Name (Roll Number):

Instructions: If you do not write your name and roll number 2 marks will be deducted. Both the questions are of multiple choice. More than one answers might be correct in both the questions. Each correct answer will give you two points and if you tick a wrong answer it will be negative two. In Question 1, if you can identify all the correct answers without ticking any wrong answer, you will get ten points, and similarly in Question 2 also.

1. Suppose $X_1, X_2, \ldots$ are independent and identically distributed random variables with $P(X_1 = 0) = P(X_1 = 1) = \dfrac{1}{2}$. If $Y_n = \max\{X_1, \ldots, X_n\}$, and let us denote $\boldsymbol{P}$ as the transition probability matrix of $\{Y_n\}$. Then which of the statements are correct.

   1. $\{Y_n\}$ is a Markov Chain.

   2. The distribution of Y_n is same for all $n \geq 1$.

   3. The state space of $\{X_n\}$ is $\{0, 1\}$.

   4. The state space of $\{Y_n\}$ is NOT $\{0, 1\}$.

   5. $\boldsymbol{P}$ does not depend on n.

   6. $\boldsymbol{P}^n$ does not depend on n.

Solutions: $p_{ij} = P(Y_{n+1} = j | Y_n = i) = P(\max\{X_1, \ldots, X_{n+1}\} = j | \max\{X_1, \ldots, X_n\} = i) = P(\max\{i, X_{n+1}\} = j | \max\{X_1, \ldots, X_n\} = i)$. $p_{00} = \dfrac{1}{2} = p_{01}, p_{10} = 0, p_{11} = 1$. $P(Y_n = 0) = \dfrac{1}{2^n}, P(Y_n = 1) = 1 - P(Y_n = 0)$.

The correct options are: 1,3,5

2. Consider the usual two persons zero sum game as we have discussed in the class, where both the players have two units of money each, and $P(H) = P(T) = \frac{1}{2}$. Here X_n denotes the money of Player 1, after n-th game. Then which of the following statements are correct:

1. The state space of $\{X_n\}$ is $\{0, 1, 2, 3, 4, 5\}$.

2. All the states communicate with each other.

3. There are two equivalent classes.

4. There are three equivalent classes.

5. The period of state '1' is 1.

6. The period of state '2' is 2.

Solution:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The equivalent classes are: $\{0\}$, $\{4\}$, $\{1, 2, 3\}$. $d(1) = d(2) = d(3) = 2$, $d(0) = d(4) = 1$.

The correct options are: 4,6