Name DOOTKA VATI Roll. No. Solutions

Let $X_1, X_2, \ldots, X_n \stackrel{iid}{\sim} F$, where F is a 2-component $\text{Exp}(\lambda)$ mixture distribution with density

$$f(x|\lambda_1,\lambda_2,\pi_1,\pi_2) = \pi_1 f_1(x|\lambda_1) + \pi_2 f_2(x|\lambda_2), \qquad \lambda_1,\lambda_2 > 0 \text{ and } \pi_1,\pi_2 \in (0,1), \pi_1 + \pi_2 = 1.$$

Recall that: $f_c(x|\lambda_c) = \lambda_c e^{-\lambda_c x}$. Clearly, write down the steps of the EM Algorithm to obtain the maximum likelihood estimate of $\theta = (\pi_1, \pi_2, \lambda_1, \lambda_2)$.

Solution:

Assume iid latent variables Z_i such that $\Pr(Z_i = c) = \pi_c$ for c = 1, 2 and $X_i | Z_i \stackrel{iid}{\sim} \operatorname{Exp}(\lambda_c)$. With this, we can obtain the E and M steps of the EM algorithm. Given any iterate $\theta_{(k)}$, the E-step is similar to the Gaussian Mixture Model

E-step:

$$q(\theta|\theta_{(k)}) = \sum_{i=1}^{n} \sum_{c=1}^{2} \log \left\{ f_c(x_i|\lambda_c)\pi_c \right\} \underbrace{\frac{f_c(x_i|\lambda_{c,(k)})\pi_{c,(k)}}{\sum_{j=1}^{2} f_j(x_i|\lambda_{j,(k)})\pi_{j,(k)}}}_{\gamma_{c,(k)}}$$

Simplifying this, $q(olo_{ck}) = \sum_{i=1}^{2} log \pi_c + log \Lambda_c - \lambda_c \pi_i \int_{i,c,ck}^{i} V_{i,c,ck}$

$$\frac{M-step}{\partial q(\theta)\theta(\kappa)} = \sum_{i=1}^{n} \begin{bmatrix} 1 - \chi_i \end{bmatrix} Y_{i,c,c(\kappa)}$$

$$\Rightarrow \begin{cases} \frac{1}{2} & \chi_i \\ \frac{1}{2} &$$

Further
$$\frac{\partial^2 q}{\partial \lambda_c^2} = \frac{1}{|a|} \left(-\frac{8inc_{i}(k)}{\lambda_c^2} \right) < 0$$
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Now, taking derivative w.r.t Trc, I need Lagrange multiplier

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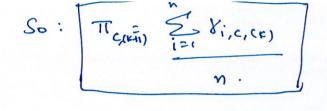
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summing for all
$$\pi_c$$
: $\Xi \pi_c = 1 = \frac{1}{2} \frac{\Sigma}{2} \frac{S_{i,c,ck}}{2} = \frac{\eta}{\eta} = 1$



Second derivative is similarly negative hence, we have maxima. (I) if maxima notchecked

EM Algo

(1) Set (A10), A200), Thos 1 T2,00) = (00)

ext: $\frac{f_{c}(x_{i}|A_{c,(k)})}{f_{c,(k)}} = \frac{f_{c}(x_{i}|A_{c,(k)})}{f_{c,(k)}} = \frac{f_{c}(x_{i}|A_{c,(k)})}{f_{c}(x_{i}|A_{c,(k)})} = \frac{f_{c}(x_{i}|A_{c,(k)}$

Set $\lambda_{(CK+1)} = \frac{\sum_{i=1}^{N} 8_{i,C_i(CK)}}{\sum_{i=1}^{N} x_i \cdot 8_{i,C_i(CK)}}$ and $\pi_{C_i(CK+1)} = \frac{\sum_{i=1}^{N} 8_{i,C_i(CK)}}{\sum_{i=1}^{N} x_i \cdot 8_{i,C_i(CK)}}$

Set Ockay)= (1,ck+1), The,(k+1))

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