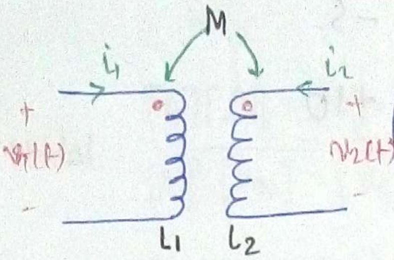
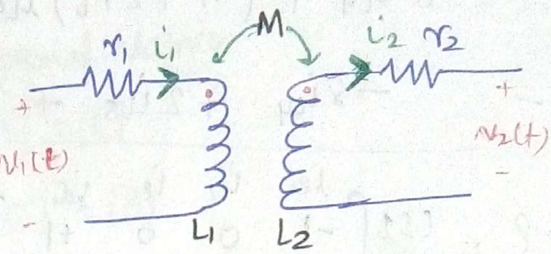
7/11/16

2-Port ~~Terminal~~ Network (4-Terminals)



$$W = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$

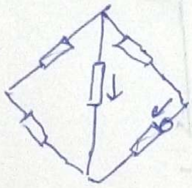


$$v_1(t) = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$v_2(t) = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

$$v_1(t) = r_1 i_1 + M\frac{d}{dt}(-i_2) + L_1\frac{di_1}{dt}$$

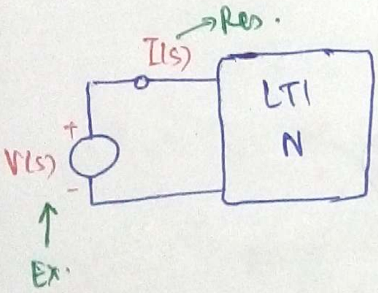
$$v_2(t) = r_2(-i_2) + L_2\frac{d}{dt}(-i_2) + M\frac{di_1}{dt}$$

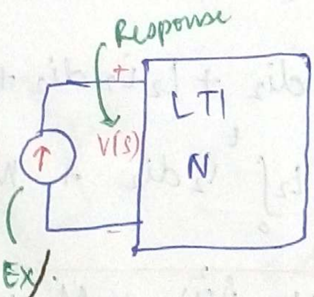


Network f^n:

$$H(s) = \frac{\text{L.T. of Response}}{\text{L.T. of Excitation}}$$

1-Port Network

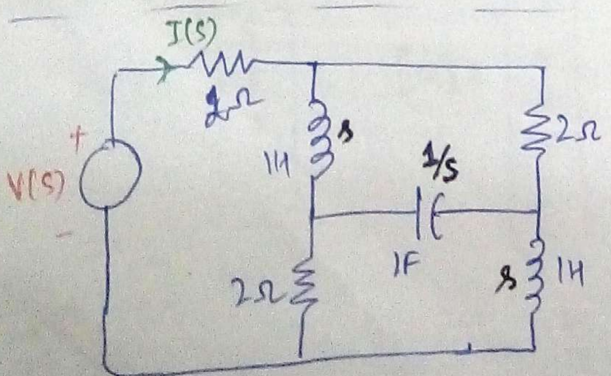




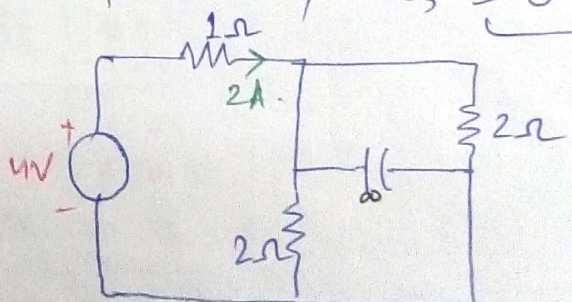
$$\frac{V(s)}{I(s)} = Z(s) = \text{driving point impedence}$$

$$Z(s)Y(s) = 1$$

$$\frac{I(s)}{V(s)} = Y(s) = \text{driving point admittance.}$$

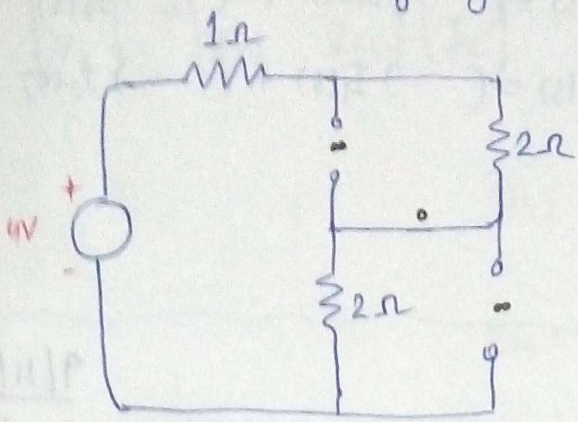


for DC steady state, $s = 0$ → Complex frequency

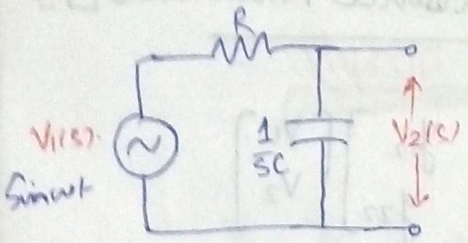


$R_{eq} = 2\,\Omega$
↓
$Z(s)$

at sinusoid of very high frequency , $s \to \infty$



$R_{eq} = 5\Omega$
$\downarrow$
$Z(s)$



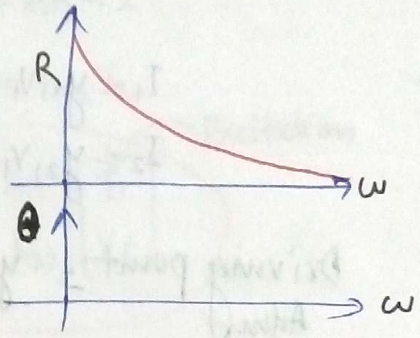
$$\frac{V_2(s)}{V_1(s)} = \frac{1/sC}{R + 1/sC}$$

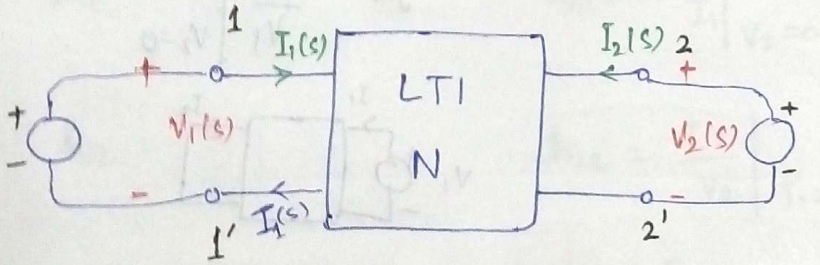
If i/p is sinusoidal

$s = j\omega$

$$\frac{V_2(j\omega)}{V_1(j\omega)} = \frac{\frac{1}{j\omega C}}{R - \frac{j}{\omega C}} = \frac{\frac{1}{\omega C}\angle -90^\circ}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}\ \angle + \tan^{-1}\left(\frac{-1/\omega C}{R}\right)}$$

$= R\angle\theta$.





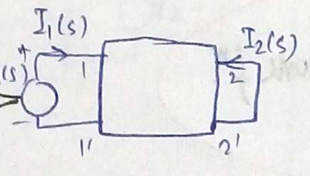
Apply voltages $V_1(s)$ and $V_2(s)$ to the port

Responses are $I_1(s)$ & $I_2(s)$

$I_1(s) = Y_{11}V_1(s) + Y_{12}V_2(s)$

$I_2(s) = Y_{21}V_1(s) + Y_{22}V_2(s)$

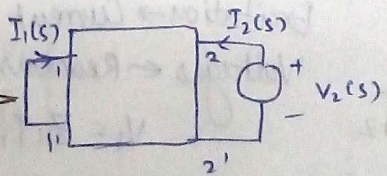
$$Y_{11} = \left.\frac{I_1(s)}{V_1(s)}\right|_{V_2(s)=0} \quad \to 2-2' \text{ shorted}$$



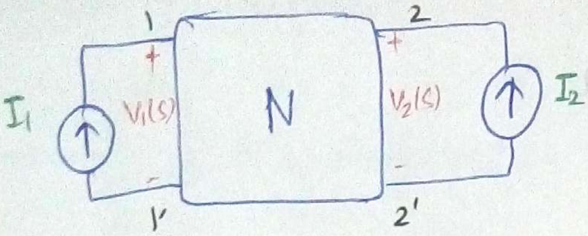
$$Y_{21} = \left.\frac{I_2(s)}{V_1(s)}\right|_{V_2(s)=0}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underbrace{\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}}_{\text{Short Ckt Admittance Matrix}} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$Y_{12} = \left.\frac{I_1(s)}{V_2(s)}\right|_{V_1(s)=0}$$



$$Y_{22} = \left.\frac{I_2(s)}{V_2(s)}\right|_{V_1(s)=0}$$

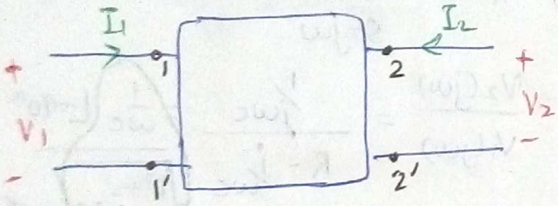


$$V_1(s) = (\quad) I_1(s) + (\quad) I_2(s)$$

$$V_2(s) = (\quad) I_1(s) + (\quad) I_2(s)$$

9/21/16

TWO PORT NETWORK



Short ckt Admittance Parameters [Y]

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

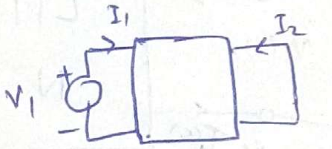
$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

Transfer Admittance y_{21} and y_{12}

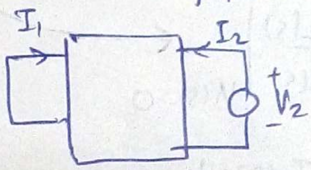
Driving point Adm. (y_{22}) $= y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$



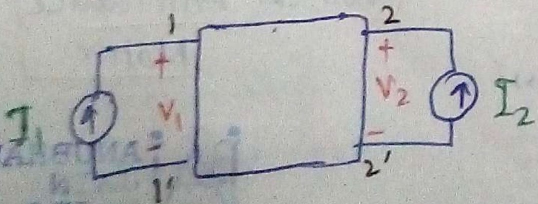
$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$



$$\frac{\text{LT of Response}}{\text{LT of Excitation}} = \text{Network } f^n.$$

Impedance Matrix



Excitation → Current

Voltages ← Response

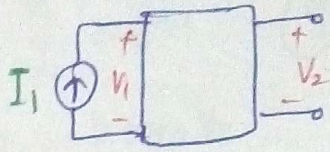
$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Scanned by CamScanner

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Open ckt $[Z]$ matrix



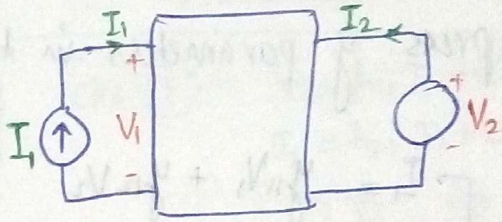
$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}.$$

Hybrid Parameters [H]

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

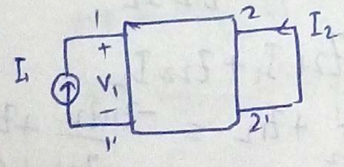


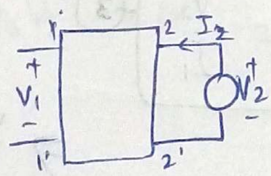
Response → Excitation

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

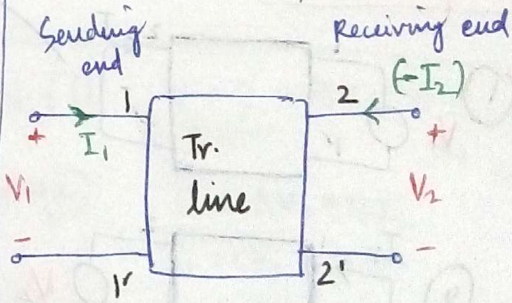




$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [H]^{-1} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

ABCD Parameters
or
Chain parameters



$$V_1 = AV_2 + BI_2$$

$$I_1 = CV_2 + DI_2.$$

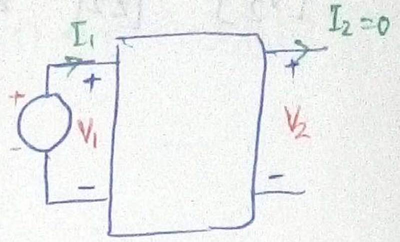
$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \quad ??$$

(We can't make $I_2 = 0$ and supply V_2 at 2-2')

$\Downarrow$

$$\frac{1}{A} = \left.\frac{V_2}{V_1}\right|_{I_2=0}.$$



$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$y \leftrightarrow h$ parameters

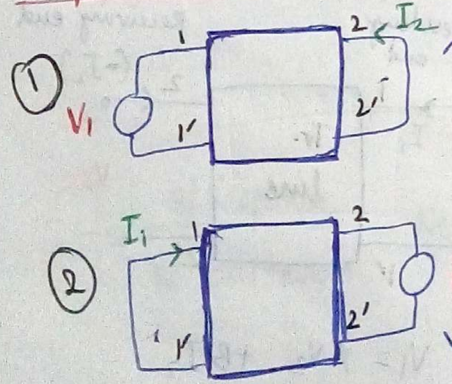
Express y parameters in terms of h parameter

$$I_1 = y_{11} V_1 + y_{12} V_2 \qquad\qquad V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \qquad\qquad I_2 = h_{21} I_1 + h_{22} V_2$$

$$\rightarrow V_1 = \frac{1}{y_{11}} I_1 - \frac{y_{12}}{y_{11}} V_2$$ $\rightarrow$ compare coefficients.

Reciprocal Networks.



Reciprocity Th^m:-

$$\boxed{\frac{V_1}{I_2} = \frac{V_2}{I_1}} \quad (3)$$

- $V_1 = Z_{11} I_1 + Z_{12} I_2$
- $0 = Z_{21} I_1 + Z_{22} I_2$

$$\Rightarrow \frac{V_1}{I_2} = Z_{11}\frac{I_1}{I_2} + Z_{12} = \frac{-Z_{11}Z_{22}}{Z_{21}} + Z_{12} \quad (1)$$

- $0 = Z_{11} I_1 + Z_{12} I_2$
- $V_2 = Z_{21} I_1 + Z_{22} I_2.$

$$\Rightarrow \frac{V_2}{I_1} = Z_{21} + Z_{22}\frac{I_2}{I_1}$$

$$\frac{V_2}{I_1} = Z_{21} + Z_{22}\left(-\frac{Z_{11}}{Z_{12}}\right) - (2)$$

From (1), (2) and (3)

$$\frac{-Z_{11}Z_{22}}{Z_{21}} + Z_{12} = Z_{21} - \frac{Z_{22}Z_{11}}{Z_{12}} \Rightarrow \boxed{Z_{12} = Z_{21}}$$

Similarly, we get

$y_{12} = y_{21}$

$h_{12} = -h_{21}$

$AD - BC = 1$

ckt ① $I_1 = y_{11} V_1 + 0$

$I_2 = y_{21} V_1 + 0$

$\frac{V_1}{I_2} = y_{21}$ — (5)

ckt ② $I_1 = 0 + y_{12} V_2$

$I_2 = 0 + y_{22} V_2$

$\frac{V_2}{I_1} = y_{12}$ — (6)

from eqn (3), (5) and (6)

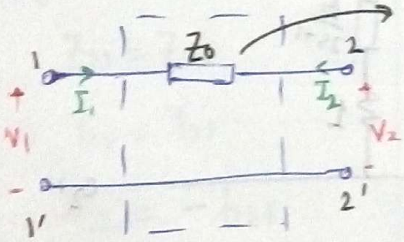
$\boxed{y_{21} = y_{12}}$

If network is symmetrical, that means Driving point Admittances are same

⇓

Symmetrical Reciprocal Network.

$Y_0 = \frac{1}{Z_0}$



* $V_1 - I_1 Z_0 = V_2$, * $I_1 = -I_2$.

$\Rightarrow I_1 Z_0 = V_1 - V_2$

$\Rightarrow$ * $I_1 = \frac{1}{Z_0} V_1 - \frac{1}{Z_0} V_2 = Y_0 V_1 - Y_0 V_2 = -I_2$.

$\Rightarrow$ * $I_2 = -Y_0 V_1 + Y_0 V_2$.

$\Rightarrow [Z] = [\times]$

$\Rightarrow [Y] = \begin{bmatrix} Y_0 & -Y_0 \\ -Y_0 & Y_0 \end{bmatrix}$

ckt ① $V_1 = h_{11} I_1 + 0$

$I_2 = h_{21} I_1 + 0$

$\Rightarrow \frac{V_1}{I_2} = \frac{h_{11}}{h_{21}}$ — (7)

ckt ② $0 = h_{11} I_1 + h_{12} V_2$

$\frac{V_2}{I_1} = \frac{-h_{11}}{h_{12}}$ — (8)

from eqn (3), (7) & (8)

$\boxed{h_{21} = -h_{12}}$

ckt ① $V_1 = 0 + B I_2 \Rightarrow \frac{V_1}{I_2} = B$ — (9)

ckt ② $0 = A V_2 + B I_2$

$I_1 = C V_2 + D I_2$

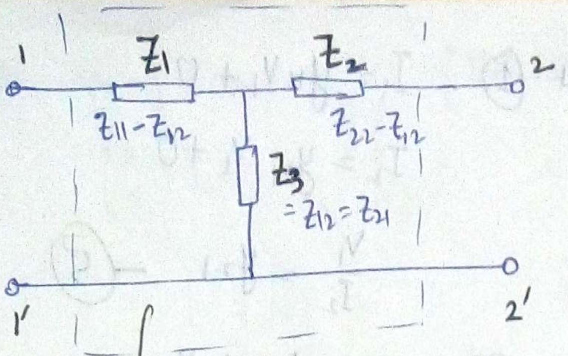
$C \frac{V_2}{I_1} = 1 - D \frac{I_2}{I_1} = 1 + \frac{DA}{B} \frac{V_2}{I_1}$

$\left(C - \frac{DA}{B}\right) \frac{V_2}{I_1} = 1$

$\frac{V_2}{I_1} = \frac{B}{BC - DA}$ — (10)

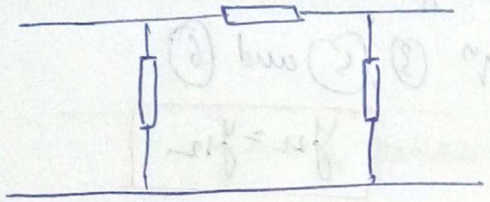
from eqn (3) (9) and (10)

$BC - DA = 1$



T-Network.

(Y-Δ conversion)



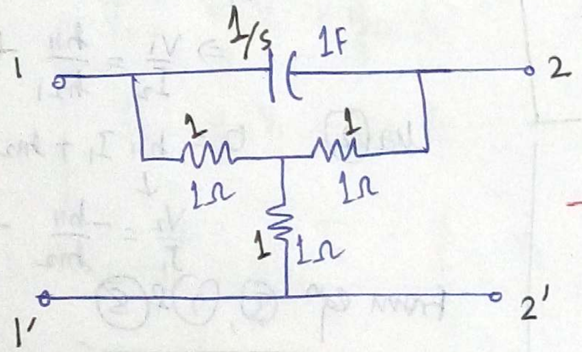
π-Network.

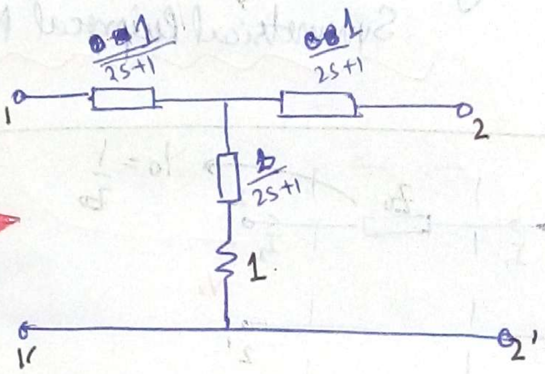
$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2.$$

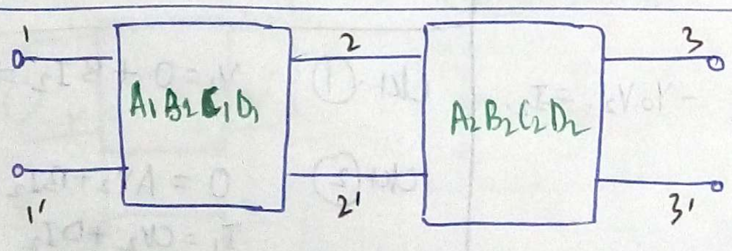
$$* \; V_1 - I_1 Z_1 - (I_1 + I_2) Z_3 = 0$$
$$\Rightarrow V_1 = (Z_1 + Z_3) I_1 + Z_3 I_2$$

$$* \; V_2 - I_2 Z_2 - (I_1 + I_2) Z_3 = 0$$
$$\Rightarrow V_2 = I_1 Z_3 + (Z_2 + Z_3) I_2$$

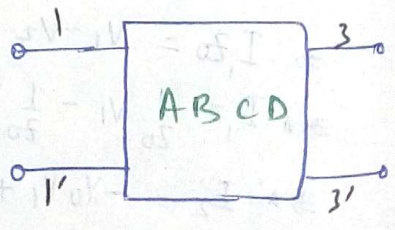
$$[Z] = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix}$$







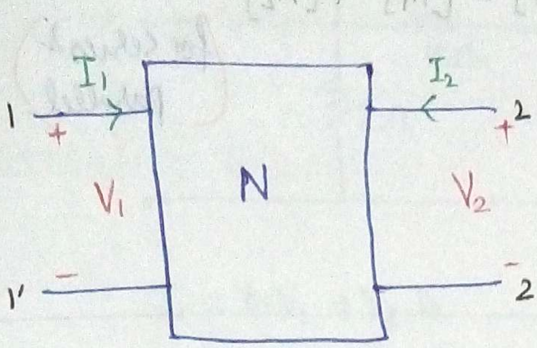
$\Rightarrow$



Class Test - 13/11/16
9am - 11.30 am
Cryptography Theory
Vikram Shila complex

2-Port Network

10/11/16



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underset{[Y]}{\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \underset{[Z]}{\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \underset{[H]}{\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \underset{[G]}{\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \underset{[T]}{\begin{bmatrix} A & B \\ C & D \end{bmatrix}} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

If Network is reciprocal :-

$Z_{12} = Z_{21}$

$Y_{12} = Y_{21}$

$h_{12} = -h_{21}$

$AD - BC = 1$

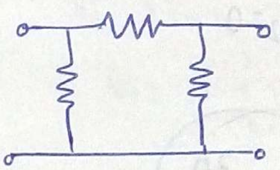
Symmetric

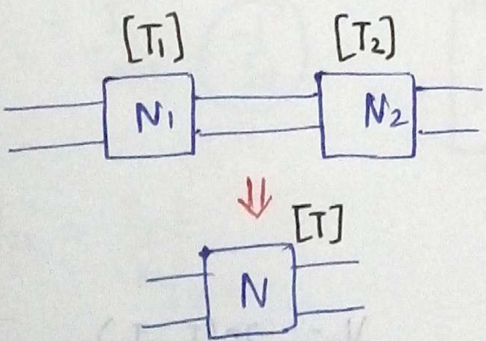
$Z_{11} = Z_{22}$

$Y_{11} = Y_{22}$

$[T] = ?$

$[H] = ?$

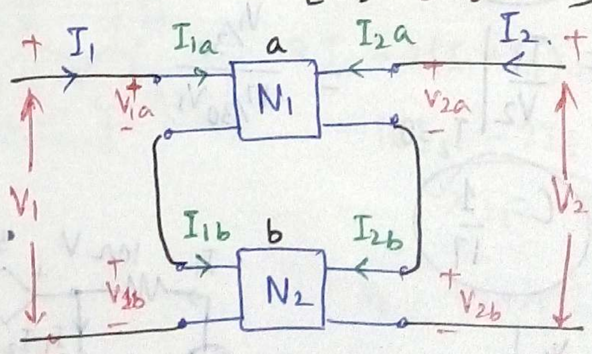


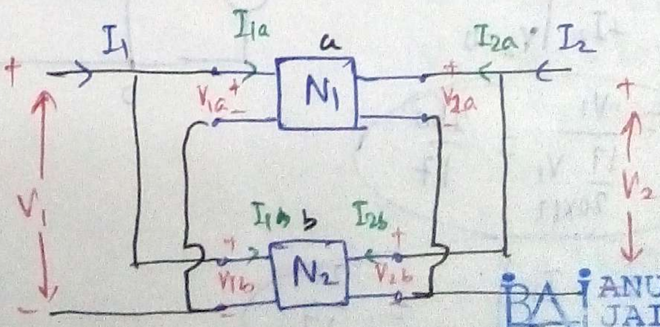


$[T] = [T_1] \cdot [T_2]$

(cascade Networks)

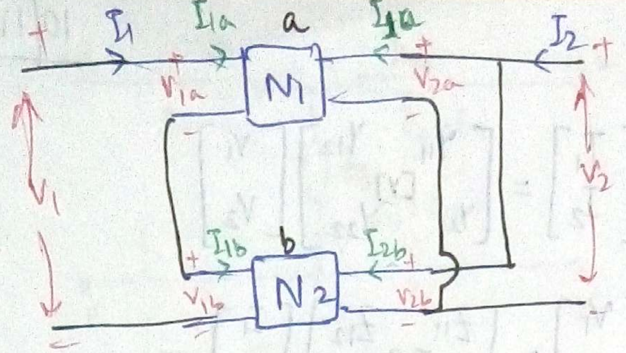
$[Z] = [Z_1] + [Z_2]$ (Series)





$[Y] = [Y_1] + [Y_2]$

(parallel)

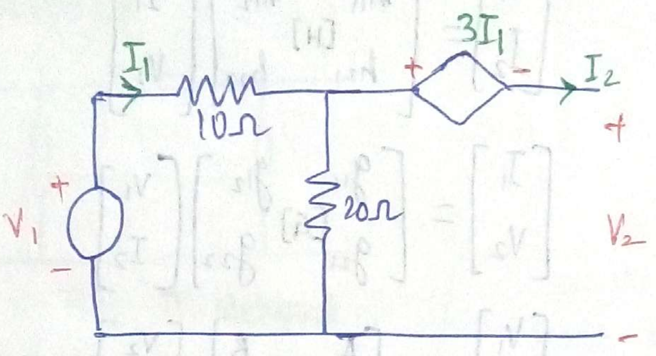


$$[H] = [H_1] + [H_2]$$

(for series & parallel)

<u>Ex.1</u>

Get A, B, C, D.



$$V_1 = AV_2 + BI_2$$

$$I_1 = CV_2 + DI_2$$

* $A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0}$ $\rightarrow V_2 = \dfrac{20}{30}V_1 - 3I_1$, $I_1 = \dfrac{V_1}{30}$

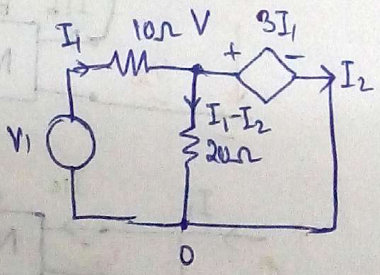
$= \dfrac{2}{3}V_1 - \dfrac{V_1}{10} = \dfrac{20-3}{30}V_1 = \dfrac{17}{30}V_1$

$\Rightarrow A = \dfrac{30}{17}$

* $C = \left.\dfrac{I_1}{V_2}\right|_{I_2=0} = \dfrac{V_1/30}{17/30\, V_1}$

$C = \dfrac{1}{17}$

* $B = \left.\dfrac{V_1}{+I_2}\right|_{V_2=0}$



$V = 20(I_1 - I_2)$

$= V_1 - 10I_1$

$= 3I_1$

$\Rightarrow V_1 = 13I_1 \Rightarrow I_1 = \dfrac{V_1}{13}$

$17I_1 = 20I_2 \Rightarrow I_2 = \dfrac{17}{20}I_1$

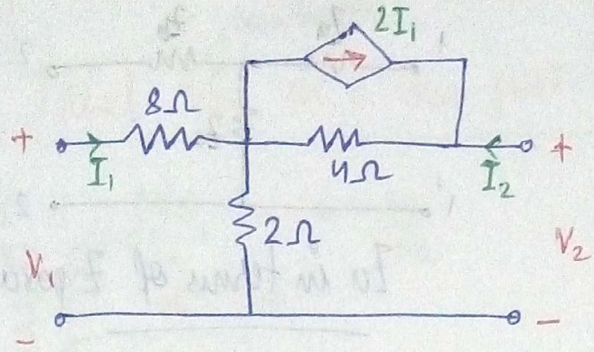
$= \dfrac{17}{20}\left(\dfrac{V_1}{13}\right)$

$B = \dfrac{V_1}{\frac{17\, V_1}{20\times 13}} = \dfrac{260}{17}$

* $D = \left.\dfrac{I_1}{I_2}\right|_{V_2=0} = \dfrac{20}{17}$.

Ex. 2

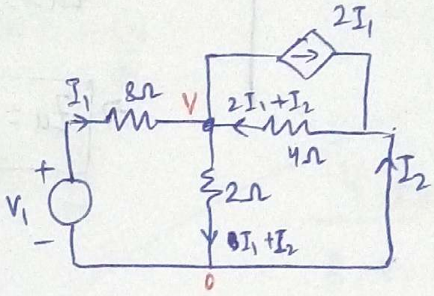
Get [Y].



$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

* $Y_{11} = \left.\dfrac{I_1}{V_1}\right|_{V_2=0}$

$= \dfrac{I_1}{20/3\, I_1} = \dfrac{3}{20}$



$V - 0 = 2(I_1+I_2)$

$= V_1 - 8I_1$

$= -4(2I_1+I_2)$

$2I_1 + 2I_2 = -8I_1 - 4I_2$

$10I_1 = -6I_2$

$5I_1 = -3I_2$

$V_1 - 8I_1 = 2\left(I_1 - \frac{5}{3}I_1\right)$

$V_1 - 8I_1 = -\frac{4}{3}I_1$

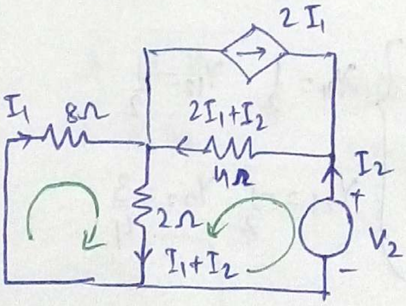
$V_1 = \left(8 - \frac{4}{3}\right)I_1 = \frac{20}{3}I_1$

* $Y_{21} = \left.\dfrac{I_2}{V_1}\right|_{V_2=0}$

$= \dfrac{-5/3\, I_1}{20/3\, I_1} = -\dfrac{1}{4}$

* $Y_{12} = \left.\dfrac{I_1}{V_2}\right|_{V_1=0}$

$= \dfrac{I_1}{-20I_1} = -\dfrac{1}{20}$



$8I_1 + 2(I_1+I_2) = 0$

$10I_1 = -2I_2$

$\Rightarrow I_2 = -5I_1$

$V_2 = 4(2I_1+I_2) + 2(I_1+I_2)$

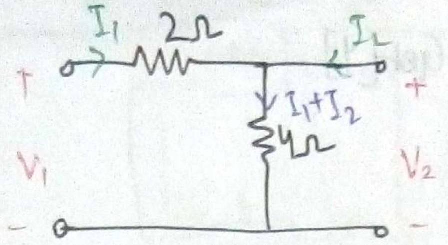
$V_2 = 10I_1 + 6I_2$

$V_2 = 10I_1 - 30I_1$

$= -20I_1$

* $Y_{22} = \left.\dfrac{I_2}{V_2}\right|_{V_1=0}$

$= \dfrac{-5I_1}{-20I_1} = \dfrac{1}{4}$



$[Z] = ?$

$[Y] = ?$

$I_1 = Y_{11} V_1 + Y_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2.$

$V_1 - 2I_1 - 4(I_1+I_2) = 0$

$V_1 = 6I_1 + 4I_2$ $\quad Z_{11}=6 \quad Z_{12}=4$

$V_2 - 4(I_1+I_2) = 0$ $\quad Z_{21}=4 \quad Z_{22}=4$

$V_1 - V_2 = 2I_1$

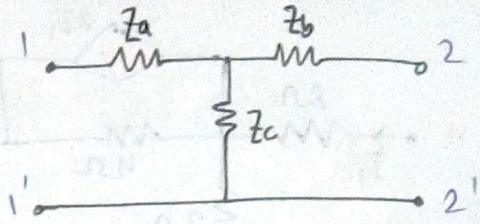
$\Rightarrow I_1 = \frac{1}{2}V_1 - \frac{1}{2}V_2$ $\quad Y_{11} = \frac{1}{2} \quad Y_{12} = \frac{-1}{2}$

$4I_2 = V_1 - 6I_1 = V_1 - (3V_1 - 3V_2)$ $\quad Y_{21} = \frac{-1}{2} \quad Y_{22} = \frac{3}{4}.$

$I_2 = \frac{-1}{2}V_1 + \frac{3}{4}V_2$

$[Z] = \begin{bmatrix} 6 & 4 \\ 4 & 4 \end{bmatrix}$ and

$[Y] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{4} \end{bmatrix}$



Z_a in terms of Z parameters?

$Z_{11} = Z_a + Z_c \qquad Z_{12} = Z_c$

$Z_{21} = Z_c \qquad Z_{22} = Z_b$

$\Rightarrow \boxed{Z_a = Z_{11} - Z_{12}}$

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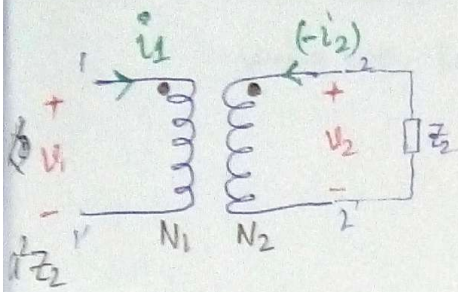
Ideal tfo as a two port network

(tfo → Transformer)



$i_2 Z_2$

$a = \frac{N_1}{N_2}$

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & -1/a \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}$$

$\frac{v_1}{N_1} = \frac{v_2}{N_2}$ or $\boxed{v_1 = a v_2}$

$i_1 = -\frac{i_2}{a}$

$N_1 i_1 = N_2(-i_2)$

Power Input to the System $= v_1 i_1 + v_2(-i_2) = 0$

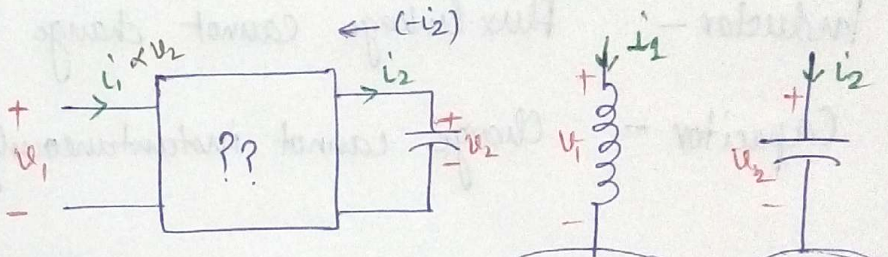
$Z_2 = \frac{v_2}{i_2}$

$\frac{v_1}{i_1} = \frac{a v_2}{(-i_2/a)} = -a^2 Z_2$

4 elements (fundamental)

(See Next Page)

R, L, C, IT (ideal tfo)



$i_1 = \frac{1}{L}\int_{-\infty}^{t} v_1 dt$

$v_2 = \frac{1}{C}\int_{-\infty}^{t} i_2 dt$

$v_1 \propto i_2$

$v_1 = +k_1 i_2$ Gyrator

Telligen

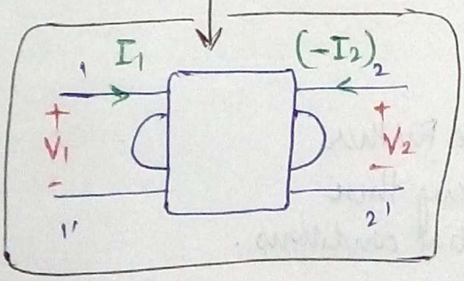
$i_1 = k_2 v_2$

$i_1 \propto v_2$

$v_1 \propto i_2$ $\Rightarrow \frac{V_1(s)}{I_1(s)} = k \frac{I_2(s)}{V_2(s)} = \frac{k}{1/cs} = kCs$

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} 0 & -k_1 \\ k_2 & 0 \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}$$

$(k_1 = \frac{1}{k_2})$



Power $\rightarrow v_1 i_1 + v_2(-i_2) = 0.$

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} 0 & k \\ 1/k & 0 \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}$$

* Non-Reciprocal Network
  (It will not follow Reciprocity ~~~~ Thm)

* If a resistance is connected at 2-2', we get a resistance only at 1-1'
* —— capacitor —— 2-2', —— ~~~~ inductor at 1-1'

ANUBHAV JAIN NOTES

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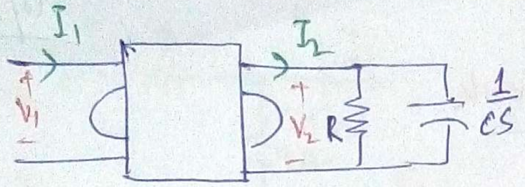


$Y(s) = \frac{1}{R} + \left(\frac{1}{cs}\right)^{-1}$

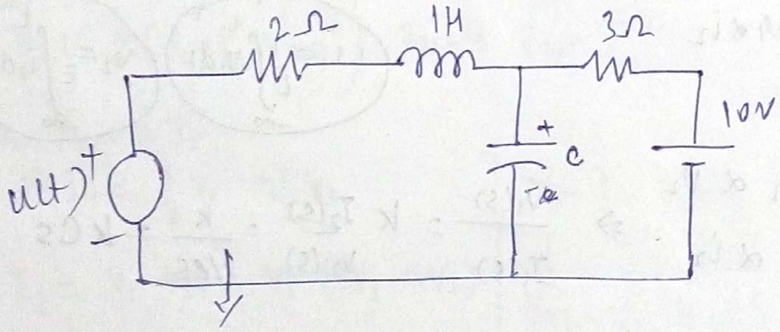
List of 5 fundamental elements:-

R , L , C , Ideal tfo , Gyrator.

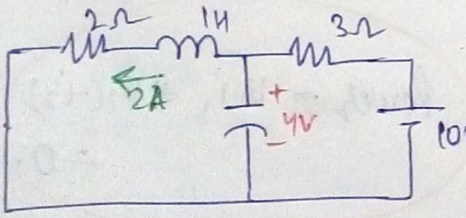
$\underbrace{\text{Ideal tfo , Gyrator}}_{\text{Lossless}}$

* Inductor - Flux linkage cannot change instantaneously

Capacitor - Charge cannot instantaneously

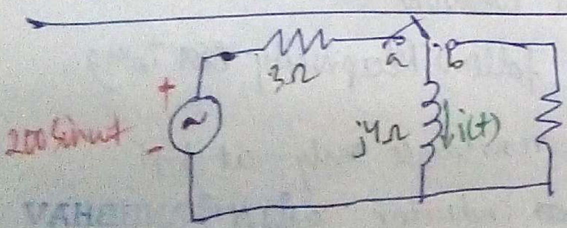


$t<0$

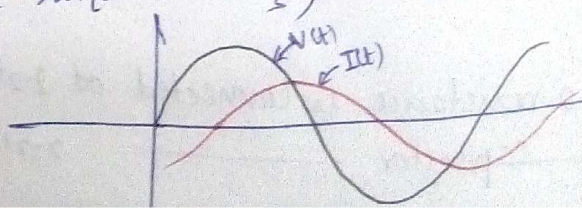


→ Solve Further taking these initial conditions.

(* Statements of Theorems can come in exam).



$i(t) = \frac{200}{5} \sin\left(\omega t - \tan^{-1}\frac{4}{3}\right)$



If switch is connected to b from a in 0 time gap,

we cannot directly tell the current through inductor until and unless we know the time at which switching happened.