Make-up Quiz-A
Applied Stochastic Process (MTH-212M/ MTH-412A)

Name (Roll Number)
Time: 15 mins.
Maximum Marks: 10
Minimum Marks: 0

Instructions: The present quiz has only one question. More than one answers might be correct in this question. Each correct answer will give you one point and if you identify a wrong answer as a correct answer it will be negative 1. If you can identify all the correct answers without ticking any wrong answer, you will get 10 points. Please do not write anything on the question paper except the answers.

1. Consider a machine which can be in two states, 'on' or 'off'. Also if the machine is 'on' today then it will be 'off' tomorrow with probability $0 < \alpha < 1$ and similarly, if it is 'off' today it will be 'on' tomorrow with probability $0 < \beta < 1$. Denote the 'on' and 'off' state by 0 and 1, respectively. Let us denote X_n as the state of the machine on day n, then $\{X_n\}$ is a Markov Chain with state space $S = \{0, 1\}$. $p_{ij} = P(X_{n+1} = j | X_n = i)$, for $0 \leq i, j \leq 1$. Then which of the following statements are correct.

1. $p_{01}^n = (1-\alpha)p_{00}^{n-1} + \beta p_{01}^{n-1}$.
2. $p_{00}^n = (1-\alpha)p_{00}^{n-1} + \beta p_{01}^{n-1}$.
3. $\lim_{n\to\infty} p_{00}^n = \dfrac{\beta}{\alpha+\beta}$.
4. $\lim_{n\to\infty} p_{11}^n = \dfrac{\beta}{\alpha+\beta}$.
5. $\lim_{n\to\infty} p_{10}^n = \alpha$.
6. $\lim_{n\to\infty} p_{01}^n = \beta$.

The correct statements are: (2) and (3).

Note that: $p_{01}^n = \alpha p_{00}^{n-1} + (1-\beta)p_{01}^{n-1}$, $p_{00}^n = (1-\alpha)p_{00}^{n-1} + \beta p_{01}^{n-1}$. The transition probability matrix is $\begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}$ and the stationary probabilities are $\beta/(\alpha+\beta)$, and $\alpha/(\alpha+\beta)$.