Quiz-1

Elementary Stochastic Process (MTH-212M/412A)

Name (Roll Number):

No extra sheet will be provided or collected, **Time 30 mins.**, **Max. Marks:20**.

1. Let $\{X_n; n \geq 1\}$ be a sequence of independent and identically distributed random variables with $P(X_1 = 1) = P(X_1 = -1) = \dfrac{1}{2}$. Let us define $Y_n = X_1 \times \ldots \times X_n$, for $n \geq 1$. (a) Find the distribution of Y_n, (b) Show that $\{Y_n\}$ is a Markov Chain, (c) Find the $\mathbf{P}$, the transition probability matrix of $\{Y_n\}$. (d) Find $\mathbf{P}^n$. [3+3+3+1=10]

Solution: By induction it easily follows that Y_n has the same distribution as X_1. Note that $Y_{n+1} = Y_n \times X_{n+1}$. In this case the state space is $\{-1, 1\}$. Hence,

$$\mathbf{P} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad \text{and} \quad \mathbf{P}^n = \mathbf{P}$$

2. Suppose $\Omega = \{1, 2, 3\}$, and $\mathcal{F}$ = the class of all subsets of Ω. Suppose X and Y are real valued functions on Ω as follows: $X(1) = 1$, $X(2) = 2$, $X(3) = 3$ and $Y(1) = Y(2) = 1$ and $Y(3) = 10$. Suppose $P(\{1\}) = 0.4$, $P(\{2\}) = 0.5$, $P(\{3\}) = 0.1$, for other sets it has been defined in such a manner so that it is probability function. (i) Find Prob$(X + Y \leq x)$ - Prob$(X + Y < x)$ for all x. (ii) Find Prob$(XY \leq y)$ - Prob$(XY < y)$ for all y. (5+5=10)

Solution:

Note that $(X+Y)(1) = 2$, $(X+Y)(2) = 3$, $(X+Y)(3) = 13$, $(XY)(1) = 1$, $(XY)(2) = 2$ and $(XY)(3) = 30$. Since, Prob$(X + Y \leq x)$ - Prob$(X + Y < x)$ = Prob$(X + Y = x)$, therefore,

$$\text{Prob}(X + Y = x) = \begin{cases} 0.4 & \text{if} \quad x = 2 \\ 0.5 & \text{if} \quad x = 3 \\ 0.1 & \text{if} \quad x = 13 \\ 0.0 & \text{if} \quad x \neq 2, 3, 13. \end{cases}$$

Further, Prob$(XY \leq x)$ - Prob$(XY < x)$ = Prob$(XY = x$, and

$$\text{Prob}(XY = x) = \begin{cases} 0.4 & \text{if} \quad x = 1 \\ 0.5 & \text{if} \quad x = 2 \\ 0.1 & \text{if} \quad x = 30 \\ 0.0 & \text{if} \quad x \neq 1, 2, 30. \end{cases}$$