OPERATIONS RESEARCH

"OR is the application of scientific methods, techniques and tools to problems involving the operations of systems so as to provide these in control of the operations with optimum solutions to the problem"

Modelling in OR:

- Classification by <u>structure</u>:
  - Iconic models
  - Analogue models
  - Mathematical (Symbolic) models.

- Classificatⁿ by <u>Purpose</u>:
  - descriptive
  - predictive
  - prescriptive
    eg: LP

- Classification by <u>Nature of Environment</u>
  - Deterministic
    eg: LP
  - Probabilistic (or Stochastic)

- Classification by <u>Behaviour</u>
  - Static
  - Dynamic

- Classificatⁿ by <u>Method of solⁿ</u>
  - Analytical
  - Simulation

- Classificatⁿ by <u>Use of Digital Computers</u>
  - Analogue & Mathematical combined
  - Function models
  - Quantitative models
  - Heuristic models

Characteristics of OR

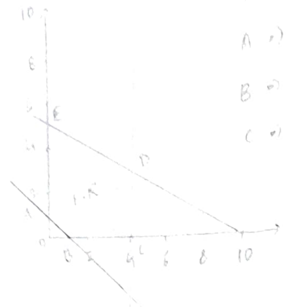
- Inter-disciplinary team approach
- Wholistic approach to the system
- Imperfections ness of solutions
- Use of scientific research
- To optimize total output

Phases of OR

1) Formulating problem
2) Constructing a mathematical model
3) Deriving solutions from the model
4) Testing the model and its solution
5) Controlling the solution
6) Implementing the solution

Scope of OR

- in Agriculture
- in Finance
- in Industry
- in Marketing
- in Personnel Management
- in Production Management
- in UC



A ⇒ (0,1) ⇒ $Z = 1$ min.

B ⇒ (1,0)

C ⇒ (4,0)

11/4/23

LPP Simplex Method

Standard form of LPP

$$Z_{max} = c_1x_1 + c_2x_2 + c_3x_3 + \dots$$

s.t.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots \quad \le = \ge b_1$$

$$a_{21}x_1 + \dots \quad \le = \ge b_2$$

$$\vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \dots \quad \le = \ge b_m$$

Conditions:

1) RHS of constraints should be positive.
2) Should be a maximizatn problem

eg: $Z_{max} = 2x_1 + 4x_2$

$Z_{min} = 3x_1 + 5x_2$

→ $Z_{max} = -3x_1 - 5x_2$

st. 1) $3x_1 + 6x_2 \le 4$

$3x_1 + 6x_2 + x_3 = 4$

2) $2x_1 + 5x_2 = 6$

$2x_1 + 5x_2 + A_1 = 6$

3) $4x_1 + 8x_2 \ge 10$

surplus variable

$4x_1 + 8x_2 - x_4 + A_2 = 10$

slack variable

$Z_{max} = 2x_1 + 4x_2 + 0x_3 + 0x_4 - MA_1 - MA_2$ → Big M method

		2	4	0	0	-M	-M	
		x_1	x_2	x_3	x_4	A_1	A_2	b_i
0	x_3	3	6	1	0	0	0	4
-M	A_1	2	5	0	0	1	0	6
-M	A_2	4	8	0	-1	0	1	10

if less than equal → slack variable

if = → add artificial variable

if ≥ → add both.

8) Use simplex method to solve the LPP

max $Z = 3x_1 + 2x_2$

s.t. $x_1 + x_2 \le 4$

$x_1 - x_2 \le 2$

and $x_1, x_2 \ge 0$

Add slack variables

$x_1 + x_2 + x_3 = 4$

$x_1 - x_2 + x_4 = 2$

$Z_{max} = 3x_1 + 2x_2 + 0x_3 + 0x_4$

IBFS (Initial Basic feasible soln) ⇒ $x_3 = 4$, $x_4 = 2$

		3	2	0	0		
		x_1	x_2	x_3	x_4	b_i	$b_i/$
0	x_3	1	1	1	0	4	4/1
0	x_4	1	-1	0	1	2	2/1
$z_j - c_j$		-3	-2	0	0		

EV (entering variable) ⇓ (highest -ve)

least +ve value is departing variable

pivot element

x_1 is entering & x_4 is departing

	x_1	x_2	x_3	x_4	b_i	b_i/x_i
x_3						
x_1	1	-1	0	1	2	

$x_{2new} = x_{3old} - x_1$

1	1	1	0	4
(−) 1	−1	0	1	2
0	2	1	−1	2

⇒

		3	2	0	0		
		x_1	x_2	x_3	x_4	b_i	$b_i/x_i = b_i/x_2$
0	x_3	0	$\boxed{2}$	1	−1	2	$2/2 = 1$
3	x_1	1	−1	0	1	2	$2/-1 = -$
	$z_j - c_j$	0	−5	0	3		

↑ EV pivot DV

−ve & indeterminate → dont consider

Need to make pivot as 1

⇒ divide $x_{2new} = \frac{x_{3old}}{2}$

$x_{1new} = x_{2new} + x_{1old}$

0	1	1/2	−1/2	1
1	−1	0	1	2
1	0	1/2	1/2	3

		3	2	0	0	
		x_1	x_2	x_3	x_4	b_i
2	x_2	0	1	1/2	−1/2	1
3	x_1	1	0	1/2	1/2	3
	$z_j - c_j$	0	0	5/2	1/2	11

All $(z_j - c_j) \geq 0$

The solution is optimal

The optimal solution is $(x_1 = 3, x_2 = 1)$

$\Rightarrow Z_{max} = 3x_1 + 2x_2 = 9 + 2 = 11$

Q) Minimize $Z = 5x_1 + 6x_2$

s.t. $2x_1 + 5x_2 \geq 15$

$3x_1 + x_2 \geq 12$

& $x_i \geq 0$

$Z_{max} = -5x_1 - 6x_2 + 0x_3 + 0x_4 - MA_1 - MA_2$

s.t. $2x_1 + 5x_2 - x_3 + A_1 = 15$

$3x_1 + x_2 - x_4 + A_2 = 12$

$x_i \geq 0$

		−5	−6	0	0	−M	−M		
		x_1	x_2	x_3	x_4	A_1	A_2	b_i	b_i/x_i
−M	A_1	2	$\boxed{5}$	−1	0	1	0	15	$15/5 = 3$ → DV
−M	A_2	3	1	0	−1	0	1	12	$b_i/x_2 = 12$
	$z_j - c_j$	−5M+5	−6M+6	M	M	0	0		

↑ EV

$A_{2new} = A_{2old} - x_2$

2/5	1	−1/5	0	1/5	0	3
3	1	0	−1	0	1	12
13/5	0	1/5	−1	−1/5	1	9

		−5	−6	0	0	−M	−M		
		x_1	x_2	x_3	x_4	~~A_1~~	A_2	b_i	b_i/x_i
−6	x_2	2/5	1	−1/5	0	~~1/5~~	0	3	15/2
−M	A_2	$\boxed{13/5}$	0	1/5	−1	~~−1/5~~	1	9	45/13
	$z_j - c_j$	$-\frac{13M}{5} + \frac{13}{5}$	0	$-\frac{M}{5} + \frac{6}{5}$	M		0		

↑ EV

			6	0	0		
		x_1	x_2	x_3	x_4	b_i	b_i/x_i
-6	x_2	0	$5/2$	$-\frac{15}{26}$	$-\frac{5}{13}$	$\frac{105}{26}$	$-\frac{315}{13}$
-5	x_1	1	0	$1/13$	$-5/13$	$\frac{45}{13}$	
		0	-9				$-\frac{540}{13}$

$x_{2_{new}} = x_{2_{old}} - x_{1_{new}}$

$\left(2/5 \quad 1 \quad -1/5 \quad 0 \quad 3\right) \times \frac{5}{2}$

$1 \quad 0 \quad 1/13 \quad -5/13 \quad \frac{45}{13}$

$0 \quad 5/2 \quad \frac{-15}{26}$

(B) Min $Z = 2x_1 + x_2$

s.t

$x_1 + 2x_2 \leq 4$

$4x_1 + 3x_2 \geq 6$

$3x_1 + x_2 = 3$

& $x_i \geq 0$

max $Z = -2x_1 - x_2$

$\Rightarrow$

$x_1 + 2x_2 + x_3 = 4$

$4x_1 + 3x_2 - x_4 + A_1 = 6$

$3x_1 + x_2 + A_2 = 3$

$Z_{max} = -2x - x_2 + 0x_3 + 0x_4 - MA_1 - MA_2$

		-2	-1	0	0	-M	-M			
		x_1	x_2	x_3	x_4	A_1	A_2	b_i	b_i/x_i	
0	x_3	1	2	1	0	0	0	4	4	
-M	A_1	4	3	0	-1	1	0	6	$6/4 = 1.5$	
-M	A_2	$\boxed{3}$	1	0	0	0	1	3	$3/3 = 1$	← DV
$Z_j - C_j$		$-7M+2$	$-4M+1$	0	M	0	0			

↓ EV

		-2	-1	0	0	-M		
		x_1	x_2	x_3	x_4	A_1	b_i	b_i/x_i
0	x_3	0	$5/3$	1	0	0	3	
-M	A_1	0	$5/12$	0	$-1/4$	$1/4$	$1/2$	
-2	x_1	1	$1/3$	0	0	0	1	
$Z_j - C_j$		0	$-\frac{5M}{12} + 1/3$	0	$M/4$	$\frac{3M}{4}$		

$\frac{1}{4} \times \quad 4 \quad 3 \quad 0 \quad -1 \quad 1 \quad 0$

$1 \quad 3/4 \quad 0 \quad -1/4 \quad 1/4 \quad 0$

$1 \quad 1/3 \quad 0 \quad 0 \quad 0 \quad 1$

$0 \quad 5/12 \quad 0 \quad -1/4 \quad 1/4 \quad 1/2$

If tie in DV $\rightarrow$ degeneracy problem

13/4/23

(4) Max $z = 3x_1 + 2x_2$

S.t $x_1 - x_2 \le 15$, $2x_1 - x_2 \le 40$

Add slack variables

$x_1 - x_2 + x_3 = 15$

$2x_1 - x_2 + x_4 = 40$

$Z_{max} = 3x_1 + 2x_2 + 0x_3 + 0x_4$

$x_1, x_2, x_3, x_4 \geq 0$

IBFS $x_3 = 15$ & $x_4 = 40$

		3	2	0	0		
		x_1	x_2	x_3	x_4	b_i	b_i/x_i
0	x_3	1	-1	1	0	15	15
0	x_4	2	-1	0	1	40	20
$z_j - c_j$		-3	-2	0	0		

↑ EV

$x_{4\,new} = x_{4\,old} - 2x_{1\,new}$

	2	-1	0	1	40	0
(-)	2	-2	2	0	30	0
	0	1	-2	1	10	

	x_1	x_2	x_3	x_4	b_i	b_i/x_i
3 x_1	1	-1	1	0	15	—
0 x_4	0	1	-2	1	10	10 → DV
$z_j - c_j$	0	-5	3	0		

(-ve not considered)

$x_{1\,new} = x_{1\,old} + x_{2\,new}$

1	-1	1	0	15
0	1	-2	$\cancel{0}$1	10
1	0	-1	1	25

	3	2	0	0		
	x_1	x_2	x_3	x_4	b_i	b_i/x_i
3 x_1	1	0	-1	1	25	—
2 x_2	0	1	-2	1	10	—
$z_j - c_j$	0	0	-7	5		

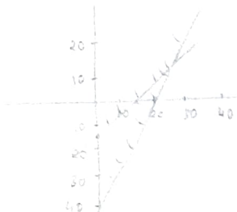
Entering variable is x_3

There is no departing variable

→ Solution is unbounded.

(0, -15) (15, 0)

(0, -40) (20, 0)



unbounded region

(5) $Z_{max} = 10x_1 + x_2 + 2x_3$

s.t

$14x_1 + x_2 - 6x_3 + x_4 = 7$

$16x_1 + x_2 - 6x_3 \leq 5$

$3x_1 - x_2 - x_3 \leq 0$

$x_1, x_2, x_3 \geq 0$

if not 1 coeff make equal by dividing

$\Rightarrow$ $Z_{max} = 10x_1 + x_2 + 2x_3 + 0x_4 + 0x_5 + 0x_6$

$14x_1 + x_2 - 6x_3 + x_4 = 7$

$16x_1 + x_2 - 6x_3 + x_5 = 5$

$3x_1 - x_2 - x_3 + x_6 = 0$

		10	1	2	0	0	0		
		x_1	x_2	x_3	x_4	x_5	x_6	b_i	b_i/x_i
0	x_4	14	1	-6	1	0	0	7	1/2
0	x_5	16	1	-6	0	1	0	5	0.3125
0	x_6	$\boxed{3}$	-1	-1	0	0	1	0	0 → DV
$z_j - c_j$		-10	-1	-2	0	0	0		

↓ EV

1 $-1/3$ $-1/3$ 0 0 $1/3$ 0

		10	1	2	0	0	0		
		x_1	x_2	x_3	x_4	x_5	x_6	b_i	b_i/x_i
0	x_4	0	$15/3$	$-4/3$	1	0	0	7	–
0	x_5	0	$19/3$	$-2/3$	0	1	0	5	–
10	x_1	1	$-1/3$	$-1/3$	0	0	$1/3$	0	0 (-ve)
		0	$-13/3$	$-16/3$	0	0	$10/3$		

No departing variable

$\Rightarrow$ Unbounded region

(6) (Infeasible solⁿ example)

$Z_{max} = 3x_1 + 4x_2$

s.t

$2x_1 + x_2 \leq 4$

$x_1 + 2x_2 \geq 12$

$x_1, x_2 \geq 0$

$\Rightarrow$ $2x_1 + x_2 + x_3 = 4$

$x_1 + 2x_2 - x_4 + A_1 = 12$

$Z_{max} = 3x_1 + 4x_2 + 0x_3 + 0x_4 - MA_1$

		3	4	0	0	-M		
		x_1	x_2	x_3	x_4	A_1	b_i	b_i/x_i
0	x_3	2	$\boxed{1}$	1	0	0	4	4 → DV
-M	A_1	1	2	0	-1	1	12	6
$z_j - c_j$		-M-3	-2M-4	0	M	0		

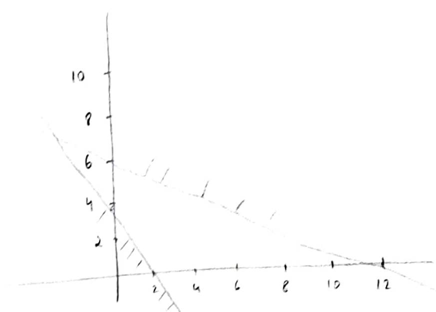
↓ EV

		3	4	0	0	-M		
		x_1	x_2	x_3	x_4	A_1	b_i	b_i/x_i
4	x_2	2	1	1	0	0	4	
-M	A_1	-3	0	-2	-1	1	4	
$z_j - c_j$		3M+5	0	2M+4	M	0		

(0, 4) (2, 0)

(12, 0) (0, 6)

all +ve but artificial variable exist. $\Rightarrow$ infeasible



10/4/23

(1) Max $z = 2x_1 + x_2$

st $3x_1 + x_2 \le 6$

$x_1 - x_2 \le 2$

$x_2 \le 3$

and $x_i \geq 0$

		x_1	x_2	x_3	x_4	x_5	b_i	b_i/x_i
0	x_3	3	1	1	0	0	6	2
0	x_4	1	-1	0	1	0	2	2
0	x_5	0	1	0	0	1	3	-
	$z_j - c_j$	-2	-1	0	0	0		
		↑ EV						

} tie → DV

⇒ degeneracy in simplex method

Method of resolving tie

- min of $\dfrac{\text{first column of the unit matrix}}{\text{corresponding elements of key (EV) column}}$

⇒ min of $\dfrac{x_3}{x_4}$ ⇒ $1/3$ for x_3 row

$0/1$ for x_4 row → DV

		2	1	0	0	0		
		x_1	x_2	x_3	x_4	x_5	b_i	b_i/x_i
0	x_3	0	4	1	-3	0	0	$0/4$ → DV
2	x_1	1	-1	0	1	0	2	$2/-1$
0	x_5	0	1	0	0	1	3	3
	$z_j - c_j$	0	-3	0	2	0		
			↑ EV					

		2	1	0	0	0		
		x_1	x_2	x_3	x_4	x_5	b_i	b_i/x_i
1	x_2	0	1	$1/4$	$-3/4$	0	0	$0/-ve$
2	x_1	1	0	$1/4$	$1/4$	0	0	0 → DV
0	x_5	0	1	0	0	1	3	-
	$z_j - c_j$	0	0	$3/4$	$-1/4$	0		
					↑ EV			

		2	1	0	0	0		
		x_1	x_2	x_3	x_4	x_5	b_i	b_i/x_i
1	x_2	+3	1	-1	0	0	0	
0	x_4	4	0	1	1	0	0	
0	x_5	0	1	0	0	1	3	
	$z_j - c_j$	1	0	1	0	0		

(2) Max $z = 2x_1 + x_2$

s.t $4x_1 + 3x_2 \le 12$

$4x_1 + x_2 \le 8$

$4x_1 - x_2 \le 8$

& $x_i \geq 0$

		2	1	0	0	0		
		x_1	x_2	x_3	x_4	x_5	b_i	b_i/x_i
0	x_3	4	3	1	0	0	12	3
0	x_4	4	1	0	1	0	8	2
0	x_5	4	-1	0	0	1	8	2 - DV
	$z_j - c_j$	-2	-1	0	0	0		
		↑ EV						

- min for $x_4 \to 0/4 \to 0$
  $x_5 \to 0/4 \to 0$ } tie

second column ⇒ $1/4$ →
& $0/4$ → DV

		2	1	0	0	0		
		x_1	x_2	x_3	x_4	x_5	b_i	b_i/x_i
0	x_3	0	4	1	0	$-1/4$	4	1
0	x_4	0	2	0	1	0	0	0 → DV
2	x_1	1	$-1/4$	0	0	$1/4$	2	–
	$z_j - c_j$	0	$-3/2$	0	0	$1/2$		
			↑ EV					

		2	1	0	0	0	
		x_1	x_2	x_3	x_4	x_5	b_i
0	x_3	0	0	1	-2	$-1/4$	4
1	x_2	0	1	0	$1/2$	0	0
2	x_1	1	0	0	-2	$1/4$	2
	$z_j - c_j$	0	0	0		$1/2$	

UNIT - 2

Transportation Problem

Types of TP

- Unbalanced T.P
- Degeneracy in T.P
- Maximization in T.P

IBFS

1) North West Corner Method (stepping stone method)
2) Least Cost Method
3) Vamp / Penalty method
4)
5) Row minima
6) Column minima

optimality check

1) MODI Method (u-v method)
2) Stepping stone method

Basic Feasible Solⁿ (BFS) - if $m+n-1$ is the no. of reallocations

(FS): set of non-neg. individual allocat's which simultaneously removes deficiencies

1) Find IBFS

	D_1	D_2	D_3	D_4	supply
S_1	15 (60)	13	10	14	60
S_2	12	14	11	15	75
S_3	13	12	14	11	25
S_4	10	11	13	12	100
	~~70~~ 10	80	60	40	

→ NW corner → S1D1
→ we can allocate max 60 (out of 60 & 70)

1) NWCM

	D_1	D_2	D_3	D_4	
S_1	15 (50)	13	10	14	~~50~~
S_2	12 (20)	14 (65)	11	15	~~75~~ 65
S_3	13	12 (15)	14 (10)	11	~~25~~ ~~10~~
S_4	10	11	13 (50)	12 (40)	~~100~~ ~~50~~ 10
	~~70~~ 20	~~80~~ ~~15~~	~~60~~ ~~50~~	~~40~~	250 / 250

If supply ≠ demand ⇒ unbalanced problem
→ add dummy row & column

$15 \times 50 + 12 \times 20 + 14 \times 65 + 12 \times 15 + 14 \times 10 + 13 \times 50 + 12 \times 40$

2) LCM (Least Cost Method)

15	13	10 (50)	14	50
17	14 (10)	11 (15)	15 (15)	25
12	17	14	11 (25)	25
10 (70)	11 (20)	12	14	100
70	80 50	60	40	

IBFS = 10×50 + 11×10 +
15×15 + 11×25 + 14×50
+ 10×70 + 11×20

= 500 + 110 + 225
+ 275 + 700 + 700
+ 220

= 2840

25
25
700
1400
500
110
225
275
220
2840

3) Penalty method

(Next 12 - 10)
13 - 10

15	13	10 (50)	14	50
12	14 (65)	11 (10)	16	25 65
13	12	14	11 (25)	25
10 (70)	11 (15)	13	12 (15)	100 30
70	~~80~~ 15	~~60~~ 10	~~40~~ 15	

P1	P2	P3	P4
(3)	-	-	-
(1)	(1)	(3)	(1)
(1)	(1)	(1)	(1)
(1)	(1)	(1)	(1)

P1	(2)	(1)	(1)	(1)
P2	(2)	(1)	(2)	(1)
P3	-	(1)	(2)	(1)
P4	-	(1)	-	(1)

10×50 + 11×10 + 14×65 + 11×25 +
12×15 + 11×15 + 10×70

IBFS = 2840

1275
60
12

14×65
70
84
910

700
500
110
275
165
910
180
2840

1)

	$v_1 = 10$	$v_2 = 11$	$v_3 = 8$	$v_4 = 12$	
$u_1 = 2$	15 3	13 0	10 (50)	14 0	50
$u_2 = 3$	12 -	14 (65)	11 (10)	16	25
$u_3 = -1$	13	12	14	11 (25)	25
$u_4 = 0$	10 (70)	11 (15)	13	12 (15)	100
	70	80	60	40	

$v_1 = 10 - 0$
$v_2 = 11 - 0$

$c_{ij} = u_i + v_j$

$c_{41} = u_4 + v_1$
$10 = 0 + v_1 \Rightarrow v_1 = 10$

$c_{34} = u_3 + v_4$
$11 = u_3 + 12 \Rightarrow u_3 = -1$

$c_{22} =$
$14 = u_2 + 11 \Rightarrow u_2 = 3$

$c_{23} = 3 + v_3 = 11 \Rightarrow v_3 = 8$

$\Delta_{ij} = c_{ij} - (u_i + v_j)$

$\Delta_{ij} \geq 0$ (check for unallocated cells)

$\Delta_{11} = 15 - (2 + 10) = 3 \geq 0$

$\Delta_{12} = 13 - 13 = 0 \geq 0$

~~$\Delta_{13} = 0$~~

$\Delta_{14} = 14 - 14 = 0$

-ve value ⇒ form a closed loop from -ve cell to other cells (allocated cells)

$\boxed{+x}$

12 - - - - 14 (65) $\boxed{65 - x}$
+x -x

(70) 10 - - - - 11 (15) $\boxed{15 + x}$
-x +x

$\boxed{70 - x}$

out of 65 - x & 70 - x
65 - x would give lesser value thus use 65

⇒ take x = 65

(for optimum (minimum) transport cost)

	$v_1=10$	$v_2=11$	$v_3=9$	$v_4=12$	
$u_1=1$	15	13	10 (50)	14	50
$u_2=2$	12 (65)	14 (0)	11 (10)	15	75
$u_3=-1$	13	12	14	11 (25)	25
$u_4=0$	10 (5)	11 (80)	13	12 (15)	100
	70	80	60	40	

Again check (-ve)

if All $\Delta_{ij} \geq 0$

solⁿ is optimal

calculate IBFS

$50\times10 + 11\times10 + 15\times12 + 11\times25$
$+ 12\times65 + 80\times11 + 50$
$= 2775$ ⇒ optimal

12×65
60
72
150
30
1
500
110
180
275
1065
880
50
790
2785

Q) 4×4 matrix,

4	6	8	13	0	50
13	11	10	8	0	70
14	4	10	13	0	30
9	11	16	8	0	50
25	35	105	20	15	200 / 185

⇒ unbalanced ⇒ add dummy rows & columns

Penalty method

						P1	P2	P3	P4	P5
4 (25)	6 (5)	8 (20)	13	0	~~50~~ ~~25~~ 20	(4)	(2)	(2)	(2)	(5)
13	11	10 (55)	8	0 (15)	70	(8)	(2)	(2)	(2)	(2)
14	4 (30)	10	13	0	30	(4)	(6)	-	-	-
9	11	16 (30)	8 (20)	0 (~~20~~)	~~50~~ 35	(8)	(1)	(1)	(3)	(8)
~~25~~	~~35~~ 5	105	20	~~15~~						

P1 (5) (2) (2) (0) (0)
P2 (5) (2) (2) (0) -
P3 (5)↑ (5), (2) (0)
P4 - (5)↑ (2) (0)

IBFS

	$v_1=4$	$v_2=6$	$v_3=8$	$v_4=0$	$v_5=-2$	
$u_1=0$	4 (25)	6 (5)	8 (20)	13 +	0 +	50
$u_2=2$	13 +	11 +	10 (55) +x	8 +	0 −x (15)	70
$u_3=-2$	14 +	4 (30)	10 +	13 +	0 +	30
$u_4=8$	9 −	11 −	16 −x (80)	8 (20)	0 − +x	50
	25	35	105	20	15	

IBFS = $4\times25 + 5\times6 + 8\times20 + 10\times55 + 30\times4 + 16\times30 + 8\times20$
= Rs 1600 /-

55+x, 15−x, 30−x, +x

$x = 15$.

Can consider u_1 or v_3 as 0.

Δ_{ij}, Highest negative for u_4v_5

	$v_1=4$	$v_2=6$	$v_3=8$	$v_4=0$	$v_5=8$	
$u_1=0$	4 (25)	6 (5)	8 (20)	13 +	0 +	50
$u_2=2$	13 +	11 +	10 (70)	8 +	0 +	70
$u_3=-2$	14 +	4 (30)	10 +	13 +	0 +	30
$u_4=8$	9 −3	11 −3	16 (15)	8 (20)	0 (15)	50
	25	35	105	20	15	

25 − x, 20 + x, +x, 15 − x

$x = 15$

	$v_1=4$	$v_2=6$	$v_3=8$	$v_4=3$	$v_5=-5$
$u_1=0$	4 (10)	6 (5)	8 (35)	13 +	0 +
$u_2=2$	13 +	11 +	10 (70)	8 +	0 +
$u_3=-2$	14 +	4 (30)	10 +	13 +	0
$u_4=5$	9 (15)	11 +	16 (~~0~~)	8 (20)	0 (15)

optimal transportⁿ cost $= 4\times10 + 5\times6 + 35\times8 + 70\times10 + 30\times4 + 15\times1 + 20\times8$

= Rs 1465/- < 1600

5)

	8	2 (100)	6 (100)	100
	10 (20)	9	9 (100)	120
	7 (80)	10	7	80
	~~100~~ 20	100	100	300 / 300

P1	(1)	(7)↑	(1)
P2	(3)↑	-	(2)
P3	(10)↑	-	(9)

P1	P2	P3
(4)	(*)	
(1)	(1)	(1)
(3)	(0)	

No. of allocations = m+n-1

4 ≠ 5

⇒ degeneracy problem

⇒ allocate a small quantity ε to least cost such that it doesn't form loop (unallocated)

	$v_1=7$	$v_2=2$	$v_3=6$
$u_1=0$	8 +	2 (100)	6 (ε)
$u_2=3$	10 (20)	9 +	9 (100)
$u_3=0$	7 (80)	10 +	7 +

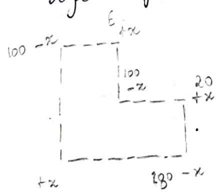
IBFS = 2×100 + 20×10 + 9×100 + 80×7

= Rs 1860 /-

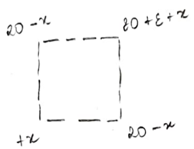
N.W Method

	$v_1=8$	$v_2=2$	$v_3=2$	
$u_1=0$	8 (100)	2 (ε) +	6 +	100
$u_2=7$	10 -5	9 (100)	9 (20)	~~120~~ 20
$u_3=5$	7 -6	10 +	7 (80)	80
	100	100	100	

Degeneracy 4 ≠ 5



	$v_1=8$	$v_2=2$	$v_3=2$
$u_1=0$	8 (20)	2 (80+ε)	6 +
$u_2=7$	10 -5	9 (20)	9 (100)
$u_3=-1$	7 (80)	10 +	7 +



	$v_1=7$	$v_2=2$	$v_3=6$
$u_1=0$	8 +	2 (100+ε)	6 (ε)
$u_2=3$	10 (20)	9 ~~(20)~~ +	9 (100)
$u_3=0$	7 (80)	10 +	7 +

IBFS = 1860 /-

16/5/23

Q) Maximize the following transportatⁿ matrix minimisatⁿ problem

				Supply
15	51	42	33	23
80	42	26	81	44
90	40	60	60	33
Demand 23	31	16	80	

Thus convert max. problem to min problem (or) profit matrix to loss matrix.

⇒ Identify highest element & subtract all from this.

⇒

				supply
75	39 (17)	48 (6)	57	~~23~~ 17
10	48 (14)	64	9 (30)	~~44~~ 14
0 (23)	50	30 (10)	30	~~33~~ 10
Demand 23	~~31~~ 17	~~16~~ 6	30	100 / 100

P1	P2	P3	P4	P5
(9)	(9)	(9)	(9)	(9)
(1)	(39)←	(16)	(16)←	
(30)←	(20)	(20)←	-	

P1	(10)	(9)	(18)	(21)
P2	ε-	(9)	(18)	(21)
P3	-	(9)	(18)	-
P4	-	(9)	(16)	-
P5		(39)	(48)	

IBFS = ₹2193

But this is for minimizatⁿ table

IBFS for original matrix (profit) is

= 17×51 + 6×42 + 42×14 + 81×30 + 90×23 + 10×60

= ₹6807

	$v_1 = 18$	$v_2 = 39$	$v_3 = 48$	$v_4 = 0$	
$u_1 = 0$	76 +	39 (13)	48 (6)	57 +	23
$u_2 = 9$	10 −	48 (14)	64 +	9 (30)	44
$u_3 = -18$	0 (23)	40 +	30 (10)	80 +	33
	23	31	16	30	

$x = 6$

$23 - x$

	$v_1 = 10$	$v_2 = 48$	$v_3 = 40$	$v_4 = 9$
$u_1 = \cancel{25}$ -9	75 (+)	39 (23)	48 (+)	57 (+)
$u_2 = 0$	10 (6)	48 (8)	64 (+)	9 (30)
$u_3 = -10$	0 (17)	50 (+)	30 (16)	30 (+)

$\Delta_{ij} \geq 0$

Optimal transportation cost = ₹ 2091 /-

Profit = ₹ 6909 /-

8) There are 3 factories A, B & C supplying goods to 4 dealers D_1, D_2, D_3 & D_4. The production capacities of these factories are 1000, 700 & 900 units resp. The req. from dealers are 900, 800, 500 & 400 units/month resp. The return cost per unit excluding transport cost are ₹ 8, ₹ 7 & ₹ 9 at the 3 factories. The following table gives the unit transport cost from factories to the dealers. Determine optimum sol^n to maximize the total returns.

	D_1	D_2	D_3	D_4	
A	2	2	2	4	8
B	3	5	3	2	7
C	4	3	2	1	9

Profit matrix = Selling price − Transport cost

Profit matrix

6	6	6	4	1000
4	2	4	5	700
5	6	7	8	900
900	800	500	400	2600 / 2600

$8 - x \longrightarrow$

$\Longrightarrow$

2 (900)	2 (100)	2	4	~~1000~~ 100
4	6 (200)	4 (500)	3	~~700~~ 200
3	2 (500)	1	0 (400)	~~900~~ 500
~~900~~	~~800~~ 700	500	400	

(P1)	1	0	1	3	(P1)	(P2)	(P3)
(P2)	1	0	1	–	0	0	0
(P3)	–	0	1	–	1	0	2
					1	1	1

	$v_1 = 2$	$v_2 = 2$	$v_3 = 0$	$v_4 = 0$
$u_1 = 0$	2 (900)	2 (100)	2 +	4 +
$u_2 = 4$	4 (−)	6 (200)	4 (500)	3 (−)
$u_3 = 0$	3 (+)	2 (500)	1	0 (400)

Q A company has 3 factories manufacturing the same product at 5 agencies in diff parts of the country. The product^n cost diff. from factory to factory & sale price differs from agency to agency. The shipping cost per unit is given in the following table, The ~~following data~~, Determine the prod & distribut^n schedule most profitable to the company.

	A_1	A_2	A_3	A_4	A_5	capacity	Prod cost/unit
F_1	3	1	5	7	4	150	20
F_2	8	7	8	3	6	200	22
F_3	4	5	3	2	7	125	18
demand	80	100	75	45	125		
sales price/unit	30	32	31	34	29		

Total cost/unit

23	21	25	27	24
30	29	30	25	28
22	23	21	20	25
sales price/unit 30	32	31	34	29

Profit = selling price − total cost

Profit matrix / Max. table

7	11	6	7	5
0	3	1	9	1
8	9	10	14	4

7	3	8	7	9	150
0	11	13	5	13	200
6	5	4	0	10	125
80	100	75	45	125	~~475~~ 425

	v_1	v_2	v_3	v_4	v_5	v_6
u_1	7	3 (100)	8	7	9 (50)	0
u_2	0 (80)	11	13	5 (45)	13 (75)	0 (ε)
u_3	6	5	4 (75)	0	10	~~500~~ 0 (50)

no. of allocations < m+n−1

⇒

7	3 (100)	8	7	9 (50)	0	50 ~~150~~
0 (80)	11	13	5 (45)	12 (75)	0	200 75 ~~100~~
6	5	4 (75)	0	10	~~500~~ (50)	~~125~~ 75
80	100	~~75~~	45	~~125~~ 75	50	

(P1)	6	2	4	5	1	50	(P1)	(P2)	(P3)	(P4)	(P5)
(P2)	6	2	4	5	1	−	3	4	4	5	1
(P3)	−	2	4	5	1	−	5	5	6	2	0
(P4)	−	2	4	−	1	−	4	4	4	1	4
(P5)	−	−	4	−	1	−					

opt sol^n = 2880

18/1/23 **Hungarian Method** — Job Allocation

Persons

Jobs	1	2	3	4
1	10	20	18	14
2	15	25	9	24
3	30	19	17	12
4	19	24	20	10

row reduction (subtract smallest ele. of row from all)

0	10	8	4
6	16	0	15
18	7	5	0
9	14	10	0

column reduction (make sure each row & each column has atleast 1 zero)

Persons

Jobs	1	2	3	4
1	0	3	8	4
2	6	9	0	15
3	18	0	5	0
4	9	7	10	0

Allocation

Row-wise allocation (first check row then column)

Person

Jobs	1	2	3	4
1	$\boxed{0}$	3	8	4
2	6	9	$\boxed{0}$	15
3	18	$\boxed{0}$	5	~~0~~
4	9	7	10	$\boxed{0}$

→ if a row has more than 1 '0' defer the decision

Optimal assignment:

J	P
1	1
2	3
3	2
4	4

N = n
no. of lines = order of matrix

optimal time is
10 + 9 + 19 + 10
= 48 hrs

Q) Using the following cost matrix shown in table below determine
1. Job assignment
2. Cost of assignment

Solve problem by hungarian method.

	1	2	3	4	5
A	2	9	3	7	1
B	6	8	7	2	4
C	4	6	5	2	3
D	4	2	7	3	5
E	5	3	6	1	2

RR

	1	2	3	4	5
A	1	8	2	6	0
B	4	6	5	0	2
C	2	4	3	0	1
D	2	0	5	1	3
E	4	2	5	0	1

	1	2	3	4	5
A	$\boxed{0}$	8	~~0~~	6	~~0~~
B	3	6	3	$\boxed{0}$	2
C	1	4	1	~~0~~	1
D	1	$\boxed{0}$	3	1	3
E	3	2	3	~~0~~	1

N ≠ n

→ Identify minimum element which is not on line

→ Subtract min. ele. from all ele. which is not on line & add min to intersectⁿ of 2 lines.

	1	2	3	4	5
A	$\boxed{0}$	9	0	7	~~0~~
B	2	6	2	$\boxed{0}$	1
C	0	4	$\boxed{0}$	~~0~~	~~0~~
D	~~0~~	$\boxed{0}$	2	1	2
E	2	2	2	~~0~~	$\boxed{0}$

N = n
5 = 5

Assignment:

P	I
1	A or C
2	D
3	C or A
4	B
5	E

optimal time: 2 + 2 + 5 + 2 + 2 = 13
or 3 + 4 + 2 + 5 + 2 + 2 = 18.

Q) A teacher has to assign 5 diff projects

23/5/23

(Q) A company has a surplus drug in each city in each of the cities A, B, C, D, E & one deficit drug in 1, 2, 3, 4, 5 & 6. The distance b/w cities in kms is given in matrix below. Find the assignment of drugs in cities in surplus 2 cities in so that total distance covered by the vehicle is minimum.

	1	2	3	4	5	6
A	12	10	15	22	18	8
B	10	18	25	15	16	12
C	11	10	3	8	5	9
D	6	14	10	13	13	12
E	8	12	11	7	13	10
F	0	0	0	0	0	0

(Add extra rows or columns to make square matrix)

row reduction :

	1	2	3	4	5	6
A	4	2	7	14	10	$\boxed{0}$
B	$\boxed{0}$	8	15	5	6	2
C	8	7	$\boxed{0}$	5	2	6
D	0	8	4	7	7	6
E	16	5	34	$\boxed{0}$	6	3
F	0	$\boxed{0}$	0	0	~~0~~	0

Allocation :

N ≠ n

5 < 6

	1	2	3	4	5	6
A	4	0	7	14	8	$\boxed{0}$
B	$\boxed{0}$	6	15	5	4	2
C	8	5	$\boxed{0}$	5	0	6
D	0	6	4	7	5	6
E	1	3	4	$\boxed{0}$	4	3
F	2	0	2	2	$\boxed{0}$	2

	1	2	3	4	5	6
A	6	$\boxed{0}$	7	16	8	0
B	0	4	13	5	2	$\boxed{0}$
C	10	5	$\boxed{0}$	7	0	6
D	$\boxed{0}$	4	2	7	3	4
E	1	1	2	$\boxed{0}$	2	1
F	4	0	2	4	$\boxed{0}$	2

N = n

6 = 6

optimal assignment = 10 + 12 + ~~10~~ 3 + 6 + ~~7~~ 7 + 0

= ~~45~~ = 38

A - 2
B - 6
C - 3
D - 1
E - 4
F - 5

(Q) **Maximization in assignment problem**

- A marketing manager has 5 salesman & there are 5 sales districts. Considering the capabilities of the salesman & the nature of districts, the estimates made by the marketing manager for the sales per month for each salesman in each district would be as follows. Find the assignment of salesman to the district, that will result in the maximum sales.

32	38	40	28	40
40	24	28	21	36
41	27	33	30	37
22	38	41	36	36
29	33	40	35	39

→ row reduction

4	10	12	0	12
19	3	7	0	15
14	0	6	3	10

Convert to loss matrix by identifying max element & subtract all elements from this element.

19	3	1	13	1
1	17	13	20	5
0	14	8	11	4
19	3	0	5	5
12	8	1	6	2

row reduction

8	2	0	12	0
0	16	12	19	4
0	14	8	11	4
19	3	0	5	5
11	7	0	5	1

column reduction

8	0	0	7	0
0	14	12	14	4
0	12	8	6	4
19	1	0	0	5
11	5	0	0	1

Allocation

8	$\boxed{0}$	0	7	0
$\boxed{0}$	14	12	14	4
0	12	8	6	4
19	1	$\boxed{0}$	0	5
11	5	0	$\boxed{0}$	1

N ≠ n

9	$\boxed{0}$	1	8	0
$\boxed{0}$	13	12	14	3
0	11	8	6	3
19	0	$\boxed{0}$	0	4
11	4	0	$\boxed{0}$	0

N ≠ n

12	$\boxed{0}$	1	8	0
$\boxed{0}$	10	9	11	0
0	8	5	3	$\boxed{0}$
22	0	$\boxed{0}$	0	4
14	4	0	$\boxed{0}$	0

~~34~~ 38 + 40 + 37

41 + 35

= 191

Q) A student has to select 1 elective in each semester & same elective should not be selected in diff sem. due to various reasons, the expected grades in each subject if selected in diff sem vary & are given below. The grade points are S = 10, A = 9, B = 8, C = 7, D = 6, E = 5, & F = 4

	1	2	3	4
I	F	E	D	C
II	E	E	C	C
III	C	D	C	A
IV	B	A	S	S

How will the student select the electives in order to maximise the total expected grade points

4	5	6	7
5	5	7	7
7	6	7	9
8	9	10	10

loss matrix

6	5	4	3
5	5	3	3
3	4	3	1
2	1	0	0

row reduction

3	2	1	0
2	2	0	0
2	3	2	0
2	1	0	0

column reduction

1	1	1	$\boxed{0}$
0	1	$\boxed{0}$	0
$\boxed{0}$	2	2	0
0	$\boxed{0}$	0	0

N = n

Max allocation for grade points = 7 + 7 + 7 + 9 = 30

Travelling Salesman problem

city

city	1	2	3	4
1	∞	2	4	3
2	5	∞	7	8
3	6	7	∞	6
4	4	3	5	∞

Q) A travelling salesman has to visit 5 cities, He wishes to start from a particular city. Visit each city once and then return to his starting point. Cost of going from one city to another is shown below. Find the least cost route.

	1	2	3	4	5
1	∞	4	10	14	2
2	12	∞	6	10	4
3	16	14	∞	8	14
4	24	8	12	∞	10
5	2	6	4	16	∞

row red^n

∞	2	8	12	0
8	∞	2	6	0
8	6	∞	0	6
16	0	4	∞	2
0	4	2	14	∞

col red^n

	1	2	3	4	5	
1	∞	2	6	12	$\boxed{0}$	1 - 5
2	8	∞	$\boxed{0}$	6	0	2 - 3
3	8	6	∞	$\boxed{0}$	6	3 - 4
4	16	$\boxed{0}$	2	∞	2	4 - 2
5	$\boxed{0}$	4	0	14	∞	5 - 1

1-5-1

2-3-4-2 } Not optimum route → Sub-route

=> instead of 0 we must allocate 2.

∞	$\boxed{2}$	6	12	0
8	∞	$\boxed{0}$	6	0
8	6	∞	$\boxed{0}$	6
16	0	2	∞	$\boxed{2}$
$\boxed{0}$	4	0	14	∞

25/5/23

Little Penalty Method

Q)

	A	B	C	D	E
A	-	10	5	4	3
B	7	-	11	3	8
C	4	11	-	5	6
D	2	7	8	-	4
E	6	3	7	4	-

=>

	A	B	C	D	E
A	-	7	0 (0+2)	1	0 (0+2)
B	4	-	6	0 (4+1)	5
C	0 (1+0)	7	-	1	2
D	0 (2+0)	5	4	-	2
E	3	0 (1+5)	2	1	-

For all zeroes

(min element in row except 0 'that')

+

(min ele in col^n except that 0)

Assign E - B (6) (highest penalty)

B - E should ∞, eliminate row & col.

(every iterat^n check if all row & columns have 0)

	A	~~B~~	C	D	E
A	-	~~0~~	0	1	0
B	4		6	0	~~4~~
C	0		-	1	2
D	0		4	-	2
~~E~~					

=>

	A	C		E
A	-	0 (0+4)	1	0 (0+2)
B	4	6	0 (4+1)	
C	0 (1+0)	-	1	2
D	0 (2+0)	4	-	2

Assign B-D (5)

	A	~~B~~ C	E
A	-	0 (0+4)	0 (0+2)
C	~~0~~ (2+0)	-	2
D	0 (2+0)	4	2

Assign A-C (4)

	A	E
C	-	2
D	0	2

=>

	A	E
C	—	0 (∞+2)
D	0 (2+∞)	~~2~~

Assign C-E & E-C is ∞.

	A
D	$\boxed{0}$

=> Assign D-A (if tie in highest penalty you can consider any!)

Optimum assignment : E-B-D-A-C-E

3 + 3 + 2 + 5 + 6 = 19

Q)

	A	B	C	D	E	F	G
A	∞	6	12	6	4	8	1
B	6	-	10	5	4	3	3
C	8	7	-	11	3	11	8
D	5	4	11	-	5	8	6
E	5	2	7	8	-	4	7
F	6	3	11	5	4	-	2
G	2	3	9	7	4	3	-

row red^n

	A	B	C	D	E	F	G
A	-	5	11	5	3	7	0
B	3	-	7	2	1	0	0
C	5	4	-	8	0	8	5
D	1	0	7	-	1	4	2
E	3	0	5	6	-	2	5
F	4	1	9	3	2	-	0
G	0	1	7	5	2	1	-

col red^n

	A	B	C	D	E	F	G
A	-	5	6	3	3	7	0 (3+0)
B	3	-	2	0 (0+1)	1	0 (0+1)	0 (0+0)
C	5	4	-	6	0 (4+1)	8	5
D	0 (1+0)	0	2	-	1	4	2
E	3	0	0 (0+2)	4	-	2	5
F	4	1	4	1	2	-	0 (1+0)
G	0 (1+1)	1	2	3	2	1	1

Assign C-E (5)

=>

	A	B	C	D	F	G
A	-	5	6	3	7	0 (3+0)
B	3	-	2	0 (0+1)	0 (0+1)	0
D	1	0	2	-	4	2
E	3	0	~~0~~	4	2	5
F	4	1	4	1	-	0 (1+1)
G	~~0~~	1	2	3	1	1

A - G

	A	B	C	D	F
B	3	-	2	0	0
D	1	0	2	-	4
E	3	0	-	4	2
F	3	0	9	0	-
G	-	0	1	2	0

→

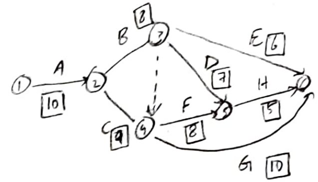
	A	B	C	D	F
B	2	-	2	0	0
D	0	0	2	-	4
E	2	0	-	4	2
F	2	0	3	0	-
G	-	0	1	2	0

D - A

① ⇒ Multiply profit for each ele. in column

6/6/23

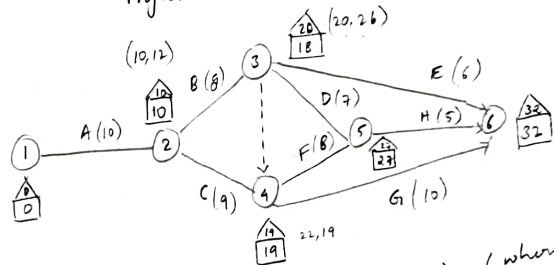
Activity	Precedence	Duration (days)
A		10
B	A	8
C	A	9
D	B	7
E	B	6
F	B, C	8
G	B, C	10
H	D, F	5



1) A - B - E 24
2) A - B - D - H 30
(~~A - B - E - F - H~~ 40
~~A - B - C - G~~) 37
3) (A - C - F - H) ✓ 32 critical path
4) A - C - G 29

Critical path ⇒ A - C - F - H
or
1 - 2 - 4 - 5 - 6

Project duration is 32 days.

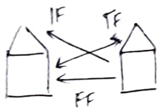


△ → latest start time (in reverse) (wherever merging consider min)
□ → latest finish time (wherever merging consider max)

critical path → no difference in △ & □ values.

Activity	Precedence	Durat^n	EST	EFT	LST	LFT	total float TF	free float FF	Independent float IF
A	–	10	0	10	0	10	0	0	0
B	A	8	10	18	12	20	2	0	0
C	BA	9	10	19	10	19	0	0	0
D	B	7	18	25	20	27	2	2	0
E	5	6	18	24	26	32	8	8	6
F	B, C	8	19	27	19	27	0	0	0
G	B, C	10	19	29	22	32	3	3	3
H	D, F	5	27	32	27	32	0	0	0

→ difference (– activity time)



⑥ FERT

(Project Evaluation & ~~Retor~~ Review Techniques)

Q) An R&D project has large no. of activities but management is interested in controlling a part of these 7 activities. The following data is available for these 7 " .

1. Draw the project network
2. Identify the critical path & find its duration
3. Calculate the variance and standard deviation of the project.
4. If the management puts a deadline of 47 days for completion of this part of the project, determine the probability that it will be completed in 47 days.
5. When should the management start these activities to get a confidence level of 99.6 for completion of these activities in the scheduled time.

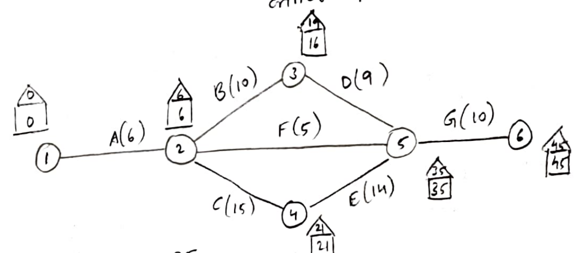
Activity	Preceding Activity	Time t_o	t_m	t_p	Variance	Expected time (t_e)
$\boxed{A}$	–	4	6	8	0.44	6
B	A	6	10	14	1.77	10
$\boxed{C}$	A	8	15	22	5.44	15
D	B	9	9	9	0	9
$\boxed{E}$	C	10	14	18	1.77	14
F	A	5	5	5	0	5
$\boxed{G}$	D, E, F	8	10	12	0.44	10

Expected time

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

$$\text{variance} = \sigma^2 = \left[\frac{t_p - t_o}{6}\right]^2$$

standard deviation $= \sqrt{\text{sum of variances of along critical path}}$



A – B – D – G = 35

A – C – E – G = 45 ⇒ critical path

A – F – G = 21

$$\text{S.D } (\sigma) = \sqrt{0.44 + 5.44 + 1.77 + 0.44} = 2.844$$

$$Z = \frac{T_s - t_e}{\sigma} = \frac{47-45}{2.844} = 0.7032$$

$\Rightarrow P = 0.76730$

Probability of completing the project in 47 days is 76.73%.

vi) 99% $\Rightarrow Z = 2.33$

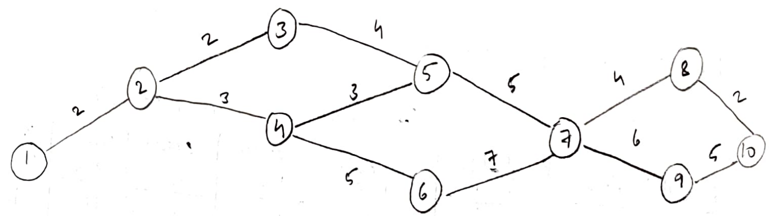
$T_s = ?$ $\quad 2.33 = \frac{T_s - 45}{2.844}$ $\Rightarrow T_s = 51.626$ days

≈ 52

Q) A project schedule has the following characteristics:

1. Construct the project network
2. Find the expected duration and variance of each activity
3. Identify the critical path
4. What is the prob of completing the project in 30 days scheduled time
5. What is the prob. of comp. 3 days earlier than expected?
6. What is the prob. that it will not be completed later than expected.
7. What's the due date 90% of chance being met.

Activity	Pr- acti	t_m	t_o	t_p	t_e	Variance
~~1-2~~	~~2,1,3~~					
1	2	2	1	3	2	0.111
2	3	2	1	3	2	
2	4	3	1	5	3	0.444
3	5	4	3	5	4	
4	5	3	2	4	3	
4	6	5	3	7	5	0.444
5	7	5	4	6	5	
6	7	7	6	8	7	~~0.027~~ 0.111
7	8	4	2	6	4	
7	9	6	4	8	6	0.444
8	10	2	1	3	2	
9	10	5	3	7	5	0.444



3. 1 - 2 - 4 - 6 - 7 - 9 - 10 = 28

4. $\sigma = 1.4135$ $\quad Z = \frac{30-28}{1.4135} = 1.414$

$P = 0.9207$

5.

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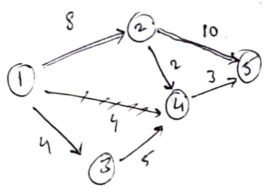
Given Indirect cost = 70.

Activity	NT	NC (Normal cost)	CT	CC	Slope	Possible days of work
$\boxed{1-2}$	8	100	6	200	50 — 1	~~2~~ ~~1~~ 0
1-3	4	150	2	350	100	2
2-4	2	50	1	90	40	1
$\boxed{2-5}$	10	100	5	400	60 — 2	~~5~~ ~~4~~ ~~3~~ ~~2~~ 1
3-4	5	100	1	200	25	4
4-5	3	80	1	100	10	~~2~~ 1
		580				

$$\text{Slope} = \frac{CC - NC}{NT - CT}$$



1-2-5 → 18

1-2-4-5 → 13

1-3-4-5 → 12

CP is 1-2-5

PD is 18 days

crash & bring possible days of work for CP to 0.

Possible days of crash is 7 days

(start from min slope (50, 60))

Parallel Paths	days	crash 1-2	crash 1-2	crash 2-5	crash 2-5	crash 2-5	crash 4-5
1-2-5 (cp)	18	17	16	15	12	11	11
1-2-4-5	13	12	11	11	11	11	10
1-3-4-5	12	12	12	12	12	12	11

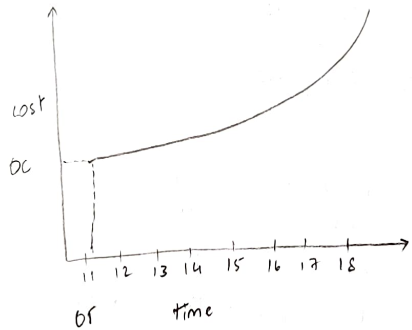
other activities should ≤ CP, if exceeds, we need to consider parallel activities.

Total cost = Direct cost (or Normal Cost) + Indirect cost + crashing cost (or Slope)

Days	Direct cost (Rs)	Indirect cost (Rs)	Slope for crashing cost	Total cost
18	580	18×70 = 1260	—	1840
17	"	17×70 = 1190	1×50 = 50	1820
16	"	16×70 = 1120	50 + 1×50 = 100	1800
15	"	1050	100 + 1×60 = 160	~~2000~~ 1790
14	"	980	160 + 1×60 = 220	1780
13	"	910	220 + 1×60 = 280	1770
12	"	840	280 + 1×60 = 340	1760
11	"	770	340 + 1×60 + 1×10 = 410	1760

optimum cost

sometimes it decreases then increases.



minimum time and corresponding cost

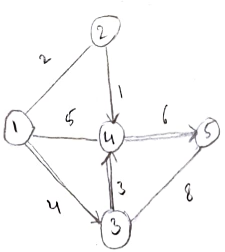
8) Do the indirect costs, NT, CT, CC, NC are given in table. find

1. The total cost for finishing the project
2. 9, 10, 11, 12 & 13 weeks
2. The least cost duration in weeks.
3. Plot a graph of duration vs total project cost

Activity	NT	NC	CT	CC	Slope = $\frac{CC-NC}{NT-CT}$		Possible days to crash (NT − CT)
1-2	2	500	1	800	300		1
1-4	5	900	3	1300	200		2
1-3	4	800	3	1000	200	II	~~1~~ 0
2-4	1	400	1	400	0		0
3-4	3	1200	2	1800	600	III	~~1~~ 0
4-5	6	700	4	900	100 → min	I	2 ~~1~~ 0
3-5	8	600	4	1200	150		~~4~~ ~~3~~ 2

$\Sigma = 5100$

Project Duration (weeks)	9	10	11	12	13
IC (Rs	6000	6150	6200	6500	7100



1 − 2 − 4 − 5 → 9

1 − 4 − 5 → 11

1 − 3 − 5 → 12

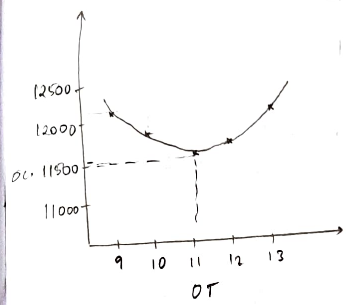
1 − 3 − 4 − 5 → 13 → CP

Project duration : 13 weeks

13/6/23

~~Project duration (week~~ Activity Parallel Paths	weeks ~~days~~	crash 4-5	crash 4-5 & 3-5	crash 1-3	crash 3-4 & 3-5
1 - 3 - 4 - 5	13	12	11	10	9
1 - 3 - 5	12	12	11	10	9
1 - 4 - 5	11	10	9	9	9
1 - 2 - 4 - 5	9	8	7	7	7

Week	Normal cost (or) Direct cost	Indirect cost	Crashing cost	Total cost
13	5100	7100	–	12200
12	5100	6500	1×100 = 100	11700
11	5100	6200	100 + 1×100 + 1×150	11650
10	5100	6150	350 + 1×200	11800
9	5100	6000	550 + 1×150 + 1×600 = 1300	12400



optimal cost = 11650

optimal time = 11 weeks

total project duration = 13 weeks

& project cost = 12200.

13/6/23

UNIT-5. GAME THEORY

Von Newmann developed game theory

A \ B	1	2	3	4	5	Row min	max of min
1	3	-1	4	6	7	-1	
2	-2	8	2	4	12	-2	
3	16	8	6	14	12	6	6
4	1	11	-4	2	1	-4	
col max	16	11	6	14	12		
minimum of max			6				

Assuming player A is winner
- If values are +ve → A will gain
- If -ve ⇒ B will gain
- to be reduced to 2x2 matrix
- Initial matrix may not be square
- Player B wants to maximise his minimum gain

→ If value of maximin & minimax is same ⇒ saddle point

⇒ both are using same strategy

Thus,
value of the game = 6

Player A strategy is A3 (3rd row)

Player B strategy is B3 (3rd col)

→ If saddle point doesn't exist ⇒ reduce to 2x2

Q)

	1	2	3	4	5	row min	maximin
1	-2	0	0	5	3	-2	
2	3	2	1	2	2	1	1 (A2)
3	-4	-3	0	-2	6	-4	
4	5	3	-4	-2	-6	-6	
col max	5	3	1	5	6		
minimax			1 (B3)				

maximin = minimax

1 = 1

⇒ saddle point

Q) What is the range of p & q values if A wins.

	B_1	B2	B3	row min	
A1	2	4	7	2	
A2	10	7	q	7	$q \geq 7$
A3	4	p	8	4	$p \geq 4$
col max	10	7	8		

$p < 7$ $q < 8$

But A wins so

$q \geq 7$ & $p \geq 8$

To reduce matrix to 2x2,

Dominance Rule

Q)

A \ B	1	2	3	4	row min	maximin
1	20	15	12	35	12	12
2	25	14	8	10	8	
3	40	2	19	5	2	
4	5	4	11	0	0	~~0~~
col max	40	15	19	35		
min max		15				

maximin ≠ minimax

A dominates.
- A needs to eliminate smaller values in rows & higher val in col.

$A_4 < A_1$ (all values)

Eliminate A_4.

A \ B	1	2	3	4
1	20	15	12	35
2	25	14	8	10
3	40	2	19	5

B1 > B2 (eliminate B1)

⇒

	2	3	4
1	15	12	35
2	14	8	10
3	2	19	5

A1 > A2 (eliminate A2)

⇒

	2	3	4
1	15	12	35
3	2	19	5

(B2 < B4 ⇒ eliminate B4)

⇒

A \ B	2	3	row min
1	15	12	
3	2	19	

Shortcut Method

		B 2	3	Difference	Interchange	Ratios
A	1	15	12	3	17	17/20
	3	2	19	17	3	3/20
Diff		13	7			
I/c		7	13			
Ratio		7/20	13/20			

Value of game $V = \frac{15 \times 17}{20} + \frac{2 \times 3}{20}$

$= \frac{261}{20} = \frac{12 \times 17}{20} + \frac{19 \times 3}{20}$

Player B $= \frac{15 \times 7}{20} + \frac{12 \times 13}{20} = \frac{261}{20}$

$= \frac{2 \times 7}{20} + \frac{19 \times 13}{20} = \frac{261}{20}$

$= 13.05$

Player A strategies are,

$\left(\frac{17}{20}, 0, \frac{3}{20}, 0\right)$

Player B strategies are

$\left(0, \frac{7}{20}, \frac{13}{20}, 0\right)$

carries mark dont forget to write

Player A gains 13.05

B loses 13.05

Actual Method

$$\begin{array}{cc|cc|c} & & \multicolumn{2}{c}{B} & \\ & & 1 & 2 & \\ \hline A & 1 & a_{11} & a_{12} & x_1 \\ & 2 & a_{21} & a_{22} & x_2 \\ \hline & & y_1 & y_2 & \end{array}$$

$x_1 = \frac{a_{22} - a_{21}}{(a_{11}+a_{22}) - (a_{21}+a_{12})}$, $x_2 = 1 - x_1$

$y_1 = \frac{a_{22} - a_{12}}{(a_{11}+a_{22}) - (a_{21}+a_{12})}$, $y_2 = 1 - y_1$

$V = \frac{a_{11}a_{22} - a_{21}a_{12}}{(a_{11}+a_{22}) - (a_{21}+a_{12})}$

(b)

							row min	Maximin
1	0	0	0	0	0	0	0	
2	4	2	0	2	1	1	0	
3	4	3	1	3	2	2	1	1
4	4	3	7	-5	1	2	-5	
5	4	3	4	-1	2	2	-1	
6	4	3	3	-2	2	2	-2	
col max	4	3	7	3	2	2		
minimax					2			

minimax ≠ maximin

Reduce

(a)

$$\begin{bmatrix} 0 & 2 & 1 & 1 \\ 1 & 3 & 2 & 2 \\ 7 & -5 & 1 & 2 \\ 4 & -1 & 2 & 2 \\ 3 & -2 & 2 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & 2 & 2 \\ 7 & -5 & 1 & 2 \\ 4 & -1 & 2 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{array}{c} 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} 4 & 3 & 1 & 3 & 2 & 2 \\ 4 & 3 & 7 & -5 & 1 & 2 \\ 4 & 3 & 4 & -1 & 2 & 2 \end{bmatrix}$$

$$\Downarrow \quad \begin{array}{c|ccc} & 3 & 4 & 5 \\ \hline 3 & 1 & 3 & 2 \\ 4 & 7 & -5 & 1 \\ 5 & 4 & -1 & 2 \end{array}$$

Compare avg of 2 row with other rows

$$\begin{bmatrix} \cancel{7} & \cancel{-5} & \cancel{1} \\ \cancel{4} & \cancel{-1} & \cancel{2} \end{bmatrix} \Rightarrow \begin{bmatrix} \ \end{bmatrix}$$

$$\begin{array}{c|cc} & 3 & 4 \\ \hline 3 & 1 & 3 \\ 4 & 7 & -5 \\ 5 & 4 & -1 \end{array} \Rightarrow \begin{array}{c|cc} & 3 & 4 \\ \hline 3 & 1 & 3 \\ 4 & \cancel{7} & \cancel{-5} \end{array} \Rightarrow \begin{array}{c|cc|c} & 3 & 4 & \\ \hline 3 & 1 & 3 & 6/7 \\ 4 & 7 & -5 & 1/7 \\ \hline & 4/7 & 3/7 & \end{array}$$

$\frac{A_3 + A_4}{2} = A_5$

$x_1 = \frac{-5-7}{-4-10} = \frac{-6}{-14} = \frac{+3}{7}$

$\frac{12}{14} = \frac{6}{7}$

$V = \frac{-5-21}{-4-10} = \frac{-26}{-14} = \frac{13}{7}$

$A\left(0, 0, \frac{6}{7}, \frac{1}{7}, 0, 0\right)$

$B\left(0, 0, \frac{4}{7}, \frac{3}{7}, 0, 0\right)$

15/6/23

Q)

	1	2	3	4	5
1	7	6	8	9	6
2	5	4	6	7	3
3					
4					
5	8	7	3	4	9

$\Rightarrow$

A \ B	1	2	3	4	5
1	7	6	8	9	6
5	8	7	3	4	9

$B_1, B_5 > B_2$

$B_4 > B_3$, Eliminate B_1, B_4, B_5

A \ B	2	3	D	I	R
1	6	8	2	4	2/3
5	7	3	4	2	1/3

D	1	5
I	5	1
R	5/6	1/6

Value of game

$V = 6 \times \frac{2}{3} + 7 \times \frac{1}{3} = \frac{19}{3}$

$A\left(\frac{2}{3}, 0, 0, 0, \frac{1}{3}\right)$

$B\left(0, \frac{5}{6}, \frac{1}{6}, 0, 0\right)$

Q)

	1	2	3	4	5	6	row min	max min
1	10	5	2	8	4	7	2	
2	13	12	15	11	6	10	6	
3	16	14	10	7	9	6	6	
4	10	11	9	8	7	5	5	
5	9	11	13	15	10	8	8	
6	11	13	15	10	12	9	9	9
col max	16	14	15	15	12	10		
minimax						10		

$\Rightarrow$

				5	6
~~13~~	~~12~~	~~15~~	~~11~~	6	10
~~16~~	~~14~~	~~10~~	~~7~~	~~9~~	~~6~~
~~10~~	~~11~~	~~9~~		~~7~~	~~5~~
~~9~~	~~11~~	~~13~~		~~10~~	~~8~~
~~11~~	~~13~~	~~15~~		12	9

	5	6
2	6	10
6	12	9
	y_1	y_2

$x_1 = \frac{-3}{15-22} = \frac{3}{7}$

$x_2 = \frac{4}{7}$

$y_1 = \frac{1}{7}$

$y_2 = \frac{6}{7}$

Value of game $= \frac{54-120}{15-22} = \frac{66}{7}$

A (0,

(Dominance with graphical)

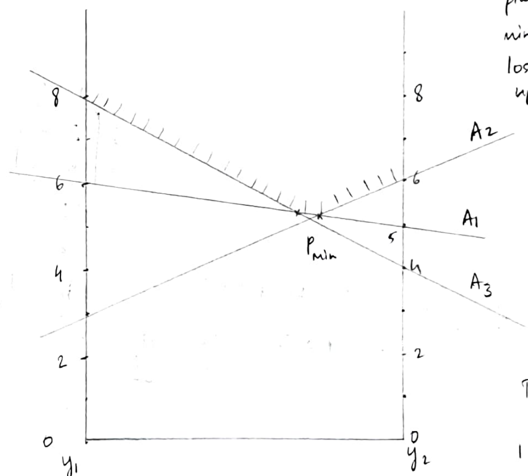
Q) Use the relation of dominance to solve the rectangular game whose square matrix for player A is given below

	1	2	3	4	row min	max min
1	19	6	7	5	5	5
2	7	3	14	6	3	
3	12	8	18	4	4	
4	~~8~~	~~7~~	~~13~~	~~-1~~	-1	
col max	19	8	18	6		
minimax				6		

$\Rightarrow$

	2	4
1	6	5
2	3	6
3	8	4
	y_1	y_2

matrix cannot be further reduced to 2x2 matrix thus use graph



player B wants to minimize maximum loss $\Rightarrow$ (consider upper boundary) min point out of 2 points P_{min} is min.

P_{min} is intersected by A_1 & A_2 lines

Thus

	2	4			
1	6	5	1	3	3/4
2	3	6	3	1	1/4
1/4	3/4	← 3	1		
		3	3		

Value $= \frac{21}{4} = \frac{16}{4} + \frac{3}{4} = 6 \times \frac{3}{4} + 3 \times \frac{1}{4}$

Player A strategies are $(\frac{3}{4}, \frac{1}{4}, 0, 0)$

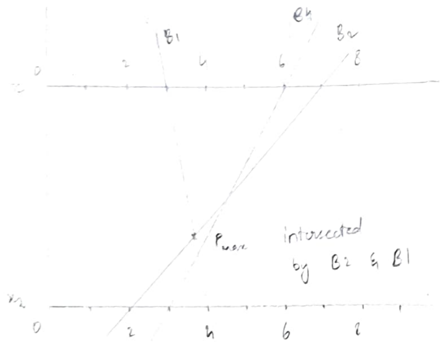
" B " " $(0, \frac{1}{4}, 0, \frac{3}{4})$

8)

	1	2	3	4	5	row min	maximin
1	~~2~~	~~6~~	~~3~~	~~5~~	~~4~~	3	3
2	3	7	4	6	5	3	
3	4	2	6	3	5	2	
4	~~1~~	~~4~~	~~2~~	~~6~~	~~3~~	1	
5	~~3~~	~~5~~	~~7~~	~~4~~	~~6~~	3	
col max	4	7	7	6	6		
minimax	4						

$\Rightarrow$

	1	2	4
2	3	7	6
3	4	2	3



or Avg & eliminate B4

	1	2			
2	3	7	4	2	2/6
3	4	2	2	4	4/6
	1	5			
	5	1			
	5/6	1/6			

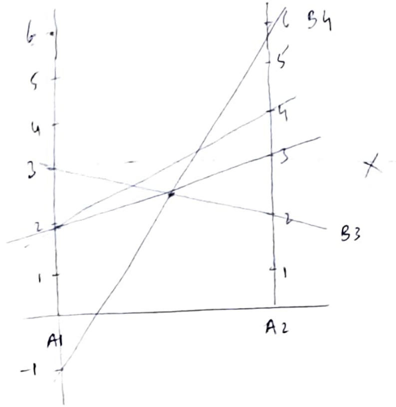
Value of game $= 3 \times \frac{2}{6} + 4 \times \frac{4}{6}$

$= \frac{6 + 16}{6} = \frac{22}{6} = \frac{11}{3}$

1Q)

		B			
		1	2	3	4
A	1	2	2	3	-1
	2	4	3	2	6

$\rightarrow$



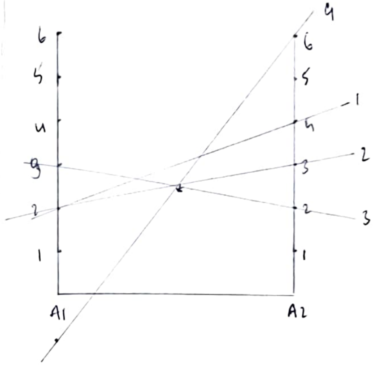
B3 & B4

	3	4		
1	3	-1	4	1/2
2	2	6	4	1/2
	1	7		
	7	1		
	7/8	1/8		

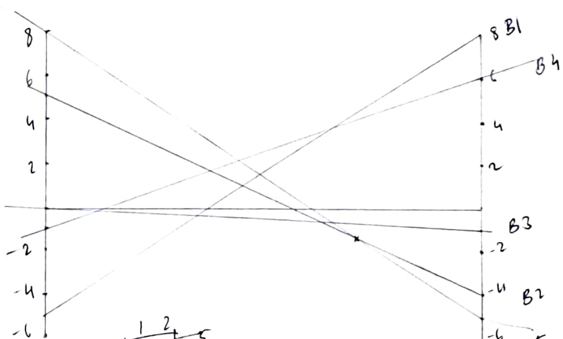
$V = \frac{3}{2} + \frac{2}{2} = \frac{5}{2}$

2Q)

	1	2	3	4	5
1	-5	5	0	-1	8
2	8	-4	-1	6	-5



2Q)



	1	2	
	-5	0	5
	8	1	7 ✗
	13	1	

B2 & B5

	2	5			
1	5	8	3	1	1/4
2	-4	-5	1	3	3/4
	9	13			
	13	9			
	13/22	9/22			

$V = \frac{5}{4} \cancel{+} \frac{\cancel{24}}{\cancel{4}}$ -12

$= \frac{29}{4}$