Forward Chaining and Backward Chaining in Al

In artificial intelligence, forward and backward chaining is one of the important topics, but before understanding forward and backward chaining lets first understand that from where these two terms came.

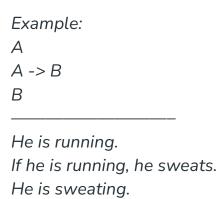
Inference engine:

The inference engine is the component of the intelligent system in artificial intelligence, which applies logical rules to the knowledge base to infer new information from known facts. The first inference engine was part of the expert system. Inference engine commonly proceeds in two modes, which are:

- a. Forward chaining
- b. Backward chaining

Forward Chaining:

Forward Chaining the Inference Engine goes through all the facts, conditions and derivations before deducing the outcome i.e When based on available data a decision is taken then the process is called as Forwarding chaining, It works from an initial state and reaches to the goal(final decision).



Backward Chaining:

In this, the inference system knows the final decision or goal, this system starts from the goal and works backwards to determine what facts must be asserted so that the goal can be achieved, i.e it works from goal(final decision) and reaches the initial state.

Example:

В

A -> B

A

He is sweating.
If he is running, he sweats.
He is running.

Forward Chaining

Difference between Forwarding Chaining and Backward Chaining:

1.	When based on available data a decision is taken then the process is called as Forward chaining.	Backward chaining starts from the goal and works backward to determine what facts must be asserted so that the goal can be achieved.
2.	Forward chaining is known as data-driven technique because we reaches to the goal using the available data.	Backward chaining is known as goal- driven technique because we start from the goal and reaches the initial state in order to extract the facts.
3.	It is a bottom-up approach.	It is a top-down approach.
4.	It applies the Breadth-First Strategy.	It applies the Depth-First Strategy.
5.	Its goal is to get the conclusion.	Its goal is to get the possible facts or the required data.

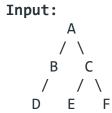
Backward Chaining

6.	Slow as it has to use all the rules.	Fast as it has to use only a few rules.
7.	It operates in forward direction i.e it works from initial state to final decision.	It operates in backward direction i.e it works from goal to reach initial state.
8.	Forward chaining is used for the planning, monitoring, control, and interpretation application.	It is used in automated inference engines, theorem proofs, proof assistants and other artificial intelligence applications.

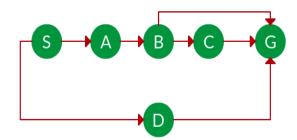
Breadth-First Search:

BFS, **Breadth-First Search**, is a vertex-based technique for finding the shortest path in the graph. It uses a <u>Queue data structure</u> that follows first in first out. In BFS, one vertex is selected at a time when it is visited and marked then its adjacent are visited and stored in the queue. It is slower than DFS.

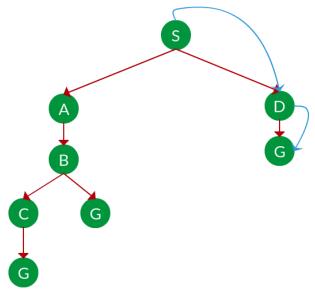
Example:



Output:



Solution. The equivalent search tree for the above graph is as follows. As BFS traverses the tree "shallowest node first", it would always pick the shallower branch until it reaches the solution (or it runs out of nodes, and goes to the next branch). The traversal is shown in blue arrows.



Path: $S \rightarrow D \rightarrow G$

= the depth of the shallowest solution.

= number of nodes in level .

Time complexity: Equivalent to the number of nodes traversed in BFS until the shallowest solution.

Space complexity: Equivalent to how large can the fringe

get.

Completeness: BFS is complete, meaning for a given search tree, BFS will come up with a solution if it exists.

Optimality: BFS is optimal as long as the costs of all edges are equal.

Depth First Search:

DFS, Depth First Search, is an edge-based technique. It uses the <u>Stack data structure</u> and performs two stages, first visited vertices are pushed into the stack, and second if there are no vertices then visited vertices are popped.

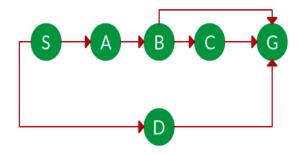
Example:

Input:

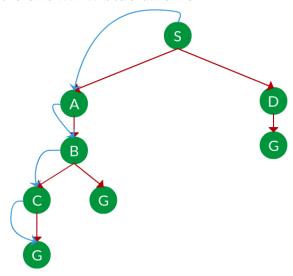


Output:

Example:



Solution. The equivalent search tree for the above graph is as follows. As DFS traverses the tree "deepest node first", it would always pick the deeper branch until it reaches the solution (or it runs out of nodes, and goes to the next branch). The traversal is shown in blue arrows.



Path: S -> A -> B -> C -> G

= the depth of the search tree = the number of levels of the search tree.

= number of nodes in level .

Time complexity: Equivalent to the number of nodes traversed in

DFS.

Space complexity: Equivalent to how large can the fringe

get.

BFS vs DFS

S. No.	Parameters	BFS	DFS
1.	Stands for	BFS stands for Breadth First Search.	DFS stands for Depth First Search.
2.	Data Structure	BFS(Breadth First Search) uses Queue data structure for finding the shortest path.	DFS(Depth First Search) uses Stack data structure.
3.	Definition	BFS is a traversal approach in which we first walk through all nodes on the same level before moving on to the next level.	DFS is also a traversal approach in which the traverse begins at the root node and proceeds through the nodes as far as possible until we reach the node with no unvisited nearby nodes.
4.	Technique	BFS can be used to find a single source shortest	In DFS, we might traverse through more

S. No.	Parameters	BFS	DFS
		path in an unweighted graph because, in BFS, we reach a vertex with a minimum number of edges from a source vertex.	edges to reach a destination vertex from a source.
5.	Conceptual Difference	BFS builds the tree level by level.	DFS builds the tree subtree by sub-tree.
6.	Approach used	It works on the concept of FIFO (First In First Out).	It works on the concept of LIFO (Last In First Out).
7.	Suitable for	BFS is more suitable for searching vertices closer to the given source.	DFS is more suitable when there are solutions away from source.
8.	Suitability for Decision- Trees	BFS considers all neighbors first and therefore not suitable for decision-making trees used in games or puzzles.	DFS is more suitable for game or puzzle problems. We make a decision, and the then explore all paths through this decision. And if this decision leads to win situation, we stop.
9.	Time Complexity	The Time complexity of BFS is O(V + E) when Adjacency List is used and O(V^2) when Adjacency Matrix is	The Time complexity of DFS is also O(V + E) when Adjacency List is used and O(V^2) when Adjacency Matrix is

S. No.	Parameters	BFS	DFS
		used, where V stands for vertices and E stands for edges.	used, where V stands for vertices and E stands for edges.
10.	Visiting of Siblings/ Children	Here, siblings are visited before the children.	Here, children are visited before the siblings.
11.	Removal of Traversed Nodes	Nodes that are traversed several times are deleted from the queue.	The visited nodes are added to the stack and then removed when there are no more nodes to visit.
12.	Backtracking	In BFS there is no concept of backtracking.	DFS algorithm is a recursive algorithm that uses the idea of backtracking
13.	Applications	BFS is used in various applications such as bipartite graphs, shortest paths, etc.	DFS is used in various applications such as acyclic graphs and topological order etc.
14.	Memory	BFS requires more memory.	DFS requires less memory.
15.	Optimality	BFS is optimal for finding the shortest path.	DFS is not optimal for finding the shortest path.

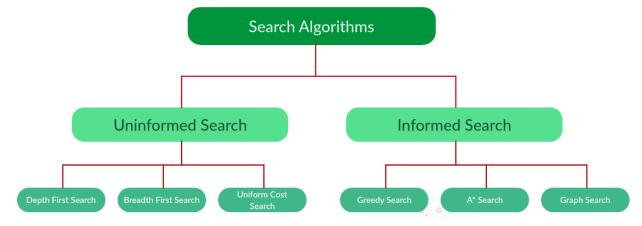
S. No.	Parameters	BFS	DFS
16.	Space complexity	In BFS, the space complexity is more critical as compared to time complexity.	DFS has lesser space complexity because at a time it needs to store only a single path from the root to the leaf node.
17.	Speed	BFS is slow as compared to DFS.	DFS is fast as compared to BFS.
18,	Tapping in loops	In BFS, there is no problem of trapping into infinite loops.	In DFS, we may be trapped in infinite loops.
19.	When to use?	When the target is close to the source, BFS performs better.	When the target is far from the source, DFS is preferable.

Artificial Intelligence is the study of building agents that act rationally. Most of the time, these agents perform some kind of search algorithm in the background in order to achieve their tasks.

- A search problem consists of:
 - A State Space. Set of all possible states where you can be.
 - A Start State. The state from where the search begins.
 - A Goal State. A function that looks at the current state returns whether or not it is the goal state.
- The **Solution** to a search problem is a sequence of actions, called the **plan** that transforms the start state to the goal state.
- This plan is achieved through search algorithms.

Types of search algorithms:

There are far too many powerful search algorithms out there to fit in a single article. Instead, this article will discuss *six* of the fundamental search algorithms, divided into *two* categories, as shown below.



Uninformed Search Algorithms:

The search algorithms in this section have no additional information on the goal node other than the one provided in the problem definition. The plans to reach the goal state from the start state differ only by the order and/or length of actions. Uninformed search is also called **Blind search**. These algorithms can only generate the successors and differentiate between the goal state and non goal state.

The following uninformed search algorithms are discussed in this section.

- 1. Depth First Search
- 2. Breadth First Search
- 3. Uniform Cost Search

Each of these algorithms will have:

- A problem **graph**, containing the start node S and the goal node G.
- A **strategy**, describing the manner in which the graph will be traversed to get to G.
- A **fringe**, which is a data structure used to store all the possible states (nodes) that you can go from the current states.
- A **tree**, that results while traversing to the goal node.

• A solution **plan**, which the sequence of nodes from S to G.

Uniform Cost Search:

UCS is different from BFS and DFS because here the costs come into play. In other words, traversing via different edges might not have the same cost. The goal is to find a path where the cumulative sum of costs is the least.

Cost of a node is defined as:

```
cost(node) = cumulative cost of all nodes from root
cost(root) = 0
```

Advantages:

- UCS is complete only if states are finite and there should be no loop with zero weight.
- UCS is optimal only if there is no negative cost.

Disadvantages:

- Explores options in every "direction".
- No information on goal location.

Informed Search Algorithms:

Here, the algorithms have information on the goal state, which helps in more efficient searching. This information is obtained by something called a *heuristic*. In this section, we will discuss the following search algorithms.

- 1. Greedy Search
- 2. A* Tree Search
- 3. A* Graph Search

Search Heuristics: In an informed search, a heuristic is a *function* that estimates how close a state is to the goal state. For example – Manhattan distance, Euclidean distance, etc. (Lesser the distance, closer the goal.) Different heuristics are used in different informed algorithms discussed below.

Greedy Search:

In greedy search, we expand the node closest to the goal node. The "closeness" is estimated by a heuristic h(x).

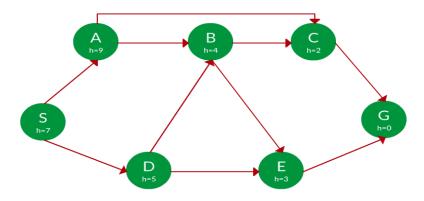
Heuristic: A heuristic h is defined as-

h(x) = Estimate of distance of node x from the goal node.

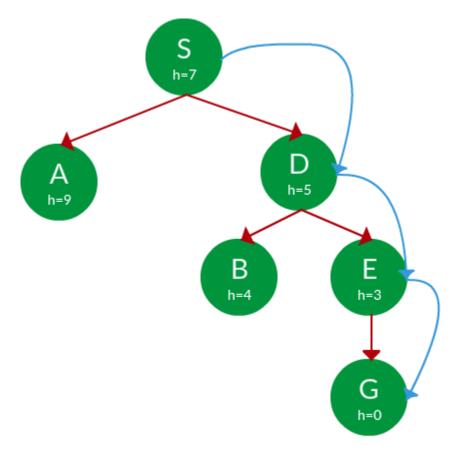
Lower the value of h(x), closer is the node from the goal.

Strategy: Expand the node closest to the goal state, *i.e.* expand the node with a lower h value.

Question. Find the path from S to G using greedy search. The heuristic values h of each node below the name of the node.



Solution. Starting from S, we can traverse to A(h=9) or D(h=5). We choose D, as it has the lower heuristic cost. Now from D, we can move to B(h=4) or E(h=3). We choose E with a lower heuristic cost. Finally, from E, we go to G(h=0). This entire traversal is shown in the search tree below, in blue.



Path: $S \rightarrow D \rightarrow E \rightarrow G$

Advantage: Works well with informed search problems, with fewer steps to

reach a goal.

Disadvantage: Can turn into unguided DFS in the worst case.

A* Tree Search:

A* Tree Search, or simply known as A* Search, combines the strengths of uniform-cost search and greedy search. In this search, the heuristic is the summation of the cost in UCS, denoted by g(x), and the cost in the greedy search, denoted by h(x). The summed cost is denoted by h(x).

Heuristic: The following points should be noted wrt heuristics in A* search.

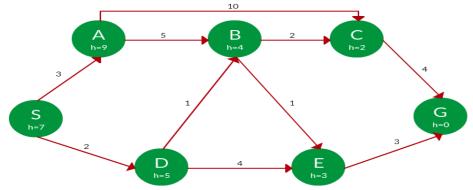
- Here, h(x) is called the **forward cost** and is an estimate of the distance of the current node from the goal node.
- And, g(x) is called the **backward cost** and is the cumulative cost of a node from the root node.
- A* search is optimal only when for all nodes, the forward cost for a node h(x) underestimates the actual cost h*(x) to reach the goal. This property of A* heuristic is called **admissibility**.

Admissibility:

Strategy: Choose the node with the lowest f(x) value.

Example:

Question. Find the path to reach from S to G using A* search.



Solution. Starting from S, the algorithm computes g(x) + h(x) for all nodes in the fringe at each step, choosing the node with the lowest sum. The entire work is shown in the table below.

Note that in the fourth set of iterations, we get two paths with equal summed cost f(x), so we expand them both in the next set. The path with a lower cost on further expansion is the chosen path.

Path	h(x)	$\mathbf{g}(\mathbf{x})$	f(x)
S	7	0	7
S -> A	9	3	12
S -> D	5	2	7

S -> D -> B	4	2 + 1 = 3 7
$S \rightarrow D \rightarrow E$	3	2 + 4 = 6 9
$S \rightarrow D \rightarrow B \rightarrow C$	2	3 + 2 = 5 7
$S \rightarrow D \rightarrow B \rightarrow E$	3	3+1=4 7
$S \rightarrow D \rightarrow B \rightarrow C \rightarrow G$	0	5+4=9 9
$S \rightarrow D \rightarrow B \rightarrow E \rightarrow$		
G	0	4 + 3 = 7 7

Path: S -> D -> B -> E -> G **Cost:** 7

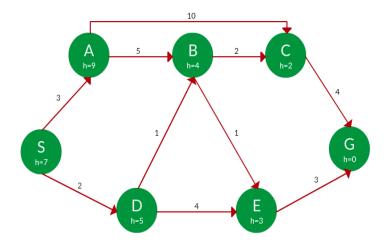
A* graph search:

- A* tree search works well, except that it takes time re-exploring the branches it has already explored. In other words, if the same node has expanded twice in different branches of the search tree, A* search might explore both of those branches, thus wasting time
- A* Graph Search, or simply Graph Search, removes this limitation by adding this rule: do not expand the same node more than once.
- Heuristic. Graph search is optimal only when the forward cost between two successive nodes A and B, given by h(A) – h (B), is less than or equal to the backward cost between those two nodes g(A -> B). This property of the graph search heuristic is called consistency.

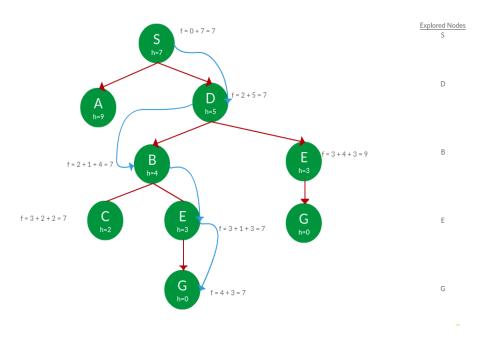
Consistency:

Example:

Question. Use graph searches to find paths from S to G in the following graph.



the **Solution.** We solve this question pretty much the same way we solved last question, but in this case, we keep a track of nodes explored so that we don't re-explore them.



Path: S -> D -> B -> E -> G

Cost: 7

Hill climbing:

Hill climbing is a simple optimization algorithm used in Artificial Intelligence (AI) to find the best possible solution for a given problem. It belongs to the family of local search algorithms and is often used in optimization problems where the goal is to find the best solution from a set of possible solutions.

- In Hill Climbing, the algorithm starts with an initial solution and then iteratively makes small changes to it in order to improve the solution. These changes are based on a heuristic function that evaluates the quality of the solution. The algorithm continues to make these small changes until it reaches a local maximum, meaning that no further improvement can be made with the current set of moves.
- There are several variations of Hill Climbing, including steepest ascent Hill Climbing, first-choice Hill Climbing, and simulated annealing. In steepest ascent Hill Climbing, the algorithm evaluates all the possible moves from the current solution and selects the one that leads to the best improvement. In first-choice Hill Climbing, the algorithm randomly selects a move and accepts it if it leads to an improvement, regardless of whether it is the best move. Simulated annealing is a probabilistic variation of Hill Climbing that allows the algorithm to occasionally accept worse moves in order to avoid getting stuck in local maxima.

Hill Climbing can be useful in a variety of optimization problems, such as scheduling, route planning, and resource allocation. However, it has some limitations, such as the tendency to get stuck in local maxima and the lack of diversity in the search space. Therefore, it is often combined with other optimization techniques, such as genetic algorithms or simulated annealing, to overcome these limitations and improve the search results.

Advantages of Hill Climbing algorithm:

- 1. Hill Climbing is a simple and intuitive algorithm that is easy to understand and implement.
- 2. It can be used in a wide variety of optimization problems, including those with a large search space and complex constraints.

- 3. Hill Climbing is often very efficient in finding local optima, making it a good choice for problems where a good solution is needed quickly.
- 4. The algorithm can be easily modified and extended to include additional heuristics or constraints.

Disadvantages of Hill Climbing algorithm:

- 1. Hill Climbing can get stuck in local optima, meaning that it may not find the global optimum of the problem.
- 2. The algorithm is sensitive to the choice of initial solution, and a poor initial solution may result in a poor final solution.
- 3. Hill Climbing does not explore the search space very thoroughly, which can limit its ability to find better solutions.
- 4. It may be less effective than other optimization algorithms, such as genetic algorithms or simulated annealing, for certain types of problems.

Hill Climbing is a heuristic search used for mathematical optimization problems in the field of Artificial Intelligence. Given a large set of inputs and a good heuristic function, it tries to find a sufficiently good solution to the problem. This solution may not be the global optimal maximum.

- In the above definition, **mathematical optimization problems** imply that hill-climbing solves the problems where we need to maximize or minimize a given real function by choosing values from the given inputs. Example-<u>Travelling salesman problem</u> where we need to minimize the distance traveled by the salesman.
- 'Heuristic search' means that this search algorithm may not find the optimal solution to the problem. However, it will give a good solution in a **reasonable time**.
- A heuristic function is a function that will rank all the possible alternatives at any branching step in the search algorithm based on the available information. It helps the algorithm to select the best route out of possible routes.

Features of Hill Climbing

1. Variant of generating and test algorithm:

It is a variant of generating and testing algorithms. The generate and test algorithm is as follows :

- Generate possible solutions.
- Test to see if this is the expected solution.
- If the solution has been found quit else go to step 1.

2. Uses the Greedy approach:

At any point in state space, the search moves in that direction only which optimizes the cost of function with the hope of finding the optimal solution at the end.

Types of Hill Climbing

1. Simple Hill climbing:

It examines the neighboring nodes one by one and selects the first neighboring node which optimizes the current cost as the next node.

2. Steepest-Ascent Hill climbing:

It first examines all the neighboring nodes and then selects the node closest to the solution state as of the next node.

3. Stochastic hill climbing:

It does not examine all the neighboring nodes before deciding which node to select. It just selects a neighboring node at random and decides (based on the amount of improvement in that neighbor) whether to move to that neighbor or to examine another.

What is Dynamic Programming?

Dynamic Programming is mainly an optimization over plain <u>recursion</u>. Wherever we see a recursive solution that has repeated calls for same inputs, we can optimize it using Dynamic Programming. The idea is to simply store the results of subproblems, so that we do not have to re-compute them when needed later. This simple optimization reduces time complexities from exponential to polynomial.

For example, if we write simple recursive solution for <u>Fibonacci Numbers</u>, we get exponential time complexity and if we optimize it by storing solutions of subproblems, time complexity reduces to linear.

Let's understand this approach through an example.

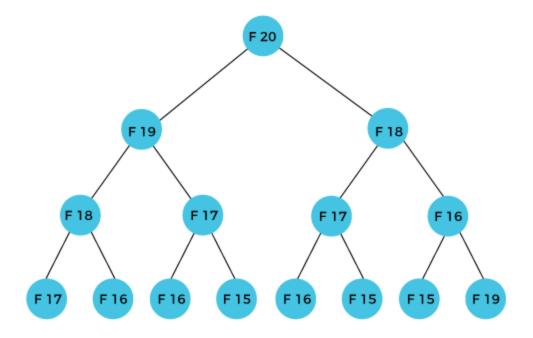
The numbers in the above series are not randomly calculated. Mathematically, we could write each of the terms using the below formula:

$$F(n) = F(n-1) + F(n-2)$$

With the base values F(0) = 0, and F(1) = 1. To calculate the other numbers, we follow the above relationship. For example, F(2) is the sum f(0) and f(1), which is equal to 1.

How can we calculate F(20)?

The F(20) term will be calculated using the nth formula of the Fibonacci series. The below figure shows that how F(20) is calculated.



As we can observe in the above figure that F(20) is calculated as the sum of F(19) and F(18). In the dynamic programming approach, we try to divide the problem into the similar subproblems. We are following this approach in the above case where F(20) into the similar subproblems, i.e., F(19) and F(18). If we recap the definition of dynamic programming that it says the similar subproblem should not be computed more than once. Still, in the above case, the subproblem is calculated twice. In the above example, F(18) is calculated two times; similarly, F(17) is also calculated twice. However, this technique is quite useful as it solves the similar subproblems, but we need to be cautious while storing the results because we are not particular about storing the result that we have computed once, then it can lead to a wastage of resources.

What is the Greedy-Best-first search algorithm?

Greedy Best-First Search is an AI search algorithm that attempts to find the most promising path from a given starting point to a goal. It prioritizes paths that appear to be the most promising, regardless of whether or not they are actually the shortest path. The algorithm works by evaluating the cost of each possible path and then expanding the path with the lowest cost. This process is repeated until the goal is reached.

The algorithm works by using a heuristic function to determine which path is the most promising. The heuristic function takes into account the cost of the current path and the estimated cost of the remaining paths. If the cost of the current

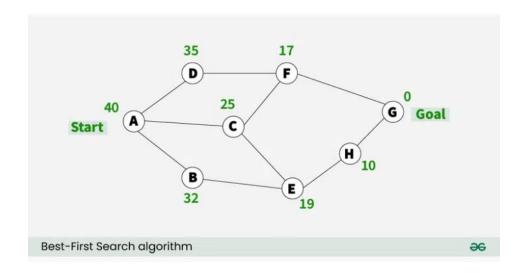
path is lower than the estimated cost of the remaining paths, then the current path is chosen. This process is repeated until the goal is reached.

How Greedy Best-First Search Works?

- Greedy Best-First Search works by evaluating the cost of each possible path and then expanding the path with the lowest cost. This process is repeated until the goal is reached.
- The algorithm uses a heuristic function to determine which path is the most promising.
- The heuristic function takes into account the cost of the current path and the estimated cost of the remaining paths.
- If the cost of the current path is lower than the estimated cost of the remaining paths, then the current path is chosen. This process is repeated until the goal is reached.

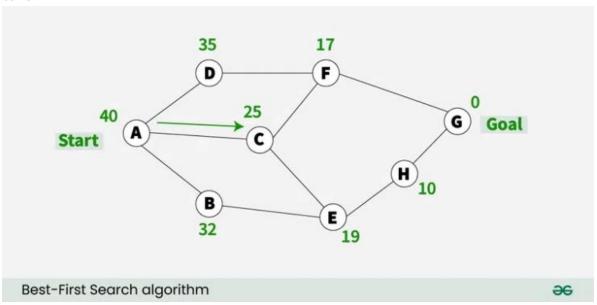
An example of the best-first search algorithm is below graph, suppose we have to find the path from A to G

The values in red color represent the heuristic value of reaching the goal node G from current node

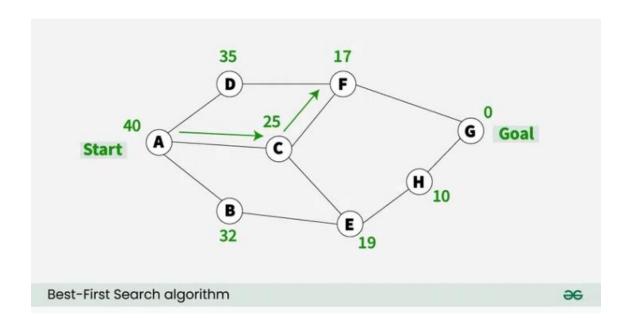


1) We are starting from A, so from A there are direct path to node B(with heuristics value of 32), from A to C (with heuristics value of 25) and from A to D(with heuristics value of 35).

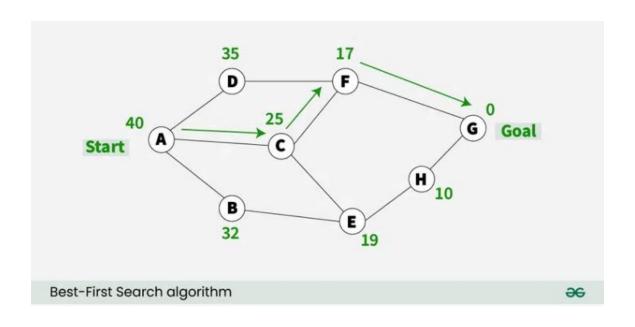
2) So as per best first search algorithm choose the path with lowest heuristics value, currently C has lowest value among above node. So we will go from A to C.



3) Now from C we have direct paths as C to F(with heuristics value of 17) and C to E(with heuristics value of 19) , so we will go from C to F



4) Now from F we have direct path to go to the goal node G (with heuristics value of 0), so we will go from F to G.



5) So now the goal node G has been reached and the path we will follow is A->C->F->G.

Advantages of Greedy Best-First Search:

• **Simple and Easy to Implement:** Greedy Best-First Search is a relatively straightforward algorithm, making it easy to implement.

- Fast and Efficient: Greedy Best-First Search is a very fast algorithm, making it ideal for applications where speed is essential.
- Low Memory Requirements: Greedy Best-First Search requires only a small amount of memory, making it suitable for applications with limited memory.
- **Flexible:** Greedy Best-First Search can be adapted to different types of problems and can be easily extended to more complex problems.
- **Efficiency:** If the heuristic function used in Greedy Best-First Search is good to estimate, how close a node is to the solution, this algorithm can be a very efficient and find a solution quickly, even in large search spaces.

Disadvantages of Greedy Best-First Search:

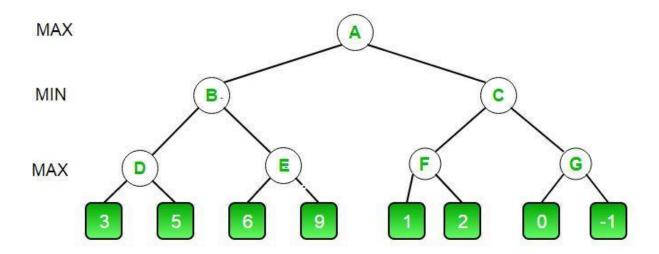
- Inaccurate Results: Greedy Best-First Search is not always guaranteed to find the optimal solution, as it is only concerned with finding the most promising path.
- Local Optima: Greedy Best-First Search can get stuck in local optima, meaning that the path chosen may not be the best possible path.
- **Heuristic Function:** Greedy Best-First Search requires a heuristic function in order to work, which adds complexity to the algorithm.
- Lack of Completeness: Greedy Best-First Search is not a complete algorithm, meaning it may not always find a solution if one is exists. This can happen if the algorithm gets stuck in a cycle or if the search space is a too much complex.

Alpha-Beta pruning:

Alpha-Beta pruning is not actually a new algorithm, but rather an optimization technique for the minimax algorithm. It reduces the computation time by a huge factor. This allows us to search much faster and even go into deeper levels in the game tree. It cuts off branches in the game tree which need not be searched because there already exists a better move available. It is called Alpha-Beta pruning because it passes 2 extra parameters in the minimax function, namely alpha and beta.

Alpha is the best value that the **maximizer** currently can guarantee at that level or above.

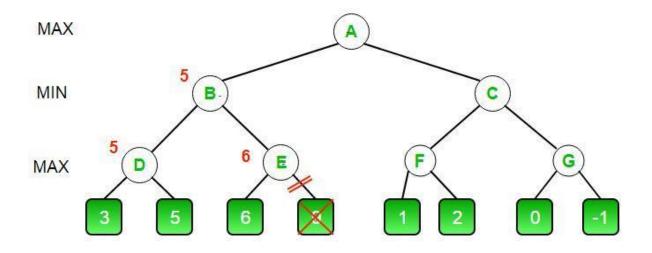
Beta is the best value that the **minimizer** currently can guarantee at that level or below.



- The initial call starts from A. The value of alpha here is -INFINITY and the value of beta is +INFINITY. These values are passed down to subsequent nodes in the tree. At A the maximizer must choose max of B and C, so A calls B first
- At B it the minimizer must choose min of D and E and hence calls D first.
- At **D**, it looks at its left child which is a leaf node. This node returns a value of 3. Now the value of alpha at **D** is max(-INF, 3) which is 3.
- To decide whether its worth looking at its right node or not, it checks the condition beta<=alpha. This is false since beta = +INF and alpha = 3. So it continues the search.
- **D** now looks at its right child which returns a value of 5.At **D**, alpha = max(3, 5) which is 5. Now the value of node **D** is 5
- **D** returns a value of 5 to **B**. At **B**, beta = min(+INF, 5) which is 5. The minimizer is now guaranteed a value of 5 or lesser. **B** now calls **E** to see if he can get a lower value than 5.

- At E the values of alpha and beta is not -INF and +INF but instead -INF and 5 respectively, because the value of beta was changed at B and that is what B passed down to E
- Now E looks at its left child which is 6. At E, alpha = max(-INF, 6) which is 6. Here the condition becomes true. beta is 5 and alpha is 6.
 So beta<=alpha is true. Hence it breaks and E returns 6 to B
- Note how it did not matter what the value of E's right child is. It could have been +INF or -INF, it still wouldn't matter, We never even had to look at it because the minimizer was guaranteed a value of 5 or lesser. So as soon as the maximizer saw the 6 he knew the minimizer would never come this way because he can get a 5 on the left side of B. This way we didn't have to look at that 9 and hence saved computation time.
- E returns a value of 6 to B. At B, beta = min(5, 6) which is 5. The value of node B is also 5

So far this is how our game tree looks. The 9 is crossed out because it was never computed.



- **B** returns 5 to **A**. At **A**, alpha = max(-INF, 5) which is 5. Now the maximizer is guaranteed a value of 5 or greater. **A** now calls **C** to see if it can get a higher value than 5.
- At C, alpha = 5 and beta = +INF. C calls F

- At F, alpha = 5 and beta = +INF. F looks at its left child which is a 1.
 alpha = max(5, 1) which is still 5.
- F looks at its right child which is a 2. Hence the best value of this node is 2. Alpha still remains 5
- **F** returns a value of 2 to **C**. At **C**, beta = min(+INF, 2). The condition beta <= alpha becomes true as beta = 2 and alpha = 5. So it breaks and it does not even have to compute the entire sub-tree of **G**.
- The intuition behind this break-off is that, at C the minimizer was guaranteed a value of 2 or lesser. But the maximizer was already guaranteed a value of 5 if he choose B. So why would the maximizer ever choose C and get a value less than 2? Again you can see that it did not matter what those last 2 values were. We also saved a lot of computation by skipping a whole sub-tree.
- **C** now returns a value of 2 to **A**. Therefore the best value at **A** is max(5, 2) which is a 5.
- Hence the optimal value that the maximizer can get is 5 This is how our final game tree looks like. As you can see **G** has been crossed out as it was never computed.

