QUIZ-3A

APPLIED STOCHASTIC PROCESS (MTH-212M/ MTH-412A)

Name (Roll Number):
Maximum Marks: 20
Minimum Marks: 0
Time: 30 Mins

Instructions: If you do not write your name and roll number 2 marks will be deducted. Both the questions are of multiple choice. More than one answers might be correct in both the questions. Each correct answer will give you two points and if you tick a wrong answer it will be negative two. In Question 1, if you can identify all the correct answers without ticking any wrong answer, you will get ten points, and similarly in Question 2 also. We are using the same notations as we have used in the class.

1. Suppose a finite Markov Chain with state space $\{1, 2, \ldots, M\}$ have the following transition probability matrix; $\boldsymbol{P} = ((p_{ij}))$, where $p_{ij} = \frac{1}{M}$, for $1 \leq i, j \leq M$. Then which of the following statements are correct.

1. All the states are recurrent.
2. There are M equivalent classes.
3. There is only one equivalent class.
4. There exists at least one transient state.
5. For $M = 10$, $\sum\limits_{n=2}^{\infty} f_{11}^n \leq 0.95$.
6. For $M = 20$, $\sum\limits_{n=2}^{\infty} f_{11}^n \geq 0.90$.

Solutions: In this case $p_{ij}^n = \frac{1}{M}$, for $1 \leq i, j \leq M$ for all $n \geq 1$. Clearly $\sum_{n=1}^{\infty} p_{ii}^n = \infty$, for $1 \leq i \leq M$. Therefore, $\sum_{n=1}^{\infty} f_{ii}^n = 1$, for $1 \leq i \leq M$. Here all the states are recurrent, and all the states communicate with each other. $f_{11}^1 = \frac{1}{M}$.

The correct answers are: 1,3,5,6

2. Suppose a finite Markov Chain with state space $\{1, 2, \ldots, M\}$, where $M > 2$, have the following transition probability matrix;

$$\boldsymbol{P} = \begin{bmatrix} \frac{1}{M} & \frac{1}{M} & \cdots & \frac{1}{M} & \frac{1}{M} \\ 0 & \frac{1}{M-1} & \cdots & \frac{1}{M-1} & \frac{1}{M-1} \\ 0 & 0 & \frac{1}{M-2} & \cdots & \frac{1}{M-2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}.$$

Let us define the matrix $\boldsymbol{Q} = \lim_{n\to\infty} \boldsymbol{P}^n$.

Then which of the following statements are correct.

1. All the states are recurrent.
2. There are M equivalent classes.
3. There are two equivalent classes.
4. There exists at least one transient state.
5. For $M = 100$, $\sum\limits_{n=1}^{\infty} p_{1M}^n \leq 100$.
6. The matrix $\boldsymbol{Q}$ is a doubly stochastic matrix.

Solution: Note that $p_{kk}^n = 1/(M-k+1)^n$, for $k = 1, \ldots, M$. Hence, all the states are transient except the state 'M', as $\sum_{n=1}^{\infty} p_{kk}^n < \infty$, for $k = 1, \ldots, M-1$, and $\sum_{n=1}^{\infty} p_{MM}^n = \infty$. $\lim_{n\to\infty} p_{kk}^n = 0$ if $k = 1, \ldots, M-1$, and it is 1, for $k = M$.

The correct answers are: 2,4,