

Name DOOTIKA VATS

Roll. No.

Instructions:

- (a) Show all mathematical details required to implement an algorithm. Marks will not be given unless all work is shown.
- (b) You are **only** allowed to draw from Uniform[0, 1]. Direct samples from any other distribution are not allowed.

Q: Consider a target density

$$f(x) = \frac{e^x}{e-1} \quad 0 \leq x \leq 1.$$

Write down the steps for both the inverse transform and an accept-reject algorithm to obtain one sample from this target density.

(1) Inverse Transform (7/10)

$$F(x) = \int_{-\infty}^x f(t) dt = \int_0^x \frac{e^t}{e-1} dt = \frac{1}{e-1} [e^t]_0^x = \frac{e^x - 1}{e-1} \quad (1)$$

$$\text{Let } F(x) = u = \frac{e^x - 1}{e-1} \Rightarrow (e-1)u + 1 = e^x \\ \Rightarrow x = \log((e-1)u + 1) = F^{-1}(u) \quad (1)$$

Steps① Draw $U \sim U(0, 1)$ ② Set $x = \log((e-1)U + 1)$ (1)

No marks for this box, if algorithm is not written properly.

(2) Accept-Reject (7/10)

Support of target is $[0, 1]$. Consider proposal $U(0, 1)$

$$g(x) = 1; \quad 0 \leq x \leq 1$$

② for choosing appropriate proposal

We need to find "c":

$$c = \sup_{x \in X} \frac{f(x)}{g(x)} = \sup_{x \in [0, 1]} \frac{e^x}{(e-1) \cdot 1} = \frac{e}{e-1}$$

[since e^x is \uparrow fn of x]

③ for finding correct "c" for their proposal.

Algorithm

① Draw $U_1 \sim U(0,1)$

② Draw $U_2 \sim U(0,1)$ independently.

③ Is $U_1 \leq \frac{f(U_2)}{c g(U_2)} = \frac{e^{U_2}}{\frac{e}{e-1} \cdot (e-1)} = \frac{e^{U_2}}{e}$

} ok, if not solve

then set $X = U_2$

② for writing algorithm property

④ Else goto ①