega$, $R_2 = 200 \text{ k}\Omega$

and

$$\frac{R_3}{R_1} = 5 \text{ so } \underline{R_3 = 250 \text{ k}\Omega}$$

c. $\bar{P} = \frac{1}{2} \frac{V_P^2}{R_L}$

or

$$V_P = \sqrt{2R_L \bar{P}} = \sqrt{2(20)(10)} = 20 \text{ V}$$

So peak values of output voltages are

$$|v_{o1}| = |v_{o2}| = 10 \text{ V}$$

$$\text{Peak load current} = \frac{20}{20} = 1 \text{ A}$$

15.62

a. $v_{o1} = \left(1 + \frac{R_2}{R_1}\right) v_I$

$$v_{o2} = -\frac{R_4}{R_3} \cdot v_{o1} = -\frac{R_4}{R_3} \left(1 + \frac{R_2}{R_1}\right) v_I$$

$$v_L = v_{o1} - v_{o2} = \left(1 + \frac{R_2}{R_1}\right) \left(1 + \frac{R_4}{R_3}\right) v_I$$

so

$$A_v = \left(1 + \frac{R_2}{R_1}\right) \left(1 + \frac{R_4}{R_3}\right)$$

b. Want $\left(1 + \frac{R_2}{R_1}\right) v_I = |v_{o1}| = |v_{o2}| \Rightarrow \underline{R_4 = R_3}$

Then

$$\left(1 + \frac{R_2}{R_1}\right) (2) = 15 \Rightarrow \underline{\frac{R_2}{R_1} = 6.5}$$

c. $\bar{P} = \frac{1}{2} \cdot \frac{V_P^2}{R_L}$

or

$$V_P = \sqrt{2R_L \bar{P}} = \sqrt{2(8)(50)} = 28.3 \text{ V}$$

= peak-to-peak load voltage

Then

$$|v_{o1}| = |v_{o2}| = 14.15 \text{ V}$$

$$\text{Load current} = \frac{28.3}{8} = 3.54 \text{ A}$$

15.63

$$\text{Line regulation} = \frac{\Delta V_0}{\Delta V^+}$$

Now

$$\Delta I = \frac{\Delta V^+}{R_1} \text{ and } \Delta V_Z = r_Z \cdot \Delta I \text{ and } \Delta V_0 = 10 \Delta V_Z$$

So

$$\Delta V_0 = 10 \cdot r_Z \cdot \frac{\Delta V^+}{R_1}$$

So

$$\text{Line regulation} = \frac{\Delta V_0}{\Delta V^+} = \frac{10(15)}{9300}$$

$$\Rightarrow 1.61\%$$

15.64

$$R_{0f} = -\frac{\Delta V_0}{\Delta I_0}$$

$$\text{So } R_{0f} = \frac{-(-10 \times 10^{-3})}{1}$$

or

$$\underline{R_{0f} = 10 \text{ m}\Omega}$$

15.65

For $V_0 = 8 \text{ V}$

$$V^+(\text{min}) = V_0 + I_0(\text{max})R_{11} + V_{BE11} \\ + V_{BE10} + V_{EB3}$$

This assumes $V_{BC3} = 0$.

Then

$$V^+(\text{min}) = 8 + (0.1)(1.9) + 0.6 + 0.6 + 0.6$$

$$\underline{V^+(\text{min}) = 9.99 \text{ V}}$$

15.66

a. $I_{C3} = I_{C5} = \frac{V_z - 3V_{BE}(\text{npn})}{R_1 + R_2 + R_3}$

$$I_{C3} = I_{C5} = \frac{6.3 - 3(0.6)}{0.576 + 3.4 + 3.9} = 0.571 \text{ mA}$$

$$I_{C8} = \frac{1}{2} \left(\frac{0.6}{2.84} \right) = 0.106 \text{ mA}$$

Neglecting current in Q_3 , total collector current and emitter current in Q_5 is

$$0.571 + 0.106 = 0.677$$

Now

$$I_{Z2}R_4 + V_{EB4} = V_{EB3}$$

$$V_{EB4} = V_T \ln \left(\frac{I_{Z2}}{I_S} \right)$$

$$V_{EB3} = V_T \ln \left(\frac{I_{C5}}{2I_S} \right)$$

Then $I_{Z2}R_4 = V_T \ln \left(\frac{I_{C5}}{2I_{Z2}} \right)$

$$R_4 = \frac{0.026}{0.25} \cdot \ln \left(\frac{0.677}{2(0.25)} \right)$$

or

$$\underline{R_4 = 31.5 \Omega}$$

b. From Example 15.16, $V_{B7} = 3.43 \text{ V}$. Then

$$\left(\frac{R_{13}}{R_{12} + R_{13}} \right) V_0 = V_{B8} = V_{B7}$$

or

$$\left(\frac{2.23}{2.23 + R_{12}} \right) (12) = 3.43$$

$$3.43(2.23 + R_{12}) = (2.23)(12)$$

which yields

$$\underline{R_{12} = 5.57 \text{ k}\Omega}$$

15.67

$$\text{Line regulation} = \frac{\Delta V_0}{\Delta V^+}$$

Now

$$\Delta V_{B7} = \Delta I_{C3} \cdot R_1$$

$$\text{and } \left(\frac{R_{13}}{R_{12} + R_{13}} \right) (\Delta V_0) = \Delta V_{B7} = \Delta I_{C3} R_1$$

$$\text{and } \Delta I_{C3} = \frac{\Delta V_Z}{R_1 + R_2 + R_3} = \frac{\Delta I_Z \cdot r_Z}{R_1 + R_2 + R_3}$$

$$\text{and } \Delta I_Z = \frac{\Delta V^+}{r_0} \text{ where } r_0 = \frac{V_A}{I_Z}$$

Then

$$(0.4288)(\Delta V_0) = \Delta I_{C3}(3.9)$$

$$= (3.9) \Delta I_Z \left(\frac{0.015}{7.876} \right)$$

$$r_0 = \frac{50}{0.571} = 87.6 \text{ k}\Omega$$

Then

$$(0.4288)(\Delta V_0) = (0.00743) \left(\frac{\Delta V^+}{87.6} \right)$$

So

$$\underline{\frac{\Delta V_0}{\Delta V^+} = 0.0198\%}$$

15.68

a. $I_Z = \frac{25 - 5}{R_1 + r_Z} = 10$

$$\text{So } R_1 + r_Z = \frac{20}{10} = 2 \text{ k}\Omega = R_1 + 0.01 \\ \Rightarrow \underline{R_1 = 1.99 \text{ k}\Omega}$$

b. In the ideal case;

$$\left(\frac{R_3 + R_4}{R_2 + R_3 + R_4} \right) V_0 = V_Z$$

$$\left(\frac{2+1}{2+1+1} \right) V_0 = 5 \Rightarrow V_0 = 6.67 \text{ V}$$

and

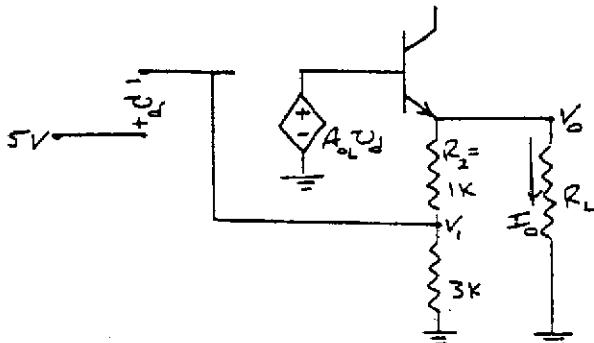
$$\left(\frac{R_4}{R_2 + R_3 + R_4} \right) V_0 = V_Z$$

$$\left(\frac{1}{2+1+1} \right) V_0 = 5 \Rightarrow V_0 = 20 \text{ V}$$

So

$$\underline{6.67 < V_0 < 20 \text{ V}}$$

c.



15.69

$$I_E = \frac{V_Z}{R_2} = \frac{5.6}{5} = 1.12 \text{ mA}$$

$$I_O = \frac{\beta}{1 + \beta} \cdot I_E = \left(\frac{100}{101} \right) (1.12)$$

 $\Rightarrow I_O = 1.109 \text{ mA}$ Load current

For

$$V_{BC} = 0 \Rightarrow V_0 = 20 - V_Z - 0.6 \\ = 20 - 5.6 - 0.6$$

or

$$V_0 = 13.8 \text{ V}$$

$$V_1 = \frac{3}{4} \cdot V_0 \text{ so } V_d = 5 - \frac{3}{4} \cdot V_0$$

$$\text{and } V_0 = A_{oL} V_d - V_{BE}$$

$$\text{and } V_{BE} = V_T \ln \left(\frac{I_O}{I_S} \right)$$

Now

$$V_0 = A_{oL} \left(5 - \frac{3}{4} \cdot V_0 \right) - V_{BE}$$

$$V_0 \left(1 + \frac{3}{4} \cdot A_{oL} \right) = 5A_{oL} - V_{BE}$$

$$V_0 = \frac{5A_{oL} - V_{BE}}{1 + \frac{3}{4} \cdot A_{oL}}$$

Then

$$R_L = \frac{V_0}{I_O} = \frac{13.8}{1.109} \Rightarrow R_L = 12.4 \text{ k}\Omega$$

So

$$0 \leq R_L \leq 12.4 \text{ k}\Omega$$

$$\text{Load regulation} = \frac{V_0(\text{NL}) - V_0(\text{FL})}{V_0(\text{NL})}$$

$$= \frac{\frac{5A_{oL} - V_{BE}(\text{NL})}{(1 + \frac{3}{4}A_{oL})} - \frac{5A_{oL} - V_{BE}(\text{FL})}{(1 + \frac{3}{4}A_{oL})}}{\frac{5A_{oL} - V_{BE}(\text{NL})}{(1 + \frac{3}{4}A_{oL})}}$$

$$= \frac{V_{BE}(\text{FL}) - V_{BE}(\text{NL})}{5A_{oL} - V_{BE}(\text{NL})} = \frac{V_T \ln \left(\frac{I_O(\text{FL})}{I_O(\text{NL})} \right)}{5A_{oL} - V_{BE}(\text{NL})}$$

$$I_O(\text{FL}) = 1 \text{ A}, \quad I_O(\text{NL}) = \frac{V_0}{4 \text{ k}\Omega} = \frac{6.67}{4 \text{ k}\Omega} = 1.67 \text{ mA}$$

$$\text{Load regulation} = \frac{(0.026) \ln \left(\frac{1}{1.67 \times 10^{-3}} \right)}{5(10^4) - 0.7}$$

$$\Rightarrow 3.33 \times 10^{-4} \%$$

Chapter 16

Exercise Solutions

E16.1

a. Driver in nonsaturation:

$$i_D = \frac{V_{DD} - v_O}{R_D} = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_L [2(v_I - V_{TN})v_O - v_O^2]$$

$$\frac{5 - (0.15)}{R_D} = \frac{35}{2}(5)[2(5 - 0.8)(0.15) - (0.15)^2]$$

$$\frac{4.85}{R_D} = 87.5[1.2375]$$

$\Rightarrow R_D = 44.8 \text{ k}\Omega$

b. From Equation (16-10):

$$\left(\frac{0.035}{2}\right)(5)(44.8)(V_{I_t} - 0.8)^2 + (V_{I_t} - 0.8) - 5 = 0$$

$$3.920(V_{I_t} - 0.8)^2 + (V_{I_t} - 0.8) - 5 = 0$$

$$V_{I_t} - 0.8 = \frac{-1 \pm \sqrt{1 + 4(3.92)(5)}}{2(3.92)}$$

$$V_{I_t} - 0.8 = 1.0 \Rightarrow V_{I_t} = 1.8 \text{ V}$$

$$V_{O_t} = 1.0 \text{ V}$$

E16.2

a. i. $v_I = 0 \Rightarrow v_O = 4 \text{ V}$ ii. $v_I = 4 \text{ V}$, driver in nonsaturation

$$\left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_L (v_{Ox} - V_{TN})^2$$

$$= \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_D [2(v_{Ox} - V_{TN})v_{DSD} - v_{DSD}^2]$$

$$2(5 - v_0 - 1)^2 = (16)[2(4 - 1)v_0 - v_0^2]$$

$$16 - 8v_0 + v_0^2 = 8(6v_0 - v_0^2)$$

$$9v_0^2 - 56v_0 + 16 = 0$$

$$v_0 = \frac{56 \pm \sqrt{(56)^2 - 4(9)(16)}}{2(9)}$$

$$v_0 = 0.30 \text{ V}$$

b. $P = i_D \cdot V_{DD}$

$$i_D = \frac{35}{2}(2)(5 - 0.30 - 1)^2 = 479 \mu\text{A}$$

$$P = (0.479)(5)$$

$$\Rightarrow P = 2.4 \text{ mW}$$

E16.3

$$P = i_D \cdot V_{DD} \Rightarrow i_D = \frac{750}{5} = 150 \mu\text{A}$$

$$150 = \frac{35}{2} \left(\frac{W}{L}\right)_L (5 - 0.2 - 0.8)^2$$

$$150 = 280 \left(\frac{W}{L}\right)_L \Rightarrow \left(\frac{W}{L}\right)_L = 0.536$$

$$i_D = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_D [2(v_I - V_{TN})v_O - v_O^2]$$

$$150 = \frac{35}{2} \left(\frac{W}{L}\right)_D [2(4.2 - 0.8)(0.2) - (0.2)^2]$$

$$150 = 23.1 \cdot \left(\frac{W}{L}\right)_D \Rightarrow \left(\frac{W}{L}\right)_D = 6.49$$

E16.4

a. Load in saturation; driver in nonsaturation:

$$\left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_L (v_{Ox} - V_{TN})^2$$

$$= \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_D [2(v_{Ox} - V_{TN})v_{DSD} - v_{DSD}^2]$$

$$2(-[-1.5])^2 = (6)[2(5 - 0.7)v_0 - v_0^2]$$

$$4.5 = 6(8.6v_0 - v_0^2)$$

$$6v_0^2 - 51.6v_0 + 4.5 = 0$$

$$v_0 = \frac{51.6 \pm \sqrt{(51.6)^2 - 4(6)(4.5)}}{2(6)}$$

$$v_0 = 0.0881 \text{ V}$$

b. Load

$$v_{Ox} = V_{DD} + V_{TN} \quad \text{Equation (16.26(b))}$$

$$= 5 - 1.5 \Rightarrow v_{Ox} = 3.5 \text{ V}$$

From Equation (16-28(b)) :

$$\sqrt{\frac{K_O}{K_L}} (v_I - V_{TN}) = -V_{TN}$$

$$\sqrt{\frac{6}{2}} (v_{I_t} - 0.7) = -(-1.5) = 1.5$$

Load:

$$v_{I_t} = 1.57 \text{ V}$$

$$v_{O_t} = 3.5 \text{ V}$$

Driver:

$$v_{I_t} = 1.57 \text{ V}$$

$$v_{O_t} = 0.87 \text{ V}$$

$$c. i_D = \frac{35}{2} \cdot (2)(1.5)^2 = 78.75 \mu\text{A}$$

$$P = I_D \cdot V_{DD} = (78.75)(5)$$

$$\Rightarrow P = 394 \mu\text{W}$$

E16.5

$$P = i_D \cdot V_{DD} \Rightarrow I_D = \frac{350}{5} = 70 \mu A$$

$$i_D = \left(\frac{k'}{2} \right) \left(\frac{W}{L} \right)_L (-V_{TN})^2$$

$$70 = \frac{35}{2} \cdot \left(\frac{W}{L} \right)_L (2)^2 \Rightarrow \underline{\underline{\left(\frac{W}{L} \right)_L = 1}}$$

$$i_D = \frac{35}{2} \cdot \left(\frac{W}{L} \right)_D [2(5 - 0.8)(0.05) - (0.05)^2]$$

$$70 = 7.31 \cdot \left(\frac{W}{L} \right)_D \Rightarrow \underline{\underline{\left(\frac{W}{L} \right)_D = 9.58}}$$

E16.6

From Equation (16-35):

$$V_{IH} = 0.85 + \frac{5 - 0.85}{16} \cdot \left\{ \frac{1 + 2(16)}{\sqrt{1 + 3(16)}} - 1 \right\}$$

$$\Rightarrow \underline{\underline{V_{IH} = 1.81 \text{ V}}}$$

Then from Equation (16-34):

$$V_{OLU} = \frac{(5 - 0.85) + 16(1.81 - 0.85)}{(1 + 2(16))}$$

$$V_{OLU} = 0.591 \text{ V}$$

$$V_{IL} = 0.85$$

$$V_{OHU} = 4.15$$

So

$$NM_L = V_{IL} - V_{OLU} = 0.85 - 0.591$$

$$\Rightarrow \underline{\underline{NM_L = 0.259 \text{ V}}}$$

$$NM_H = V_{OHU} - V_{IH} = 4.15 - 1.81$$

$$\Rightarrow \underline{\underline{NM_H = 2.34 \text{ V}}}$$

E16.7

From Equation (16-38):

$$V_{IL} = 1 + \frac{1.7}{\sqrt{3}(6)} \Rightarrow \underline{\underline{V_{IL} = 1.31 \text{ V}}}$$

Then from Equation (16-37):

$$V_{OHU} = (5 - 1.7) + (5)(1.31 - 1) \\ = 4.85 \text{ V}$$

From Equation (16-41):

$$V_{IH} = 1 + \frac{2(1.7)}{\sqrt{3}(5)} \Rightarrow \underline{\underline{V_{IH} = 1.88 \text{ V}}}$$

Then from Equation (16-40):

$$V_{OLU} = \frac{1.88 - 1}{2} \Rightarrow \underline{\underline{V_{OLU} = 0.44 \text{ V}}}$$

$$NM_L = V_{IL} - V_{OLU} = 1.31 - 0.44$$

$$\Rightarrow \underline{\underline{NM_L = 0.87 \text{ V}}}$$

$$NM_H = V_{OHU} - V_{IH} = 4.85 - 1.88$$

$$\Rightarrow \underline{\underline{NM_H = 2.97 \text{ V}}}$$

E16.8

- a. i. $A = \text{logic 1} = 10 \text{ V}$, $B = \text{logic 0}$
 " " driver in nonsaturation, "B" driver off

$$\left(\frac{k'}{2} \right) \left(\frac{W}{L} \right)_L (-V_{TN})^2 \\ = \left(\frac{k'}{2} \right) \left(\frac{W}{L} \right)_D [2(v_i - V_{TN})V_{OL} - V_{OL}^2]$$

$$2(3)^2 = (10)[2(10 - 1.5)V_{OL} - V_{OL}^2]$$

$$9 = 5(17V_{OL} - V_{OL}^2)$$

$$5V_{OL}^2 - 85V_{OL} + 9 = 0$$

$$V_{OL} = \frac{85 \pm \sqrt{(85)^2 - 4(5)(9)}}{2(5)} \\ \Rightarrow \underline{\underline{V_{OL} = 0.107 \text{ V}}}$$

- ii. $A = B = \text{logic 1}$

$$\left(\frac{k'}{2} \right) \left(\frac{W}{L} \right)_L (-V_{TN})^2 \\ = 2 \left(\frac{k'}{2} \right) \left(\frac{W}{L} \right)_D [2(v_i - V_{TN})V_{OL} - V_{OL}^2]$$

$$2(3)^2 = (2)(10)[2(10 - 1.5)V_{OL} - V_{OL}^2]$$

$$9 = 10(17V_{OL} - V_{OL}^2)$$

$$10V_{OL}^2 - 170V_{OL} + 9 = 0$$

$$V_{OL} = \frac{170 \pm \sqrt{(170)^2 - 4(10)(9)}}{2(10)} \\ \Rightarrow \underline{\underline{V_{OL} = 0.0531 \text{ V}}}$$

- b. Both cases.

$$i_D = \frac{35}{2} \cdot (2)(3)^2 = 315 \mu A \Rightarrow P = i_D \cdot V_{DD} \\ \Rightarrow \underline{\underline{P = 3.15 \text{ mW}}}$$

E16.9

$$P = i_D \cdot V_{DD} \Rightarrow i_D = \frac{800}{5} = 160 \mu A$$

$$i_D = 160 = \frac{35}{2} \cdot \left(\frac{W}{L} \right)_L (1.4)^2 = 34.3 \left(\frac{W}{L} \right)_L \\ \Rightarrow \underline{\underline{\left(\frac{W}{L} \right)_L = 4.66}}$$

$$i_D = 160 = \frac{35}{2} \cdot \left(\frac{W}{L} \right)_D [2(5 - 0.8)(0.12) - (0.12)^2] \\ \Rightarrow \underline{\underline{\left(\frac{W}{L} \right)_D = 9.20}}$$

E16.10

$$P = i_D \cdot V_{DD} \Rightarrow i_D = \frac{800}{5} = 160 \mu A$$

$$i_D = 160 = \frac{35}{2} \cdot \left(\frac{W}{L} \right)_L (1.4)^2 \\ \Rightarrow \underline{\underline{\left(\frac{W}{L} \right)_L = 4.66}}$$

$$i_D = 160 \mu A$$

$$\begin{aligned} &= \frac{35}{2} \cdot \frac{1}{3} \cdot \left(\frac{W}{L} \right)_D [2(5 - 0.8)(0.12) - (0.12)^2] \\ &\Rightarrow \underline{\underline{\left(\frac{W}{L} \right)_D = 27.6}} \end{aligned}$$

E16.11

a. From the load transistor:

$$\begin{aligned} I_{DL} &= \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right)_L (V_{GS0} - V_{TN0})^2 \\ &= \frac{35}{2} (0.5) (5 - 0.15 - 0.7)^2 \end{aligned}$$

or

$$I_{DL} = 150.7 \mu A$$

Maximum v_0 occurs when either A or B is high and C is high. For the two NMOS in series, the effective k_N is cut in half, so

$$I_{DL} = \frac{1}{2} \left[\left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right)_D \right] [2(V_{GS0} - V_{TN0})V_{DS} - V_{DS}^2]$$

or

$$150.7 = \frac{1}{2} \left[\frac{35}{2} \cdot \left(\frac{W}{L} \right)_D \right] [2(5 - 0.7)(0.15) - (0.15)^2]$$

which yields

$$\underline{\underline{\left(\frac{W}{L} \right)_D = 13.6}}$$

$$b. P = i_D \cdot V_{DD} = (150.7)(5) \Rightarrow \underline{\underline{P = 753 \mu W}}$$

E16.12

a. $v_0(\max)$ occurs when $A = B = 1$ and $C = D = 0$ or $A = B = 0$ and $C = D = 1$

$$\left(\frac{W}{L} \right)_L (-V_{TN0})^2 = \frac{1}{2} \left(\frac{W}{L} \right)_D [2(v_i - V_{TN0})v_o - v_o^2]$$

$$(0.5)(1.2)^2 = \frac{1}{2} \cdot \left(\frac{W}{L} \right)_D [2(5 - 0.7)(0.15) - (0.15)^2]$$

$$0.72 = (0.634) \left(\frac{W}{L} \right)_D \Rightarrow \underline{\underline{\left(\frac{W}{L} \right)_D = 1.14}}$$

$$b. i_D = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right)_L (-V_{TN0})^2 = \left(\frac{35}{2} \right) (0.5) [(-1.2)]^2$$

$$i_D = 12.6 \mu A$$

$$P = i_D \cdot V_{DD} = (12.6)(5) \Rightarrow \underline{\underline{P = 63 \mu W}}$$

E16.13

$$a. K_n/K_p = 1$$

$$V_{It} = \frac{10 - 2 + (1)(2)}{1+1} \Rightarrow V_{It} = 5 V$$

$$V_{OPt} = V_u + |V_{TP}| = 5 + 2 \Rightarrow V_{OPt} = 7 V$$

$$V_{ONt} = V_u - V_{TN} = 5 - 2 \Rightarrow V_{ONt} = 3 V$$

$$b. K_n/K_p = 0.5$$

$$\Rightarrow V_{It} = \frac{10 - 2 + \sqrt{0.5}(2)}{1 + \sqrt{0.5}} \Rightarrow V_{It} = 5.52 V$$

$$V_{OPt} = 7.52 V$$

$$V_{ONt} = 3.52 V$$

$$c. K_n/K_p = 2$$

$$\Rightarrow V_{It} = \frac{10 - 2 + \sqrt{2}(2)}{1 + \sqrt{2}} \Rightarrow V_{It} = 4.49 V$$

$$V_{OPt} = 6.49 V$$

$$V_{ONt} = 2.49 V$$

E16.14

$$a. K_n = K_p = 50 \mu A/V^2$$

$$V_{It} = 2.5 V$$

$$i_D(\max) = K_n (V_{It} - V_{TN})^2 = 50(2.5 - 0.8)^2$$

$$\Rightarrow \underline{\underline{i_D(\max) = 145 \mu A}}$$

$$b. K_n = K_p = 200 \mu A/V^2$$

$$V_{It} = 2.5 V$$

$$i_D(\max) = (200)(2.5 - 0.8)^2$$

$$\Rightarrow \underline{\underline{i_D(\max) = 578 \mu A}}$$

E16.15

$$P = f \cdot C_L \cdot V_{DD}^2$$

$$(0.10 \times 10^{-6}) = f(0.5 \times 10^{-12})(3)^2$$

$$f = 2.22 \times 10^4 \text{ Hz} \Rightarrow \underline{\underline{f = 22.2 \text{ kHz}}}$$

E16.16

$$a. K_n/K_p = 200/80 = 2.5$$

$$\Rightarrow V_{IL} = \frac{10 - 2 + \sqrt{2.5}(2)}{1 + \sqrt{2.5}} \Rightarrow V_{IL} = 4.32 V$$

$$V_{OPt} = 6.32 V$$

$$V_{ONt} = 2.32 V$$

$$b. V_{IL} = 2 + \frac{10 - 2 - 2}{2.5 - 1} \cdot \left[2\sqrt{\frac{2.5}{2.5 + 3}} - 1 \right]$$

$$\Rightarrow \underline{\underline{V_{IL} = 3.39 V}}$$

$$V_{OHV} = \frac{1}{2} \{(1 + 2.5)(3.39) + 10 - (2.5)(2) + 2\}$$

$$V_{OHV} = 9.43 V$$

$$V_{IH} = 2 + \frac{10 - 2 - 2}{2.5 - 1} \cdot \left[\frac{2(2.5)}{\sqrt{3(2.5) + 1}} - 1 \right]$$

$$\Rightarrow \underline{\underline{V_{IH} = 4.86 V}}$$

$$V_{OLU} = \frac{(4.86)(1 + 2.5) - 10 - (2.5)(2) + 2}{2(2.5)}$$

$$V_{OLU} = 0.802 V$$

c. $NM_L = V_{IL} - V_{oLU} = 3.39 - 0.802$

$$\Rightarrow NM_L = 2.59 \text{ V}$$

$$NM_H = V_{oHU} - V_{IH} = 9.43 - 4.86$$

$$\Rightarrow NM_H = 4.57 \text{ V}$$

E16.17

a. $V_{It} = \frac{5 - 2 + (1)(6.8)}{1 + 1}$

$$V_{It} = 1.9 \text{ V}$$

$$V_{oPt} = 3.9 \text{ V}$$

$$V_{oNt} = 1.1 \text{ V}$$

b. $V_{IL} = 0.8 + \frac{3}{8} \cdot [5 - 2 - 0.8]$

$$\Rightarrow V_{IL} = 1.63 \text{ V}$$

$$V_{oHU} = \frac{1}{2} \{2(1.63) + 5 - 0.8 + 2\}$$

$$V_{oHU} = 4.73 \text{ V}$$

$$V_{IH} = 0.8 + \frac{5}{8}(5 - 2 - 0.8)$$

$$\Rightarrow V_{IH} = 2.18 \text{ V}$$

$$V_{oLU} = \frac{1}{2} \{2(2.18) - 5 - 0.8 + 2\}$$

$$V_{oLU} = 0.275 \text{ V}$$

c. $NM_L = V_{IL} - V_{oLU} = 1.63 - 0.275$

$$\Rightarrow NM_L = 1.35 \text{ V}$$

$$NM_H = V_{oHU} - V_{IH} = 4.73 - 2.18$$

$$\Rightarrow NM_H = 2.55 \text{ V}$$

E16.18

a. $A = 0 \Rightarrow M_{PA}$ (assume zero resistance)

$$K_n = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_N$$

$$K_p = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_P = \frac{1}{2} \left(\frac{k'_p}{2}\right) \cdot 8 \left(\frac{W}{L}\right)_P$$

$$K_p = 2K_n \Rightarrow \frac{K_p}{K_n} = \frac{1}{2}$$

From Equation (16-55),

$$V_{It} = \frac{5 - 1 + \sqrt{0.5(1)}}{1 + \sqrt{0.5}} \Rightarrow V_{It} = 2.76 \text{ V}$$

$$V_{oPt} = 3.76 \text{ V}$$

$$V_{oNt} = 1.76 \text{ V}$$

b. From Equation (16-58(b))

$$i_D(\text{peak}) = K_n(V_n - V_m)^2$$

$$50 = \frac{35}{2} \left(\frac{W}{L}\right)_N (2.76 - 1)^2 = 54.2 \left(\frac{W}{L}\right)_N$$

$$\Rightarrow \left(\frac{W}{L}\right)_N = 0.923$$

$$\left(\frac{W}{L}\right)_P = (8)(0.923) = 7.38$$

E16.19

$$\text{Want } K_{n,\text{eff}} = K_{p,\text{eff}}$$

$$\left(\frac{k'_n}{2}\right) \cdot \frac{1}{2} \left(\frac{W}{L}\right)_N = \left(\frac{k'_p}{2}\right) \cdot 2 \left(\frac{W}{L}\right)_P = \frac{1}{2} \cdot \frac{k'_n}{2} \cdot 2 \left(\frac{W}{L}\right)_P$$

$$\Rightarrow \left(\frac{W}{L}\right)_N = 2 \left(\frac{W}{L}\right)_P$$

E16.20

$$\text{Want } K_{n,\text{eff}} = K_{p,\text{eff}}$$

$$\left(\frac{k'_n}{2}\right) \cdot 3 \left(\frac{W}{L}\right)_N = \left(\frac{k'_p}{2}\right) \cdot \frac{1}{3} \left(\frac{W}{L}\right)_P = \frac{1}{2} \cdot \frac{k'_n}{2} \cdot \frac{1}{3} \left(\frac{W}{L}\right)_P$$

$$3 \left(\frac{W}{L}\right)_N = \frac{1}{6} \cdot \left(\frac{W}{L}\right)_P$$

Or

$$\Rightarrow \frac{(W/L)_P}{(W/L)_N} = 18$$

E16.21

$$\text{Want } K_{n,\text{eff}} = K_{p,\text{eff}}$$

$$\left(\frac{k'_n}{2}\right) \cdot \frac{1}{3} \left(\frac{W}{L}\right)_N = \left(\frac{k'_p}{2}\right) \cdot 3 \left(\frac{W}{L}\right)_P = \frac{1}{2} \cdot \frac{k'_n}{2} \cdot 3 \left(\frac{W}{L}\right)_P$$

$$\frac{1}{3} \left(\frac{W}{L}\right)_N = \frac{3}{2} \cdot \left(\frac{W}{L}\right)_P$$

Or

$$\Rightarrow \frac{(W/L)_P}{(W/L)_N} = \frac{2}{9}$$

E16.22

NMOS:

$$M_{NA}, M_{NB} \text{ in series} \Rightarrow \left(\frac{W}{L}\right) = 2$$

$$M_{ND}, M_{NE} \text{ in parallel} \Rightarrow \left(\frac{W}{L}\right) = 1$$

$$M_{NC} \text{ in series with } M_{ND} \parallel M_{NE} \Rightarrow \left(\frac{W}{L}\right) = 2$$

$$\text{Effective composite } \left(\frac{W}{L}\right) = 1 \text{ for each side.}$$

PMOS:

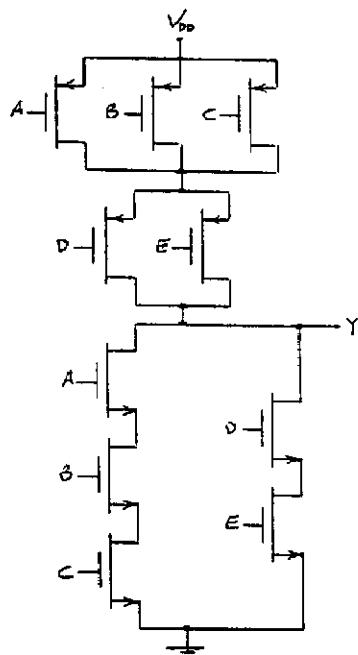
Want the effective composite $\left(\frac{W}{L}\right)$ of each side to be 2.

M_{PA}, M_{PC} in series $\Rightarrow \left(\frac{W}{L}\right) = 4$

M_{PA}, M_{PB} in parallel $\Rightarrow \left(\frac{W}{L}\right)_B = 4$

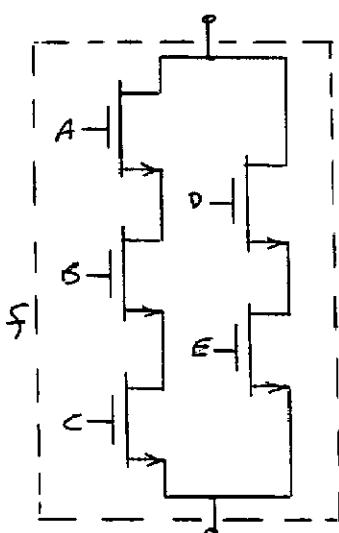
M_{PD}, M_{PE} in series $\Rightarrow \left(\frac{W}{L}\right) = 8$

E16.23



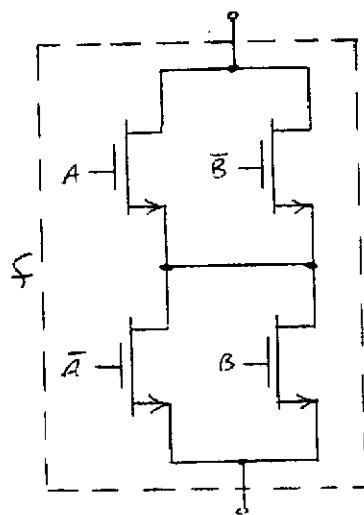
E16.24

The NMOS part of the circuit is:



E16.25

The NMOS part of the circuit is:



E16.26

- $v_I = \phi = 5 \text{ V} \Rightarrow v_o = 4 \text{ V}$
- $v_I = 3 \text{ V}, \phi = 5 \text{ V} \Rightarrow v_o = 3 \text{ V}$
- $v_I = 4.2 \text{ V}, \phi = 5 \text{ V} \Rightarrow v_o = 4 \text{ V}$
- $v_I = 5 \text{ V}, \phi = 3 \text{ V} \Rightarrow v_o = 2 \text{ V}$

E16.27

(a) $v_I = 8 \text{ V}, \phi = 10 \text{ V} \Rightarrow v_{GSD} = 8 \text{ V}$
 M_D in nonsaturation

$$K_D [2(v_{GSD} - V_{TN})v_o - v_o^2]$$

$$K_L [V_{DD} - v_o - V_{TL}]^2$$

$$\frac{K_D}{K_L} [2(8-2)(0.5) - (0.5)^2] = [10 - 0.5 - 2]^2$$

$$\Rightarrow \frac{K_D}{K_L} = 9.78$$

(b) $v_I = \phi = 8 \text{ V} \Rightarrow v_{GSD} = 6 \text{ V}$

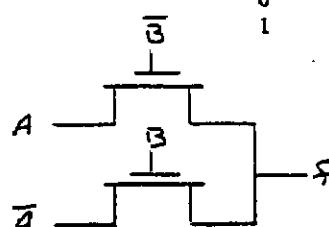
$$\frac{K_D}{K_L} [2(6-2)(0.5) - (0.5)^2] = [10 - 0.5 - 2]^2$$

$$\Rightarrow \frac{K_D}{K_L} = 15$$

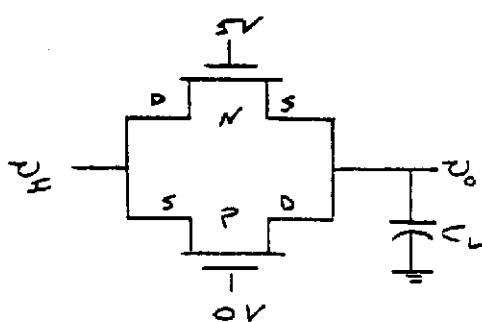
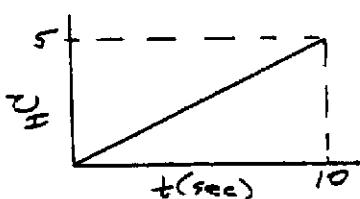
E16.28

Exclusive-OR

| A | B | f |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 0 |



E16.29

NMOS conducting for $0 \leq v_I \leq 4.2$ V⇒ NMOS Conducting: $0 \leq t \leq 8.4$ sNMOS Cutoff: $8.4 \leq t \leq 10$ sPMOS cutoff for $0 \leq v_I \leq 1.2$ V⇒ PMOS Cutoff: $0 \leq t \leq 2.4$ sPMOS Conducting: $2.4 \leq t \leq 10$ s

E16.30

(a) $1K \Rightarrow 32 \times 32$ arrayEach row and column requires a 5-bit word \Rightarrow
6 transistors per row and column, \Rightarrow
 $32 \times 6 + 32 \times 6 = 384$ transistors plus buffer
transistors.(b) $4K \Rightarrow 64 \times 64$ arrayEach row and column requires a 6-bit word \Rightarrow
7 transistors per row and column \Rightarrow
 $64 \times 7 + 64 \times 7 = 896$ transistors plus buffer
transistors.(c) $16K \Rightarrow 128 \times 128$ arrayEach row and column requires a 7-bit word \Rightarrow
8 transistors per row and column \Rightarrow
 $128 \times 8 + 128 \times 8 = 2048$ transistors plus buffer
transistors.

E16.31

 $16K \Rightarrow 16384$ cellsTotal Power = $125\text{ mW} = (2.5)I_r$ $\Rightarrow I_r = 50\text{ mA}$ Then, for each cell, $I = \frac{50\text{ mA}}{16384} \Rightarrow I = 3.05\text{ }\mu\text{A}$ Now, $I \equiv \frac{V_{DD}}{R}$ or $R = \frac{V_{DD}}{I} = \frac{2.5}{3.05} \Rightarrow$ $R = 0.82\text{ M}\Omega$

E16.32

From Equation (16.93)

$$\frac{(W/L)_{sd}}{(W/L)_{sr}} = \frac{2(V_{DD}V_{TN}) - 3V_{TN}^2}{(V_{DD} - 2V_{TN})^2}$$

$$= \frac{2(2.5)(0.4) - 3(0.4)^2}{(2.5 - 2(0.4))^2} = 0.526$$

From Equation (16.95)

$$\frac{(W/L)_p}{(W/L)_{ns}} = \frac{k'_n}{k'_p} \cdot \frac{2(V_{DD}V_{TN}) - 3V_{TN}^2}{(V_{DD} + V_{TP})^2}$$

$$= (2.5) \left[\frac{2(2.5)(0.4) - 3(0.4)^2}{(2.5 - 0.4)^2} \right] = 1.31$$

So $\left(\frac{W}{L}\right)$ of transmission gate device must be< 0.526 times the $\left(\frac{W}{L}\right)$ of the NMOS transistors in
the inverter cell. The $\left(\frac{W}{L}\right)$ of the PMOS transistors
must be < 1.31 times the $\left(\frac{W}{L}\right)$ of the transmission
gate devices. Then the $\left(\frac{W}{L}\right)$ of the PMOS devices
must be < 0.689 times $\left(\frac{W}{L}\right)$ of NMOS devices in
cell.

E16.33

Initial voltage across the storage capacitor
 $= V_{DD} - V_{TN} = 3 - 0.5 = 2.5V$.

Now

$$-I = C \frac{dV}{dt} \text{ or } V = -\frac{I}{C} \cdot t + K$$

where $K = 2.5V$, $t = 1.5\text{ ms}$, $V = \frac{2.5}{2} = 1.25V$, and
 $C = 0.05\text{ pF}$. Then

$$1.25 = 2.5 - \frac{I(1.5 \times 10^{-3})}{(0.05 \times 10^{-12})} \Rightarrow$$

$$I = 4.17 \times 10^{-11} \text{ A} \Rightarrow I = 41.7 \text{ pA}$$

Chapter 16

Problem Solutions

16.1

$$(a) \Delta V_{TN} = \frac{\sqrt{2e N_a}}{C_\infty} [\sqrt{2\phi_p + V_{SB}} - \sqrt{2\phi_p}]$$

$$C_\infty = \frac{\epsilon_\infty}{t_\infty} = \frac{(3.9)(8.85 \times 10^{-14})}{450 \times 10^{-9}} = 7.67 \times 10^{-8}$$

$$\sqrt{2e N_a}$$

$$= [2(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(8 \times 10^{15})]^{1/2}$$

$$= 5.15 \times 10^{-4}$$

Then

$$\Delta V_{TN} = \frac{5.15 \times 10^{-4}}{7.67 \times 10^{-8}} [\sqrt{2(0.343) + V_{SB}} - \sqrt{2(0.343)}]$$

For $V_{SB} = 1V$:

$$\Delta V_{TN} = 0.671 [\sqrt{1.686} - \sqrt{0.686}] \Rightarrow \underline{\Delta V_{TN} = 0.316V}$$

For $V_{SB} = 2V$:

$$\Delta V_{TN} = 0.671 [\sqrt{2.686} - \sqrt{0.686}] \Rightarrow \underline{\Delta V_{TN} = 0.544V}$$

(b) For $V_{GS} = 2.5V$, $V_{DS} = 5V$, transistor biased in the saturation region.

$$I_D = K_n (V_{GS} - V_{TN})^2$$

For $V_{SB} = 0$,

$$I_D = 0.2(2.5 - 0.8)^2 = 0.578 \text{ mA}$$

For $V_{SB} = 1$,

$$I_D = 0.2(2.5 - [0.8 + 0.316])^2 = 0.383 \text{ mA}$$

For $V_{SB} = 2$,

$$I_D = 0.2(2.5 - [0.8 + 0.544])^2 = 0.267 \text{ mA}$$

16.2

$$(a) I_D = \frac{V_{DD} - v_o}{R_D} = K_n [2(V_{GS} - V_{TN})v_o - v_o^2]$$

$$\frac{5 - (0.1)}{40 \times 10^3} = K_n [2(5 - 0.8)(0.1) - (0.1)^2]$$

$$\text{or } K_n = 1.476 \times 10^{-4} \text{ A/V}^2 = \frac{8 \times 10^{-5}}{2} \left(\frac{W}{L}\right)$$

$$\text{So that } \left(\frac{W}{L}\right) = 3.69$$

b. From Equation (16.10),

$$K_n R_D [V_H - V_{TN}]^2 + [V_H - V_{TN}] - V_{DD} = 0$$

$$(0.1476)(40)[V_{f_t} - 0.8]^2 + [V_{f_t} - 0.8] - 5 = 0$$

$$\text{or } [V_{f_t} - 0.8] = \frac{-1 \pm \sqrt{(1)^2 + 4(0.1476)(40)(5)}}{2(0.1476)(40)}$$

$$\text{or } [V_{f_t} - 0.8] = 0.839$$

So that $V_{f_t} = 1.64 \text{ V}$

$$P = I_D(\max) \cdot V_{DD}$$

$$\text{and } I_D(\max) = \frac{5 - (0.1)}{40} = 0.1225 \text{ mA}$$

$$\text{or } P = 0.6125 \text{ mW}$$

16.3

a. From Equation (16.10), the transistor point is found from

$$K_n R_D (V_H - V_{TN})^2 + (V_H - V_{TN}) - V_{DD} = 0$$

$$K_n = 50 \mu A/V^2, R_D = 20 k\Omega, V_{DD} = 0.8V$$

$$(0.05)(20)(V_H - V_{TN})^2 + (V_H - V_{TN}) - 5 = 0$$

$$V_H - V_{TN} = \frac{-1 \pm \sqrt{1 + 4(0.05)(20)(5)}}{2(0.05)(20)}$$

$$= 1.79 \text{ V So } V_{f_t} = 2.59 \text{ V}$$

$$V_{o_t} = 1.79 \text{ V}$$

Output voltage for $v_I = 5 \text{ V}$ is determined from Equation (16.12):

$$v_0 = 5 - (0.05)(20)[2(5 - 0.8)v_0 - v_0^2]$$

$$v_0^2 - 9.4v_0 + 5 = 0$$

$$\text{So } v_0 = \frac{9.4 \pm \sqrt{(9.4)^2 - 4(1)(5)}}{2(1)} = 0.566 \text{ V}$$

b. For $R_D = 200 \text{ k}\Omega$,

$$(V_H - V_{TN}) = \frac{-1 \pm \sqrt{1 + 4(0.05)(200)(5)}}{2(0.05)(200)}$$

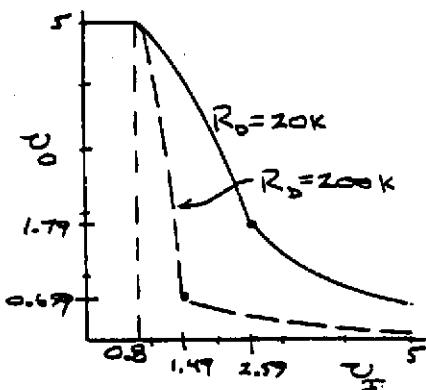
$$= 0.659 \text{ V So } V_{f_t} = 1.46 \text{ V}$$

$$V_{o_t} = 0.659 \text{ V}$$

$$v_0 = 5 - (0.05)(200)[2(5 - 0.8)v_0 - v_0^2]$$

$$\text{or } 10v_0^2 - 85v_0 + 5 = 0$$

$$v_0 = \frac{85 \pm \sqrt{(85)^2 - 4(10)(5)}}{2(10)} = 0.0592 \text{ V}$$



16.4

$$P = i_D \cdot V_{DD}$$

$$1 = i_D(5) \Rightarrow i_D = 0.2 \text{ mA}$$

Now

$$i_D = K_n [2(v_i - V_{TN})v_o - v_o^2]$$

$$0.2 = K_n [2(5 - 0.8)(0.2) - (0.2)^2]$$

$$\text{or } K_n = 0.122 \text{ mA/V}^2 = \left(\frac{0.080}{2}\right) \left(\frac{W}{L}\right) \Rightarrow$$

$$\left(\frac{W}{L}\right) = 3.05$$

Also

$$v_o = V_{DD} - K_n R_D [2(v_i - V_{TN})v_o - v_o^2]$$

$$0.2 = 5 - (0.122)R_D [2(5 - 0.8)(0.2) - (0.2)^2] \Rightarrow$$

$$R_D = 24 \text{ k}\Omega$$

16.5

From Equation (16.21):

$$V_{Ie} = \frac{10 - 2 + 2 \left(1 + \sqrt{\frac{200}{50}}\right)}{1 + \sqrt{\frac{200}{50}}}$$

$$\text{or } V_{Ie} = 4.67 \text{ V}$$

$$V_{Oe} = V_H - V_{TN} = 4.67 - 2 \Rightarrow V_{Oe} = 2.67 \text{ V}$$

From Equation (16.23):

$$200[2(8 - 2)v_0 - v_0^2] = 50[10 - v_0 - 2]^2$$

$$4[12v_0 - v_0^2] = [8 - v_0]^2 = [64 - 16v_0 + v_0^2]$$

$$5v_0^2 - 64v_0 + 64 = 0$$

$$v_0 = \frac{64 \pm \sqrt{(64)^2 - 4(5)(64)}}{2(5)}$$

$$\text{or } v_0 = 1.09 \text{ V}$$

16.6

(a) From Equation (16.23)

$$\frac{K_D}{K_L} \left[2(3 - 0.5)(0.25) - (0.25)^2 \right] = (3 - 0.25 - 0.5)^2$$

$$\Rightarrow \frac{K_D}{K_L} = 4.26$$

$$(b) \frac{K_D}{K_L} \left[2(2.5 - 0.5)(0.25) - (0.25)^2 \right] = (3 - 0.25 - 0.5)^2$$

$$\Rightarrow \frac{K_D}{K_L} = 5.06$$

$$(c) i_D = K_L (V_{GS} - V_{TNL})^2 = K_L (V_{DD} - v_o - V_{TNL})^2 \\ = \left(\frac{0.080}{2}\right) (1)(3 - 0.25 - 0.5)^2 \Rightarrow$$

$$i_D = 0.203 \text{ mA}$$

$$P = i_D \cdot V_{DD} = (0.203)(3) \Rightarrow P = 0.608 \text{ mW}$$

for both parts (a) and (b).

16.7

$$(a) P = 0.4 \text{ mW} = i_D \cdot V_{DD} = i_D(3) \Rightarrow$$

$$i_D = 0.133 \text{ mA}$$

$$i_D = K_L (V_{DD} - v_o - V_{TNL})^2$$

$$0.133 = \left(\frac{0.080}{2}\right) \left(\frac{W}{L}\right)_L (3 - 0.1 - 0.5)^2 = (0.2304) \left(\frac{W}{L}\right)_L$$

$$\text{So } \left(\frac{W}{L}\right)_L = 0.577$$

$$\frac{K_D}{K_L} \left[2(2.5 - 0.5)(0.1) - (0.1)^2 \right] = (3 - 0.1 - 0.5)^2$$

$$\Rightarrow \frac{K_D}{K_L} = 14.8 \text{ so that } \left(\frac{W}{L}\right)_D = 8.54$$

$$V_H = \frac{3 - 0.5 + 0.5(1 + \sqrt{14.8})}{1 + \sqrt{14.8}}$$

or

$$V_H = 1.02 \text{ V}, V_{Oe} = 0.52 \text{ V}$$

$$(b) NM_L = V_H - V_{OeU}$$

$$NM_H = V_{OeU} - V_H$$

From Equation (16.35)

$$V_H = 0.5 + \frac{(3 - 0.5)}{14.8} \left\{ \frac{(1 + 2(14.8))}{\sqrt{1 + 3(14.8)}} - 1 \right\} \Rightarrow$$

$$V_H = 1.10 \text{ V}$$

$$V_{OeU} = 3.0 - 0.5 = 2.5 \text{ V}$$

$$NM_H = V_{OeU} - V_H = 2.5 - 1.10 \Rightarrow NM_H = 1.40 \text{ V}$$

$$V_{IL} = V_{TNH} = 0.5 \text{ V}$$

$$V_{OL} = \frac{(V_{DD} - V_{ML}) + \frac{K_D}{K_L}(V_I - V_{TND})}{1 + 2\left(\frac{K_D}{K_L}\right)} = \frac{(3 - 0.5) + 14.8(1.1 - 0.5)}{1 + 2(14.8)} \Rightarrow$$

$$V_{OL} = 0.372 V$$

$$\text{Then } NM_L = V_u - V_{OL} = 0.5 - 0.372 \Rightarrow NM_L = 0.128 V$$

16.8

We have

$$\begin{aligned} \frac{K_D}{K_L} [2(v_I - V_{TND})v_o - v_o^2] &= (V_{DD} - v_o - V_{ML})^2 \\ \frac{(W/L)_D}{(W/L)_L} [2(V_{DD} - V_{ML} - V_{TN})(0.08V_{DD}) - (0.08V_{DD})^2] &= (V_{DD} - 0.08V_{DD} - V_{ML})^2 \\ \frac{(W/L)_D}{(W/L)_L} [2(V_{DD} - 2(0.2)V_{DD})(0.08V_{DD}) - 0.0064V_{DD}^2] &= [(0.92 - 0.2)V_{DD}]^2 = 0.5184V_{DD}^2 \\ \frac{(W/L)_D}{(W/L)_L} [0.096] &= 0.5184 \Rightarrow \frac{(W/L)_D}{(W/L)_L} = 5.4 \end{aligned}$$

16.9

$$V_{OH} = V_s - V_{os} = \text{Logic 1}$$

So

- (a) $V_s = 4 V \Rightarrow V_{OH} = 3 V$
- (b) $V_s = 5 V \Rightarrow V_{OH} = 4 V$
- (c) $V_s = 6 V \Rightarrow V_{OH} = 5 V$
- (d) $V_s = 7 V \Rightarrow V_{OH} = 6 V$

 For $v_I = V_{OH}$

$$K_D [2(v_I - V_r)v_o - v_o^2] = K_L [V_s - v_o - V_r]^2$$

Then

- (a) $(1)[2(3-1)V_{os} - V_{os}^2] = (0.4)[4 - V_{os} - 1]^2 \Rightarrow V_{os} = 0.657 V$
- (b) $(1)[2(4-1)V_{os} - V_{os}^2] = (0.4)[5 - V_{os} - 1]^2 \Rightarrow V_{os} = 0.791 V$
- (c) $(1)[2(5-1)V_{os} - V_{os}^2] = (0.4)[6 - V_{os} - 1]^2 \Rightarrow V_{os} = 0.935 V$
- (d) $(1)[2(6-1)V_{os} - V_{os}^2] = (0.4)[7 - V_{os} - 1]^2 \Rightarrow V_{os} = 1.08 V$

16.10

$$\begin{aligned} \text{a. For load } V_{os} &= V_{DD} + V_{ML} = 5 - 2 = 3 V \\ \sqrt{\frac{K_D}{K_L}} \cdot (V_I - V_{TND}) &= -V_{ML} \\ \sqrt{\frac{500}{100}} (V_I - 0.8) &= -(-2) \\ \Rightarrow V_I &= 1.69 V \\ V_{os} &= 3 V \end{aligned} \quad \left. \begin{array}{l} \text{Load} \\ \text{Driver} \end{array} \right\}$$

$$\text{Driver: } V_{os} = V_u - V_{TND} = 1.69 - 0.8 = 0.89 V$$

$$\left. \begin{array}{l} V_I = 1.69 V \\ V_{os} = 0.89 V \end{array} \right\} \text{Driver}$$

b. From Equation (16.29(b)):

$$\begin{aligned} \frac{500}{100} \cdot [2(5 - 0.8)v_0 - v_0^2] &= [-(-2)]^2 \\ 5v_0^2 - 42v_0 + 4 &= 0 \\ v_0 = \frac{42 \pm \sqrt{(42)^2 - 4(5)(4)}}{2(5)} &\Rightarrow v_0 = 0.0963 V \end{aligned}$$

$$\begin{aligned} \text{c. } i_D &= K_L (-V_{ML})^2 = 100[-(-2)]^2 \Rightarrow \\ i_D &= 400 \mu A \end{aligned}$$

16.11

$$\left(\frac{500}{50} \right) [2(3 - 0.5)(0.1) - (0.1)^2] = (-V_{ML})^2$$

So

$$(-V_{ML})^2 = 4.9 \Rightarrow V_{ML} = -2.21 V$$

16.12

$$(a) P = i_D \cdot V_{DD}$$

$$150 = i_D \cdot (3) \Rightarrow i_D = 50 \mu A$$

$$i_D = K_L (-V_{ML})^2$$

$$50 = \left(\frac{80}{2} \right) \left(\frac{W}{L} \right)_L [-(-1)]^2 \Rightarrow \left(\frac{W}{L} \right)_L = 1.25$$

$$\frac{K_D}{K_L} [2(3 - 0.5)(0.1) - (0.1)^2] = [-(-1)]^2$$

$$\frac{K_D}{K_L} = \frac{(W/L)_D}{(W/L)_L} = 2.04 \Rightarrow \left(\frac{W}{L} \right)_D = 2.55$$

For the Load:

$$V_{os} = V_{DD} + V_{ML} = 3 - 1 \Rightarrow V_{os} = 2 V$$

$$\sqrt{2.04}(V_u - 0.5) = [-(-1)] \Rightarrow V_u = 1.20 V$$

For the Driver:

$$\begin{aligned} V_{os} &= V_u - V_{TND} = 1.20 - 0.5 \Rightarrow V_{os} = 0.70 V \\ V_u &= 1.20 V \end{aligned}$$

(b) $NM_L = V_L - V_{OL}$
 $NM_H = V_{OH} - V_{IH}$

$$V_L = 0.5 + \frac{[-(-1)]}{\sqrt{(2.04)(1+2.04)}} = 0.902 V$$

$$V_{IH} = 0.5 + \frac{2[-(-1)]}{\sqrt{3(2.04)}} = 1.31 V$$

Then $V_{OL} = (3-1)+(2.04)(0.902-0.5) = 2.82 V$

$$V_{OL} = \frac{(1.31-0.5)}{2} = 0.405 V$$

$$NM_L = 0.902 - 0.405 \Rightarrow NM_L = 0.497 V$$

$$NM_H = 2.82 - 1.31 \Rightarrow NM_H = 1.51 V$$

16.13

a. From Equation (16.29(b)):

$$\left(\frac{W}{L}\right)_D [2(2.5 - 0.5)(0.05) - (0.05)^2] \\ = \left(\frac{W}{L}\right)_L [-(-1)]^2$$

$$\left(\frac{W}{L}\right)_L = 1$$

Then $\left(\frac{W}{L}\right)_D = 5.06$

b. $i_D = \left(\frac{80}{2}\right)(1)[-(-1)]^2$

or $i_D = 40 \mu A$

$$P = i_D \cdot V_{DD} = (40)(2.5)$$

$$\Rightarrow P = 100 \mu W$$

16.14

a. i. $v_T = 0.5 V \Rightarrow i_D = 0 \Rightarrow P = 0$

ii. $v_T = 5 V$, From Equation (16.12),

$$v_0 = 5 - (0.1)(20)[2(5 - 1.5)v_0 - v_0^2]$$

$$2v_0^2 - 15v_0 + 5 = 0$$

$$v_0 = \frac{15 \pm \sqrt{(15)^2 - 4(2)(5)}}{2(2)} \Rightarrow v_0 = 0.35 V$$

$$i_D = \frac{5 - 0.35}{20} = 0.2325 mA$$

$$P = i_D \cdot V_{DD} = (0.2325)(5) \Rightarrow P = 1.16 mW$$

b. i. $v_T = 0.25 V \Rightarrow i_D = 0 \Rightarrow P = 0$

ii. $v_T = 4.3 V$, From Equation (16.23),

$$100[2(4.3 - 0.7)v_0 - v_0^2] = 10[5 - v_0 - 0.7]^2$$

$$10[7.2v_0 - v_0^2] = 18.49 - 8.6v_0 + v_0^2$$

Then

$$11v_0^2 - 80.6v_0 + 18.49 = 0$$

$$v_0 = \frac{80.6 \pm \sqrt{(80.6)^2 - 4(11)(18.49)}}{2(11)}$$

$$\Rightarrow v_0 = 0.237 V$$

Then

$$i_D = 10[5 - 0.237 - 0.7]^2 = 165 \mu A$$

$$P = i_D \cdot V_{DD} = (165)(5) \Rightarrow P = 825 \mu W$$

c. i. $v_T = 0.03 V \Rightarrow i_D = 0 \Rightarrow P = 0$

ii. $v_T = 5 V$

$$i_D = K_L(-V_{TH})^2 = (10)[-(-2)]^2 = 40 \mu A$$

$$P = i_D \cdot V_{DD} = (40)(5) \Rightarrow P = 200 \mu W$$

16.15

From Equation (16.35)

$$V_{IH} = 0.8 + \frac{5 - (0.8)}{10} \cdot \left\{ \frac{1 + 2(10)}{\sqrt{1 + 3(10)}} - 1 \right\}$$

$$V_{IH} = 0.8 + 0.42 \cdot \left\{ \frac{21}{5.57} - 1 \right\}$$

or

$$V_{IH} = 1.96 V$$

 M_{DS2} in non-saturation region. Then

$$K_D [2(v_{DS2} - V_{TH})v_{DS2} - v_{DS2}^2]$$

$$= K_L [V_{DD} - v_{DS2} - V_{TH}]^2$$

$$v_{DS2} = v_{O2} \text{ and } v_{GS2} = V_{IH} = 1.96$$

$$10[2(1.96 - 0.8)v_{O2} - v_{O2}^2] = [5 - v_{O2} - 0.8]^2$$

$$23.2v_{O2} - 10v_{O2}^2 = 17.64 - 8.4v_{O2} + v_{O2}^2$$

or

$$11v_{O2}^2 - 31.6v_{O2} + 17.64 = 0$$

$$v_{O2} = \frac{31.6 \pm \sqrt{(31.6)^2 - 4(11)(17.64)}}{2(11)}$$

$$\Rightarrow v_{O2} = 0.758 V$$

Now M_{D1} in saturation region. Then

$$K_D [v_I - V_{TN}]^2 = K_L [V_{DD} - v_{O1} - V_{TN}]^2$$

$$\sqrt{10} \cdot (v_I - 0.8) = 5 - 1.96 - 0.8 = 2.24$$

$$\text{Then } v_I = 1.51 \text{ V}$$

16.16

a. From Equation (16.41),

$$V_{IH} = 0.8 + \frac{2[-(-2)]}{\sqrt{3(4)}} \Rightarrow V_{IH} = 1.95 \text{ V} = v_{O1}$$

M_{D2} in non-saturation and M_{L2} in saturation.

$$K_D [2(v_{O1} - V_{TN})v_{O2} - v_{O2}^2] = K_L (-V_{TN})^2$$

$$4[2(1.95 - 0.8)v_{O2} - v_{O2}^2] = (1)[-(-2)]^2$$

$$4v_{O2}^2 - 9.2v_{O2} + 4 = 0$$

$$v_{O2} = \frac{9.2 \pm \sqrt{(9.2)^2 - 4(4)(4)}}{2(4)} \Rightarrow v_{O2} = 0.582 \text{ V}$$

Both M_{D1} and M_{L1} in saturation region. From Equation (16.2B(b)).

$$\sqrt{4} \cdot (v_I - 0.8) = -(-2)$$

$$\text{or } v_I = 1.8 \text{ V}$$

b. $V_{IL} = 0.8 + \frac{(+2)}{\sqrt{4(1+4)}} = 1.25 \text{ V} = v_{O1}$

M_{D2} in saturation, M_{L2} in non-saturation

$$K_D [v_{O1} - V_{TN}]^2$$

$$= K_L [2(-V_{TN})(5 - v_{O2}) - (5 - v_{O2})^2]$$

$$4(1.25 - 0.8)^2 = 2(2)(5 - v_{O2}) - (5 - v_{O2})^2$$

$$(5 - v_{O2})^2 - 4(5 - v_{O2}) + 0.81 = 0$$

$$5 - v_{O2} = \frac{4 \pm \sqrt{(4)^2 - 4(1)(0.81)}}{2(1)} = 0.214 \text{ V}$$

so

$$v_{O2} = 4.786 \text{ V}$$

To find v_I :

$$4(v_{O1} - 0.8)^2 = (1)(-(-2))^2$$

$$v_{O1} - 0.8 = 1$$

$$v_{O1} = 1.8 \text{ V}$$

c. $V_{IH} = 1.95 \text{ V}, V_{IL} = 1.25 \text{ V}$

16.17

a. i. Neglecting the body effect,

$$v_0 = V_{DD} - V_{TH}$$

Assume $V_{DD} = 5 \text{ V}$, then $v_0 = 4.2 \text{ V}$

ii. Taking the body effect into account:
From Problem 16.1,

$$V_{TN} = V_{TN0} + 0.671 [\sqrt{0.686 + V_{SB}} - \sqrt{0.686}]$$

$$\text{and } V_{SB} = v_0$$

Then

$$v_0 = 5 - [0.8 + 0.671 (\sqrt{0.686 + v_0} - \sqrt{0.686})]$$

$$v_0 = 4.756 - 0.671 \sqrt{0.686 + v_0}$$

$$0.671 \sqrt{0.686 + v_0} = 4.756 - v_0$$

$$0.450(0.686 + v_0) = 22.62 - 9.51v_0 + v_0^2$$

$$v_0^2 - 9.96v_0 + 22.3 = 0$$

$$v_0 = \frac{9.96 \pm \sqrt{(9.96)^2 - 4(22.3)}}{2} \Rightarrow v_0 = 3.40 \text{ V}$$

b. PSpice results similar to Figure 16.18(a).

16.18

Results similar to Figure 16.18(b).

16.19

a. M_X on, M_Y cutoff.

From Equation (16.29(b)):

$$\frac{K_D}{K_L} [2(5 - 0.8)(0.2) - (0.2)^2] = [-(-2)]^2$$

$$\text{or } \frac{K_D}{K_L} = 2.44$$

b. For $v_X = v_Y = .5 \text{ V}$

$$2(2.44)[2(5 - 0.8)v_0 - v_0^2] = [-(-2)]^2$$

$$4.88v_0^2 - 41.0v_0 + 4 = 0$$

$$v_0 = \frac{41 \pm \sqrt{(41)^2 - 4(4.88)(4)}}{2(4.88)}$$

or

$$v_0 = 0.0987 \text{ V}$$

c. $i_D = \left(\frac{80}{2}\right)(1)(-(-2))^2 = 160 \mu\text{A}$

$$P = (160)(5) \Rightarrow P = 800 \mu\text{W}$$

for both parts (a) and (b).

16.20

- (a) Maximum value of v_o in low state- when only one input is high, then,

$$\frac{K_D}{K_L} [2(3 - 0.5)(0.1) - (0.1)^2] = [-(-1)]^2$$

$$\frac{K_D}{K_L} = 2.04$$

(b) $P = i_D \cdot V_{DD}$

$$0.1 = i_D (3) \Rightarrow i_D = 33.3 \mu A$$

$$i_D = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right)_L (-V_{TN})^2$$

$$33.3 = \left(\frac{80}{2} \right) \left(\frac{W}{L} \right)_L [-(-1)]^2 \Rightarrow \left(\frac{W}{L} \right)_L = 0.8325$$

Then $\left(\frac{W}{L} \right)_D = 1.70$

(c) $3(2.04)[2(3 - 0.5)v_o - v_o^2] = [-(-1)]^2 \Rightarrow v_o = 0.0329 V$

16.21

a. $P = i_D \cdot V_{DD}$

$$250 = i_D (5) \Rightarrow i_D = 50 \mu A$$

$$i_D = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right)_{M1} [-V_{TN1}]^2$$

$$50 = \left(\frac{60}{2} \right) \left(\frac{W}{L} \right)_{M1} [-(-2)]^2$$

So that $\left(\frac{W}{L} \right)_{M1} = 0.417$

$$\frac{K_D}{K_L} [2(v_i - V_{TN})v_o - v_o^2] = [-V_{TN}]^2$$

$$\frac{K_D}{K_L} [2(5 - 0.8)(0.15) - (0.15)^2] = [-(-2)]^2$$

$$\text{or } \frac{K_D}{K_L} = 3.23 \Rightarrow \left(\frac{W}{L} \right)_{M1} = 1.35$$

b. For $v_x = v_y = 0 \Rightarrow v_{o1} = 5 \text{ and } v_{o3} = 4.2$

Then

$$K_{D2}[2(v_{o1} - V_{TN})v_{o2} - v_{o2}^2]$$

$$+ K_{D3}[2(v_{o3} - V_{TN})v_{o2} - v_{o2}^2] = K_{L2}[-V_{TN2}]^2$$

$$K_{D2} \propto 8, K_{D3} \propto 8, K_{L2} \propto 1$$

$$8[2(5 - 0.8)v_{o2} - v_{o2}^2] + 8[2(4.2 - 0.8)v_{o2} - v_{o2}^2] = (1)[-(-2)]^2$$

$$67.2v_{o2} - 8v_{o2}^2 + 54.4v_{o2} - 8v_{o2}^2 = 4$$

Then

$$16v_{o2}^2 - 121.6v_{o2} + 4 = 0$$

$$v_{o2} = \frac{121.6 \pm \sqrt{(121.6)^2 - 4(16)(4)}}{2(16)}$$

So $v_{o2} = 0.0330 V$

16.22

- b. We can write

$$K_s [2(v_x - V_{TN})v_{DSX} - v_{DSX}^2]$$

$$= K_p [2(v_y - V_{DSX} - V_{TN})v_{DSY} - v_{DSY}^2]$$

$$\text{where } v_0 = v_{DSX} + v_{DSY}$$

We have

$$v_x = v_y = 9.2 V, V_{DD} = 10 V, V_{TN} = 0.8 V$$

As a good first approximation, neglect the v_{DSX}^2 and v_{DSY}^2 terms. Let $v_0 \approx 2v_{DSX}$. Then from the first and third terms in the above equation,

$$9[2(9.2 - 0.8)v_{DSX}] \approx (1)(10 - 2v_{DSX} - 0.8)^2$$

$$(151.2)v_{DSX} \approx 84.64 - 36.8v_{DSX}$$

So that $v_{DSX} = 0.450 V$

From the first and second terms of the above equation,

$$9[2(9.2 - 0.8)v_{DSX}] \approx 9[2(9.2 - v_{DSX} - 0.8)v_{DSY}]$$

or

$$(16.8)(0.45) = 2(9.2 - 0.45 - 0.8)v_{DSY}$$

which yields $v_{DSY} = 0.475 V$

Then $v_0 = v_{DSX} + v_{DSY} = 0.450 + 0.475$

or $v_0 = 0.925 V$

We have $v_{GSX} = 9.2 V$

and $v_{GSY} = 9.2 - v_{DSX} = 9.2 - 0.45$

or $v_{GSY} = 8.75 V$

b. Since v_0 is close to ground potential, the body effect will have minimal effect on the results.

From a PSpice analysis:

For part (a) :

$$v_{DSX} = 0.462 V, v_{DSY} = 0.491 V, v_0 = 0.9536 V,$$

$$v_{GSX} = 9.2 V, \text{ and } v_{GSY} = 8.738 V$$

For part (b) :

$$v_{DSX} = 0.441 V, v_{DSY} = 0.475 V, v_0 = 0.9154 V,$$

$$v_{GSX} = 9.2 V, \text{ and } v_{GSY} = 8.759 V$$

Chapter 16: Problem Solutions

16.23

a. We can write

$$\begin{aligned} K_s [2(v_x - V_{DSR})v_{DSR} - v_{DSR}^2] \\ = K_s [2(v_r - v_{DSR} - V_{DSY})v_{DSY} - v_{DSY}^2] \\ = K_t [-V_{DSY}]^2 \end{aligned}$$

From the first and third terms, (neglect v_{DSX}^2).

$$4[2(5 - 0.8)v_{DSX}] = (1)[-(-1.5)]^2$$

$$\text{or } v_{DSX} = 0.067 \text{ V}$$

From the second and third terms, (neglect v_{DSY}^2),

$$4[2(5 - 0.067 - 0.8)v_{DSY}] = (1)[-(-1.5)]^2$$

$$\text{or } v_{DSY} = 0.068 \text{ V}$$

Now

$$v_{GSX} = 5, v_{GSY} = 5 - 0.067 \Rightarrow v_{GSY} = 4.933 \text{ V}$$

$$\text{and } v_0 = v_{DSX} + v_{DSY} \Rightarrow v_0 = 0.135 \text{ V}$$

Since v_0 is close to ground potential, the body-effect has little effect on the results.

16.24

Complement of (B AND C) OR $A \Rightarrow \overline{(B \cdot C) + A}$

16.25

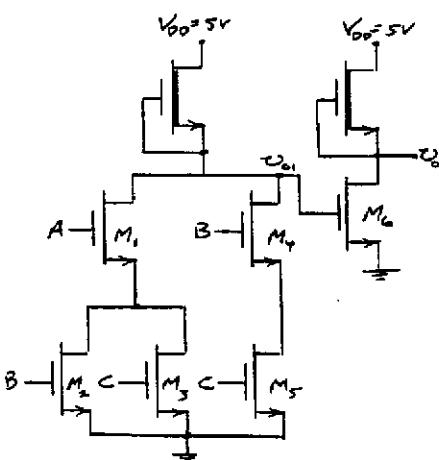
Considering a truth table, we find

| A | B | Y |
|-----|-----|-----|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

which shows that the circuit performs the exclusive-OR function.

16.26

(a) Carry-out = $A \bullet (B+C) + B \bullet C$



(b) For $v_{o1} = Low = 0.2 \text{ V}$

$$\frac{K_p}{K_t} [2(5 - 0.8)(0.2) - (0.2)^2] = [-(-1.5)]^2 \Rightarrow$$

$$\text{For } \left(\frac{W}{L}\right)_L = 1, \text{ then } \left(\frac{W}{L}\right)_D = 1.37$$

$$\text{So, for } M_6: \left(\frac{W}{L}\right)_6 = 1.37$$

To achieve the required composite conduction parameter,

$$\text{For } M_1 - M_3: \left(\frac{W}{L}\right)_{1-3} = 2.74$$

16.28

a. From Equation (16.55),

$$v_I = V_{It} = \frac{5 - 0.8 + 0.8}{1+1} = V_{It} = 2.5 \text{ V}$$

$$p - \text{channel}, V_{OpI} = 2.5 - (-0.8) \Rightarrow V_{OpI} = 3.3 \text{ V}$$

$$n - \text{channel}, V_{ONI} = 2.5 - 0.8 \Rightarrow V_{ONI} = 1.7 \text{ V}$$

c. For $v_I = 2 \text{ V}$, NMOS in saturation and PMOS in nonsaturation. From Equation (16.49),

$$(2 - 0.8)^2 = [2(5 - 2 - 0.8)(5 - v_0) - (5 - v_0)^2]$$

$$1.44 = 4.4(5 - v_0) - (5 - v_0)^2$$

$$\text{So } (5 - v_0)^2 - 4.4(5 - v_0) + 1.44 = 0$$

$$(5 - v_0) = \frac{4.4 \pm \sqrt{(4.4)^2 - 4(1)(1.44)}}{2}$$

or

$$5 - v_0 = 0.356 \Rightarrow v_0 = 4.64 \text{ V}$$

By symmetry, for $v_I = 3 \text{ V}$, $v_0 = 0.356 \text{ V}$

16.29

$$(a) K_n = \left(\frac{80}{2}\right)(2) = 80 \mu A/V^2$$

$$K_p = \left(\frac{40}{2}\right)(4) = 80 \mu A/V^2$$

$$(i) V_h = \frac{V_{DD} + V_{TP} + \sqrt{\frac{K_n}{K_p} \cdot V_{TN}}}{1 + \sqrt{\frac{K_n}{K_p}}} = \frac{3.3 - 0.4 + (1)(0.4)}{1+1} = 1.65 \text{ V}$$

$$V_h = 1.65 \text{ V}$$

PMOS:

$$V_{\alpha} = V_h - V_{TN} = 1.65 - (-0.4) \Rightarrow V_{\alpha} = 2.05 \text{ V}$$

NMOS:

$$V_{\alpha} = V_h - V_{TN} = 1.65 - (0.4) \Rightarrow V_{\alpha} = 1.25 \text{ V}$$

(iii) For $v_o = 0.4 V$: NMOS: Non-sat; PMOS: Sat

$$K_n [2(V_{GSN} - V_{TN})V_{DS} - V_{DS}^2] = K_p [V_{GSP} + V_{TP}]^2$$

$$2(v_i - 0.4)(0.4) - (0.4)^2 = (3.3 - v_i - 0.4)^2 \Rightarrow$$

$$v_i = 1.89 V$$

For $v_o = 2.9 V$, By symmetry

$$v_i = 1.65 - (1.89 - 1.65) \Rightarrow v_i = 1.41 V$$

$$(b) K_n = \left(\frac{80}{2}\right)(2) = 80 \mu A/V^2$$

$$K_p = \left(\frac{40}{2}\right)(2) = 40 \mu A/V^2$$

$$(i) V_n = \frac{3.3 - 0.4 + \sqrt{\frac{80}{40} \cdot (0.4)}}{1 + \sqrt{\frac{80}{40}}} \Rightarrow V_n = 1.44 V$$

PMOS:

$$V_{o1} = 1.44 - (-0.4) \Rightarrow V_{o1} = 1.84 V$$

NMOS:

$$V_{o1} = 1.44 - 0.4 \Rightarrow V_{o1} = 1.04 V$$

(iii) For $v_o = 0.4 V$

$$(80)[2(v_i - 0.4)(0.4) - (0.4)^2] = (40)[3.3 - v_i - 0.4]^2$$

$$\Rightarrow v_i = 1.62 V$$

For $v_o = 2.9 V$: NMOS: Sat, PMOS: Non-sat

$$(80)[v_i - 0.4]^2 = (40)[2(3.3 - v_i - 0.4)(0.4) - (0.4)^2]$$

$$\Rightarrow v_i = 1.16 V$$

16.30

a. For $v_{o1} = 0.6 < V_{TN} \Rightarrow v_{o2} = 5 V$

N_1 in nonsaturation and P_1 in saturation. From Equation (16.57),

$$[2(v_i - 0.8)(0.6) - (0.6)^2] = [5 - v_i - 0.8]^2$$

$$1.2v_i - 1.32 = 17.64 - 8.4v_i + v_i^2$$

or

$$v_i^2 - 9.6v_i + 18.96 = 0$$

$$v_i = \frac{9.6 \pm \sqrt{(9.6)^2 - 4(1)(18.96)}}{2}$$

or

$$v_i = 2.78 V$$

b. $V_{oN1} \leq v_{o2} \leq V_{oP1}$

From symmetry, $V_{I1} = 2.5 V$

$$V_{oP1} = 2.5 + 0.8 = 3.3 V$$

$$\text{and } V_{oN1} = 2.5 - 0.8 = 1.7 V$$

$$\text{So } 1.7 \leq v_{o2} \leq 3.3 V$$

16.31

$$a. V_{oN1} \leq v_{o1} \leq V_{oP1}$$

By symmetry, $V_{I1} = 2.5 V$

$$V_{oP1} = 2.5 + 0.8 = 3.3 V$$

$$\text{and } V_{oN1} = 2.5 - 0.8 = 1.7 V$$

$$\text{So } 1.7 \leq v_{o1} \leq 3.3 V$$

b. For $v_{o2} = 0.6 < V_{TN} \Rightarrow v_{o3} = 5 V$

N_2 in nonsaturation and P_2 in saturation. From Equation (16.57),

$$[2(v_{I2} - 0.8)(0.6) - (0.6)^2] = [5 - v_{I2} - 0.8]^2$$

$$1.2v_{I2} - 1.32 = 17.64 - 8.4v_{I2} + v_{I2}^2$$

or

$$v_{I2}^2 - 9.6v_{I2} + 18.96 = 0$$

$$\text{So } v_{I2} = v_{o1} = 2.78 V$$

For $v_{o1} = 2.78$, both N_1 and P_1 in saturation. Then

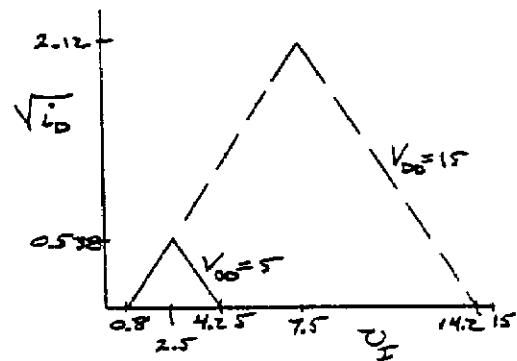
$$v_I = 2.5 V$$

16.32

$$a. \sqrt{i_{peak}} = \sqrt{K_n(v_i - V_{TN})}$$

$$\sqrt{i_{peak}} = \sqrt{0.1 \cdot (2.5 - 0.8)} = 0.538 (\text{mA})^{1/2}$$

$$b. \sqrt{i_{peak}} = \sqrt{0.1 \cdot (7.5 - 0.8)} = 2.12 (\text{mA})^{1/2}$$



16.33

$$(a) K_n = \left(\frac{50}{2}\right)(2) = 50 \mu A/V^2$$

$$K_p = \left(\frac{25}{2}\right)(4) = 50 \mu A/V^2$$

$$I_{D,peak} = K_n(v_i - V_{TN})^2 = 50(2.5 - 0.8)^2$$

$$\text{or } I_{D,peak} = 144.5 \mu A$$

$$(b) K_n = 50 \mu A/V^2, K_p = 25 \mu A/V^2$$

From Equation (16.55),

$$V_{in} = \frac{5 - 0.8 + \sqrt{\frac{50}{25}(0.8)}}{1 + \sqrt{\frac{50}{25}}} = 2.21 V$$

Then

$$I_{D,peak} = K_n(V_{in} - V_{TN})^2 = 50(2.21 - 0.8)^2$$

$$\text{or } I_{D,peak} = 99.4 \mu A$$

16.34

$$a. P = fC_L V_{DD}^2$$

$$\text{For } V_{DD} = 5 V, P = (10 \times 10^6)(0.2 \times 10^{-12})(5)^2$$

$$\text{or } P = 50 \mu W$$

$$\text{For } V_{DD} = 15 V, P = (10 \times 10^6)(0.2 \times 10^{-12})(15)^2$$

$$\text{or } P = 450 \mu W$$

$$b. \text{ For } V_{DD} = 5 V, P = (10 \times 10^6)(0.2 \times 10^{-12})(5)^2$$

$$\text{or } P = 50 \mu W$$

16.35

$$(a) P = fC_L V_{DD}^2 = (150 \times 10^6)(0.4 \times 10^{-12})(5)^2$$

$$= 1.5 \times 10^{-3} W/\text{inverter}$$

$$\text{Total power: } P_T = (2 \times 10^6)(1.5 \times 10^{-3}) \Rightarrow$$

$$P_T = 3000 W!!!!$$

$$(b) \text{ For } f = 300 \text{ MHz}$$

$$1.5 \times 10^{-3} = (300 \times 10^6)(0.4 \times 10^{-12})V_{DD}^2 \Rightarrow$$

$$\underline{V_{DD} = 3.54 V}$$

16.36

$$(a) \text{ For } v_i \equiv V_{DD}, \text{ NMOS in nonsaturation}$$

$$i_D = K_n [2(v_i - V_{TN})v_{DS} - v_{DS}^2] \text{ and } v_{DS} \equiv 0$$

$$\text{So } \frac{1}{r_{ds}} = \frac{di_D}{dv_{DS}} \cong K_n [2(V_{DD} - V_{TN})]$$

Or

$$r_{ds} = \frac{1}{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_n \cdot 2(V_{DD} - V_{TN})}$$

or

$$r_{ds} = \frac{1}{k'_n \left(\frac{W}{L}\right)_n \cdot (V_{DD} - V_{TN})}$$

For $v_i \equiv 0$, PMOS in nonsaturation

$$i_D = K_p [2(V_{DD} - v_i + V_{TP})v_{SD} - v_{SD}^2]$$

and $v_{SD} \equiv 0$ for $v_i \equiv 0$.

So

$$\frac{1}{r_{ds}} = \frac{di_D}{dv_{SD}} \cong \left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_p \cdot 2(V_{DD} + V_{TP})$$

or

$$r_{ds} = \frac{1}{k'_p \left(\frac{W}{L}\right)_p \cdot (V_{DD} + V_{TP})}$$

$$(b) \text{ For } \left(\frac{W}{L}\right)_n = 2, \left(\frac{W}{L}\right)_p = 4$$

$$r_{ds} = \frac{1}{(50)(2)(5 - 0.8)} \Rightarrow r_{ds} = 2.38 k\Omega$$

$$r_{ds} = \frac{1}{(25)(4)(5 - 0.8)} \Rightarrow r_{ds} = 2.38 k\Omega$$

$$\text{For } \left(\frac{W}{L}\right)_p = 2,$$

$$r_{ds} = \frac{1}{(25)(2)(5 - 0.8)} \Rightarrow r_{ds} = 4.76 k\Omega$$

Now, for NMOS:

$$v_{ds} = i_d r_{ds} \text{ or } i_d = \frac{v_{ds}}{r_{ds}} = \frac{0.5}{2.38} \Rightarrow \underline{i_d = 0.21 mA}$$

For PMOS:

$$\text{For } r_{ds} = 2.38 k\Omega,$$

$$i_d = \frac{v_{ds}}{r_{ds}} = \frac{0.5}{2.38} \Rightarrow \underline{i_d = 0.21 mA}$$

$$\text{For } r_{ds} = 4.76 k\Omega,$$

$$i_d = \frac{v_{ds}}{r_{ds}} = \frac{0.5}{4.76} \Rightarrow \underline{i_d = 0.105 mA}$$

16.37

From Equation (16.73)

$$V_{IL} = 1.5 + \frac{3}{8} \cdot (10 - 1.5 - 1.5) \Rightarrow \underline{V_{IL} = 4.125 V}$$

and Equation (16.72)

$$V_{IH} = \frac{1}{2} \cdot [2(4.125) + 10 - 1.5 + 1.5]$$

$$\text{or } \underline{V_{IH} = 9.125 V}$$

From Equation (16.79)

$$V_{IH} = 1.5 + \frac{5}{8} \cdot (10 - 1.5 - 1.5) \Rightarrow \underline{V_{IH} = 5.875 V}$$

and Equation (16.78)

$$V_{IL} = \frac{1}{2} \cdot [2(5.875) - 10 - 1.5 + 1.5]$$

$$\text{or } \underline{V_{IL} = 0.875 V}$$

Now

$$NM_L = V_{IL} - V_{OLU} = 4.125 - 0.875$$

$$\Rightarrow NM_L = 3.25 \text{ V}$$

$$NM_H = V_{OHU} - V_{IH} = 9.125 - 5.875$$

$$\Rightarrow NM_H = 3.25 \text{ V}$$

16.38

From Equation (16.71)

$$V_{IL} = 1.5 + \frac{(10 - 1.5 - 1.5)}{\left(\frac{100}{50} - 1\right)} \left[2 \sqrt{\frac{\frac{100}{50}}{\frac{100}{50} + 3}} - 1 \right]$$

$$= 1.5 + 7[2(0.632) - 1]$$

or

$$V_{IL} = 3.348 \text{ V}$$

From Equation (16.70)

$$V_{OHU} = \frac{1}{2} \cdot \left\{ \left(1 + \frac{100}{50} \right) (3.348) + 10 - \left(\frac{100}{50} \right) (1.5) + 1.5 \right\}$$

$$\text{or } V_{OHU} = 9.272 \text{ V}$$

From Equation (16.77)

$$V_{IH} = 1.5 + \frac{(10 - 1.5 - 1.5)}{\left(\frac{100}{50} - 1\right)} \left[\frac{2\left(\frac{100}{50}\right)}{\sqrt{3\left(\frac{100}{50}\right) + 1}} - 1 \right]$$

$$= 1.5 + 7[1.51 - 1]$$

or

$$V_{IH} = 5.07 \text{ V}$$

From Equation (16.76)

$$V_{OLU} = \frac{(5.07)\left(1 + \frac{100}{50}\right) - 10 - \left(\frac{100}{50}\right)(1.5) + 1.5}{2\left(\frac{100}{50}\right)}$$

$$\text{or } V_{OLU} = 0.9275 \text{ V}$$

$$\text{Now } NM_L = V_{IL} - V_{OLU} = 3.348 - 0.9275$$

$$\text{or } NM_L = 2.42 \text{ V}$$

$$NM_H = V_{OHU} - V_{IH} = 9.272 - 5.07$$

$$\text{or } NM_H = 4.20 \text{ V}$$

16.39

$$\text{a. } v_A = v_B = 5 \text{ V}$$

$$N_1 \text{ and } N_2 \text{ on, so } v_{DS1} \approx v_{DS2} \approx 0 \text{ V}$$

$$P_1 \text{ and } P_2 \text{ off}$$

So we have a $P_3 - N_1$ CMOS inverter. By symmetry, $v_C = 2.5 \text{ V}$ (Transition Point).

$$\text{b. For } v_A = v_B = v_C \equiv v_i$$

$$\text{Want } K_{n,eff} = K_{p,eff}$$

$$\frac{k'_n}{2} \cdot \left(\frac{W}{3L}\right)_n = \frac{k'_p}{2} \cdot \left(\frac{3W}{L}\right)_p$$

$$\text{With } k'_n = 2k'_p, \text{ then}$$

$$\frac{2}{2} \cdot \frac{1}{3} \cdot \left(\frac{W}{L}\right)_n = \frac{1}{2} \cdot 3 \cdot \left(\frac{W}{L}\right)_p$$

$$\text{Or } \left(\frac{W}{L}\right)_n = \frac{9}{2} \cdot \left(\frac{W}{L}\right)_p$$

c. We have

$$K_n = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_n = \left(\frac{2k'_p}{2}\right) \left(\frac{9}{2}\right) \left(\frac{W}{L}\right)_p$$

$$K_p = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_p$$

Then from Equation (16.55)

$$V_n = \frac{5 + (-0.8) + \sqrt{\frac{K_n}{K_p} \cdot (0.8)}}{1 + \sqrt{\frac{K_n}{K_p}}}$$

Now

$$\frac{K_n}{K_p} = (2) \left(\frac{9}{2}\right) = 9$$

Then

$$V_n = \frac{5 + (-0.8) + 3(0.8)}{1+3} \Rightarrow V_n = 1.65V$$

16.40

By definition, NMOS is on if gate voltage is 5 V and is off if gate voltage is 0 V.

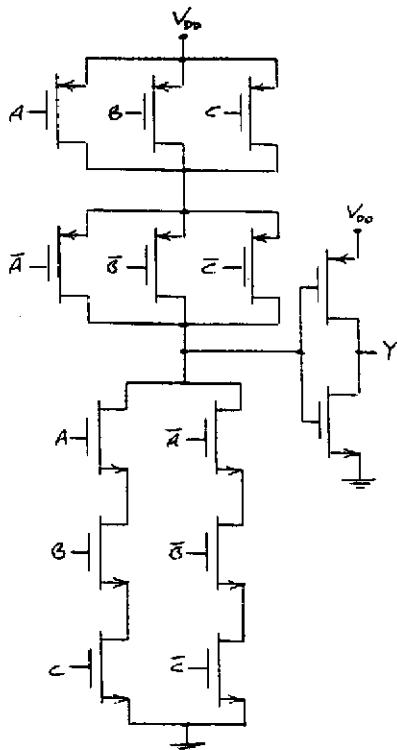
| State | N_1 | N_2 | N_3 | N_4 | N_5 | v_0 |
|-------|-------|-------|-------|-------|-------|-------|
| 1 | off | on | off | on | on | 0 |
| 2 | off | off | on | on | off | 0 |
| 3 | on | on | off | off | on | 5 |
| 4 | on | on | off | on | on | 0 |

Logic function $(v_x \text{ OR } v_y) \otimes (v_x \text{ AND } v_z)$

Exclusive OR of $(v_x \text{ OR } v_y)$ with $(v_x \text{ AND } v_z)$

16.41

(a) A classic design is shown:



$\bar{A}, \bar{B}, \bar{C}$ signals supplied through inverters.

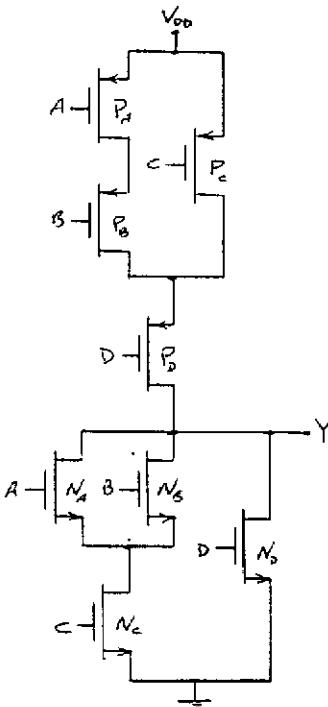
(b) For Inverters, $\left(\frac{W}{L}\right)_n = 1$ and $\left(\frac{W}{L}\right)_p = 2$

For PMOS in Logic function, let $\left(\frac{W}{L}\right)_p = 1$, then

for NMOS in Logic function, $\left(\frac{W}{L}\right)_n = 2.25$

16.42

(a) A classic design is shown:



$$(b) \left(\frac{W}{L}\right)_{ND} = 1, \left(\frac{W}{L}\right)_{NA,NB,NC} = 2$$

$$\left(\frac{W}{L}\right)_{PA,PB} = 8, \left(\frac{W}{L}\right)_{PC,PD} = 4$$

16.43

$$\overline{(A \text{ OR } B) \text{ AND } C}$$

16.44

Let $\left(\frac{W}{L}\right)_p = 1$ for each PMOS: Composite PMOS

$\left(\frac{W}{L}\right)_p = 5$. Want composite $\left(\frac{W}{L}\right)_p = 2.5$ for NMOS,

So that $\left(\frac{W}{L}\right)_n = 5(2.5) = 12.5$ for each NMOS.

16.45

(a) Let $\left(\frac{W}{L}\right)_n = 1$ for each NMOS. Composite $\left(\frac{W}{L}\right)$

of NMOS = 6. Want composite $\left(\frac{W}{L}\right)$ of PMOS =

12. Then $\left(\frac{W}{L}\right)_p = 6(12) = 72$ for each PMOS. Let

$\left(\frac{W}{L}\right)_n = 1$ and $\left(\frac{W}{L}\right)_p = 2$ for each transistor in inverter.

(a) For 3-input NOR:

Let $\left(\frac{W}{L}\right)_n = 1$ for each NMOS. Composite $\left(\frac{W}{L}\right)$ of

NMOS = 3. Want composite $\left(\frac{W}{L}\right)$ of PMOS = 6.

Then $\left(\frac{W}{L}\right)_p = 3(6) = 18$ for each PMOS.

For 2-input NAND:

Let $\left(\frac{W}{L}\right)_p = 1$ for each PMOS. Composite $\left(\frac{W}{L}\right)$ of

PMOS = 2. Want composite $\left(\frac{W}{L}\right)$ of NMOS = 1.

Then $\left(\frac{W}{L}\right)_n = 2$ for each NMOS.

Sizes of PMOS transistors in (b) are substantially less than those in (a).

16.46

By definition:

NMOS off if gate voltage = 0

NMOS on if gate voltage = 5 V

PMOS off if gate voltage = 5 V

PMOS on if gate voltage = 0

| State | N_1 | P_1 | N_A | N_B | N_C | v_{o1} | N_2 | P_2 | v_{o2} |
|-------|-------|-------|-------|-------|-------|----------|-------|-------|----------|
| 1 | off | on | off | off | off | 5 | on | off | 0 |
| 2 | on | off | on | off | off | 5 | on | off | 0 |
| 3 | off | on | off | off | off | 5 | on | off | 0 |
| 4 | on | off | off | off | on | 5 | on | off | 0 |
| 5 | off | on | off | off | off | 5 | on | off | 0 |
| 6 | on | off | off | on | on | 0 | off | on | 5 |

Logic function is

$$v_{o2} = (v_A \text{ OR } v_B) \text{ AND } v_C$$

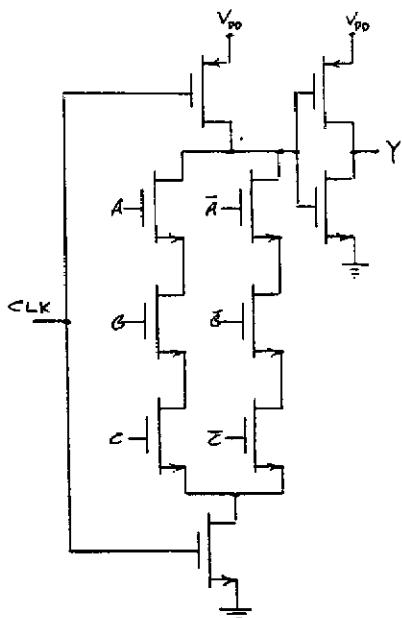
16.47

| State | v_{o1} | v_{o2} | v_{o3} |
|-------|----------|----------|----------|
| 1 | 5 | 5 | 0 |
| 2 | 0 | 0 | 5 |
| 3 | 5 | 5 | 0 |
| 4 | 5 | 0 | 5 |
| 5 | 5 | 5 | 0 |
| 6 | 0 | 5 | 0 |

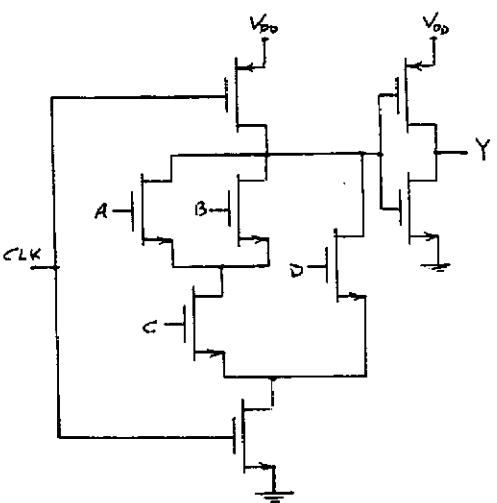
Logic function:

$$v_{o3} = (v_X \text{ OR } v_Z) \text{ AND } v_Y$$

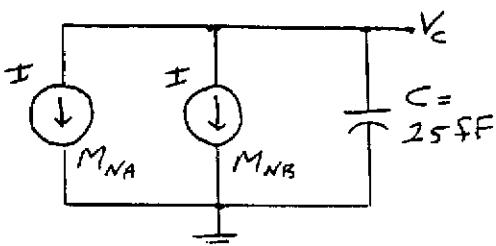
16.48



16.49



16.50



$$2I = -C \frac{dV_C}{dt}$$

So

$$\Delta V_C = -\frac{1}{C} (2I) \cdot t$$

 For $\Delta V_C = -0.5V$

$$-0.5 = -\frac{2(2 \times 10^{-12}) \cdot t}{25 \times 10^{-15}} \Rightarrow t = 3.125 \text{ ms}$$

16.51

(a)

$$(i) v_o = 0$$

$$(ii) v_o = 4.2V$$

$$(iii) v_o = 2.5V$$

(b)

$$(i) v_o = 0$$

$$(ii) v_o = 3.2V$$

$$(iii) v_o = 2.5V$$

16.52

Neglect the body effect.

$$a. v_{o1}(\text{logic 1}) = 4.2V, v_{o2}(\text{logic 1}) = 5V$$

$$b. v_I = 5V \Rightarrow v_{GS1} = 4.2V$$

M_1 in nonsaturation and M_2 in saturation. From Equation (16.23)

$$\begin{aligned} \left(\frac{W}{L}\right)_D [2(v_{GS1} - V_{THD})v_{o1} - v_{o1}^2] \\ = \left(\frac{W}{L}\right)_L (V_{DD} - v_{o1} - V_{THL}) \\ \left(\frac{W}{L}\right)_D [2(4.2 - 0.8)(0.1) - (0.1)^2] \\ = (1)[5 - 0.1 - 0.8]^2 \end{aligned}$$

Or

$$\left(\frac{W}{L}\right)_D (0.67) = 16.81 \Rightarrow \underline{\left(\frac{W}{L}\right)_D = 25.1}$$

Now

$$v_{o1} = 4.2V \Rightarrow v_{GS3} = 4.2V$$

M_3 in nonsaturation and M_4 in saturation. From Equation (16.29(b)).

$$\left(\frac{W}{L}\right)_D [2(v_{GS3} - V_{THD})v_{o2} - v_{o2}^2] = \left(\frac{W}{L}\right)_L [-V_{THL}]^2$$

$$\left(\frac{W}{L}\right)_D [2(4.2 - 0.8)(0.1) - (0.1)^2] = (2)[-(-1.5)]^2$$

$$\left(\frac{W}{L}\right)_D (0.67) = 2.25$$

$$\text{Or } \underline{\left(\frac{W}{L}\right)_D = 3.36}$$

16.53

For $\phi = 1, \bar{\phi} = 0$, then $Y = B$. And for $\phi = 0, \bar{\phi} = 1$, then $Y = A$.

A multiplexer.

16.54

| A | B | Y |
|---|---|------|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0, 1 |

⇒ indeterminate

Without the top transistor, the circuit performs the exclusive-NOR function.

16.55

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 0 |

Exclusive-OR function.

16.56

This circuit is referred to as a radioless circuit. Identical minimum-sized transistors can be used throughout.

When ϕ_1 is low, C_3 is charged to V_{DD} . Then when ϕ_1 is high and ϕ_2 is low, M_6 turns on. If $A = B = 0$, then M_3 and M_4 are off so C_1 remains charged and $v_{o1} = \text{high}$. When ϕ_2 goes high, then v_{o1} is applied to the gates of M_9 and M_{10} . The circuit performs the OR logic function.

16.57

This circuit is referred to as a two-phase ratioed circuit. The same width-to-length ratios between the driver and load transistors must be maintained as discussed previously with the enhancement load inverter.

When ϕ_1 is high, v_{o1} becomes the complement of v_I . When ϕ_2 goes high, then v_o becomes the complement of v_{o1} or is the same as v_I . The circuit is a shift register.

16.58

Let $Q = 0$ and $\bar{Q} = 1$; as S increases, \bar{Q} decreases.

When \bar{Q} reaches the transition point of the $M_3 - M_4$ inverter, the flip-flop will change state.

From Equation (16.28(b)),

$$V_n = \sqrt{\frac{K_L}{K_D}} \cdot (-V_{TNL}) + V_{TND}$$

where $K_L = K_6$ and $K_D = K_5$.

Then

$$V_n = \sqrt{\frac{30}{100}} \cdot [-(2)] + 1 \Rightarrow V_n = \bar{Q} = 2.095 V$$

This is the region where both M_1 and M_3 are biased in the saturation region. Then

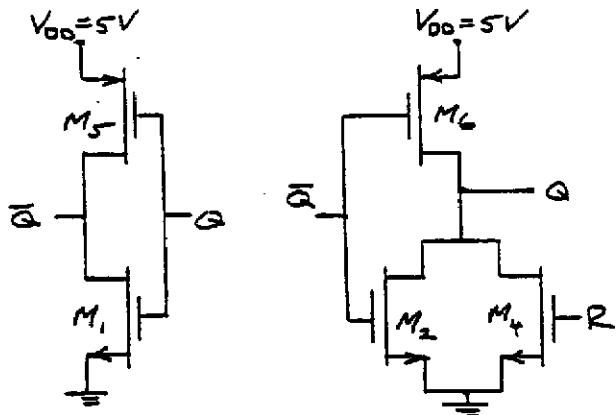
$$S = \sqrt{\frac{K_3}{K_1}} \cdot (-V_{TNL}) + V_{TND} = \sqrt{\frac{30}{200}} \cdot [-(2)] + 1$$

or $S = 1.77 V$

This analysis neglects the effect of M_2 starting to turn on at the same time.

16.59

Let $v_Y = R$, $v_X = S$, $v_{D1} = Q$, and $v_{D2} = \bar{Q}$. Assume $V_{THN} = 0.5 V$ and $V_{THP} = -0.5 V$. For $S = 0$, we have the following:



If we want the switching to occur for $R = 2.5 V$, then because of the nonsymmetry between the two circuits, we cannot have Q and \bar{Q} both equal to 2.5 V.

Set $R = Q = 2.5 V$ and assume \bar{Q} goes low.

For the $M_1 - M_3$ inverter, M_1 is nonsaturation and M_3 is saturation. Then

$$K_p[2(2.5 - 0.5)\bar{Q} - \bar{Q}^2] = K_s[2.5 - 0.5]^2$$

Or

$$4\bar{Q} - \bar{Q}^2 = 4 \left(\frac{K_p}{K_s} \right)$$

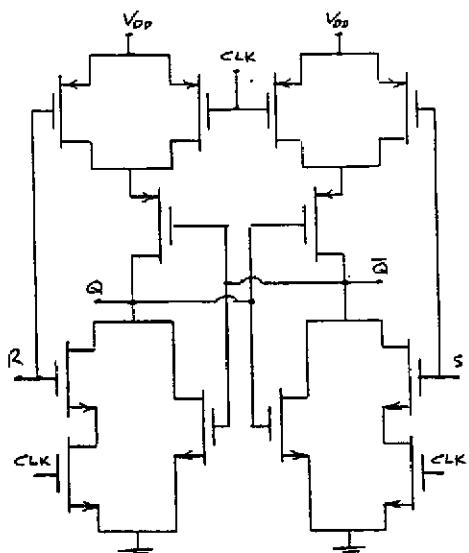
For the other circuit, $M_2 - M_4$ in saturation and M_6 in nonsaturation. Then

$$\begin{aligned} K_p(2.5 - 0.5)^2 + K_s(\bar{Q} - 0.5)^2 \\ = K_p[2(5 - \bar{Q} - 0.5)(2.5) - (2.5^2)] \end{aligned}$$

Combining these equations and neglecting the \bar{Q}^3 term, we find

$$\underline{\bar{Q} = 1.4 V} \text{ and } \underline{\frac{K_p}{K_s} = 0.9}$$

16.60



16.61

- a. Positive edge triggered flip-flop when $CLK = 1$, output of first inverter is D and then $Q = \bar{D} = D$.
- b. For example, put a CMOS transmission gate between the output and the gate of M_1 driven by a \overline{CLK} pulse.

16.62

For $J = 1$, $K = 0$, and $\text{CLK} = 1$; this makes $Q = 1$ and $\bar{Q} = 0$.

For $J = 0$, $K = 1$, and $\text{CLK} = 1$, and if $Q = 1$, then the circuit is driven so that $Q = 0$ and $\bar{Q} = 1$.

If initially, $Q = 0$, then the circuit is driven so that there is no change and $Q = 0$ and $\bar{Q} = 1$.

$J = 1$, $K = 1$, and $\text{CLK} = 1$, and if $Q = 1$, then the circuit is driven so that $Q = 0$.

If initially, $Q = 0$, then the circuit is driven so that $Q = 1$.

So if $J = K = 1$, the output changes state.

16.63

For $J = \nu_X = 1$, $K = \nu_Y = 0$, and $\text{CLK} = \nu_Z = 1$, then $\nu_0 = 0$.

For $J = \nu_X = 0$, $K = \nu_Y = 1$, and $\text{CLK} = \nu_Z = 1$, then $\nu_0 = 1$.

Now consider $J = K = \text{CLK} = 1$. With $\nu_X = \nu_Z = 1$, the output is always $\nu_0 = 0$. So the output does not change state when $J = K = \text{CLK} = 1$. This is not actually a $J - K$ flip-flop.

16.64

$64 K \Rightarrow 65,536$ transistors arranged in a 256×256 array.

(a) Each column and row decoder required 8 inputs.

(b)

(i) Address = 01011110 so input
 $= a_7\bar{a}_6a_5\bar{a}_4\bar{a}_3\bar{a}_2\bar{a}_1a_0$

(ii) Address = 11101111 so input
 $= \bar{a}_7\bar{a}_6\bar{a}_5a_4\bar{a}_3\bar{a}_2\bar{a}_1\bar{a}_0$

(c)

(i) Address = 00100111 so input
 $= a_7a_6\bar{a}_5a_4a_3\bar{a}_2\bar{a}_1\bar{a}_0$

(ii) Address = 01111011 so input
 $= a_7\bar{a}_6\bar{a}_5\bar{a}_4\bar{a}_3a_2\bar{a}_1\bar{a}_0$

16.65

Put 128 words in a 8×16 array, which means 8 row (or column) address lines and 16 column (or row) address lines.

16.66

Assume the address line is initially uncharged, then

$$I = C \frac{dV_c}{dt} \text{ or } V_c = \frac{1}{C} \int I dt = \frac{I}{C} \cdot t$$

$$\text{Then } t = \frac{V_c \cdot C}{I} = \frac{(2.7)(5.8 \times 10^{-12})}{250 \times 10^{-6}} \Rightarrow$$

$$t = 6.26 \times 10^{-8} \text{ s} \Rightarrow 62.6 \text{ ns}$$

16.67

$$(a) \frac{S-0.1}{1} = \left(\frac{35}{2} \right) \left(\frac{W}{L} \right) [2(5-0.7)(0.1)-(0.1)^2]$$

$$\text{or } \left(\frac{W}{L} \right) = 0.329$$

(b) $16 K \Rightarrow 16,384$ cells

$$i_D \equiv \frac{2}{1} = 2 \mu A$$

$$\text{Power per cell} = (2 \mu A)(2 V) = 4 \mu W$$

$$\text{Total Power} = P_T = (4 \mu W)(16,384) \Rightarrow$$

$$P_T = 65.5 \text{ mW}$$

$$\text{Standby current} = (2 \mu A)(16,384) \Rightarrow I_T = 32.8 \text{ mA}$$

16.68

$16 K \Rightarrow 16,384$ cells

$$P_T = 200 \text{ mW} \Rightarrow \text{Power per cell}$$

$$= \frac{200}{16,384} \Rightarrow 12.2 \mu W$$

$$i_D = \frac{P}{V_{DD}} = \frac{12.2}{2.5} = 4.88 \mu A \equiv \frac{V_{DD}}{R} = \frac{2.5}{R} \Rightarrow$$

$$R = 0.512 M\Omega$$

If we want $v_o = 0.1 V$ for a logic 0, then

$$i_D = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) [2(V_{DD} - V_{TN})v_o - v_o^2]$$

$$4.88 = \left(\frac{35}{2} \right) \left(\frac{W}{L} \right) [2(2.5 - 0.7)(0.1) - (0.1)^2]$$

$$\text{So } \left(\frac{W}{L} \right) = 0.797$$

16.69

$$Q = 0, \bar{Q} = 1$$

$$\text{So } \overline{D} = \text{Logic 1} = 5V$$

A very short time after the row has been addressed, D remains charged at $V_{DD} = 5V$. Then M_{P3} , M_A , and M_{N1} begin to conduct and D decreases. In steady-state, all three transistors are biased in the nonsaturation region. Then

$$K_{P3} [2(V_{GS3} + V_{TP3})V_{SD3} - V_{SD3}^2]$$

$$= K_{N1} [2(V_{GS1} - V_{TN1})V_{DS1} - V_{DS1}^2]$$

$$= K_{N1} [2(V_{GS1} - V_{TN1})V_{DS1} - V_{DS1}^2]$$

$$K_{P3} [2(V_{DD} + V_{TP3})(V_{DD} - D) - (V_{DD} - D)^2]$$

$$= K_{N1} [2(V_{DD} - Q - V_{TN1})(D - Q) - (D - Q)^2]$$

$$= K_{N1} [2(V_{DD} - V_{TN1})Q - Q^2] \quad (1)$$

Equating the first and third terms:

$$\begin{aligned} \left(\frac{20}{2}\right)(1)[2(5-0.8)(5-D)-(5-D)^2] \\ = \left(\frac{40}{2}\right)(2)[2(5-0.8)Q-Q^2] \end{aligned} \quad (2)$$

As a first approximation, neglect the $(5-D)^2$ and Q^2 terms. We find

$$Q = 1.25 - 0.25D \quad (3)$$

Then, equating the first and second terms of Equation (1):

$$\begin{aligned} \left(\frac{20}{2}\right)(1)[2(5-0.8)(5-D)-(5-D)^2] \\ = \left(\frac{40}{2}\right)(1)[2(5-Q-0.8)(D-Q)-(D-Q)^2] \end{aligned}$$

Substituting Equation (3), we find as a first approximation: $D = 2.14 V$

Substituting this value of D into equation (2), we find

$$8.4(5-2.14)-(5-2.14)^2 = 4[8.4Q-Q^2]$$

We find $\underline{Q = 0.50 V}$

Using this value of Q , we can find a second approximation for D by equating the second and third terms of equation (1). We have

$$\begin{aligned} 20[2(4.2-Q)(D-Q)-(D-Q)^2] \\ = 40[2(4.2Q)-Q^2] \end{aligned}$$

Using $Q = 0.50 V$, we find $\underline{D = 1.79 V}$

16.70

Initially M_{N1} and M_A turn on.

M_{N1} , Nonsat; M_A , sat.

$$\begin{aligned} K_{n1}[V_{DD} - Q - V_{TN1}]^2 &= K_{n1}[2(V_{DD} - V_{TN1})Q - Q^2] \\ \left(\frac{40}{2}\right)(1)[5-Q-0.8]^2 &= \left(\frac{40}{2}\right)(2)[2(5-0.8)Q-Q^2] \end{aligned}$$

which yields

$$\underline{Q = 0.771 V}$$

Initially M_{P2} and M_B turn on

Both biased in nonsaturation region

$$\begin{aligned} K_{p2}[2(V_{DD} + V_{TP2})(V_{DD} - \bar{Q}) - (V_{DD} - \bar{Q})^2] \\ = K_{p2}[2(V_{DD} - V_{TP2})\bar{Q} - \bar{Q}^2] \\ \left(\frac{20}{2}\right)(4)[2(5-0.8)(5-\bar{Q})-(5-\bar{Q})^2] \\ = \left(\frac{40}{2}\right)(1)[2(5-0.8)\bar{Q}-\bar{Q}^2] \end{aligned}$$

which yields $\underline{\bar{Q} = 3.78 V}$

Note: (W/L) ratios do not satisfy Equation (16.95)

16.71

For Logic 1, v_1 :

$$(5)(0.05) + (4)(1) = (1+0.05)v_1 \Rightarrow \underline{v_1 = 4.0476 V}$$

v_2 :

$$(5)(0.025) + (4)(1) = (1+1.025)v_2 \Rightarrow \underline{v_2 = 4.0244 V}$$

For Logic 0, v_1 :

$$(0)(0.05) + (4)(1) = (1+0.05)v_1 \Rightarrow \underline{v_1 = 3.8095 V}$$

v_2 :

$$(0)(0.025) + (4)(1) = (1+0.025)v_2 \Rightarrow \underline{v_2 = 3.9024 V}$$

Chapter 17

Exercise Solutions

E17.1

a. $i_E = \frac{-0.7 - (-5)}{R_E} = 1 \text{ mA} \Rightarrow R_E = 4.3 \text{ k}\Omega$
 $i_{C1} = i_{C2} = 0.5 \text{ mA} = \frac{5 - 3.5}{R_{C1}}$
 $\Rightarrow R_{C1} = R_{C2} = 3 \text{ k}\Omega$

b. i. $v_1 = 1 \text{ V}$

$i_E = \frac{(1 - 0.7) - (-5)}{4.3} \Rightarrow i_E = 1.23 \text{ mA}$
 $i_{C1} = i_E = v_{o1} = 5 - (1.23)(3)$
 $\Rightarrow v_{o1} = 1.31 \text{ V}$
 $v_{o2} = 5 \text{ V}$

ii. $v_f = -1 \text{ V}$

$i_E = 1 \text{ mA} \Rightarrow v_{o2} = 5 - (1)(3)$
 $\Rightarrow v_{o2} = 2 \text{ V}$
 $v_{o1} = 5 \text{ V}$

E17.2

a. Q_R on

$i_E = \frac{1.5 - 0.7 - (-3.5)}{R_E} = 2 \text{ mA}$
 $\Rightarrow R_E = 2.15 \text{ k}\Omega$
 $i_{CR} \approx i_E = 2 \text{ mA} = \frac{3.5 - 2}{R_{C2}}$
 $\Rightarrow R_{C2} = 0.75 \text{ k}\Omega$

b. $v_X = v_Y = 2 \text{ V} \Rightarrow Q_1$ and Q_2 on

$i_E = \frac{2 - 0.7 - (-3.5)}{R_E} = \frac{4.8}{2.15} \Rightarrow i_E = 2.23 \text{ mA}$
 $R_{C1} = \frac{3.5 - 2}{i_{CXY}} = \frac{1.5}{2.23} \Rightarrow R_{C1} = 0.673 \text{ k}\Omega$

E17.3

logic 1 = -0.7 V

 Q_1 and Q_2 on when $v_X = v_Y = -0.7 \text{ V}$

$i_E = \frac{-0.7 - 0.7 - (-5.2)}{R_E} = 2.5$
 $\Rightarrow R_E = 1.52 \text{ k}\Omega$

$v_{NOR} = -1.5 \Rightarrow R_{C1} = \frac{0 - (-1.5 + 0.7)}{2.5}$
 $\Rightarrow R_{C1} = 320 \Omega$

$V_R = \frac{-1.5 - 0.7}{2} \Rightarrow V_R = -1.1 \text{ V}$

$Q_R \text{ on} \Rightarrow i_E = \frac{-1.1 - 0.7 - (-5.2)}{1.52} = 2.24 \text{ mA}$
 $R_{C2} = \frac{0 - (-1.5 + 0.7)}{2.24} \Rightarrow R_{C2} = 357 \Omega$
 $R_3 = R_4 = \frac{-0.7 - (-5.2)}{2.5} \Rightarrow R_3 = R_4 = 1.8 \text{ k}\Omega$

E17.4

$P(i_{CXY} + i_{CR} + i_3 + i_4)(5.2)$

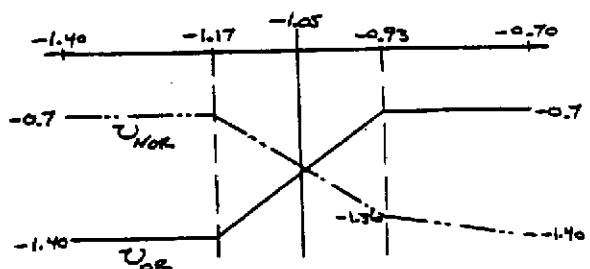
a. $v_X = v_Y = \text{logic 1}$
 $\Rightarrow i_{CXY} = 3.22 \text{ mA}$
 $i_{CR} = 0$
 $i_3 = \frac{-0.7 + 5.2}{1.5} = 3 \text{ mA}$
 $i_4 = \frac{-1.4 + 5.2}{1.5} = 2.53 \text{ mA}$

$P = (3.22 + 0 + 3 + 2.53)(5.2)$
 $\Rightarrow P = 45.5 \text{ mW}$

b. $v_X = v_Y = \text{logic 0}$
 $\Rightarrow i_{CXY} = 0$
 $i_{CR} = 2.92 \text{ mA}$
 $i_3 = 2.53 \text{ mA}$
 $i_4 = 3 \text{ mA}$

$P = (0 + 2.92 + 2.53 + 3)(5.2)$
 $\Rightarrow P = 43.9 \text{ mW}$

E17.5



$NM_H = -0.70 - (-0.93) \Rightarrow NM_H = 0.23 \text{ V}$

$NM_L = -1.17 - (-1.40) \Rightarrow NM_L = 0.23 \text{ V}$

E17.6

$$\begin{aligned}
 P &= I_Q \cdot V_{CC} \Rightarrow 0.2 = I_Q(1.7) \\
 \Rightarrow I_Q &= 118 \mu A \\
 Q_R \text{ on} \Rightarrow v_0 &= 1.7 - I_Q R_C = 1.7 - 0.4 \\
 \Rightarrow R_C &= \frac{0.4}{0.118} \Rightarrow R_C = 3.39 \text{ k}\Omega \\
 V_R &= \frac{1.7 + 1.3}{2} \Rightarrow V_R = 1.5 \text{ V}
 \end{aligned}$$

E17.7

| State | A | B | C | Q_{01} | Q_{02} | Q_{03} | Q_1 | Q_2 | Q_R | v_o |
|-------|---|---|---|----------|----------|----------|-------|-------|-------|-------|
| 1 | 0 | 0 | 0 | off | off | off | on | on | on | 0 |
| 2 | 1 | 0 | 0 | "on" | off | off | off | on | on | 0 |
| 3 | 0 | 1 | 0 | off | on | off | off | on | "off" | 1 |
| 4 | 0 | 0 | 1 | off | off | on | on | off | on | 0 |
| 5 | 1 | 1 | 0 | on | on | off | off | on | off | 1 |
| 6 | 1 | 0 | 1 | on | off | on | on | off | "off" | 1 |
| 7 | 0 | 1 | 1 | off | on | on | on | off | on | 0 |
| 8 | 1 | 1 | 1 | on | on | on | on | off | off | 1 |

$\underbrace{(A \text{ AND } C)}_{\text{true for states 6 and 8}} \text{ OR } \underbrace{(B \text{ AND } \bar{C})}_{\text{true for states 3 and 5}}$
 Output goes high for these 4 states

E17.8

| A | B | C | v_o |
|---|---|---|-------|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 |

$$\Rightarrow (A \oplus B) \oplus C$$

E17.9

$$\begin{aligned}
 i_C &= \frac{5 - 0.1}{1} = 4.9 \text{ mA} \\
 i_B &= \frac{4.9}{30} = 0.163 \text{ mA} = \frac{v_T - 0.7}{R_B} = \frac{4.3}{R_B} \\
 \Rightarrow R_B &= 26.4 \text{ k}\Omega
 \end{aligned}$$

E17.10

$$\begin{aligned}
 (a) \quad v_x &= v_T = 5V \\
 v_1 &= V_{BE}(\text{sat}) + 2V_T = 0.8 + 2(0.7) = 2.2V
 \end{aligned}$$

$$i_1 = \frac{5 - 2.2}{4} = 0.70 \text{ mA}$$

$$i_{RC} = \frac{V_{CC} - V_{CE}(\text{sat})}{R_C} = \frac{5 - 0.1}{4} = 1.23 \text{ mA}$$

$$P = (i_1 + i_{RC})V_{CC} = (0.70 + 1.23)(5)$$

$$\text{or } P = 9.65 \text{ mW}$$

$$(b) \quad v_x = v_T = 0 \Rightarrow v_1 = 0.70V$$

$$i_1 = \frac{V_{CC} - v_1}{R_1} = \frac{5 - 0.70}{4} = 1.08 \text{ mA}$$

$$P = i_1 \cdot V_{CC} = (1.08)(5) \Rightarrow P = 5.4 \text{ mW}$$

E17.11

$$a. \quad \text{Inputs high } v_1 = 0.8 + 0.7 + 0.8 = 2.3 \text{ V}$$

$$i_1 = i_{B1} = \frac{5 - 2.3}{4} = 0.675 \text{ mA}$$

$$v_{C1} = 0.8 + 0.7 + 0.7 = 1.6 \text{ V}$$

$$i_2 = \frac{5 - 1.6}{2} = 1.7 \text{ mA}$$

$$i_3 = 1.7 + 0.675 = 2.38 \text{ mA}$$

$$i_4 = \frac{0.8}{10} = 0.08 \text{ mA}$$

$$i_{B0} = 2.38 - 0.08 = 2.3 \text{ mA}$$

$$i_{RC} = \frac{5 - 0.1}{4} = 1.23 \text{ mA}$$

$$i'_L = \frac{5 - 0.8}{4} = 1.05 \text{ mA}$$

$$\beta i_{B0} = N i'_L + i_{RC}$$

$$(30)(2.3) = N(1.05) + 1.23 \Rightarrow N = 64$$

$$b. \quad I_{C,\text{rated}} = 20 \text{ mA}$$

$$20 = N i'_L + i_{RC} = N(1.05) + 1.23$$

$$\Rightarrow N = 17$$

Test Your Understanding

Chapter 17: Exercise Solutions

E17.12

a. $v_X = v_Y = 0.1 \Rightarrow v_1 = 0.8$

$$i_1 = \frac{5 - 0.8}{6} \Rightarrow i_1 = 0.7 \text{ mA}$$

$$i_2 = i_R = i_B = i_{RC} = 0$$

$$v_0 = 5 \text{ V}$$

b. Same as part (a).

c. $v_X = v_Y = 5 \text{ V} \Rightarrow v_1 = 0.8 + 0.7 + 0.7 = 2.2 \text{ V}$

$$i_1 = i_2 = \frac{5 - 2.2}{6} \Rightarrow i_1 = i_2 = 0.467 \text{ mA}$$

$$i_R = \frac{0.8}{15} \Rightarrow i_R = 0.053 \text{ mA}$$

$$i_B = i_2 - i_R \Rightarrow i_B = 0.414 \text{ mA}$$

$$i_{RC} = \frac{5 - 0.1}{5} \Rightarrow i_{RC} = 0.98 \text{ mA}$$

$$v_0 = 0.1 \text{ V}$$

$$i_{BX} = i_{BY} = i_1 \cdot \beta_R = (0.45)(0.1) = 0.045 \text{ mA}$$

$$i_{B2} = |i_{C1}| = i_1 + i_{BX} + i_{BY} = 0.45 + 2(0.045)$$

$$i_{B2} = |i_{C1}| = 0.54 \text{ mA}$$

$$v_{C2} = 0.8 + 0.1 = 0.9$$

$$i_2 = i_{C2} = \frac{5 - 0.9}{1.5} \Rightarrow i_2 = i_{C2} = 2.73 \text{ mA}$$

$$i_{B2} = 2.73 + 0.54 = 3.27 \text{ mA}$$

$$i_4 = \frac{0.8}{1.5} = 0.533 \text{ mA}$$

$$i_{B0} = 3.27 - 0.533 \Rightarrow i_{B0} = 2.74 \text{ mA}$$

$$i_3 = i_{C0} = \frac{5 - 0.1}{2.2} \Rightarrow i_3 = i_{C0} = 2.23 \text{ mA}$$

For Q_2 :

$$\frac{i_{C2}}{i_{B2}} = \frac{2.73}{0.54} = 5.06 < \beta_F \Rightarrow Q_2 \text{ in saturation}$$

For Q_0 :

$$\frac{i_{C0}}{i_{B0}} = \frac{2.23}{2.74} = 0.81 < \beta_F \Rightarrow Q_0 \text{ in saturation}$$

E17.13

a. $v_X = v_Y = 0.1 \text{ V}$

$$i_1 = \frac{5 - 0.8}{8} = 0.525 \text{ mA}$$

$$P = i_1(5 - 0.1) = (0.525)(4.9) \Rightarrow P = 2.57 \text{ mW}$$

b. $v_X = v_Y = 5 \text{ V}, v_1 = 2.3 \text{ V}$

$$i_1 = \frac{5 - 2.3}{8} = 0.338 \text{ mA}$$

$$v_{C1} = 1.6 \text{ V} \Rightarrow i_2 = \frac{5 - 1.6}{3.6} = 0.944 \text{ mA}$$

$$i_{RC} = \frac{5 - 0.1}{6} = 0.817 \text{ mA}$$

$$P = (i_1 + i_2 + i_{RC})(V_{CC})$$

$$= (0.338 + 0.944 + 0.817)(5)$$

$$P = 10.3 \text{ mW}$$

E17.15

a. Output low:

$$i'_L = \frac{5 - (0.1 + 0.8)}{6} = 0.683$$

$$i_{CO(\max)} = \beta_F \cdot i_{B0} = N i'_L + i_3$$

$$(20)(2.74) = N(0.683) + 2.23$$

$$\Rightarrow N = 76$$

b. Output high:

Q'_1 inverse active

$$i'_1 = \frac{5 - 2.3}{6} = 0.45 \text{ mA}$$

$$i'_L = \beta_R \cdot i'_1 = (0.1)(0.45) = 0.045 \text{ mA}$$

$$v_0(\min) = 2.4 \text{ V} \Rightarrow i_L(\max) = \frac{5 - 2.4}{2.2} = N \cdot i'_L$$

$$1.18 = N(0.045)$$

$$\Rightarrow N = 26$$

E17.14

a. $v_X = v_Y = 0.1 \text{ V}$

$$v_{B1} = 0.1 + 0.8 = 0.9 \text{ V}$$

$$i_{B1} = i_1 = \frac{5 - 0.9}{6} \Rightarrow i_1 = i_{B1} = 0.663 \text{ mA}$$

$$i_{C1} \approx 0$$

$$i_{B2} = i_{C2} = 0$$

$$i_{B0} = i_{C0} = 0$$

b. $v_X = v_Y = 3.6 \text{ V}$

$$v_{B1} = 0.8 + 0.8 + 0.7 = 2.3 \text{ V}$$

$$i_1 = i_{B1} = \frac{5 - 2.3}{6} \Rightarrow i_1 = i_{B1} = 0.45 \text{ mA}$$

E17.16

a. $v_X = v_Y = 0.1 \text{ V} \Rightarrow v_0(\max) = 3.6 \text{ V}$

Q'_1 inverse active

$$i'_1 = \frac{5 - 2.3}{6} = 0.45 \text{ mA}$$

$$i'_L = \beta_R \cdot i'_1 = (0.1)(0.45) = 0.045$$

$$i_{B3} = \frac{0.1}{2} = 0.05 \text{ mA}$$

$$i_L = (1 + \beta_F)i_{B3} = (21)(0.05) = 1.05 \text{ mA}$$

$$i_L = N i'_L \Rightarrow 1.05 = N(0.045) \Rightarrow N = 23$$

b. $v_X = v_Y = 3.6 \text{ V}$, Output low:

$$i_1 = \frac{5 - 2.3}{6} = 0.45 \text{ mA}$$

$$i_{B2} = (1 + 2\beta_R)i_1$$

$$i_{B2} = 0.54 \text{ mA}$$

$$v_{B3} = 0.8 + 0.1 = 0.9$$

$$i_2 = \frac{5 - 0.9}{2} = 2.05 \text{ mA}$$

$$i_{B0} = 2.05 + 0.54 - \frac{0.8}{1.5} \Rightarrow i_{B0} = 2.06 \text{ mA}$$

$$i_L' = \frac{5 - 0.9}{6} = 0.683 \text{ mA}$$

$$\beta_F \cdot i_{B0} = N \cdot i_L' \Rightarrow (20)(2.06) = N(0.683)$$

$$\Rightarrow N = 60$$

E17.17

a. $v_X = v_Y = 3.6 \text{ V}$

$$i_{B1} = \frac{5 - 2.3}{4} = 0.675 \text{ mA} = i_{B1} = |i_{C1}| = i_{B2}$$

$$v_{B4} = 0.8 + 0.1 = 0.9 \text{ V}$$

$$i_2 = \frac{5 - 0.9}{1.6} = 2.56 \text{ mA}$$

$$i_{B4} = \frac{0.2}{(1 + \beta_F)(4)} = \frac{0.2}{(31)(4)} \Rightarrow i_{B4} = 1.61 \mu\text{A}$$

$$i_{C4} = 48.3 \mu\text{A}$$

$$i_{B3} = i_{C3} = 0$$

$$\Rightarrow i_{C2} \approx i_2 = 2.56 \text{ mA}$$

$$i_{B0} = 2.45 + 0.675 - \frac{0.8}{1} \Rightarrow i_{B0} = 2.44 \text{ mA}$$

One load:

$$i_L' = i_{C0} = \frac{5 - 0.9}{4} \Rightarrow i_{C0}(\text{max}) = 1.03 \text{ mA}$$

b. $v_X = v_Y = 0.1 \text{ V}$

$$i_{B1} = \frac{5 - 0.9}{4} \Rightarrow i_{B1} = 1.03 \text{ mA}$$

$$|i_{C1}| = i_{B2} = i_{C2} = 0$$

$$i_{B0} = i_{C0} = 0$$

Output high, $\beta_R = 0 \Rightarrow i_{B3} = i_{C3} = 0$

$$5 = i_{B4}(1.6) + 0.7 + (31)i_{B4}(4)$$

$$\Rightarrow i_{B4} = 0.0342 \text{ mA}$$

$$i_{C4} = 1.03 \text{ mA}$$

E17.18

Q_1 in saturation

$$i_{B1} = \frac{5 - 0.9}{6} \Rightarrow i_{B1} = 0.683 \text{ mA}$$

$$|i_{C1}| = i_{B2} = i_{C2} = 0$$

$$i_{B0} = i_{C0} = 0$$

$$v_{B4} = 0.1 + 0.7 = 0.8 \text{ V}$$

$$i_{B4} = \frac{0.1}{(21)(4)} \Rightarrow i_{B4} = 1.19 \mu\text{A}$$

$$i_{C4} = 23.8 \mu\text{A}$$

$$i_{B3} = i_{C3} = 0$$

E17.19

a. $i_{RC} = \frac{5 - 0.4}{2.25} = 2.04 \text{ mA}$

$$i_C' = \frac{2 + 2.04}{1 + \frac{1}{10}} \Rightarrow i_C' = 3.67 \text{ mA}$$

$$i_B' = \frac{3.67}{10} = 0.367 \text{ mA}$$

$$i_D = 2 - 0.367 \Rightarrow i_D = 1.63 \text{ mA}$$

b. $i_D = 0 \Rightarrow i_B' = i_B = 2 \text{ mA}$

$$i_C' = \beta i_B' = (10)(2) = 20 \text{ mA} = i_{RC} + i_L$$

$$i_L = 20 - 2.04 \Rightarrow i_L(\text{max}) \approx 18 \text{ mA}$$

Chapter 17

Problem Solutions

17.1

- a. $v_I = -1.5 \text{ V}$; Q_1 off, Q_2 on

$$i_E = \frac{-0.7 - (-3.5)}{5} \Rightarrow i_E = 0.56 \text{ mA}$$

$$i_{C1} = 0 \Rightarrow v_{O1} = 3.5 \text{ V}$$

$$i_{C2} = i_E \Rightarrow v_{O2} = 3.5 - i_E R_{C2} = 3.5 - (0.56)(2)$$

or

$$v_{O2} = 2.38 \text{ V}$$

- b. $v_I = 1.0 \text{ V}$; Q_1 on, Q_2 off

$$i_E = \frac{(1 - 0.7) - (-3.5)}{5} \Rightarrow i_E = 0.76 \text{ mA}$$

$$i_{C1} = 0 \Rightarrow v_{O1} = 3.5 \text{ V}$$

- c. logic 0 at v_{O2} (low level) = 2.38 V

Then

$$v_{O1} = 2.38 = 3.5 - (0.76)R_{C1}$$

or

$$R_{C1} = 1.47 \text{ k}\Omega$$

17.2

- (a) Q_1 on, $v_E = -1.2 - 0.7 = -1.9 \text{ V}$

$$i_E = i_{C1} = \frac{-1.9 - (-5.2)}{2.5} = 1.32 \text{ mA}$$

$$v_2 = -1 \text{ V} = -i_{C2}R_{C2} = -(1.32)(R_{C2})$$

$$R_{C2} = 0.758 \text{ k}\Omega$$

- (b) Q_1 on, $v_E = -0.7 - 0.7 = -1.40 \text{ V}$

$$i_E = i_{C1} = \frac{-1.4 - (-5.2)}{2.5} = 1.52 \text{ mA}$$

$$v_1 = -1 \text{ V} = -i_{C1}R_{C1} = -(1.52)(R_{C1})$$

$$R_{C1} = 0.658 \text{ k}\Omega$$

- (c) For $v_{in} = -0.7 \text{ V}$, Q_1 on, Q_2 off

$$\Rightarrow v_{O1} = -0.7 \text{ V}$$

$$v_{O2} = -1 - 0.7 \Rightarrow v_{O2} = -1.7 \text{ V}$$

For $v_{in} = -1.7 \text{ V}$, Q_1 off, Q_2 on

$$\Rightarrow v_{O2} = -0.7 \text{ V}$$

$$v_{O1} = -1 - 0.7 \Rightarrow v_{O1} = -1.7 \text{ V}$$

- (d) (i) For $v_{in} = -0.7 \text{ V}$, $i_E = 1.52 \text{ mA}$

$$i_{C4} = \frac{-1.7 - (-5.2)}{3} = 1.17 \text{ mA}$$

$$i_{C3} = \frac{-0.7 - (-5.2)}{3} = 1.5 \text{ mA}$$

$$P = (i_E + i_{C4} + i_{C3})(5.2) = (1.52 + 1.17 + 1.5)(5.2) \\ \text{or } P = 21.8 \text{ mW}$$

- (ii) For $v_{in} = -1.7 \text{ V}$, $i_E = 1.32 \text{ mA}$

$$i_{C4} = \frac{-0.7 - (-5.2)}{3} = 1.5 \text{ mA}$$

$$i_{C3} = \frac{-1.7 - (-5.2)}{3} = 1.17 \text{ mA}$$

$$P = (1.32 + 1.5 + 1.17)(5.2) \\ \text{or } P = 20.7 \text{ mW}$$

17.3

$$a. I_3 = \frac{3.7 - 0.7}{0.67 + 1.33} = 1.5 \text{ mA}$$

$$V_R = I_3 R_4 + V_T = (1.5)(1.33) + 0.7$$

or

$$V_R = 2.70 \text{ V}$$

- b. logic 1 level = $3.7 - 0.7 \Rightarrow 3.0 \text{ V}$

For $v_X = v_Y = \text{logic 1}$,

$$i_E = \frac{3 - 0.7}{0.8} = 2.875 \text{ mA} = i_{RC1}$$

$$v_{B3} = 3.7 - (2.875)(0.21) = 3.10 \text{ V} \\ \Rightarrow v_{O1}(\text{logic 0}) = 2.4 \text{ V}$$

For $v_X = v_Y = \text{logic 0}$, Q_R on

$$i_E = \frac{2.7 - 0.7}{0.8} = 2.5 \text{ mA} = i_{RC2}$$

$$v_{B4} = 3.7 - (2.5)(0.24) = 3.1 \text{ V} \\ \Rightarrow v_{O2}(\text{logic 0}) = 2.4 \text{ V}$$

17.4

$$V_R = \frac{\text{logic 1} + \text{logic 0}}{2} = \frac{1 + 0}{2} = 0.5 \text{ V}$$

For $i_2 = 1 \text{ mA}$

$$R_S = \frac{0.5 - (-2.3)}{1} \Rightarrow R_S = 2.8 \text{ k}\Omega$$

For Q_R on,

$$i_E = \frac{V_R - V_{BS} - (-2.3)}{R_S}$$

or

$$R_E = \frac{0.5 - 0.7 + 2.3}{1} \Rightarrow R_E = 2.1 \text{ k}\Omega$$

$$V_{B2} = V_R + V_{BE2} = 0.5 + 0.7 = 1.2 \text{ V}$$

$$i_1 = \frac{1.2 - 1.4 - (-2.3)}{R_2}$$

or

$$R_2 = \frac{1.2 - 1.4 + 2.3}{1} \Rightarrow R_2 = 2.1 \text{ k}\Omega$$

$$R_1 = \frac{1.7 - 1.2}{1} \Rightarrow R_1 = 0.5 \text{ k}\Omega$$

$$i_3 = \frac{1 - (-2.3)}{R_3} = 3 \Rightarrow R_3 = 1.1 \text{ k}\Omega$$

$$i_4 = \frac{0 - (-2.3)}{R_4} = 3 \Rightarrow R_4 = 0.767 \text{ k}\Omega$$

For Q_R on,

$$v_{oR} = \text{logic 0} = 0 \text{ V} \Rightarrow v_{B3} = 0.7 \text{ V}$$

$$i_E = i_{CR} = 1 \text{ mA}$$

So

$$R_{C2} = \frac{1.7 - 0.7}{1} \Rightarrow R_{C2} = 1 \text{ k}\Omega$$

For $v_I = \text{logic 1} = 1 \text{ V}$,

$$i_E = \frac{1 - 0.7 - (-2.3)}{2.1} = 1.238 \text{ mA}$$

For $v_{NO_R} = \text{logic 0} = 0 \text{ V}$,

$$v_{B4} = 0.7 \text{ V}$$

Then

$$R_{C1} = \frac{1.7 - 0.7}{1.238} \Rightarrow R_{C1} = 0.808 \text{ k}\Omega$$

17.5

Maximum i_E for $v_I = \text{logic 1} = 3.3 \text{ V}$

$$\text{Then } i_E = 5 \text{ mA} = \frac{3.3 - 0.7}{R_E} \Rightarrow R_E = 0.52 \text{ k}\Omega$$

For $v_{O2} = \text{logic 1} = 3.3 \text{ V}$

$$i_{E3} = \frac{3.3 - 0}{R_3} = 5 \text{ mA}$$

or

$$R_3 = 0.66 \text{ k}\Omega$$

By symmetry,

$$R_2 = 0.66 \text{ k}\Omega$$

For Q_1 on,

$$i_E = i_{RC1} = 5 \text{ mA} = \frac{4 - (2.7 + 0.7)}{R_{C1}}$$

So

$$R_{C1} = 0.12 \text{ k}\Omega$$

For Q_R on,

$$i_E = \frac{3 - 0.7}{0.52} = 4.423 \text{ mA} = i_{RC1}$$

$$\text{and } i_{RC2} = 4.423 = \frac{4 - (2.7 + 0.7)}{R_{C2}}$$

$$\Rightarrow R_{C1} = 0.136 \text{ k}\Omega$$

17.6

Neglecting base currents:

$$(a) I_{E1} = 0, I_{E3} = 0$$

$$I_{E5} = \frac{5 - 0.7}{2.5} \Rightarrow I_{E5} = 1.72 \text{ mA}$$

$$Y = 0.7 \text{ V}$$

$$(b) I_{E1} = \frac{5 - 0.7}{18} \Rightarrow I_{E1} = 0.239 \text{ mA}$$

$$I_{E3} = 0$$

$$I_{E5} = \frac{5 - 0.7}{2.5} \Rightarrow I_{E5} = 1.72 \text{ mA}$$

$$Y = 0.7 \text{ V}$$

$$(c) I_{E1} = I_{E3} = \frac{5 - 0.7}{18} \Rightarrow I_{E1} = I_{E3} = 0.239 \text{ mA}$$

$$I_{E5} = 0, Y = 5 \text{ V}$$

(d) Same as (c).

17.7

$$(a) V_R = -(1)(1) - 0.7 \Rightarrow V_R = -1.7 \text{ V}$$

$$(b) Q_R \text{ off, then } v_{o1} = \text{Logic 1} = -0.7 \text{ V}$$

$$Q_R \text{ on, then } v_{o1} = -(1)(2) - 0.7 \Rightarrow$$

$$v_{o1} = \text{Logic 0} = -2.7 \text{ V}$$

$$Q_A / Q_B \text{ off, then } v_{o2} = \text{Logic 1} = -0.7 \text{ V}$$

$$Q_A / Q_B \text{ on, then } v_{o2} = -(1)(2) - 0.7 \Rightarrow$$

$$v_{o2} = \text{Logic 0} = -2.7 \text{ V}$$

$$(c) A = B = \text{Logic 0} = -2.7 \text{ V}, Q_R \text{ on,}$$

$$V_E = -1.7 - 0.7 \Rightarrow V_E = -2.4 \text{ V}$$

$$A = B = \text{Logic 1} = -0.7 \text{ V}, Q_A / Q_B \text{ on,}$$

$$V_E = -0.7 - 0.7 \Rightarrow V_E = -1.4 \text{ V}$$

$$(d) A = B = \text{Logic 1} = -0.7 \text{ V}, Q_A / Q_B \text{ on,}$$

$$i_{C3} = \frac{-2.7 - (-5.2)}{1.5} = 1.67 \text{ mA}$$

$$i_{C2} = \frac{-0.7 - (-5.2)}{1.5} = 3 \text{ mA}$$

$$P = (1.67 + 1 + 1 + 3)(5.2) \Rightarrow P = 39.9 \text{ mW}$$

$$A = B = \text{Logic 0} = -2.7 \text{ V}$$

$$i_{C3} = 3 \text{ mA}, i_{C2} = 1.67 \text{ mA}$$

$$P = 39.9 \text{ mW}$$

17.8

a. AND logic function

b. logic 0 = 0 V

$$Q_3 \text{ on, } i = \frac{5 - (1.6 + 0.7)}{1.2} = 2.25 \text{ mA}$$

$$V_2 = (2.25)(0.8) \Rightarrow \text{logic 1} = 1.8 \text{ V}$$

c. $i_{E1} = \frac{5 - 0.7}{2.6} \Rightarrow i_{E1} = 1.65 \text{ mA}$

$$i_{E2} = \frac{5 - (0.7 + 0.7)}{1.2} \Rightarrow i_{E2} = 3 \text{ mA}$$

$$i_{C1} = 0, \quad i_{C2} = i_{E2} = 3 \text{ mA}$$

$$V_2 = 0$$

d. $i_{E1} = \frac{5 - (1.8 + 0.7)}{2.6} \Rightarrow i_{E1} = 0.962 \text{ mA}$

$$i_{E2} = \frac{5 - (1.6 + 0.7)}{1.2} \Rightarrow i_{E2} = 2.25 \text{ mA}$$

$$i_{C1} = 0, \quad i_{C2} = i_{E2} = 2.25 \text{ mA}$$

$$V_2 = 1.8 \text{ V}$$

17.9

a. $V_R = \frac{3.5 + 3.1}{2} - 0.7 \Rightarrow V_R = 2.6 \text{ V}$

b. For Q_1 on, $v_X = v_Y = \text{logic 1} = 3.5 \text{ V}$

$$i_E = \frac{3.5 - (0.7 + 0.7)}{12} = 0.175 \text{ mA}$$

$$\text{Want } i_{RC1} = \frac{0.175}{2} = \frac{0.4}{R_{C1}} \Rightarrow R_{C1} = 4.57 \text{ k}\Omega$$

c. For Q_2 on, $i_E = \frac{2.6 - 0.7}{12} = 0.158 \text{ mA}$

$$\text{Want } i_{RC2} = \frac{0.158}{2} = \frac{0.4}{R_{C2}} \Rightarrow R_{C2} = 5.06 \text{ k}\Omega$$

d. For $v_Y = \text{logic 1} = 3.5 \text{ V}$

$$i_{R1} = \frac{3.5 - 0.7}{8} = 0.35 \text{ mA}, \quad i_R = 0.175 \text{ mA}$$

$$P = (i_{R1} + i_R)(V_{CC}) = (0.35 + 0.175)(3.5) \Rightarrow P = 1.84 \text{ mW}$$

17.10

a. logic 1 = 0.2 V

logic 0 = -0.2 V

b. $i_E = \frac{(0 - 0.7) - (-3.1)}{R_E} = 0.8$

So

$$R_E = 3 \text{ k}\Omega$$

c. Want $i_{R1} = \frac{0.8}{2} = \frac{0.4}{R_1} \Rightarrow R_1 = 1 \text{ k}\Omega$

d. For $v_X = v_Y = \text{logic 1} = 0.2 \text{ V}$

$$i_E = \frac{(0.2 - 0.7) - (-3.1)}{3} = 0.867 \text{ mA}$$

$$i_{R2} = \frac{0.4}{1} \Rightarrow i_{R2} = 0.4 \text{ mA}$$

$$i_{D2} = 0.467 \text{ mA}$$

e. $i_E = 0.867 \text{ mA}$

$$i_3 = \frac{0.2 - (-3.1)}{3.3} = 1 \text{ mA}$$

$$i_4 = \frac{-0.2 - (-3.1)}{3.3} = 0.879 \text{ mA}$$

$$P = (0.867 + 1 + 0.879)(0.9 - (-3.1))$$

or

$$P = 11.0 \text{ mW}$$

17.11

a. $i_1 = \frac{(-0.9 - 0.7) - (-3)}{1} \Rightarrow i_1 = 1.4 \text{ mA}$

$$i_3 = \frac{(-0.2 - 0.7) - (-3)}{15} \Rightarrow i_3 = 0.14 \text{ mA}$$

$$i_4 = \frac{(-0.2 - 0.7) - (-3)}{15} \Rightarrow i_4 = 0.14 \text{ mA}$$

$$i_2 + i_D = i_1 + i_3 = 1.4 + 0.14 = 1.54 \text{ mA}$$

$$i_2 = \frac{0.4}{0.5} \Rightarrow i_2 = 0.8 \text{ mA}$$

$$i_D = 0.74 \text{ mA}$$

$$v_0 = -0.4 \text{ V}$$

b. $i_1 = 1.4 \text{ mA}$

$$i_3 = \frac{(0 - 0.7) - (-3)}{15} \Rightarrow i_3 = 0.153 \text{ mA}$$

$$i_4 = i_3 \Rightarrow i_4 = 0.153 \text{ mA}$$

$$i_2 + i_D = i_4 \Rightarrow i_2 = 0.153 \text{ mA}$$

$$i_D = 0$$

$$v_0 = -(0.153)(0.5) \Rightarrow v_0 = -0.0765 \text{ V}$$

c. $i_1 = \frac{(0 - 0.7 - 0.7) - (-3)}{1} \Rightarrow i_1 = 1.6 \text{ mA}$

$$i_3 = \frac{(-0.2 - 0.7) - (-3)}{15} \Rightarrow i_3 = 0.14 \text{ mA}$$

$$i_4 = i_3 \Rightarrow i_4 = 0.14 \text{ mA}$$

$$i_2 + i_D = i_3 \Rightarrow i_2 = 0.14 \text{ mA}$$

$$i_D = 0.0$$

$$v_0 = -(0.14)(0.5) \Rightarrow v_0 = -0.07 \text{ V}$$

d. $i_1 = \frac{(0 - 0.7 - 0.7) - (-3)}{1} \Rightarrow i_1 = 1.6 \text{ mA}$

$$i_3 = \frac{(0 - 0.7) - (-3)}{15} \Rightarrow i_3 = 0.153 \text{ mA}$$

$$i_4 = i_3 \Rightarrow i_4 = 0.153 \text{ mA}$$

$$i_2 + i_D = i_1 + i_4 = 1.6 + 0.153 = 1.753 \text{ mA}$$

$$i_2 = \frac{0.4}{0.5} \Rightarrow i_2 = 0.8 \text{ mA}$$

$$i_D = 0.953 \text{ mA}$$

$$v_0 = -0.40 \text{ V}$$

17.12

a. L $A = B = C = D = 0 \Rightarrow Q_1 = Q_2$ cutoff

So

$$V_{DD} = 2I_E R_1 + V_{SS} + I_B R_2$$

and

$$I_B = \frac{I_E}{1 + \beta_p} = \frac{I_E}{51}$$

Then

$$2.5 = 2I_E(2) + 0.7 + \frac{I_E}{51} \cdot (15)$$

$$2.5 - 0.7 = I_E \cdot \left(4 + \frac{15}{51}\right)$$

So

$$I_E = 0.419 \text{ mA}$$

and

$$Y = 2.5 - 2(0.419)(2) \Rightarrow Y = 0.824 \text{ V}$$

ii. $A = C = 0, B = D = 2.5 \text{ V}$

Now

$$v_{B1} = v_{B2} = 2.5 - 0.7 = 1.8 \text{ V}$$

and

$$Y = v_{B1} + 0.7 \Rightarrow Y = 2.5 \text{ V}$$

b. $Y = (A \text{ OR } B) \text{ AND } (C \text{ OR } D)$

17.13

a. logic 1 = 0 V

logic 0 = -0.4 V

b. $v_{o1} = \overline{A \text{ OR } B}$

$v_{o2} = \overline{C \text{ OR } D}$

$v_{o3} = v_{o1} \text{ OR } v_{o2}$

or

$v_{o3} = (A \text{ OR } B) \text{ AND } (C \text{ OR } D)$

17.14

- a. For CLOCK = high, I_{DC} flows through the left side of the circuit. If D is high, I_{DC} flows through the left R resistor pulling \overline{Q} low. If D is low, I_{DC} flows through the right R resistor pulling Q low.

For CLOCK = low, I_{DC} flows through the right side of the circuit maintaining Q and \overline{Q} in their previous state.

b. $P = (I_{DC} + 0.5I_{DC} + 0.1I_{DC} + 0.1I_{DC})(3)$

$$P = 1.7I_{DC}(3) = (1.7)(50)(3) \Rightarrow P = 255 \mu\text{W}$$

17.15

i. For $v_X = v_Y = 0.1 \text{ V} \Rightarrow v' = 0.8 \text{ V}$

$$i_1 = \frac{5 - 2.2}{8} \Rightarrow i_1 = 0.525 \text{ mA}$$

$$i_3 = i_4 = 0$$

ii. For $v_X = v_Y = 5 \text{ V}$,

$$v' = 0.8 + 0.7 + 0.7 \Rightarrow v' = 2.2 \text{ V}$$

$$i_1 = \frac{5 - 2.2}{8} \Rightarrow i_1 = 0.35 \text{ mA}$$

$$i_4 = i_1 - \frac{0.8}{15} \Rightarrow i_4 = 0.297 \text{ mA}$$

$$i_3 = \frac{5 - 0.1}{2.4} \Rightarrow i_3 = 2.04 \text{ mA}$$

17.16

- a. For $v_X = v_Y = 5 \text{ V}$, both Q_1 and Q_2 driven into saturation.

$$v_1 = 0.8 + 0.7 + 0.8 \Rightarrow v_1 = 2.3 \text{ V}$$

$$i_1 = \frac{5 - 2.3}{4} \Rightarrow i_1 = i_{B1} = 0.675 \text{ mA}$$

$$i_2 = \frac{5 - (0.8 + 0.7 + 0.1)}{2} \Rightarrow i_2 = 1.7 \text{ mA}$$

$$i_4 = i_{B1} + i_2 \Rightarrow i_4 = 2.375 \text{ mA}$$

$$i_5 = \frac{0.8}{10} \Rightarrow i_5 = 0.08 \text{ mA}$$

$$i_{B2} = i_4 - i_5 \Rightarrow i_{B2} = 2.295 \text{ mA}$$

$$i_3 = \frac{5 - 0.1}{4} \Rightarrow i_3 = 1.225 \text{ mA}$$

$$v_0 = 0.1 \text{ V}$$

b. $i'_L = \frac{5 - (0.1 + 0.7)}{4} = 1.05 \text{ mA}$

$$i_C(\text{max}) = \beta i_{B2} = N i'_L + i_5$$

$$(20)(2.295) = N(1.05) + 1.225$$

So

$$N = 42$$

17.17

D_x and D_y off, Q_1 forward active mode

$$v_1 = 0.8 + 0.7 + 0.7 = 2.2 \text{ V}$$

$$5 = i_1 R_1 + i_2 R_2 + v_1 \text{ and } i_1 = (1 + \beta) i_2$$

$$\text{So } 5 - 2.2 = i_2 [(1 + \beta) R_1 + R_2]$$

Assume $\beta = 25$

$$i_2 = \frac{5 - 2.2}{(26)(1.75) + 2} \Rightarrow i_2 = 0.0589 \text{ mA}$$

$$i_1 = (1 + \beta) i_2 = (26)(0.0589) \Rightarrow i_1 = 1.53 \text{ mA}$$

$$i_3 = \beta i_2 \Rightarrow i_3 = 1.47 \text{ mA}$$

$$i_{B2} = i_2 + i_3 - \frac{0.8}{5} = 0.0589 + 1.47 - 0.16 \Rightarrow$$

$$i_{B2} = 1.37 \text{ mA}$$

Q_2 in saturation

$$i_{C2} = \frac{5 - 0.1}{6} \Rightarrow i_{C2} = 0.817 \text{ mA}$$

17.18

- a. i. $v_X = v_Y = 0.1$ V, so Q_1 in saturation.

$$i_1 = \frac{5 - (0.1 + 0.8)}{6} \Rightarrow i_1 = 0.683 \text{ mA}$$

$$\Rightarrow i_{B2} = i_2 = i_4 = i_{B3} = i_3 = 0$$

- ii. $v_X = v_Y = 5$ V, so Q_1 in inverse active mode.
Assume Q_2 and Q_3 in saturation.

$$i_1 = \frac{5 - (0.8 + 0.8 + 0.7)}{6} \Rightarrow i_1 = i_{B2} = 0.45 \text{ mA}$$

$$i_2 = \frac{5 - (0.8 + 0.1)}{2} \Rightarrow i_2 = 2.05 \text{ mA}$$

$$i_4 = \frac{0.8}{1.5} \Rightarrow i_4 = 0.533 \text{ mA}$$

$$i_{B3} = (i_{B2} + i_2) - i_4 = 0.45 + 2.05 - 0.533$$

or

$$i_{B3} = 1.97 \text{ mA}$$

$$i_3 = \frac{5 - 0.1}{2.2} \Rightarrow i_3 = 2.23 \text{ mA}$$

- b. For Q_3 :

$$\frac{i_3}{i_{B3}} = \frac{2.23}{1.97} = 1.13 < \beta$$

For Q_2 :

$$\frac{i_2}{i_{B2}} = \frac{2.05}{0.45} = 4.56 < \beta$$

Since $(I_C/I_B) < \beta$, then each transistor is in saturation.

17.19

- (a) $v_X = v_Y = \text{Logic 1}$

$$v' = 0.8 + 2(0.7) = 2.2 \text{ V}$$

$$i_1 = \frac{5 - 2.2}{8} = 0.35 \text{ mA}$$

$$i_4 = i_1 - \frac{0.8}{15} = 0.35 - 0.0533 = 0.2967 \text{ mA}$$

$$i_3 = \frac{5 - 0.1}{2.4} = 2.04 \text{ mA}$$

$$i'_L = \frac{5 - (0.1 + 0.7)}{8} = 0.525 \text{ mA}$$

Assume $\beta = 25$

$$\text{Then } (25)(0.2967) = 2.04 + N(0.525)$$

$$\text{So } N = 10.2 \Rightarrow N = 10$$

(b) Now

$$5 = 2.04 + N(0.525)$$

$$\text{So } N = 5.64 \Rightarrow N = 5$$

17.20

- a. For $v_X = v_Y = 5$ V, Q_1 in inverse active mode.

$$i_{B1} = \frac{5 - (0.8 + 0.8 + 0.7)}{5} = 0.45 \text{ mA}$$

$$i_{B2} = i_{B1} + 2\beta_F i_{B1} = 0.45(1 + 2[0.1]) = 0.54 \text{ mA}$$

$$i_{C2} = \frac{5 - (0.8 + 0.1)}{2} = 2.05 \text{ mA}$$

$$i_{B3} = (i_{B2} + i_{C2}) - \frac{0.8}{1.5} = 0.54 + 2.05 - 0.533$$

or

$$i_{B3} = 2.06 \text{ mA}$$

Now

$$i'_L = \frac{5 - (0.1 + 0.8)}{6} = 0.683 \text{ mA}$$

Then

$$i_{C3}(\max) = \beta_F i_{B3} = N i'_L$$

$$\text{or } (20)(2.06) = N(0.683)$$

$$\Rightarrow N = 60$$

- b. From above, for v_0 high, $I'_L = (0.1)(0.45) = 0.045 \text{ mA}$. Now

$$I'_L(\max) = (1 + \beta_F) \left(\frac{5 - 4.9}{R_2} \right) = \frac{(21)(0.1)}{2} = 1.05 \text{ mA}$$

So

$$I_L(\max) = N I'_L$$

$$\text{or } 1.05 = N(0.045)$$

$$\Rightarrow N = 23$$

17.21

- (a) $V_{in} = 0.1$ V: Q_1 , Sat: Q_s , Q_o , Cutoff

$$i_1 = \frac{5 - (0.1 + 0.8)}{4} = 1.025 \text{ mA}$$

$$P = i_1(5 - 0.1) = (1.025)(4.9) \Rightarrow$$

$$P = 5.02 \text{ mW}$$

- (b) $V_{in} = 5$ V, Q_1 , Inverse Active; Q_s , Q_o , Saturation

$$v_{B1} = 0.7 + 0.8 + 0.7 = 2.2 \text{ V}$$

$$i_1 = \frac{5 - 2.2}{4} = 0.7 \text{ mA}$$

$$i_{out} = \beta_R \cdot i_1 = (0.1)(0.7) = 0.07 \text{ mA}$$

$$V_{out} = 0.7 + 0.1 = 0.8 \text{ V}$$

$$i_2 = \frac{5 - 0.8}{1} = 4.2 \text{ mA}$$

$$P = (i_1 + i_{out} + i_2)(5) = (0.7 + 0.07 + 4.2)(5) \Rightarrow$$

$$P = 24.9 \text{ mW}$$

17.22

a. $v_X = v_Y = v_Z = 0.1 \text{ V}$

$$i_{B1} = \frac{5 - (0.1 + 0.8)}{3.9} \Rightarrow i_{B1} = 1.05 \text{ mA}$$

Then

$$i_{C1} = i_{B1} = i_{C2} = i_{B3} = i_{C3} = 0$$

b. $v_X = v_Y = v_Z = 5 \text{ V}$

$$i_{B1} = \frac{5 - (0.8 + 0.8 + 0.7)}{3.9} \Rightarrow i_{B1} = 0.692 \text{ mA}$$

Then

$$i_{C1} = i_{B2} = i_{B1}(1 + 3\beta_R) = (0.692)(1 + 3[0.5]) \\ \Rightarrow i_{C1} = i_{B2} = 1.73 \text{ mA}$$

$$i_{C2} = \frac{5 - (0.1 + 0.8)}{2} \Rightarrow i_{C2} = 2.05 \text{ mA}$$

$$i_{B3} = i_{B1} + i_{C2} - \frac{0.8}{0.8} = 1.73 + 2.05 - 1.0 \\ \Rightarrow i_{B3} = 2.78 \text{ mA}$$

$$i_{R3} = \frac{5 - 0.1}{2.4} = 2.04 \text{ mA}$$

$$i'_L = \frac{5 - (0.1 + 0.8)}{3.9} = 1.05 \text{ mA}$$

$$i_{C3} = i_{R3} + 5i'_L = 2.04 + (5)(1.05) \\ \Rightarrow i_{C3} = 7.29 \text{ mA}$$

17.23

a. $v_X = v_Y = v_Z = 2.8 \text{ V}$, Q_1 biased in the inverse active mode.

$$i_{B1} = \frac{2.8 - (0.8 + 0.8 + 0.7)}{2} \Rightarrow i_{B1} = 0.25 \text{ mA}$$

$$i_{B2} = i_{B1}(1 + 3\beta_R) = 0.25(1 + 3[0.3])$$

$$\Rightarrow i_{B2} = 0.475 \text{ mA}$$

$$v_{C2} = 0.8 + 0.1 = 0.9 \text{ V}$$

$$i_{B4} = \frac{0.9 - (0.7 + 0.1)}{(1 + \beta_F)(0.5)} = \frac{0.1}{(101)(0.5)}$$

$$= 0.00198 \text{ mA (Negligible)}$$

$$i_{R2} = \frac{5 - 0.9}{0.9} = 4.56 \text{ mA}$$

$$\Rightarrow i_{C2} = 4.56 \text{ mA}$$

$$i_{B3} = i_{B2} + i_{C2} - \frac{0.8}{1} = 0.475 + 4.56 - 0.8 \\ \Rightarrow i_{B3} = 4.235 \text{ mA}$$

b. $v_X = v_Y = v_Z = 0.1 \text{ V}$

$$i_{B1} = \frac{5 - (0.1 + 0.8)}{2} \Rightarrow i_{B1} = 2.05 \text{ mA}$$

From part (a),

$$i'_L = \beta_R \cdot i_{B1} = (0.3)(0.25) = 0.075 \text{ mA}$$

Then

$$i_{B4} = \frac{5i'_L}{1 + \beta_F} = \frac{5(0.075)}{101} \Rightarrow i_{B4} = 0.00371 \text{ mA}$$

17.24

a. $v_X = v_Y = v_Z = 0.1 \text{ V}$

$$i_{B1} = \frac{5 - (0.1 + 0.8)}{R_{B1}} + i_{B3}$$

where

$$i_{B3} = \frac{(2 - 0.7) - (0.9)}{R_{B2}} = \frac{0.4}{1} \\ \Rightarrow i_{B3} = 0.4 \text{ mA}$$

Then

$$i_{B1} = \frac{1.1}{1} + 0.4 \Rightarrow i_{B1} = 1.5 \text{ mA}$$

$$i_{B2} = 0 = i_{C2}$$

 Q_3 in saturation $i_{C3} = 5i'_L$ For v_0 high,

$$i'_{B1} = 0.8 + 0.7 = 1.5 \text{ V} \Rightarrow Q'_3 \text{ off}$$

$$i'_{B1} = \frac{2 - 1.5}{1} = 0.5 \text{ mA}$$

$$i'_L = \beta_R i'_{B1} = (0.2)(0.5) = 0.1 \text{ mA}$$

Then

$$i_{C3} = 0.5 \text{ mA}$$

b. $v_X = v_Y = v_Z = 2 \text{ V}$

From part (a),

$$\Rightarrow i_{B1} = 0.5 \text{ mA}$$

$$i_{B3} = 0 = i_{C3}$$

$$i_{B2} = i_{B1}(1 + 3\beta_R) = (0.5)(1 + 3[0.2])$$

$$i_{B2} = 0.8 \text{ mA}$$

$$i_{C2} = 5i'_L, \text{ and from part (a), } i'_L = 1.5 \text{ mA}$$

So

$$i_{C2} = 7.5 \text{ mA}$$

Chapter 17: Problem Solutions

17.25

$$(a) I_B + I_D = \frac{5.8 - 0.7}{10} = 0.51 \text{ mA}$$

$$I_C - I_D = \frac{5 - (0.7 - 0.3)}{1} = 4.6 \text{ mA}$$

Now

$$I_D = 0.51 - I_B = 0.51 - \frac{I_C}{\beta} = 0.51 - \frac{I_C}{50}$$

Then

$$I_C - I_D = I_C - \left(0.51 - \frac{I_C}{50}\right) = I_C \left(1 + \frac{1}{50}\right) - 0.51 = 4.6$$

$$\text{So } I_C = 5.01 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{5.01}{50} \Rightarrow I_B = 0.1002 \text{ mA}$$

$$I_D = 0.51 - 0.1002 \Rightarrow I_D = 0.4098 \text{ mA}$$

$$V_{CE} = 0.4 \text{ V}$$

$$(b) I_D = 0, V_{CE} = V_{CE(sat)} = 0.1 \text{ V}$$

$$I_B = \frac{5.8 - 0.8}{10} \Rightarrow I_B = 0.5 \text{ mA}$$

$$I_C = \frac{5 - 0.1}{1} \Rightarrow I_C = 4.9 \text{ mA}$$

17.26

$$a. \nu_X = \nu_Y = 0.4 \text{ V}$$

$$\nu_{B1} = 0.4 + 0.7 \Rightarrow \nu_{B1} = 1.1 \text{ V}$$

$$i_{B1} = \frac{5 - 1.1}{2.8} \Rightarrow i_{B1} = 1.39 \text{ mA}$$

$$\nu_{B2} = 0.4 + 0.4 \Rightarrow \nu_{B2} = 0.8 \text{ V}$$

$$i_{B2} = i_{C2} = i_{B0} = i_{C0} = i_{B3} = i_{C3} \\ = i_{B4} = i_{C4} = 0 \text{ (No load)}$$

$$S = i_{B4} R_2 + V_{BE} + (1 + \beta) i_{B4} R_4$$

$$i_{B4} = \frac{5 - 0.7}{0.76 + (31)(3.5)} \\ \Rightarrow i_{B4} = 0.0394 \text{ mA}$$

$$i_{C4} = \beta i_{B4}$$

$$\Rightarrow i_{C4} = 1.18 \text{ mA}$$

$$\nu_{B4} = 5 - (0.0394)(0.76)$$

$$\Rightarrow \nu_{B4} = 4.97 \text{ V}$$

$$b. \nu_X = \nu_Y = 3.6 \text{ V}$$

$$\nu_{B1} = 0.7 + 0.7 + 0.3 \Rightarrow \nu_{B1} = 1.7 \text{ V}$$

$$\nu_{B2} = 1.4 \text{ V}$$

$$\nu_{B0} = 0.7 \text{ V}$$

$$\nu_{C2} = 1.1 \text{ V}$$

$$i_{B1} = \frac{5 - 1.7}{2.8}$$

$$\Rightarrow i_{B1} = 1.18 \text{ mA}$$

$$i_{B2} = i_{B1}(1 + 2\beta_R) = 1.18(1 + 2[0.1])$$

$$\Rightarrow i_{B2} = 1.42 \text{ mA}$$

$$i_{B4} = \frac{1.1 - 0.7}{(31)(3.5)} \\ \Rightarrow i_{B4} = 0.00369 \text{ mA}$$

$$i_{R2} = \frac{5 - 1.1}{0.76} = 5.13 \text{ mA} \\ \Rightarrow i_{C2} \approx 5.13 \text{ mA}$$

$$i_{B0} \approx i_{B2} + i_{C2} \\ i_{B0} = 6.55 \text{ mA}$$

17.27

- a. Assuming the output transistor Q_2 is a Schottky transistor, then

$$\nu_0 = 0.4 \text{ V}, i_L' = \frac{2.5 - (0.4 + 0.3)}{R_{B1}} = 0.5$$

Then

$$R_{B1} = 3.6 \text{ k}\Omega$$

Then

$$i_{B1} = \frac{2.5 - (0.7 + 0.8)}{R_{B1}} = \frac{10}{3.6} = 0.278 \text{ mA}$$

$$i_{B2} = 0.5 \text{ mA}, i_{E1} = 0.5 + \frac{0.7}{0.7} = 1.50 \text{ mA}$$

$$i_{B1} = i_{B2} + i_{C1} \Rightarrow i_{C1} = 1.50 - 0.278 = 1.222 \text{ mA}$$

$$\text{and } i_{C1} = \frac{2.5 - (0.7 + 0.1)}{R_{C1}} = 1.222 \text{ mA} \\ \Rightarrow R_{C1} = 1.39 \text{ k}\Omega$$

$$b. \nu_X = \nu_Y = 0.4 \text{ V}, \nu_{B1} = 0.7 \text{ V}$$

$$\nu_{C2} = 2.5 - 0.7 \Rightarrow \nu_{C2} = 1.8 \text{ V}$$

All transistor currents are zero.

$$c. \nu_{B1} = 1.5 \text{ V}, \nu_{C1} = 0.8 \text{ V}$$

Currents calculated in part (a).

$$d. i_{B2} = 0.5 \text{ mA}, i_L' = 0.5 \text{ mA}$$

$$i_{C2(\max)} = \beta i_{B2} = N i_L' \text{ or } (50)(0.5) = N(0.5)$$

So

$$N = 50$$

17.28

- a. For $\nu_X = \nu_Y = 3.6 \text{ V}$

$$\nu_{B1} = 3(0.7) = 2.1$$

$$\Rightarrow i_{B1} = \frac{5 - 2.1}{10} = 0.29 \text{ mA}$$

$$\nu_{C1} = 0.7 + 0.7 + 0.4 = 1.8 \text{ V}$$

$$\Rightarrow i_{C1} = \frac{5 - 1.8}{10} = 0.32 \text{ mA}$$

$$i_{B2} = i_{B1} + i_{C1} - \frac{1.4}{15} = 0.29 + 0.32 - 0.0933$$

So

$$i_{B1} = 0.517 \text{ mA}$$

$$v_{C2} = 0.7 + 0.4 = 1.1 \text{ V}$$

$$i_{C2} = \frac{5 - 1.1}{4.1} = 0.951 \text{ mA}$$

$$i_{B3} = i_{B1} + i_{C2} - \frac{0.7}{4} = 0.517 + 0.951 - 0.175$$

$$\text{or } i_{B3} = 1.293 \text{ mA}$$

$$\text{For } v_0 = 0.4 \text{ V, } v'_{B1} = 0.4 + 0.7 = 1.1 \text{ V}$$

Then

$$i'_{B1} = \frac{1.1 - 0.7}{(31)(15)} = 0.00086 \text{ mA}$$

$$i'_L = \frac{5 - 1.1}{10} - 0.00086 \text{ or } i'_L \approx 0.39 \text{ mA}$$

$$\text{So } i_{C3}(\text{max}) = \beta i_{B3} = N i'_L$$

$$(30)(1.293) = N(0.39)$$

$$\Rightarrow N = 99$$

$$\text{b. } P = (0.29 + 0.32 + 0.951)(5) + (99)(0.39)(0.4)$$

$$P = 7.805 + 15.444 \text{ or } P = 23.2 \text{ mW}$$

(Assuming 99 load circuits which is unreasonably large.)

17.29

a. Assume no load. For $v_X = \text{logic 0} = 0.4 \text{ V}$

$$i_{B1} = \frac{5 - (0.4 + 0.7)}{40} = 0.0975 \text{ mA}$$

Essentially all of this current goes to ground from V_{CC} .

$$P = i_{B1} \cdot V_{CC} = (0.0975)(5)$$

$$\Rightarrow P = 0.4875 \text{ mW}$$

$$\text{b. } i_{R1} = \frac{5 - (3)(0.7)}{40} = 0.0725 \text{ mA}$$

$$i_{R2} = \frac{5 - (0.7 + 0.7 + 0.4)}{50} = 0.064 \text{ mA}$$

$$i_{R3} = \frac{5 - (0.7 + 0.4)}{15} = 0.26 \text{ mA}$$

$$P = (0.0725 + 0.064 + 0.26)(5)$$

$$P = 1.95 \text{ mW}$$

c. For $v_0 = 0, v_{C1} = 0.7 + 0.4 = 1.1 \text{ V}$

$$i_{R7} = \frac{5 - 1.1}{0.050} \Rightarrow i_{R7} = 78 \text{ mA} \approx i_{SC}$$

17.30

(a) $v_I = v_O = 2.5 \text{ V}$; A transient situation

$$v_{DS}(M_N) = 2.5 - 0.7 = 1.8 \text{ V}$$

$$v_{GS}(M_N) = 2.5 - 0.7 = 1.8 \text{ V} \Rightarrow M_N \text{ in saturation}$$

$$v_{SD}(M_P) = 5 - (2.5 + 0.7) = 1.8 \text{ V}$$

$$v_{SG}(M_P) = 2.5 - 2.5 = 2.5 \text{ V} \Rightarrow M_P \text{ in saturation}$$

$$i_{DN} = K_n(v_{GSN} - V_{TN})^2 = (0.1)(1.8 - 0.8)^2 \Rightarrow$$

$$i_{DN} = 0.1 \text{ mA}$$

$$i_{DP} = K_p(v_{SGP} + V_{TP})^2 = (0.1)(2.5 - 0.8)^2 \Rightarrow$$

$$i_{DP} = 0.289 \text{ mA}$$

$$i_{C1} = \beta i_{DP} = (50)(0.289) \Rightarrow i_{C1} = 14.45 \text{ mA}$$

$$i_{C2} = \beta i_{DN} = (50)(0.1) \Rightarrow i_{C2} = 5 \text{ mA}$$

Difference between i_{C1} and $i_{DN} + i_{C2}$ is a load current.

(b) Assume $i_{C1} = 14.45 \text{ mA}$ is a constant

$$V_C = \frac{1}{C} \int i_{C1} dt = \frac{i_{C1} \cdot t}{C} \Rightarrow t = \frac{(V_C)(C)}{i_{C1}}$$

$$t = \frac{(5)(15 \times 10^{-12})}{14.45 \times 10^{-3}} \Rightarrow t = 5.19 \text{ ns}$$

$$(c) t = \frac{(5)(15 \times 10^{-12})}{0.289 \times 10^{-3}} \Rightarrow t = 260 \text{ ns}$$

17.31

Let $R_1 = R_2 = 10 \text{ k}\Omega$

$$(a) i_{DN} = 0.1 \text{ mA}, i_{DP} = 0.289 \text{ mA}$$

(Same as Problem 17.30)

$$i_{R1} = \frac{0.7}{10} = 0.07 \text{ mA} \Rightarrow i_{B1} = 0.289 - 0.07 = 0.219 \text{ mA}$$

$$i_{C1} = (50)(0.219) \Rightarrow i_{C1} = 10.95 \text{ mA}$$

$$i_{R2} = \frac{0.7}{10} = 0.07 \text{ mA} \Rightarrow i_{B2} = 0.1 - 0.07 = 0.03 \text{ mA}$$

$$i_{C2} = (50)(0.03) \Rightarrow i_{C2} = 1.5 \text{ mA}$$

$$(b) t = \frac{(5)(15 \times 10^{-12})}{10.95 \times 10^{-3}} \Rightarrow t = 6.85 \text{ ns}$$

(c) $t = 260 \text{ ns}$ (Same as Problem 17.30)