Matrices:

Consider the matrix:

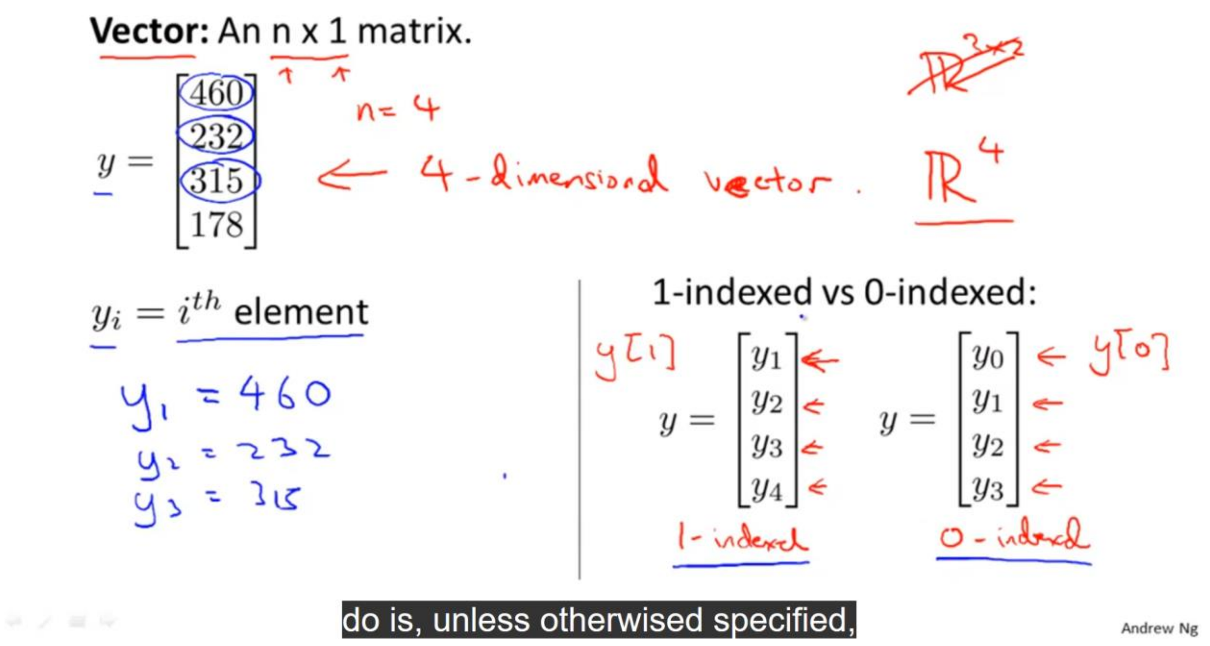
$$\begin{bmatrix} 1 & 5 \\ 2 & 6 \\ 3 & 7 \\ 4 & 8 \end{bmatrix}$$

Above matrix can be represented as $\mathbb{R}^{42}$

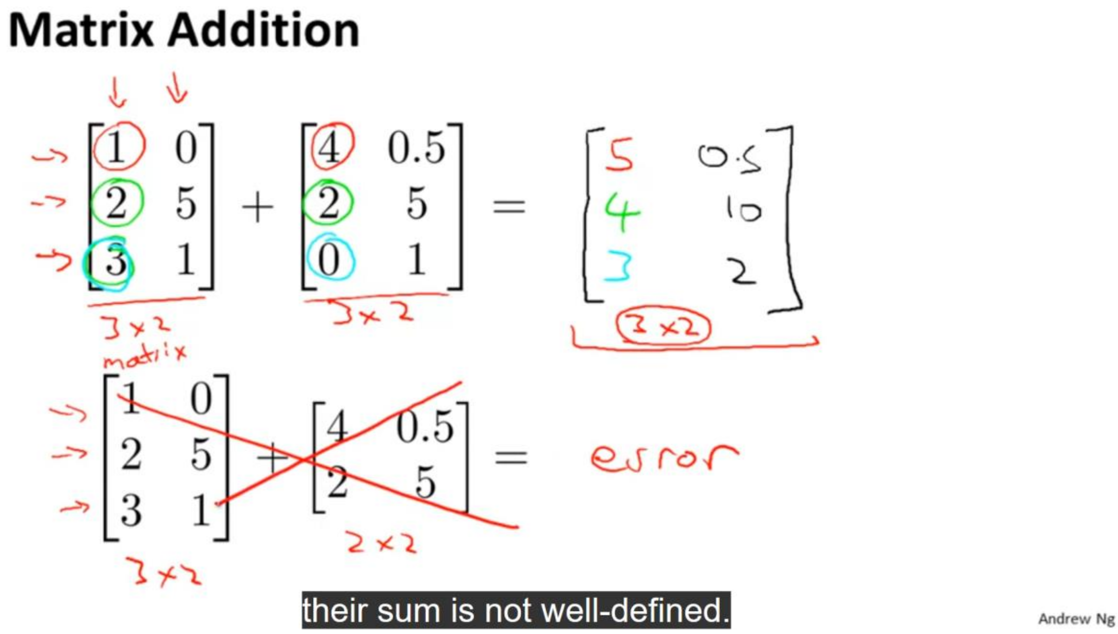
This symbol means that the matrix is an element of the family of matrices having 4 rows and 2 columns.

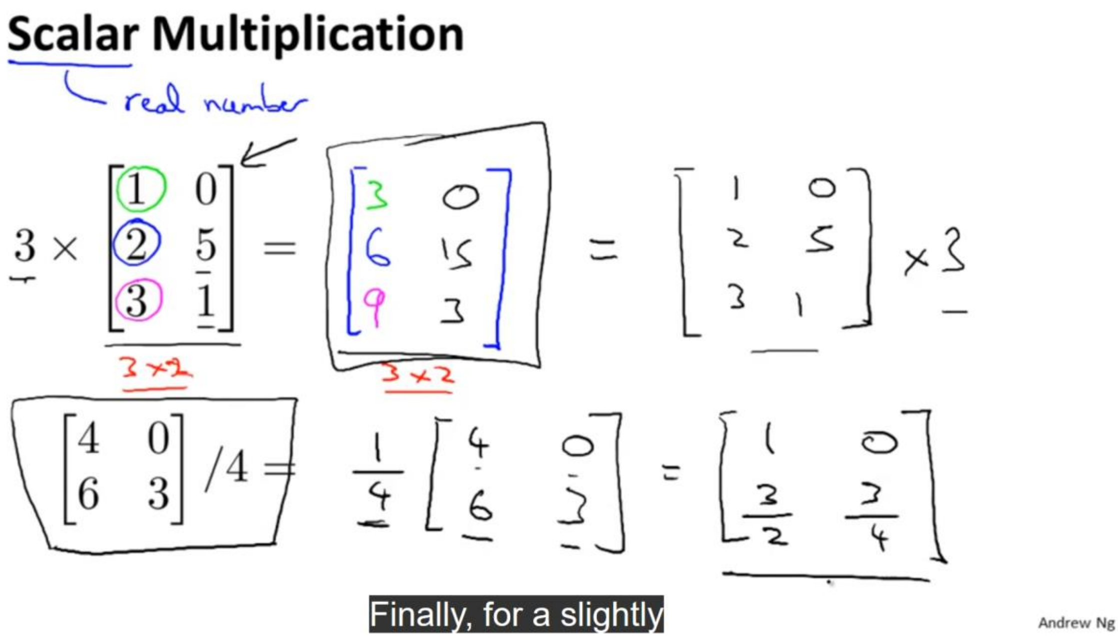
Vectors:

Vectors are matrices with n rows but only one column.



MATRIX ADDITION:





Combination of Operands

$$\boxed{3 \times \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} - \boxed{\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} / 3}$$

Scalar multiplication

Scalar division

$$= \begin{bmatrix} 3 \\ 12 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ \frac{2}{3} \end{bmatrix}$$

matrix subtraction / Vector subtraction

matrix addition / vector addition

$$= \begin{bmatrix} 2 \\ 12 \\ 10\frac{1}{3} \end{bmatrix}$$

3x1 matrix

3-dimensional vector

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Matrix vector Multiplication:

Example

$$\begin{bmatrix} 1 & 2 & 1 & 5 \\ 0 & 3 & 0 & 4 \\ -1 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} \boxed{14} \\ \boxed{13} \\ \boxed{-7} \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ -7 \end{bmatrix}$$

$\boxed{3} \times 4$ 4×1 3×1

$1 \times 1 + 2 \times 3 + 1 \times 2 + 5 \times 1 = 14$

$0 \times 1 + 3 \times 3 + 0 \times 2 + 4 \times 1 = 13$

$-1 \times 1 + (-2) \times 3 + 0 \times 2 + 0 \times 1 = -7$

So that's how you multiply a matrix and a vector.

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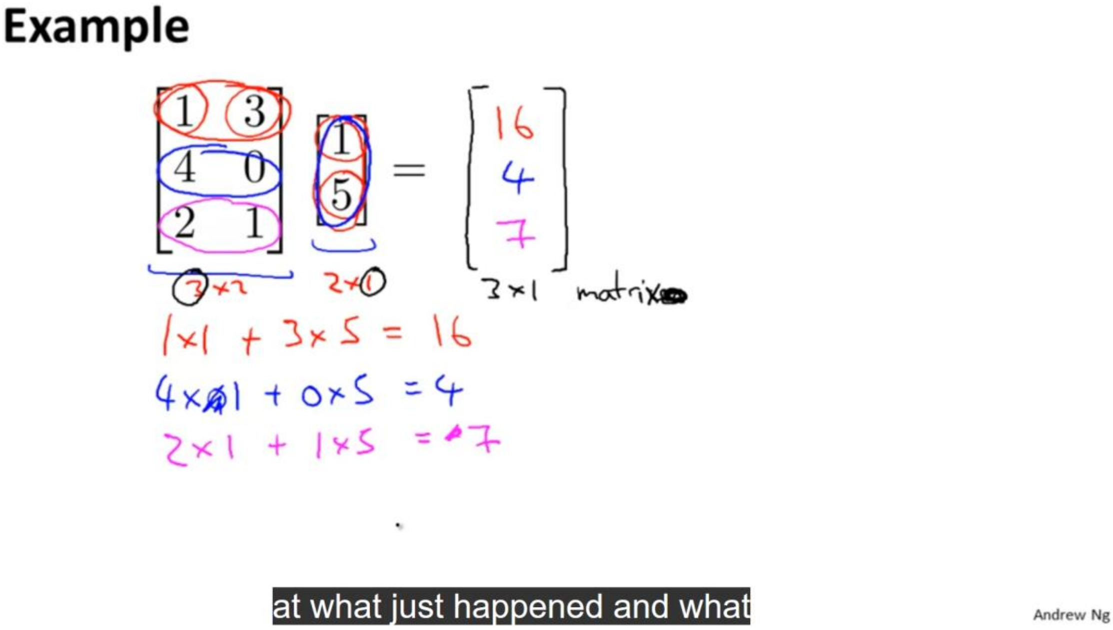
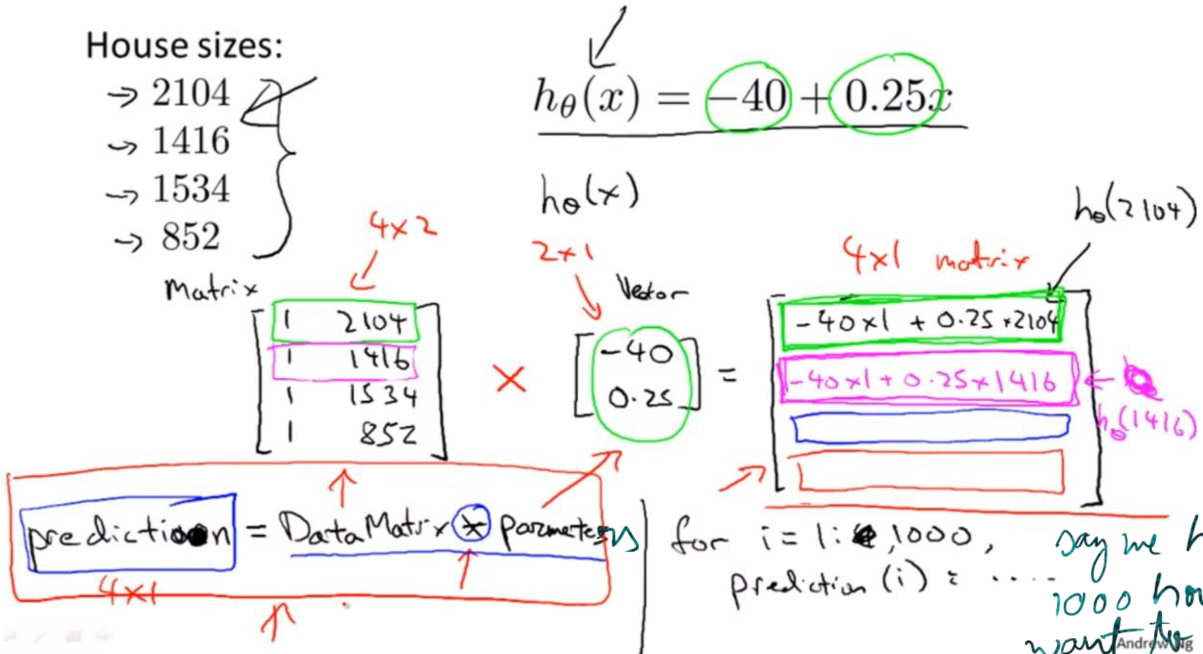


Predicting house prices using matrix vector multiplication:



say we have 1000 houses & want to predict their price w/o using matrices the task beco-mes tedious.

Multiplying two MATRICES:

Example

$$\begin{bmatrix}1 & 3 & 2\\ 4 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\0\\5\end{bmatrix}\begin{bmatrix}3\\1\\2\end{bmatrix} = \begin{bmatrix}11 & 10\\ 9 & 14\end{bmatrix}$$

$$\begin{bmatrix}1 & 3 & 2\\ 4 & 0 & 1\end{bmatrix} \times \begin{bmatrix}1\\0\\5\end{bmatrix} = \begin{bmatrix}11\\9\end{bmatrix}$$

$$\begin{bmatrix}1 & 3 & 2\\ 4 & 0 & 1\end{bmatrix} \times \begin{bmatrix}3\\1\\2\end{bmatrix} = \begin{bmatrix}10\\14\end{bmatrix}$$

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Eg 2:

$$\underset{2\times 2}{\begin{bmatrix}1 & 3\\ 2 & 5\end{bmatrix}}\ \underset{2\times 2}{\begin{bmatrix}0\\3\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}} = \underset{2\times 2}{\begin{bmatrix}9 & 7\\ 15 & 12\end{bmatrix}}$$

$$\begin{bmatrix}1 & 3\\ 2 & 5\end{bmatrix}\begin{bmatrix}0\\3\end{bmatrix} = \begin{bmatrix}1\times 0 + 3\times 3\\ 2\times 0 + 5\times 3\end{bmatrix} = \cancel{\begin{bmatrix}9\\15\end{bmatrix}}\ \begin{bmatrix}9\\15\end{bmatrix}$$

$$\begin{bmatrix}1 & 3\\ 2 & 5\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}1\times 1 + 3\times 2\\ 2\times 1 + 5\times 2\end{bmatrix} = \begin{bmatrix}7\\12\end{bmatrix}$$

Predicting Housing prices using 3 equations simultaneously:

House sizes:

$$\begin{Bmatrix} 2104 \\ 1416 \\ 1534 \\ 852 \end{Bmatrix}$$

Have 3 competing hypotheses:

1. $h_\theta(x) = -40 + 0.25x$
2. $h_\theta(x) = 200 + 0.1x$
3. $h_\theta(x) = -150 + 0.4x$

Matrix $\qquad$ Matrix

$$\begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix} \times \begin{bmatrix} \boxed{\begin{matrix} -40 \\ 0.25 \end{matrix}} & \boxed{\begin{matrix} 200 \\ 0.1 \end{matrix}} & \boxed{\begin{matrix} -150 \\ 0.4 \end{matrix}} \end{bmatrix} = \begin{bmatrix} \boxed{\begin{matrix} 486 \\ 314 \\ 344 \\ 173 \end{matrix}} & \boxed{\begin{matrix} 410 \\ 342 \\ 353 \\ 285 \end{matrix}} & \boxed{\begin{matrix} 692 \\ 416 \\ 464 \\ 191 \end{matrix}} \end{bmatrix}$$

Prediction of first h_θ $\qquad$ Predictions of 2^{nd} h_θ

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Properties of Matrix Multiplication:

1. Matrix multiplication is not cummutative

$3 \times 5 = 5 \times 3$ $\qquad$ "Commutative"

Let A and B be matrices. Then in general,

$A \times B \neq B \times A$. (not commutative.)

E.g. $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$

$\neq$

$\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$

$\underset{m \times n}{A} \times \underset{n \times m}{B}$

$A \times B$ is $m \times m$

$B \times A$ is $n \times n$

Matrix multiplication is Assosciative:

$3 \times 5 \times 2$ $\qquad 3 \times (5 \times 2) = (3 \times 5) \times 2$ $\qquad$ "Associative"

$3 \times 10 = 30 = 15 \times 2$

let A, B & C be 3 matrices

$A \times (B \times C)$

$(A \times B) \times C$

same

$A \times B \times C.$

Let $D = B \times C.$ Compute $A \times D.$ $\qquad A \times (B \times C)$

Let $E = A \times B.$ Compute $E \times C.$ $\qquad (A \times B) \times C$

Same

Identity matrix:

1 is identity. $\qquad \boxed{1 \times z = z \times 1 = z}$ for any z

Identity Matrix

Denoted I (or $\boxed{I_{n\times n}}$).

Examples of identity matrices:

$$[1] \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1x1 $\qquad$ 2 x 2 $\qquad$ 3 x 3 $\qquad$ 4 x 4

Informally:

$$\begin{bmatrix} 1 & & 0 \\ & 1 & \\ & & 1 \\ 0 & & \ddots & \\ & & & 1 \end{bmatrix}$$

Informal representation of identity matrix

For any matrix A,

$$A \cdot \boxed{I} = \boxed{I} \cdot A = A$$

$m \times n \qquad n \times n \qquad m \times m \qquad m \times n \qquad m \times n$

$I_{n \times n}$

Note:

$AB \neq BA$ in general

$AI = IA$ ✓

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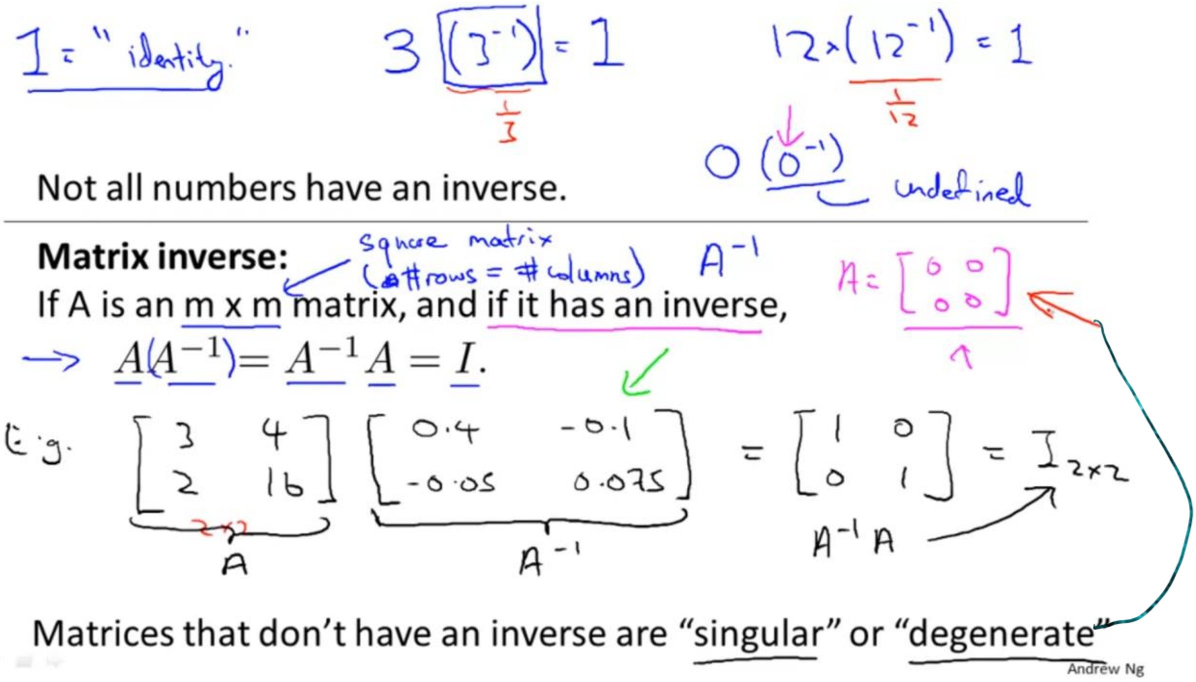
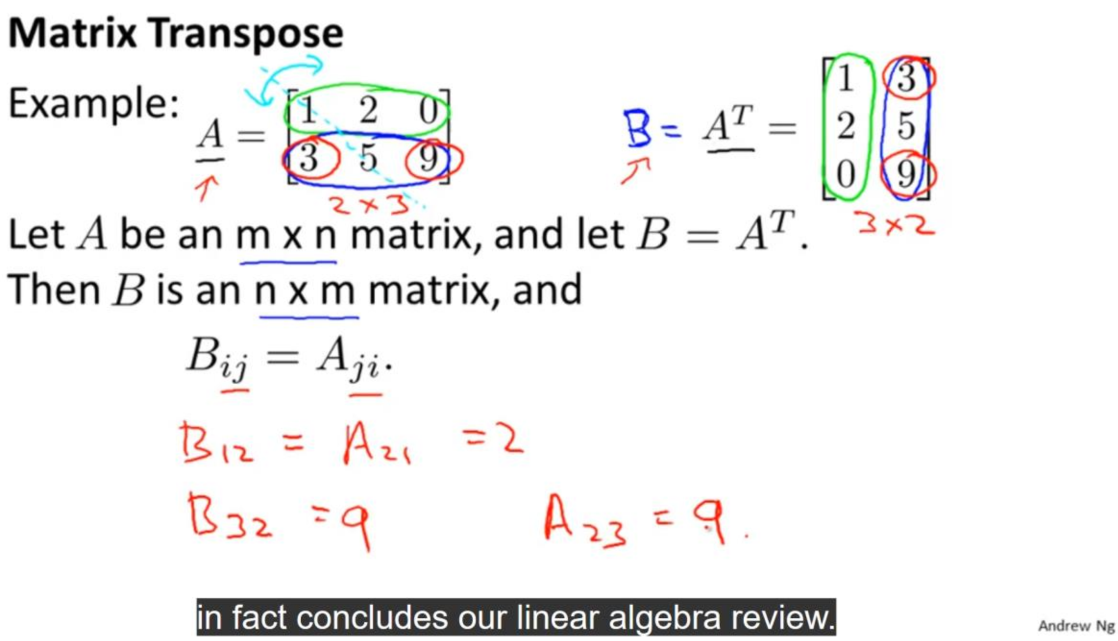
Inverse of a Matrix:

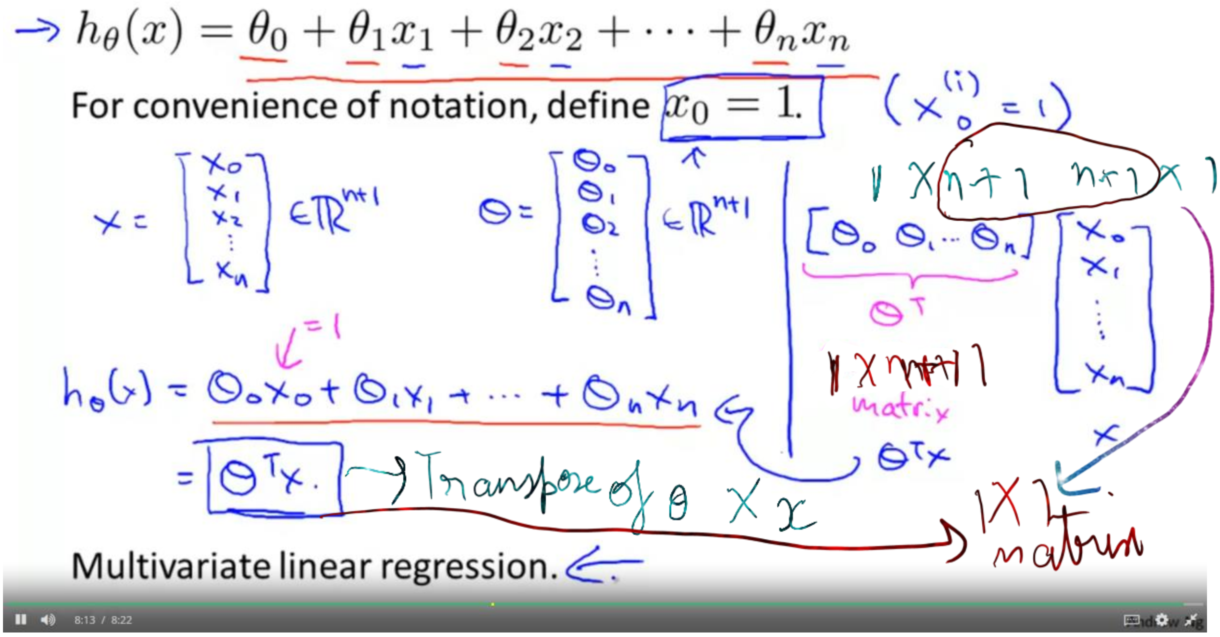


Transpose of a Matrix:

In transpose , rows become columns and columns become rows.



Multivariate Linear Regression:



Transpose of θ = $[\theta_0, \theta_1, \theta_2 \ldots\ldots \theta_n]$

We assume $x_0 = 1$ so that we can form a matrix of x of n+1 terms from x_0 to x_{n+1} .

Multiple Features

Note: [7:25 - θ^T is a 1 by (n+1) matrix and not an (n+1) by 1 matrix]

Linear regression with multiple variables is also known as "multivariate linear regression".

We now introduce notation for equations where we can have any number of input variables.

$x_j^{(i)}$ = value of feature j in the i^{th} training example
$x^{(i)}$ = the input (features) of the i^{th} training example
m = the number of training examples
n = the number of features

feature means columns & hence j represents columns whereas i represents rows. So x^4 represents 4th row whereas x^4_1 represents data there in 1st column of 4th row.

The multivariable form of the hypothesis function accommodating these multiple features is as follows:

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \cdots + \theta_n x_n$$

In order to develop intuition about this function, we can think about θ_0 as the basic price of a house, θ_1 as the price per square meter, θ_2 as the price per floor, etc. x_1 will be the number of square meters in the house, x_2 the number of floors, etc.

Using the definition of matrix multiplication, our multivariable hypothesis function can be concisely represented as:

$$h_\theta(x) = \begin{bmatrix} \theta_0 & \theta_1 & \ldots & \theta_n \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix} = \theta^T x$$

Remark: Note that for convenience reasons in this course we assume $x_0^{(i)} = 1 \text{ for } (i \in 1, ..., m)$. This allows us to do matrix operations with theta and x. Hence making the two vectors 'θ' and $x^{(i)}$ match each other element-wise (that is, have the same number of elements: n+1).]