MTH210: Lab 2

Accept-Reject Method for Discrete Random Variables

1. The file `BinomAR.R` contains partial code for the implementation of the Accept-Reject method for a Binomial(n, p) problem (as discussed in class). The function `draw_binom()` is left incomplete at some place. Complete the function and run the line

   ```
   draw_binom(n = 10, p = .25)
   ```

   If written correctly, the above line will return

   - an $X \sim \text{Binom}(10, .25)$
   - the number of times the algorithm looped

   Change the values of n and p and observe what happens. What happens when n is very large?

2. Go over the rest of the code in `BinomAR.R` and observe what happens when the simulation is repeated many times. Further, observe how the performance of the upper bound c changes when different Geometric proposals are used. All of this is implemented in the code.

3. Implement Problem 8 from Section 3.4 in the notes in R. In this problem, the target distribution is the "truncated Poisson distribution'' with pmf:

   $$\Pr(X = i) = \frac{e^{-\lambda}\lambda^i/i!}{\sum_{j=0}^{m} e^{-\lambda}\lambda^j/j!} \quad i = 0, 1, 2, \dots, m\,.$$

   What is a reasonable proposal for this target distribution and the respective value of c? Implement in R with $m = 30$ and $\lambda = 20$.

4. Consider a Geometric(p) target distribution. We are aware than a Poisson target distribution cannot be used as a valid proposal distribution for accept-reject. "Verify" this claim by writing code that calculates the bound c.

   As it turns out, for the Geometric(p) distribution, it is extremely challenging to find a valid proposal distribution.