Tutorial-4

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1) $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n^2)$

$a \geq 1, b \geq 1$

On Comparing

$a=3, b=2, f(n)=n^2$

Now, $C = \log_b a = \log_2 3$

$= 1.584$

$n^c = n^{1.584} < n^2$

$f(n) > n^c$

$T(n) = \theta(n^2)$

2) $T(n) = 4T(n/2) + n^2$

$\rightarrow a \geq 1, b \geq 1$

$a=4, b=2, f(n)=n^2$

$c = \log_2 4 = 2$

$n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \theta(n^2 \log_2 n)$

3) $T(n) = T(n/2) + 2^n$

$a=1, b=2$

$f(n) = 2^n$

$c = \log_b a = \log_2 c = 0$

$n^c = n^b = 1$

$f(n) > n$

$T(n) = \theta(2^n)$

4) $T(n) = 2^n T(n/2) + n^n$

$a = 2^n$

$b=2, f(n) = n^2$

$c = \log_b a = \log_2 2^n$

$= n$

$n^c + n^n$

$f(n) = n^c$

$T(n) = \theta(n^2 \log_2 n)$

5) $T(n) = 16T(n/4) + n$

$a=16, b=4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2\log_4 k$

$= 2^4$

$n^c \Rightarrow n^2$

$f(n) < n^c$

$\therefore T(n) = \theta(n^2)$

6) $T(n) = 2T(n/2) + n\log n$

$a=2, b=2$

$f(n) = n \log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$n \log n > n$

$f(n) > n^c$

$T(n) = \theta(n \log n)$

7. $T(n) = 2T(n/2) + n/\log n$

$a=2$, $b=2$, $f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$\frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \theta(n)$

8. $T(n) = 2T(n/4) + n^{0.51}$

$a=2$, $b=4$, $f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \theta(n^{0.51})$

9. $T(n) = 0.5T(n/2) + 1/n$

$a = 0.5$, $b=2$

$a \geq 1$ but here $a < 0.5$

So we can't apply Master's Method.

10. $T(n) = 16T(n/4) + n!$

$a=16$, $b=4$, $f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \theta(n!)$

11) $4T(\frac{n}{2}) + \log n$

$a=4$, $b=2$, $f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = \log n$

$\therefore \log n < n^2$

$f(n) < n^c$

$T(n) = \theta(n^c)$

$= \theta(n^2)$

12) $T(n) = sqrt(n)T(n/2) + \log n$

$a = \sqrt{n}$, $b=2$

$c = \log_2 \sqrt{n} = \frac{1}{2}\log_2 n$

$\therefore \frac{1}{2}\log_2 n < \log n$

$\therefore f(n) > n^c$

$T(n) = \theta(\log(n))$

13) $T(n) = 3T(n/2) + n$

$a=3$, $b=2$

$c = \log_2 3 = 1.584$

$n^c = n^{1.58}$

$n < n^{1.58} \Rightarrow f(n) < n^c$

$T(n) = \theta(n^{1.58})$

14) $T(n) = 3T(n/3) + sqrt(n)$

$a=3$, $b=3$

$c = \log_3 3 = 1$

$n^c = n$

$sqrt(n) < n$

$n^c > f(n)$

$T(n) = \theta(n)$

15) $T(n) = 4T(n/2) + n$

$a=4,\ b=2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$n^c > f(n)$

$\boxed{T(n) = \theta(n^2)}$

16) $T(n) = 3T\left(\frac{n}{4}\right) + n\log n$

$a=3,\ b=4$

$c = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$n^c < f(n)$

$\boxed{T(n) = \theta(n\log n)}$

17 $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

$a=3,\ b=3$

$c = \log_3 3 = 1$

$n^c = n$

$n^c > f(n)$

$\boxed{T(n) = \theta(n)}$

18: $T(n) = 6T\left(\frac{n}{3}\right) + n^2\log n$

$a=6,\ b=3$

$c = \log_3 6 = 1.6309$

$n^c < n^2\log n$

$\boxed{T(n) = \theta(n^2\log n)}$

19: $T(n) = 4T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

$a=4,\ b=2,\ f(n) = \frac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$\boxed{T(n) = \theta(n^2)}$

20: $T(n) = 64T\left(\frac{n}{8}\right) - n^2\log n$

$a=64,\ b=8$

$c = \log_8 64 = 2$

$n^c = n^2$

$n^2\log n > n^2$

$\boxed{T(n) = \theta(n^2\log n)}$

21) $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$a=7,\ b=3$

$c = \log_3 7 = 1.7712$

$n^c = n^{1.77}$

$n^c < f(n)$

$\boxed{T(n) = \theta(n^2)}$

22) $T(n) = T\left(\frac{n}{2}\right) + n(2 - \cos x)$

$a=1,\ b=2$

$c = \log_2 1 = 0$

$n^c = 1$

$n(2-\cos x) > n^c$

$\boxed{T(n) = \theta(n(2-\cos x))}$