Tutorial - 1

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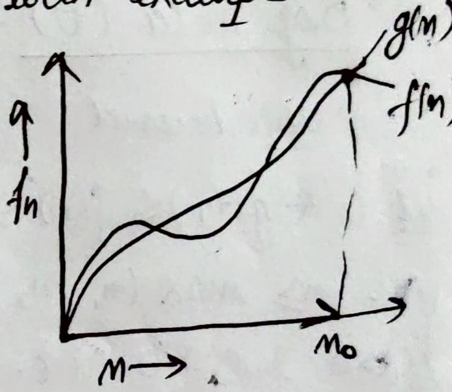
Ans 1 What do you understand by Asymptotic Notation define different asymptotic notation with example.

i) Big O(n)



$$f(n) = O(g(n))$$

if $f(n) \le g(n) * C \quad \forall n \ge n_0$

for some constant, $c > 0$

g(n) is tight upper bound of f(n).

eg- $f(n) = n^2 + n$

$g(n) = n^3$

$n^2 + n \le c * n^3$

$n^2 + n = O(n^3)$

ii) Big Omega (Ω)

when $f(n) = \Omega(g(n))$ means $(g(n))$ is "tight" lowerbound of f(n) i.e. f(n) can go beyond g(n) i.e. $f(n) = \Omega g(n)$.

iff $f(n) \geq c \cdot g(n)$

$\forall$ $n_2 > n_0$ & c = constant > 0

eg: $f(n) = n^3 + 4n^2$

$g(n) = n^2$

i.e. $f(n) \geq c * g(n)$

$n^3 + 4n^2 = \Omega(n^2)$

iii) **Big Theta (θ)** - when $f(n) = \theta(g(n))$ gives the tight upperbound & lowerbound both. i.e. $f(n) = \theta(g(n))$.

iff $c_1 * g(n_1) \leq f(n) \leq c_2 * g(n_2)$ for all $n \geq \max(n_1, n_2)$, some constant $c_1 > 0$ & $c_2 > 0$. i.e. $f(n)$ can never go beyond $c_2 g(n)$ & will never come down of $c_1 g(n)$.

eg. $3n+2 = \theta(n)$ as $3n+i \geq 3n$

$3n+2 \leq 4n$ for n, $c_1 = 3$, $c_2 = 4$ & $n_0 = 2$

iv) **Small-oh (o)**- $f(n) = o(g(n))$

$g(n)$ is upperbound of $f(n)$.

$f(n) < g(n)$ $\forall$ $n > n_0$ & $c > 0$

v) **Small-omega (ω)**- $f(n) = \omega(g(n))$

$g(n)$ is lower bound of $f(n)$.

$f(n) > g(n)$ $\forall$ $n > n_0$ & $c > 0$

Ans. 2 Complexity of ;

```
for (i=1 to n)
{    i=i*2;
}
```

$i = 1, 2, 4, 8 \ ---- \ n.$

$a = 1 \ , \ r = 2$

k^{th} term of G.P., $t_k = a * r^{k-1}$

$$n = 1 * 2^{k-1}$$

$$n = \frac{2^k}{2}$$

$$2n = 2^k$$

$$\log_2(2n) = k \log_2(2)$$

$$(\log_2 2) + (\log_2 n) = k \log_2(2)$$

$$1 + \log_2 n = k$$

$$k = 1 + \log_2(n)$$

$$\boxed{\text{Complexity} = O(\log n)}$$

Ans. 3 $T(n) = 3T(n-1)$

$T(1) = 1$

$T(n) = 3T(n-1) \longrightarrow ①$

put $n = n-1$ in eq^n ①

$T(n-1) = 3T(n-2)$

put value of $T(n-1)$ in eq ①

$T(n) = 3[3T(n-2)]$

$T(n) = 3^2 T(n-2) \longrightarrow ②$

Put $n = n-2$ in eq - 1

$T(n-2) = 3T(n-3)$

put value of $T(n-2)$ in eq^n II

$T(n) = 3^2(3T(n-3)) = 3^3 T(n-3)$. — III

from I, II & III

$T(n) = 3^k [T(n-k)]$ —— 4

$T(1) = 1$

$n-k = 1$

$k = n-1$

Put value of k in eq - 4

$T(n) = 3^{n-1}[T(1)]$

$T(n) = 3^{n-1}$

$\boxed{\text{complexity} = O(3^n)}$

Ques 4 $T(n) = 2T(n-1) - 1$

$T(1) = 1$

let, $T(n) = 2T(n-1) - 1 \longrightarrow 1$

put $n = n-1$ in eq 1

$T(n-1) = 2T(n-2) - 1$

put value in eq 1

$T(n) = 2(2T(n-2) - 1) - 1 = 2^2 T(n-2) - 2 - 1 \longrightarrow 2$

put $n = n-2$ in eq - 1

$T(n-2) = 2T(n-3) - 1$

$T(n) = 2^2(2T(n-3) - 1) - 2 - 1$

$T(n) = 2^3 T(n-3) - 2^2 - 2^1 - 2^0 \longrightarrow$ 3

~~put~~ ~~nesece~~

$T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} - \cdots - 2^1 - 2^0$

$n - k = 1$

$\boxed{k = n-1}$

Put k in above eq^n

$T(n) = 2^{n-1} T(1) - 2^{n-2} - 2^{n-3} - \cdots - 2^0.$

$= 2^n \left[\frac{1}{2} - \frac{1}{2^2} - \frac{1}{2^3} - \cdots \frac{1}{2^n} \right]$

$= 2^{n-1} - 2^n \left[\frac{1}{2^2} + \frac{1}{2^3} + \cdots \frac{1}{2^n} \right]$

$= 2^{n-1} - 2^n \left[\frac{1}{4} \frac{\left(1 - \left(\frac{1}{2}\right)^k\right.}{1/2} \right]$

$= 2^{n-1} - 2^{n+1} \left[\frac{1}{4} \left(1 - \left(\frac{1}{2}\right)^k \right] \right.$

$= 2^{n-1} - 2^{n+1} \left[\frac{1}{4} \left(1 - \left(\frac{1}{2}\right)^{n-1} \right] \right.$

$= 2^{n-1} - 2^{n-1} \left[1 - \left(\frac{1}{2}\right)^{n-1} \right]$

$= 2^{n-1} - 2^{n-1} \left[\frac{2^{n-1} - 1}{2^{n-1}} \right]$

$= 2^{n-1} - 2^{n-1} + 1 \qquad = 1$

$T(n) = O(1)$

Ques-5 what should be time complexity of

```
int i=1, s=1;
while (s<=n)
{ i++;
  s=s+i;
  printf("#");
}
```

$i = 1, 2, 3, 4 \ldots\ldots$

$s = 1 + 3 + 6 + 10 \ldots\ldots + n \rightarrow (1)$

also $s = 1 + 3 + 6 + 10 + \ldots\ldots T_{n-1} + T_n \rightarrow 2)$

$s = 0 + 1 + 3 + 6 + 10 + \ldots$

$0 = 1 + 2 + 3 + 4 + \ldots\ldots n - T_n$

$T_k = 1 + 2 + 3 + \ldots\ldots + k$

$T_k = \frac{1}{2} k (k+1)$

for k iterations

$1 + 2 + 3 + \ldots\ldots k <= n$

$\frac{k(k+1)}{2} <= n$

$\frac{k^2+k}{2} <= n$

$O(k)^2 <= n$

$k = O(\sqrt{n})$

$\boxed{T(k) = O(\sqrt{n})}$

ques 6 Time Complexity of

```
void f (int n)
{
  int i, count = 0;
  for (i=1 ; i*i<=n; ++i)
}
```

As $i^2 = n$

$i = \sqrt{n}$

$i = 1, 2, 3, 4 --- \sqrt{n}$

$\sum_{i=1} 1 + 2 + 3 + 4 + --- \sqrt{n}$

$T(n) = \frac{\sqrt{n} * (\sqrt{n} + 1)}{2}$

$T(n) = \frac{n * \sqrt{n}}{2}$

$\boxed{T(n) = O(n)}$

ques-7 Time complexity of

```
void f(int n)
{
  int i, j, k, count = 0;
  for (int i= n/2 ; i<=n; ++i)
    for (j=1 ; j<=n ; j=j*2)
      for( k=1; k<= n; k=k*2)
            count++;
}
```

since, for $k = k^2$

$k = 1, 2, 4, 8 \ldots n$

$\because$ series is in G.P.

So, $a = 1$, $r = 2$

$$\frac{a(r^n - 1)}{r-1}$$

$$= \frac{1(2^k - 1)}{1}$$

$$n = 2^k - 1$$

$$n+1 = 2^k$$

$$\log_2(n) = k$$

i	j	k
1	$\log(n)$	$\log(n) * \log(n)$
2	$\log(n)$	⋮
⋮	⋮	⋮
n	$\log(n)$	$\log(n) * \log(n)$

T.C. $\Rightarrow O(n * \log n * \log n)$

$\Rightarrow O(n \log^2(n))$ — ans

Ques. 8 Time Complexity of

```
void function (int n)
{
    if (n==1) return;
    for (i=1 to n) {
    for (j= 1 to n) {
        printf ("*");
    }
}
    function (n-3);
}
```

(8) for (i= 1 to n)

we get j= n times every turn

$\therefore$ $i*j = n^2$

k^{th}, Now, $T(n) = n^2 + T(n-3)$;

$T(n-3) = (n-3)^2 + T(n-6)$;

$T(n-6) = (n-6)^2 + T(n-9)$;

and $T(1) = 1$

Now, Substitute each value in T(n)

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + --- + 1$

Let

$k^n - 3k = 1$

$k = (n-1)/3$ total times $= k+1$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + - - +1$

$T(n) \simeq k n^2$

$T(n) \simeq (k-1)/3 * n^2$

so, $T(n) = O(n^3)$ Ans

Ques-9 Time complexity of :-

```
void function (int n)
{
    for (int i=1 to n) {
    for ( int j=1 ; j<= n ; j = j+i) {
        printf("*");
    }
  }
}
```

for $i=1$ $\quad j = 1+2+ \cdots \quad (n \geq j+i)$

$i=2$ $\quad j = 1+3+5 \cdots \quad (n \geq j+i)$

$i=3$ $\quad j = 1+4+7+ \cdots \quad (n \geq j+i)$

n^{th} term of AP is

$$T(n) = a + d * m$$

$$T(n) = 1 + d * m$$

$$(n-1)/d = m$$

for $i=1$ $\quad (n-1)/1$ times

$i=2$ $\quad (n-1)/2$ times

$i=n-1$

we get,

$$T(n) = i_1 j_1 + i_2 j_2 + \cdots i_{n-1} j_{n-1}.$$

$$= \frac{(n-1)}{1} + \frac{(n-2)}{2} + \frac{(n-3)}{3} + \cdots 1$$

$$= n + \frac{n}{2} + \frac{n}{3} + \cdots \frac{n}{n-1} - n \times 1$$

$$= n\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{n-1}\right] - n * 1$$

$$= n \times \log n - n + 1$$

since $\int \frac{1}{x} = \log x$

$$\boxed{T(n) = O(n \log n)}$$ Ans

Ques10 -

For the function n^{k} & c^n, what is the asymptotic relationship b/w these f^n's?

Assume that $k>=1$ & $c>1$ are constants. Find out the value of c & no. of which relationship holds.

As given n^k and c^n

Relationship b/w n^k & c^n is

$$n^k = O(c^n)$$

$$n^k \leq a(c^n)$$

$\forall$ $n \geq n_0$ & constant, $a>0$

for $n_0 = 1$; $c=2$

$\Rightarrow$ $1^k < a^2$

$\Rightarrow$ $n_0 = 1$ & $c=2$ Ans