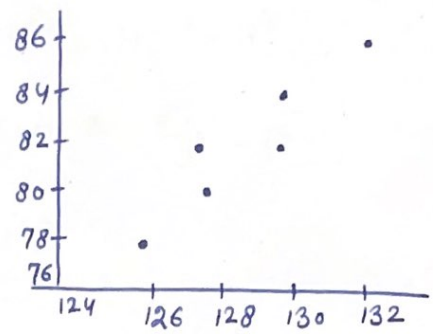
PCA EXAMPLE

Question:

A	B
126	78
128	80
128	82
130	82
130	84
132	86



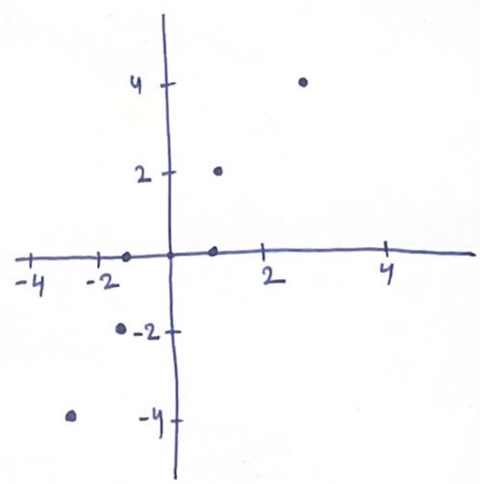
Solution:

Step 1 Centralize Data

mean (A) = 129

mean (B) = 82

A	B
126 - 129 = -3	78 - 82 = -4
128 - 129 = -1	80 - 82 = -2
128 - 129 = -1	82 - 82 = 0
130 - 129 = 1	82 - 82 = 0
130 - 129 = 1	84 - 82 = 2
132 - 129 = 3	86 - 82 = 4



Step 2 Covariance Matrix

$$var(A) = \frac{1}{n-1} \sum (A_i - mean(A))^2$$

$$\Rightarrow \frac{1}{6-1} \left((-3)^2 + (-1)^2 + (-1)^2 + 1^2 + 1^2 + 3^2\right)$$

$$\Rightarrow \frac{1}{5}(9+1+1+1+1+9) \Rightarrow \frac{22}{5} = \boxed{4.4}$$

$$var(B) = \frac{1}{n-1} \sum (B_i - mean(B))^2$$

$$\Rightarrow \frac{1}{5}\left((-4)^2 + (-2)^2 + 0^2 + 0^2 + 2^2 + 4^2\right)$$

$$\Rightarrow \frac{1}{5}(16+4+4+16) = \boxed{8}$$

$$cov(A,B) = cov(B,A) = \frac{1}{n-1} \sum (A_i - mean(A))(B_i - mean(B))$$

$$\Rightarrow \frac{1}{5}\left((-3)(-4) + (-1).(-2) + (-1).0 + 1.0 + 1.2 + 3.4\right)$$

$$\Rightarrow \boxed{5.6}$$

Hence,

covariance matrix is

$$\begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 4.4 & 5.6 \\ 5.6 & 8.0 \end{bmatrix} \end{array}$$

Step 3 <u>Calculate Eigen Values of Covariance Matrix</u>

$$\det\left(\begin{bmatrix} 4.4 & 5.6 \\ 5.6 & 8 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} 4.4 & 5.6 \\ 5.6 & 8 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0 \quad , \det\begin{bmatrix} 4.4-\lambda & 5.6 \\ 5.6 & 8-\lambda \end{bmatrix} = 0$$

$$(4.4-\lambda)(8-\lambda) - (5.6)(5.6) = 0$$

Solving this will give 2 eigen values

$$\lambda_1 = 12.08$$

$$\lambda_2 = 0.32$$

Step 4 Calculate Eigen Vectors Corresponding to these Eigen values:

<u>a.</u> Eigen vector corresponding to 12.08

$$\begin{bmatrix} 4.4 & 5.6 \\ 5.6 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 12.08\begin{bmatrix} x \\ y \end{bmatrix}$$

$$4.4x + 5.6y = 12.08x$$

$$5.6x + 8.0y = 12.08y$$

Solving these equations will give

$$y = 1.37x$$

Hence,
Eigen vector corresponding to 12.08 is

$$V_1 = \begin{bmatrix} 1 \\ 1.37 \end{bmatrix}$$

Narmalize this vectar v_1 to unit uectar by

$$\sqrt{1^2+(1.37)^2} = 1.69$$

$$v_1 = \begin{bmatrix} \frac{1}{1.69} \\ \frac{1.37}{1.69} \end{bmatrix} = \begin{bmatrix} 0.59 \\ 0.81 \end{bmatrix}$$

b. Simarly compute eigen uectar carrusponds to eigen halue 0.32.

$$v_2 = \begin{bmatrix} -0.81 \\ 0.59 \end{bmatrix}$$

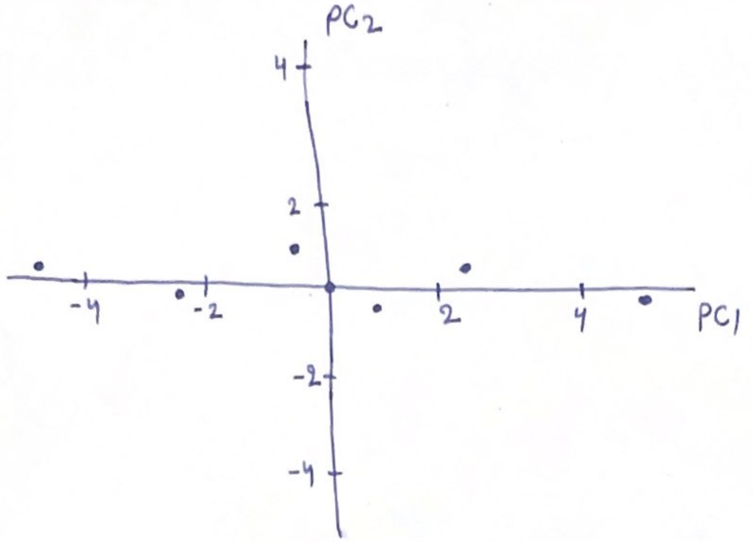
Step ⑤ Ordur the eigen uectar in decreasing ardur of their eigen nalues :

$$\begin{bmatrix} 0.59 & -0.81 \\ 0.81 & 0.59 \end{bmatrix}$$

Step ⑥ Calculate principal Components

$$\begin{bmatrix} -3 & -4 \\ -1 & -2 \\ -1 & 0 \\ 1 & 0 \\ 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0.59 & -0.81 \\ 0.81 & 0.59 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -5.0 & 0.1 \\ -2.2 & -0.4 \\ -0.6 & 0.8 \\ 0.6 & -0.8 \\ 2.2 & 0.4 \\ 5.0 & -0.1 \end{bmatrix}$$



Now we can see that variance is low in PC2 direction. Therefore, we can discard PC2.

$$\% \text{ variance of } PC_1 = \frac{12.08}{12.08 + 0.32} \Rightarrow 97.4\%$$