

MTH210: Lab 3

Generating Continuous Random Variables

(New code that you write for this assignment can be saved in new *.R files and pushed back to the repository.)

1. The file `BetaAR.R` contains partial code to implement an AR algorithm for a $\text{Beta}(4, 3)$ target. Complete the code and analyse the results.
2. Write R code for Problem 7 in Exercises from Section 4 of the notes.
3. The file `circleAR.R` contains partial code to implement the accept-reject sampler to draw from the uniform distribution over the circle. Complete the code.
4. Taking inspiration from `circleAR.R`, implement Problem 16 from Section 4 Exercises of the notes.
5. Modify the `BetaAR.R` appropriately so that it can implement an AR algorithm for $\text{Beta}(2, .1)$.
6. Using only $U(0, 1)$ draws, draw samples from $\text{Gamma}(4, 3)$ using Accept-Reject and an exponential proposal. Compare the performance of the sampler using the optimal exponential proposal, versus $\lambda = 2$.
7. **Suppose $Y = \sum_{i=1}^5 X_i$ where $X_i \sim \text{Weibull}(\alpha_i, \lambda)$. Here density of $\text{Weibull}(\alpha, \lambda)$ is**

$$f(x) = \alpha \lambda^{-\alpha} x^{\alpha-1} e^{-\lambda x^\alpha}, \quad x > 0.$$

Using only $U(0, 1)$ draws, estimate $E(Y^2)$. Assume $\alpha_i = i$ and $\lambda = 5$.