

Assignment 1

AI1110: Probability and Random Variables
Indian Institute of Technology Hyderabad

Anshika Gupta
CS22BTECH11007

12.13.6.7: Question. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

Answer: $\frac{625}{23328}$.

Solution:

Probability of obtaining a six in the throw of a die = $\frac{1}{6}$

Let X be a random variable representing the number of sixes obtained in the first five throws of the die.

$$x \in \{0, 1, 2, 3, 4, 5\}$$

$$P(X = x) = {}^5C_x \times \left(\frac{1}{6}\right)^x \times \left(\frac{5}{6}\right)^{(5-x)}$$

where x is the possible value of X .

$$P(X = x) = \begin{cases} \frac{3125}{7776} & x = 0 \\ \frac{3125}{7776} & x = 1 \\ \frac{625}{3888} & x = 2 \\ \frac{125}{3888} & x = 3 \\ \frac{25}{7776} & x = 4 \\ \frac{1}{7776} & x = 5 \end{cases}$$

Let E_1 : Event that two sixes are obtained in first five throws of the die

E_2 : Event that a six is obtained at the sixth throw of the die

Event that both E_1 and E_2 occur is favourable

$$P(E_1 E_2) = P(E_1) \times P(E_2)$$

[\because The events E_1 and E_2 are independent]

$$P(E_1 E_2) = P(X = x) \times \frac{1}{6}$$

$$P(E_1 E_2) = \frac{625}{23328}$$