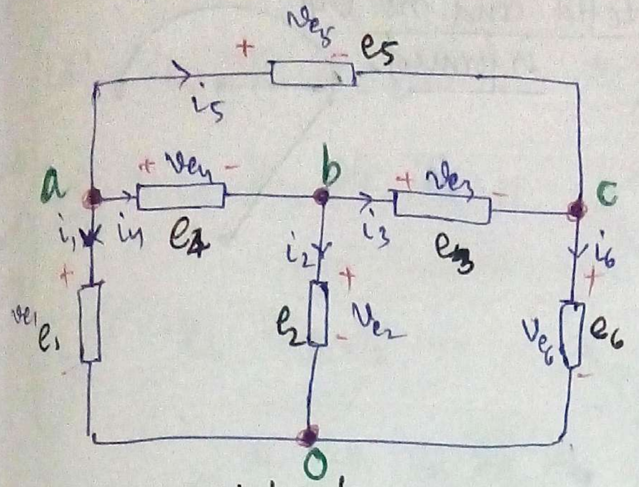
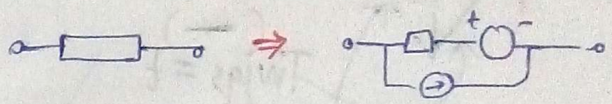
Graph Theory Applied to Network Analysis

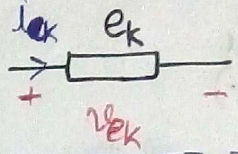


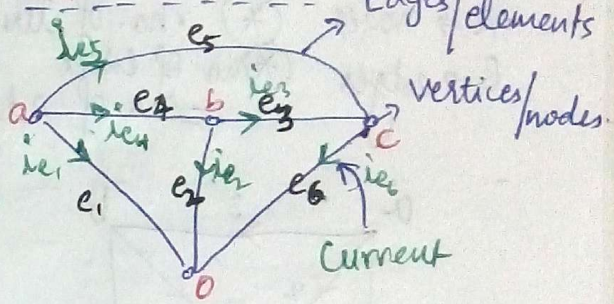
Network

4 Nodes

Elements:-







Directed Graph

KCL at node a:-

$$i_{e_1} + i_{e_4} + i_{e_5} = 0$$

node b:- $$i_{e_2} + i_{e_3} - i_{e_4} = 0$$

node c:- $$i_{e_6} - i_{e_3} - i_{e_5} = 0$$

extra → node O:- $$-(i_{e_1} + i_{e_2} + i_{e_6}) = 0$$ → this can be manufactured from linear combination of above 3 equations. So, we usually don't write it.

$$\begin{array}{c|c}
 & \begin{array}{cccccc} i_{e_1} & i_{e_2} & i_{e_3} & i_{e_4} & i_{e_5} & i_{e_6} \end{array} \\
\begin{array}{c} \text{KCL at a} \\ \text{KCL at b} \\ \text{KCL at c} \end{array} & \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{bmatrix} \\
\text{KCL at O} & \begin{array}{cccccc} -1 & -1 & 0 & 0 & 0 & -1 \end{array}
\end{array}
\begin{bmatrix} i_{e_1} \\ i_{e_2} \\ i_{e_3} \\ i_{e_4} \\ i_{e_5} \\ i_{e_6} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

at any node, arrow away ⇒ +1

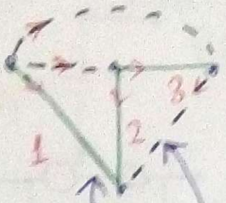
$$\boxed{[A]\,[i_e] = [0]}$$

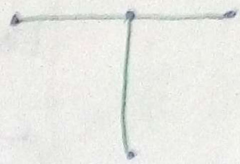
$(n-1)\times e$ $\quad e\times 1 \quad$ $(n-1)\times 1$

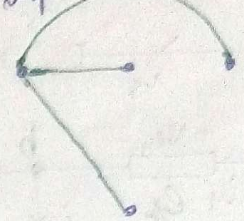
$$[A]^t\,[v_N] = [v_e]$$

Tree of Graph

→ All nodes are connected and no loop is formed.







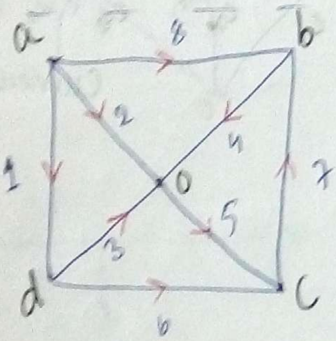
(—) Twigs = t = n-1

Links (----)

$n \to$ nodes (*) no. of twigs = $t = n-1$

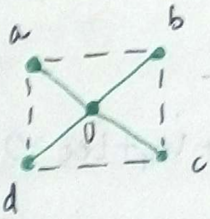
$e \to$ edges (*) no. of links = no. of edges − twigs = $e - t$

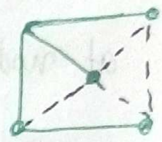
$= e - n + 1$



$n = 5$

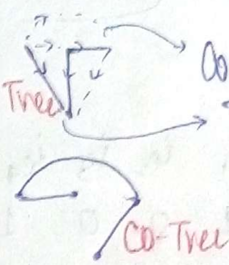
$e = 8$





$t = 4$ $(n-1)$

$l = 4$ $(e-t)$



Graph (elements/edges)

Co-tree (links)

Tree (twigs)

[A]

$$
[A] = \begin{array}{c|cccccccc}
 & i_{e1} & i_{e2} & i_{e3} & i_{e4} & i_{e5} & i_{e6} & i_{e7} & i_{e8} \\
\hline
a & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
b & 0 & 0 & 0 & 1 & 0 & 0 & -1 & -1 \\
c & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 0 \\
d & -1 & 0 & -1 & 0 & 0 & 1 & 0 & 0
\end{array}
$$

Relationship between node voltages and element voltages

$v_{ao} = v_{e1}$

$v_{bo} = v_{e2}$

$v_{bo} - v_{co} = v_{e3}$

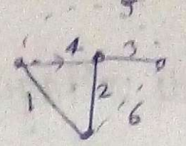
$$
\begin{array}{ccc}
v_{ao} & v_{bo} & v_{co}
\end{array}
$$

$$
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 1 & -1 \\
1 & -1 & 0 \\
1 & 0 & -1 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} v_{ao} \\ v_{bo} \\ v_{co} \end{bmatrix}
=
\begin{bmatrix} v_{e1} \\ v_{e2} \\ v_{e3} \\ v_{e4} \\ v_{e5} \\ v_{e6} \end{bmatrix}
$$

$e \times (n-1)$ $e \times 1$

$$\boxed{[A]^T [v_N] = [v_e]}$$

Loop Analysis



Fundamental loop

L4: $v_{e_4}+v_{e_2}-v_{e_1}=0$

L5: $v_{e_5}-v_{e_3}+v_{e_2}-v_{e_1}=0$

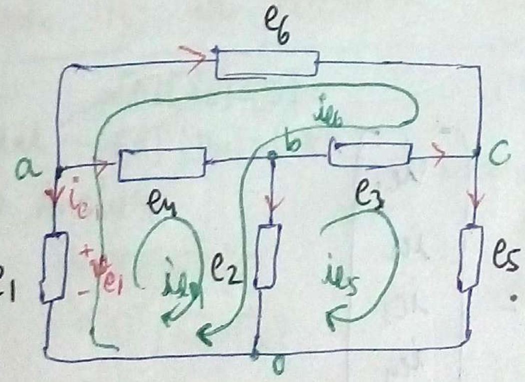
L6: $v_{e_6}-v_{e_2}+v_{e_3}=0$

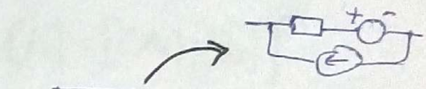
$$
\begin{array}{c}
 \\ L4 \\ L5 \\ L6
\end{array}
\begin{array}{c}
\begin{array}{cccccc} v_{e_1} & v_{e_2} & v_{e_3} & v_{e_4} & v_{e_5} & v_{e_6} \end{array} \\
\begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{bmatrix}
\end{array}
\begin{bmatrix} v_{e_1} \\ v_{e_2} \\ v_{e_3} \\ v_{e_4} \\ v_{e_5} \\ v_{e_6} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$\boxed{[B]\,[v_e] = [0]}$$

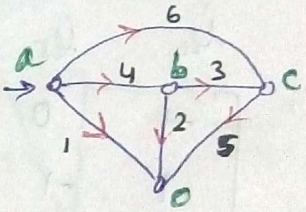
Graph Theory - in Network Analysis

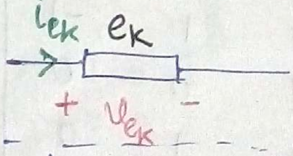
27/10/16





Graph →



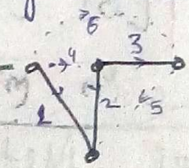


Let n = total no. of nodes
e = total no of elements

$$[i_e] = \begin{bmatrix} i_{e_1} \\ i_{e_2} \\ \vdots \\ i_{e_6} \end{bmatrix}$$

Tree → Subset of graph where all the nodes are shown but only selected edges are shown.

Tree-



Twigs $= t$
$= n-1$

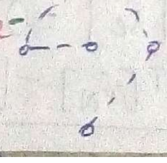
KCL at nodes:-

Nodal Analysis

$$[A]\,[i_e] = [0]$$

$(n-1)\times e$ $e\times 1$

CoTree-



Links

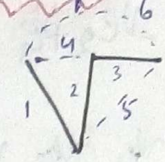
$l = e-t$
$= e-n+1$

Nodal Analysis: Relationship of node voltages with element voltages

$$[A]^T [v_N] = [v_e]$$

$[A]^T$: $e \times (n-1)$ (t); $[v_N]$: $(n-1)\times 1$; $[v_e]$: $e \times 1$

Loops dictated by links are called fundamental loops.



links → fundamental loops

KVL in the F. loops:- (Move along dirⁿ of arrow ⇒ +)

L4: $v_{e_4} + v_{e_2} - v_{e_1} = 0$

L5: $v_{e_5} - v_{e_2} + v_{e_3} = 0$

L6: $v_{e_6} - v_{e_3} + v_{e_2} - v_{e_1} = 0$

twig

$$\begin{array}{c} \\ L_4 \\ L_5 \\ L_6 \end{array}\begin{array}{c} \begin{matrix} v_{e_1} & v_{e_2} & v_{e_3} & v_{e_4} & v_{e_5} & v_{e_6} \end{matrix} \\ \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ -1 & 1 & -1 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} v_{e_1} \\ v_{e_2} \\ v_{e_3} \\ v_{e_4} \\ v_{e_5} \\ v_{e_6} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

link components

$$[B][v_e] = [0]$$

$[B]$: $\cancel{(e-n+1)} \times e$ → $l \times e$; $[v_e]$: $e \times 1$; $[0]$: $\cancel{(e-n+1)} \times 1$ → $l \times 1$

$e-n+1$ = no. of links → l

* Relating loop currents $[i_l]$ with element currents $[i_e]$

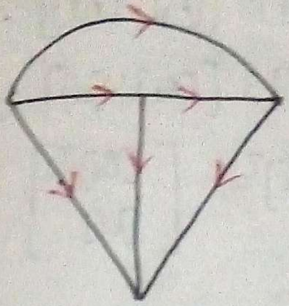
$$i_{e_1} = -i_{l_4} - i_{l_6}$$
$$i_{e_2} = i_{l_4} - i_{l_5} + i_{l_6}$$

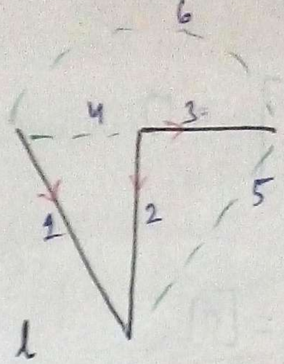
$$\begin{array}{c} \begin{matrix} i_{l_4} & i_{l_5} & i_{l_6} \end{matrix} \\ \begin{bmatrix} -1 & 0 & -1 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} i_{l_4} \\ i_{l_5} \\ i_{l_6} \end{bmatrix} = \begin{bmatrix} i_{e_1} \\ i_{e_2} \\ i_{e_3} \\ i_{e_4} \\ i_{e_5} \\ i_{e_6} \end{bmatrix}$$

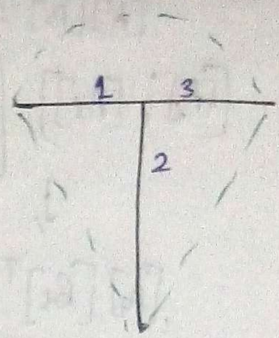
$$\boxed{[B]^T [i_l] = [i_e]}$$

$[B]^T$: $e \times \cancel{(e-n+1)}$; $[i_l]$: $\cancel{(e-n+1)} \times 1$; $[i_e]$: $e \times 1$

Nodal Method	Loop Analysis
$[A][i_e] = 0$	$[B][v_e] = 0$
$[A]^T [v_N] = [v_e]$	$[B]^T [i_l] = [i_e]$







$$[A] = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \quad t \qquad\qquad l \\ \begin{array}{ccc|ccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & -1 \end{array}\right] \end{array}$$

$$[A] = \Big[\underbrace{[A_t]}_{t \times t} \,\vdots\, \underbrace{[A_l]}_{t \times l}\Big]$$

$$[B] = \Big[\underbrace{[B_t]}_{l \times t} \,\vdots\, \underbrace{[B_l]}_{l \times l}\Big]$$

31/10/16

Graph Th. in Network Analysis

$[A] = \big[[A_t] \,\vdots\, [A_l]\big]$

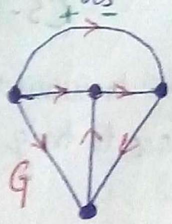
- Nodal
  - $[A][i_e] = [0]$
  - $[A]^T [v_N] = [v_e]$
- Loop Analysis
  - $[B][v_e] = [0]$
  - $[B]^T [i_L] = [i_e]$
- Cut-set Analysis ?

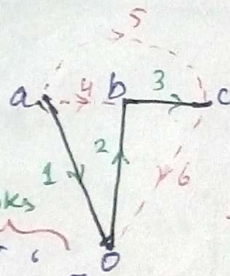
$[B_t]^T = -[A_t]^{-1}[A_L]$

n

e

G





—— Tree

Twig → $t = n-1$

- - - Co-Tree

Links → $l = e - t$

$$[A] = \begin{array}{c} \overbrace{\begin{array}{ccc} \text{Twigs} \\ 1 & 2 & 3 \end{array}} \; \overbrace{\begin{array}{ccc} \text{Links} \\ 4 & 5 & 6 \end{array}} \\ \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{array}\right] \end{array}$$

Loop Method :-

$[B][v_e] = [0]$

$$\begin{array}{c} \\ L4 \\ L5 \\ L6 \end{array} \begin{array}{c} \begin{array}{ccc|ccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \left[\begin{array}{ccc|ccc} -1 & -1 & 0 & 1 & 0 & 0 \\ -1 & -1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \end{array} \begin{bmatrix} v_{e_1} \\ v_{e_2} \\ v_{e_3} \\ v_{e_4} \\ v_{e_5} \\ v_{e_6} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$[B_t]$ — $l \times t$ ← Not necessarily sq. matrix.

$[B_l] = [U] = I_{l=3}$ — $l \times l$ unitary matrix.

* $\boxed{[A] \text{ and } [B] \text{ are orthogonal Matrices. } \quad [A][B]^T = [0]}$

$$[A][B]^T = [0]$$

$$\left[[A_t] \,\vdots\, [A_l] \right] \begin{bmatrix} [B_t]^T \\ [U] \end{bmatrix} = 0$$

$$\downarrow$$

$$[A_t][B_t]^T + [A_l] = [0]$$

$$\therefore \; [B_t]^T = -[A_t]^{-1}[A_l]$$

$$[B_t] = \{ \quad \}^T$$

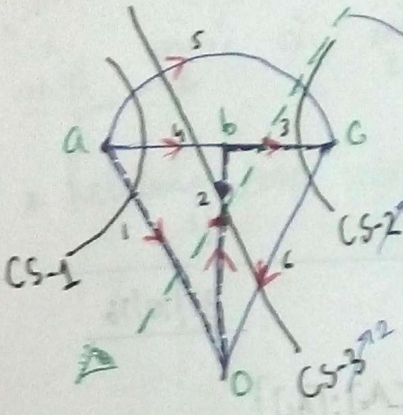
$$[A] = \left[[A_t] \,\vdots\, [A_l] \right]$$

$$[B] = \left[[B_t] \,\vdots\, [U] \right]$$

$$[B]^T = \begin{bmatrix} [B_t]^T \\ [U] \end{bmatrix}$$

* Loops formed by links are called fundamental loops.

Cut Set Method & Analysis of networks



Current entering line = Current leaving line.

$$i_1 + i_2 + i_3 + i_5 = 0$$

Fundamental Cut Set

CS-1 $\quad i_{e_1} + i_{e_4} + i_{e_5} = 0$

CS-~~2~~ 3 $\quad i_{e_3} + i_{e_5} - i_{e_6} = 0$

CS-~~3~~ 2 $\quad i_{e_2} + i_{e_4} + i_{e_5} = i_{e_6}$

A cut including exactly one twig

Its better to number cutset acc to twig no.

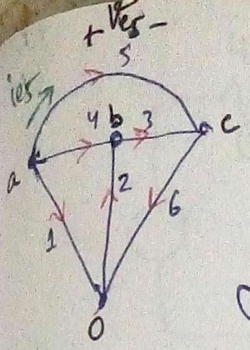
$$\begin{array}{c} \\ \text{CS-1} \\ \text{CS-2} \\ \text{CS-3} \end{array} \begin{array}{c} \begin{array}{cccccc} i_{e_1} & i_{e_2} & i_{e_3} & i_{e_4} & i_{e_5} & i_{e_6} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 & 1 & -1 \end{bmatrix} \end{array} \begin{bmatrix} i_{e_1} \\ i_{e_2} \\ i_{e_3} \\ i_{e_4} \\ i_{e_5} \\ i_{e_6} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

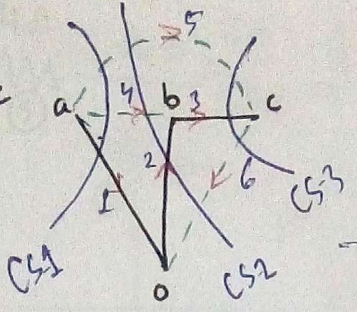
$$[Q]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array} \right]$$

$$[Q_t]_{t \times t} \quad [Q_l]_{t \times l} = \left[[U] \,\vdots\, [Q_l] \right]$$

Scanned by CamScanner





2/11/16

Summary :-

Nodal: $[A][i_e]=[0]$

$[A]^T[v_N]=[v_e]$

Loop: $[B][v_e]=[0]$

$[B]^T[i_l]=[i_e]$

$[A]=\left[[A_t]\,\vdots\,[A_l]\right]$

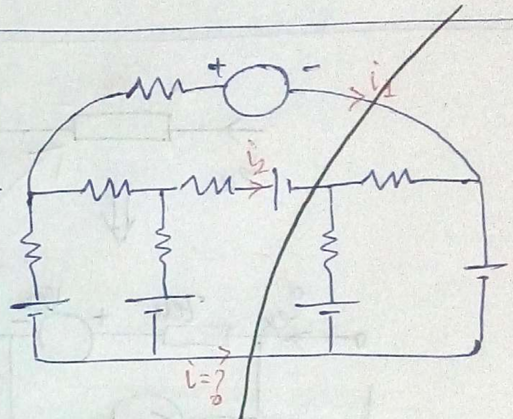
$[B]=\left[[B_t]\,\vdots\,[U]\right]$

$[A][B]^T=[0]$ $\therefore$ orthogonal

$[B_t]^T=-[A_t]^{-1}[A_l]$

$$i_e=\begin{bmatrix} i_{e1}\\ i_{e2}\\ \vdots \\ i_{e6}\end{bmatrix} \qquad [v_N]=\begin{bmatrix} v_{ao}\\ v_{bo}\\ v_{co}\end{bmatrix}$$

Example of cutset :-



$i_1+i_2+i=0$

$\Rightarrow i=-i_1-i_2$

Cutset :-

Write down KCLs at the cut sets.

CS1 $\quad i_{e1}+i_{e4}+i_{e5}=0$

CS2 $\quad i_{e2}+i_{e4}+i_{e5}=i_{e6}$

CS3 $\quad i_{e3}+i_{e5}=i_{e6}$

$$\begin{array}{c} CS1\\ CS2\\ CS3\end{array}\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0\\ 0 & 1 & 0 & 1 & 1 & -1\\ 0 & 0 & 1 & 0 & 1 & -1\end{bmatrix}\begin{bmatrix} i_{e1}\\ i_{e2}\\ \vdots\\ i_{e6}\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$[Q][i_e]=[0]$

$[Q]=\left[\underset{[Q_t]}{[U]}\,\vdots\,[Q_l]\right]$

$$\begin{array}{ccc} v_{e1t} & v_{e2t} & v_{e3t}\end{array}$$
$$\begin{bmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 1 & 0\\ 1 & 1 & 1\\ 0 & -1 & -1\end{bmatrix}\begin{bmatrix} v_{e1t}\\ v_{e2t}\\ v_{e3t}\end{bmatrix}=\begin{bmatrix} v_{e1}\\ v_{e2}\\ \vdots\\ v_{e6}\end{bmatrix}$$

$\circ\ v_{e1t}+v_{e2t}=v_{e4}$

$[Q]^T[v_{et}]=[v_e]$

* $[B]$ & $[Q]$ are orthogonal.

$\boxed{[B][Q]^T=[0]}$

$\left[[B_t]\,\vdots\,[U]\right]\left[[U]\,\vdots\,[Q_l]\right]^T=[0]$

$$\left[[B_t] \mid [U]\right] \begin{bmatrix} [U] \\ [Q_l]^T \end{bmatrix} = [0]$$

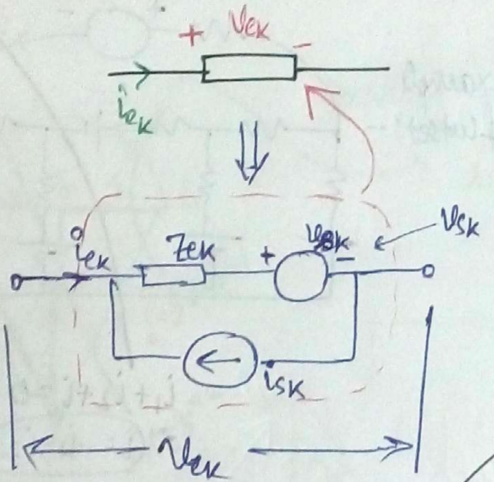
$$[B_t] + [Q_l]^T = [0]$$

$$\Rightarrow [B_t] = -[Q_l]^T$$

$$[B_t]^T = -[Q_l]$$

$$-[A_t]^{-1}[A_l] = -[Q_l]$$

$$\Rightarrow \boxed{[Q_l] = [A_t]^{-1}[A_l]}$$



For k^{th} element

By Loop analysis :-

$$v_{ek} = v_{sk} + Z_{ek}(i_{sk} + i_{ek})$$

or $v_{ek} = Z_{ek} i_{ek} + v_{sk} + Z_{ek} i_{sk}$

$$\begin{bmatrix} v_{e1} \\ v_{e2} \\ \vdots \\ v_{e6} \end{bmatrix} = \begin{bmatrix} Z_{e1} & 0 & \cdots & \\ 0 & Z_{e2} & & \\ \vdots & & \ddots & \\ & & & Z_{e6} \end{bmatrix} \begin{bmatrix} i_{e1} \\ i_{e2} \\ \vdots \\ i_{e6} \end{bmatrix} + \begin{bmatrix} v_{s1} \\ v_{s2} \\ \vdots \\ v_{s6} \end{bmatrix} + \begin{bmatrix} Z_{e1} & 0 & \cdots & \\ 0 & Z_{e2} & & \\ \vdots & & \ddots & \\ & & & Z_{e6} \end{bmatrix} \begin{bmatrix} i_{s1} \\ i_{s2} \\ \vdots \\ i_{s3} \end{bmatrix}$$

diagonal matrix

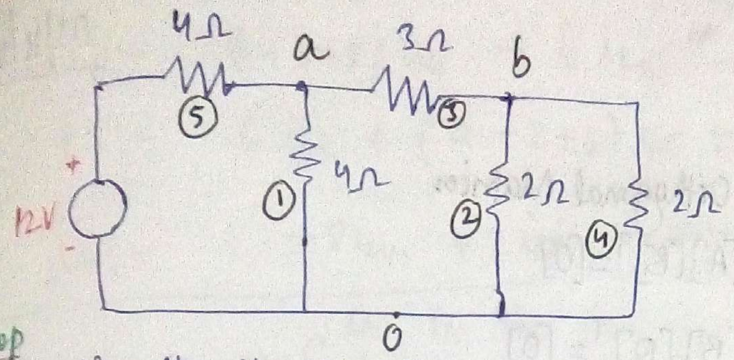
$$[v_e] = [Z_e][i_e] + [v_s] + [Z_e][i_s]$$

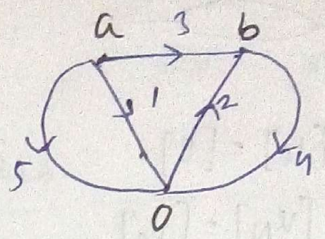
Premultiply both sides by $[B]$

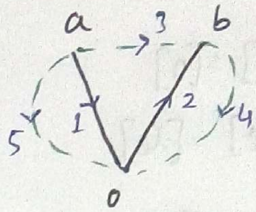
$$\Rightarrow [B][v_e] = [B][Z_e][i_e] + [B][v_s] + [B][Z_e][i_s] = [0]$$

$$[B][Z_e][i_e] = -[B][v_s] - [B][Z_e][i_s]$$

$$\underbrace{[B][Z_e][B]^T}_{Z_{loop}}[i_l] = -[B][v_s] - [B][Z_e][i_s]$$





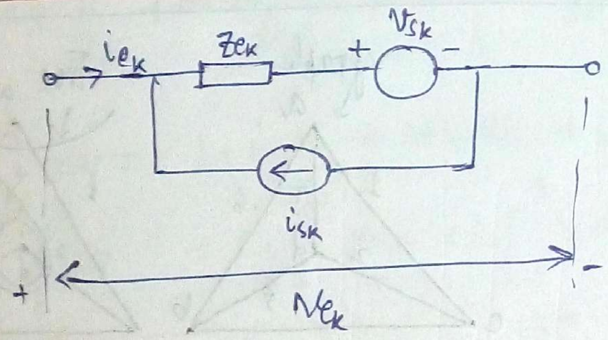


By loop analysis

$$\begin{array}{c|ccccc} & v_{e1} & v_{e2} & v_{e3} & v_{e4} & v_{e5} \\ \hline L3 & -1 & -1 & 1 & 0 & 0 \\ L4 & 0 & 1 & 0 & 1 & 0 \\ L5 & -1 & 0 & 0 & 0 & 1 \end{array} = [B]$$

$$[Z_e] = \begin{bmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix} \qquad [v_s] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 12 \end{bmatrix} \qquad [i_s] = [0]$$

By nodal method



$$i_{ek} + i_{sk} = \frac{v_{ek} - v_{sk}}{Z_{ek}}$$

$$= Y_{ek} v_{ek} - Y_{ek} v_{sk}$$

$$\Rightarrow i_{ek} = Y_{ek} v_{ek} - i_{sk} - Y_{ek} v_{ek}$$

$$[i_e] = [Y_e][v_e] - [i_s] - [Y_e][v_s]$$

Premultiply both sides by [A].

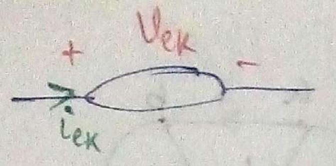
$$\Rightarrow [A][i_e] = [A][Y_e][v_e] - [A][i_s] - [A][Y_e][v_s] = [0]$$

$$[A][Y_e]\underbrace{[v_e]} = [A][i_s] + [A][Y_e][v_s]$$

$$\boxed{[A][Y_e][A]^T[v_N] = [A][i_s] + [A][Y_e][v_s]}$$

$$[A] = \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & -1 & -1 & 1 & 0 \end{bmatrix} \end{array}$$

03/11/16

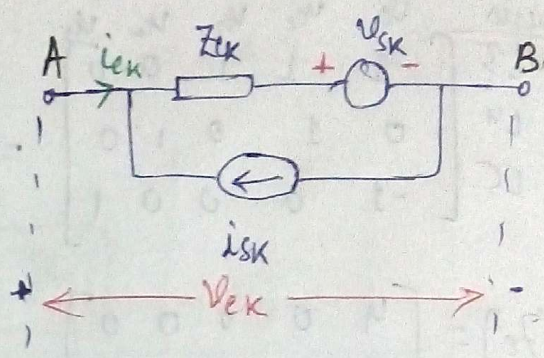


Nodal
$[A][i_e] = [0]$
$[A]^T[v_N] = [v_e]$

Mesh
$[B][v_e] = [0]$
$[B]^T[i_\ell] = [i_e]$

Cut-set
$[Q][i_e] = [0]$
$[Q]^T[v_{et}] = [v_e]$

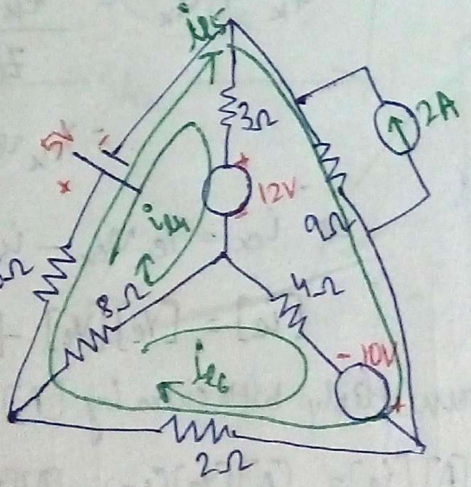
Orthogonal Matrices
$\rightarrow [A][B]^T = [0]$
$\rightarrow [B][Q]^T = [0]$

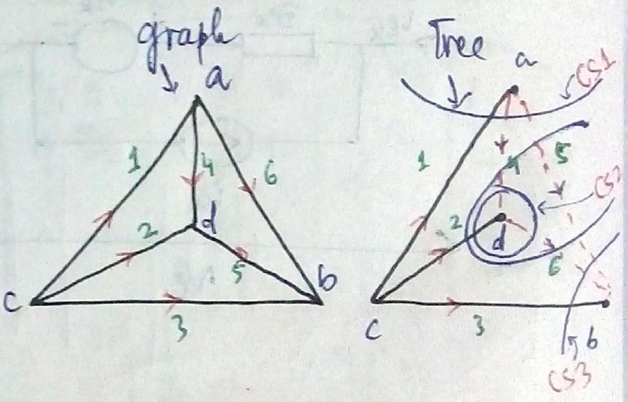


Nodal $[A][Y][A]^T[v_N] = [A][i_s] + [A][Y][v_s]$

CutSet $[Q][Y][Q]^T[v_N] = [Q][i_s] + [Q][Y][v_s]$

Loop $[B][Z][B]^T[i_\ell] = -[B][v_s] - [B][Z][i_s]$





Mesh Analysis

$$[v_s] = \begin{bmatrix} 5 \\ 0 \\ 0 \\ 12 \\ 0 \\ -10 \end{bmatrix} \quad [i_s] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 0 \end{bmatrix} \quad [Z] = \begin{bmatrix} 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

$$[B] = \begin{array}{c c} & \begin{array}{cccccc} v_{e_1} & v_{e_2} & v_{e_3} & v_{e_4} & v_{e_5} & v_{e_6} \end{array} \\ \begin{array}{c} L4 \\ L5 \\ L6 \end{array} & \begin{bmatrix} 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

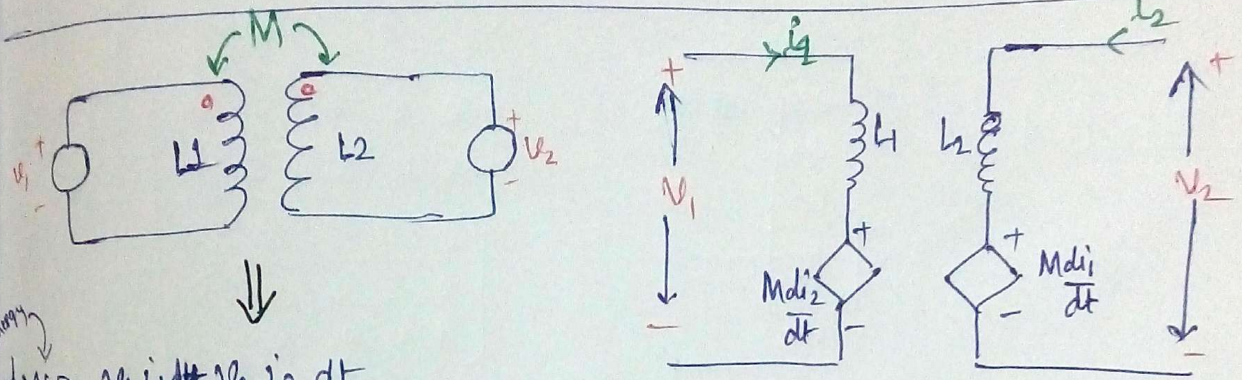
CT- Sunday 13/11/16 1-5hrs.

$L4:- \quad (6+3+8) i_{L4} + 6 i_{L5} - 8 i_{L6} = -5-12$

$L5:- \quad 6 i_{L4} + (9+2+6) i_{L5} + 2 i_{L6} = -5$

$L6:- \quad -8 i_{L4} + 2 i_{L5} + (8+4+2) i_{L6} = +10$

$$Q = ? = \begin{array}{c} \\ CS1 \\ CS2 \\ CS3 \end{array} \begin{array}{c} \begin{array}{cccccc} i_{e1} & i_{e2} & i_{e3} & i_{e4} & i_{e5} & i_{e6} \end{array} \\ \begin{bmatrix} -1 & 0 & 0 & +1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & -1 & -1 \end{bmatrix} \end{array}$$



Energy

$dw = v_1 i_1 dt + v_2 i_2 dt$

$dw = \left(L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}\right) i_1 \, dt + \left(L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}\right) i_2 \, dt$

$dw = L_1 i_1 di_1 + M i_1 di_2 + L_2 i_2 di_2 + M i_2 di_1$

$\int_0^t dw = L_1 \int_0^t i_1 di_1 + L_2 \int_0^t i_2 di_2 + M \int_0^t d(i_1 i_2)$

$$\boxed{w = \frac{1}{2} L_1 i_1^2(t) + \frac{1}{2} L_2 i_2^2(t) + M i_1(t) i_2(t)}$$