SDM

Statistic and Discrete Mathematics

Graph Theory : UNIT 1

Graph : A graph is a pair (V,E) where V is (*) non-empty set and E is a set of unordered pair of elements taken from set V. Here elements of V are called vertices and elements of E are called edges. Set $V \to$ vertex set
set $E \to$ edge set.

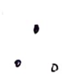
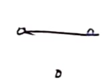
Graph (V,E) is denoted by $G(V,E)$ or G.

Note: ① E can be empty, where V has to be non-empty.

Types of Graph :

⓪ **Null Graph:** A graph containing no edges is called null graph.

A null graph with only one vertex is called trivial graph.

G_1 : G_2 : G_3 :  G_4 : 

$G_1, G_2, G_3 \to$ null graphs

$G_4 \to$ trivial graph

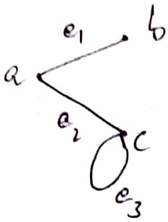
② **Finite graph:** Vertex & edge set are finite

[In infinite graph either V or E is infinite]

Note: ① No. of vertices of a graph G is called its order & no. of edges of G is called its size.

② Vertices usually denoted by $v_1, v_2 \ldots$ & edges, $e_1, e_2 \ldots$

③ If v_i & v_j are 2 vertices of G and if e_k is an edge joining v_i & v_j, then v_i & v_j are called end vertices of e_k.

G: 

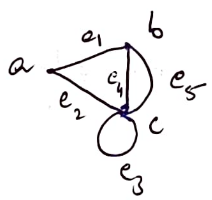
end vertices of e_1 are a & b

end vertices of e_2 are a & c.

④ A loop is an edge joining a vertex to itself

eg: e_3.

⑤ If there are two or more edges with same vertices then such edges are called multiple or parallel edges.

G: 

eg: e_4 & e_5

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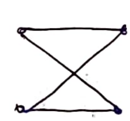
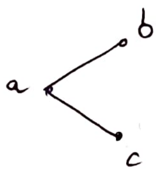
③ <u>Simple Graph</u>: A graph with no multiple edges & no self loops is called simple graph

eg: G_1:  G_2: 

④ <u>Labeled / Unlabeled Graphs</u>:

If we assign labels to vertices of G, then G is called labeled graph. If we dont assign label to vertices of G, then it is unlabeled.

G_1: 

labeled.

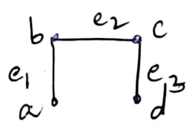
G_2: 

<u>Note</u>: ① When a vertex v of a graph G is an ~~ed~~ end vertex of an edge, we say edge e is <u>incident</u> on the vertex v.

② Two end vertices are coincident if edge is a loop.

③ Two non-parallel edges are said to be <u>adjacent</u> edges if they are incident on common vertex.

④ Two vertices are said to be adjacent vertices if there is an edge joining them



e_1 is incident on a & b
e_1 is adjacent to e_2
b is adjacent to a & c.

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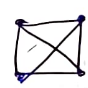
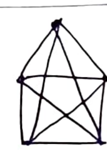
⑤ <u>Complete Graph</u>:

K_2: K_3:  K_4:  K_5: 

A simple graph of order $n \geq 2$ in which there is an edge b/w every pair of vertices is called complete graph. (denoted by K_n)

<u>Note</u>: K_1 : • → complete graph

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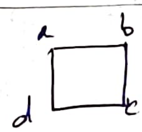
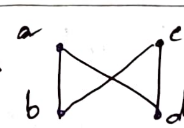
⑥ <u>Bipartite Graph</u>: G: 

→ 

A simple graph G is such that its vertex set V is the union of two of its mutually disjoint non-empty subsets, V_1 & V_2 which are such that each edge in G joins a vertex in V_1 & a vertex in V_2.

Every edge of G joins a vertex in V_1 to V_2.

$V_1 = \{a, c\}$

$V_2 = \{b, d\}$

$V = a, b, c, d$

$V_1 \cup V_2 = V$ & $V_1 \cap V_2 = \phi$

Complete bipartite graph

A bipartite graph $G = (V_1, V_2, E)$ is called a complete bipartite graph if there is an edge b/w every vertex of V_1 & every vertex of V_2.

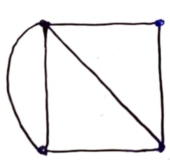
A complete bipartite graph is denoted by $K_{r,s}$ where V_1 has r vertices, V_2 has s vertices & $r \leq s$

Note: No. of vertices in $K_{r,s}$ = $r + s$

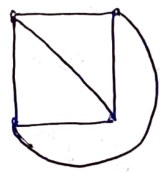
No. of edges in $K_{r,s}$ = rs

Q) Which of the following graph is complete graph?

G_1:



G_2:



→ G_1 is not a simple graph

∴ It is not a complete graph.

G_2 is complete graph as it is simple and every vertex is ~~(connected)~~ adjacent to remaining vertices of G_2.

② ST a complete graph with n vertices, K has ____ edges.

→ $\frac{n(n-1)}{2}$

[$\because$ choosing 2 out of n $\quad {}^nC_2 = \frac{n(n-1)}{2}$]

In a complete graph, there exists one edge b/w every pair of vertices. So no. of edges in a complete graph is = no. of pairs of vertices.
If there are n vertices, then no. of pair of vertices is ${}^nC_2$.

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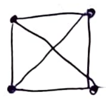
③ Show that a simple graph of order $n=4$ & size $m=7$ and a complete graph of order $n=4$ and size $m=5$ cannot exist.

→ i) Maximum no. of edges in simple graph is ${}^nC_2$.



⇒ For $n=4$, $m=6$,

As $m=7>6$, A simple graph with $n=4$, $m=7$ does not exist.

ii) In complete graph, no. of edges $=\frac{n(n-1)}{2}$

when $n=4$, $m=6$.

As $m=5<6$, A complete graph with $n=4$, $m=5$ does not exist.

4) a) How many edges & vertices are there in complete bipartite graph $K_{7,11}$?

b) If $K_{r,13}$ has 78 edges, find r.

→ a) No. of vertices in $K_{7,11} = 7 + 11 = 18$

No. of edges in $K_{7,11} = 7(11) = 77$

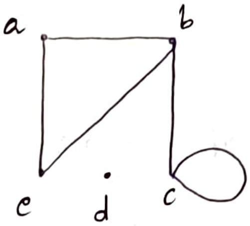
b) No. of edges in $K_{r,13} = 13r$

$78 = 13r$

$r = \underline{6}$

Degree of a Vertex:

G:



deg a = 2   deg d = 0

deg b = 3   deg e = 2

deg c = 3

Degree of a vertex v in G is no. of edges incident on v. It is denoted by deg(v) or d(v)

Regular Graph: A graph in which degree of every vertex is same is called regular graph. If degree of every vertex in G is k, then it is called k regular graph.

eg:

Note: Is Regular graph a complete graph? No.

Is complete graph a regular graph? ✓

⊛ 3 regular graph is called cubic graph.

Hand Shaking Property: [First theorem of Graph theory]

Statement: The sum of degrees of all the vertices in a graph is an even no. and this no. is = to twice the no. of edges in the graph.

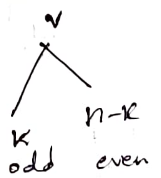
If several people shake hand, then total no. of handshake must be even because just 2 hands are involved in each handshake

Problem: In every graph, the no. of vertices of odd degree is even.

→ let G be a graph with n vertices, out of which k vertices are of odd degree and remaining n−k vertices are of even degrees.

Let $v_1, v_2 \ldots v_k$ be vertices of odd deg &

$v_{k+1}, v_{k+2} \ldots v_n$ be vertices of even deg.



TPT k is even

By HSP,
$$\sum_{i=1}^{n} d(v_i) = 2m$$

$$\sum_{i=1}^{k} d(v_i) + \sum_{i=k+1}^{n} d(v_i) = 2m$$

$$\sum_{i=1}^{k} d(v_i) = 2m - \sum_{i=k+1}^{n} d(v_i) \quad \text{——— (1)}$$

$\sum_{i=k+1}^{n} d(v_i)$ is an even no. [Sum of even no. is even]

RHS of ① is even [Even − Even = Even no.]

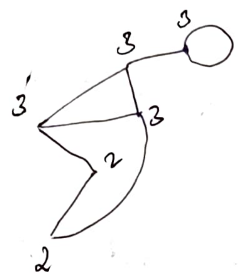
$\sum_{i=1}^{k} d(v_i) = \text{Even no.}$ ——— ②

$d(v_1) + d(v_2) + \dots + d(v_n) = \text{even no.}$

[∵ odd no's should be added even no. of times to get even no.]

⇒ even no. of terms on LHS, of ②

⇒ k is even



$2+2+3+3+3+3 = 16 = 2(8)$ HSP ✓

Q. Can there be a graph with 12 vertices such that 2 of the vertices have deg. 3 each, & remaining 10 vertices have deg. 4 each

→ [Note: If any of deg, order, size is given better to use HSP]

By HSP

$\sum_{i=1}^{12} d(v_i) = 2m$

$3+3+\underbrace{4+4+\dots+4}_{10 \text{ times}} =$

$= 6 + 4(10)$

$= 46 = 2(23)$

Since sum of deg is even no., we can construct graph with given data

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⑤ Determine order of graph G in following cases.

i) G is a cubic graph with 9 edges.

ii) G has 10 edges with 2 vertices of deg 4 and all other vertices of deg. 3

iii) G is a regular graph with 15 edges.

→ Let order of graph G be n.

By HSP, $\sum_{i=1}^{n} d(v_i) = 2m$

i) $\underbrace{3+3+\cdots+3}_{n \text{ times}} = 2(9)$

$\Rightarrow 3n = 18 \Rightarrow n = 6$

ii) $4+4+\underbrace{3+\cdots+3}_{n-2} = 2(10)$

$\Rightarrow 8 + 3(n-2) = 20$

$\Rightarrow n-2 = 4 \Rightarrow n = 6$

iii) Let G be k regular graph

$\underbrace{k+k+\cdots+k}_{n \text{ times}} = 2(15)$

$nk = 30$

$\Rightarrow n = \frac{30}{k}$; $k = 1, 2, 3, 5, 6, 10, 15, 30$

$n = 30, 15, 10, 6, 5, 3, 2, 1$

Q The deg of every vertex of graph G of order 2
& size 62 is either 3, 4, 5 or 6.
There are 2 vertices of deg 4 & 11 vertices
of degree 6. How many vertices of G have d
5 ?

→

$$4+4 + \underbrace{6+..+6}_{11 \text{ times}} + \underbrace{3+...+3}_{2k} + \underbrace{5+...+5}_{2l} = 2(62)$$

$2k + 2l + 11 + 2 = 25 \Rightarrow 13 + 2(l+k) = 25$

$= l + k = 6$

$8 + 66 + 6k + 10l = 124$

$8 + 66 + 36 + 4l = 124$

$2l = \frac{14}{2}$

$(2l) = 7$

$$\begin{array}{r} 124 \\ 110 \\ \hline 14 \end{array} \qquad \begin{array}{r} 8 \\ 66 \\ 36 \\ \hline 110 \end{array}$$

Q Let G be a graph of order 9 such that
each vertex has deg 5 or 6. P.T atleast
5 vertices have deg 6 (or) atleast 6 vertices
have deg 5.

→ Given, order of graph = 9

Let x vertices be of deg 5 and $9-x$ be
vertices be of deg 6.

By HSP,

$$\sum_{i=1}^{9} d(v_i) = 2m$$

$$\underbrace{5+5+\dots+5}_{x \text{ times}} + \underbrace{6+6+\dots+6}_{(9-x) \text{ times}} = 2m$$

$$5x + 6(9-x) = 2m$$

$$54 - x = (\text{even})$$

$$x = 54 - (\text{even}) \Rightarrow x = (\text{even})$$

$x = 0, 2, 4, 6, 8$

x deg 5	$9-x$ deg 6
0	9
2	7
4	5
6	3
8	1

In all above cases we observe that $x \geq 6$
(or) $y \geq 5 \Rightarrow$ atleast 6 vertices are of deg 5
(or) atleast 5 vertices are of deg 6.

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9) let G be a bipartite graph of order 22, with partition sets V_1 & V_2, where $|V_1| = 12$, suppose that every vertex in V_1 has degree 3, while every vertices of V_2 has degree 2 or 4. How many vertices of G has degree 2 & deg 4?

$3(12) \overset{?}{=} 2x + 4y = 2m$

$\cancel{36}$ $\cancel{=}$ $36 + 2x + 4(10-x) = (36)2$

$40 - 2x = 36$

$x = 2$

$10 - x = \underline{\underline{8}}$

Q) S.T the hypercube Q_3 is a bipartite graph but not a complete bipartite graph.

→ Q_2 : 

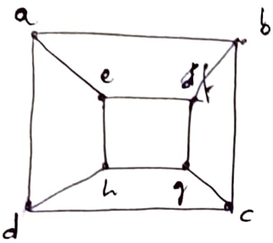
2^2 vertices

degree of every vertex : 2

Q_3 : 

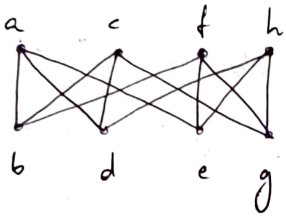
2^3 vertices

degree : 3

Q_3 : 

$V_1 = \{a, c, f, h\}$

$V_2 = \{b, d, e, g\}$

G : 

As $V_1 \cup V_2 = V$, $V_1 \cap V_2 = \phi$ and every edge of Q_3 joins a vertex of V_1 to a vertex of V_2, $\therefore$ Q_3 is bipartite graph.

Total no. of edges to be present for G to be complete bipartite $= 4 \times 4 = 16$

Since there are only 12 edges, Q_3 is not a complete bipartite graph.

Q) Prove that K-dimensional hypercube Q_K has $K2^{K-1}$ edges.

→ In an hypercube Q_K degree of every vertex is K & no. of vertices in Q_K is 2^K.

By HSP, $\sum_{i=1}^{2^k} = 2m$

$\Rightarrow \underbrace{k + k + k \ldots + k}_{2^k} = 2m \quad ; \quad k2^k = 2m$

$\Rightarrow \underline{m = k2^{k-1}}$

1) What is dimension of hypercube with 524288 edges

2) How many vertices are there in hypercube with 4980736 edges

→ Let Q_k be hypercube with dimensions k.

No. of edges in $Q_k = k2^{k-1}$

No. of vertices in $Q_k = 2^k$.

① No. of edges = 524288

$k2^{k-1} = 524288$

$k2^{k-1} = 2^{19}$

$2^4 2^{15} = 2^{19}$

$16\,2^{16-1} = 2^{19} \Rightarrow \underline{k = 16}$

No. of vertices = 2^{16}

2	524288
2	262144
2	131072
2	65536
2	32768
2	16384
2	8192
2^3	4096
2	2048
	1024

2) →

$k2^{k-1} = 4980736$

4980736

$19 \times 2^{18} = 4980736$

$\underline{k = 19}$

No. of vertices = 2^{19}.

Theorem 2 :

Statement : For a graph with n vertices, m edges, if δ is the minimum and Δ is maximum of edges degree of vertices, then,

$$\delta \leq \frac{2m}{n} \leq \Delta$$

Proof: Let G be a graph, with n vertices & m edges.

Let $d_1, d_2, \ldots, d_n$ be degree of vertices, $v_1, v_2, \ldots, v_n$ resp.

$\delta = \min\{d_1, d_2, \ldots, d_n\}$, $\Delta = \max\{d_1, d_2 \ldots d_n\}$

$\delta \leq d_1$, $\delta \leq d_2$, $\ldots$, $\delta \leq d_n$

$$\delta + \delta + \ldots + \delta \leq d_1 + d_2 + \ldots + d_n$$

$$n\delta \leq 2m \qquad [\because \sum d_i = 2m]$$

$$\Rightarrow \boxed{\delta \leq \frac{2m}{n}}$$

$\Delta \geq d_1$, $\Delta \geq d_2$ $\ldots$, $\Delta \geq d_n$

$$n\Delta \geq d_1 + d_2 + \ldots + d_n$$

$$n\Delta \geq 2m$$

$$\boxed{\Delta \geq \frac{2m}{n}}$$

$$\therefore \delta \leq \frac{2m}{n} \leq \Delta$$

Directed Graph [Digraph]

A graph $D=(V,E)$ with (vert) where V is non-empty set and E is set of ordered pairs of elements taken from set V. V is called the vertex set. E is called edge set.

Every directed edge of a digraph is determined by 2 vertices, a vertex from which it begins (called initial vertex) and a vertex at which it ends (terminal vertex)

eg:

a → initial vertex
b → terminal vertex

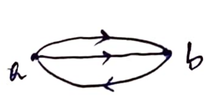
(= ab)

(= ba)

Note: ① For a directed edge, initial & terminal vertex need not be different.

A directed edge which begins & ends at same vertex is called a loop.

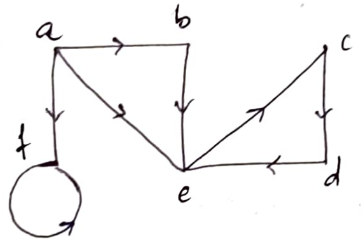
② Two or more directed edges, having same initial & terminal vertex are called parallel or multiple edges.



In degree & Out degree of a vertex

If v is a vertex of digraph D, the no. of edges for which v is initial vertex is called out degree of v, denoted by $d^+(v)$ or $od(v)$ & no. of edges for which v is terminal vertex is called in-degree of v denoted by $d^-(v)$ or $id(v)$

eg: D :



	a	b	c	d	e	f	
In-degree	0	1	1	1	3	2	8
Out-degree	3	1	1	1	1	1	8

Note : Sum of in-deg. (~~vertices~~) = Sum of out-deg (~~vertices~~)

First Theorem of Digraph

Statement : If In every digraph D, the sum of out-deg of all vertices = sum of in-deg. of all vertices, each sum being equal to no. of edges in D.

Proof: Let D be digraph with n vertices $v_1, v_2 \ldots, v_n$ & m edges.

Let r_1 be no. of edges going out of v_1
r_2 " " " " of v_2
⋮
r_n " " of v_n

$d^+(v_1) = r_1$, $d^+(v_2) = r_2$, ..., $d^+(v_n) = r_n$

Since every edge terminates at some vertex &

since there are m edges, we say

$r_1 + r_2 + \dots + r_n = m$

$$\boxed{\sum_{i=1}^{n} d^+(v_i) = m}$$

|||^{ly} let s_1 be no. of edges coming into v_1

s_2 " " " v_2

⋮

s_n " " " v_n

$d^-(v_1) = s_1$, $d^-(v_2) = s_2$, ... $d^-(v_n) = s_n$

$\Rightarrow$ $s_1 + s_2 + s_3 + \dots + s_n = m$

$$\boxed{\sum_{i=1}^{n} d^-(v_i) = m}$$

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(8) let D be a digraph with odd no. of vertices if each vertex of D has an odd out-degree Prove that D has an odd no. of vertices with odd in-degree.

→ ~~let~~ n (no. of vertices) = odd

since out-degree of every vertex is odd,

sum, $\sum_{i=1}^{n} d^+(v_i) = \text{odd}$ [∵ adding odd no. odd no. of times]

By First theorem of digraph

$$\sum_{i=1}^{n} d^+(v_i) = m = \sum_{i=1}^{n} d^-(v_i)$$

m is an odd no.

$$\sum_{i=1}^{n} d^-(v_i) = odd$$

out of n vertices, let k vertices $(v_1, v_2, \ldots v_k)$ be of odd in-degree & $n-k$ vertices $(v_{k+1}, v_{k+2}, \ldots, v_n)$ be of even in-degree.

$$\sum_{i=1}^{k} d^-(v_i) + \underbrace{\sum_{i=k+1}^{n} d^-(v_i)}_{even} = odd.$$

The second sum on LHS is an even no.

$$\Rightarrow \sum_{i=1}^{k} d^-(v_i) = odd - even = odd$$

$\Rightarrow$ In order to make sum to be odd, odd no. should be added odd no. of times.

This $\Rightarrow$ no. of terms with odd in-degree is odd [$\because$ k is odd]

Isomorphism :

Consider 2 graphs $G=(V,E)$ & $G'=(V',E')$.

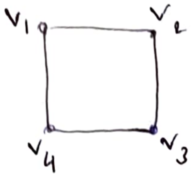
Suppose there exists a funⁿ $f: V \to V'$ & f is one to one correspondance ($\to$ means one to one & one to one onto funⁿ) & for all vertices a, b in G, ab is an edge in G iff $f(a)f(b)$ is an edge in G'. Then G is said to be isomorphic to G', $\Rightarrow$ $\boxed{G \cong G'}$

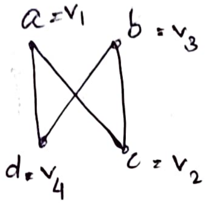
Q) Check whether following graphs are isomorphic or not.

1) G_1 :  G_2 : 

→ G_1 has 4 edges, 4 vertices & deg sequence is 2,2,2,2

G_2 " " " "





Vertex correspondance

$v_1 \leftrightarrow a$
$v_2 \leftrightarrow c$
$v_3 \leftrightarrow b$
$v_4 \leftrightarrow d$

edge correspondance

$v_1v_2 \leftrightarrow ac$
$v_2v_3 \leftrightarrow cb$
$v_3v_4 \leftrightarrow bd$
$v_4v_1 \leftrightarrow da$

These represent one to one correspondance b/w edges of 2 graph under which adjacent vertices in 1st graph corresponds to adjacent vertices in 2nd graph & vice-versa.

$\therefore G_1 \cong G_2$.

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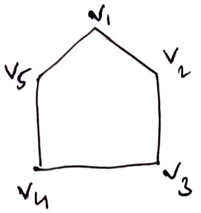
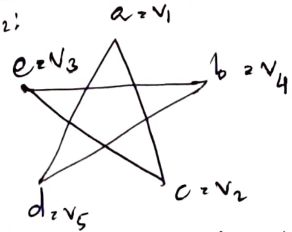
2) G_1: 

G_2: 

G_1 has 5 edges, 5 vertices & degree sequence 2,2,2,2,2.

G_2 " " " " "

Vertex correspondance

$v_1 \leftrightarrow a$
$v_2 \leftrightarrow c$
$v_3 \leftrightarrow e$
$v_4 \leftrightarrow b$
$v_5 \leftrightarrow d$

edge correspondance

$v_1v_2 \leftrightarrow ac$
$v_2v_3 \leftrightarrow ce$
$v_3v_4 \leftrightarrow eb$
$v_4v_5 \leftrightarrow bd$
$v_5v_1 \leftrightarrow da$

These represent one to one correspondance b/w edges of 2 graph under which adjacent vertices in graph 1 corresponds to adjacent vertices ~~to~~ of graph

$\boxed{\therefore G_1 \cong G_2}$

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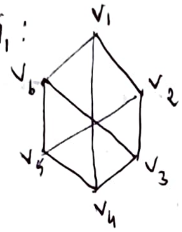
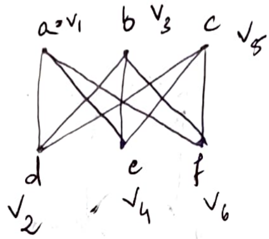
③ G_1:



G_2:



G_1 has 6 vertices & 9 edges, &

G_2 has 6 vertices & 9 edges.

Deg. seq. of G_1 is 3,3,3,3,3,3

Deg. seq. of G_2 is 3,3,3,3,3,3.

For vertex correspondance,

$a = v_1$, $b = v_3$, $c = v_5$, $d = v_2$, $e = v_4$, $f = v_6$.

Edge correspondance,

$v_1v_2 \leftrightarrow ad$

$v_1v_6 \leftrightarrow af$

$v_6v_5 \leftrightarrow fc$

$v_5v_4 \leftrightarrow ce$

$v_4v_3 \leftrightarrow eb$

$v_3v_2 \leftrightarrow bd$

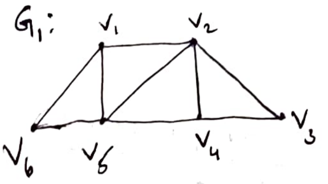
$v_6v_3 \leftrightarrow fb$

$v_5v_2 \leftrightarrow cd$

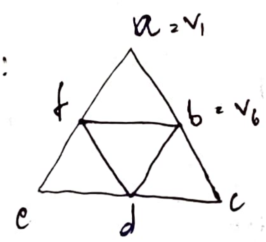
$v_1v_4 \leftrightarrow ae$

$\therefore G_1 \cong G_2$

④ G_1:



G_2:



G_1 has 6 vertices, 9 edges

G_2 has 6 vertices, 9 edges.

Deg seq. of G_1 is

2, 2, 3, 3, 4, 4

Deg seq in G_2 is

2, 2, 2, 4, 4, 4.

vertex correspondance,

v_1

v_2

v_3

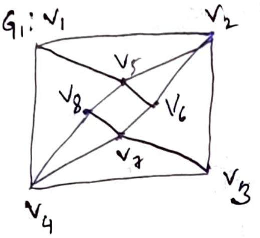
v_4

v_5

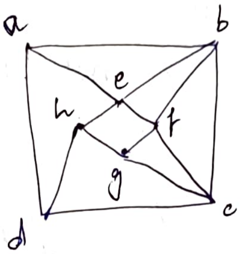
As deg. seq. of G_1 & G_2 are not same, ~~to~~ G_1 is not isomorphic to G_2 as there cannot be 1-1 correspondance.

---

(5) G_1:



G_2:



G_1 has 8 vertices, 14 edges,

G_2 has 8 vertices, 14 edges.

Deg seq. in G_1 is 3, 3, 3, 3, 4, 4, 4, 4

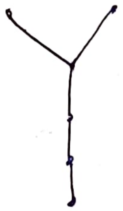
Deg seq in G_2 is 3, 3, 3, 3, 4, 4, 4, 4

A deg. 3 vertex in G_1 is adjacent to 3 vertices with deg. 4. whereas in G_2 such a vertex does not exist. Hence there cannot be 1-1 correspon-dance b/w the vertices of G_1 & G_2.

$\therefore$ G_1 is not isomorphic to G_2

---

(6) G_1:



G_2:

G_1 has 6 vertices, 5 edges

G_2 has 6 vertices & 5 edges

Deg. seq. in G_1 is 1, 1, 1, 2, 2, 3

Deg seq. in G_2 is 1, 1, 1, 2, 2, 3.

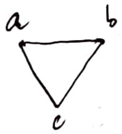
A deg 3 vertex in G_1 is adjacent to 2, & deg 1 vertex & 1 deg 2 vertex whereas a deg 3 vertex in G_2 is adjacent to 2 deg 2 vertex & 1 deg 1 vertex.

$\therefore$ G_1 is not isomorphic to G_2.

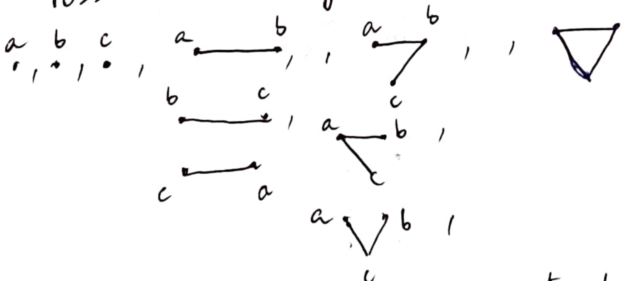
<u>Sub-graph</u>: Given 2 graphs G & G_1 we say that G_1 is sub-graph of G if following condit, hold.

1. All the vertices & all the edges of G_1 are in G.
2. Each edge of G_1 has same end vertices in G as in G_1.

Ex:

G : 

Possible sub-graphs,



... etc

(subgraph may or may not have all edges of original graph, but there must strictly be no new edges)

Note:

a) A subgraph of a graph is also a graph.

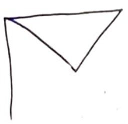
b) Every graph is a sub-graph of itself.

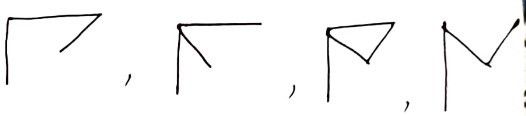
c) Every simple graph of n-vertices is a subgraph of complete graph K_n.

d) A single vertex in a graph G is a subgraph of G.

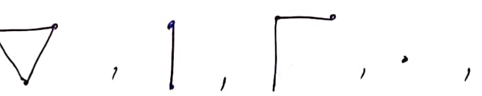
e) A single edge in a graph, together with its end vertices is a subgraph of G.

Spanning Subgraph: (when vertex-set of subgraph is same as original graph)

eg:
G: 

spanning subgraphs of G : 

subgraphs of G which are not spanning,



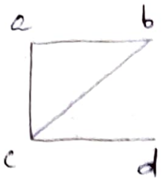
Given a graph $G(V,E)$, if there is a subgraph $G_1(V_1,E_1)$ of G such that $V = V_1$, then G_1 is spanning sub-graph of G.

Induced Subgraph:

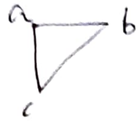
Given a graph $G(V,E)$, suppose there is a subgraph $G_1(V_1,E_1)$ of G such that every edge ~~AB~~ of ab of G where $a,b \in V_1$ is an edge of G_1 also, then G_1 is called induced subgraph of G denoted by $\langle V_1 \rangle$.

$\langle v_1 \rangle$ (Induced by v_1) [∵ For the vertices included in subgraph all corresponding edges must be included]

eg:
G :

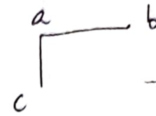


G_1 :



, G_2 :

, G_3 :



→ not induced

[∵ bc edge is missing]

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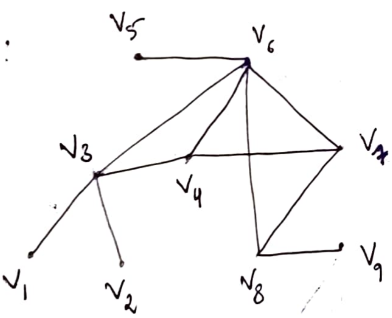
Q Consider the given graph

1) Verify that G_1 is an induced subgraph of G. Is this a spanning subgraph of G ?

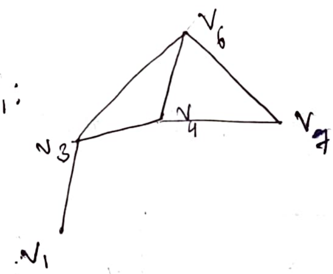
2) Draw the subgraph G_2 of G induced by the set $V_2 = \{v_3, v_4, v_6, v_8, v_9\}$

→

G :



G_1 :



→

1) $V_1 = \{v_1, v_3, v_4, v_7, v_6\} \subseteq V$

& edges in G_1 are edges in G

Each edge in G_1 has same end vertices as in G

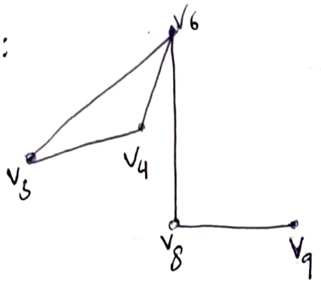
∴ G_1 is a subgraph of G [∵ $G_1 \subseteq G$]

Two vertices in G_1 are adjacent whenever they are adjacent in G_1.

∴ G_1 is induced subgraph of G.

As $V_1 \neq V$, G_1 is not spanning subgraph of G.

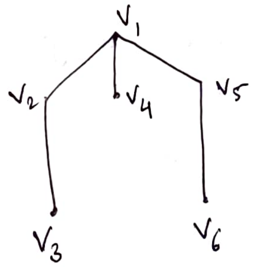
c) G_2 :



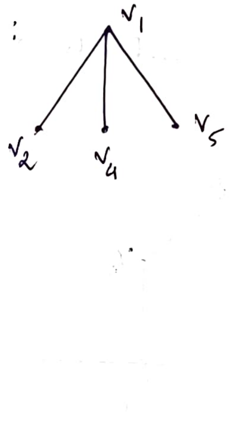
Q) Consider graph G.

i) Verify that graph G_1 & G_2 shown below are induced subgraph of G whereas graph G_3 is not an induced subgraph of G.

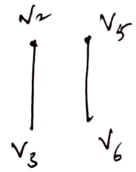
G :



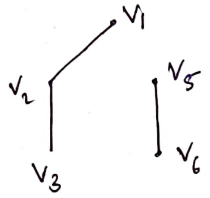
G_1 :



G_2 :



G_3 :



→ $V_1 = \{v_1, v_2, v_4, v_5\} \subseteq V$

$V_2 = \{v_2, v_3, v_5, v_6\} \subseteq V$

$V_3 = \{v_1, v_2, v_3, v_5, v_6\} \subseteq V$

} ⇒ Vertex set of G_1, G_2, G_3 are subsets of V

& edge set of G_1, G_2, G_3 are subsets of edge set of G.

Each edge in G_1, G_2, G_3 has same end vertices as in G.

$\therefore G_1, G_2, G_3 \subseteq G$

~~Two~~ G_1 & G_2 are induced subgraphs of G as whenever two vertices in G_1, G_2 are adjacent they are adjacent in G.