QUIZ-2A

ELEMENTARY STOCHASTIC PROCESS (MTH-212A)

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**Time:** 20 mins.

**Maximum Marks:** 15

**Minimum Marks:** 0

**Instructions:** Both the questions are of multiple choice. More than one answers might be correct in both the questions. Each correct answer will give you one point and if you tick a wrong answer it will be negative 1. In Question 1, if you can identify all the correct answers without ticking any wrong answer, you will get eight points, and similarly in Question 2, you will get 7 points. No extra sheet will be provided.

1. Suppose $X_1, X_2, \ldots$ are independent and identically distributed random variables with $P(X_1 = 0) = P(X_1 = 1) = \dfrac{1}{2}$. If $Y_n = \max\{X_1, \ldots, X_n\}$, and let us denote $\boldsymbol{P}$ as the transition probability matrix of $\{Y_n\}$. Then which of the statements are correct.

$Y_n = \max\{Y_{n-1}, X_n\}$

Period calculation

$d(0) = \gcd\{n : P_{00}^n > 0\} = 1$

1. $\lim_{n\to\infty} p_{00}^n = 1$
2. $\lim_{n\to\infty} p_{10}^n = 0$ $\quad \because P_{10}^n = 0 \ \forall n \geq 1$
3. The period of state $\{0\}$ is 2.
4. The period of state $\{1\}$ is 1.
5. The state $\{0\}$ is reachable from state $\{1\}$.

$$P = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 1/2 & 1/2 \\ 1 & 0 & 1 \end{array}$$

$d(1) = 1$ ( $\because P_{11}^1 > 0$) && $P_{00}^1 > 0$

$$P^2 = \begin{bmatrix} 1/4 & 3/4 \\ 0 & 1 \end{bmatrix}$$

The correct answers are: [2],[4]

Thus. $P_{00}^n = \frac{1}{2^n} \Rightarrow \lim \frac{1}{2^n} = \underline{\underline{0}}$ $\quad P^3 = \begin{bmatrix} \frac{1}{2^3} & 1 - \frac{1}{2^3} \\ 0 & 1 \end{bmatrix}$

**Solutions of 1:** $\boldsymbol{P} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 1 \end{bmatrix}$, $\boldsymbol{P}^n = \begin{bmatrix} \frac{1}{2^n} & 1 - \frac{1}{2^n} \\ 0 & 1 \end{bmatrix}$. $\{1\}$ and $\{0\}$ do not communicate with each other, $d(0) = d(1) = 1$.

2. Let $\{X_n\}$ be a Markov Chain with the following transition probability matrix

$$
P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}
\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \end{array}
$$

$d(i) = 1$
$\forall\, i \in \{1, 2, 0, 3\}$

$$
P^2 = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \cdot & - & & - \\ \cdot & & - & \end{bmatrix} = P
$$

thus $P^n = P$

$0 \to 0$ ; $0 \to 2$

$1 \to 1$ ; $1 \to 3$

$2 \to 0$ ; $2 \to 2$

$\{0, 2\}$ & $\{1, 3\}$
equivalence class

Then which of the statements are correct.

1. $\boldsymbol{P}^n$ does not depend on $n$.
2. $\boldsymbol{P}^n$ depends on $n$.
3. There is only one equivalent class.
4. All the states do not communicate with each other.
5. The periods of all the states are NOT same.

The correct answers are: [1],[4]

**Solution:** $\boldsymbol{P}^n = \boldsymbol{P}$. If we deonote the states as $\{1, 2, 3, 4\}$, then there are two equivalent classes $\{1, 3\}$ and $\{2, 4\}$. All the states have period 1.