Linear Transformation in Linear Space

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1 Linear Transformation

Let V and W be an n dimensional vector space over a field $\mathbb{F}$. Let $T : V \to W$ be a function with V as its domain and its range contained in W.

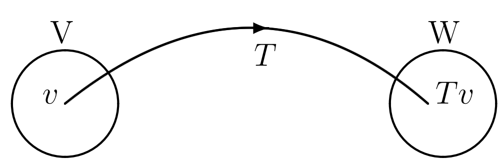
$$T(V) \subset W$$

We also assume T is linear in the sense that

$$T(v_1 + v_2) = T(v_1) + T(v_2)$$

$$T(\alpha v_1) = \alpha T(v_1)$$

$\forall \ v_1, v_2 \in V$ and $\alpha \in \mathbb{F}$.

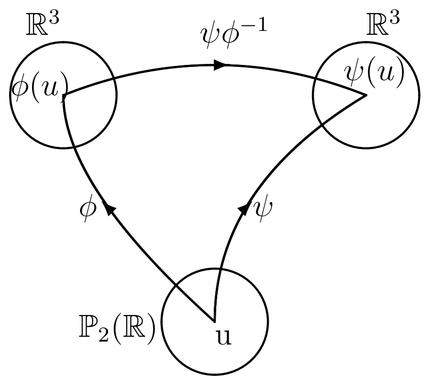


Let $L(V, W)$ denote the set of linear transformation from V to W. If $T \in L(V, W)$, T is defined if we prescribe the action of T on a basis of V.

Let $\mathcal{B} = v_1, v_2, ..., v_n$ be a basis of V. Then $v \in V$ given by $v = x_1v_1 + x_2v_2 + ... + x_nv_n$, $\forall \ x_i \in \mathbb{F}$

$$T(v) = T(x_1v_1 + x_2v_2 + ... + x_nv_n)$$

$= x_1T(v_1) + x_2T(v_2) + ... + x_nT(v_n)$
If we know every $T(v_i)$ we will get T(v).



2 Matrix Representation of Linear Transformation