TUTORIAL - 4

① $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

$T(n) = aT\left(\frac{n}{b}\right) + F(n)$

$a \geq 1,\ b > 1$

Comparing

$a = 3,\ b = 2,\ f(n) = n^2$

Now

$c = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$F(n) > n^c$

② $T(n) = 4T(n/2) + n^2$

$a > 1,\ b > 1$

$a = 4,\ b = 2,\ f(n) = n^2$

$T(n) = \Theta(n^2 \log_2 n)$

$c = \log_2 4 = 2$

$n^c = n^2 = f(n) = n^2$

$T(n) = \Theta(n^2 \log_2 n)$

③ $T(n) = T(n/2) + 2^n$

let $a = 1,\ b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 C = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \Theta(2^n)$

Q④ $T(n) = 2^n T(n/2) + n^n$

$a = 2^n$

$b = 2$, $f(n) = n^n$

$C = \log_b a = \log_2 2^n$

⑤ $T(n) = 16T(n/4) + n$

$a = 16$, $b = 4$

$F(n) = n$

$c = \log_n 16 = \log_n (4)^2 = 2$

$n^c = n^2$

$F(n) < n^c$

$T(n) = \Theta(n^2)$

⑥ $T(n) = 2T(n/2) + n \log n$

$a = 2$, $b = 2$

$F(n) = n \log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

Since, $n \log n > m$

$F(n) > n^c$

$T(n) = \Theta(n \log n)$

⑦ $T(n) = 2T(n/2) + n/\log n$

$a = 2$, $b = 2$, $F(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n = n$

Since $\frac{n}{\log n} < n$

$F(n) < n^c$

$T(n) = \Theta(n)$

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(7) $T(n) = 2T\left(\frac{n}{2}\right) + n/\log n$

$a=2, b=2, f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n = n$

Since $\frac{n}{\log n} < n$

$F(n) < n^c$

$T(n) = \Theta(n)$

(9) $T(n) = 0.5T(n/2) + 1/n$

$a = 0.5, b = 2$

Since acc to master theorem $a \geq 1$ but here a is 0.5 so we cannot apply master theorem.

(11) $T(n) = 4T(n/2) + \log n$

$a = 4, b = 2, f(n) = \log(n)$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$F(n) = \log(n)$

$\log n < n^2$

$f(n) < n^2$

$T(n) = \Theta(n^2)$

$\Theta(n^2)$.

(13) $T(n) = 3T(n/2) + n$

$a=3, b=2, f(n) = n$

$c = \log_b a = \log_2 3 = 1.53$

$n^c = n^{1.5849}$

$n < n^{1.5849}$

$f(n) < n^c$ $T(n) = \Theta(n)^{1.58}$

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(15) $T(n) = 4T(n/2) + n$

$a = 4, b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n$

$n < n^2$ (for any constant)

$f(n) < n^c$

$T(n) = \Theta(n^2)$

(17) $T(n) = 3T(n/3) + n/2$

$a = 3, b = 3$

$c = \log_b a = \log_3 3 = 1$

$f(n) = n/2$

$n^c = n^1 = n$

As $n/2 < n$

$f(n) < n^c$

$T(n) = \Theta(n)$

(19) $T(n) = 4T(n/2) + n \log n$

$a = 4, b = 2, f(n) = n \log n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$T(n) = \Theta(n^2)$

(21) $T(n) = 7T(n/3) + n^2$

$a = 7, b = 3, f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7$

$n^c = n^{1.771}$

$n^{1.77} < n^2$

$T(n) = \Theta(n^2)$

(22) $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, b = 2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$n(2 - \cos n) > n^c$

$T(n) = \Theta(n(2 - \cos n))$