Home Work 5
MTH 212M/MTH 412A (2024)
Applied Stochastic Process

1. Consider the following transition probability matrix

$$\mathbf{P} = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 & 0 & \dots \\ 1/2 & 0 & 1/2 & 0 & 0 & 0 & \dots \\ 1/2 & 0 & 0 & 1/2 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{bmatrix}.$$

   Prove that the Markov Chain is an irreducible aperiodic recurrent Markov Chain. Find $\lim p_{00}^{(n)}$.

2. Consider the following transition probability matrix

$$\mathbf{P} = \begin{bmatrix} 2/3 & 1/3 & 0 & 0 & 0 & 0 & \dots \\ 2/3 & 0 & 1/3 & 0 & 0 & 0 & \dots \\ 2/3 & 0 & 0 & 1/3 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{bmatrix}.$$

   Prove that the Markov Chain is an irreducible aperiodic recurrent Markov Chain. Find $\lim p_{00}^{(n)}$.

3. Consider the following transition probability matrix

$$\mathbf{P} = \begin{bmatrix} p & 1-p & 0 & 0 & 0 & 0 & \dots \\ p & 0 & 1-p & 0 & 0 & 0 & \dots \\ p & 0 & 0 & 1-p & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{bmatrix}.$$

   Here $0 < p < 1$. Find the values of p such that the Markov Chain is an irreducible aperiodic recurrent Markov Chain. Find $\lim p_{00}^{(n)}$.

4. Consider the following transition probability matrix

$$\mathbf{P} = \begin{bmatrix} 1-p & p & 0 & 0 & 0 & 0 & \dots \\ 1-p^2 & 0 & p^2 & 0 & 0 & 0 & \dots \\ 1-p^3 & 0 & 0 & p^3 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{bmatrix}.$$

   Here $0 < p < 1$. Find the values of p such that the Markov Chain is an irreducible aperiodic recurrent Markov Chain. Find $\lim p_{00}^{(n)}$.